

Chapter #7

Data Structure - Sorting Techniques

Sorting refers to arranging data in a particular format. Sorting algorithm specifies the way to arrange data in a particular order. Most common orders are in numerical or lexicographical order.

The importance of sorting lies in the fact that data searching can be optimized to a very high level, if data is stored in a sorted manner. Sorting is also used to represent data in more readable formats. Following are some of the examples of sorting in real-life scenarios –

- **Telephone Directory** – The telephone directory stores the telephone numbers of people sorted by their names, so that the names can be searched easily.
- **Dictionary** – The dictionary stores words in an alphabetical order so that searching of any word becomes easy.

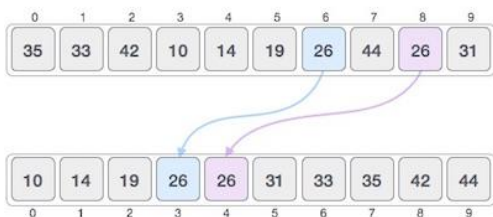
In-place Sorting and Not-in-place Sorting

Sorting algorithms may require some extra space for comparison and temporary storage of few data elements. These algorithms do not require any extra space and sorting is said to happen in-place, or for example, within the array itself. This is called **in-place sorting**. Bubble sort is an example of in-place sorting.

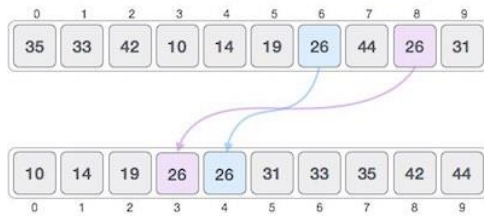
However, in some sorting algorithms, the program requires space which is more than or equal to the elements being sorted. Sorting which uses equal or more space is called **not-in-place sorting**. Merge-sort is an example of not-in-place sorting.

Stable and Not Stable Sorting

If a sorting algorithm, after sorting the contents, does not change the sequence of similar content in which they appear, it is called **stable sorting**.



If a sorting algorithm, after sorting the contents, changes the sequence of similar content in which they appear, it is called **unstable sorting**.



Stability of an algorithm matters when we wish to maintain the sequence of original elements, like in a tuple for example.

Adaptive and Non-Adaptive Sorting Algorithm

A sorting algorithm is said to be adaptive, if it takes advantage of already 'sorted' elements in the list that is to be sorted. That is, while sorting if the source list has some element already sorted, adaptive algorithms will take this into account and will try not to re-order them.

A non-adaptive algorithm is one which does not take into account the elements which are already sorted. They try to force every single element to be re-ordered to confirm their sortedness.

Important Terms

Some terms are generally coined while discussing sorting techniques, here is a brief introduction to them –

Increasing Order

A sequence of values is said to be in **increasing order**, if the successive element is greater than the previous one. For example, 1, 3, 4, 6, 8, 9 are in increasing order, as every next element is greater than the previous element.

Decreasing Order

A sequence of values is said to be in **decreasing order**, if the successive element is less than the current one. For example, 9, 8, 6, 4, 3, 1 are in decreasing order, as every next element is less than the previous element.

Non-Increasing Order

A sequence of values is said to be in **non-increasing order**, if the successive element is less than or equal to its previous element in the sequence. This order occurs when the sequence contains duplicate values. For example, 9, 8, 6, 3, 3, 1 are in non-increasing order, as every next element is less than or equal to (in case of 3) but not greater than any previous element.

Non-Decreasing Order

A sequence of values is said to be in **non-decreasing order**, if the successive element is greater than or equal to its previous element in the sequence. This order occurs when the sequence contains duplicate values. For example, 1, 3, 3, 6, 8, 9 are in non-decreasing order, as every next element is greater than or equal to (in case of 3) but not less than the previous one.

Data Structure - Bubble Sort Algorithm

Bubble sort is a simple sorting algorithm. This sorting algorithm is comparison-based algorithm in which each pair of adjacent elements is compared and the elements are swapped if they are not in order. This algorithm is not suitable for large data sets as its average and worst case complexity are of $O(n^2)$ where **n** is the number of items.

How Bubble Sort Works?

We take an unsorted array for our example. Bubble sort takes $O(n^2)$ time so we're keeping it short and precise.



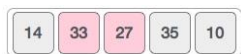
Bubble sort starts with very first two elements, comparing them to check which one is greater.



