If $f(x)$ is continuous over an interval $[a, b]$, then there is at least one point $c \in [a, b]$ such that $f(c) = \frac{1}{b-a}\int_a^b f(x)dx.$

- **Fundamental Theorem of Calculus Part 1**

  If $f(x)$ is continuous over an interval $[a, b]$, and the function $F(x)$ is defined by $F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x).$

- **Fundamental Theorem of Calculus Part 2**

  If *f* is continuous over the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then $\int_a^b f(x)dx = F(b) - F(a).$

- **Net Change Theorem**

  $F(b) = F(a) + \int_a^b F'(x)dx$ or $\int_a^b F'(x)dx = F(b) - F(a)$

- **Substitution with Indefinite Integrals**

  $\int f[g(x)]g'(x)dx = \int f(u)du = F(u) + C = F(g(x)) + C$

- **Substitution with Definite Integrals**

  $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$

- **Integrals of Exponential Functions**

  $\int e^x dx = e^x + C$

  $\int a^x dx = \frac{a^x}{\ln a} + C$

- **Integration Formulas Involving Logarithmic Functions**

  $\int x^{-1} dx = \ln|x| + C$

  $\int \ln x\, dx = x\ln x - x + C = x(\ln x - 1) + C$

  $\int \log_a x\, dx = \frac{x}{\ln a}(\ln x - 1) + C$

- **Integrals That Produce Inverse Trigonometric Functions**

  $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$

  $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

  $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C$

KEY CONCEPTS

5.1 Approximating Areas

- The use of sigma (summation) notation of the form $\sum_{i=1}^{n} a_i$ is useful for expressing long sums of values in compact form.