Online and out-of-core learning

an FTRL perspective

Kristian Holsheimer | 2018-03-27

Booking.com

Q What to do when your dataset doesn't fit in memory?

… especially if you don't have access to a large data cluster.

What?

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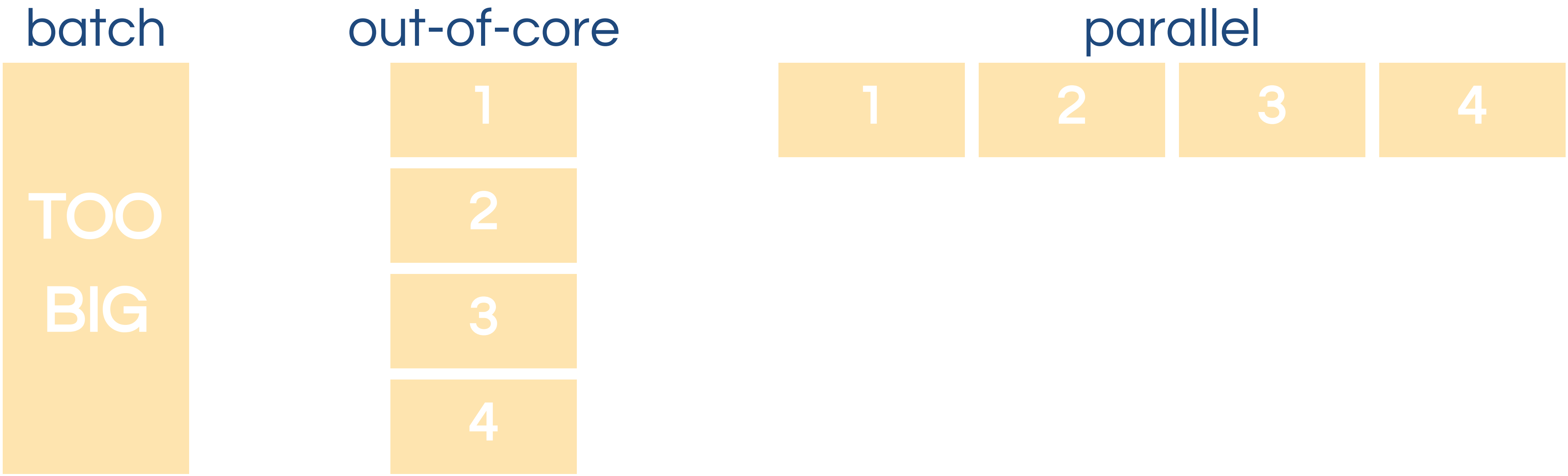
A Train on smaller chunks and then combine models / parameters



What?

Q **What to do when your dataset doesn’t fit in memory?**
A Train on smaller chunks and then combine models / parameters

Parallel approach

- appears superior
- received a lot of attention
- needs a cluster of many machines (and infrastructure etc)
- feasible for large companies

Out-of-core approach this talk

- appears inferior
- hasn’t received the attention it deserves
- runs on a single machine
- fits small startups *and* large companies

Questions.

What is **out-of-core** learning?

What is **online** learning?

How do we apply it to ordinary **batch** learning?

Why would we expect this to work?

Can we do **better** than the naive approach?

Online-to-batch learning spotted **in the wild**?
→ *Vowpal Wabbit*

Program.

