□

Notice how we started in this proof with one side of the equality, and through a series of steps ended up with the other side. This is a good method to follow, when you're trying to prove two things are equal.

Exercise 4.2.2. Prove that addition on complex numbers is associative. ◊

Now that we have addition worked out, let's do multiplication. We observe that the complex number $a + bi$ looks just like the polynomial $a + bx$, except the imaginary i replaces the unknown x. So we'll take a cue from polynomial multiplication, and multiply complex numbers just like polynomial factors, using the FOIL (first, outside, inside, last) method. Better yet, with complex numbers it's more convenient to use FLOI (first, last, outside, inside) instead. The product of z and w is

$$(a + bi)(c + di) = ac + bdi^2 + adi + bci = (ac - bd) + (ad + bc)i.$$

Question: How did we get rid of the i^2 in the final equality? Answer: Remember, we defined $i^2 = -1$, and we just made the substitution.

A bevy of nice properties follow from this definition:

Example 4.2.3. Complex multiplication is commutative. This may be proved as follows. (Note that here we are combining statement-reason and paragraph proof formats. It's OK to mix and match formats, as long as you get the job done!)

$(a + bi)(c + di) = (ac - bd) + (bc + ad)i$ (FLOI)

On the other hand:

$(c + di)(a + bi) = (ca - db) + (cb + da)i$ (FLOI)

$= (ac - bd) + (bc + ad)i$ (commutativity of real multiplication)

Since we obtain the same expression for $(a+bi)(c+di)$ and $(c+di)(a+bi)$, it follows that $(a + bi)(c + di) = (c + di)(a + bi)$. ♦

Similar proofs can be given for other multiplicative properties:

Exercise 4.2.4. Prove the associative law for multiplication of ‘complex numbers. (Follow the style of Example 4.2.3). ◊