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# 1 Contest

## 1.1 bash config

```
#copy first argument to clipborad ! ONLY WORK ON
XORG !
alias clip="xclip -sel clip"

# compile the $1 parameter, if a $2 is provided
# the name will be the the binary output, if
# none is provided the binary name will be
# 'a.out'
comp() {
    echo ">> COMPILING $1 <<" 1>&2
    if [ $# -gt 1 ]; then
        outfile="${2}"
    else
        outfile="a.out"
    fi
    time g++ -std=c++20 \
        -O2 \
        -g3 \
        -Wall \
        -fsanitize=address,undefined \
        -fno-sanitize-recover \
        -D LOCAL \
        -o "${outfile}" \
        "$1"
    if [ $? -ne 0 ]; then
        echo ">> FAILED <<" 1>&2
        return 1
    fi
    echo ">> DONE <<" 1>&2
}

# run the binary given in $1, if none is
# given it will try to run the 'a.out'
# binary
run() {
    to_run=./a.out
    if [ -n "$1" ]; then
        to_run="$1"
    fi
    time $to_run
}

# just comp and run your cpp file
# accpets <in1 >out and everything else
comprun() {
    comp "$1" "a" && run ./a ${@:2}
}

testall() {
    comp "$1" generator
    comp "$2" brute
    comp "$3" main
    input_counter=1
    while true; do
        echo "$input_counter"
        run ./generator >input
        run ./main <input >main_output.txt
        run ./brute <input >brute_output.txt
        diff brute_output.txt main_output.txt
        if [ $? -ne 0 ]; then
            echo "Outputs differ at input $input_counter"
            echo "Brute file output:"
            cat brute_output.txt
            echo "Main file output:"
            cat main_output.txt
            echo "input used: "
            cat input
        fi
        break
    fi
    touch_macro() {
        cat "$1"/template.cpp >>"$2"
        cat "$1"/run.cpp >>"$2"
        cp "$1"/debug.cpp .
    }

    # Creates a contest with hame $2
    # Copies the macro and debug file from $1
    # Already creates files a...z .cpp and .py
    prepare_contest() {
        mkdir "$2"
        cd "$2"
        for i in {a..z}; do
            touch_macro $1 $i.cpp
        done
    }
}
```

```
        break
    fi
    ((input_counter++))
done
}

touch_macro() {
    cat "$1"/template.cpp >>"$2"
    cat "$1"/run.cpp >>"$2"
    cp "$1"/debug.cpp .
}

# Creates a contest with hame $2
# Copies the macro and debug file from $1
# Already creates files a...z .cpp and .py
prepare_contest() {
    mkdir "$2"
    cd "$2"
    for i in {a..z}; do
        touch_macro $1 $i.cpp
    done
}
}
```

## 1.2 debug

```
template <typename T>
concept Printable = requires(T t) {
    { std::cout << t
    } -> std::same_as<std::ostream &>;
};

template <Printable T>
void __print(const T &x) {
    cerr << x;
}

template <size_t T>
void __print(const bitset<T> &x) {
    cerr << x;
}

template <typename A, typename B>
void __print(const pair<A, B> &p);
template <typename... A>
void __print(const tuple<A...> &t);
template <typename T>
void __print(stack<T> s);
template <typename T>
void __print(queue<T> q);
template <typename T, typename... U>
void __print(priority_queue<T, U...> q);
template <typename A>
void __print(const A &x) {
    bool first = true;
    cerr << '{';
    for (const auto &i : x) {
        cerr << (first ? "" : ","), __print(i);
        first = false;
    }
    cerr << '}'
}

template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '{';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << '}'
}

template <typename... A>
void __print(const tuple<A...> &t) {
    cerr << '{';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << '}'
}

template <typename T>
void __print(stack<T> s) {
    cerr << '{';
    while (!s.empty()) {
        __print(s.top());
        s.pop();
    }
    cerr << '}'
}

template <typename T>
void __print(queue<T> q) {
    cerr << '{';
    while (!q.empty()) {
        __print(q.front());
        q.pop();
    }
    cerr << '}'
}

template <typename T, typename... U>
void __print(priority_queue<T, U...> q) {
    cerr << '{';
    while (!q.empty()) {
        __print(q.top());
        q.pop();
    }
    cerr << '}'
}

template <typename Head, typename... Tail>
void __print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";
    __print(T...);
}

#define dbg(x...) \
    cerr << "[" << #x << "]" = [{"; \
    _print(x)
```

```
bool first = true;
cerr << '{';
apply(
    [&first](const auto &...args) {
        ((cerr << (first ? "" : ","),
        __print(args), first = false),
        ...);
    },
    t);
cerr << '}'
}

template <typename T>
void __print(stack<T> s) {
    vector<T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    }
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
}

template <typename T>
void __print(queue<T> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.front();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}

template <typename T, typename... U>
void __print(priority_queue<T, U...> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.top();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}

void _print() { cerr << "]\n"; }
template <typename Head, typename... Tail>
void _print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";
    __print(T...);
}

#define dbg(x...) \
    cerr << "[" << #x << "]" = [{"; \
    _print(x)
```

