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1 Data structures

1.1 Disjoint Sparse Table

```
Answers queries of any monoid operation (i.e. has identity element and is associative)
Build: O(N \log N), Query: O(1)
#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct DisjointSparseTable {
 using Operation = T (*)(T, T);
  vector < vector < T >> st;
  Operation f;
 T identity;
 static constexpr int log2_floor(unsigned long long i) noexcept {
    return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
 // Lazy loading constructor. Needs to call build!
  DisjointSparseTable(Operation op, const T neutral = T())
   : st(), f(op), identity(neutral) {}
 DisjointSparseTable(vector <T > v) : DisjointSparseTable(v, F(min(a, b))) {}
  DisjointSparseTable(vector<T> v, Operation op, const T neutral = T())
   : st(), f(op), identity(neutral) {
    build(v);
  void build(vector<T> v) {
    st.resize(log2_floor(v.size()) + 1,
              vector < T > (111 << (log2_floor(v.size()) + 1)));</pre>
    v.resize(st[0].size(), identity);
    for (int level = 0; level < (int)st.size(); ++level) {</pre>
      for (int block = 0; block < (1 << level); ++block) {</pre>
        const auto 1 = block << (st.size() - level);</pre>
        const auto r = (block + 1) << (st.size() - level);</pre>
        const auto m = 1 + (r - 1) / 2;
        st[level][m] = v[m];
        for (int i = m + 1; i < r; i++)</pre>
          st[level][i] = f(st[level][i - 1], v[i]);
        st[level][m - 1] = v[m - 1];
        for (int i = m - 2; i >= 1; i--)
          st[level][i] = f(st[level][i + 1], v[i]);
    }
 }
 T query(int 1, int r) const {
    if (1 > r) return identity;
    if (1 == r) return st.back()[1];
    const auto k = log2_floor(l ^ r);
    const auto level = (int)st.size() - 1 - k:
    return f(st[level][1], st[level][r]);
```

1.2 Dsu

};

```
struct DSU {
    vi ps;
    vi size;
    DSU(int N) : ps(N + 1), size(N + 1, 1) { iota(all(ps), 0); }
    int find_set(int x) { return ps[x] == x ? x : ps[x] = find_set(ps[x]); }
    bool same_set(int x, int y) { return find_set(x) == find_set(y); }
    void union_set(int x, int y) {
        if (same_set(x, y)) return;

        int px = find_set(x);
        int py = find_set(y);

        if (size[px] < size[py]) swap(px, py);

        ps[py] = px;
        size[px] += size[py];
    }
};</pre>
```

1.3 Ordered Set

If you need an ordered **multi**set you may add an id to each value. Using greater_equal, or less_equal is considered undefined behavior.

- order of key (k): Number of items strictly smaller/greater than k.
- find by order(k): K-th element in a set (counting from zero).

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set =
   tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
```

1.4 SegTree Point Update (dynamic function)

```
Answers queries of any monoid operation (i.e. has identity element and is associative)
Build: O(N), Query: O(log N)

#define F(expr) [](auto a, auto b) { return expr; }

template <typename T>

struct SegTree {
  using Operation = T (*)(T, T);

int N;
  vector <T> ns;
  Operation operation;
  T identity;

SegTree(int n, Operation op = F(a + b), T neutral = T())
  : N(n), ns(2 * N, neutral), operation op = F(a + b), T neutral = T())

SegTree(const vector <T> &v, Operation op = F(a + b), T neutral = T())
```

```
: SegTree((int)v.size(), op, neutral) {
    copy(v.begin(), v.end(), ns.begin() + N);
    for (int i = N - 1; i > 0; --i) ns[i] = operation(ns[2 * i], ns[2 * i +
   1]);
  T query(size_t i) const { return ns[i + N]; }
  T querv(size t 1. size t r) const {
    auto a = 1 + N, b = r + N;
    auto ans = identity;
    while (a <= b) {</pre>
      if (a \& 1) ans = operation(ans, ns[a++]);
      if (not(b \& 1)) ans = operation(ans, ns[b--]);
      a /= 2:
      b /= 2;
    return ans:
  void update(size_t i, T value) { update_set(i, operation(ns[i + N], value));
    }
  void update_set(size_t i, T value) {
    auto a = i + N;
    ns[a] = value:
    while (a >>= 1) ns[a] = operation(ns[2 * a], ns[2 * a + 1]);
  }
};
```

1.5 Segtree Range Max Query Range Max Update

```
template <typename T = 11>
struct SegTree {
 int N;
 T nu, nq;
 vector <T> st, lazy;
 SegTree(const vector <T> &xs)
   : N(len(xs)).
     nu(numeric_limits <T>::min()),
     ng(numeric_limits <T>::min()),
     st(4 * N + 1, nu),
     lazy(4 * N + 1, nu) {
   for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);</pre>
 void update(int 1, int r, T value) { update(1, 0, N - 1, 1, r, value); }
 T query(int 1, int r) { return query(1, 0, N - 1, 1, r); }
 void update(int node, int nl, int nr, int ql, int qr, T v) {
   propagation(node, nl, nr);
```

```
if (ql > nr or qr < nl) return;
    st[node] = max(st[node], v):
    if (ql <= nl and nr <= qr) {</pre>
     if (nl < nr) {</pre>
        lazy[left(node)] = max(lazy[left(node)], v);
        lazy[right(node)] = max(lazy[right(node)], v);
      return:
    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);
    st[node] = max(st[left(node)], st[right(node)]);
  T query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);
    if (ql > nr or qr < nl) return nq;</pre>
    if (gl <= nl and nr <= gr) return st[node]:
    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);
    return max(x, v);
  }
  void propagation(int node, int nl, int nr) {
   if (lazy[node] != nu) {
      st[node] = max(st[node], lazy[node]);
      if (nl < nr) {
        lazy[left(node)] = max(lazy[left(node)], lazy[node]);
        lazy[right(node)] = max(lazy[right(node)], lazy[node]);
      lazy[node] = nu;
  int left(int p) { return p << 1; }</pre>
  int right(int p) { return (p << 1) + 1; }</pre>
  int mid(int 1, int r) { return (r - 1) / 2 + 1; }
};
int main() {
  int n:
  cin >> n:
  vector < array < int , 3>> xs(n);
  for (int i = 0; i < n; ++i) {</pre>
   for (int j = 0; j < 3; ++j) {
      cin >> xs[i][j];
  }
  vi aux(n. 0):
  SegTree < int > st(aux):
```

```
for (int i = 0; i < n; ++i) {
   int a = min(i + xs[i][1], n);
   int b = min(i + xs[i][2], n);
   st.update(i, i, st.query(i, i) + xs[i][0]);
   int cur = st.query(i, i);
   st.update(a, b, cur);
}
cout << st.query(0, n) << '\n';</pre>
```

1.6 SegTree Range Min Query Point Assign Update

```
template <typename T = 11>
struct SegTree {
  int n:
  T nu, nq;
  vector <T> st;
  SegTree(const vector <T> &v)
    : n(len(v)), nu(0), nq(numeric_limits < T > :: max()), st(n * 4 + 1, nu) {
    for (int i = 0; i < n; ++i) update(i, v[i]);</pre>
  void update(int p, T v) { update(1, 0, n - 1, p, v); }
  T query(int 1, int r) { return query(1, 0, n - 1, 1, r); }
  void update(int node, int nl, int nr, int p, T v) {
    if (p < nl or p > nr) return;
    if (nl == nr) {
      st[node] = v:
      return;
    }
    update(left(node), nl, mid(nl, nr), p, v);
    update(right(node), mid(nl, nr) + 1, nr, p, v);
    st[node] = min(st[left(node)], st[right(node)]);
  T query(int node, int nl, int nr, int ql, int qr) {
    if (ql <= nl and qr >= nr) return st[node];
    if (nl > qr or nr < ql) return nq;</pre>
    if (nl == nr) return st[node];
    return min(query(left(node), nl, mid(nl, nr), ql, qr),
               query(right(node), mid(nl, nr) + 1, nr, ql, qr));
  int left(int p) { return p << 1; }</pre>
  int right(int p) { return (p << 1) + 1; }</pre>
  int mid(int 1, int r) { return (r - 1) / 2 + 1; }
}:
```

1.7 SegTree Range Min Query Range Sum Update