Out-of-core learning

Online learning

Online-to-batch conversion - naive

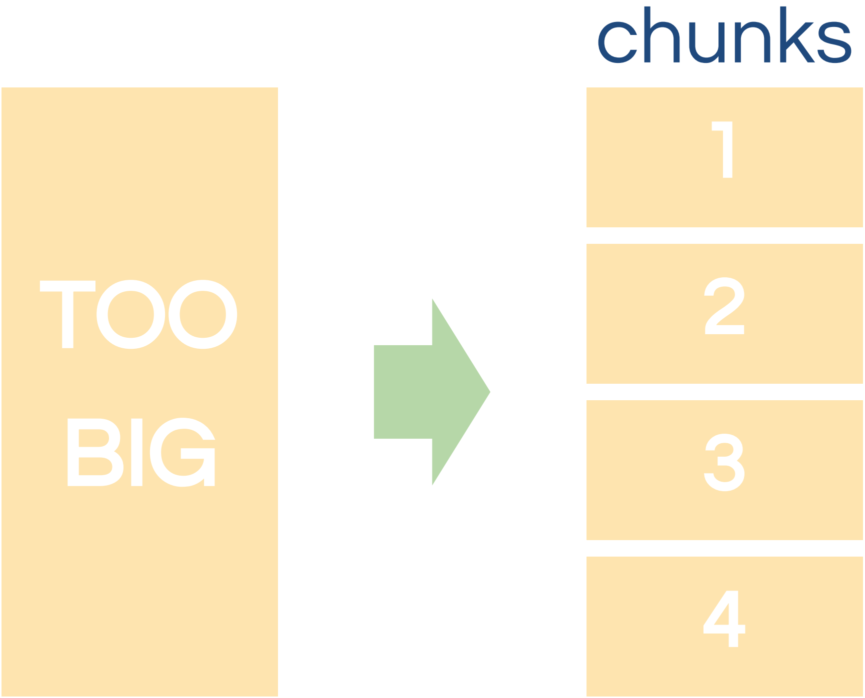
FTRL - Follow The (Regularized) Leader

Online-to-batch conversion - a closer look

Business applications at Booking.com

Out-of-core learning

Out-of-core learning.



pseudo code

```
for x, y in chunks:
    model.update(x, y)
```

Out-of-core learning.

Dataset

Suppose we have a fixed distribution:

$$\mathcal{D} = X \times Y$$

Training data consists of i.i.d. samples from D:

$$(x, y) \sim \mathcal{D}$$

Therefore, training data set is an unordered sequence:

$$\mathcal{D}_{\text{train}} = (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$$

Out-of-core learning.

Objective

Minimize expected loss:

$$L(\theta) = \mathop{\mathbb{E}}_{x,y \sim \mathcal{D}} \ell(x, y, \theta)$$

where l is the **single-datapoint loss**, e.g. for linear regression:

$$\ell(x, y, \theta) = \frac{1}{2}(\theta \cdot x - y)^2$$

Out-of-core learning.

Gradient descent

$$\theta \leftarrow \theta - \eta\, g(\theta)$$

$$g(\theta) = \mathop{\mathbb{E}}_{x,y \sim \mathcal{D}} \nabla_\theta\, \ell(x, y, \theta)$$

Ordinary (batch) gradient descent

$$g(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\theta\, \ell(x_i, y_i, \theta)$$

Stochastic gradient descent

$$g(\theta) \approx \frac{1}{n} \sum_{i=1}^{n} \nabla_\theta\, \ell(x_i, y_i, \theta)$$

Online learning

Online learning set-up.

The data

- Data is **no longer i.i.d.**
- Sequence of observations is **ordered**

$$\mathcal{O}_t = (x_1, y_1), (x_2, y_2), \ \ldots \ , (x_{t-1}, y_{t-1})$$

The objective

- Predict the next y_t given the next x_t as well as all observations $\mathcal{O}_t$ so far:

$$\hat{y}_t = h(x_t, \theta_t)$$

Online learning.

Remember batch loss:

$$\mathrm{Loss}(\theta) \;=\; \frac{1}{N}\sum_{n=1}^{N}\ell(x_n, y_n, \theta)$$

Instead, we **minimize Regret** (assume game *"not too unfair"*)

$$\mathrm{Regret}(\vartheta, \theta_*) \;=\; \sum_{t=1}^{T}\Big(\ell(x_t, y_t, \theta_t) - \ell(x_t, y_t, \theta_*)\Big)$$

where

$$\begin{aligned}\vartheta &= \{\theta_1, \ldots, \theta_T\} \\ \theta_* &= \arg\min_{\theta} \sum_{t=1}^{T} \ell(x_t, y_t, \theta)\end{aligned}$$

Online learning.

