$$\frac{dy}{dt} = -4.03.$$

Since both of these numbers are negative, we can see that our initial trajectory is headed down and to the left—slightly more to the left than down. However, we have no guarantee that the trajectory will continue in this direction.

Before we proceed further with our analysis, let us determine what happens on the nullclines themselves. That is, we will examine the case when $dx/dt = 0$ or $dy/dt = 0$. If $dx/dt = 0$, then any solution crossing this x-nullcline must be moving vertically. There can be no right or left movement at this point in the phase portrait. We can indicate this fact along with the direction of vertical movement by drawing a vertical slash with an arrow on the x-nullcline (Figure 2.2.9). For example, the point $(0.5, 0.25)$ lies on the x-nullcline and

$$\frac{dy}{dt}(0.5, 0.25) < 0.$$

Therefore, the trajectory that crosses the R-nullcline at $(0.5, 0.25)$ is moving down. As we move along the x-nullcline, the direction of this arrow can only change when we cross an y-nullcline.

Similarly, there can be no vertical motion on an y-nullcline. We will indicate the direction of horizontal motion by drawing a horizontal line with an arrow pointing to the right if $dx/dt > 0$ on the y-nullcline and an arrow pointing to the left if $dx/dt < 0$ on the y-nullcline.

We can also determine the basic direction of the solution curve by checking what happens at a point in each of the regions bounded by the nullclines. For example, at the point $(0.1, 0.1)$ we find that

$$\frac{d}{dt}x(0.1, 0.1) = 0.7$$
$$\frac{d}{dt}y(0.1, 0.1) = 0.7.$$

Thus, the general direction of any solution curve in this region is up and right (Figure 2.2.9).

What happens to the initial condition $(1.4, 1.3)$? We see three possible scenarios if we follow the nullclines for large values of t.

- Only species x survives and species y becomes extinct.
- Only species y survives and species x becomes extinct.
- There are essentially equal numbers of species x and y.

□

Activity 2.2.2 Plotting Direction Fields with Nullclines. Consider the competing species model

$$\frac{dx}{dt} = x(1 - x) - \beta xy$$
$$\frac{dy}{dt} = y(1 - y) - \beta xy,$$

where the species interact weakly, say $\beta = 1/2$.

(a) Find the x and y-nullclines for this system.

(b) Find all equilibrium points for this system.