```
template <typename t = 11>
struct SegTree {
  int n;
```

```
t nu:
vector <t> st, lazy;
SegTree(const vector<t> &xs)
 : n(len(xs)),
    nu(0).
   nq(numeric_limits <t>::max()),
    st(4 * n, nu),
   lazv(4 * n. nu) {
 for (int i = 0: i < len(xs): ++i) update(i, i, xs[i]):
SegTree(int n): n(n), st(4 * n, nu), lazy(4 * n, nu) {}
void update(int 1, int r, 11 value) { update(1, 0, n - 1, 1, r, value); }
t querv(int 1. int r) { return querv(1, 0, n - 1, 1, r): }
void update(int node, int nl, int nr, int ql, int qr, ll v) {
  propagation(node, nl, nr);
 if (ql > nr or qr < nl) return;</pre>
 if (ql <= nl and nr <= qr) {</pre>
    st[node] += (nr - nl + 1) * v:
   if (nl < nr) {</pre>
     lazv[left(node)] += v:
      lazy[right(node)] += v;
   return;
  update(left(node), nl, mid(nl, nr), ql, qr, v);
  update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);
 st[node] = min(st[left(node)], st[right(node)]);
t query(int node, int nl, int nr, int ql, int qr) {
  propagation(node, nl, nr);
 if (ql > nr or qr < nl) return nq;
 if (ql <= nl and nr <= qr) return st[node];</pre>
 t x = query(left(node), nl, mid(nl, nr), ql, qr);
 t y = query(right(node), mid(n1, nr) + 1, nr, q1, qr);
  return min(x, y);
void propagation(int node, int nl, int nr) {
 if (lazy[node]) {
    st[node] += lazy[node];
    if (nl < nr) {
```

```
lazy[left(node)] += lazy[node];
    lazy[right(node)] += lazy[node];
}

lazy[node] = nu;
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - 1) / 2 + 1; }
;</pre>
```

1.8 SegTree Range Sum Query Range Sum Update

```
template <tvpename T = 11>
struct SegTree {
 int N;
 vector <T> st, lazy;
 T nu = 0:
 T nq = 0:
 SegTree(const vector<T> &xs) : N(len(xs)), st(4 * N, nu), lazy(4 * N, nu) {
   for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);</pre>
 SegTree(int n): N(n), st(4 * N, nu), lazy(4 * N, nu) {}
 void update(int 1, int r, 11 value) { update(1, 0, N - 1, 1, r, value); }
 T query(int 1, int r) { return query(1, 0, N - 1, 1, r); }
 void update(int node, int nl, int nr, int ql, int qr, ll v) {
   propagation(node, nl, nr);
   if (ql > nr or qr < nl) return;</pre>
   if (ql <= nl and nr <= qr) {</pre>
      st[node] += (nr - nl + 1) * v;
     if (nl < nr) {
        lazv[left(node)] += v;
        lazy[right(node)] += v;
     return:
   update(left(node), nl, mid(nl, nr), ql, qr, v);
   update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);
   st[node] = st[left(node)] + st[right(node)]:
 }
 T query(int node, int nl, int nr, int ql, int qr) {
   propagation(node, nl, nr);
   if (ql > nr or qr < nl) return nq;</pre>
```

```
if (ql <= nl and nr <= qr) return st[node];</pre>
    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);
    return x + v;
  void propagation(int node, int nl, int nr) {
    if (lazv[node]) {
      st[node] += (nr - nl + 1) * lazy[node];
      if (n1 < nr) {
        lazy[left(node)] += lazy[node];
        lazy[right(node)] += lazy[node];
      lazy[node] = nu;
  }
  int left(int p) { return p << 1; }</pre>
  int right(int p) { return (p << 1) + 1; }</pre>
  int mid(int 1, int r) { return (r - 1) / 2 + 1; }
};
```

1.9 Sparse Table Range Min Query

```
Build: O(NlogN), Query: O(1)
int fastlog2(11 x) {
 ull i = x;
  return i ? builtin clzll(1) - builtin clzll(i) : -1:
template <typename T>
class SparseTable {
public:
 int N:
 int K:
  vector < vector < T >> st;
  SparseTable(vector <T> vs)
   : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector < T > (N + 1)) {
    copy(vs.begin(), vs.end(), st[0].begin());
    for (int i = 1; i <= K; ++i)</pre>
      for (int j = 0; j + (1 << i) <= N; ++j)
        st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
 T RMQ(int 1, int r) { // [1, r], 0 indexed
    int i = fastlog2(r - l + 1);
    return min(st[i][1], st[i][r - (1 << i) + 1]);</pre>
};
```

2 Dynamic programming

if (st[i][m] != -1) return st[i][m]:

auto res = dp(i - 1, m, M, cs);

2.1 Edit Distance

```
O(N*M)
int edit_distance(const string &a, const string &b) {
  int n = a.size();
  int m = b.size();
  vector < vi > dp(n + 1, vi(m + 1, 0));
  int ADD = 1, DEL = 1, CHG = 1;
  for (int i = 0: i <= n: ++i) {
    dp[i][0] = i * DEL;
  for (int i = 1: i <= m: ++i) {
    dp[0][i] = ADD * i;
  for (int i = 1: i \le n: ++i) {
    for (int j = 1; j <= m; ++j) {
      int add = dp[i][j - 1] + ADD;
      int del = dp[i - 1][j] + DEL;
      int chg = dp[i - 1][j - 1] + (a[i - 1] == b[j - 1] ? 0 : 1) * CHG;
      dp[i][j] = min({add, del, chg});
   }
  }
  return dp[n][m];
     Kadane
Find the maximum subarray sum in a given a rray.
int kadane(const vi &as) {
  vi s(len(as));
  s[0] = as[0]:
  for (int i = 1; i < len(as); ++i) s[i] = max(as[i], s[i - 1] + as[i]);
  return *max_element(all(s));
}
     Knapsack (value)
Finds the maximum points possible
const int MAXN{2010}, MAXM{2010};
ll st[MAXN][MAXM]:
11 dp(int i, int m, int M, const vii &cs) {
  if (i < 0) return 0;</pre>
```

```
auto [w, v] = cs[i];
  if (w \le m) res = max(res, dp(i - 1, m - w, M, cs) + v);
  st[i][m] = res;
  return res;
ll knapsack(int M, const vii &cs) {
  memset(st, -1, sizeof st):
  return dp((int)cs.size() - 1, M, M, cs);
2.4 Knapsack (elements)
Finds the maximum posisble points carry and which elements to achieve it
const int MAXN{2010}, MAXM{2010};
11 st[MAXN][MAXM];
char ps[MAXN][MAXM];
pair < 11. vi > knapsack(int M. const vii &cs) {
  int N = len(cs) - 1;
  for (int i = 0; i \le N; ++i) st[i][0] = 0;
  for (int m = 0; m <= M; ++m) st[0][m] = 0;</pre>
  for (int i = 1: i <= N: ++i) {</pre>
    for (int m = 1: m <= M: ++m) {
      st[i][m] = st[i - 1][m];
      ps[i][m] = 0;
      auto [w, v] = cs[i];
      if (w \le m \text{ and } st[i - 1][m - w] + v > st[i][m]) {
        st[i][m] = st[i - 1][m - w] + v;
        ps[i][m] = 1;
   }
  }
  int m = M:
  for (int i = N; i >= 1; --i) {
   if (ps[i][m]) {
      is.push_back(i);
      m -= cs[i].first;
  }
  reverse(all(is)):
  // max value. items
  return {st[N][M], is};
```

2.5 Longest Increasing Subsequence (LIS)

Finds the length of the longest subsequence in

 $O(n \log n)$

```
int LIS(const vi& as) {
  const ll oo = 1e18;
  int n = len(as);
  vll lis(n + 1, oo);
  lis[0] = -oo;

auto ans = 0;

for (int i = 0; i < n; ++i) {
   auto it = lower_bound(all(lis), as[i]);
   auto pos = (int)(it - lis.begin());

  ans = max(ans, pos);
  lis[pos] = as[i];
}

return ans;
}</pre>
```

2.6 Money Sum (Bottom Up)

Find every possible sum using the given values only once.

```
set < int > money_sum(const vi &xs) {
  using vc = vector < char >:
  using vvc = vector<vc>;
  int _m = accumulate(all(xs), 0);
  int _n = xs.size();
  vvc _dp(_n + 1, vc(_m + 1, 0));
  set < int > _ans;
  dp[0][xs[0]] = 1:
  for (int i = 1; i < _n; ++i) {</pre>
    for (int j = 0; j <= _m; ++j) {
      if (j == 0 or _dp[i - 1][j]) {
        _{dp}[i][i + xs[i]] = 1;
        _{dp[i][j]} = 1;
   }
  for (int i = 0; i < _n; ++i)
    for (int j = 0; j <= _m; ++j)
      if (_dp[i][j]) _ans.insert(j);
  return ans:
}
```

2.7 Travelling Salesman Problem

```
using vi = vector<int>;
vector<vi> dist;
```

```
vector < vi > memo;
/* 0 ( N^2 * 2^N )*/
int tsp(int i, int mask, int N) {
   if (mask == (1 << N) - 1) return dist[i][0];
   if (memo[i][mask] != -1) return memo[i][mask];
   int ans = INT_MAX << 1;
   for (int j = 0; j < N; ++j) {
      if (mask & (1 << j)) continue;
      auto t = tsp(j, mask | (1 << j), N) + dist[i][j];
      ans = min(ans, t);
   }
   return memo[i][mask] = ans;
}</pre>
```

${f 3}$ Geometry

3.1 Convex Hull

Given a set of points find the smallest convex polygon that contains all the given points.

Time: $O(N \log N)$

By default it removes the collinear points, set the boolean to true if you don't want that

