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Listening to the coefficient of restitution

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A pedagogical method is described for measurement of the coefficient of restitution for collisions between a bouncing ball and a horizontal surface. Indirect measurement using only the sound made by the bouncing ball facilitates the rapid collection of data, allowing introductory laboratory students more time to concentrate on theory, method, data reduction, analysis, and the physical implications of their results. The coefficient of restitution is found to decrease systematically with increasing relative approach speed.

INTRODUCTION

The reader may be familiar with a straightforward method of measuring the coefficient of restitution for collisions between a steel ball and a steel block that has been standard introductory laboratory fare for many years.[1] The experiment involves repetitively dropping a steel ball from a known height into a vertical glass tube, and measuring the estimated heights of its first few rebounds from a hardened steel block at the bottom of the tube. The ball must be retrieved from the tube after each trial, including the unsuccessful trials in which the ball touches the sides of the tube while dropping or rebounding. The coefficient of restitution is readily calculated from the observed heights, but the procedure itself is time-consuming if reproducible results are to be obtained.

At Rutgers in Newark, we have adopted another method, whose chief role in our laboratory program is to exemplify the concept of indirect measurement by looking for clues to the behavior of a physical system in such "incidental" manifestations as the noise it makes. The procedure is simple and less tedious than the method described above. The analysis is somewhat more complicated than that required by the earlier method; it has been found useful as an example of the use of graphical techniques and as an introduction to the use of an interactive computer terminal.

METHOD

A ball is dropped from a known height h_0 onto a smooth horizontal surface; we wish to measure the coefficient of restitution ϵ for collisions between the ball and the surface. This quantity is defined for collisions between two bodies as the ratio of relative speed of separation to relative speed of approach:

$$\epsilon = |\mathbf{v}_2' - \mathbf{v}_1'| \Big/ |\mathbf{v}_2 - \mathbf{v}_1|,$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities of the two bodies before collision, and the primed variables represent the corresponding velocities after collision. ϵ varies from zero for a completely inelastic collision (in which the bodies do not separate) to unity for a perfectly elastic collision; it can exceed unity if enough internal energy is converted to kinetic energy during the collision. In our experiment, ϵ is initially assumed constant, but it is later found to be a function of approach speed.

Our objective is to determine ϵ as a function of the elapsed time between bounces, which is measured with a pen recorder from the sound of the bounces. We begin by calculating the vertical speed v_n (the vertical component of the velocity) at which the ball rises from its nth bounce. Neglecting air resistance, we may expect the ball to come down into its next bounce $(n + 1)$ at the same vertical speed. Since the net force acting on the ball between bounces is that of gravity, the downward acceleration is g (980 cm/sec^2). If we measure time from the top of the ball's trajectory, when the vertical speed is instantaneously zero, then the vertical speed at a later time t is of magnitude

$$v_{\text{vert}} = gt.$$

The flight time from the top of the trajectory to the surface is one-half of the total flight time T_n between the nth and $(n + 1)$th bounces. Hence, using the notation illustrated in Fig. 1,

$$v_n = gT_n/2 \quad (n = 1,2,3, \ldots). \tag{1}$$

Assuming that ϵ is constant, we may express this speed in terms of the coefficient of restitution and the speed v_0 at which the ball first strikes the surface after being released:

$$v_n = \epsilon v_{n-1} = \epsilon^n v_0 \quad (n = 1,2,3, \ldots). \tag{2}$$

Equating the two expressions for v_n in Eqs. (1) and (2), we have

$$gT_n/2 = \epsilon^n v_0 \quad (n = 1,2,3, \ldots),$$

whence

$$T_n = \epsilon^n(2v_0/g) \quad (n = 1,2,3, \ldots).$$

Finally, by taking the common logarithm of both sides we obtain

$$\log T_n = n \log\epsilon + \log(2v_0/g) \quad (n = 1,2,3, \ldots). \tag{3}$$

This is our "working" equation. Evidently, if we plot $\log T_n$ as a function of n, we can expect a straight line whose slope is $\log\epsilon$ and whose ordinate intercept is $\log(2v_0/g)$. The slope and intercept are then sufficient to determine ϵ, v_0, and (hence) v_n for all but the last bounce considered. The only data required are the flight times (the T_n's) between bounces. We can also calculate the initial height h_0 from which the ball was dropped, as well as the height h_n reached after the nth bounce. Still neglecting air resistance, we have

$$mgh_n = mv_n{}^2/2 \quad (n = 0,1,2, \ldots)$$

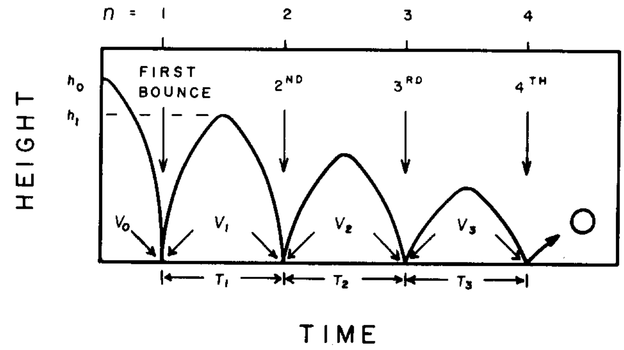

Fig. 1. Schematic representation of the first few collisions, illustrating the notation used in the derivation.

