

Contents

1	Data structures	2
1.1	Disjoint Sparse Table	2
1.2	Dsu	2
1.3	Ordered Set	2
1.4	SegTree Point Update (dynamic function) . .	2
1.5	Segtree Range Max Query Range Max Update	3
1.6	SegTree Range Min Query Point Assign Update	4
1.7	SegTree Range Xor Query Point Assign Update	4
1.8	SegTree Range Min Query Range Sum Update	4
1.9	SegTree Range Sum Query Range Sum Update	5
1.10	Sparse Table Range Min Query	6
2	Dynamic programming	6
2.1	Edit Distance	6
2.2	Kadane	6
2.3	Knapsack (value)	6
2.4	Knapsack (elements)	6
2.5	Longest Increasing Subsequence (LIS)	7
2.6	Money Sum (Bottom Up)	7
2.7	Travelling Salesman Problem	7
3	Geometry	7
3.1	Convex Hull	7
3.2	Determinant	8
3.3	Equals	8
3.4	Line	8
3.5	Point Struct And Utils (2d)	8
3.6	Segment	9
4	Graphs	9
4.1	2 SAT	9
4.2	Cycle Distances	10
4.3	SCC (struct)	10

4.4	Bellman-Ford (find negative cycle)	10
4.5	Bellman Ford	11
4.6	Binary Lifting	11
4.7	Check Bipartite	11
4.8	Dijkstra (k Shortest Paths)	11
4.9	Dijkstra (restore Path)	12
4.10	Dijkstra	12
4.11	Disjoint Edges Path (Maxflow)	12
4.12	Euler Path (directed)	13
4.13	Euler Path (undirected)	13
4.14	Floyd Warshall	14
4.15	Graph Cycle (directed)	14
4.16	Graph Cycle (undirected)	15
4.17	Kruskal	15
4.18	Lowest Common Ancestor	15
4.19	Tree Maximum Distance	16
4.20	Maximum Flow (Edmonds-Karp)	17
4.21	Minimum Cut (unweighted)	17
4.22	Small to Large	18
4.23	Sum every node distance	18
4.24	Topological Sorting	19
4.25	Tree Diameter	19
5	Math	20
5.1	GCD (with factorization)	20
5.2	GCD	20
5.3	LCM (with factorization)	20
5.4	LCM	20
5.5	Arithmetic Progression Sum	20
5.6	Binomial MOD	20
5.7	Binomial	20
5.8	Euler phi $\varphi(n)$ (in range)	20
5.9	Euler phi $\varphi(n)$	21
5.10	Factorial Factorization	21

5.11	Factorial	21
5.12	Factorization (Pollard Rho)	21
5.13	Factorization	22
5.14	Fast Fourier Transform	22
5.15	Fast pow	22
5.16	Gauss Elimination	22
5.17	Integer Mod	23
5.18	Is prime	23
5.19	Number of Divisors $\tau(n)$	23
5.20	Power Sum	24
5.21	Sieve list primes	24
5.22	Sum of Divisors $\sigma(n)$	24
6	Problems	24
6.1	Hanoi Tower	24
7	Searching	24
7.1	Meet in the middle	24
7.2	Ternary Search Recursive	25
8	Strings	25
8.1	Count Distinct Anagrams	25
8.2	Hash Range Query	25
8.3	Longest Palindrome	25
8.4	Rabin Karp	25
8.5	String Psum	26
8.6	Suffix Automaton (complete)	26
8.7	Z-function get occurence positions	27
9	Settings and macros	27
9.1	short-macro.cpp	27
9.2	.vimrc	28
9.3	degug.cpp	28
9.4	.bashrc	29
9.5	macro.cpp	29

1 Data structures

1.1 Disjoint Sparse Table

Answers queries of any monoid operation (i.e. has identity element and is associative)

Build: $O(N \log N)$, Query: $O(1)$

```
#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct DisjointSparseTable {
    using Operation = T (*)(T, T);

    vector<vector<T>> st;
    Operation f;
    T identity;

    static constexpr int log2_floor(unsigned long long i) noexcept {
        return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
    }

    // Lazy loading constructor. Needs to call build!
    DisjointSparseTable(Operation op, const T neutral = T())
        : st(), f(op), identity(neutral) {}

    DisjointSparseTable(vector<T> v) : DisjointSparseTable(v, F(min(a, b))) {}

    DisjointSparseTable(vector<T> v, Operation op, const T neutral = T())
        : st(), f(op), identity(neutral) {
        build(v);
    }

    void build(vector<T> v) {
        st.resize(log2_floor(v.size()) + 1,
            vector<T>(1ll << (log2_floor(v.size()) + 1)));
        v.resize(st[0].size(), identity);
        for (int level = 0; level < (int)st.size(); ++level) {
            for (int block = 0; block < (1 << level); ++block) {
                const auto l = block << (st.size() - level);
                const auto r = (block + 1) << (st.size() - level);
                const auto m = l + (r - l) / 2;

                st[level][m] = v[m];
                for (int i = m + 1; i < r; i++)
                    st[level][i] = f(st[level][i - 1], v[i]);
                st[level][m - 1] = v[m - 1];
                for (int i = m - 2; i >= l; i--)
                    st[level][i] = f(st[level][i + 1], v[i]);
            }
        }
    }

    T query(int l, int r) const {
        if (l > r) return identity;
        if (l == r) return st.back()[l];

        const auto k = log2_floor(l ^ r);
        const auto level = (int)st.size() - 1 - k;
        return f(st[level][l], st[level][r]);
    }
};
```

```
};
```

1.2 Dsu

```
struct DSU {
    vi ps;
    vi size;
    DSU(int N) : ps(N + 1), size(N + 1, 1) { iota(all(ps), 0); }
    int find_set(int x) { return ps[x] == x ? x : ps[x] = find_set(ps[x]); }
    bool same_set(int x, int y) { return find_set(x) == find_set(y); }
    void union_set(int x, int y) {
        if (same_set(x, y)) return;

        int px = find_set(x);
        int py = find_set(y);

        if (size[px] < size[py]) swap(px, py);

        ps[py] = px;
        size[px] += size[py];
    }
};
```

1.3 Ordered Set

If you need an ordered **multiset** you may add an id to each value. Using `greater_equal`, or `less_equal` is considered undefined behavior.

- `order_of_key(k)` : Number of items strictly smaller/greater than `k`.
- `find_by_order(k)` : `K`-th element in a set (counting from zero).

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set =
    tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
```

1.4 SegTree Point Update (dynamic function)

Answers queries of any monoid operation (i.e. has identity element and is associative)

Build: $O(N)$, Query: $O(\log N)$

```
#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct SegTree {
    using Operation = T (*)(T, T);

    int N;
    vector<T> ns;
    Operation operation;
    T identity;

    SegTree(int n, Operation op = F(a + b), T neutral = T())
        : N(n), ns(2 * N, neutral), operation(op), identity(neutral) {}

    SegTree(const vector<T> &v, Operation op = F(a + b), T neutral = T())
```

```

: SegTree((int)v.size(), op, neutral) {
copy(v.begin(), v.end(), ns.begin() + N);

for (int i = N - 1; i > 0; --i) ns[i] = operation(ns[2 * i], ns[2 * i +
1]);
}

T query(size_t i) const { return ns[i + N]; }

T query(size_t l, size_t r) const {
auto a = l + N, b = r + N;
auto ans = identity;

while (a <= b) {
if (a & 1) ans = operation(ans, ns[a++]);
if (not(b & 1)) ans = operation(ans, ns[b--]);

a /= 2;
b /= 2;
}

return ans;
}

void update(size_t i, T value) { update_set(i, operation(ns[i + N], value));
}

void update_set(size_t i, T value) {
auto a = i + N;

ns[a] = value;
while (a >= 1) ns[a] = operation(ns[2 * a], ns[2 * a + 1]);
}
};

```

1.5 Segtree Range Max Query Range Max Update

```

template <typename T = ll>
struct SegTree {
int N;
T nu, nq;
vector<T> st, lazy;
SegTree(const vector<T> &xs)
: N(len(xs)),
nu(numeric_limits<T>::min()),
nq(numeric_limits<T>::min()),
st(4 * N + 1, nu),
lazy(4 * N + 1, nu) {
for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
}

void update(int l, int r, T value) { update(1, 0, N - 1, l, r, value); }

T query(int l, int r) { return query(1, 0, N - 1, l, r); }

void update(int node, int nl, int nr, int ql, int qr, T v) {
propagation(node, nl, nr);

```

```

if (ql > nr or qr < nl) return;

st[node] = max(st[node], v);
if (ql <= nl and nr <= qr) {
if (nl < nr) {
lazy[left(node)] = max(lazy[left(node)], v);
lazy[right(node)] = max(lazy[right(node)], v);
}
return;
}
update(left(node), nl, mid(nl, nr), ql, qr, v);
update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

st[node] = max(st[left(node)], st[right(node)]);
}

T query(int node, int nl, int nr, int ql, int qr) {
propagation(node, nl, nr);

if (ql > nr or qr < nl) return nq;

if (ql <= nl and nr <= qr) return st[node];

T x = query(left(node), nl, mid(nl, nr), ql, qr);
T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

return max(x, y);
}

void propagation(int node, int nl, int nr) {
if (lazy[node] != nu) {
st[node] = max(st[node], lazy[node]);

if (nl < nr) {
lazy[left(node)] = max(lazy[left(node)], lazy[node]);
lazy[right(node)] = max(lazy[right(node)], lazy[node]);
}

lazy[node] = nu;
}
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

int main() {
int n;
cin >> n;
vector<array<int, 3>> xs(n);
for (int i = 0; i < n; ++i) {
for (int j = 0; j < 3; ++j) {
cin >> xs[i][j];
}
}
vi aux(n, 0);
SegTree<int> st(aux);

```

```

for (int i = 0; i < n; ++i) {
    int a = min(i + xs[i][1], n);
    int b = min(i + xs[i][2], n);
    st.update(i, i, st.query(i, i) + xs[i][0]);
    int cur = st.query(i, i);
    st.update(a, b, cur);
}

cout << st.query(0, n) << '\n';
}

```

1.6 SegTree Range Min Query Point Assign Update

```

template <typename T = ll>
struct SegTree {
    int n;
    T nu, nq;
    vector<T> st;
    SegTree(const vector<T> &v)
        : n(len(v)), nu(0), nq(numeric_limits<T>::max()), st(n * 4 + 1, nu) {
        for (int i = 0; i < n; ++i) update(i, v[i]);
    }
    void update(int p, T v) { update(1, 0, n - 1, p, v); }
    T query(int l, int r) { return query(1, 0, n - 1, l, r); }

    void update(int node, int nl, int nr, int p, T v) {
        if (p < nl or p > nr) return;

        if (nl == nr) {
            st[node] = v;
            return;
        }

        update(left(node), nl, mid(nl, nr), p, v);
        update(right(node), mid(nl, nr) + 1, nr, p, v);

        st[node] = min(st[left(node)], st[right(node)]);
    }

    T query(int node, int nl, int nr, int ql, int qr) {
        if (ql <= nl and qr >= nr) return st[node];
        if (nl > qr or nr < ql) return nq;
        if (nl == nr) return st[node];

        return min(query(left(node), nl, mid(nl, nr), ql, qr),
            query(right(node), mid(nl, nr) + 1, nr, ql, qr));
    }

    int left(int p) { return p << 1; }
    int right(int p) { return (p << 1) + 1; }
    int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

