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# 1 Data structures

## 1.1 Segtree Lazy (Atcoder)

```
struct Node {
    // need an empty constructor with the neutral node
    Node() : {}
};

struct Lazy {
    // need an empty constructor with the neutral lazy
    Lazy() : {}
};

// how to merge two nodes
Node op(Node a, Node b) {}

// how to apply the lazy into a node
Node mapping(Lazy a, Node b, int, int) {}

// how to merge two lazy
Lazy comp(Lazy a, Lazy b) {}

template <typename T, auto op, typename L, auto mapping,
          auto composition>
struct SegTreeLazy {
    static_assert(
        is_convertible_v<decltype(op), function<T(T, T)>>,
        "op must be a function T(T, T)");
    static_assert(
        is_convertible_v<decltype(mapping),
            function<T(L, T, int, int)>>,
        "mapping must be a function T(L, T, int, int)");
    static_assert(is_convertible_v<decltype(composition),
        function<L(L, L)>>,
        "composition must be a function L(L, L)");

    int N, size, height;
    const T eT;
    const L eL;
    vector<T> d;
    vector<L> lz;

    SegTreeLazy(const T &eT_ = T(), const L &eL_ = L())
```

```
        : SegTreeLazy(0, eT_, eL_) {}
    explicit SegTreeLazy(int n, const T &eT_ = T(),
        const L &eL_ = L())
        : SegTreeLazy(vector<T>(n, eT_), eT_, eL_) {}
    explicit SegTreeLazy(const vector<T> &v,
        const T &eT_ = T(),
        const L &eL_ = L())
        : N(int(v.size())), eT(eT_), eL(eL_) {
        size = 1;
        height = 0;
        while (size < N) size <= 1, height++;
        d = vector<T>(2 * size, eT);
        lz = vector<L>(size, eL);
        for (int i = 0; i < N; i++) d[size + i] = v[i];
        for (int i = size - 1; i >= 1; i--) {
            update(i);
        }
    }

    void set(int p, T x) {
        assert(0 <= p && p < N);
        p += size;
        for (int i = height; i >= 1; i--) push(p >> i);
        d[p] = x;
        for (int i = 1; i <= height; i++) update(p >> i);
    }

    T get(int p) {
        assert(0 <= p && p < N);
        p += size;
        for (int i = height; i >= 1; i--) push(p >> i);
        return d[p];
    }

    T query(int l, int r) {
        assert(0 <= l && l <= r && r < N);

        l += size;
        r += size;

        for (int i = height; i >= 1; i--) {
            if (((l >> i) << i) != l) push(l >> i);
            if (((r + 1) >> i) << i) != (r + 1)) push(r >> i);
        }
    }
};
```

```

T sml = eT, smr = eT;
while (l <= r) {
    if (l & 1) sml = op(sml, d[l++]);
    if (!(r & 1)) smr = op(d[r--], smr);
    l >>= 1;
    r >>= 1;
}

return op(sml, smr);
}

T query_all() { return d[1]; }

void update(int p, L f) {
    assert(0 <= p && p < N);
    p += size;
    for (int i = height; i >= 1; i--) push(p >> i);
    d[p] = mapping(f, d[p]);
    for (int i = 1; i <= height; i++) update(p >> i);
}

void update(int l, int r, L f) {
    assert(0 <= l && l <= r && r < N);

    l += size;
    r += size;

    for (int i = height; i >= 1; i--) {
        if (((l >> i) << i) != l) push(l >> i);
        if (((r + 1) >> i) << i) != (r + 1)) push(r >> i);
    }

    {
        int l2 = l, r2 = r;
        while (l <= r) {
            if (l & 1) all_apply(l++, f);
            if (!(r & 1)) all_apply(r--, f);
            l >>= 1;
            r >>= 1;
        }
        l = l2;
        r = r2;
    }

    for (int i = 1; i <= height; i++) {
        if (((l >> i) << i) != l) update(l >> i);
    }
}

```

```

        if (((r + 1) >> i) << i) != (r + 1)) update(r >> i);
    }
}

pair<int, int> node_range(int k) const {
    int remain = height;
    for (int kk = k; kk >>= 1; --remain)
        ;
    int fst = k << remain;
    int lst = min(fst + (1 << remain) - 1, size + N - 1);
    return {fst - size, lst - size};
}

private:
void update(int k) { d[k] = op(d[2 * k], d[2 * k + 1]); }
void all_apply(int k, L f) {
    auto [fst, lst] = node_range(k);
    d[k] = mapping(f, d[k], fst, lst);
    if (k < size) lz[k] = composition(f, lz[k]);
}

void push(int k) {
    all_apply(2 * k, lz[k]);
    all_apply(2 * k + 1, lz[k]);
    lz[k] = eL;
}

};

```

## 1.2 Bitree 2D

Given a 2d array allow you to sum *val* to the position  $(x, y)$  and find the sum of the rectangle with left top corner  $(x_1, y_1)$  and right bottom corner  $(x_2, y_2)$

**Update and query 1 indexed !**

Time: update  $O(\log n^2)$ , query  $O(\log n^2)$

```

struct Bit2d {
    int n;
    vll2d bit;
    Bit2d(int ni) : n(ni), bit(n + 1, vll(n + 1)) {}
    Bit2d(int ni, vll2d &xs) : n(ni), bit(n + 1, vll(n + 1)) {
        for (int i = 1; i <= n; i++) {
            for (int j = 1; j <= n; j++) {
                update(i, j, xs[i][j]);
            }
        }
    }

    void update(int x, int y, ll val) {
        for (; x <= n; x += (x & (-x))) {

```

```

        for (int i = y; i <= n; i += (i & (-i))) {
            bit[x][i] += val;
        }
    }
}

11 sum(int x, int y) {
    11 ans = 0;

    for (int i = x; i; i -= (i & (-i))) {
        for (int j = y; j; j -= (j & (-j))) {
            ans += bit[i][j];
        }
    }
    return ans;
}

11 query(int x1, int y1, int x2, int y2) {
    return sum(x2, y2) - sum(x2, y1 - 1) - sum(x1 - 1, y2) +
        sum(x1 - 1, y1 - 1);
}

};

```

### 1.3 Bitree

```

template <typename T>
struct BITree {
    int N;
    vector<T> v;

    BITree(int n) : N(n), v(n + 1, 0) {}

    void update(int i, const T& x) {
        if (i == 0) return;
        for (; i <= N; i += i & -i) v[i] += x;
    }

    T range_sum(int i, int j) {
        return range_sum(j) - range_sum(i - 1);
    }

    T range_sum(int i) {
        T sum = 0;
        for (; i > 0; i -= i & -i) sum += v[i];
        return sum;
    }
};

```

### 1.4 Disjoint Sparse Table

Answers queries of any monoid operation (i.e. has identity element and is associative)  
 Build:  $O(N \log N)$ , Query:  $O(1)$

```

#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct DisjointSparseTable {
    using Operation = T (*)(T, T);

    vector<vector<T>> st;
    Operation f;
    T identity;

    static constexpr int log2_floor(
        unsigned long long i) noexcept {
        return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
    }

    // Lazy loading constructor. Needs to call build!
    DisjointSparseTable(Operation op, const T neutral = T())
        : st(), f(op), identity(neutral) {}

    DisjointSparseTable(vector<T> v)
        : DisjointSparseTable(v, F(min(a, b))) {}

    DisjointSparseTable(vector<T> v, Operation op,
        const T neutral = T())
        : st(), f(op), identity(neutral) {
        build(v);
    }

    void build(vector<T> v) {
        st.resize(log2_floor(v.size()) + 1,
            vector<T>(1ll << (log2_floor(v.size()) + 1)));
        v.resize(st[0].size(), identity);
        for (int level = 0; level < (int)st.size(); ++level) {
            for (int block = 0; block < (1 << level); ++block) {
                const auto l = block << (st.size() - level);
                const auto r = (block + 1) << (st.size() - level);
                const auto m = l + (r - l) / 2;

                st[level][m] = v[m];
                for (int i = m + 1; i < r; i++)
                    st[level][i] = f(st[level][i - 1], v[i]);
                st[level][m - 1] = v[m - 1];
            }
        }
    }
};

```

```

        for (int i = m - 2; i >= 1; i--)
            st[level][i] = f(st[level][i + 1], v[i]);
    }
}

T query(int l, int r) const {
    if (l > r) return identity;
    if (l == r) return st.back()[l];

    const auto k = log2_floor(l ^ r);
    const auto level = (int)st.size() - 1 - k;
    return f(st[level][l], st[level][r]);
}
};

```

## 1.5 Dsu

```

struct DSU {
    vi ps, sz;

    // vector<unordered_set<int>> sts;

    DSU(int N) : ps(N + 1), sz(N, 1) /*, sts(N) */ {
        iota(all(ps), 0);
        // for (int i = 0; i < N; i++) sts[i].insert(i);
    }
    int find_set(int x) {
        return ps[x] == x ? x : ps[x] = find_set(ps[x]);
    }
    int size(int u) { return sz[find_set(u)]; }
    bool same_set(int x, int y) {
        return find_set(x) == find_set(y);
    }
    void union_set(int x, int y) {
        if (same_set(x, y)) return;

        int px = find_set(x);
        int py = find_set(y);

        if (sz[px] < sz[py]) swap(px, py);

        ps[py] = px;
        sz[px] += sz[py];
        // sts[px].merge(sts[py]);
    }
};

```

```

    }
};

```

## 1.6 Ordered Set

If you need an ordered **multiset** you may add an id to each value. Using `greater_equal`, or `less_equal` is considered undefined behavior.

- **order\_of\_key(k)** : Number of items strictly smaller/greater than k .
- **find\_by\_order(k)** : K-th element in a set (counting from zero).

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
                        tree_order_statistics_node_update>;

```

## 1.7 Prefix Sum 2D

Given an 2d array with  $n$  lines and  $m$  columns, find the sum of the subarray that have the left upper corner at  $(x1, y1)$  and right bottom corner at  $(x2, y2)$ .

Time: build  $O(n \cdot m)$ , query  $O(1)$ .

```

struct psum2d {
    vll2d s;
    vll2d psum;
    psum2d(vll2d &grid, int n, int m)
        : s(n + 1, vll(m + 1)), psum(n + 1, vll(m + 1)) {
        for (int i = 1; i <= n; i++)
            for (int j = 1; j <= m; j++)
                s[i][j] = s[i][j - 1] + grid[i][j];

        for (int i = 1; i <= n; i++)
            for (int j = 1; j <= m; j++)
                psum[i][j] = psum[i - 1][j] + s[i][j];
    }

    ll query(int x1, int y1, int x2, int y2) {
        ll ans = psum[x2][y2] + psum[x1 - 1][y1 - 1];
        ans -= psum[x2][y1 - 1] + psum[x1 - 1][y2];
        return ans;
    }
};

```

## 1.8 SegTree Range Sum Query Range PA sum/set Update

Makes arithmetic progression updates in range and sum queries.

Considering  $PA(A, R) = [A + R, A + 2R, A + 3R, \dots]$

- **update\_set(l, r, A, R):** sets [l, r] to  $PA(A, R)$
- **update\_add(l, r, A, R):** sum  $PA(A, R)$  in [l, r]
- **query(l, r):** sum in range [l, r]

**0 indexed !**

Time: build  $O(n)$ , updates and queries  $O(\log n)$

```
const ll oo = 1e18;
struct SegTree {
    struct Data {
        ll sum;
        ll set_a, set_r, add_a, add_r;
        Data()
            : sum(0), set_a(oo), set_r(0), add_a(0), add_r(0) {}
    };
    int n;
    vector<Data> seg;
    SegTree(int n_) : n(n_), seg(vector<Data>(4 * n)) {}

    void prop(int p, int l, int r) {
        int sz = r - l + 1;
        ll &sum = seg[p].sum, &set_a = seg[p].set_a,
            &set_r = seg[p].set_r, &add_a = seg[p].add_a,
            &add_r = seg[p].add_r;

        if (set_a != oo) {
            set_a += add_a, set_r += add_r;
            sum = set_a * sz + set_r * sz * (sz + 1) / 2;
            if (l != r) {
                int m = (l + r) / 2;

                seg[2 * p].set_a = set_a;
                seg[2 * p].set_r = set_r;
                seg[2 * p].add_a = seg[2 * p].add_r = 0;

                seg[2 * p + 1].set_a = set_a + set_r * (m - l + 1);
                seg[2 * p + 1].set_r = set_r;
                seg[2 * p + 1].add_a = seg[2 * p + 1].add_r = 0;
            }
            set_a = oo, set_r = 0;
            add_a = add_r = 0;
        } else if (add_a or add_r) {
            sum += add_a * sz + add_r * sz * (sz + 1) / 2;
```

```
        if (l != r) {
            int m = (l + r) / 2;

            seg[2 * p].add_a += add_a;
            seg[2 * p].add_r += add_r;

            seg[2 * p + 1].add_a += add_a + add_r * (m - l + 1);
            seg[2 * p + 1].add_r += add_r;
        }
        add_a = add_r = 0;
    }
}

int inter(pii a, pii b) {
    if (a.first > b.first) swap(a, b);
    return max(0, min(a.second, b.second) - b.first + 1);
}

ll set(int a, int b, ll aa, ll rr, int p, int l, int r) {
    prop(p, l, r);
    if (b < l or r < a) return seg[p].sum;
    if (a <= l and r <= b) {
        seg[p].set_a = aa;
        seg[p].set_r = rr;
        prop(p, l, r);
        return seg[p].sum;
    }
    int m = (l + r) / 2;
    int tam_l = inter({l, m}, {a, b});
    return seg[p].sum = set(a, b, aa, rr, 2 * p, l, m) +
        set(a, b, aa + rr * tam_l, rr,
            2 * p + 1, m + 1, r);
}

void update_set(int l, int r, ll aa, ll rr) {
    set(l, r, aa, rr, 1, 0, n - 1);
}

ll add(int a, int b, ll aa, ll rr, int p, int l, int r) {
    prop(p, l, r);
    if (b < l or r < a) return seg[p].sum;
    if (a <= l and r <= b) {
        seg[p].add_a += aa;
        seg[p].add_r += rr;
        prop(p, l, r);
        return seg[p].sum;
    }
    int m = (l + r) / 2;
```

```

    int tam_l = inter({l, m}, {a, b});
    return seg[p].sum = add(a, b, aa, rr, 2 * p, l, m) +
        add(a, b, aa + rr * tam_l, rr,
            2 * p + 1, m + 1, r);
}

void update_add(int l, int r, ll aa, ll rr) {
    add(l, r, aa, rr, 1, 0, n - 1);
}

ll query(int a, int b, int p, int l, int r) {
    prop(p, l, r);
    if (b < l or r < a) return 0;
    if (a <= l and r <= b) return seg[p].sum;
    int m = (l + r) / 2;
    return query(a, b, 2 * p, l, m) +
        query(a, b, 2 * p + 1, m + 1, r);
}

ll query(int l, int r) {
    return query(l, r, 1, 0, n - 1);
}
};

```

## 1.9 SegTree Point Update (dynamic function)

Answers queries of any monoid operation (i.e. has identity element and is associative)  
 Build:  $O(N)$ , Query:  $O(\log N)$

```

#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct SegTree {
    using Operation = T (*)(T, T);

    int N;
    vector<T> ns;
    Operation operation;
    T identity;

    SegTree(int n, Operation op = F(a + b), T neutral = T())
        : N(n),
          ns(2 * N, neutral),
          operation(op),
          identity(neutral) {}

    SegTree(const vector<T> &v, Operation op = F(a + b),
            T neutral = T())
        : SegTree((int)v.size(), op, neutral) {
        copy(v.begin(), v.end(), ns.begin() + N);
    }
};

```

```

for (int i = N - 1; i > 0; --i)
    ns[i] = operation(ns[2 * i], ns[2 * i + 1]);
}

T query(size_t i) const { return ns[i + N]; }

T query(size_t l, size_t r) const {
    auto a = l + N, b = r + N;
    auto ans = identity;
    // Non-associative operations needs to be processed
    // backwards
    stack<T> st;
    while (a <= b) {
        if (a & 1) ans = operation(ans, ns[a++]);
        if (not(b & 1)) st.push(ns[b--]);

        a >>= 1;
        b >>= 1;
    }

    for (; !st.empty(); st.pop())
        ans = operation(ans, st.top());

    return ans;
}

void update(size_t i, T value) {
    update_set(i, operation(ns[i + N], value));
}

void update_set(size_t i, T value) {
    auto a = i + N;

    ns[a] = value;
    while (a >>= 1)
        ns[a] = operation(ns[2 * a], ns[2 * a + 1]);
}
};

```

## 1.10 Segtree Range Max Query Point Max Assign Update (dynamic)

Answers range queries in ranges until  $10^9$  (maybe more)  
 Time: query and update  $O(n \cdot \log n)$



```

struct node;
node *newNode();

struct node {
    node *left, *right;
    int lv, rv;
    ll val;

    node() : left(NULL), right(NULL), val(-oo) {}

    inline void init(int l, int r) {
        lv = l;
        rv = r;
    }

    inline void extend() {
        if (!left) {
            int m = (lv + rv) / 2;
            left = newNode();
            right = newNode();
            left->init(lv, m);
            right->init(m + 1, rv);
        }
    }

    ll query(int l, int r) {
        if (r < lv || rv < l) {
            return 0;
        }

        if (l <= lv && rv <= r) {
            return val;
        }

        extend();
        return max(left->query(l, r), right->query(l, r));
    }

    void update(int p, ll newVal) {
        if (lv == rv) {
            val = max(val, newVal);
            return;
        }

        extend();

```

```

        (p <= left->rv ? left : right)->update(p, newVal);
        val = max(left->val, right->val);
    }
};

const int BUFFSZ(1e7);
node *newNode() {
    static int bufSize = BUFFSZ;
    static node buf[(int)BUFFSZ];
    assert(bufSize);
    return &buf[--bufSize];
}

struct SegTree {
    int n;
    node *root;
    SegTree(int _n) : n(_n) {
        root = newNode();
        root->init(0, n);
    }

    ll query(int l, int r) { return root->query(l, r); }
    void update(int p, ll v) { root->update(p, v); }
};

1.11 Segtree Range Max Query Range Max Update

template <typename T = ll>
struct SegTree {
    int N;
    T nu, nq;
    vector<T> st, lazy;
    SegTree(const vector<T> &xs)
        : N(len(xs)),
          nu(numeric_limits<T>::min()),
          nq(numeric_limits<T>::min()),
          st(4 * N + 1, nu),
          lazy(4 * N + 1, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    void update(int l, int r, T value) {
        update(1, 0, N - 1, l, r, value);
    }

    T query(int l, int r) { return query(1, 0, N - 1, l, r); }

```

```

void update(int node, int nl, int nr, int ql, int qr,
            T v) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return;

    st[node] = max(st[node], v);
    if (ql <= nl and nr <= qr) {
        if (nl < nr) {
            lazy[left(node)] = max(lazy[left(node)], v);
            lazy[right(node)] = max(lazy[right(node)], v);
        }
        return;
    }
    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

    st[node] = max(st[left(node)], st[right(node)]);
}

T query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return max(x, y);
}

void propagation(int node, int nl, int nr) {
    if (lazy[node] != nu) {
        st[node] = max(st[node], lazy[node]);

        if (nl < nr) {
            lazy[left(node)] =
                max(lazy[left(node)], lazy[node]);
            lazy[right(node)] =
                max(lazy[right(node)], lazy[node]);
        }
    }
}

```

```

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

int main() {
    int n;
    cin >> n;
    vector<array<int, 3>> xs(n);
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < 3; ++j) {
            cin >> xs[i][j];
        }
    }
    vi aux(n, 0);
    SegTree<int> st(aux);
    for (int i = 0; i < n; ++i) {
        int a = min(i + xs[i][1], n);
        int b = min(i + xs[i][2], n);
        st.update(i, i, st.query(i, i) + xs[i][0]);
        int cur = st.query(i, i);
        st.update(a, b, cur);
    }

    cout << st.query(0, n) << '\n';
}

```

## 1.12 SegTree Range Min Query Point Assign Update

