Chapter 3
Gate-Level Minimization

Karnaugh Maps

POS Simplification

Don't Care Conditions

NAND and NOR

Exclusive OR

Four-Variable Map

- **A** four-variable map **holds 16 minterms for four variables.**
  - **Again, we mark the squares of the minterms that belong to a given function.**
  - **Note that the sequence is not arranged in a binary way.**
  - **The sequence used is a Gray code and allows only one bit to change from column to column and row to row.**

wx \ yz	00	01	11	10
00	w'x'y'z'	w'x'y'z	w'x'yz	w'x'yz'
01	w'xy'z'	w'xy'z	w'xyz	w'xyz'
11	wxy'z'	wxy'z	wxyz	wxyz'
10	wx'y'z'	wx'y'z	wx'yz	wx'yz'

wx \ yz	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

4-Variable Map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

(a)

$wx \backslash yz$	00	01	11	10
00	$w'x'y'z'$	$w'x'y'z$	$w'x'yz$	$w'x'yz'$
01	$w'xy'z'$	$w'xy'z$	$w'xyz$	$w'xyz'$
11	$wxy'z'$	$wxy'z$	$wxyz$	$wxyz'$
10	$wx'y'z'$	$wx'y'z$	$wx'yz$	$wx'yz'$

(b)

Fig. 3-8 Four-variable Map

4-Variable Map Patterns

- **The number of adjacent squares that may be combined always represent a number that is a power of 2 such as 1, 2, 4, 8, and 16.**
  - **One square represents one minterm with four literals.**
  - **Two adjacent squares represents a term of three literals.**
  - **Four adjacent squares represents a term of two literals.**
  - **Eight adjacent squares represents a term of one literal.**
  - **Sixteen adjacent squares represents the entire map and produces a function that is always equal to 1.**

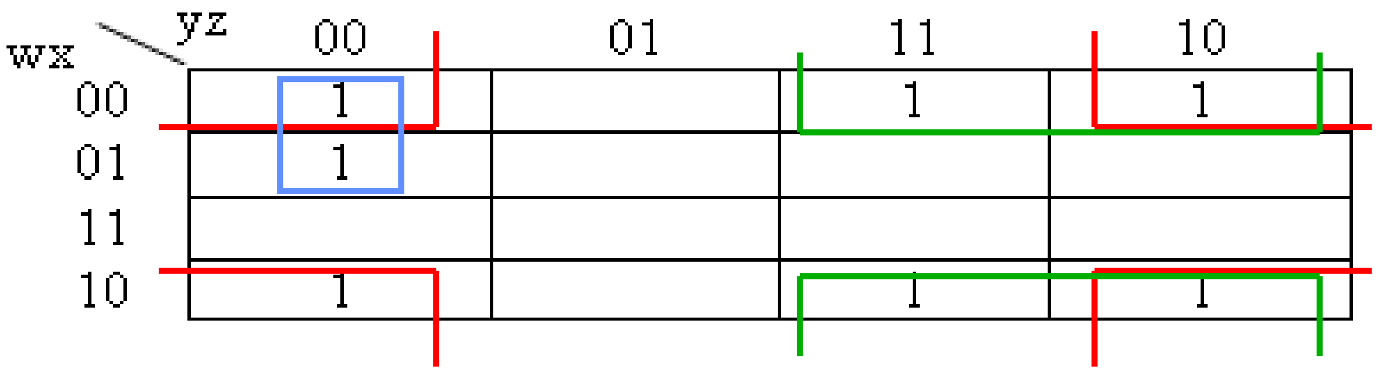
Minimization Example

wx \ yz	00	01	11	10
00			1	1
01			1	1
11	1		1	1
10	1		1	1

$$F(w, x, y, z) = \sum(2, 3, 6, 7, 8, 10, 11, 12, 14, 15)$$

- **The eight adjacent squares can be combined to form the one literal term y.**
- **Four adjacent squares can be combined to form the two literal term wz'.**
- **F = y+ wz'**

Another Example



$$F(w, x, y, z) = \sum(0, 2, 3, 4, 8, 10, 11)$$

- **Four adjacent corners can be combined to form the two literal term x’z’.**
- **Four adjacent squares can be combined to form the two literal term x’y.**
- **The remaining 1 is combined with a single adjacent 1 to obtain the three literal term w’y’z’.**
- **F = x’z’ + x’y + w’y’z’**

