CURVE AND SURFACE DESIGN AND ANIMATION

In computer graphics, we often need to draw different types of objects onto the screen. Objects are not flat all the time and we need to draw curves many times to draw an object.

Types of Curves

A curve is an infinitely large set of points. Each point has two neighbors except endpoints. Curves can be broadly classified into three categories explicit, implicit, and parametric curves.

Implicit Curves

Implicit curve representations define the set of points on a curve by employing a procedure that can test to see if a point in on the curve. Usually, an implicit curve is defined by an implicit function of the form −

$$f(x, y) = 0$$

It can represent multivalued curves (multiple y values for an x value). A common example is the circle, whose implicit representation is

$$x^2 + y^2 - R^2 = 0$$

Explicit Curves

A mathematical function $y = f(x)$ can be plotted as a curve. Such a function is the explicit representation of the curve. The explicit representation is not general, since it cannot represent vertical lines and is also single-valued. For each value of x, only a single value of y is normally computed by the function.

Parametric Curves

Curves having parametric form are called parametric curves. The explicit and implicit curve representations can be used only when the function is known. In practice the parametric curves are used. A two-dimensional parametric curve has the following form:

$$P(t) = f(t), g(t) \text{ or } P(t) = x(t), y(t)$$

The functions f and g become the (x, y) coordinates of any point on the curve, and the points are obtained when the parameter t is varied over a certain interval [a, b], normally [0, 1].

Bezier Curves

Bezier curve is discovered by the French engineer Pierre Bézier. These curves can be generated under the control of other points. Approximate tangents by using control points are used to generate curve. The Bezier curve can be represented mathematically as:

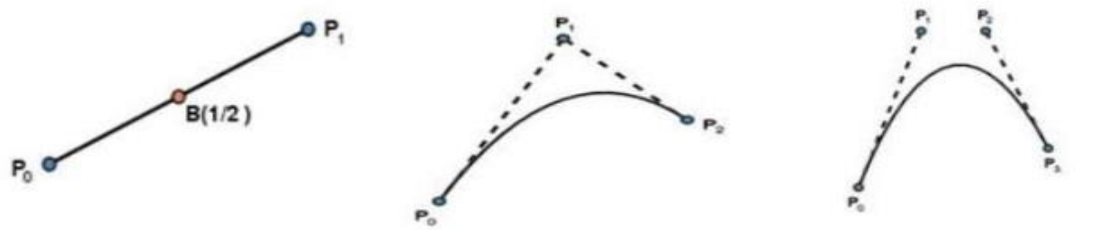
$$\sum_{k=0}^{n} P_i B_i^n(t)$$

Where p_i is the set of points and $B_i^n(t)$ represents the Bernstein polynomials which are given by −

$$B_i^n(t) = \binom{n}{i}(1-t)^{n-i}t^i$$

Where **n** is the polynomial degree, **i** is the index, and **t** is the variable.

The simplest Bézier curve is the straight line from the point P_0 to P_1. A quadratic Bezier curve is determined by three control points. A cubic Bezier curve is determined by four control points.



Simple Bezier Curve Quadratic Bazier Curve Cubic Bazier Curve

Properties of Bezier Curves

Bezier curves have the following properties:

- They generally follow the shape of the control polygon, which consists of the segments joining the control points.
- They always pass through the first and last control points.
- They are contained in the convex hull of their defining control points.
- The degree of the polynomial defining the curve segment is one less that the number of defining polygon point. Therefore, for 4 control points, the degree of the polynomial is 3, i.e. cubic polynomial.
- A Bezier curve generally follows the shape of the defining polygon.
- The direction of the tangent vector at the end points is same as that of the vector determined by first and last segments.
- The convex hull property for a Bezier curve ensures that the polynomial smoothly follows the control points.
- No straight line intersects a Bezier curve more times than it intersects its control polygon.
- They are invariant under an affine transformation.
- Bezier curves exhibit global control means moving a control point alters the shape of the whole curve.
- A given Bezier curve can be subdivided at a point t=t0 into two Bezier segments which join together at the point corresponding to the parameter value t=t0.

B-Spline Curves

The Bezier-curve produced by the Bernstein basis function has limited flexibility.

- First, the number of specified polygon vertices fixes the order of the resulting polynomial which defines the curve.
- The second limiting characteristic is that the value of the blending function is nonzero for all parameter values over the entire curve.

The B-spline basis contains the Bernstein basis as the special case. The B-spline basis is non-global.