```

template <typename T = ll>
struct SegTree {
    int n;
    T nu, nq;
    vector<T> st;
    SegTree(const vector<T> &v)
        : n(len(v)),
          nu(0),
          nq(numeric_limits<T>::max()),
          st(n * 4 + 1, nu) {
        for (int i = 0; i < n; ++i) update(i, v[i]);
    }
    void update(int p, T v) { update(1, 0, n - 1, p, v); }
}

```

```

T query(int l, int r) { return query(1, 0, n - 1, l, r); }

void update(int node, int nl, int nr, int p, T v) {
    if (p < nl or p > nr) return;

    if (nl == nr) {
        st[node] = v;
        return;
    }

    update(left(node), nl, mid(nl, nr), p, v);
    update(right(node), mid(nl, nr) + 1, nr, p, v);

    st[node] = min(st[left(node)], st[right(node)]);
}

T query(int node, int nl, int nr, int ql, int qr) {
    if (ql <= nl and qr >= nr) return st[node];
    if (nl > qr or nr < ql) return nq;
    if (nl == nr) return st[node];

    return min(
        query(left(node), nl, mid(nl, nr), ql, qr),
        query(right(node), mid(nl, nr) + 1, nr, ql, qr));
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

### 1.13 Segtree Range Sum Query Point Sum Update (dynamic)

Answers range queries in ranges until  $10^9$  (maybe more)

Time: query and update  $O(n \cdot \log n)$

```

struct node;
node *newNode();

struct node {
    node *left, *right;
    int lv, rv;
    ll val;

    node() : left(NULL), right(NULL), val(0) {}

    inline void init(int l, int r) {

```

```

        lv = l;
        rv = r;
    }

    inline void extend() {
        if (!left) {
            int m = (rv - lv) / 2 + lv;
            left = newNode();
            right = newNode();
            left->init(lv, m);
            right->init(m + 1, rv);
        }
    }

    ll query(int l, int r) {
        if (r < lv || rv < l) {
            return 0;
        }

        if (l <= lv && rv <= r) {
            return val;
        }

        extend();
        return left->query(l, r) + right->query(l, r);
    }

    void update(int p, ll newVal) {
        if (lv == rv) {
            val += newVal;
            return;
        }

        extend();
        (p <= left->rv ? left : right)->update(p, newVal);
        val = left->val + right->val;
    }
};

const int BUFFSZ(1.3e7);
node *newNode() {
    static int bufSize = BUFFSZ;
    static node buf[(int)BUFFSZ];
    // assert(bufSize);
    return &buf[--bufSize];
}

```

```
}
```

```
struct SegTree {
    int n;
    node *root;
    SegTree(int _n) : n(_n) {
        root = newNode();
        root->init(0, n);
    }
    ll query(int l, int r) { return root->query(l, r); }
    void update(int p, ll v) { root->update(p, v); }
};
```

### 1.14 SegTree Range Xor query Point Assign Update

```
template <typename T = ll>
struct SegTree {
    int n;
    T nu, nq;
    vector<T> st;
    SegTree(const vector<T> &v)
        : n(len(v)), nu(0), nq(0), st(n * 4 + 1, nu) {
        for (int i = 0; i < n; ++i) update(i, v[i]);
    }
    void update(int p, T v) { update(1, 0, n - 1, p, v); }
    T query(int l, int r) { return query(1, 0, n - 1, l, r); }

    void update(int node, int nl, int nr, int p, T v) {
        if (p < nl or p > nr) return;

        if (nl == nr) {
            st[node] = v;
            return;
        }

        update(left(node), nl, mid(nl, nr), p, v);
        update(right(node), mid(nl, nr) + 1, nr, p, v);

        st[node] = st[left(node)] ^ st[right(node)];
    }

    T query(int node, int nl, int nr, int ql, int qr) {
        if (ql <= nl and qr >= nr) return st[node];
        if (nl > qr or nr < ql) return nq;
        if (nl == nr) return st[node];
    }
};
```

```
        return query(left(node), nl, mid(nl, nr), ql, qr) ^
               query(right(node), mid(nl, nr) + 1, nr, ql, qr);
    }
};
```

```
int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + l; }
};
```

### 1.15 SegTree Range Min Query Range Sum Update

```
template <typename t = ll>
struct SegTree {
    int n;
    t nu;
    t nq;
    vector<t> st, lazy;
    SegTree(const vector<t> &xs)
        : n(len(xs)),
          nu(0),
          nq(numeric_limits<t>::max()),
          st(4 * n, nu),
          lazy(4 * n, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    SegTree(int n) : n(n), st(4 * n, nu), lazy(4 * n, nu) {}

    void update(int l, int r, ll value) {
        update(1, 0, n - 1, l, r, value);
    }

    t query(int l, int r) { return query(1, 0, n - 1, l, r); }

    void update(int node, int nl, int nr, int ql, int qr,
                ll v) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return;

        if (ql <= nl and nr <= qr) {
            st[node] += (nr - nl + 1) * v;

            if (nl < nr) {
```

```

        lazy[left(node)] += v;
        lazy[right(node)] += v;
    }

    return;
}

update(left(node), nl, mid(nl, nr), ql, qr, v);
update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

st[node] = min(st[left(node)], st[right(node)]);
}

t query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    t x = query(left(node), nl, mid(nl, nr), ql, qr);
    t y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return min(x, y);
}

void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
        st[node] += lazy[node];

        if (nl < nr) {
            lazy[left(node)] += lazy[node];
            lazy[right(node)] += lazy[node];
        }

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

## 1.16 SegTree Range Sum Query Range Sum Update

```

template <typename T = ll>
struct SegTree {
    int N;
    T nu;
    T nq;
    vector<T> st, lazy;
    SegTree(const vector<T> &xs)
        : N(len(xs)),
          nu(0),
          nq(0),
          st(4 * N, nu),
          lazy(4 * N, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    SegTree(int n)
        : N(n), nu(0), nq(0), st(4 * N, nu), lazy(4 * N, nu) {}

    void update(int l, int r, ll value) {
        update(1, 0, N - 1, l, r, value);
    }

    T query(int l, int r) { return query(1, 0, N - 1, l, r); }

    void update(int node, int nl, int nr, int ql, int qr,
                ll v) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return;

        if (ql <= nl and nr <= qr) {
            st[node] += (nr - nl + 1) * v;

            if (nl < nr) {
                lazy[left(node)] += v;
                lazy[right(node)] += v;
            }

            return;
        }

        update(left(node), nl, mid(nl, nr), ql, qr, v);
        update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

        st[node] = st[left(node)] + st[right(node)];
    }
};

```

```

}

T query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return x + y;
}

void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
        st[node] += (nr - nl + 1) * lazy[node];

        if (nl < nr) {
            lazy[left(node)] += lazy[node];
            lazy[right(node)] += lazy[node];
        }

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

## 1.17 Sparse Table

Answer the range query defined at the function `op`.  
 Build:  $O(N \log N)$ , Query:  $O(1)$

```

template <typename T>
struct SparseTable {
    vector<T> v;
    int n;
    static const int b = 30;
    vi mask, t;

    int op(int x, int y) { return v[x] < v[y] ? x : y; }
    int msb(int x) {

```

```

        return __builtin_clz(1) - __builtin_clz(x);
    }
};

SparseTable() {}
SparseTable(const vector<T>& v_)
    : v(v_), n(v.size()), mask(n), t(n) {
    for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
        at = (at << 1) & ((1 << b) - 1);
        while (at and op(i, i - msb(at & -at)) == i)
            at ^= at & -at;
    }
    for (int i = 0; i < n / b; i++)
        t[i] = b * i + b - 1 - msb(mask[b * i + b - 1]);
    for (int j = 1; (1 << j) <= n / b; j++)
        for (int i = 0; i + (1 << j) <= n / b; i++)
            t[n / b * j + i] =
                op(t[n / b * (j - 1) + i],
                  t[n / b * (j - 1) + i + (1 << (j - 1))]);
    }

    int small(int r, int sz = b) {
        return r - msb(mask[r] & ((1 << sz) - 1));
    }

    T query(int l, int r) {
        if (r - l + 1 <= b) return small(r, r - l + 1);
        int ans = op(small(l + b - 1), small(r));
        int x = l / b + 1, y = r / b - 1;
        if (x <= y) {
            int j = msb(y - x + 1);
            ans = op(ans, op(t[n / b * j + x],
                             t[n / b * j + y - (1 << j) + 1]));
        }
        return ans;
    }
};

```

## 2 Dynamic programming

### 2.1 Binary Knapsack (bottom up)

Given  $N$  items, each with its own value  $V_i$  and weight  $W_i$  and a maximum knapsack weight  $W$ , compute the maximum value of the items that we can carry, if we can either ignore or take a particular item.

Assume that  $1 \leq n \leq 1000$ ,  $1 \leq S \leq 10000$ .

Time and space:  $O(N * W)$

the vectors  $VS$  and  $WS$  starts at one, so it need an empty value at index 0.

```

const int MAXN(2010), MAXM(2010);
ll st[MAXN + 1][MAXM + 1];

```

```

char ps[MAXN + 1][MAXM + 1];
pair<ll, vi> knapsack(int M, const vll &VS, const vi &WS) {
    memset(st, 0, sizeof(st));
    memset(ps, 0, sizeof(ps));
    int N = len(VS) - 1; // ELEMENTS START AT INDEX 1 !

    for (int i = 0; i <= N; ++i) st[i][0] = 0;

    for (int m = 0; m <= M; ++m) st[0][m] = 0;

    for (int i = 1; i <= N; ++i) {
        for (int m = 1; m <= M; ++m) {
            st[i][m] = st[i - 1][m];
            ps[i][m] = 0;
            int w = WS[i];
            ll v = VS[i];

            if (w <= m and st[i - 1][m - w] + v > st[i][m]) {
                st[i][m] = st[i - 1][m - w] + v;
                ps[i][m] = 1;
            }
        }
    }

    int m = M;
    vi is;
    for (int i = N; i >= 1; --i) {
        if (ps[i][m]) {
            is.emplace_back(i - 1);
            m -= WS[i];
        }
    }

    return {st[N][M], is};
}

```

## 2.2 Binary Knapsack (top down)

Given  $N$  items, each with its own value  $V_i$  and weight  $W_i$  and a maximum knapsack weight  $W$ , compute the maximum value of the items that we can carry, if we can either ignore or take a particular item.

Assume that  $1 \leq n \leq 1000$ ,  $1 \leq S \leq 10000$ .

Time and space:  $O(N * W)$

the bottom up version is 5 times faster !

```

const int MAXN(2000), MAXM(2000);
ll memo[MAXN][MAXM + 1];
char choosen[MAXN][MAXM + 1];

```

```

ll knapSack(int u, int w, vll &VS, vi &WS) {
    if (u < 0) return 0;
    if (memo[u][w] != -1) return memo[u][w];

    ll a = 0, b = 0;
    a = knapSack(u - 1, w, VS, WS);
    if (WS[u] <= w)
        b = knapSack(u - 1, w - WS[u], VS, WS) + VS[u];
    if (b > a) {
        choosen[u][w] = true;
    }
    return memo[u][w] = max(a, b);
}

pair<ll, vi> knapSack(int W, vll &VS, vi &WS) {
    memset(memo, -1, sizeof(memo));
    memset(choosen, 0, sizeof(choosen));
    int n = len(VS);
    ll v = knapSack(n - 1, W, VS, WS);
    ll cw = W;
    vi choosed;
    for (int i = n - 1; i >= 0; i--) {
        if (choosen[i][cw]) {
            cw -= WS[i];
            choosed.emplace_back(i);
        }
    }
    return {v, choosed};
}

```

## 2.3 Digits

Finds the number of digits between 1 and  $x$  that don't have 4 or 13 as substring.

```

ll memo[20][30][2];
ll dp(int p, int d, bool l, const vi &digits) {
    if (p == len(digits)) return 0;

    if (memo[p][d][l] != -1ll) {
        return memo[p][d][l];
    }

    ll tot = 0ull;

    int k = l and d == digits[p] ? digits[p + 1ull] : 9ull;
    for (int i = 0; i <= k; i++) {
        if (i == 4) continue;

```

```

    if (d == 1 and i == 3) continue;
    tot += dp(p + 1, i, 1 and d == digits[p], digits);
}

return memo[p][d][1] = tot;
}

vi get_digits(ll x) {
    vi digits;

    while (x) {
        digits.emplace_back(x % 10ull);
        x /= 10ull;
    }

    reverse(all(digits));
    return digits;
}

ll dp(ll x) {
    auto digits = get_digits(x);
    memset(memo, -1, sizeof memo);

    for (ll i = 0; i <= 9; i++) {
        memo[len(digits) - 1][i][0] = 1ull;
        memo[len(digits) - 1][i][1] = i <= digits.back();
    }

    ll tot = 0;
    for (int i = 0; i <= digits[0]; i++) {
        if (i == 4) continue;
        tot += dp(0, i, i == digits[0], digits);
    }

    return tot - 1ull;
}

```

## 2.4 Edit Distance

$O(N * M)$

```

int edit_distance(const string &a, const string &b) {
    int n = a.size();
    int m = b.size();
    vector<vi> dp(n + 1, vi(m + 1, 0));

```

```

int ADD = 1, DEL = 1, CHG = 1;
for (int i = 0; i <= n; ++i) {
    dp[i][0] = i * DEL;
}
for (int i = 1; i <= m; ++i) {
    dp[0][i] = ADD * i;
}

for (int i = 1; i <= n; ++i) {
    for (int j = 1; j <= m; ++j) {
        int add = dp[i][j - 1] + ADD;
        int del = dp[i - 1][j] + DEL;
        int chg = dp[i - 1][j - 1] +
            (a[i - 1] == b[j - 1] ? 0 : 1) * CHG;
        dp[i][j] = min({add, del, chg});
    }
}

return dp[n][m];
}

```

## 2.5 Kadane

Find the maximum subarray sum in a given array.

```

int kadane(const vi &as) {
    vi s(len(as));
    s[0] = as[0];

    for (int i = 1; i < len(as); ++i)
        s[i] = max(as[i], s[i - 1] + as[i]);

    return *max_element(all(s));
}

```

## 2.6 Longest Increasing Subsequence (LIS)

Finds the length of the longest subsequence in

$O(n \log n)$

```

int LIS(const vi& as) {
    const ll oo = 1e18;
    int n = len(as);
    vll lis(n + 1, oo);
    lis[0] = -oo;

```



```

auto ans = 0;

for (int i = 0; i < n; ++i) {
    auto it = lower_bound(all(lis), as[i]);
    auto pos = (int)(it - lis.begin());

    ans = max(ans, pos);
    lis[pos] = as[i];
}

return ans;
}

```

## 2.7 Money Sum (Bottom Up)

Find every possible sum using the given values only once.

```

set<int> money_sum(const vi &xs) {
    using vc = vector<char>;
    using vvc = vector<vc>;
    int _m = accumulate(all(xs), 0);
    int _n = xs.size();
    vvc _dp(_n + 1, vc(_m + 1, 0));
    set<int> _ans;
    _dp[0][xs[0]] = 1;
    for (int i = 1; i < _n; ++i) {
        for (int j = 0; j <= _m; ++j) {
            if (j == 0 or _dp[i - 1][j]) {
                _dp[i][j + xs[i]] = 1;
                _dp[i][j] = 1;
            }
        }
    }

    for (int i = 0; i < _n; ++i)
        for (int j = 0; j <= _m; ++j)
            if (_dp[i][j]) _ans.insert(j);
    return _ans;
}

```

## 2.8 Travelling Salesman Problem

```

using vi = vector<int>;
vector<vi> dist;

```

```

vector<vi> memo;
/* 0 ( N^2 * 2^N )*/
int tsp(int i, int mask, int N) {
    if (mask == (1 << N) - 1) return dist[i][0];
    if (memo[i][mask] != -1) return memo[i][mask];
    int ans = INT_MAX << 1;
    for (int j = 0; j < N; ++j) {
        if (mask & (1 << j)) continue;
        auto t = tsp(j, mask | (1 << j), N) + dist[i][j];
        ans = min(ans, t);
    }
    return memo[i][mask] = ans;
}

```

## 3 Geometry

### 3.1 Convex Hull

Given a set of points find the smallest convex polygon that contains all the given points.

Time:  $O(N \log N)$

By default it removes the collinear points, set the boolean to true if you don't want that

```

struct pt {
    double x, y;
    int id;
};

int orientation(pt a, pt b, pt c) {
    double v = a.x * (b.y - c.y) + b.x * (c.y - a.y) +
              c.x * (a.y - b.y);
    if (v < 0) return -1; // clockwise
    if (v > 0) return +1; // counter-clockwise
    return 0;
}

bool cw(pt a, pt b, pt c, bool include_collinear) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}

bool collinear(pt a, pt b, pt c) {
    return orientation(a, b, c) == 0;
}

void convex_hull(vector<pt>& pts,
                bool include_collinear = false) {
    pt p0 = *min_element(all(pts), [](pt a, pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
}

```

```

});
sort(all(pts), [&p0](const pt& a, const pt& b) {
    int o = orientation(p0, a, b);
    if (o == 0)
        return (p0.x - a.x) * (p0.x - a.x) +
               (p0.y - a.y) * (p0.y - a.y) <
               (p0.x - b.x) * (p0.x - b.x) +
               (p0.y - b.y) * (p0.y - b.y);
    return o < 0;
});
if (include_collinear) {
    int i = len(pts) - 1;
    while (i >= 0 && collinear(p0, pts[i], pts.back())) i--;
    reverse(pts.begin() + i + 1, pts.end());
}

vector<pt> st;
for (int i = 0; i < len(pts); i++) {
    while (st.size() > 1 && !cw(st[len(st) - 2], st.back(),
                               pts[i], include_collinear))
        st.pop_back();
    st.push_back(pts[i]);
}

pts = st;
}

```

### 3.2 Determinant

```

#include "Point.cpp"

template <typename T>
T D(const Point<T> &P, const Point<T> &Q,
    const Point<T> &R) {
    return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
           (R.x * Q.y + R.y * P.x + Q.x * P.y);
}

```

### 3.3 Equals

```

template <typename T>
bool equals(T a, T b) {
    const double EPS{1e-9};
    if (is_floating_point<T>::value)
        return fabs(a - b) < EPS;
}

```

```

else
    return a == b;
}

```

### 3.4 Line

```

#include <bits/stdc++.h>

#include "point-struct-and-utils.cpp"
using namespace std;

struct line {
    ld a, b, c;
};

// the answer is stored in the third parameter (pass by
// reference)
void pointsToLine(const point &p1, const point &p2,
                  line &l) {
    if (fabs(p1.x - p2.x) < EPS)
        // vertical line
        l = {1.0, 0.0, -p1.x};
    // default values
    else
        l = {-(ld)(p1.y - p2.y) / (p1.x - p2.x), 1.0,
            -(ld)(l.a * p1.x) - p1.y};
}

```

### 3.5 Point Struct And Utils (2d)

```

#include <bits/stdc++.h>
using namespace std;
using ld = long double;

struct point {
    ld x, y;
    int id;
    point(ld x = 0.0, ld y = 0.0, int id = -1)
        : x(x), y(y), id(id) {}

    point& operator+=(const point& t) {
        x += t.x;
        y += t.y;
        return *this;
    }

    point& operator-=(const point& t) {

```

```

    x -= t.x;
    y -= t.y;
    return *this;
}
point& operator*=(ld t) {
    x *= t;
    y *= t;
    return *this;
}
point& operator/=(ld t) {
    x /= t;
    y /= t;
    return *this;
}
point operator+(const point& t) const {
    return point(*this) += t;
}
point operator-(const point& t) const {
    return point(*this) -= t;
}
point operator*(ld t) const { return point(*this) *= t; }
point operator/(ld t) const { return point(*this) /= t; }
};

ld dot(point& a, point& b) { return a.x * b.x + a.y * b.y; }

ld norm(point& a) { return dot(a, a); }

ld abs(point a) { return sqrt(norm(a)); }

ld proj(point a, point b) { return dot(a, b) / abs(b); }

ld angle(point a, point b) {
    return acos(dot(a, b) / abs(a) / abs(b));
}