## 1.3 run

```
void run();

int32_t main() {
#ifdef LOCAL
    fastio;
#endif
    int T = 1;
    cin >> T;
    rep(t, 0, T) {
        dbg(t);
    }
}
```

```

    run();
}
}
void run() {}

```

## 1.4 short-template

```

#include <bits/stdc++.h>
using namespace std;
#define fastio \
    ios_base::sync_with_stdio(0); \
    cin.tie(0);
void run() {}
int32_t main(void) {
    fastio;
    int t;
    t = 1;
    // cin >> t;
    while (t--> 0) run();
}

```

## 1.5 template

```

#include <bits/stdc++.h>
using namespace std;
#ifdef LOCAL
#include "debug.cpp"
#else
#define dbg(...)
#endif
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(0); \
    cin.tie(0);
#define int long long
#define all(j) j.begin(), j.end()
#define rall(j) j.rbegin(), j.rend()
#define len(j) (int)j.size()
#define rep(i, a, b) \
    for (common_type_t<decltype(a), decltype(b)> i = (a); \
         i < (b); i++)
#define rrep(i, a, b) \
    for (common_type_t<decltype(a), decltype(b)> i = (a); \
         i > (b); i--)
#define trav(a,x) for (auto& a : x)
#define pb push_back
#define pf push_front
#define ppb pop_back
#define ppf pop_front
#define eb emplace_back
#define lb lower_bound
#define ub upper_bound
#define emp emplace
#define ins insert
#define divc(a,b) ((a)+(b)-1ll)/(b)
using str = string;
using ll = long long;
using ull = unsigned long long;
using ld = long double;
using vll = vector<ll>;
using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;

```

```

using vi2d = vector<vi>;
using pii = pair<int, int>;
using vpil = vector<pii>;
using vc = vector<char>;
using vs = vector<str>;
template <typename T>
using pqmn =
    priority_queue<T, vector<T>, greater<T>>;
template <typename T>
using pqmx = priority_queue<T, vector<T>>;
template <typename T, typename U>
inline bool chmax(T &a, U const &b) {
    return (a < b ? a = b, 1 : 0);
}
template <typename T, typename U>
inline bool chmin(T &a, U const &b) {
    return (a > b ? a = b, 1 : 0);
}

```

## 1.6 vim config

```

set sta nu rnu sc cindent
set ts=2 sw=2
set bg=dark ruler clipboard=unnamed,unnamedplus,
    timeoutlen=100
colorscheme default
syntax on
" Takes the hash of the selected text and put
" in the vim clipboard
function! HashSelectedText()
    " Yank the selected text to the unnamed
    register
    normal! gvy
    " Use the system() function to call sha256sum
    with the yanked text
    let l:hash = system('echo ' . shellescape(@@) .
    ' | sha256sum')
    " Yank the hash into Vim's unnamed register
    let @" = l:hash
endfunction

