Student Information

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Answer 1

1.1

Table 1: a) Tautology

p	q	r	¬ r	p → q	p ∧ ¬ r	(p → q) ↔ (p ∧ ¬ r)	¬ (q ∧ r)	RESULT
T	T	T	F	T	F	F	F	T
T	T	F	T	T	T	T	T	T
T	F	T	F	F	F	T	T	T
T	F	F	T	F	T	F	T	T
F	T	T	F	T	F	F	F	T
F	T	F	T	T	F	F	T	T
F	F	T	F	T	F	F	T	T
F	F	F	T	T	F	F	T	T

RESULT: $\mathbf{((p \rightarrow q) \leftrightarrow (p \wedge \neg r)) \rightarrow \neg (p \wedge r)}$

Table 2: b) Contradiction

p	q	¬ p	p ∨ q	p → q	(p ∨ q) ∧ (p → q)	q → ¬ p	RESULT
T	T	F	T	T	T	F	F
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	F
F	F	T	F	T	F	T	F

RESULT: $\mathbf{\neg((p \vee q) \wedge (p \rightarrow q) \vee (q \rightarrow \neg p))}$

1.2

a) This argument is **invalid**. Let $D = \{-1, 1\}$ be a domain for $P(x)$ and $Q(x)$. Suppose $P(x) : x < 0$ and $Q(x) : x > 0$. Note that $P(x)$ is true for $x = -1$ and $Q(x)$ is true for $x = 1$ in domain D. So, there exists at least one x which $P(x)$ is true and there exists at least some other x which $Q(x)$ is true. Note that first and second quantifiers' scopes are different on the left hand side so that one can choose different x values. Thus, the argument $\exists x P(x) \wedge \exists x Q(x)$ is valid for different choices of x. However, since on the right hand side, one quantifier's scope is $(P(x) \wedge Q(x))$, one can choose only one x for both $P(x)$ and $Q(x)$. Therefore, the argument $\exists x(P(x) \wedge Q(x))$ is invalid for $x = -1$ and $x = 1$. In conclusion, the argument $\exists x P(x) \wedge \exists x Q(x) \rightarrow \exists x(P(x) \wedge Q(x))$ is invalid.

b) This argument is **valid**. This argument is invalid if one can prove that the left hand side is true but the right hand side is false. Choose some arbitrary constant c in the domain of $P(x)$ and suppose the left hand side is true. This means for all choices of x, $P(x)$ is true; moreover, $P(c)$ is true. So, there exist at least one x that $P(x)$ is true. Thus, the argument $\exists x P(x)$ is true. Hence, if the left hand side is true, the right hand side must be true and the argument $\forall x P(x) \rightarrow \exists x P(x)$ is valid.

Answer 2

1.	$(\neg p \vee p) \rightarrow ((p \wedge \neg q) \rightarrow r)$	Premise
2.	$T \rightarrow ((p \wedge \neg q) \rightarrow r)$	Negation Law
3.	$\neg T \vee ((p \wedge \neg q) \rightarrow r)$	Table 7/Line 1
4.	$F \vee ((p \wedge \neg q) \rightarrow r)$	Negation
5.	$(p \wedge \neg q) \rightarrow r$	Identity Law
6.	$\neg(p \wedge \neg q) \vee r$	Table 7/Line 1
7.	$(\neg p \vee q) \vee r$	De Morgan's Law
8.	$\neg p \vee q \vee r$	Associative Law
9.	$q \vee r \vee \neg p$	Commutative Law
10.	$(q \vee r) \vee \neg p$	Associative Law

Answer 3

1. $\forall x(W(x) \rightarrow Has\_CS\_Degree(x))$
2. $\forall x \forall y((Phd(x) \wedge Phd(y) \wedge W(x) \wedge W(y) \wedge (x \neq y)) \rightarrow Knows(x, y))$
3. $\forall x((W(x) \wedge (x \neq Cenk)) \rightarrow Older(Cenk, x))$
4. $\forall x((W(x) \wedge (x \neq Selen)) \rightarrow Phd(x))$
5. $\neg(\forall x \forall y((W(x) \wedge W(y) \wedge (x \neq y)) \rightarrow Knows(x, y)))$
6. $(\exists x Phd(x) \wedge \exists y Phd(y) \wedge (x \neq y)) \rightarrow \forall z((z \neq x) \wedge (z \neq y) \rightarrow \neg Phd(z))$
7. $(\exists x \exists y \exists z((x \neq y \neq z \neq Gizem) \wedge Older(x, Gizem) \wedge Older(y, Gizem) \wedge Older(z, Gizem)))$
8. $\exists x(Phd(x) \wedge W(x)) \rightarrow (\forall y((y \neq x) \rightarrow \neg(Phd(y) \wedge W(y))))$

Answer 4

1.	$(p \to r) \vee (q \to r)$	premise
2.	$p \to r$	assumed
3.	$p \wedge q$	assumed
4.	p	$\wedge e, 3$
5.	r	$\to e, 2, 4$
6.	$(p \wedge q) \to r$	$\to i, 3-5$
7.	$q \to r$	assumed
8.	$p \wedge q$	assumed
9.	q	$\wedge e, 8$
10.	r	$\to e, 7, 9$
11.	$(p \wedge q) \to r$	$\to i, 8-10$
12.	$(p \wedge q) \to r$	$\vee e, 1, 2-6, 7-11$

Answer 5

1.	$(\neg p \vee \neg q)$	premise
2.	$p \wedge q$	assumed
3.	p	$\wedge e, 2$
4.	q	$\wedge e, 2$
5.	$\neg p$	assumed
6.	$\perp$	$\neg e, 5, 3$
7.	r	*lemma* " $\perp \vdash X$"
8.	$\neg q$	assumed
9.	$\perp$	$\neg e, 8, 4$
10.	r	*lemma* " $\perp \vdash X$"
11.	r	$\vee e, 1, 5-7, 8-10$
12.	$(p \wedge q) \to r$	$\to i, 2-11$

Proof for *lemma* " $\perp \vdash X$" **:**

1.	$\perp$	premise
2.	$\neg A$	assumed
3.	$\perp$	*copy*1
4.	$\neg\neg A$	$\neg i, 2-3$
5.	A	$\neg\neg e, 4$

Answer 6

1.	$\forall x(P(x) \to (Q(x) \to R(x)))$	premise
2.	$\exists x P(x)$	premise
3.	$\forall x(\neg R(x))$	premise
4.	$P(c)$	assumed
5.	$(P(c) \to (Q(c) \to R(c)))$	$\forall e, 1$
6.	$Q(c) \to R(c)$	$\to e, 5, 4$
7.	$\neg R(c)$	$\forall e, 3$
8.	$Q(c)$	assumed
9.	$R(c)$	$\to e, 6, 8$
10.	$\perp$	$\neg e, 9, 7$
11.	$\neg Q(c)$	$\neg i, 8-10$
12.	$\exists x(\neg Q(x))$	$\exists i, 11$
13.	$\exists x(\neg Q(x))$	$\exists e, 2, 4-12$