

- (a) A point  $a \in G$  is an **interior point** of  $G$  if some open disk with center  $a$  is a subset of  $G$ .
- (b) A point  $b \in \mathbb{C}$  is a **boundary point** of  $G$  if every open disk centered at  $b$  contains a point in  $G$  and also a point that is not in  $G$ .
- (c) A point  $c \in \mathbb{C}$  is an **accumulation point** of  $G$  if every open disk centered at  $c$  contains a point of  $G$  different from  $c$ .
- (d) A point  $d \in G$  is an **isolated point** of  $G$  if some open disk centered at  $d$  contains no point of  $G$  other than  $d$ .

The idea is that if you don't move too far from an interior point of  $G$  then you remain in  $G$ ; but at a boundary point you can make an arbitrarily small move and get to a point inside  $G$  and you can also make an arbitrarily small move and get to a point outside  $G$ .

**Definition.** A set is **open** if all its points are interior points. A set is **closed** if it contains all its boundary points.

**Example 1.8.** For  $r > 0$  and  $a \in \mathbb{C}$ , the sets  $\{z \in \mathbb{C} : |z - a| < r\} = D[a, r]$  and  $\{z \in \mathbb{C} : |z - a| > r\}$  are open. The **closed disk**

$$\overline{D}[a, r] := \{z \in \mathbb{C} : |z - a| \leq r\}$$

is an example of a closed set. □

A given set might be neither open nor closed. The complex plane  $\mathbb{C}$  and the **empty set**  $\emptyset$  are (the only sets that are) both open and closed.

**Definition.** The **boundary**  $\partial G$  of a set  $G$  is the set of all boundary points of  $G$ . The **interior** of  $G$  is the set of all interior points of  $G$ . The **closure** of  $G$  is the set  $G \cup \partial G$ .

**Example 1.9.** The closure of the open disk  $D[a, r]$  is  $\overline{D}[a, r]$ . The boundary of  $D[a, r]$  is the circle  $C[a, r]$ . □

**Definition.** The set  $G$  is **bounded** if  $G \subseteq D[0, r]$  for some  $r$ .

One notion that is somewhat subtle in the complex domain is the idea of *connectedness*. Intuitively, a set is connected if it is “in one piece.” In  $\mathbb{R}$  a set is connected if and only if it is an interval, so there is little reason to discuss the matter. However, in the plane there is a vast variety of connected subsets.