18. Determine the numerical value of

a) $\displaystyle\sum_{n=1}^{5}\left(\frac{1}{n}-\frac{1}{n+1}\right)$ b) $\displaystyle\sum_{n=1}^{500}\left(\frac{1}{n}-\frac{1}{n+1}\right)$

19. Express the following sums using summation notation.

a) $1^2+2^2+3^2+\cdots+n^2$.

b) $2^1+2^2+2^3+\cdots+2^m$.

c) $f(s_1)\Delta s+f(s_2)\Delta s+\cdots+f(s_{12})\Delta s$.

d) $y_1^2\,\Delta y_1+y_2^2\,\Delta y_2+\cdots+y_n^2\,\Delta y_n$.

20. Express each of the following as a sum written out term-by-term. (There is no need to calculate the numerical value, even when that can be done.)

a) $\displaystyle\sum_{l=3}^{n-1}a_l$ b) $\displaystyle\sum_{j=0}^{4}\frac{j+1}{j^2+1}$ c) $\displaystyle\sum_{k=1}^{5}H(x_k)\,\Delta x_k$.

21. Acquire experimental evidence for the claim

$$\left(\sum_{k=1}^{n}k\right)^2=\sum_{k=1}^{n}k^3$$

by determining the numerical values of both sides of the equation for $n=2$, 3, 4, 5, and 6.

22. Let $g(u)=25-u^2$ and suppose the interval $[0,2]$ has been divided into 4 equal subintervals Δu and u_j is the left endpoint of the j-th interval. Determine the numerical value of the Riemann sum

$$\sum_{j=1}^{4}g(u_j)\,\Delta u\,.$$

Length and area

23. Using Riemann sums with equal subintervals, estimate the length of the parabola $y=x^2$ over the interval $0\le x\le 1$. Obtain a sequence of estimates that stabilize to four decimal places. How many subintervals did you need? (Compare your result here with the earlier result on page 96.)