```

### 3.6 Polygon Lattice Points (Pick's Theorem)

Given a polygon with  $N$  points finds the number of lattice points inside and on boundaries. Time :  $O(N)$

```

ll cross(ll x1, ll y1, ll x2, ll y2) {
    return x1 * y2 - x2 * y1;
}

```

```

ll polygonArea(vector<p11>& pts) {

```

```

    ll ats = 0;
    for (int i = 2; i < len(pts); i++)
        ats += cross(pts[i].first - pts[0].first,
                     pts[i].second - pts[0].second,
                     pts[i - 1].first - pts[0].first,
                     pts[i - 1].second - pts[0].second);
    return abs(ats / 2ll);
}

ll boundary(vector<p11>& pts) {
    ll ats = pts.size();
    for (int i = 0; i < len(pts); i++) {
        ll deltax =
            (pts[i].first - pts[(i + 1) % pts.size()].first);
        ll deltay =
            (pts[i].second - pts[(i + 1) % pts.size()].second);
        ats += abs(__gcd(deltax, deltay)) - 1;
    }
    return ats;
}

p11 latticePoints(vector<p11>& pts) {
    ll bounds = boundary(pts);
    ll area = polygonArea(pts);
    ll inside = area + 1ll - bounds / 2ll;

    return {inside, bounds};
}

```

### 3.7 Segment

```

#include "Line.cpp"
#include "Point.cpp"
#include "equals.cpp"

```

```

template <typename T>
struct segment {
    Point<T> A, B;

    bool contains(const Point<T> &P) const;

    Point<T> closest(const Point<T> &p) const;
};

```

```

template <typename T>

```

```

bool segment<T>::contains(const Point<T> &P) const {
    // verifica se P é est contido na reta
    double dAB = Point<T>::dist(A, B),
           dAP = Point<T>::dist(A, P),
           dPB = Point<T>::dist(P, B);

    return equals(dAP + dPB, dAB);
}

template <typename T>
Point<T> segment<T>::closest(const Point<T> &P) const {
    Line<T> R(A, B);
    auto Q = R.closest(P);

    if (this->contains(Q)) return Q;

    auto distA = Point<T>::dist(P, A);
    auto distB = Point<T>::dist(P, B);

    if (distA <= distB)
        return A;
    else
        return B;
}

```

### 3.8 Template Line

```

#include "template-point.cpp"

template <typename T>
struct Line {
    T a, b, c;

    Line(T av, T bv, T cv) : a(av), b(bv), c(cv) {}

    Line(const Point<T> &P, const Point<T> &Q)
        : a(P.y - Q.y),
          b(Q.x - P.x),
          c(P.x * Q.y - Q.x * P.y) {}

    // verify if a point belongs to the line
    bool contains(const Point<T> &P) {
        return equals(a * P.x + b * P.y + c, 0);
    }
}

```

```

// shortest distance between P and a point Q that belongs
// to this line
double distance(const Point<T> &P) const {
    return fabs(a * P.x + b * P.y + c) / hypot(a, b);
}

// the closest point in this line to the given point
Point<T> closest(const Point<T> &P) const {
    auto den = (a * a) + (b * b);

    auto x = (b * (b * P.x - a * P.y) - a * c) / den;
    auto y = (a * (-b * P.x + a * P.y) - b * c) / den;

    return Point<T>{x, y};
}
};

```

### 3.9 Template Point

```

template <typename T>
struct Point {
    T x, y;

    Point(T xv = 0, T yv = 0) : x(xv), y(yv) {}

    double distance(const Point<T> &P) const {
        return hypot(static_cast<double>(P.x - this->x),
                     static_cast<double>(P.y - this->y));
    }
};

```

### 3.10 Template Segment

```

#include "equals.cpp"
#include "template-line.cpp"
#include "template-point.cpp"

template <typename T>
struct Segment {
    Point<T> A, B;

    Segment(const Point<T> &a, const Point<T> &b)
        : A(a), B(b) {}

    /*
     * Verify if a given point P belongs to the segment,

```

```

* considering that P belongs to the line defined with A
* and B
*/
bool contains(const Point<T> &P) const {
    return equals(A.x, B.x)
        ? min(A.y, B.y) <= P.y and P.y <= max(A.y, B.y)
        : min(A.x, B.x) <= P.x and
            P.x <= max(A.x, B.x);
}

/*
* Verify if P belongs to the segment AB,
* even if P don't belong to the line defined with A and B
* */
bool contains2(const Point<T> &P) const {
    double dAB = dist(A, B), dAP = dist(A, P),
           dPB = dist(P, B);
    return equals(dAP + dPB, dAB);
}

/*
* Find the closest point in P that belongs to the segment
* */
Point<T> closest(const Point<T> &P) {
    Line<T> r(A, B);
    auto Q = r.closest(P);

    if (this->contains(Q)) return Q;

    auto distA = P.distance(A);
    auto distB = P.distance(B);

    return distA <= distB ? A : B;
}

double distToClosest(const Point<T> &P) {
    return closest(P).distance(P);
}
};

```

## 4 Graphs

### 4.1 2 SAT

```

struct SAT2 {

```

```

    ll n;
    vll2d adj, adj_t;
    vc used;
    vll order, comp;
    vc assignment;
    bool solvable;
    SAT2(ll _n)
        : n(2 * _n),
          adj(n),
          adj_t(n),
          used(n),
          order(n),
          comp(n, -1),
          assignment(n / 2) {}
    void dfs1(int v) {
        used[v] = true;
        for (int u : adj[v]) {
            if (!used[u]) dfs1(u);
        }
        order.push_back(v);
    }

    void dfs2(int v, int cl) {
        comp[v] = cl;
        for (int u : adj_t[v]) {
            if (comp[u] == -1) dfs2(u, cl);
        }
    }

    bool solve_2SAT() {
        // find and label each SCC
        for (int i = 0; i < n; ++i) {
            if (!used[i]) dfs1(i);
        }
        reverse(all(order));
        ll j = 0;
        for (auto &v : order) {
            if (comp[v] == -1) dfs2(v, j++);
        }

        assignment.assign(n / 2, false);
        for (int i = 0; i < n; i += 2) {
            // x and !x belong to the same SCC
            if (comp[i] == comp[i + 1]) {
                solvable = false;
            }
        }
    }
};

```

```

        return false;
    }

    assignment[i / 2] = comp[i] > comp[i + 1];
}
solvable = true;
return true;
}

void add_disjunction(int a, bool na, int b, bool nb) {
    a = (2 * a) ^ na;
    b = (2 * b) ^ nb;
    int neg_a = a ^ 1;
    int neg_b = b ^ 1;
    adj[neg_a].push_back(b);
    adj[neg_b].push_back(a);
    adj_t[b].push_back(neg_a);
    adj_t[a].push_back(neg_b);
}
};

```

## 4.2 Cycle Distances

Given a vertex  $s$  finds the longest cycle that end's in  $s$ , note that the vector **dist** will contain the distance that each vertex  $u$  needs to reach  $s$ .

Time:  $O(N)$

```

using adj = vector<vector<pair<int, ll>>>;
ll cycleDistances(int u, int n, int s, vc &vis, adj &g,
                  vll &dist) {

    vis[u] = 1;

    for (auto [v, d] : g[u]) {
        if (v == s) {
            dist[u] = max(dist[u], d);
            continue;
        }

        if (vis[v] == 1) {
            continue;
        }

        if (vis[v] == 2) {
            dist[u] = max(dist[u], dist[v] + d);
        } else {
            ll d2 = cycleDistances(v, n, s, vis, g, dist);
            if (d2 != -oo) {

```

```

                dist[u] = max(dist[u], d2 + d);
            }
        }
    }
    vis[u] = 2;
    return dist[u];
}

```

## 4.3 SCC (struct)

Able to find the component of each node and the total of SCC in  $O(V * E)$  and build the SCC graph ( $O(V * E)$ ).

```

struct SCC {
    ll N;
    int totsc;
    vll2d adj, tadj;
    vll todo, comps, comp;
    vector<set<ll>> sccadj;
    vchar vis;
    SCC(ll _N)
        : N(_N),
          totsc(0),
          adj(_N),
          tadj(_N),
          comp(_N, -1),
          sccadj(_N),
          vis(_N) {}

    void add_edge(ll x, ll y) { adj[x].eb(y), tadj[y].eb(x); }

    void dfs(ll x) {
        vis[x] = 1;
        for (auto &y : adj[x])
            if (!vis[y]) dfs(y);
        todo.pb(x);
    }

    void dfs2(ll x, ll v) {
        comp[x] = v;
        for (auto &y : tadj[x])
            if (comp[y] == -1) dfs2(y, v);
    }

    void gen() {
        for (ll i = 0; i < N; ++i)
            if (!vis[i]) dfs(i);
        reverse(all(todo));
    }
};

```

```

for (auto &x : todo)
    if (comp[x] == -1) {
        dfs2(x, x);
        comps.pb(x);
        totsc++;;
    }
}

void genSCCGraph() {
    for (ll i = 0; i < N; ++i) {
        for (auto &j : adj[i]) {
            if (comp[i] != comp[j]) {
                sccadj[comp[i]].insert(comp[j]);
            }
        }
    }
}
};

```

#### 4.4 Bellman-Ford (find negative cycle)

Given a directed graph find a negative cycle by running  $n$  iterations, and if the last one produces a relaxation then there is a cycle.

Time:  $O(V \cdot E)$

```

const ll oo = 2500 * 1e9;

using graph = vector<vector<pair<int, ll>>>;
vi negative_cycle(graph &g, int n) {
    vll d(n, oo);
    vi p(n, -1);
    int x = -1;
    d[0] = 0;
    for (int i = 0; i < n; i++) {
        x = -1;
        for (int u = 0; u < n; u++) {
            for (auto &[v, l] : g[u]) {
                if (d[u] + l < d[v]) {
                    d[v] = d[u] + l;
                    p[v] = u;
                    x = v;
                }
            }
        }
    }
}

```

```

if (x == -1)
    return {};
else {
    for (int i = 0; i < n; i++) x = p[x];
    vi cycle;
    for (int v = x;; v = p[v]) {
        cycle.pb(v);
        if (v == x and len(cycle) > 1) break;
    }
    reverse(all(cycle));
    return cycle;
}
}

```

#### 4.5 Bellman Ford

Find shortest path from a single source to all other nodes. Can detect negative cycles.

Time:  $O(V \cdot E)$

```

bool bellman_ford(const vector<vector<pair<int, ll>>> &g,
                  int s, vector<ll> &dist) {
    int n = (int)g.size();
    dist.assign(n, LLONG_MAX);

    vector<int> count(n);
    vector<char> in_queue(n);
    queue<int> q;

    dist[s] = 0;
    q.push(s);
    in_queue[s] = true;

    while (not q.empty()) {
        int cur = q.front();
        q.pop();
        in_queue[cur] = false;

        for (auto [to, w] : g[cur]) {
            if (dist[cur] + w < dist[to]) {
                dist[to] = dist[cur] + w;
                if (not in_queue[to]) {
                    q.push(to);
                    in_queue[to] = true;
                    count[to]++;
                    if (count[to] > n) return false;
                }
            }
        }
    }
}

```

```

    }
}
}

return true;
}

```

## 4.6 BFS 01

Similar to a Dijkstra given a weighted graph finds the distance from source  $s$  to every other node (SSSP).

Applicable only when the weight of the edges  $\in \{0, x\}$

Time:  $O(V + E)$

```

vector<pair<ll, int>> adj[maxn];
ll dists[maxn];
int s, n;
void bfs_01() {
    fill(dists, dists + n, oo);
    dist[s] = 0;

    deque<int> q;
    q.emplace_back(s);

    while (not q.empty()) {
        auto u = q.front();
        q.pop_front();

        for (auto [v, w] : adj[u]) {
            if (dist[v] <= dist[u] + w) continue;
            dist[v] = dist[u] + w;
            w ? q.emplace_back(v) : q.emplace_front(v);
        }
    }
}

```

## 4.7 Binary Lifting/Jumping

Given a function/successor graph answers queries of the form which is the node after  $k$  moves starting from  $u$ .

Time: build  $O(N \cdot \text{MAXLOG2})$ , query  $O(\text{MAXLOG2})$ .

```

const int MAXN(2e5), MAXLOG2(30);
int bl[MAXN][MAXLOG2 + 1];
int N;

int jump(int u, ll k) {
    for (int i = 0; i <= MAXLOG2; i++) {

```

```

        if (k & (1ll << i)) u = bl[u][i];
    }
    return u;
}

void build() {
    for (int i = 1; i <= MAXLOG2; i++) {
        for (int j = 0; j < N; j++) {
            bl[j][i] = bl[bl[j][i - 1]][i - 1];
        }
    }
}

```

## 4.8 Block Cut Tree

```

// O(n + m)
struct BlockCutTree {
    vector<vector<int>> blocks, tree;
    vector<vector<pair<int, int>>> block_edges;
    vector<int> articulation, pos;

    BlockCutTree(const vector<vector<int>> &g)
        : articulation(g.size()), pos(g.size()) {
        int t = 0;
        vector<int> id(g.size(), -1);
        stack<int> s1;
        stack<pair<int, int>> s2;
        for (int i = 0; i < (int)g.size(); i++)
            if (id[i] == -1) dfs(g, i, -1, t, id, s1, s2);

        tree.resize(blocks.size());
        for (int i = 0; i < (int)g.size(); i++)
            if (articulation[i])
                pos[i] = (int)tree.size(), tree.emplace_back();

        for (int i = 0; i < (int)blocks.size(); i++) {
            for (auto j : blocks[i]) {
                if (not articulation[j])
                    pos[j] = i;
                else
                    tree[i].push_back(pos[j]),
                    tree[pos[j]].push_back(i);
            }
        }
    }
}

```



```
private:
int dfs(const vector<vector<int>> &g, int i, int p,
        int &t, vector<int> &id, stack<int> &s1,
        stack<pair<int, int>> &s2) {
    int lo = id[i] = t++;
    s1.push(i);

    if (p != -1) s2.emplace(i, p);
    for (auto j : g[i])
        if (j != p and id[j] != -1) s2.emplace(i, j);

    for (auto j : g[i])
        if (j != p) {
            if (id[j] == -1) {
                int val = dfs(g, j, i, t, id, s1, s2);
                lo = min(lo, val);

                if (val >= id[i]) {
                    articulation[i]++;
                    blocks.emplace_back(1, i);
                    for (; blocks.back().back() != j; s1.pop())
                        blocks.back().push_back(s1.top());

                    block_edges.emplace_back(1, s2.top());
                    s2.pop();
                    for (; block_edges.back().back() !=
                        make_pair(j, i);
                        s2.pop())
                        block_edges.back().push_back(s2.top());
                }
            } else {
                lo = min(lo, id[j]);
            }
        }

    if (p == -1 and articulation[i]) --articulation[i];
    return lo;
}
};
```

## 4.9 Check Bipartite

$O(V)$

```
vi2d G;
int N, M;
```

```
bool check() {
    vi side(N, -1);
    queue<int> q;
    for (int st = 0; st < N; st++) {
        if (side[st] == -1) {
            q.emplace(st);
            side[st] = 0;
            while (not q.empty()) {
                int u = q.front();
                q.pop();
                for (auto v : G[u]) {
                    if (side[v] == -1) {
                        side[v] = side[u] ^ 1;
                        q.push(v);
                    } else if (side[u] == side[v])
                        return false;
                }
            }
        }
    }
    return true;
}
```

## 4.10 Dijkstra (k Shortest Paths)

```
const ll oo = 1e9 * 1e5 + 1;
using adj = vector<vector<pll>>;
vector<priority_queue<ll>> dijkstra(
    const vector<vector<pll>> &g, int n, int s, int k) {
    priority_queue<pll, vector<pll>, greater<pll>> pq;

    vector<priority_queue<ll>> dist(n);
    dist[0].emplace(0);
    pq.emplace(0, s);
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();

        if (not dist[v].empty() and dist[v].top() < d1)
            continue;

        for (auto [d2, u] : g[v]) {
            if (len(dist[u]) < k) {
                pq.emplace(d2 + d1, u);
                dist[u].emplace(d2 + d1);
            }
        }
    }
}
```

```

    } else {
        if (dist[u].top() > d1 + d2) {
            dist[u].pop();
            dist[u].emplace(d1 + d2);
            pq.emplace(d2 + d1, u);
        }
    }
}
}
return dist;
}

```

## 4.11 Dijkstra

Finds the shortest path from  $s$  to every other node, and keep the 'parent' tracking.  
Time:  $O(E \cdot \log V)$

```

pair<vll, vi> dijkstra(const vector<vector<pll>> &g, int n,
                     int s) {
    priority_queue<pll, vector<pll>, greater<pll>> pq;
    vll dist(n, oo);
    vi p(n, -1);
    pq.emplace(0, s);
    dist[s] = 0;
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();
        if (dist[v] < d1) continue;

        for (auto [d2, u] : g[v]) {
            if (dist[u] > d1 + d2) {
                dist[u] = d1 + d2;
                p[u] = v;
                pq.emplace(dist[u], u);
            }
        }
    }
    return {dist, p};
}

```

## 4.12 Disjoint Edges Path (Maxflow)

Given a directed graph find's every path with disjoint edges that starts at  $s$  and ends at  $t$   
Time:  $O(E \cdot V^2)$

```

struct DisjointPaths {
    int n;
    vi2d g, capacity;

```

```

    vector<vc> isedge;

```

```

    DisjointPaths(int _n)
        : n(_n), g(n), capacity(n, vi(n)), isedge(n, vc(n)) {}

```

```

    void add(int u, int v, int w = 1) {
        g[u].emplace_back(v);
        g[v].emplace_back(u);
        capacity[u][v] += w;
        isedge[u][v] = true;
    }

```

```

// finds the new flow to insert

```

```

int bfs(int s, int t, vi &parent) {
    fill(all(parent), -1);
    parent[s] = -2;
    queue<pair<int, int>> q;
    q.push({oo, s});

    while (!q.empty()) {
        auto [flow, cur] = q.front();
        q.pop();

        for (auto next : g[cur]) {
            if (parent[next] == -1 and capacity[cur][next]) {
                parent[next] = cur;
                ll new_flow = min(flow, capacity[cur][next]);
                if (next == t) return new_flow;
                q.push({new_flow, next});
            }
        }
    }

    return 0;
}

```

```

int maxflow(int s, int t) {
    int flow = 0;
    vi parent(n);
    int new_flow;

    while ((new_flow = bfs(s, t, parent))) {
        flow += new_flow;
        int cur = t;
        while (cur != s) {

```

```

        int prev = parent[cur];
        capacity[prev][cur] -= new_flow;
        capacity[cur][prev] += new_flow;
        cur = prev;
    }
}

return flow;
}

// build the distinct routes based in the capacity set by
// maxflow
void dfs(int u, int t, vc2d &vis, vi &route,
        vi2d &routes) {
    route.eb(u);
    if (u == t) {
        routes.emplace_back(route);
        route.pop_back();
        return;
    }

    for (auto &v : g[u]) {
        if (capacity[u][v] == 0 and isedge[u][v] and
            not vis[u][v]) {
            vis[u][v] = true;
            dfs(v, t, vis, route, routes);
            route.pop_back();
            return;
        }
    }
}

vi2d disjoint_paths(int s, int t) {
    int mf = maxflow(s, t);
    vi2d routes;
    vi route;
    vc2d vis(n, vc(n));
    for (int i = 0; i < mf; i++)
        dfs(s, t, vis, route, routes);
    return routes;
}
};

```

### 4.13 Euler Path (directed)

Given a **directed** graph finds a path that visits every edge exactly once.