```
struct pt {
double x, y;
 int id;
};
int orientation(pt a, pt b, pt c) {
 double v = a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y - b.y);
 if (v < 0) return -1; // clockwise
 if (v > 0) return +1; // counter-clockwise
 return 0:
bool cw(pt a, pt b, pt c, bool include_collinear) {
 int o = orientation(a, b, c);
 return o < 0 || (include_collinear && o == 0);</pre>
bool collinear(pt a, pt b, pt c) { return orientation(a, b, c) == 0; }
void convex_hull(vector < pt > & pts, bool include_collinear = false) {
  pt p0 = *min_element(all(pts), [](pt a, pt b) {
    return make_pair(a.y, a.x) < make_pair(b.y, b.x);</pre>
  sort(all(pts), [&p0](const pt& a, const pt& b) {
   int o = orientation(p0, a, b);
   if (o == 0)
      return (p0.x - a.x) * (p0.x - a.x) + (p0.y - a.y) * (p0.y - a.y) <
             (p0.x - b.x) * (p0.x - b.x) + (p0.y - b.y) * (p0.y - b.y);
    return o < 0:
  }):
  if (include_collinear) {
   int i = len(pts) - 1;
    while (i >= 0 && collinear(p0, pts[i], pts.back())) i--;
    reverse(pts.begin() + i + 1, pts.end());
  vector <pt> st;
```

```
for (int i = 0; i < len(pts); i++) {</pre>
    while (st.size() > 1 &&
           !cw(st[len(st) - 2], st.back(), pts[i], include_collinear))
      st.pop_back();
    st.push_back(pts[i]);
  pts = st;
     Determinant
#include "Point.cpp"
template <typename T>
T D(const Point <T > &P, const Point <T > &Q, const Point <T > &R) {
  return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
         (R.x * Q.y + R.y * P.x + Q.x * P.y);
    Equals
template <typename T>
bool equals(T a, T b) {
  const double EPS{1e-9};
  if (is_floating_point <T>::value)
    return fabs(a - b) < EPS;</pre>
    return a == b;
}
3.4 Line
#include <bits/stdc++.h>
#include "point-struct-and-utils.cpp"
using namespace std;
struct line {
  ld a, b, c;
}:
// the answer is stored in the third parameter (pass by reference)
void pointsToLine(const point &p1, const point &p2, line &1) {
  if (fabs(p1.x - p2.x) < EPS)
   // vertical line
   1 = \{1.0, 0.0, -p1.x\};
  // default values
  else
    1 = \{-(1d)(p1.y - p2.y) / (p1.x - p2.x), 1.0, -(1d)(1.a * p1.x) - p1.y\};
     Point Struct And Utils (2d)
#include <bits/stdc++.h>
using namespace std;
```

```
using ld = long double;
struct point {
 ld x, y;
  int id;
  point(1d = 0.0, 1d = 0.0, 1d = 0.0, 1d = -1): 1d = -1): 1d = -1): 1d = -1
  point& operator+=(const point& t) {
   x += t.x:
   v += t.v:
   return *this;
  point& operator -= (const point& t) {
   x \rightarrow t.x;
   y -= t.y;
   return *this;
  point& operator*=(ld t) {
   y *= t;
   return *this;
  point& operator/=(ld t) {
   x /= t;
   y /= t;
   return *this;
  point operator+(const point& t) const { return point(*this) += t; }
  point operator-(const point& t) const { return point(*this) -= t; }
  point operator*(ld t) const { return point(*this) *= t; }
 point operator/(ld t) const { return point(*this) /= t; }
};
ld dot(point& a, point& b) { return a.x * b.x + a.y * b.y; }
ld norm(point& a) { return dot(a, a); }
ld abs(point a) { return sqrt(norm(a)); }
ld proj(point a, point b) { return dot(a, b) / abs(b); }
ld angle(point a, point b) { return acos(dot(a, b) / abs(a) / abs(b)); }
ld cross(point a, point b) { return a.x * b.y - a.y * b.x; }
3.6 Segment
#include "Line.cpp"
#include "Point.cpp"
#include "equals.cpp"
template <typename T>
struct segment {
 Point <T> A, B;
  bool contains(const Point < T > & P) const;
  Point <T> closest(const Point <T> &p) const;
```

```
};
template <typename T>
bool segment <T>::contains(const Point <T> &P) const {
  // verifica se P áest contido na reta
  double dAB = Point <T >:: dist(A, B), dAP = Point <T >:: dist(A, P),
         dPB = Point < T > :: dist(P, B);
  return equals(dAP + dPB, dAB);
template <typename T>
Point <T > segment <T > :: closest (const Point <T > &P) const {
  Line \langle T \rangle R(A, B);
  auto Q = R.closest(P);
  if (this->contains(0)) return 0:
  auto distA = Point <T>::dist(P, A);
  auto distB = Point <T>::dist(P, B);
  if (distA <= distB)</pre>
    return A:
  else
    return B;
     Graphs
4.1 2 SAT
struct SAT2 {
  11 n:
  vll2d adj, adj_t;
  vc used;
  vll order, comp;
  vc assignment;
  bool solvable;
  SAT2(11 n)
    : n(2 * _n),
      adj(n),
      adi t(n).
      used(n),
      order(n).
      comp(n, -1),
      assignment(n / 2) {}
  void dfs1(int v) {
    used[v] = true;
    for (int u : adj[v]) {
      if (!used[u]) dfs1(u):
    }
    order.push_back(v);
  void dfs2(int v. int cl) {
    comp[v] = cl;
    for (int u : adj_t[v]) {
```

```
if (comp[u] == -1) dfs2(u, cl);
    }
  }
  bool solve_2SAT() {
    // find and label each SCC
    for (int i = 0; i < n; ++i) {</pre>
      if (!used[i]) dfs1(i);
    reverse(all(order)):
    11 j = 0;
    for (auto &v : order) {
      if (comp[v] == -1) dfs2(v, j++);
    assignment.assign(n / 2, false);
    for (int i = 0; i < n; i += 2) {
      // x and !x belong to the same SCC
      if (comp[i] == comp[i + 1]) {
        solvable = false;
        return false;
      assignment[i / 2] = comp[i] > comp[i + 1];
    solvable = true:
    return true;
  }
  void add_disjunction(int a, bool na, int b, bool nb) {
    a = (2 * a) ^na;
    b = (2 * b) ^ nb;
    int neg_a = a ^ 1;
    int neg_b = b ^1;
    adj[neg_a].push_back(b);
    adj[neg_b].push_back(a);
    adj_t[b].push_back(neg_a);
    adj_t[a].push_back(neg_b);
  }
};
      SCC (struct)
Able to find the component of each node and the total of SCC in O(V*E) and build the SCC graph
(O(V*E)).
struct SCC {
  11 N:
  int totscc;
  vll2d adj, tadj;
  vll todo, comps, comp;
  vector < set < 11 >> sccadj;
  vchar vis:
  SCC(11 _N)
    : N(N), totscc(0), adj(N), tadj(N), comp(N, -1), sccadj(N), vis(N)
  void add_edge(11 x, 11 y) { adj[x].eb(y), tadj[y].eb(x); }
```

```
void dfs(ll x) {
    vis[x] = 1:
    for (auto &y : adj[x])
     if (!vis[y]) dfs(y);
    todo.pb(x);
  void dfs2(11 x, 11 v) {
    comp[x] = v;
    for (auto &y : tadj[x])
      if (comp[y] == -1) dfs2(y, v);
 }
 void gen() {
    for (ll i = 0; i < N; ++i)</pre>
      if (!vis[i]) dfs(i);
    reverse(all(todo));
    for (auto &x : todo)
     if (comp[x] == -1) {
        dfs2(x, x);
        comps.pb(x);
        totscc++;
 void genSCCGraph() {
   for (11 i = 0; i < N; ++i) {</pre>
      for (auto &j : adj[i]) {
        if (comp[i] != comp[j]) {
          sccadj[comp[i]].insert(comp[j]);
     }
};
     Bellman Ford
```

Find shortest path from a single source to all other nodes. Can detect negative cycles. Time: O(V * E)

```
bool bellman_ford(const vector<vector<pair<int, ll>>> &g, int s,
                  vector<ll> &dist) {
 int n = (int)g.size();
 dist.assign(n, LLONG_MAX);
 vector < int > count(n):
 vector < char > in_queue(n);
 queue < int > q;
 dist[s] = 0;
 q.push(s);
 in_queue[s] = true;
 while (not q.empty()) {
    int cur = q.front();
   q.pop();
    in_queue[cur] = false;
    for (auto [to, w] : g[cur]) {
```

