Decompose Restaurant:
Heuristic Search+Linear Programming

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0)Set Up:

Variable:
$\{n^w_{j_0.k}\}$number of words in Restaurant j_0,serving dish k, $k = 1, 2, ..., K_p(= K' + n_{j_0..}); w = 1, 2, ..., W$

Statistics:
$k_{set} = \{k : \exists t \ s.t. \ k_{j_0t} = k\}$
$m_{j_0k} = 1 - \delta(\sum_{w=1}^{W} n^w_{j_0.k})$
$m_{j_0.} = \sum_{k=1}^{K} m_{j_0k}$
$n_{j_0.k} = \sum_{w=1}^{W} n^w_{j_0.k}$

Linear Constraints:
$\sum_{k=1}^{K} n^w_{j_0.k} = n^w_{j_0..}$, for each w

Nonlinear Object Funciton:
$L_{decom-res}(j_0, n^w_{j_0.k})$

$=$(t-term)$\sum_{k\in\mathbf{k_{set}}}[log(\Gamma(\mathbf{n_{j_0.k}})] + \mathbf{m_{j_0.}}log\alpha$

$+$(k-term)$log\frac{1}{\Gamma(m'_{..}+\mathbf{m_{j_0.}}+\gamma)} + \sum_{k\in\mathbf{k_{set}}}[log(\frac{\Pi_{w=1}^{W}\Gamma(\lambda_0+n'^{w}_{..k}+\mathbf{n^w_{j_0.k}})}{\Gamma(n'_{..k}+\mathbf{n_{j_0.k}}+W\lambda_0)}) + log(\frac{\Gamma(W\lambda_0)}{\Gamma(\lambda_0)^W}) + log(\Gamma(m'_{.k} + \mathbf{m_{j_0k}}) + log\gamma]$

1)Linear Programming:

If we know:

1) $m_{j_0.} = card(k_{set})$ get rid of $log\frac{1}{\Gamma(m'_{..}+\mathbf{m_{j_0.}}+\gamma)} + \mathbf{m_{j_0.}}log\alpha$
2) $\vec{n}_{j_0t}$ get rid of $\sum_{k\in k_{set}}[log(\Gamma(\mathbf{n_{j_0.k}}) + log(\frac{1}{\Gamma(n'_{..k}+\mathbf{n_{j_0.k}}+W\lambda_0)})]$
3) k_{set} get rid of $log(\frac{\Gamma(W\lambda_0)}{\Gamma(\lambda_0)^W}) + log(\Gamma(m'_{.k} + \mathbf{m_{j_0k}}) + log\gamma]$

Then the Object function becomes:
$L'_{decom-res} = \sum_{k\in k_{set}}[log(\Pi_{w=1}^{W}\Gamma(\lambda_0 + n'^{w}_{..k} + \mathbf{n^w_{j_0.k}}))]$
Which is a perfect Binary Integer Programming problem:

Variable:
Binary:$\{q_{kwn}\}, k \in k_{set}; w = 1, 2, ..., W; n = 1, 2, ..., n^w_{j_0..}$

Statistics:
$\sum_{n=1}^{n^w_{j_0..}} q_{kwn} = n^w_{j_0.k}$, for each w,k
Coeff: $l_{kwn} = log(\lambda_0 + n'^w_{..k} + n - 1)$

Linear Constraints:
$q_{kwn} \in \{0, 1\}$
$q_{kwn} \geq q_{kwn'}$,for n≤n'
$\sum_{k \in k_{set}} \sum_{n=1}^{n^w_{j_0..}} q_{kwn} = n^w_{j_0..}$, for each w

Linear Object Funciton!!!:
$L_{decom-res}(j_0, n^w_{j_0.k}) = \sum_{k \in k_{set}} \sum_{w=1}^{W} \sum_{n=1}^{n^w_{j_0..}} q_{kwn} l_{kwn}$

2)Modern Heuristic:

Amazingly, our previous randomized search greedily approximate the needed statistics for Linear Programming:

1) $m_{j_0.}$ sample new tables(upper bounder for $m_{j_0.}$)+merge-table(Local change $m_{j_0.}$ by 1)
2) $\vec{n}_{j_0 t}$ local-table(Local change $\vec{n}_{j_0 t}$ by 1)
3) k_{set} search-dish(Local change k_{set} by 1 element)