Time:  $O(E)$

```

vector<int> euler_cycle(vector<vector<int>> &g, int u) {
    vector<int> res;

    stack<int> st;
    st.push(u);
    while (!st.empty()) {
        auto cur = st.top();
        if (g[cur].empty()) {
            res.push_back(cur);
            st.pop();
        } else {
            auto next = g[cur].back();
            st.push(next);

            g[cur].pop_back();
        }
    }

    for (auto &x : g)
        if (!x.empty()) return {};

    return res;
}

vector<int> euler_path(vector<vector<int>> &g, int first) {
    {
        int n = (int)g.size();
        vector<int> in(n), out(n);
        for (int i = 0; i < n; i++)
            for (auto x : g[i]) in[x]++, out[i]++;

        int a = 0, b = 0, c = 0;
        for (int i = 0; i < n; i++)
            if (in[i] == out[i])
                c++;
            else if (in[i] - out[i] == 1)
                b++;
            else if (in[i] - out[i] == -1)
                a++;
    }
}

```

```

    if (c != n - 2 or a != 1 or b != 1) return {};
}

auto res = euler_cycle(g, first);
if (res.empty()) return res;

reverse(all(res));
return res;
}

```

#### 4.14 Euler Path (undirected)

Given a **undirected** graph finds a path that visits every edge exactly once.

Time:  $O(E)$

```

vector<int> euler_cycle(vector<vector<int>> &g, int u) {
    vector<int> res;
    multiset<pair<int, int>> vis;

    stack<int> st;
    st.push(u);
    while (!st.empty()) {
        auto cur = st.top();

        while (!g[cur].empty()) {
            auto it = vis.find(make_pair(cur, g[cur].back()));
            if (it == vis.end()) break;
            g[cur].pop_back();
            vis.erase(it);
        }

        if (g[cur].empty()) {
            res.push_back(cur);
            st.pop();
        } else {
            auto next = g[cur].back();
            st.push(next);

            vis.emplace(next, cur);
            g[cur].pop_back();
        }
    }

    for (auto &x : g)
        if (!x.empty()) return {};
}

```

```

    return res;
}

vector<int> euler_path(vector<vector<int>> &g, int first) {
    int n = (int)g.size();
    int v1 = -1, v2 = -1;
    {
        bool bad = false;
        for (int i = 0; i < n; i++)
            if (g[i].size() & 1) {
                if (v1 == -1)
                    v1 = i;
                else if (v2 == -1)
                    v2 = i;
                else
                    bad = true;
            }

        if (bad or (v1 != -1 and v2 == -1)) return {};
    }

    if (v2 != -1) {
        // insert cycle
        g[v1].push_back(v2);
        g[v2].push_back(v1);
    }

    auto res = euler_cycle(g, first);
    if (res.empty()) return res;

    if (v1 != -1) {
        for (int i = 0; i + 1 < (int)res.size(); i++) {
            if ((res[i] == v1 and res[i + 1] == v2) ||
                (res[i] == v2 and res[i + 1] == v1)) {
                vector<int> res2;
                for (int j = i + 1; j < (int)res.size(); j++)
                    res2.push_back(res[j]);
                for (int j = 1; j <= i; j++) res2.push_back(res[j]);
                res = res2;
                break;
            }
        }
    }
}

```

```

    reverse(all(res));
    return res;
}

```

## 4.15 Find Articulation/Cut Points

Given an **undirected** graph find it's articulation points.

**articulation point (or cut vertex):** is defined as a **vertex** which, when removed along with associated edges, increases the number of connected components in the graph.

A vertex  $u$  can be an articulation point if and only if has at least 2 adjacent vertex

Time:  $O(N + M)$

```

const int MAXN(100);
int N;
vi2d G;
int timer;
int tin[MAXN], low[MAXN];
set<int> cpoints;

int dfs(int u, int p = -1) {
    int cnt = 0;
    low[u] = tin[u] = timer++;
    for (auto v : G[u]) {
        if (not tin[v]) {
            cnt++;
            dfs(v, u);

            if (low[v] >= tin[u]) cpoints.insert(u);
            low[u] = min(low[u], low[v]);
        } else if (v != p)
            low[u] = min(low[u], tin[v]);
    }

    return cnt;
}

void getCutPoints() {
    memset(low, 0, sizeof(low));
    memset(tin, 0, sizeof(tin));
    cpoints.clear();

    timer = 1;
    for (int i = 0; i < N; i++) {
        if (tin[i]) continue;
        int cnt = dfs(i);
        if (cnt == 1) cpoints.erase(i);
    }
}

```

```

    }
}

```

## 4.16 Find Bridges (online)

```

// O((n+m)*log(n))
struct BridgeFinder {
    // 2ecc = 2 edge conected component
    // cc = conected component
    vector<int> parent, dsu_2ecc, dsu_cc, dsu_cc_size;
    int bridges, lca_iteration;
    vector<int> last_visit;

    BridgeFinder(int n)
        : parent(n, -1),
          dsu_2ecc(n),
          dsu_cc(n),
          dsu_cc_size(n, 1),
          bridges(0),
          lca_iteration(0),
          last_visit(n) {
        for (int i = 0; i < n; i++) {
            dsu_2ecc[i] = i;
            dsu_cc[i] = i;
        }
    }

    int find_2ecc(int v) {
        if (v == -1) return -1;
        return dsu_2ecc[v] == v
            ? v
            : dsu_2ecc[v] = find_2ecc(dsu_2ecc[v]);
    }

    int find_cc(int v) {
        v = find_2ecc(v);
        return dsu_cc[v] == v ? v
            : dsu_cc[v] = find_cc(dsu_cc[v]);
    }

    void make_root(int v) {
        v = find_2ecc(v);
        int root = v;
        int child = -1;
        while (v != -1) {

```

```

    int p = find_2ecc(parent[v]);
    parent[v] = child;
    dsu_cc[v] = root;
    child = v;
    v = p;
}
dsu_cc_size[root] = dsu_cc_size[child];
}

void merge_path(int a, int b) {
    ++lca_iteration;
    vector<int> path_a, path_b;
    int lca = -1;
    while (lca == -1) {
        if (a != -1) {
            a = find_2ecc(a);
            path_a.push_back(a);
            if (last_visit[a] == lca_iteration) {
                lca = a;
                break;
            }
            last_visit[a] = lca_iteration;
            a = parent[a];
        }
        if (b != -1) {
            b = find_2ecc(b);
            path_b.push_back(b);
            if (last_visit[b] == lca_iteration) {
                lca = b;
                break;
            }
            last_visit[b] = lca_iteration;
            b = parent[b];
        }
    }

    for (auto v : path_a) {
        dsu_2ecc[v] = lca;
        if (v == lca) break;
        --bridges;
    }
    for (auto v : path_b) {
        dsu_2ecc[v] = lca;
        if (v == lca) break;
        --bridges;
    }
}

```

```

    }
}

void add_edge(int a, int b) {
    a = find_2ecc(a);
    b = find_2ecc(b);

    if (a == b) return;

    int ca = find_cc(a);
    int cb = find_cc(b);

    if (ca != cb) {
        ++bridges;
        if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
            swap(a, b);
            swap(ca, cb);
        }
        make_root(a);
        parent[a] = dsu_cc[a] = b;
        dsu_cc_size[cb] += dsu_cc_size[a];
    } else {
        merge_path(a, b);
    }
}
};

```

## 4.17 Find Bridges

Find every bridge in a **undirected** connected graph.

**bridge:** A bridge is defined as an **edge** which, when removed, increases the number of connected components in the graph.

Time:  $O(N + M)$

```

const int MAXN(50);
vi2d G(MAXN);
int tin[MAXN];
int low[MAXN];
char vis[MAXN];
int timer;
int N, M;
vector<pii> bridges;

void dfs(int u, int p = -1) {
    vis[u] = true;
    tin[u] = low[u] = timer++;
}

```

```

for (auto v : G[u]) {
    if (v == p) continue;
    if (vis[v]) {
        low[u] = min(low[u], tin[v]);
    } else {
        dfs(v, u);
        low[u] = min(low[u], low[v]);
        if (low[v] > tin[u]) {
            bridges.emplace_back(u, v);
        }
    }
}
}
}

void getBridges() {
    timer = 0;

    memset(vis, 0, sizeof(vis));
    memset(tin, -1, sizeof(tin));
    memset(low, -1, sizeof(low));
    bridges.clear();

    for (int i = 0; i < N; i++) {
        if (not vis[i]) dfs(i);
    }
}

```

## 4.18 Find Centroid

Given a tree (don't forget to make it 'undirected'), find its centroids.  
Time:  $O(V)$

```

void dfs(int u, int p, int n, vi2d &g, vi &sz,
         vi &centroid) {
    sz[u] = 1;

    bool iscentroid = true;
    for (auto v : g[u])
        if (v != p) {
            dfs(v, u, n, g, sz, centroid);
            if (sz[v] > n / 2) iscentroid = false;
            sz[u] += sz[v];
        }

    if (n - sz[u] > n / 2) iscentroid = false;
    if (iscentroid) centroid.eb(u);
}

```

```

vi getCentroid(vi2d &g, int n) {
    vi centroid;
    vi sz(n);
    dfs(0, -1, n, g, sz, centroid);
    return centroid;
}

```

## 4.19 Floyd Warshall

Simply finds the minimal distance for each node to every other node.  $O(V^3)$

```

vector<vll> floyd_warshall(const vector<vll> &adj, ll n) {
    auto dist = adj;

    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            for (int k = 0; k < n; ++k) {
                dist[j][k] =
                    min(dist[j][k], dist[j][i] + dist[i][k]);
            }
        }
    }
    return dist;
}

```

## 4.20 Functional/Successor Graph

Given a functional graph find the vertex after  $k$  moves starting at  $u$  and also the distance between  $u$  and  $v$ , if it's impossible to reach  $v$  starting at  $u$  returns -1.

Time: build  $O(N \cdot \text{MAXLOG2})$ , kth  $O(\text{MAXLOG2})$ , dist  $O(\text{MAXLOG2})$

```

const int MAXN(2'000'000), MAXLOG2(24);
int N;
vi2d succ(MAXN, vi(MAXLOG2 + 1));
vi dst(MAXN, 0);

int vis[MAXN];
void dfsbuild(int u) {
    if (vis[u]) return;
    vis[u] = 1;
    int v = succ[u][0];
    dfsbuild(v);
    dst[u] = dst[v] + 1;
}

void build() {
    for (int i = 0; i < N; i++) {

```

```

    if (not vis[i]) dfsbuild(i);
}

for (int k = 1; k <= MAXLOG2; k++) {
    for (int i = 0; i < N; i++) {
        succ[i][k] = succ[succ[i][k - 1]][k - 1];
    }
}
}

```

```

int kth(int u, ll k) {
    if (k <= 0) return u;
    for (int i = 0; i <= MAXLOG2; i++)
        if ((1ll << i) & k) u = succ[u][i];
    return u;
}

```

```

int dist(int u, int v) {
    int cu = kth(u, dst[u]);
    if (kth(u, dst[u] - dst[v]) == v)
        return dst[u] - dst[v];
    else if (kth(cu, dst[cu] - dst[v]) == v)
        return dst[u] + (dst[cu] - dst[v]);
    else
        return -1;
}

```

## 4.21 Graph Cycle (directed)

Given a directed graph finds a cycle (or not).

Time :  $O(E)$

```

bool dfs(int v, vi2d &adj, vc &visited, vi &parent,
         vc &color, int &cycle_start, int &cycle_end) {
    color[v] = 1;
    for (int u : adj[v]) {
        if (color[u] == 0) {
            parent[u] = v;
            if (dfs(u, adj, visited, parent, color, cycle_start,
                    cycle_end))
                return true;
        } else if (color[u] == 1) {
            cycle_end = v;
            cycle_start = u;
            return true;
        }
    }
}

```

```

}
color[v] = 2;
return false;
}

```

```

vi find_cycle(vi2d &g, int n) {
    vc visited(n);
    vi parent(n);
    vc color(n);
    int cycle_start, cycle_end;
    color.assign(n, 0);
    parent.assign(n, -1);
    cycle_start = -1;

    for (int v = 0; v < n; v++) {
        if (color[v] == 0 && dfs(v, g, visited, parent, color,
                                cycle_start, cycle_end))
            break;
    }

    if (cycle_start == -1) {
        return {};
    } else {
        vector<int> cycle;
        cycle.push_back(cycle_start);
        for (int v = cycle_end; v != cycle_start; v = parent[v])
            cycle.push_back(v);
        cycle.push_back(cycle_start);
        reverse(cycle.begin(), cycle.end());
        return cycle;
    }
}

```

## 4.22 Graph Cycle (undirected)

Detects if a graph contains a cycle. If path parameter is not null, it will contain the cycle if one exists.

Time:  $O(V + E)$

```

void graph_cycles(const vector<vector<int>> &g, int u,
                 int p, vector<int> &ps,
                 vector<int> &color, int &cn,
                 vector<vector<int>> &cycles) {

    if (color[u] == 2) {
        return;
    }
}

```



```

if (color[u] == 1) {
    cn++;
    int cur = p;
    cycles.emplace_back();
    auto &v = cycles.back();
    v.push_back(cur);
    while (cur != u) {
        cur = ps[cur];
        v.push_back(cur);
    }
    reverse(all(v));
    return;
}

ps[u] = p;
color[u] = 1;
for (auto v : g[u]) {
    if (v != p)
        graph_cycles(g, v, u, ps, color, cn, cycles);
}

color[u] = 2;
}

vector<vector<int>> graph_cycles(
    const vector<vector<int>> &g) {
    vector<int> ps(g.size(), -1), color(g.size());
    int cn = 0;
    vector<vector<int>> cycles;
    for (int i = 0; i < (int)g.size(); i++)
        graph_cycles(g, i, -1, ps, color, cn, cycles);
    return cycles;
}

```

## 4.23 Heavy Light Decomposition

```

struct HeavyLightDecomposition {
    vector<int> parent, depth, size, heavy, head, pos;

    using SegT = int;
    static SegT op(SegT a, SegT b) { return max(a, b); }
    SegTree<SegT, op> seg;

    HeavyLightDecomposition(const vector<vector<int>> &g,
        const vector<int> &v,

```

```

        int root = 0)
    : parent(g.size()),
      depth(g.size()),
      size(g.size()),
      heavy(g.size(), -1),
      head(g.size()),
      pos(g.size()),
      seg((int)g.size()) {
        dfs(g, root);
        int cur_pos = 0;
        decompose(g, root, root, cur_pos);

        for (int i = 0; i < (int)g.size(); i++) {
            seg.set(pos[i], v[i]);
        }
    }

    SegT query_path(int a, int b) const {
        int res = 0;
        for (; head[a] != head[b]; b = parent[head[b]]) {
            if (depth[head[a]] > depth[head[b]]) swap(a, b);
            res = op(res, seg.query(pos[head[b]], pos[b]));
        }
        if (depth[a] > depth[b]) swap(a, b);
        return op(res, seg.query(pos[a], pos[b]));
    }

    SegT query_subtree(int a) const {
        return seg.query(pos[a], pos[a] + size[a] - 1);
    }

    void set(int a, int x) { seg.set(pos[a], x); }

private:
    void dfs(const vector<vector<int>> &g, int u) {
        size[u] = 1;
        int mx_child_size = 0;
        for (auto x : g[u])
            if (x != parent[u]) {
                parent[x] = u;
                depth[x] = depth[u] + 1;
                dfs(g, x);
                size[u] += size[x];
                if (size[x] > mx_child_size)
                    mx_child_size = size[x], heavy[u] = x;
            }
    }

```

```

    }
}

void decompose(const vector<vector<int>> &g, int u, int h,
               int &cur_pos) {
    head[u] = h;
    pos[u] = cur_pos++;
    if (heavy[u] != -1) decompose(g, heavy[u], h, cur_pos);

    for (auto x : g[u])
        if (x != parent[u] and x != heavy[u]) {
            decompose(g, x, x, cur_pos);
        }
}
};

```

## 4.24 Kruskal

Find the minimum spanning tree of a graph.

Time:  $O(E \log E)$

can be used to find the maximum spanning tree by changing the comparison operator in the sort

```

struct UFDS {
    vector<int> ps, sz;
    int components;

    UFDS(int n) : ps(n + 1), sz(n + 1, 1), components(n) {
        iota(all(ps), 0);
    }

    int find_set(int x) {
        return (x == ps[x] ? x : (ps[x] = find_set(ps[x])));
    }

    bool same_set(int x, int y) {
        return find_set(x) == find_set(y);
    }

    void union_set(int x, int y) {
        x = find_set(x);
        y = find_set(y);

        if (x == y) return;

        if (sz[x] < sz[y]) swap(x, y);

        ps[y] = x;
    }
}

```

```

        sz[x] += sz[y];

        components--;
    }
};

vector<tuple<ll, int, int>> kruskal(
    int n, vector<tuple<ll, int, int>> &edges) {
    UFDS udfs(n);
    vector<tuple<ll, int, int>> ans;

    sort(all(edges));
    for (auto [a, b, c] : edges) {
        if (udfs.same_set(b, c)) continue;

        ans.emplace_back(a, b, c);
        udfs.union_set(b, c);
    }

    return ans;
}

```

## 4.25 Lowest Common Ancestor (Binary Lifting)

Finds the LCA between two nodes using binary lifting

Time: build  $O(N \cdot \text{MAXLOG2})$  query  $O(\text{MAXLOG2})$

```

const int MAXLOG2 = 20, MAXN(2'00'000);
int N;
int G[MAXN];
int depth[MAXN];
int up[MAXN][MAXLOG2 + 1];
vi GT[MAXN];

void build(int u = 0) {
    for (int i = 1; i <= MAXLOG2; i++)
        up[u][i] = up[up[u][i - 1]][i - 1];

    for (int v : GT[u])
        if (v != up[u][0]) {
            depth[v] = depth[up[v][0] = u] + 1;
            build(v);
        }
}

int jump(int u, ll k) {

```

```

for (ll i = 0; i <= MAXLOG2; i++)
    if (k & (1ll << i)) u = up[u][i];

return u;
}

int lca(int a, int b) {
    if (depth[a] < depth[b]) swap(a, b);

    a = jump(a, depth[a] - depth[b]);

    if (b == a) return a;

    for (int i = MAXLOG2; i >= 0; i--) {
        int at = up[a][i], bt = up[b][i];
        if (at != bt) a = at, b = bt;
    }

    return up[a][0];
}

```

## 4.26 Lowest Common Ancestor

Given two nodes of a tree find their lowest common ancestor, or their distance  
 Build :  $O(V)$ , Queries:  $O(1)$

```

template <typename T>
struct SparseTable {
    vector<T> v;
    int n;
    static const int b = 30;
    vi mask, t;

    int op(int x, int y) { return v[x] < v[y] ? x : y; }
    int msb(int x) {
        return __builtin_clz(1) - __builtin_clz(x);
    }

    SparseTable() {}
    SparseTable(const vector<T>& v_)
        : v(v_), n(v.size()), mask(n), t(n) {
        for (int i = 0, at = 0; i < n; mask[i++] = at != 1) {
            at = (at << 1) & ((1 << b) - 1);
            while (at and op(i, i - msb(at & -at)) == i)
                at ^= at & -at;
        }
        for (int i = 0; i < n / b; i++)

```

```

        t[i] = b * i + b - 1 - msb(mask[b * i + b - 1]);
        for (int j = 1; (1 << j) <= n / b; j++)
            for (int i = 0; i + (1 << j) <= n / b; i++)
                t[n / b * j + i] =
                    op(t[n / b * (j - 1) + i],
                       t[n / b * (j - 1) + i + (1 << (j - 1))]);
    }
    int small(int r, int sz = b) {
        return r - msb(mask[r] & ((1 << sz) - 1));
    }
    T query(int l, int r) {
        if (r - l + 1 <= b) return small(r, r - l + 1);
        int ans = op(small(l + b - 1), small(r));
        int x = l / b + 1, y = r / b - 1;
        if (x <= y) {
            int j = msb(y - x + 1);
            ans = op(ans, op(t[n / b * j + x],
                             t[n / b * j + y - (1 << j) + 1]));
        }
        return ans;
    }
};

```

```

struct LCA {
    SparseTable<int> st;
    int n;
    vi v, pos, dep;

    LCA(const vi2d& g, int root) : n(len(g)), pos(n) {
        dfs(root, 0, -1, g);
        st = SparseTable<int>(vector<int>(all(dep)));
    }

    void dfs(int i, int d, int p, const vi2d& g) {
        v.eb(len(dep)) = i, pos[i] = len(dep), dep.eb(d);
        for (auto j : g[i])
            if (j != p) {
                dfs(j, d + 1, i, g);
                v.eb(len(dep)) = i, dep.eb(d);
            }
    }

    int lca(int a, int b) {
        int l = min(pos[a], pos[b]);
        int r = max(pos[a], pos[b]);

```

```

    return v[st.query(l, r)];
}
int dist(int a, int b) {
    return dep[pos[a]] + dep[pos[b]] -
           2 * dep[pos[lca(a, b)]];
}
};

```

## 4.27 Maximum Flow (Edmonds-Karp)

Finds the **maximum flow** in a graph network, given the **source**  $s$  and the **sink**  $t$ .  
Time:  $O(V \cdot E^2)$

```

struct maxflow {
    int n;
    vi2d g;
    vll2d cps;
    vi ps;
    vector<vector<char>> isedge;

    maxflow(int _n)
        : n(_n),
          g(n),
          cps(n, vll(n)),
          ps(n),
          isedge(n, vc(n)) {}

    void add(int u, int v, ll c, bool set = true) {
        g[u].emplace_back(v);
        g[v].emplace_back(u);
        cps[u][v] = cps[u][v] * (!set) + c;
        isedge[u][v] = true;
    }

    ll bfs(int s, int t) {
        fill(all(ps), -1);
        ps[s] = -2;
        queue<pair<ll, int>> q;
        q.emplace(0, s);

        while (!q.empty()) {
            auto [flow, cur] = q.front();
            q.pop();

            for (auto next : g[cur]) {
                if (ps[next] == -1 and cps[cur][next]) {

```

```

                    ps[next] = cur;
                    ll new_flow = min(flow, cps[cur][next]);
                    if (next == t) return new_flow;
                    q.emplace(new_flow, next);
                }
            }
        }

        return 0;
    }

    ll flow(int s, int t) {
        ll flow = 0;
        ll new_flow;

        while ((new_flow = bfs(s, t))) {
            flow += new_flow;
            int cur = t;
            while (cur != s) {
                int prev = ps[cur];
                cps[prev][cur] -= new_flow;
                cps[cur][prev] += new_flow;
                cur = prev;
            }
        }

        return flow;
    }

    vector<pii> get_used() {
        vector<pii> used;
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
                if (isedge[i][j] and cps[i][j] > 0)
                    used.emplace_back(i, j);
            }
        }
        return used;
    }
};

```

## 4.28 Minimum Cost Flow

Given a network find the minimum cost to achieve a flow of at most  $f$ . Works with **directed** and **undirected** graphs

- **add(u, v, w, c):** adds an edge from  $u$  to  $v$  with capacity  $w$  and cost  $c$ .
- **flow(s, t, f):** return a pair ( $flow, cost$ ) with the maximum flow until  $f$  with source at  $s$  and sink at  $t$ , with the minimum cost possible.

