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1 Data structures

1.1 Disjoint Sparse Table

Answers queries of any monoid operation (i.e. has identity element and is associative)

Build: $O(N \log N)$, Query: $O(1)$

```
#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct DisjointSparseTable {
    using Operation = T (*)(T, T);

    vector<vector<T>> st;
    Operation f;
    T identity;

    static constexpr int log2_floor(unsigned long long i) noexcept {
        return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
    }

    // Lazy loading constructor. Needs to call build!
    DisjointSparseTable(Operation op, const T neutral = T())
        : st(), f(op), identity(neutral) {}

    DisjointSparseTable(vector<T> v) : DisjointSparseTable(v, F(min(a, b))) {}

    DisjointSparseTable(vector<T> v, Operation op, const T neutral = T())
        : st(), f(op), identity(neutral) {
        build(v);
    }

    void build(vector<T> v) {
        st.resize(log2_floor(v.size()) + 1,
            vector<T>(1ll << (log2_floor(v.size()) + 1)));
        v.resize(st[0].size(), identity);
        for (int level = 0; level < (int)st.size(); ++level) {
            for (int block = 0; block < (1 << level); ++block) {
                const auto l = block << (st.size() - level);
                const auto r = (block + 1) << (st.size() - level);
                const auto m = l + (r - l) / 2;

                st[level][m] = v[m];
                for (int i = m + 1; i < r; i++)
                    st[level][i] = f(st[level][i - 1], v[i]);
                st[level][m - 1] = v[m - 1];
                for (int i = m - 2; i >= l; i--)
                    st[level][i] = f(st[level][i + 1], v[i]);
            }
        }
    }

    T query(int l, int r) const {
        if (l > r) return identity;
        if (l == r) return st.back()[l];

        const auto k = log2_floor(l ^ r);
        const auto level = (int)st.size() - 1 - k;
        return f(st[level][l], st[level][r]);
    }
};
```

```
};
```

1.2 Dsu

```
struct DSU {
    vi ps;
    vi size;
    DSU(int N) : ps(N + 1), size(N + 1, 1) { iota(all(ps), 0); }
    int find_set(int x) { return ps[x] == x ? x : ps[x] = find_set(ps[x]); }
    bool same_set(int x, int y) { return find_set(x) == find_set(y); }
    void union_set(int x, int y) {
        if (same_set(x, y)) return;

        int px = find_set(x);
        int py = find_set(y);

        if (size[px] < size[py]) swap(px, py);

        ps[py] = px;
        size[px] += size[py];
    }
};
```

1.3 Ordered Set

If you need an ordered **multiset** you may add an id to each value. Using `greater_equal`, or `less_equal` is considered undefined behavior.

- `order_of_key(k)` : Number of items strictly smaller/greater than `k`.
- `find_by_order(k)` : `K`-th element in a set (counting from zero).

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set =
    tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
```

1.4 SegTree Point Update (dynamic function)

Answers queries of any monoid operation (i.e. has identity element and is associative)

Build: $O(N)$, Query: $O(\log N)$

```
#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct SegTree {
    using Operation = T (*)(T, T);

    int N;
    vector<T> ns;
    Operation operation;
    T identity;

    SegTree(int n, Operation op = F(a + b), T neutral = T())
        : N(n), ns(2 * N, neutral), operation(op), identity(neutral) {}

    SegTree(const vector<T> &v, Operation op = F(a + b), T neutral = T())
```

```

: SegTree((int)v.size(), op, neutral) {
copy(v.begin(), v.end(), ns.begin() + N);

for (int i = N - 1; i > 0; --i) ns[i] = operation(ns[2 * i], ns[2 * i +
1]);
}

T query(size_t i) const { return ns[i + N]; }

T query(size_t l, size_t r) const {
auto a = l + N, b = r + N;
auto ans = identity;

while (a <= b) {
if (a & 1) ans = operation(ans, ns[a++]);
if (not(b & 1)) ans = operation(ans, ns[b--]);

a /= 2;
b /= 2;
}

return ans;
}

void update(size_t i, T value) { update_set(i, operation(ns[i + N], value));
}

void update_set(size_t i, T value) {
auto a = i + N;

ns[a] = value;
while (a >= 1) ns[a] = operation(ns[2 * a], ns[2 * a + 1]);
}
};

```

1.5 Segtree Range Max Query Range Max Update

```

template <typename T = ll>
struct SegTree {
int N;
T nu, nq;
vector<T> st, lazy;
SegTree(const vector<T> &xs)
: N(len(xs)),
nu(numeric_limits<T>::min()),
nq(numeric_limits<T>::min()),
st(4 * N + 1, nu),
lazy(4 * N + 1, nu) {
for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
}

void update(int l, int r, T value) { update(1, 0, N - 1, l, r, value); }

T query(int l, int r) { return query(1, 0, N - 1, l, r); }

void update(int node, int nl, int nr, int ql, int qr, T v) {
propagation(node, nl, nr);

```

```

if (ql > nr or qr < nl) return;

st[node] = max(st[node], v);
if (ql <= nl and nr <= qr) {
if (nl < nr) {
lazy[left(node)] = max(lazy[left(node)], v);
lazy[right(node)] = max(lazy[right(node)], v);
}
return;
}
update(left(node), nl, mid(nl, nr), ql, qr, v);
update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

st[node] = max(st[left(node)], st[right(node)]);
}

T query(int node, int nl, int nr, int ql, int qr) {
propagation(node, nl, nr);

if (ql > nr or qr < nl) return nq;

if (ql <= nl and nr <= qr) return st[node];

T x = query(left(node), nl, mid(nl, nr), ql, qr);
T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

return max(x, y);
}

void propagation(int node, int nl, int nr) {
if (lazy[node] != nu) {
st[node] = max(st[node], lazy[node]);

if (nl < nr) {
lazy[left(node)] = max(lazy[left(node)], lazy[node]);
lazy[right(node)] = max(lazy[right(node)], lazy[node]);
}

lazy[node] = nu;
}
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

int main() {
int n;
cin >> n;
vector<array<int, 3>> xs(n);
for (int i = 0; i < n; ++i) {
for (int j = 0; j < 3; ++j) {
cin >> xs[i][j];
}
}
vi aux(n, 0);
SegTree<int> st(aux);

```

```

for (int i = 0; i < n; ++i) {
    int a = min(i + xs[i][1], n);
    int b = min(i + xs[i][2], n);
    st.update(i, i, st.query(i, i) + xs[i][0]);
    int cur = st.query(i, i);
    st.update(a, b, cur);
}

cout << st.query(0, n) << '\n';
}

```

1.6 SegTree Range Min Query Point Assign Update

```

template <typename T = ll>
struct SegTree {
    int n;
    T nu, nq;
    vector<T> st;
    SegTree(const vector<T> &v)
        : n(len(v)), nu(0), nq(numeric_limits<T>::max()), st(n * 4 + 1, nu) {
        for (int i = 0; i < n; ++i) update(i, v[i]);
    }
    void update(int p, T v) { update(1, 0, n - 1, p, v); }
    T query(int l, int r) { return query(1, 0, n - 1, l, r); }

    void update(int node, int nl, int nr, int p, T v) {
        if (p < nl or p > nr) return;

        if (nl == nr) {
            st[node] = v;
            return;
        }

        update(left(node), nl, mid(nl, nr), p, v);
        update(right(node), mid(nl, nr) + 1, nr, p, v);

        st[node] = min(st[left(node)], st[right(node)]);
    }

    T query(int node, int nl, int nr, int ql, int qr) {
        if (ql <= nl and qr >= nr) return st[node];
        if (nl > qr or nr < ql) return nq;
        if (nl == nr) return st[node];

        return min(query(left(node), nl, mid(nl, nr), ql, qr),
                    query(right(node), mid(nl, nr) + 1, nr, ql, qr));
    }

    int left(int p) { return p << 1; }
    int right(int p) { return (p << 1) + 1; }
    int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