A B-spline curve is defined as a linear combination of control points Pi and B-spline basis function N i, Ni, k (t) given by

$$C(t) = \sum_{i=0}^{n} P_i N_{i,k}(t), \qquad n \geq k-1,$$
$$t \,\epsilon\, [tk-1, tn+1]$$

Where,

- {p i pi: i=0, 1, 2….n} are the control points
- k is the order of the polynomial segments of the B-spline curve. Order k means that the curve is made up of piecewise polynomial segments of degree k - 1,
- the N i,k (t) Ni,k(t) are the “normalized B-spline blending functions”. They are described by the order k and by a non-decreasing sequence of real numbers normally called the “knot sequence”.

$$t_i : i = 0, \ldots n + K$$

The N_i, k functions are described as follows −

$$N_{i,1}(t) = \begin{cases} 1, & if\, u \,\epsilon\, [t_i, t_{i+1}) \\ 0, & Otherwise \end{cases}$$

and if k > 1,

$$N_{i,k}(t) = \frac{t - t_i}{t_{i+k-1}} N_{i,k-1}(t) + \frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} N_{i+1,k-1}(t)$$

and

$$t \,\epsilon\, [t_{k-1}, t_{n+1})$$

Properties of B-spline Curve

B-spline curves have the following properties −

- The sum of the B-spline basis functions for any parameter value is 1.
- Each basis function is positive or zero for all parameter values.
- Each basis function has precisely one maximum value, except for k=1.

- The maximum order of the curve is equal to the number of vertices of defining polygon.
- The degree of B-spline polynomial is independent on the number of vertices of defining polygon.
- B-spline allows the local control over the curve surface because each vertex affects the shape of a curve only over a range of parameter values where its associated basis function is nonzero.
- The curve exhibits the variation diminishing property.
- The curve generally follows the shape of defining polygon.
- Any affine transformation can be applied to the curve by applying it to the vertices of defining polygon.
- The curve line within the convex hull of its defining polygon.

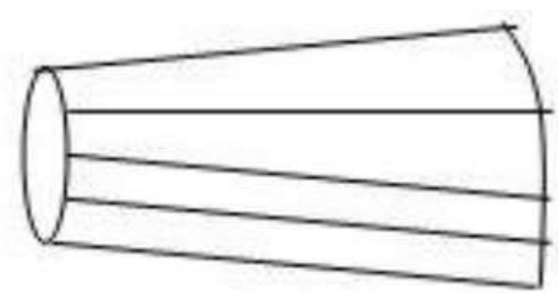
Polygon Surfaces

Objects are represented as a collection of surfaces. 3D object representation is divided into two categories.

- Boundary Representations (B-reps) − It describes a 3D object as a set of surfaces that separates the object interior from the environment.
- Space–partitioning representations − It is used to describe interior properties, by partitioning the spatial region containing an object into a set of small, non-overlapping, contiguous solids (usually cubes).

The most commonly used boundary representation for a 3D graphics object is a set of surface polygons that enclose the object interior. Many graphics system use this method. Set of polygons are stored for object description. This simplifies and speeds up the surface rendering and display of object since all surfaces can be described with linear equations.

The polygon surfaces are common in design and solid-modeling applications, since their **wireframe display** can be done quickly to give general indication of surface structure. Then realistic scenes are produced by interpolating shading patterns across polygon surface to illuminate.



A 3D object represented by polygons

Advantages

- It can be used to model almost any object.
- They are easy to represent as a collection of vertices.
- They are easy to transform.
- They are easy to draw on computer screen.

Disadvantages

- Curved surfaces can only be approximately described.
- It is difficult to simulate some type of objects like hair or liquid.

- Animation means giving life to any object in computer graphics. It has the power of injecting energy and emotions into the most seemingly inanimate objects. Computer-assisted animation and computer-generated animation are two categories of computer animation. It can be presented via film or video.
- The basic idea behind animation is to play back the recorded images at the rates fast enough to fool the human eye into interpreting them as continuous motion. Animation can make a series of dead images come alive. Animation can be used in many areas like entertainment, computer aided-design, scientific visualization, training, education, e-commerce, and computer art.

Animation Techniques

Animators have invented and used a variety of different animation techniques. Basically there are six animation techniques which we would discuss one by one in this section.

Traditional Animation (frame by frame)

Traditionally most of the animation was done by hand. All the frames in an animation had to be drawn by hand. Since each second of animation requires 24 frames (film), the amount of efforts required to create even the shortest of movies can be tremendous.

Keyframing

In this technique, a storyboard is laid out and then the artists draw the major frames of the animation. Major frames are the ones in which prominent changes take place. They are the key points of animation. Keyframing requires that the animator specifies critical or key positions for the objects. The computer then automatically fills in the missing frames by smoothly interpolating between those positions.

Procedural

In a procedural animation, the objects are animated by a procedure a set of rules not by keyframing. The animator specifies rules and initial conditions and runs simulation. Rules are often based on physical rules of the real world expressed by mathematical equations.

Behavioral

In behavioral animation, an autonomous character determines its own actions, at least to a certain extent. This gives the character some ability to improvise, and frees the animator from the need to specify each detail of every character's motion.