```

## 2 Data Structures

### 2.1 SQRT decomposition

#### 2.1.1 two-sequence-queries

```

using ll = long long;
const ll MOD = 998244353;
inline ll sum(const ll a, const ll b) {
    return (a + b) % MOD;
}
ll sub(const ll a, const ll b) {
    return (a - b + MOD) % MOD;
}
inline ll mul(const ll a, const ll b) {
    return (a * b) % MOD;
}
struct SqrtDecomposition {
    struct t_sqrt {
        int l, r;
        ll x, y;
        ll prod;
        ll sum_as, sum_bs;
    };
};

```

```

t_sqrt() {
    l = numeric_limits<int>::max();
    r = numeric_limits<int>::min();
    x = y = prod = sum_as = sum_bs = 0;
};
int sqrtLen;
vector<t_sqrt> blocks;
vector<ll> as, bs;
SqrtDecomposition(const vector<ll> &as,
    const vector<ll> &bs) {
    int n = as.size();
    sqrtLen = (int)sqrt(n + .0) + 1;
    blocks.resize(sqrtLen + 6.66);
    as = as_;
    bs = bs_;
    for (int i = 0; i < n; i++) {
        auto &bi = blocks[i / sqrtLen];
        bi.l = min(bi.l, i);
        bi.r = max(bi.r, i);
        bi.sum_as = sum(bi.sum_as, as[i]);
        bi.sum_bs = sum(bi.sum_bs, bs[i]);
        bi.prod = sum(bi.prod, mul(as[i], bs[i]));
    }
    // adds x to a[i], and y to b[i], in range [l,
    // r]
    void update(int l, int r, ll x, ll y) {
        auto apply1 = [&](int idx, ll x,
            ll y) -> void {
            auto &block = blocks[idx / sqrtLen];
            block.prod =
                sub(block.prod, mul(as[idx], bs[idx]));
            block.sum_as = sub(block.sum_as, as[idx]);
            block.sum_bs = sub(block.sum_bs, bs[idx]);
            as[idx] = sum(as[idx], x);
            bs[idx] = sum(bs[idx], y);
            block.prod =
                sum(block.prod, as[idx] * bs[idx]);
            block.sum_as = sum(block.sum_as, as[idx]);
            block.sum_bs = sum(block.sum_bs, bs[idx]);
        };
        auto apply2 = [&](int idx, ll x,
            ll y) -> void {
            blocks[idx].x = sum(blocks[idx].x, x);
            blocks[idx].y = sum(blocks[idx].y, y);
        };
        int cl = l / sqrtLen, cr = r / sqrtLen;
        if (cl == cr) {
            for (int i = l; i <= r; i++) {
                apply1(i, x, y);
            }
        } else {
            for (int i = l; i <= (cl + 1) * sqrtLen - 1;
                i++) {
                apply1(i, x, y);
            }
            for (int i = cl + 1; i <= cr - 1; i++) {
                apply2(i, x, y);
            }
            for (int i = cr * sqrtLen; i <= r; i++) {
                apply1(i, x, y);
            }
        }
    }
}

```

```

}
// sum of a[i]*b[i] in range [l r]
ll query(int l, int r) {
    auto eval1 = [&](int idx) -> ll {
        auto &block = blocks[idx / sqrtLen];
        return mul(sum(as[idx], +block.x),
                    sum(bs[idx], block.y));
    };
    auto eval2 = [&](int idx) -> ll {
        auto &block = blocks[idx];
        ll ret = 0;
        ret = sum(
            sum(ret,
                mul(mul(block.x, block.y),
                    sum(sub(block.r, block.l), 1))),
            block.prod);
        ret = sum(ret, block.y * block.sum_as);
        ret = sum(ret, block.x * block.sum_bs);
        return ret;
    };
    ll ret = 0;
    int cl = l / sqrtLen, cr = r / sqrtLen;
    if (cl == cr) {
        for (int i = l; i <= r; i++) {
            ret = sum(ret, eval1(i));
        }
    } else {
        for (int i = l; i <= (cl + 1) * sqrtLen - 1; i++) {
            ret = sum(eval1(i), ret);
        }
        for (int i = cl + 1; i <= cr - 1; i++) {
            ret = sum(ret, eval2(i));
        }
        for (int i = cr * sqrtLen; i <= r; i++) {
            ret = sum(ret, eval1(i));
        }
    }
    return ret;
}
};

