4.1.2 Unreal, but unavoidable

Mathematicians have known Proposition 4.1.1 for thousands of years, and for a long time that settled the question. Unfortunately, that nasty $\sqrt{-1}$ kept popping up in all sorts of inconvenient places. For example, about 400 years ago, it was very fashionable to study the roots of cubic polynomials such as $x^3 - 15x - 4 = 0$. A mathematician named Bombelli came up with a formula for a solution that eventually simplified to: $x = (2 + \sqrt{-1}) + (2 - \sqrt{-1})$. By canceling out the $\sqrt{-1}$ terms, he got the correct solution $x = 4$. But how can you cancel something that doesn't exist?

Since mathematicians couldn't completely avoid those embarrassing $\sqrt{-1}$'s, they decided to put up with them as best they could. They called $\sqrt{-1}$ an *imaginary* number, just to emphasize that it wasn't up to par with the *real* numbers. They also used the symbol i to represent $\sqrt{-1}$, to make it less conspicuous (and easier to write). Finally, they created a larger set of numbers that included both real and imaginary numbers, called the *complex numbers.* [4]

Definition 4.1.7. The ***complex numbers*** are defined as

$$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\},$$

where $i^2 = -1$. If $z = a + bi$, then a is the ***real part*** of z and b is the ***imaginary part*** of z. (Note that the imaginary part of a complex number is a *real number*. It is the coefficient of i in the expression $z = a + bi$.) △

Examples of complex numbers include

- $1 + i$
- $5.387 - 6.432i$
- $\frac{1}{2} - \frac{\sqrt{3}}{2}i$
- $3i$ (equal to $0 + 3i$)
- 7.42 (equal to $7.42 + 0i$).
- 0 (equal to $0 + 0i$).

[4] The web site http://math.fullerton.edu/mathews/n2003/ComplexNumberOrigin.html gives more information about the origin of complex numbers.