

Contents

1 Data structures

1.1	Segtree Lazy (Atcoder)	2
1.2	Bitree 2D	3
1.3	Bitree	4
1.4	Disjoint Sparse Table	4
1.5	Dsu	5
1.6	Ordered Set	5
1.7	Prefix Sum 2D	5
1.8	SegTree Range Sum Query Range PA sum/set Update	6
1.9	SegTree Point Update (dynamic function)	7
1.10	Segtree Range Max Query Point Max Assign Update (dynamic)	7
1.11	Segtree Range Max Query Range Max Update	8
1.12	SegTree Range Min Query Point Assign Update	9
1.13	Segtree Range Sum Query Point Sum Update (dynamic)	10
1.14	SegTree Range Xor query Point Assign Update	11
1.15	SegTree Range Min Query Range Sum Update	11
1.16	SegTree Range Sum Query Range Sum Update	12
1.17	Sparse Table	13

2 Dynamic programming

2.1	Binary Knapsack (bottom up)	13
2.2	Binary Knapsack (top down)	14
2.3	Digits	14
2.4	Edit Distance	15
2.5	Kadane	15
2.6	Longest Increasing Subsequence (LIS)	15
2.7	Money Sum (Bottom Up)	16
2.8	Travelling Salesman Problem	16

3 Geometry

3.1	Convex Hull	16
3.2	Determinant	17
3.3	Equals	17
3.4	Line	17
3.5	Point Struct And Utils (2d)	17
3.6	Polygon Lattice Points (Pick's Theorem)	18
3.7	Segment	18
3.8	Template Line	19
3.9	Template Point	19
3.10	Template Segment	19

4 Graphs

4.1	2 SAT	20
4.2	Cycle Distances	21
4.3	SCC (struct)	21
4.4	Array Cycle	22
4.5	Bellman-Ford (find negative cycle)	23
4.6	Bellman Ford	23
4.7	BFS 01	24
4.8	Block Cut Tree	24
4.9	Check Bipartite	25
4.10	Dijkstra (k Shortest Paths)	25
4.11	Dijkstra	26
4.12	Disjoint Edges Path (Maxflow)	26
4.13	Euler Path (directed)	27
4.14	Euler Path (undirected)	27
4.15	Find Articulation/Cut Points	28
4.16	Find Bridges (online)	29
4.17	Find Bridges	30
4.18	Find Centroid	31
4.19	Floyd Warshall	31
4.20	Graph Cycle (directed)	31
4.21	Graph Cycle (undirected)	32
4.22	Heavy Light Decomposition	32
4.23	Kruskal	33
4.24	Maximum Flow (Edmonds-Karp)	34
4.25	Minimum Cost Flow	34
4.26	Minimum Cut (unweighted)	36
4.27	Prim (MST)	37
4.28	Sum every node distance	38
4.29	Topological Sorting (Tarjan)	38

5 Math

5.1	GCD (with factorization)	38
5.2	GCD	39
5.3	LCM (with factorization)	39
5.4	LCM	39
5.5	Arithmetic Progression Sum	39
5.6	Binomial MOD	39
5.7	Binomial	39
5.8	Euler phi $\varphi(n)$ (in range)	39
5.9	Euler phi $\varphi(n)$	40
5.10	Factorial Factorization	40
5.11	Factorial	40
5.12	Factorization (Pollard Rho)	40
5.13	Factorization	41

5.14	Fast Fourier Transform	41	9.3	Hash Range Query	54
5.15	Fast pow	41	9.4	K-th digit in digit string	55
5.16	Gauss Elimination	42	9.5	Longest Palindrome Substring (Manacher)	55
5.17	Integer Mod	42	9.6	Longest Palindrome	56
5.18	Is prime	43	9.7	Rabin Karp	56
5.19	N Choose K (elements)	44	9.8	String Psum	57
5.20	Number Of Divisors (sieve)	44	9.9	Suffix Automaton (complete)	57
5.21	Number of Divisors $\tau(n)$	44	9.10	Trie	58
5.22	Power Sum	44	9.11	Z-function get occurence positions	59
5.23	Sieve list primes	44			
5.24	Sum of Divisors $\sigma(n)$	44	10	Trees	59
6	Primitives	45	10.1	Binary Lifting (struct)	59
6.1	Bigint	45	10.2	Binary Lifting	60
6.2	Integer Mod	49	10.3	Isomorphism	60
6.3	Matrix	50	10.4	Lowest Common Ancestor	61
7	Problems	52	10.5	Tree Maximum Distance	62
7.1	Hanoi Tower	52	10.6	Small to Large	63
8	Searching	53	10.7	Tree Diameter	64
8.1	Meet in the middle	53	10.8	Tree Flatten	64
8.2	Ternary Search Recursive	53	11	Settings and macros	64
9	Strings	53	11.1	.vimrc	64
9.1	Count Distinct Anagrams	53	11.2	macro.cpp	64
9.2	Double Hash Range Query	54	11.3	.bashrc	65
			11.4	short-macro.cpp	65
			11.5	debug.cpp	66

1 Data structures

1.1 Segtree Lazy (Atcoder)

```
struct Node {
    // need an empty constructor with the neutral node
    Node() : {}
};

struct Lazy {
    // need an empty constructor with the neutral lazy
    Lazy() : {}
};

// how to merge two nodes
Node op(Node a, Node b) {}

// how to apply the lazy into a node
Node mapping(Lazy a, Node b, int, int) {}

// how to merge two lazy
Lazy comp(Lazy a, Lazy b) {}

template <typename T, auto op, typename L, auto mapping,
          auto composition>
struct SegTreeLazy {
    static_assert(
        is_convertible_v<decltype(op), function<T(T, T)>>,
        "op must be a function T(T, T)");
    static_assert(
        is_convertible_v<decltype(mapping),
        function<T(L, T, int, int)>>,
        "mapping must be a function T(L, T, int, int)");
    static_assert(is_convertible_v<decltype(composition),
        function<L(L, L)>>,
        "composition must be a function L(L, L)");

    int N, size, height;
    const T eT;
    const L eL;
    vector<T> d;
    vector<L> lz;

    SegTreeLazy(const T &eT_ = T(), const L &eL_ = L())
```

```
        : SegTreeLazy(0, eT_, eL_) {}
    explicit SegTreeLazy(int n, const T &eT_ = T(),
        const L &eL_ = L())
        : SegTreeLazy(vector<T>(n, eT_), eT_, eL_) {}
    explicit SegTreeLazy(const vector<T> &v,
        const T &eT_ = T(),
        const L &eL_ = L())
        : N(int(v.size())), eT(eT_), eL(eL_) {
        size = 1;
        height = 0;
        while (size < N) size <= 1, height++;
        d = vector<T>(2 * size, eT);
        lz = vector<L>(size, eL);
        for (int i = 0; i < N; i++) d[size + i] = v[i];
        for (int i = size - 1; i >= 1; i--) {
            update(i);
        }
    }

    void set(int p, T x) {
        assert(0 <= p && p < N);
        p += size;
        for (int i = height; i >= 1; i--) push(p >> i);
        d[p] = x;
        for (int i = 1; i <= height; i++) update(p >> i);
    }

    T get(int p) {
        assert(0 <= p && p < N);
        p += size;
        for (int i = height; i >= 1; i--) push(p >> i);
        return d[p];
    }

    T query(int l, int r) {
        assert(0 <= l && l <= r && r < N);

        l += size;
        r += size;

        for (int i = height; i >= 1; i--) {
            if (((l >> i) << i) != l) push(l >> i);
            if (((r + 1) >> i) << i) != (r + 1)) push(r >> i);
        }
    }
};
```

```

T sml = eT, smr = eT;
while (l <= r) {
    if (l & 1) sml = op(sml, d[l++]);
    if (!(r & 1)) smr = op(d[r--], smr);
    l >>= 1;
    r >>= 1;
}

return op(sml, smr);
}

T query_all() { return d[1]; }

void update(int p, L f) {
    assert(0 <= p && p < N);
    p += size;
    for (int i = height; i >= 1; i--) push(p >> i);
    d[p] = mapping(f, d[p]);
    for (int i = 1; i <= height; i++) update(p >> i);
}

void update(int l, int r, L f) {
    assert(0 <= l && l <= r && r < N);

    l += size;
    r += size;

    for (int i = height; i >= 1; i--) {
        if (((l >> i) << i) != l) push(l >> i);
        if (((r + 1) >> i) << i) != (r + 1)) push(r >> i);
    }

    {
        int l2 = l, r2 = r;
        while (l <= r) {
            if (l & 1) all_apply(l++, f);
            if (!(r & 1)) all_apply(r--, f);
            l >>= 1;
            r >>= 1;
        }
        l = l2;
        r = r2;
    }

    for (int i = 1; i <= height; i++) {
        if (((l >> i) << i) != l) update(l >> i);
    }
}

```

```

        if (((r + 1) >> i) << i) != (r + 1)) update(r >> i);
    }
}

pair<int, int> node_range(int k) const {
    int remain = height;
    for (int kk = k; kk >>= 1; --remain)
        ;
    int fst = k << remain;
    int lst = min(fst + (1 << remain) - 1, size + N - 1);
    return {fst - size, lst - size};
}

private:
void update(int k) { d[k] = op(d[2 * k], d[2 * k + 1]); }
void all_apply(int k, L f) {
    auto [fst, lst] = node_range(k);
    d[k] = mapping(f, d[k], fst, lst);
    if (k < size) lz[k] = composition(f, lz[k]);
}

void push(int k) {
    all_apply(2 * k, lz[k]);
    all_apply(2 * k + 1, lz[k]);
    lz[k] = eL;
}

};

```

1.2 Bitree 2D

Given a 2d array allow you to sum *val* to the position (x, y) and find the sum of the rectangle with left top corner (x_1, y_1) and right bottom corner (x_2, y_2)

Update and query 1 indexed !

Time: update $O(\log n^2)$, query $O(\log n^2)$

```

struct Bit2d {
    int n;
    vll2d bit;
    Bit2d(int ni) : n(ni), bit(n + 1, vll(n + 1)) {}
    Bit2d(int ni, vll2d &xs) : n(ni), bit(n + 1, vll(n + 1)) {
        for (int i = 1; i <= n; i++) {
            for (int j = 1; j <= n; j++) {
                update(i, j, xs[i][j]);
            }
        }
    }

    void update(int x, int y, ll val) {
        for (; x <= n; x += (x & (-x))) {

```

```

        for (int i = y; i <= n; i += (i & (-i))) {
            bit[x][i] += val;
        }
    }
}

11 sum(int x, int y) {
    11 ans = 0;

    for (int i = x; i; i -= (i & (-i))) {
        for (int j = y; j; j -= (j & (-j))) {
            ans += bit[i][j];
        }
    }
    return ans;
}

11 query(int x1, int y1, int x2, int y2) {
    return sum(x2, y2) - sum(x2, y1 - 1) - sum(x1 - 1, y2) +
        sum(x1 - 1, y1 - 1);
}

};

```

1.3 Bitree

```

template <typename T>
struct BITree {
    int N;
    vector<T> v;

    BITree(int n) : N(n), v(n + 1, 0) {}

    void update(int i, const T& x) {
        if (i == 0) return;
        for (; i <= N; i += i & -i) v[i] += x;
    }

    T range_sum(int i, int j) {
        return range_sum(j) - range_sum(i - 1);
    }

    T range_sum(int i) {
        T sum = 0;
        for (; i > 0; i -= i & -i) sum += v[i];
        return sum;
    }
};

```

1.4 Disjoint Sparse Table

Answers queries of any monoid operation (i.e. has identity element and is associative)
 Build: $O(N \log N)$, Query: $O(1)$

```

#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct DisjointSparseTable {
    using Operation = T (*)(T, T);

    vector<vector<T>> st;
    Operation f;
    T identity;

    static constexpr int log2_floor(
        unsigned long long i) noexcept {
        return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
    }

    // Lazy loading constructor. Needs to call build!
    DisjointSparseTable(Operation op, const T neutral = T())
        : st(), f(op), identity(neutral) {}

    DisjointSparseTable(vector<T> v)
        : DisjointSparseTable(v, F(min(a, b))) {}

    DisjointSparseTable(vector<T> v, Operation op,
        const T neutral = T())
        : st(), f(op), identity(neutral) {
        build(v);
    }

    void build(vector<T> v) {
        st.resize(log2_floor(v.size()) + 1,
            vector<T>(1ll << (log2_floor(v.size()) + 1)));
        v.resize(st[0].size(), identity);
        for (int level = 0; level < (int)st.size(); ++level) {
            for (int block = 0; block < (1 << level); ++block) {
                const auto l = block << (st.size() - level);
                const auto r = (block + 1) << (st.size() - level);
                const auto m = l + (r - l) / 2;

                st[level][m] = v[m];
                for (int i = m + 1; i < r; i++)
                    st[level][i] = f(st[level][i - 1], v[i]);
                st[level][m - 1] = v[m - 1];
            }
        }
    }
};

```

```

        for (int i = m - 2; i >= 1; i--)
            st[level][i] = f(st[level][i + 1], v[i]);
    }
}

T query(int l, int r) const {
    if (l > r) return identity;
    if (l == r) return st.back()[l];

    const auto k = log2_floor(l ^ r);
    const auto level = (int)st.size() - 1 - k;
    return f(st[level][l], st[level][r]);
}
};

```

1.5 Dsu

```

struct DSU {
    vi ps, sz;

    // vector<unordered_set<int>> sts;

    DSU(int N) : ps(N + 1), sz(N, 1) /*, sts(N) */ {
        iota(all(ps), 0);
        // for (int i = 0; i < N; i++) sts[i].insert(i);
    }
    int find_set(int x) {
        return ps[x] == x ? x : ps[x] = find_set(ps[x]);
    }
    int size(int u) { return sz[find_set(u)]; }
    bool same_set(int x, int y) {
        return find_set(x) == find_set(y);
    }
    void union_set(int x, int y) {
        if (same_set(x, y)) return;

        int px = find_set(x);
        int py = find_set(y);

        if (sz[px] < sz[py]) swap(px, py);

        ps[py] = px;
        sz[px] += sz[py];
        // sts[px].merge(sts[py]);
    }
};

```

```

    }
};

```

1.6 Ordered Set

If you need an ordered **multiset** you may add an id to each value. Using `greater_equal`, or `less_equal` is considered undefined behavior.

- **order_of_key(k)** : Number of items strictly smaller/greater than k .
- **find_by_order(k)** : K-th element in a set (counting from zero).

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
                        tree_order_statistics_node_update>;

```

1.7 Prefix Sum 2D

Given an 2d array with n lines and m columns, find the sum of the subarray that have the left upper corner at $(x1, y1)$ and right bottom corner at $(x2, y2)$.

Time: build $O(n \cdot m)$, query $O(1)$.

```

struct psum2d {
    vll2d s;
    vll2d psum;
    psum2d(vll2d &grid, int n, int m)
        : s(n + 1, vll(m + 1)), psum(n + 1, vll(m + 1)) {
        for (int i = 1; i <= n; i++)
            for (int j = 1; j <= m; j++)
                s[i][j] = s[i][j - 1] + grid[i][j];

        for (int i = 1; i <= n; i++)
            for (int j = 1; j <= m; j++)
                psum[i][j] = psum[i - 1][j] + s[i][j];
    }

    ll query(int x1, int y1, int x2, int y2) {
        ll ans = psum[x2][y2] + psum[x1 - 1][y1 - 1];
        ans -= psum[x2][y1 - 1] + psum[x1 - 1][y2];
        return ans;
    }
};

```

1.8 SegTree Range Sum Query Range PA sum/set Update

Makes arithmetic progression updates in range and sum queries.

Considering $PA(A, R) = [A + R, A + 2R, A + 3R, \dots]$

- **update_set(l, r, A, R):** sets [l, r] to $PA(A, R)$
- **update_add(l, r, A, R):** sum $PA(A, R)$ in [l, r]
- **query(l, r):** sum in range [l, r]

0 indexed !

Time: build $O(n)$, updates and queries $O(\log n)$

```
const ll oo = 1e18;
struct SegTree {
    struct Data {
        ll sum;
        ll set_a, set_r, add_a, add_r;
        Data()
            : sum(0), set_a(oo), set_r(0), add_a(0), add_r(0) {}
    };
    int n;
    vector<Data> seg;
    SegTree(int n_) : n(n_), seg(vector<Data>(4 * n)) {}

    void prop(int p, int l, int r) {
        int sz = r - l + 1;
        ll &sum = seg[p].sum, &set_a = seg[p].set_a,
            &set_r = seg[p].set_r, &add_a = seg[p].add_a,
            &add_r = seg[p].add_r;

        if (set_a != oo) {
            set_a += add_a, set_r += add_r;
            sum = set_a * sz + set_r * sz * (sz + 1) / 2;
            if (l != r) {
                int m = (l + r) / 2;

                seg[2 * p].set_a = set_a;
                seg[2 * p].set_r = set_r;
                seg[2 * p].add_a = seg[2 * p].add_r = 0;

                seg[2 * p + 1].set_a = set_a + set_r * (m - l + 1);
                seg[2 * p + 1].set_r = set_r;
                seg[2 * p + 1].add_a = seg[2 * p + 1].add_r = 0;
            }
            set_a = oo, set_r = 0;
            add_a = add_r = 0;
        } else if (add_a or add_r) {
            sum += add_a * sz + add_r * sz * (sz + 1) / 2;
```

```
        if (l != r) {
            int m = (l + r) / 2;

            seg[2 * p].add_a += add_a;
            seg[2 * p].add_r += add_r;

            seg[2 * p + 1].add_a += add_a + add_r * (m - l + 1);
            seg[2 * p + 1].add_r += add_r;
        }
        add_a = add_r = 0;
    }
}

int inter(pii a, pii b) {
    if (a.first > b.first) swap(a, b);
    return max(0, min(a.second, b.second) - b.first + 1);
}

ll set(int a, int b, ll aa, ll rr, int p, int l, int r) {
    prop(p, l, r);
    if (b < l or r < a) return seg[p].sum;
    if (a <= l and r <= b) {
        seg[p].set_a = aa;
        seg[p].set_r = rr;
        prop(p, l, r);
        return seg[p].sum;
    }
    int m = (l + r) / 2;
    int tam_l = inter({l, m}, {a, b});
    return seg[p].sum = set(a, b, aa, rr, 2 * p, l, m) +
        set(a, b, aa + rr * tam_l, rr,
            2 * p + 1, m + 1, r);
}

void update_set(int l, int r, ll aa, ll rr) {
    set(l, r, aa, rr, 1, 0, n - 1);
}

ll add(int a, int b, ll aa, ll rr, int p, int l, int r) {
    prop(p, l, r);
    if (b < l or r < a) return seg[p].sum;
    if (a <= l and r <= b) {
        seg[p].add_a += aa;
        seg[p].add_r += rr;
        prop(p, l, r);
        return seg[p].sum;
    }
    int m = (l + r) / 2;
```

```

    int tam_l = inter({l, m}, {a, b});
    return seg[p].sum = add(a, b, aa, rr, 2 * p, l, m) +
        add(a, b, aa + rr * tam_l, rr,
            2 * p + 1, m + 1, r);
}

void update_add(int l, int r, ll aa, ll rr) {
    add(l, r, aa, rr, 1, 0, n - 1);
}

ll query(int a, int b, int p, int l, int r) {
    prop(p, l, r);
    if (b < l or r < a) return 0;
    if (a <= l and r <= b) return seg[p].sum;
    int m = (l + r) / 2;
    return query(a, b, 2 * p, l, m) +
        query(a, b, 2 * p + 1, m + 1, r);
}

ll query(int l, int r) {
    return query(l, r, 1, 0, n - 1);
}
};

```

1.9 SegTree Point Update (dynamic function)

Answers queries of any monoid operation (i.e. has identity element and is associative)
 Build: $O(N)$, Query: $O(\log N)$

