9. Among a group of n people, is it possible for everyone to be friends with an odd number of people in the group? If so, what can you say about n?

10. Your friend has challenged you to create a convex polyhedron containing 9 triangles and 6 pentagons.

(a) Is it possible to build such a polyhedron using *only* these shapes? Explain.

(b) You decide to also include one heptagon (seven-sided polygon). How many vertices does your new convex polyhedron contain?

(c) Assuming you are successful in building your new 16-faced polyhedron, could every vertex be the joining of the same number of faces? Could each vertex join either 3 or 4 faces? If so, how many of each type of vertex would there be?

11. Is there a convex polyhedron that requires 5 colors to properly color the vertices of the polyhedron? Explain.

12. How many edges does the graph $K_{n,n}$ have? For which values of n does the graph contain an Euler circuit? For which values of n is the graph planar?

13. The graph G has 6 vertices with degrees $1, 2, 2, 3, 3, 5$. How many edges does G have? If G was planar how many faces would it have? Does G have an Euler trail?

14. What is the smallest number of colors you need to properly color the vertices of K_7. Can you say whether K_7 is planar based on your answer?

15. What is the smallest number of colors you need to properly color the vertices of $K_{3,4}$? Can you say whether $K_{3,4}$ is planar based on your answer?

16. Prove that $K_{3,4}$ is not planar. Do this using Euler's formula, not just by appealing to the fact that $K_{3,3}$ is not planar.

17. A dodecahedron is a regular convex polyhedron made up of 12 regular pentagons.

(a) Suppose you color each pentagon with one of three colors. Prove that there must be two adjacent pentagons colored identically.

(b) What if you use four colors?

(c) What if instead of a dodecahedron you colored the faces of a cube?

18. Decide whether the following statements are true or false. Prove your answers.

(a) If two graphs G_1 and G_2 have the same chromatic number, then they are isomorphic.

(b) If two graphs G_1 and G_2 have the same number of vertices and edges and have the same chromatic number, then they are isomorphic.

(c) If two graphs are isomorphic, then they have the same chromatic number.