Student Information

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Answer 1

Let $\Sigma = \{0, 1\}$. A_k is clearly finite since A_k has a partitioning $S_1, S_2 ... S_k$ where each S_i is the set of strings of length i. The cardinality of S_i is i^2. All S_i's are finite. Union of these finite sets are also finite. B is a language and it is countably infinite. The set of strings Σ^* is countably infinite and 2^{Σ^*} is uncountable by theorem 1.5.2 on page 28. So, C is uncountable.

a.

The difference of an uncountable set and a finite set is uncountable.

b.

$$L^* = \Sigma^*$$

1) $L \subseteq \Sigma^* \rightarrow L^* \subseteq \Sigma^{**} = \Sigma^*$. Therefore, $L^* \subseteq \Sigma^*$.
2) $(\{0, 1\} \subseteq L) \rightarrow (\{0, 1\}^* \subseteq L^*) \rightarrow (\Sigma^* \subseteq L^*)$

So, $B^* = \{0, 1\}^*$. Thus, the intersection of $2^{\Sigma^*}, \Sigma^*$ and A_7 is A_7 and it is finite. $|A_7| = 49$

c.

Countably infinite since $\bigcup C = \{0, 1\}^*$, $A_2 \times K$ is the set of tuples and $\bigcup C \cap (A_2 \times K) = \{\}$. So, $\bigcup C - (A_2 \times K) = \bigcup C$.

d.

Countable and its cardinality is 0 since $\bigcup C = \bigcup_{k=1}^{\infty} A_k = \{0, 1\}^*$.

Answer 2

a.

For s, there can be 4 different initial states. For F, there can be as many as the power set of K, which is 2^4. For δ, the transition function is from $K \times \Sigma$ to K. There are 2^3 tuples in $K \times \Sigma$. These tuples can map to only one state. So, there are $4^8 = 2^{16}$ different delta functions. Therefore, $2^2 * 2^4 * 2^5 = 2^{22}$ different DFA can be constructed.

b.

For s, there can be 4 different initial states. For F, there can be as many as the power set of K, which is 2^4. For Δ, the transition function is $\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$. There are $4.3.4 = 48$ different tuples and the transition function can be any subset of these tuples. So, there are 2^{48} different transition functions. Therefore, $2^2 * 2^4 * 2^{48} = 2^{54}$ different NFA can be constructed.

c.

In NFA's, one configuration can yield to many different configurations but in DFA's, one configuration can yield to only another configuration. Also, in NFA's, machine can pass to another state without reading an input symbol. Because of these reasons, the number of transition functions and the number of M's are clearly different.

d.

The number of different FA's are equal to the number of different languages over

Answer 3

a.

$$0^*(100^*100^*100^*100^*)^*0^* \cup 0^*(0^*010^*010^*010^*01)^*0^*$$

b.

$$(0 \cup 1)(01 \cup 00)^* \cup (10 \cup 00)^*(0 \cup 1)$$

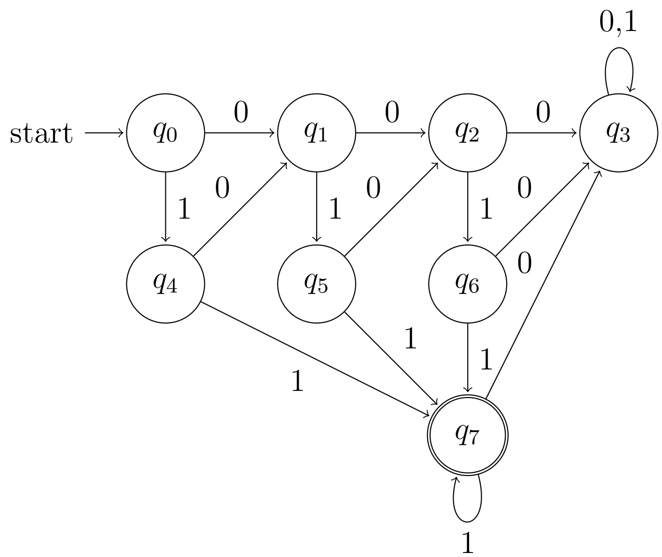
c.

