CENG 384 - Signals and Systems for Computer Engineers
Spring 2020
Written Assignment 2

Özalp, Zeynep
e2237691@ceng.metu.edu.tr

Yıldırım, Hilmi Cihan
e2237949@ceng.metu.edu.tr

March 9, 2020

---

1. (a) **memory**
y[0]=y[-1]+y[-2]+.. so we need memory because the output y[5] depends upon the past value of x[-1] so we need memory
stability
we have to find bounded inputs give or give not bounded outputs so
it is unstable because if B¡x[n]¡B y[n] goes infinity because there is infity sum of B's
Linearity
Let us apply superposition property to determine linearity
y1[n] =x1[n-1]+x1[n-2]+...
y2[n] =x2[n-1]+x2[n-2]+...
now let us consider a third input x3[n] such that it is linear combination of x1[n] and x2[n]
x3[n]=ax1[n]+bx2[n]
therefore ,the output y3[n] is given as
y3[n]=x3[n-1]+x3[n-2]+...
y3[n] = ax1[n-1]+bx2[n-1]+ax1[n-2]+bx2[n-2]...
y3[n] = a(x1[n-1]+x1[n-2]+...)+b(x2[n-1]+x2[n-2]+...)
y3[n] = y1[n]+y2[n]
From the above expressions we conclude system is linear we can easily see that both additivity and homogeneity proporteis hold.
Invertibilty
A system is invertible if distinct inputs lead distinct outputs
we can easily see that system performs summation of inputs .For different inputs the outputs are different. so system is invertible
Time -Invariance
To check that this system is time invariant, we must determine whether the timeinvariance property holds for any input and any time shift n0 Thus, let x1[n-n0] be an arbitrary input to this system, and let
y1[n]= x1[n-1]+x1[n-2]+..

x2[n]=x1[n-n0]
y2[n]=x2[n-1]+x2[n-2]+... =x1[n-n0-1]+x1[n-n0-2]+...
so
y2[n]=y1[n-n0] therefore the system is time invariant.
Conculusion
the system is has memory, unstable, linear, invertible and time invariant

(b) **memory**
A system is said to be memory less if its output for each value of the independent variable at a given time is dependent only on the input at that same time.

y(1)=y(5) so system **need memory** because is not dependent only on the input at that same time.
stability
let us consider $|x(t)| < \infty$
$|y(t)| = |ty(2t + 3)|$
$|y(t)| \leq |t||y(2t + 3)|$
even if $|x(t)| < \infty$ the magnitude of the output depends upon the variable n,which states that the output is **unstable**
Linearity
y1(t) = t x1(2t+3)

y2(t) = t x2(2t+3)

x3(t) = ax1(t)+bx2(t)

y3(t) = tx3(2t+1)
y3(t) = atx1(2t+3)+btx2(2t+3) = ay1(t)+by2(t) so this system is linear

Invertibilty

A system is invertible if distinct inputs lead distinct outputs
this system is not invertible because for example if x(1)=5 and x(5)=1 then y(1)=1*5 and y(5)=5*1 so system give same output for different inputs system is **not invetible**.

Time -Invariance

To check that this system is time invariant, we must determine whether the timeinvariance property holds for any input and any time shift n0 Thus, let x1[n-n0] be an arbitrary input to this system, and let
x(t-t0) $\rightarrow$ y(n)=nx(n-n0) but y(n-n0)=(n-n0)x(n-n0)tehy are not equal so system is **not time invariant**

2. (a) $y(t) = \int_{-\infty}^{\infty}(x(\tau) - 5y(\tau))d\tau$
   Differentiate both sides.
   $\dfrac{dy(t)}{dt} = x(t) - 5y(t)$
   $\dfrac{dy(t)}{dt} + 5y(t) = x(t)$
   $y'(t) + 5y(t) = x(t)$

