SPHERA v.8.0 (RSE SpA): documentation
Appendix

This file belongs to SPHERA's guide: "SPHERA v.8.0 (RSE SpA): documentation"

1. BODY DYNAMICS: CLARIFICATIONS ON GRAVITY FORCE, SLIDING FRICTION FORCE, BODY-BOUNDARY NORMAL REACTION FORCE UNDER SLIDING, NORMAL COEFFICIENT OF RESTITUTION (MASTER CODE BRANCH ON 12JUN2017)

Please refer to the notation symbols of the main documentation file.

1.1. Gravity

Gravity is always active.

The expressions "gravity deactivated" (input file template) only refers to the activation of drag and reaction forces, which temporarily balances gravity components, during an impingement or in case of sliding. The approximations refer to drag and reaction forces, not to gravity.

1.2. Sliding friction force

1.2.1. Aerial stage (non-negative value for the input friction angle and body-frontier interactions)

The simulated and exact formulation (*) for the sliding friction force are equal:

$$\underline{T}_S^* = -\underline{G} \cdot \underline{n}_w \mu_{sf} \underline{s}_w = -\underline{G} \cdot \underline{n}_w \tan\left(\varphi_{dry}\right) \underline{s}_w \tag{1.1}$$

where φ is the sliding friction angle, the subscript "$_{dry}$" refers to dry conditions, $\underline{s}_w$ is the unit vector parallel to the local frontiers and with direction opposite to the velocity vector of the body barycentre (projected on the local DEM). The overall normal of the neighbouring frontiers is the unit vector aligned with the vector sum of the neighbouring normals.

The coefficient of sliding friction (μ_{sf}) reads:

$$\mu_{sf} = \tan\left(\varphi_{dry}\right) \tag{1.2}$$

The absolute value for the exact formulation reads:

$$\left|\underline{T}_S^*\right| = mg \cos\left(\alpha_{DTM}\right) \tan\left(\varphi_{dry}\right) \tag{1.3}$$

where the cosinus of the slope angle is obtained by means of a dot product:

$$\cos \alpha_{DTM} = \left|\underline{n}_w \cdot \underline{k}\right| \tag{1.4}$$

The direction of the sliding friction force is the opposite to the velocity direction of the centre of mass of the computational body.

The following limiter applies to the sliding friction force:

$$\left|\underline{T}_S^*\right|_{\max} = \frac{\left|\underline{u}_{\tan}\right| m}{dt} \tag{1.5}$$

where $\underline{u}_{\tan}$ is the maximum particle velocity (all over the computational body) tangential to the interacting local frontier elements.

Present approximations: a unique friction angle applies to all the body-frontier interactions; the vector sum of the normal reaction force under sliding and the sliding friction force provide no contribution to the body torque (nevertheless the limiter for the sliding friction force depends on the velocity of the solid particles interacting with the frontiers).

1.2.2. Aerial stage (negative value for the input friction angle or body-body interactions)

The sliding friction force ($\underline{T}_S$: drag force on a body under body-boundary interactions) is approximately represented by means of a tangential force, which balances the tangential component of gravity:

$$\underline{T}_S = -\left[\underline{G} - \left(\underline{G} \cdot \underline{n}_w\right)\underline{n}_w\right] \tag{1.6}$$

Its absolute value is:

$$\left|\underline{T}_S\right| = mg \sin\left(\alpha_{DTM}\right) \tag{1.7}$$

where α_{DTM} is the slope angle and $\underline{k}$ the unit vector aligned with the vertical axis:

$$\sin \alpha_{DTM} = \left|\underline{n}_w \times \underline{k}\right| \tag{1.8}$$

The removal of the gravity force component which is parallel to the bottom is equivalent to introducing an approximated sliding friction force, where the slope angle approximates the sliding friction angle.

$$\begin{gathered} \underline{T}_S = \underline{T}_S^* \Rightarrow \\ \Rightarrow mg \sin \alpha_{DTM} = mg \cos\left(\alpha_{DTM}\right) \tan\left(\varphi_{dry}\right) \Rightarrow \\ \Rightarrow \varphi_{dry} \cong \alpha_{DTM} \end{gathered} \tag{1.9}$$

This approximation might be acceptable, especially in case the sliding friction angle is unavailable.

1.2.3. Submerged stage

The sliding friction force is negligible (hypothesis on the inertial fluid flows, Amicarelli et al., 2015, CAF):

$$\underline{T}_S = \underline{0} \tag{1.10}$$

The exact formulation reads:

$$\underline{T}_S^* = -\left(\underline{G} + \underline{P}_F\right) \cdot \underline{n}_w \tan\left(\varphi_{wet}\right) \underline{s}_w \tag{1.11}$$

1.3. Body-boundary normal reaction force under sliding (at null normal velocity)

1.3.1. Aerial stage

The exact formulation is correctly represented:

$$\underline{P}_S = \underline{P}_S^* = -\left(\underline{G} \cdot \underline{n}_w\right)\underline{n}_w, \qquad \underline{u} \cdot \underline{n}_w = 0 \tag{1.12}$$

This term is added to the body-boundary normal force in the main file of this documentation -Eq.(6.32)-. The overall normal of the neighbouring frontiers is the unit vector aligned with the vector sum of the neighbouring normals.

1.3.2. Submerged stage

The normal reaction is formally null.

$$\underline{P}_S = \underline{0}, \qquad \underline{u} \cdot \underline{n}_w = 0 \tag{1.13}$$

According to Monaghan (2005), the normal forces of the boundary force particles (6.32) dynamically restore the normal reaction force (body-boundary interactions), despite some body spurious oscillations (normal to the frontier) in the interaction zone (noise amplitude comparable with the spatial resolution).

$$\underline{P_S^*} = -\left(\underline{G} + \underline{P_F}\right) \cdot \underline{n_w}, \qquad \underline{u} \cdot \underline{n_w} = 0 \tag{1.14}$$

1.4. Normal restitution coefficient (R_n)

The simulated normal restitution coefficient is equal to unity if all the following conditions are satisfied: homogeneous velocity of the solid particles belonging to the impinging body, impingement with a single frontier element, isolated impingement, body axes aligned with the frontier axes. Otherwise (real cases), the scheme for body-boundary force is dissipative and represents R_n<1. The equivalent value of the restitution coefficient might be estimated a posteriori.

2. BULK MODULUS ASSIGNMENT (MASTER CODE BRANCH ON 14MAR2018)

Please refer to the notation symbols of the main documentation file.

Eq.7.21 of the main documentation file has to be replaced with the following formula:

$$K = \rho \frac{\partial p}{\partial \rho} = \rho c_{ref}^2 = \rho \left(A_{WC} U_{scale} \right)^2 \tag{2.1}$$

The sound speed (c_{ref}) should be at least 10 times higher than the maximum velocity in the fluid (WC approach). This position (constant A_{WC} equal to 10) provides a maximum relative error on density of 1%, whereas the assumption A_{WC}=4.5 increases the density relative error to 5% (Monaghan, 2005).

The velocity scale U_{scale} reads:

$$U_{scale} = \max\left(\left| \underline{u} \right|_{\max}, \sqrt{2 g Y_{\max}} \right) \tag{2.2}$$

where Y_{max} is the maximum water depth and $\left| \underline{u} \right|_{\max}$ is the maximum absolute value of velocity (the maxima operate both over the whole simulated time and the whole 3D domain space).