c.

$$\begin{aligned} 2x_1 - x_2 + 3x_3 + 3x_4 &= 8 \\ -x_1 + x_2 + 2x_4 &= -1 \\ 2x_2 + 2x_3 + 6x_4 &= 4 \\ 3x_1 + 2x_2 - x_3 - 3x_4 &= 0 \end{aligned}$$

3. Include an example of an appropriate matrix as you justify your responses to the following questions.

a. Suppose a linear system having six equations and three variables is consistent. Can you guarantee that the solution is unique? Can you guarantee that there are infinitely many solutions?

b. Suppose that a linear system having three equations and six variables is consistent. Can you guarantee that the solution is unique? Can you guarantee that there are infinitely many solutions?

c. Suppose that a linear system is consistent and has a unique solution. What can you guarantee about the pivot positions in the augmented matrix?

4. Determine whether the following statements are true or false and provide a justification for your response.

a. If the coefficient matrix of a linear system has a pivot in the rightmost column, then the system is inconsistent.

b. If a linear system has two equations and four variables, then it must be consistent.

c. If a linear system having four equations and three variables is consistent, then the solution is unique.

d. Suppose that a linear system has four equations and four variables and that the coefficient matrix has four pivots. Then the linear system is consistent and has a unique solution.

e. Suppose that a linear system has five equations and three variables and that the coefficient matrix has a pivot position in every column. Then the linear system is consistent and has a unique solution.

5. We began our explorations in Section 1.1 by noticing that the solution spaces of linear systems with more equations seem to be smaller. Let's reexamine this idea using what we know about pivot positions.

a. Remember that the solution space of a single linear equation in three variables is a plane. Can two planes ever intersect in a single point? What are the possible ways in which two planes can intersect? How can our understanding of pivot positions help answer these questions?

b. Suppose that a consistent linear system has more variables than equations. By considering the possible pivot positions, what can you say with certainty about the solution space?

c. If a linear system has many more equations than variables, why is it reasonable to expect the system to be inconsistent?