1.7 SegTree Range Min Query Range Sum Update

```

template <typename t = ll>
struct SegTree {
    int n;

```

```

    t nu;
    t nq;
    vector<t> st, lazy;
    SegTree(const vector<t> &xs)
        : n(len(xs)),
          nu(0),
          nq(numeric_limits<t>::max()),
          st(4 * n, nu),
          lazy(4 * n, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    SegTree(int n) : n(n), st(4 * n, nu), lazy(4 * n, nu) {}

    void update(int l, int r, ll value) { update(1, 0, n - 1, l, r, value); }

    t query(int l, int r) { return query(1, 0, n - 1, l, r); }

    void update(int node, int nl, int nr, int ql, int qr, ll v) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return;

        if (ql <= nl and nr <= qr) {
            st[node] += (nr - nl + 1) * v;

            if (nl < nr) {
                lazy[left(node)] += v;
                lazy[right(node)] += v;
            }

            return;
        }

        update(left(node), nl, mid(nl, nr), ql, qr, v);
        update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

        st[node] = min(st[left(node)], st[right(node)]);
    }

    t query(int node, int nl, int nr, int ql, int qr) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return nq;

        if (ql <= nl and nr <= qr) return st[node];

        t x = query(left(node), nl, mid(nl, nr), ql, qr);
        t y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

        return min(x, y);
    }

    void propagation(int node, int nl, int nr) {
        if (lazy[node]) {
            st[node] += lazy[node];

            if (nl < nr) {

```

```

        lazy[left(node)] += lazy[node];
        lazy[right(node)] += lazy[node];
    }

    lazy[node] = nu;
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

1.8 SegTree Range Sum Query Range Sum Update

```

template <typename T = ll>
struct SegTree {
    int N;
    vector<T> st, lazy;
    T nu = 0;
    T nq = 0;
    SegTree(const vector<T> &xs) : N(len(xs)), st(4 * N, nu), lazy(4 * N, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    SegTree(int n) : N(n), st(4 * N, nu), lazy(4 * N, nu) {}

    void update(int l, int r, ll value) { update(1, 0, N - 1, l, r, value); }

    T query(int l, int r) { return query(1, 0, N - 1, l, r); }

    void update(int node, int nl, int nr, int ql, int qr, ll v) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return;

        if (ql <= nl and nr <= qr) {
            st[node] += (nr - nl + 1) * v;

            if (nl < nr) {
                lazy[left(node)] += v;
                lazy[right(node)] += v;
            }

            return;
        }

        update(left(node), nl, mid(nl, nr), ql, qr, v);
        update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

        st[node] = st[left(node)] + st[right(node)];
    }

    T query(int node, int nl, int nr, int ql, int qr) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return nq;
    }
};

```

```

    if (ql <= nl and nr <= qr) return st[node];

    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return x + y;
}

void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
        st[node] += (nr - nl + 1) * lazy[node];

        if (nl < nr) {
            lazy[left(node)] += lazy[node];
            lazy[right(node)] += lazy[node];
        }

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

1.9 Sparse Table Range Min Query

Build: $O(N \log N)$, Query: $O(1)$

```

int fastlog2(ll x) {
    ull i = x;
    return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
}

template <typename T>
class SparseTable {
public:
    int N;
    int K;
    vector<vector<T>> st;
    SparseTable(vector<T> vs)
        : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector<T>(N + 1)) {
        copy(vs.begin(), vs.end(), st[0].begin());

        for (int i = 1; i <= K; ++i)
            for (int j = 0; j + (1 << i) <= N; ++j)
                st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
    }

    T RMQ(int l, int r) { // [l, r], 0 indexed
        int i = fastlog2(r - l + 1);
        return min(st[i][l], st[i][r - (1 << i) + 1]);
    }
};

```

2 Dynamic programming

2.1 Edit Distance

$O(N * M)$

```
int edit_distance(const string &a, const string &b) {
    int n = a.size();
    int m = b.size();
    vector<vi> dp(n + 1, vi(m + 1, 0));

    int ADD = 1, DEL = 1, CHG = 1;
    for (int i = 0; i <= n; ++i) {
        dp[i][0] = i * DEL;
    }
    for (int i = 1; i <= m; ++i) {
        dp[0][i] = ADD * i;
    }

    for (int i = 1; i <= n; ++i) {
        for (int j = 1; j <= m; ++j) {
            int add = dp[i][j - 1] + ADD;
            int del = dp[i - 1][j] + DEL;
            int chg = dp[i - 1][j - 1] + (a[i - 1] == b[j - 1] ? 0 : 1) * CHG;
            dp[i][j] = min({add, del, chg});
        }
    }

    return dp[n][m];
}
```

2.2 Kadane

Find the maximum subarray sum in a given array.

```
int kadane(const vi &as) {
    vi s(len(as));
    s[0] = as[0];

    for (int i = 1; i < len(as); ++i) s[i] = max(as[i], s[i - 1] + as[i]);

    return *max_element(all(s));
}
```

2.3 Knapsack (value)

Finds the maximum points possible

```
const int MAXN{2010}, MAXM{2010};

ll st[MAXN][MAXM];

ll dp(int i, int m, int M, const vii &cs) {
    if (i < 0) return 0;

    if (st[i][m] != -1) return st[i][m];

    auto res = dp(i - 1, m, M, cs);
```

```
    auto [w, v] = cs[i];

    if (w <= m) res = max(res, dp(i - 1, m - w, M, cs) + v);

    st[i][m] = res;
    return res;
}

ll knapsack(int M, const vii &cs) {
    memset(st, -1, sizeof st);

    return dp((int)cs.size() - 1, M, M, cs);
}
```

2.4 Knapsack (elements)

Finds the maximum possible points carry and which elements to achieve it

```
const int MAXN{2010}, MAXM{2010};
ll st[MAXN][MAXM];
char ps[MAXN][MAXM];

pair<ll, vi> knapsack(int M, const vii &cs) {
    int N = len(cs) - 1;

    for (int i = 0; i <= N; ++i) st[i][0] = 0;

    for (int m = 0; m <= M; ++m) st[0][m] = 0;

    for (int i = 1; i <= N; ++i) {
        for (int m = 1; m <= M; ++m) {
            st[i][m] = st[i - 1][m];
            ps[i][m] = 0;
            auto [w, v] = cs[i];

            if (w <= m and st[i - 1][m - w] + v > st[i][m]) {
                st[i][m] = st[i - 1][m - w] + v;
                ps[i][m] = 1;
            }
        }
    }

    int m = M;
    vi is;
    for (int i = N; i >= 1; --i) {
        if (ps[i][m]) {
            is.push_back(i);
            m -= cs[i].first;
        }
    }

    reverse(all(is));

    // max value, items
    return {st[N][M], is};
}
```

2.5 Longest Increasing Subsequence (LIS)

Finds the length of the longest subsequence in

$O(n \log n)$

```
.
int LIS(const vi& as) {
    const ll oo = 1e18;
    int n = len(as);
    vll lis(n + 1, oo);
    lis[0] = -oo;

    auto ans = 0;

    for (int i = 0; i < n; ++i) {
        auto it = lower_bound(all(lis), as[i]);
        auto pos = (int)(it - lis.begin());

        ans = max(ans, pos);
        lis[pos] = as[i];
    }

    return ans;
}
```

2.6 Money Sum (Bottom Up)

Find every possible sum using the given values only once.

```
set<int> money_sum(const vi &xs) {
    using vc = vector<char>;
    using vvc = vector<vc>;
    int _m = accumulate(all(xs), 0);
    int _n = xs.size();
    vvc _dp(_n + 1, vc(_m + 1, 0));
    set<int> _ans;
    _dp[0][xs[0]] = 1;
    for (int i = 1; i < _n; ++i) {
        for (int j = 0; j <= _m; ++j) {
            if (j == 0 or _dp[i - 1][j]) {
                _dp[i][j + xs[i]] = 1;
                _dp[i][j] = 1;
            }
        }
    }

    for (int i = 0; i < _n; ++i)
        for (int j = 0; j <= _m; ++j)
            if (_dp[i][j]) _ans.insert(j);
    return _ans;
}
```

2.7 Travelling Salesman Problem

```
using vi = vector<int>;
vector<vi> dist;
```

```
vector<vi> memo;
/* 0 ( N^2 * 2^N )*/
int tsp(int i, int mask, int N) {
    if (mask == (1 << N) - 1) return dist[i][0];
    if (memo[i][mask] != -1) return memo[i][mask];
    int ans = INT_MAX << 1;
    for (int j = 0; j < N; ++j) {
        if (mask & (1 << j)) continue;
        auto t = tsp(j, mask | (1 << j), N) + dist[i][j];
        ans = min(ans, t);
    }
    return memo[i][mask] = ans;
}
```

3 Geometry

3.1 Convex Hull

Given a set of points find the smallest convex polygon that contains all the given points.

