

For a somewhat more abstract example, consider the sum

$$a_1 + a_2 + a_3 + \cdots + a_n,$$

which we can express as

$$\sum_{k=1}^n a_k.$$

We use the capital letter *sigma* \sum from the Greek alphabet to denote a sum. For this reason, summation notation is sometimes referred to as **sigma notation**. You should regard \sum as an instruction telling you to **sum** the numbers of the indicated form as the index k runs through the integers, starting at the integer displayed below the \sum and ending at the integer displayed above it. Notice that changing the index k to some other letter has no effect on the sum. For example,

Sigma notation

$$\sum_{k=1}^{20} k^3 = \sum_{j=1}^{20} j^3,$$

since both expressions give the sum of the cubes of the first twenty positive integers. Other aspects of summation notation will be covered in the exercises.

Summation notation allows us to write the Riemann sum

$$f(x_1) \cdot \Delta x_1 + \cdots + f(x_n) \cdot \Delta x_n$$

more efficiently as

$$\sum_{k=1}^n f(x_k) \cdot \Delta x_k.$$

Be sure not to get tied into one particular way of using these symbols. For example, you should instantly recognize

$$\sum_{i=1}^m \Delta t_i g(t_i)$$

as a Riemann sum. In what follows we will commonly use summation notation when working with Riemann sums. The important thing to remember is that summation notation is only a “shorthand” to express a Riemann sum in a more compact form.