(a) The circle $C_1^+(0)$.

(b) The circle $C_1^+(4)$.

11. Evaluate $\int_{C_1^+(1+i)} (z^4+4)^{-1}\,dz$.

12. Evaluate $\int_C z^{-1}(z-1)^{-1}\exp z\,dz$ along the following contours:

    (a) The circle $C_{\frac{1}{2}}^+(0)$.

    (b) The circle $C_2^+(0)$.

13. Evaluate $\int_C (z^2+1)^{-1}\sin z\,dz$ along the following contours:

    (a) The circle $C_1^+(i)$.

    (b) The circle $C_1^+(-i)$.

14. Evaluate $\int_{C_1^+(i)} (z^2+1)^{-2}\,dz$.

15. Evaluate $\int_C (z^2+1)^{-1}\,dz$ along the following contours:

    (a) The circle $C_1^+(i)$.

    (b) The circle $C_1^+(-i)$.

16. Let $P(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3$. Evaluate $\int_{C_1^+(0)} P(z) z^{-n}\,dz$, where n is a positive integer.

17. Let z_1 and z_2 be two complex numbers that lie interior to the simple closed contour C with positive orientation. Evaluate

$$\int_C (z-z_1)^{-1}(z-z_2)^{-1}\,dz.$$

18. Let f be analytic in the simply connected domain D and let z_1 and z_2 be two complex numbers that lie interior to the simple closed contour C having positive orientation that lies in D. Show that

$$\frac{f(z_2)-f(z_1)}{z_2-z_1} = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-z_1)(z-z_2)}\,dz.$$

    What happens when $z_2 \to z_1$? Why?

19. The *Legendre polynomial* $P_n(z)$ is defined by

$$P_n(z) = \frac{1}{2^n n!}\frac{d^n}{dz^n}[(z^2-1)^n].$$

    Use Cauchy 's integral formula to show that

$$P_n(z) = \frac{1}{2\pi i}\int_C \frac{(\xi^2-1)^n}{2^n(\xi-z)^{n+1}}\,d\xi,$$

    where C is a simple closed contour having positive orientation and z lies inside C.

20. Discuss the importance of being able to define an analytic function $f(z)$ with the contour integral in Formula (6.44). How does this definition differ from other definitions of a function that you have learned?