Simplest approach: **Online Gradient Descent**

$$\begin{aligned} \theta_{t+1} &= \theta_t - \eta\, g_t \\ g_t &= \nabla_\theta\, \ell(x_t, y_t, \theta_t) \end{aligned}$$

where η can be simple learning rate schedule:

$$\eta = \frac{\alpha}{\sqrt{t}}$$

or adaptive per-coordinate learning rate (AdaGrad):

$$\eta_i = \frac{\alpha}{\sqrt{\sum_{s=1}^{t} g_{i,t}^2}}$$

Online learning.

When to use online learning?

- when consecutive observations are not i.i.d.
- when you care about errors you make early in the learning process
- underlying “state of the environment” changes over time

  *e.g. binary classification for **spam detection***

Online-to-batch conversion - naive.

Simplest approach: **Take the last iterate**

- treat the i.i.d. dataset as an ordered sequence
- train as you would do for online learning
- take weights at round t=T and treat them as optimal set of weights

However

- why would final weights at t=T also be optimal for full dataset (at all previous t<T)?
- too much variance in weight updates?

Remember: **Online Gradient Descent**

$$\begin{aligned} \theta_{t+1} &= \theta_t - \eta\, g_t \\ g_t &= \nabla_\theta\, \ell(x_t, y_t, \theta_t) \end{aligned}$$

Follow the (regularized) leader

Why do we expect online learning to work for batch learning?

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$$\ell_s(\theta) = \ell(x_s, y_s, \theta)$$

Follow The (Regularized) Leader.

The idea

- Adjust course "now", but in a way that would **also minimize prior loss**
- Update based on **total loss so far**

$$\theta_{t+1} = \arg\min_{\theta} \sum_{s=1}^{t} \ell_s(\theta)$$

- Add regularization:

$$\theta_{t+1} = \arg\min_{\theta} \sum_{s=1}^{t} \ell_s(\theta) + \frac{1}{2\eta}||\theta||^2$$

arxiv.org/abs/1403.3465

[Shalev-Shwartz survey]()

Follow The Regularized Leader.

e.g. for least squares:

$$g_t = x_t\,(\theta_t \cdot x_t - y_t)$$

Useful simplification: linearization

Recall power-series expansion (Taylor):

$$f(a) = f(b) + (a-b)\,f'(b) + \frac{1}{2}(a-b)^2 f''(b) + \ldots$$

Use subgradient formulation (i.e. "linear upper bound")

$$\begin{aligned}
\mathrm{Regret}_T(\vartheta, \theta_*) &= \sum_{t=1}^{T}\left(\ell_t(\theta_t) - \ell_t(\theta_*) + \frac{\|\theta_t\|^2 - \|\theta_*\|^2}{2\eta}\right) \\
&\leq \sum_{t=1}^{T}\left((\theta_t - \theta_*)\cdot g_t + \frac{\|\theta_t\|^2 - \|\theta_*\|^2}{2\eta}\right)
\end{aligned}$$

Follow The Regularized Leader.

Online Gradient Descent from FTRL

$$
\begin{aligned}
\theta_{t+1} &= \arg\min_{\theta} \text{Regret}_t(\theta, \theta_*) \\
&\approx \arg\min_{\theta} \sum_{s=1}^{t} \theta \cdot g_s + \frac{1}{2\eta}\|\theta\|^2 + \text{const} \\
&= -\eta \sum_{s=1}^{t} g_s \\
&= \theta_t - \eta\, g_t
\end{aligned}
$$

Follow The Regularized Leader.

