Representation:$(t_{ji}, k_{jt}) \Rightarrow (z_{ji}, m_{ji})$

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0.Notation

t_{ji}: table assignment of Customer i in restaurant j
k_{jt}: dish assignment of Table t in restaurant j
$l_{t_{ji}}$: the i th customer in restaurant j is sitting at the l lth table serving dish k
n^{l}_{jtk}: number of customers in restaurant j sitting at table t, which is the l lth table serving dish k
m_{jk}: number of tables in restaurant j serving dish k.
. means marginalization
$m_{.k}$: number of tables in dish k
$m_{j.}$: number of tables in Restaurant j
$n_{jt.}$: number of customers in Restaurant j table t
$n_{j..}$: number of tables in Restaurant j
$n_{..k}$: number of customers in dish k
$n^{w}_{..k}$ number of tables in dish k with word w

1.CRF (t_{ji}, k_{jt})

$$
\begin{aligned}
&P(x_{ji}, t_{ji}, k_{jt}|\alpha, \gamma, \lambda)\\
&= (table\ term)\Pi_{j=1}^{J}\{\frac{\Gamma(\alpha)}{\Gamma(n_{j..}+\alpha)}[\Pi_{t=1}^{m_{j.}}\Gamma(n_{jt.})]\alpha^{m_{j.}}\}\\
&(dish\ term)\frac{\Gamma(\gamma)\Pi_{k=1}^{K}\Gamma(m_{.k})}{\Gamma(m_{..}+\gamma)}\Pi_{k=1}^{K}[\frac{\Pi_{w=1}^{W}\Gamma(\lambda_0+n^{w}_{..k})}{\Gamma(n_{..k}+W\lambda_0)}\frac{\Gamma(W\lambda_0)}{\Gamma(\lambda_0)^{W}}]\gamma^{K}\\
&= C\times\Pi_{j=1}^{J}\Pi_{t=1}^{m_{j.}}\Gamma(n_{jt.})\\
&= C\times\Pi_{j=1}^{J}\Pi_{k=1}^{K}[\Pi_{l=1}^{m_{jk}}\Gamma(n^{l}_{j.k})]
\end{aligned}
$$

2.CRF (z_{ji}, m_{jk})

Notice that $(t_{ji}, k_{jt}) = (z_{ji}, l_{t_{ji}})$
Below, we are going to have a more compact assignment:sum over $l_{t_{ji}} \rightarrow \vec{n}_{j.k} \rightarrow m_{jk}$

$$
\begin{aligned}
&P(z_{ji}, m_{jk}|\alpha, \gamma, \lambda)\\
&= \sum_{partition} P(x_{ji}, t_{ji}, k_{jt}|\alpha, \gamma, \lambda)\\
&= C\times\Pi_{j=1}^{J}\Pi_{k=1}^{K}\{\tilde{\sum_{\vec{n}^{l}_{j.k}}}[\begin{pmatrix} & n_{j.k} & \\ n^{1}_{jk.} & ... & n^{l}_{jk.}\end{pmatrix}\Pi_{l=1}^{m_{jk}}\Gamma(n^{l}_{j.k})]\}
\end{aligned}
$$

