SPHERA v.8.0 (RSE SpA): documentation
Appendix

This file belongs to SPHERA's guide: "SPHERA v.8.0 (RSE SpA): documentation"

1. BODY DYNAMICS: CLARIFICATIONS ON GRAVITY FORCE, SLIDING FRICTION FORCE, BODY-BOUNDARY NORMAL REACTION FORCE UNDER SLIDING, NORMAL COEFFICIENT OF RESTITUTION (MASTER CODE BRANCH ON 15MAY2017)

This section reports some clarifications on the meaning of the following approximated configuration of SPHERA: (*imping_body_grav*=1, *imping_body_grav_dry*=0).
Please refer to the notation symbols of the main documentation file.

1.1. Gravity

Gravity is always active.
The expressions "gravity deactivated" (input file template) only refers to the activation of drag and reaction forces, which temporarily balances gravity components, during an impingement or in case of sliding. The approximations refer to drag and reaction forces, not to gravity.

1.2. Sliding friction force

1.2.1. Aerial stage

The sliding friction force ($\underline{T}_S$: drag force on a body under body-boundary interactions) is approximately represented by means of a tangential force, which balances the tangential component of gravity:

$$\underline{T}_S = -\left[\underline{G} - \left(\underline{G} \cdot \underline{n}_w\right)\underline{n}_w\right] \tag{1.1}$$

Its absolute value is:

$$\left|\underline{T}_S\right| = mg \sin\left(\alpha_{DTM}\right) \tag{1.2}$$

where α_{DTM} is the slope angle and $\underline{k}$ the unit vector aligned with the vertical axis:

$$\sin \alpha_{DTM} = \left|\underline{n}_w \times \underline{k}\right| \tag{1.3}$$

The exact formulation (*) for the sliding friction force reads:

$$\underline{T}_S^* = -\underline{G} \cdot \underline{n}_w \mu_{sf}\, \underline{s}_w = -\underline{G} \cdot \underline{n}_w \tan\left(\varphi_{dry}\right)\underline{s}_w \tag{1.4}$$

where φ is the sliding friction angle, the subscript "$_{dry}$" refers to dry conditions, $\underline{s}_w$ is the unit vector parallel to the frontier and pointing downward (it might also be horizontal), whereas the coefficient of sliding friction (μ_{sf}) reads:

$$\mu_{sf} = \tan\left(\varphi_{dry}\right) \tag{1.5}$$

The absolute value for the exact formulation reads:

$$\left|\underline{T}_S^*\right| = mg \cos\left(\alpha_{DTM}\right)\tan\left(\varphi_{dry}\right) \tag{1.6}$$

where the cosinus of the slope angle is obtained by means of a dot product:

$$\cos \alpha_{DTM} = \left|\underline{n}_w \cdot \underline{k}\right| \tag{1.7}$$

The removal of the gravity force component which is parallel to the bottom is equivalent to introducing an approximated sliding friction force, where the slope angle approximates the sliding friction angle.

$$\underline{T_S} = \underline{T_S^*} \Rightarrow$$
$$\Rightarrow mg \sin \alpha_{DTM} = mg \cos(\alpha_{DTM}) \tan(\varphi_{dry}) \Rightarrow \quad (1.8)$$
$$\Rightarrow \varphi_{dry} \cong \alpha_{DTM}$$

This approximation might be acceptable, especially in case the sliding friction angle is unavailable.

1.2.2. Submerged stage

The sliding friction force is negligible (hypothesis on the inertial fluid flows, Amicarelli et al., 2015, CAF):

$$\underline{T_S} = \underline{0} \quad (1.9)$$

The exact formulation reads:

$$\underline{T_S^*} = -\left(\underline{G} + \underline{P_F}\right) \cdot \underline{n_w} \tan(\varphi_{wet}) \underline{s_w} \quad (1.10)$$

1.3. Body-boundary normal reaction force under sliding (at null normal velocity)

1.3.1. Aerial stage

The exact formulation is correctly represented:

$$\underline{P_S} = \underline{P_S^*} = -\left(\underline{G} \cdot \underline{n_w}\right)\underline{n_w}, \qquad \underline{u} \cdot \underline{n_w} = 0 \quad (1.11)$$

This term is added to the body-boundary normal force in the main file of this documentation -Eq.(6.32)-.

1.3.2. Submerged stage

The normal reaction is formally null.

$$\underline{P_S} = \underline{0}, \qquad \underline{u} \cdot \underline{n_w} = 0 \quad (1.12)$$

Acording to Monaghan (2005), the normal forces of the boundary force particles (6.32) dynamically restore the normal reaction force (body-boundary interactions), despite some body spurious obscillations (normal to the frontier) in the interaction zone (noise amplitude comparable with the spatial resolution).

$$\underline{P_S^*} = -\left(\underline{G} + \underline{P_F}\right) \cdot \underline{n_w}, \qquad \underline{u} \cdot \underline{n_w} = 0 \quad (1.13)$$

1.4. Normal restitution coefficient (R_n)

The simulated normal restitution coefficient is equal to unity if all the following conditions are satisfied: homogeneous velocity of the solid particles belonging to the impinging body, impingement with a single frontier element, isolated impingement, body axes aligned with the frontier axes. Otherwise (real cases), the scheme for body-boundary force is dissipative and represents R_n<1. The equivalent value of the restitution coefficient might be estimated a posteriori.