Time: $O(N \log N)$

By default it removes the collinear points, set the boolean to true if you don't want that

```
struct pt {
    double x, y;
    int id;
};

int orientation(pt a, pt b, pt c) {
    double v = a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y - b.y);
    if (v < 0) return -1; // clockwise
    if (v > 0) return +1; // counter-clockwise
    return 0;
}

bool cw(pt a, pt b, pt c, bool include_collinear) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}

bool collinear(pt a, pt b, pt c) { return orientation(a, b, c) == 0; }

void convex_hull(vector<pt>& pts, bool include_collinear = false) {
    pt p0 = *min_element(all(pts), [](pt a, pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
    sort(all(pts), [&p0](const pt& a, const pt& b) {
        int o = orientation(p0, a, b);
        if (o == 0)
            return (p0.x - a.x) * (p0.x - a.x) + (p0.y - a.y) * (p0.y - a.y) <
                (p0.x - b.x) * (p0.x - b.x) + (p0.y - b.y) * (p0.y - b.y);
        return o < 0;
    });
    if (include_collinear) {
        int i = len(pts) - 1;
        while (i >= 0 && collinear(p0, pts[i], pts.back())) i--;
        reverse(pts.begin() + i + 1, pts.end());
    }

    vector<pt> st;
```

```

for (int i = 0; i < len(pts); i++) {
    while (st.size() > 1 &&
           !cw(st[len(st) - 2], st.back(), pts[i], include_collinear))
        st.pop_back();
    st.push_back(pts[i]);
}

pts = st;
}

```

3.2 Determinant

```
#include "Point.cpp"
```

```

template <typename T>
T D(const Point<T> &P, const Point<T> &Q, const Point<T> &R) {
    return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
           (R.x * Q.y + R.y * P.x + Q.x * P.y);
}

```

3.3 Equals

```

template <typename T>
bool equals(T a, T b) {
    const double EPS{1e-9};
    if (is_floating_point<T>::value)
        return fabs(a - b) < EPS;
    else
        return a == b;
}

```

3.4 Line

```
#include <bits/stdc++.h>
```

```
#include "point-struct-and-utils.cpp"
using namespace std;
```

```

struct line {
    ld a, b, c;
};

```

```

// the answer is stored in the third parameter (pass by reference)
void pointsToLine(const point &p1, const point &p2, line &l) {
    if (fabs(p1.x - p2.x) < EPS)
        // vertical line
        l = {1.0, 0.0, -p1.x};
    // default values
    else
        l = {-(ld)(p1.y - p2.y) / (p1.x - p2.x), 1.0, -(ld)(l.a * p1.x) - p1.y};
}

```

3.5 Point Struct And Utils (2d)

```
#include <bits/stdc++.h>
using namespace std;
```

```
using ld = long double;
```

```

struct point {
    ld x, y;
    int id;
    point(ld x = 0.0, ld y = 0.0, int id = -1) : x(x), y(y), id(id) {}

    point& operator+=(const point& t) {
        x += t.x;
        y += t.y;
        return *this;
    }
    point& operator-=(const point& t) {
        x -= t.x;
        y -= t.y;
        return *this;
    }
    point& operator*=(ld t) {
        x *= t;
        y *= t;
        return *this;
    }
    point& operator/=(ld t) {
        x /= t;
        y /= t;
        return *this;
    }
    point operator+(const point& t) const { return point(*this) += t; }
    point operator-(const point& t) const { return point(*this) -= t; }
    point operator*(ld t) const { return point(*this) *= t; }
    point operator/(ld t) const { return point(*this) /= t; }
};

```

```
ld dot(point& a, point& b) { return a.x * b.x + a.y * b.y; }
```

```
ld norm(point& a) { return dot(a, a); }
```

```
ld abs(point a) { return sqrt(norm(a)); }
```

```
ld proj(point a, point b) { return dot(a, b) / abs(b); }
```

```
ld angle(point a, point b) { return acos(dot(a, b) / abs(a) / abs(b)); }
```

```
ld cross(point a, point b) { return a.x * b.y - a.y * b.x; }
```

3.6 Segment

```
#include "Line.cpp"
#include "Point.cpp"
#include "equals.cpp"
```

```

template <typename T>
struct segment {
    Point<T> A, B;

```

```
    bool contains(const Point<T> &P) const;
```

```
    Point<T> closest(const Point<T> &p) const;
```



```

};

template <typename T>
bool segment<T>::contains(const Point<T> &P) const {
    // verifica se P áest contido na reta
    double dAB = Point<T>::dist(A, B), dAP = Point<T>::dist(A, P),
        dPB = Point<T>::dist(P, B);

    return equals(dAP + dPB, dAB);
}

template <typename T>
Point<T> segment<T>::closest(const Point<T> &P) const {
    Line<T> R(A, B);
    auto Q = R.closest(P);

    if (this->contains(Q)) return Q;

    auto distA = Point<T>::dist(P, A);
    auto distB = Point<T>::dist(P, B);

    if (distA <= distB)
        return A;
    else
        return B;
}

```

4 Graphs

4.1 2 SAT

```

struct SAT2 {
    ll n;
    vll2d adj, adj_t;
    vc used;
    vll order, comp;
    vc assignment;
    bool solvable;
    SAT2(ll _n)
        : n(2 * _n),
          adj(n),
          adj_t(n),
          used(n),
          order(n),
          comp(n, -1),
          assignment(n / 2) {}
    void dfs1(int v) {
        used[v] = true;
        for (int u : adj[v]) {
            if (!used[u]) dfs1(u);
        }
        order.push_back(v);
    }

    void dfs2(int v, int c1) {
        comp[v] = c1;
        for (int u : adj_t[v]) {

```

```

            if (comp[u] == -1) dfs2(u, c1);
        }
    }

    bool solve_2SAT() {
        // find and label each SCC
        for (int i = 0; i < n; ++i) {
            if (!used[i]) dfs1(i);
        }
        reverse(all(order));
        ll j = 0;
        for (auto &v : order) {
            if (comp[v] == -1) dfs2(v, j++);
        }

        assignment.assign(n / 2, false);
        for (int i = 0; i < n; i += 2) {
            // x and !x belong to the same SCC
            if (comp[i] == comp[i + 1]) {
                solvable = false;
                return false;
            }

            assignment[i / 2] = comp[i] > comp[i + 1];
        }
        solvable = true;
        return true;
    }

    void add_disjunction(int a, bool na, int b, bool nb) {
        a = (2 * a) ^ na;
        b = (2 * b) ^ nb;
        int neg_a = a ^ 1;
        int neg_b = b ^ 1;
        adj[neg_a].push_back(b);
        adj[neg_b].push_back(a);
        adj_t[b].push_back(neg_a);
        adj_t[a].push_back(neg_b);
    }
};

```

4.2 SCC (struct)

Able to find the component of each node and the total of SCC in $O(V * E)$ and build the SCC graph ($O(V * E)$).

```

struct SCC {
    ll N;
    int totsc;
    vll2d adj, tadj;
    vll todo, comps, comp;
    vector<set<ll>> sccadj;
    vchar vis;
    SCC(ll _N)
        : N(_N), totsc(0), adj(_N), tadj(_N), comp(_N, -1), sccadj(_N), vis(_N)
        {}

    void add_edge(ll x, ll y) { adj[x].eb(y), tadj[y].eb(x); }

```

```

void dfs(ll x) {
    vis[x] = 1;
    for (auto &y : adj[x])
        if (!vis[y]) dfs(y);
    todo.pb(x);
}

void dfs2(ll x, ll v) {
    comp[x] = v;
    for (auto &y : tadj[x])
        if (comp[y] == -1) dfs2(y, v);
}

void gen() {
    for (ll i = 0; i < N; ++i)
        if (!vis[i]) dfs(i);
    reverse(all(todo));
    for (auto &x : todo)
        if (comp[x] == -1) {
            dfs2(x, x);
            comps.pb(x);
            totscscc++;
        }
}

void genSCCGraph() {
    for (ll i = 0; i < N; ++i) {
        for (auto &j : adj[i]) {
            if (comp[i] != comp[j]) {
                sccadj[comp[i]].insert(comp[j]);
            }
        }
    }
}

};

