What Cannot Be Learned With Bounded Memory?

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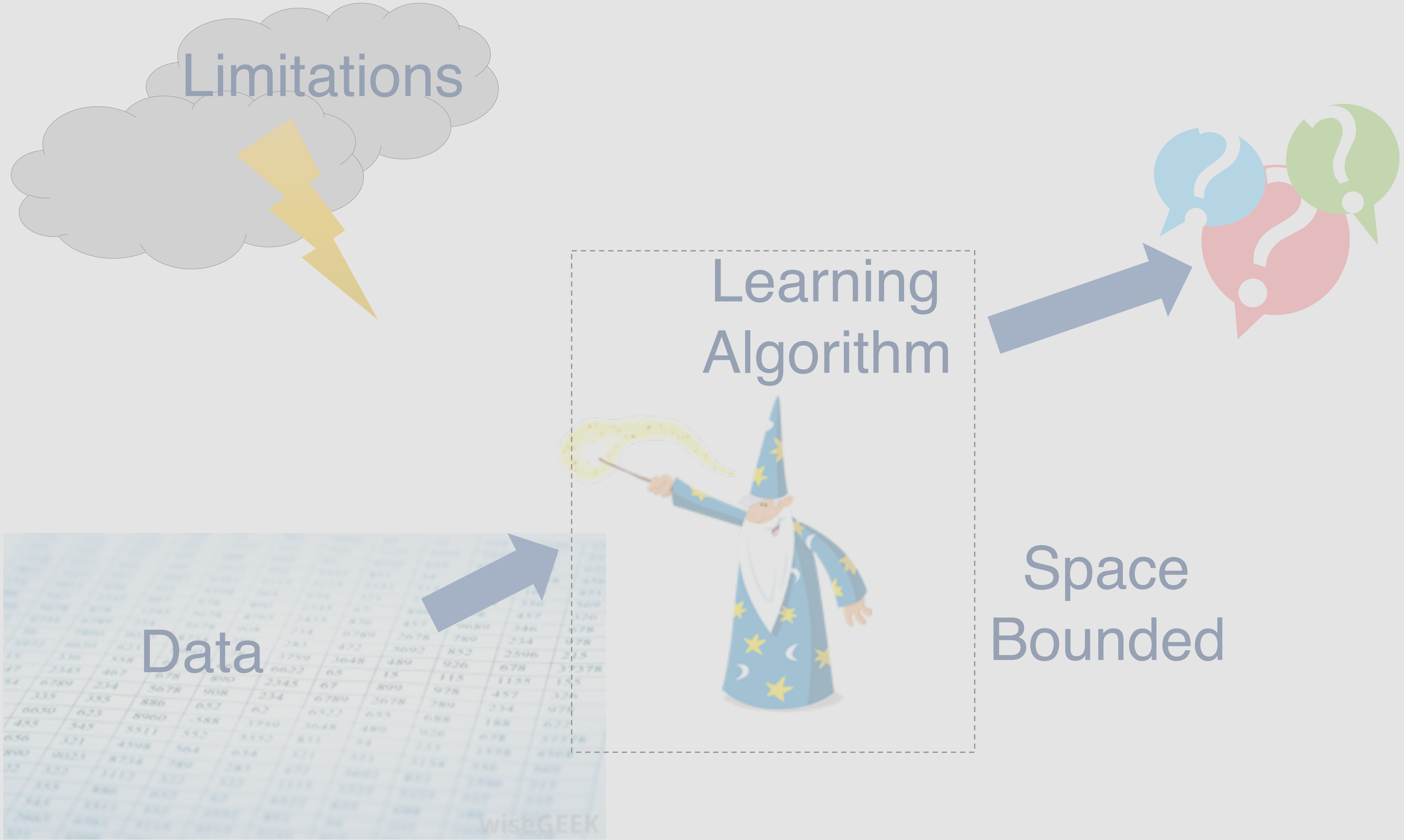

Limitations
Learning Algorithm
Space Bounded
Data
wiseGEEK

Learning with Bounded Space: Motivation

- Natural question - next step after time constraints
- We are in the middle of the big data era
- (Artificial) Neural Networks can be viewed as a bounded space algorithm
- Number of neutrons in the nervous system is bounded

Plan

- Definitions
  - What is (PAC) learning?
  - What is online learning?
  - What is bounded memory learning?
- Problem Formulation
- Main Theorem and a Surprising Conclusion

Supervised Learning: Example

labelled examples

h

PAC Learning (Valiant, 1984)