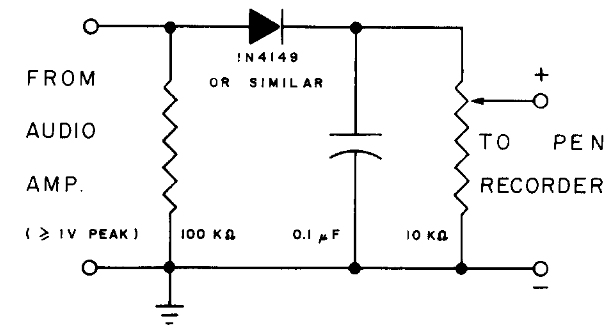

Fig. 3. Peak-detector circuit for use with a dc voltage or current pen recorder. The 100 kΩ resistor may be omitted if the audio-amplifier output is dc coupled.

for a ball of mass m; this yields

$$h_n = v_n^2/2g \quad (n = 0,1,2, \ldots). \tag{4}$$

PROCEDURE

The sound made by a bouncing ball is detected by a microphone, amplified, and used to produce transverse deflections on a pen recorder operating at a (known) constant speed, or an X-Y recorder with one axis swept at a suitable constant rate. The bounces are identified as the beginnings of the deflections, and the time intervals between bounces may be readily determined from the distances along the chart between successive deflections.

Figure 2 shows a portion of the chart obtained when a solid plastic ball of 1.56-cm diameter was dropped from a height of 100.0 cm onto a smooth massive stone laboratory bench top. In this example, an inexpensive dynamic microphone was used with an audio-frequency decade amplifier connected to the Y input of an X-Y plotter whose X axis was swept at 4.00 cm/sec. It should be emphasized that this apparatus was chosen because it was available. Almost any microphone and audio-frequency amplifier will be found suitable. If the pen recorder chosen measures only dc voltage or current, a peak-detector circuit such as that shown in Fig. 3 may be used at its input.

Since the data may be obtained very quickly, several teams of students can run the experiment independently on a single apparatus in a fairly short time. Various materials may be found suitable for collisions, and appropriate alterations in the conditions of the experiment can be made as questions arise. For example, cardboard can be placed beneath a glass plate, increasing the amount of deflection of the plate and consequently the loudness of the bounces.

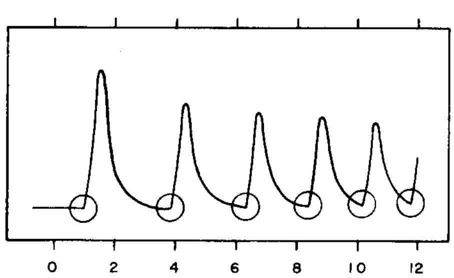

Fig. 2. A portion of the recorder chart for the example discussed. The collision points are circled.

Students may then verify their suspicions about the change in coefficient of restitution to be expected with a noisier collision, recognizing that the energy dissipated as noise must come from the kinetic energy of the ball just before it collides. The method can also be adapted for use in studying "slow motion" collisions between a glider on an inclined air track and a stationary bumper.

ANALYSIS

When a plot of Eq. (3) is prepared from the time-interval data obtained as described above, considerable random error will often be evident, especially for collisions on an uneven surface such as a concrete floor. (Here we have a clear example of random variation which cannot possibly be attributed to "human error.") Some judgment is required in choosing the "best" straight line among the plotted points; accordingly, the experiment may be used to motivate a discussion of the method of least squares.

As a check on the appropriateness of their choice and on the correctness of their interpretation of the straight line so obtained, our students use an interactive computer program (COREST) prepared by the author for use with a teletype or typewriter terminal.[2] The input to the program includes the chart speed and the positions along the recorder chart of the deflections corresponding to the successive bounces, measured from an arbitrarily chosen origin. The program calculates the times elapsed between bounces and determines ϵ and v_0, using the method of least squares.

