b) Find an error bound for this estimate, and use it to construct the best possible lower and upper bounds

$$A \leq \int_0^{\pi/2} \sin x \, dx \leq B.$$

12. In the text (page 373), a Riemann sum using left endpoints on 1000 equal subintervals produces an estimate of 6.226083 for the value of

$$\int_1^3 \sqrt{1+x^3} \, dx.$$

a) Is the true value of the integral larger or smaller than this estimate? Explain your answer, and do so without referring to the fact that the true value of the integral is known to be 6.229959....

b) Find an error bound for this estimate.

c) Find the upper and lower bounds for the value of the integral that are determined by this estimate.

d) According to these bounds, how many digits of the value of the integral are now known for certain?

The average value of a function

In the exercises for section 1 we saw that the average staffing level for a job is

$$\text{average staffing} = \frac{\text{total staff-hours}}{\text{hours worked}}.$$

If $S(t)$ represents the number of staff working at time t, then the total staff-hours accumulated between $t = a$ and $t = b$ hours is

$$\text{total staff-hours} = \int_a^b S(t) \, dt \quad \text{staff-hours.}$$

Therefore the average staffing is

$$\text{average staffing} = \frac{1}{b-a} \int_a^b S(t) \, dt \quad \text{staff.}$$