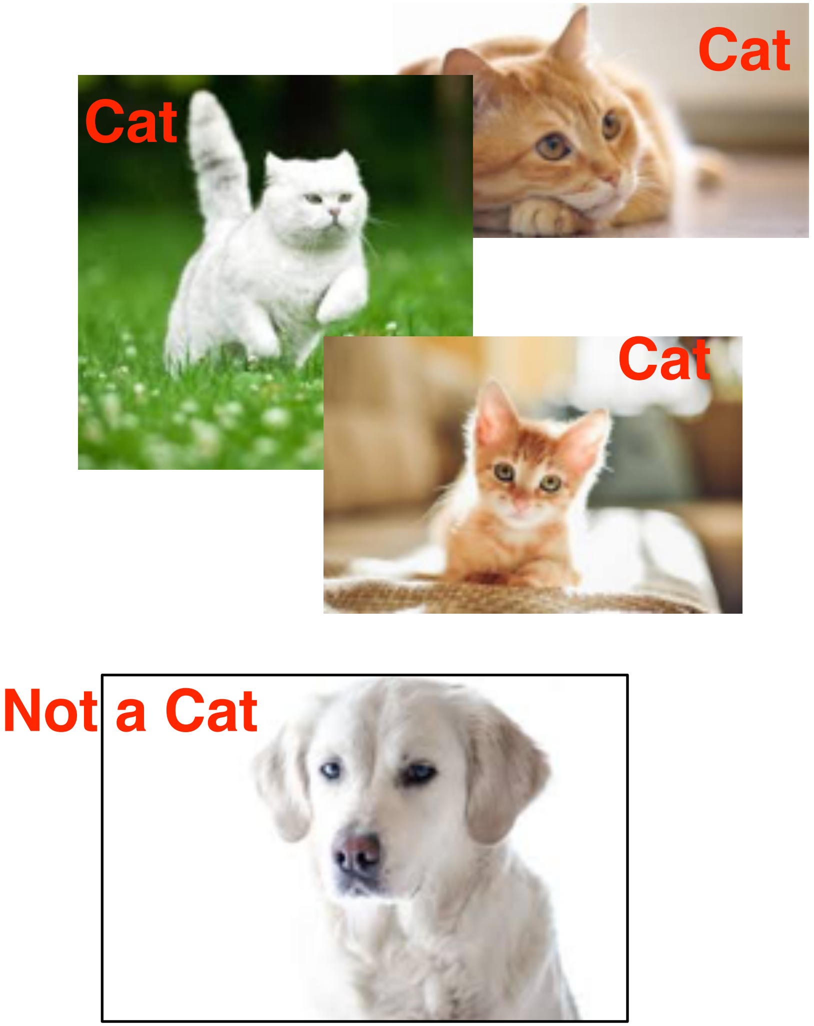
Hypothesis class H={h:X→{0,1}} is PAC-learnable if there is a learner s.t.
for any h, for any distribution over X, with probability > 0.99, the learner will come up with an approximation h' with $P_x(h'(x) \neq h(x)) < 0.01$.

