

# Sample-Efficient Optimization in the Latent Space of Deep Generative Models via Weighted Retraining

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- 会议: NIPS2020
- 地址: <https://arxiv.org/abs/2006.09191>
- 代码: <https://github.com/cambridge-mlg/weighted-retraining>

## 论文主要内容

### 摘要

许多实际问题涉及在复杂、高维、结构化(graph、sequence、sets)空间内优化一个昂贵的黑盒函数。机器学习方法对这样的问题很有效，但是已有的方法非常缺少sample efficiency。本文提出使用一个deep generative model来将优化的空间变成低维、连续的latent manifold。通过在优化迭代中周期**retaining** generative model、**weighting** data points实现。

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## 研究内容

### Motivation

- *Latent space optimization* (LSO) 方法
  - 1: 构建一个(deep)生成模型，用来将低维连续隐空间的tensors映射到输入空间的数据 manifold
  - 2: 在latent space上优化目标函数（使用surrogate model）
  - 问题：
    - 一般生成模型使用pretrained，没有针对下游任务专门精心定制（从优化任务中decouple）。
    - 给优化造成不必要的困难
- 基于此提出：对数据分布weighting、周期retaining生成模型来消除decouple

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# 方法

## 介绍

## 背景

### Sample-Efficient Black Box Optimization

- $\mathcal{X}$  是input space (这篇文章针对于高维的、结构化的)
- $f : \mathcal{X} \mapsto \mathbb{R}$  是优化的目标函数。  $f$  是黑盒的, 即不知道解析式、无法获得梯度信息。  
evaluate的代价很高(昂贵)
- problem: 通过evaluate  $f$  次数尽量少优化。  $\mathcal{D}_M \equiv \{\mathbf{x}_i, f(\mathbf{x}_i)\}_{i=1}^M$  为evaluate M次的一个序列

### Model-based Optimization

- 使用一个objective/surrogate model  $h_{\mathcal{X}} : \mathcal{X} \mapsto \mathbb{R}$  来近似  $f$  . 优化  $h$  要比  $f$  容易的多。  
 $f(\mathbf{x}) \approx h(\mathbf{x})$
- 但由于  $\mathcal{X}$  空间的复杂, 使得优化  $h_{\mathcal{X}}$  也变得困难
- $h_{\mathcal{X}}$  需要用样本  $\{(\mathbf{x}, f(\mathbf{x}))\}$  来拟合

### Latent Space Optimization (LSO)

- 建立一个生成模型:  $g : \mathcal{Z} \mapsto \mathcal{X}$  . (可以使用VAE、GAN)
- 使用一个latent objective/surrogate model  $h : \mathcal{Z} \mapsto \mathbb{R}$  来近似  $f$  .  $f(g(\mathbf{z})) \approx h(\mathbf{z})$
- 如果  $\mathcal{Z}$  选择低维、连续空间, 如  $\mathbb{R}^n$  , 优化  $h_{\mathcal{Z}}$  就变简单了
- $h_{\mathcal{Z}}$  需要用样本  $\{(\mathbf{z}, f(g(\mathbf{z})))\}$  来拟合, 所以需要有一个解码器  $q : \mathcal{X} \mapsto \mathcal{Z}$

### LSO NOT working well

- 首先生成模型有个特点: 生成模型  $g : \mathcal{Z} \mapsto \mathcal{X}$  , SOTA的模型如VAE、GAN训练中需要一个prior  $p(\mathbf{z})$ 
  - 这意味着尽管  $g$  是定义在整个  $\mathcal{Z}$  上的, 但几乎全部概率集中在一个子空间内  $\mathcal{Z}' \subset \mathcal{Z}$  ,  
 $\mathcal{Z}'$  称为隐空间的可行域。在这个可行域以外的sample一般结果很差或者invalid structure
  - 所以一般都会在可行域  $\mathcal{Z}'$  内进行优化。 *bounded optimization*
- 对很多优化问题, DGM训练数据大部分是low-score(sub optimal)=>致使可行域大部分都是low-score点 (使求解问题变得“大海捞针”)、把稀有的high-score的点放到DGM training distribution外