In this case, value 33 is greater than 14, so it is already in sorted locations. Next, we compare 33 with 27.



We find that 27 is smaller than 33 and these two values must be swapped.



The new array should look like this –



Next we compare 33 and 35. We find that both are in already sorted positions.



Data Structure and Algorithms Notes

Then we move to the next two values, 35 and 10.



We know then that 10 is smaller 35. Hence they are not sorted.



We swap these values. We find that we have reached the end of the array. After one iteration, the array should look like this –



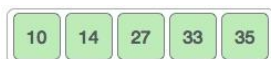
To be precise, we are now showing how an array should look like after each iteration. After the second iteration, it should look like this –



Notice that after each iteration, at least one value moves at the end.



And when there's no swap required, bubble sorts learns that an array is completely sorted.



Now we should look into some practical aspects of bubble sort.

Algorithm

We assume **list** is an array of **n** elements. We further assume that **swap** function swaps the values of the given array elements.

```
begin BubbleSort(list)

  for all elements of list
    if list[i] > list[i+1]
      swap(list[i], list[i+1])
    end if
```

```
end for

return list

end BubbleSort
```

Pseudocode

We observe in algorithm that Bubble Sort compares each pair of array element unless the whole array is completely sorted in an ascending order. This may cause a few complexity issues like what if the array needs no more swapping as all the elements are already ascending.

To ease-out the issue, we use one flag variable **swapped** which will help us see if any swap has happened or not. If no swap has occurred, i.e. the array requires no more processing to be sorted, it will come out of the loop.

Pseudocode of BubbleSort algorithm can be written as follows –

```
procedure bubbleSort( list : array of items )

    loop = list.count;

    for i = 0 to loop-1 do:
        swapped = false

        for j = 0 to loop-1 do:

            /* compare the adjacent elements */
            if list[j] > list[j+1] then
                /* swap them */
                swap( list[j], list[j+1] )
                swapped = true
            end if
```

```
end for

/*if no number was swapped that means
array is sorted now, break the loop.*/

if(not swapped) then
    break
end if

end for

end procedure return list
```

Implementation

One more issue we did not address in our original algorithm and its improvised pseudocode, is that, after every iteration the highest values settles down at the end of the array. Hence, the next iteration need not include already sorted elements. For this purpose, in our implementation, we restrict the inner loop to avoid already sorted values.

Data Structure and Algorithms Insertion Sort

This is an in-place comparison-based sorting algorithm. Here, a sub-list is maintained which is always sorted. For example, the lower part of an array is maintained to be sorted. An element which is to be 'insert'ed in this sorted sub-list, has to find its appropriate place and then it has to be inserted there. Hence the name, **insertion sort**.

The array is searched sequentially and unsorted items are moved and inserted into the sorted sub-list (in the same array). This algorithm is not suitable for large data sets as its average and worst case complexity are of $O(n^2)$, where **n** is the number of items.

How Insertion Sort Works?

We take an unsorted array for our example.



Insertion sort compares the first two elements.

Data Structure and Algorithms Notes



It finds that both 14 and 33 are already in ascending order. For now, 14 is in sorted sub-list.



Insertion sort moves ahead and compares 33 with 27.



And finds that 33 is not in the correct position.



It swaps 33 with 27. It also checks with all the elements of sorted sub-list. Here we see that the sorted sub-list has only one element 14, and 27 is greater than 14. Hence, the sorted sub-list remains sorted after swapping.



By now we have 14 and 27 in the sorted sub-list. Next, it compares 33 with 10.



These values are not in a sorted order.



So we swap them.



However, swapping makes 27 and 10 unsorted.



Hence, we swap them too.



Data Structure and Algorithms Notes

Again we find 14 and 10 in an unsorted order.



We swap them again. By the end of third iteration, we have a sorted sub-list of 4 items.



This process goes on until all the unsorted values are covered in a sorted sub-list. Now we shall see some programming aspects of insertion sort.

Algorithm

Now we have a bigger picture of how this sorting technique works, so we can derive simple steps by which we can achieve insertion sort.

