Possible Choices of Complex Co-ordinates for a Surface
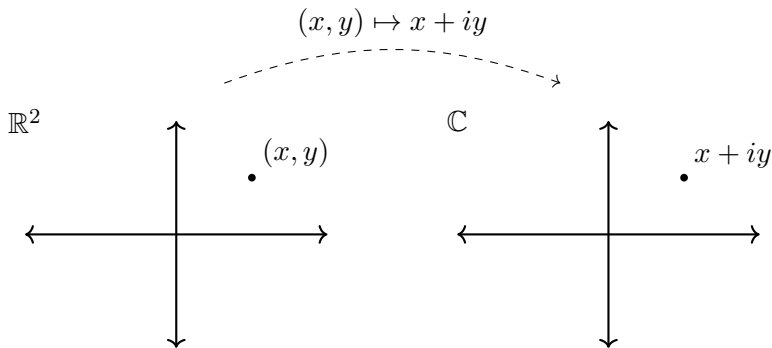## Moduli Spaces of Riemann Surfaces

Zhaoshen Zhai
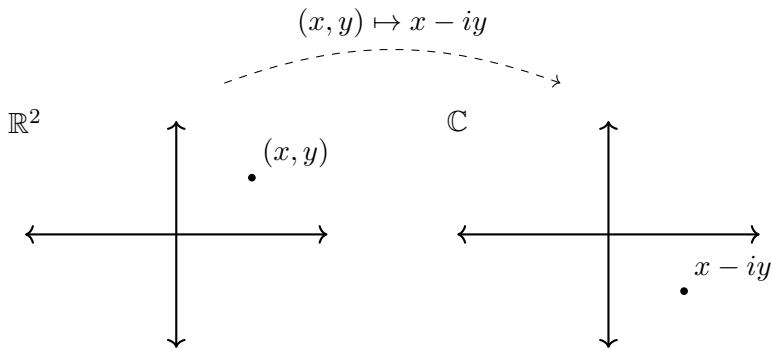
McGill University

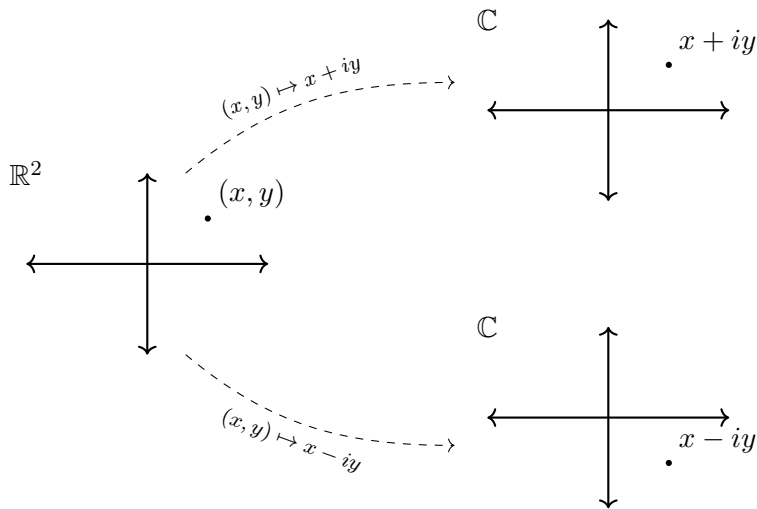April 23, 2023

How many complex coordinates on $\mathbb{R}^2$?

$(x, y) \mapsto x + iy$

$\mathbb{R}^2$

(x, y)

$\mathbb{C}$

$x + iy$

How many complex coordinates on $\mathbb{R}^2$?

$$(x, y) \mapsto x - iy$$

$\mathbb{R}^2$

(x, y)

