Poisson Summation Formula

$$X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{T}\right) \tag{1}$$

$$X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right) \tag{2}$$

Analysis Equation

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{3}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{4}$$

Synthesis Equation

$$a_k = \frac{1}{T} \int_{<T>} x(t) e^{-jk\frac{2\pi}{T}t} dt \tag{5}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t} \tag{6}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{7}$$

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\frac{2\pi}{N}n} \tag{8}$$

$$x[n] = \sum_{k=<N>} a_k e^{jk\frac{2\pi}{N}n} \tag{9}$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \tag{10}$$

Symmetry Properties
If $x(t)$ is real, then $|X(j\omega)|$ is even and $\angle X(j\omega)$ is odd. Moreover, if $x(t)$ is real and even, then $X(j\omega)$ must be purely real and even, and if $x(t)$ is real and odd, $X(j\omega)$ is purely imaginary and odd.

Duality Property

$$\mathcal{F}\{X(t)\} = 2\pi x(-\omega). \tag{11}$$

Modulation Property

$$x(t)\cos(\omega_0 t) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2}\left(X(j(\omega - \omega_0)) + X(j(\omega - \omega_0))\right) \tag{12}$$

LTI System Invertibility Criterion
An LTI system with frequency response $H(j\omega)$ (CT) or $H(e^{j\omega})$ (DT) is invertible if and only if $H(j\omega) \neq 0$ for all ω (CT) or $H(e^{j\omega}) \neq 0$ for all $\omega \in [-\pi, \pi]$ (DT).

LTI System Input/Output

$$Y(j\omega) = H(j\omega)X(j\omega) \tag{13}$$

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) \tag{14}$$

Differential Equations
For a system described by:

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

The frequency response is:

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\sum_{k=0}^{M} b_k (j\omega)^k}{\sum_{k=0}^{N} a_k (j\omega)^k} \tag{15}$$

Difference Equations
For a system described by:

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

The frequency response is:

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{\sum_{k=0}^{N} a_k e^{-j\omega k}} \tag{16}$$

Table 1: Properties of Fourier Transforms

Property	**CT Time Domain**	**CT Frequency Domain**	**DT Time Domain**	**DT Frequency Domain**
Linearity	$Ax_1(t) + Bx_2(t)$	$AX_1(j\omega) + BX_2(j\omega)$	$Ax_1[n] + Bx_2[n]$	$AX_1(e^{j\omega}) + BX_2(e^{j\omega})$
Time Shifting	$x(t - t_0)$	$X(j\omega)e^{-j\omega t_0}$	$x[n - n_0]$	$X(e^{j\omega})e^{-j\omega n_0}$
Frequency Shifting	$x(t)e^{j\omega_0 t}$	$X(j(\omega - \omega_0))$	$x[n]e^{j\omega_0 n}$	$X(e^{j(\omega-\omega_0)})$
Time Scaling	$x(at)$, $a \neq 0$	$\frac{1}{\lvert a\rvert} X\left(\frac{j\omega}{a}\right)$	—	—
Time Reversal	$x(-t)$	$X(-j\omega)$	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*(t)$	$X^*(-j\omega)$	$x^*[n]$	$X^*(e^{-j\omega})$
Differentiation	$\frac{d^n}{dt^n} x(t)$	$(j\omega)^n X(j\omega)$	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$	$\sum_{k=-\infty}^{n} x[k]$	$\frac{X(e^{j\omega})}{1-e^{-j\omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
Convolution	$(x_1 * x_2)(t)$	$X_1(j\omega)X_2(j\omega)$	$(x_1 * x_2)[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}(X_1 * X_2)(j\omega)$	$x_1[n]x_2[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta$
Parseval's Theorem	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2 d\omega$	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$

Table 2: Fourier Transform Pairs

CT Time Domain $x(t)$	**CT Fourier Transform** $X(j\omega)$	**DT Time Domain** $x[n]$	**DT Fourier Transform** $X(e^{j\omega})$
$\delta(t)$	1	$\delta[n]$	1
1	$2\pi\, \delta(\omega)$	1	$2\pi\, \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$u(t)$	$\pi\, \delta(\omega) + \frac{1}{j\omega}$	$u[n]$	$\frac{1}{1-e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$e^{-at}u(t), \quad \Re(a) > 0$	$\dfrac{1}{a + j\omega}$	$a^n u[n], \quad \lvert a\rvert < 1$	$\dfrac{1}{1 - ae^{-j\omega}}$
$\cos(\omega_0 t)$	$\pi \Big[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\Big]$	$\cos(\omega_0 n)$	$\pi\, \sum_{k=-\infty}^{\infty} \Big[\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)\Big]$
$\sin(\omega_0 t)$	$\dfrac{\pi}{j} \Big[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\Big]$	$\sin(\omega_0 n)$	$\dfrac{\pi}{j}\, \sum_{k=-\infty}^{\infty} \Big[\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)\Big]$
$\begin{cases} 1, & \lvert t\rvert \leq T/2 \\ 0, & \lvert t\rvert > T/2 \end{cases}$	$2\frac{\sin\left(\frac{\omega T}{2}\right)}{\omega}$	$\begin{cases} 1, & 0 \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{\sin\Big(\omega(N+1)/2\Big)}{\sin\Big(\omega/2\Big)}\, e^{-j\omega N/2}$