1.7 SegTree Range Xor Query Point Assign Update

```

template <typename T = ll>
struct SegTree {
    int n;

```

```

    T nu, nq;
    vector<T> st;
    SegTree(const vector<T> &v) : n(len(v)), nu(0), nq(0), st(n * 4 + 1, nu) {
        for (int i = 0; i < n; ++i) update(i, v[i]);
    }
    void update(int p, T v) { update(1, 0, n - 1, p, v); }
    T query(int l, int r) { return query(1, 0, n - 1, l, r); }

    void update(int node, int nl, int nr, int p, T v) {
        if (p < nl or p > nr) return;

        if (nl == nr) {
            st[node] = v;
            return;
        }

        update(left(node), nl, mid(nl, nr), p, v);
        update(right(node), mid(nl, nr) + 1, nr, p, v);

        st[node] = st[left(node)] ^ st[right(node)];
    }

    T query(int node, int nl, int nr, int ql, int qr) {
        if (ql <= nl and qr >= nr) return st[node];
        if (nl > qr or nr < ql) return nq;
        if (nl == nr) return st[node];

        return query(left(node), nl, mid(nl, nr), ql, qr) ^
            query(right(node), mid(nl, nr) + 1, nr, ql, qr);
    }

    int left(int p) { return p << 1; }
    int right(int p) { return (p << 1) + 1; }
    int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

1.8 SegTree Range Min Query Range Sum Update

```

template <typename t = ll>
struct SegTree {
    int n;
    t nu;
    t nq;
    vector<t> st, lazy;
    SegTree(const vector<t> &xs)
        : n(len(xs)),
          nu(0),
          nq(numeric_limits<t>::max()),
          st(4 * n, nu),
          lazy(4 * n, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    SegTree(int n) : n(n), st(4 * n, nu), lazy(4 * n, nu) {}

    void update(int l, int r, ll value) { update(1, 0, n - 1, l, r, value); }

    t query(int l, int r) { return query(1, 0, n - 1, l, r); }

```

```

void update(int node, int nl, int nr, int ql, int qr, ll v) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return;

    if (ql <= nl and nr <= qr) {
        st[node] += (nr - nl + 1) * v;

        if (nl < nr) {
            lazy[left(node)] += v;
            lazy[right(node)] += v;
        }

        return;
    }

    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

    st[node] = min(st[left(node)], st[right(node)]);
}

t query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    t x = query(left(node), nl, mid(nl, nr), ql, qr);
    t y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return min(x, y);
}

void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
        st[node] += lazy[node];

        if (nl < nr) {
            lazy[left(node)] += lazy[node];
            lazy[right(node)] += lazy[node];
        }

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

1.9 SegTree Range Sum Query Range Sum Update

```

template <typename T = ll>
struct SegTree {

```

```

    int N;
    vector<T> st, lazy;
    T nu = 0;
    T nq = 0;
    SegTree(const vector<T> &xs) : N(len(xs)), st(4 * N, nu), lazy(4 * N, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    SegTree(int n) : N(n), st(4 * N, nu), lazy(4 * N, nu) {}

    void update(int l, int r, ll value) { update(1, 0, N - 1, l, r, value); }

    T query(int l, int r) { return query(1, 0, N - 1, l, r); }

    void update(int node, int nl, int nr, int ql, int qr, ll v) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return;

        if (ql <= nl and nr <= qr) {
            st[node] += (nr - nl + 1) * v;

            if (nl < nr) {
                lazy[left(node)] += v;
                lazy[right(node)] += v;
            }

            return;
        }

        update(left(node), nl, mid(nl, nr), ql, qr, v);
        update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

        st[node] = st[left(node)] + st[right(node)];
    }

    T query(int node, int nl, int nr, int ql, int qr) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return nq;

        if (ql <= nl and nr <= qr) return st[node];

        T x = query(left(node), nl, mid(nl, nr), ql, qr);
        T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

        return x + y;
    }

    void propagation(int node, int nl, int nr) {
        if (lazy[node]) {
            st[node] += (nr - nl + 1) * lazy[node];

            if (nl < nr) {
                lazy[left(node)] += lazy[node];
                lazy[right(node)] += lazy[node];
            }
        }
    }
};

```

```

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

1.10 Sparse Table Range Min Query

Build: $O(N \log N)$, Query: $O(1)$

```

int fastlog2(ll x) {
    ull i = x;
    return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
}

template <typename T>
class SparseTable {
public:
    int N;
    int K;
    vector<vector<T>> st;
    SparseTable(vector<T> vs)
        : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector<T>(N + 1)) {
        copy(vs.begin(), vs.end(), st[0].begin());

        for (int i = 1; i <= K; ++i)
            for (int j = 0; j + (1 << i) <= N; ++j)
                st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << i)]);
    }
    T RMQ(int l, int r) { // [l, r], 0 indexed
        int i = fastlog2(r - l + 1);
        return min(st[i][l], st[i][r - (1 << i) + 1]);
    }
};

```

2 Dynamic programming

2.1 Edit Distance

$O(N * M)$

```

int edit_distance(const string &a, const string &b) {
    int n = a.size();
    int m = b.size();
    vector<vi> dp(n + 1, vi(m + 1, 0));

    int ADD = 1, DEL = 1, CHG = 1;
    for (int i = 0; i <= n; ++i) {
        dp[i][0] = i * DEL;
    }
    for (int i = 1; i <= m; ++i) {
        dp[0][i] = ADD * i;
    }

    for (int i = 1; i <= n; ++i) {
        for (int j = 1; j <= m; ++j) {

```

```

            int add = dp[i][j - 1] + ADD;
            int del = dp[i - 1][j] + DEL;
            int chg = dp[i - 1][j - 1] + (a[i - 1] == b[j - 1] ? 0 : 1) * CHG;
            dp[i][j] = min({add, del, chg});
        }
    }

    return dp[n][m];
}

```

2.2 Kadane

Find the maximum subarray sum in a given array.

```

int kadane(const vi &as) {
    vi s(len(as));
    s[0] = as[0];

    for (int i = 1; i < len(as); ++i) s[i] = max(as[i], s[i - 1] + as[i]);

    return *max_element(all(s));
}

```

2.3 Knapsack (value)

Finds the maximum points possible

```
const int MAXN{2010}, MAXM{2010};
```

```
ll st[MAXN][MAXM];
```

```

ll dp(int i, int m, int M, const vii &cs) {
    if (i < 0) return 0;

    if (st[i][m] != -1) return st[i][m];

    auto res = dp(i - 1, m, M, cs);
    auto [w, v] = cs[i];

    if (w <= m) res = max(res, dp(i - 1, m - w, M, cs) + v);

    st[i][m] = res;
    return res;
}

ll knapsack(int M, const vii &cs) {
    memset(st, -1, sizeof st);

    return dp((int)cs.size() - 1, M, M, cs);
}

```

2.4 Knapsack (elements)

Finds the maximum possible points carry and which elements to achieve it

```
const int MAXN{2010}, MAXM{2010};
ll st[MAXN][MAXM];
```

```

char ps[MAXN][MAXM];

pair<ll, vi> knapsack(int M, const vii &cs) {
    int N = len(cs) - 1;

    for (int i = 0; i <= N; ++i) st[i][0] = 0;

    for (int m = 0; m <= M; ++m) st[0][m] = 0;

    for (int i = 1; i <= N; ++i) {
        for (int m = 1; m <= M; ++m) {
            st[i][m] = st[i - 1][m];
            ps[i][m] = 0;
            auto [w, v] = cs[i];

            if (w <= m and st[i - 1][m - w] + v > st[i][m]) {
                st[i][m] = st[i - 1][m - w] + v;
                ps[i][m] = 1;
            }
        }
    }

    int m = M;
    vi is;
    for (int i = N; i >= 1; --i) {
        if (ps[i][m]) {
            is.push_back(i);
            m -= cs[i].first;
        }
    }

    reverse(all(is));

    // max value, items
    return {st[N][M], is};
}

```

2.5 Longest Increasing Subsequence (LIS)

Finds the length of the longest subsequence in

$O(n \log n)$

```

int LIS(const vi& as) {
    const ll oo = 1e18;
    int n = len(as);
    vll lis(n + 1, oo);
    lis[0] = -oo;

    auto ans = 0;

    for (int i = 0; i < n; ++i) {
        auto it = lower_bound(all(lis), as[i]);
        auto pos = (int)(it - lis.begin());

        ans = max(ans, pos);
        lis[pos] = as[i];
    }
}

```

```

}

return ans;
}

2.6 Money Sum (Bottom Up)

Find every possible sum using the given values only once.

set<int> money_sum(const vi &xs) {
    using vc = vector<char>;
    using vvc = vector<vc>;
    int _m = accumulate(all(xs), 0);
    int _n = xs.size();
    vvc _dp(_n + 1, vc(_m + 1, 0));
    set<int> _ans;
    _dp[0][xs[0]] = 1;
    for (int i = 1; i < _n; ++i) {
        for (int j = 0; j <= _m; ++j) {
            if (j == 0 or _dp[i - 1][j]) {
                _dp[i][j + xs[i]] = 1;
                _dp[i][j] = 1;
            }
        }
    }

    for (int i = 0; i < _n; ++i)
        for (int j = 0; j <= _m; ++j)
            if (_dp[i][j]) _ans.insert(j);
    return _ans;
}

```

2.7 Travelling Salesman Problem

```

using vi = vector<int>;
vector<vi> dist;
vector<vi> memo;
/* 0 ( N^2 * 2^N ) */
int tsp(int i, int mask, int N) {
    if (mask == (1 << N) - 1) return dist[i][0];
    if (memo[i][mask] != -1) return memo[i][mask];
    int ans = INT_MAX << 1;
    for (int j = 0; j < N; ++j) {
        if (mask & (1 << j)) continue;
        auto t = tsp(j, mask | (1 << j), N) + dist[i][j];
        ans = min(ans, t);
    }
    return memo[i][mask] = ans;
}

```

3 Geometry

3.1 Convex Hull

Given a set of points find the smallest convex polygon that contains all the given points.

Time: $O(N \log N)$

By default it removes the collinear points, set the boolean to true if you don't want that

