LECTURE # 4
BICONDITIONAL

If p and q are statement variables, the biconditional of p and q is
"**p if, and only if, q**" and is denoted **p↔q**. *if and only if* abbreviated **iff.**
The double headed arrow " ↔" is the **biconditional operator.**
TRUTH TABLE FOR
p↔q.

p	q	p ↔q
T	T	T
T	F	F
F	T	F
F	F	T

EXAMPLES:
True or false?
1."**1+1 = 3** if and only if **earth is flat**"
TRUE
2. "**Sky is blue** iff **1 = 0**"
FALSE3. "**Milk is white** iff **birds lay eggs**"
TRUE
4. "**33 is divisible by 4** if and only if **horse has four legs**"
FALSE
5. "**$x > 5$ iff $x^2 > 25$**"
FALSE

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

p	q	p↔q	p→q	q→p	(p→q)∧(q→p)
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

same truth values

REPHRASING BICONDITIONAL:
p↔q is also expressed as:
"p is necessary and sufficient for q"

"if p then q, and conversely"
"p is equivalent to q"

EXERCISE:

Rephrase the following propositions in the form "p if and only if q" in English.

1.If it is hot outside you buy an ice cream cone, and if you buy an ice cream cone it is hot outside.

Sol You buy an ice cream cone if and only if it is hot outside.

2.For you to win the contest it is necessary and sufficient that you have the only winning ticket.

Sol You win the contest if and only if you hold the only winning ticket.

3.If you read the news paper every day, you will be informed and conversely.

Sol You will be informed if and only if you read the news paper every day.**4.It rains if it is a weekend day, and it is a weekend day if it rains.**

Sol It rains if and only if it is a weekend day.

5.The train runs late on exactly those days when I take it.

Sol The train runs late if and only if it is a day I take the train.

6.This number is divisible by 6 precisely when it is divisible by both 2 and 3.

Sol This number is divisible by 6 if and only if it is divisible by both 2 and 3.

TRUTH TABLE FOR
$(p\to q) \leftrightarrow (\sim q\to \sim p)$

p	q	$p\to q$	$\sim q$	$\sim p$	$\sim q\to \sim p$	$(p\to q) \leftrightarrow (\sim q\to \sim p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

TRUTH TABLE FOR
$(p\leftrightarrow q)\leftrightarrow(r\leftrightarrow q)$

p	q	r	$p\leftrightarrow q$	$r\leftrightarrow q$	$(p\leftrightarrow q)\leftrightarrow(r\leftrightarrow q)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	T
T	F	F	F	T	F
F	T	T	F	T	F
F	T	F	F	F	T
F	F	T	T	F	F
F	F	F	T	T	T

TRUTH TABLE FOR
$p \wedge \sim r \leftrightarrow q \vee r$
Here $p \wedge \sim r \leftrightarrow q \vee r$ means $(p \wedge (\sim r)) \leftrightarrow (q \vee r)$

p	q	r	~r	$p\wedge\sim r$	$q\vee r$	$p \wedge \sim r \leftrightarrow q\vee r$
T	T	T	F	F	T	F
T	T	F	T	T	T	T
T	F	T	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	T	F	T	F	T	F
F	F	T	F	F	T	F
F	F	F	T	F	F	T

LOGICAL EQUIVALENCE
INVOLVING BICONDITIONAL

Show that **$\sim p\leftrightarrow q$** and **$p\leftrightarrow\sim q$** are logically equivalent

p	q	~p	~q	$\sim p\leftrightarrow q$	$p\leftrightarrow\sim q$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

same truth values

<u>EXERCISE:</u>

Show that **$\sim(p\oplus q)$** and **$p\leftrightarrow q$** are logically equivalent

p	q	$p\oplus q$	$\sim(p\oplus q)$	$p\leftrightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

same truth values

LAWS OF LOGIC:

1.Commutative Law: $p \leftrightarrow q \equiv q \leftrightarrow p$

2.Implication Laws: $p \to q \equiv \sim p \vee q$

$\equiv \sim(p \wedge \sim q)$

3.Exportation Law: $(p \wedge q) \to r \equiv p \to (q \to r)$

4.Equivalence: $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

5.Reductio ad absurdum $p \to q \equiv (p \wedge \sim q) \to c$

APPLICATION:

Rewrite the statement forms without using the symbols $\to$ or $\leftrightarrow$

1.$p\wedge\sim q\to r$ **2. $(p\to r)\leftrightarrow(q \to r)$**

SOLUTION

1.$p\wedge\sim q\to r \equiv (p\wedge\sim q)\to r$ order of operations

$\equiv\sim(p\wedge\sim q) \vee r$ implication law

2.$(p\to r)\leftrightarrow(q \to r) \equiv (\sim p \vee r)\leftrightarrow(\sim q \vee r)$ implication law

$\equiv [(\sim p \vee r) \to(\sim q \vee r)] \wedge [(\sim q \vee r) \to(\sim p \vee r)]$

equivalence of biconditional

$\equiv [\sim(\sim p \vee r) \vee (\sim q \vee r)] \wedge [\sim(\sim q \vee r) \vee (\sim p \vee r)]$

implication law

Rewrite the statement form **$\sim p \vee q \to r \vee \sim q$** to a logically equivalent form that uses only **~** and **$\wedge$**

SOLUTION

STATEMENT	**REASON**
$\sim p \vee q \to r \vee \sim q$	Given statement form
$\equiv (\sim p \vee q) \to (r \vee \sim q)$	Order of operations
$\equiv \sim[(\sim p \vee q) \wedge \sim (r \vee \sim q)]$	Implication law $p\to q \equiv \sim(p\wedge\sim q)$
$\equiv \sim[\sim(p \wedge \sim q) \wedge (\sim r \wedge q)]$	De Morgan's law

Show that **$\sim(p\to q) \to p$** is a tautology without using truth tables.

SOLUTION

STATEMENT	**REASON**
$\sim(p\to q) \to p$	Given statement form
$\equiv \sim[\sim(p \wedge \sim q)] \to p$	Implication law $p\to q \equiv \sim(p \wedge \sim q)$
$\equiv (p \wedge \sim q) \to p$	Double negation law
$\equiv \sim(p \wedge \sim q) \vee p$	Implication law $p\to q \equiv \sim p \vee q$
$\equiv (\sim p \vee q) \vee p$	De Morgan's law

$\equiv (q \vee \sim p) \vee p$	Commutative law of
$\vee \equiv q \vee (\sim p \vee p)$	Associative law of
$\vee \equiv q \vee t$	Negation law
$\equiv t$	Universal bound law

EXERCISE:

Suppose that p and q are statements so that $p \rightarrow q$ is false. Find the truth values of each of the following:

1.$\sim p \rightarrow q$

2.$p \vee q$

3.$q \leftrightarrow p$

SOLUTION

1.TRUE

2.TRUE

3.FALSE