Time :  $O(N \cdot M + f \cdot m \log n)$

```
template <typename T>
struct mcmf {
    struct edge {
        int to, rev, flow, cap;
        bool res; // if it's a reverse edge
        T cost; // cost per unity of flow
        edge()
        : to(0),
          rev(0),
          flow(0),
          cap(0),
          cost(0),
          res(false) {}
        edge(int to_, int rev_, int flow_, int cap_, T cost_,
             bool res_)
        : to(to_),
          rev(rev_),
          flow(flow_),
          cap(cap_),
          res(res_),
          cost(cost_) {}
    };

    vector<vector<edge>> g;
    vector<int> par_idx, par;
    T inf;
    vector<T> dist;

    mcmf(int n)
    : g(n),
      par_idx(n),
      par(n),
      inf(numeric_limits<T>::max() / 3) {}

    void add(int u, int v, int w, T cost) {
        edge a = edge(v, g[v].size(), 0, w, cost, false);
        edge b = edge(u, g[u].size(), 0, 0, -cost, true);

        g[u].push_back(a);
        g[v].push_back(b);
    }
};
```

```
vector<T> spfa(int s) { // don't code it if there isn't
                        // negative cycles

    deque<int> q;
    vector<bool> is_inside(g.size(), 0);
    dist = vector<T>(g.size(), inf);

    dist[s] = 0;
    q.push_back(s);
    is_inside[s] = true;

    while (!q.empty()) {
        int v = q.front();
        q.pop_front();
        is_inside[v] = false;

        for (int i = 0; i < g[v].size(); i++) {
            auto [to, rev, flow, cap, res, cost] = g[v][i];
            if (flow < cap and dist[v] + cost < dist[to]) {
                dist[to] = dist[v] + cost;

                if (is_inside[to]) continue;
                if (!q.empty() and dist[to] > dist[q.front()])
                    q.push_back(to);
                else
                    q.push_front(to);
                is_inside[to] = true;
            }
        }
    }
    return dist;
}

bool dijkstra(int s, int t, vector<T>& pot) {
    priority_queue<pair<T, int>, vector<pair<T, int>>,
                  greater<>>
    q;
    dist = vector<T>(g.size(), inf);
    dist[s] = 0;
    q.emplace(0, s);
    while (q.size()) {
        auto [d, v] = q.top();
        q.pop();
        if (dist[v] < d) continue;
        for (int i = 0; i < g[v].size(); i++) {
            auto [to, rev, flow, cap, res, cost] = g[v][i];
```

```

        cost += pot[v] - pot[to];
        if (flow < cap and dist[v] + cost < dist[to]) {
            dist[to] = dist[v] + cost;
            q.emplace(dist[to], to);
            par_idx[to] = i, par[to] = v;
        }
    }
}
return dist[t] < inf;
}

pair<int, T> min_cost_flow(int s, int t, int flow = inf) {
    vector<T> pot(g.size(), 0);
    pot = spfa(s); // comment this line if there isn't
                  // negative cycles

    int f = 0;
    T ret = 0;
    while (f < flow and dijkstra(s, t, pot)) {
        for (int i = 0; i < g.size(); i++)
            if (dist[i] < inf) pot[i] += dist[i];

        int mn_flow = flow - f, u = t;
        while (u != s) {
            mn_flow =
                min(mn_flow, g[par[u]][par_idx[u]].cap -
                    g[par[u]][par_idx[u]].flow);
            u = par[u];
        }

        ret += pot[t] * mn_flow;

        u = t;
        while (u != s) {
            g[par[u]][par_idx[u]].flow += mn_flow;
            g[u][g[par[u]][par_idx[u]].rev].flow -= mn_flow;
            u = par[u];
        }

        f += mn_flow;
    }

    return make_pair(f, ret);
}
};

```

## 4.29 Minimum Cut (unweighted)

After build the **direct**/**undirected** graph find the minimum of edges needed to be removed to make the sink  $t$  unreachable from the source  $s$ .

Time:  $O(V \cdot E^2)$

```

struct Mincut {
    int n;
    vi2d g;
    vii edges;
    vll2d capacity;
    vi ps, vis;

    Mincut(int _n)
        : n(_n), g(n), capacity(n, vll(n)), ps(n), vis(n) {}

    void add(int u, int v, ll c = 1, bool directed = false,
             bool set = false) {
        edges.emplace_back(u, v);
        g[u].emplace_back(v);

        if (not set)
            capacity[u][v] += c;
        else
            capacity[u][v] = c;

        if (not directed) {
            g[v].emplace_back(u);

            if (not set)
                capacity[v][u] += c;
            else
                capacity[v][u] = c;
        }
    }

    ll bfs(int s, int t) {
        fill(all(ps), -1);
        ps[s] = -2;
        queue<pair<ll, int>> q;
        q.push({0, s});

        while (!q.empty()) {
            auto [flow, cur] = q.front();
            q.pop();

            for (auto next : g[cur]) {

```

```

        if (ps[next] == -1 and capacity[cur][next]) {
            ps[next] = cur;
            ll new_flow = min(flow, capacity[cur][next]);
            if (next == t) return new_flow;
            q.push({new_flow, next});
        }
    }
}

return 0ll;
}

ll maxflow(int s, int t) {
    ll flow = 0;
    ll new_flow;

    while ((new_flow = bfs(s, t))) {
        flow += new_flow;
        int cur = t;
        while (cur != s) {
            int prev = ps[cur];
            capacity[prev][cur] -= new_flow;
            capacity[cur][prev] += new_flow;
            cur = prev;
        }
    }

    return flow;
}

void dfs(int u) {
    vis[u] = true;

    for (auto v : g[u]) {
        if (capacity[u][v] > 0 and not vis[v]) {
            dfs(v);
        }
    }
}

vii mincut(int s, int t) {
    maxflow(s, t);
    fill(all(vis), 0);
    dfs(s);

    vii removed;

```

```

        for (auto &[u, v] : edges) {
            if ((vis[u] and not vis[v]) or
                (vis[v] and not vis[u]))
                removed.emplace_back(u, v);
        }

    return removed;
}

};

4.30 Prim (MST)

Given a graph with  $N$  vertex finds the minimum spanning tree, if there is no such tree returns inf, it starts using the edges that connect with each  $s_i \in s$ , if none is provided than it starts with the edges of node 0.
Time:  $O(V \log E)$ 

const int MAXN(1'000'000);
int N;
vector<pair<ll, int>> G[MAXN];
ll prim(vi s = vi(1, 0)) {
    priority_queue<pair<ll, int>, vector<pair<ll, int>>,
        greater<pair<ll, int>>>
        pq;
    vector<char> ingraph(MAXN);
    int ingraphcnt(0);
    for (auto si : s) {
        ingraphcnt++;
        ingraph[si] = true;
        for (auto &[w, v] : G[si]) pq.emplace(w, v);
    }

    ll mstcost = 0;
    while (ingraphcnt < N and !pq.empty()) {
        ll w;
        int v;

        do {
            tie(w, v) = pq.top();
            pq.pop();
        } while (not pq.empty() and ingraph[v]);

        mstcost += w, ingraph[v] = true, ingraphcnt++;

        for (auto &[w2, v2] : G[v]) {
            pq.emplace(w2, v2);
        }
    }
}

```

```

}

return ingraphcnt == N ? mstcost : oo;
}

```

### 4.31 Small to Large

Answer queries of the form "How many vertices in the subtree of vertex  $v$  have property  $P$ ?"  
 \* this implementation answers how many distinct  $values[i]$  are in the subtree starting at  $u$ .  
 Build:  $O(N)$ , Query:  $O(N \log N)$

```

struct SmallToLarge {
    int n;
    vi2d tree, vis_chlds;
    vi sizes, values, ans;
    set<int> cnt;

    SmallToLarge(vi2d &g, vi &v)
        : tree(g),
          vis_chlds(len(g)),
          sizes(len(g)),
          values(v),
          ans(len(g)) {
        get_size(0);
        dfs(0);
    }

    inline void add_value(int u) { cnt.insert(values[u]); }

    inline void remove_value(int u) { cnt.erase(values[u]); }

    inline void update_ans(int u) { ans[u] = len(cnt); }

    void dfs(int u, int p = -1, bool keep = true) {
        int mx = -1;
        for (auto x : tree[u]) {
            if (x == p) continue;

            if (mx == -1 or sizes[mx] < sizes[x]) mx = x;

            for (auto x : tree[u]) {
                if (x != p and x != mx) dfs(x, u, false);
            }

            if (mx != -1) {

```

```

                dfs(mx, u, true);
                swap(vis_chlds[u], vis_chlds[mx]);
            }

            vis_chlds[u].push_back(u);
            add_value(u);

            for (auto x : tree[u]) {
                if (x != p and x != mx) {
                    for (auto y : vis_chlds[x]) {
                        add_value(y);
                        vis_chlds[u].push_back(y);
                    }
                }
            }

            update_ans(u);

            if (!keep) {
                for (auto x : vis_chlds[u]) remove_value(x);
            }
        }
    }

    void get_size(int u, int p = -1) {
        sizes[u] = 1;
        for (auto x : tree[u])
            if (x != p) {
                get_size(x, u);
                sizes[u] += sizes[x];
            }
    }
};

```

### 4.32 Successor Graph-(struct)

```

struct SuccessorGraph {
    vector<vector<int>> paths;
    vector<int> path_num, pos;
    vector<char> is_cycle;

    SuccessorGraph(const vector<int> &v)
        : path_num(v.size()), pos(v.size()) {
        paths.reserve(v.size());
        is_cycle.reserve(v.size());
    }

```



```

vector<char> vis(v.size());
for (auto i : topological_order(v)) {
    if (vis[i]) continue;

    vector<int> path;
    int cur;
    for (cur = i; not vis[cur]; cur = v[cur]) {
        path.push_back(cur);
        vis[cur] = 1;
    }

    int cycle_start = 0;
    for (; cycle_start < (int)path.size() and
        path[cycle_start] != cur;
        cycle_start++);

    if (cycle_start > 0) {
        paths.emplace_back();
        for (int j = 0; j < cycle_start; j++) {
            paths.back().push_back(path[j]);
            pos[path[j]] = j;
            path_num[path[j]] = (int)paths.size() - 1;
        }
        paths.back().push_back(cur);
        is_cycle.push_back(false);
    }

    if (cycle_start < (int)path.size()) {
        paths.emplace_back();
        for (int j = cycle_start; j < (int)path.size();
            j++) {
            paths.back().push_back(path[j]);
            pos[path[j]] = j - cycle_start;
            path_num[path[j]] = (int)paths.size() - 1;
        }
        is_cycle.push_back(true);
    }
}

const vector<int> &path(int cur) const {
    return paths[path_num[cur]];
}

```

```

int kth_pos(int cur, ll k) const {
    while (not is_cycle[path_num[cur]]) {
        auto &p = path(cur);
        int remain = (int)p.size() - pos[cur] - 1;
        if (k <= remain) return p[pos[cur] + k];
        cur = p.back();
        k -= remain;
    }

    auto &p = path(cur);
    return p[(pos[cur] + k) % p.size()];
}

// {element, number_of_moves}
pair<int, int> go_to_cycle(int cur) const {
    int moves = 0;
    while (not is_cycle[path_num[cur]]) {
        auto &p = path(cur);
        moves += (int)p.size() - pos[cur] - 1;
        cur = p.back();
    }
    return {cur, moves};
}

// min cost to reach dest from cur
int reach(int cur, int dest) const {
    int moves = 0;
    while (not is_cycle[path_num[cur]] and
        path_num[cur] != path_num[dest]) {
        auto &p = path(cur);
        moves += (int)p.size() - pos[cur] - 1;
        cur = p.back();
    }

    if (path_num[cur] != path_num[dest]) return -1;

    if (pos[cur] <= pos[dest])
        return moves + pos[dest] - pos[cur];

    if (not is_cycle[path_num[cur]]) return -1;

    return moves + pos[dest] + (int)path(cur).size() -
        pos[cur];
}

```

```
private:
    void topological_order(const vector<int> &g,
                          vector<char> &vis,
                          vector<int> &order, int u) {
        vis[u] = true;
        if (not vis[g[u]])
            topological_order(g, vis, order, g[u]);
        order.push_back(u);
    }

    vector<int> topological_order(const vector<int> &g) {
        vector<char> vis(g.size(), false);
        vector<int> order;
        for (auto i = 0; i < (int)g.size(); i++)
            if (not vis[i]) topological_order(g, vis, order, i);
        reverse(order.begin(), order.end());
        return order;
    }
};
```

### 4.33 Sum every node distance

Given a **tree**, for each node  $i$  find the sum of distance from  $i$  to every other node.  
**don't forget to set the tree as undirected, that's needed to choose an arbitrary root**  
 Time:  $O(N)$

```
void getRoot(int u, int p, vi2d &g, vll &d, vll &cnt) {
    for (int i = 0; i < len(g[u]); i++) {
        int v = g[u][i];
        if (v == p) continue;
        getRoot(v, u, g, d, cnt);
        d[u] += d[v] + cnt[v];
        cnt[u] += cnt[v];
    }
}

void dfs(int u, int p, vi2d &g, vll &cnt, vll &ansd,
         int n) {
    for (int i = 0; i < len(g[u]); i++) {
        int v = g[u][i];
        if (v == p) continue;

        ansd[v] = ansd[u] - cnt[v] + (n - cnt[v]);
        dfs(v, u, g, cnt, ansd, n);
    }
}
```

```
vll fromToAll(vi2d &g, int n) {
    vll d(n);
    vll cnt(n, 1);
    getRoot(0, -1, g, d, cnt);

    vll ansdist(n);
    ansdist[0] = d[0];

    dfs(0, -1, g, cnt, ansdist, n);
    return ansdist;
}
```

### 4.34 Topological Labelling (Kahn)

The same thing as topological sorting but over every possible order gives lexicographically smaller  
 Time:  $O(E + V \cdot \log V)$

```
const int MAXN(1'00'000);
int OUTCNT[MAXN];
vi2d GIN(MAXN);
int N;

vi toposort() {
    vi order;
    priority_queue<int> q;

    for (int i = 0; i < N; i++)
        if (!OUTCNT[i]) q.emplace(i);

    while (!q.empty()) {
        auto u = q.top();
        q.pop();
        order.emplace_back(u);
        for (auto v : GIN[u]) {
            OUTCNT[v]--;
            if (OUTCNT[v] == 0) q.emplace(v);
        }
    }

    reverse(all(order));
    return len(order) == N ? order : vi();
}
```

### 4.35 Topological Sorting (Kahn)

Finds the topological sorting in a **DAG**, if the given graph is not a **DAG** than an empty vector is returned,  
 need to 'initialize' the **INCNT** as you build the graph.

Time:  $O(V + E)$

```
const int MAXN(2'00'000);
int INCNT[MAXN];
vi2d GOUT(MAXN);
int N;

vi toposort() {
    vi order;
    queue<int> q;

    for (int i = 0; i < N; i++)
        if (!INCNT[i]) q.emplace(i);

    while (!q.empty()) {
        auto u = q.front();
        q.pop();
        order.emplace_back(u);
        for (auto v : GOUT[u]) {
            INCNT[v]--;
            if (INCNT[v] == 0) q.emplace(v);
        }
    }

    return len(order) == N ? order : vi();
}
```

## 4.36 Topological Sorting (Tarjan)

Finds a the topological order for the graph, if there is no such order it means the graph is cyclic, then it returns an empty vector

$O(V + E)$

```
const int maxn(1'00'000);
int n, m;
vi g[maxn];

int not_found = 0, found = 1, processed = 2;
int state[maxn];

bool dfs(int u, vi &order) {
    if (state[u] == processed) return true;
    if (state[u] == found) return false;

    state[u] = found;

    for (auto v : g[u]) {
```

```
        if (not dfs(v, order)) return false;
    }

    state[u] = processed;
    order.emplace_back(u);

    return true;
}

vi topo_sort() {
    vi order;
    memset(state, 0, sizeof state);

    for (int u = 0; u < n; u++) {
        if (state[u] == not_found and not dfs(u, order))
            return {};
    }

    reverse(all(order));
    return order;
}
```

## 4.37 Tree Diameter (DP)

```
const int MAXN(2'00'000);
int N;
vi2d G(MAXN);
int toleaf[MAXN], maxdist[MAXN];

void dfs(int u, int p = -1) {
    int ds1, ds2;
    ds1 = ds2 = -1;
    for (auto v : G[u]) {
        if (v == p) continue;
        if (ds1 > ds2) swap(ds1, ds2);
        dfs(v, u);

        ds1 = max(ds1, toleaf[v]);
    }

    toleaf[u] = max(ds1, ds2) + 1;

    maxdist[u] = 2 + ds1 + ds2;
}
```

```

int diameter(int root, int n) {
    dfs(root);

    int d = 0;

    for (int u = 0; u < n; ++u) d = max(d, maxdist[u]);

    return d;
}

```

## 4.38 Tree Isomorphism

Two trees are considered **isomorphic** if the hash given by *thash()* is the same.

