- τ takes $F \to E$, and σ takes $E \to D$; hence $\sigma\tau$ takes $F \to D$.
- τ takes $D \to F$, and σ takes $F \to A$; hence $\sigma\tau$ takes $D \to A$.

We have finished a cycle: (AFD). Let us check where the other letters B, C, E go:

- τ takes $B \to D$, and σ takes $D \to B$; hence $\sigma\tau$ takes $B \to B$.
- Neither τ nor σ affects C; hence $\sigma\tau$ takes $C \to C$.
- τ takes $E \to A$, and σ takes $A \to E$; hence $\sigma\tau$ takes $E \to E$.

Since B, C, E are unaffected by $\sigma\tau$, we conclude that $\sigma\tau = (AFD)$. ♦

Exercise 14.3.17. Given that $\delta = (135)$, $\sigma = (347)$, and $\rho = (567)$ are permutations in S_7, compute the following:

(a) $\delta\sigma$ (c) $\delta\rho$ (e) $\sigma\rho$

(b) $\sigma\delta$ (d) $\rho\delta$ (f) $\rho\sigma$

◊

14.3.3 Product of disjoint cycles

Definition 14.3.18. Two cycles are ***disjoint*** if their parentheses contain no elements in common. Formally, two cycles $(a_1, a_2, \ldots, a_k)$ and $(b_1, b_2, \ldots, b_l)$, are ***disjoint*** if $a_i \neq b_j, \forall i, j$ such that $1 \leq i \leq k$ and $1 \leq j \leq l$. △

For example, the cycles (135) and (27) are disjoint, whereas the cycles (135) and (347) are not.

Example 14.3.19. Given $\sigma = (135)$, $\tau = (27)$, $\sigma, \tau \in S_7$; let us compute $\sigma\tau$.

Notice right away that every number affected by τ is unaffected by σ; and vice versa. Since the two cycles always remain separate, it is appropriate to represent $\sigma\tau$ as $(135)(27)$, because the cycles don't reduce any farther. ♦