**Example 4.23.** We'd like to compute

$$\int_\gamma \frac{dz}{z^2 - 2z}$$

where $\gamma$ is the unit circle, oriented counter-clockwise. (Try computing it from first principles.) We use a partial fractions expansion to write

$$\int_\gamma \frac{dz}{z^2 - 2z} = \frac{1}{2}\int_\gamma \frac{dz}{z-2} - \frac{1}{2}\int_\gamma \frac{dz}{z}.$$

The first integral on the right-hand side is zero by Corollary 4.20 applied to the function $f(z) = \frac{1}{z-2}$ (note that $f$ is holomorphic in $\mathbb{C} \setminus \{2\}$ and $\gamma$ is $(\mathbb{C} \setminus \{2\})$-contractible). The second integral is $2\pi i$ by Exercise 4.4, and so

$$\int_\gamma \frac{dz}{z^2 - 2z} = -\pi i. \qquad \square$$

Sometimes Corollary 4.20 itself is known as Cauchy's Theorem. See Exercise 4.26 for a related formulation of Corollary 4.20, with a proof based on Green's Theorem.

## 4.4 Cauchy's Integral Formula

We recall our notations

$$C[a,r] = \{z \in \mathbb{C} : |z-a| = r\}$$
$$D[a,r] = \{z \in \mathbb{C} : |z-a| < r\}$$
$$\overline{D}[a,r] = \{z \in \mathbb{C} : |z-a| \le r\}$$

for the circle, open disk, and closed disk, respectively, with center $a \in \mathbb{C}$ and radius $r > 0$. Unless stated otherwise, we orient $C[a,r]$ counter-clockwise.

**Theorem 4.24.** If $f$ is holomorphic in an open set containing $\overline{D}[w,R]$ then

$$f(w) = \frac{1}{2\pi i}\int_{C[w,R]} \frac{f(z)}{z-w}\, dz.$$

This is *Cauchy's Integral Formula* for the case that the integration path is a circle; we will prove the general statement at the end of this chapter. However, already this special case is worth meditating over: the data on the right-hand side of Theorem 4.24