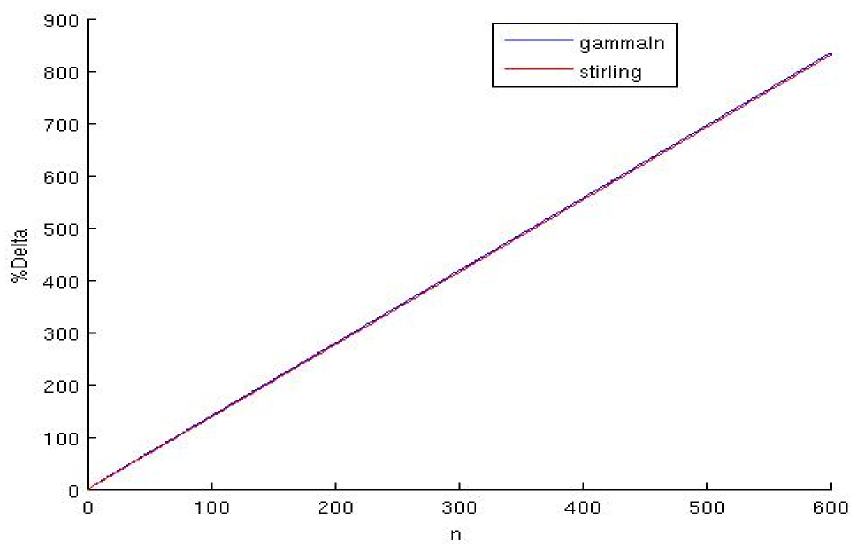
$= C \times \Pi_{j=1}^{J}\Pi_{k=1}^{K}\left[\begin{array}{c} n_{j.k} \\ m_{jk} \end{array}\right]$

$(\tilde{\sum_{\vec{n}^l_{j.k}}}$:with constriants that $\sum \vec{n}^l_{j.k} = n_{j.k})$

1. Given $\vec{n}^l_{j.k}$, we sum over $l_{t_{ji}}$ configuration:
   We get the factor $\left(\begin{array}{ccc} & n_{j.k} & \\ n^1_{jk.} & ... & n^l_{jk.} \end{array}\right)$ (number of combination to assign $n_{j.k}$ elements to m_{jk} identifiable groups with size $\vec{n}^l_{j.k}$)
2. Given m_{jk}, we sum over $\vec{n}^l_{j.k}$
   Then $\tilde{\sum_{\vec{n}^l_{j.k}}}\left(\begin{array}{ccc} & n_{j.k} & \\ n^1_{jk.} & ... & n^l_{jk.} \end{array}\right)[\Pi_{l=1}^{m_{jk}}\Gamma(n^l_{j.k})]$ exactly counts the number of permutations of $n_{j.k}$ elements with m_{jk} disjoint cycles.

3.Will it solve the problem?

1. Suppose in the previous (t_{ji}, k_{jt}) representation, $m_{jk} = \{0, 1\}$, then:(modify s.t.$\Gamma(0) = 1$)
   $P(x_{ji}, t_{ji}, k_{jt}|\alpha, \gamma, \lambda) = C \times \Pi_{j=1}^{J}\Pi_{k=1}^{K}\Gamma(n_{j.k})$
   $P(z_{ji}, m_{jk}|\alpha, \gamma, \lambda) = C \times \Pi_{j=1}^{J}\Pi_{k=1}^{K}\left[\begin{array}{c} n_{j.k} \\ m_{jk} \end{array}\right]$
2. Similar size:
   $\Gamma(n_{j.k}) < max(\left[\begin{array}{c} n_{j.k} \\ . \end{array}\right]) = \left[\begin{array}{c} n_{j.k} \\ * \end{array}\right] < \Gamma(n_{j.k} + 1)$
3. Penalty for splitting the table into 2:
   For CRF(t_{ji}, k_{jt}):$\Delta(1 \to 2) = log\Gamma(n) - 2log\Gamma(\frac{n}{2}) \approx nlog(n) - 2(\frac{n}{2})log(\frac{n}{2}) = nlog(2)$
   For CRF(z_{ji}, m_{jk}):$\Delta(1 \to 2) = log(\left[\begin{array}{c} n \\ * \end{array}\right]) - 2log(\left[\begin{array}{c} \frac{n}{2} \\ * \end{array}\right])$
   There is no simple asymptotic approximation, but we can plot it numerically
   We can see from the picture below, the penalty for splitting the table into 2 are similar.



Conclusion:

1. $\begin{bmatrix} n_{j.k} \\ * \end{bmatrix}$ behaves similarly to $\Gamma(n_{j.k})$

2. If the likelihood term is hard to improve, the restaurant still prefers one "dish" configuration which has more partitions.

3. However, the degenate configuration now is sevaral tables serving the same dish instead of one big table serving that dish alone before.