Student Information

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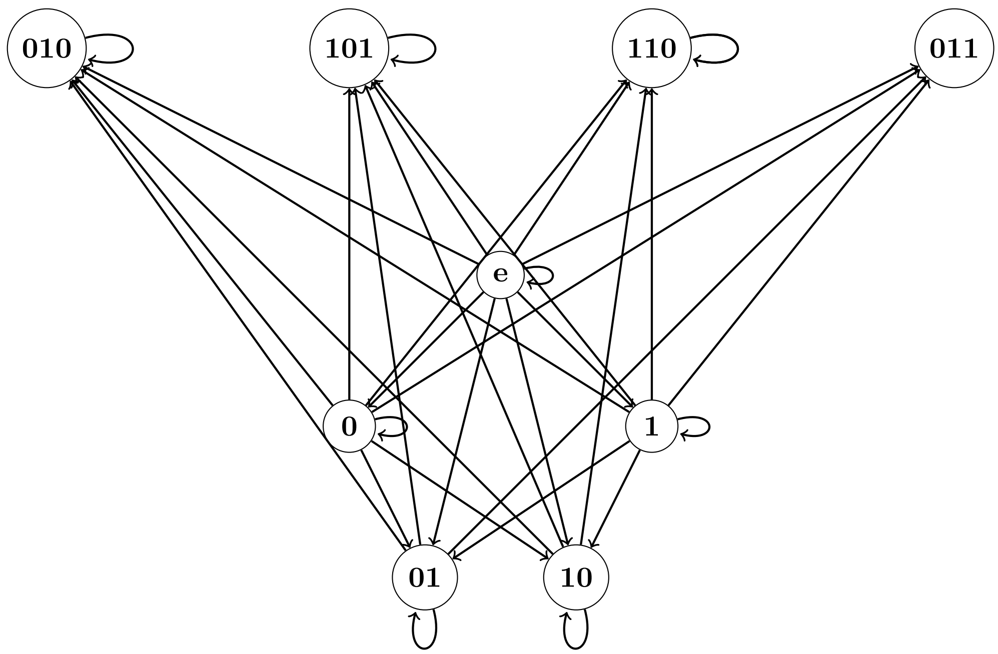
Answer 1

a

Note that the symbol "e" is used as empty string.



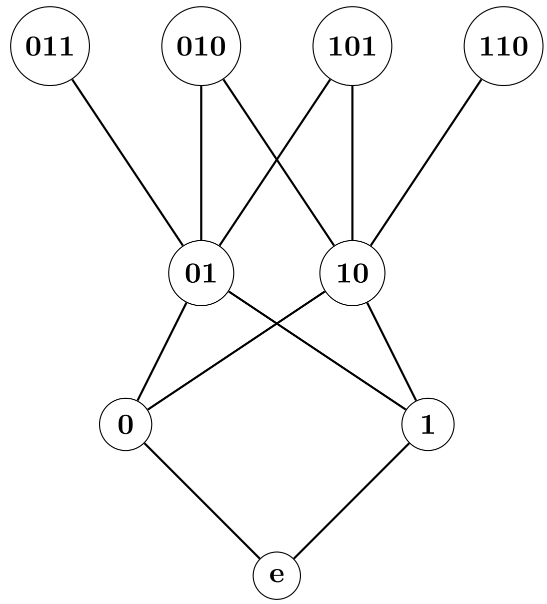
b

Since every string is a subset of itself, this relation is reflexive on S. Assume that aRb and bRa for $a, b \in S$. This is possible if and only if $a = b$. So, R is antisymmetric. Clearly, $aRb \wedge bRc \rightarrow aRc$ for $a, b, c \in S$ since if a is a substring of b and b is a substring of c, a is a substring of c. Thus, (S, R) is a poset.

c

For every $a, b \in S$, if $a\alpha b$ then the relation α is a total order. For example, between 010 and 110 there is no relation. Thus, R is not a total order.

d



Minimal: e. Maximal: 010,101,110.

e

Not a lattice since least upper bound of 010 and 101 does not exist in S.

Answer 2

a

Initial Vertex	Terminal Vertex
a	a,b,d
b	c,d
c	b
d	c
e	b,f
f	b,e,f
g	c,f

b

Order of the vertices is a,b,c,d,e,f,g.

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

c

Vertex	Indegrees	Outdegrees
a	1	3
b	3	1
c	3	1
d	2	1
e	2	1
f	2	3
g	0	2

d

1. abcbd
2. adcbc
3. ebdcb
4. efbdc
5. gfebd
6. gfbdc

e

1. dcbd
2. bdcb
3. cbdc
4. feff

f

Graph G is weakly connected if and only if there is a path between every two vertices when the directions of the edges are disregarded. Vertex b is connected to all vertices except g but vertex g is connected to f and c. Clearly, there is a path between every two vertices.

g

1. Vertex a.
2. Vertex g.
3. Subgraph consisting vertices b,c,d and edges (b,c), (c,b), (d,c), (b,d).
4. Subgraph consisting vertices e, f and edges (e,f), (f,e).

h

1. Between a and b: adcb, abcb.
1. Between a and c: aabc, aadc, abdc.
2. Between a and d there is 1 path : aabd.
3. Between b and d there is 1 path : bcbd.
4. Between d and c there is 1 path : dcbc.

