(b)

$$\begin{aligned} x' &= 2x \\ y' &= y/2 \end{aligned}$$

(c)

$$\begin{aligned} x' &= x/2 \\ y' &= 2y \end{aligned}$$

(d)

$$\begin{aligned} x' &= x + y \\ y' &= x - y \end{aligned}$$

2.2.2 Modified Predator-Prey System

Let us recall the modified predator-prey system that we developed in the last section. That is, we will assume that the prey in our model has logistic growth,

$$\begin{aligned} \frac{dR}{dt} &= aR(1 - R/N) - bRF, \\ \frac{dF}{dt} &= -cF + dRF, \end{aligned}$$

where N is the carrying capacity. In order to investigate the geometric properties of our system, we will rewrite our system in vector form. For each value of t, we will use $\mathbf{x}(t)$ to denote the vector-valued function

$$\mathbf{x}(t) = \begin{pmatrix} R(t) \\ F(t) \end{pmatrix}.$$

This vector-valued function, $\mathbf{x}(t)$ corresponds to our solution curve $(R(t), F(t))$ in the RF-plane. Now we can write our predator-prey model as a single vector equation,

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} R'(t) \\ F'(t) \end{pmatrix} = \begin{pmatrix} aR(1 - R/N) - bRF, \\ -cF + dRF \end{pmatrix}.$$

We can view the right side of the equation as a **vector field**. The specific example that we examined was

$$\begin{aligned} \frac{dR}{dt} &= 2R\left(1 - \frac{R}{10}\right) - RF, \\ \frac{dF}{dt} &= -5F + RF. \end{aligned}$$

The vector field form of this system is

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 2R(1 - R/10) - RF \\ -5F + RF \end{pmatrix}.$$

We can associate a vector in the RF-plane for each value of R and F. For example, if we let $(R, F) = (10, 10)$, we have $(R', F') = (-100, 50)$. At this particular point, the population of rabbits is falling rapidly while the number of foxes is climbing very quickly, We can represent this vector in the phase plane