$$1^*((00)(01 \cup 10 \cup 1)^*(00))^*1^*(e \cup 0)1^*$$

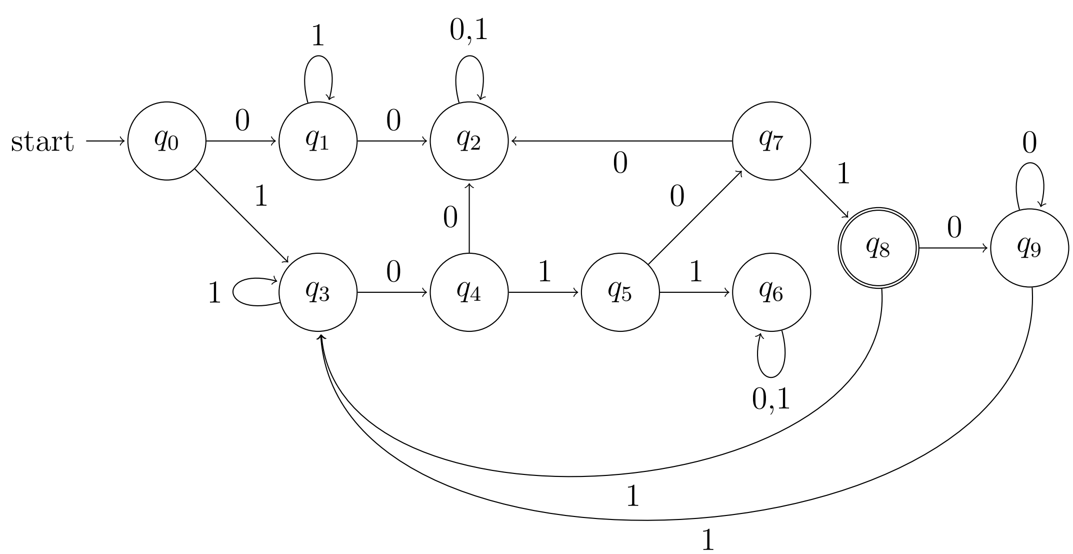
Answer 4

a.

$M = (K, \Sigma, \delta, s, F)$
$K = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$, $\Sigma = \{0, 1\}$, $s = q_0$, $F = q_7$



b.



Answer 5

a.

$$(q_0, abaa) \vdash_N (q_2, abaa) \vdash_N (q_4, baa) \vdash_N (q_4, aa) \vdash_N (q_1, a) \vdash_N (q_1, e)$$

$w_1 \in L(N)$.

b.

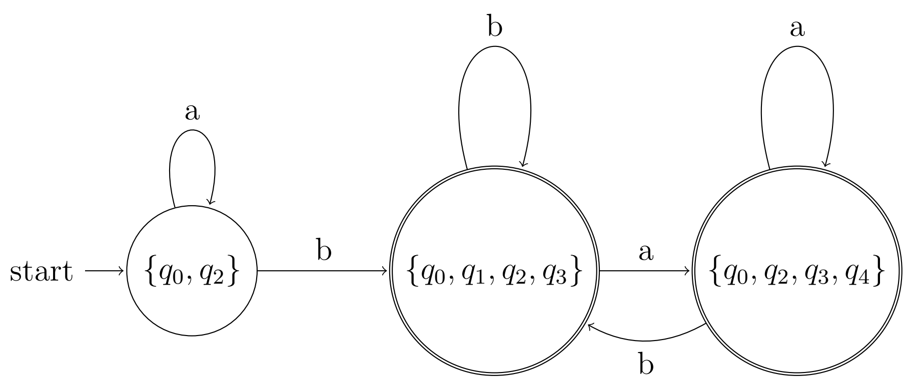
$$(q_0, babb) \vdash_N (q_1, abb) \vdash_N (q_1, bb)$$
$$(q_0, babb) \vdash_N (q_2, babb) \vdash_N (q_2, abb) \vdash_N (q_2, bb) \vdash_N (q_2, b) \vdash_N (q_2, e)$$
$$(q_0, babb) \vdash_N (q_2, babb) \vdash_N (q_2, abb) \vdash_N (q_4, bb) \vdash_N (q_4, b) \vdash_N (q_4, e)$$
$$(q_0, babb) \vdash_N (q_3, abb) \vdash_N (q_1, bb)$$
$$(q_0, babb) \vdash_N (q_3, abb) \vdash_N (q_4, abb) \vdash_N (q_1, bb)$$
$$(q_0, babb) \vdash_N (q_3, abb) \vdash_N (q_4, abb) \vdash_N (q_2, bb) \vdash_N (q_2, b) \vdash_N (q_2, e)$$

These are all possible configurations for w_2. Thus, $w_2 \notin L(N)$.

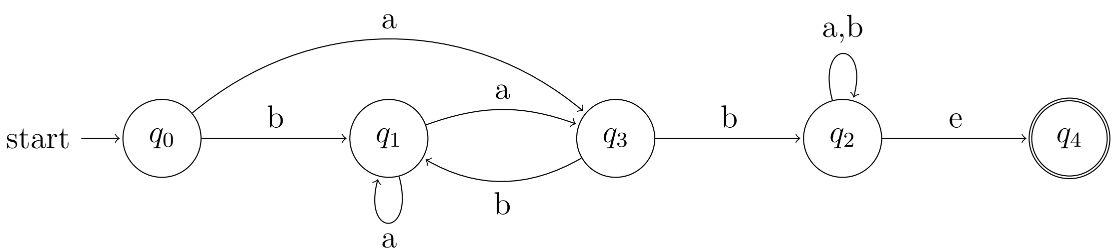
Answer 6

$E(q_0) = \{q_0, q_2\}$
$E(q_1) = \{q_1\}$
$E(q_2) = \{q_2\}$
$E(q_3) = \{q_3\}$
$E(q_4) = \{q_3, q_4\}$

