

Annealed ME

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2010.6.24

0) Notations

Hierarchical Dirichlet Process Model with Dirichlet-Multinomial:

i) Formula

n_{jtk} number of customers in table t, restaurant j, with dish k

m_{jk} number of tables in restaurant j, with dish k

$n_{..k}$ number of customers in dish k

$n_{..k}^w$ number of occurrence of word w in dish k

$n_{j.}$ number of customers in Restaurant j

$n_{jt.}$ number of customers in table t in Restaurant j

$m_{.}$ number of tables in total

$m_{.k}$ number of tables in dish k

J Restaurants, K dishes

a) Goal: (Marginalize θ and Search over z .)

Maximize log Probability $L = \log p(x, z | \lambda)$

=

$$\begin{aligned} & \text{(t-term)} \log \frac{\Gamma(\gamma)}{\Gamma(m_{..} + \gamma)} + \sum_{j=1}^J \{ \log \frac{\Gamma(\alpha)}{\Gamma(n_{j.} + \alpha)} + \sum_{t=1}^{m_{j.}} [\log(\Gamma(n_{jt.})) + \log \alpha] \} \\ & + \text{(k-term)} \sum_{k=1}^K [\log \left(\frac{\prod_{w=1}^W \Gamma(\lambda_0 + n_{..k}^w)}{\Gamma(n_{..k} + W\lambda_0)} \right) + \log \left(\frac{\Gamma(W\lambda_0)}{\Gamma(\lambda_0)^W} \right) + \log(\Gamma(m_{.k})) + \log \gamma] \end{aligned}$$

(underlined part come from Hierarchical Dirichlet Process)

b) Annealing: Maximize the annealed log Probability:

$$L'(\text{Temperature}) = \text{Temperature} * (\text{t-term}) + (\text{k-term})$$

1) ME algorithm:

i) Unified Backbone

(1) Annealing:(n:number of iterations; p:annealing power)

(i) For iter=1:n

(ii) Temperature= $(\frac{iter}{n})^p$

(iii) Decompose Restaurants(Temperature)

(iv) **Merge Dish(Temperature)**

(v) End

(2) Running for Convergence:

(i) While L doesn't increase any more

(ii) Decompose Restaurants(1)

(iii) **Merge Dish(1)**

(iv) End

ii) Decompose Restaurants(Temperature)

For j=Randperm(J)

(A) Rough reconfiguration for Restaurant j:

(i) Make Restaurant j into one table t_0 where customers following uniform distribution:

(%Thus the Probability $P(t_{ji} = t_0) = \frac{1}{W}$)

(ii) Possible Dish={Nonempty dishes}

(iii) While Possible Dish is not empty:

(a) For each dish $k \in \text{Possible Dish}$, propose to form a new table t_k out of t_0 with dish k and calculate **the change for these two dishes Δk** :

(%For each customer i in t_0 , sample $t_{ji} \in \{t_0, t_k\} \sim \{\frac{1}{W}, \frac{n_{\cdot,k}^w + \phi}{n_{\cdot,k} + W\phi}\}$)

(%Propose to form table t_k with customers whose $t_{ji} = t_k$) Sample a proposal t_{k*} according to the weight and make the new table:

(%Sample a proposal $\{t_{k_1}, \dots, t_{k_K}\} \sim e^{r_{proposal}\{\Delta k_1, \dots, \Delta k_K\}}$)

(% $r_{proposal} > 0$, the more decrease of Δk , the less propable to form table t_k)

(b) Possible Dish=Possible Dish \ k_*

(iv) If there are still customers left in t_0 , make it a new table with a new dish

(B) Refined reconfiguration for Restaurant j:TKM(j,**Temperature**):

(%Divide the change of L between present Restaurant j config and its previous config into t-term, k-term change:
 $\Delta L = \Delta K + \Delta T$)

(C) Decision:

If($\Delta K + \text{Temperature} * \Delta T < 0$):

Accept the new configuration

else:

Restore Previous Config

End

iii) Merge Dish(Temperature)

Dish List= $\{Nonempty\ dishes\}$

While Dish List is not empty:

1) Randomly pick a dish $k \in$ Dish List

2) Dish List=Dish List $\setminus k$

3) Merge dish k to the dish \in Dish List which increase $\mathbf{L' (Temperature)}$ mostly
(if cannot increase $\mathbf{L' (Temperature)}$, then leave it alone)

End

iv) TKM(Restaurant index,Temperature)

While $\mathbf{L' (Temperature)}$ doesn't increase any more:

(A) Local Search Table:

For $t_1 = \text{Randperm}(m_j)$:

For $t_2 = \text{Randperm}(m_j \setminus t_1)$:

While local move can be made:

Greedily move one customer at a time from t_1 to t_2 if the move increases L

End

End

End

(B) Local Search Dish:

For $t_1 = \text{Randperm}(m_j)$:

