4. Explain why Wallis's view of complex numbers results in $-\sqrt{-1}$ being represented by the same point as is $\sqrt{-1}$.

5. Use Bombelli's technique to get all solutions to the following depressed cubics.

   (a) $x^3 - 30x - 36 = 0$.

   (b) $x^3 - 87x - 130 = 0$.

   (c) $x^3 - 60x - 32 = 0$.

6. Use Cardano's technique (of substituting $z = x - \frac{a_2}{3}$) to solve the following cubics.

   (a) $z^3 - 6z^2 - 3z + 18 = 0$.

   (b) $z^3 + 3z^2 - 24z + 28 = 0$.

7. Is it possible to modify slightly Wallis's picture of complex numbers so that it is consistent with the representation used today? To help you answer this question, refer to the article by Alec Norton and Benjamin Lotto, "Complex Roots Made Visible," *The College Mathematics Journal*, 15(3), June 1984, pp. 248–249.

8. Investigate library or web resources and write up a detailed description explaining why the solution to the depressed cubic, Equation (1.3), is valid.

1.2 The Algebra of Complex Numbers, Part I

We have shown that complex numbers came to be viewed as ordered pairs of real numbers. That is, a complex number z is defined to be

$$z = (x, y), \tag{1.7}$$

where x and y are both real numbers.

The reason we say *ordered* pair is because we are thinking of a point in the plane. The point $(2, 3)$, for example, is not the same as $(3, 2)$. The *order* in which we write x and y in Equation (1.7) makes a difference. Clearly, then, two complex numbers are equal if and only if their x coordinates are equal *and* their y coordinates are equal. In other words,

$$(x, y) = (u, v) \quad \text{iff} \quad x = u \quad \textit{and} \quad y = v.$$

(Throughout this text, "iff" means *if and only if.*)

A meaningful number system requires a method for combining ordered pairs. The definition of algebraic operations must be consistent so that the sum, difference, product, and quotient of any two ordered pairs will again be an ordered pair. The key to defining how these numbers should be manipulated is to follow Gauss's lead and equate (x, y) with $x + iy$. Then, if $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$ are arbitrary complex numbers, we have

$$\begin{aligned} z_1 + z_2 &= (x_1, y_1) + (x_2, y_2) \\ &= (x_1 + iy_1) + (x_2 + iy_2) \\ &= (x_1 + x_2) + i(y_1 + y_2) \\ &= (x_1 + x_2,\ y_1 + y_2). \end{aligned}$$

Thus, the following definitions should make sense.