edges to be vertical, i.e. laying on the y-axis, then the height of the last vertex is

$$\sum_{i \in J} \ell_i - \sum_{i \notin J} \ell_i \tag{1}$$

where $J \subseteq \{1, \ldots, k\}$ is the set of indices of the edges pointed positively (with respect to the y-axis). Let a_j be the number of subsets $J \subseteq \{1, \ldots, k\}$ such that $|J| = j$ and

$$\sum_{i \in J} \ell_i - \sum_{i \notin J} \ell_i \leq -|h|. \tag{2}$$

Similarly, let b_j be the number of subsets $J \subseteq \{1, \ldots, k\}$ such that $|J| = j$ and

$$\sum_{i \in J} \ell_i - \sum_{i \notin J} \ell_i > |h|. \tag{3}$$

Furthermore, let A_h be motion space of the robotic arm constricted to the horizontal line $y = h$. With this notation, we can state the first result of this paper.

Theorem 1. *$H_j(A_h; \mathbb{Z})$ is free abelian with rank $a_j + b_{j+1}$.*

Thus, we will reduce a topological problem to a combinatorial problem of determining a_j and b_j from (ℓ_i).

For the second case, let $\gamma : [0, 1] \to \mathbb{R}^2$ be an embedding of an interval into the plane such that

$$|\gamma(0)| = |\gamma(1)| = \sum_{i=1}^{k} \ell_i \tag{4}$$

and that for each $J \subseteq \{1, \ldots, k\}$, γ only intersects transversely with the circles centered at the origin with radius $|r_J|$, where

$$r_J = \sum_{i \in J} \ell_i - \sum_{i \notin J} \ell_i \tag{5}$$

which we call "circles of a critical radius." We define Γ to be the motion space of the robotic arm constricted to γ. Then for each subset $J \subseteq \{1, \ldots, k\}$, we introduce the concept of the multiplier μ_J, defined to be half the number of times the γ intersects the circle of radius $|r_J|$. As such, we redefine

$$a_j = \sum \mu_J \tag{6}$$

over all $J \subseteq \{1, \ldots, k\}$ where $|J| = j$ and $r_J < 0$; furthermore,

$$b_j = \sum \mu_J \tag{7}$$

over all $J \subseteq \{1, \ldots, k\}$ where $|J| = j$ and $r_J > 0$. Observe, even if there is a J with $r_J = 0$, no intersection with the origin can be transverse. Now, we can state the second result of this paper.

Theorem 2. *$H_j(\Gamma; \mathbb{Z})$ is free abelian with rank $a_j + b_{j+1}$.*