In these two examples, the density function is constant and does not depend on the particular outcome. It is often the case that experiments in which the coordinates are chosen *at random* can be described by *constant* density functions, and, as in Section 1.2, we call such density functions *uniform* or *equiprobable.* Not all experiments are of this type, however.

Example 2.12 (Example 2.9 continued) In the second dart game experiment, we choose for our sample space the unit interval on the real line and for our density the function

$$f(r) = \begin{cases} 2r, & \text{if } 0 < r < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then the probability that the dart lands at distance r, $a \le r \le b$, from the center of the target is given by

$$\begin{aligned} P([a,b]) &= \int_a^b 2r\,dr \\ &= b^2 - a^2\ . \end{aligned}$$

Here again, since the density is small when r is near 0 and large when r is near 1, we see that in this experiment the dart is more likely to land near the rim of the target than near the center. In terms of the bar graph of Example 2.9, the heights of the bars approximate the density function, while the areas of the bars approximate the probabilities of the subintervals (see Figure 2.12). $\square$

We see in this example that, unlike the case of discrete sample spaces, the value $f(x)$ of the density function for the outcome x is *not* the probability of x occurring (we have seen that this probability is always 0) and in general $f(x)$ is *not a probability at all.* In this example, if we take $\lambda = 2$ then $f(3/4) = 3/2$, which being bigger than 1, cannot be a probability.

Nevertheless, the density function f does contain all the probability information about the experiment, since the probabilities of all events can be derived from it. In particular, the probability that the outcome of the experiment falls in an interval $[a, b]$ is given by

$$P([a,b]) = \int_a^b f(x)\,dx\ ,$$

that is, by the *area* under the graph of the density function in the interval $[a, b]$. Thus, there is a close connection here between probabilities and areas. We have been guided by this close connection in making up our bar graphs; each bar is chosen so that its *area,* and not its height, represents the relative frequency of occurrence, and hence estimates the probability of the outcome falling in the associated interval.

In the language of the calculus, we can say that the probability of occurrence of an event of the form $[x, x + dx]$, where dx is small, is approximately given by

$$P([x, x + dx]) \approx f(x)dx\ ,$$

that is, by the area of the rectangle under the graph of f. Note that as $dx \to 0$, this probability $\to 0$, so that the probability $P(\{x\})$ of a single point is again 0, as in Example 2.7.