```

#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct SegTree {
    using Operation = T (*)(T, T);

    int N;
    vector<T> ns;
    Operation operation;
    T identity;

    SegTree(int n, Operation op = F(a + b), T neutral = T())
        : N(n),
          ns(2 * N, neutral),
          operation(op),
          identity(neutral) {}

    SegTree(const vector<T> &v, Operation op = F(a + b),
            T neutral = T())
        : SegTree((int)v.size(), op, neutral) {
        copy(v.begin(), v.end(), ns.begin() + N);
    }
};

```

```

for (int i = N - 1; i > 0; --i)
    ns[i] = operation(ns[2 * i], ns[2 * i + 1]);
}

T query(size_t i) const { return ns[i + N]; }

T query(size_t l, size_t r) const {
    auto a = l + N, b = r + N;
    auto ans = identity;
    // Non-associative operations needs to be processed
    // backwards
    stack<T> st;
    while (a <= b) {
        if (a & 1) ans = operation(ans, ns[a++]);
        if (not(b & 1)) st.push(ns[b--]);

        a >>= 1;
        b >>= 1;
    }

    for (; !st.empty(); st.pop())
        ans = operation(ans, st.top());

    return ans;
}

void update(size_t i, T value) {
    update_set(i, operation(ns[i + N], value));
}

void update_set(size_t i, T value) {
    auto a = i + N;

    ns[a] = value;
    while (a >>= 1)
        ns[a] = operation(ns[2 * a], ns[2 * a + 1]);
}
};

```

1.10 Segtree Range Max Query Point Max Assign Update (dynamic)

Answers range queries in ranges until 10^9 (maybe more)
 Time: query and update $O(n \cdot \log n)$


```

struct node;
node *newNode();

struct node {
    node *left, *right;
    int lv, rv;
    ll val;

    node() : left(NULL), right(NULL), val(-oo) {}

    inline void init(int l, int r) {
        lv = l;
        rv = r;
    }

    inline void extend() {
        if (!left) {
            int m = (lv + rv) / 2;
            left = newNode();
            right = newNode();
            left->init(lv, m);
            right->init(m + 1, rv);
        }
    }

    ll query(int l, int r) {
        if (r < lv || rv < l) {
            return 0;
        }

        if (l <= lv && rv <= r) {
            return val;
        }

        extend();
        return max(left->query(l, r), right->query(l, r));
    }

    void update(int p, ll newVal) {
        if (lv == rv) {
            val = max(val, newVal);
            return;
        }

        extend();

```

```

        (p <= left->rv ? left : right)->update(p, newVal);
        val = max(left->val, right->val);
    }
};

const int BUFFSZ(1e7);
node *newNode() {
    static int bufSize = BUFFSZ;
    static node buf[(int)BUFFSZ];
    assert(bufSize);
    return &buf[--bufSize];
}

struct SegTree {
    int n;
    node *root;
    SegTree(int _n) : n(_n) {
        root = newNode();
        root->init(0, n);
    }

    ll query(int l, int r) { return root->query(l, r); }
    void update(int p, ll v) { root->update(p, v); }
};

1.11 Segtree Range Max Query Range Max Update

template <typename T = ll>
struct SegTree {
    int N;
    T nu, nq;
    vector<T> st, lazy;
    SegTree(const vector<T> &xs)
        : N(len(xs)),
          nu(numeric_limits<T>::min()),
          nq(numeric_limits<T>::min()),
          st(4 * N + 1, nu),
          lazy(4 * N + 1, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    void update(int l, int r, T value) {
        update(1, 0, N - 1, l, r, value);
    }

    T query(int l, int r) { return query(1, 0, N - 1, l, r); }

```

```

void update(int node, int nl, int nr, int ql, int qr,
            T v) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return;

    st[node] = max(st[node], v);
    if (ql <= nl and nr <= qr) {
        if (nl < nr) {
            lazy[left(node)] = max(lazy[left(node)], v);
            lazy[right(node)] = max(lazy[right(node)], v);
        }
        return;
    }
    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

    st[node] = max(st[left(node)], st[right(node)]);
}

T query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return max(x, y);
}

void propagation(int node, int nl, int nr) {
    if (lazy[node] != nu) {
        st[node] = max(st[node], lazy[node]);

        if (nl < nr) {
            lazy[left(node)] =
                max(lazy[left(node)], lazy[node]);
            lazy[right(node)] =
                max(lazy[right(node)], lazy[node]);
        }
    }
}

```

```

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

int main() {
    int n;
    cin >> n;
    vector<array<int, 3>> xs(n);
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < 3; ++j) {
            cin >> xs[i][j];
        }
    }
    vi aux(n, 0);
    SegTree<int> st(aux);
    for (int i = 0; i < n; ++i) {
        int a = min(i + xs[i][1], n);
        int b = min(i + xs[i][2], n);
        st.update(i, i, st.query(i, i) + xs[i][0]);
        int cur = st.query(i, i);
        st.update(a, b, cur);
    }

    cout << st.query(0, n) << '\n';
}

```

1.12 SegTree Range Min Query Point Assign Update

```

template <typename T = ll>
struct SegTree {
    int n;
    T nu, nq;
    vector<T> st;
    SegTree(const vector<T> &v)
        : n(len(v)),
          nu(0),
          nq(numeric_limits<T>::max()),
          st(n * 4 + 1, nu) {
        for (int i = 0; i < n; ++i) update(i, v[i]);
    }
    void update(int p, T v) { update(1, 0, n - 1, p, v); }
}

```

```

T query(int l, int r) { return query(1, 0, n - 1, l, r); }

void update(int node, int nl, int nr, int p, T v) {
    if (p < nl or p > nr) return;

    if (nl == nr) {
        st[node] = v;
        return;
    }

    update(left(node), nl, mid(nl, nr), p, v);
    update(right(node), mid(nl, nr) + 1, nr, p, v);

    st[node] = min(st[left(node)], st[right(node)]);
}

T query(int node, int nl, int nr, int ql, int qr) {
    if (ql <= nl and qr >= nr) return st[node];
    if (nl > qr or nr < ql) return nq;
    if (nl == nr) return st[node];

    return min(
        query(left(node), nl, mid(nl, nr), ql, qr),
        query(right(node), mid(nl, nr) + 1, nr, ql, qr));
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

1.13 Segtree Range Sum Query Point Sum Update (dynamic)

Answers range queries in ranges until 10^9 (maybe more)

Time: query and update $O(n \cdot \log n)$

```

struct node;
node *newNode();

struct node {
    node *left, *right;
    int lv, rv;
    ll val;

    node() : left(NULL), right(NULL), val(0) {}

    inline void init(int l, int r) {

```

```

        lv = l;
        rv = r;
    }

    inline void extend() {
        if (!left) {
            int m = (rv - lv) / 2 + lv;
            left = newNode();
            right = newNode();
            left->init(lv, m);
            right->init(m + 1, rv);
        }
    }

    ll query(int l, int r) {
        if (r < lv || rv < l) {
            return 0;
        }

        if (l <= lv && rv <= r) {
            return val;
        }

        extend();
        return left->query(l, r) + right->query(l, r);
    }

    void update(int p, ll newVal) {
        if (lv == rv) {
            val += newVal;
            return;
        }

        extend();
        (p <= left->rv ? left : right)->update(p, newVal);
        val = left->val + right->val;
    }
};

const int BUFFSZ(1.3e7);
node *newNode() {
    static int bufSize = BUFFSZ;
    static node buf[(int)BUFFSZ];
    // assert(bufSize);
    return &buf[--bufSize];
}

```

```

}

struct SegTree {
    int n;
    node *root;
    SegTree(int _n) : n(_n) {
        root = newNode();
        root->init(0, n);
    }
    ll query(int l, int r) { return root->query(l, r); }
    void update(int p, ll v) { root->update(p, v); }
};

```

1.14 SegTree Range Xor query Point Assign Update

```

template <typename T = ll>
struct SegTree {
    int n;
    T nu, nq;
    vector<T> st;
    SegTree(const vector<T> &v)
        : n(len(v)), nu(0), nq(0), st(n * 4 + 1, nu) {
        for (int i = 0; i < n; ++i) update(i, v[i]);
    }
    void update(int p, T v) { update(1, 0, n - 1, p, v); }
    T query(int l, int r) { return query(1, 0, n - 1, l, r); }

    void update(int node, int nl, int nr, int p, T v) {
        if (p < nl or p > nr) return;

        if (nl == nr) {
            st[node] = v;
            return;
        }

        update(left(node), nl, mid(nl, nr), p, v);
        update(right(node), mid(nl, nr) + 1, nr, p, v);

        st[node] = st[left(node)] ^ st[right(node)];
    }

    T query(int node, int nl, int nr, int ql, int qr) {
        if (ql <= nl and qr >= nr) return st[node];
        if (nl > qr or nr < ql) return nq;
        if (nl == nr) return st[node];

```

```

        return query(left(node), nl, mid(nl, nr), ql, qr) ^
            query(right(node), mid(nl, nr) + 1, nr, ql, qr);
    }

    int left(int p) { return p << 1; }
    int right(int p) { return (p << 1) + 1; }
    int mid(int l, int r) { return (r - l) / 2 + l; }
};

```

1.15 SegTree Range Min Query Range Sum Update

```

template <typename t = ll>
struct SegTree {
    int n;
    t nu;
    t nq;
    vector<t> st, lazy;
    SegTree(const vector<t> &xs)
        : n(len(xs)),
          nu(0),
          nq(numeric_limits<t>::max()),
          st(4 * n, nu),
          lazy(4 * n, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    SegTree(int n) : n(n), st(4 * n, nu), lazy(4 * n, nu) {}

    void update(int l, int r, ll value) {
        update(1, 0, n - 1, l, r, value);
    }

    t query(int l, int r) { return query(1, 0, n - 1, l, r); }

    void update(int node, int nl, int nr, int ql, int qr,
        ll v) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return;

        if (ql <= nl and nr <= qr) {
            st[node] += (nr - nl + 1) * v;

            if (nl < nr) {

```

```

        lazy[left(node)] += v;
        lazy[right(node)] += v;
    }

    return;
}

update(left(node), nl, mid(nl, nr), ql, qr, v);
update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

st[node] = min(st[left(node)], st[right(node)]);
}

t query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    t x = query(left(node), nl, mid(nl, nr), ql, qr);
    t y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return min(x, y);
}

void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
        st[node] += lazy[node];

        if (nl < nr) {
            lazy[left(node)] += lazy[node];
            lazy[right(node)] += lazy[node];
        }

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

1.16 SegTree Range Sum Query Range Sum Update

```

template <typename T = ll>
struct SegTree {
    int N;
    T nu;
    T nq;
    vector<T> st, lazy;
    SegTree(const vector<T> &xs)
        : N(len(xs)),
          nu(0),
          nq(0),
          st(4 * N, nu),
          lazy(4 * N, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    SegTree(int n)
        : N(n), nu(0), nq(0), st(4 * N, nu), lazy(4 * N, nu) {}

    void update(int l, int r, ll value) {
        update(1, 0, N - 1, l, r, value);
    }

    T query(int l, int r) { return query(1, 0, N - 1, l, r); }

    void update(int node, int nl, int nr, int ql, int qr,
                ll v) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return;

        if (ql <= nl and nr <= qr) {
            st[node] += (nr - nl + 1) * v;

            if (nl < nr) {
                lazy[left(node)] += v;
                lazy[right(node)] += v;
            }

            return;
        }

        update(left(node), nl, mid(nl, nr), ql, qr, v);
        update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

        st[node] = st[left(node)] + st[right(node)];
    }
};

```

```

}

T query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return x + y;
}

void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
        st[node] += (nr - nl + 1) * lazy[node];

        if (nl < nr) {
            lazy[left(node)] += lazy[node];
            lazy[right(node)] += lazy[node];
        }

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

1.17 Sparse Table

Answer the range query defined at the function `op`.
 Build: $O(N \log N)$, Query: $O(1)$

```

template <typename T>
struct SparseTable {
    vector<T> v;
    int n;
    static const int b = 30;
    vi mask, t;

    int op(int x, int y) { return v[x] < v[y] ? x : y; }
    int msb(int x) {

```

```

        return __builtin_clz(1) - __builtin_clz(x);
    }
};

SparseTable() {}
SparseTable(const vector<T>& v_)
    : v(v_), n(v.size()), mask(n), t(n) {
    for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
        at = (at << 1) & ((1 << b) - 1);
        while (at and op(i, i - msb(at & -at)) == i)
            at ^= at & -at;
    }
    for (int i = 0; i < n / b; i++)
        t[i] = b * i + b - 1 - msb(mask[b * i + b - 1]);
    for (int j = 1; (1 << j) <= n / b; j++)
        for (int i = 0; i + (1 << j) <= n / b; i++)
            t[n / b * j + i] =
                op(t[n / b * (j - 1) + i],
                  t[n / b * (j - 1) + i + (1 << (j - 1))]);
    }

    int small(int r, int sz = b) {
        return r - msb(mask[r] & ((1 << sz) - 1));
    }

    T query(int l, int r) {
        if (r - l + 1 <= b) return small(r, r - l + 1);
        int ans = op(small(l + b - 1), small(r));
        int x = l / b + 1, y = r / b - 1;
        if (x <= y) {
            int j = msb(y - x + 1);
            ans = op(ans, op(t[n / b * j + x],
                             t[n / b * j + y - (1 << j) + 1]));
        }
        return ans;
    }
};

```

2 Dynamic programming

2.1 Binary Knapsack (bottom up)

Given N items, each with its own value V_i and weight W_i and a maximum knapsack weight W , compute the maximum value of the items that we can carry, if we can either ignore or take a particular item.

Assume that $1 \leq n \leq 1000$, $1 \leq S \leq 10000$.

Time and space: $O(N * W)$

the vectors VS and WS starts at one, so it need an empty value at index 0.

```

const int MAXN(2010), MAXM(2010);
ll st[MAXN + 1][MAXM + 1];

```

```

char ps[MAXN + 1][MAXM + 1];
pair<ll, vi> knapsack(int M, const vll &VS, const vi &WS) {
    memset(st, 0, sizeof(st));
    memset(ps, 0, sizeof(ps));
    int N = len(VS) - 1; // ELEMENTS START AT INDEX 1 !

    for (int i = 0; i <= N; ++i) st[i][0] = 0;

    for (int m = 0; m <= M; ++m) st[0][m] = 0;

    for (int i = 1; i <= N; ++i) {
        for (int m = 1; m <= M; ++m) {
            st[i][m] = st[i - 1][m];
            ps[i][m] = 0;
            int w = WS[i];
            ll v = VS[i];

            if (w <= m and st[i - 1][m - w] + v > st[i][m]) {
                st[i][m] = st[i - 1][m - w] + v;
                ps[i][m] = 1;
            }
        }
    }

    int m = M;
    vi is;
    for (int i = N; i >= 1; --i) {
        if (ps[i][m]) {
            is.emplace_back(i - 1);
            m -= WS[i];
        }
    }

    return {st[N][M], is};
}

```

2.2 Binary Knapsack (top down)

Given N items, each with its own value V_i and weight W_i and a maximum knapsack weight W , compute the maximum value of the items that we can carry, if we can either ignore or take a particular item.

Assume that $1 \leq n \leq 1000$, $1 \leq S \leq 10000$.

Time and space: $O(N * W)$

the bottom up version is 5 times faster !

```

const int MAXN(2000), MAXM(2000);
ll memo[MAXN][MAXM + 1];
char choosen[MAXN][MAXM + 1];

```

```

ll knapSack(int u, int w, vll &VS, vi &WS) {
    if (u < 0) return 0;
    if (memo[u][w] != -1) return memo[u][w];

    ll a = 0, b = 0;
    a = knapSack(u - 1, w, VS, WS);
    if (WS[u] <= w)
        b = knapSack(u - 1, w - WS[u], VS, WS) + VS[u];
    if (b > a) {
        choosen[u][w] = true;
    }
    return memo[u][w] = max(a, b);
}

pair<ll, vi> knapSack(int W, vll &VS, vi &WS) {
    memset(memo, -1, sizeof(memo));
    memset(choosen, 0, sizeof(choosen));
    int n = len(VS);
    ll v = knapSack(n - 1, W, VS, WS);
    ll cw = W;
    vi choosed;
    for (int i = n - 1; i >= 0; i--) {
        if (choosen[i][cw]) {
            cw -= WS[i];
            choosed.emplace_back(i);
        }
    }
    return {v, choosed};
}

```

2.3 Digits

Finds the number of digits between 1 and x that don't have 4 or 13 as substring.

```

ll memo[20][30][2];
ll dp(int p, int d, bool l, const vi &digits) {
    if (p == len(digits)) return 0;

    if (memo[p][d][l] != -1) {
        return memo[p][d][l];
    }

    ll tot = 0;

    int k = l and d == digits[p] ? digits[p + 1] : 9;
    for (int i = 0; i <= k; i++) {
        if (i == 4) continue;

```

```

    if (d == 1 and i == 3) continue;
    tot += dp(p + 1, i, 1 and d == digits[p], digits);
}

return memo[p][d][1] = tot;
}

vi get_digits(ll x) {
    vi digits;

    while (x) {
        digits.emplace_back(x % 10ull);
        x /= 10ull;
    }

    reverse(all(digits));
    return digits;
}

ll dp(ll x) {
    auto digits = get_digits(x);
    memset(memo, -1, sizeof memo);

    for (ll i = 0; i <= 9; i++) {
        memo[len(digits) - 1][i][0] = 1ull;
        memo[len(digits) - 1][i][1] = i <= digits.back();
    }

    ll tot = 0;
    for (int i = 0; i <= digits[0]; i++) {
        if (i == 4) continue;
        tot += dp(0, i, i == digits[0], digits);
    }

    return tot - 1ull;
}

```

2.4 Edit Distance

$O(N * M)$

```

int edit_distance(const string &a, const string &b) {
    int n = a.size();
    int m = b.size();
    vector<vi> dp(n + 1, vi(m + 1, 0));

```

```

int ADD = 1, DEL = 1, CHG = 1;
for (int i = 0; i <= n; ++i) {
    dp[i][0] = i * DEL;
}
for (int i = 1; i <= m; ++i) {
    dp[0][i] = ADD * i;
}

for (int i = 1; i <= n; ++i) {
    for (int j = 1; j <= m; ++j) {
        int add = dp[i][j - 1] + ADD;
        int del = dp[i - 1][j] + DEL;
        int chg = dp[i - 1][j - 1] +
            (a[i - 1] == b[j - 1] ? 0 : 1) * CHG;
        dp[i][j] = min({add, del, chg});
    }
}

return dp[n][m];
}

```

2.5 Kadane

Find the maximum subarray sum in a given array.

```

int kadane(const vi &as) {
    vi s(len(as));
    s[0] = as[0];

    for (int i = 1; i < len(as); ++i)
        s[i] = max(as[i], s[i - 1] + as[i]);

    return *max_element(all(s));
}

```

2.6 Longest Increasing Subsequence (LIS)

Finds the length of the longest subsequence in

$O(n \log n)$

```

int LIS(const vi& as) {
    const ll oo = 1e18;
    int n = len(as);
    vll lis(n + 1, oo);
    lis[0] = -oo;

```



```

auto ans = 0;

for (int i = 0; i < n; ++i) {
    auto it = lower_bound(all(lis), as[i]);
    auto pos = (int)(it - lis.begin());

    ans = max(ans, pos);
    lis[pos] = as[i];
}

return ans;
}

```

2.7 Money Sum (Bottom Up)

Find every possible sum using the given values only once.

```

set<int> money_sum(const vi &xs) {
    using vc = vector<char>;
    using vvc = vector<vc>;
    int _m = accumulate(all(xs), 0);
    int _n = xs.size();
    vvc _dp(_n + 1, vc(_m + 1, 0));
    set<int> _ans;
    _dp[0][xs[0]] = 1;
    for (int i = 1; i < _n; ++i) {
        for (int j = 0; j <= _m; ++j) {
            if (j == 0 or _dp[i - 1][j]) {
                _dp[i][j + xs[i]] = 1;
                _dp[i][j] = 1;
            }
        }
    }

    for (int i = 0; i < _n; ++i)
        for (int j = 0; j <= _m; ++j)
            if (_dp[i][j]) _ans.insert(j);
    return _ans;
}

```

2.8 Travelling Salesman Problem

```

using vi = vector<int>;
vector<vi> dist;

```

```

vector<vi> memo;
/* 0 ( N^2 * 2^N ) */
int tsp(int i, int mask, int N) {
    if (mask == (1 << N) - 1) return dist[i][0];
    if (memo[i][mask] != -1) return memo[i][mask];
    int ans = INT_MAX << 1;
    for (int j = 0; j < N; ++j) {
        if (mask & (1 << j)) continue;
        auto t = tsp(j, mask | (1 << j), N) + dist[i][j];
        ans = min(ans, t);
    }
    return memo[i][mask] = ans;
}

```

3 Geometry

3.1 Convex Hull

Given a set of points find the smallest convex polygon that contains all the given points.

