Poisson Summation Formula

$$X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{T}\right) \tag{1}$$

$$X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right) \tag{2}$$

Analysis Equation

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{3}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{4}$$

Synthesis Equation

$$a_k = \frac{1}{T} \int_{<T>} x(t) e^{-jk\frac{2\pi}{T}t} dt \tag{5}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t} \tag{6}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{7}$$

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\frac{2\pi}{N}n} \tag{8}$$

$$x[n] = \sum_{k=<N>} a_k e^{jk\frac{2\pi}{N}n} \tag{9}$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \tag{10}$$

Symmetry Properties

If $x(t)$ is real, then $|X(j\omega)|$ is even and $\angle X(j\omega)$ is odd. Moreover, if $x(t)$ is real and even, then $X(j\omega)$ must be purely real and even, and if $x(t)$ is real and odd, $X(j\omega)$ is purely imaginary and odd.

Duality Property

$$\mathcal{F}\{X(t)\} = 2\pi x(-\omega). \tag{11}$$

Modulation Property

$$x(t)\cos(\omega_0 t) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2}\left(X(j(\omega-\omega_0)) + X(j(\omega+\omega_0))\right) \tag{12}$$

$$x[n]\cos(\omega_0 n) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2}\left(X(e^{j(\omega-\omega_0)}) + X(e^{j(\omega+\omega_0)})\right) \tag{13}$$

LTI System Invertibility Criterion

An LTI system with frequency response $H(j\omega)$ (CT) or $H(e^{j\omega})$ (DT) is invertible if and only if $H(j\omega) \neq 0$ for all ω (CT) or $H(e^{j\omega}) \neq 0$ for all $\omega \in [-\pi, \pi]$ (DT).

LTI System Input/Output

$$Y(j\omega) = H(j\omega)X(j\omega) \tag{14}$$

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) \tag{15}$$

Differential Equations

For a system described by:

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \tag{16}$$

The frequency response is:

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\sum_{k=0}^{M} b_k (j\omega)^k}{\sum_{k=0}^{N} a_k (j\omega)^k} \tag{17}$$

Difference Equations

For a system described by:

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k] \tag{18}$$

The frequency response is:

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{\sum_{k=0}^{N} a_k e^{-j\omega k}} \tag{19}$$

Table 1: Properties of Fourier Transforms

Property	**CT Time Domain**	**CT Frequency Domain**	**DT Time Domain**	**DT Frequency Domain**
Linearity	$Ax_1(t) + Bx_2(t)$	$AX_1(j\omega) + BX_2(j\omega)$	$Ax_1[n] + Bx_2[n]$	$AX_1(e^{j\omega}) + BX_2(e^{j\omega})$
Time Shifting	$x(t - t_0)$	$X(j\omega)e^{-j\omega t_0}$	$x[n - n_0]$	$X(e^{j\omega})e^{-j\omega n_0}$
Frequency Shifting	$x(t)e^{j\omega_0 t}$	$X(j(\omega - \omega_0))$	$x[n]e^{j\omega_0 n}$	$X(e^{j(\omega-\omega_0)})$
Time Scaling	$x(at)$, $a \neq 0$	$\frac{1}{\vert a\vert} X\left(\frac{j\omega}{a}\right)$	—	—
Time Reversal	$x(-t)$	$X(-j\omega)$	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*(t)$	$X^*(-j\omega)$	$x^*[n]$	$X^*(e^{-j\omega})$
Differentiation	$\frac{d^n}{dt^n}x(t)$	$(j\omega)^n X(j\omega)$	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$	$\sum_{k=-\infty}^{n} x[k]$	$\frac{X(e^{j\omega})}{1-e^{-j\omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
Convolution	$(x_1 * x_2)(t)$	$X_1(j\omega)X_2(j\omega)$	$(x_1 * x_2)[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}(X_1 * X_2)(j\omega)$	$x_1[n]x_2[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta$
Parseval's Theorem	$\int_{-\infty}^{\infty} \vert x(t)\vert^2 dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \vert X(j\omega)\vert^2 d\omega$	$\sum_{n=-\infty}^{\infty} \vert x[n]\vert^2$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} \vert X(e^{j\omega})\vert^2 d\omega$