```

struct pt {
    double x, y;
    int id;
};

int orientation(pt a, pt b, pt c) {
    double v = a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y - b.y);
    if (v < 0) return -1; // clockwise
    if (v > 0) return +1; // counter-clockwise
    return 0;
}

bool cw(pt a, pt b, pt c, bool include_collinear) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}

bool collinear(pt a, pt b, pt c) { return orientation(a, b, c) == 0; }

void convex_hull(vector<pt>& pts, bool include_collinear = false) {
    pt p0 = *min_element(all(pts), [](pt a, pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
    sort(all(pts), [&p0](const pt& a, const pt& b) {
        int o = orientation(p0, a, b);
        if (o == 0)
            return (p0.x - a.x) * (p0.x - a.x) + (p0.y - a.y) * (p0.y - a.y) <
                (p0.x - b.x) * (p0.x - b.x) + (p0.y - b.y) * (p0.y - b.y);
        return o < 0;
    });
    if (include_collinear) {
        int i = len(pts) - 1;
        while (i >= 0 && collinear(p0, pts[i], pts.back())) i--;
        reverse(pts.begin() + i + 1, pts.end());
    }

    vector<pt> st;
    for (int i = 0; i < len(pts); i++) {
        while (st.size() > 1 &&
            !cw(st[len(st) - 2], st.back(), pts[i], include_collinear))
            st.pop_back();
        st.push_back(pts[i]);
    }

    pts = st;
}

```

3.2 Determinant

```

#include "Point.cpp"

template <typename T>
T D(const Point<T> &P, const Point<T> &Q, const Point<T> &R) {
    return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
        (R.x * Q.y + R.y * P.x + Q.x * P.y);
}

```

3.3 Equals

```

template <typename T>
bool equals(T a, T b) {
    const double EPS{1e-9};
    if (is_floating_point<T>::value)
        return fabs(a - b) < EPS;
    else
        return a == b;
}

```

3.4 Line

```

#include <bits/stdc++.h>

#include "point-struct-and-utils.cpp"
using namespace std;

struct line {
    ld a, b, c;
};

// the answer is stored in the third parameter (pass by reference)
void pointsToLine(const point &p1, const point &p2, line &l) {
    if (fabs(p1.x - p2.x) < EPS)
        // vertical line
        l = {1.0, 0.0, -p1.x};
    // default values
    else
        l = {-(ld)(p1.y - p2.y) / (p1.x - p2.x), 1.0, -(ld)(l.a * p1.x) - p1.y};
}

```

3.5 Point Struct And Utils (2d)

```

#include <bits/stdc++.h>
using namespace std;
using ld = long double;

struct point {
    ld x, y;
    int id;
    point(ld x = 0.0, ld y = 0.0, int id = -1) : x(x), y(y), id(id) {}

    point& operator+=(const point& t) {
        x += t.x;
        y += t.y;
        return *this;
    }

    point& operator-=(const point& t) {
        x -= t.x;
        y -= t.y;
        return *this;
    }

    point& operator*=(ld t) {
        x *= t;
        y *= t;
        return *this;
    }
}

```



```

}
point& operator/=(ld t) {
    x /= t;
    y /= t;
    return *this;
}
point operator+(const point& t) const { return point(*this) += t; }
point operator-(const point& t) const { return point(*this) -= t; }
point operator*(ld t) const { return point(*this) *= t; }
point operator/(ld t) const { return point(*this) /= t; }
};

ld dot(point& a, point& b) { return a.x * b.x + a.y * b.y; }

ld norm(point& a) { return dot(a, a); }

ld abs(point a) { return sqrt(norm(a)); }

ld proj(point a, point b) { return dot(a, b) / abs(b); }

ld angle(point a, point b) { return acos(dot(a, b) / abs(a) / abs(b)); }

ld cross(point a, point b) { return a.x * b.y - a.y * b.x; }

```

3.6 Segment

```

#include "Line.cpp"
#include "Point.cpp"
#include "equals.cpp"

template <typename T>
struct segment {
    Point<T> A, B;

    bool contains(const Point<T> &P) const;

    Point<T> closest(const Point<T> &p) const;
};

template <typename T>
bool segment<T>::contains(const Point<T> &P) const {
    // verifica se P está contido na reta
    double dAB = Point<T>::dist(A, B), dAP = Point<T>::dist(A, P),
           dPB = Point<T>::dist(P, B);

    return equals(dAP + dPB, dAB);
}

template <typename T>
Point<T> segment<T>::closest(const Point<T> &P) const {
    Line<T> R(A, B);
    auto Q = R.closest(P);

    if (this->contains(Q)) return Q;

    auto distA = Point<T>::dist(P, A);
    auto distB = Point<T>::dist(P, B);

```

```

    if (distA <= distB)
        return A;
    else
        return B;
}

```

4 Graphs

4.1 2 SAT

```

struct SAT2 {
    ll n;
    vll2d adj, adj_t;
    vc used;
    vll order, comp;
    vc assignment;
    bool solvable;
    SAT2(ll _n)
        : n(2 * _n),
          adj(n),
          adj_t(n),
          used(n),
          order(n),
          comp(n, -1),
          assignment(n / 2) {}
    void dfs1(int v) {
        used[v] = true;
        for (int u : adj[v]) {
            if (!used[u]) dfs1(u);
        }
        order.push_back(v);
    }

    void dfs2(int v, int cl) {
        comp[v] = cl;
        for (int u : adj_t[v]) {
            if (comp[u] == -1) dfs2(u, cl);
        }
    }

    bool solve_2SAT() {
        // find and label each SCC
        for (int i = 0; i < n; ++i) {
            if (!used[i]) dfs1(i);
        }
        reverse(all(order));
        ll j = 0;
        for (auto &v : order) {
            if (comp[v] == -1) dfs2(v, j++);
        }

        assignment.assign(n / 2, false);
        for (int i = 0; i < n; i += 2) {
            // x and !x belong to the same SCC
            if (comp[i] == comp[i + 1]) {
                solvable = false;
                return false;
            }

```

```

    }

    assignment[i / 2] = comp[i] > comp[i + 1];
}
solvable = true;
return true;
}

void add_disjunction(int a, bool na, int b, bool nb) {
    a = (2 * a) ^ na;
    b = (2 * b) ^ nb;
    int neg_a = a ^ 1;
    int neg_b = b ^ 1;
    adj[neg_a].push_back(b);
    adj[neg_b].push_back(a);
    adj_t[b].push_back(neg_a);
    adj_t[a].push_back(neg_b);
}
};

```

4.2 Cycle Distances

Given a vertex s finds the longest cycle that end's in s , note that the vector **dist** will contain the distance that each vertex u needs to reach s .

Time: $O(N)$

```

using adj = vector<vector<pair<int, ll>>>;
ll cycleDistances(int u, int n, int s, vc &vis, adj &g, vll &dist) {
    vis[u] = 1;

    for (auto [v, d] : g[u]) {
        if (v == s) {
            dist[u] = max(dist[u], d);
            continue;
        }

        if (vis[v] == 1) {
            continue;
        }

        if (vis[v] == 2) {
            dist[u] = max(dist[u], dist[v] + d);
        } else {
            ll d2 = cycleDistances(v, n, s, vis, g, dist);
            if (d2 != -oo) {
                dist[u] = max(dist[u], d2 + d);
            }
        }
    }
    vis[u] = 2;
    return dist[u];
}

```

4.3 SCC (struct)

Able to find the component of each node and the total of SCC in $O(V * E)$ and build the SCC graph ($O(V * E)$).

```

struct SCC {
    ll N;
    int totsc;
    vll2d adj, tadj;
    vll todo, comps, comp;
    vector<set<ll>> sccadj;
    vchar vis;
    SCC(ll _N)
        : N(_N), totsc(0), adj(_N), tadj(_N), comp(_N, -1), sccadj(_N), vis(_N)
    {}

    void add_edge(ll x, ll y) { adj[x].eb(y), tadj[y].eb(x); }

    void dfs(ll x) {
        vis[x] = 1;
        for (auto &y : adj[x])
            if (!vis[y]) dfs(y);
        todo.pb(x);
    }
    void dfs2(ll x, ll v) {
        comp[x] = v;
        for (auto &y : tadj[x])
            if (comp[y] == -1) dfs2(y, v);
    }
    void gen() {
        for (ll i = 0; i < N; ++i)
            if (!vis[i]) dfs(i);
        reverse(all(todo));
        for (auto &x : todo)
            if (comp[x] == -1) {
                dfs2(x, x);
                comps.pb(x);
                totsc++;
            }
    }

    void genSCCGraph() {
        for (ll i = 0; i < N; ++i) {
            for (auto &j : adj[i]) {
                if (comp[i] != comp[j]) {
                    sccadj[comp[i]].insert(comp[j]);
                }
            }
        }
    }
};

```

4.4 Bellman-Ford (find negative cycle)

Given a directed graph find a negative cycle by running n iterations, and if the last one produces a relaxation than there is a cycle.

Time: $O(V * E)$

```

const ll oo = 2500 * 1e9;

using graph = vector<vector<pair<int, ll>>>;
vi negative_cycle(graph &g, int n) {
    vll d(n, oo);

```

```

vi p(n, -1);
int x = -1;
d[0] = 0;
for (int i = 0; i < n; i++) {
    x = -1;
    for (int u = 0; u < n; u++) {
        for (auto &[v, l] : g[u]) {
            if (d[u] + l < d[v]) {
                d[v] = d[u] + l;
                p[v] = u;
                x = v;
            }
        }
    }
}

if (x == -1)
    return {};
else {
    for (int i = 0; i < n; i++) x = p[x];
    vi cycle;
    for (int v = x;; v = p[v]) {
        cycle.eb(v);
        if (v == x and len(cycle) > 1) break;
    }
    reverse(all(cycle));
    return cycle;
}
}

```

4.5 Bellman Ford

Find shortest path from a single source to all other nodes. Can detect negative cycles.
Time: $O(V * E)$

```

bool bellman_ford(const vector<vector<pair<int, ll>>> &g, int s,
                 vector<ll> &dist) {
    int n = (int)g.size();
    dist.assign(n, LLONG_MAX);

    vector<int> count(n);
    vector<char> in_queue(n);
    queue<int> q;

    dist[s] = 0;
    q.push(s);
    in_queue[s] = true;

    while (not q.empty()) {
        int cur = q.front();
        q.pop();
        in_queue[cur] = false;

        for (auto [to, w] : g[cur]) {
            if (dist[cur] + w < dist[to]) {
                dist[to] = dist[cur] + w;
                if (not in_queue[to]) {
                    q.push(to);

```

```

                in_queue[to] = true;
                count[to]++;
                if (count[to] > n) return false;
            }
        }
    }

    return true;
}

```

4.6 Binary Lifting

$far[h][i]$ = the node that is 2^h distance from node i

Build : $O(N * \log N)$

sometimes is useful invert the order of loops

```

const int maxlog = 20;
int far[maxlog + 1][n + 1];
int n;
for (int h = 1; h <= maxlog; h++) {
    for (int i = 1; i <= n; i++) {
        far[h][i] = far[h - 1][far[h - 1][i]];
    }
}

```

4.7 Check Bipartite

$O(V)$

