(b)

$$\begin{aligned} x' &= -3\sin x + y \\ y' &= 4x + \cos y - 1 \end{aligned}$$

(c)

$$\begin{aligned} x' &= -3\sin x + y \\ y' &= 4x + 3\cos y - 3 \end{aligned}$$

All three systems have an equilibrium solution at $(0,0)$. Which two systems have phase portraits with the same "local picture" near $(0,0)$? Justify your answer.

8. Let us consider an epidemic model for a city. We make the following additional assumptions about our model.

- The city has a constant birth rate rate of α persons per day. All new born babies are susceptible to the disease.
- Infected people either recover or die after a certain number of days. If an individual recovers, he or she is immune.

If we let $S(t)$ be the number of susceptible individuals at time t and $I(t)$ be the number of infected individuals at time t, our assumptions give rise to the following system of differential equations,

$$\begin{aligned} \frac{dS}{dt} &= -\alpha SI + \beta \\ \frac{dI}{dt} &= -\gamma I + \alpha SI. \end{aligned}$$

The constant α is determined by the probability of an interaction between a susceptible individual and an infected individual, and γ is the rate at which infected individuals are removed from the population. If

$$\begin{aligned} \frac{dS}{dt} &= -\alpha SI + \beta = 0 \\ \frac{dI}{dt} &= -\gamma I + \alpha SI = 0, \end{aligned}$$

then both the susceptible and infected populations do not change. This will occur at

$$\begin{aligned} S_0 &= \frac{\gamma}{\alpha} \\ I_0 &= \frac{\beta}{\gamma}. \end{aligned}$$

We are interested in the behavior of solutions near (S_0, I_0). If solutions approach this equilibrium point, then the disease will become endemic to the population.

9. Consider the predator-prey system modeled by the following equations,

$$\begin{aligned} \frac{dx}{dt} &= ax - \alpha xy = x(a - \alpha y) \\ \frac{dy}{dt} &= -by + \beta xy = y(-b + \beta x). \end{aligned}$$