```

4.3 Bellman Ford

Find shortest path from a single source to all other nodes. Can detect negative cycles.
Time: $O(V * E)$

```

bool bellman_ford(const vector<vector<pair<int, ll>>> &g, int s,
                 vector<ll> &dist) {
    int n = (int)g.size();
    dist.assign(n, LLONG_MAX);

    vector<int> count(n);
    vector<char> in_queue(n);
    queue<int> q;

    dist[s] = 0;
    q.push(s);
    in_queue[s] = true;

    while (not q.empty()) {
        int cur = q.front();
        q.pop();
        in_queue[cur] = false;

        for (auto [to, w] : g[cur]) {

```

```

            if (dist[cur] + w < dist[to]) {
                dist[to] = dist[cur] + w;
                if (not in_queue[to]) {
                    q.push(to);
                    in_queue[to] = true;
                    count[to]++;
                    if (count[to] > n) return false;
                }
            }
        }
    }

    return true;
}

```

4.4 Binary Lifting

$far[h][i]$ = the node that is 2^h distance from node i

Build : $O(N * \log N)$

sometimes is useful invert the order of loops

```

const int maxlog = 20;
int far[maxlog + 1][n + 1];
int n;
for (int h = 1; h <= maxlog; h++) {
    for (int i = 1; i <= n; i++) {
        far[h][i] = far[h - 1][far[h - 1][i]];
    }
}

```

4.5 Check Bipartite

$O(V)$

```

bool checkBipartite(const ll n, const vector<vll> &adj) {
    ll s = 0;
    queue<ll> q;
    q.push(s);
    vll color(n, INF);
    color[s] = 0;
    bool isBipartite = true;
    while (!q.empty() && isBipartite) {
        ll u = q.front();
        q.pop();
        for (auto &v : adj[u]) {
            if (color[v] == INF) {
                color[v] = 1 - color[u];
                q.push(v);
            } else if (color[v] == color[u]) {
                return false;
            }
        }
    }
    return true;
}

```

4.6 Dijkstra

Finds the minimum distance from s to every other node in

$$O(E * \log E)$$

time.

```
vll dijkstra(const vector<vector<pll>> &g, int n, int s) {
    priority_queue<pll, vector<pll>, greater<pll>> pq;
    vll dist(n + 1, oo);
    vector<char> vis(n + 1);
    pq.emplace(0, s);
    dist[s] = 0;
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();
        if (vis[v]) continue;
        vis[v] = true;

        for (auto [d2, u] : g[v]) {
            if (dist[u] > d1 + d2) {
                dist[u] = d1 + d2;
                pq.emplace(dist[u], u);
            }
        }
    }
    return dist;
}
```

4.7 Euler Path

Find a path that visits every edge exactly once.

Time: $O(E)$

graphs with sets are undirected, graphs with vectors are directed

// Directed Edges

```
vector<int> euler_cycle(vector<vector<int>> &g, int u) {
    vector<int> res;
```

```
    stack<int> st;
    st.push(u);
    while (!st.empty()) {
        auto cur = st.top();
        if (g[cur].empty()) {
            res.push_back(cur);
            st.pop();
        } else {
            auto next = g[cur].back();
            st.push(next);

            g[cur].pop_back();
        }
    }
```

```
    for (auto &x : g)
        if (!x.empty()) return {};

    return res;
```

```
}
```

// Directed Edges

```
vector<int> euler_path(vector<vector<int>> &g, int first) {
    {
        int n = (int)g.size();
        vector<int> in(n), out(n);
        for (int i = 0; i < n; i++)
            for (auto x : g[i]) in[x]++, out[i]++;
```

```
        int a = 0, b = 0, c = 0;
        for (int i = 0; i < n; i++)
            if (in[i] == out[i])
                c++;
            else if (in[i] - out[i] == 1)
                b++;
            else if (in[i] - out[i] == -1)
                a++;
```

```
        if (c != n - 2 or a != 1 or b != 1) return {};
    }
}
```

```
auto res = euler_cycle(g, first);
if (res.empty()) return res;
```

```
reverse(all(res));
return res;
```

```
}
```

// Undirected Edges

```
vector<int> euler_cycle(vector<set<int>> &g, int u) {
    vector<int> res;
```

```
    stack<int> st;
    st.push(u);
    while (!st.empty()) {
        auto cur = st.top();
        if (g[cur].empty()) {
            res.push_back(cur);
            st.pop();
        } else {
            auto next = *g[cur].begin();
            st.push(next);

            g[cur].erase(next);
            g[next].erase(cur);
        }
    }
```

```
    for (auto &x : g)
        if (!x.empty()) return {};
}
```

```
    return res;
```

```
}
```

// Undirected edges

```
vector<int> euler_path(vector<set<int>> &g, int first) {
    int n = (int)g.size();
```

```

int v1 = -1, v2 = -1;
{
    bool bad = false;
    for (int i = 0; i < n; i++)
        if (g[i].size() & 1) {
            if (v1 == -1)
                v1 = i;
            else if (v2 == -1)
                v2 = i;
            else
                bad = true;
        }

    if (bad or (v1 != -1 and v2 == -1)) return {};
}

if (v1 != -1) {
    // insert cycle
    g[v1].insert(v2);
    g[v2].insert(v1);
}

auto res = euler_cycle(g, first);
if (res.empty()) return res;

if (v1 != -1) {
    for (int i = 0; i + 1 < (int)res.size(); i++) {
        if ((res[i] == v1 and res[i + 1] == v2) ||
            (res[i] == v2 and res[i + 1] == v1)) {
            vector<int> res2;
            for (int j = i + 1; j < (int)res.size(); j++) res2.push_back(res[j]);
            for (int j = 1; j <= i; j++) res2.push_back(res[j]);
            res = res2;
            break;
        }
    }
}

reverse(all(res));
return res;
}

```

4.8 Floyd Warshall

Simply finds the minimal distance for each node to every other node. $O(V^3)$

```

vector<vll> floyd_warshall(const vector<vll> &adj, ll n) {
    auto dist = adj;

    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            for (int k = 0; k < n; ++k) {
                dist[j][k] = min(dist[j][k], dist[j][i] + dist[i][k]);
            }
        }
    }
    return dist;
}

```

4.9 Graph Cycle

Detects if a graph contains a cycle. If path parameter is not null, it will contain the cycle if one exists.

Time: $O(V + E)$

```

bool has_cycle(const vector<vector<int>> &g, int s, vector<char> &vis,
               vector<char> &in_path, vector<int> *path = nullptr) {
    vis[s] = in_path[s] = 1;
    if (path != nullptr) path->push_back(s);
    for (auto x : g[s]) {
        if (!vis[x] && has_cycle(g, x, vis, in_path, path))
            return true;
        else if (in_path[x]) {
            if (path != nullptr) path->push_back(x);
            return true;
        }
    }
    in_path[s] = 0;
    if (path != nullptr) path->pop_back();
    return false;
}

```

4.10 Kruskal

Find the minimum spanning tree of a graph.

Time: $O(E \log E)$

can be used to find the maximum spanning tree by changing the comparison operator in the sort

```

struct UFDS {
    vector<int> ps, sz;
    int components;

    UFDS(int n) : ps(n + 1), sz(n + 1, 1), components(n) { iota(all(ps), 0); }

    int find_set(int x) { return (x == ps[x] ? x : (ps[x] = find_set(ps[x]))); }

    bool same_set(int x, int y) { return find_set(x) == find_set(y); }

    void union_set(int x, int y) {
        x = find_set(x);
        y = find_set(y);

        if (x == y) return;

        if (sz[x] < sz[y]) swap(x, y);

        ps[y] = x;
        sz[x] += sz[y];

        components--;
    }
};

vector<tuple<ll, int, int>> kruskal(int n, vector<tuple<ll, int, int>> &edges)
{
    UFDS ufds(n);
    vector<tuple<ll, int, int>> ans;
}

```

```

sort(all(edges));
for (auto [a, b, c] : edges) {
    if (ufds.same_set(b, c)) continue;

    ans.emplace_back(a, b, c);
    ufds.union_set(b, c);
}

return ans;
}

```

4.11 Lowest Common Ancestor

Given two nodes find the lowest common ancestor of both.