```

## 2.2 Segment tree (dynamic)

### 2.2.1 Range Max Query Point Max Assignment

**Description:** Answers range queries in ranges until  $10^9$  (maybe more)  
**Time:** Query and update  $O(n \cdot \log n)$

```

struct node;
node *newNode();
struct node {
    node *left, *right;
    int lv, rv;
    ll val;
    node() : left(NULL), right(NULL), val(-oo) {}
    inline void init(int l, int r) {
        lv = l;
        rv = r;
    }
    inline void extend() {
        if (!left) {
            int m = (lv + rv) / 2;
            left = newNode();
            right = newNode();
        }
    }
};

```

```

        left->init(lv, m);
        right->init(m + 1, rv);
    }
}
ll query(int l, int r) {
    if (r < lv || rv < l) {
        return 0;
    }
    if (l <= lv && rv <= r) {
        return val;
    }
    extend();
    return max(left->query(l, r),
               right->query(l, r));
}
void update(int p, ll newVal) {
    if (lv == rv) {
        val = max(val, newVal);
        return;
    }
    extend();
    (p <= left->rv ? left : right)
        ->update(p, newVal);
    val = max(left->val, right->val);
}
};
const int BUFFSZ(1e7);
node *newNode() {
    static int bufSize = BUFFSZ;
    static node buf[(int)BUFFSZ];
    assert(bufSize);
    return &buf[--bufSize];
}
struct SegTree {
    int n;
    node *root;
    SegTree(int _n) : n(_n) {
        root = newNode();
        root->init(0, n);
    }
    ll query(int l, int r) {
        return root->query(l, r);
    }
    void update(int p, ll v) { root->update(p, v); }
};

```

### 2.2.2 Range Sum Query Point Sum Update

**Description:** Answers range queries in ranges until  $10^9$  (maybe more)  
**Time:** Query and update in  $O(n \cdot \log n)$

```

struct node;
node *newNode();
struct node {
    node *left, *right;
    int lv, rv;
    ll val;
    node() : left(NULL), right(NULL), val(0) {}
    inline void init(int l, int r) {
        lv = l;
        rv = r;
    }
    inline void extend() {
        if (!left) {
            int m = (rv - lv) / 2 + lv;
            left = newNode();
        }
    }
};

```

```

        right = newNode();
        left->init(lv, m);
        right->init(m + 1, rv);
    }
}
ll query(int l, int r) {
    if (r < lv || rv < l) {
        return 0;
    }
    if (l <= lv && rv <= r) {
        return val;
    }
    extend();
    return left->query(l, r) + right->query(l, r);
}
void update(int p, ll newVal) {
    if (lv == rv) {
        val += newVal;
        return;
    }
    extend();
    (p <= left->rv ? left : right)
        ->update(p, newVal);
    val = left->val + right->val;
}
};
const int BUFFSZ(1.3e7);
node *newNode() {
    static int bufSize = BUFFSZ;
    static node buf[(int)BUFFSZ];
    // assert(bufSize);
    return &buf[--bufSize];
}
struct SegTree {
    int n;
    node *root;
    SegTree(int _n) : n(_n) {
        root = newNode();
        root->init(0, n);
    }
    ll query(int l, int r) {
        return root->query(l, r);
    }
    void update(int p, ll v) { root->update(p, v); }
};

```

## 2.3 Segment tree point update range query

### 2.3.1 Query GCD (bottom up)

```

using ll = long long;
struct Node {
    ll value;
    bool undef;
    Node()
        : value(1), undef(1){}; // Neutral element
    Node(ll v) : value(v), undef(0){};
};
inline Node combine(const Node &nl,
                   const Node &nr) {
    if (nl.undef) return nr;
    if (nr.undef) return nl;
    Node m;
    m.value = gcd(nl.value, nr.value);
};

