

CTFT Analysis Equation

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \quad (1)$$

DTFT Analysis Equation

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \quad (2)$$

CTFT Synthesis Equation

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)e^{j\omega t} d\omega \quad (3)$$

DTFT Synthesis Equation

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega \quad (4)$$

Fourier Series to Transform

If the Fourier series of $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad (5)$$

then the Fourier transform is

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0). \quad (6)$$

Symmetry Properties

If $x(t)$ is real, then $|X(j\omega)|$ is even and $\angle X(j\omega)$ is odd. Moreover, if $x(t)$ is real and even, then $X(j\omega)$ must be purely real and even, and if $x(t)$ is real and odd, $X(j\omega)$ is purely imaginary and odd.

Duality Property

$$\mathcal{F}\{X(t)\} = 2\pi x(-\omega). \quad (7)$$

Modulation Property

$$x(t) \cos(\omega_0 t) \xrightarrow{\mathcal{F}} \frac{1}{2} (X(j(\omega - \omega_0)) + X(j(\omega + \omega_0))) \quad (8)$$

LTI System Invertibility Criterion

An LTI system with frequency response $H(j\omega)$ (CT) or $H(e^{j\omega})$ (DT) is invertible if and only if $H(j\omega) \neq 0$ for all ω (CT) or $H(e^{j\omega}) \neq 0$ for all $\omega \in [-\pi, \pi]$ (DT).

Table 1: Properties of Fourier Transforms

Property	CT Time Domain	CT Frequency Domain	DT Time Domain	DT Frequency Domain
Linearity	$Ax_1(t) + Bx_2(t)$	$AX_1(j\omega) + BX_2(j\omega)$	$Ax_1[n] + Bx_2[n]$	$AX_1(e^{j\omega}) + BX_2(e^{j\omega})$
Time Shifting	$x(t - t_0)$	$X(j\omega)e^{-j\omega t_0}$	$x[n - n_0]$	$X(e^{j\omega})e^{-j\omega n_0}$
Frequency Shifting	$x(t)e^{j\omega_0 t}$	$X(j(\omega - \omega_0))$	$x[n]e^{j\omega_0 n}$	$X(e^{j(\omega - \omega_0)})$
Time Scaling	$x(at), a \neq 0$	$\frac{1}{ a } X\left(\frac{j\omega}{a}\right)$	—	—
Time Reversal	$x(-t)$	$X(-j\omega)$	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*(t)$	$X^*(-j\omega)$	$x^*[n]$	$X^*(e^{-j\omega})$
Differentiation	$\frac{d^n}{dt^n} x(t)$	$(j\omega)^n X(j\omega)$	$x[n] - x[n - 1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
Integration	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$	$\sum_{k=-\infty}^n x[k]$	$\frac{X(e^{j\omega})}{1 - e^{-j\omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
Convolution	$(x_1 * x_2)(t)$	$X_1(j\omega)X_2(j\omega)$	$(x_1 * x_2)[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi} (X_1 * X_2)(j\omega)$	$x_1[n]x_2[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\theta})X_2(e^{j(\omega - \theta)}) d\theta$
Parseval's Theorem	$\int_{-\infty}^{\infty} x(t) ^2 dt$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) ^2 d\omega$	$\sum_{n=-\infty}^{\infty} x[n] ^2$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) ^2 d\omega$

Table 2: Common Fourier Transform Pairs

CT Time Domain $x(t)$	CT Fourier Transform $X(j\omega)$	DT Time Domain $x[n]$	DT Fourier Transform $X(e^{j\omega})$
$\delta(t)$	1	$\delta[n]$	1
1	$2\pi \delta(\omega)$	1	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$u(t)$	$\pi \delta(\omega) + \frac{1}{j\omega}$	$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$e^{-at}u(t), \Re(a) > 0$	$\frac{1}{a + j\omega}$	$a^n u[n], a < 1$	$\frac{1}{1 - ae^{-j\omega}}$
$\cos(\omega_0 t)$	$\pi \left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0) \right]$	$\cos(\omega_0 n)$	$\pi \sum_{k=-\infty}^{\infty} \left[\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k) \right]$
$\sin(\omega_0 t)$	$\frac{\pi}{j} \left[\delta(\omega - \omega_0) - \delta(\omega + \omega_0) \right]$	$\sin(\omega_0 n)$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} \left[\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k) \right]$
$\begin{cases} 1, & t \leq T/2 \\ 0, & t > T/2 \end{cases}$	$T \operatorname{sinc}\left(\frac{\omega T}{2\pi}\right)$	$\begin{cases} 1, & 0 \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin(\omega(N+1)/2)}{\sin(\omega/2)} e^{-j\omega N/2}$