Time: $O(N \log N)$

By default it removes the collinear points, set the boolean to true if you don't want that

```

struct pt {
    double x, y;
    int id;
};

int orientation(pt a, pt b, pt c) {
    double v = a.x * (b.y - c.y) + b.x * (c.y - a.y) +
              c.x * (a.y - b.y);
    if (v < 0) return -1; // clockwise
    if (v > 0) return +1; // counter-clockwise
    return 0;
}

bool cw(pt a, pt b, pt c, bool include_collinear) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}

bool collinear(pt a, pt b, pt c) {
    return orientation(a, b, c) == 0;
}

void convex_hull(vector<pt>& pts,
                bool include_collinear = false) {
    pt p0 = *min_element(all(pts), [](pt a, pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
}

```

```

});
sort(all(pts), [&p0](const pt& a, const pt& b) {
    int o = orientation(p0, a, b);
    if (o == 0)
        return (p0.x - a.x) * (p0.x - a.x) +
               (p0.y - a.y) * (p0.y - a.y) <
               (p0.x - b.x) * (p0.x - b.x) +
               (p0.y - b.y) * (p0.y - b.y);
    return o < 0;
});
if (include_collinear) {
    int i = len(pts) - 1;
    while (i >= 0 && collinear(p0, pts[i], pts.back())) i--;
    reverse(pts.begin() + i + 1, pts.end());
}

vector<pt> st;
for (int i = 0; i < len(pts); i++) {
    while (st.size() > 1 && !cw(st[len(st) - 2], st.back(),
                               pts[i], include_collinear))
        st.pop_back();
    st.push_back(pts[i]);
}

pts = st;
}

```

3.2 Determinant

```

#include "Point.cpp"

template <typename T>
T D(const Point<T> &P, const Point<T> &Q,
    const Point<T> &R) {
    return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
           (R.x * Q.y + R.y * P.x + Q.x * P.y);
}

```

3.3 Equals

```

template <typename T>
bool equals(T a, T b) {
    const double EPS{1e-9};
    if (is_floating_point<T>::value)
        return fabs(a - b) < EPS;
}

```

```

else
    return a == b;
}

```

3.4 Line

```

#include <bits/stdc++.h>

#include "point-struct-and-utils.cpp"
using namespace std;

struct line {
    ld a, b, c;
};

// the answer is stored in the third parameter (pass by
// reference)
void pointsToLine(const point &p1, const point &p2,
                  line &l) {
    if (fabs(p1.x - p2.x) < EPS)
        // vertical line
        l = {1.0, 0.0, -p1.x};
    // default values
    else
        l = {-(ld)(p1.y - p2.y) / (p1.x - p2.x), 1.0,
            -(ld)(l.a * p1.x) - p1.y};
}

```

3.5 Point Struct And Utils (2d)

```

#include <bits/stdc++.h>
using namespace std;
using ld = long double;

struct point {
    ld x, y;
    int id;
    point(ld x = 0.0, ld y = 0.0, int id = -1)
        : x(x), y(y), id(id) {}

    point& operator+=(const point& t) {
        x += t.x;
        y += t.y;
        return *this;
    }

    point& operator-=(const point& t) {

```

```

    x -= t.x;
    y -= t.y;
    return *this;
}
point& operator*=(ld t) {
    x *= t;
    y *= t;
    return *this;
}
point& operator/=(ld t) {
    x /= t;
    y /= t;
    return *this;
}
point operator+(const point& t) const {
    return point(*this) += t;
}
point operator-(const point& t) const {
    return point(*this) -= t;
}
point operator*(ld t) const { return point(*this) *= t; }
point operator/(ld t) const { return point(*this) /= t; }
};

ld dot(point& a, point& b) { return a.x * b.x + a.y * b.y; }

ld norm(point& a) { return dot(a, a); }

ld abs(point a) { return sqrt(norm(a)); }

ld proj(point a, point b) { return dot(a, b) / abs(b); }

ld angle(point a, point b) {
    return acos(dot(a, b) / abs(a) / abs(b));
}

```

3.6 Polygon Lattice Points (Pick's Theorem)

Given a polygon with N points finds the number of lattice points inside and on boundaries. Time : $O(N)$

```

ll cross(ll x1, ll y1, ll x2, ll y2) {
    return x1 * y2 - x2 * y1;
}

```

```

ll polygonArea(vector<pll>& pts) {

```

```

    ll ats = 0;
    for (int i = 2; i < len(pts); i++)
        ats += cross(pts[i].first - pts[0].first,
                     pts[i].second - pts[0].second,
                     pts[i - 1].first - pts[0].first,
                     pts[i - 1].second - pts[0].second);
    return abs(ats / 2ll);
}

ll boundary(vector<pll>& pts) {
    ll ats = pts.size();
    for (int i = 0; i < len(pts); i++) {
        ll deltax =
            (pts[i].first - pts[(i + 1) % pts.size()].first);
        ll deltay =
            (pts[i].second - pts[(i + 1) % pts.size()].second);
        ats += abs(__gcd(deltax, deltay)) - 1;
    }
    return ats;
}

pll latticePoints(vector<pll>& pts) {
    ll bounds = boundary(pts);
    ll area = polygonArea(pts);
    ll inside = area + 1ll - bounds / 2ll;

    return {inside, bounds};
}

```

3.7 Segment

```

#include "Line.cpp"
#include "Point.cpp"
#include "equals.cpp"

```

```

template <typename T>
struct segment {
    Point<T> A, B;

    bool contains(const Point<T> &P) const;

    Point<T> closest(const Point<T> &p) const;
};

```

```

template <typename T>

```

```

bool segment<T>::contains(const Point<T> &P) const {
    // verifica se P é est contido na reta
    double dAB = Point<T>::dist(A, B),
           dAP = Point<T>::dist(A, P),
           dPB = Point<T>::dist(P, B);

    return equals(dAP + dPB, dAB);
}

template <typename T>
Point<T> segment<T>::closest(const Point<T> &P) const {
    Line<T> R(A, B);
    auto Q = R.closest(P);

    if (this->contains(Q)) return Q;

    auto distA = Point<T>::dist(P, A);
    auto distB = Point<T>::dist(P, B);

    if (distA <= distB)
        return A;
    else
        return B;
}

```

3.8 Template Line

```

#include "template-point.cpp"

template <typename T>
struct Line {
    T a, b, c;

    Line(T av, T bv, T cv) : a(av), b(bv), c(cv) {}

    Line(const Point<T> &P, const Point<T> &Q)
        : a(P.y - Q.y),
          b(Q.x - P.x),
          c(P.x * Q.y - Q.x * P.y) {}

    // verify if a point belongs to the line
    bool contains(const Point<T> &P) {
        return equals(a * P.x + b * P.y + c, 0);
    }
}

```

```

// shortest distance between P and a point Q that belongs
// to this line
double distance(const Point<T> &P) const {
    return fabs(a * P.x + b * P.y + c) / hypot(a, b);
}

// the closest point in this line to the given point
Point<T> closest(const Point<T> &P) const {
    auto den = (a * a) + (b * b);

    auto x = (b * (b * P.x - a * P.y) - a * c) / den;
    auto y = (a * (-b * P.x + a * P.y) - b * c) / den;

    return Point<T>{x, y};
}
};

```

3.9 Template Point

```

template <typename T>
struct Point {
    T x, y;

    Point(T xv = 0, T yv = 0) : x(xv), y(yv) {}

    double distance(const Point<T> &P) const {
        return hypot(static_cast<double>(P.x - this->x),
                     static_cast<double>(P.y - this->y));
    }
};

```

3.10 Template Segment

```

#include "equals.cpp"
#include "template-line.cpp"
#include "template-point.cpp"

template <typename T>
struct Segment {
    Point<T> A, B;

    Segment(const Point<T> &a, const Point<T> &b)
        : A(a), B(b) {}

    /*
     * Verify if a given point P belongs to the segment,

```

```

* considering that P belongs to the line defined with A
* and B
*/
bool contains(const Point<T> &P) const {
    return equals(A.x, B.x)
        ? min(A.y, B.y) <= P.y and P.y <= max(A.y, B.y)
        : min(A.x, B.x) <= P.x and
            P.x <= max(A.x, B.x);
}

/*
* Verify if P belongs to the segment AB,
* even if P don't belong to the line defined with A and B
* */
bool contains2(const Point<T> &P) const {
    double dAB = dist(A, B), dAP = dist(A, P),
           dPB = dist(P, B);
    return equals(dAP + dPB, dAB);
}

/*
* Find the closest point in P that belongs to the segment
* */
Point<T> closest(const Point<T> &P) {
    Line<T> r(A, B);
    auto Q = r.closest(P);

    if (this->contains(Q)) return Q;

    auto distA = P.distance(A);
    auto distB = P.distance(B);

    return distA <= distB ? A : B;
}

double distToClosest(const Point<T> &P) {
    return closest(P).distance(P);
}
};

```

4 Graphs

4.1 2 SAT

```
struct SAT2 {
```

```

ll n;
vll2d adj, adj_t;
vc used;
vll order, comp;
vc assignment;
bool solvable;
SAT2(ll _n)
    : n(2 * _n),
      adj(n),
      adj_t(n),
      used(n),
      order(n),
      comp(n, -1),
      assignment(n / 2) {}
void dfs1(int v) {
    used[v] = true;
    for (int u : adj[v]) {
        if (!used[u]) dfs1(u);
    }
    order.push_back(v);
}

void dfs2(int v, int cl) {
    comp[v] = cl;
    for (int u : adj_t[v]) {
        if (comp[u] == -1) dfs2(u, cl);
    }
}

bool solve_2SAT() {
    // find and label each SCC
    for (int i = 0; i < n; ++i) {
        if (!used[i]) dfs1(i);
    }
    reverse(all(order));
    ll j = 0;
    for (auto &v : order) {
        if (comp[v] == -1) dfs2(v, j++);
    }

    assignment.assign(n / 2, false);
    for (int i = 0; i < n; i += 2) {
        // x and !x belong to the same SCC
        if (comp[i] == comp[i + 1]) {
            solvable = false;

```

```

        return false;
    }

    assignment[i / 2] = comp[i] > comp[i + 1];
}
solvable = true;
return true;
}

void add_disjunction(int a, bool na, int b, bool nb) {
    a = (2 * a) ^ na;
    b = (2 * b) ^ nb;
    int neg_a = a ^ 1;
    int neg_b = b ^ 1;
    adj[neg_a].push_back(b);
    adj[neg_b].push_back(a);
    adj_t[b].push_back(neg_a);
    adj_t[a].push_back(neg_b);
}
};

```

4.2 Cycle Distances

Given a vertex s finds the longest cycle that end's in s , note that the vector **dist** will contain the distance that each vertex u needs to reach s .

Time: $O(N)$

```

using adj = vector<vector<pair<int, ll>>>;
ll cycleDistances(int u, int n, int s, vc &vis, adj &g,
                  vll &dist) {

    vis[u] = 1;

    for (auto [v, d] : g[u]) {
        if (v == s) {
            dist[u] = max(dist[u], d);
            continue;
        }

        if (vis[v] == 1) {
            continue;
        }

        if (vis[v] == 2) {
            dist[u] = max(dist[u], dist[v] + d);
        } else {
            ll d2 = cycleDistances(v, n, s, vis, g, dist);
            if (d2 != -oo) {

```

```

                dist[u] = max(dist[u], d2 + d);
            }
        }
    }
    vis[u] = 2;
    return dist[u];
}

```

4.3 SCC (struct)

Able to find the component of each node and the total of SCC in $O(V * E)$ and build the SCC graph ($O(V * E)$).

```

struct SCC {
    ll N;
    int totsc;
    vll2d adj, tadj;
    vll todo, comps, comp;
    vector<set<ll>> sccadj;
    vchar vis;
    SCC(ll _N)
        : N(_N),
          totsc(0),
          adj(_N),
          tadj(_N),
          comp(_N, -1),
          sccadj(_N),
          vis(_N) {}

    void add_edge(ll x, ll y) { adj[x].eb(y), tadj[y].eb(x); }

    void dfs(ll x) {
        vis[x] = 1;
        for (auto &y : adj[x])
            if (!vis[y]) dfs(y);
        todo.pb(x);
    }

    void dfs2(ll x, ll v) {
        comp[x] = v;
        for (auto &y : tadj[x])
            if (comp[y] == -1) dfs2(y, v);
    }

    void gen() {
        for (ll i = 0; i < N; ++i)
            if (!vis[i]) dfs(i);
        reverse(all(todo));
    }
}

```

```

for (auto &x : todo)
    if (comp[x] == -1) {
        dfs2(x, x);
        comps.pb(x);
        totsc++;;
    }
}

void genSCCGraph() {
    for (ll i = 0; i < N; ++i) {
        for (auto &j : adj[i]) {
            if (comp[i] != comp[j]) {
                sccadj[comp[i]].insert(comp[j]);
            }
        }
    }
}
};

```

4.4 Array Cycle

```

struct ArrayCycle {
    vector<vector<int>> paths;
    vector<int> path_num, pos;
    vector<char> is_cycle;

    ArrayCycle(const vector<int> &v)
        : path_num(v.size()), pos(v.size()) {
        paths.reserve(v.size());
        is_cycle.reserve(v.size());

        vector<char> vis(v.size());
        for (auto i : topological_order(v)) {
            if (vis[i]) continue;

            vector<int> path;
            int cur;
            for (cur = i; not vis[cur]; cur = v[cur]) {
                path.push_back(cur);
                vis[cur] = 1;
            }

            {
                int cycle_start = 0;
                for (; cycle_start < (int)path.size() and

```

```

                path[cycle_start] != cur;
                cycle_start++)
            ;

            if (cycle_start > 0) {
                paths.emplace_back();
                for (int j = 0; j < cycle_start; j++) {
                    paths.back().push_back(path[j]);
                    pos[path[j]] = j;
                    path_num[path[j]] = (int)paths.size() - 1;
                }
                paths.back().push_back(cur);
                is_cycle.push_back(false);
            }

            if (cycle_start < (int)path.size()) {
                paths.emplace_back();
                for (int j = cycle_start; j < (int)path.size();
                    j++) {
                    paths.back().push_back(path[j]);
                    pos[path[j]] = j - cycle_start;
                    path_num[path[j]] = (int)paths.size() - 1;
                }
                is_cycle.push_back(true);
            }
        }
    }

    const vector<int> &path(int cur) const {
        return paths[path_num[cur]];
    }

    int kth_pos(int cur, ll k) const {
        while (not is_cycle[path_num[cur]]) {
            auto &p = path(cur);
            int remain = (int)p.size() - pos[cur] - 1;
            if (k <= remain) return p[pos[cur] + k];
            cur = p.back();
            k -= remain;
        }

        auto &p = path(cur);
        return p[(pos[cur] + k) % p.size()];
    }
}

```

```

// {element, number_of_moves}
pair<int, int> go_to_cycle(int cur) const {
    int moves = 0;
    while (not is_cycle[path_num[cur]]) {
        auto &p = path(cur);
        moves += (int)p.size() - pos[cur] - 1;
        cur = p.back();
    }
    return {cur, moves};
}

void topological_order(const vector<int> &g,
                     vector<char> &vis,
                     vector<int> &order, int u) {
    vis[u] = true;
    if (not vis[g[u]])
        topological_order(g, vis, order, g[u]);
    order.push_back(u);
}

vector<int> topological_order(const vector<int> &g) {
    vector<char> vis(g.size(), false);
    vector<int> order;
    for (auto i = 0; i < (int)g.size(); i++)
        if (not vis[i]) topological_order(g, vis, order, i);
    reverse(order.begin(), order.end());
    return order;
}
};

```

4.5 Bellman-Ford (find negative cycle)

Given a directed graph find a negative cycle by running n iterations, and if the last one produces a relaxation than there is a cycle.

Time: $O(V \cdot E)$

```

const ll oo = 2500 * 1e9;

using graph = vector<vector<pair<int, ll>>>;
vi negative_cycle(graph &g, int n) {
    vll d(n, oo);
    vi p(n, -1);
    int x = -1;
    d[0] = 0;
    for (int i = 0; i < n; i++) {
        x = -1;

```

```

        for (int u = 0; u < n; u++) {
            for (auto &[v, l] : g[u]) {
                if (d[u] + l < d[v]) {
                    d[v] = d[u] + l;
                    p[v] = u;
                    x = v;
                }
            }
        }
    }

    if (x == -1)
        return {};
    else {
        for (int i = 0; i < n; i++) x = p[x];
        vi cycle;
        for (int v = x;; v = p[v]) {
            cycle.eb(v);
            if (v == x and len(cycle) > 1) break;
        }
        reverse(all(cycle));
        return cycle;
    }
}

```

4.6 Bellman Ford

Find shortest path from a single source to all other nodes. Can detect negative cycles.

Time: $O(V \cdot E)$

```

bool bellman_ford(const vector<vector<pair<int, ll>>> &g,
                 int s, vector<ll> &dist) {
    int n = (int)g.size();
    dist.assign(n, LLONG_MAX);

    vector<int> count(n);
    vector<char> in_queue(n);
    queue<int> q;

    dist[s] = 0;
    q.push(s);
    in_queue[s] = true;

    while (not q.empty()) {
        int cur = q.front();
        q.pop();

```



```

in_queue[cur] = false;

for (auto [to, w] : g[cur]) {
    if (dist[cur] + w < dist[to]) {
        dist[to] = dist[cur] + w;
        if (not in_queue[to]) {
            q.push(to);
            in_queue[to] = true;
            count[to]++;
            if (count[to] > n) return false;
        }
    }
}

return true;
}

```

4.7 BFS 01

Similar to a Dijkstra given a weighted graph finds the distance from source s to every other node (SSSP).