In short,

- FTRL attempts to minimize Regret *explicitly*, albeit greedy
- FTRL is a general framework that encompasses many algorithms (incl. OGD)
- FTRL picks weights at each round to get closer to the optimum θ_* (thus good candidate for batch optimization)

If there's time, will explain the well-known *Adaptive FTRL-Proximal* algorithm

Online-to-batch conversion

How to use **online learning** as a tool for ordinary **batch learning**?

Online-to-batch conversion.

arxiv.org/abs/1109.5647

arxiv.org/abs/1212.1824

Some choices:

Standard averaging:

$$\theta = \frac{1}{T}\sum_{t=1}^{T}\theta_t \qquad \mathrm{Loss}(\theta) - \mathrm{Loss}(\theta_*) = \mathcal{O}\left(\frac{\log T}{\sqrt{T}}\right)$$

Take the last $k = \alpha T$ iterates:

$$\theta = \frac{1}{k}\sum_{t=1}^{k}\theta_{t+T-k} \qquad \mathrm{Loss}(\theta) - \mathrm{Loss}(\theta_*) = \mathcal{O}\left(\frac{1}{\sqrt{T}}\right)$$

Or simply the last iterate:

$$\theta = \theta_T \qquad \mathrm{Loss}(\theta) - \mathrm{Loss}(\theta_*) = \mathcal{O}\left(\frac{\log T}{\sqrt{T}}\right)$$

Online-to-batch conversion.

Take-home message:

Taking the last iterate $\theta = \theta_T$ is sub-optimal … but only **marginally** so.

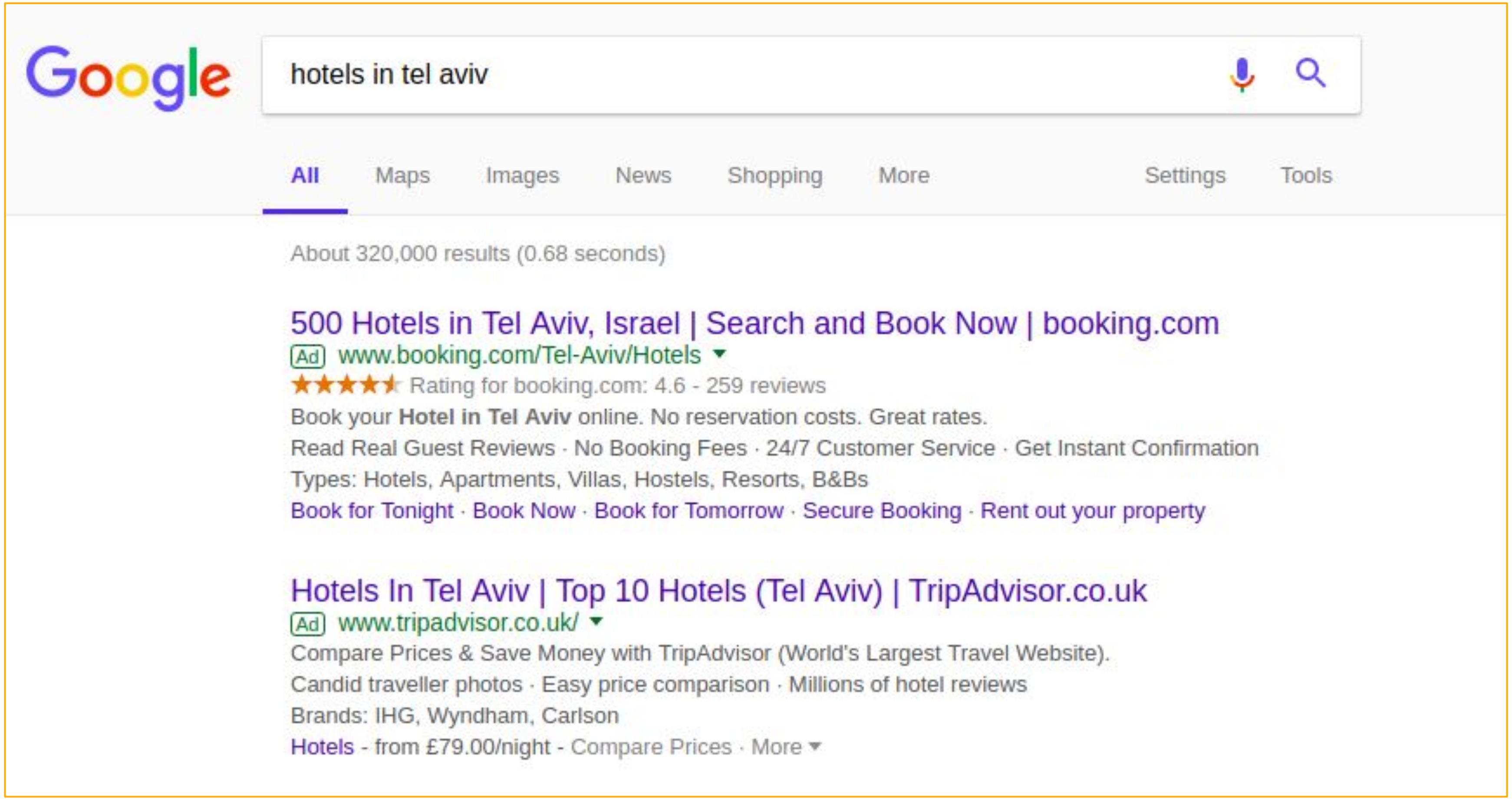
Ask yourself:

