for $U(s)$ and $V(s)$, we obtain

$$U(s) = \frac{s+1}{s^2} \quad \text{and} \quad V(s) = \frac{2s+1}{s^2}.$$

(b) Find $u(t)$ and $v(t)$.

(c) Solve the linear system

$$\begin{aligned} u' + 4u - 6v &= 0 \\ v' + 3u - 5v &= 0 \\ u(0) = 3, \quad v(0) &= 2 \end{aligned}$$

using Laplace transforms.

(d) Solve the linear system

$$\begin{aligned} w' - y &= 0 \\ w + y' + z &= 1 \\ w - y + z' &= 2\sin t \\ w(0) = 1, \quad y(0) = 1, \quad z(0) &= 1 \end{aligned}$$

using Laplace transforms.

6.3 Delta Functions and Forcing

Objectives

- To understand **Impulse forcing**, a term used to describe a very quick push or pull on a system, such as the blow of a hammer or the force of an explosion, and that an impulse function can be described by **Dirac delta function**, $\delta(t)$, which has the properties

$$\begin{aligned} \delta(t) &= 0, \qquad t \neq 0; \\ \int_{-\infty}^{\infty} \delta(t)\, dt &= 1. \end{aligned}$$

- To understand that we can use the Dirac delta function to solve initial value problems such as

$$\begin{aligned} \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 26y &= \delta_4(t) \\ y(0) &= 1 \\ y'(0) &= 0, \end{aligned}$$

or

$$\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = g(t),$$

where $g(t)$ is a function that is very large in a very short time interval.