Assign t_1 to the dish which increase L most(allow it to have new dish)

End

(C) Merge table:

For $t_1 = \text{Randperm}(m_j)$:

Merge table t_1 to the table in j which increase $\mathbf{L' (Temperature)}$ most

(if cannot increase $\mathbf{L' (Temperature)}$, then leave it alone)

End

End

2) Annealing

Things to play with:

1. Parameters to tune:
 - (a) Annealing Scheme: n: number of iterations; p: annealing power
 - (b) $r_{proposal}$: constant or annealed? (we may want it peaky in the beginning and allow more variability later on)
2. Functions to Anneal: ("Merge table" is annealed, "Local-Search-Dish" doesn't change t-term, thus no anneal needed)
 - 1) Anneal "Local-table" and "Merge dish"?

P.S. The tests are done on a 5 by 5 matrix (10 bars) with 40 restaurants, because the problem becomes easier with more restaurants.

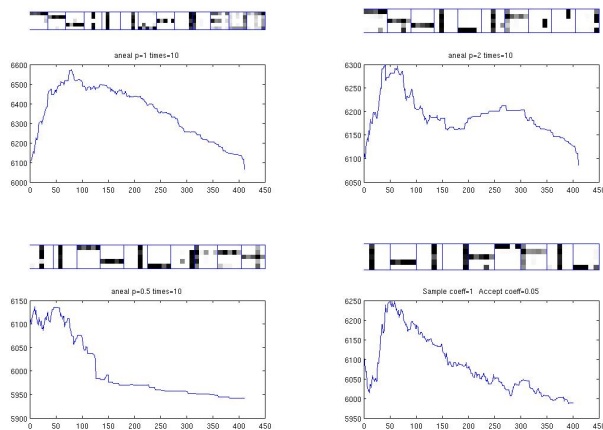


Figure 1: Down-Right is previous no-annealing result, the annealed ones (constant $r_{proposal}$) do not look good

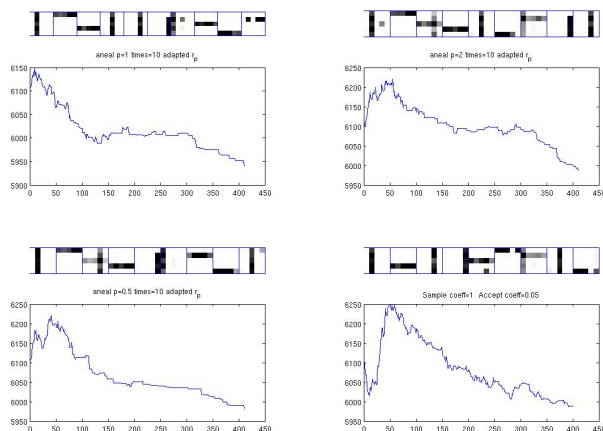


Figure 2: Using annealed $r_{proposal}$, $p=1$ is even better than the no-annealed one (down-right) by finding right number bars

i) Tuning parameters:

1) Annealing Scheme: $n=10$; $p \in \{0.5, 1, 2\}$;

2) Annealed $r_{proposal}$;

Simply, I set $r_{proposal} = \frac{1}{T}$, where we want harder proposal assignment in the beginning and softer one later on.

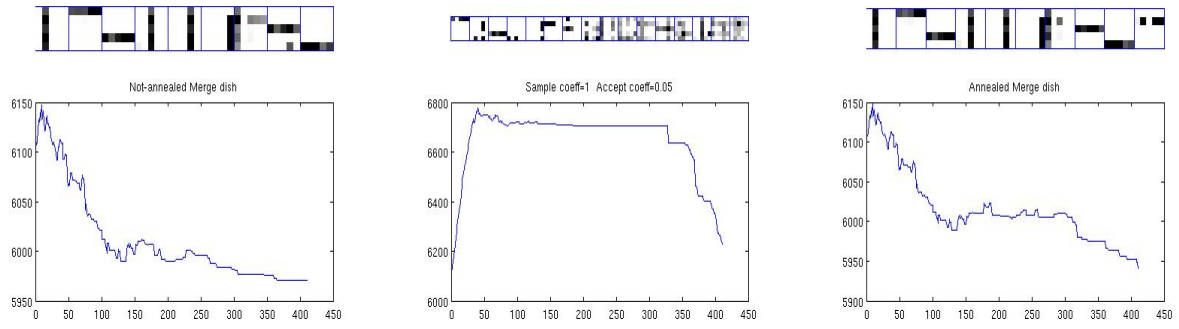


Figure 3: (left)no anneal local-table,merge-dish;(mid)anneal local-table only(right)anneal merge-dish only

ii)Annealed Local-table and Merge Dish:

From Figure 3, we can see that:

- 1) annealing merge dish(right) can be also useful to prevent mixture of bars;
- 2) annealing local table(left) is harmful to prevent tables from communicating with each other;