Build : $O(V)$, Query: $O(1)$

```

int fastlog2(ll x) {
    ull i = x;
    return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
}

template <typename T>
class SparseTable {
public:
    int N;
    int K;
    vector<vector<T>> st;
    SparseTable(vector<T> vs)
        : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector<T>(N + 1)) {
        copy(vs.begin(), vs.end(), st[0].begin());

        for (int i = 1; i <= K; ++i)
            for (int j = 0; j + (1 << i) <= N; ++j)
                st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
    }
    SparseTable() {}
    T RMQ(int l, int r) {
        int i = fastlog2(r - l + 1);
        return min(st[i][l], st[i][r - (1 << i) + 1]);
    }
};

class LCA {
public:
    int p;
    int n;
    vi first;
    vector<char> visited;
    vi vertices;
    vi height;
    SparseTable<int> st;

    LCA(const vector<vi> &g)
        : p(0), n((int)g.size()), first(n + 1), visited(n + 1, 0), height(n + 1) {
        build_dfs(g, 1, 1);
        st = SparseTable<int>(vertices);
    }

    void build_dfs(const vector<vi> &g, int u, int hi) {
        visited[u] = true;

```

```

        height[u] = hi;
        first[u] = vertices.size();
        vertices.push_back(u);
        for (auto uv : g[u]) {
            if (!visited[uv]) {
                build_dfs(g, uv, hi + 1);
                vertices.push_back(u);
            }
        }
    }

    int lca(int a, int b) {
        int l = min(first[a], first[b]);
        int r = max(first[a], first[b]);
        return st.RMQ(l, r);
    }
};

```

4.12 Tree Maximum Distance

Returns the maximum distance from every node to any other node in the tree. $O(6V) = O(V)$

```

pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
    // 0(V)
    // 0 indexed
    ll mostDistantNode = root;
    ll nodeDistance = 0;
    queue<pll> q;
    vector<char> vis(n);
    q.emplace(root, 0);
    vis[root] = true;
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (dist > nodeDistance) {
            nodeDistance = dist;
            mostDistantNode = node;
        }
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return {mostDistantNode, nodeDistance};
}

ll twoNodesDist(const vector<vll> &adj, ll n, ll a, ll b) {
    queue<pll> q;
    vector<char> vis(n);
    q.emplace(a, 0);
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (node == b) return dist;
        for (auto u : adj[node]) {
            if (!vis[u]) {

```

```

        vis[u] = true;
        q.emplace(u, dist + 1);
    }
}
}
return -1;
}

tuple<ll, ll, ll> tree_diameter(const vector<vll> &adj, ll n) {
    // returns two points of the diameter and the diameter itself
    auto [node1, dist1] = mostDistantFrom(adj, n, 0); // 0(V)
    auto [node2, dist2] = mostDistantFrom(adj, n, node1); // 0(V)
    auto diameter = twoNodesDist(adj, n, node1, node2); // 0(V)
    return make_tuple(node1, node2, diameter);
}

vll everyDistanceFromNode(const vector<vll> &adj, ll n, ll root) {
    // Single Source Shortest Path, from a given root
    queue<pair<ll, ll>> q;
    vll ans(n, -1);
    ans[root] = 0;
    q.emplace(root, 0);
    while (!q.empty()) {
        auto [u, d] = q.front();
        q.pop();

        for (auto w : adj[u]) {
            if (ans[w] != -1) continue;
            ans[w] = d + 1;
            q.emplace(w, d + 1);
        }
    }
    return ans;
}

vll maxDistances(const vector<vll> &adj, ll n) {
    auto [node1, node2, diameter] = tree_diameter(adj, n); // 0(3V)
    auto distances1 = everyDistanceFromNode(adj, n, node1); // 0(V)
    auto distances2 = everyDistanceFromNode(adj, n, node2); // 0(V)
    vll ans(n);
    for (int i = 0; i < n; ++i)
        ans[i] = max(distances1[i], distances2[i]); // 0(V)
    return ans;
}

```

4.13 Small to Large

Answer queries of the form "How many vertices in the subtree of vertex v have property P ?"

Build: $O(N)$, Query: $O(N \log N)$

```

struct SmallToLarge {
    vector<vector<int>> tree, vis_chlds;
    vector<int> sizes, values, ans;
    set<int> cnt;

    SmallToLarge(vector<vector<int>> &&g, vector<int> &&v)
        : tree(g), vis_chlds(g.size()), sizes(g.size()), values(v), ans(g.size())
    {

```

```

        update_sizes(0);
    }

    inline void add_value(int u) { cnt.insert(values[u]); }

    inline void remove_value(int u) { cnt.erase(values[u]); }

    inline void update_ans(int u) { ans[u] = (int)cnt.size(); }

    void dfs(int u, int p = -1, bool keep = true) {
        int mx = -1;
        for (auto x : tree[u]) {
            if (x == p) continue;

            if (mx == -1 or sizes[mx] < sizes[x]) mx = x;
        }

        for (auto x : tree[u]) {
            if (x != p and x != mx) dfs(x, u, false);
        }

        if (mx != -1) {
            dfs(mx, u, true);
            swap(vis_chlds[u], vis_chlds[mx]);
        }

        vis_chlds[u].push_back(u);
        add_value(u);

        for (auto x : tree[u]) {
            if (x != p and x != mx) {
                for (auto y : vis_chlds[x]) {
                    add_value(y);
                    vis_chlds[u].push_back(y);
                }
            }
        }

        update_ans(u);

        if (!keep) {
            for (auto x : vis_chlds[u]) remove_value(x);
        }
    }

    void update_sizes(int u, int p = -1) {
        sizes[u] = 1;
        for (auto x : tree[u]) {
            if (x != p) {
                update_sizes(x, u);
                sizes[u] += sizes[x];
            }
        }
    }
};

```

4.14 Topological Sorting

Assumes that :

- vertices index $[0, n - 1]$
- is a DAG (else it returns an empty vector)

$O(V)$

```
enum class state { not_visited, processing, done };
bool dfs(const vector<vll> &adj, ll s, vector<state> &states, vll &order) {
    states[s] = state::processing;
    for (auto &v : adj[s]) {
        if (states[v] == state::not_visited) {
            if (not dfs(adj, v, states, order)) return false;
        } else if (states[v] == state::processing)
            return false;
    }
    states[s] = state::done;
    order.pb(s);
    return true;
}
vll topologicalSorting(const vector<vll> &adj) {
    ll n = len(adj);
    vll order;
    vector<state> states(n, state::not_visited);
    for (int i = 0; i < n; ++i) {
        if (states[i] == state::not_visited) {
            if (not dfs(adj, i, states, order)) return {};
        }
    }
    reverse(all(order));
    return order;
}
```

4.15 Tree Diameter

Finds the length of the diameter of the tree in $O(V)$, it's easy to recover the nodes at the point of the diameter .

```
p11 mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
    // 0 indexed
    ll mostDistantNode = root;
    ll nodeDistance = 0;
    queue<p11> q;
    vector<char> vis(n);
    q.emplace(root, 0);
    vis[root] = true;
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (dist > nodeDistance) {
            nodeDistance = dist;
            mostDistantNode = node;
        }
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
}
```

```
    }
}
return {mostDistantNode, nodeDistance};
}
ll twoNodesDist(const vector<vll> &adj, ll n, ll a, ll b) {
    // 0 indexed
    queue<p11> q;
    vector<char> vis(n);
    q.emplace(a, 0);
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (node == b) {
            return dist;
        }
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return -1;
}
ll tree_diameter(const vector<vll> &adj, ll n) {
    // 0 indexed !!!
    auto [node1, dist1] = mostDistantFrom(adj, n, 0); // 0(V)
    auto [node2, dist2] = mostDistantFrom(adj, n, node1); // 0(V)
    auto diameter = twoNodesDist(adj, n, node1, node2); // 0(V)
    return diameter;
}
```

5 Math

5.1 GCD (with factorization)

$O(\sqrt{n})$ due to factorization.

```
ll gcd_with_factorization(ll a, ll b) {
    map<ll, ll> fa = factorization(a);
    map<ll, ll> fb = factorization(b);
    ll ans = 1;
    for (auto fai : fa) {
        ll k = min(fai.second, fb[fai.first]);
        while (k-- > 0) ans *= fai.first;
    }
    return ans;
}
```

