**Exercise 4.2.5.** Prove the distributive law for complex arithmetic: that is, if $u, w$, and $z$ are complex numbers, then $(u)(w + z) = uw + uz$. ♢

Two arithmetic operations down, two to go! Let's consider subtraction of complex numbers. We may define $z - w$ using complex addition and multiplication as: $z - w = z + (-1) \cdot w$.

**Exercise 4.2.6.** Given that $z = a+bi$ and $w = c+di$ use the above definition of subtraction to derive an expression for $z-w$ in terms of $a, b, c, d$. Express your answer as (Real part) + (Imaginary part)$i$. ♢

Division is a little more complicated. First we consider division of a complex number by a real number. In this case we can define division as multiplication by the reciprocal, just as with real numbers:

$$\frac{a+bi}{c} = (a+bi) \cdot \frac{1}{c} = a \cdot \frac{1}{c} + (bi) \cdot \frac{1}{c} = \frac{a}{c} + \cdot \frac{b}{c} i,$$

where we have used the distributive, associative, and commutative properties of complex multiplication.

Now let's try to make sense of the ratio of two complex numbers:

$$\frac{w}{z} = \frac{c+di}{a+bi}.$$

This notation suggests that it should be true that

$$\frac{w}{z} = (c+di) \cdot \frac{1}{a+bi}.$$

But what is $1/(a + bi)$? To understand this, let's go back to arithmetic with real numbers. If we have an ordinary real number $r$, then $1/r$ is the *multiplicative inverse* of $r$: that is, $r \cdot 1/r = 1/r \cdot r = 1$. We also write $1/r$ as $r^{-1}$. By analogy, to make sense of $1/z = 1/(a + bi)$, we need to find a complex number $z^{-1}$ such that $z^{-1} \cdot z = z \cdot z^{-1} = 1$.

**Exercise 4.2.7.** Given that $z = a + bi$ is a complex number and $z \neq 0$ (recall that 0 is the same as $0 + 0i$). Show that the complex number

$$w = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} i.$$

satisfies $zw = wz = 1$, where $z = a + bi$. (*Hint*) ♢