Example. Evaluate $\displaystyle\int_0^{\ln 2} e^{-u}\,du$.

For the antiderivative, we can choose $F(u) = -e^{-u}$. Then

$$\begin{aligned}\int_0^{\ln 2} e^{-u}\,du &= F(\ln 2) - F(0) \\ &= -e^{-\ln 2} - (-e^0) = -(1/2) - (-1) = 1/2\end{aligned}$$

Example. Evaluate $\displaystyle\int_2^3 \ln t\,dt$.

For the antiderivative, we can choose $F(t) = t\ln t - t$. Using the product rule, we can show that this is indeed an antiderivative:

$$F'(t) = t \cdot \frac{1}{t} + 1 \cdot \ln t - 1 = 1 + \ln t - 1 = \ln t$$

Thus,

$$\begin{aligned}\int_2^3 \ln t\,dt &= F(3) - F(2) \\ &= 3 \cdot \ln 3 - 3 - (2 \cdot \ln 2 - 2) \\ &= \ln 3^3 - \ln 2^2 - 1 = \ln 27 - \ln 4 - 1 = \ln(27/4) - 1\end{aligned}$$

While formulas make it possible to get exact values, they do present us with problems of their own. For instance, we need to know that $t\ln t - t$ is an antiderivative of $\ln t$. This is not obvious. In fact, there is no guarantee that the antiderivative of a function given by a formula will have a formula! The antiderivatives of $\cos(x^2)$ and $\sin(x)/x$ do not have formulas, for instance. Many techniques have been devised to find the formula for an antiderivative. In chapter 11 of Calculus II we will survey some of those that are most frequently used.

Parameters

In chapter 4.2 we considered differential equations that involved parameters (see pages 214–218). It also happens that integrals can involve parameters. However, parameters complicate numerical work. If we calculate the value of an integral numerically, by making estimates with Riemann sums, we must first fix the value of any parameters that appear. This makes it difficult to

When integrals depend on parameters . . .