- Step 1** – If it is the first element, it is already sorted. return 1;
- Step 2** – Pick next element
- Step 3** – Compare with all elements in the sorted sub-list
- Step 4** – Shift all the elements in the sorted sub-list that is greater than the value to be sorted
- Step 5** – Insert the value
- Step 6** – Repeat until list is sorted

Pseudocode

```
procedure insertionSort( A : array of items )  
    int holePosition  
    int valueToInsert  
  
    for i = 1 to length(A) inclusive do:  
  
        /* select value to be inserted */  
        valueToInsert = A[i]  
        holePosition = i  
  
        /*locate hole position for the element to be inserted */
```



```
while holePosition > 0 and A[holePosition-1] > valueToInsert do:  
    A[holePosition] = A[holePosition-1]  
    holePosition = holePosition - 1  
end while  
  
/* insert the number at hole position */  
A[holePosition] = valueToInsert  
  
end for  
  
end procedure
```

Data Structure and Algorithms Selection Sort

Selection sort is a simple sorting algorithm. This sorting algorithm is an in-place comparison-based algorithm in which the list is divided into two parts, the sorted part at the left end and the unsorted part at the right end. Initially, the sorted part is empty and the unsorted part is the entire list.

The smallest element is selected from the unsorted array and swapped with the leftmost element, and that element becomes a part of the sorted array. This process continues moving unsorted array boundary by one element to the right.

This algorithm is not suitable for large data sets as its average and worst case complexities are of $O(n^2)$, where n is the number of items.

How Selection Sort Works?

Consider the following depicted array as an example.



For the first position in the sorted list, the whole list is scanned sequentially. The first position where 14 is stored presently, we search the whole list and find that 10 is the lowest value.



Data Structure and Algorithms Notes

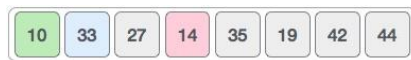
So we replace 14 with 10. After one iteration 10, which happens to be the minimum value in the list, appears in the first position of the sorted list.



For the second position, where 33 is residing, we start scanning the rest of the list in a linear manner.



We find that 14 is the second lowest value in the list and it should appear at the second place. We swap these values.



After two iterations, two least values are positioned at the beginning in a sorted manner.



The same process is applied to the rest of the items in the array.

Following is a pictorial depiction of the entire sorting process –



Now, let us learn some programming aspects of selection sort.

Algorithm

Step 1 – Set MIN to location 0
Step 2 – Search the minimum element in the list
Step 3 – Swap with value at location MIN
Step 4 – Increment MIN to point to next element
Step 5 – Repeat until list is sorted

Pseudocode

procedure selection sort

list : array of items

n : size of list

for i = 1 to n - 1

/* set current element as minimum*/

min = i

/* check the element to be minimum */

for j = i+1 to n

if list[j] < list[min] then

min = j;

end if

end for

/* swap the minimum element with the current element*/

if indexMin != i then

swap list[min] and list[i]

end if

end for

end procedure

Data Structures - Merge Sort Algorithm

Merge sort is a sorting technique based on divide and conquer technique. With worst-case time complexity being $O(n \log n)$, it is one of the most respected algorithms.

Merge sort first divides the array into equal halves and then combines them in a sorted manner.

How Merge Sort Works?

To understand merge sort, we take an unsorted array as the following –



We know that merge sort first divides the whole array iteratively into equal halves unless the atomic values are achieved. We see here that an array of 8 items is divided into two arrays of size 4.



This does not change the sequence of appearance of items in the original. Now we divide these two arrays into halves.



We further divide these arrays and we achieve atomic value which can no more be divided.



Now, we combine them in exactly the same manner as they were broken down. Please note the color codes given to these lists.

We first compare the element for each list and then combine them into another list in a sorted manner. We see that 14 and 33 are in sorted positions. We compare 27 and 10 and in the target list of 2 values we put 10 first, followed by 27. We change the order of 19 and 35 whereas 42 and 44 are placed sequentially.



In the next iteration of the combining phase, we compare lists of two data values, and merge them into a list of found data values placing all in a sorted order.



After the final merging, the list should look like this –



Now we should learn some programming aspects of merge sorting.

Algorithm

Merge sort keeps on dividing the list into equal halves until it can no more be divided. By definition, if it is only one element in the list, it is sorted. Then, merge sort combines the smaller sorted lists keeping the new list sorted too.

Step 1 – if it is only one element in the list it is already sorted, return.

Step 2 – divide the list recursively into two halves until it can no more be divided.

Step 3 – merge the smaller lists into new list in sorted order.