Another Example

- $F = A'BC' + A'CD' + ABC + AB'C'D' + ABC' + AB'C$

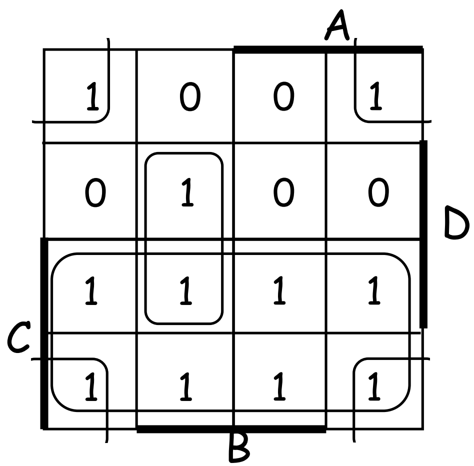
AB \ CD	00	01	11	10
00	0	0	0	1
01	1	1	0	1
11	1	1	1	1
10	1	0	1	1

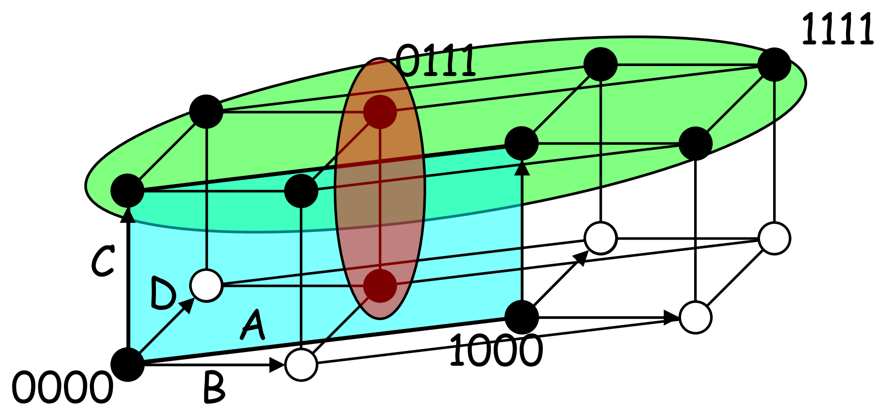
$F = BC' + CD' + AC + AD'$

Another Example

- **F(A,B,C,D) = Σm(0,2,5,8,9,10,11,12,13,14,15)**
  - **F =** C + A'BD + B'D'

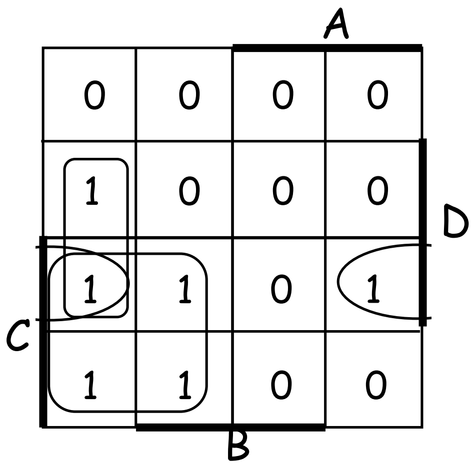
C + A'BD + B'D'



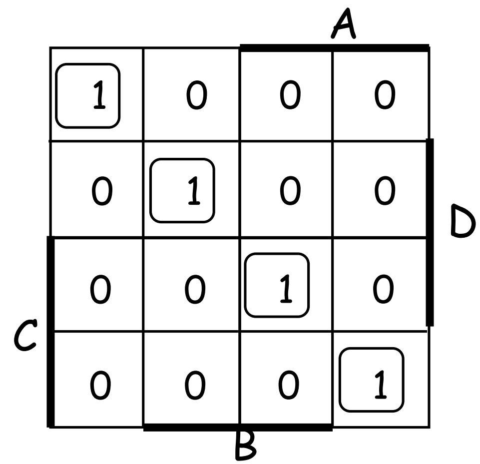


Solution set can be considered as a coordinate System!

