Combinatoric Optimization Problem

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2011.1.16

0)Set Up:

中餐馆有K道菜供选择，共J家连锁店，顾客可分成W种（e.g.年龄）
每家餐馆中，每桌只有一道菜（k相同），人种不限（w任意）

Constants:
α, γ, λ

Variable:
$\{n_{jk}^w\}$:在第j家连锁店 jÆ$1, 2, ..., J$，点了第k道菜$k = 1, 2, ..., K$的第w种顾客的人数$w = 1, 2, ..., W$ ***Statistics:***
$n_{jk} = \sum_{w=1}^{W} n_{jk}^w$
$n_{j.} = \sum_{k=1}^{K} n_{jk}$

$\{m_{jk}\}$:在第j家连锁店 jÆ$1, 2, ..., J$，点了第k道菜$k = 1, 2, ..., K$的桌子数量
$m_{j.} = \sum_{k=1}^{K} m_{jk}$
$m_{..} = \sum_{j=1}^{J} m_{j.}$

Linear Constraints:
$\sum_{k=1}^{K} n_{jk}^w = n_{j.}^w$, for each w

Nonlinear Object Funciton:

$$
\begin{aligned}
F &= \sum_{j=1}^{J}\{log\frac{\Gamma(\alpha)}{\Gamma(n_{j..}+\alpha)} + \sum_{k=1}^{K}[log(\Gamma(n_{jk}) + log\alpha]\} \\
&+ log\frac{\Gamma(\gamma)}{\Gamma(m_{..}+\gamma)} + \sum_{k=1}^{K}[log(\frac{\Pi_{w=1}^{W}\Gamma(\lambda + n^{w}_{..k})}{\Gamma(n_{..k} + W\lambda_0)}) + log(\frac{\Gamma(W\lambda)}{\Gamma(\lambda)^W}) + log(\Gamma(m_{.k}) + log\gamma]
\end{aligned}
$$

Γ*: gamma function,*$\Gamma(n) = (n-1)!$ *for integers*
GOAL:
Clustering problem: find the partition n_{jk}^w that maximizes F

2)Modern Heuristic:

1) relaxed linear programming doesn't seem to fit
2) convex optimization doesn't either, though F is a convex function. (The derivative of gamma function is digamma function, which is hard for analytic or numerical)
3) So, we choose modern heuristic methods to **SEARCH** for n_{jk}^{w}:

i) Big Move: since we have 3 coordinates(j,k,w), we can change only one j or k or w
ii)Small Move: within the big move above, we can local move,split move and merge move

3)Additional:

i) It has some probability meanings and we can use Gibbs sampling and K-means++ for randomization
ii) Instead of searching the partition n_{jk}^{w}, we can try to search configuration k_{ji}, the dish that the i th customer in the j th restaurant is served, which is easier to search.