

to its axis with velocity v relatively to system K. Its ends possess in K the co-ordinates

$$x_1 = vt, \quad y_1 = \frac{l}{\phi(v)}, \quad z_1 = 0$$

and

$$x_2 = vt, \quad y_2 = 0, \quad z_2 = 0.$$

The length of the rod measured in K is therefore $l/\phi(v)$; and this gives us the meaning of the function $\phi(v)$. From reasons of symmetry it is now evident that the length of a given rod moving perpendicularly to its axis, measured in the stationary system, must depend only on the velocity and not on the direction and the sense of the motion. The length of the moving rod measured in the stationary system does not change, therefore, if v and $-v$ are interchanged. Hence follows that $l/\phi(v) = l/\phi(-v)$, or

$$\phi(v) = \phi(-v).$$

It follows from this relation and the one previously found that $\phi(v) = 1$, so that the transformation equations which have been found become

$$\begin{aligned} \tau &= \beta(t - vx/c^2), \\ \xi &= \beta(x - vt), \\ \eta &= y, \\ \zeta &= z, \end{aligned}$$

where

$$\beta = 1/\sqrt{1 - v^2/c^2}.$$

§ 4. Physical Meaning of the Equations Obtained in Respect to Moving Rigid Bodies and Moving Clocks

We envisage a rigid sphere⁶ of radius R, at rest relatively to the moving system k , and with its centre at the origin of co-ordinates of k . The equation of the surface of this sphere moving relatively to the system K with velocity v is

$$\xi^2 + \eta^2 + \zeta^2 = R^2.$$

The equation of this surface expressed in x, y, z at the time $t = 0$ is

$$\frac{x^2}{(\sqrt{1 - v^2/c^2})^2} + y^2 + z^2 = R^2.$$

⁶That is, a body possessing spherical form when examined at rest.