14. a) What is the average value of the function $px - x^2$ on the interval $[0, 1]$? The average depends on the parameter p.
b) For which value of p will that average be zero?

6.5 Chapter Summary

The Main Ideas

- A **Riemann sum** for the function $f(x)$ on the interval $[a, b]$ is a sum of the form
$$f(x_1) \cdot \Delta x_1 + f(x_2) \cdot \Delta x_2 + \cdots + f(x_n) \cdot \Delta x_n,$$
where the interval $[a, b]$ has been subdivided into n subintervals whose lengths are $\Delta x_1, \Delta x_2, \ldots, \Delta x_n$, and each x_k is a sampling point in the k-th subinterval (for each k from 1 to n).

- Riemann sums can be used to approximate a variety of quantities expressed as **products** where one factor varies with the other.

- Riemann sums give more accurate **approximations** as the lengths $\Delta x_1, \Delta x_2, \ldots, \Delta x_n$ are made small.

- If the Riemann sums for a function $f(x)$ on an interval $[a, b]$ converge, the limit is called the **integral** of $f(x)$ on $[a, b]$, and it is denoted
$$\int_a^b f(x)\, dx.$$

- The **units** of $\int_a^b f(x)\, dx$ equal the product of the units of $f(x)$ and the units of x.

- **The Fundamental Theorem of Calculus.**
The solution $y = A(x)$ of the initial value problem
$$y' = f(x) \qquad y(a) = 0$$
is the **accumulation function**
$$A(X) = \int_a^X f(x)\, dx.$$