Based on the previous exercise, we are able to define z^{-1} for the complex number $z = a + bi$:

$$z^{-1} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i = \frac{a-bi}{a^2+b^2},$$

where the second equality follows from the distributive law. We finally arrive at the formula for dividing two complex numbers:

$$\frac{c+di}{a+bi} = (c+di) \cdot \frac{a-bi}{a^2+b^2},$$

or alternatively

$$\frac{c+di}{a+bi} = \frac{a-bi}{a^2+b^2} \cdot (c+di).$$

(These formulas holds as long as $a + bi \neq 0$).

It seems obvious that we should be able to write this formula more compactly as

$$\frac{c+di}{a+bi} = \frac{(c+di)(a-bi)}{a^2+b^2},$$

and in fact we can. This is because the distributive and associative laws once again comes to our rescue. Starting with the first expression above for $(c + di)/(a + bi)$ we have:

$$\begin{aligned}
\frac{c+di}{a+bi} &= (c+di) \cdot \frac{a-bi}{a^2+b^2} && \text{(from above)} \\
&= (c+di) \cdot \left((a-bi) \cdot \frac{1}{a^2+b^2}\right) && \text{(distributive law)} \\
&= ((c+di) \cdot (a-bi)) \cdot \frac{1}{a^2+b^2} && \text{(associative law)} \\
&= \frac{(c+di) \cdot (a-bi)}{a^2+b^2} && \text{(definition of division).}
\end{aligned}$$

We summarize the formulas for complex addition, multiplication, and division below:

- Addition: $(a + bi) + (c + di) = (a + c) + (b + d)i$
- Multiplication: $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$
- Division: $\dfrac{c+di}{a+bi} = \dfrac{(c+di)(a-bi)}{a^2+b^2}$

Exercise 4.2.8. Evaluate each of the following.