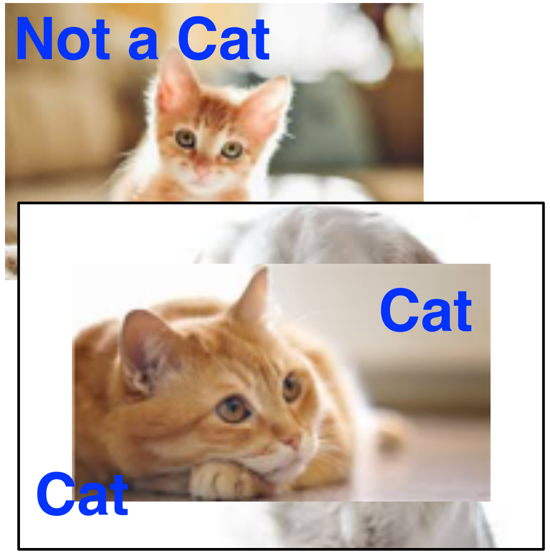
Hypothesis class
H={h:X→{0,1}}

Learner

h' close to h

Online Learning: Example





h^2

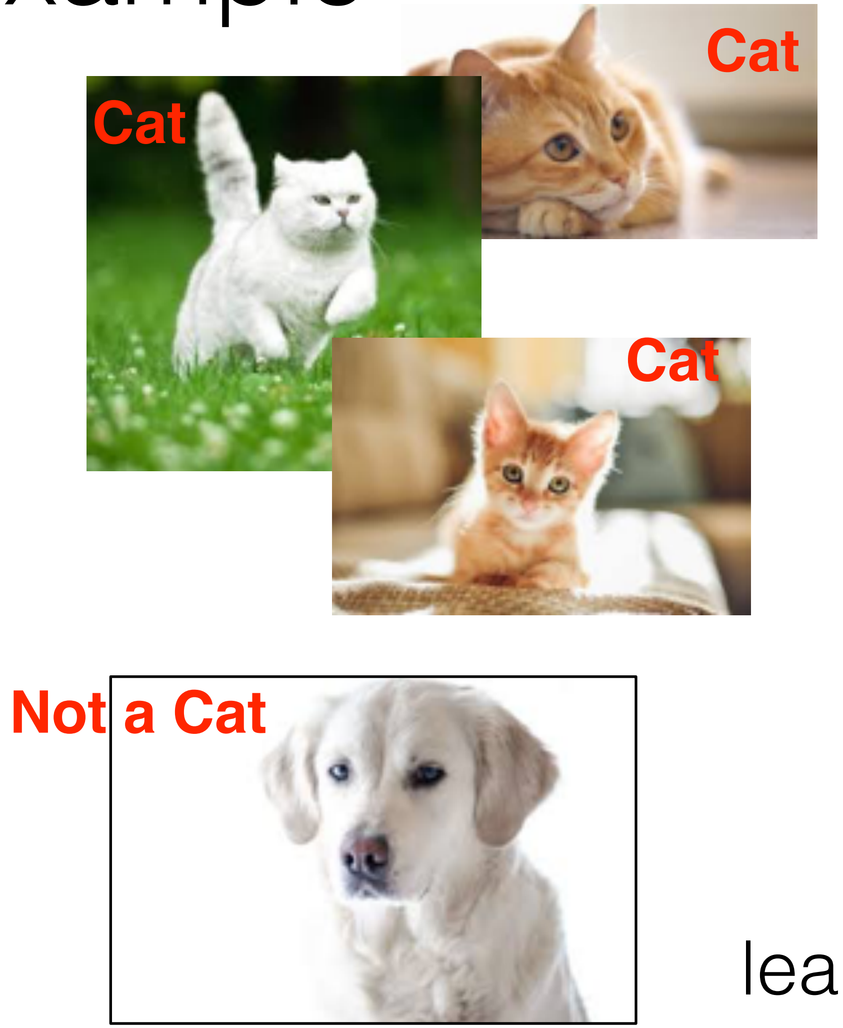
2

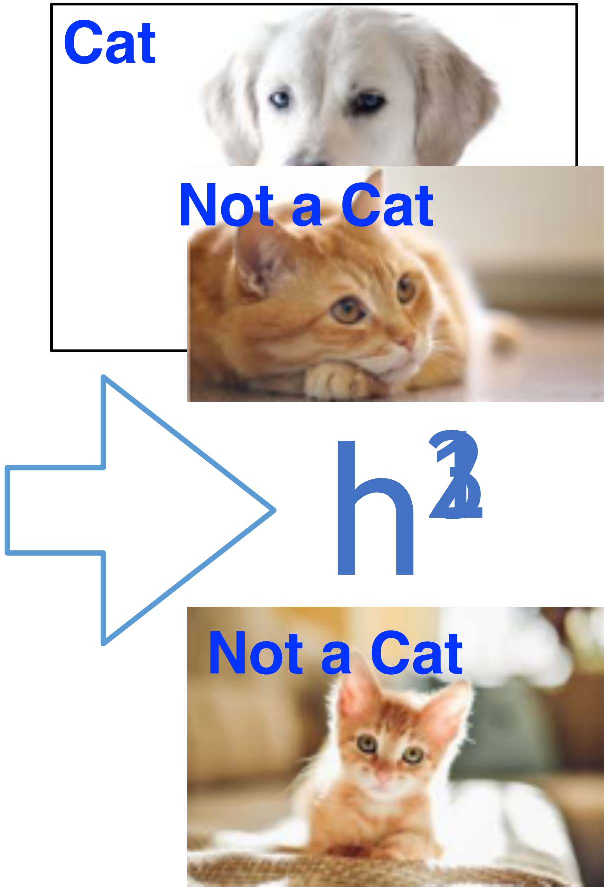
The Online-Learning Framework

- For t=1,2,…
  - An example x^t is given
  - Learner predicts label y^t
  - True label y^t is revealed