Table 2: Fourier Transform Pairs

CT Time Domain $x(t)$	**CT Fourier Transform** $X(j\omega)$	**DT Time Domain** $x[n]$	**DT Fourier Transform** $X(e^{j\omega})$
$\delta(t)$	1	$\delta[n]$	1
1	$2\pi\,\delta(\omega)$	1	$2\pi\,\sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$u(t)$	$\pi\,\delta(\omega) + \frac{1}{j\omega}$	$u[n]$	$\frac{1}{1-e^{-j\omega}} + \pi\sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$e^{-at}u(t), \quad \Re(a) > 0$	$\dfrac{1}{a + j\omega}$	$a^n u[n], \quad \vert a\vert < 1$	$\dfrac{1}{1 - ae^{-j\omega}}$
$e^{-a\vert t\vert}$, $a > 0$	$\dfrac{2a}{a^2 + \omega^2}$	$a^{\vert n\vert}, \quad \vert a\vert < 1$	$\dfrac{1 - a^2}{1 - 2a\cos\omega + a^2}$
$\cos(\omega_0 t)$	$\pi\Big[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\Big]$	$\cos(\omega_0 n)$	$\pi\,\sum_{k=-\infty}^{\infty}\Big[\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)\Big]$
$\sin(\omega_0 t)$	$\dfrac{\pi}{j}\Big[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\Big]$	$\sin(\omega_0 n)$	$\dfrac{\pi}{j}\,\sum_{k=-\infty}^{\infty}\Big[\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)\Big]$
$\begin{cases} 1, & \vert t\vert \le T/2 \\ 0, & \vert t\vert > T/2 \end{cases}$	$2\frac{\sin\left(\frac{\omega T}{2}\right)}{\omega}$	$\begin{cases} 1, & 0 \le n \le N \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{\sin\Big(\omega(N+1)/2\Big)}{\sin\Big(\omega/2\Big)}\, e^{-j\omega N/2}$

Table 3: Properties of Fourier Series

Property	**CT Time Domain**	**CT Frequency Domain** (a_k)	**DT Time Domain**	**DT Frequency Domain** (a_k)
Linearity	$Ax_1(t) + Bx_2(t)$	$Aa_k + Bb_k$	$A\,x_1[n] + B\,x_2[n]$	$A\,a_k + B\,b_k$
Time Shifting	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0}$	$x[n - n_0]$	$a_k\,e^{-j\,\frac{2\pi}{N}kn_0}$
Frequency Shifting	$x(t)e^{jM\omega_0 t}$	a_{k-M}	$x[n]\,e^{j\,\frac{2\pi}{N}Mn}$	$a_{(k-M) \bmod N}$
Time Reversal	$x(-t)$	a_{-k}	$x[-n]$	a_{-k} (indices mod N)
Conjugation	$x^*(t)$	a_{-k}^*	$x^*[n]$	a_{-k}^* (indices mod N)
Periodic Convolution	$(x * y)(t)$	$Ta_k b_k$	$(x \circledast y)[n]$	$Na_k\,b_k$
Multiplication	$x(t)y(t)$	$\sum_{l=-\infty}^{\infty} a_l b_{k-l}$	$x[n]\,y[n]$	$\sum_{m=0}^{N-1} a_m\,b_{(k-m) \bmod N}$
Differentiation	$\frac{d}{dt}x(t)$	$jk\omega_0 a_k$	$x[n] - x[n-1]$	$a_k\Big(1 - e^{-j\,\frac{2\pi}{N}k}\Big)$
Integration	$\int x(t)dt$	$\frac{a_k}{jk\omega_0}$ ($a_0 = 0$)	—	—
Parseval's Theorem	$\frac{1}{T}\int_T \vert x(t)\vert^2 dt$	$\sum_{k=-\infty}^{\infty} \vert a_k\vert^2$	$\frac{1}{N}\sum_{n=0}^{N-1} \vert x[n]\vert^2$	$\sum_{k=0}^{N-1} \vert a_k\vert^2$
Symmetry (Real)	$x(t)$ real	$a_k = a_{-k}^*$	$x[n]$ real	$a_k = a_{-k}^*$
Symmetry (Real+Even)	$x(t)$ real and even	a_k real and even	$x[n]$ real and even	a_k real and even
Symmetry (Real+Odd)	$x(t)$ real and odd	a_k purely imaginary and odd	$x[n]$ real and odd	a_k purely imaginary and odd