```

bool checkBipartite(const ll n, const vector<vll> &adj) {
    ll s = 0;
    queue<ll> q;
    q.push(s);
    vll color(n, INF);
    color[s] = 0;
    bool isBipartite = true;
    while (!q.empty() && isBipartite) {
        ll u = q.front();
        q.pop();
        for (auto &v : adj[u]) {
            if (color[v] == INF) {
                color[v] = 1 - color[u];
                q.push(v);
            } else if (color[v] == color[u]) {
                return false;
            }
        }
    }
    return true;
}

```

4.8 Dijkstra (k Shortest Paths)

```

const ll oo = 1e9 * 1e5 + 1;
using adj = vector<vector<pll>>;
vector<priority_queue<ll>> dijkstra(const vector<vector<pll>> &g, int n, int s
,
                                int k) {
    priority_queue<pll, vector<pll>, greater<pll>> pq;

    vector<priority_queue<ll>> dist(n);
    dist[0].emplace(0);
    pq.emplace(0, s);
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();

        if (not dist[v].empty() and dist[v].top() < d1) continue;

        for (auto [d2, u] : g[v]) {
            if (len(dist[u]) < k) {
                pq.emplace(d2 + d1, u);
                dist[u].emplace(d2 + d1);
            } else {
                if (dist[u].top() > d1 + d2) {
                    dist[u].pop();
                    dist[u].emplace(d1 + d2);
                    pq.emplace(d2 + d1, u);
                }
            }
        }
    }
    return dist;
}

```

4.9 Dijkstra (restore Path)

```

pair<vll, vi> dijkstra(const vector<vector<pll>> &g, int n, int s) {
    priority_queue<pll, vector<pll>, greater<pll>> pq;
    vll dist(n, oo);
    vi p(n, -1);
    pq.emplace(0, s);
    dist[s] = 0;
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();
        if (dist[v] < d1) continue;

        for (auto [d2, u] : g[v]) {
            if (dist[u] > d1 + d2) {
                dist[u] = d1 + d2;
                p[u] = v;
                pq.emplace(dist[u], u);
            }
        }
    }
    return {dist, p};
}

```

4.10 Dijkstra

Finds the minimum distance from s to every other node in

$$O(E * \log E)$$

time.

```

vll dijkstra(const vector<vector<pll>> &g, int n, int s) {
    priority_queue<pll, vector<pll>, greater<pll>> pq;
    vll dist(n + 1, oo);
    pq.emplace(0, s);
    dist[s] = 0;
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();
        if (dist[v] < d1) continue;

        for (auto [d2, u] : g[v]) {
            if (dist[u] > d1 + d2) {
                dist[u] = d1 + d2;
                pq.emplace(dist[u], u);
            }
        }
    }
    return dist;
}

```

4.11 Disjoint Edges Path (Maxflow)

Given a directed graph find's every path with distinct edges that starts at s and ends at t

When building the graph, if there is an edge (u, v) is necessary to also add the transposed edge (v, u) but only need to add the capacity $c(u, v)$, and mark $isedge(u, v)$ as true.

Time : $O(E \cdot V^2)$

```

ll bfs(int s, int t, vi2d &g, vll2d &capacity, vi &parent) {
    fill(all(parent), -1);
    parent[s] = -2;
    queue<pair<ll, int>> q;
    q.push({0, s});

    while (!q.empty()) {
        auto [flow, cur] = q.front();
        q.pop();

        for (auto next : g[cur]) {
            if (parent[next] == -1 and capacity[cur][next]) {
                parent[next] = cur;
                ll new_flow = min(flow, capacity[cur][next]);
                if (next == t) return new_flow;
                q.push({new_flow, next});
            }
        }
    }

    return 0ll;
}

ll maxflow(int s, int t, int n, vi2d &g, vll2d &capacity) {

```

```

ll flow = 0;
vi parent(n);
ll new_flow;

while ((new_flow = bfs(s, t, g, capacity, parent))) {
    flow += new_flow;
    int cur = t;
    while (cur != s) {
        int prev = parent[cur];
        capacity[prev][cur] -= new_flow;
        capacity[cur][prev] += new_flow;
        cur = prev;
    }
}

return flow;
}

void dfs(int u, int t, vi2d &g, vc2d &vis, vc2d &isedge, vll2d &capacity,
        vi &route, vi2d &routes) {
    route.pb(u);
    if (u == t) {
        routes.emplace_back(route);
        route.pop_back();
        return;
    }
    for (auto &v : g[u]) {
        if (capacity[u][v] == 0 and isedge[u][v] and not vis[u][v]) {
            vis[u][v] = true;
            dfs(v, t, g, vis, isedge, capacity, route, routes);
            route.pop_back();
            return;
        }
    }
}

vi2d disjoint_paths(vi2d &g, vll2d &capacity, vc2d &isedge, int s, int t,
                    int n) {
    ll mf = maxflow(s, t, n, g, capacity);
    vi2d routes;
    vi route;
    vc2d vis(n + 1, vc(n + 1));
    for (int i = 0; i < (int)mf; i++)
        dfs(s, t, g, vis, isedge, capacity, route, routes);
    return routes;
}

```

4.12 Euler Path (directed)

Given a **directed** graph finds a path that visits every edge exactly once.
Time: $O(E)$

```

vector<int> euler_cycle(vector<vector<int>> &g, int u) {
    vector<int> res;

    stack<int> st;
    st.push(u);
    while (!st.empty()) {

```

```

        auto cur = st.top();
        if (g[cur].empty()) {
            res.push_back(cur);
            st.pop();
        } else {
            auto next = g[cur].back();
            st.push(next);

            g[cur].pop_back();
        }
    }

    for (auto &x : g)
        if (!x.empty()) return {};

    return res;
}

vector<int> euler_path(vector<vector<int>> &g, int first) {
    {
        int n = (int)g.size();
        vector<int> in(n), out(n);
        for (int i = 0; i < n; i++)
            for (auto x : g[i]) in[x]++, out[i]++;

        int a = 0, b = 0, c = 0;
        for (int i = 0; i < n; i++)
            if (in[i] == out[i])
                c++;
            else if (in[i] - out[i] == 1)
                b++;
            else if (in[i] - out[i] == -1)
                a++;

        if (c != n - 2 or a != 1 or b != 1) return {};
    }

    auto res = euler_cycle(g, first);
    if (res.empty()) return res;

    reverse(all(res));
    return res;
}

```

4.13 Euler Path (undirected)

Given a **undirected** graph finds a path that visits every edge exactly once.
Time: $O(E)$

```

vector<int> euler_cycle(vector<vector<int>> &g, int u) {
    vector<int> res;
    multiset<pair<int, int>> vis;

    stack<int> st;
    st.push(u);
    while (!st.empty()) {
        auto cur = st.top();

```

```

while (!g[cur].empty()) {
    auto it = vis.find(make_pair(cur, g[cur].back()));
    if (it == vis.end()) break;
    g[cur].pop_back();
    vis.erase(it);
}

if (g[cur].empty()) {
    res.push_back(cur);
    st.pop();
} else {
    auto next = g[cur].back();
    st.push(next);

    vis.emplace(next, cur);
    g[cur].pop_back();
}
}

for (auto &x : g)
    if (!x.empty()) return {};

return res;
}

vector<int> euler_path(vector<vector<int>> &g, int first) {
    int n = (int)g.size();
    int v1 = -1, v2 = -1;
    {
        bool bad = false;
        for (int i = 0; i < n; i++)
            if (g[i].size() & 1) {
                if (v1 == -1)
                    v1 = i;
                else if (v2 == -1)
                    v2 = i;
                else
                    bad = true;
            }

        if (bad or (v1 != -1 and v2 == -1)) return {};
    }

    if (v2 != -1) {
        // insert cycle
        g[v1].push_back(v2);
        g[v2].push_back(v1);
    }

    auto res = euler_cycle(g, first);
    if (res.empty()) return res;

    if (v1 != -1) {
        for (int i = 0; i + 1 < (int)res.size(); i++) {
            if ((res[i] == v1 and res[i + 1] == v2) ||
                (res[i] == v2 and res[i + 1] == v1)) {
                vector<int> res2;

```

```

                for (int j = i + 1; j < (int)res.size(); j++) res2.push_back(res[j]);
                for (int j = 1; j <= i; j++) res2.push_back(res[j]);
                res = res2;
                break;
            }
        }
    }

    reverse(all(res));
    return res;
}

4.14 Floyd Warshall

Simply finds the minimal distance for each node to every other node.  $O(V^3)$ 

vector<vll> floyd_warshall(const vector<vll> &adj, ll n) {
    auto dist = adj;

    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            for (int k = 0; k < n; ++k) {
                dist[j][k] = min(dist[j][k], dist[j][i] + dist[i][k]);
            }
        }
    }

    return dist;
}

4.15 Graph Cycle (directed)

Given a directed graph finds a cycle (or not).
Time :  $O(E)$ 

bool dfs(int v, vi2d &adj, vc &visited, vi &parent, vc &color, int &
    cycle_start,
    int &cycle_end) {
    color[v] = 1;
    for (int u : adj[v]) {
        if (color[u] == 0) {
            parent[u] = v;
            if (dfs(u, adj, visited, parent, color, cycle_start, cycle_end))
                return true;
        } else if (color[u] == 1) {
            cycle_end = v;
            cycle_start = u;
            return true;
        }
    }
    color[v] = 2;
    return false;
}

vi find_cycle(vi2d &g, int n) {
    vc visited(n);
    vi parent(n);
    vc color(n);

```

```

int cycle_start, cycle_end;
color.assign(n, 0);
parent.assign(n, -1);
cycle_start = -1;

for (int v = 0; v < n; v++) {
    if (color[v] == 0 &&
        dfs(v, g, visited, parent, color, cycle_start, cycle_end))
        break;
}

if (cycle_start == -1) {
    return {};
} else {
    vector<int> cycle;
    cycle.push_back(cycle_start);
    for (int v = cycle_end; v != cycle_start; v = parent[v]) cycle.push_back(v);
    cycle.push_back(cycle_start);
    reverse(cycle.begin(), cycle.end());
    return cycle;
}
}

```

4.16 Graph Cycle (undirected)

Detects if a graph contains a cycle. If path parameter is not null, it will contain the cycle if one exists.

Time: $O(V + E)$

```

bool has_cycle(const vector<vector<int>> &g, int s, vector<char> &vis,
              vector<char> &in_path, vector<int> *path = nullptr) {
    vis[s] = in_path[s] = 1;
    if (path != nullptr) path->push_back(s);
    for (auto x : g[s]) {
        if (!vis[x] && has_cycle(g, x, vis, in_path, path))
            return true;
        else if (in_path[x]) {
            if (path != nullptr) path->push_back(x);
            return true;
        }
    }
    in_path[s] = 0;
    if (path != nullptr) path->pop_back();
    return false;
}

```

4.17 Kruskal

Find the minimum spanning tree of a graph.

Time: $O(E \log E)$

can be used to find the maximum spanning tree by changing the comparison operator in the sort

```

struct UFDS {
    vector<int> ps, sz;
    int components;

    UFDS(int n) : ps(n + 1), sz(n + 1, 1), components(n) { iota(all(ps), 0); }
}

```

```

int find_set(int x) { return (x == ps[x] ? x : (ps[x] = find_set(ps[x]))); }

bool same_set(int x, int y) { return find_set(x) == find_set(y); }

void union_set(int x, int y) {
    x = find_set(x);
    y = find_set(y);

    if (x == y) return;

    if (sz[x] < sz[y]) swap(x, y);

    ps[y] = x;
    sz[x] += sz[y];

    components--;
}

vector<tuple<ll, int, int>> kruskal(int n, vector<tuple<ll, int, int>> &edges)
{
    UFDS ufds(n);
    vector<tuple<ll, int, int>> ans;

    sort(all(edges));
    for (auto [a, b, c] : edges) {
        if (ufds.same_set(b, c)) continue;

        ans.emplace_back(a, b, c);
        ufds.union_set(b, c);
    }

    return ans;
}

```

4.18 Lowest Common Ancestor

Given two nodes of a tree find their lowest common ancestor, or their distance

Build : $O(V)$, Queries: $O(1)$

0 indexed !

