Chapter 4

Second-Order Linear Equations

4.1 Homogeneous Linear Equations

Objectives

- To understand that a second-order linear differential equation with constant coefficients is an equation of the form

  $$ax'' + bx' + cx = 0.$$

  and can be solved by examining the roots of the characteristic polynomial $ar^2 + br + c = 0$.

- To understand that simple harmonic oscillator can be modeled by the equation

  $$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0,$$

  where $m > 0$, $k > 0$, and $b \geq 0$.

A differential equation of the form

$$a(t)x'' + b(t)x' + c(t)x = g(t)$$

is called a **second-order linear differential equation**. We will first consider the case

$$ax'' + bx' + cx = 0,$$

where a, b, and c are constants and $a \neq 0$. An equation of this form is said to be **homogeneous** with **constant coefficients**. We already know how to solve such equations since we can rewrite them as a system of first-order linear equations. Thus, we can find the general solution of a homogeneous second-order linear differential equation with constant coefficients by computing the eigenvalues and eigenvectors of the matrix of the corresponding system.

4.1.1 RLC Circuits

Recall the RC circuits that we studied earlier (see Section 1.3). Such circuits contained a voltage source, a capacitor, and a resistor. A battery or generator is an example of a voltage source, and a toaster or an electric stove is an