sequence by explicitly giving the function for the n-th term. For example,

$$a_n = \frac{n(n+1)}{2}.$$

Alternatively, we could define a sequence recursively by saying how to get from one term to the next. This is especially useful for the Fibonacci sequence:

$$f_n = f_{n-1} + f_{n-2}; f_1 = 1, f_2 = 1.$$

Much of our effort in understanding sequences will go into taking a recursive definition and finding a closed formula for the sequence. We will study this, and everything else sequence related in Chapter 4.

0.2.5 Relations

How are the numbers 2 and 6 related? Oh, I know: $2 < 6$. Also, 6 is a multiple of 2. The two numbers are also both even. And here is another fact: they are *not equal*. All four of these are examples of **relations**: less than, multiple of, both even, not equal. And there are many more (infinitely many) relations that might or might not hold of a pair of two numbers.

The examples above are all **binary relations** in that they relate two elements. You could also consider relations between more than two elements. For example, we could consider the relation "Pythagorean triple" that holds of three numbers precisely if they are the side lengths of a right triangle. So the relation is true of the triple $(3,4,5)$, but not of $(4,5,6)$ (since $3^2 + 4^2 = 5^2$ but $4^2 + 5^2 \neq 6^2$).

Notice that we can talk about a pair or triple *satisfying* a relation. We might say that a pair *belongs* to the relation. The careful and formal way to define a relation is as a *set* of ordered pairs (or triples, etc.). Consider the (infinite) set of all ordered pairs (a, b) such that $a < b$. Every element of this set contains numbers for which the relation "less than" is true, and every pair of numbers for which the relation is true is a pair in the set. So we can say that this set of pairs ***is*** the relation.

Relations can have some standard properties, and deciding whether a particular relation has a given property can often help us understand the relation better. The less-than relation is, for example, **irreflexive** because there are no elements that are less than themselves. It is also **antisymmetric** since there are no distinct numbers a and b such that $a < b$ and $b < a$. It is also **transitive** since if $a < b$ and $b < c$ then it must follow that $a < c$. These are just a few examples of relation properties though.

Why would we care about these properties? It turns out that some groups of properties happen together frequently, and for such collections of properties, we can prove general results about the relations that satisfy them. So if we can prove that a given relation is **reflexive**, **symmetric**, and **transitive** (whatever those mean), then we know the relation is an **equivalence relation**, and therefore we know it has a bunch of other properties. A large portion of discrete mathematics is about studying particular types of relations. One of my favorites is a relation that gives us a graph.