Applicable only when the weight of the edges $\in \{0, x\}$

Time: $O(V + E)$

```

vector<pair<ll, int>> adj[maxn];
ll dists[maxn];
int s, n;
void bfs_01() {
    fill(dists, dists + n, oo);
    dist[s] = 0;

    deque<int> q;
    q.emplace_back(s);

    while (not q.empty()) {
        auto u = q.front();
        q.pop_front();

        for (auto [v, w] : adj[u]) {
            if (dist[v] <= dist[u] + w) continue;
            dist[v] = dist[u] + w;
            w ? q.emplace_back(v) : q.emplace_front(v);
        }
    }
}

```

4.8 Block Cut Tree

```

// O(n + m)
struct BlockCutTree {
    vector<vector<int>> blocks, tree;
    vector<vector<pair<int, int>>> block_edges;
    vector<int> articulation, pos;

    BlockCutTree(const vector<vector<int>> &g)
        : articulation(g.size()), pos(g.size()) {
        int t = 0;
        vector<int> id(g.size(), -1);
        stack<int> s1;
        stack<pair<int, int>> s2;
        for (int i = 0; i < (int)g.size(); i++)
            if (id[i] == -1) dfs(g, i, -1, t, id, s1, s2);

        tree.resize(blocks.size());
        for (int i = 0; i < (int)g.size(); i++)
            if (articulation[i])
                pos[i] = (int)tree.size(), tree.emplace_back();

        for (int i = 0; i < (int)blocks.size(); i++) {
            for (auto j : blocks[i]) {
                if (not articulation[j])
                    pos[j] = i;
                else
                    tree[i].push_back(pos[j]),
                    tree[pos[j]].push_back(i);
            }
        }
    }

private:
    int dfs(const vector<vector<int>> &g, int i, int p,
            int &t, vector<int> &id, stack<int> &s1,
            stack<pair<int, int>> &s2) {
        int lo = id[i] = t++;
        s1.push(i);

        if (p != -1) s2.emplace(i, p);
        for (auto j : g[i])
            if (j != p and id[j] != -1) s2.emplace(i, j);

        for (auto j : g[i])

```

```

if (j != p) {
    if (id[j] == -1) {
        int val = dfs(g, j, i, t, id, s1, s2);
        lo = min(lo, val);

        if (val >= id[i]) {
            articulation[i]++;
            blocks.emplace_back(1, i);
            for (; blocks.back().back() != j; s1.pop())
                blocks.back().push_back(s1.top());

            block_edges.emplace_back(1, s2.top());
            s2.pop();
            for (; block_edges.back().back() !=
                make_pair(j, i);
                s2.pop())
                block_edges.back().push_back(s2.top());
        }
    } else {
        lo = min(lo, id[j]);
    }
}

if (p == -1 and articulation[i]) --articulation[i];
return lo;
}
};

```

4.9 Check Bipartite

$O(V)$

```

vi2d G;
int N, M;

bool check() {
    vi side(N, -1);
    queue<int> q;
    for (int st = 0; st < N; st++) {
        if (side[st] == -1) {
            q.emplace(st);
            side[st] = 0;
            while (not q.empty()) {
                int u = q.front();
                q.pop();
                for (auto v : G[u]) {
                    if (side[v] == -1) {

```

```

                        side[v] = side[u] ^ 1;
                        q.push(v);
                    } else if (side[u] == side[v])
                        return false;
                }
            }
        }
    }
    return true;
}

```

4.10 Dijkstra (k Shortest Paths)

```

const ll oo = 1e9 * 1e5 + 1;
using adj = vector<vector<pll>>;
vector<priority_queue<ll>> dijkstra(
    const vector<vector<pll>> &g, int n, int s, int k) {
    priority_queue<pll, vector<pll>, greater<pll>> pq;

    vector<priority_queue<ll>> dist(n);
    dist[0].emplace(0);
    pq.emplace(0, s);
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();

        if (not dist[v].empty() and dist[v].top() < d1)
            continue;

        for (auto [d2, u] : g[v]) {
            if (len(dist[u]) < k) {
                pq.emplace(d2 + d1, u);
                dist[u].emplace(d2 + d1);
            } else {
                if (dist[u].top() > d1 + d2) {
                    dist[u].pop();
                    dist[u].emplace(d1 + d2);
                    pq.emplace(d2 + d1, u);
                }
            }
        }
    }
    return dist;
}

```

4.11 Dijkstra

Finds the shortest path from s to every other node, and keep the 'parent' tracking.

Time: $O(E \cdot \log V)$

```
pair<vll, vi> dijkstra(const vector<vector<pll>> &g, int n,
                      int s) {
    priority_queue<pll, vector<pll>, greater<pll>> pq;
    vll dist(n, oo);
    vi p(n, -1);
    pq.emplace(0, s);
    dist[s] = 0;
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();
        if (dist[v] < d1) continue;

        for (auto [d2, u] : g[v]) {
            if (dist[u] > d1 + d2) {
                dist[u] = d1 + d2;
                p[u] = v;
                pq.emplace(dist[u], u);
            }
        }
    }
    return {dist, p};
}
```

4.12 Disjoint Edges Path (Maxflow)

Given a directed graph find's every path with disjoint edges that starts at s and ends at t

Time : $O(E \cdot V^2)$

```
struct DisjointPaths {
    int n;
    vi2d g, capacity;
    vector<vc> isedge;

    DisjointPaths(int _n)
        : n(_n), g(n), capacity(n, vi(n)), isedge(n, vc(n)) {}

    void add(int u, int v, int w = 1) {
        g[u].emplace_back(v);
        g[v].emplace_back(u);
        capacity[u][v] += w;
        isedge[u][v] = true;
    }
}
```

```
// finds the new flow to insert
int bfs(int s, int t, vi &parent) {
    fill(all(parent), -1);
    parent[s] = -2;
    queue<pair<int, int>> q;
    q.push({oo, s});

    while (!q.empty()) {
        auto [flow, cur] = q.front();
        q.pop();

        for (auto next : g[cur]) {
            if (parent[next] == -1 and capacity[cur][next]) {
                parent[next] = cur;
                ll new_flow = min(flow, capacity[cur][next]);
                if (next == t) return new_flow;
                q.push({new_flow, next});
            }
        }
    }

    return 0;
}

int maxflow(int s, int t) {
    int flow = 0;
    vi parent(n);
    int new_flow;

    while ((new_flow = bfs(s, t, parent))) {
        flow += new_flow;
        int cur = t;
        while (cur != s) {
            int prev = parent[cur];
            capacity[prev][cur] -= new_flow;
            capacity[cur][prev] += new_flow;
            cur = prev;
        }
    }

    return flow;
}

// build the distinct routes based in the capacity set by
```

```
// maxflow
void dfs(int u, int t, vc2d &vis, vi &route,
        vi2d &routes) {
    route.eb(u);
    if (u == t) {
        routes.emplace_back(route);
        route.pop_back();
        return;
    }

    for (auto &v : g[u]) {
        if (capacity[u][v] == 0 and isedge[u][v] and
            not vis[u][v]) {
            vis[u][v] = true;
            dfs(v, t, vis, route, routes);
            route.pop_back();
            return;
        }
    }
}

vi2d disjoint_paths(int s, int t) {
    int mf = maxflow(s, t);
    vi2d routes;
    vi route;
    vc2d vis(n, vc(n));
    for (int i = 0; i < mf; i++)
        dfs(s, t, vis, route, routes);
    return routes;
}
};
```

4.13 Euler Path (directed)

Given a **directed** graph finds a path that visits every edge exactly once.
Time: $O(E)$

```
vector<int> euler_cycle(vector<vector<int>> &g, int u) {
    vector<int> res;

    stack<int> st;
    st.push(u);
    while (!st.empty()) {
        auto cur = st.top();
        if (g[cur].empty()) {
```

```
            res.push_back(cur);
            st.pop();
        } else {
            auto next = g[cur].back();
            st.push(next);

            g[cur].pop_back();
        }
    }

    for (auto &x : g)
        if (!x.empty()) return {};

    return res;
}

vector<int> euler_path(vector<vector<int>> &g, int first) {
    {
        int n = (int)g.size();
        vector<int> in(n), out(n);
        for (int i = 0; i < n; i++)
            for (auto x : g[i]) in[x]++, out[i]++;

        int a = 0, b = 0, c = 0;
        for (int i = 0; i < n; i++)
            if (in[i] == out[i])
                c++;
            else if (in[i] - out[i] == 1)
                b++;
            else if (in[i] - out[i] == -1)
                a++;

        if (c != n - 2 or a != 1 or b != 1) return {};
    }

    auto res = euler_cycle(g, first);
    if (res.empty()) return res;

    reverse(all(res));
    return res;
}
```

4.14 Euler Path (undirected)

Given a **undirected** graph finds a path that visits every edge exactly once.

Time: $O(E)$

```
vector<int> euler_cycle(vector<vector<int>> &g, int u) {
    vector<int> res;
    multiset<pair<int, int>> vis;

    stack<int> st;
    st.push(u);
    while (!st.empty()) {
        auto cur = st.top();

        while (!g[cur].empty()) {
            auto it = vis.find(make_pair(cur, g[cur].back()));
            if (it == vis.end()) break;
            g[cur].pop_back();
            vis.erase(it);
        }

        if (g[cur].empty()) {
            res.push_back(cur);
            st.pop();
        } else {
            auto next = g[cur].back();
            st.push(next);

            vis.emplace(next, cur);
            g[cur].pop_back();
        }
    }

    for (auto &x : g)
        if (!x.empty()) return {};

    return res;
}

vector<int> euler_path(vector<vector<int>> &g, int first) {
    int n = (int)g.size();
    int v1 = -1, v2 = -1;
    {
        bool bad = false;
        for (int i = 0; i < n; i++)
            if (g[i].size() & 1) {
                if (v1 == -1)
```

```
                    v1 = i;
                else if (v2 == -1)
                    v2 = i;
                else
                    bad = true;
            }

        if (bad or (v1 != -1 and v2 == -1)) return {};
    }

    if (v2 != -1) {
        // insert cycle
        g[v1].push_back(v2);
        g[v2].push_back(v1);
    }

    auto res = euler_cycle(g, first);
    if (res.empty()) return res;

    if (v1 != -1) {
        for (int i = 0; i + 1 < (int)res.size(); i++) {
            if ((res[i] == v1 and res[i + 1] == v2) ||
                (res[i] == v2 and res[i + 1] == v1)) {
                vector<int> res2;
                for (int j = i + 1; j < (int)res.size(); j++)
                    res2.push_back(res[j]);
                for (int j = 1; j <= i; j++) res2.push_back(res[j]);
                res = res2;
                break;
            }
        }
    }

    reverse(all(res));
    return res;
}
```

4.15 Find Articulation/Cut Points

Given an **undirected** graph find its articulation points.

articulation point (or cut vertex): is defined as a **vertex** which, when removed along with associated edges, increases the number of connected components in the graph.

A vertex u can be an articulation point if and only if it has at least 2 adjacent vertices.

Time: $O(N + M)$

```

const int MAXN(100);
int N;
vi2d G;
int timer;
int tin[MAXN], low[MAXN];
set<int> cpoints;

int dfs(int u, int p = -1) {
    int cnt = 0;
    low[u] = tin[u] = timer++;
    for (auto v : G[u]) {
        if (not tin[v]) {
            cnt++;
            dfs(v, u);

            if (low[v] >= tin[u]) cpoints.insert(u);
            low[u] = min(low[u], low[v]);
        } else if (v != p)
            low[u] = min(low[u], tin[v]);
    }

    return cnt;
}

void getCutPoints() {
    memset(low, 0, sizeof(low));
    memset(tin, 0, sizeof(tin));
    cpoints.clear();

    timer = 1;
    for (int i = 0; i < N; i++) {
        if (tin[i]) continue;
        int cnt = dfs(i);
        if (cnt == 1) cpoints.erase(i);
    }
}

```

4.16 Find Bridges (online)

```

// O((n+m)*log(n))
struct BridgeFinder {
    // 2ecc = 2 edge conected component
    // cc = conected component
    vector<int> parent, dsu_2ecc, dsu_cc, dsu_cc_size;
    int bridges, lca_iteration;
    vector<int> last_visit;
}

```

```

BridgeFinder(int n)
: parent(n, -1),
  dsu_2ecc(n),
  dsu_cc(n),
  dsu_cc_size(n, 1),
  bridges(0),
  lca_iteration(0),
  last_visit(n) {
    for (int i = 0; i < n; i++) {
        dsu_2ecc[i] = i;
        dsu_cc[i] = i;
    }
}

int find_2ecc(int v) {
    if (v == -1) return -1;
    return dsu_2ecc[v] == v
        ? v
        : dsu_2ecc[v] = find_2ecc(dsu_2ecc[v]);
}

int find_cc(int v) {
    v = find_2ecc(v);
    return dsu_cc[v] == v ? v
        : dsu_cc[v] = find_cc(dsu_cc[v]);
}

void make_root(int v) {
    v = find_2ecc(v);
    int root = v;
    int child = -1;
    while (v != -1) {
        int p = find_2ecc(parent[v]);
        parent[v] = child;
        dsu_cc[v] = root;
        child = v;
        v = p;
    }
    dsu_cc_size[root] = dsu_cc_size[child];
}

void merge_path(int a, int b) {
    ++lca_iteration;
    vector<int> path_a, path_b;
}

```

```

int lca = -1;
while (lca == -1) {
    if (a != -1) {
        a = find_2ecc(a);
        path_a.push_back(a);
        if (last_visit[a] == lca_iteration) {
            lca = a;
            break;
        }
        last_visit[a] = lca_iteration;
        a = parent[a];
    }
    if (b != -1) {
        b = find_2ecc(b);
        path_b.push_back(b);
        if (last_visit[b] == lca_iteration) {
            lca = b;
            break;
        }
        last_visit[b] = lca_iteration;
        b = parent[b];
    }
}

for (auto v : path_a) {
    dsu_2ecc[v] = lca;
    if (v == lca) break;
    --bridges;
}

for (auto v : path_b) {
    dsu_2ecc[v] = lca;
    if (v == lca) break;
    --bridges;
}
}

void add_edge(int a, int b) {
    a = find_2ecc(a);
    b = find_2ecc(b);

    if (a == b) return;

    int ca = find_cc(a);
    int cb = find_cc(b);

```

```

    if (ca != cb) {
        ++bridges;
        if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
            swap(a, b);
            swap(ca, cb);
        }
        make_root(a);
        parent[a] = dsu_cc[a] = b;
        dsu_cc_size[cb] += dsu_cc_size[a];
    } else {
        merge_path(a, b);
    }
}
};

```

4.17 Find Bridges

Find every bridge in a **undirected** connected graph.

bridge: A bridge is defined as an **edge** which, when removed, increases the number of connected components in the graph.

Time: $O(N + M)$

```

const int MAXN(50);
vi2d G(MAXN);
int tin[MAXN];
int low[MAXN];
char vis[MAXN];
int timer;
int N, M;
vector<pii> bridges;

void dfs(int u, int p = -1) {
    vis[u] = true;
    tin[u] = low[u] = timer++;

    for (auto v : G[u]) {
        if (v == p) continue;
        if (vis[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] > tin[u]) {
                bridges.emplace_back(u, v);
            }
        }
    }
}

```

```

}
void getBridges() {
    timer = 0;

    memset(vis, 0, sizeof(vis));
    memset(tin, -1, sizeof(tin));
    memset(low, -1, sizeof(low));
    bridges.clear();

    for (int i = 0; i < N; i++) {
        if (not vis[i]) dfs(i);
    }
}

```

4.18 Find Centroid

Given a tree (don't forget to make it 'undirected'), find its centroids.

Time: $O(V)$

```

void dfs(int u, int p, int n, vi2d &g, vi &sz,
        vi &centroid) {
    sz[u] = 1;

    bool iscentroid = true;
    for (auto v : g[u])
        if (v != p) {
            dfs(v, u, n, g, sz, centroid);
            if (sz[v] > n / 2) iscentroid = false;
            sz[u] += sz[v];
        }

    if (n - sz[u] > n / 2) iscentroid = false;
    if (iscentroid) centroid.eb(u);
}

vi getCentroid(vi2d &g, int n) {
    vi centroid;
    vi sz(n);
    dfs(0, -1, n, g, sz, centroid);
    return centroid;
}

```

4.19 Floyd Warshall

Simply finds the minimal distance for each node to every other node. $O(V^3)$

```

vector<vll> floyd_warshall(const vector<vll> &adj, ll n) {
    auto dist = adj;

```

```

    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            for (int k = 0; k < n; ++k) {
                dist[j][k] =
                    min(dist[j][k], dist[j][i] + dist[i][k]);
            }
        }
    }
    return dist;
}

```

4.20 Graph Cycle (directed)

Given a directed graph finds a cycle (or not).

Time: $O(E)$

```

bool dfs(int v, vi2d &adj, vc &visited, vi &parent,
        vc &color, int &cycle_start, int &cycle_end) {
    color[v] = 1;
    for (int u : adj[v]) {
        if (color[u] == 0) {
            parent[u] = v;
            if (dfs(u, adj, visited, parent, color, cycle_start,
                    cycle_end))
                return true;
        } else if (color[u] == 1) {
            cycle_end = v;
            cycle_start = u;
            return true;
        }
    }
    color[v] = 2;
    return false;
}

vi find_cycle(vi2d &g, int n) {
    vc visited(n);
    vi parent(n);
    vc color(n);
    int cycle_start, cycle_end;
    color.assign(n, 0);
    parent.assign(n, -1);
    cycle_start = -1;

    for (int v = 0; v < n; v++) {

```



```

    if (color[v] == 0 && dfs(v, g, visited, parent, color,
                           cycle_start, cycle_end))
        break;
}

if (cycle_start == -1) {
    return {};
} else {
    vector<int> cycle;
    cycle.push_back(cycle_start);
    for (int v = cycle_end; v != cycle_start; v = parent[v])
        cycle.push_back(v);
    cycle.push_back(cycle_start);
    reverse(cycle.begin(), cycle.end());
    return cycle;
}
}

```

4.21 Graph Cycle (undirected)

Detects if a graph contains a cycle. If path parameter is not null, it will contain the cycle if one exists.
Time: $O(V + E)$

```

void graph_cycles(const vector<vector<int>> &g, int u,
                 int p, vector<int> &ps,
                 vector<int> &color, int &cn,
                 vector<vector<int>> &cycles) {
    if (color[u] == 2) {
        return;
    }

    if (color[u] == 1) {
        cn++;
        int cur = p;
        cycles.emplace_back();
        auto &v = cycles.back();
        v.push_back(cur);
        while (cur != u) {
            cur = ps[cur];
            v.push_back(cur);
        }
        reverse(all(v));
        return;
    }

    ps[u] = p;

```

```

    color[u] = 1;
    for (auto v : g[u]) {
        if (v != p)
            graph_cycles(g, v, u, ps, color, cn, cycles);
    }

    color[u] = 2;
}

vector<vector<int>> graph_cycles(
    const vector<vector<int>> &g) {
    vector<int> ps(g.size(), -1), color(g.size());
    int cn = 0;
    vector<vector<int>> cycles;
    for (int i = 0; i < (int)g.size(); i++)
        graph_cycles(g, i, -1, ps, color, cn, cycles);
    return cycles;
}

```

4.22 Heavy Light Decomposition

```

struct HeavyLightDecomposition {
    vector<int> parent, depth, size, heavy, head, pos;

    using SegT = int;
    static SegT op(SegT a, SegT b) { return max(a, b); }
    SegTree<SegT, op> seg;

    HeavyLightDecomposition(const vector<vector<int>> &g,
                           const vector<int> &v,
                           int root = 0)
        : parent(g.size()),
          depth(g.size()),
          size(g.size()),
          heavy(g.size(), -1),
          head(g.size()),
          pos(g.size()),
          seg((int)g.size()) {
        dfs(g, root);
        int cur_pos = 0;
        decompose(g, root, root, cur_pos);

        for (int i = 0; i < (int)g.size(); i++) {
            seg.set(pos[i], v[i]);
        }
    }
}

```

```

}

SegT query_path(int a, int b) const {
    int res = 0;
    for (; head[a] != head[b]; b = parent[head[b]]) {
        if (depth[head[a]] > depth[head[b]]) swap(a, b);
        res = op(res, seg.query(pos[head[b]], pos[b]));
    }
    if (depth[a] > depth[b]) swap(a, b);
    return op(res, seg.query(pos[a], pos[b]));
}

SegT query_subtree(int a) const {
    return seg.query(pos[a], pos[a] + size[a] - 1);
}

void set(int a, int x) { seg.set(pos[a], x); }

private:
void dfs(const vector<vector<int>> &g, int u) {
    size[u] = 1;
    int mx_child_size = 0;
    for (auto x : g[u])
        if (x != parent[u]) {
            parent[x] = u;
            depth[x] = depth[u] + 1;
            dfs(g, x);
            size[u] += size[x];
            if (size[x] > mx_child_size)
                mx_child_size = size[x], heavy[u] = x;
        }
}

void decompose(const vector<vector<int>> &g, int u, int h,
               int &cur_pos) {
    head[u] = h;
    pos[u] = cur_pos++;
    if (heavy[u] != -1) decompose(g, heavy[u], h, cur_pos);

    for (auto x : g[u])
        if (x != parent[u] and x != heavy[u]) {
            decompose(g, x, x, cur_pos);
        }
}
};

```

4.23 Kruskal

Find the minimum spanning tree of a graph.

