For periodic functions $x_1(t)$, $x_2(t)$ with periods T_1 and T_2 respectively, the period of $x_1(t)+x_2(t)$ is $LCM(T_1,T_2)$.
For periodic functions $x_1[n]$, $x_2[n]$ with periods N_1 and N_2 respectively, the period of $x_1[n]+x_2[n]$ is $LCM(N_1,N_2)$.

$$\begin{aligned} E &= \int_{t_1}^{t_2} |x(t)|^2 dt \\ &= \int_{t_1}^{t_2} (x_{Re}(t)^2 + x_{Im}(t)^2) dt. \end{aligned}$$

$$\begin{aligned} E &= \sum_{n=n_1}^{n_2} |x[n]|^2 \\ &= \sum_{n=n_1}^{n_2} (x_{Re}[n]^2 + x_{Im}[n]^2). \end{aligned}$$

$$P_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 dt.$$

$$P_{avg} = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2.$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$\begin{aligned} x(t) &= x_{even}(t) + x_{odd}(t) \\ &= \frac{x(t)+x(-t)}{2} + \frac{x(t)-x(-t)}{2} \end{aligned}$$

$$\begin{aligned} x[n] &= x_{even}[n] + x_{odd}[n] \\ &= \frac{x[n]+x[-n]}{2} + \frac{x[n]-x[-n]}{2} \end{aligned}$$

$$\begin{aligned} x_k(t) &= e^{jk\omega_0 t}, k \in \mathbb{Z} \\ \omega_0 &= \frac{2\pi}{T}, \text{ period T} \\ x_k[n] &= e^{jk\omega_0 n}, k \in \mathbb{Z} \\ \omega_0 &= \frac{2\pi}{N}, \text{ period N} \end{aligned}$$

- A *memoryless* system depends only on input at time t (or n for DT systems).
- A *linear* system satisfies $S\{ax_1(t)+bx_2(t)\} = aS\{x_1(t)\} + bS\{x_2(t)\}$.
- A system is *time invariant* if a time shift (t_0 or n_0) in the input results in an identical shift in the output.
- A *linear time invariant*'s impulse response h is found by calculating $S(\delta(t))$ (or $S(\delta[n])$). $y(t) = x(t) * h(t)$ (or $y[n] = x[n] * h[n]$).
- A system is *invertible* if there exists an inverse S_2 such that $S_2(S_1(x(t))) = S_1(S_2(x(t))) = x(t)$.
- A system is *causal* if its output at time t (index n) depends only on present and past inputs.
- A system is *stable* if there exist constants $B, M > 0$ such that $|x(t)| < B\ \forall t \implies |y(t)| < M\ \forall t$.

- $x(t)$ real $\to a_k = a^*_{-k}$
- For a memoryless system, the impulse response is the delta function times a scalar.

Property	**Time Domain**		**Frequency Domain**
Linearity	$Ax_1(t) + Bx_2(t)$	$\overset{FS}{\leftrightarrow}$	$Aa_k + Bb_k$
Even Symmetry	$x(t)$ even	$\overset{FS}{\leftrightarrow}$	a_k even
Odd Symmetry	$x(t)$ odd	$\overset{FS}{\leftrightarrow}$	a_k odd
Time Shifting	$x(t - t_0)$	$\overset{FS}{\leftrightarrow}$	$a_k e^{-jk\omega_0 t_0}$
Frequency Shifting	$x(t)e^{jn\omega_0 t}$	$\overset{FS}{\leftrightarrow}$	a_{k-n}
Time Reversal	$x(-t)$	$\overset{FS}{\leftrightarrow}$	a_{-k}
Conjugation	$x^*(t)$	$\overset{FS}{\leftrightarrow}$	a^*_{-k}
Periodic Convolution	$(x * y)(t)$	$\overset{FS}{\leftrightarrow}$	$a_k b_k$
Multiplication	$x(t)y(t)$	$\overset{FS}{\leftrightarrow}$	$\sum_{n=-\infty}^{\infty} a_n b_{k-n}$
Differentiation	$\frac{d}{dt}x(t)$	$\overset{FS}{\leftrightarrow}$	$jk\omega_0 a_k$
Integration	$\int x(t)dt$	$\overset{FS}{\leftrightarrow}$	$\frac{a_k}{jk\omega_0}$ if $a_0 = 0)$
Parseval's Theorem	$\frac{1}{T}\int_T \lvert x(t)\rvert^2 dt$	$\overset{FS}{\leftrightarrow}$	$\sum_{k=-\infty}^{\infty} \lvert a_k\rvert^2$

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Property	**Time Domain**		**Frequency Domain**
Linearity	$A\,x_1[n] + B\,x_2[n]$	$\overset{DTFS}{\leftrightarrow}$	$A\,a_k + B\,b_k$
Even Symmetry	$x[n]$ even (i.e., $x[n] = x[-n]$)	$\overset{DTFS}{\leftrightarrow}$	a_k even
Odd Symmetry	$x[n]$ odd (i.e., $x[n] = -x[-n]$)	$\overset{DTFS}{\leftrightarrow}$	a_k odd
Time Shifting	$x[n - n_0]$	$\overset{DTFS}{\leftrightarrow}$	$a_k\, e^{-j\frac{2\pi}{N}kn_0}$
Frequency Shifting	$x[n]\, e^{j\frac{2\pi}{N}n_0 n}$	$\overset{DTFS}{\leftrightarrow}$	$a_{(k-n_0) \bmod N}$
Time Reversal	$x[-n]$	$\overset{DTFS}{\leftrightarrow}$	a_{-k} (indices mod N)
Conjugation	$x^*[n]$	$\overset{DTFS}{\leftrightarrow}$	a^*_{-k}
Circular Convolution	$(x \circledast y)[n]$	$\overset{DTFS}{\leftrightarrow}$	$a_k\, b_k$
Multiplication	$x[n]\, y[n]$	$\overset{DTFS}{\leftrightarrow}$	$\frac{1}{N}\sum_{m=0}^{N-1} a_m\, b_{(k-m) \bmod N}$
Difference Operator	$x[n] - x[n-1]$	$\overset{DTFS}{\leftrightarrow}$	$a_k\left(1 - e^{-j\frac{2\pi}{N}k}\right)$
Parseval's Theorem	$\frac{1}{N}\sum_{n=0}^{N-1} \lvert x[n]\rvert^2$	$\overset{DTFS}{\leftrightarrow}$	$\sum_{k=0}^{N-1} \lvert a_k\rvert^2$

- For causal systems, the impulse response must be 0 for values of n (t) less than 0.
- For a stable LTI system, the impulse response must be absolutely summable. That is,

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty. \tag{1}$$

Or for continuous time, absolutely integrable

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty. \tag{2}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \tag{3}$$

$$a_k = \frac{1}{T}\int_{<T>} x(t) e^{-jk\omega_0 t} dt \tag{4}$$

$$x[n] = \sum_{k=<N>} a_k x_k[n] \tag{5}$$

$$a_k = \frac{1}{N}\sum_{n=<N>} x[n] e^{-jk\frac{2\pi}{N}n}, \tag{6}$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \tag{7}$$

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]. \tag{8}$$