30 Suppose that the number of years a car will run is exponentially distributed with parameter $\mu = 1/4$. If Prosser buys a used car today, what is the probability that it will still run after 4 years?

31 Let U be a uniformly distributed random variable on $[0, 1]$. What is the probability that the equation

$$x^2 + 4Ux + 1 = 0$$

has two distinct real roots x_1 and x_2?

32 Write a program to simulate the random variables whose densities are given by the following, making a suitable bar graph of each and comparing the exact density with the bar graph.

(a) $f_X(x) = e^{-x}$ on $[0, \infty)$ (but just do it on $[0, 10]$).

(b) $f_X(x) = 2x$ on $[0, 1]$.

(c) $f_X(x) = 3x^2$ on $[0, 1]$.

(d) $f_X(x) = 4|x - 1/2|$ on $[0, 1]$.

33 Suppose we are observing a process such that the time between occurrences is exponentially distributed with $\lambda = 1/30$ (i.e., the average time between occurrences is 30 minutes). Suppose that the process starts at a certain time and we start observing the process 3 hours later. Write a program to simulate this process. Let T denote the length of time that we have to wait, after we start our observation, for an occurrence. Have your program keep track of T. What is an estimate for the average value of T?

34 Jones puts in two new lightbulbs: a 60 watt bulb and a 100 watt bulb. It is claimed that the lifetime of the 60 watt bulb has an exponential density with average lifetime 200 hours ($\lambda = 1/200$). The 100 watt bulb also has an exponential density but with average lifetime of only 100 hours ($\lambda = 1/100$). Jones wonders what is the probability that the 100 watt bulb will outlast the 60 watt bulb.

If X and Y are two independent random variables with exponential densities $f(x) = \lambda e^{-\lambda x}$ and $g(x) = \mu e^{-\mu x}$, respectively, then the probability that X is less than Y is given by

$$P(X < Y) = \int_0^\infty f(x)(1 - G(x))\, dx,$$

where $G(x)$ is the cumulative distribution function for $g(x)$. Explain why this is the case. Use this to show that

$$P(X < Y) = \frac{\lambda}{\lambda + \mu}$$

and to answer Jones's question.