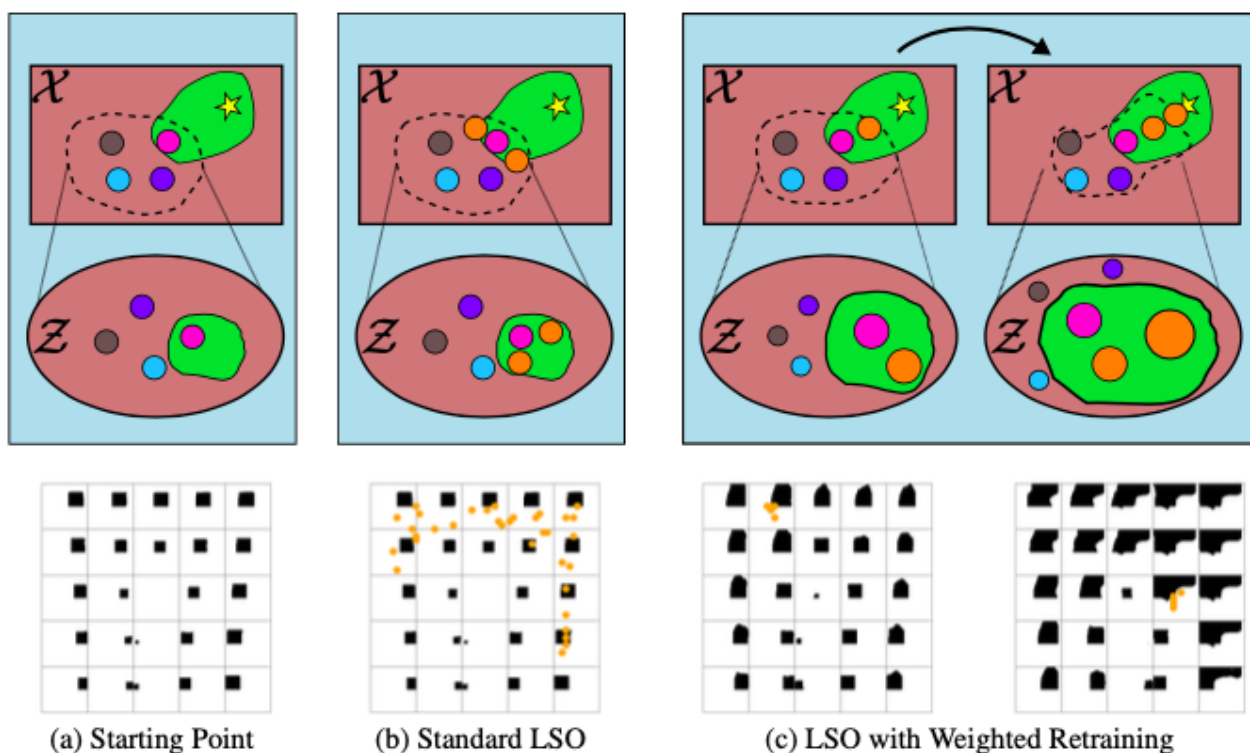


Figure 1: Schematic illustrating LSO with and without weighted retraining. The cartoon illustrates the input/latent space of the generative model (**top**). The latent manifold from Section 6.2's 2D shape area maximization task is shown for comparison (**bottom**). Each image in the manifold shows the result of decoding a latent point on a uniform square grid in a 2D latent space; images are centered on the original grid points. **Red/green regions correspond to points with low/high objective function values respectively.** The yellow star is the global optimum in  $\mathcal{X}$ . Coloured circles are data points; their radius represents their weight. The dashed line surrounds the region of  $\mathcal{X}$  modelled by  $g$  (i.e.  $g(\mathcal{Z})$ , the image of  $\mathcal{Z}$ ). **(a)** The status of the generative model  $g$  at the start of optimization. **(b)** The result of standard LSO with  $g$  fixed, which queries the points in orange. It is only able to find points close to the training data used to learn  $\mathcal{Z}$ , resulting in slow and incomplete exploration of  $\mathcal{X}$ . **(c)** The result midway (left) and at the end (right) of LSO with our proposed approach, which weights data points according to their objective function value and retrains  $g$  to incorporate newly queried data. This continually adjusts  $\mathcal{Z}$  to focus on modelling the most promising regions of  $\mathcal{X}$ , speeding up the optimization and allowing for substantial extrapolation beyond the initial training data.

