One other important distinction is that the objective function in the Chebyshev approximation problem (1.6) is not differentiable; the objective in the least-squares problem (1.4) is quadratic, and therefore differentiable.

The Chebyshev approximation problem (1.6) can be solved by solving the linear program

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & a_i^T x - t \le b_i, \quad i = 1, \ldots, k \\ & -a_i^T x - t \le -b_i, \quad i = 1, \ldots, k, \end{array} \tag{1.7}$$

with variables $x \in \mathbf{R}^n$ and $t \in \mathbf{R}$. (The details will be given in chapter 6.) Since linear programs are readily solved, the Chebyshev approximation problem is therefore readily solved.

Anyone with a working knowledge of linear programming would recognize the Chebyshev approximation problem (1.6) as one that can be reduced to a linear program. For those without this background, though, it might not be obvious that the Chebyshev approximation problem (1.6), with its nondifferentiable objective, can be formulated and solved as a linear program.

While recognizing problems that can be reduced to linear programs is more involved than recognizing a least-squares problem, it is a skill that is readily acquired, since only a few standard tricks are used. The task can even be partially automated; some software systems for specifying and solving optimization problems can automatically recognize (some) problems that can be reformulated as linear programs.

1.3 Convex optimization

A convex optimization problem is one of the form

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \le b_i, \quad i = 1, \ldots, m, \end{array} \tag{1.8}$$

where the functions $f_0, \ldots, f_m : \mathbf{R}^n \to \mathbf{R}$ are convex, *i.e.*, satisfy

$$f_i(\alpha x + \beta y) \le \alpha f_i(x) + \beta f_i(y)$$

for all $x,\ y \in \mathbf{R}^n$ and all $\alpha,\ \beta \in \mathbf{R}$ with $\alpha + \beta = 1$, $\alpha \ge 0$, $\beta \ge 0$. The least-squares problem (1.4) and linear programming problem (1.5) are both special cases of the general convex optimization problem (1.8).

1.3.1 Solving convex optimization problems

There is in general no analytical formula for the solution of convex optimization problems, but (as with linear programming problems) there are very effective methods for solving them. Interior-point methods work very well in practice, and in some cases can be proved to solve the problem to a specified accuracy with a number of