

We now have to prove that any ray of light, measured in the moving system, is propagated with the velocity c , if, as we have assumed, this is the case in the stationary system; for we have not as yet furnished the proof that the principle of the constancy of the velocity of light is compatible with the principle of relativity.

At the time $t = \tau = 0$, when the origin of the co-ordinates is common to the two systems, let a spherical wave be emitted therefrom, and be propagated with the velocity c in system K . If (x, y, z) be a point just attained by this wave, then

$$x^2 + y^2 + z^2 = c^2 t^2.$$

Transforming this equation with the aid of our equations of transformation we obtain after a simple calculation

$$\xi^2 + \eta^2 + \zeta^2 = c^2 \tau^2.$$

The wave under consideration is therefore no less a spherical wave with velocity of propagation c when viewed in the moving system. This shows that our two fundamental principles are compatible.⁵

In the equations of transformation which have been developed there enters an unknown function ϕ of v , which we will now determine.

For this purpose we introduce a third system of co-ordinates K' , which relatively to the system k is in a state of parallel translatory motion parallel to the axis of X , such that the origin of co-ordinates of system k moves with velocity $-v$ on the axis of X . At the time $t = 0$ let all three origins coincide, and when $t = x = y = z = 0$ let the time t' of the system K' be zero. We call the co-ordinates, measured in the system K' , x', y', z' , and by a twofold application of our equations of transformation we obtain

$$\begin{aligned} t' &= \phi(-v)\beta(-v)(\tau + v\xi/c^2) &= \phi(v)\phi(-v)t, \\ x' &= \phi(-v)\beta(-v)(\xi + v\tau) &= \phi(v)\phi(-v)x, \\ y' &= \phi(-v)\eta &= \phi(v)\phi(-v)y, \\ z' &= \phi(-v)\zeta &= \phi(v)\phi(-v)z. \end{aligned}$$

Since the relations between x', y', z' and x, y, z do not contain the time t , the systems K and K' are at rest with respect to one another, and it is clear that the transformation from K to K' must be the identical transformation. Thus

$$\phi(v)\phi(-v) = 1.$$

We now inquire into the signification of $\phi(v)$. We give our attention to that part of the axis of Y of system k which lies between $\xi = 0, \eta = 0, \zeta = 0$ and $\xi = 0, \eta = l, \zeta = 0$. This part of the axis of Y is a rod moving perpendicularly

⁵The equations of the Lorentz transformation may be more simply deduced directly from the condition that in virtue of those equations the relation $x^2 + y^2 + z^2 = c^2 t^2$ shall have as its consequence the second relation $\xi^2 + \eta^2 + \zeta^2 = c^2 \tau^2$.