Chapter 2
Boolean Algebra and Logic Gates

Definitions

Theorems

Functions

Canonical and Standard Forms

Operations

Gates

Integrated Circuits

Mathematical Systems

- Mathematical systems **can be defined with:**
  - **A set of** elements **containing a set of objects with common properties.**
  - **A set of** operators **that can be performed on any subset of the elements.**
  - **A set of** axioms **or** postulates **forming a basis from which we can deduce rules, theorems and properties of the system.**

Set Notations

- **The following notations are being used in this class:**
  - $x \in S$ **indicates that x is an element of the set S.**
  - $y \notin S$ **indicates that y is not an element of the set S.**
  - **A = {1, 2, 3, 4} indicates that set A exists with a finite number of elements (1, 2, 3, 4).**

Basic Postulates

- **The basic postulates of a mathematical system are:**
  - Closure. **A set S is closed w.r.t a binary operator if this operation only produces results that are within the set of elements defined by the system.**
  - Associative Law. **A binary operator is said to be associative when:**
    - » **(x * y) * z = x * (y * z)**
  - Commutative Law. **A binary operator is said to be commutative when:**
    - » **x * y = y * x**
  - Identity Element. **A set is said to have an identity element with respect to a binary operation if there exists an element, e, that is a member of the set with the property:**
    - » **e * x = x * e = x for every element of the set**
      - **Additive identity is 0 and multiplicative identity is 1**
  - **Note: * + and . are binary operators**

Basic Postulates

- **Inverse. For a set with an identity element with respect to a binary operation, the set is said to have an inverse if for every element of the set the following property holds:**
  - » **$x * y = e$**
    - **The additive inverse of element a is –a and it defines subtraction, since a + (–a) = 0. Multiplicative inverse of a is 1/a and defines division, since a.1/a = 1**
- **Distributive Law. * is said to be distributive over . when**
  - » **$x * (y \cdot z) = (x * y) \cdot (x * z)$**
- **Note: * + and . are binary operators. Binary operator + defines addition and binary operator . defines multiplication**

Two-Valued Boolean Algebra

- **Two-value Boolean algebra is defined by the:**
  - **The set of two elements B={0, 1}**
  - **The operators of AND (·) and OR (+)**
  - Huntington Postulates **are satisfied**

Huntington Postulates

- **Boolean algebra has the following postulates:**
  1. Closure.
     a) **with respect to the binary operation OR (+)**
     b) **with respect to the binary operation AND (·)**
  2. Identity.
     a) **with respect to OR (+) is 0:**
        - ❖ $x + 0 = 0 + x = x$, for $x = 1$ or $x = 0$
     b) **with respect to AND (·) is 1:**
        - ❖ $x \cdot 1 = 1 \cdot x = x$, for $x = 1$ or $x = 0$
  3. Commutative Law.
     a) **With respect to OR (+):**
        - ❖ $x + y = y + x$
     b) **With respect to AND (·):**
        - ❖ $x \cdot y = y \cdot x$

Huntington Postulates Continued…

- **Boolean algebra has the following postulates:**

4. Distributive Law.

   a) with respect to the binary operation OR (+):

   - ❖ $x + (y \cdot z) = (x + y) \cdot (x + z)$ **+ is distributive over .**

   b) with respect to the binary operation AND (·):

   - ❖ $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$ **. is distributive over +**

5. Complement.

   a) $x + x' = 1$, for $x = 1$ or $x = 0$

   b) $x \cdot x' = 0$, for $x = 1$ or $x = 0$

6. Membership.

   - ❖ **There exists at least two elements, x and y, of the set such that $x \neq y$.**
     - ❖ $0 \neq 1$

Notes on Huntington Postulates

- **The associative law is not listed but it can be derived from the existing postulates for both operations.**
- **The distributive law of + over . i.e.,**

$$x+(y \cdot z) = (x + y) \cdot (x + z)$$

  is valid for Boolean algebra but not for ordinary algebra.
- **Boolean algebra doesn't have inverses (additive or multiplicative) therefore there are no operations related to subtraction or division.**
- **Boolean algebra deals with only two elements, 0 and 1**

Operator Tables

- A two-valued Boolean algebra is defined on a set of two elements B={0,1}, with rules for the two binary operators + and . as shown in the following operator tables:

- AND Operation

x	y	x·y
0	0	0
0	1	0
1	0	0
1	1	1

- OR Operation

x	y	x+y
0	0	0
0	1	1
1	0	1
1	1	1

- NOT Operation

x	x'
0	1
1	0

Proving the Distributive Law

x	y	z	$y + z$	$x \cdot (y + z)$	$x \cdot y$	$x \cdot z$	$(x \cdot y) + (x \cdot z)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1