

- b. Some students might get nothing?
 - c. Each student gets at least 1 dollar but no more than 8 dollars?
17. How many functions $f : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$ are there satisfying:
- a. $f(1) = 1$ or $f(2) = 2$ (or both)?
 - b. $f(1) \neq 1$ or $f(2) \neq 2$ (or both)?
 - c. $f(1) \neq 1$ and $f(2) \neq 2$, and f is injective?
 - d. f is surjective, but $\forall x \in \{1, 2, \dots, 4\}, f(x) \neq x$?
18. How many functions map $\{1, 2, 3, 4, 5\}$ onto $\{1, 2, 3\}$ (i.e., how many *surjections* are there)?
19. To thank your math professor for doing such an amazing job all semester, you decide to bake your professor cookies. You know how to make 13 different types of cookies.
- a. If you want to give your professor 8 different types of cookies, how many different combinations of cookie type can you select? Explain your answer.
 - b. To keep things interesting, you decide to make a different number of each type of cookie. If again you want to select 8 cookie types, how many ways can you select the cookie types and decide for which there will be the most, second most, etc. Explain your answer.
 - c. You change your mind again. This time you decide you will make a total of 23 cookies. Each cookie could be any one of the 13 types of cookies you know how to bake (and it's okay if you leave some types out). How many choices do you have? Explain.
 - d. You realize that the previous plan did not account for presentation. This time, you once again want to make 23 cookies, each one could be any one of the 13 types of cookies. However, now you plan to shape the cookies into the numerals 1, 2, ..., 23. How many choices do you have for which types of cookies to bake into which numerals? Explain.
 - e. The only flaw with the last plan is that your professor might not get to sample all 13 different varieties of cookies. How many choices do you have for which types of cookies to make into which numerals, given that each type of cookie should be present at least once? Explain.
20. For which of the parts of the previous problem (Exercise 3.9.19) does it make sense to interpret the counting question as counting some number of functions? Say what the domain and codomain should be, and whether you are counting all functions, injections, surjections, or something else.