- If we divide the interval $a \le t \le b$ into $a \le t \le c$ and $c \le t \le b$ and integrate $f(t)$ over these subintervals by using Definition 6.1, then we get

$$\int_a^b f(t)\,dt = \int_a^c f(t)\,dt + \int_c^b f(t)\,dt. \tag{6.4}$$

- Similarly, if $c + id$ denotes a complex constant, then

$$\int_a^b (c+id)f(t)\,dt = (c+id)\int_a^b f(t)\,dt. \tag{6.5}$$

- If the limits of integration are reversed, then

$$\int_a^b f(t)\,dt = -\int_b^a f(t)\,dt. \tag{6.6}$$

- The integral of the product fg becomes

$$\begin{aligned}\int_a^b f(t)g(t)\,dt = &\int_a^b \big[u(t)p(t) - v(t)q(t)\big]\,dt \\ &+ i\int_a^b \big[u(t)q(t) + v(t)p(t)\big]\,dt.\end{aligned} \tag{6.7}$$

Example 6.3. Let us verify Property (6.5). We start by writing

$$(c+id)f(t) = (c+id)\big(u(t) + iv(t)\big) = cu(t) - dv(t) + i\big[cv(t) + du(t)\big].$$

Using Definition 6.1, we write the left side of Equation 6.5 as

$$c\int_a^b u(t)\,dt - d\int_a^b v(t)\,dt + ic\int_a^b v(t)\,dt + id\int_a^b u(t)\,dt.$$

which is equivalent to

$$(c+id)\left[\int_a^b u(t)\,dt + i\int_a^b v(t)\,dt\right].$$

It is worthwhile to point out the similarity between Equation (6.2) and its counterpart in calculus. Suppose that U and V are differentiable on $a < t < b$ and $F(t) = U(t) + iV(t)$. Since $F'(t) = U'(t) + iV'(t) = u(t) + iv(t) = f(t)$, Equation (6.2) takes on the familiar form

$$\int_a^b f(t)\,dt = F(t)\Big|_{t=a}^{t=b} = F(b) - F(a). \tag{6.8}$$

where $F'(t) = f(t)$. We can view Equation (6.8) as an extension of the fundamental theorem of calculus. In Section 6.5 we show how to generalize this extension to analytic functions of a complex variable. For now, we simply note an important case of Equation (6.8):

$$\int_a^b f'(t)\,dt = f(b) - f(a). \tag{6.9}$$