Time: $O(E \log E)$

can be used to find the maximum spanning tree by changing the comparison operator in the sort

```

struct UFDS {
    vector<int> ps, sz;
    int components;

    UFDS(int n) : ps(n + 1), sz(n + 1, 1), components(n) {
        iota(all(ps), 0);
    }

    int find_set(int x) {
        return (x == ps[x] ? x : (ps[x] = find_set(ps[x])));
    }

    bool same_set(int x, int y) {
        return find_set(x) == find_set(y);
    }

    void union_set(int x, int y) {
        x = find_set(x);
        y = find_set(y);

        if (x == y) return;

        if (sz[x] < sz[y]) swap(x, y);

        ps[y] = x;
        sz[x] += sz[y];

        components--;
    }
};

vector<tuple<ll, int, int>> kruskal(
    int n, vector<tuple<ll, int, int>> &edges) {
    UFDS ufds(n);
    vector<tuple<ll, int, int>> ans;

    sort(all(edges));
    for (auto [a, b, c] : edges) {
        if (ufds.same_set(b, c)) continue;

        ans.emplace_back(a, b, c);
    }
}

```

```

    udfs.union_set(b, c);
}

return ans;
}

```

4.24 Maximum Flow (Edmonds-Karp)

Finds the **maximum flow** in a graph network, given the **source** s and the **sink** t .
Time: $O(V \cdot E^2)$

```

struct maxflow {
    int n;
    vi2d g;
    vll2d cps;
    vi ps;
    vector<vector<char>> isedge;

    maxflow(int _n)
        : n(_n),
          g(n),
          cps(n, vll(n)),
          ps(n),
          isedge(n, vc(n)) {}

    void add(int u, int v, ll c, bool set = true) {
        g[u].emplace_back(v);
        g[v].emplace_back(u);
        cps[u][v] = cps[u][v] * (!set) + c;
        isedge[u][v] = true;
    }

    ll bfs(int s, int t) {
        fill(all(ps), -1);
        ps[s] = -2;
        queue<pair<ll, int>> q;
        q.emplace(0, s);

        while (!q.empty()) {
            auto [flow, cur] = q.front();
            q.pop();

            for (auto next : g[cur]) {
                if (ps[next] == -1 and cps[cur][next]) {
                    ps[next] = cur;
                    ll new_flow = min(flow, cps[cur][next]);

```

```

                    if (next == t) return new_flow;
                    q.emplace(new_flow, next);
                }
            }
        }

        return 0;
    }

    ll flow(int s, int t) {
        ll flow = 0;
        ll new_flow;

        while ((new_flow = bfs(s, t))) {
            flow += new_flow;
            int cur = t;
            while (cur != s) {
                int prev = ps[cur];
                cps[prev][cur] -= new_flow;
                cps[cur][prev] += new_flow;
                cur = prev;
            }
        }

        return flow;
    }

    vector<pii> get_used() {
        vector<pii> used;
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
                if (isedge[i][j] and cps[i][j] > 0)
                    used.emplace_back(i, j);
            }
        }
        return used;
    }
};

```

4.25 Minimum Cost Flow

Given a network find the minimum cost to achieve a flow of at most f . Works with **directed** and **undirected** graphs

- **add(u, v, w, c):** adds an edge from u to v with capacity w and cost c .
- **flow(s, t, f):** return a pair $(flow, cost)$ with the maximum flow until f with source at s and sink at t , with the minimum cost possible.

Time : $O(N \cdot M + f \cdot m \log n)$

```
template <typename T>
struct mcmf {
    struct edge {
        int to, rev, flow, cap;
        bool res; // if it's a reverse edge
        T cost; // cost per unity of flow
        edge()
            : to(0),
              rev(0),
              flow(0),
              cap(0),
              cost(0),
              res(false) {}
        edge(int to_, int rev_, int flow_, int cap_, T cost_,
             bool res_)
            : to(to_),
              rev(rev_),
              flow(flow_),
              cap(cap_),
              res(res_),
              cost(cost_) {}
    };

    vector<vector<edge>> g;
    vector<int> par_idx, par;
    T inf;
    vector<T> dist;

    mcmf(int n)
        : g(n),
          par_idx(n),
          par(n),
          inf(numeric_limits<T>::max() / 3) {}

    void add(int u, int v, int w, T cost) {
        edge a = edge(v, g[v].size(), 0, w, cost, false);
        edge b = edge(u, g[u].size(), 0, 0, -cost, true);

        g[u].push_back(a);
        g[v].push_back(b);
    }

    vector<T> spfa(int s) { // don't code it if there isn't
```

```
        // negative cycles
        deque<int> q;
        vector<bool> is_inside(g.size(), 0);
        dist = vector<T>(g.size(), inf);

        dist[s] = 0;
        q.push_back(s);
        is_inside[s] = true;

        while (!q.empty()) {
            int v = q.front();
            q.pop_front();
            is_inside[v] = false;

            for (int i = 0; i < g[v].size(); i++) {
                auto [to, rev, flow, cap, res, cost] = g[v][i];
                if (flow < cap and dist[v] + cost < dist[to]) {
                    dist[to] = dist[v] + cost;

                    if (is_inside[to]) continue;
                    if (!q.empty() and dist[to] > dist[q.front()])
                        q.push_back(to);
                    else
                        q.push_front(to);
                    is_inside[to] = true;
                }
            }
        }
        return dist;
    }

    bool dijkstra(int s, int t, vector<T>& pot) {
        priority_queue<pair<T, int>, vector<pair<T, int>>,
                       greater<>>
            q;
        dist = vector<T>(g.size(), inf);
        dist[s] = 0;
        q.emplace(0, s);
        while (q.size()) {
            auto [d, v] = q.top();
            q.pop();
            if (dist[v] < d) continue;
            for (int i = 0; i < g[v].size(); i++) {
                auto [to, rev, flow, cap, res, cost] = g[v][i];
                cost += pot[v] - pot[to];
                if (flow < cap and dist[v] + cost < dist[to]) {
```

```

        dist[to] = dist[v] + cost;
        q.emplace(dist[to], to);
        par_idx[to] = i, par[to] = v;
    }
}
}
return dist[t] < inf;
}

pair<int, T> min_cost_flow(int s, int t, int flow = inf) {
    vector<T> pot(g.size(), 0);
    pot = spfa(s); // comment this line if there isn't
                  // negative cycles

    int f = 0;
    T ret = 0;
    while (f < flow and dijkstra(s, t, pot)) {
        for (int i = 0; i < g.size(); i++)
            if (dist[i] < inf) pot[i] += dist[i];

        int mn_flow = flow - f, u = t;
        while (u != s) {
            mn_flow =
                min(mn_flow, g[par[u]][par_idx[u]].cap -
                    g[par[u]][par_idx[u]].flow);
            u = par[u];
        }

        ret += pot[t] * mn_flow;

        u = t;
        while (u != s) {
            g[par[u]][par_idx[u]].flow += mn_flow;
            g[u][g[par[u]][par_idx[u]].rev].flow -= mn_flow;
            u = par[u];
        }

        f += mn_flow;
    }

    return make_pair(f, ret);
}
};

```

4.26 Minimum Cut (unweighted)

After build the **direct**/**undirected** graph find the minimum of edges needed to be removed to make the sink t unreachable from the source s .

Time: $O(V \cdot E^2)$

```

struct Mincut {
    int n;
    vi2d g;
    vii edges;
    vll2d capacity;
    vi ps, vis;

    Mincut(int _n)
        : n(_n), g(n), capacity(n, vll(n)), ps(n), vis(n) {}

    void add(int u, int v, ll c = 1, bool directed = false,
             bool set = false) {
        edges.emplace_back(u, v);
        g[u].emplace_back(v);

        if (not set)
            capacity[u][v] += c;
        else
            capacity[u][v] = c;

        if (not directed) {
            g[v].emplace_back(u);

            if (not set)
                capacity[v][u] += c;
            else
                capacity[v][u] = c;
        }
    }

    ll bfs(int s, int t) {
        fill(all(ps), -1);
        ps[s] = -2;
        queue<pair<ll, int>> q;
        q.push({0, s});

        while (!q.empty()) {
            auto [flow, cur] = q.front();
            q.pop();

            for (auto next : g[cur]) {

```

```

        if (ps[next] == -1 and capacity[cur][next]) {
            ps[next] = cur;
            ll new_flow = min(flow, capacity[cur][next]);
            if (next == t) return new_flow;
            q.push({new_flow, next});
        }
    }
}

return 0ll;
}

ll maxflow(int s, int t) {
    ll flow = 0;
    ll new_flow;

    while ((new_flow = bfs(s, t))) {
        flow += new_flow;
        int cur = t;
        while (cur != s) {
            int prev = ps[cur];
            capacity[prev][cur] -= new_flow;
            capacity[cur][prev] += new_flow;
            cur = prev;
        }
    }

    return flow;
}

void dfs(int u) {
    vis[u] = true;

    for (auto v : g[u]) {
        if (capacity[u][v] > 0 and not vis[v]) {
            dfs(v);
        }
    }
}

vii mincut(int s, int t) {
    maxflow(s, t);
    fill(all(vis), 0);
    dfs(s);

    vii removed;

```

```

        for (auto &[u, v] : edges) {
            if ((vis[u] and not vis[v]) or
                (vis[v] and not vis[u]))
                removed.emplace_back(u, v);
        }

        return removed;
    }
};

4.27 Prim (MST)

Given a graph with  $n$  vertex finds the minimum spanning tree, if there is no such three returns inf
Time:  $O(V \log E)$ 

const int maxn(1'00'000);
vector<pair<ll, int>> g[maxn];
int n;

ll prim(int s = 0) {
    priority_queue<pair<ll, int>, vector<pair<ll, int>>,
        greater<pair<ll, int>>>
        pq;
    vector<char> ingraph(maxn);
    ingraph[s] = true;
    int ingraphcnt = 1;
    for (auto &[w, v] : g[s]) pq.emplace(w, v);

    ll mstcost = 0;
    while (ingraphcnt < n) {
        ll w;
        int v;

        do {
            tie(w, v) = pq.top();
            pq.pop();
        } while (not pq.empty() and ingraph[v]);

        mstcost += w, ingraph[v] = true, ingraphcnt++;

        for (auto &[w2, v2] : g[v]) {
            pq.emplace(w2, v2);
        }
    }

    return ingraphcnt == n ? mstcost : oo;
}

```

```
}
```

4.28 Sum every node distance

Given a **tree**, for each node i find the sum of distance from i to every other node.
don't forget to set the tree as undirected, that's needed to choose an arbitrary root
 Time: $O(N)$

```
void getRoot(int u, int p, vi2d &g, vll &d, vll &cnt) {
    for (int i = 0; i < len(g[u]); i++) {
        int v = g[u][i];
        if (v == p) continue;
        getRoot(v, u, g, d, cnt);
        d[u] += d[v] + cnt[v];
        cnt[u] += cnt[v];
    }
}

void dfs(int u, int p, vi2d &g, vll &cnt, vll &ansd,
        int n) {
    for (int i = 0; i < len(g[u]); i++) {
        int v = g[u][i];
        if (v == p) continue;

        ansd[v] = ansd[u] - cnt[v] + (n - cnt[v]);
        dfs(v, u, g, cnt, ansd, n);
    }
}

vll fromToAll(vi2d &g, int n) {
    vll d(n);
    vll cnt(n, 1);
    getRoot(0, -1, g, d, cnt);

    vll ansdist(n);
    ansdist[0] = d[0];

    dfs(0, -1, g, cnt, ansdist, n);
    return ansdist;
}
```

4.29 Topological Sorting (Tarjan)

Finds a the topological order for the graph, if there is no such order it means the graph is cyclic, then it returns an empty vector
 $O(V + E)$

```
const int maxn(1'00'000);
int n, m;
vi g[maxn];

int not_found = 0, found = 1, processed = 2;
int state[maxn];

bool dfs(int u, vi &order) {
    if (state[u] == processed) return true;
    if (state[u] == found) return false;

    state[u] = found;

    for (auto v : g[u]) {
        if (not dfs(v, order)) return false;
    }

    state[u] = processed;
    order.emplace_back(u);

    return true;
}

vi topo_sort() {
    vi order;
    memset(state, 0, sizeof state);

    for (int u = 0; u < n; u++) {
        if (state[u] == not_found and not dfs(u, order))
            return {};
    }

    reverse(all(order));
    return order;
}
```

5 Math

5.1 GCD (with factorization)

$O(\sqrt{n})$ due to factorization.

```
ll gcd_with_factorization(ll a, ll b) {
    map<ll, ll> fa = factorization(a);
    map<ll, ll> fb = factorization(b);
```

```

11 ans = 1;
for (auto fai : fa) {
    11 k = min(fai.second, fb[fai.first]);
    while (k--) ans *= fai.first;
}
return ans;
}

```

5.2 GCD

```

11 gcd(11 a, 11 b) { return b ? gcd(b, a % b) : a; }

```

5.3 LCM (with factorization)

$O(\sqrt{n})$ due to factorization.

```

11 lcm_with_factorization(11 a, 11 b) {
    map<11, 11> fa = factorization(a);
    map<11, 11> fb = factorization(b);
    11 ans = 1;
    for (auto fai : fa) {
        11 k = max(fai.second, fb[fai.first]);
        while (k--) ans *= fai.first;
    }
    return ans;
}

```

5.4 LCM

```

11 gcd(11 a, 11 b) { return b ? gcd(b, a % b) : a; }
11 lcm(11 a, 11 b) { return a / gcd(a, b) * b; }

```

5.5 Arithmetic Progression Sum

- s : first term
- d : common difference
- n : number of terms

```

11 arithmeticProgressionSum(11 s, 11 d, 11 n) {
    return (s + (s + d * (n - 1))) * n / 211;
}

```

5.6 Binomial MOD

Precompute every factorial until $maxn$ ($O(maxn)$) allowing to answer the $\binom{n}{k}$ in $O(\log mod)$ time, due to the fastpow. Note that it needs $O(maxn)$ in memory.

```

const 11 MOD = 1e9 + 7;
const 11 maxn = 2 * 1e6;
v11 fats(maxn + 1, -1);
void precompute() {
    fats[0] = 1;
    for (11 i = 1; i <= maxn; i++) {
        fats[i] = (fats[i - 1] * i) % MOD;
    }
}

11 fpow(11 a, 11 n, 11 mod = LLONG_MAX) {
    if (n == 011) return 111;
    if (n == 111) return a;
    11 x = fpow(a, n / 211, mod) % mod;
    return ((x * x) % mod * (n & 111 ? a : 111)) % mod;
}

11 binommod(11 n, 11 k) {
    11 upper = fats[n];
    11 lower = (fats[k] * fats[n - k]) % MOD;
    return (upper * fpow(lower, MOD - 211, MOD)) % MOD;
}

```

5.7 Binomial

$O(nm)$ time, $O(m)$ space
Equal to n choose k

```

11 binom(11 n, 11 k) {
    if (k > n) return 0;
    v11 dp(k + 1, 0);
    dp[0] = 1;
    for (11 i = 1; i <= n; i++)
        for (11 j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
    return dp[k];
}

```

5.8 Euler phi $\varphi(n)$ (in range)

Computes the number of positive integers less than n that are coprimes with n , in the range $[1, n]$, in $O(N \log N)$.

```

const int MAX = 1e6;
vi range_phi(int n) {
    bitset<MAX> sieve;
    vi phi(n + 1);

```



```

iota(phi.begin(), phi.end(), 0);
sieve.set();

for (int p = 2; p <= n; p += 2) phi[p] /= 2;
for (int p = 3; p <= n; p += 2) {
    if (sieve[p]) {
        for (int j = p; j <= n; j += p) {
            sieve[j] = false;
            phi[j] /= p;
            phi[j] *= (p - 1);
        }
    }
}

return phi;
}

```

5.9 Euler phi $\varphi(n)$

Computes the number of positive integers less than n that are coprimes with n , in $O(\sqrt{N})$.

```

int phi(int n) {
    if (n == 1) return 1;

    auto fs = factorization(n); // a vector of pair or a map
    auto res = n;

    for (auto [p, k] : fs) {
        res /= p;
        res *= (p - 1);
    }

    return res;
}

```

5.10 Factorial Factorization

Computes the factorization of $n!$ in $\pi(N) * \log n$

```

// O(logN)
ll E(ll n, ll p) {
    ll k = 0, b = p;
    while (b <= n) {
        k += n / b;
        b *= p;
    }
}

```

```

return k;
}

// O(pi(N)*logN)
map<ll, ll> factorial_factorization(ll n,
                                   const vll &primes) {

    map<ll, ll> fs;
    for (const auto &p : primes) {
        if (p > n) break;
        fs[p] = E(n, p);
    }
    return fs;
}

```

5.11 Factorial

```

const ll MAX = 18;
vll fv(MAX, -1);
ll factorial(ll n) {
    if (fv[n] != -1) return fv[n];
    if (n == 0) return 1;
    return n * factorial(n - 1);
}

```

5.12 Factorization (Pollard Rho)

Factorizes a number into its prime factors in $O(n^{\frac{1}{4}} * \log(n))$.

```

ll mul(ll a, ll b, ll m) {
    ll ret = a * b - (ll)((ld)1 / m * a * b + 0.5) * m;
    return ret < 0 ? ret + m : ret;
}

ll pow(ll a, ll b, ll m) {
    ll ans = 1;
    for (; b > 0; b /= 2ll, a = mul(a, a, m)) {
        if (b % 2ll == 1) ans = mul(ans, a, m);
    }
    return ans;
}

bool prime(ll n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;
}

```

```

ll r = __builtin_ctzll(n - 1), d = n >> r;
for (int a :
    {2, 325, 9375, 28178, 450775, 9780504, 795265022}) {
    ll x = pow(a, d, n);
    if (x == 1 or x == n - 1 or a % n == 0) continue;

    for (int j = 0; j < r - 1; j++) {
        x = mul(x, x, n);
        if (x == n - 1) break;
    }
    if (x != n - 1) return 0;
}
return 1;
}

```

```

ll rho(ll n) {
    if (n == 1 or prime(n)) return n;
    auto f = [n](ll x) { return mul(x, x, n) + 1; };

```

```

    ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
    while (t % 40 != 0 or gcd(prd, n) == 1) {
        if (x == y) x = ++x0, y = f(x);
        q = mul(prd, abs(x - y), n);
        if (q != 0) prd = q;
        x = f(x), y = f(f(y)), t++;
    }
    return gcd(prd, n);
}

```

```

vll fact(ll n) {
    if (n == 1) return {};
    if (prime(n)) return {n};
    ll d = rho(n);
    vll l = fact(d), r = fact(n / d);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}

```

5.13 Factorization

Computes the factorization of n in $O(\sqrt{n})$.

```

map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;