Total = 8.

Answer 3

a

Theorem in lecture notes of section 3: A graph has a Euler path if it is connected and there is either 0 or 2 vertices of odd degree. Clearly, the graph is connected.
deg(a)=2, deg(b)=6, deg(c)=4, deg(d)=4, deg(e)=6, deg(f)=6, deg(g)=4, deg(h)=4, deg(i)=4, deg(j)=6. Therefore, G has a Euler path.

b

Since the graph is connected and every vertex has a even degree, the graph has an Euler circuit by Theorem 1 on page 696.

c

One example of Hamiltonian path of G: abfehicdgkj. So, G has a Hamiltonian path.

d

The number of vertices is 11. Not all of vertices have a degree greater than 11/2 and the greatest number of sum of the degrees of two non adjacent vertices is 10. So, there is no Hamiltonian circuit with respect to Theorem 3 and 4 on page 701.

Answer 4

a

There are m+n vertices. Let use the symbols M and N to represent disjoint sets of m and n vertices, respectively. For each vertex in M, there is an n edges to every vertex in N. Since there are m vertices in M, the number of edges is m times n in total.

b

Since there is no edge between the elements of M and N, the Hamiltonian path have a zigzag shape.

1. Assume that $n > m$

a) Assume we begin with a vertex in N. Then, we have to visit a vertex in M, then in N, and goes like a zigzag shape until there are n-m unvisited vertices in N. Since there are no edges between these vertices, there are no Hamiltonian circuit or path.
b) Assume we begin with a vertex in M. Again we have a zigzag shape. Since we begin with a vertex in M, path will end at one of vertex in N but there are still n-m unvisited vertices in N. Since there are no edges between these vertices, there are no Hamiltonian circuit or path.

2. Assume that $m > n$.

a) Assume we begin with a vertex in M. Again we have a zigzag shape. Since we begin with a vertex in M, path will end at one of vertex in N but there are still m-n unvisited vertices in M. Since there are no edges between these vertices, there are no Hamiltonian circuit or path.
a) Assume we begin with a vertex in N. Again we have a zigzag shape. There remains m-n unvisited vertices in M. Since there are no edges between these vertices, there are no Hamiltonian circuit or path.

Answer 5

a

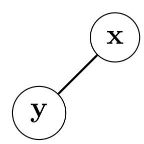
The table shows visited vertices, selected vertex in each step and current distances of vertices to vertex s. The distances are initially infinity. Distance from s to s is zero (no need to show).

Visited Vertices	Selected Vertex	u	v	w	x	y	z	t
	s	4	5	3	∞	∞	∞	∞
s	w	4	5	3	11	∞	15	∞
s, w	u	4	5	3	11	15	15	∞
s, w, u	v	4	5	3	7	11	15	∞
s, w, u, v	x	4	5	3	7	8	13	∞
s, w, u, v, x	y	4	5	3	7	8	12	17
s, w, u, v, x, y	z	4	5	3	7	8	12	15
s, w, u, v, x, y, z	t	4	5	3	7	8	12	15

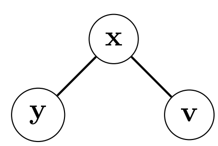
Shortest path from s to t is svxyzt and its weight is 15.

b

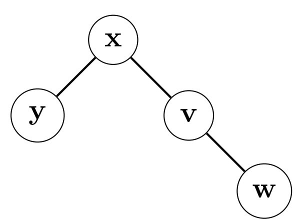
The root is x. Add y since its weight is minimal, 1.

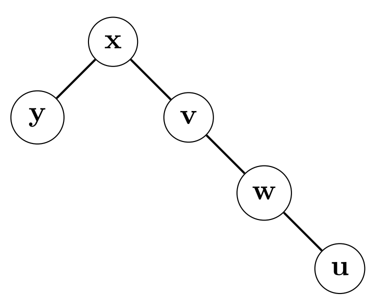


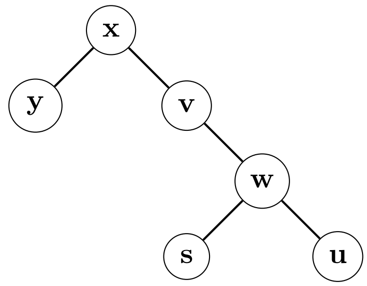
Now, we have vertices x and y. Add v since its weight is smallest among u, w, z, t.