```
if (dist[cur] + w < dist[to]) {</pre>
        dist[to] = dist[cur] + w;
        if (not in_queue[to]) {
          q.push(to);
          in_queue[to] = true;
          count[to]++;
          if (count[to] > n) return false;
  return true;
4.4 Binary Lifting
far[h][i] = the node that is 2^h distance from node i
Build : O(N * \log N)
sometimes is useful invert the order of loops
const int maxlog = 20;
int far[maxlog + 1][n + 1];
for (int h = 1; h <= maxlog; h++) {</pre>
  for (int i = 1: i <= n: i++) {
    far[h][i] = far[h - 1][far[h - 1][i]];
 }
}
      Check Bipartitie
4.5
O(V)
bool checkBipartite(const ll n, const vector<vll> &adj) {
  11 s = 0:
  queue <11> q;
  q.push(s);
  vll color(n, INF);
  color[s] = 0;
  bool isBipartite = true;
  while (!q.empty() && isBipartite) {
    11 u = q.front();
    q.pop();
    for (auto &v : adj[u]) {
      if (color[v] == INF) {
        color[v] = 1 - color[u];
        q.push(v);
      } else if (color[v] == color[u]) {
        return false;
  return true;
```

4.6 Dijkstra

```
Finds the minimum distance from s to every other node in
```

```
O(E * \log E)
```

```
time.
vll dijkstra(const vector<vector<pll>>> &g, int n, int s) {
 priority_queue<pll, vector<pll>, greater<pll>> pq;
 vll dist(n + 1, oo):
 vector < char > vis(n + 1);
 pq.emplace(0, s);
 dist[s] = 0;
 while (!pq.empty()) {
    auto [d1, v] = pq.top();
    pq.pop();
   if (vis[v]) continue;
    vis[v] = true:
    for (auto [d2, u] : g[v]) {
      if (dist[u] > d1 + d2) {
        dist[u] = d1 + d2;
        pq.emplace(dist[u], u);
   }
  return dist;
```

4.7 Euler Path

return res:

```
Find a path that visits every edge exactly once. Time: O(E) graphs with sets are undirected, graphs with vectors are directed
```

```
// Directed Edges
vector<int> euler_cycle(vector<vector<int>> &g, int u) {
 vector<int> res;
  stack<int> st:
  st.push(u);
 while (!st.empty()) {
    auto cur = st.top();
    if (g[cur].empty()) {
     res.push_back(cur);
      st.pop();
    } else {
      auto next = g[cur].back();
      st.push(next);
      g[cur].pop_back();
 for (auto &x : g)
    if (!x.empty()) return {};
```

```
}
// Directed Edges
vector<int> euler_path(vector<vector<int>> &g, int first) {
    int n = (int)g.size();
    vector < int > in(n), out(n);
    for (int i = 0; i < n; i++)</pre>
      for (auto x : g[i]) in[x]++, out[i]++;
    int a = 0, b = 0, c = 0;
    for (int i = 0; i < n; i++)</pre>
      if (in[i] == out[i])
        c++;
      else if (in[i] - out[i] == 1)
      else if (in[i] - out[i] == -1)
    if (c != n - 2 or a != 1 or b != 1) return {};
  auto res = euler_cycle(g, first);
  if (res.empty()) return res;
  reverse(all(res));
  return res;
// Undirected Edges
vector < int > euler_cycle(vector < set < int >> &g, int u) {
  vector < int > res;
  stack<int> st:
  st.push(u);
  while (!st.empty()) {
    auto cur = st.top();
    if (g[cur].empty()) {
      res.push_back(cur);
      st.pop();
    } else {
      auto next = *g[cur].begin();
      st.push(next);
      g[cur].erase(next);
      g[next].erase(cur);
  }
  for (auto &x : g)
    if (!x.empty()) return {};
  return res;
// Undirected edges
vector < int > euler_path(vector < set < int >> &g, int first) {
  int n = (int)g.size();
```

```
int v1 = -1, v2 = -1;
    bool bad = false;
    for (int i = 0; i < n; i++)
      if (g[i].size() & 1) {
        if (v1 == -1)
          v1 = i:
        else if (v2 == -1)
          v2 = i:
        else
          bad = true;
    if (bad or (v1 != -1 and v2 == -1)) return \{\};
  if (v1 != -1) {
    // insert cycle
    g[v1].insert(v2);
   g[v2].insert(v1);
  auto res = euler cvcle(g. first):
  if (res.empty()) return res;
  if (v1 != -1) {
    for (int i = 0; i + 1 < (int)res.size(); i++) {</pre>
      if ((res[i] == v1 and res[i + 1] == v2) ||
          (res[i] == v2 \text{ and } res[i + 1] == v1)) {
        vector<int> res2:
        for (int j = i + 1; j < (int)res.size(); <math>j++) res2.push_back(res[i]);
        for (int j = 1; j <= i; j++) res2.push_back(res[j]);</pre>
        res = res2:
        break:
      }
    }
  reverse(all(res)):
  return res;
     Flovd Warshall
Simply finds the minimal distance for each node to every other node. O(V^3)
vector<vll> floyd_warshall(const vector<vll> &adj, ll n) {
  auto dist = adj;
  for (int i = 0: i < n: ++i) {
    for (int i = 0: i < n: ++i) {
      for (int k = 0; k < n; ++k) {
        dist[j][k] = min(dist[j][k], dist[j][i] + dist[i][k]);
      }
    }
```

return dist;

4.9 Graph Cycle

```
Detects if a graph contains a cycle. If path parameter is not null, it will contain the cycle if one exists.
Time: O(V+E)
bool has_cycle(const vector<vector<int>> &g, int s, vector<char> &vis,
                vector < char > &in_path, vector < int > *path = nullptr) {
  vis[s] = in_path[s] = 1;
  if (path != nullptr) path->push_back(s);
  for (auto x : g[s]) {
    if (!vis[x] && has_cycle(g, x, vis, in_path, path))
      return true;
    else if (in_path[x]) {
      if (path != nullptr) path->push_back(x);
      return true:
    }
  }
  in_path[s] = 0;
  if (path != nullptr) path->pop_back();
 return false:
4.10 Kruskal
Find the minimum spanning tree of a graph.
can be used to find the maximum spanning tree by changing the comparison operator in the sort
struct UFDS {
  vector < int > ps, sz;
  int components;
  UFDS(int n): ps(n + 1), sz(n + 1, 1), components(n) { iota(all(ps), 0); }
  int find_set(int x) { return (x == ps[x] ? x : (ps[x] = find_set(ps[x]))); }
  bool same_set(int x, int y) { return find_set(x) == find_set(y); }
  void union_set(int x, int y) {
    x = find set(x):
    v = find_set(v);
    if (x == v) return:
    if (sz[x] < sz[y]) swap(x, y);
    ps[y] = x;
    sz[x] += sz[y];
    components --;
 }
};
vector < tuple < ll, int, int >> kruskal (int n, vector < tuple < ll, int, int >> & edges)
    {
```

UFDS ufds(n):

vector<tuple<11, int, int>> ans;

```
sort(all(edges));
  for (auto [a, b, c] : edges) {
    if (ufds.same_set(b, c)) continue;
    ans.emplace_back(a, b, c);
    ufds.union_set(b, c);
  return ans;
4.11 Lowest Common Ancestor
Given two nodes find the lowest common ancestor of both.
Build : O(V), Query: O(1)
int fastlog2(11 x) {
  ull i = x;
  return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
template <typename T>
class SparseTable {
public:
  int N;
  int K;
  vector < vector < T >> st;
  SparseTable(vector<T> vs)
    : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector < T > (N + 1)) {
    copy(vs.begin(), vs.end(), st[0].begin());
    for (int i = 1; i <= K; ++i)
      for (int j = 0; j + (1 << i) <= N; ++j)
        st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
  SparseTable() {}
  T RMQ(int 1, int r) {
    int i = fastlog2(r - 1 + 1);
    return min(st[i][1], st[i][r - (1 << i) + 1]);</pre>
}:
class LCA {
 public:
 int p;
  int n;
  vi first:
  vector < char > visited;
  vi vertices:
  vi height;
  SparseTable < int > st;
  LCA(const vector < vi> &g)
    : p(0), n((int)g.size()), first(n + 1), visited(n + 1, 0), height(n + 1) {
    build_dfs(g, 1, 1);
    st = SparseTable < int > (vertices);
```

void build_dfs(const vector < vi > &g, int u, int hi) {

visited[u] = true:

```
height[u] = hi;
    first[u] = vertices.size();
    vertices.push_back(u);
    for (auto uv : g[u]) {
     if (!visited[uv]) {
        build_dfs(g, uv, hi + 1);
        vertices.push_back(u);
   }
  }
  int lca(int a, int b) {
    int l = min(first[a], first[b]);
   int r = max(first[a], first[b]);
    return st.RMQ(1, r);
 }
};
       Tree Maximum Distance
Returns the maximum distance from every node to any other node in the tree. O(6V) = O(V)
pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
 // O(V)
  // 0 indexed
  11 mostDistantNode = root;
  11 nodeDistance = 0:
  queue <pll> q;
  vector < char > vis(n);
  q.emplace(root, 0);
  vis[root] = true;
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
   if (dist > nodeDistance) {
      nodeDistance = dist;
      mostDistantNode = node:
    for (auto u : adj[node]) {
     if (!vis[u]) {
        vis[u] = true;
        q.emplace(u, dist + 1);
   }
  return {mostDistantNode, nodeDistance};
11 twoNodesDist(const vector < vll> &adj, ll n, ll a, ll b) {
  queue <pll> q;
  vector < char > vis(n):
  q.emplace(a, 0);
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
    if (node == b) return dist:
   for (auto u : adj[node]) {
      if (!vis[u]) {
```

```
vis[u] = true:
        q.emplace(u, dist + 1);
    }
  }
  return -1;
tuple < 11. 11. 11> tree diameter (const vector < vll> & adi. 11 n) {
  // returns two points of the diameter and the diameter itself
  auto [node1, dist1] = mostDistantFrom(adj, n, 0);
  auto [node2, dist2] = mostDistantFrom(adj, n, node1); // O(V)
  auto diameter = twoNodesDist(adj, n, node1, node2);
  return make_tuple(node1, node2, diameter);
}
vll evervDistanceFromNode(const vector<vll> &adi. ll n. ll root) {
  // Single Source Shortest Path, from a given root
  queue <pair <11, 11>> q;
  vll ans(n, -1);
  ans[root] = 0;
  q.emplace(root, 0);
  while (!a.emptv()) {
    auto [u, d] = q.front();
    q.pop();
    for (auto w : adj[u]) {
      if (ans[w] != -1) continue:
      ans[w] = d + 1;
      q.emplace(w, d + 1);
  return ans;
vll maxDistances(const vector < vll > & adj, ll n) {
  auto [node1, node2, diameter] = tree_diameter(adj, n); // 0(3V)
  auto distances1 = everyDistanceFromNode(adj, n, node1); // O(V)
  auto distances2 = everyDistanceFromNode(adi, n. node2): // O(V)
  vll ans(n):
  for (int i = 0: i < n: ++i)
    ans[i] = max(distances1[i], distances2[i]); // O(V)
  return ans;
4.13 Small to Large
Answer queries of the form "How many vertices in the subtree of vertex v have property P?"
Build: O(N), Query: O(N \log N)
struct SmallToLarge {
  vector < vector < int >> tree, vis_childs;
  vector < int > sizes, values, ans;
  set < int > cnt:
  SmallToLarge(vector < vector < int >> &&g. vector < int > &&v)
    : tree(g), vis_childs(g.size()), sizes(g.size()), values(v), ans(g.size())
     {
```

```
update_sizes(0);
 }
  inline void add value(int u) { cnt.insert(values[u]): }
  inline void remove_value(int u) { cnt.erase(values[u]); }
  inline void update_ans(int u) { ans[u] = (int)cnt.size(); }
  void dfs(int u, int p = -1, bool keep = true) {
   int mx = -1;
   for (auto x : tree[u]) {
     if (x == p) continue;
      if (mx == -1 \text{ or sizes}[mx] < sizes[x]) mx = x:
    for (auto x : tree[u]) {
      if (x != p and x != mx) dfs(x, u, false);
    if (mx != -1) {
      dfs(mx, u, true):
      swap(vis_childs[u], vis_childs[mx]);
    vis_childs[u].push_back(u);
    add value(u):
    for (auto x : tree[u]) {
     if (x != p and x != mx) {
        for (auto y : vis_childs[x]) {
          add_value(y);
          vis_childs[u].push_back(v);
    update ans(u):
    if (!keep) {
      for (auto x : vis_childs[u]) remove_value(x);
  }
  void update_sizes(int u, int p = -1) {
    sizes[u] = 1:
    for (auto x : tree[u]) {
     if (x != p) {
        update sizes(x, u):
        sizes[u] += sizes[x];
   }
 }
};
```

4.14 Topological Sorting

```
Assumes that:
   • vertices index [0, n-1]
   • is a DAG (else it returns an empty vector)
enum class state { not_visited, processing, done };
bool dfs(const vector<vll> &adj, ll s, vector<state> &states, vll &order) {
  states[s] = state::processing;
  for (auto &v : adj[s]) {
    if (states[v] == state::not_visited) {
      if (not dfs(adj, v, states, order)) return false;
   } else if (states[v] == state::processing)
      return false;
  states[s] = state::done;
  order.pb(s);
  return true:
vll topologicalSorting(const vector<vll> &adj) {
  ll n = len(adj);
  vll order:
  vector < state > states(n, state::not_visited);
  for (int i = 0; i < n; ++i) {</pre>
    if (states[i] == state::not_visited) {
      if (not dfs(adi, i, states, order)) return {}:
  reverse(all(order));
  return order;
```

4.15 Tree Diameter

Finds the length of the diameter of the tree in O(V), it's easy to recover the nodes at the point of the diameter.

```
pll mostDistantFrom(const vector <vll> &adj, ll n, ll root) {
 // 0 indexed
 11 mostDistantNode = root;
 11 nodeDistance = 0;
 queue <pll> q;
 vector < char > vis(n);
 q.emplace(root, 0);
 vis[root] = true;
 while (!q.empty()) {
   auto [node, dist] = q.front();
   q.pop();
   if (dist > nodeDistance) {
      nodeDistance = dist:
      mostDistantNode = node;
   for (auto u : adj[node]) {
      if (!vis[u]) {
        vis[u] = true:
        q.emplace(u, dist + 1);
```

```
return {mostDistantNode, nodeDistance};
11 twoNodesDist(const vector < vll> &adj, ll n, ll a, ll b) {
 // O indexed
  queue < pll > q;
  vector < char > vis(n);
  q.emplace(a, 0);
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
    if (node == b) {
      return dist;
    for (auto u : adj[node]) {
      if (!vis[u]) {
        vis[u] = true:
        q.emplace(u, dist + 1);
   }
  }
  return -1:
11 tree_diameter(const vector < vll > & adj, ll n) {
  // 0 indexed !!!
  auto [node1, dist1] = mostDistantFrom(adj, n, 0);
  auto [node2, dist2] = mostDistantFrom(adj, n, node1); // O(V)
  auto diameter = twoNodesDist(adj, n, node1, node2);
  return diameter;
    Math
5.1 GCD (with factorization)
O(\sqrt{n}) due to factorization.
```

```
11 gcd_with_factorization(ll a, ll b) {
  map<11, ll> fa = factorization(a);
  map<11, ll> fb = factorization(b);
  ll ans = 1;
  for (auto fai : fa) {
     ll k = min(fai.second, fb[fai.first]);
     while (k--) ans *= fai.first;
  }
  return ans;
}
```

5.2 GCD

ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }

5.3 LCM (with factorization)

 $O(\sqrt{n})$ due to factorization.

```
11 lcm_with_factorization(ll a, ll b) {
  map<ll, ll> fa = factorization(a);
  map<ll, ll> fb = factorization(b);
  ll ans = 1;
  for (auto fai : fa) {
    ll k = max(fai.second, fb[fai.first]);
    while (k--) ans *= fai.first;
  }
  return ans;
}
```

5.4 LCM

```
11 gcd(11 a, 11 b) { return b ? gcd(b, a % b) : a; }
11 lcm(11 a, 11 b) { return a / gcd(a, b) * b; }
```

5.5 Arithmetic Progression Sum

- \bullet s: first term
- \bullet d: common difference
- \bullet n: number of terms

```
11 arithmeticProgressionSum(11 s, 11 d, 11 n) {
   return (s + (s + d * (n - 1))) * n / 211;
}
```

5.6 Binomial MOD

Precompute every factorial until maxn (O(maxn)) allowing to answer the $\binom{n}{k}$ in $O(\log mod)$ time, due to the fastpow. Note that it needs O(maxn) in memory.

```
const 11 MOD = 1e9 + 7;
const 11 \text{ maxn} = 2 * 1e6:
vll fats(maxn + 1, -1);
void precompute() {
  fats[0] = 1;
  for (ll i = 1: i <= maxn: i++) {
    fats[i] = (fats[i - 1] * i) % MOD;
 }
}
11 fpow(ll a, ll n, ll mod = LLONG_MAX) {
  if (n == 011) return 111:
  if (n == 111) return a;
  11 x = fpow(a, n / 211, mod) \% mod;
  return ((x * x) % mod * (n & 111 ? a : 111)) % mod;
ll binommod(ll n, ll k) {
  11 upper = fats[n];
  11 lower = (fats[k] * fats[n - k]) % MOD;
  return (upper * fpow(lower, MOD - 211, MOD)) % MOD;
}
```

5.7 Binomial

```
O(nm) time, O(m) space
Equal to n choose k

11 binom(ll n, ll k) {
   if (k > n) return 0;
   v1l dp(k + 1, 0);
   dp[0] = 1;
   for (ll i = 1; i <= n; i++)
      for (ll j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
   return dp[k];
}
```

5.8 Euler phi $\varphi(n)$ (in range)

Computes the number of positive integers less than n that are coprimes with n, in the range [1, n], in $O(N \log N)$.