Time:  $O(V \cdot \log V)$

```
map<vi, int> mphash;
```

```

struct Tree {
    int n;
    vi2d g;
    vi sz, cs;

    Tree(int n_) : n(n_), g(n), sz(n) {}

    void add_edge(int u, int v) {
        g[u].emplace_back(v);
        g[v].emplace_back(u);
    }

    void dfs_centroid(int v, int p) {
        sz[v] = 1;
        bool cent = true;
        for (int u : g[v])
            if (u != p) {
                dfs_centroid(u, v);
                sz[v] += sz[u];
                cent &= not(sz[u] > n / 2);
            }
        if (cent and n - sz[v] <= n / 2) cs.push_back(v);
    }

    int fhash(int v, int p) {
        vi h;
        for (int u : g[v])
            if (u != p) h.push_back(fhash(u, v));
        sort(all(h));
        if (!mphash.count(h)) mphash[h] = mphash.size();
    }
}

```

```

        return mphash[h];
    }

    ll thash() {
        cs.clear();
        dfs_centroid(0, -1);
        if (cs.size() == 1) return fhash(cs[0], -1);
        ll h1 = fhash(cs[0], cs[1]), h2 = fhash(cs[1], cs[0]);
        return (min(h1, h2) << 3011) + max(h1, h2);
    }
};

```

## 4.39 Tree Maximum Distance

Returns the maximum distance from every node to any other node in the tree.

$O(6V) = O(V)$

```

pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
    // 0(V)
    // 0 indexed
    ll mostDistantNode = root;
    ll nodeDistance = 0;
    queue<pll> q;
    vector<char> vis(n);
    q.emplace(root, 0);
    vis[root] = true;
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (dist > nodeDistance) {
            nodeDistance = dist;
            mostDistantNode = node;
        }
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return {mostDistantNode, nodeDistance};
}

ll twoNodesDist(const vector<vll> &adj, ll n, ll a, ll b) {
    queue<pll> q;
    vector<char> vis(n);
}

```

```

q.emplace(a, 0);
while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
    if (node == b) return dist;
    for (auto u : adj[node]) {
        if (!vis[u]) {
            vis[u] = true;
            q.emplace(u, dist + 1);
        }
    }
}
return -1;
}

tuple<ll, ll, ll> tree_diameter(const vector<vll> &adj,
                               ll n) {
    // returns two points of the diameter and the diameter
    // itself
    auto [node1, dist1] = mostDistantFrom(adj, n, 0); // O(V)
    auto [node2, dist2] =
        mostDistantFrom(adj, n, node1); // O(V)
    auto diameter =
        twoNodesDist(adj, n, node1, node2); // O(V)
    return make_tuple(node1, node2, diameter);
}

vll everyDistanceFromNode(const vector<vll> &adj, ll n,
                           ll root) {
    // Single Source Shortest Path, from a given root
    queue<pair<ll, ll>> q;
    vll ans(n, -1);
    ans[root] = 0;
    q.emplace(root, 0);
    while (!q.empty()) {
        auto [u, d] = q.front();
        q.pop();

        for (auto w : adj[u]) {
            if (ans[w] != -1) continue;
            ans[w] = d + 1;
            q.emplace(w, d + 1);
        }
    }
    return ans;
}

```

```

}

vll maxDistances(const vector<vll> &adj, ll n) {
    auto [node1, node2, diameter] =
        tree_diameter(adj, n); // O(3V)
    auto distances1 =
        everyDistanceFromNode(adj, n, node1); // O(V)
    auto distances2 =
        everyDistanceFromNode(adj, n, node2); // O(V)
    vll ans(n);
    for (int i = 0; i < n; ++i)
        ans[i] = max(distances1[i], distances2[i]); // O(V)
    return ans;
}

```

## 4.40 Tree Flatten

```

void tree_flatten(const vector<vector<int>> &g, int u,
                  int p, vector<int> &pre, vector<int> &pos,
                  int &idx) {
    ++idx;
    pre.push_back(u);
    for (auto x : g[u])
        if (x != p) tree_flatten(g, x, u, pre, pos, idx);
    pos[u] = idx;
}

pair<vector<int>, vector<int>> tree_flatten(
    const vector<vector<int>> &g, int root = 0) {
    vector<int> first(g.size()), last(g.size()), pre;
    int timer = -1;
    tree_flatten(g, root, -1, pre, last, timer);
    for (int i = 0; i < (int)g.size(); i++) first[pre[i]] = i;
    return {first, last};
}

```

## 5 Math

### 5.1 GCD (with factorization)

$O(\sqrt{n})$  due to factorization.

```

ll gcd_with_factorization(ll a, ll b) {
    map<ll, ll> fa = factorization(a);
    map<ll, ll> fb = factorization(b);
    ll ans = 1;

```

```

for (auto fai : fa) {
    ll k = min(fai.second, fb[fai.first]);
    while (k--) ans *= fai.first;
}
return ans;
}

```

## 5.2 GCD

```

ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }

```

## 5.3 LCM (with factorization)

$O(\sqrt{n})$  due to factorization.

```

ll lcm_with_factorization(ll a, ll b) {
    map<ll, ll> fa = factorization(a);
    map<ll, ll> fb = factorization(b);
    ll ans = 1;
    for (auto fai : fa) {
        ll k = max(fai.second, fb[fai.first]);
        while (k--) ans *= fai.first;
    }
    return ans;
}

```

## 5.4 LCM

```

ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
ll lcm(ll a, ll b) { return a / gcd(a, b) * b; }

```

## 5.5 Arithmetic Progression Sum

- $s$  : first term
- $d$  : common difference
- $n$  : number of terms

```

ll arithmeticProgressionSum(ll s, ll d, ll n) {
    return (s + (s + d * (n - 1))) * n / 2ll;
}

```

## 5.6 Binomial MOD

Precompute every factorial until  $maxn$  ( $O(maxn)$ ) allowing to answer the  $\binom{n}{k}$  in  $O(\log mod)$  time, due to the fastpow. Note that it needs  $O(maxn)$  in memory.

```

const ll MOD = 1e9 + 7;
const ll maxn = 2 * 1e6;
vll fats(maxn + 1, -1);
void precompute() {
    fats[0] = 1;
    for (ll i = 1; i <= maxn; i++) {
        fats[i] = (fats[i - 1] * i) % MOD;
    }
}

```

```

ll fpow(ll a, ll n, ll mod = LLONG_MAX) {
    if (n == 0ll) return 1ll;
    if (n == 1ll) return a;
    ll x = fpow(a, n / 2ll, mod) % mod;
    return ((x * x) % mod * (n & 1ll ? a : 1ll)) % mod;
}

```

```

ll binommod(ll n, ll k) {
    ll upper = fats[n];
    ll lower = (fats[k] * fats[n - k]) % MOD;
    return (upper * fpow(lower, MOD - 2ll, MOD)) % MOD;
}

```

## 5.7 Binomial

$O(nm)$  time,  $O(m)$  space  
Equal to  $n$  choose  $k$

```

ll binom(ll n, ll k) {
    if (k > n) return 0;
    vll dp(k + 1, 0);
    dp[0] = 1;
    for (ll i = 1; i <= n; i++)
        for (ll j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
    return dp[k];
}

```

## 5.8 Chinese Remainder Theorem

Finds the solution  $x$  to the  $n$  modular equations.

$$\begin{aligned}
 x &\equiv a_1 \pmod{m_1} \\
 &\dots \\
 x &\equiv a_n \pmod{m_n}
 \end{aligned} \tag{1}$$

The  $m_i$  don't need to be coprime, if there is no solution then it returns -1.

```

template <typename T = ll>

```

```

struct crt {
    T a, m;

    crt() : a(0), m(1) {}
    crt(T a_, T m_) : a(a_), m(m_) {}
    crt operator*(crt C) {
        auto [g, x, y] = ext_gcd(m, C.m);
        if ((a - C.a) % g != 0) a = -1;
        if (a == -1 or C.a == -1) return crt(-1, 0);
        T lcm = m / g * C.m;
        T ans = a + (x * (C.a - a) / g % (C.m / g)) * m;
        return crt((ans % lcm + lcm) % lcm, lcm);
    }
};

```

```

template <typename T = ll>
struct Congruence {
    T a, m;
};

```

```

template <typename T = ll>
T chinese_remainder_theorem(
    const vector<Congruence<T>> &equations) {
    crt<T> ans;

    for (auto &[a_, m_] : equations) {
        ans = ans * crt<T>(a_, m_);
    }

    return ans.a;
}

```

## 5.9 Euler phi $\varphi(n)$ (in range)

Computes the number of positive integers less than  $n$  that are coprimes with  $n$ , in the range  $[1, n]$ , in  $O(N \log N)$ .

```

const int MAX = 1e6;
vi range_phi(int n) {
    bitset<MAX> sieve;
    vi phi(n + 1);

    iota(phi.begin(), phi.end(), 0);
    sieve.set();

    for (int p = 2; p <= n; p += 2) phi[p] /= 2;

```

```

    for (int p = 3; p <= n; p += 2) {
        if (sieve[p]) {
            for (int j = p; j <= n; j += p) {
                sieve[j] = false;
                phi[j] /= p;
                phi[j] *= (p - 1);
            }
        }
    }

    return phi;
}

```

## 5.10 Euler phi $\varphi(n)$

Computes the number of positive integers less than  $n$  that are coprimes with  $n$ , in  $O(\sqrt{N})$ .

```

int phi(int n) {
    if (n == 1) return 1;

    auto fs = factorization(n); // a vector of pair or a map
    auto res = n;

    for (auto [p, k] : fs) {
        res /= p;
        res *= (p - 1);
    }

    return res;
}

```

## 5.11 Factorial Factorization

Computes the factorization of  $n!$  in  $\pi(N) * \log n$

```

// O(logN)
ll E(ll n, ll p) {
    ll k = 0, b = p;
    while (b <= n) {
        k += n / b;
        b *= p;
    }
    return k;
}

// O(pi(N)*logN)

```

```
map<ll, ll> factorial_factorization(ll n,
                                   const vll &primes) {
    map<ll, ll> fs;
    for (const auto &p : primes) {
        if (p > n) break;
        fs[p] = E(n, p);
    }
    return fs;
}
```

## 5.12 Factorial

```
const ll MAX = 18;
vll fv(MAX, -1);
ll factorial(ll n) {
    if (fv[n] != -1) return fv[n];
    if (n == 0) return 1;
    return n * factorial(n - 1);
}
```

## 5.13 Factorization (Pollard Rho)

Factorizes a number into its prime factors in  $O(n^{\frac{1}{4}} * \log(n))$ .

```
ll mul(ll a, ll b, ll m) {
    ll ret = a * b - (ll)((ld)1 / m * a * b + 0.5) * m;
    return ret < 0 ? ret + m : ret;
}
```

```
ll pow(ll a, ll b, ll m) {
    ll ans = 1;
    for (; b > 0; b /= 211, a = mul(a, a, m)) {
        if (b % 211 == 1) ans = mul(ans, a, m);
    }
    return ans;
}
```

```
bool prime(ll n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;

    ll r = __builtin_ctzll(n - 1), d = n >> r;
    for (int a :
         {2, 325, 9375, 28178, 450775, 9780504, 795265022}) {
        ll x = pow(a, d, n);
```

```
        if (x == 1 or x == n - 1 or a % n == 0) continue;

        for (int j = 0; j < r - 1; j++) {
            x = mul(x, x, n);
            if (x == n - 1) break;
        }
        if (x != n - 1) return 0;
    }
    return 1;
}
```

```
ll rho(ll n) {
    if (n == 1 or prime(n)) return n;
    auto f = [n](ll x) { return mul(x, x, n) + 1; };

    ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
    while (t % 40 != 0 or gcd(prd, n) == 1) {
        if (x == y) x = ++x0, y = f(x);
        q = mul(prd, abs(x - y), n);
        if (q != 0) prd = q;
        x = f(x), y = f(f(y)), t++;
    }
    return gcd(prd, n);
}
```

```
vll fact(ll n) {
    if (n == 1) return {};
    if (prime(n)) return {n};
    ll d = rho(n);
    vll l = fact(d), r = fact(n / d);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}
```

## 5.14 Factorization

Computes the factorization of n in  $O(\sqrt{n})$ .

```
map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;
        for (; n % i == 0; count++, n /= i)
            ;
        if (count) ans[i] = count;
    }
}
```



```

    if (n > 1) ans[n]++;
    return ans;
}

```

## 5.15 Fast Fourier Transform

```

template <bool invert = false>
void fft(vector<complex<double>>& xs) {
    int N = (int)xs.size();

    if (N == 1) return;

    vector<complex<double>> es(N / 2), os(N / 2);

    for (int i = 0; i < N / 2; ++i) es[i] = xs[2 * i];

    for (int i = 0; i < N / 2; ++i) os[i] = xs[2 * i + 1];

    fft<invert>(es);
    fft<invert>(os);

    auto signal = (invert ? 1 : -1);
    auto theta = 2 * signal * acos(-1) / N;
    complex<double> S{1}, S1{cos(theta), sin(theta)};

    for (int i = 0; i < N / 2; ++i) {
        xs[i] = (es[i] + S * os[i]);
        xs[i] /= (invert ? 2 : 1);

        xs[i + N / 2] = (es[i] - S * os[i]);
        xs[i + N / 2] /= (invert ? 2 : 1);

        S *= S1;
    }
}

```

## 5.16 Fast pow

Computes  $a^b \pmod m$  in  $O(\log N)$ .

```

ll fpow(ll a, ll b, ll m) {
    ll ret = 1;
    while (b) {
        if (b & 1) ret = (ret * a) % m;
        b >>= 1;
        a = (a * a) % m;
    }
}

```

```

    }
    return ret;
}

ll fpow(ll a, ll b, ll m) {
    if (!b) return 1;
    ll ans = fpow2((a * a) % m, b / 2ll, m);
    return b & 1 ? (a * ans) % m : ans;
}

```

## 5.17 Find Multiplicative Inverse

```

ll inv(ll a, ll m) {
    return a > 1ll ? m - inv(m % a, a) * m / a : 1ll;
}

```

## 5.18 Gauss Elimination

```

template <size_t Dim>
struct GaussianElimination {
    vector<ll> basis;
    size_t size;

    GaussianElimination() : basis(Dim + 1), size(0) {}

    void insert(ll x) {
        for (ll i = Dim; i >= 0; i--) {
            if ((x & 1ll << i) == 0) continue;

            if (!basis[i]) {
                basis[i] = x;
                size++;
                break;
            }

            x ^= basis[i];
        }
    }

    void normalize() {
        for (ll i = Dim; i >= 0; i--)
            for (ll j = i - 1; j >= 0; j--)
                if (basis[i] & 1ll << j) basis[i] ^= basis[j];
    }
}

```

```

bool check(ll x) {
    for (ll i = Dim; i >= 0; i--) {
        if ((x & 1ll << i) == 0) continue;

        if (!basis[i]) return false;

        x ^= basis[i];
    }

    return true;
}

auto operator[](ll k) { return at(k); }

ll at(ll k) {
    ll ans = 0;
    ll total = 1ll << size;
    for (ll i = Dim; ~i; i--) {
        if (!basis[i]) continue;

        ll mid = total >> 1ll;
        if ((mid < k and (ans & 1ll << i) == 0) ||
            (k <= mid and (ans & 1ll << i)))
            ans ^= basis[i];

        if (mid < k) k -= mid;

        total >>= 1ll;
    }
    return ans;
}

ll at_normalized(ll k) {
    ll ans = 0;
    k--;
    for (size_t i = 0; i <= Dim; i++) {
        if (!basis[i]) continue;
        if (k & 1) ans ^= basis[i];
        k >>= 1;
    }
    return ans;
}
};

```

## 5.19 Integer Mod

```

const ll INF = 1e18;
const ll mod = 998244353;
template <ll MOD = mod>
struct Modular {
    ll value;
    static const ll MOD_value = MOD;

    Modular(ll v = 0) {
        value = v % MOD;
        if (value < 0) value += MOD;
    }

    Modular(ll a, ll b) : value(0) {
        *this += a;
        *this /= b;
    }

    Modular& operator+=(Modular const& b) {
        value += b.value;
        if (value >= MOD) value -= MOD;
        return *this;
    }

    Modular& operator-=(Modular const& b) {
        value -= b.value;
        if (value < 0) value += MOD;
        return *this;
    }

    Modular& operator*=(Modular const& b) {
        value = (ll)value * b.value % MOD;
        return *this;
    }

    friend Modular mexp(Modular a, ll e) {
        Modular res = 1;
        while (e) {
            if (e & 1) res *= a;
            a *= a;
            e >>= 1;
        }
        return res;
    }

    friend Modular inverse(Modular a) {
        return mexp(a, MOD - 2);
    }

    Modular& operator/=(Modular const& b) {

```

```

    return *this *= inverse(b);
}
friend Modular operator+(Modular a, Modular const b) {
    return a += b;
}
Modular operator++(int) {
    return this->value = (this->value + 1) % MOD;
}
Modular operator++() {
    return this->value = (this->value + 1) % MOD;
}
friend Modular operator-(Modular a, Modular const b) {
    return a -= b;
}
friend Modular operator-(Modular const a) {
    return 0 - a;
}
Modular operator--(int) {
    return this->value = (this->value - 1 + MOD) % MOD;
}
Modular operator--() {
    return this->value = (this->value - 1 + MOD) % MOD;
}
friend Modular operator*(Modular a, Modular const b) {
    return a *= b;
}
friend Modular operator/(Modular a, Modular const b) {
    return a /= b;
}
friend std::ostream& operator<<(std::ostream& os,
                                Modular const& a) {
    return os << a.value;
}
friend bool operator==(Modular const& a,
                        Modular const& b) {
    return a.value == b.value;
}
friend bool operator!=(Modular const& a,
                        Modular const& b) {
    return a.value != b.value;
}
};

```

## 5.20 N Choose K (elements)

process every possible combination of  $K$  elements from  $N$  elements, those index marked as 1 in the index vector says which elements are chosen at that moment.