- 生成模型的目标和实际的目标不一致
  - 生成模型：为了学习latent space能尽可能接近训练数据的分布
  - 实际的目标：为了学习latent space能易于优化算法在latent space上高效优化
- 随着新的数据点的加入，没有shift latent space。整个过程固定了  $g$

## LSO with Weighted Retraining

- weighting
  - 简单的在DGM训练中丢掉low-score 点仅仅在数据点多的时候有效(X)
  - 让high-score在latent space上有更高的概率，low-score在latent space上更低的概率。使用所有data训练学习representation防止过拟合
    - 具体做法1：为每个data points赋予一个大于等于0的权重。score越高，权重越大。所有数据权重求和为1
    - 具体做法2: 权重作为采样概率来构成minibatch训练

- 一种rank可选方式，k是超参：

$$w(\mathbf{x}; \mathcal{D}, k) \propto \frac{1}{kN + \text{rank}_{f, \mathcal{D}}(\mathbf{x})}, \quad \text{rank}_{f, \mathcal{D}}(\mathbf{x}) = |\{\mathbf{x}_i : f(\mathbf{x}_i) > f(\mathbf{x}), \mathbf{x}_i \in \mathcal{D}\}|, \quad (1)$$

- Retraining
  - fine-tuning
  - 若新的data带有high-score，那么会赋予大的权重同时给旧data小的权重。对training distribution影响较大。因此latent space会较大程度改变
  - 若新的data带有low-score，那么会赋予小的权重，同时旧data权重改变不大。基本不改变training distribution。因此latent space不怎么变化

### A.3.1 PyTorch (weighted sampling)

#### Standard Training

```
from torch.utils.data import *

dataloader = DataLoader(data)
for batch in dataloader:
    # ...
```

#### Weighted Training

```
from torch.utils.data import *
sampler = WeightedRandomSampler(
    weights, len(data))
dataloader = DataLoader(data, sampler=sampler)
for batch in dataloader:
    # ...
```

### A.3.2 PyTorch (direct application of weights)

#### Standard Training

```
criterion = nn.MSELoss()
outputs = model(inputs)
loss = criterion(outputs, targets)

loss.backward()
```

#### Weighted Training

```
criterion = nn.MSELoss(reduction=None)
outputs = model(inputs)
loss = criterion(outputs, targets)
loss = torch.mean(loss * weights)
loss.backward()
```

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### Algorithm 1 Latent Space Optimization with Weighted Retraining (changes highlighted in blue)

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- 1: **Input:** Data  $\mathcal{D} = \{(\mathbf{x}_i, f(\mathbf{x}_i))\}_{i=1}^N$ , query budget  $M$ , objective function  $f(\mathbf{x})$ , latent objective model  $h(\mathbf{z})$ , generative/inverse model  $g(\mathbf{z})/q(\mathbf{x})$ , **retrain frequency  $r$ , weighting function  $w(\mathbf{x})$**
  - 2: **for**  $1, \dots, M/r$  **do**
  - 3:   Train generative model  $g$  and inverse model  $q$  on data  $\mathcal{D}$  **weighted by  $\mathcal{W} = \{w(\mathbf{x})\}_{\mathbf{x} \in \mathcal{D}}$**
  - 4:   **for**  $1, \dots, r$  **do**
  - 5:     Compute latent variables  $\mathcal{Z} = \{\mathbf{z} = q(\mathbf{x})\}_{\mathbf{x} \in \mathcal{D}}$
  - 6:     Fit objective model  $h$  to  $\mathcal{Z}$  and  $\mathcal{D}$ , and optimize  $h$  to obtain new latent query point  $\tilde{\mathbf{z}}$
  - 7:     Obtain corresponding input  $\tilde{\mathbf{x}} = g(\tilde{\mathbf{z}})$ , evaluate  $f(\tilde{\mathbf{x}})$  and set  $\mathcal{D} \leftarrow \mathcal{D} \cup \{(\tilde{\mathbf{x}}, f(\tilde{\mathbf{x}}))\}$
  - 8:   **end for**
  - 9: **end for**
  - 10: **Output:** Augmented dataset  $\mathcal{D}$
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## 实验