```
const int MAX = 1e6;
vi range_phi(int n) {
  bitset<MAX> sieve;
  vi phi(n + 1);

  iota(phi.begin(), phi.end(), 0);
  sieve.set();

  for (int p = 2; p <= n; p += 2) phi[p] /= 2;
  for (int p = 3; p <= n; p += 2) {
    if (sieve[p]) {
      for (int j = p; j <= n; j += p) {
        sieve[j] = false;
        phi[j] /= p;
        phi[j] *= (p - 1);
    }
  }
}
return phi;
}</pre>
```

5.9 Euler phi $\varphi(n)$

Computes the number of positive integers less than n that are coprimes with n, in $O(\sqrt{N})$

```
int phi(int n) {
  if (n == 1) return 1;

auto fs = factorization(n); // a vctor of pair or a map
  auto res = n;

for (auto [p, k] : fs) {
   res /= p;
   res *= (p - 1);
  }

return res;
}
```

5.10 Factorial Factorization

```
Computes the factorization of n! in \pi(N) * \log n
```

```
// O(logN)
11 E(ll n, ll p) {
    ll k = 0, b = p;
    while (b <= n) {
        k += n / b;
        b *= p;
    }
    return k;
}

// O(pi(N)*logN)
map<ll, ll> factorial_factorization(ll n, const vll &primes) {
    map<ll, ll> fs;
    for (const auto &p : primes) {
        if (p > n) break;
        fs[p] = E(n, p);
    }
    return fs;
}
```

5.11 Factorial

```
const l1 MAX = 18;
vll fv(MAX, -1);
ll factorial(ll n) {
   if (fv[n] != -1) return fv[n];
   if (n == 0) return 1;
   return n * factorial(n - 1);
}
```

5.12 Factorization (Pollard Rho)

```
Factorizes a number into its prime factors in O(n^{(\frac{1}{4})} * \log(n)).

11 mul(11 a, 11 b, 11 m) {
    11 ret = a * b - (11)((1d)1 / m * a * b + 0.5) * m;
    return ret < 0 ? ret + m : ret;
}

11 pow(11 a, 11 b, 11 m) {
    11 ans = 1;
    for (; b > 0; b /= 211, a = mul(a, a, m)) {
        if (b % 211 == 1) ans = mul(ans, a, m);
    }
    return ans;
}

bool prime(11 n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;

11 r = __builtin_ctzll(n - 1), d = n >> r;
    for (int a : {2, 325, 9375, 28178, 450775, 9780504, 795265022}) {
```

```
ll x = pow(a, d, n);
    if (x == 1 \text{ or } x == n - 1 \text{ or a } \% n == 0) continue;
   for (int j = 0; j < r - 1; j++) {
     x = mul(x, x, n);
     if (x == n - 1) break;
   if (x != n - 1) return 0;
 return 1:
11 rho(11 n) {
 if (n == 1 or prime(n)) return n;
  auto f = [n](11 x) \{ return mul(x, x, n) + 1; \};
  11 x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
  while (t % 40 != 0 or gcd(prd, n) == 1) {
   if (x == y) x = ++x0, y = f(x);
    q = mul(prd, abs(x - y), n);
   if (q != 0) prd = q;
   x = f(x), y = f(f(y)), t++;
  return gcd(prd, n);
vll fact(ll n) {
 if (n == 1) return {}:
 if (prime(n)) return {n};
 ll d = rho(n);
 vll l = fact(d), r = fact(n / d);
 1.insert(1.end(), r.begin(), r.end());
 return 1;
}
5.13 Factorization
Computes the factorization of n in O(\sqrt{n}).
map<11. 11> factorization(11 n) {
  map<11. 11> ans:
 for (11 i = 2; i * i <= n; i++) {
   11 count = 0:
   for (; n % i == 0; count++, n /= i)
   if (count) ans[i] = count;
 }
 if (n > 1) ans[n]++;
 return ans:
}
5.14 Fast Fourrier Transform
template <bool invert = false>
void fft(vector<complex<double>>& xs) {
```

17

int N = (int)xs.size();

```
if (N == 1) return:
  vector < complex < double >> es(N / 2). os(N / 2);
  for (int i = 0; i < N / 2; ++i) es[i] = xs[2 * i];
  for (int i = 0; i < N / 2; ++i) os[i] = xs[2 * i + 1];
  fft < invert > (es):
  fft < invert > (os);
  auto signal = (invert ? 1 : -1);
  auto theta = 2 * signal * acos(-1) / N;
  complex <double > S{1}, S1{cos(theta), sin(theta)};
  for (int i = 0; i < N / 2; ++i) {
    xs[i] = (es[i] + S * os[i]):
    xs[i] /= (invert ? 2 : 1);
    xs[i + N / 2] = (es[i] - S * os[i]);
    xs[i + N / 2] /= (invert ? 2 : 1);
    S *= S1;
5.15 Fast pow
Computes a^n in O(\log N).
11 fpow(11 a, int n, 11 mod = LLONG_MAX) {
  if (n == 0) return 1;
  if (n == 1) return a:
  11 x = fpow(a, n / 2, mod) \% mod;
  return ((x * x) % mod * (n & 1 ? a : 111)) % mod;
5.16 Gauss Elimination
template <size_t Dim>
struct GaussianElimination {
  vector < 11 > basis;
  size t size:
  GaussianElimination() : basis(Dim + 1), size(0) {}
  void insert(ll x) {
    for (11 i = Dim: i >= 0: i--) {
      if ((x & 111 << i) == 0) continue:</pre>
      if (!basis[i]) {
        basis[i] = x;
        size++;
        break:
      }
```

```
x ^= basis[i]:
    }
    void normalize() {
      for (11 i = Dim; i >= 0; i--)
        for (11 j = i - 1; j >= 0; j--)
          if (basis[i] & 111 << j) basis[i] ^= basis[j];</pre>
    }
    bool check(ll x) {
      for (11 i = Dim; i >= 0; i--) {
        if ((x & 111 << i) == 0) continue;</pre>
        if (!basis[i]) return false:
        x ^= basis[i]:
      return true;
    auto operator[](11 k) { return at(k): }
    11 at(11 k) {
      11 \text{ ans} = 0:
      11 total = 111 << size;</pre>
      for (11 i = Dim: ~i: i--) {
        if (!basis[i]) continue;
        11 mid = total >> 111:
        if ((mid < k and (ans & 111 << i) == 0) ||</pre>
             (k <= mid and (ans & 111 << i)))
          ans ^= basis[i]:
        if (mid < k) k -= mid;</pre>
        total >>= 111;
      return ans;
    }
    ll at_normalized(ll k) {
      11 \text{ ans} = 0:
      for (size_t i = 0; i <= Dim; i++) {</pre>
        if (!basis[i]) continue;
        if (k & 1) ans ^= basis[i]:
        k >>= 1:
      return ans;
  };
  5.17 Integer Mod
  const ll INF = 1e18;
  const 11 mod = 998244353:
18
```

```
template <11 MOD = mod>
struct Modular {
 ll value:
 static const 11 MOD value = MOD:
 Modular(11 v = 0)  {
   value = v % MOD:
   if (value < 0) value += MOD;</pre>
 Modular(ll a. ll b) : value(0) {
   *this += a;
   *this /= b:
 Modular& operator += (Modular const& b) {
   value += b.value;
   if (value >= MOD) value -= MOD;
   return *this:
 Modular& operator -= (Modular const& b) {
   value -= b.value;
   if (value < 0) value += MOD:
   return *this:
 Modular& operator*=(Modular const& b) {
   value = (11)value * b.value % MOD:
   return *this;
 friend Modular mexp(Modular a, 11 e) {
   Modular res = 1:
   while (e) {
     if (e & 1) res *= a;
     a *= a:
     e >>= 1:
   }
   return res;
 friend Modular inverse (Modular a) { return mexp(a, MOD - 2); }
 Modular& operator/=(Modular const& b) { return *this *= inverse(b): }
 friend Modular operator+(Modular a, Modular const b) { return a += b; }
 Modular operator++(int) { return this->value = (this->value + 1) % MOD; }
 Modular operator++() { return this->value = (this->value + 1) % MOD; }
 friend Modular operator-(Modular a, Modular const b) { return a -= b; }
 friend Modular operator - (Modular const a) { return 0 - a; }
 Modular operator -- (int) {
   return this->value = (this->value - 1 + MOD) % MOD:
 Modular operator -- () { return this -> value = (this -> value - 1 + MOD) % MOD; }
 friend Modular operator*(Modular a. Modular const b) { return a *= b: }
 friend Modular operator/(Modular a, Modular const b) { return a /= b; }
 friend std::ostream& operator << (std::ostream& os, Modular const& a) {</pre>
   return os << a.value:
 friend bool operator == (Modular const& a. Modular const& b) {
   return a.value == b.value:
```