Time :  $O(\binom{N}{K} \cdot O(\text{process}))$

```

void process(vi &index) {
    for (int i = 0; i < len(index); i++) {
        if (index[i]) cout << i << " \n"[i == len(index) - 1];
    }
}

void n_choose_k(int n, int k) {
    vi index(n);
    fill(index.end() - k, index.end(), 1);

    do {
        process(index);
    } while (next_permutation(all(index)));
}

```

## 5.21 Number Of Divisors (sieve)

```

ll divisors(ll n) {
    ll ans = 1;
    for (auto p : primes) {
        if (p * p * p > n) break;

        int count = 1;
        while (n % p == 0) {
            n /= p;
            count++;
        }

        ans *= count;
    }

    if (is_prime[n])
        ans *= 2;
    else if (is_prime_square[n])
        ans *= 3;
    else if (n != 1)
        ans *= 4;

    return ans;
}

```

## 5.22 Number of Divisors $\tau(n)$

Find the total of divisors of  $N$  in  $O(\sqrt{N})$

```
ll number_of_divisors(ll n) {
    ll res = 0;

    for (ll d = 1; d * d <= n; ++d) {
        if (n % d == 0) res += (d == n / d ? 1 : 2);
    }

    return res;
}
```

## 5.23 Power Sum

Calculates  $K^0 + K^1 + \dots + K^n$

```
ll powersum(ll n, ll k) {
    return (fastpow(n, k + 1) - 1) / (k - 1);
}
```

## 5.24 Sieve list primes

List every prime until  $MAXN$ ,  $O(N \log N)$  in time and  $O(MAXN)$  in memory.

```
const ll MAXN = 1e5;
vll list_primes(ll n) {
    vll ps;
    bitset<MAXN> sieve;
    sieve.set();
    sieve.reset(1);
    for (ll i = 2; i <= n; ++i) {
        if (sieve[i]) ps.push_back(i);
        for (ll j = i * 2; j <= n; j += i) {
            sieve.reset(j);
        }
    }
    return ps;
}
```

## 5.25 Sum of Divisors $\sigma(n)$

Computes the sum of each divisor of  $n$  in  $O(\sqrt{n})$ .

```
ll sum_of_divisors(long long n) {
    ll res = 0;
```

```
    for (ll d = 1; d * d <= n; ++d) {
        if (n % d == 0) {
            ll k = n / d;

            res += (d == k ? d : d + k);
        }
    }

    return res;
}
```

## 6 Primitives

### 6.1 Bigint

```
const int maxn = 1e2 + 14, lg = 15;
const int base = 1000000000;
const int base_digits = 9;
struct bigint {
    vi a;
    int sign;

    int size() {
        if (a.empty()) return 0;
        int ans = (a.size() - 1) * base_digits;
        int ca = a.back();
        while (ca) ans++, ca /= 10;
        return ans;
    }

    bigint operator^(const bigint &v) {
        bigint ans = 1, a = *this, b = v;
        while (!b.isZero()) {
            if (b % 2) ans *= a;
            a *= a, b /= 2;
        }
        return ans;
    }

    string to_string() {
        stringstream ss;
        ss << *this;
        string s;
        ss >> s;
        return s;
    }
}
```

```

int sumof() {
    string s = to_string();
    int ans = 0;
    for (auto c : s) ans += c - '0';
    return ans;
}
/*</arpa>*/
bigint() : sign(1) {}

bigint(long long v) { *this = v; }

bigint(const string &s) { read(s); }

void operator=(const bigint &v) {
    sign = v.sign;
    a = v.a;
}

void operator=(long long v) {
    sign = 1;
    a.clear();
    if (v < 0) sign = -1, v = -v;
    for (; v > 0; v = v / base) a.push_back(v % base);
}

bigint operator+(const bigint &v) const {
    if (sign == v.sign) {
        bigint res = v;

        for (int i = 0, carry = 0;
             i < (int)max(a.size(), v.a.size()) || carry;
             ++i) {
            if (i == (int)res.a.size()) res.a.push_back(0);
            res.a[i] += carry + (i < (int)a.size() ? a[i] : 0);
            carry = res.a[i] >= base;
            if (carry) res.a[i] -= base;
        }
        return res;
    }
    return *this - (-v);
}

bigint operator-(const bigint &v) const {
    if (sign == v.sign) {
        if (abs() >= v.abs()) {

```

```

        bigint res = *this;
        for (int i = 0, carry = 0;
             i < (int)v.a.size() || carry; ++i) {
            res.a[i] -=
                carry + (i < (int)v.a.size() ? v.a[i] : 0);
            carry = res.a[i] < 0;
            if (carry) res.a[i] += base;
        }
        res.trim();
        return res;
    }
    return -(v - *this);
}

return *this + (-v);
}

void operator*=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = 0, carry = 0; i < (int)a.size() || carry;
         ++i) {
        if (i == (int)a.size()) a.push_back(0);
        long long cur = a[i] * (long long)v + carry;
        carry = (int)(cur / base);
        a[i] = (int)(cur % base);
        // asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) :
        // "A"(cur), "c"(base));
    }
    trim();
}

bigint operator*(int v) const {
    bigint res = *this;
    res *= v;
    return res;
}

void operator*=(long long v) {
    if (v < 0) sign = -sign, v = -v;
    if (v > base) {
        *this =
            *this * (v / base) * base + *this * (v % base);
        return;
    }
    for (int i = 0, carry = 0; i < (int)a.size() || carry;
         ++i) {

```

```

    if (i == (int)a.size()) a.push_back(0);
    long long cur = a[i] * (long long)v + carry;
    carry = (int)(cur / base);
    a[i] = (int)(cur % base);
    // asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) :
    // "A"(cur), "c"(base));
}
trim();
}

bigint operator*(long long v) const {
    bigint res = *this;
    res *= v;
    return res;
}

friend pair<bigint, bigint> divmod(const bigint &a1,
                                const bigint &b1) {

    int norm = base / (b1.a.back() + 1);
    bigint a = a1.abs() * norm;
    bigint b = b1.abs() * norm;
    bigint q, r;
    q.a.resize(a.a.size());

    for (int i = a.a.size() - 1; i >= 0; i--) {
        r *= base;
        r += a.a[i];
        int s1 =
            r.a.size() <= b.a.size() ? 0 : r.a[b.a.size()];
        int s2 = r.a.size() <= b.a.size() - 1
            ? 0
            : r.a[b.a.size() - 1];
        int d = ((long long)base * s1 + s2) / b.a.back();
        r -= b * d;
        while (r < 0) r += b, --d;
        q.a[i] = d;
    }

    q.sign = a1.sign * b1.sign;
    r.sign = a1.sign;
    q.trim();
    r.trim();
    return make_pair(q, r / norm);
}

```

```

bigint operator/(const bigint &v) const {
    return divmod(*this, v).first;
}

bigint operator%(const bigint &v) const {
    return divmod(*this, v).second;
}

void operator/=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = (int)a.size() - 1, rem = 0; i >= 0; --i) {
        long long cur = a[i] + rem * (long long)base;
        a[i] = (int)(cur / v);
        rem = (int)(cur % v);
    }
    trim();
}

bigint operator/(int v) const {
    bigint res = *this;
    res /= v;
    return res;
}

int operator%(int v) const {
    if (v < 0) v = -v;
    int m = 0;
    for (int i = a.size() - 1; i >= 0; --i)
        m = (a[i] + m * (long long)base) % v;
    return m * sign;
}

void operator+=(const bigint &v) { *this = *this + v; }
void operator-=(const bigint &v) { *this = *this - v; }
void operator*=(const bigint &v) { *this = *this * v; }
void operator/=(const bigint &v) { *this = *this / v; }

bool operator<(const bigint &v) const {
    if (sign != v.sign) return sign < v.sign;
    if (a.size() != v.a.size())
        return a.size() * sign < v.a.size() * v.sign;
    for (int i = a.size() - 1; i >= 0; i--)
        if (a[i] != v.a[i])
            return a[i] * sign < v.a[i] * sign;
    return false;
}

```

```

}

bool operator>(const bigint &v) const {
    return v < *this;
}

bool operator<=(const bigint &v) const {
    return !(v < *this);
}

bool operator>=(const bigint &v) const {
    return !(*this < v);
}

bool operator==(const bigint &v) const {
    return !(*this < v) && !(v < *this);
}

bool operator!=(const bigint &v) const {
    return *this < v || v < *this;
}

void trim() {
    while (!a.empty() && !a.back()) a.pop_back();
    if (a.empty()) sign = 1;
}

bool isZero() const {
    return a.empty() || (a.size() == 1 && !a[0]);
}

bigint operator-() const {
    bigint res = *this;
    res.sign = -sign;
    return res;
}

bigint abs() const {
    bigint res = *this;
    res.sign *= res.sign;
    return res;
}

long long longValue() const {
    long long res = 0;
    for (int i = a.size() - 1; i >= 0; i--)
        res = res * base + a[i];
    return res * sign;
}

```

```

friend bigint gcd(const bigint &a, const bigint &b) {
    return b.isZero() ? a : gcd(b, a % b);
}

friend bigint lcm(const bigint &a, const bigint &b) {
    return a / gcd(a, b) * b;
}

void read(const string &s) {
    sign = 1;
    a.clear();
    int pos = 0;
    while (pos < (int)s.size() &&
           (s[pos] == '-' || s[pos] == '+')) {
        if (s[pos] == '-') sign = -sign;
        ++pos;
    }
    for (int i = s.size() - 1; i >= pos; i -= base_digits) {
        int x = 0;
        for (int j = max(pos, i - base_digits + 1); j <= i;
             j++)
            x = x * 10 + s[j] - '0';
        a.push_back(x);
    }
    trim();
}

friend istream &operator>>(istream &stream, bigint &v) {
    string s;
    stream >> s;
    v.read(s);
    return stream;
}

friend ostream &operator<<(ostream &stream,
                           const bigint &v) {
    if (v.sign == -1) stream << '-';
    stream << (v.a.empty() ? 0 : v.a.back());
    for (int i = (int)v.a.size() - 2; i >= 0; --i)
        stream << setw(base_digits) << setfill('0') << v.a[i];
    return stream;
}

static vector<int> convert_base(const vector<int> &a,
                                int old_digits,

```

```

        int new_digits) {
vector<long long> p(max(old_digits, new_digits) + 1);
p[0] = 1;
for (int i = 1; i < (int)p.size(); i++)
    p[i] = p[i - 1] * 10;
vector<int> res;
long long cur = 0;
int cur_digits = 0;
for (int i = 0; i < (int)a.size(); i++) {
    cur += a[i] * p[cur_digits];
    cur_digits += old_digits;
    while (cur_digits >= new_digits) {
        res.push_back((int)(cur % p[new_digits]));
        cur /= p[new_digits];
        cur_digits -= new_digits;
    }
}
res.push_back((int)cur);
while (!res.empty() && !res.back()) res.pop_back();
return res;
}

typedef vector<long long> vll;

static vll karatsubaMultiply(const vll &a, const vll &b) {
    int n = a.size();
    vll res(n + n);
    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                res[i + j] += a[i] * b[j];
        return res;
    }

    int k = n >> 1;
    vll a1(a.begin(), a.begin() + k);
    vll a2(a.begin() + k, a.end());
    vll b1(b.begin(), b.begin() + k);
    vll b2(b.begin() + k, b.end());

    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);

    for (int i = 0; i < k; i++) a2[i] += a1[i];
    for (int i = 0; i < k; i++) b2[i] += b1[i];

```

```

    vll r = karatsubaMultiply(a2, b2);
    for (int i = 0; i < (int)a1b1.size(); i++)
        r[i] -= a1b1[i];
    for (int i = 0; i < (int)a2b2.size(); i++)
        r[i] -= a2b2[i];

    for (int i = 0; i < (int)r.size(); i++)
        res[i + k] += r[i];
    for (int i = 0; i < (int)a1b1.size(); i++)
        res[i] += a1b1[i];
    for (int i = 0; i < (int)a2b2.size(); i++)
        res[i + n] += a2b2[i];
    return res;
}

bigint operator*(const bigint &v) const {
    vector<int> a6 = convert_base(this->a, base_digits, 6);
    vector<int> b6 = convert_base(v.a, base_digits, 6);
    vll a(a6.begin(), a6.end());
    vll b(b6.begin(), b6.end());
    while (a.size() < b.size()) a.push_back(0);
    while (b.size() < a.size()) b.push_back(0);
    while (a.size() & (a.size() - 1))
        a.push_back(0), b.push_back(0);
    vll c = karatsubaMultiply(a, b);
    bigint res;
    res.sign = sign * v.sign;
    for (int i = 0, carry = 0; i < (int)c.size(); i++) {
        long long cur = c[i] + carry;
        res.a.push_back((int)(cur % 1000000));
        carry = (int)(cur / 1000000);
    }
    res.a = convert_base(res.a, 6, base_digits);
    res.trim();
    return res;
}
};

```

## 6.2 Integer Mod

```

const ll INF = 1e18;
const ll mod = 998244353;
template <ll MOD = mod>
struct Modular {

```



```

ll value;
static const ll MOD_value = MOD;

Modular(ll v = 0) {
    value = v % MOD;
    if (value < 0) value += MOD;
}

Modular(ll a, ll b) : value(0) {
    *this += a;
    *this /= b;
}

Modular& operator+=(Modular const& b) {
    value += b.value;
    if (value >= MOD) value -= MOD;
    return *this;
}

Modular& operator--(Modular const& b) {
    value -= b.value;
    if (value < 0) value += MOD;
    return *this;
}

Modular& operator*=(Modular const& b) {
    value = (ll)value * b.value % MOD;
    return *this;
}

friend Modular mexp(Modular a, ll e) {
    Modular res = 1;
    while (e) {
        if (e & 1) res *= a;
        a *= a;
        e >>= 1;
    }
    return res;
}

friend Modular inverse(Modular a) {
    return mexp(a, MOD - 2);
}

Modular& operator/=(Modular const& b) {
    return *this *= inverse(b);
}

friend Modular operator+(Modular a, Modular const b) {
    return a += b;
}

```

```

}
Modular operator++(int) {
    return this->value = (this->value + 1) % MOD;
}

Modular operator++() {
    return this->value = (this->value + 1) % MOD;
}

friend Modular operator-(Modular a, Modular const b) {
    return a -= b;
}

friend Modular operator-(Modular const a) {
    return 0 - a;
}

Modular operator--(int) {
    return this->value = (this->value - 1 + MOD) % MOD;
}

Modular operator--() {
    return this->value = (this->value - 1 + MOD) % MOD;
}

friend Modular operator*(Modular a, Modular const b) {
    return a *= b;
}

friend Modular operator/(Modular a, Modular const b) {
    return a /= b;
}

friend std::ostream& operator<<(std::ostream& os,
                                Modular const& a) {
    return os << a.value;
}

friend bool operator==(Modular const& a,
                        Modular const& b) {
    return a.value == b.value;
}

friend bool operator!=(Modular const& a,
                        Modular const& b) {
    return a.value != b.value;
}
};

```

## 6.3 Matrix

```

template <typename T>
struct Matrix {
    vector<vector<T>>> d;
}

```

```

Matrix() : Matrix(0) {}
Matrix(int n) : Matrix(n, n) {}
Matrix(int n, int m)
    : Matrix(vector<vector<T>>(n, vector<T>(m))) {}
Matrix(const vector<vector<T>> &v) : d(v) {}

constexpr int n() const { return (int)d.size(); }
constexpr int m() const {
    return n() ? (int)d[0].size() : 0;
}

void rotate() { *this = rotated(); }

Matrix<T> rotated() const {
    Matrix<T> res(m(), n());
    for (int i = 0; i < m(); i++) {
        for (int j = 0; j < n(); j++) {
            res[i][j] = d[n() - j - 1][i];
        }
    }
    return res;
}

Matrix<T> pow(int power) const {
    assert(n() == m());

    auto res = Matrix<T>::identity(n());
    auto b = *this;
    while (power) {
        if (power & 1) res *= b;
        b *= b;
        power >>= 1;
    }
    return res;
}

Matrix<T> submatrix(int start_i, int start_j,
                    int rows = INT_MAX,
                    int cols = INT_MAX) const {
    rows = min(rows, n() - start_i);
    cols = min(cols, m() - start_j);
    if (rows <= 0 or cols <= 0) return {};

    Matrix<T> res(rows, cols);

```

```

        for (int i = 0; i < rows; i++)
            for (int j = 0; j < cols; j++)
                res[i][j] = d[i + start_i][j + start_j];
    return res;
}

Matrix<T> translated(int x, int y) const {
    Matrix<T> res(n(), m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            if (i + x < 0 or i + x >= n() or j + y < 0 or
                j + y >= m())
                continue;
            res[i + x][j + y] = d[i][j];
        }
    }
    return res;
}

static Matrix<T> identity(int n) {
    Matrix<T> res(n);
    for (int i = 0; i < n; i++) res[i][i] = 1;
    return res;
}

vector<T> &operator[](int i) { return d[i]; }
const vector<T> &operator[](int i) const { return d[i]; }
Matrix<T> &operator+=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x += value;
    }
    return *this;
}

Matrix<T> operator+(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x + value;
    }
    return res;
}

Matrix<T> &operator-=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x -= value;
    }
    return *this;
}

```

```

}
Matrix<T> operator-(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x - value;
    }
    return res;
}
Matrix<T> &operator*=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x *= value;
    }
    return *this;
}
Matrix<T> operator*(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x * value;
    }
    return res;
}
Matrix<T> &operator/=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x /= value;
    }
    return *this;
}
Matrix<T> operator/(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x / value;
    }
    return res;
}
Matrix<T> &operator+=(const Matrix<T> &o) {
    assert(n() == o.n() and m() == o.m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            d[i][j] += o[i][j];
        }
    }
    return *this;
}
Matrix<T> operator+(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());

```

```

    auto res = *this;
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            res[i][j] = res[i][j] + o[i][j];
        }
    }
    return res;
}
Matrix<T> &operator-=(const Matrix<T> &o) {
    assert(n() == o.n() and m() == o.m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            d[i][j] -= o[i][j];
        }
    }
    return *this;
}
Matrix<T> operator-(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());
    auto res = *this;
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            res[i][j] = res[i][j] - o[i][j];
        }
    }
    return res;
}
Matrix<T> &operator*=(const Matrix<T> &o) {
    *this = *this * o;
    return *this;
}
Matrix<T> operator*(const Matrix<T> &o) const {
    assert(m() == o.n());
    Matrix<T> res(n(), o.m());
    for (int i = 0; i < res.n(); i++) {
        for (int j = 0; j < res.m(); j++) {
            auto &x = res[i][j];
            for (int k = 0; k < m(); k++) {
                x += (d[i][k] * o[k][j]);
            }
        }
    }
    return res;
}

```

```

friend istream &operator>>(istream &is, Matrix<T> &mat) {
    for (auto &row : mat)
        for (auto &x : row) is >> x;
    return is;
}

friend ostream &operator<<(ostream &os,
                           const Matrix<T> &mat) {
    bool frow = 1;
    for (auto &row : mat) {
        if (not frow) os << '\n';
        bool first = 1;
        for (auto &x : row) {
            if (not first) os << ' ';
            os << x;
            first = 0;
        }

        frow = 0;
    }
    return os;
}

auto begin() { return d.begin(); }
auto end() { return d.end(); }
auto rbegin() { return d.rbegin(); }
auto rend() { return d.rend(); }

auto begin() const { return d.begin(); }
auto end() const { return d.end(); }
auto rbegin() const { return d.rbegin(); }
auto rend() const { return d.rend(); }
};

```

## 7 Problems

### 7.1 Hanoi Tower

Let  $T_n$  be the total of moves to solve a hanoi tower, we know that  $T_n \geq 2 \cdot T_{n-1} + 1$ , for  $n > 0$ , and  $T_0 = 0$ . By induction it's easy to see that  $T_n = 2^n - 1$ , for  $n > 0$ .

The following algorithm finds the necessary steps to solve the game for 3 stacks and  $n$  disks.

```

void move(int a, int b) { cout << a << ' ' << b << endl; }
void solve(int n, int s, int e) {
    if (n == 0) return;
    if (n == 1) {
        move(s, e);
        return;
    }
}

```

```

}
solve(n - 1, s, 6 - s - e);
move(s, e);
solve(n - 1, 6 - s - e, e);
}

```

## 8 Searching

### 8.1 Meet in the middle

Answers the query how many subsets of the vector  $xs$  have sum equal  $x$ .