- Goal: minimize number of mistakes

Online Learning with Bounded Memory: Example



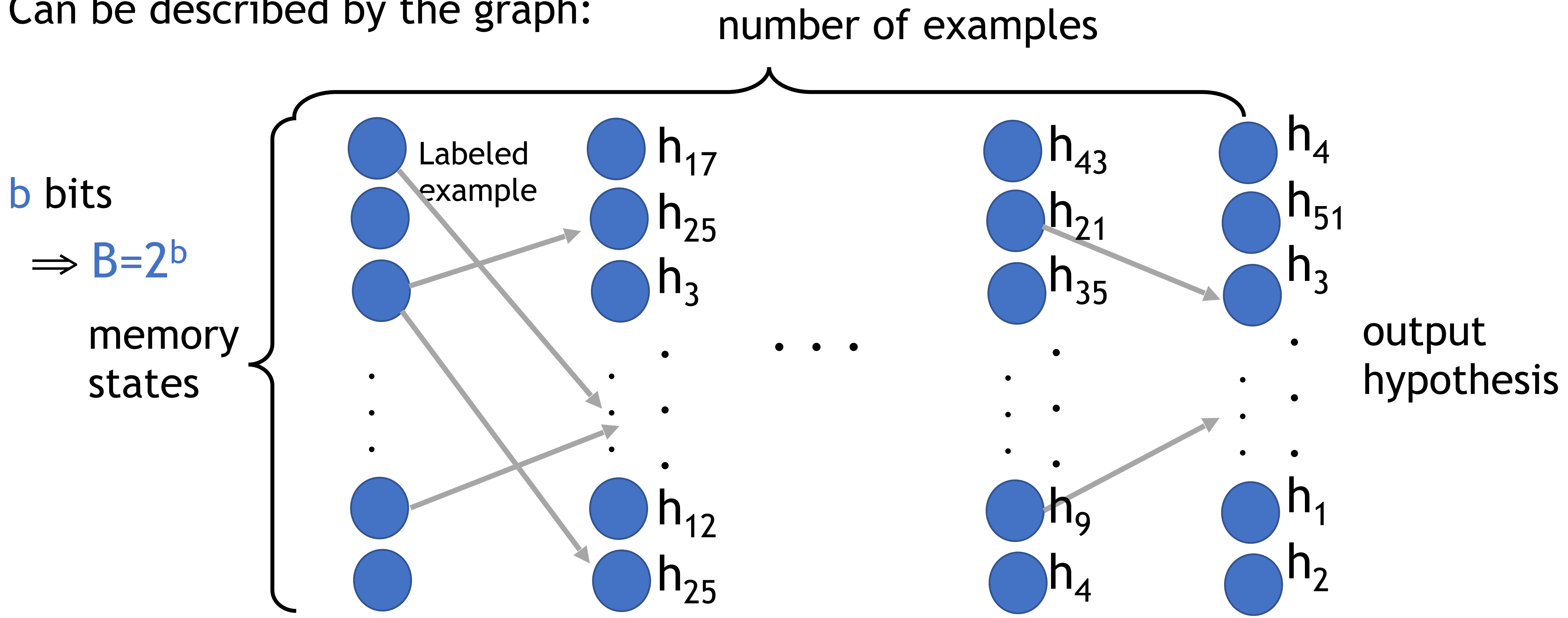


learning with bounded memory is hard

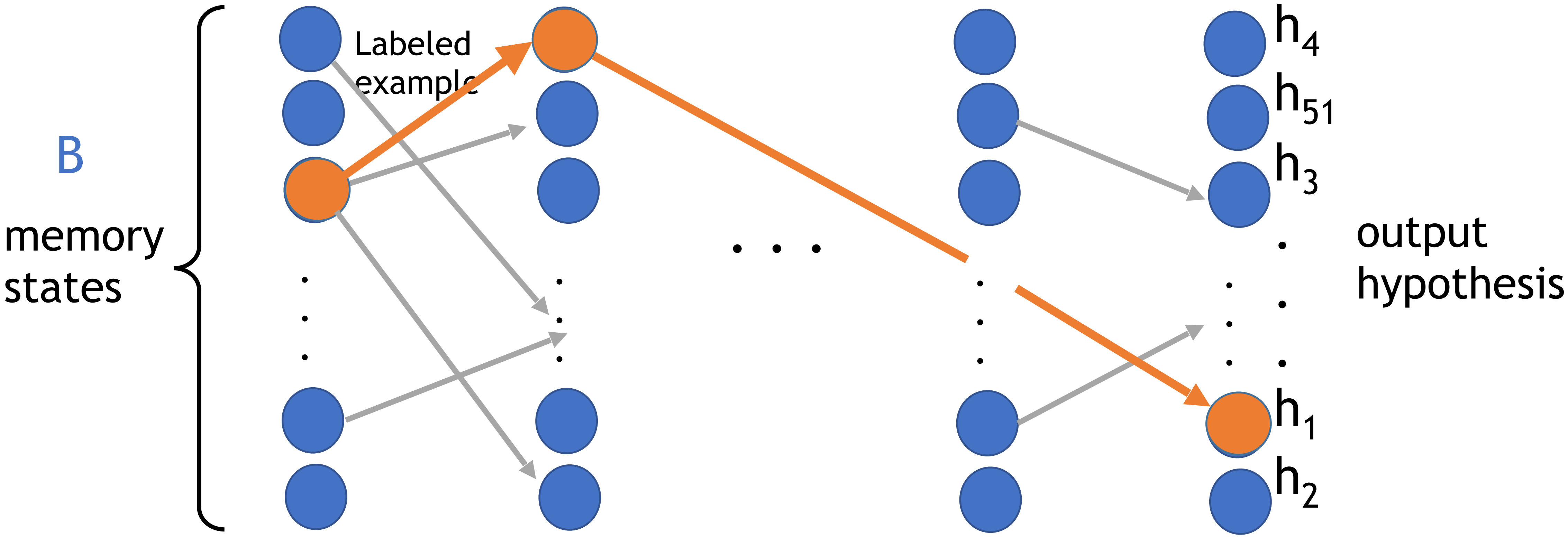
What Cannot be Learned with Bounded Memory?

Learning with Bounded Memory

Fix an algorithm A.
Can be described by the graph:



Learning with Bounded Memory



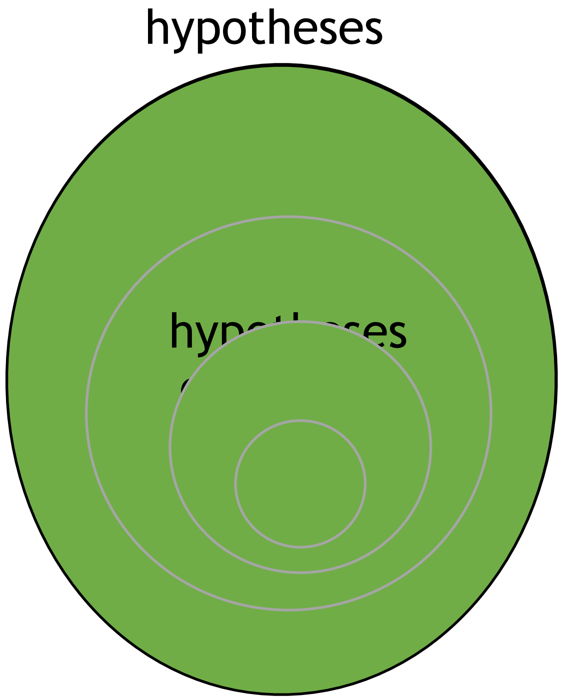
Unbounded Learner

Initially, all h in H are possible.

Repeat: Given example (x,b), rule out h' where $h'(x) \neq b$.

Claim: With high probability, after seeing $O(\log|H|)$ random examples any h' not ruled out satisfies $P_z(h'(z)=h(z))>0.99$.

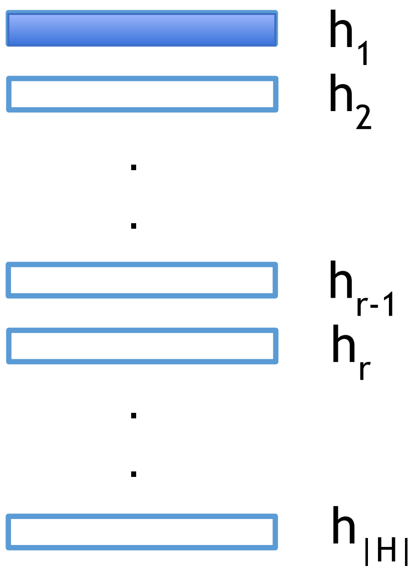
But this requires a lot of memory! Must store received examples in memory, which requires $\min\{\Theta(\log|H|\log|X|), |H|\}$ memory bits.



Bounded Learner

Initially, h’ is the first function in H.