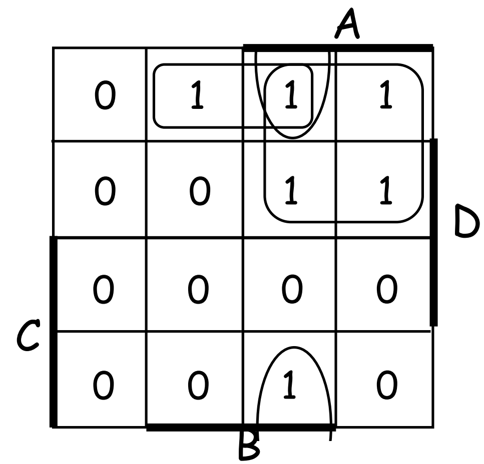
Another Example



K-map for LT



K-map for EQ



K-map for GT

$LT = A'B'D + A'C + B'CD$

$EQ = A'B'C'D' + A'BC'D + ABCD + AB'CD'$

$GT = BC'D' + AC' + ABD'$

Example 3-5

- $F(w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$
- Sol:
  - 1 is marked in each minterm that represents the function
  - Find the possible adjacent squares and mark them with rectangles
  - We combine eight adjacent squares to get a single literal term y’
  - The top two 1’s on the right are combined with the top two 1,son the left to give the term w’z’
  - We combine the single square left on right with three adjecent squares that are already used to give the term xz’
  - The logical sum of these three terms gives:

    F = y’ + w’z’+xz’

wx \ yz	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

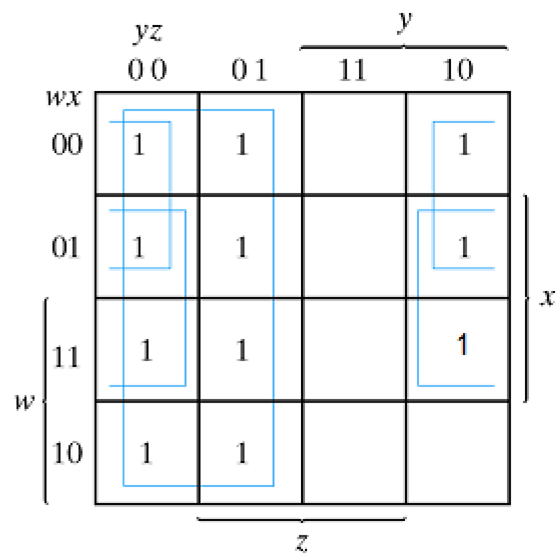


Fig. 3-9 Map for Example 3-5; $F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14) = y' + w'z' + xz'$

Example 3-6

- – F = A'B'C'+B'CD'+A'BCD'+AB'C'

- Sol:
  - – Each of three literal term in map is represented by two squares and four literal term in map is represented by one square
  - – We combine the 1's in the four corners to give the term B'D'
  - – The two left hand 1's in the top row are combined with two 1's in the bottom row to give the term B'C'
  - – The remaining 1's may be combined in the two-square area to give the term A'CD'
  - – The logical sum of these three terms gives:

    F = B'D' + B'C'+ A'CD'

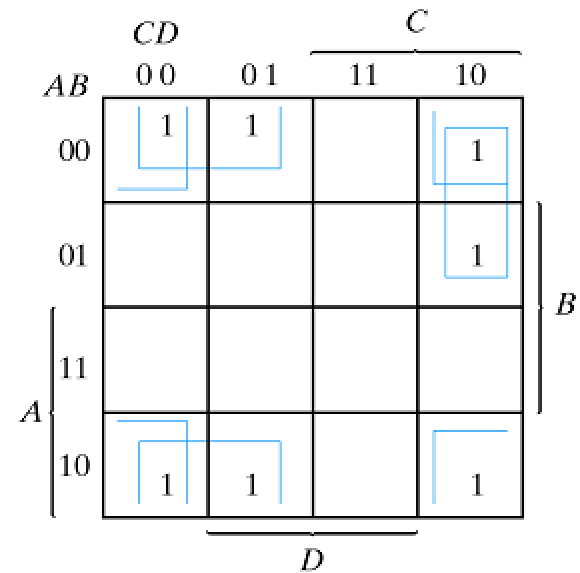


Fig.3-10 Map for Example 3-6; $A'B'C' + B'CD' + A'BCD' + AB'C' = B'D' + B'C' + A'CD'$