Time:  $O(N \cdot 2^{\frac{N}{2}})$

```

vll get_subset_sums(int l, int r, vll &a) {
    int len = r - l + 1;
    vll res;

    for (int i = 0; i < (1 << len); i++) {
        ll sum = 0;
        for (int j = 0; j < len; j++) {
            if (i & (1 << j)) {
                sum += a[l + j];
            }
        }
        res.push_back(sum);
    }
    return res;
};

ll count(vll &xs, ll x) {
    int n = len(xs);
    vll left = get_subset_sums(0, n / 2 - 1, xs);
    vll right = get_subset_sums(n / 2, n - 1, xs);
    sort(all(left));
    sort(all(right));
    ll ans = 0;
    for (ll i : left) {
        auto start_index =
            lower_bound(right.begin(), right.end(), x - i) -
            right.begin();
        auto end_index =
            upper_bound(right.begin(), right.end(), x - i) -
            right.begin();
        ans += end_index - start_index;
    }
}

```

```
    return ans;
}
```

## 8.2 Ternary Search Recursive

```
const double eps = 1e-6;

// IT MUST BE AN UNIMODAL FUNCTION
double f(int x) { return x * x + 2 * x + 4; }

double ternary_search(double l, double r) {
    if (fabs(f(l) - f(r)) < eps)
        return f((l + (r - l) / 2.0));

    auto third = (r - l) / 3.0;
    auto m1 = l + third;
    auto m2 = r - third;

    // change the signal to find the maximum point.
    return m1 < m2 ? ternary_search(m1, r)
        : ternary_search(l, m2);
}
```

## 9 Strings

### 9.1 Count Distinct Anagrams

```
const ll MOD = 1e9 + 7;
const int maxn = 1e6;
vll fs(maxn + 1);
void precompute() {
    fs[0] = 1;
    for (ll i = 1; i <= maxn; i++) {
        fs[i] = (fs[i - 1] * i) % MOD;
    }
}

ll fpow(ll a, int n, ll mod = LLONG_MAX) {
    if (n == 0) return 1;
    if (n == 1) return a;
    ll x = fpow(a, n / 2, mod) % mod;
    return ((x * x) % mod * (n & 1 ? a : 1)) % mod;
}

ll distinctAnagrams(const string &s) {
```

```
    precompute();
    vi hist('z' - 'a' + 1, 0);
    for (auto &c : s) hist[c - 'a']++;
    ll ans = fs[len(s)];
    for (auto &q : hist) {
        ans = (ans * fpow(fs[q], MOD - 2, MOD)) % MOD;
    }
    return ans;
}
```

### 9.2 Double Hash Range Query

```
const ll MOD = 1000027957;
const int MOD2 = 1000015187;

struct Hash {
    const ll P = 31;
    int n;
    string s;
    vll h, h2, hi, hi2, p, p2;
    Hash() {}
    Hash(string _s)
        : s(_s),
          n(len(_s)),
          h(n),
          h2(n),
          hi(n),
          hi2(n),
          p(n),
          p2(n) {
        for (int i = 0; i < n; i++)
            p[i] = (i ? P * p[i - 1] : 1) % MOD;
        for (int i = 0; i < n; i++)
            p2[i] = (i ? P * p2[i - 1] : 1) % MOD2;
        for (int i = 0; i < n; i++)
            h[i] = (s[i] + (i ? h[i - 1] : 0) * P) % MOD;
        for (int i = 0; i < n; i++)
            h2[i] = (s[i] + (i ? h2[i - 1] : 0) * P) % MOD2;
        for (int i = n - 1; i >= 0; i--)
            hi[i] =
                (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % MOD;
        for (int i = n - 1; i >= 0; i--)
            hi2[i] =
                (s[i] + (i + 1 < n ? hi2[i + 1] : 0) * P) % MOD2;
    }
}
```

```

pii query(int l, int r) {
    ll hash =
        (h[r] - (l ? h[l - 1] * p[r - l + 1] % MOD : 0));
    ll hash2 =
        (h2[r] - (l ? h2[l - 1] * p2[r - l + 1] % MOD2 : 0));

    return {(hash < 0 ? hash + MOD : hash),
            (hash2 < 0 ? hash2 + MOD2 : hash2)};
}

pii query_inv(int l, int r) {
    ll hash =
        (hi[l] -
         (r + 1 < n ? hi[r + 1] * p[r - l + 1] % MOD : 0));
    ll hash2 =
        (hi2[l] -
         (r + 1 < n ? hi2[r + 1] * p2[r - l + 1] % MOD2 : 0));
    return {(hash < 0 ? hash + MOD : hash),
            (hash2 < 0 ? hash2 + MOD2 : hash2)};
}
};

```

### 9.3 Hash Range Query

```

struct Hash {
    const ll P = 31;
    const ll mod = 1e9 + 7;
    string s;
    int n;
    vll h, hi, p;
    Hash() {}
    Hash(string s) : s(s), n(s.size()), h(n), hi(n), p(n) {
        for (int i = 0; i < n; i++)
            p[i] = (i ? P * p[i - 1] : 1) % mod;
        for (int i = 0; i < n; i++)
            h[i] = (s[i] + (i ? h[i - 1] : 0) * P) % mod;
        for (int i = n - 1; i >= 0; i--)
            hi[i] =
                (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % mod;
    }
    ll query(int l, int r) {
        ll hash =
            (h[r] - (l ? h[l - 1] * p[r - l + 1] % mod : 0));
        return hash < 0 ? hash + mod : hash;
    }
    ll query_inv(int l, int r) {

```

```

        ll hash =
            (hi[l] -
             (r + 1 < n ? hi[r + 1] * p[r - l + 1] % mod : 0));
        return hash < 0 ? hash + mod : hash;
    }
};

```

### 9.4 K-th digit in digit string

Find the k-th digit in a *digit string*, only works for  $1 \leq k \leq 10^{18}$  !  
 Time: precompute  $O(1)$ , query  $O(1)$

```

using vull = vector<ull>;
vull pow10;
vector<array<ull, 4>> memo;
void precompute(int maxpow = 18) {
    ull qtd = 1;
    ull start = 1;
    ull end = 9;
    ull curlenght = 9;
    ull startstr = 1;
    ull endstr = 9;

    for (ull i = 0, j = 1ll; (int)i < maxpow; i++, j *= 10ll)
        pow10.eb(j);

    for (ull i = 0; i < maxpow - 1ull; i++) {
        memo.push_back({start, end, startstr, endstr});

        start = end + 1ll;
        end = end + (9ll * pow10[qtd]);
        curlenght = end - start + 1ull;

        qtd++;
        startstr = endstr + 1ull;
        endstr = (endstr + 1ull) + (curlenght)*qtd - 1ull;
    }
}

char kthDigit(ull k) {
    int qtd = 1;
    for (auto [s, e, ss, es] : memo) {
        if (k >= ss and k <= es) {
            ull pos = k - ss;
            ull index = pos / qtd;
            ull nmr = s + index;
            int i = k - ss - qtd * index;

```

```

        return ((nmr / pow10[qtd - i - 1]) % 10) + '0';
    }
    qtd++;
}

return 'X';
}

```

## 9.5 Longest Palindrome Substring (Manacher)

Finds the longest palindrome substring, manacher returns a vector where the  $i$ -th position is how much is possible to grow the string to the left and the right of  $i$  and keep it a palindrome.

Time:  $O(N)$

```

vi manacher(string s) {
    string t2;
    for (auto c : s) t2 += string("#") + c;
    t2 = t2 + '#';
    int n = t2.size();
    t2 = "$" + t2 + "^";
    vi p(n + 2);
    int l = 1, r = 1;
    for (int i = 1; i <= n; i++) {
        p[i] = max(0, min(r - i, p[l + (r - i)]));
        while (t2[i - p[i]] == t2[i + p[i]]) {
            p[i]++;
        }
        if (i + p[i] > r) {
            l = i - p[i], r = i + p[i];
        }
        p[i]--;
    }
    return vi(begin(p) + 1, end(p) - 1);
}

string longest_palindrome(const string &s) {
    vi xs = manacher(s);

    string s2;
    for (auto c : s) s2 += string("#") + c;
    s2 = s2 + '#';

    int mpos = 0;
    for (int i = 0; i < len(xs); i++) {
        if (xs[i] > xs[mpos]) {
            mpos = i;
        }
    }
}

```

```

}

string ans;
int k = xs[mpos];
for (int i = mpos - k; i <= mpos + k; i++) {
    if (s2[i] != '#') {
        ans += s2[i];
    }
}
return ans;
}

void run() {
    string s;
    cin >> s;
    auto ans = longest_palindrome(s);
    cout << ans << endl;
}

```

## 9.6 Longest Palindrome

```

string longest_palindrome(const string &s) {
    int n = (int)s.size();
    vector<array<int, 2>> dp(n);

    pii odd(0, -1), even(0, -1);
    pii ans;
    for (int i = 0; i < n; i++) {
        int k = 0;
        if (i > odd.second)
            k = 1;
        else
            k = min(dp[odd.first + odd.second - i][0],
                    odd.second - i + 1);
        while (i - k >= 0 and i + k < n and
                s[i - k] == s[i + k])
            k++;
        dp[i][0] = k--;
        if (i + k > odd.second) odd = {i - k, i + k};
        if (2 * dp[i][0] - 1 > ans.second)
            ans = {i - k, 2 * dp[i][0] - 1};

        k = 0;
        if (i <= even.second)
            k = min(dp[even.first + even.second - i + 1][1],
                    even.second - i + 1);
    }
}

```

```

while (i - k - 1 >= 0 and i + k < n and
      s[i - k - 1] == s[i + k])
    k++;
dp[i][1] = k--;
if (i + k > even.second) even = {i - k - 1, i + k};
if (2 * dp[i][1] > ans.second)
    ans = {i - k - 1, 2 * dp[i][1]};
}
return s.substr(ans.first, ans.second);
}

```

## 9.7 Rabin Karp

```

size_t rabin_karp(const string &s, const string &p) {
    if (s.size() < p.size()) return 0;

    auto n = s.size(), m = p.size();
    const ll p1 = 31, p2 = 29, q1 = 1e9 + 7, q2 = 1e9 + 9;
    const ll p1_1 = fpow(p1, q1 - 2, q1),
            p1_2 = fpow(p1, m - 1, q1);
    const ll p2_1 = fpow(p2, q2 - 2, q2),
            p2_2 = fpow(p2, m - 1, q2);

    pair<ll, ll> hs, hp;
    for (int i = (int)m - 1; ~i; --i) {
        hs.first = (hs.first * p1) % q1;
        hs.first = (hs.first + (s[i] - 'a' + 1)) % q1;
        hs.second = (hs.second * p2) % q2;
        hs.second = (hs.second + (s[i] - 'a' + 1)) % q2;

        hp.first = (hp.first * p1) % q1;
        hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
        hp.second = (hp.second * p2) % q2;
        hp.second = (hp.second + (p[i] - 'a' + 1)) % q2;
    }

    size_t occ = 0;
    for (size_t i = 0; i < n - m; i++) {
        int fi = s[i] - 'a' + 1;
        int fm = s[i + m] - 'a' + 1;

        hs.first = (hs.first - fi + q1) % q1;
        hs.first = (hs.first * p1_1) % q1;

```

```

        hs.first = (hs.first + fm * p1_2) % q1;
        hs.second = (hs.second - fi + q2) % q2;
        hs.second = (hs.second * p2_1) % q2;
        hs.second = (hs.second + fm * p2_2) % q2;
    }
    occ += hs == hp;

    return occ;
}

```

## 9.8 String Psum

```

struct strPsum {
    ll n;
    ll k;
    vector<vll> psum;
    strPsum(const string &s)
        : n(s.size()), k(100), psum(k, vll(n + 1)) {
        for (ll i = 1; i <= n; ++i) {
            for (ll j = 0; j < k; ++j) {
                psum[j][i] = psum[j][i - 1];
            }
            psum[s[i - 1]][i]++;
        }
    }

    ll qtd(ll l, ll r, char c) { // [0,n-1]
        return psum[c][r + 1] - psum[c][l];
    }
}

```

## 9.9 Suffix Automaton (complete)

```

struct state {
    int len, link, cnt, firstpos;
    // this can be optimized using a vector with the alphabet
    // size
    map<char, int> next;
    vi inv_link;
};

struct SuffixAutomaton {
    vector<state> st;
    int sz = 0;
    int last;
    vc cloned;
}

```



```

SuffixAutomaton(const string &s, int maxlen)
: st(maxlen * 2), cloned(maxlen * 2) {
    st[0].len = 0;
    st[0].link = -1;
    sz++;
    last = 0;
    for (auto &c : s) add_char(c);

    // precompute for count occurrences
    for (int i = 1; i < sz; i++) {
        st[i].cnt = !cloned[i];
    }
    vector<pair<state, int>> aux;
    for (int i = 0; i < sz; i++) {
        aux.push_back({st[i], i});
    }

    sort(all(aux), [](const pair<state, int> &a,
                     const pair<state, int> &b) {
        return a.fst.len > b.fst.len;
    });

    for (auto &[stt, id] : aux) {
        if (stt.link != -1) {
            st[stt.link].cnt += st[id].cnt;
        }
    }

    // for find every occurende position
    for (int v = 1; v < sz; v++) {
        st[st[v].link].inv_link.push_back(v);
    }
}

void add_char(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    st[cur].firstpos = st[cur].len - 1;
    int p = last;
    // follow the suffix link until find a transition to c
    while (p != -1 and !st[p].next.count(c)) {
        st[p].next[c] = cur;
        p = st[p].link;
    }
    // there was no transition to c so create and leave

```

```

    if (p == -1) {
        st[cur].link = 0;
        last = cur;
        return;
    }

    int q = st[p].next[c];
    if (st[p].len + 1 == st[q].len) {
        st[cur].link = q;
    } else {
        int clone = sz++;
        cloned[clone] = true;
        st[clone].len = st[p].len + 1;
        st[clone].next = st[q].next;
        st[clone].link = st[q].link;
        st[clone].firstpos = st[q].firstpos;
        while (p != -1 and st[p].next[c] == q) {
            st[p].next[c] = clone;
            p = st[p].link;
        }
        st[q].link = st[cur].link = clone;
    }
    last = cur;
}

bool checkOccurrence(const string &t) { // O(len(t))
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return false;
        cur = st[cur].next[c];
    }
    return true;
}

ll totalSubstrings() { // distinct, O(len(s))
    ll tot = 0;
    for (int i = 1; i < sz; i++) {
        tot += st[i].len - st[st[i].link].len;
    }
    return tot;
}

// count occurrences of a given string t
int countOccurrences(const string &t) {
    int cur = 0;
    for (auto &c : t) {

```

```

        if (!st[cur].next.count(c)) return 0;
        cur = st[cur].next[c];
    }
    return st[cur].cnt;
}

// find the first index where t appears a substring
// O(len(t))
int firstOccurence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return -1;
        cur = st[cur].next[c];
    }
    return st[cur].firstpos - len(t) + 1;
}

vi everyOccurence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return {};
        cur = st[cur].next[c];
    }
    vi ans;
    getEveryOccurence(cur, len(t), ans);
    return ans;
}

void getEveryOccurence(int v, int P_length, vi &ans) {
    if (!cloned[v]) ans.pb(st[v].firstpos - P_length + 1);
    for (int u : st[v].inv_link)
        getEveryOccurence(u, P_length, ans);
}
};

```

## 9.10 Trie

- build with the size of the alphabet (*sigma*) and the first char (*norm*)
- *insert(s)* insert the string in the trie  $O(|s| * \text{sigma})$
- *erase(s)* remove the string from the trie  $O(|s|)$
- *find(s)* return the last node from the string s, 0 if not found  $O(|s|)$

```

struct trie {
    vi2d to;
    vi end, pref;
    int sigma;

```

```

    char norm;

    trie(int sigma_ = 26, char norm_ = 'a')
        : sigma(sigma_), norm(norm_) {
        to = {vector<int>(sigma)};
        end = {0}, pref = {0};
    }

    int next(int node, char key) {
        return to[node][key - norm];
    }

    void insert(const string &s) {
        int x = 0;
        for (auto c : s) {
            int &nxt = to[x][c - norm];
            if (!nxt) {
                nxt = len(to);
                to.push_back(vi(sigma));
                end.emplace_back(0), pref.emplace_back(0);
            }
            x = nxt, pref[x]++;
        }
        end[x]++, pref[0]++;
    }

    void erase(const string &s) {
        int x = 0;
        for (char c : s) {
            int &nxt = to[x][c - norm];
            x = nxt, pref[x]--;
            if (!pref[x]) nxt = 0;
        }
        end[x]--, pref[0]--;
    }

    int find(const string &s) {
        int x = 0;
        for (auto c : s) {
            x = to[x][c - norm];
            if (!x) return 0;
        }
        return x;
    }
};

```

## 9.11 Z-function get occurrence positions

$O(\text{len}(s) + \text{len}(p))$

```
vi getOccPos(string &s, string &p) {
    // Z-function
    char delim = '#';
    string t{p + delim + s};
    vi zs(len(t));

    int l = 0, r = 0;
    for (int i = 1; i < len(t); i++) {
        if (i <= r) zs[i] = min(zs[i - l], r - i + 1);
        while (zs[i] + i < len(t) and t[zs[i]] == t[i + zs[i]])
            zs[i]++;
        if (r < i + zs[i] - 1) l = i, r = i + zs[i] - 1;
    }

    // Iterate over the results of Z-function to get ranges
    vi ans;
    int start = len(p) + 1 + 1 - 1;
    for (int i = start; i < len(zs); i++) {
        if (zs[i] == len(p)) {
            int l = i - start;
            ans.emplace_back(l);
        }
    }
    return ans;
}
```

## 10 Settings and macros

### 10.1 .bashrc

```
cpp() {
    g++ -std=c++20 -fsanitize=address,undefined -Wall $1 && time
    ./a.out
}
```

```
cpp() {
    echo ">> COMPILING <<" 1>&2
    g++ -std=c++17 \
        -O2 \
        -g \
        -g3 \
        -Wextra \
```

```
-Wshadow \
-Wformat=2 \
-Wconversion \
-fsanitize=address,undefined \
-fno-sanitize-recover \
-Wfatal-errors \
$1
```

```
if [ $? -ne 0 ]; then
    echo ">> FAILED <<" 1>&2
    return 1
fi
echo ">> DONE <<" 1>&2
time ./a.out ${@:2}
}
```

```
prepare() {
    cp debug.cpp ./
    for i in {a..z}
    do
        cp macro.cpp $i.cpp
        touch $i.py
    done

    for i in {1..10}
    do
        touch in${i}
        touch out${i}
        touch ans${i}
    done
}
```

### 10.2 .vimrc

```
set ts=4 sw=4 sta nu rnu sc cindent
set bg=dark ruler clipboard=unnamed,unnamedplus, timeoutlen=100
colorscheme default
```

```
nnoremap <C-j> :botright belowright term bash <CR>
syntax on
```

### 10.3 short-macro.cpp

```
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
```

```

#define fastio \
    ios_base::sync_with_stdio(false); \
    cin.tie(0); \
    cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using pii = pair<int, int>;
#define all(a) a.begin(), a.end()

void run() {}
int32_t main(void) {
    fastio;
    int t;
    t = 1;
    // cin >> t;
    while (t--) run();
}

```

## 10.4 macro.cpp

```

#include <bits/stdc++.h>
using namespace std;
#ifdef LOCAL
#include "debug.cpp"
#else
#define dbg(...) 42
#endif
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(false); \
    cin.tie(0); \
    cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using ull = unsigned long long;
using ld = long double;
using vll = vector<ll>;
using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
using vii = vector<pii>;
using vc = vector<char>;

```

```

#define all(a) a.begin(), a.end()
#define pb(__x) push_back(__x)
#define mp(__a, __b) make_pair(__a, __b)
#define eb(__x) emplace_back(__x)
int lg2(ll x) {
    return __builtin_clzll(1) - __builtin_clzll(x);
}

// vector<string> dir({"LU", "U", "RU", "R", "RD", "D",
// "LD", "L"}); int dx[] = {-1, -1, -1, 0, 1, 1, 1, 0}; int
// dy[] = {-1, 0, 1, 1, 1, 0, -1, -1};
vector<string> dir({"U", "R", "D", "L"});
int dx[] = {-1, 0, 1, 0};
int dy[] = {0, 1, 0, -1};

const ll oo = 1e18;
int T(1);
const int MAXN(1'000'000);

auto solve() {}

int32_t main(void) {
#ifdef LOCAL
    fastio;
#endif

    // cin >> T;

    for (int i = 1; i <= T; i++) {
        solve();
    }
}

```

## 10.5 debug.cpp

```

#include <bits/stdc++.h>
using namespace std;
/***** Debug Code *****/
template <typename T>
concept Printable = requires(T t) {
    { std::cout << t } -> std::same_as<std::ostream &>;
};
template <Printable T>
void __print(const T &x) {
    cerr << x;
}

```

```

}
template <size_t T>
void __print(const bitset<T> &x) {
    cerr << x;
}
template <typename A, typename B>
void __print(const pair<A, B> &p);
template <typename... A>
void __print(const tuple<A...> &t);
template <typename T>
void __print(stack<T> s);
template <typename T>
void __print(queue<T> q);
template <typename T, typename... U>
void __print(priority_queue<T, U...> q);
template <typename A>
void __print(const A &x) {
    bool first = true;
    cerr << '{';
    for (const auto &i : x) {
        cerr << (first ? "" : ","), __print(i);
        first = false;
    }
    cerr << '}';
}
template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '(';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << ')';
}
template <typename... A>
void __print(const tuple<A...> &t) {
    bool first = true;
    cerr << '(';
    apply(
        [&first](const auto &...args) {
            ((cerr << (first ? "" : ","), __print(args), first
= false), ...);
        },
        t);
    cerr << ')';
}

```

```

}
template <typename T>
void __print(stack<T> s) {
    vector<T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    }
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
}
template <typename T>
void __print(queue<T> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.front();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
template <typename T, typename... U>
void __print(priority_queue<T, U...> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.top();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
void _print() { cerr << "]\n"; }
template <typename Head, typename... Tail>
void _print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";
    _print(T...);
}

#define dbg(x...) \
    cerr << "[" << #x << "]" = ["; \
    _print(x)

```