   (b) $y'(t) + 5y(t) = e^{-t} + e^{-3t}$
   First, homogenous solution:
   $\lambda + 5 = 0 \Rightarrow \lambda = -5$
   $y_H(t) = Ke^{-5t}$
   For particular solution, since the system is linear, find particular solution to $x_1(t) = e^{-t}$ and $x_2(t) = e^{-3t}$ separately. Then combine.
   1) For $x_1(t) = e^{-t}$, assume $y_{P1} = A_1e^{-t}$
   $-A_1e^{-t} + 5A_1e^{-t} = e^{-t} \Rightarrow A_1 = 1/4$
   $y_{P1} = (1/4)e^{-t}$
   $y_{G1} = Ke^{-5t} + (1/4)e^{-t}$
   2) For $x_2(t) = e^{-3t}$, assume $y_{P2} = A_2e^{-3t}$
   $-3A_2e^{-3t} + 5A_2e^{-3t} = A_2e^{-3t} \Rightarrow A_2 = 1/2$
   $y_{P2} = (1/2)e^{-3t}$
   $y_{G2} = Ke^{-5t} + (1/2)e^{-3t}$
   So, general solution is $y(t) = 2Ke^{-5t} + (1/4)e^{-t} + (1/2)e^{-3t}$
   $y(0) = 2K + 3/4 = 0 \Rightarrow K = -3/8$
   $y(t) = -(3/4)e^{-5t} + (1/4)e^{-t} + (1/2)e^{-3t}$

3. (a) y[n] = x[n] * h[n]= $\sum\limits_{k=-\infty}^{\infty} x[k]h[n-k]$

   x[n] = $\sum\limits_{k=-\infty}^{\infty} x[k]\delta[n-k]$
   according to above equations from book
   y[n] = $\sum\limits_{k=-\infty}^{\infty} x[k]\delta[n-1-k]$+ 2 $\sum\limits_{k=-\infty}^{\infty} x[k]\delta[n+1-k]$
   x[n-1]= $\sum\limits_{k=-\infty}^{\infty} x[k]\delta[n-1-k]$
   x[n+1]= $\sum\limits_{k=-\infty}^{\infty} x[k]\delta[n+1-k]$
   so
   y[n] = x[n-1]+2x[n+1]
   y[n] = $2\delta[n-1] + \delta[n] + 2(2\delta[n+1] + \delta[n+2])$

   (b) $\dfrac{d}{dt}u(t-t0) = \delta(t-t0)$ from book therefore
   $\dfrac{d}{dt}x(t) = \delta(t-1) + \delta(t+1)$

   y(t) = $(\delta(t-1) + \delta(t+1))h(t)$
   from distribution property of convolution
   y(t) = $\delta(t-1) * h(t) + \delta(t+1) * h(t))$
   from book x(t)$\delta(t-t0) = x(t-t0)$ therefore
   y(t) = h(t-1)+h(t+1)

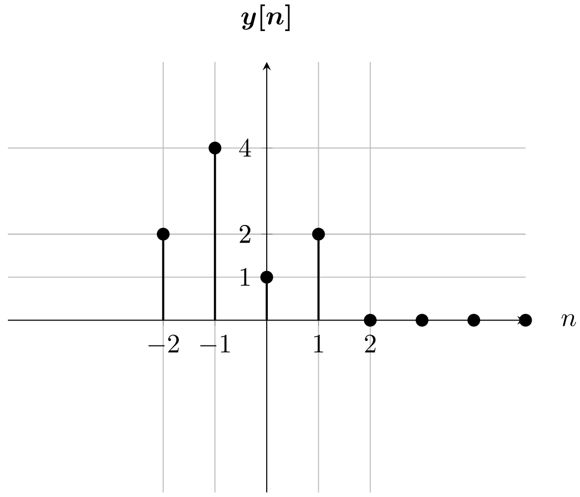

Figure 1: n vs. $x[n]$.

y(t) $= e^{-t+1}sin(t-1)u(t-1) + e^{-t-1}sin(t+1)u(t+1)$
y(t)=0 for t< -1
y(t) $= e^{-t-1}sin(t+1)u(t+1)$ for -1≤t<1
y(t) $= e^{-t+1}sin(t-1)u(t-1) + e^{-t-1}sin(t+1)u(t+1)$ for 1≤t