```

```

    m.undef = false;
    return m;
}

template <typename T = Node, auto F = combine>
struct SegTree {
    int n;
    vector<T> st;
    SegTree(int _n) : n(_n), st(n << 1) {}
    void assign(int p, const T &k) {
        for (st[p += n] = k; p >>= 1;)
            st[p] = F(st[p << 1], st[p << 1 | 1]);
    }
    T query(int l, int r) {
        T ans1, ansr;
        for (l += n, r += n + 1; l < r;
             l >>= 1, r >>= 1) {
            if (l & 1) ans1 = F(ans1, st[l++]);
            if (r & 1) ansr = F(st[--r], ansr);
        }
        return F(ans1, ansr);
    }
};

```

### 2.3.2 Query hash (top down)

```

const ll MOD = 1'000'000'009;
const ll P = 31;
const int MAXN = 2'000'000;

ll pows[MAXN + 1];
void computePows() {
    pows[0] = 1;
    for (int i = 1; i <= MAXN; i++) {
        pows[i] = (pows[i - 1] * P) % MOD;
    }
}

struct Node {
    ll hash;
    Node() : hash(-1){}; // Neutral element
    Node(ll v) : hash(v){};
};

inline Node combine(Node &vl, Node &vr, int nl,
                    int nr, int ql, int qr) {
    if (vl.hash == -1) return vr;
    if (vr.hash == -1) return vl;
    Node vm;
    int nm = midpoint(nl, nr);
    int lsize = min(nm, qr) - max(nl, ql) + 1;
    vm.hash = (vl.hash +
                ((vr.hash * pows[lsize]) % MOD)) %
                MOD;
    return vm;
}

template <typename T = Node, auto F = combine>
struct SegTree {
    int n;
    vector<T> st;
    SegTree(int n) : n(n), st(n << 2) {}
    void assign(int p, const T &v) {
        assign(1, 0, n - 1, p, v);
    }
    void assign(int node, int l, int r, int p,
                const T &v) {
        if (l == r) {
            st[node] = v;
            return;
        }
    }
};

```

```

}
int m = midpoint(l, r);
if (p <= m)
    assign(node << 1, l, m, p, v);
else
    assign(node << 1 | 1, m + 1, r, p, v);
st[node] = F(st[node << 1], st[node << 1 | 1],
             l, r, l, r);
}

inline T query(int l, int r) {
    return query(1, 0, n - 1, l, r);
}

inline T query(int node, int nl, int nr, int l,
                int r) const {
    if (r < nl or nr < l) return T();
    if (l <= nl and nr <= r) return st[node];
    int m = midpoint(nl, nr);
    auto a = query(node << 1, nl, m, l, r);
    auto b = query(node << 1 | 1, m + 1, nr, l, r);
    return F(a, b, nl, nr, l, r);
}
};

```

### 2.3.3 Query max subarray sum (bottom up)

```

struct Node {
    ll tot, suf, pref, best;
    // Neutral element
    Node()
        : tot(-oo),
          suf(-oo),
          pref(-oo),
          best(-oo) {} // Neutral element
    // for assign
    Node(ll x) {
        tot = x, suf = x, pref = x,
        best = max(0ll, x);
    }
};

Node combine(Node &nl, Node &nr) {
    if (nl.tot == -oo) return nr;
    if (nr.tot == -oo) return nl;
    Node m;
    m.tot = nl.tot + nr.tot;
    m.pref = max({nl.pref, nl.tot + nr.pref});
    m.suf = max({nr.suf, nr.tot + nl.suf});
    m.best = max({nl.best, nr.best, nl.suf + nr.pref});
    return m;
}

```

### 2.3.4 Query min (bottom up)

```

struct Node {
    ll value;
    Node()
        : value(numeric_limits<
                ll>::max()){}; // Neutral element
    Node(ll v) : value(v){};
};

Node combine(Node &l, Node &r) {
    Node m;