Is it worth the trouble to do a proper online-to-batch conversion?

… or will I train my model slightly longer?

Neat examples

Paid search results on Google.



Paid search results on Google.

Context

- Compute bids for Google AdWords
- We have a **very large number** of distinct keywords
- Runs daily

Old setup

- ML framework: pyspark.ml
- Training time: **90 minutes**
- Prediction time: **2 hours**

Current setup

- ML framework: Vowpal Wabbit
- Training time: **10 minutes** (single instance)
- Prediction time: **2 minutes** (parallelized on Hadoop)

Spam detection.

Context

- Customer Care
- Detect spam specific to Booking.com (e.g. auto-replies, receipt notification, etc.)
- Want to maximize precision/recall at fixed human capacity

Approach

- bag-of-words
- ML framework: Vowpal Wabbit
- feature engineering done by Vowpal Wabbit (n-grams, skip-grams, “tf-idf”, etc.)

Missing children.

Context

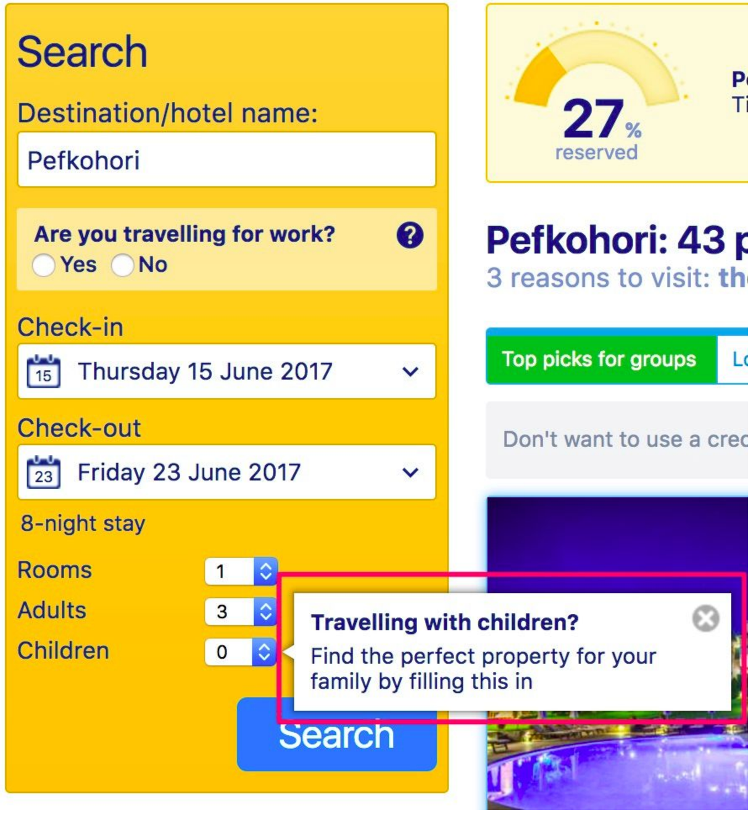
30% of the searches done by ‘Family with children’ guests do not specify number of children!

