## Objectives

- To understand how the righthand side of the system

$$\frac{dx}{dt} = f(x, y)$$
$$\frac{dy}{dt} = g(x, y)$$

can be viewed as a vector field, $(f(x, y), g(x, y))$, which can be plotted in the $x, y$-plane.

- To understand and be able to use nullclines and phase plane analysis to sketch solution curves for the system

$$\frac{dx}{dt} = f(x, y)$$
$$\frac{dy}{dt} = g(x, y).$$

We can use **direction fields** in the phase plane to represent **autonomous systems**

$$\frac{dx}{dt} = f(x, y), \tag{2.2.1}$$
$$\frac{dy}{dt} = g(x, y). \tag{2.2.2}$$

Equation (2.2.1) tells us how a solution curve changes in the $x$ direction, while equation (2.2.2) tells us how a solution curve changes in the $y$ direction.

### 2.2.1 Direction Fields

**Example 2.2.1** Consider the differential equation for a simple harmonic oscillator that we developed in Section 1.1,

$$mx'' + kx = 0.$$

If we assume that $k$ and $m$ are both equal to one and let $x' = v$, we can rewrite this equation as the first order system,

$$x' = v,$$
$$v' = -x.$$

The direction field is relatively easy to understand. After plotting only few vectors, we can very quickly see that the vectors are tangent to circles centered at the origin (Figure 2.2.2). Since the solutions to the undamped harmonic oscillator $x'' + x = 0$ are of the form

$$x(t) = A\cos t + B\sin t$$

for arbitrary constants $A$ and $B$, this should not be too surprising.