(e) Is the contrapositive of the original statement true or false? Prove your answer.

(f) Is the converse of the original statement true or false? Prove your answer.

(g) Is the negation of the original statement true or false? Prove your answer.

3. For each of the statements below, say what method of proof you should use to prove them. Then say how the proof starts and how it ends. Bonus points for filling in the middle.

(a) There are no integers x and y such that x is a prime greater than 5 and $x = 6y + 3$.

(b) For all integers n, if n is a multiple of 3, then n can be written as the sum of consecutive integers.

(c) For all integers a and b, if $a^2 + b^2$ is odd, then a or b is odd.

4. Consider the statement: for all integers n, if n is even then $8n$ is even.

(a) Prove the statement. What sort of proof are you using?

(b) Is the converse true? Prove or disprove.

5. The game TENZI comes with 40 six-sided dice (each numbered 1 to 6). Suppose you roll all 40 dice.

(a) Prove that there will be at least seven dice that land on the same number.

(b) How many dice would you have to roll before you were guaranteed that some four of them would all match or all be different? Prove your answer.

6. Prove that for all integers n, it is the case that n is even if and only if $3n$ is even. That is, prove both implications: if n is even, then $3n$ is even, and if $3n$ is even, then n is even.

7. Prove that $\sqrt{3}$ is irrational.

8. Consider the statement: for all integers a and b, if a is even and b is a multiple of 3, then ab is a multiple of 6.

(a) Prove the statement. What sort of proof are you using?

(b) State the converse. Is it true? Prove or disprove.

9. Prove the statement: For all integers n, if $5n$ is odd, then n is odd. Clearly state the style of proof you are using.

10. Prove the statement: For all integers a, b, and c, if $a^2 + b^2 = c^2$, then a or b is even.

11. Suppose that you would like to prove the following implication:

For all numbers n, if n is prime then n is solitary.