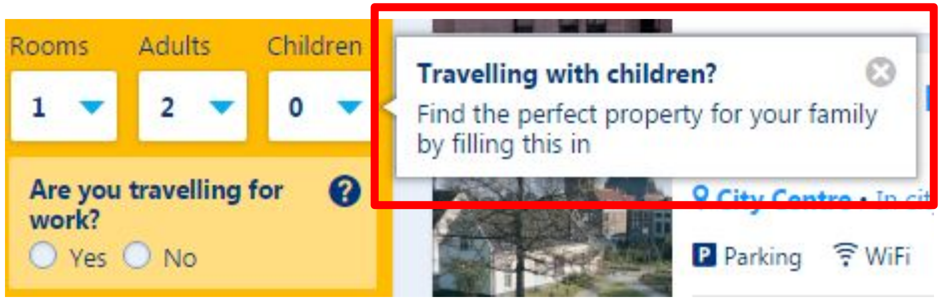
Hypothesis

They forgot their children

Missing children.

Actions taken





Missing children.

Target

At the Stay Review Form users tell us if they are a Family, a Group, Solo or a Couple

Build a Machine Learning Model that guesses the Traveller Type

Use information like Location, Destination, Filter Usage, etc.

Application

Apply the treatment only when the model says the user is most likely a Family.

Some details

Multiclass classification problem

Used 1 year data (200M examples / 200k features)

Missing children.

Outcome

A/B experiment a big success

Bonus material

Adaptive FTRL-proximal - “the FTRL optimizer”

Follow The Regularized Leader.

ordinary L1 regularization

Adaptive FTRL-proximal

$$\theta_{t+1} = \arg\min_{\theta} \sum_{s=1}^{t} \theta \cdot g_s + \lambda_1 \|\theta\|_1 + \frac{\sigma_s}{2} \|\theta - \theta_s\|_2^2$$

proximal

adaptive

Follow The Regularized Leader.

$$z_t = \sum_{s=1}^{t} g_s - \sigma_s \theta_s$$

$$\eta_t = \frac{1}{\sum_{s=1}^{t} \sigma_s}$$

Adaptive FTRL-proximal

$$\begin{aligned}
\theta_{t+1} &= \arg\min_{\theta} \sum_{s=1}^{t} \theta \cdot g_s + \lambda_1 \|\theta\|_1 + \frac{\sigma_s}{2} \|\theta - \theta_s\|_2^2 \\
&= \arg\min_{\theta} \theta \cdot z_s + \lambda_1 \|\theta\|_1 + \frac{1}{2\eta_t} \|\theta\|_2^2 + \text{const} \\
&= \begin{cases} -\eta_t \left(z_t - \operatorname{sign}(z_t)\, \lambda_1 \right) & \text{for } |z_t| > \lambda_1 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

Follow The Regularized Leader.

Adaptive learning rate schedule

- Adagrad:

$$\eta_{t,i} = \frac{\alpha}{\sqrt{\sum_{s=1}^{t} g_{s,i}^2}}$$

- FTRL-proximal:

$$\eta_{t,i} = \frac{\alpha}{\sqrt{\sum_{s=1}^{t} g_{s,i}^2} + \beta + \alpha\lambda_2}$$

The end.

