Approximation: from Discrete to Continuous

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2010.8.3

0)GOAL:

(Marginalize θ and Search over z:)
Maximize log Probability $L = logp(x, z|\lambda)$
$=$
(t-term)$\underline{\sum_{j=1}^{J}\{log\frac{\Gamma(\alpha)}{\Gamma(n_{j..}+\alpha)} + \sum_{t=1}^{m_{j.}}[log(\Gamma(n_{jt.}) + log\alpha]\}}$

+(k-term)$\underline{log\frac{\Gamma(\gamma)}{\Gamma(m_{..}+\gamma)}} + \sum_{k=1}^{K}[log(\frac{\Pi_{w=1}^{W}\Gamma(\lambda_0+n^{w}_{..k})}{\Gamma(n_{..k}+W\lambda_0)}) + log(\frac{\Gamma(W\lambda_0)}{\Gamma(\lambda_0)^W}) + \underline{log(\Gamma(m_{.k}) + log\gamma]}$

1)$log(\Gamma(x))$ approximation:

Stirling Appoximation: $\Gamma(n+1) = n! \approx\sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$

a) L$\rightarrow$ $L_{decom-res}$

$L_{decom-res}(j_0, n^{w}_{j_0.k})$

=(t-term)$\sum_{k=1}^{\mathbf{K}}[(1-\delta(\sum_{\mathbf{w}}\mathbf{n^{w}_{j_0.k}}))log(\Gamma(\sum_{\mathbf{w}}\mathbf{n^{w}_{j_0.k}})] + \sum_{\mathbf{k}}(\mathbf{1}-\delta(\sum_{\mathbf{w}}\mathbf{n^{w}_{j_0.k}})\mathbf{log}\alpha$

+(k-term)$log\frac{1}{\Gamma(m'_{..}+\mathbf{m_{j_0.}}+\gamma)}+\sum_{k=1}^{\mathbf{K}}[(1-\delta(\mathbf{n_{j_0.k}}))log(\frac{\Pi_{w=1}^{W}\Gamma(\lambda_0+n'^{w}_{..k}+\mathbf{n^{w}_{j_0.k}})}{\Gamma(n'_{..k}+\mathbf{n_{j_0.k}}+W\lambda_0)})+log(\frac{\Gamma(W\lambda_0)}{\Gamma(\lambda_0)^W})+log(\Gamma(m'_{.k}+\mathbf{m_{j_0k}})+$
$log\gamma]$

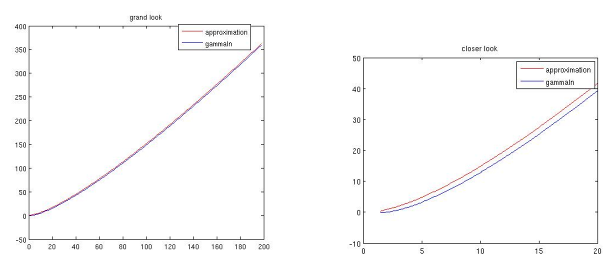

Figure 1: left: Grand view; right: look closely