A portion of the COREST output for our example is shown in Table I, and the results are plotted in Fig. 4.[3] In this instance the initial height and the positions of the chart deflections were measured to within 1 mm, estimating tenths of a millimeter. Since the distances between successive deflections ranged from 2.94 cm at the beginning of the run to 0.38 cm at the end, the flight times and corresponding logarithms in Table I, as well as all of the values in Table II not in parentheses, are known to three significant figures for the lower. Ten repeated trials made with the same apparatus yielded values of ϵ having a standard deviation of 5.3×10^{-4}.

As a further check, the initial height h_0 may be calculated from the student's (or the computer's) value for v_0, using Eq. (4). In the example, h_0 as calculated from the value of v_0 shown in Table I is 58.2 cm, in considerable disagreement with the actual value of 100.0 cm; moreover, little random variation is noticeable in Fig. 4, and a suspicious curving

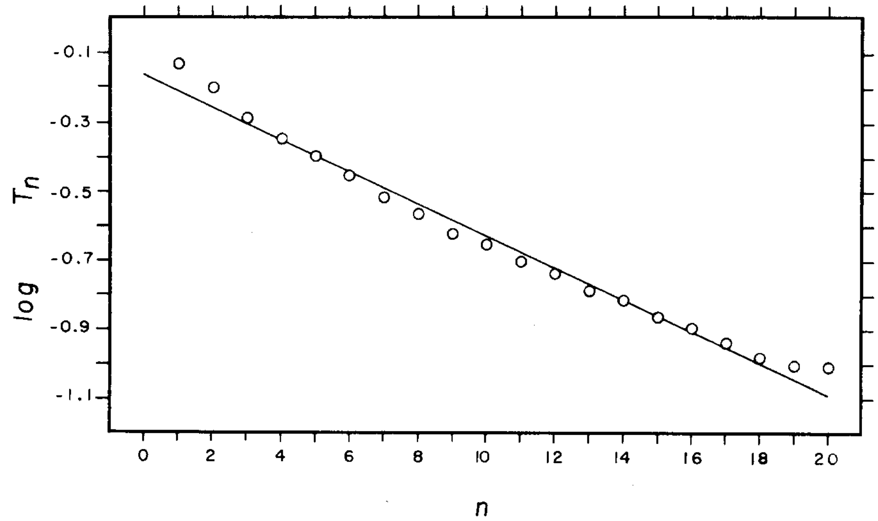

Fig. 4. Logarithm of flight time as a function of bounce number for the example discussed. The line shown represents the result of the least-squares calculation performed by COREST.

tendency is clearly evident in the "straight line." We conclude that a reappraisal of our assumptions is in order.

LISTENING MORE CLOSELY

At this point the experiment becomes open-ended, in the sense that the analysis may proceed in various directions according to the order in which various possibilities occur to the investigators. The time-axis calibration of the chart recorder is often suggested as a possible source of error, but it has been found to be correct to within 0.1% for our apparatus.

Our method depends on the assumption that air resistance is negligible. This assumption is supported by the results of a series of measurements of the flight times of various objects dropped from a height of 100.0 cm, using a solenoid-operated clamp, a piezoelectric transducer (attached to the landing platform), and a digital timer. The only significant error was found (not surprisingly) in the case of a Ping-Pong ball.

We are now forced to suspect our basic assumption that the coefficient of restitution is a constant. Fortunately, this assumption can be tested without additional data.

We now postulate that the coefficient of restitution may be a function of approach speed, and calculate ϵ separately for each collision directly from the definition, in the form

$$\epsilon_n = v_n/v_{n-1} \quad (n = 2,3,4,\dots), \tag{5}$$

plotting the results as a function of approach speed (see Fig. 5). To facilitate this analysis, the COREST printout includes a supplementary table (Table II) showing the speeds and their ratios as calculated using Eqs. (1) and (5). An additional point may be obtained if the initial height h_0 is known, using conservation of energy. For $n = 0$, Eq. (4) may be rewritten as

$$v_0 = (2gh_0)^{1/2},$$

and substitution in Eq. (5) yields

$$\epsilon_1 = v_1/(2gh_0)^{1/2}. \tag{6}$$

Table I. Part of COREST printout for the example discussed. As part of their analysis, students are expected to decide how many of the decimal places shown are actually significant.

