the derivative of the second function times the first function, all divided by the square of the second function.

- We used the limit definition of the derivative to develop formulas that allow us to find derivatives without resorting to the definition of the derivative. These formulas can be used singly or in combination with each other.

3.4 Derivatives as Rates of Change

- Using $f(a+h) \approx f(a) + f'(a)h$, it is possible to estimate $f(a+h)$ given $f'(a)$ and $f(a)$.
- The rate of change of position is velocity, and the rate of change of velocity is acceleration. Speed is the absolute value, or magnitude, of velocity.
- The population growth rate and the present population can be used to predict the size of a future population.
- Marginal cost, marginal revenue, and marginal profit functions can be used to predict, respectively, the cost of producing one more item, the revenue obtained by selling one more item, and the profit obtained by producing and selling one more item.

3.5 Derivatives of Trigonometric Functions

- We can find the derivatives of sin x and cos x by using the definition of derivative and the limit formulas found earlier. The results are

$$\frac{d}{dx}\sin x = \cos x \frac{d}{dx}\cos x = -\sin x.$$

- With these two formulas, we can determine the derivatives of all six basic trigonometric functions.

3.6 The Chain Rule

- The chain rule allows us to differentiate compositions of two or more functions. It states that for $h(x) = f(g(x))$,

$$h'(x) = f'(g(x))g'(x).$$

  In Leibniz's notation this rule takes the form

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

- We can use the chain rule with other rules that we have learned, and we can derive formulas for some of them.
- The chain rule combines with the power rule to form a new rule:

$$\text{If } h(x) = (g(x))^n, \text{ then } h'(x) = n(g(x))^{n-1}g'(x).$$

- When applied to the composition of three functions, the chain rule can be expressed as follows: If $h(x) = f(g(k(x)))$, then $h'(x) = f'(g(k(x))g'(k(x))k'(x)$.

3.7 Derivatives of Inverse Functions

- The inverse function theorem allows us to compute derivatives of inverse functions without using the limit definition of the derivative.
- We can use the inverse function theorem to develop differentiation formulas for the inverse trigonometric functions.

3.8 Implicit Differentiation

- We use implicit differentiation to find derivatives of implicitly defined functions (functions defined by equations).
- By using implicit differentiation, we can find the equation of a tangent line to the graph of a curve.

3.9 Derivatives of Exponential and Logarithmic Functions

- On the basis of the assumption that the exponential function $y = b^x$, $b > 0$ is continuous everywhere and