2. Which of the graphs in the previous question contain Euler trails or circuits? Which of the graphs are planar?

3. Draw a graph that has an Euler circuit but is not planar.

4. Draw a graph that does not have an Euler trail and is also not planar.

5. Consider the graph $G = (V, E)$ with $V = \{a, b, c, d, e, f, g\}$ and $E = \{ab, ac, af, bg, cd, ce\}$ (here we are using the shorthand for edges: ab really means $\{a, b\}$, for example).

(a) Is the graph G isomorphic to $G' = (V', E')$ with $V' = \{t, u, v, w, x, y, z\}$ and $E' = \{tz, uv, uy, uz, vw, vx\}$? If so, give the isomorphism. If not, explain how you know.

(b) Find a graph G'' with 7 vertices and 6 edges which is NOT isomorphic to G, or explain why this is not possible.

(c) Write down the *degree sequence* for G. That is, write down the degrees of all the vertices, in decreasing order.

(d) Find a connected graph G''' with the same degree sequence of G which is NOT isomorphic to G, or explain why this is not possible.

(e) What kind of graph is G? Is G complete? Bipartite? A tree? A cycle? A path? A wheel?

(f) Is G planar?

(g) What is the chromatic number of G? Explain.

(h) Does G have an Euler trail or circuit? Explain.

6. If a graph has 10 vertices and 10 edges and contains an Euler circuit, must it be planar? How many faces would it have?

7. Suppose G is a graph with n vertices, each having degree 5.

(a) For which values of n does this make sense?

(b) For which values of n does the graph have an Euler trail?

(c) What is the smallest value of n for which the graph might be planar? (tricky)

8. At a school dance, 6 girls and 4 boys take turns dancing (as couples) with each other.

(a) How many couples dance if every girl dances with every boy?

(b) How many couples dance if everyone dances with everyone else (regardless of gender)?

(c) Explain what graphs can be used to represent these situations.