## Results

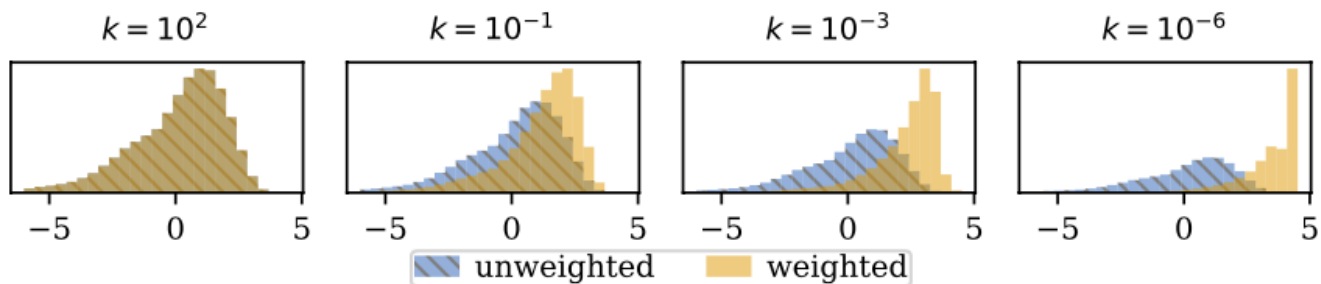


Figure 2: Histogram of objective function values for the ZINC dataset (see Section 6) with uniform weighting (in blue) as well as rank weighting from Equation (1) for different  $k$  values (in orange). Large  $k$  approaches uniform weighting, while small  $k$  places most weight on high-scoring points.

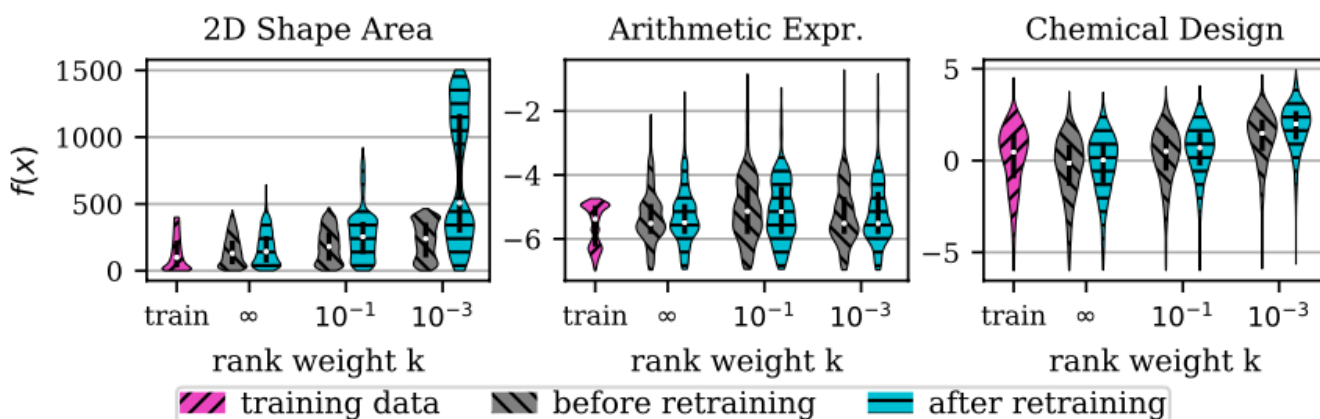


Figure 3: Objective value distribution for the training set and samples from the DGM's prior for all three tasks for different  $k$  values, before and after weighted retraining (see Section 6.2).