```

template <typename T>
struct SparseTable {
    vector<T> v;
    int n;
    static const int b = 30;
    vi mask, t;

    int op(int x, int y) { return v[x] < v[y] ? x : y; }
    int msb(int x) { return __builtin_clz(1) - __builtin_clz(x); }
    SparseTable() {}
    SparseTable(const vector<T>& v_) : v(v_), n(v.size()), mask(n), t(n) {
        for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
            at = (at << 1) & ((1 << b) - 1);
            while (at and op(i, i - msb(at & -at)) == i) at ^= at & -at;
        }
        for (int i = 0; i < n / b; i++)

```

```

    t[i] = b * i + b - 1 - msb(mask[b * i + b - 1]);
    for (int j = 1; (1 << j) <= n / b; j++)
        for (int i = 0; i + (1 << j) <= n / b; i++)
            t[n / b * j + i] =
                op(t[n / b * (j - 1) + i], t[n / b * (j - 1) + i + (1 << (j - 1))]);
}
int small(int r, int sz = b) { return r - msb(mask[r] & ((1 << sz) - 1)); }
T query(int l, int r) {
    if (r - l + 1 <= b) return small(r, r - l + 1);
    int ans = op(small(l + b - 1), small(r));
    int x = l / b + 1, y = r / b - 1;
    if (x <= y) {
        int j = msb(y - x + 1);
        ans = op(ans, op(t[n / b * j + x], t[n / b * j + y - (1 << j) + 1]));
    }
    return ans;
}
};

struct LCA {
    SparseTable<int> st;
    int n;
    vi v, pos, dep;

    LCA(const vi2d& g, int root) : n(len(g)), pos(n) {
        dfs(root, 0, -1, g);
        st = SparseTable<int>(vector<int>(all(dep)));
    }

    void dfs(int i, int d, int p, const vi2d& g) {
        v.pb(len(dep)) = i, pos[i] = len(dep), dep.pb(d);
        for (auto j : g[i])
            if (j != p) {
                dfs(j, d + 1, i, g);
                v.pb(len(dep)) = i, dep.pb(d);
            }
    }

    int lca(int a, int b) {
        int l = min(pos[a], pos[b]);
        int r = max(pos[a], pos[b]);
        return v[st.query(l, r)];
    }

    int dist(int a, int b) {
        return dep[pos[a]] + dep[pos[b]] - 2 * dep[pos[lca(a, b)]];
    }
};

```

4.19 Tree Maximum Distance

Returns the maximum distance from every node to any other node in the tree. $O(6V) = O(V)$

```

pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
    // 0(V)
    // 0 indexed
    ll mostDistantNode = root;
    ll nodeDistance = 0;
    queue<pll> q;

```

```

    vector<char> vis(n);
    q.emplace(root, 0);
    vis[root] = true;
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (dist > nodeDistance) {
            nodeDistance = dist;
            mostDistantNode = node;
        }
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return {mostDistantNode, nodeDistance};
}

```

```

ll twoNodesDist(const vector<vll> &adj, ll n, ll a, ll b) {
    queue<pll> q;
    vector<char> vis(n);
    q.emplace(a, 0);
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (node == b) return dist;
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return -1;
}

```

```

tuple<ll, ll, ll> tree_diameter(const vector<vll> &adj, ll n) {
    // returns two points of the diameter and the diameter itself
    auto [node1, dist1] = mostDistantFrom(adj, n, 0); // 0(V)
    auto [node2, dist2] = mostDistantFrom(adj, n, node1); // 0(V)
    auto diameter = twoNodesDist(adj, n, node1, node2); // 0(V)
    return make_tuple(node1, node2, diameter);
}

```

```

vll everyDistanceFromNode(const vector<vll> &adj, ll n, ll root) {
    // Single Source Shortest Path, from a given root
    queue<pair<ll, ll>> q;
    vll ans(n, -1);
    ans[root] = 0;
    q.emplace(root, 0);
    while (!q.empty()) {
        auto [u, d] = q.front();
        q.pop();

        for (auto w : adj[u]) {
            if (ans[w] != -1) continue;

```



```

        ans[w] = d + 1;
        q.emplace(w, d + 1);
    }
}
return ans;
}

vll maxDistances(const vector<vll> &adj, ll n) {
    auto [node1, node2, diameter] = tree_diameter(adj, n);    // O(3V)
    auto distances1 = everyDistanceFromNode(adj, n, node1);    // O(V)
    auto distances2 = everyDistanceFromNode(adj, n, node2);    // O(V)
    vll ans(n);
    for (int i = 0; i < n; ++i)
        ans[i] = max(distances1[i], distances2[i]);    // O(V)
    return ans;
}

```

4.20 Maximum Flow (Edmonds-Karp)

Finds the **maximum flow** in a graph network, given the **source** s and the **sink** t .

When building the graph, if there is an edge (u, v) is necessary to also add the transposed edge (v, u) but only need to add the capacity $c(u, v)$.

Time: $O(V \cdot E^2)$

```

const ll oo = 1e17;

ll bfs(int s, int t, vi2d &g, vll2d &capacity, vi &parent) {
    fill(all(parent), -1);
    parent[s] = -2;
    queue<pair<ll, int>> q;
    q.push({oo, s});

    while (!q.empty()) {
        auto [flow, cur] = q.front();
        q.pop();

        for (auto next : g[cur]) {
            if (parent[next] == -1 and capacity[cur][next]) {
                parent[next] = cur;
                ll new_flow = min(flow, capacity[cur][next]);
                if (next == t) return new_flow;
                q.push({new_flow, next});
            }
        }
    }

    return 0ll;
}

ll maxflow(int s, int t, int n, vi2d &g, vll2d &capacity) {
    ll flow = 0;
    vi parent(n);
    ll new_flow;

    while ((new_flow = bfs(s, t, g, capacity, parent))) {
        flow += new_flow;
        int cur = t;
        while (cur != s) {

```

```

            int prev = parent[cur];
            capacity[prev][cur] -= new_flow;
            capacity[cur][prev] += new_flow;
            cur = prev;
        }
    }

    return flow;
}

```

4.21 Minimum Cut (unweighted)

Given the edges of a directed/undirected graph find the minum of edges that needs to be removed to make the sink t unreachable from the source s .

Time: $O(V \cdot E^2)$

```

const ll oo = 1e17;

ll bfs(int s, int t, vi2d &g, vll2d &capacity, vi &parent) {
    fill(all(parent), -1);
    parent[s] = -2;
    queue<pair<ll, int>> q;
    q.push({oo, s});

    while (!q.empty()) {
        auto [flow, cur] = q.front();
        q.pop();

        for (auto next : g[cur]) {
            if (parent[next] == -1 and capacity[cur][next]) {
                parent[next] = cur;
                ll new_flow = min(flow, capacity[cur][next]);
                if (next == t) return new_flow;
                q.push({new_flow, next});
            }
        }
    }

    return 0ll;
}

ll maxflow(int s, int t, int n, vi2d &g, vll2d &capacity) {
    ll flow = 0;
    vi parent(n);
    ll new_flow;

    while ((new_flow = bfs(s, t, g, capacity, parent))) {
        flow += new_flow;
        int cur = t;
        while (cur != s) {
            int prev = parent[cur];
            capacity[prev][cur] -= new_flow;
            capacity[cur][prev] += new_flow;
            cur = prev;
        }
    }

    return flow;
}

```

```

}
void dfs(int u, vi2d &g, vll2d &capacity, vc &visited) {
    visited[u] = true;

    for (auto v : g[u]) {
        if (capacity[u][v] > 0 and not visited[v]) {
            dfs(v, g, capacity, visited);
        }
    }
}

vii mincut(vii &edges, int s, int t, int n, bool directed = false) {
    vll2d capacity(n, vll(n));
    vi2d g(n);
    for (auto &[u, v] : edges) {
        g[u].eb(v);
        capacity[u][v] += 1;
        if (not directed) {
            g[v].eb(u);
            capacity[v][u] += 1;
        }
    }

    maxflow(0, n - 1, n, g, capacity);
    vc vis(n);
    dfs(0, g, capacity, vis);

    vii removed;
    for (auto &[u, v] : edges) {
        if ((vis[u] and not vis[v]) or (vis[v] and not vis[u]))
            removed.emplace_back(u, v);
    }

    return removed;
}

```

4.22 Small to Large

Answer queries of the form "How many vertices in the subtree of vertex v have property P ?"
 Build: $O(N)$, Query: $O(N \log N)$

```

struct SmallToLarge {
    vector<vector<int>> tree, vis_chlds;
    vector<int> sizes, values, ans;
    set<int> cnt;

    SmallToLarge(vector<vector<int>> &&g, vector<int> &&v)
        : tree(g), vis_chlds(g.size()), sizes(g.size()), values(v), ans(g.size())
        {
            update_sizes(0);
        }

    inline void add_value(int u) { cnt.insert(values[u]); }

    inline void remove_value(int u) { cnt.erase(values[u]); }

    inline void update_ans(int u) { ans[u] = (int)cnt.size(); }
}

```

```

void dfs(int u, int p = -1, bool keep = true) {
    int mx = -1;
    for (auto x : tree[u]) {
        if (x == p) continue;

        if (mx == -1 or sizes[mx] < sizes[x]) mx = x;
    }

    for (auto x : tree[u]) {
        if (x != p and x != mx) dfs(x, u, false);
    }

    if (mx != -1) {
        dfs(mx, u, true);
        swap(vis_chlds[u], vis_chlds[mx]);
    }

    vis_chlds[u].push_back(u);
    add_value(u);

    for (auto x : tree[u]) {
        if (x != p and x != mx) {
            for (auto y : vis_chlds[x]) {
                add_value(y);
                vis_chlds[u].push_back(y);
            }
        }
    }

    update_ans(u);

    if (!keep) {
        for (auto x : vis_chlds[u]) remove_value(x);
    }
}

void update_sizes(int u, int p = -1) {
    sizes[u] = 1;
    for (auto x : tree[u]) {
        if (x != p) {
            update_sizes(x, u);
            sizes[u] += sizes[x];
        }
    }
}
};

```

4.23 Sum every node distance

Given a **tree**, for each node i find the sum of distance from i to every other node.
 don't forget to set the tree as undirected, that's needed to choose an arbitrary root
 Time: $O(N)$

