changing. The second derivative is the "rate of change of the velocity." A straight line has constant slope (constant velocity), so its second derivative is zero:

$$f(t) = 5t \quad \text{has} \quad df/dt = 5 \quad \text{and} \quad d^2f/dt^2 = 0.$$

The parabola $y = x^2$ has slope $2x$ (linear) which has slope 2 (constant). Similarly

$$f(t) = \tfrac{1}{2}at^2 \quad \text{has} \quad df/dt = at \quad \text{and} \quad d^2f/dt^2 = a.$$

There stands the notation d^2f/dt^2 (or d^2y/dx^2) for the second derivative. A short form is f'' or y''. (This is pronounced f *double prime* or y *double prime*). Example: The second derivative of $y = x^3$ is $y'' = 6x$.

In the distance-velocity problem, f'' is *acceleration*. It tells how fast v is changing, while v tells how fast f is changing. Where df/dt was distance/time, the second derivative is distance/(time)^2. The acceleration due to gravity is about 32 ft/sec^2 or 9.8 m/sec^2, which means that v increases by 32 ft/sec in one second. It does not mean that the distance increases by 32 feet!

The graph of $y = \sin t$ increases at the start. Its derivative $\cos t$ is positive. However the second derivative is $-\sin t$. ***The curve is bending down while going up***. The arch is "***concave down***" because $y'' = -\sin t$ is negative.

At $t = \pi$ the curve reaches zero and goes negative. The second derivative becomes positive. *Now the curve bends upward.* The lower arch is "***concave up***."

$y'' > 0$ means that y' increases so y bends upward (concave up)

$y'' < 0$ means that y' decreases so y bends down (concave down).

Chapter 3 studies these things properly—here we get an advance look for $\sin t$.

The remarkable fact about the sine and cosine is that $y'' = -y$. That is unusual and special: *acceleration* $= -$ *distance*. The greater the distance, the greater the force pulling back:

$$y = \sin t \quad \text{has} \quad dy/dt = +\cos t \quad \text{and} \quad d^2y/dt^2 = -\sin t = -y.$$

$$y = \cos t \quad \text{has} \quad dy/dt = -\sin t \quad \text{and} \quad d^2y/dt^2 = -\cos t = -y.$$

Question Does $d^2y/dt^2 < 0$ mean that the distance $y(t)$ is decreasing?

Answer No. Absolutely not! It means that dy/dt is decreasing, not necessarily y. At the start of the sine curve, y is still increasing but $y'' < 0$.

Sines and cosines give *simple harmonic motion*—up and down, forward and back, out and in, tension and compression. Stretch a spring, and the restoring force pulls it back. Push a swing up, and gravity brings it down. These motions are controlled by a ***differential equation***:

$$\frac{d^2y}{dt^2} = -y. \tag{12}$$

All solutions are combinations of the sine and cosine: $y = A \sin t + B \cos t$.

This is not a course on differential equations. But you have to see the purpose of calculus. It models events by equations. It models oscillation by equation (12). Your heart fills and empties. Balls bounce. Current alternates. The economy goes up and down:

$$\text{high prices} \to \text{high production} \to \text{low prices} \to \cdots$$

We can't live without oscillations (or differential equations).