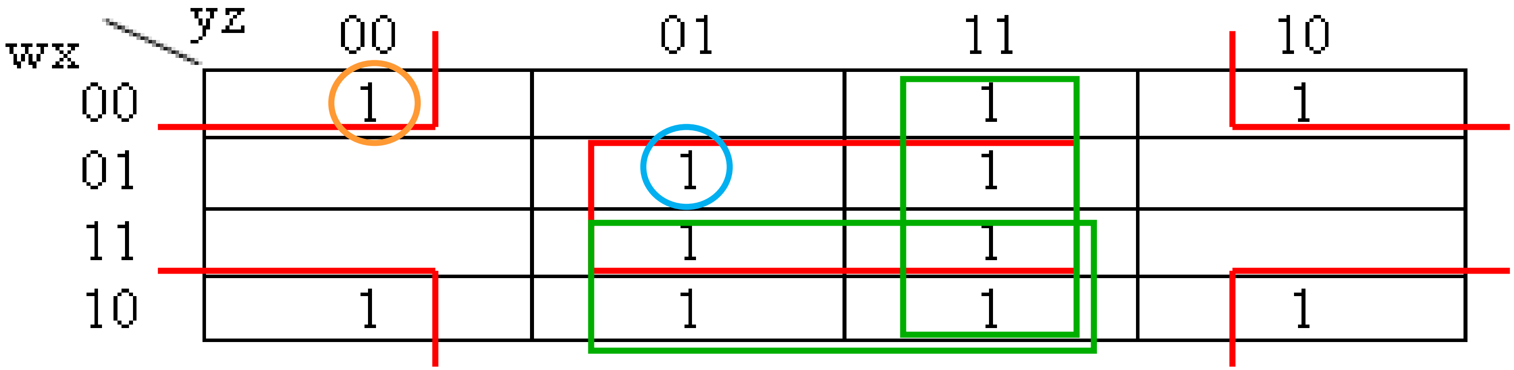
Prime Implicants

- **A** prime implicant **is a product term obtained by combining the maximum possible number of adjacent squares in the map.**
  - **A single 1 on a map represents a prime implicant if it is not adjacent to any other 1.**
  - **Two adjacent 1's form a prime implicant, provided they are not within a group of four adjacent squares.**
  - **Four adjacent 1's form a prime implicant if they are not within a group of eight adjacent squares, and so on.**
- **If a minterm in a square is covered by only one prime implicant, that prime implicant is said to be** essential**.**
  - **They are found by looking at each square marked with a 1 and checking the number of prime implicants that cover it. Those with only one prime implicant are essential.**

Finding Simplified Expressions

- The procedure for finding simplified expressions is
  - determine all essential prime implicants first
  - determine the expression from the logical sum of the essential prime implicants with other prime implicants needed to cover the remaining minterms
- There may be more than one simplified expression.