```

```

        for (; n % i == 0; count++, n /= i)
            ;
        if (count) ans[i] = count;
    }
    if (n > 1) ans[n]++;
    return ans;
}

```

5.14 Fast Fourier Transform

```

template <bool invert = false>
void fft(vector<complex<double>>& xs) {
    int N = (int)xs.size();

    if (N == 1) return;

    vector<complex<double>> es(N / 2), os(N / 2);

    for (int i = 0; i < N / 2; ++i) es[i] = xs[2 * i];

    for (int i = 0; i < N / 2; ++i) os[i] = xs[2 * i + 1];

    fft<invert>(es);
    fft<invert>(os);

    auto signal = (invert ? 1 : -1);
    auto theta = 2 * signal * acos(-1) / N;
    complex<double> S{1}, S1{cos(theta), sin(theta)};

    for (int i = 0; i < N / 2; ++i) {
        xs[i] = (es[i] + S * os[i]);
        xs[i] /= (invert ? 2 : 1);

        xs[i + N / 2] = (es[i] - S * os[i]);
        xs[i + N / 2] /= (invert ? 2 : 1);

        S *= S1;
    }
}

```

5.15 Fast pow

Computes a^n in $O(\log N)$.

```

ll fpow(ll a, int n, ll mod = LLONG_MAX) {
    if (n == 0) return 1;

```

```

if (n == 1) return a;
ll x = fpow(a, n / 2, mod) % mod;
return ((x * x) % mod * (n & 1 ? a : 1ll)) % mod;
}

```

5.16 Gauss Elimination

```

template <size_t Dim>
struct GaussianElimination {
    vector<ll> basis;
    size_t size;

    GaussianElimination() : basis(Dim + 1), size(0) {}

    void insert(ll x) {
        for (ll i = Dim; i >= 0; i--) {
            if ((x & 1ll << i) == 0) continue;

            if (!basis[i]) {
                basis[i] = x;
                size++;
                break;
            }

            x ^= basis[i];
        }
    }

    void normalize() {
        for (ll i = Dim; i >= 0; i--)
            for (ll j = i - 1; j >= 0; j--)
                if (basis[i] & 1ll << j) basis[i] ^= basis[j];
    }

    bool check(ll x) {
        for (ll i = Dim; i >= 0; i--) {
            if ((x & 1ll << i) == 0) continue;

            if (!basis[i]) return false;

            x ^= basis[i];
        }

        return true;
    }
}

```

```

auto operator[] (ll k) { return at(k); }

```

```

ll at(ll k) {
    ll ans = 0;
    ll total = 1ll << size;
    for (ll i = Dim; ~i; i--) {
        if (!basis[i]) continue;

        ll mid = total >> 1ll;
        if ((mid < k and (ans & 1ll << i) == 0) ||
            (k <= mid and (ans & 1ll << i)))
            ans ^= basis[i];

        if (mid < k) k -= mid;

        total >>= 1ll;
    }
    return ans;
}

ll at_normalized(ll k) {
    ll ans = 0;
    k--;
    for (size_t i = 0; i <= Dim; i++) {
        if (!basis[i]) continue;
        if (k & 1) ans ^= basis[i];
        k >>= 1;
    }
    return ans;
}
};

```

5.17 Integer Mod

```

const ll INF = 1e18;
const ll mod = 998244353;
template <ll MOD = mod>
struct Modular {
    ll value;
    static const ll MOD_value = MOD;

    Modular(ll v = 0) {
        value = v % MOD;
        if (value < 0) value += MOD;
    }
}

```

```

}
Modular(11 a, 11 b) : value(0) {
    *this += a;
    *this /= b;
}

Modular& operator+=(Modular const& b) {
    value += b.value;
    if (value >= MOD) value -= MOD;
    return *this;
}

Modular& operator--(Modular const& b) {
    value -= b.value;
    if (value < 0) value += MOD;
    return *this;
}

Modular& operator*=(Modular const& b) {
    value = (11)value * b.value % MOD;
    return *this;
}

friend Modular mexp(Modular a, 11 e) {
    Modular res = 1;
    while (e) {
        if (e & 1) res *= a;
        a *= a;
        e >>= 1;
    }
    return res;
}

friend Modular inverse(Modular a) {
    return mexp(a, MOD - 2);
}

Modular& operator/=(Modular const& b) {
    return *this *= inverse(b);
}

friend Modular operator+(Modular a, Modular const b) {
    return a += b;
}

Modular operator++(int) {
    return this->value = (this->value + 1) % MOD;
}

Modular operator++() {
    return this->value = (this->value + 1) % MOD;
}

```

```

}
friend Modular operator-(Modular a, Modular const b) {
    return a -= b;
}

friend Modular operator-(Modular const a) {
    return 0 - a;
}

Modular operator--(int) {
    return this->value = (this->value - 1 + MOD) % MOD;
}

Modular operator--() {
    return this->value = (this->value - 1 + MOD) % MOD;
}

friend Modular operator*(Modular a, Modular const b) {
    return a *= b;
}

friend Modular operator/(Modular a, Modular const b) {
    return a /= b;
}

friend std::ostream& operator<<(std::ostream& os,
                                Modular const& a) {
    return os << a.value;
}

friend bool operator==(Modular const& a,
                        Modular const& b) {
    return a.value == b.value;
}

friend bool operator!=(Modular const& a,
                        Modular const& b) {
    return a.value != b.value;
}
};

```

5.18 Is prime

$O(\sqrt{N})$

```

bool isprime(11 n) {
    if (n < 2) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    for (11 i = 3; i * i < n; i += 2)
        if (n % i == 0) return false;
    return true;
}

```

5.19 N Choose K (elements)

process every possible combination of K elements from N elements, those index marked as 1 in the index vector says which elements are chosen at that moment.

Time : $O(\binom{N}{K} \cdot O(\text{process}))$

```
void process(vi &index) {
    for (int i = 0; i < len(index); i++) {
        if (index[i]) cout << i << " \n"[i == len(index) - 1];
    }
}

void n_choose_k(int n, int k) {
    vi index(n);
    fill(index.end() - k, index.end(), 1);

    do {
        process(index);
    } while (next_permutation(all(index)));
}
```

5.20 Number Of Divisors (sieve)

```
ll divisors(ll n) {
    ll ans = 1;
    for (auto p : primes) {
        if (p * p * p > n) break;

        int count = 1;
        while (n % p == 0) {
            n /= p;
            count++;
        }

        ans *= count;
    }

    if (is_prime[n])
        ans *= 2;
    else if (is_prime_square[n])
        ans *= 3;
    else if (n != 1)
        ans *= 4;

    return ans;
}
```

5.21 Number of Divisors $\tau(n)$

Find the total of divisors of N in $O(\sqrt{N})$

```
ll number_of_divisors(ll n) {
    ll res = 0;

    for (ll d = 1; d * d <= n; ++d) {
        if (n % d == 0) res += (d == n / d ? 1 : 2);
    }

    return res;
}
```

5.22 Power Sum

Calculates $K^0 + K^1 + \dots + K^n$

```
ll powersum(ll n, ll k) {
    return (fastpow(n, k + 1) - 1) / (k - 1);
}
```

5.23 Sieve list primes

List every prime until MAXN , $O(N \log N)$ in time and $O(\text{MAXN})$ in memory.

```
const ll MAXN = 1e5;
vll list_primes(ll n) {
    vll ps;
    bitset<MAXN> sieve;
    sieve.set();
    sieve.reset(1);
    for (ll i = 2; i <= n; ++i) {
        if (sieve[i]) ps.push_back(i);
        for (ll j = i * 2; j <= n; j += i) {
            sieve.reset(j);
        }
    }
    return ps;
}
```

5.24 Sum of Divisors $\sigma(n)$

Computes the sum of each divisor of n in $O(\sqrt{n})$.

```
ll sum_of_divisors(long long n) {
    ll res = 0;

    for (ll d = 1; d * d <= n; ++d) {
```

```

    if (n % d == 0) {
        ll k = n / d;

        res += (d == k ? d : d + k);
    }
}

return res;
}

```

6 Primitives

6.1 Bigint

```

const int maxn = 1e2 + 14, lg = 15;
const int base = 1000000000;
const int base_digits = 9;
struct bigint {
    vi a;
    int sign;

    int size() {
        if (a.empty()) return 0;
        int ans = (a.size() - 1) * base_digits;
        int ca = a.back();
        while (ca) ans++, ca /= 10;
        return ans;
    }

    bigint operator^(const bigint &v) {
        bigint ans = 1, a = *this, b = v;
        while (!b.isZero()) {
            if (b % 2) ans *= a;
            a *= a, b /= 2;
        }
        return ans;
    }

    string to_string() {
        stringstream ss;
        ss << *this;
        string s;
        ss >> s;
        return s;
    }

    int sumof() {

```

```

        string s = to_string();
        int ans = 0;
        for (auto c : s) ans += c - '0';
        return ans;
    }

    /*</arpa>*/
    bigint() : sign(1) {}

    bigint(long long v) { *this = v; }

    bigint(const string &s) { read(s); }

    void operator=(const bigint &v) {
        sign = v.sign;
        a = v.a;
    }

    void operator=(long long v) {
        sign = 1;
        a.clear();
        if (v < 0) sign = -1, v = -v;
        for (; v > 0; v = v / base) a.push_back(v % base);
    }

    bigint operator+(const bigint &v) const {
        if (sign == v.sign) {
            bigint res = v;

            for (int i = 0, carry = 0;
                 i < (int)max(a.size(), v.a.size()) || carry;
                 ++i) {
                if (i == (int)res.a.size()) res.a.push_back(0);
                res.a[i] += carry + (i < (int)a.size() ? a[i] : 0);
                carry = res.a[i] >= base;
                if (carry) res.a[i] -= base;
            }
            return res;
        }
        return *this - (-v);
    }

    bigint operator-(const bigint &v) const {
        if (sign == v.sign) {
            if (abs() >= v.abs()) {
                bigint res = *this;

```

```

        for (int i = 0, carry = 0;
            i < (int)v.a.size() || carry; ++i) {
            res.a[i] -=
                carry + (i < (int)v.a.size() ? v.a[i] : 0);
            carry = res.a[i] < 0;
            if (carry) res.a[i] += base;
        }
        res.trim();
        return res;
    }
    return -(v - *this);
}
return *this + (-v);
}

void operator*=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = 0, carry = 0; i < (int)a.size() || carry;
        ++i) {
        if (i == (int)a.size()) a.push_back(0);
        long long cur = a[i] * (long long)v + carry;
        carry = (int)(cur / base);
        a[i] = (int)(cur % base);
        // asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) :
        // "A"(cur), "c"(base));
    }
    trim();
}

bigint operator*(int v) const {
    bigint res = *this;
    res *= v;
    return res;
}

void operator*=(long long v) {
    if (v < 0) sign = -sign, v = -v;
    if (v > base) {
        *this =
            *this * (v / base) * base + *this * (v % base);
        return;
    }
    for (int i = 0, carry = 0; i < (int)a.size() || carry;
        ++i) {
        if (i == (int)a.size()) a.push_back(0);

```

```

        long long cur = a[i] * (long long)v + carry;
        carry = (int)(cur / base);
        a[i] = (int)(cur % base);
        // asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) :
        // "A"(cur), "c"(base));
    }
    trim();
}

bigint operator*(long long v) const {
    bigint res = *this;
    res *= v;
    return res;
}

friend pair<bigint, bigint> divmod(const bigint &a1,
                                   const bigint &b1) {
    int norm = base / (b1.a.back() + 1);
    bigint a = a1.abs() * norm;
    bigint b = b1.abs() * norm;
    bigint q, r;
    q.a.resize(a.a.size());

    for (int i = a.a.size() - 1; i >= 0; i--) {
        r *= base;
        r += a.a[i];
        int s1 =
            r.a.size() <= b.a.size() ? 0 : r.a[b.a.size()];
        int s2 = r.a.size() <= b.a.size() - 1
            ? 0
            : r.a[b.a.size() - 1];
        int d = ((long long)base * s1 + s2) / b.a.back();
        r -= b * d;
        while (r < 0) r += b, --d;
        q.a[i] = d;
    }

    q.sign = a1.sign * b1.sign;
    r.sign = a1.sign;
    q.trim();
    r.trim();
    return make_pair(q, r / norm);
}

bigint operator/(const bigint &v) const {

```

```

    return divmod(*this, v).first;
}

bigint operator%(const bigint &v) const {
    return divmod(*this, v).second;
}

void operator/=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = (int)a.size() - 1, rem = 0; i >= 0; --i) {
        long long cur = a[i] + rem * (long long)base;
        a[i] = (int)(cur / v);
        rem = (int)(cur % v);
    }
    trim();
}

bigint operator/(int v) const {
    bigint res = *this;
    res /= v;
    return res;
}

int operator%(int v) const {
    if (v < 0) v = -v;
    int m = 0;
    for (int i = a.size() - 1; i >= 0; --i)
        m = (a[i] + m * (long long)base) % v;
    return m * sign;
}

void operator+=(const bigint &v) { *this = *this + v; }
void operator-=(const bigint &v) { *this = *this - v; }
void operator*=(const bigint &v) { *this = *this * v; }
void operator/=(const bigint &v) { *this = *this / v; }

bool operator<(const bigint &v) const {
    if (sign != v.sign) return sign < v.sign;
    if (a.size() != v.a.size())
        return a.size() * sign < v.a.size() * v.sign;
    for (int i = a.size() - 1; i >= 0; i--)
        if (a[i] != v.a[i])
            return a[i] * sign < v.a[i] * sign;
    return false;
}

```

```

bool operator>(const bigint &v) const {
    return v < *this;
}

bool operator<=(const bigint &v) const {
    return !(v < *this);
}

bool operator>=(const bigint &v) const {
    return !(*this < v);
}

bool operator==(const bigint &v) const {
    return !(*this < v) && !(v < *this);
}

bool operator!=(const bigint &v) const {
    return *this < v || v < *this;
}

void trim() {
    while (!a.empty() && !a.back()) a.pop_back();
    if (a.empty()) sign = 1;
}

bool isZero() const {
    return a.empty() || (a.size() == 1 && !a[0]);
}

bigint operator-() const {
    bigint res = *this;
    res.sign = -sign;
    return res;
}

bigint abs() const {
    bigint res = *this;
    res.sign *= res.sign;
    return res;
}

long long longValue() const {
    long long res = 0;
    for (int i = a.size() - 1; i >= 0; i--)
        res = res * base + a[i];
    return res * sign;
}

```



```

friend bigint gcd(const bigint &a, const bigint &b) {
    return b.isZero() ? a : gcd(b, a % b);
}

friend bigint lcm(const bigint &a, const bigint &b) {
    return a / gcd(a, b) * b;
}

void read(const string &s) {
    sign = 1;
    a.clear();
    int pos = 0;
    while (pos < (int)s.size() &&
           (s[pos] == '-' || s[pos] == '+')) {
        if (s[pos] == '-') sign = -sign;
        ++pos;
    }
    for (int i = s.size() - 1; i >= pos; i -= base_digits) {
        int x = 0;
        for (int j = max(pos, i - base_digits + 1); j <= i;
             j++)
            x = x * 10 + s[j] - '0';
        a.push_back(x);
    }
    trim();
}

friend istream &operator>>(istream &stream, bigint &v) {
    string s;
    stream >> s;
    v.read(s);
    return stream;
}

friend ostream &operator<<(ostream &stream,
                           const bigint &v) {
    if (v.sign == -1) stream << '-';
    stream << (v.a.empty() ? 0 : v.a.back());
    for (int i = (int)v.a.size() - 2; i >= 0; --i)
        stream << setw(base_digits) << setfill('0') << v.a[i];
    return stream;
}

static vector<int> convert_base(const vector<int> &a,
                                int old_digits,
                                int new_digits) {

```

```

    vector<long long> p(max(old_digits, new_digits) + 1);
    p[0] = 1;
    for (int i = 1; i < (int)p.size(); i++)
        p[i] = p[i - 1] * 10;
    vector<int> res;
    long long cur = 0;
    int cur_digits = 0;
    for (int i = 0; i < (int)a.size(); i++) {
        cur += a[i] * p[cur_digits];
        cur_digits += old_digits;
        while (cur_digits >= new_digits) {
            res.push_back((int)(cur % p[new_digits]));
            cur /= p[new_digits];
            cur_digits -= new_digits;
        }
    }
    res.push_back((int)cur);
    while (!res.empty() && !res.back()) res.pop_back();
    return res;
}

typedef vector<long long> vll;

static vll karatsubaMultiply(const vll &a, const vll &b) {
    int n = a.size();
    vll res(n + n);
    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                res[i + j] += a[i] * b[j];
        return res;
    }

    int k = n >> 1;
    vll a1(a.begin(), a.begin() + k);
    vll a2(a.begin() + k, a.end());
    vll b1(b.begin(), b.begin() + k);
    vll b2(b.begin() + k, b.end());

    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);

    for (int i = 0; i < k; i++) a2[i] += a1[i];
    for (int i = 0; i < k; i++) b2[i] += b1[i];

```

```

vll r = karatsubaMultiply(a2, b2);
for (int i = 0; i < (int)a1b1.size(); i++)
    r[i] -= a1b1[i];
for (int i = 0; i < (int)a2b2.size(); i++)
    r[i] -= a2b2[i];

for (int i = 0; i < (int)r.size(); i++)
    res[i + k] += r[i];
for (int i = 0; i < (int)a1b1.size(); i++)
    res[i] += a1b1[i];
for (int i = 0; i < (int)a2b2.size(); i++)
    res[i + n] += a2b2[i];
return res;
}

bigint operator*(const bigint &v) const {
    vector<int> a6 = convert_base(this->a, base_digits, 6);
    vector<int> b6 = convert_base(v.a, base_digits, 6);
    vll a(a6.begin(), a6.end());
    vll b(b6.begin(), b6.end());
    while (a.size() < b.size()) a.push_back(0);
    while (b.size() < a.size()) b.push_back(0);
    while (a.size() & (a.size() - 1))
        a.push_back(0), b.push_back(0);
    vll c = karatsubaMultiply(a, b);
    bigint res;
    res.sign = sign * v.sign;
    for (int i = 0, carry = 0; i < (int)c.size(); i++) {
        long long cur = c[i] + carry;
        res.a.push_back((int)(cur % 1000000));
        carry = (int)(cur / 1000000);
    }
    res.a = convert_base(res.a, 6, base_digits);
    res.trim();
    return res;
}
};

```

6.2 Integer Mod

```

const ll INF = 1e18;
const ll mod = 998244353;
template <ll MOD = mod>
struct Modular {
    ll value;

```

```

static const ll MOD_value = MOD;

Modular(ll v = 0) {
    value = v % MOD;
    if (value < 0) value += MOD;
}

Modular(ll a, ll b) : value(0) {
    *this += a;
    *this /= b;
}

Modular& operator+=(Modular const& b) {
    value += b.value;
    if (value >= MOD) value -= MOD;
    return *this;
}

Modular& operator-=(Modular const& b) {
    value -= b.value;
    if (value < 0) value += MOD;
    return *this;
}

Modular& operator*=(Modular const& b) {
    value = (ll)value * b.value % MOD;
    return *this;
}

friend Modular mexp(Modular a, ll e) {
    Modular res = 1;
    while (e) {
        if (e & 1) res *= a;
        a *= a;
        e >>= 1;
    }
    return res;
}

friend Modular inverse(Modular a) {
    return mexp(a, MOD - 2);
}

Modular& operator/=(Modular const& b) {
    return *this *= inverse(b);
}

friend Modular operator+(Modular a, Modular const b) {
    return a += b;
}