Thanks!

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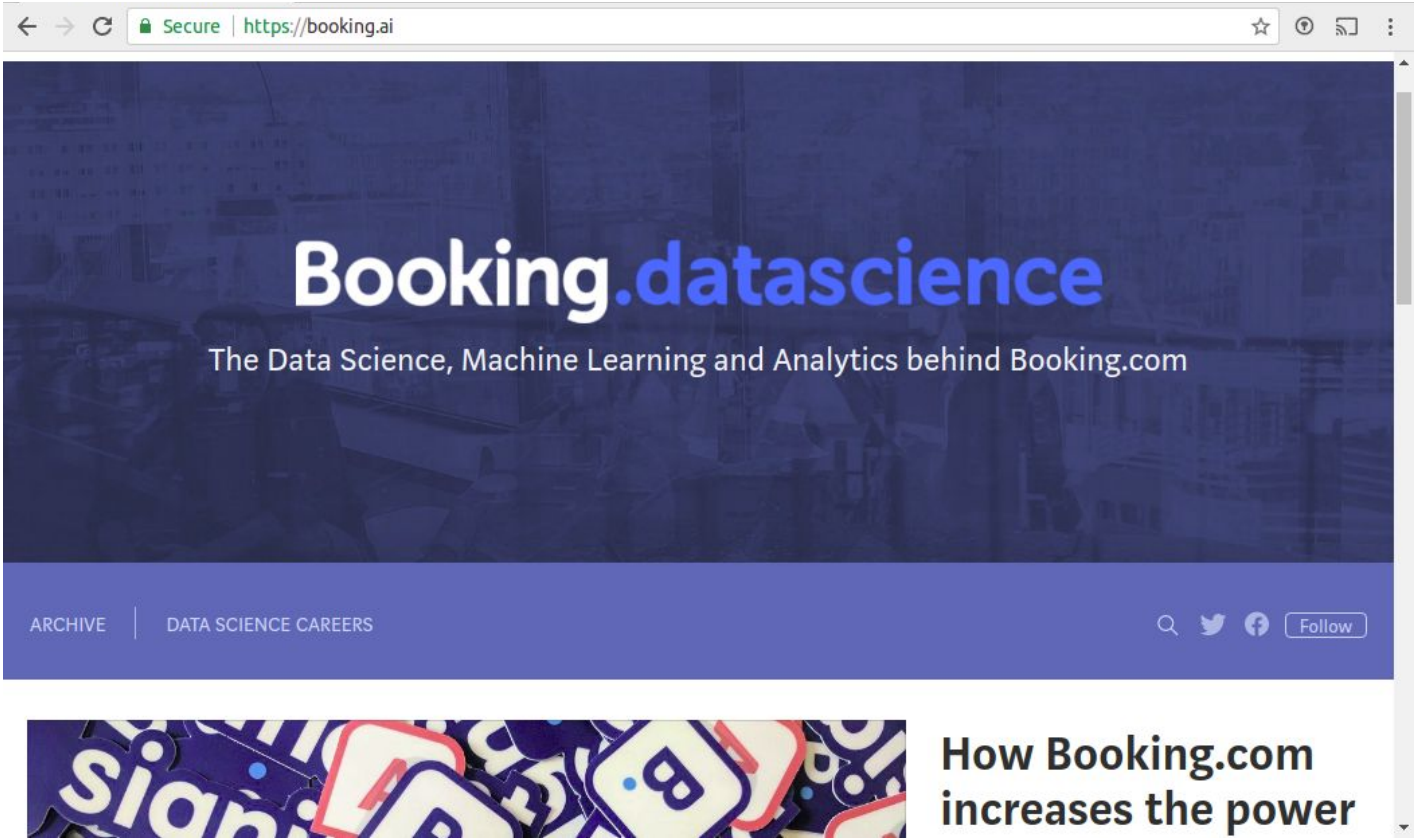
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