Coefficient of restitution: 0.89909
Initial landing speed: 337.77 cm/sec

Bounce number, n	Flight time (sec)	$\log T_n$	Takeoff speed (cm/sec)	Height reached (cm)
1	0.7350	−0.134	303.69	47.04
2	0.6300	−0.201	273.04	38.03
3	0.5150	−0.288	245.49	30.74
4	0.4475	−0.349	220.72	24.85
5	0.4000	−0.398	198.45	20.09
6	0.3500	−0.456	178.42	16.24
7	0.3050	−0.516	160.42	13.13
8	0.2725	−0.565	144.23	10.61
9	0.2400	−0.620	129.68	8.58
10	0.2225	−0.653	116.59	6.93
11	0.1975	−0.704	104.83	5.60
12	0.1825	−0.739	94.25	4.53
13	0.1625	−0.789	84.74	3.66
14	0.1525	−0.817	76.19	2.96
15	0.1375	−0.862	68.50	2.39
16	0.1275	−0.895	61.59	1.93
17	0.1150	−0.939	55.37	1.56
18	0.1050	−0.979	49.78	1.26
19	0.0975	−1.011	44.76	1.02
20	0.0950	−1.022	40.24	0.83

Table II. Part of COREST printout, showing coefficient of restitution as a function of approach speed for the example discussed. Supplementary values (in parentheses) were calculated using Eqs. (1) and (6). As before, students are expected to decide how many of the decimal places shown are actually significant.

n	v_n (cm/sec)	ϵ_n	n	v_n (cm/sec)	ϵ_n
0	(442.78)	—			
1	360.25	(0.814)	11	96.80	0.888
2	308.78	0.857	12	91.90	0.949
3	252.42	0.817	13	79.65	0.867
4	219.34	0.869	14	74.75	0.938
5	196.05	0.894	15	67.39	0.902
6	171.55	0.875	16	62.49	0.927
7	149.49	0.871	17	56.37	0.902
8	133.56	0.893	18	51.46	0.913
9	117.63	0.881	19	47.79	0.929
10	109.05	0.927	20	46.56	0.974

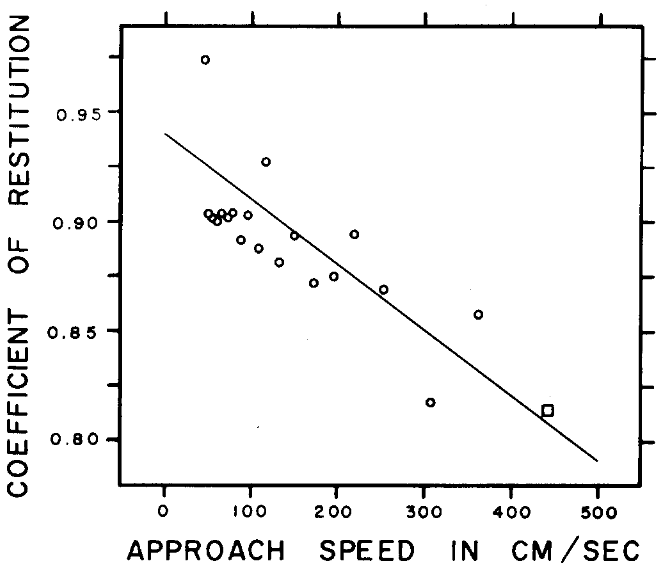

Fig. 5. Coefficient of restitution as a function of relative speed of approach for the example discussed. The line shown represents a linear least-squares fit. The last point on the right (□) was determined from Eqs. (1) and (6), using conservation of energy.

The results of these calculations are plotted in Fig. 5, in which it appears that the coefficient of restitution decreases systematically with increasing approach speed. The worst-case deviation from the least-squares fit included in Fig. 5 to emphasize the systematic variation in ϵ is about 5%; however, it should be borne in mind that the points shown do not represent averages, but are the results of a single 6-sec run.

CONCLUSION

The experiment described above has been found useful in introductory laboratory courses. The procedure is fast, convenient, and amenable to variation. Analysis of the data includes useful practice in dealing with motion under constant acceleration and the conservation of energy. The student makes use of graphical techniques and of least-squares calculations performed, if desired, with the aid of an interactive computer terminal.

A built-in check (the calculation of h_0 from v_0) shows that at least one of the preliminary assumptions must be invalid, and investigation of the assumptions one by one leads to the conclusion that the coefficient of restitution decreases systematically with increasing approach speed.

At present writing further experiments are being conducted in hopes of determining more precisely how ϵ varies with approach speed, and perhaps learning something about the mechanism of the dependence.

[1]V. E. Eaton, M. J. Martin, R. S. Minor, R. J. Stephenson, and M. W. White, *Selective Experiments in Physics* (Central Scientific, Chicago, 1941), No. 71990-M97b.

[2]The program is written in FORTRAN and can easily be modified for batch operation. A program listing and sample run may be obtained from the author.

[3]The value of g used in the program is that given for Hoboken, NJ (980.269 cm/sec^2) in the *Handbook of Chemistry and Physics*, 34th ed. (Chemical Rubber, Cleveland, 1952), p. 2867.