Chapter 2
Boolean Algebra and Logic Gates

Definitions

Theorems

Functions

Canonical and Standard Forms

Operations

Gates

Integrated Circuits

Canonical Forms

- A canonical form is a standard method for representing Boolean functions.
- The two canonical forms that are used are:
  - Sum of Minterms
  - Product of Maxterms
- These forms are sometimes considered the “brute force” method of representing functions as they seldom represent a function in a minimized form.

Minterms

- **Any given binary variable can be represented in two forms:**
  - **x, its normal form, and**
  - **x', its complement**
- **If we consider two binary variables and the AND operation, there are four combinations of the variables:**
  - **xy**
  - **xy'**
  - **x'y**
  - **x'y'**
- **Each of the above four AND terms is called a minterm or a standard product.**
- **n variables can be combined to form 2^n minterms.**

Minterms Expressed

				Minterms	
x	**y**	**z**		**Term**	**Designation**
0	0	0		$x'y'z'$	m_0
0	0	1		$x'y'z$	m_1
0	1	0		$x'yz'$	m_2
0	1	1		$x'yz$	m_3
1	0	0		$xy'z'$	m_4
1	0	1		$x'yz'$	m_5
1	1	0		xyz'	m_6
1	1	1		xyz	m_7

Maxterms

- **Any given binary variable can be represented in two forms:**
  - **x, its normal form, and**
  - **x', its complement**
- **If we consider two binary variables and the OR operation, there are four combinations of the variables:**
  - **x + y**
  - **x + y'**
  - **x' + y**
  - **x' + y'**
- **Each of the above four OR terms is called a maxterm or a standard sum.**
- **n variables can be combined to form 2^n maxterms.**
- Each maxterm is the complement of its corresponding minterm and vice-versa.

Maxterms Expressed

				Maxterms	
x	**y**	**z**		**Term**	**Designation**
0	0	0		$x + y + z$	M_0
0	0	1		$x + y + z'$	M_1
0	1	0		$x + y' + z$	M_2
0	1	1		$x + y' + z'$	M_3
1	0	0		$x' + y + z$	M_4
1	0	1		$x' + y + z'$	M_5
1	1	0		$x' + y' + z$	M_6
1	1	1		$x' + y' + z'$	M_7

Truth Table to Expression (Sum of Minterms)

- Any Boolean function can be expressed as a sum of minterms or sum of products (i.e. the ORing of terms).
  - You can form the function algebraically by forming a minterm for each combination of the variables that produces a 1 in the function. (Each row with output of 1 becomes a product term)
  - Sum (OR) product terms together.

x	y	z	G
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

xyz + xyz’ + x’yz

Sum of Minterms Example

x	y	z		Function F_1	Required Minterms
0	0	0		1	x'y'z'
0	0	1		0	
0	1	0		0	
0	1	1		1	x'yz
1	0	0		1	xy'z'
1	0	1		0	
1	1	0		0	
1	1	1		0	

$$F_1 = x'y'z' + x'yz + xy'z'$$

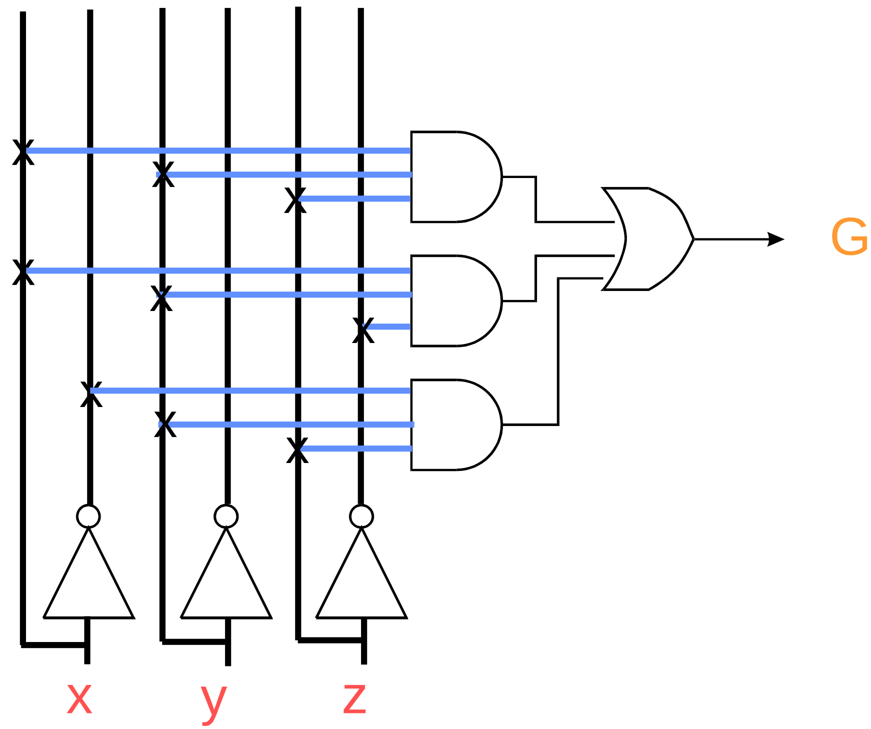
$$= m_0+m_3+m_4$$

$$= \sum(0,3,4)$$

Equivalent Representations of Circuits

- **All three formats are equivalent**
- **Number of 1's in truth table output column equals AND terms for Sum-of-Products (SOP)**