```

```

Modular operator++(int) {
    return this->value = (this->value + 1) % MOD;
}
Modular operator++() {
    return this->value = (this->value + 1) % MOD;
}
friend Modular operator--(Modular a, Modular const b) {
    return a -= b;
}
friend Modular operator--(Modular const a) {
    return 0 - a;
}
Modular operator--(int) {
    return this->value = (this->value - 1 + MOD) % MOD;
}
Modular operator--() {
    return this->value = (this->value - 1 + MOD) % MOD;
}
friend Modular operator*(Modular a, Modular const b) {
    return a *= b;
}
friend Modular operator/(Modular a, Modular const b) {
    return a /= b;
}
friend std::ostream& operator<<(std::ostream& os,
                                Modular const& a) {
    return os << a.value;
}
friend bool operator==(Modular const& a,
                        Modular const& b) {
    return a.value == b.value;
}
friend bool operator!=(Modular const& a,
                        Modular const& b) {
    return a.value != b.value;
}
};

```

6.3 Matrix

```

template <typename T>
struct Matrix {
    vector<vector<T>> d;

```

```

Matrix() : Matrix(0) {}
Matrix(int n) : Matrix(n, n) {}
Matrix(int n, int m)
    : Matrix(vector<vector<T>>(n, vector<T>(m))) {}
Matrix(const vector<vector<T>> &v) : d(v) {}

constexpr int n() const { return (int)d.size(); }
constexpr int m() const {
    return n() ? (int)d[0].size() : 0;
}

void rotate() { *this = rotated(); }

Matrix<T> rotated() const {
    Matrix<T> res(m(), n());
    for (int i = 0; i < m(); i++) {
        for (int j = 0; j < n(); j++) {
            res[i][j] = d[n() - j - 1][i];
        }
    }
    return res;
}

Matrix<T> pow(int power) const {
    assert(n() == m());

    auto res = Matrix<T>::identity(n());
    auto b = *this;
    while (power) {
        if (power & 1) res *= b;
        b *= b;
        power >>= 1;
    }
    return res;
}

Matrix<T> submatrix(int start_i, int start_j,
                    int rows = INT_MAX,
                    int cols = INT_MAX) const {
    rows = min(rows, n() - start_i);
    cols = min(cols, m() - start_j);
    if (rows <= 0 or cols <= 0) return {};

    Matrix<T> res(rows, cols);
    for (int i = 0; i < rows; i++)

```

```

        for (int j = 0; j < cols; j++)
            res[i][j] = d[i + start_i][j + start_j];
    return res;
}

Matrix<T> translated(int x, int y) const {
    Matrix<T> res(n(), m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            if (i + x < 0 or i + x >= n() or j + y < 0 or
                j + y >= m())
                continue;
            res[i + x][j + y] = d[i][j];
        }
    }
    return res;
}

static Matrix<T> identity(int n) {
    Matrix<T> res(n);
    for (int i = 0; i < n; i++) res[i][i] = 1;
    return res;
}

vector<T> &operator[](int i) { return d[i]; }
const vector<T> &operator[](int i) const { return d[i]; }
Matrix<T> &operator+=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x += value;
    }
    return *this;
}

Matrix<T> operator+(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x + value;
    }
    return res;
}

Matrix<T> &operator-=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x -= value;
    }
    return *this;
}

```

```

Matrix<T> operator-(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x - value;
    }
    return res;
}

Matrix<T> &operator*=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x *= value;
    }
    return *this;
}

Matrix<T> operator*(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x * value;
    }
    return res;
}

Matrix<T> &operator/=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x /= value;
    }
    return *this;
}

Matrix<T> operator/(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x / value;
    }
    return res;
}

Matrix<T> &operator+=(const Matrix<T> &o) {
    assert(n() == o.n() and m() == o.m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            d[i][j] += o[i][j];
        }
    }
    return *this;
}

Matrix<T> operator+(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());
    auto res = *this;

```

```

    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            res[i][j] = res[i][j] + o[i][j];
        }
    }
    return res;
}

Matrix<T> &operator+=(const Matrix<T> &o) {
    assert(n() == o.n() and m() == o.m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            d[i][j] += o[i][j];
        }
    }
    return *this;
}

Matrix<T> operator-(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());
    auto res = *this;
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            res[i][j] = res[i][j] - o[i][j];
        }
    }
    return res;
}

Matrix<T> &operator*=(const Matrix<T> &o) {
    *this = *this * o;
    return *this;
}

Matrix<T> operator*(const Matrix<T> &o) const {
    assert(m() == o.n());
    Matrix<T> res(n(), o.m());
    for (int i = 0; i < res.n(); i++) {
        for (int j = 0; j < res.m(); j++) {
            auto &x = res[i][j];
            for (int k = 0; k < m(); k++) {
                x += (d[i][k] * o[k][j]);
            }
        }
    }
    return res;
}

friend istream &operator>>(istream &is, Matrix<T> &mat) {

```

```

    for (auto &row : mat)
        for (auto &x : row) is >> x;
    return is;
}

friend ostream &operator<<(ostream &os,
                           const Matrix<T> &mat) {

    bool frow = 1;
    for (auto &row : mat) {
        if (not frow) os << '\n';
        bool first = 1;
        for (auto &x : row) {
            if (not first) os << ' ';
            os << x;
            first = 0;
        }

        frow = 0;
    }
    return os;
}

auto begin() { return d.begin(); }
auto end() { return d.end(); }
auto rbegin() { return d.rbegin(); }
auto rend() { return d.rend(); }

auto begin() const { return d.begin(); }
auto end() const { return d.end(); }
auto rbegin() const { return d.rbegin(); }
auto rend() const { return d.rend(); }
};

```

7 Problems

7.1 Hanoi Tower

Let T_n be the total of moves to solve a hanoi tower, we know that $T_n \geq 2 \cdot T_{n-1} + 1$, for $n > 0$, and $T_0 = 0$. By induction it's easy to see that $T_n = 2^n - 1$, for $n > 0$.

The following algorithm finds the necessary steps to solve the game for 3 stacks and n disks.

```

void move(int a, int b) { cout << a << ' ' << b << endl; }
void solve(int n, int s, int e) {
    if (n == 0) return;
    if (n == 1) {
        move(s, e);
        return;
    }
}

```

```

    solve(n - 1, s, 6 - s - e);
    move(s, e);
    solve(n - 1, 6 - s - e, e);
}

```

8 Searching

8.1 Meet in the middle

Answers the query how many subsets of the vector xs have sum equal x .

Time: $O(N \cdot 2^{\frac{N}{2}})$

```

vll get_subset_sums(int l, int r, vll &a) {
    int len = r - l + 1;
    vll res;

    for (int i = 0; i < (1 << len); i++) {
        ll sum = 0;
        for (int j = 0; j < len; j++) {
            if (i & (1 << j)) {
                sum += a[l + j];
            }
        }
        res.push_back(sum);
    }
    return res;
};

ll count(vll &xs, ll x) {
    int n = len(xs);
    vll left = get_subset_sums(0, n / 2 - 1, xs);
    vll right = get_subset_sums(n / 2, n - 1, xs);
    sort(all(left));
    sort(all(right));
    ll ans = 0;
    for (ll i : left) {
        auto start_index =
            lower_bound(right.begin(), right.end(), x - i) -
            right.begin();
        auto end_index =
            upper_bound(right.begin(), right.end(), x - i) -
            right.begin();
        ans += end_index - start_index;
    }
    return ans;
}

```

8.2 Ternary Search Recursive

```

const double eps = 1e-6;

// IT MUST BE AN UNIMODAL FUNCTION
double f(int x) { return x * x + 2 * x + 4; }

double ternary_search(double l, double r) {
    if (fabs(f(l) - f(r)) < eps)
        return f((l + (r - l) / 2.0));

    auto third = (r - l) / 3.0;
    auto m1 = l + third;
    auto m2 = r - third;

    // change the signal to find the maximum point.
    return m1 < m2 ? ternary_search(m1, r)
        : ternary_search(l, m2);
}

```

9 Strings

9.1 Count Distinct Anagrams

```

const ll MOD = 1e9 + 7;
const int maxn = 1e6;
vll fs(maxn + 1);
void precompute() {
    fs[0] = 1;
    for (ll i = 1; i <= maxn; i++) {
        fs[i] = (fs[i - 1] * i) % MOD;
    }
}

ll fpow(ll a, int n, ll mod = LLONG_MAX) {
    if (n == 0) return 1;
    if (n == 1) return a;
    ll x = fpow(a, n / 2, mod) % mod;
    return ((x * x) % mod * (n & 1 ? a : 1)) % mod;
}

ll distinctAnagrams(const string &s) {
    precompute();
    vi hist('z' - 'a' + 1, 0);
    for (auto &c : s) hist[c - 'a']++;
}

```

```

11 ans = fs[len(s)];
for (auto &q : hist) {
    ans = (ans * fpow(fs[q], MOD - 2, MOD)) % MOD;
}
return ans;
}

```

9.2 Double Hash Range Query

```

const ll MOD = 1000027957;
const int MOD2 = 1000015187;

struct Hash {
    const ll P = 31;
    int n;
    string s;
    vll h, h2, hi, hi2, p, p2;
    Hash() {}
    Hash(string _s)
        : s(_s),
          n(len(_s)),
          h(n),
          h2(n),
          hi(n),
          hi2(n),
          p(n),
          p2(n) {
        for (int i = 0; i < n; i++)
            p[i] = (i ? P * p[i - 1] : 1) % MOD;
        for (int i = 0; i < n; i++)
            p2[i] = (i ? P * p2[i - 1] : 1) % MOD2;
        for (int i = 0; i < n; i++)
            h[i] = (s[i] + (i ? h[i - 1] : 0) * P) % MOD;
        for (int i = 0; i < n; i++)
            h2[i] = (s[i] + (i ? h2[i - 1] : 0) * P) % MOD2;
        for (int i = n - 1; i >= 0; i--)
            hi[i] =
                (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % MOD;
        for (int i = n - 1; i >= 0; i--)
            hi2[i] =
                (s[i] + (i + 1 < n ? hi2[i + 1] : 0) * P) % MOD2;
    }
    pii query(int l, int r) {
        ll hash =
            (h[r] - (l ? h[l - 1] * p[r - l + 1] % MOD : 0));

```

```

11 hash2 =
    (h2[r] - (l ? h2[l - 1] * p2[r - l + 1] % MOD2 : 0));

    return {(hash < 0 ? hash + MOD : hash),
            (hash2 < 0 ? hash2 + MOD2 : hash2)};
}
pii query_inv(int l, int r) {
    ll hash =
        (hi[l] -
         (r + 1 < n ? hi[r + 1] * p[r - l + 1] % MOD : 0));
    ll hash2 =
        (hi2[l] -
         (r + 1 < n ? hi2[r + 1] * p2[r - l + 1] % MOD2 : 0));
    return {(hash < 0 ? hash + MOD : hash),
            (hash2 < 0 ? hash2 + MOD2 : hash2)};
}
};

```

9.3 Hash Range Query

```

struct Hash {
    const ll P = 31;
    const ll mod = 1e9 + 7;
    string s;
    int n;
    vll h, hi, p;
    Hash() {}
    Hash(string s) : s(s), n(s.size()), h(n), hi(n), p(n) {
        for (int i = 0; i < n; i++)
            p[i] = (i ? P * p[i - 1] : 1) % mod;
        for (int i = 0; i < n; i++)
            h[i] = (s[i] + (i ? h[i - 1] : 0) * P) % mod;
        for (int i = n - 1; i >= 0; i--)
            hi[i] =
                (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % mod;
    }
    ll query(int l, int r) {
        ll hash =
            (h[r] - (l ? h[l - 1] * p[r - l + 1] % mod : 0));
        return hash < 0 ? hash + mod : hash;
    }
    ll query_inv(int l, int r) {
        ll hash =
            (hi[l] -
             (r + 1 < n ? hi[r + 1] * p[r - l + 1] % mod : 0));

```

```

    return hash < 0 ? hash + mod : hash;
}
};

```

9.4 K-th digit in digit string

Find the k-th digit in a *digit string*, only works for $1 \leq k \leq 10^{18}$!
Time: precompute $O(1)$, query $O(1)$

```

using ull = vector<ull>;
ull pow10;
vector<array<ull, 4>> memo;
void precompute(int maxpow = 18) {
    ull qtd = 1;
    ull start = 1;
    ull end = 9;
    ull curlenght = 9;
    ull startstr = 1;
    ull endstr = 9;

    for (ull i = 0, j = 1ll; (int)i < maxpow; i++, j *= 10ll)
        pow10.eb(j);

    for (ull i = 0; i < maxpow - 1ull; i++) {
        memo.push_back({start, end, startstr, endstr});

        start = end + 1ll;
        end = end + (9ll * pow10[qtd]);
        curlenght = end - start + 1ull;

        qtd++;
        startstr = endstr + 1ull;
        endstr = (endstr + 1ull) + (curlenght)*qtd - 1ull;
    }
}

char kthDigit(ull k) {
    int qtd = 1;
    for (auto [s, e, ss, es] : memo) {
        if (k >= ss and k <= es) {
            ull pos = k - ss;
            ull index = pos / qtd;
            ull nmr = s + index;
            int i = k - ss - qtd * index;

            return ((nmr / pow10[qtd - i - 1]) % 10) + '0';
        }
        qtd++;
    }
}

```

```

}

return 'X';
}

```

9.5 Longest Palindrome Substring (Manacher)

Finds the longest palindrome substring, manacher returns a vector where the i-th position is how much is possible to grow the string to the left and the right of i and keep it a palindrome.
Time: $O(N)$

```

vi manacher(string s) {
    string t2;
    for (auto c : s) t2 += string("#") + c;
    t2 = t2 + '#';
    int n = t2.size();
    t2 = "$" + t2 + "^";
    vi p(n + 2);
    int l = 1, r = 1;
    for (int i = 1; i <= n; i++) {
        p[i] = max(0, min(r - i, p[l + (r - i)]));
        while (t2[i - p[i]] == t2[i + p[i]]) {
            p[i]++;
        }
        if (i + p[i] > r) {
            l = i - p[i], r = i + p[i];
        }
        p[i]--;
    }
    return vi(begin(p) + 1, end(p) - 1);
}

string longest_palindrome(const string &s) {
    vi xs = manacher(s);

    string s2;
    for (auto c : s) s2 += string("#") + c;
    s2 = s2 + '#';

    int mpos = 0;
    for (int i = 0; i < len(xs); i++) {
        if (xs[i] > xs[mpos]) {
            mpos = i;
        }
    }

    string ans;
}

```



```

int k = xs[mpos];
for (int i = mpos - k; i <= mpos + k; i++) {
    if (s2[i] != '#') {
        ans += s2[i];
    }
}
return ans;
}

void run() {
    string s;
    cin >> s;
    auto ans = longest_palindrome(s);
    cout << ans << endl;
}

```

9.6 Longest Palindrome

```

string longest_palindrome(const string &s) {
    int n = (int)s.size();
    vector<array<int, 2>> dp(n);

    pii odd(0, -1), even(0, -1);
    pii ans;
    for (int i = 0; i < n; i++) {
        int k = 0;
        if (i > odd.second)
            k = 1;
        else
            k = min(dp[odd.first + odd.second - i][0],
                    odd.second - i + 1);
        while (i - k >= 0 and i + k < n and
                s[i - k] == s[i + k])
            k++;
        dp[i][0] = k--;
        if (i + k > odd.second) odd = {i - k, i + k};
        if (2 * dp[i][0] - 1 > ans.second)
            ans = {i - k, 2 * dp[i][0] - 1};

        k = 0;
        if (i <= even.second)
            k = min(dp[even.first + even.second - i + 1][1],
                    even.second - i + 1);
        while (i - k - 1 >= 0 and i + k < n and
                s[i - k - 1] == s[i + k])
            k++;
    }
}

```

```

dp[i][1] = k--;
if (i + k > even.second) even = {i - k - 1, i + k};
if (2 * dp[i][1] > ans.second)
    ans = {i - k - 1, 2 * dp[i][1]};
}
return s.substr(ans.first, ans.second);
}

```

9.7 Rabin Karp

```

size_t rabin_karp(const string &s, const string &p) {
    if (s.size() < p.size()) return 0;

    auto n = s.size(), m = p.size();
    const ll p1 = 31, p2 = 29, q1 = 1e9 + 7, q2 = 1e9 + 9;
    const ll p1_1 = fpow(p1, q1 - 2, q1),
              p1_2 = fpow(p1, m - 1, q1);
    const ll p2_1 = fpow(p2, q2 - 2, q2),
              p2_2 = fpow(p2, m - 1, q2);

    pair<ll, ll> hs, hp;
    for (int i = (int)m - 1; ~i; --i) {
        hs.first = (hs.first * p1) % q1;
        hs.first = (hs.first + (s[i] - 'a' + 1)) % q1;
        hs.second = (hs.second * p2) % q2;
        hs.second = (hs.second + (s[i] - 'a' + 1)) % q2;

        hp.first = (hp.first * p1) % q1;
        hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
        hp.second = (hp.second * p2) % q2;
        hp.second = (hp.second + (p[i] - 'a' + 1)) % q2;
    }

    size_t occ = 0;
    for (size_t i = 0; i < n - m; i++) {
        occ += (hs == hp);

        int fi = s[i] - 'a' + 1;
        int fm = s[i + m] - 'a' + 1;

        hs.first = (hs.first - fi + q1) % q1;
        hs.first = (hs.first * p1_1) % q1;
        hs.first = (hs.first + fm * p1_2) % q1;
        hs.second = (hs.second - fi + q2) % q2;
        hs.second = (hs.second * p2_1) % q2;
    }
}

```

```

    hs.second = (hs.second + fm * p2_2) % q2;
}
occ += hs == hp;

return occ;
}

```

9.8 String Psum

```

struct strPsum {
    ll n;
    ll k;
    vector<vll> psum;
    strPsum(const string &s)
        : n(s.size()), k(100), psum(k, vll(n + 1)) {
        for (ll i = 1; i <= n; ++i) {
            for (ll j = 0; j < k; ++j) {
                psum[j][i] = psum[j][i - 1];
            }
            psum[s[i - 1]][i]++;
        }
    }

    ll qtd(ll l, ll r, char c) { // [0,n-1]
        return psum[c][r + 1] - psum[c][l];
    }
}

```

9.9 Suffix Automaton (complete)

```

struct state {
    int len, link, cnt, firstpos;
    // this can be optimized using a vector with the alphabet
    // size
    map<char, int> next;
    vi inv_link;
};

struct SuffixAutomaton {
    vector<state> st;
    int sz = 0;
    int last;
    vc cloned;

    SuffixAutomaton(const string &s, int maxlen)
        : st(maxlen * 2), cloned(maxlen * 2) {
        st[0].len = 0;
    }
}

```

```

st[0].link = -1;
sz++;
last = 0;
for (auto &c : s) add_char(c);

// precompute for count occurrences
for (int i = 1; i < sz; i++) {
    st[i].cnt = !cloned[i];
}
vector<pair<state, int>> aux;
for (int i = 0; i < sz; i++) {
    aux.push_back({st[i], i});
}

sort(all(aux), [](const pair<state, int> &a,
                  const pair<state, int> &b) {
    return a.fst.len > b.fst.len;
});

for (auto &[stt, id] : aux) {
    if (stt.link != -1) {
        st[stt.link].cnt += st[id].cnt;
    }
}

// for find every occurende position
for (int v = 1; v < sz; v++) {
    st[st[v].link].inv_link.push_back(v);
}
}

```

```

void add_char(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    st[cur].firstpos = st[cur].len - 1;
    int p = last;
    // follow the suffix link until find a transition to c
    while (p != -1 and !st[p].next.count(c)) {
        st[p].next[c] = cur;
        p = st[p].link;
    }
    // there was no transition to c so create and leave
    if (p == -1) {
        st[cur].link = 0;
        last = cur;
    }
}

