Example 3.31

The radius of a right circular cylinder is decreasing at the rate of 3 cm/min, while the height is increasing at the rate of 2 cm/min. Find the rate of change of the volume of the cylinder when the radius is 8 cm and the height is 6 cm.

Solution: Let r, h, and V be the radius, height, and volume, respectively, of the cylinder. Then $V = \pi r^2 h$. Since $\frac{dr}{dt} = -3$ cm/min and $\frac{dh}{dt} = 2$ cm/min, then by the Product Rule:

$$\frac{dV}{dt} = \frac{d}{dt}(\pi r^2 h) = \left(2\pi r \cdot \frac{dr}{dt}\right) h + \pi r^2 \cdot \frac{dh}{dt} \quad \Rightarrow \quad \left.\frac{dV}{dt}\right|_{\substack{r=8\\h=6}} = 2\pi(8)(-3)(6) + \pi(8^2)(2) = -160\pi \ \frac{\text{cm}^3}{\text{min}}$$

Exercises

A

1. A stone is dropped into still water. If the radius of the circular outer ripple increases at the rate of 4 ft/s, how fast is the area of the circle of disturbed water increasing when the radius is 10 ft?

2. The radius of a sphere decreases at a rate of 3 mm/hr. Determine how fast the volume and surface area of the sphere are changing when the radius is 5 mm.

3. A kite 80 ft above level ground moves horizontally at a rate of 4 ft/s away from the person flying it. How fast is the string being released at the instant when 100 ft of string have been released?

4. A 10-ft ladder is leaning against a wall on level ground. If the bottom of the ladder is dragged away from the wall at the rate of 5 ft/s, how fast will the top of the ladder descend at the instant when it is 8 ft from the ground?

5. A person 6 ft tall is walking at a rate of 6 ft/s away from a light which is 15 ft above the ground. At what rate is the end of the person's shadow moving along the ground away from the light?

6. An object moves along the curve $y = x^3$ in the xy-plane. At what points on the curve are the x and y coordinates of the object changing at the same rate?

7. The radius of a right circular cone is decreasing at the rate of 4 cm/min, while the height is increasing at the rate of 3 cm/min. Find the rate of change of the volume of the cone when the radius is 6 cm and the height is 7 cm.

8. Two boats leave the same dock at the same time, one goes north at 25 mph and the other goes east at 30 mph. How fast is the distance between the boats changing when they are 100 miles apart?

9. Repeat Exercise 8 with the angle between the boats being 110°.

B

10. An angle θ changes with time. For what values of θ do $\sin\theta$ and $\tan\theta$ change at the same rate?

11. Repeat Example 3.30 but with the ground making a 100° angle with the pole to the left of the pole.

C

12. An upright cylindrical tank full of water is tipped over at a constant angular speed. Assume that the height of the tank is at least twice its radius. Show that at the instant the tank has been tipped 45°, water is leaving the tank twice as fast as it did at the instant the tank was first tipped. *(Hint: Think of how the water looks inside the tank as it is being tipped.)*