```

void getRoot(int u, int p, vi2d &g, vll &d, vll &cnt) {
    for (int i = 0; i < len(g[u]); i++) {
        int v = g[u][i];
        if (v == p) continue;
    }
}

```

```

    getRoot(v, u, g, d, cnt);
    d[u] += d[v] + cnt[v];
    cnt[u] += cnt[v];
}
}

void dfs(int u, int p, vi2d &g, vll &cnt, vll &ansd, int n) {
    for (int i = 0; i < len(g[u]); i++) {
        int v = g[u][i];
        if (v == p) continue;

        ansd[v] = ansd[u] - cnt[v] + (n - cnt[v]);
        dfs(v, u, g, cnt, ansd, n);
    }
}

vll fromToAll(vi2d &g, int n) {
    vll d(n);
    vll cnt(n, 1);
    getRoot(0, -1, g, d, cnt);

    vll ansdist(n);
    ansdist[0] = d[0];

    dfs(0, -1, g, cnt, ansdist, n);
    return ansdist;
}

```

4.24 Topological Sorting

Assumes that :

- vertices index $[0, n - 1]$
- is a DAG (else it returns an empty vector)

$O(V)$

```

enum class state { not_visited, processing, done };
bool dfs(const vector<vll> &adj, ll s, vector<state> &states, vll &order) {
    states[s] = state::processing;
    for (auto &v : adj[s]) {
        if (states[v] == state::not_visited) {
            if (not dfs(adj, v, states, order)) return false;
        } else if (states[v] == state::processing)
            return false;
    }
    states[s] = state::done;
    order.pb(s);
    return true;
}

vll topologicalSorting(const vector<vll> &adj) {
    ll n = len(adj);
    vll order;
    vector<state> states(n, state::not_visited);
    for (int i = 0; i < n; ++i) {
        if (states[i] == state::not_visited) {
            if (not dfs(adj, i, states, order)) return {};
        }
    }
    reverse(all(order));
}

```

```

    return order;
}

```

4.25 Tree Diameter

Finds the length of the diameter of the tree in $O(V)$, it's easy to recover the nodes at the point of the diameter .

```

pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
    // 0 indexed
    ll mostDistantNode = root;
    ll nodeDistance = 0;
    queue<pll> q;
    vector<char> vis(n);
    q.emplace(root, 0);
    vis[root] = true;
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (dist > nodeDistance) {
            nodeDistance = dist;
            mostDistantNode = node;
        }
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return {mostDistantNode, nodeDistance};
}

ll twoNodesDist(const vector<vll> &adj, ll n, ll a, ll b) {
    // 0 indexed
    queue<pll> q;
    vector<char> vis(n);
    q.emplace(a, 0);
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (node == b) {
            return dist;
        }
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return -1;
}

ll tree_diameter(const vector<vll> &adj, ll n) {
    // 0 indexed !!!
    auto [node1, dist1] = mostDistantFrom(adj, n, 0); // 0(V)
    auto [node2, dist2] = mostDistantFrom(adj, n, node1); // 0(V)
    auto diameter = twoNodesDist(adj, n, node1, node2); // 0(V)
    return diameter;
}

```

```
}

```

5 Math

5.1 GCD (with factorization)

$O(\sqrt{n})$ due to factorization.

```
ll gcd_with_factorization(ll a, ll b) {
    map<ll, ll> fa = factorization(a);
    map<ll, ll> fb = factorization(b);
    ll ans = 1;
    for (auto fai : fa) {
        ll k = min(fai.second, fb[fai.first]);
        while (k--) ans *= fai.first;
    }
    return ans;
}
```

5.2 GCD

```
ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
```

5.3 LCM (with factorization)

$O(\sqrt{n})$ due to factorization.

```
ll lcm_with_factorization(ll a, ll b) {
    map<ll, ll> fa = factorization(a);
    map<ll, ll> fb = factorization(b);
    ll ans = 1;
    for (auto fai : fa) {
        ll k = max(fai.second, fb[fai.first]);
        while (k--) ans *= fai.first;
    }
    return ans;
}
```

5.4 LCM

```
ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
ll lcm(ll a, ll b) { return a / gcd(a, b) * b; }
```

5.5 Arithmetic Progression Sum

- s : first term
- d : common difference
- n : number of terms

```
ll arithmeticProgressionSum(ll s, ll d, ll n) {
    return (s + (s + d * (n - 1))) * n / 2ll;
}
```

5.6 Binomial MOD

Precompute every factorial until $maxn$ ($O(maxn)$) allowing to answer the $\binom{n}{k}$ in $O(\log mod)$ time, due to the fastpow. Note that it needs $O(maxn)$ in memory.

```
const ll MOD = 1e9 + 7;
const ll maxn = 2 * 1e6;
vll fats(maxn + 1, -1);
void precompute() {
    fats[0] = 1;
    for (ll i = 1; i <= maxn; i++) {
        fats[i] = (fats[i - 1] * i) % MOD;
    }
}

ll fpow(ll a, ll n, ll mod = LLONG_MAX) {
    if (n == 0ll) return 1ll;
    if (n == 1ll) return a;
    ll x = fpow(a, n / 2ll, mod) % mod;
    return ((x * x) % mod * (n & 1ll ? a : 1ll)) % mod;
}

ll binommod(ll n, ll k) {
    ll upper = fats[n];
    ll lower = (fats[k] * fats[n - k]) % MOD;
    return (upper * fpow(lower, MOD - 2ll, MOD)) % MOD;
}
```

5.7 Binomial

$O(nm)$ time, $O(m)$ space

Equal to n choose k

```
ll binom(ll n, ll k) {
    if (k > n) return 0;
    vll dp(k + 1, 0);
    dp[0] = 1;
    for (ll i = 1; i <= n; i++)
        for (ll j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
    return dp[k];
}
```

5.8 Euler phi $\varphi(n)$ (in range)

Computes the number of positive integers less than n that are coprimes with n , in the range $[1, n]$, in $O(N \log N)$.

```
const int MAX = 1e6;
vi range_phi(int n) {
    bitset<MAX> sieve;
    vi phi(n + 1);

    iota(phi.begin(), phi.end(), 0);
    sieve.set();

    for (int p = 2; p <= n; p += 2) phi[p] /= 2;
    for (int p = 3; p <= n; p += 2) {
        if (sieve[p]) {
```

```

    for (int j = p; j <= n; j += p) {
        sieve[j] = false;
        phi[j] /= p;
        phi[j] *= (p - 1);
    }
}

return phi;
}

```

5.9 Euler phi $\varphi(n)$

Computes the number of positive integers less than n that are coprimes with n , in $O(\sqrt{N})$.

```

int phi(int n) {
    if (n == 1) return 1;

    auto fs = factorization(n); // a vector of pair or a map
    auto res = n;

    for (auto [p, k] : fs) {
        res /= p;
        res *= (p - 1);
    }

    return res;
}

```

5.10 Factorial Factorization

Computes the factorization of $n!$ in $\pi(N) * \log n$

```

// O(logN)
ll E(ll n, ll p) {
    ll k = 0, b = p;
    while (b <= n) {
        k += n / b;
        b *= p;
    }
    return k;
}

// O(pi(N)*logN)
map<ll, ll> factorial_factorization(ll n, const vll &primes) {
    map<ll, ll> fs;
    for (const auto &p : primes) {
        if (p > n) break;
        fs[p] = E(n, p);
    }
    return fs;
}

```

5.11 Factorial

```

const ll MAX = 18;
vll fv(MAX, -1);
ll factorial(ll n) {
    if (fv[n] != -1) return fv[n];
    if (n == 0) return 1;
    return n * factorial(n - 1);
}

```

5.12 Factorization (Pollard Rho)

Factorizes a number into its prime factors in $O(n^{(\frac{1}{4})} * \log(n))$.

```

ll mul(ll a, ll b, ll m) {
    ll ret = a * b - (ll)((ld)1 / m * a * b + 0.5) * m;
    return ret < 0 ? ret + m : ret;
}

ll pow(ll a, ll b, ll m) {
    ll ans = 1;
    for (; b > 0; b /= 2ll, a = mul(a, a, m)) {
        if (b % 2ll == 1) ans = mul(ans, a, m);
    }
    return ans;
}

bool prime(ll n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;

    ll r = __builtin_ctzll(n - 1), d = n >> r;
    for (int a : {2, 325, 9375, 28178, 450775, 9780504, 795265022}) {
        ll x = pow(a, d, n);
        if (x == 1 or x == n - 1 or a % n == 0) continue;

        for (int j = 0; j < r - 1; j++) {
            x = mul(x, x, n);
            if (x == n - 1) break;
        }
        if (x != n - 1) return 0;
    }
    return 1;
}

ll rho(ll n) {
    if (n == 1 or prime(n)) return n;
    auto f = [n](ll x) { return mul(x, x, n) + 1; };

    ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
    while (t % 40 != 0 or gcd(prd, n) == 1) {
        if (x == y) x = ++x0, y = f(x);
        q = mul(prd, abs(x - y), n);
        if (q != 0) prd = q;
        x = f(x), y = f(f(y)), t++;
    }
    return gcd(prd, n);
}

vll fact(ll n) {

```

```

if (n == 1) return {};
if (prime(n)) return {n};
ll d = rho(n);
vll l = fact(d), r = fact(n / d);
l.insert(l.end(), r.begin(), r.end());
return l;
}

```

5.13 Factorization

Computes the factorization of n in $O(\sqrt{n})$.

```

map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;
        for (; n % i == 0; count++, n /= i)
            ;
        if (count) ans[i] = count;
    }
    if (n > 1) ans[n]++;
    return ans;
}

```

5.14 Fast Fourier Transform

```

template <bool invert = false>
void fft(vector<complex<double>>& xs) {
    int N = (int)xs.size();

    if (N == 1) return;

    vector<complex<double>> es(N / 2), os(N / 2);

    for (int i = 0; i < N / 2; ++i) es[i] = xs[2 * i];

    for (int i = 0; i < N / 2; ++i) os[i] = xs[2 * i + 1];

    fft<invert>(es);
    fft<invert>(os);

    auto signal = (invert ? 1 : -1);
    auto theta = 2 * signal * acos(-1) / N;
    complex<double> S{1}, S1{cos(theta), sin(theta)};

    for (int i = 0; i < N / 2; ++i) {
        xs[i] = (es[i] + S * os[i]);
        xs[i] /= (invert ? 2 : 1);

        xs[i + N / 2] = (es[i] - S * os[i]);
        xs[i + N / 2] /= (invert ? 2 : 1);

        S *= S1;
    }
}

```

5.15 Fast pow

Computes a^n in $O(\log N)$.

