24. Using Riemann sums, obtain a sequence of estimates for the area under each of the following curves. Continue until the first four decimal places stabilize in your estimates.

a) $y = x^2$ over $[0, 1]$ b) $y = x^2$ over $[0, 3]$ c) $y = x \sin x$ over $[0, \pi]$

25. What is the area under the curve $y = \exp(-x^2)$ over the interval $[0, 1]$? Give an estimate that is accurate to four decimal places. Sketch the curve and shade the area.

26. a) Estimate, to four decimal place accuracy, the length of the graph of the natural logarithm function $y = \ln x$ over the interval $[1, e]$.

b) Estimate, to four decimal place accuracy, the length of the graph of the exponential function $y = \exp(x)$ over the interval $[0, 1]$.

27. a) What is the length of the hyperbola $y = 1/x$ over the interval $[1, 4]$? Obtain an estimate that is accurate to four decimal places.

b) What is the area under the hyperbola over the same interval? Obtain an estimate that is accurate to four decimal places.

28. The graph of $y = \sqrt{4 - x^2}$ is a semicircle whose radius is 2. The circumference of the whole circle is 4π, so the length of the part of the circle in the first quadrant is exactly π.

a) Using left endpoint Riemann sums, estimate the length of the graph $y = \sqrt{4 - x^2}$ over the interval $[0, 2]$ in the first quadrant. How many subintervals did you need in order to get an estimate that has the value 3.14159...?

b) There is a technical problem that makes it impossible to use *right* endpoint Riemann sums. What is the problem?