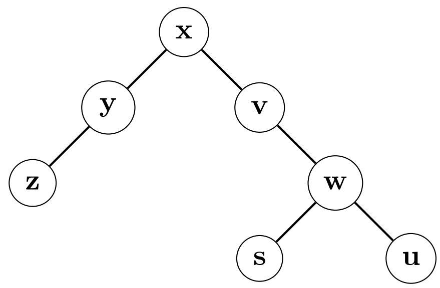


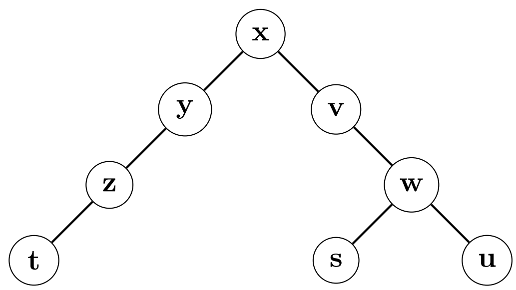
Now we have vertices x, y and v. Add w since its weight is smallest. This procedure goes like this.





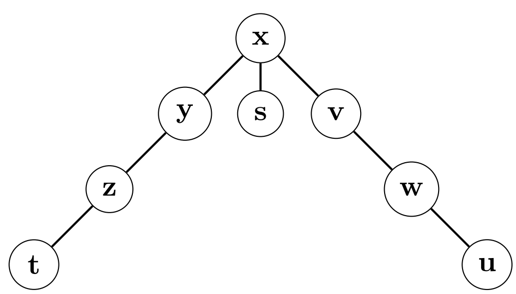




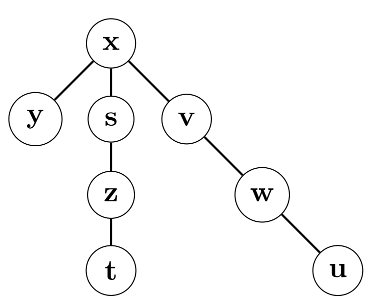


c

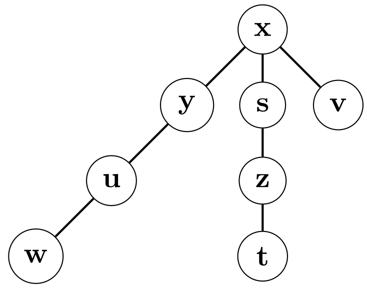
- After adding (s,x,1):



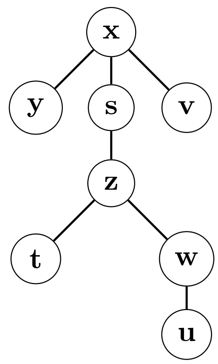
- After adding (t,u,6): Nothing changes.
- After adding (s,z,-6):



- After adding (u,y,3):



- After adding (w,z,-1):



d

Yes. Since we construct a minimum spanning tree, we can use the edges of it. So, the shortest path from s to t is szt has weight -3+3=0.

Answer 6

a

13 vertices, 12 edges, height is 4.

b

Post-order traversal: w, s, m, t, q, x, n, y, u, z, v, r, p.

c

In-order traversal: s, w, q, m, t, p, x, u, n, y, r, v, z.

d

Pre-order traversal: p, q, s, w, t, m, r, u, x, y, n, v, z.

e

No. In a full binary tree every node has two children except the ones at height 0.

f

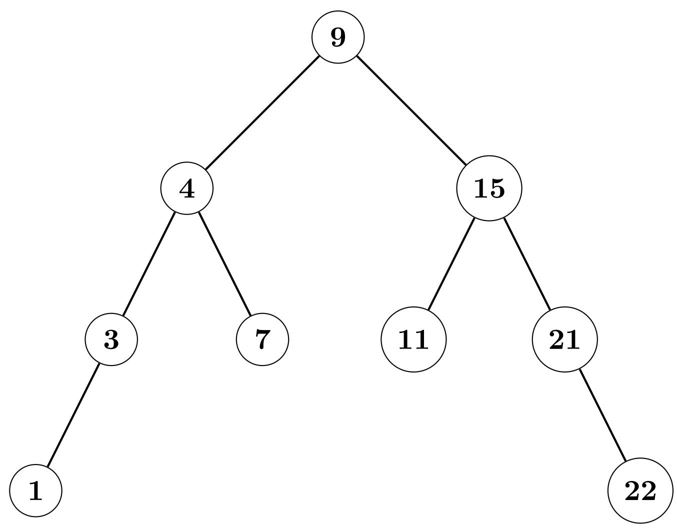
No since x:41 should be in the sub-tree of p:42.

g

By the definition on page 748, in a full m-ary tree, every internal vertex has exactly m children. Let N be the number of vertices.

$$N = 1 + 3 + 3^2 + 3^3 + ... = \sum_{k=0}^{h} 3^k = \frac{3^{h+1} - 1}{3 - 1} = \frac{3^{h+1} - 1}{2}$$

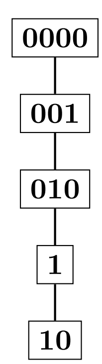
h



i

For 2: 9, 4, 3, 1. Unsuccessful search.
For 22: 9, 15, 21, 22. Successful search.

j

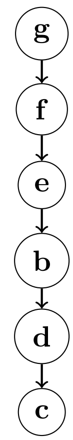


k

For 001: 0000, 001. Successful search.
For 011: 0000, 001, 010, 1. Unsuccessful search.

l

First tree:



Second tree: