a car to pass on a highway, or the times between emissions of particles from a radioactive source, are simulated by a sequence of random numbers, each of which is chosen by computing $(-1/\lambda)\log(\mathrm{rnd})$, where $1/\lambda$ is the average time between cars or emissions. Write a program to simulate the times between cars when the average time between cars is 30 seconds. Have your program compute an area bar graph for these times by breaking the time interval from 0 to 120 into 24 subintervals. On the same pair of axes, plot the function $f(x) = (1/30)e^{-(1/30)x}$. Does the function fit the bar graph well?

10 In Exercise 9, the distribution came "out of a hat." In this problem, we will again consider an experiment whose outcomes are not equally likely. We will determine a function $f(x)$ which can be used to determine the probability of certain events. Let T be the right triangle in the plane with vertices at the points $(0,0)$, $(1,0)$, and $(0,1)$. The experiment consists of picking a point at random in the interior of T, and recording only the x-coordinate of the point. Thus, the sample space is the set $[0,1]$, but the outcomes do not seem to be equally likely. We can simulate this experiment by asking a computer to return two random real numbers in $[0,1]$, and recording the first of these two numbers if their sum is less than 1. Write this program and run it for 10,000 trials. Then make a bar graph of the result, breaking the interval $[0,1]$ into 10 intervals. Compare the bar graph with the function $f(x) = 2 - 2x$. Now show that there is a constant c such that the height of T at the x-coordinate value x is c times $f(x)$ for every x in $[0,1]$. Finally, show that

$$\int_0^1 f(x)\,dx = 1\ .$$

How might one use the function $f(x)$ to determine the probability that the outcome is between .2 and .5?

11 Here is another way to pick a chord *at random* on the circle of unit radius. Imagine that we have a card table whose sides are of length 100. We place coordinate axes on the table in such a way that each side of the table is parallel to one of the axes, and so that the center of the table is the origin. We now place a circle of unit radius on the table so that the center of the circle is the origin. Now pick out a point (x_0, y_0) at random in the square, and an angle θ at random in the interval $(-\pi/2, \pi/2)$. Let $m = \tan\theta$. Then the equation of the line passing through (x_0, y_0) with slope m is

$$y = y_0 + m(x - x_0)\ ,$$

and the distance of this line from the center of the circle (i.e., the origin) is

$$d = \left|\frac{y_0 - m x_0}{\sqrt{m^2+1}}\right|\ .$$

We can use this distance formula to check whether the line intersects the circle (i.e., whether $d < 1$). If so, we consider the resulting chord a *random* chord.