$\delta(E(q_0), a) = E(q_0) = \{q_0, q_2\}$
$\delta(E(q_0), b) = E(q_0) \cup E(q_1) \cup E(q_3) = \{q_0, q_1, q_2, q_3\}$
$\delta(\{q_0, q_1, q_2, q_3\}, a) = E(q_0) \cup E(q_4) = \{q_0, q_2, q_3, q_4\}$
$\delta(\{q_0, q_1, q_2, q_3\}, b) = E(q_0) \cup E(q_1) \cup E(q_3) = \{q_0, q_1, q_2, q_3\}$
$\delta(\{q_0, q_2, q_3, q_4\}, a) = E(q_0) \cup E(q_4) = \{q_0, q_2, q_3, q_4\}$
$\delta(\{q_0, q_2, q_3, q_4\}, b) = E(q_0) \cup E(q_1) \cup E(q_3) = \{q_0, q_1, q_2, q_3\}$

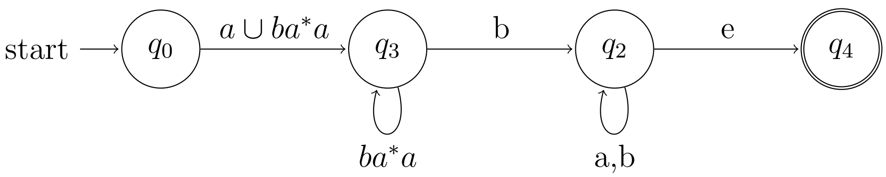


Answer 7



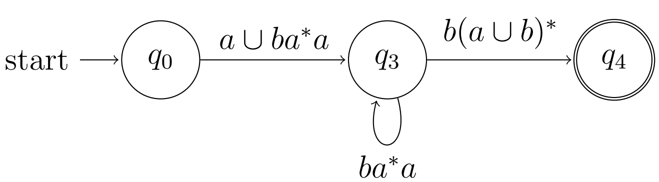
a) Eliminate q_1

$R(0,3,1) = ba^*a$
$R(3,3,1) = ba^*a$



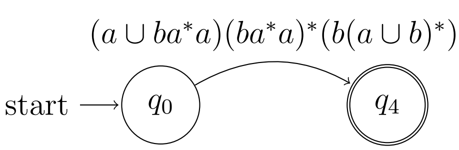
b) Eliminate q_2

$R(3,4,2) = b(a \cup b)^*e = b(a \cup b)^*$



c) Eliminate q_3

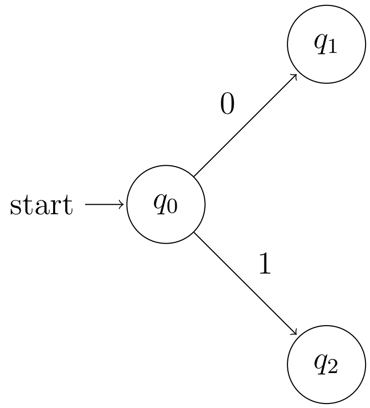
$R(0,4,3) = (a \cup ba^*a)(ba^*a)^*(b(a \cup b)^*)$



Regular expression is $(a \cup ba^*a)(ba^*a)^*(b(a \cup b)^*)$.

Answer 8

Assume that $M_L = (K, \Sigma, \delta, q_0, F)$ and $M_H = (K', \Sigma, \delta', s, F')$. I could not draw M_L in a single picture as in our textbook on page 77 so I can just show the beginning part of it.



The starting state of M_H must be q_1 and the accepting states are the ones who have outgoing transition with symbol 1 to any of final states of M_L. If $0w1 \in L$, then $(q_0, 0w1) \vdash^*_{M_L} (f, e)$ for some $f \in F$ if and only if $(q_1, w1) \vdash^*_{M_L} (f, e)$ and $(s, 0w) \vdash^*_{M_L} (f', 1)$ for some $f' \in F'$. Hence, M_L accepts $0w1$ if and only if M_H accepts w. Thus, $L(M_H) = H$.

Answer 9

a.

Assume L is regular. There is an integer $p \geq 1$ such that any string $w \in L$ with $|w| \geq p$ can be written as $w = xyz$ such that $y \neq e$, $|xy| \leq p$ and $xy^k z \in L$ for each $k \geq 0$.
Let

$$x = 1^t 0^i$$

$$y = 0^j$$

$$z = 0^{p!-i-j} 1^{2(p+1)!}$$

for $0 < j \leq p$. Then,

$$xy^k z = 1^t 0^{p!+(k-1)j} 1^{2(p+1)!}$$

If $p! + (k-1)j = (p+1)!$ then $n = m$. Let's see if the first equation holds for any k.

$$k - 1 = \frac{p!p}{j}$$

Since $j \leq p$, we can divide $p!$ by j and there exists an integer k. This is a contradiction. Thus, L is not regular.