there are 4 rising sequences; they are (1), $(2, 3, 4)$, $(5, 6)$, and (7). It is easy to see that an ordering is the result of a riffle shuffle applied to the identity ordering if and only if it has no more than two rising sequences. (If the ordering has two rising sequences, then these rising sequences correspond to the two stacks induced by the cut, and if the ordering has one rising sequence, then it is the identity ordering.) Thus, the sample space of orderings obtained by applying a riffle shuffle to the identity ordering is naturally described as the set of all orderings with at most two rising sequences.

It is now easy to assign a probability distribution to this sample space. Each ordering with two rising sequences is assigned the value

$$\frac{b(n, 1/2, k)}{\binom{n}{k}} = \frac{1}{2^n} ,$$

and the identity ordering is assigned the value

$$\frac{n+1}{2^n} .$$

There is another way to view a riffle shuffle. We can imagine starting with a deck cut into two stacks as before, with the same probabilities assignment as before i.e., the binomial distribution. Once we have the two stacks, we take cards, one by one, off of the bottom of the two stacks, and place them onto one stack. If there are k_1 and k_2 cards, respectively, in the two stacks at some point in this process, then we make the assumption that the probabilities that the next card to be taken comes from a given stack is proportional to the current stack size. This implies that the probability that we take the next card from the first stack equals

$$\frac{k_1}{k_1 + k_2} ,$$

and the corresponding probability for the second stack is

$$\frac{k_2}{k_1 + k_2} .$$

We shall now show that this process assigns the uniform probability to each of the possible interleavings of the two stacks.

Suppose, for example, that an interleaving came about as the result of choosing cards from the two stacks in some order. The probability that this result occurred is the product of the probabilities at each point in the process, since the choice of card at each point is assumed to be independent of the previous choices. Each factor of this product is of the form

$$\frac{k_i}{k_1 + k_2} ,$$

where $i = 1$ or 2, and the denominator of each factor equals the number of cards left to be chosen. Thus, the denominator of the probability is just $n!$. At the moment when a card is chosen from a stack that has i cards in it, the numerator of the