```
friend bool operator!=(Modular const& a, Modular const& b) {
    return a.value != b.value;
};
5.18 Is prime
O(\sqrt{N})
bool isprime(ll n) {
  if (n < 2) return false;</pre>
  if (n == 2) return true;
 if (n % 2 == 0) return false;
  for (11 i = 3; i * i < n; i += 2)
   if (n % i == 0) return false:
  return true;
5.19 Number of Divisors \tau(n)
Find the total of divisors of N in O(\sqrt{N})
ll number_of_divisors(ll n) {
 ll res = 0:
  for (11 d = 1; d * d <= n; ++d) {
    if (n % d == 0) res += (d == n / d ? 1 : 2):
  return res:
5.20 Power Sum
Calculates K^0 + K^1 + ... + K^n
ll powersum(ll n, ll k) { return (fastpow(n, k + 1) - 1) / (n - 1); }
5.21 Sieve list primes
List every prime until MAXN, O(N \log N) in time and O(MAXN) in memory.
const ll MAXN = 1e5:
vll list_primes(ll n) {
  bitset < MAXN > sieve:
  sieve.set():
  sieve.reset(1):
  for (11 i = 2: i <= n: ++i) {
    if (sieve[i]) ps.push_back(i);
    for (11 j = i * 2; j <= n; j += i) {
      sieve.reset(j);
  return ps;
```

5.22 Sum of Divisors $\sigma(n)$

```
Computes the sum of each divisor of n in O(\sqrt{n})
11 sum_of_divisors(long long n) {
  11 \text{ res} = 0:
  for (11 d = 1; d * d <= n; ++d) {
    if (n % d == 0) {
      11 k = n / d;
      res += (d == k ? d : d + k):
    }
  }
  return res;
```

Searching

Meet in the middle

```
Answers the query how many subsets of the vector xs have sum equal x.
Time: O(N \cdot 2^{\frac{N}{2}})
vll get_subset_sums(int 1, int r, vll &a) {
  int len = r - l + 1;
  vll res:
  for (int i = 0; i < (1 << len); i++) {
    11 sum = 0:
    for (int j = 0; j < len; j++) {
      if (i & (1 << j)) {</pre>
         sum += a[1 + j];
      }
    res.push_back(sum);
  return res;
};
11 count(vll &xs, ll x) {
  int n = len(xs);
  vll left = get_subset_sums(0, n / 2 - 1, xs);
  vll right = get_subset_sums(n / 2, n - 1, xs);
  sort(all(left));
  sort(all(right)):
  11 \text{ ans} = 0;
  for (ll i : left) {
      lower_bound(right.begin(), right.end(), x - i) - right.begin();
    auto end index =
      upper_bound(right.begin(), right.end(), x - i) - right.begin();
    ans += end_index - start_index;
  return ans;
```

6.2 Ternary Search Recursive

```
const double eps = 1e-6;
// IT MUST BE AN UNIMODAL FUNCTION
double f(int x) { return x * x + 2 * x + 4; }
double ternary_search(double 1, double r) {
  if (fabs(f(1) - f(r)) < eps) return f((1 + (r - 1) / 2.0));
  auto third = (r - 1) / 3.0;
  auto m1 = 1 + third:
  auto m2 = r - third;
 // change the signal to find the maximum point.
 return m1 < m2 ? ternary_search(m1, r) : ternary_search(1, m2);</pre>
```

Strings

7.1 Count Distinct Anagrams

```
const 11 MOD = 1e9 + 7:
const int maxn = 1e6;
vll fs(maxn + 1);
void precompute() {
 fs[0] = 1;
 for (ll i = 1: i <= maxn: i++) {
    fs[i] = (fs[i - 1] * i) % MOD:
 }
}
11 fpow(11 a, int n, 11 mod = LLONG_MAX) {
 if (n == 0) return 1:
 if (n == 1) return a;
 11 x = fpow(a, n / 2, mod) \% mod:
 return ((x * x) % mod * (n & 1 ? a : 111)) % mod;
11 distinctAnagrams(const string &s) {
  precompute();
  vi hist('z' - 'a' + 1, 0);
  for (auto &c : s) hist[c - 'a']++;
 11 ans = fs[len(s)];
  for (auto &q : hist) {
    ans = (ans * fpow(fs[q], MOD - 2, MOD)) % MOD;
 }
  return ans;
```

7.2 Hash Range Query

```
struct Hash {
  const 11 P = 31;
  const 11 \mod = 1e9 + 7:
  string s;
  int n:
```

```
vll h, hi, p;
  Hash() {}
  Hash(string s) : s(s), n(s.size()), h(n), hi(n), p(n) {
    for (int i = 0; i < n; i++) p[i] = (i ? P * p[i - 1] : 1) % mod;
    for (int i = 0; i < n; i++) h[i] = (s[i] + (i ? h[i - 1] : 0) * P) % mod;
    for (int i = n - 1; i >= 0; i--)
      hi[i] = (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % mod;
  11 query(int 1, int r) {
    ll hash = (h[r] - (1?h[1 - 1] * p[r - 1 + 1] % mod : 0)):
    return hash < 0 ? hash + mod : hash;</pre>
  ll query_inv(int 1, int r) {
    ll hash = (hi[1] - (r + 1 < n ? hi[r + 1] * p[r - 1 + 1] % mod : 0));
    return hash < 0 ? hash + mod : hash:</pre>
};
    Longest Palindrome
string longest_palindrome(const string &s) {
  int n = (int)s.size();
  vector < array < int , 2>> dp(n);
  pii odd(0, -1), even(0, -1);
  pii ans:
  for (int i = 0; i < n; i++) {
    int k = 0:
    if (i > odd.second)
      k = 1:
    else
      k = \min(dp \lceil odd.first + odd.second - i \rceil \lceil 0 \rceil, odd.second - i + 1):
    while (i - k \ge 0 \text{ and } i + k < n \text{ and } s[i - k] == s[i + k]) k++;
    dp[i][0] = k--:
    if (i + k > odd.second) odd = \{i - k, i + k\};
    if (2 * dp[i][0] - 1 > ans.second) ans = \{i - k, 2 * dp[i][0] - 1\};
    k = 0;
    if (i <= even.second)</pre>
      k = min(dp[even.first + even.second - i + 1][1], even.second - i + 1);
    while (i - k - 1) = 0 and i + k < n and s[i - k - 1] == s[i + k] +;
    dp[i][1] = k--;
    if (i + k > even.second) even = \{i - k - 1, i + k\};
    if (2 * dp[i][1] > ans.second) ans = \{i - k - 1, 2 * dp[i][1]\};
  return s.substr(ans.first, ans.second);
7.4 Rabin Karp
size_t rabin_karp(const string &s, const string &p) {
  if (s.size() < p.size()) return 0;</pre>
  auto n = s.size(), m = p.size();
  const 11 p1 = 31, p2 = 29, q1 = 1e9 + 7, q2 = 1e9 + 9;
  const ll p1_1 = fpow(p1, q1 - 2, q1), p1_2 = fpow(p1, m - 1, q1);
  const 11 p2_1 = fpow(p2, q2 - 2, q2), p2_2 = fpow(p2, m - 1, q2);
```

```
for (int i = (int)m - 1; ~i; --i) {
    hs.first = (hs.first * p1) % q1;
   hs.first = (hs.first + (s[i] - 'a' + 1)) % q1;
   hs.second = (hs.second * p2) % q2;
    hs.second = (hs.second + (s[i] - 'a' + 1)) % q2;
    hp.first = (hp.first * p1) % q1;
    hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
   hp.second = (hp.second * p2) % q2;
   hp.second = (hp.second + (p[i] - 'a' + 1)) % q2;
  size_t occ = 0;
  for (size t i = 0: i < n - m: i++) {
    occ += (hs == hp);
    int fi = s[i] - 'a' + 1;
    int fm = s[i + m] - 'a' + 1;
   hs.first = (hs.first - fi + q1) % q1;
   hs.first = (hs.first * p1_1) % q1;
   hs.first = (hs.first + fm * p1 2) % q1:
   hs.second = (hs.second - fi + q2) \% q2;
   hs.second = (hs.second * p2_1) % q2;
   hs.second = (hs.second + fm * p2_2) % q2;
  occ += hs == hp:
  return occ;
     String Psum
struct strPsum {
 11 n:
  vector < vll> psum;
  strPsum(const string \&s) : n(s.size()), k(100), psum(k, vll(n + 1)) {
    for (ll i = 1; i <= n; ++i) {
      for (11 j = 0; j < k; ++j) {
        psum[j][i] = psum[j][i - 1];
      psum[s[i - 1]][i]++;
  }
 ll qtd(ll l, ll r, char c) { // [0,n-1]
    return psum[c][r + 1] - psum[c][1];
 }
}
     Suffix Automaton (complete)
struct state {
  int len, link, cnt, firstpos;
  // this can be optimized using a vector with the alphabet size
  map < char , int > next;
```

pair < ll, ll > hs, hp;