5.2 GCD

```
ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
```

5.3 LCM (with factorization)

$O(\sqrt{n})$ due to factorization.

```

11 lcm_with_factorization(11 a, 11 b) {
    map<11, 11> fa = factorization(a);
    map<11, 11> fb = factorization(b);
    11 ans = 1;
    for (auto fai : fa) {
        11 k = max(fai.second, fb[fai.first]);
        while (k--) ans *= fai.first;
    }
    return ans;
}

```

5.4 LCM

```

11 gcd(11 a, 11 b) { return b ? gcd(b, a % b) : a; }
11 lcm(11 a, 11 b) { return a / gcd(a, b) * b; }

```

5.5 Arithmetic Progression Sum

- s : first term
- d : common difference
- n : number of terms

```

11 arithmeticProgressionSum(11 s, 11 d, 11 n) {
    return (s + (s + d * (n - 1))) * n / 211;
}

```

5.6 Binomial

$O(nm)$ time, $O(m)$ space
 Equal to n choose k

```

11 binom(11 n, 11 k) {
    if (k > n) return 0;
    v11 dp(k + 1, 0);
    dp[0] = 1;
    for (11 i = 1; i <= n; i++)
        for (11 j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
    return dp[k];
}

```

5.7 Euler phi $\varphi(n)$ (in range)

Computes the number of positive integers less than n that are coprimes with n , in the range $[1, n]$, in $O(N \log N)$.

```

const int MAX = 1e6;
vi range_phi(int n) {
    bitset<MAX> sieve;
    vi phi(n + 1);

    iota(phi.begin(), phi.end(), 0);
    sieve.set();

    for (int p = 2; p <= n; p += 2) phi[p] /= 2;
    for (int p = 3; p <= n; p += 2) {

```

```

        if (sieve[p]) {
            for (int j = p; j <= n; j += p) {
                sieve[j] = false;
                phi[j] /= p;
                phi[j] *= (p - 1);
            }
        }
    }

    return phi;
}

```

5.8 Euler phi $\varphi(n)$

Computes the number of positive integers less than n that are coprimes with n , in $O(\sqrt{N})$.

```

int phi(int n) {
    if (n == 1) return 1;

    auto fs = factorization(n); // a vector of pair or a map
    auto res = n;

    for (auto [p, k] : fs) {
        res /= p;
        res *= (p - 1);
    }

    return res;
}

```

5.9 Factorial Factorization

Computes the factorization of $n!$ in $\pi(N) * \log n$

```

// O(logN)
11 E(11 n, 11 p) {
    11 k = 0, b = p;
    while (b <= n) {
        k += n / b;
        b *= p;
    }
    return k;
}

// O(pi(N)*logN)
map<11, 11> factorial_factorization(11 n, const v11 &primes) {
    map<11, 11> fs;
    for (const auto &p : primes) {
        if (p > n) break;
        fs[p] = E(n, p);
    }
    return fs;
}

```


5.10 Factorial

```
const ll MAX = 18;
vll fv(MAX, -1);
ll factorial(ll n) {
    if (fv[n] != -1) return fv[n];
    if (n == 0) return 1;
    return n * factorial(n - 1);
}
```

5.11 Factorization (Pollard Rho)

Factorizes a number into its prime factors in $O(n^{\frac{1}{4}} * \log(n))$.

```
ll mul(ll a, ll b, ll m) {
    ll ret = a * b - (ll)((ld)1 / m * a * b + 0.5) * m;
    return ret < 0 ? ret + m : ret;
}

ll pow(ll a, ll b, ll m) {
    ll ans = 1;
    for (; b > 0; b /= 2ll, a = mul(a, a, m)) {
        if (b % 2ll == 1) ans = mul(ans, a, m);
    }
    return ans;
}

bool prime(ll n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;

    ll r = __builtin_ctzll(n - 1), d = n >> r;
    for (int a : {2, 325, 9375, 28178, 450775, 9780504, 795265022}) {
        ll x = pow(a, d, n);
        if (x == 1 or x == n - 1 or a % n == 0) continue;

        for (int j = 0; j < r - 1; j++) {
            x = mul(x, x, n);
            if (x == n - 1) break;
        }
        if (x != n - 1) return 0;
    }
    return 1;
}

ll rho(ll n) {
    if (n == 1 or prime(n)) return n;
    auto f = [n](ll x) { return mul(x, x, n) + 1; };

    ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
    while (t % 40 != 0 or gcd(prd, n) == 1) {
        if (x == y) x = ++x0, y = f(x);
        q = mul(prd, abs(x - y), n);
        if (q != 0) prd = q;
        x = f(x), y = f(f(y)), t++;
    }
    return gcd(prd, n);
}
```

```
}

vll fact(ll n) {
    if (n == 1) return {};
    if (prime(n)) return {n};
    ll d = rho(n);
    vll l = fact(d), r = fact(n / d);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}
```

5.12 Factorization

Computes the factorization of n in $O(\sqrt{n})$.

```
map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;
        for (; n % i == 0; count++, n /= i)
            ;
        if (count) ans[i] = count;
    }
    if (n > 1) ans[n]++;
    return ans;
}
```

5.13 Fast Fourier Transform

```
template <bool invert = false>
void fft(vector<complex<double>>& xs) {
    int N = (int)xs.size();

    if (N == 1) return;

    vector<complex<double>> es(N / 2), os(N / 2);

    for (int i = 0; i < N / 2; ++i) es[i] = xs[2 * i];

    for (int i = 0; i < N / 2; ++i) os[i] = xs[2 * i + 1];

    fft<invert>(es);
    fft<invert>(os);

    auto signal = (invert ? 1 : -1);
    auto theta = 2 * signal * acos(-1) / N;
    complex<double> S{1}, S1{cos(theta), sin(theta)};

    for (int i = 0; i < N / 2; ++i) {
        xs[i] = (es[i] + S * os[i]);
        xs[i] /= (invert ? 2 : 1);

        xs[i + N / 2] = (es[i] - S * os[i]);
        xs[i + N / 2] /= (invert ? 2 : 1);

        S *= S1;
    }
}
```

```

}
}

```

5.14 Fast pow

Computes a^n in $O(\log N)$.

```

ll fpow(ll a, int n, ll mod = LLONG_MAX) {
    if (n == 0) return 1;
    if (n == 1) return a;
    ll x = fpow(a, n / 2, mod) % mod;
    return ((x * x) % mod * (n & 1 ? a : 1)) % mod;
}

```

5.15 Gauss Elimination

```

template <size_t Dim>
struct GaussianElimination {
    vector<ll> basis;
    size_t size;

    GaussianElimination() : basis(Dim + 1), size(0) {}

    void insert(ll x) {
        for (ll i = Dim; i >= 0; i--) {
            if ((x & 1ll << i) == 0) continue;

            if (!basis[i]) {
                basis[i] = x;
                size++;
                break;
            }

            x ^= basis[i];
        }
    }

    void normalize() {
        for (ll i = Dim; i >= 0; i--)
            for (ll j = i - 1; j >= 0; j--)
                if (basis[i] & 1ll << j) basis[i] ^= basis[j];
    }

    bool check(ll x) {
        for (ll i = Dim; i >= 0; i--) {
            if ((x & 1ll << i) == 0) continue;

            if (!basis[i]) return false;

            x ^= basis[i];
        }

        return true;
    }

    auto operator[](ll k) { return at(k); }
}

```

```

ll at(ll k) {
    ll ans = 0;
    ll total = 1ll << size;
    for (ll i = Dim; ~i; i--) {
        if (!basis[i]) continue;

        ll mid = total >> 1ll;
        if ((mid < k and (ans & 1ll << i) == 0) ||
            (k <= mid and (ans & 1ll << i)))
            ans ^= basis[i];

        if (mid < k) k -= mid;

        total >>= 1ll;
    }
    return ans;
}

ll at_normalized(ll k) {
    ll ans = 0;
    k--;
    for (size_t i = 0; i <= Dim; i++) {
        if (!basis[i]) continue;
        if (k & 1) ans ^= basis[i];
        k >>= 1;
    }
    return ans;
}
};

