the interval $[1, 3]$ using 40, 400, 4000, and 40000 equally-spaced subintervals. How many digits in this sequence have stabilized?

b) Comment on the efficiency of right endpoint Riemann sums as compared to left endpoint and to midpoint Riemann sums—at least as far as the function $\sqrt{1 + x^3}$ is concerned.

8. Calculate left endpoint Riemann sums for the function

$$f(x) = \sqrt{1 - x^2} \quad \text{on the interval } [-1, 1].$$

Use 20 and 50 equally-spaced subintervals. Compare your values with the estimates for the area of a semicircle given on page 356.

9. a) Calculate left endpoint Riemann sums for the function

$$f(x) = \sqrt{1 + \cos^2 x} \quad \text{on the interval } [0, \pi].$$

Use 4 and 20 equally-spaced subintervals. Compare your values with the estimates for the length of the graph of $y = \sin x$ between 0 and π, given on page 358.

b) What is the limiting value of the Riemann sums, as the number of subintervals becomes infinite? Find the limit to 11 decimal places accuracy.

10. Calculate left endpoint Riemann sums for the function

$$f(x) = \cos(x^2) \quad \text{on the interval } [0, 4],$$

using 100, 1000, and 10000 equally-spaced subintervals.

[Answer: With 10000 equally-spaced intervals, the left endpoint Riemann sum has the value .59485189.]

11. Calculate left endpoint Riemann sums for the function

$$f(x) = \frac{\cos x}{1 + x^2} \text{ on the interval } [2, 3],$$

using 10, 100, and 1000 equally-spaced subintervals. The Riemann sums are all negative; why? (A suggestion: sketch the graph of f. What does that tell you about the signs of the terms in a Riemann sum for f?)