x	y	z	G
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1



G = xyz + xyz' + x'yz

Truth Table to Expression (Product of Maxterms)

- **Any Boolean function can be expressed as a product of maxterms or product of sums (i.e. the ANDing of terms).**
  - **You can form the function algebraically by forming a maxterm for each combination of the variables that produces a 0 in the function. (Each row with output of 0 becomes a standard sums)**
  - **AND these maxterms together.**

Product of Maxterms Example

x	y	z		Function F_1	Required Maxterms
0	0	0		1	
0	0	1		0	$x + y + z'$
0	1	0		0	$x + y' + z$
0	1	1		1	
1	0	0		1	
1	0	1		0	$x' + y + z'$
1	1	0		0	$x' + y' + z$
1	1	1		0	$x' + y' + z'$

$$F_1 = (x + y + z')(x + y' + z)(x' + y + z')(x' + y' + z)(x' + y' + z')$$

$$= M_1 M_2 M_5 M_6 M_7$$

$$= \prod(1,2, 5, 6, 7)$$

Minterms and Maxterms

- Each variable in a Boolean expression is a literal
- Boolean variables can appear in normal (x) or complement form (x')
- Each AND combination of terms is a minterm
- Each OR combination of terms is a maxterm
- Example:

Minterms

x	y	z	Minterm	
0	0	0	x'y'z'	m_0
0	0	1	x'y'z	m_1
			…	
1	0	0	xy'z'	m_4
			…	
1	1	1	xyz	m_7

Maxterms

x	y	z	Maxterm	
0	0	0	x+y+z	M_0
0	0	1	x+y+z'	M_1
			…	
1	0	0	x'+y+z	M_4
			…	
1	1	1	x'+y'+z'	M_7

Obtaining Sum of Minterms Form

A	B	C	F = A'B + B' + C	Required Minterms	Required Designations
0	0	0	1	A'B'C'	m_0
0	0	1	1	A'B'C	m_1
0	1	0	1	A'BC'	m_2
0	1	1	1	A'BC	m_3
1	0	0	1	AB'C'	m_4
1	0	1	1	AB'C	m_5
1	1	0	0		
1	1	1	1	ABC	m_7

F = A'B'C' + A'B'C + A'BC' + A'BC + AB'C' + AB'C+ ABC

$= m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_7$

$F(A, B, C) = \sum(0, 1, 2, 3, 4, 5, 7)$

Obtaining Product of Maxterms

A	B	C		F = A'B + B'C	Required Maxterms	Required Designations
0	0	0		0	A + B + C	M_0
0	0	1		1		
0	1	0		1		
0	1	1		1		
1	0	0		0	A' + B + C	M_4
1	0	1		1		
1	1	0		0	A' + B' + C	M_6
1	1	1		0	A' + B' + C'	M_7

$$F = (A + B + C)(A' + B + C)(A' + B' + C)(A' + B' + C')$$

$$= M_0 \cdot M_4 \cdot M_6 \cdot M_7$$

$$F(A, B, C) = \prod(0, 4, 6, 7)$$

Canonical Form Conversion

- A function represented as Sum of minterms can be represented as the Product of maxterms of the remaining terms.
- The complement of a function expressed in sum of minterms equals the sum of minterms missing from the original function
  - $F(A, B, C) = \sum(0, 3,4) = m_0+m_3+m_4$
  - $F'(A, B, C) = \sum(1,2,5,6,7)= m_1+m_2+m_5+m_6+m_7$
- Now if we take the complement of F' using DeMorgan's theorem, we obtain F in the product of maxterms form:
  - $(F')' = (m_1+m_2+m_5+m_6+m_7)'$
  - $F = m_1' . m_2' . m_5' . m_6' . m_7'$ [Complement of minterms]
  - $= M_1M_2M_5M_6M_7$ [maxterms]
  - $= \prod(1,2, 5, 6, 7)$
- This implies the following relation:
  - $m'j = Mj$
- So sum of minterms: $\sum(0,3,4)$ = product of maxterms: $\prod(1,2, 5, 6, 7)$