4. (a) The product of $x(\tau)$ *and* $h(t-\tau)$ is non zero for only $0 < \tau < t$
For $t \geq 0$: $\int_0^t x(\tau)h(t-\tau)d\tau = \int_0^t e^{-\tau}e^{-2t}e^{2\tau}d\tau = e^{-2t}\int_0^t e^{\tau}d\tau = e^{-2t}(e^t - 1) = (e^{-t} - e^{-2t})$
For $t < 0$: y(t)=0
Thus, $y(t) = (e^{-t} - e^{-2t})u(t)$

   (b) The product of $x(\tau)$ *and* $h(t-\tau)$ is nonzero for only $0 \leq \tau \leq 1$. Also, because $x(\tau) = 1$ for $0 \leq \tau \leq 1$, we do not need to write it in the product.
$\int_0^1 x(\tau)h(t-\tau)d\tau = y(t) = \int_0^1 e^{3t-3\tau}d\tau = e^{3t}\int_0^1 e^{-3\tau}d\tau = e^{3t}\dfrac{e^{-3}-1}{-3} = \dfrac{-e^{3t-3}+e^{3t}}{3}$
Thus,

$$y(t) = \begin{cases} \dfrac{-e^{3t-3}+e^{3t}}{3} & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

5. (a)

   (b) we assume y(t)=Ae^{rt}
so
$y'(t) = Are^{rt}$
$y''(t) = Ar^2e^{rt}$
$y'''(t) = Ar^3e^{rt}$
so
$r^3e^{rt} - 3(r^2e^{rt}) + 4(re^{rt}) - 2(e^{rt})$
$Ae^{rt}(r^3 - 3r^2 + 4r - 2) = 0$ e^{rt}cannot be zero so $(r^3 - 3r^2 + 4r - 2) = 0$ need to be true
so r1=1
r2=1-j
r3=1+j
y(t)=c1 e^t+e^t (c2sin(t)+c3cos(t))
y(0)=c1+c3=3
y'(t)=c1 e^t+e^t (c2sin(t)+c3cos(t))+e^t (c2cos(t)-c3sin(t))
y'(0)=c1+c2+c3=1
y"(t)=c1 e^t+e^t (c2sin(t)+c3cos(t))+e^t (c2cos(t)-c3sin(t)) +e^t (c2cos(t)+c3 -sin(t)) +e^t (c2-sin(t)-c3cos(t))
y"(0)=c1+2c2=2
c1=6
c2=-2
c3=-3

6. (a) $w[n] - \dfrac{1}{2}w[n-1] = x[n]$
Take the Fourier Transform of both sides to be in frequency domain.
$W(e^{jw}) - \dfrac{1}{2}e^{-jw}W(e^{jw}) = X(e^{jw})$
$W(e^{jw})(1 - \dfrac{1}{2}e^{-jw}) = X(e^{jw})$

$$\frac{W(e^{jw})}{X(e^{jw})} = H_0(e^{jw}) = \frac{1}{1 - \dfrac{1}{2}e^{-jw}}$$

Now, take the reverse fourier to be in time domain.

$h_0[n] = (\dfrac{1}{2})^n u[n]$

(b) $h_0[k]h_0[n-k]$ is positive only for $0 \le k \le n$.

$h_0[n] * h_0[n] = \Sigma_0^n h_0[k]h_0[n-k] = \Sigma_0^n (\dfrac{1}{2})^n$

$h[n] = n(\dfrac{1}{2})^n u[n]$

(c) $X(e^{jw})H_0(e^{jw})H_0(e^{jw}) = W(e^{jw})$

$$\frac{Y(e^{jw})}{X(e^{jw})} = (H_0(e^{jw}))^2 = \frac{1}{1 - e^{-jw} - \frac{1}{4}e^{-2jw}}$$

$Y(e^{jw}) - e^{-jw}Y(e^{jw}) - \frac{1}{4}e^{-2jw}Y(e^{jw}) = X(e^{jw})$

Take the inverse fourier to be in time domain.

$y[n] - y[n-1] - \frac{1}{4}y[n-2] = x[n]$