perpendicular lines multiply to give -1.) Example 2 has $m = 12$, so the normal line has slope $-1/12$:

$$\textit{tangent line}: y - 6 = 12(x - 2) \qquad \textit{normal line}: y - 6 = -\tfrac{1}{12}(x - 2).$$

Light rays travel in the normal direction. So do brush fires—they move perpendicular to the fire line. Use the point-slope form! The tangent is $y = 12x - 18$, the normal is not $y = -\frac{1}{12}x - 18$.

EXAMPLE 3 You are on a roller-coaster whose track follows $y = x^2 + 4$. You see a friend at (0, 0) and want to get there quickly. Where do you step off?

Solution Your path will be the tangent line (at high speed). The problem is ***to choose*** $x = a$ ***so the tangent line passes through*** $x = 0$, $y = 0$. When you step off at $x = a$,

the height is $y = a^2 + 4$ and the slope is $2a$

the equation of the tangent line is $y - (a^2 + 4) = 2a(x - a)$

this line goes through (0, 0) if $-(a^2 + 4) = -2a^2$ or $a = \pm 2$.

The same problem is solved by spacecraft controllers and baseball pitchers. Releasing a ball at the right time to hit a target 60 feet away is an amazing display of calculus. Quarterbacks with a moving target should read Chapter 4 on related rates.

Here is a better example than a roller-coaster. Stopping at a red light wastes gas. It is smarter to slow down early, and then accelerate. When a car is waiting in front of you, the timing needs calculus:

EXAMPLE 4 How much must you slow down when a red light is 72 meters away? In 4 seconds it will be green. The waiting car will accelerate at 3 meters/sec^2. You cannot pass the car.

Strategy Slow down immediately to the speed V at which you will just catch that car. (If you wait and brake later, your speed will have to go below V.) At the catch-up time T, the cars have the same speed and same distance. ***Two conditions***, so the distance functions in Figure 2.6d are tangent.

Solution At time T, the other car's speed is $3(T - 4)$. That shows the delay of 4 seconds. Speeds are equal when $3(T - 4) = V$ or $T = \frac{1}{3}V + 4$. Now require equal distances. Your distance is V times T. The other car's distance is $72 + \frac{1}{2}at^2$:

$$72 + \tfrac{1}{2} \cdot 3(T - 4)^2 = VT \quad \text{becomes} \quad 72 + \tfrac{1}{2} \cdot \tfrac{1}{3}V^2 = V(\tfrac{1}{3}V + 4).$$

The solution is $V = 12$ meters/second. This is 43 km/hr or 27 miles per hour.

Without the other car, you only slow down to $V = 72/4 = 18$ meters/second. As the light turns green, you go through at 65 km/hr or 40 miles per hour. Try it.

THE SECANT LINE CONNECTING TWO POINTS ON A CURVE

Instead of the tangent line through one point, consider the ***secant line through two points***. For the tangent line the points came together. Now spread them apart. The point-slope form of a linear equation is replaced by the ***two-point form***.

The equation of the curve is still $y = f(x)$. The first point remains at $x = a$, $y = f(a)$. The other point is at $x = c$, $y = f(c)$. The secant line goes between them, and we want its equation. This time we don't start with the slope—but m is easy to find.