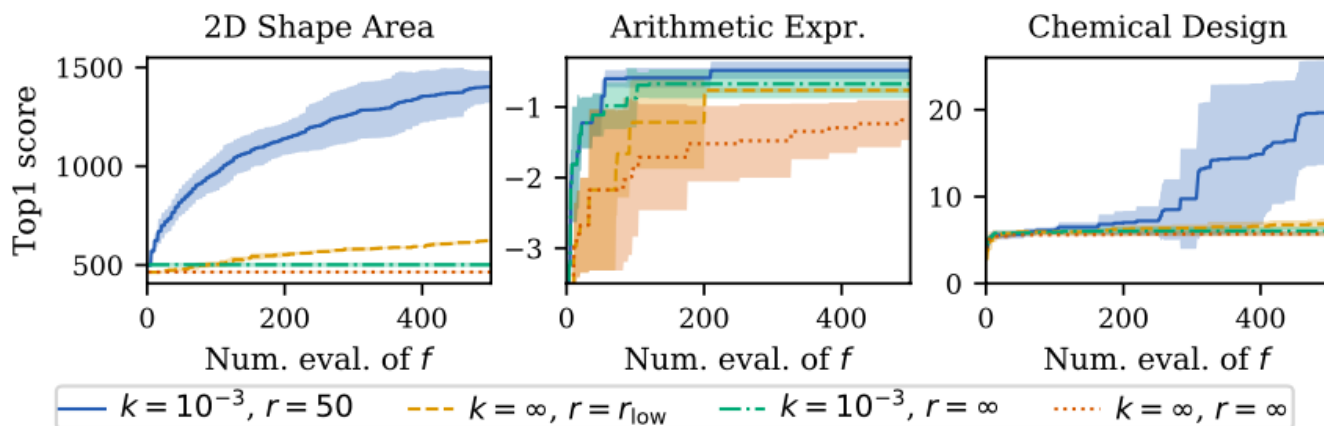


Figure 4: Top1 optimization performance of weighted retraining for all tasks, for different  $k$  values (i.e.  $k \in \{10^{-3}, \infty\}$ ) and retraining frequencies (i.e.  $r_{\text{low}} = 5$  for the 2D shape area task, and  $r_{\text{low}} = 50$  for the other two tasks). Shaded area corresponds to standard deviation.

·  $r$ 是retraining 周期



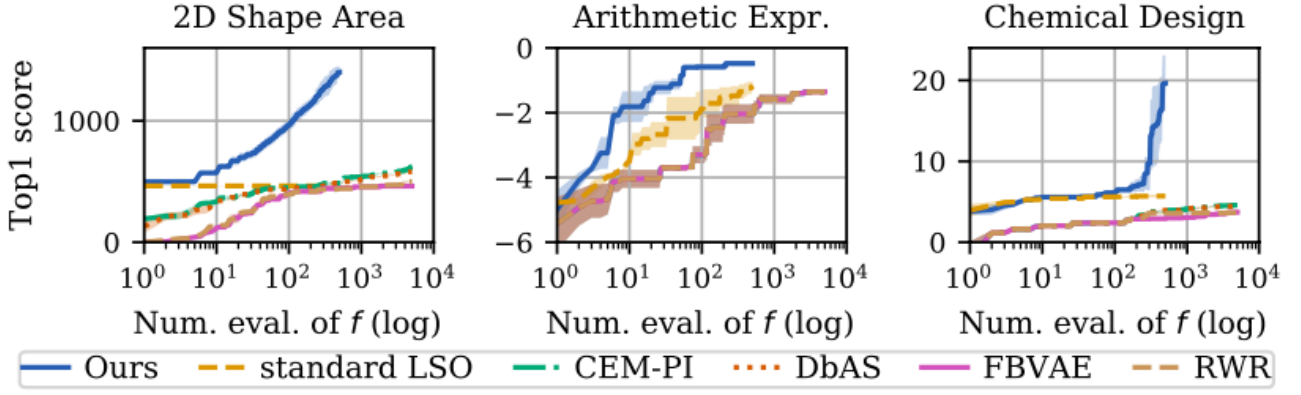


Figure 5: Comparison of weighted retraining, LSO, CEM-PI, DbAS, FBVAE and RWR. Our approach significantly outperforms all baselines, achieving both better sample-efficiency and final performance.

## Independence from Dataset Size

$$\begin{aligned}
 \sum_{r=q_1 N}^{q_2 N} w(\mathbf{x}_r; \mathcal{D}, k) &= \sum_{r=q_1 N}^{q_2 N} \frac{1}{kN + r} \\
 &= \sum_{r=1}^{(k+q_2)N} \frac{1}{r} - \sum_{r'=1}^{(k+q_1)N-1} \frac{1}{r'} \\
 &\approx (\ln((k+q_2)N - 1) + \gamma) - (\ln((k+q_1)N - 1) + \gamma) \\
 &= \ln \frac{(k+q_2)N}{(k+q_1)N - 1} \approx \ln \frac{(k+q_2)N}{(k+q_1)N} = \ln \frac{(k+q_2)}{(k+q_1)}
 \end{aligned}$$

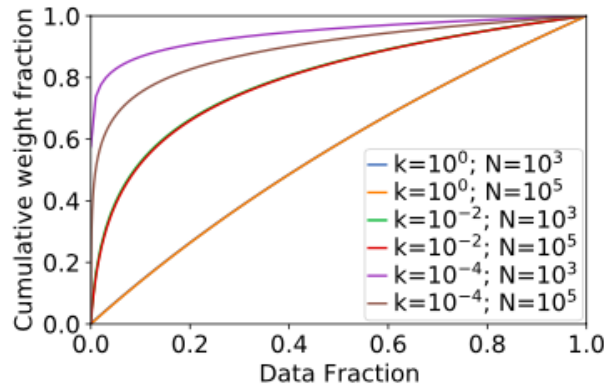


Figure 6: Cumulative distribution of rank weights (sorted highest to lowest), showing a distribution that is independent of  $N$  if  $kN > 1$ .