Pseudocode

We shall now see the pseudocodes for merge sort functions. As our algorithms point out two main functions – divide & merge.

Merge sort works with recursion and we shall see our implementation in the same way.

```
procedure mergesort( var a as array )
    if ( n == 1 ) return a

    var l1 as array = a[0] ... a[n/2]
    var l2 as array = a[n/2+1] ... a[n]

    l1 = mergesort( l1 )
    l2 = mergesort( l2 )

    return merge( l1, l2 )
end procedure
```

```
procedure merge( var a as array, var b as array )
```

```
    var c as array
```

```
    while ( a and b have elements )
```

```
        if ( a[0] > b[0] )
```

```
            add b[0] to the end of c
```

```
            remove b[0] from b
```

```
        else
```

```
            add a[0] to the end of c
```

```
            remove a[0] from a
```

```
        end if
```

```
    end while
```

```
    while ( a has elements )
```

```
        add a[0] to the end of c
```

```
        remove a[0] from a
```

```
    end while
```

```
    while ( b has elements )
```

```
        add b[0] to the end of c
```

```
        remove b[0] from b
```

```
    end while
```

```
    return c
```

```
end procedure
```

Data Structure and Algorithms - Shell Sort

Shell sort is a highly efficient sorting algorithm and is based on insertion sort algorithm. This algorithm avoids large shifts as in case of insertion sort, if the smaller value is to the far right and has to be moved to the far left.

This algorithm uses insertion sort on a widely spread elements, first to sort them and then sorts the less widely spaced elements. This spacing is termed as **interval**. This interval is calculated based on Knuth's formula as –

Knuth's Formula

$$h = h * 3 + 1$$

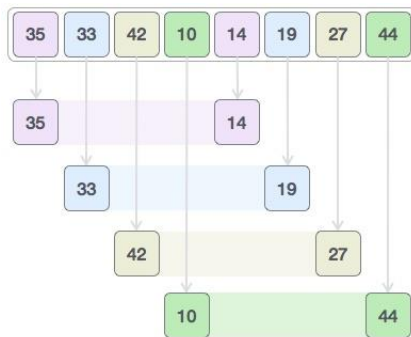
where –

h is interval with initial value 1

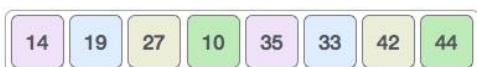
This algorithm is quite efficient for medium-sized data sets as its average and worst case complexity are of $O(n)$, where **n** is the number of items.

How Shell Sort Works?

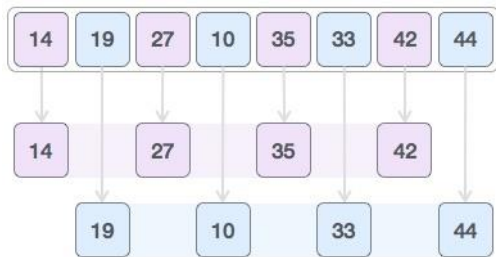
Let us consider the following example to have an idea of how shell sort works. We take the same array we have used in our previous examples. For our example and ease of understanding, we take the interval of 4. Make a virtual sub-list of all values located at the interval of 4 positions. Here these values are {35, 14}, {33, 19}, {42, 27} and {10, 44}



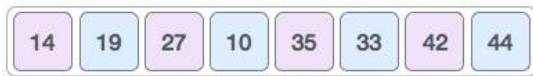
We compare values in each sub-list and swap them (if necessary) in the original array. After this step, the new array should look like this –



Then, we take interval of 2 and this gap generates two sub-lists - {14, 27, 35, 42}, {19, 10, 33, 44}

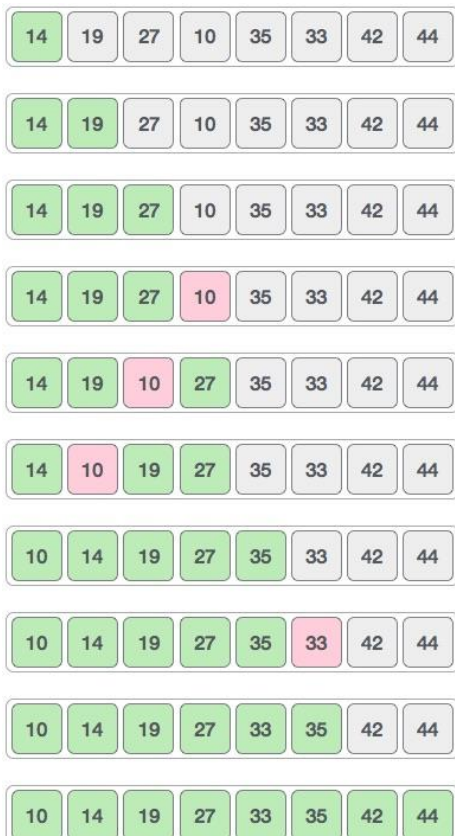


We compare and swap the values, if required, in the original array. After this step, the array should look like this –