```

ll fpow(ll a, int n, ll mod = LLONG_MAX) {
    if (n == 0) return 1;
    if (n == 1) return a;
    ll x = fpow(a, n / 2, mod) % mod;
    return ((x * x) % mod * (n & 1 ? a : 1)) % mod;
}

```

5.16 Gauss Elimination

```

template <size_t Dim>
struct GaussianElimination {
    vector<ll> basis;
    size_t size;

    GaussianElimination() : basis(Dim + 1), size(0) {}

    void insert(ll x) {
        for (ll i = Dim; i >= 0; i--) {
            if ((x & 1ll << i) == 0) continue;

            if (!basis[i]) {
                basis[i] = x;
                size++;
                break;
            }

            x ^= basis[i];
        }
    }

    void normalize() {
        for (ll i = Dim; i >= 0; i--)
            for (ll j = i - 1; j >= 0; j--)
                if (basis[i] & 1ll << j) basis[i] ^= basis[j];
    }

    bool check(ll x) {
        for (ll i = Dim; i >= 0; i--) {
            if ((x & 1ll << i) == 0) continue;

            if (!basis[i]) return false;

            x ^= basis[i];
        }

        return true;
    }

    auto operator[](ll k) { return at(k); }

    ll at(ll k) {
        ll ans = 0;
        ll total = 1ll << size;
        for (ll i = Dim; ~i; i--) {

```

```

    if (!basis[i]) continue;

    ll mid = total >> 1ll;
    if ((mid < k and (ans & 1ll << i) == 0) ||
        (k <= mid and (ans & 1ll << i)))
        ans ^= basis[i];

    if (mid < k) k -= mid;

    total >>= 1ll;
}
return ans;
}

ll at_normalized(ll k) {
    ll ans = 0;
    k--;
    for (size_t i = 0; i <= Dim; i++) {
        if (!basis[i]) continue;
        if (k & 1) ans ^= basis[i];
        k >>= 1;
    }
    return ans;
}
};

```

5.17 Integer Mod

```

const ll INF = 1e18;
const ll mod = 998244353;
template <ll MOD = mod>
struct Modular {
    ll value;
    static const ll MOD_value = MOD;

    Modular(ll v = 0) {
        value = v % MOD;
        if (value < 0) value += MOD;
    }

    Modular(ll a, ll b) : value(0) {
        *this += a;
        *this /= b;
    }

    Modular& operator+=(Modular const& b) {
        value += b.value;
        if (value >= MOD) value -= MOD;
        return *this;
    }

    Modular& operator--(Modular const& b) {
        value -= b.value;
        if (value < 0) value += MOD;
        return *this;
    }

    Modular& operator*=(Modular const& b) {
        value = (ll)value * b.value % MOD;
        return *this;
    }
}

```

```

friend Modular mexp(Modular a, ll e) {
    Modular res = 1;
    while (e) {
        if (e & 1) res *= a;
        a *= a;
        e >>= 1;
    }
    return res;
}

friend Modular inverse(Modular a) { return mexp(a, MOD - 2); }

Modular& operator/=(Modular const& b) { return *this *= inverse(b); }
friend Modular operator+(Modular a, Modular const b) { return a += b; }
Modular operator++(int) { return this->value = (this->value + 1) % MOD; }
Modular operator++() { return this->value = (this->value + 1) % MOD; }
friend Modular operator-(Modular a, Modular const b) { return a -= b; }
friend Modular operator--(Modular const a) { return 0 - a; }
Modular operator--(int) {
    return this->value = (this->value - 1 + MOD) % MOD;
}

Modular operator--() { return this->value = (this->value - 1 + MOD) % MOD; }
friend Modular operator*(Modular a, Modular const b) { return a *= b; }
friend Modular operator/(Modular a, Modular const b) { return a /= b; }
friend std::ostream& operator<<(std::ostream& os, Modular const& a) {
    return os << a.value;
}

friend bool operator==(Modular const& a, Modular const& b) {
    return a.value == b.value;
}

friend bool operator!=(Modular const& a, Modular const& b) {
    return a.value != b.value;
}
};

```

5.18 Is prime

$O(\sqrt{N})$

```

bool isprime(ll n) {
    if (n < 2) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    for (ll i = 3; i * i < n; i += 2)
        if (n % i == 0) return false;
    return true;
}

```

5.19 Number of Divisors $\tau(n)$

Find the total of divisors of N in $O(\sqrt{N})$

```

ll number_of_divisors(ll n) {
    ll res = 0;

    for (ll d = 1; d * d <= n; ++d) {
        if (n % d == 0) res += (d == n / d ? 1 : 2);
    }
}

```

```

}

return res;
}

```

5.20 Power Sum

Calculates $K^0 + K^1 + \dots + K^n$

```

ll powersum(ll n, ll k) { return (fastpow(n, k + 1) - 1) / (k - 1); }

```

5.21 Sieve list primes

List every prime until MAXN, $O(N \log N)$ in time and $O(MAXN)$ in memory.

```

const ll MAXN = 1e5;
vll list_primes(ll n) {
    vll ps;
    bitset<MAXN> sieve;
    sieve.set();
    sieve.reset(1);
    for (ll i = 2; i <= n; ++i) {
        if (sieve[i]) ps.push_back(i);
        for (ll j = i * 2; j <= n; j += i) {
            sieve.reset(j);
        }
    }
    return ps;
}

```

5.22 Sum of Divisors $\sigma(n)$

Computes the sum of each divisor of n in $O(\sqrt{n})$.

```

ll sum_of_divisors(long long n) {
    ll res = 0;

    for (ll d = 1; d * d <= n; ++d) {
        if (n % d == 0) {
            ll k = n / d;

            res += (d == k ? d : d + k);
        }
    }

    return res;
}

```

6 Problems

6.1 Hanoi Tower

Let T_n be the total of moves to solve a hanoi tower, we know that $T_n \geq 2 \cdot T_{n-1} + 1$, for $n > 0$, and $T_0 = 0$. By induction it's easy to see that $T_n = 2^n - 1$, for $n > 0$. The following algorithm finds the necessary steps to solve the game for 3 stacks and n disks.

```

void move(int a, int b) { cout << a << ' ' << b << endl; }
void solve(int n, int s, int e) {
    if (n == 0) return;
    if (n == 1) {
        move(s, e);
        return;
    }
    solve(n - 1, s, 6 - s - e);
    move(s, e);
    solve(n - 1, 6 - s - e, e);
}

```

7 Searching

7.1 Meet in the middle

Answers the query how many subsets of the vector xs have sum equal x .

Time: $O(N \cdot 2^{\frac{N}{2}})$

```

vll get_subset_sums(int l, int r, vll &a) {
    int len = r - l + 1;
    vll res;

    for (int i = 0; i < (1 << len); i++) {
        ll sum = 0;
        for (int j = 0; j < len; j++) {
            if (i & (1 << j)) {
                sum += a[l + j];
            }
        }
        res.push_back(sum);
    }
    return res;
};

ll count(vll &xs, ll x) {
    int n = len(xs);
    vll left = get_subset_sums(0, n / 2 - 1, xs);
    vll right = get_subset_sums(n / 2, n - 1, xs);
    sort(all(left));
    sort(all(right));
    ll ans = 0;
    for (ll i : left) {
        auto start_index =
            lower_bound(right.begin(), right.end(), x - i) - right.begin();
        auto end_index =
            upper_bound(right.begin(), right.end(), x - i) - right.begin();
        ans += end_index - start_index;
    }
    return ans;
}

```


7.2 Ternary Search Recursive

```
const double eps = 1e-6;

// IT MUST BE AN UNIMODAL FUNCTION
double f(int x) { return x * x + 2 * x + 4; }

double ternary_search(double l, double r) {
    if (fabs(f(l) - f(r)) < eps) return f((l + (r - l) / 2.0));

    auto third = (r - l) / 3.0;
    auto m1 = l + third;
    auto m2 = r - third;

    // change the signal to find the maximum point.
    return m1 < m2 ? ternary_search(m1, r) : ternary_search(l, m2);
}
```

8 Strings

8.1 Count Distinct Anagrams

```
const ll MOD = 1e9 + 7;
const int maxn = 1e6;
vll fs(maxn + 1);
void precompute() {
    fs[0] = 1;
    for (ll i = 1; i <= maxn; i++) {
        fs[i] = (fs[i - 1] * i) % MOD;
    }
}

ll fpow(ll a, int n, ll mod = LLONG_MAX) {
    if (n == 0) return 1;
    if (n == 1) return a;
    ll x = fpow(a, n / 2, mod) % mod;
    return ((x * x) % mod * (n & 1 ? a : 1)) % mod;
}

ll distinctAnagrams(const string &s) {
    precompute();
    vi hist('z' - 'a' + 1, 0);
    for (auto &c : s) hist[c - 'a']++;
    ll ans = fs[len(s)];
    for (auto &q : hist) {
        ans = (ans * fpow(fs[q], MOD - 2, MOD)) % MOD;
    }
    return ans;
}
```

8.2 Hash Range Query

```
struct Hash {
    const ll P = 31;
    const ll mod = 1e9 + 7;
    string s;
    int n;
```

```
vll h, hi, p;
Hash() {}
Hash(string s) : s(s), n(s.size()), h(n), hi(n), p(n) {
    for (int i = 0; i < n; i++) p[i] = (i ? P * p[i - 1] : 1) % mod;
    for (int i = 0; i < n; i++) h[i] = (s[i] + (i ? h[i - 1] : 0) * P) % mod;
    for (int i = n - 1; i >= 0; i--)
        hi[i] = (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % mod;
}

ll query(int l, int r) {
    ll hash = (h[r] - (l ? h[l - 1] * p[r - l + 1] % mod : 0));
    return hash < 0 ? hash + mod : hash;
}

ll query_inv(int l, int r) {
    ll hash = (hi[l] - (r + 1 < n ? hi[r + 1] * p[r - l + 1] % mod : 0));
    return hash < 0 ? hash + mod : hash;
}
};
```

8.3 Longest Palindrome

