9. Consider functions $f : \{1,2,3,4,5,6,7,8\} \to \{a,b,c,d,e,f\}$. How many functions have the property that $f(1) \neq c$ or $f(2) \neq f$, or both?

10. Consider sets A and B with $|A| = 8$ and $|B| = 5$. How many functions $f : A \to B$ are surjective?

11. Let $A = \{1,2,3,4,5,6,7\}$. How many injective functions $f : A \to A$ have the property that for each $x \in A$, $f(x) \neq x$?

3.8.6 Additional Exercises

1. Based on the previous question, give a combinatorial proof for the identity:

$$\binom{n}{k} = \binom{n+k-1}{k} - \sum_{j=1}^{n}(-1)^{j+1}\binom{n}{j}\binom{n+k-(2j+1)}{k-2j}.$$

2. Illustrate how the counting of derangements works by writing all permutations of $\{1,2,3,4\}$ and then crossing out those which are not derangements. Keep track of the permutations you cross out more than once, using PIE.

3. Let d_n be the number of derangements of n objects. For example, using the techniques of this section, we find

$$d_3 = 3! - \left(\binom{3}{1}2! - \binom{3}{2}1! + \binom{3}{3}0!\right).$$

We can use the formula for $\binom{n}{k}$ to write this all in terms of factorials. After simplifying, for d_3 we would get

$$d_3 = 3!\left(1 - \frac{1}{1} + \frac{1}{2} - \frac{1}{6}\right).$$

Generalize this to find a nicer formula for d_n. Bonus: For large n, approximately what fraction of all permutations are derangements? Use your knowledge of Taylor series from calculus.