7. Let $y = cu^n$, where c and n are constants. Show that

$$\frac{dy}{y} = n\,\frac{du}{u}\,.$$

8. Obviously the derivative of the constant π^2 is not 2π. But is $d(\pi^2) = 2\pi\,d(\pi)$ true? Explain.

B

9. The *continuity relation* for an ideal gas is

$$\frac{PM}{\sqrt{T}} = \text{constant}$$

where P and T are the pressure and temperature, respectively, of the gas, and M is the *Mach number*. Show that

$$\frac{dP}{P} + \frac{dM}{M} = \frac{dT}{2T}\,.$$

10. For an ideal gas, satisfying the equation $PV = RT$ as before, the *Gibbs energy* G is defined as $G = H - TS$, where H and S are the *enthalpy* and *entropy*, respectively, of the gas.

(a) Show that

$$d\left(\frac{G}{RT}\right) = \frac{1}{RT}\,dG - \frac{G}{RT^2}\,dT\,.$$

(b) One of the *fundamental property relations* for an ideal gas (which you do not need to prove) is

$$dG = V\,dP - S\,dT\,.$$

Use this and part (a) to show that

$$d\left(\frac{G}{RT}\right) = \frac{V}{RT}\,dP - \frac{H}{RT^2}\,dT\,.$$

11. The derivative of the volume $\pi r^2 h$ of a right circular cylinder of radius r and height h, as a function of r, equals its lateral surface area $2\pi rh$. Use the notion of a differential as an infinitesimal change to explain why this makes sense geometrically.

12. The derivative of the volume $\frac{4\pi}{3}r^3$ of a sphere of radius r, as a function of r, equals its surface area $4\pi r^2$. Use the notion of a differential as an infinitesimal change to explain why this makes sense geometrically.

13. In *quantum calculus* the *q-differential* of a function $f(x)$ is

$$d_q f(x) = f(qx) - f(x)\,,$$

and the *q-derivative* of $f(x)$ is

$$D_q f(x) = \frac{d_q f(x)}{d_q x} = \frac{f(qx) - f(x)}{qx - x} = \frac{f(qx) - f(x)}{(q-1)x}\,.$$

(a) Show that for all positive integers n,

$$D_q\left(x^n\right) = [n]x^{n-1}\,,$$

where $[n] = 1 + q + q^2 + \cdots + q^{n-1}$.

(b) Use part (a) to show that for all positive integers n,

$$\lim_{q\to 1} D_q\left(x^n\right) = \frac{d}{dx}\left(x^n\right)\,.$$