```
string longest_palindrome(const string &s) {
    int n = (int)s.size();
    vector<array<int, 2>> dp(n);

    pii odd(0, -1), even(0, -1);
    pii ans;
    for (int i = 0; i < n; i++) {
        int k = 0;
        if (i > odd.second)
            k = 1;
        else
            k = min(dp[odd.first + odd.second - i][0], odd.second - i + 1);
        while (i - k >= 0 and i + k < n and s[i - k] == s[i + k]) k++;
        dp[i][0] = k--;
        if (i + k > odd.second) odd = {i - k, i + k};
        if (2 * dp[i][0] - 1 > ans.second) ans = {i - k, 2 * dp[i][0] - 1};

        k = 0;
        if (i <= even.second)
            k = min(dp[even.first + even.second - i + 1][1], even.second - i + 1);
        while (i - k - 1 >= 0 and i + k < n and s[i - k - 1] == s[i + k]) k++;
        dp[i][1] = k--;
        if (i + k > even.second) even = {i - k - 1, i + k};
        if (2 * dp[i][1] > ans.second) ans = {i - k - 1, 2 * dp[i][1]};
    }
    return s.substr(ans.first, ans.second);
}
```

8.4 Rabin Karp

```
size_t rabin_karp(const string &s, const string &p) {
    if (s.size() < p.size()) return 0;

    auto n = s.size(), m = p.size();
    const ll p1 = 31, p2 = 29, q1 = 1e9 + 7, q2 = 1e9 + 9;
    const ll p1_1 = fpow(p1, q1 - 2, q1), p1_2 = fpow(p1, m - 1, q1);
    const ll p2_1 = fpow(p2, q2 - 2, q2), p2_2 = fpow(p2, m - 1, q2);
```

```

pair<ll, ll> hs, hp;
for (int i = (int)m - 1; ~i; --i) {
    hs.first = (hs.first * p1) % q1;
    hs.first = (hs.first + (s[i] - 'a' + 1)) % q1;
    hs.second = (hs.second * p2) % q2;
    hs.second = (hs.second + (s[i] - 'a' + 1)) % q2;

    hp.first = (hp.first * p1) % q1;
    hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
    hp.second = (hp.second * p2) % q2;
    hp.second = (hp.second + (p[i] - 'a' + 1)) % q2;
}

```

```

size_t occ = 0;
for (size_t i = 0; i < n - m; i++) {
    occ += (hs == hp);

    int fi = s[i] - 'a' + 1;
    int fm = s[i + m] - 'a' + 1;

    hs.first = (hs.first - fi + q1) % q1;
    hs.first = (hs.first * p1_1) % q1;
    hs.first = (hs.first + fm * p1_2) % q1;
    hs.second = (hs.second - fi + q2) % q2;
    hs.second = (hs.second * p2_1) % q2;
    hs.second = (hs.second + fm * p2_2) % q2;
}
occ += hs == hp;

```

```

return occ;
}

```

8.5 String Psum

```

struct strPsum {
    ll n;
    ll k;
    vector<vll> psum;
    strPsum(const string &s) : n(s.size()), k(100), psum(k, vll(n + 1)) {
        for (ll i = 1; i <= n; ++i) {
            for (ll j = 0; j < k; ++j) {
                psum[j][i] = psum[j][i - 1];
            }
            psum[s[i - 1]][i]++;
        }
    }

    ll qtd(ll l, ll r, char c) { // [0,n-1]
        return psum[c][r + 1] - psum[c][l];
    }
}

```

8.6 Suffix Automaton (complete)

```

struct state {
    int len, link, cnt, firstpos;
    // this can be optimized using a vector with the alphabet size
    map<char, int> next;

```

```

    vi inv_link;
};

struct SuffixAutomaton {
    vector<state> st;
    int sz = 0;
    int last;
    vc cloned;

    SuffixAutomaton(const string &s, int maxlen)
        : st(maxlen * 2), cloned(maxlen * 2) {
        st[0].len = 0;
        st[0].link = -1;
        sz++;
        last = 0;
        for (auto &c : s) add_char(c);

        // precompute for count occurrences
        for (int i = 1; i < sz; i++) {
            st[i].cnt = !cloned[i];
        }
        vector<pair<state, int>> aux;
        for (int i = 0; i < sz; i++) {
            aux.push_back({st[i], i});
        }

        sort(all(aux), [](const pair<state, int> &a, const pair<state, int> &b) {
            return a.fst.len > b.fst.len;
        });

        for (auto &[stt, id] : aux) {
            if (stt.link != -1) {
                st[stt.link].cnt += st[id].cnt;
            }
        }

        // for find every occurende position
        for (int v = 1; v < sz; v++) {
            st[st[v].link].inv_link.push_back(v);
        }
    }

    void add_char(char c) {
        int cur = sz++;
        st[cur].len = st[last].len + 1;
        st[cur].firstpos = st[cur].len - 1;
        int p = last;
        // follow the suffix link until find a transition to c
        while (p != -1 and !st[p].next.count(c)) {
            st[p].next[c] = cur;
            p = st[p].link;
        }
        // there was no transition to c so create and leave
        if (p == -1) {
            st[cur].link = 0;
            last = cur;
            return;
        }
    }
}

```

```

int q = st[p].next[c];
if (st[p].len + 1 == st[q].len) {
    st[cur].link = q;
} else {
    int clone = sz++;
    cloned[clone] = true;
    st[clone].len = st[p].len + 1;
    st[clone].next = st[q].next;
    st[clone].link = st[q].link;
    st[clone].firstpos = st[q].firstpos;
    while (p != -1 and st[p].next[c] == q) {
        st[p].next[c] = clone;
        p = st[p].link;
    }
    st[q].link = st[cur].link = clone;
}
}
last = cur;
}

bool checkOccurrence(const string &t) { // O(len(t))
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return false;
        cur = st[cur].next[c];
    }
    return true;
}

ll totalSubstrings() { // distinct, O(len(s))
    ll tot = 0;
    for (int i = 1; i < sz; i++) {
        tot += st[i].len - st[st[i].link].len;
    }
    return tot;
}

// count occurrences of a given string t
int countOccurrences(const string &t) {
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return 0;
        cur = st[cur].next[c];
    }
    return st[cur].cnt;
}

// find the first index where t appears a substring O(len(t))
int firstOccurrence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return -1;
        cur = st[cur].next[c];
    }
    return st[cur].firstpos - len(t) + 1;
}

vi everyOccurrence(const string &t) {
    int cur = 0;
    for (auto c : t) {

```

```

        if (!st[cur].next.count(c)) return {};
        cur = st[cur].next[c];
    }
    vi ans;
    getEveryOccurrence(cur, len(t), ans);
    return ans;
}

void getEveryOccurrence(int v, int P_length, vi &ans) {
    if (!cloned[v]) ans.pb(st[v].firstpos - P_length + 1);
    for (int u : st[v].inv_link) getEveryOccurrence(u, P_length, ans);
}
};

```

8.7 Z-function get occurrence positions

$O(len(s) + len(p))$

```

vi getOccPos(string &s, string &p) {
    // Z-function
    char delim = '#';
    string t{p + delim + s};
    vi zs(len(t));

    int l = 0, r = 0;
    for (int i = 1; i < len(t); i++) {
        if (i <= r) zs[i] = min(zs[i - l], r - i + 1);
        while (zs[i] + i < len(t) and t[zs[i]] == t[i + zs[i]]) zs[i]++;
        if (r < i + zs[i] - 1) l = i, r = i + zs[i] - 1;
    }

    // Iterate over the results of Z-function to get ranges
    vi ans;
    int start = len(p) + 1 + 1 - 1;
    for (int i = start; i < len(zs); i++) {
        if (zs[i] == len(p)) {
            int l = i - start;
            ans.emplace_back(l);
        }
    }
    return ans;
}

```

9 Settings and macros

9.1 short-macro.cpp

```

#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(false); \
    cin.tie(0); \
    cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using pii = pair<int, int>;

```

```
#define all(a) a.begin(), a.end()
```

```
void run() {}
int32_t main(void) {
    fastio;
    int t;
    t = 1;
    // cin >> t;
    while (t--> 0) run();
}
```

9.2 .vimrc

```
set ts=4 sw=4 sta nu rnu sc cindent
set bg=dark ruler clipboard=unnamed,unnamedplus, timeoutlen=100
colorscheme default
```

```
nnoremap <C-j> :botright belowright term bash <CR>
syntax on
```

9.3 degug.cpp

```
#include <bits/stdc++.h>
using namespace std;
/***** Debug Code *****/
template <typename T>
concept Printable = requires(T t) {
    { std::cout << t } -> std::same_as<std::ostream &>;
};
template <Printable T>
void __print(const T &x) {
    cerr << x;
}
template <size_t T>
void __print(const bitset<T> &x) {
    cerr << x;
}
template <typename A, typename B>
void __print(const pair<A, B> &p);
template <typename... A>
void __print(const tuple<A...> &t);
template <typename T>
void __print(stack<T> s);
template <typename T>
void __print(queue<T> q);
template <typename T, typename... U>
void __print(priority_queue<T, U...> q);
template <typename A>
void __print(const A &x) {
    bool first = true;
    cerr << '{';
    for (const auto &i : x) {
        cerr << (first ? "" : ", "), __print(i);
        first = false;
    }
    cerr << '}';
}
```

```
template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '(';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << ')';
}
template <typename... A>
void __print(const tuple<A...> &t) {
    bool first = true;
    cerr << '(';
    apply(
        [&first](const auto &...args) {
            ((cerr << (first ? "" : ", "), __print(args), first = false), ...);
        },
        t);
    cerr << ')';
}
template <typename T>
void __print(stack<T> s) {
    vector<T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    }
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
}
template <typename T>
void __print(queue<T> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.front();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
template <typename T, typename... U>
void __print(priority_queue<T, U...> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.top();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
void _print() { cerr << "]\n"; }
template <typename Head, typename... Tail>
void _print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";
    _print(T...);
}
```

```

#define dbg(x...) \
    cerr << "[" << #x << "]" = ["; \
    _print(x)

```

9.4 .bashrc

```

cpp() {
    echo ">> COMPILING <<" 1>&2
    g++ -std=c++17 \
        -O2 \
        -g \
        -g3 \
        -Wextra \
        -Wshadow \
        -Wformat=2 \
        -Wconversion \
        -fsanitize=address,undefined \
        -fno-sanitize-recover \
        -Wfatal-errors \
        $1

    if [ $? -ne 0 ]; then
        echo ">> FAILED <<" 1>&2
        return 1
    fi
    echo ">> DONE <<" 1>&2
    time ./a.out ${@:2}
}

prepare() {
    for i in {a..z}
    do
        cp macro.cpp $i.cpp
        touch $i.py
    done

    for i in {1..10}
    do
        touch in${i}
        touch out${i}
        touch ans${i}
    done
}

```

```

}

```

9.5 macro.cpp

```

#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(false); \
    cin.tie(0); \
    cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using ull = unsigned long long;
using ld = long double;
using vll = vector<ll>;
using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
using vii = vector<pii>;
using vc = vector<char>;
#define all(a) a.begin(), a.end()
#define snd second
#define fst first
#define pb(___x) push_back(___x)
#define mp(___a, ___b) make_pair(___a, ___b)
#define eb(___x) emplace_back(___x)

const ll oo = 1e18;

void run() {}
int32_t main(void) {
    fastio;
    int t;
    t = 1;
    // cin >> t;
    while (t--) run();
}

```