$\mathbb{C}$

$x - iy$

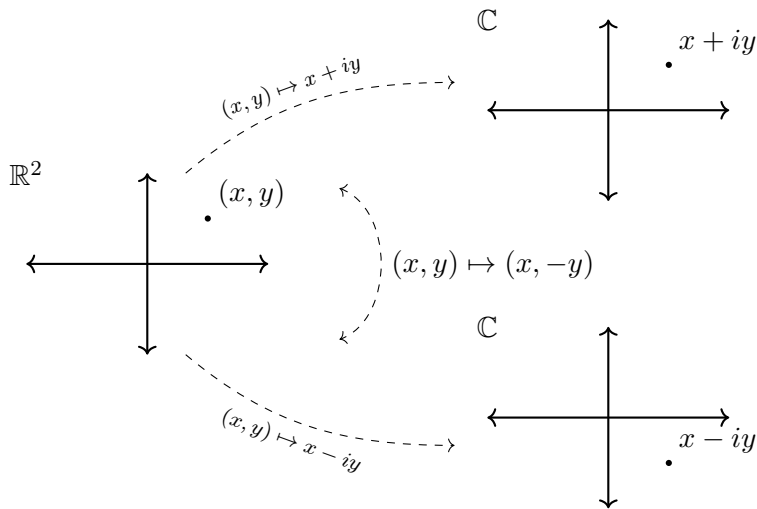# How many complex coordinates on $\mathbb{R}^2$?
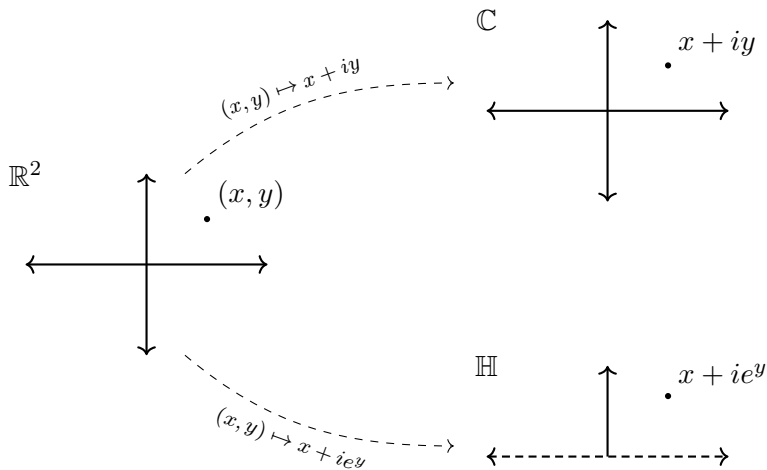
How many complex coordinates on $\mathbb{R}^2$?

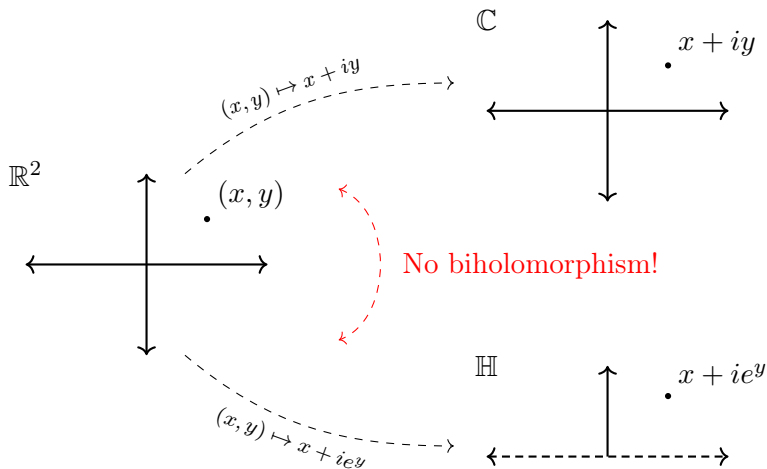How many complex coordinates on $\mathbb{R}^2$?

~~How many complex coordinates on $\mathbb{R}^2$?~~
How many complex coordinates on $\mathbb{R}^2$ *up to biholomorphism*?

$\mathbb{C}$ and $\mathbb{H}$

$\mathbb{C}$

$x + iy$

$(x,y) \mapsto x + iy$

$\mathbb{R}^2$

(x, y)

No biholomorphism!

$\mathbb{H}$

$x + ie^y$

$(x,y) \mapsto x + ie^y$

What about for manifolds?

What about for manifolds?

Definition
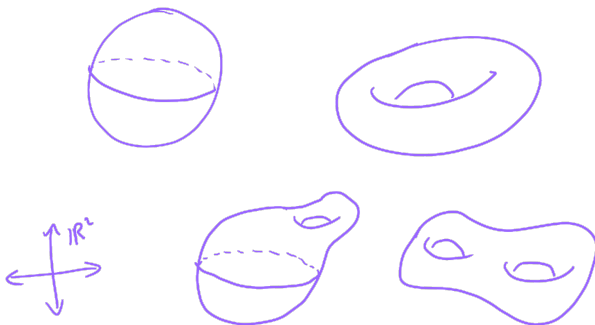
A *Riemann surface* is a connected 1-dimensional complex manifold.

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Definition

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Theorem (Classification)

Every compact Riemann surface is classified by its genus.

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