```

```

    return;
}

int q = st[p].next[c];
if (st[p].len + 1 == st[q].len) {
    st[cur].link = q;
} else {
    int clone = sz++;
    cloned[clone] = true;
    st[clone].len = st[p].len + 1;
    st[clone].next = st[q].next;
    st[clone].link = st[q].link;
    st[clone].firstpos = st[q].firstpos;
    while (p != -1 and st[p].next[c] == q) {
        st[p].next[c] = clone;
        p = st[p].link;
    }
    st[q].link = st[cur].link = clone;
}
last = cur;
}

bool checkOccurrence(const string &t) { // O(len(t))
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return false;
        cur = st[cur].next[c];
    }
    return true;
}

ll totalSubstrings() { // distinct, O(len(s))
    ll tot = 0;
    for (int i = 1; i < sz; i++) {
        tot += st[i].len - st[st[i].link].len;
    }
    return tot;
}

// count occurrences of a given string t
int countOccurrences(const string &t) {
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return 0;
        cur = st[cur].next[c];
    }
}

```

```

    return st[cur].cnt;
}

// find the first index where t appears a substring
// O(len(t))
int firstOccurrence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return -1;
        cur = st[cur].next[c];
    }
    return st[cur].firstpos - len(t) + 1;
}

vi everyOccurrence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return {};
        cur = st[cur].next[c];
    }
    vi ans;
    getEveryOccurrence(cur, len(t), ans);
    return ans;
}

void getEveryOccurrence(int v, int P_length, vi &ans) {
    if (!cloned[v]) ans.pb(st[v].firstpos - P_length + 1);
    for (int u : st[v].inv_link)
        getEveryOccurrence(u, P_length, ans);
}
};

```

9.10 Trie

- build with the size of the alphabet (*sigma*) and the first char (*norm*)
- *insert(s)* insert the string in the trie $O(|s| * \text{sigma})$
- *erase(s)* remove the string from the trie $O(|s|)$
- *find(s)* return the last node from the string s, 0 if not found $O(|s|)$

```

struct trie {
    vi2d to;
    vi end, pref;
    int sigma;
    char norm;

    trie(int sigma_ = 26, char norm_ = 'a')

```

```

: sigma(sigma_), norm(norm_) {
to = {vector<int>(sigma)};
end = {0}, pref = {0};
}

int next(int node, char key) {
return to[node][key - norm];
}

void insert(const string &s) {
int x = 0;
for (auto c : s) {
int &nxt = to[x][c - norm];
if (!nxt) {
nxt = len(to);
to.push_back(vi(sigma));
end.emplace_back(0), pref.emplace_back(0);
}
x = nxt, pref[x]++;
}
end[x]++, pref[0]++;
}

void erase(const string &s) {
int x = 0;
for (char c : s) {
int &nxt = to[x][c - norm];
x = nxt, pref[x]--;
if (!pref[x]) nxt = 0;
}
end[x]--, pref[0]--;
}

int find(const string &s) {
int x = 0;
for (auto c : s) {
x = to[x][c - norm];
if (!x) return 0;
}
return x;
}
};

```

9.11 Z-function get occurrence positions

$O(\text{len}(s) + \text{len}(p))$

```
vi getOccPos(string &s, string &p) {
```

```

// Z-function
char delim = '#';
string t{p + delim + s};
vi zs(len(t));

int l = 0, r = 0;
for (int i = 1; i < len(t); i++) {
if (i <= r) zs[i] = min(zs[i - 1], r - i + 1);
while (zs[i] + i < len(t) and t[zs[i]] == t[i + zs[i]])
zs[i]++;
if (r < i + zs[i] - 1) l = i, r = i + zs[i] - 1;
}

// Iterate over the results of Z-function to get ranges
vi ans;
int start = len(p) + 1 + 1 - 1;
for (int i = start; i < len(zs); i++) {
if (zs[i] == len(p)) {
int l = i - start;
ans.emplace_back(l);
}
}
return ans;
}

```

10 Trees

10.1 Binary Lifting (struct)

```

struct BinaryLifting {
vector<int> far, level, parent;

BinaryLifting(const vector<vector<int>> &g, int root = 0)
: far(g.size(), -1), level(g.size()), parent(g.size()) {
level[root] = 1;
vector<int> q{root};
q.reserve(g.size());
for (int u = 0; u < (int)q.size(); u++) {
for (auto x : g[u])
if (x != parent[u]) {
parent[x] = u;
level[x] = level[u] + 1;

int p1 = u;

```

```

        int p2 = far[u];

        if (p2 > -1 and far[p2] > -1 and
            level[p1] - level[p2] ==
            level[p2] - level[far[p2]])
            far[x] = far[p2];
        else
            far[x] = p1;

        q.push_back(x);
    }
}

int kth_parent(int node, int k) const {
    while (node >= 0 and k > 0) {
        if (far[node] > -1 and
            level[node] - k <= level[far[node]]) {
            k -= level[node] - level[far[node]];
            node = far[node];
        } else {
            k--;
            node = parent[node];
        }
    }

    return node;
}

int lca(int u, int v) const {
    if (level[u] < level[v]) swap(u, v);

    while (level[v] < level[u]) {
        if (far[u] > -1 and level[v] <= level[far[u]])
            u = far[u];
        else
            u = parent[u];
    }

    if (u == v) return u;

    while (parent[u] != parent[v]) {
        if (far[u] > -1 and far[v] > -1 and
            far[u] != far[v]) {
            u = far[u];

```

```

            v = far[v];
        } else {
            u = parent[u];
            v = parent[v];
        }
    }

    return parent[u];
}
};

```

10.2 Binary Lifting

```

/*
 * far[h][i] = the node that 2^h far from node i
 * sometimes is useful invert the order of loops
 * time : O(nlogn)
 * */
const int maxlog = 20;
int far[maxlog + 1][n + 1];
int n;
for (int h = 1; h <= maxlog; h++) {
    for (int i = 1; i <= n; i++) {
        far[h][i] = far[h - 1][far[h - 1][i]];
    }
}

```

10.3 Isomorphism

Two trees are considered **isomorphic** if the hash given by *thash()* is the same.
Time: $O(V \cdot \log V)$

```

map<vector<int>, int> mphash;

struct Tree {
    int n;
    vi2d g;
    vi sz, cs;

    Tree(int n_) : n(n_), g(n), sz(n) {}

    void add_edge(int u, int v) {
        g[u].emplace_back(v);
        g[v].emplace_back(u);
    }

    void dfs_centroid(int v, int p) {

```

```

sz[v] = 1;
bool cent = true;
for (int u : g[v])
    if (u != p) {
        dfs_centroid(u, v);
        sz[v] += sz[u];
        cent &= not(sz[u] > n / 2);
    }
if (cent and n - sz[v] <= n / 2) cs.push_back(v);
}

int fhash(int v, int p) {
    vi h;
    for (int u : g[v])
        if (u != p) h.push_back(fhash(u, v));
    sort(all(h));
    if (!mphash.count(h)) mphash[h] = mphash.size();
    return mphash[h];
}

ll thash() {
    cs.clear();
    dfs_centroid(0, -1);
    if (cs.size() == 1) return fhash(cs[0], -1);
    ll h1 = fhash(cs[0], cs[1]), h2 = fhash(cs[1], cs[0]);
    return (min(h1, h2) << 3011) + max(h1, h2);
}
};

```

10.4 Lowest Common Ancestor

Given two nodes of a tree find their lowest common ancestor, or their distance

Build : $O(V)$, Queries: $O(1)$

0 indexed !

```

template <typename T>
struct SparseTable {
    vector<T> v;
    int n;
    static const int b = 30;
    vi mask, t;

    int op(int x, int y) { return v[x] < v[y] ? x : y; }
    int msb(int x) {
        return __builtin_clz(1) - __builtin_clz(x);
    }
}

```

```

SparseTable() {}
SparseTable(const vector<T>& v_)
    : v(v_), n(v.size()), mask(n), t(n) {
    for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
        at = (at << 1) & ((1 << b) - 1);
        while (at and op(i, i - msb(at & -at)) == i)
            at ^= at & -at;
    }
    for (int i = 0; i < n / b; i++)
        t[i] = b * i + b - 1 - msb(mask[b * i + b - 1]);
    for (int j = 1; (1 << j) <= n / b; j++)
        for (int i = 0; i + (1 << j) <= n / b; i++)
            t[n / b * j + i] =
                op(t[n / b * (j - 1) + i],
                  t[n / b * (j - 1) + i + (1 << (j - 1))]);
}

int small(int r, int sz = b) {
    return r - msb(mask[r] & ((1 << sz) - 1));
}

T query(int l, int r) {
    if (r - l + 1 <= b) return small(r, r - l + 1);
    int ans = op(small(l + b - 1), small(r));
    int x = l / b + 1, y = r / b - 1;
    if (x <= y) {
        int j = msb(y - x + 1);
        ans = op(ans, op(t[n / b * j + x],
                          t[n / b * j + y - (1 << j) + 1]));
    }
    return ans;
}
};

```

```

struct LCA {
    SparseTable<int> st;
    int n;
    vi v, pos, dep;

    LCA(const vi2d& g, int root) : n(len(g)), pos(n) {
        dfs(root, 0, -1, g);
        st = SparseTable<int>(vector<int>(all(dep)));
    }

    void dfs(int i, int d, int p, const vi2d& g) {
        v.pb(len(dep)) = i, pos[i] = len(dep), dep.pb(d);
        for (auto j : g[i])

```

```

    if (j != p) {
        dfs(j, d + 1, i, g);
        v.eb(len(dep)) = i, dep.eb(d);
    }
}

int lca(int a, int b) {
    int l = min(pos[a], pos[b]);
    int r = max(pos[a], pos[b]);
    return v[st.query(l, r)];
}

int dist(int a, int b) {
    return dep[pos[a]] + dep[pos[b]] -
        2 * dep[pos[lca(a, b)]];
}
};

```

10.5 Tree Maximum Distance

Returns the maximum distance from every node to any other node in the tree. $O(6V) = O(V)$

```

pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
    // O(V)
    // 0 indexed
    ll mostDistantNode = root;
    ll nodeDistance = 0;
    queue<pll> q;
    vector<char> vis(n);
    q.emplace(root, 0);
    vis[root] = true;
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (dist > nodeDistance) {
            nodeDistance = dist;
            mostDistantNode = node;
        }
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return {mostDistantNode, nodeDistance};
}

```

```

ll twoNodesDist(const vector<vll> &adj, ll n, ll a, ll b) {
    queue<pll> q;
    vector<char> vis(n);
    q.emplace(a, 0);
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (node == b) return dist;
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return -1;
}

tuple<ll, ll, ll> tree_diameter(const vector<vll> &adj,
                                ll n) {
    // returns two points of the diameter and the diameter
    // itself
    auto [node1, dist1] = mostDistantFrom(adj, n, 0); // O(V)
    auto [node2, dist2] =
        mostDistantFrom(adj, n, node1); // O(V)
    auto diameter =
        twoNodesDist(adj, n, node1, node2); // O(V)
    return make_tuple(node1, node2, diameter);
}

vll everyDistanceFromNode(const vector<vll> &adj, ll n,
                           ll root) {
    // Single Source Shortest Path, from a given root
    queue<pair<ll, ll>> q;
    vll ans(n, -1);
    ans[root] = 0;
    q.emplace(root, 0);
    while (!q.empty()) {
        auto [u, d] = q.front();
        q.pop();

        for (auto w : adj[u]) {
            if (ans[w] != -1) continue;
            ans[w] = d + 1;
        }
    }
}

```

```

        q.emplace(w, d + 1);
    }
}
return ans;
}

vll maxDistances(const vector<vll> &adj, ll n) {
    auto [node1, node2, diameter] =
        tree_diameter(adj, n); // O(3V)
    auto distances1 =
        everyDistanceFromNode(adj, n, node1); // O(V)
    auto distances2 =
        everyDistanceFromNode(adj, n, node2); // O(V)
    vll ans(n);
    for (int i = 0; i < n; ++i)
        ans[i] = max(distances1[i], distances2[i]); // O(V)
    return ans;
}

```

10.6 Small to Large

Answer queries of the form "How many vertices in the subtree of vertex v have property P ?"
 * this implementation answers how many distinct $values[i]$ are in the subtree starting at u .
 Build: $O(N)$, Query: $O(N \log N)$

```

struct SmallToLarge {
    int n;
    vi2d tree, vis_childs;
    vi sizes, values, ans;
    set<int> cnt;

    SmallToLarge(vi2d &g, vi &v)
        : tree(g),
          vis_childs(len(g)),
          sizes(len(g)),
          values(v),
          ans(len(g)) {
        get_size(0);
        dfs(0);
    }

    inline void add_value(int u) { cnt.insert(values[u]); }

    inline void remove_value(int u) { cnt.erase(values[u]); }

    inline void update_ans(int u) { ans[u] = len(cnt); }
}

```

```

void dfs(int u, int p = -1, bool keep = true) {
    int mx = -1;
    for (auto x : tree[u]) {
        if (x == p) continue;

        if (mx == -1 or sizes[mx] < sizes[x]) mx = x;
    }

    for (auto x : tree[u]) {
        if (x != p and x != mx) dfs(x, u, false);
    }

    if (mx != -1) {
        dfs(mx, u, true);
        swap(vis_childs[u], vis_childs[mx]);
    }

    vis_childs[u].push_back(u);
    add_value(u);

    for (auto x : tree[u]) {
        if (x != p and x != mx) {
            for (auto y : vis_childs[x]) {
                add_value(y);
                vis_childs[u].push_back(y);
            }
        }
    }

    update_ans(u);

    if (!keep) {
        for (auto x : vis_childs[u]) remove_value(x);
    }
}

void get_size(int u, int p = -1) {
    sizes[u] = 1;
    for (auto x : tree[u])
        if (x != p) {
            get_size(x, u);
            sizes[u] += sizes[x];
        }
}

```



```
};
```

10.7 Tree Diameter

```
const int MAXN(2'00'000);
int N;
vi2d G(MAXN);
int toleaf[MAXN], maxdist[MAXN];

void dfs(int u, int p = -1) {
    int ds1, ds2;
    ds1 = ds2 = -1;
    for (auto v : G[u]) {
        if (v == p) continue;
        if (ds1 > ds2) swap(ds1, ds2);
        dfs(v, u);

        ds1 = max(ds1, toleaf[v]);
    }

    toleaf[u] = max(ds1, ds2) + 1;

    maxdist[u] = 2 + ds1 + ds2;
}

int diameter(int root, int n) {
    dfs(root);

    int d = 0;

    for (int u = 0; u < n; ++u) d = max(d, maxdist[u]);

    return d;
}
```

10.8 Tree Flatten

```
void tree_flatten(const vector<vector<int>> &g, int u,
                  int p, vector<int> &pre, vector<int> &pos,
                  int &idx) {
    ++idx;
    pre.push_back(u);
    for (auto x : g[u])
        if (x != p) tree_flatten(g, x, u, pre, pos, idx);
    pos[u] = idx;
```

```
}
```

```
pair<vector<int>, vector<int>> tree_flatten(
    const vector<vector<int>> &g, int root = 0) {
    vector<int> first(g.size()), last(g.size()), pre;
    int timer = -1;
    tree_flatten(g, root, -1, pre, last, timer);
    for (int i = 0; i < (int)g.size(); i++) first[pre[i]] = i;
    return {first, last};
}
```

11 Settings and macros

11.1 .vimrc

```
set ts=4 sw=4 sta nu rnu sc cindent
set bg=dark ruler clipboard=unnamed,unnamedplus, timeoutlen=100
colorscheme default
```

```
nnoremap <C-j> :botright belowright term bash <CR>
syntax on
```

11.2 macro.cpp

```
#include <bits/stdc++.h>
using namespace std;
#ifdef LOCAL
#include "debug.cpp"
#else
#define dbg(...) 42
#endif
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(false); \
    cin.tie(0); \
    cout.tie(0);
#define len(__x) (int)__x.size()
using ll = long long;
using ull = unsigned long long;
using ld = long double;
using vll = vector<ll>;
using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
```

```

using vii = vector<pii>;
using vc = vector<char>;
#define all(a) a.begin(), a.end()
#define pb(___x) push_back(___x)
#define mp(___a, ___b) make_pair(___a, ___b)
#define eb(___x) emplace_back(___x)

// vector<string> dir({"LU", "U", "RU", "R", "RD", "D",
// "LD", "L"}); int dx[] = {-1, -1, -1, 0, 1, 1, 1, 0}; int
// dy[] = {-1, 0, 1, 1, 1, 0, -1, -1};
vector<string> dir({"U", "R", "D", "L"});
int dx[] = {-1, 0, 1, 0};
int dy[] = {0, 1, 0, -1};

const ll oo = 1e18;
int T(1);

auto solve() {}

int32_t main(void) {
#ifdef LOCAL
    fastio;
#endif

    // cin >> T;

    for (int i = 1; i <= T; i++) {
        solve();
    }
}

```

11.3 .bashrc

```

cpp() {
    g++ -std=c++20 -fsanitize=address,undefined -Wall $1 && time
    ./a.out
}

cpp() {
    echo ">> COMPILING <<" 1>&2
    g++ -std=c++17 \
        -O2 \
        -g \
        -g3 \
        -Wextra \

```

```

-Wshadow \
-Wformat=2 \
-Wconversion \
-fsanitize=address,undefined \
-fno-sanitize-recover \
-Wfatal-errors \
$1

if [ $? -ne 0 ]; then
    echo ">> FAILED <<" 1>&2
    return 1
fi
echo ">> DONE <<" 1>&2
time ./a.out ${@:2}
}

prepare() {
    cp debug.cpp ./
    for i in {a..z}
    do
        cp macro.cpp $i.cpp
        touch $i.py
    done

    for i in {1..10}
    do
        touch in${i}
        touch out${i}
        touch ans${i}
    done
}

```

11.4 short-macro.cpp

```

#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(false); \
    cin.tie(0); \
    cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using pii = pair<int, int>;
#define all(a) a.begin(), a.end()

```

```

void run() {}
int32_t main(void) {
    fastio;
    int t;
    t = 1;
    // cin >> t;
    while (t--) run();
}

```

11.5 debug.cpp

```

#include <bits/stdc++.h>
using namespace std;
/***** Debug Code *****/
template <typename T>
concept Printable = requires(T t) {
    { std::cout << t } -> std::same_as<std::ostream &>;
};
template <Printable T>
void __print(const T &x) {
    cerr << x;
}
template <size_t T>
void __print(const bitset<T> &x) {
    cerr << x;
}
template <typename A, typename B>
void __print(const pair<A, B> &p);
template <typename... A>
void __print(const tuple<A...> &t);
template <typename T>
void __print(stack<T> s);
template <typename T>
void __print(queue<T> q);
template <typename T, typename... U>
void __print(priority_queue<T, U...> q);
template <typename A>
void __print(const A &x) {
    bool first = true;
    cerr << '{';
    for (const auto &i : x) {
        cerr << (first ? "" : ","), __print(i);
        first = false;
    }
}

```

```

    }
    cerr << '>';
}
template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '(';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << ')';
}
template <typename... A>
void __print(const tuple<A...> &t) {
    bool first = true;
    cerr << '(';
    apply(
        [&first](const auto &...args) {
            ((cerr << (first ? "" : ","), __print(args), first
= false), ...);
        },
        t);
    cerr << ')';
}
template <typename T>
void __print(stack<T> s) {
    vector<T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    }
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
}
template <typename T>
void __print(queue<T> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.front();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
template <typename T, typename... U>

```

```

void __print(priority_queue<T, U...> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.top();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
void _print() { cerr << "]\n"; }

```

```

template <typename Head, typename... Tail>
void _print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";
    _print(T...);
}

#define dbg(x...) \
    cerr << "[" << #x << "]" = ["; \
    _print(x)

```