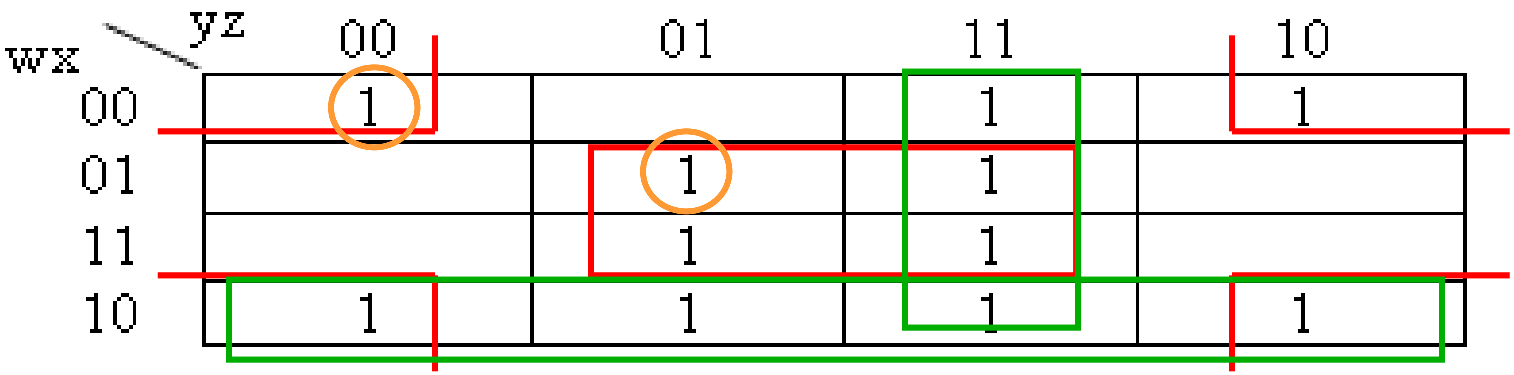
Example of Prime Implicants



$$F(w, x, y, z) = \sum(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$$

- **Two essential prime implicants (caused by m_0 and m_5)**
  - **This gives us two terms: x'z' and xz**
- **Finding prime implicants for the remainders results in expression:**
  - **F = xz + x'z' + yz + wz**

Example of Prime Implicants



$$F(w, x, y, z) = \sum(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$$

- **Two essential prime implicants (caused by m_0 and m_5)**
  - **This gives us two terms: x'z' and xz**
- **Finding prime implicants for the remainders results in this expression:**
  - **F = xz + x'z' + yz + wx'**

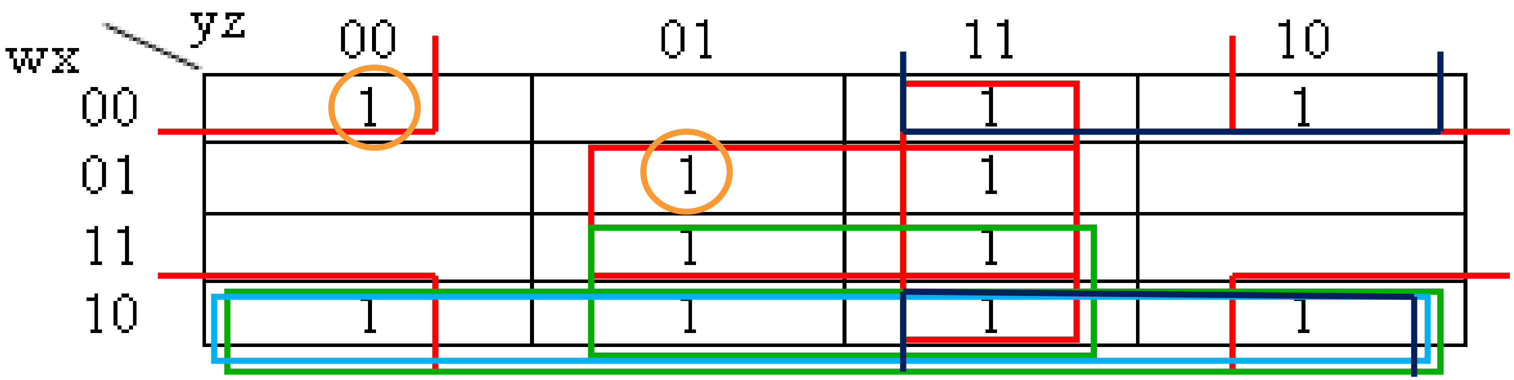
Example of Prime Implicants

wx \ yz	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

$$F(w, x, y, z) = \sum(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$$

- **Two essential prime implicants (caused by m_0 and m_5)**
  - **This gives us two terms: x'z' and xz**
- **Finding prime implicants for the remainders results in this expression:**
  - **$F = xz + x'z' + x'y + wz$**

Example of Prime Implicants



$$F(w, x, y, z) = \sum(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$$

- **Two essential prime implicants (caused by m_0 and m_5)**
  - **This gives us two terms: x'z' and xz**
- **Finding prime implicants for the remainders results in four expressions:**
  - **F = xz + x'z' + yz + wz**
  - **F = xz + x'z' + yz + wx'**
  - **F = xz + x'z' + x'y + wz**
  - **F = xz + x'z' + x'y + wx'**