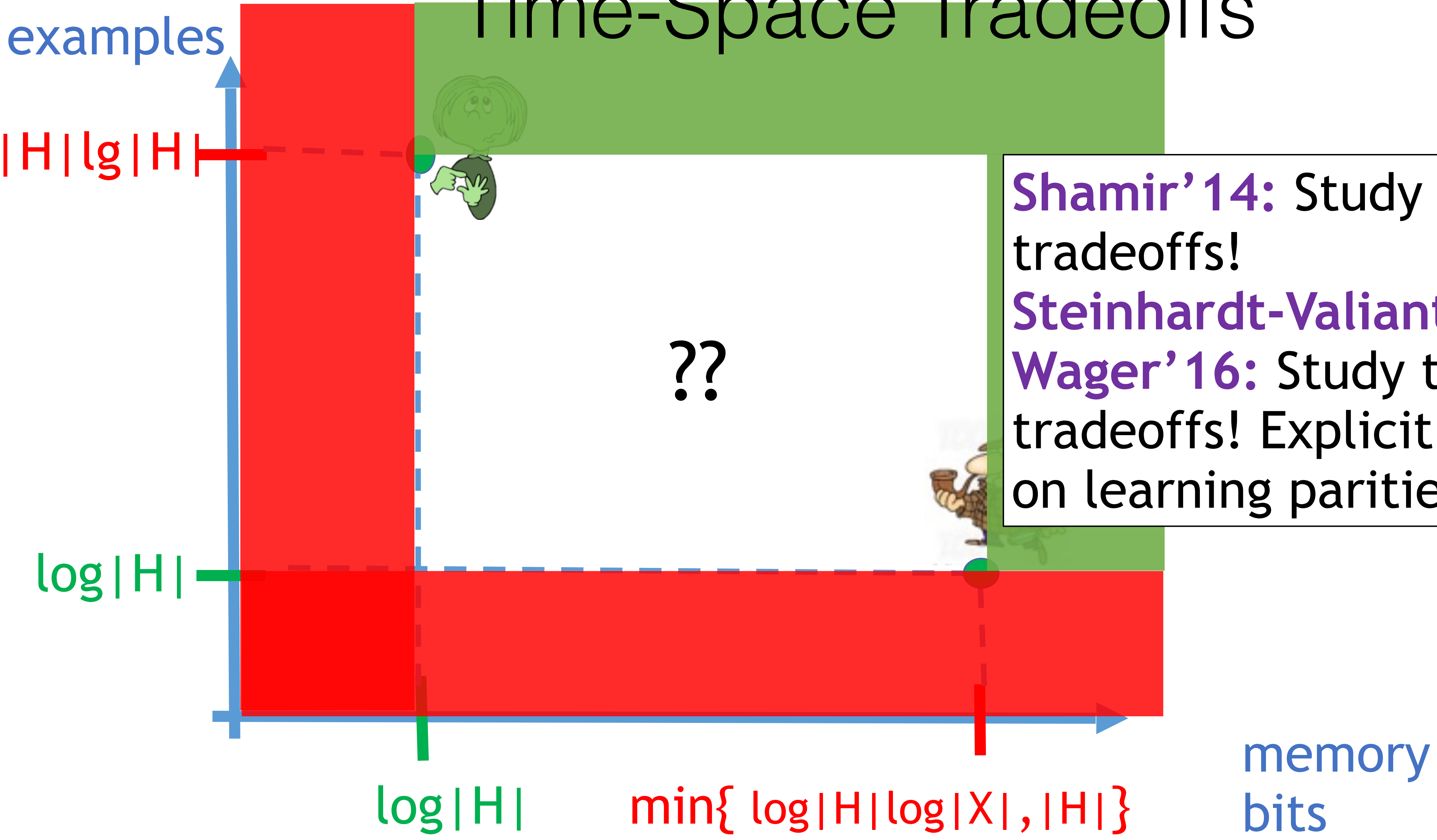
Repeat: Given example (x,b), if $h'(x) \neq b$, then let h’ be the next function in H.



Claim: With high probability, after seeing $O(|H| \log|H|)$ random examples, h’ satisfies $P_z(h'(z)=h(z))>0.99$.

This requires only a minimal number of $\log|H|$ memory bits.