Performance Based (Motion Capture)

Another technique is Motion Capture, in which magnetic or vision-based sensors record the actions of a human or animal object in three dimensions. A computer then uses these data to animate the object.

This technology has enabled a number of famous athletes to supply the actions for characters in sports video games. Motion capture is pretty popular with the animators mainly because some of the commonplace human actions can be captured with relative ease. However, there can be serious discrepancies between the shapes or dimensions of the subject and the graphical character and this may lead to problems of exact execution.

Physically Based (Dynamics)

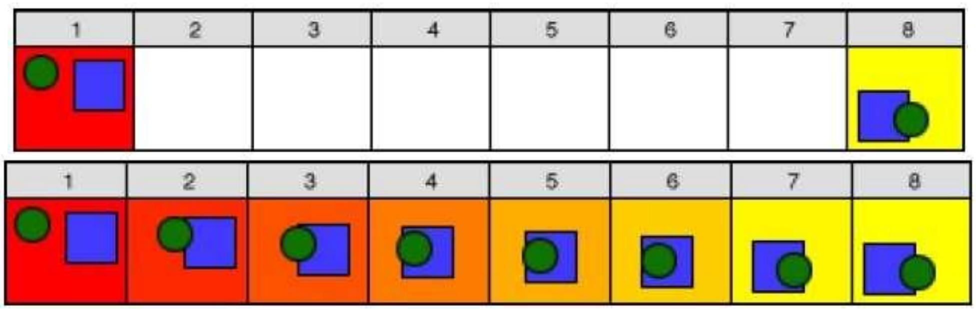
Unlike key framing and motion picture, simulation uses the laws of physics to generate motion of pictures and other objects. Simulations can be easily used to produce slightly different sequences while maintaining physical realism. Secondly, real-time simulations allow a higher degree of interactivity where the real person can maneuver the actions of the simulated character.

In contrast the applications based on key-framing and motion select and modify motions form a pre-computed library of motions. One drawback that simulation suffers from is the expertise and time required to handcraft the appropriate controls systems.

Key Framing

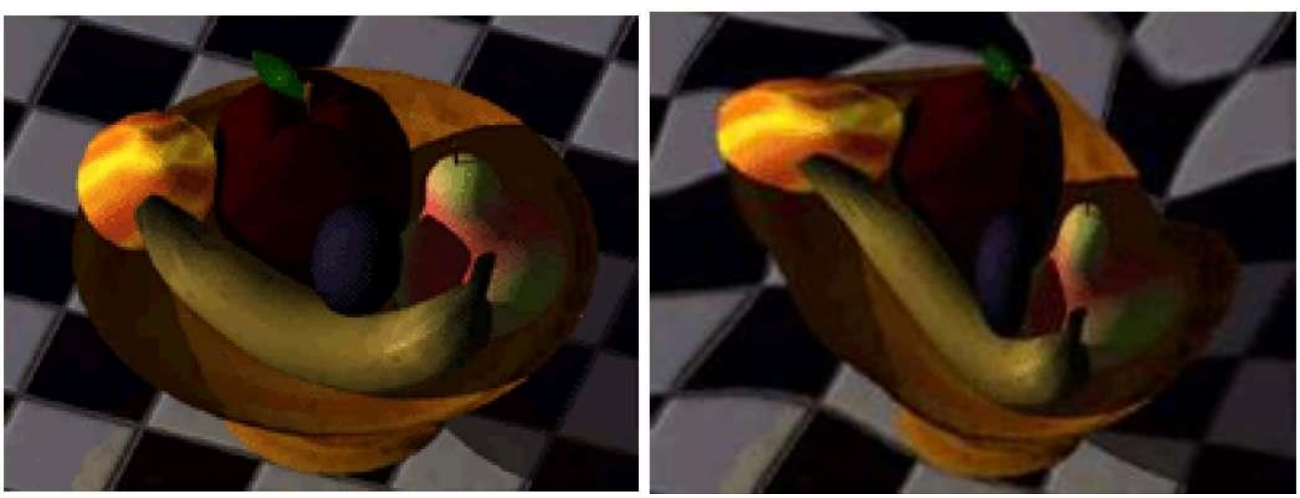
A keyframe is a frame where we define changes in animation. Every frame is a keyframe when we create frame by frame animation. When someone creates a 3D animation on a computer, they usually don’t specify the exact position of any given object on every single frame. They create keyframes.

Keyframe are important frames during which an object changes its size, direction, shape or other properties. The computer then figures out all the in-between frames and saves an extreme amount of time for the animator. The following illustrations depict the frames drawn by user and the frames generated by computer.



Morphing

The transformation of object shapes from one form to another form is called morphing. It is one of the most complicated transformations.



A morph looks as if two images melt into each other with a very fluid motion. In technical terms, two images are distorted and a fade occurs between them.