because each equation in the following system

$$\begin{aligned} x &= -4 \\ y &= \ \ 2. \end{aligned}$$

has only one variable, it prescribes a specific value for that variable. We therefore see that there is exactly one solution, which is $(x, y) = (-4, 2)$. We call such a system *decoupled*.

Observation 1.2.2 Second, there is a process that can be used to find solutions to certain types of linear systems. For instance, let's consider the system

$$\begin{aligned} x + 2y - 2z &= -4 \\ -y + \ \ z &= \ \ \ 3 \\ 3z &= \ \ \ 3. \end{aligned}$$

Multiplying both sides of the last equation by $1/3$ gives us

$$\begin{aligned} x + 2y - 2z &= -4 \\ -y + \ \ z &= \ \ \ 3 \\ z &= \ \ \ 1. \end{aligned}$$

Any solution to this linear system must then have $z = 1$.

Once we know that, we can substitute $z = 1$ into the first and second equations and simplify to obtain a new system of equations having the same solutions:

$$\begin{aligned} x + 2y &= -2 \\ -y &= \ \ 2. \end{aligned}$$

The second equation, after multiplying both sides by -1, tells us that $y = -2$. We can then substitute this value into the first equation to determine that $x = 2$.

In this way, we arrive at a decoupled system, which shows that there is exactly one solution, namely $(x, y, z) = (2, -2, 1)$.

Our original system,

$$\begin{aligned} x + 2y - 2z &= -4 \\ -y + \ \ z &= \ \ \ 3 \\ 3z &= \ \ \ 3, \end{aligned}$$

is called a *triangular* system due to the shape formed by the coefficients. As this example demonstrates, triangular systems are easily solved by this process, which is called *back substitution*.

Observation 1.2.3 We can use substitution in a more general way to solve linear systems. For example, a natural approach to the system

$$\begin{aligned} x + 2y &= \ 1 \\ 2x + 3y &= 3. \end{aligned}$$