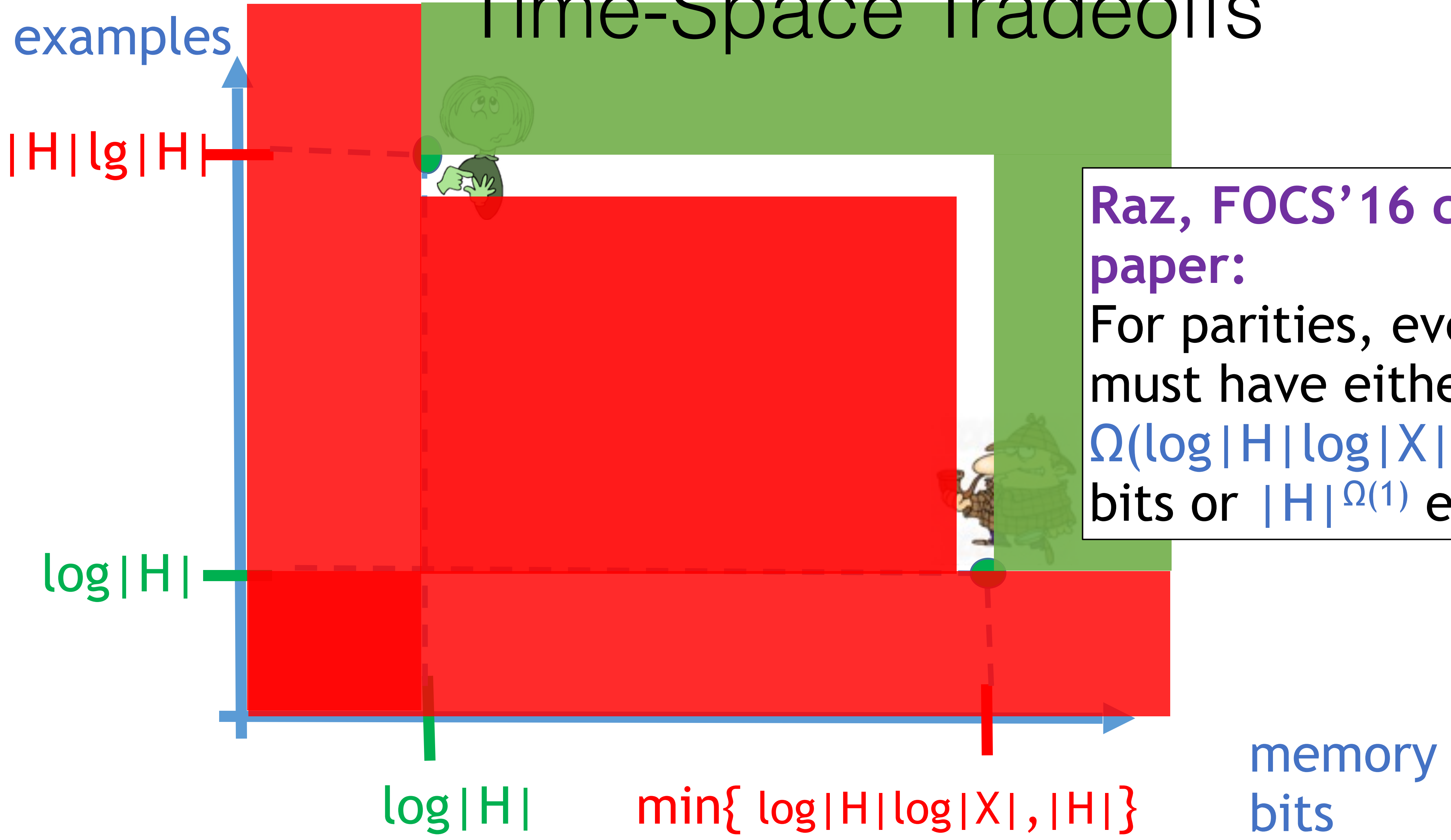
Time-Space Tradeoffs



Shamir'14: Study time-space tradeoffs!

Steinhardt-Valiant-Wager'16: Study time-space tradeoffs! Explicit conjecture on learning parities.

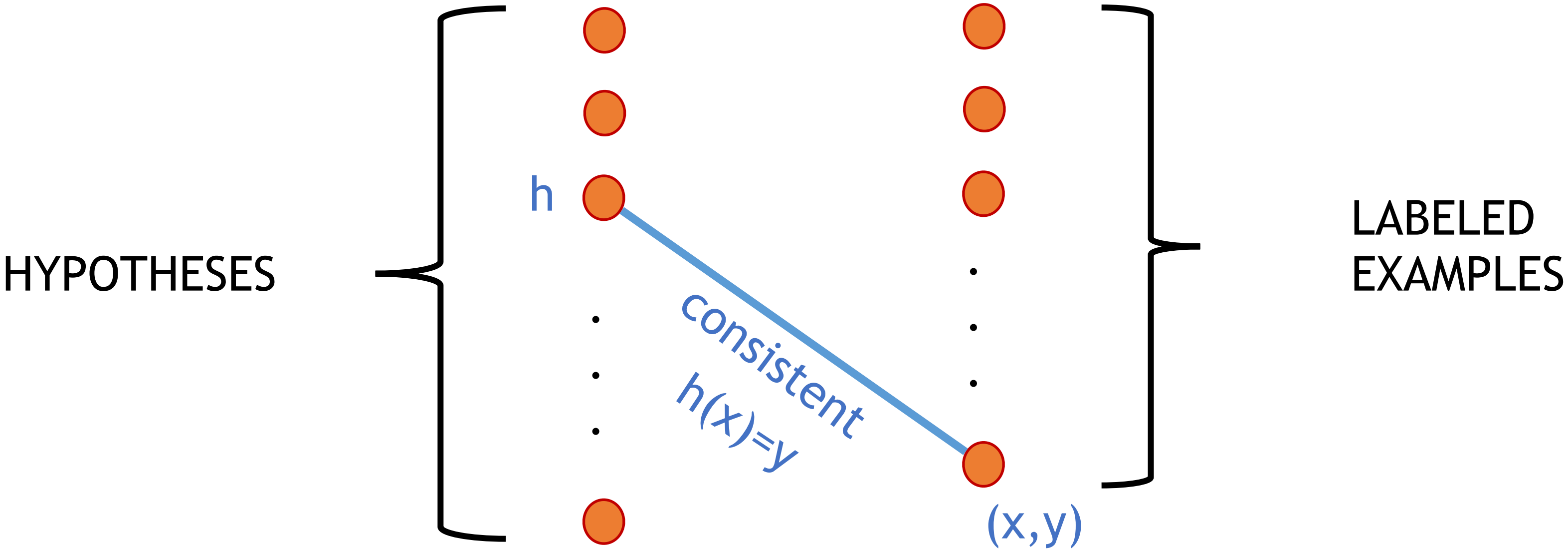
Time-Space Tradeoffs



Raz, FOCS'16 co-best paper:
For parities, every learner must have either $\Omega(\log|H|\log|X|)$ memory bits or $|H|^{\Omega(1)}$ examples.

How Can We Prove Lower Bounds For General Hypotheses Classes?

Hypotheses Graph



Mixing

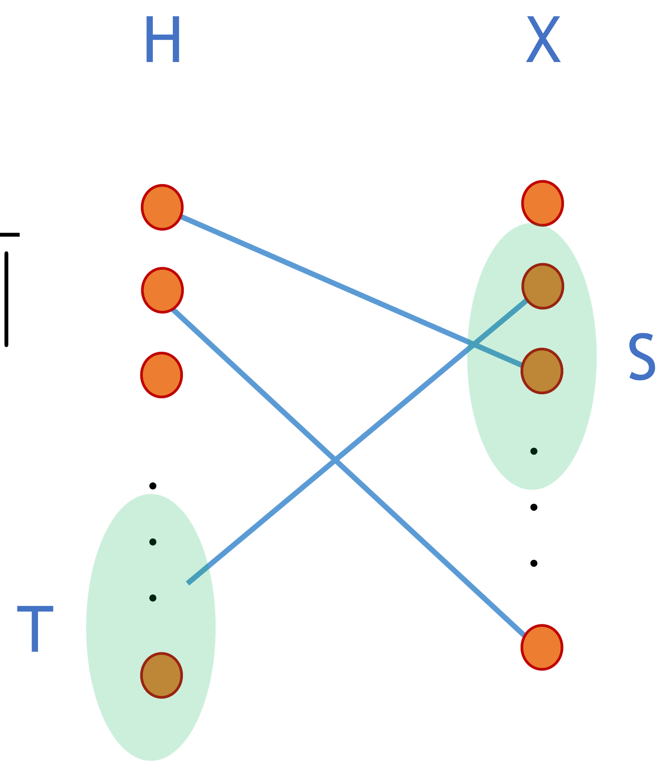
$$p = \frac{E(H,X)}{|H||X|}$$

D-Mixing: For every set S of hypotheses, every set T of labeled examples,

$$\big|E(S,T) - p|S||T|\big| \leq D\sqrt{|S||T|}$$

number of edges between S and T

expected number of edges between S and T



Examples:

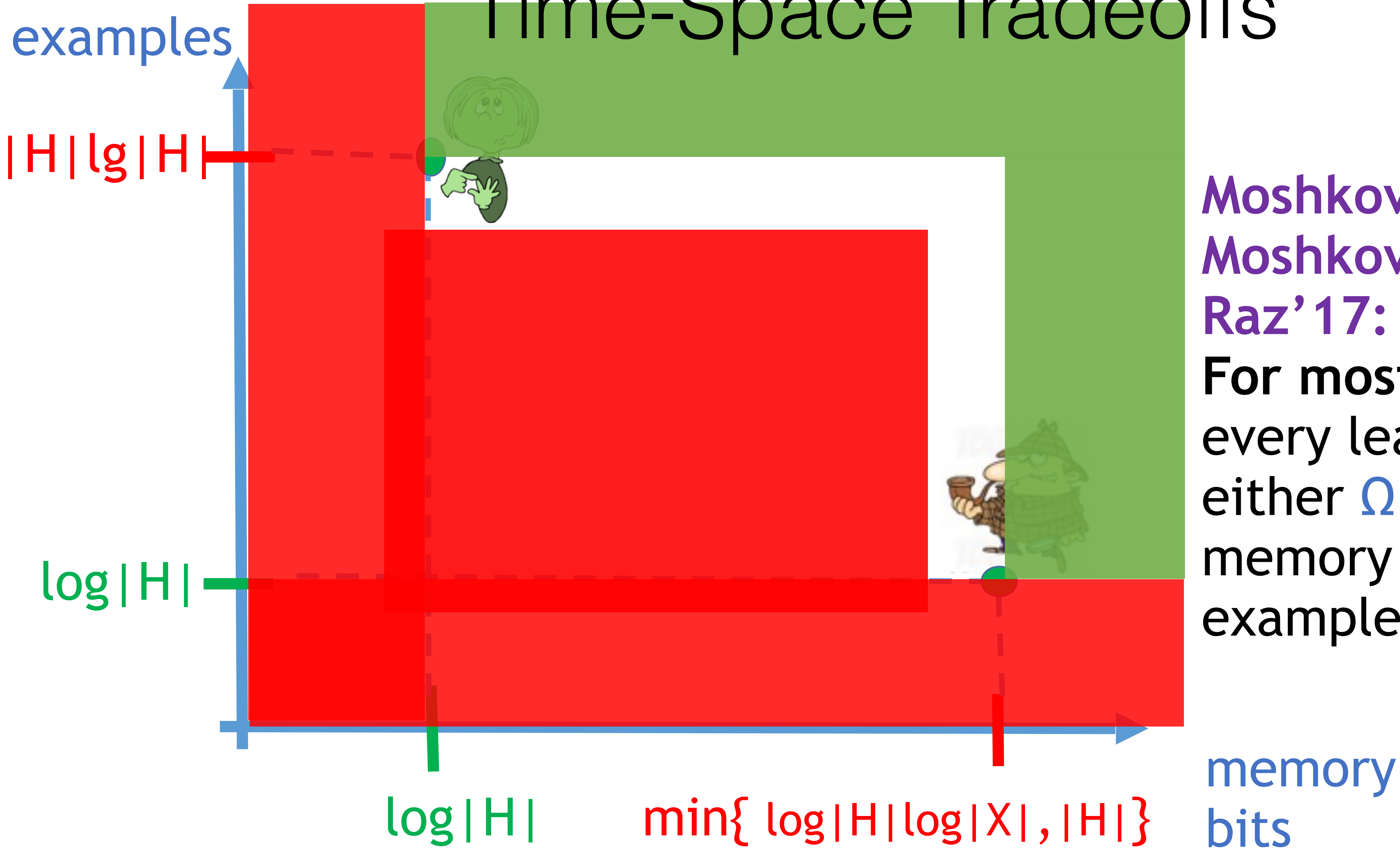
- Parities are O(√|X|)-mixing
- Almost surely G(n,m,0.5) is O(√n)-mixing (for n>m)

Our Theorem: From Mixing To Lower Bounds

Main Thm: If the hypotheses graph is $O(\sqrt{|X|})$-mixing, then either $B=\Omega(\log^2|H|)$ memory bits or $|H|^{\Omega(1)}$ examples are needed to learn.

- A pseudorandomness **sufficient condition** for unlearnability with bounded memory.
- A new combinatorial **framework** for proving lower bounds on space bounded learning.

Time-Space Tradeoffs



Moshkovitz-Moshkovitz'17a+b; Raz'17:

For most classes H, every learner must have either $\Omega(\log^2|H|)$ memory bits or $|H|^{\Omega(1)}$ examples.

The Low Certainty Framework

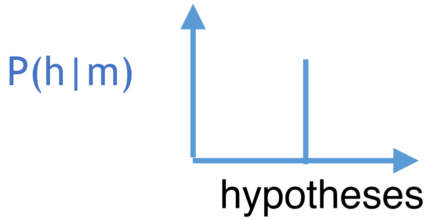
Certainty

We'll pick the underlying hypothesis at random.