Finally, we sort the rest of the array using interval of value 1. Shell sort uses insertion sort to sort the array.

Following is the step-by-step depiction –



We see that it required only four swaps to sort the rest of the array.

Algorithm

Following is the algorithm for shell sort.

Step 1 – Initialize the value of h
Step 2 – Divide the list into smaller sub-list of equal interval h
Step 3 – Sort these sub-lists using **insertion sort**
Step 3 – Repeat until complete list is sorted

Pseudocode

Following is the pseudocode for shell sort.

```
procedure shellSort()
    A : array of items

    /* calculate interval */
    while interval < A.length / 3 do:
        interval = interval * 3 + 1
    end while

    while interval > 0 do:

        for outer = interval; outer < A.length; outer ++ do:

            /* select value to be inserted */
            valueToInsert = A[outer]
            inner = outer;

            /* shift element towards right */
            while inner > interval - 1 && A[inner - interval] >= valueToInsert do:
                A[inner] = A[inner - interval]
                inner = inner - interval
```

```
end while

/* insert the number at hole position */
A[inner] = valueToInsert

end for

/* calculate interval*/
interval = (interval - 1) / 3;

end while

end procedure
```

Data Structure and Algorithms - Quick Sort

Quick sort is a highly efficient sorting algorithm and is based on partitioning of array of data into smaller arrays. A large array is partitioned into two arrays one of which holds values smaller than the specified value, say pivot, based on which the partition is made and another array holds values greater than the pivot value.

Quick sort partitions an array and then calls itself recursively twice to sort the two resulting subarrays. This algorithm is quite efficient for large-sized data sets as its average and worst case complexity are of $O(n^2)$, where **n** is the number of items.

Partition in Quick Sort

Following animated representation explains how to find the pivot value in an array.

Unsorted Array



The pivot value divides the list into two parts. And recursively, we find the pivot for each sub-lists until all lists contains only one element.

Quick Sort Pivot Algorithm

Based on our understanding of partitioning in quick sort, we will now try to write an algorithm for it, which is as follows.

```
Step 1 – Choose the highest index value has pivot
Step 2 – Take two variables to point left and right of the list excluding pivot
Step 3 – left points to the low index
Step 4 – right points to the high
Step 5 – while value at left is less than pivot move right
Step 6 – while value at right is greater than pivot move left
Step 7 – if both step 5 and step 6 does not match swap left and right
Step 8 – if  $\text{left} \geq \text{right}$ , the point where they met is new pivot
```

Quick Sort Pivot Pseudocode

The pseudocode for the above algorithm can be derived as –

```
function partitionFunc(left, right, pivot)
    leftPointer = left
    rightPointer = right - 1

    while True do
        while A[++leftPointer] < pivot do
            //do-nothing
        end while

        while rightPointer > 0 && A[--rightPointer] > pivot do
            //do-nothing
        end while

        if leftPointer >= rightPointer
            break
        else
```

```
        swap leftPointer,rightPointer
    end if

end while

swap leftPointer,right
return leftPointer

end function
```

Quick Sort Algorithm

Using pivot algorithm recursively, we end up with smaller possible partitions. Each partition is then processed for quick sort. We define recursive algorithm for quicksort as follows –

```
Step 1 – Make the right-most index value pivot
Step 2 – partition the array using pivot value
Step 3 – quicksort left partition recursively
Step 4 – quicksort right partition recursively
```

Quick Sort Pseudocode

To get more into it, let see the pseudocode for quick sort algorithm –

```
procedure quickSort(left, right)

    if right-left <= 0
        return
    else
        pivot = A[right]
        partition = partitionFunc(left, right, pivot)
        quickSort(left,partition-1)
        quickSort(partition+1,right)
    end if

end procedure
```