```

5.16 Integer Mod

```

const ll INF = 1e18;
const ll mod = 998244353;
template <ll MOD = mod>
struct Modular {
    ll value;
    static const ll MOD_value = MOD;

    Modular(ll v = 0) {
        value = v % MOD;
        if (value < 0) value += MOD;
    }

    Modular(ll a, ll b) : value(0) {
        *this += a;
        *this /= b;
    }

    Modular& operator+=(Modular const& b) {
        value += b.value;
        if (value >= MOD) value -= MOD;
        return *this;
    }

    Modular& operator-=(Modular const& b) {
        value -= b.value;
        if (value < 0) value += MOD;
        return *this;
    }
}

```

```

Modular& operator==(Modular const& b) {
    value = (ll)value * b.value % MOD;
    return *this;
}

friend Modular mexp(Modular a, ll e) {
    Modular res = 1;
    while (e) {
        if (e & 1) res *= a;
        a *= a;
        e >>= 1;
    }
    return res;
}

friend Modular inverse(Modular a) { return mexp(a, MOD - 2); }

Modular& operator/=(Modular const& b) { return *this *= inverse(b); }
friend Modular operator+(Modular a, Modular const b) { return a += b; }
Modular operator++(int) { return this->value = (this->value + 1) % MOD; }
Modular operator++() { return this->value = (this->value + 1) % MOD; }
friend Modular operator-(Modular a, Modular const b) { return a -= b; }
friend Modular operator-(Modular const a) { return 0 - a; }
Modular operator--(int) {
    return this->value = (this->value - 1 + MOD) % MOD;
}

Modular operator--() { return this->value = (this->value - 1 + MOD) % MOD; }
friend Modular operator*(Modular a, Modular const b) { return a *= b; }
friend Modular operator/(Modular a, Modular const b) { return a /= b; }
friend std::ostream& operator<<(std::ostream& os, Modular const& a) {
    return os << a.value;
}

friend bool operator==(Modular const& a, Modular const& b) {
    return a.value == b.value;
}

friend bool operator!=(Modular const& a, Modular const& b) {
    return a.value != b.value;
}
};

```

5.17 Is prime

$O(\sqrt{N})$

```

bool isprime(ll n) {
    if (n < 2) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    for (ll i = 3; i * i < n; i += 2)
        if (n % i == 0) return false;
    return true;
}

```

5.18 Number of Divisors $\tau(n)$

Find the total of divisors of N in $O(\sqrt{N})$

```

ll number_of_divisors(ll n) {

```

```

    ll res = 0;

    for (ll d = 1; d * d <= n; ++d) {
        if (n % d == 0) res += (d == n / d ? 1 : 2);
    }

    return res;
}

```

5.19 Power Sum

Calculates $K^0 + K^1 + \dots + K^n$

```

ll powersum(ll n, ll k) { return (fastpow(n, k + 1) - 1) / (k - 1); }

```

5.20 Sieve list primes

List every prime until MAXN, $O(N \log N)$ in time and $O(MAXN)$ in memory.

```

const ll MAXN = 1e5;
vll list_primes(ll n) {
    vll ps;
    bitset<MAXN> sieve;
    sieve.set();
    sieve.reset(1);
    for (ll i = 2; i <= n; ++i) {
        if (sieve[i]) ps.push_back(i);
        for (ll j = i * 2; j <= n; j += i) {
            sieve.reset(j);
        }
    }
    return ps;
}

```

5.21 Sum of Divisors $\sigma(n)$

Computes the sum of each divisor of n in $O(\sqrt{n})$.

```

ll sum_of_divisors(long long n) {
    ll res = 0;

    for (ll d = 1; d * d <= n; ++d) {
        if (n % d == 0) {
            ll k = n / d;

            res += (d == k ? d : d + k);
        }
    }

    return res;
}

```

6 Searching

6.1 Ternary Search Recursive

```

const double eps = 1e-6;

// IT MUST BE AN UNIMODAL FUNCTION
double f(int x) { return x * x + 2 * x + 4; }

double ternary_search(double l, double r) {
    if (fabs(f(l) - f(r)) < eps) return f((l + (r - l) / 2.0));

    auto third = (r - l) / 3.0;
    auto m1 = l + third;
    auto m2 = r - third;

    // change the signal to find the maximum point.
    return m1 < m2 ? ternary_search(m1, r) : ternary_search(l, m2);
}

```

7 Strings

7.1 Hash Range Query

```

struct Hash {
    const ll P = 31;
    const ll mod = 1e9 + 7;
    string s;
    int n;
    vll h, hi, p;
    Hash() {}
    Hash(string s) : s(s), n(s.size()), h(n), hi(n), p(n) {
        for (int i = 0; i < n; i++) p[i] = (i ? P * p[i - 1] : 1) % mod;
        for (int i = 0; i < n; i++) h[i] = (s[i] + (i ? h[i - 1] : 0) * P) % mod;
        for (int i = n - 1; i >= 0; i--)
            hi[i] = (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % mod;
    }
    ll query(int l, int r) {
        ll hash = (h[r] - (l ? h[l - 1] * p[r - l + 1] % mod : 0));
        return hash < 0 ? hash + mod : hash;
    }
    ll query_inv(int l, int r) {
        ll hash = (hi[l] - (r + 1 < n ? hi[r + 1] * p[r - l + 1] % mod : 0));
        return hash < 0 ? hash + mod : hash;
    }
};

```

7.2 Longest Palindrome

```

string longest_palindrome(const string &s) {
    int n = (int)s.size();
    vector<array<int, 2>> dp(n);

    pii odd(0, -1), even(0, -1);
    pii ans;
    for (int i = 0; i < n; i++) {
        int k = 0;
        if (i > odd.second)
            k = 1;
        else
            k = min(dp[odd.first + odd.second - i][0], odd.second - i + 1);
    }
}

```

```

while (i - k >= 0 and i + k < n and s[i - k] == s[i + k]) k++;
dp[i][0] = k--;
if (i + k > odd.second) odd = {i - k, i + k};
if (2 * dp[i][0] - 1 > ans.second) ans = {i - k, 2 * dp[i][0] - 1};

k = 0;
if (i <= even.second)
    k = min(dp[even.first + even.second - i + 1][1], even.second - i + 1);
while (i - k - 1 >= 0 and i + k < n and s[i - k - 1] == s[i + k]) k++;
dp[i][1] = k--;
if (i + k > even.second) even = {i - k - 1, i + k};
if (2 * dp[i][1] > ans.second) ans = {i - k - 1, 2 * dp[i][1]};
}
return s.substr(ans.first, ans.second);
}

```

7.3 Rabin Karp

```

size_t rabin_karp(const string &s, const string &p) {
    if (s.size() < p.size()) return 0;

    auto n = s.size(), m = p.size();
    const ll p1 = 31, p2 = 29, q1 = 1e9 + 7, q2 = 1e9 + 9;
    const ll p1_1 = fpow(p1, q1 - 2, q1), p1_2 = fpow(p1, m - 1, q1);
    const ll p2_1 = fpow(p2, q2 - 2, q2), p2_2 = fpow(p2, m - 1, q2);

    pair<ll, ll> hs, hp;
    for (int i = (int)m - 1; ~i; --i) {
        hs.first = (hs.first * p1) % q1;
        hs.first = (hs.first + (s[i] - 'a' + 1)) % q1;
        hs.second = (hs.second * p2) % q2;
        hs.second = (hs.second + (s[i] - 'a' + 1)) % q2;

        hp.first = (hp.first * p1) % q1;
        hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
        hp.second = (hp.second * p2) % q2;
        hp.second = (hp.second + (p[i] - 'a' + 1)) % q2;
    }

    size_t occ = 0;
    for (size_t i = 0; i < n - m; i++) {
        occ += (hs == hp);

        int fi = s[i] - 'a' + 1;
        int fm = p[i + m] - 'a' + 1;

        hs.first = (hs.first - fi + q1) % q1;
        hs.first = (hs.first * p1_1) % q1;
        hs.first = (hs.first + fm * p1_2) % q1;
        hs.second = (hs.second - fi + q2) % q2;
        hs.second = (hs.second * p2_1) % q2;
        hs.second = (hs.second + fm * p2_2) % q2;
    }
    occ += hs == hp;

    return occ;
}

