4.29 *Maximizing probability of satisfying a linear inequality.* Let c be a random variable in $\mathbf{R}^n$, normally distributed with mean $\bar{c}$ and covariance matrix R. Consider the problem

$$\begin{array}{ll} \text{maximize} & \mathbf{prob}(c^T x \geq \alpha) \\ \text{subject to} & Fx \preceq g, \quad Ax = b. \end{array}$$

Assuming there exists a feasible point $\tilde{x}$ for which $\bar{c}^T \tilde{x} \geq \alpha$, show that this problem is equivalent to a convex or quasiconvex optimization problem. Formulate the problem as a QP, QCQP, or SOCP (if the problem is convex), or explain how you can solve it by solving a sequence of QP, QCQP, or SOCP feasibility problems (if the problem is quasiconvex).

Geometric programming

4.30 A heated fluid at temperature T (degrees above ambient temperature) flows in a pipe with fixed length and circular cross section with radius r. A layer of insulation, with thickness $w \ll r$, surrounds the pipe to reduce heat loss through the pipe walls. The design variables in this problem are T, r, and w.

The heat loss is (approximately) proportional to Tr/w, so over a fixed lifetime, the energy cost due to heat loss is given by $\alpha_1 Tr/w$. The cost of the pipe, which has a fixed wall thickness, is approximately proportional to the total material, *i.e.*, it is given by $\alpha_2 r$. The cost of the insulation is also approximately proportional to the total insulation material, *i.e.*, $\alpha_3 rw$ (using $w \ll r$). The total cost is the sum of these three costs.

The heat flow down the pipe is entirely due to the flow of the fluid, which has a fixed velocity, *i.e.*, it is given by $\alpha_4 Tr^2$. The constants α_i are all positive, as are the variables T, r, and w.

Now the problem: maximize the total heat flow down the pipe, subject to an upper limit $C_{\max}$ on total cost, and the constraints

$$T_{\min} \leq T \leq T_{\max}, \qquad r_{\min} \leq r \leq r_{\max}, \qquad w_{\min} \leq w \leq w_{\max}, \quad w \leq 0.1r.$$

Express this problem as a geometric program.

4.31 *Recursive formulation of optimal beam design problem.* Show that the GP (4.46) is equivalent to the GP

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^N w_i h_i \\ \text{subject to} & w_i/w_{\max} \leq 1, \quad w_{\min}/w_i \leq 1, \quad i = 1, \ldots, N \\ & h_i/h_{\max} \leq 1, \quad h_{\min}/h_i \leq 1, \quad i = 1, \ldots, N \\ & h_i/(w_i S_{\max}) \leq 1, \quad S_{\min} w_i/h_i \leq 1, \quad i = 1, \ldots, N \\ & 6iF/(\sigma_{\max} w_i h_i^2) \leq 1, \quad i = 1, \ldots, N \\ & (2i-1)d_i/v_i + v_{i+1}/v_i \leq 1, \quad i = 1, \ldots, N \\ & (i-1/3)d_i/y_i + v_{i+1}/y_i + y_{i+1}/y_i \leq 1, \quad i = 1, \ldots, N \\ & y_1/y_{\max} \leq 1 \\ & E w_i h_i^3 d_i/(6F) = 1, \quad i = 1, \ldots, N. \end{array}$$

The variables are w_i, h_i, v_i, d_i, y_i for $i = 1, \ldots, N$.

4.32 *Approximating a function as a monomial.* Suppose the function $f : \mathbf{R}^n \to \mathbf{R}$ is differentiable at a point $x_0 \succ 0$, with $f(x_0) > 0$. How would you find a monomial function $\hat{f} : \mathbf{R}^n \to \mathbf{R}$ such that $f(x_0) = \hat{f}(x_0)$ and for x near x_0, $\hat{f}(x)$ is very near $f(x)$?

4.33 Express the following problems as convex optimization problems.

(a) Minimize $\max\{p(x), q(x)\}$, where p and q are posynomials.

(b) Minimize $\exp(p(x)) + \exp(q(x))$, where p and q are posynomials.

(c) Minimize $p(x)/(r(x) - q(x))$, subject to $r(x) > q(x)$, where p, q are posynomials, and r is a monomial.