```
vi inv link:
struct SuffixAutomaton {
 vector < state > st:
 int sz = 0;
 int last;
 vc cloned;
  SuffixAutomaton(const string &s, int maxlen)
   : st(maxlen * 2), cloned(maxlen * 2) {
   st[0].len = 0:
   st[0].link = -1:
   sz++;
   last = 0;
   for (auto &c : s) add_char(c);
   // precompute for count occurences
   for (int i = 1: i < sz: i++) {
     st[i].cnt = !cloned[i];
   vector<pair<state, int>> aux;
   for (int i = 0; i < sz; i++) {</pre>
     aux.push back({st[i], i});
   sort(all(aux), [](const pair<state, int> &a, const pair<state, int> &b) {
    return a.fst.len > b.fst.len;
   }):
   for (auto &[stt, id] : aux) {
     if (stt.link != -1) {
        st[stt.link].cnt += st[id].cnt;
     }
   }
   // for find every occurende position
   for (int v = 1; v < sz; v++) {</pre>
      st[st[v].link].inv_link.push_back(v);
 }
 void add_char(char c) {
   int cur = sz++;
   st[cur].len = st[last].len + 1:
   st[cur].firstpos = st[cur].len - 1;
   int p = last;
   // follow the suffix link until find a transition to c
   while (p != -1 and !st[p].next.count(c)) {
      st[p].next[c] = cur;
     p = st[p].link;
   // there was no transition to c so create and leave
   if (p == -1) {
      st[cur].link = 0;
     last = cur:
     return;
```

```
int q = st[p].next[c];
  if (st[p].len + 1 == st[q].len) {
    st[cur].link = q;
 } else {
   int clone = sz++;
    cloned[clone] = true;
    st[clone].len = st[p].len + 1;
    st[clone].next = st[q].next;
    st[clone].link = st[q].link;
    st[clone].firstpos = st[a].firstpos:
    while (p != -1 and st[p].next[c] == q) {
     st[p].next[c] = clone;
     p = st[p].link;
    st[a].link = st[cur].link = clone:
 last = cur:
}
bool checkOccurrence(const string &t) { // O(len(t))
  int cur = 0:
  for (auto &c : t) {
   if (!st[cur].next.count(c)) return false:
    cur = st[cur].next[c];
  return true:
11 totalSubstrings() { // distinct, O(len(s))
 11 \text{ tot} = 0:
 for (int i = 1; i < sz; i++) {</pre>
   tot += st[i].len - st[st[i].link].len:
 return tot;
// count occurences of a given string t
int countOccurences(const string &t) {
 int cur = 0:
 for (auto &c : t) {
   if (!st[cur].next.count(c)) return 0;
    cur = st[cur].next[c];
 return st[cur].cnt;
// find the first index where t appears a substring O(len(t))
int firstOccurence(const string &t) {
 int cur = 0:
 for (auto c : t) {
   if (!st[cur].next.count(c)) return -1;
    cur = st[cur].next[c];
  return st[cur].firstpos - len(t) + 1;
vi everyOccurence(const string &t) {
 int cur = 0:
 for (auto c : t) {
```

```
if (!st[cur].next.count(c)) return {};
    cur = st[cur].next[c];
}
    vi ans;
getEveryOccurence(cur, len(t), ans);
    return ans;
}

void getEveryOccurence(int v, int P_length, vi &ans) {
    if (!cloned[v]) ans.pb(st[v].firstpos - P_length + 1);
    for (int u : st[v].inv_link) getEveryOccurence(u, P_length, ans);
}
};
```

7.7 Z-function get occurence positions

```
O(len(s) + len(p))
vi getOccPos(string &s, string &p) {
 // Z-function
  char delim = '#';
 string t{p + delim + s}:
 vi zs(len(t));
 int 1 = 0, r = 0:
 for (int i = 1; i < len(t); i++) {</pre>
   if (i \le r) zs[i] = min(zs[i - 1], r - i + 1):
    while (zs[i] + i < len(t)) and t[zs[i]] == t[i + zs[i]]) zs[i]++:
   if (r < i + zs[i] - 1) l = i, r = i + zs[i] - 1;
 // Iterate over the results of Z-function to get ranges
  vi ans:
 int start = len(p) + 1 + 1 - 1;
 for (int i = start: i < len(zs): i++) {</pre>
    if (zs[i] == len(p)) {
     int l = i - start;
      ans.emplace_back(1);
 return ans;
```

8 Settings and macros

8.1 short-macro.cpp

```
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define fastio
   ios_base::sync_with_stdio(false); \
   cin.tie(0);
   cout.tie(0);
#define len(_x) (int) __x.size()
using ll = long long;
using pii = pair<int, int>;
```

```
#define all(a) a.begin(), a.end()
void run() {}
int32_t main(void) {
 fastio:
 int t:
 t = 1;
  // cin >> t:
  while (t--) run():
8.2 .vimrc
set ts=4 sw=4 sta nu rnu sc cindent
set bg=dark ruler clipboard=unnamed.unnamedplus.timeoutlen=100
colorscheme default
nnoremap <C-j>:botright belowright term bash <CR>
syntax on
8.3 degug.cpp
#include <bits/stdc++.h>
using namespace std;
/****** Debug Code ******/
template <tvpename T>
concept Printable = requires(T t) {
    { std::cout << t } -> std::same_as<std::ostream &>;
template <Printable T>
void __print(const T &x) {
    cerr << x:
template <size_t T>
void __print(const bitset<T> &x) {
    cerr << x:
template <typename A, typename B>
void __print(const pair<A, B> &p);
template <typename... A>
void __print(const tuple < A . . . > &t);
template <tvpename T>
void __print(stack<T> s);
template <typename T>
void __print(queue < T > q);
template <typename T, typename... U>
void __print(priority_queue < T, U... > q);
template <typename A>
void __print(const A &x) {
    bool first = true:
    cerr << '{':
    for (const auto &i : x) {
        cerr << (first ? "" : ","), __print(i);</pre>
        first = false;
    cerr << '}';
```

```
template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '(';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << ')';
template <typename... A>
void print(const tuple < A... > &t) {
    bool first = true;
    cerr << '(';
    apply(
        [&first](const auto &...args) {
            ((cerr << (first ? "" : ","), __print(args), first = false), ...);
        },
        t):
    cerr << ')':
template <typename T>
void __print(stack<T> s) {
    vector <T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
template <typename T>
void __print(queue < T > q) {
    vector <T> debugVector;
    while (!q.empty()) {
        T t = q.front();
        debugVector.push_back(t);
        q.pop();
    __print(debugVector);
template <typename T, typename... U>
void __print(priority_queue < T, U... > q) {
    vector <T> debugVector;
    while (!q.empty()) {
        T t = q.top();
        debugVector.push_back(t);
        q.pop();
    __print(debugVector);
void _print() { cerr << "]\n"; }</pre>
template <typename Head, typename... Tail>
void _print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";</pre>
    _print(T...);
}
```

```
#define dbg(x...)
    cerr << "[" << #x << "] = [": \
    _print(x)
8.4 .bashrc
cpp() {
  echo ">> COMPILING <<" 1>&2
  g++-std=c++17
      -02 \
      -g \
      -g3 \
      -Wextra \
      -Wshadow \
      -Wformat=2 \
      -Wconversion \
      -fsanitize=address,undefined \
      -fno-sanitize-recover \
      -Wfatal-errors \
  if [ $? -ne 0 ]; then
      echo ">> FAILED <<" 1>&2
      return 1
  echo ">> DONE << " 1>&2
  time ./a.out ${0:2}
prepare() {
    for i in {a..z}
        cp macro.cpp $i.cpp
        touch $i.pv
    for i in {1..10}
        touch in${i}
        touch out${i}
        touch ans${i}
    done
}
8.5 macro.cpp
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define fastio
ios_base::sync_with_stdio(false); \
 cin.tie(0);
  cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using ld = long double;
using vll = vector<11>;
using pll = pair<11, 11>;
```

```
using v112d = vector<v1l>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
using vii = vector<pii>;
using vc = vector<char>;
#define all(a) a.begin(), a.end()
#define fst first
#define pb(___x) push_back(__x)
#define mp(__a, __b) make_pair(_a, __b)
#define eb(__x) emplace_back(__x)
```

```
const ll oo = 1e18;

void run() {}
int32_t main(void) {
  fastio;
  int t;
  t = 1;
  // cin >> t;
  while (t--) run();
}
```