```

7.4 String Psum

```
struct strPsum {
    ll n;
    ll k;
    vector<vll> psum;
    strPsum(const string &s) : n(s.size()), k(100), psum(k, vll(n + 1)) {
        for (ll i = 1; i <= n; ++i) {
            for (ll j = 0; j < k; ++j) {
                psum[j][i] = psum[j][i - 1];
            }
            psum[s[i - 1]][i]++;
        }
    }

    ll qtd(ll l, ll r, char c) { // [0,n-1]
        return psum[c][r + 1] - psum[c][l];
    }
}
```

7.5 Suffix Automaton (complete)

```
struct state {
    int len, link, cnt, firstpos;
    // this can be optimized using a vector with the alphabet size
    map<char, int> next;
    vi inv_link;
};

struct SuffixAutomaton {
    vector<state> st;
    int sz = 0;
    int last;
    vc cloned;

    SuffixAutomaton(const string &s, int maxlen)
        : st(maxlen * 2), cloned(maxlen * 2) {
        st[0].len = 0;
        st[0].link = -1;
        sz++;
        last = 0;
        for (auto &c : s) add_char(c);

        // precompute for count occurrences
        for (int i = 1; i < sz; i++) {
            st[i].cnt = !cloned[i];
        }
        vector<pair<state, int>> aux;
        for (int i = 0; i < sz; i++) {
            aux.push_back({st[i], i});
        }

        sort(all(aux), [](const pair<state, int> &a, const pair<state, int> &b) {
            return a.fst.len > b.fst.len;
        });

        for (auto &[stt, id] : aux) {
            if (stt.link != -1) {
                st[stt.link].cnt += st[id].cnt;
            }
        }
    }
}
```

```
    }
}

// for find every occurende position
for (int v = 1; v < sz; v++) {
    st[st[v].link].inv_link.push_back(v);
}

void add_char(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    st[cur].firstpos = st[cur].len - 1;
    int p = last;
    // follow the suffix link until find a transition to c
    while (p != -1 and !st[p].next.count(c)) {
        st[p].next[c] = cur;
        p = st[p].link;
    }
    // there was no transition to c so create and leave
    if (p == -1) {
        st[cur].link = 0;
        last = cur;
        return;
    }

    int q = st[p].next[c];
    if (st[p].len + 1 == st[q].len) {
        st[cur].link = q;
    } else {
        int clone = sz++;
        cloned[clone] = true;
        st[clone].len = st[p].len + 1;
        st[clone].next = st[q].next;
        st[clone].link = st[q].link;
        st[clone].firstpos = st[q].firstpos;
        while (p != -1 and st[p].next[c] == q) {
            st[p].next[c] = clone;
            p = st[p].link;
        }
        st[q].link = st[cur].link = clone;
    }
    last = cur;
}

bool checkOccurrence(const string &t) { // 0(len(t))
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return false;
        cur = st[cur].next[c];
    }
    return true;
}

ll totalSubstrings() { // distinct, 0(len(s))
    ll tot = 0;
    for (int i = 1; i < sz; i++) {
        tot += st[i].len - st[st[i].link].len;
    }
}
```

```

    return tot;
}

// count occurrences of a given string t
int countOccurrences(const string &t) {
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return 0;
        cur = st[cur].next[c];
    }
    return st[cur].cnt;
}

// find the first index where t appears a substring 0(len(t))
int firstOccurrence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return -1;
        cur = st[cur].next[c];
    }
    return st[cur].firstpos - len(t) + 1;
}

vi everyOccurrence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return {};
        cur = st[cur].next[c];
    }
    vi ans;
    getEveryOccurrence(cur, len(t), ans);
    return ans;
}

void getEveryOccurrence(int v, int P_length, vi &ans) {
    if (!cloned[v]) ans.pb(st[v].firstpos - P_length + 1);
    for (int u : st[v].inv_link) getEveryOccurrence(u, P_length, ans);
}
};

```

7.6 Z-function get occurrence positions

$O(len(s) + len(p))$

```

vi getOccPos(string &s, string &p) {
    // Z-function
    char delim = '#';
    string t{p + delim + s};
    vi zs(len(t));

    int l = 0, r = 0;
    for (int i = 1; i < len(t); i++) {
        if (i <= r) zs[i] = min(zs[i - l], r - i + 1);
        while (zs[i] + i < len(t) and t[zs[i]] == t[i + zs[i]]) zs[i]++;
        if (r < i + zs[i] - 1) l = i, r = i + zs[i] - 1;
    }

    // Iterate over the results of Z-function to get ranges
    vi ans;

```

```

    int start = len(p) + 1 + 1 - 1;
    for (int i = start; i < len(zs); i++) {
        if (zs[i] == len(p)) {
            int l = i - start;
            ans.emplace_back(l);
        }
    }
    return ans;
}

```

8 Settings and macros

8.1 short-macro.cpp

```

#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(false); \
    cin.tie(0); \
    cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using pii = pair<int, int>;
#define all(a) a.begin(), a.end()

void run() {}
int32_t main(void) {
    fastio;
    int t;
    t = 1;
    // cin >> t;
    while (t--) run();
}

```

8.2 .vimrc

```

set ts=4 sw=4 sta nu rnu sc cindent
set bg=dark ruler clipboard=unnamed,unnamedplus, timeoutlen=100
colorscheme default

```

```

nnoremap <C-j> :botright belowright term bash <CR>
syntax on

```

8.3 degug.cpp

```

#include <bits/stdc++.h>
using namespace std;
/***** Debug Code *****/
template <typename T>
concept Printable = requires(T t) {
    { std::cout << t } -> std::same_as<std::ostream &>;
};
template <Printable T>

```

```

void __print(const T &x) {
    cerr << x;
}
template <size_t T>
void __print(const bitset<T> &x) {
    cerr << x;
}
template <typename A, typename B>
void __print(const pair<A, B> &p);
template <typename... A>
void __print(const tuple<A...> &t);
template <typename T>
void __print(stack<T> s);
template <typename T>
void __print(queue<T> q);
template <typename T, typename... U>
void __print(priority_queue<T, U...> q);
template <typename A>
void __print(const A &x) {
    bool first = true;
    cerr << '{';
    for (const auto &i : x) {
        cerr << (first ? "" : ","), __print(i);
        first = false;
    }
    cerr << '}';
}
template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '(';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << ')';
}
template <typename... A>
void __print(const tuple<A...> &t) {
    bool first = true;
    cerr << '(';
    apply(
        [&first](const auto &...args) {
            ((cerr << (first ? "" : ","), __print(args), first = false), ...);
        },
        t);
    cerr << ')';
}
template <typename T>
void __print(stack<T> s) {
    vector<T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    }
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
}
template <typename T>

```

```

void __print(queue<T> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.front();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
template <typename T, typename... U>
void __print(priority_queue<T, U...> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.top();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
void _print() { cerr << "]\n"; }
template <typename Head, typename... Tail>
void _print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";
    _print(T...);
}

#define dbg(x...) \
    cerr << "[" << #x << "]" = ["; \
    _print(x)

```

8.4 .bashrc

```

cpp() {
    echo ">> COMPILING <<" 1>&2
    g++ -std=c++17 \
        -O2 \
        -g \
        -g3 \
        -Wextra \
        -Wshadow \
        -Wformat=2 \
        -Wconversion \
        -fsanitize=address,undefined \
        -fno-sanitize-recover \
        -Wfatal-errors \
        $1

    if [ $? -ne 0 ]; then
        echo ">> FAILED <<" 1>&2
        return 1
    fi
    echo ">> DONE << " 1>&2
    time ./a.out ${@:2}
}

prepare() {
    for i in {a..z}

```

```

do
    cp macro.cpp $i.cpp
    touch $i.py
done

for i in {1..10}
do
    touch in${i}
    touch out${i}
    touch ans${i}
done
}

```

8.5 macro.cpp

```

#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(false); \
    cin.tie(0); \
    cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using ld = long double;
using vll = vector<ll>;

```

```

using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
using vii = vector<pii>;
using vc = vector<char>;
#define all(a) a.begin(), a.end()
#define snd second
#define fst first
#define pb(___x) push_back(___x)
#define mp(___a, ___b) make_pair(___a, ___b)
#define eb(___x) emplace_back(___x)

const ll oo = 1e18;

void run() {}
int32_t main(void) {
    fastio;
    int t;
    t = 1;
    // cin >> t;
    while (t--) run();
}

```