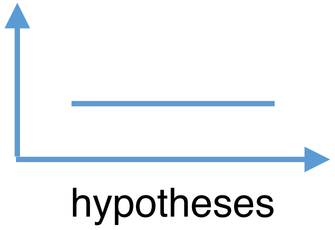
The *certainty* of a memory state m at time t:

- $P(h|m)$ - probability h correct given the algorithm in state m

high certainty:



low certainty:



- $cer^t(m)=\Sigma_h P(h|m)^2$.

The *average certainty* at time t is $cer^t=E_m[cer(m)]$.

Proof Outline

1. **Initially:** $cer^1 = O(1/|H|)$.
2. **Eventually:** Learning at time $T \Rightarrow cer^T \geq \Omega(1)$.
3. **We'll show:** For any learner with B memory states, for every time t, after removal of few hypotheses and examples of low total probability, $cer^{t+1} \leq (1+1/|H|^{\Theta(1)})cer^t$.

As a result: either B memory states or $|H|^{\Omega(1)}$ examples are needed.

The Intuition (The paper is 51 pages)

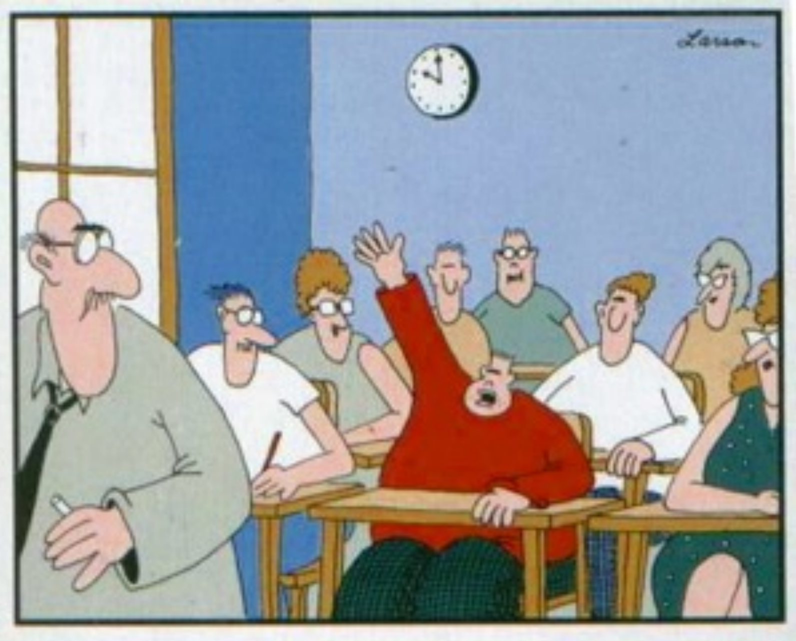
Given an example there are two things the learner can do:

(1) **Heavy step:**

- **Remember only a little about the example.** For mixing H, gain almost no information about h.

(2) **Many Step:**

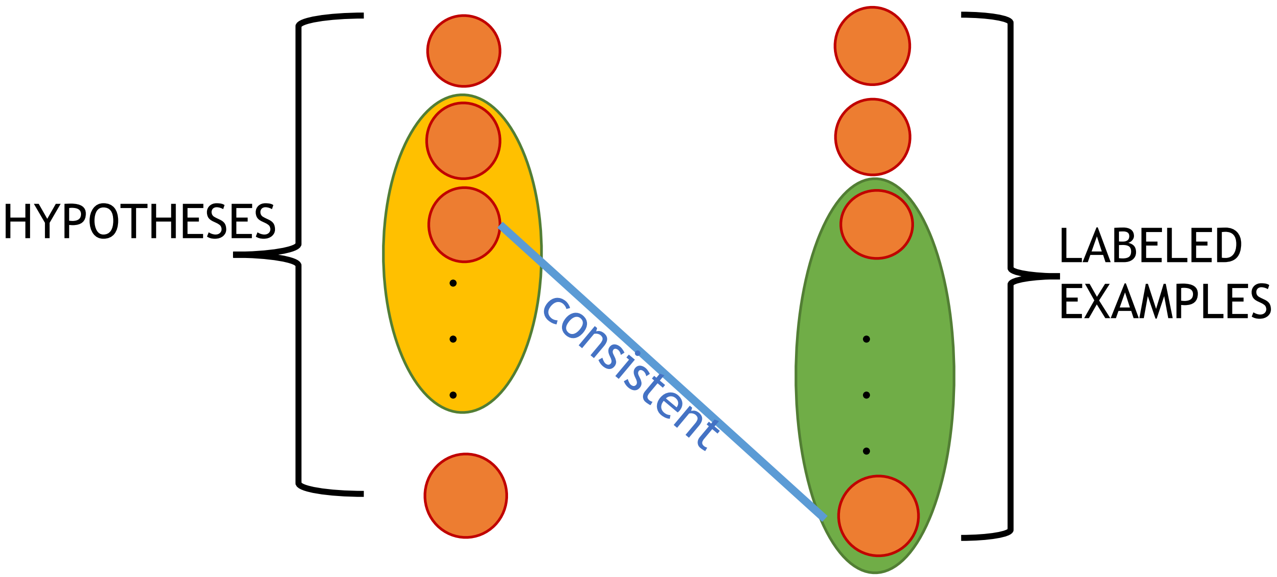
- **Remember the example in detail**, but then have to erase previous memories.



"Mr. Osborne, may I be excused? My brain is full."

Forget Some Lose All Principle

Fix a mixing hypotheses graph. Suppose that one picks uniformly at random a hypothesis h and an example x. Suppose that one stores a short string s about (x,h(x)). Then h|s is close to uniform.



Surprising Conclusion

Most problems cannot be learned with bounded memory …

Luckily, real-problems are not mixing
and can be learned with bounded memory

Thank you!