```

```

    m.value = min(l.value, r.value);
    return m;
}

template <typename T = Node, auto F = combine>
struct SegTree {
    int n;
    vector<T> st;
    SegTree(int _n) : n(_n), st(n << 1) {}
    void assign(int p, const T &k) {
        for (st[p += n] = k; p >>= 1;)
            st[p] = F(st[p << 1], st[p << 1 | 1]);
    }
    T query(int l, int r) {
        T ans1 = T(), ansr = T();
        for (l += n, r += n + 1; l < r;
             l >>= 1, r >>= 1) {
            if (l & 1) ans1 = F(ans1, st[l++]);
            if (r & 1) ansr = F(st[--r], ansr);
        }
        return F(ans1, ansr);
    }
};

```

### 2.3.5 Query sum (bottom up)

```

struct Node {
    ll value;
    Node() : value(0){}; // Neutral element
    Node(ll v) : value(v){};
};

inline Node combine(const Node &nl,
                    const Node &nr) {
    Node m;
    m.value = nl.value + nr.value;
    return m;
}

struct SegTree {
    int n;
    vector<Node> st;
    SegTree(int _n) : n(_n), st(n << 1) {}
    void assign(int p, const Node &k) {
        for (st[p += n] = k; p >>= 1;)
            st[p] = combine(st[p << 1], st[p << 1 | 1]);
    }
    Node query(int l, int r) {
        Node ans1 = Node(), ansr = Node();
        for (l += n, r += n + 1; l < r;
             l >>= 1, r >>= 1) {
            if (l & 1) ans1 = combine(ans1, st[l++]);
            if (r & 1) ansr = combine(st[--r], ansr);
        }
        return combine(ans1, ansr);
    }
};

```

## 2.4 Segment tree range update range query

### 2.4.1 Arithmetic progression sum update, query sum

**Description:** Makes arithmetic progression updates in range and sum queries.

**Usage:** Considering  $PA(A, R) = [A + R, A + 2R, A + 3R, \dots]$

- **update\_set(l, r, A, R):** sets  $[l, r]$  to  $PA(A, R)$

- **update\_add(l, r, A, R):** sum PA(A, R) in [l, r]
- **query(l, r):** sum in range [l, r]

Time: build  $O(N)$ , updates and queries  $O(\log N)$

```
const ll oo = 1e18;
struct SegTree {
    struct Data {
        ll sum;
        ll set_a, set_r, add_a, add_r;
        Data() {
            : sum(0),
              set_a(oo),
              set_r(0),
              add_a(0),
              add_r(0) {}
        };
    };
    int n;
    vector<Data> seg;
    SegTree(int n_) {
        : n(n_), seg(vector<Data>(4 * n)) {}
    };
    void prop(int p, int l, int r) {
        int sz = r - l + 1;
        ll &sum = seg[p].sum, &set_a = seg[p].set_a,
           &set_r = seg[p].set_r,
           &add_a = seg[p].add_a,
           &add_r = seg[p].add_r;
        if (set_a != oo) {
            set_a += add_a, set_r += add_r;
            sum = set_a * sz + set_r * sz * (sz + 1) / 2;
            if (l != r) {
                int m = (l + r) / 2;
                seg[2 * p].set_a = set_a;
                seg[2 * p].set_r = set_r;
                seg[2 * p].add_a = seg[2 * p].add_r = 0;
                seg[2 * p + 1].set_a =
                    set_a + set_r * (m - l + 1);
                seg[2 * p + 1].set_r = set_r;
                seg[2 * p + 1].add_a =
                    seg[2 * p + 1].add_r = 0;
            }
            set_a = oo, set_r = 0;
            add_a = add_r = 0;
        } else if (add_a or add_r) {
            sum += add_a * sz + add_r * sz * (sz + 1) / 2;
            if (l != r) {
                int m = (l + r) / 2;
                seg[2 * p].add_a += add_a;
                seg[2 * p].add_r += add_r;
                seg[2 * p + 1].add_a +=
                    add_a + add_r * (m - l + 1);
                seg[2 * p + 1].add_r += add_r;
            }
            add_a = add_r = 0;
        }
    };
    int inter(pii a, pii b) {
        if (a.first > b.first) swap(a, b);
        return max(
            0, min(a.second, b.second) - b.first + 1);
    };
    ll set(int a, int b, ll aa, ll rr, int p, int l,
           int r) {
        prop(p, l, r);
        if (b < l or r < a) return seg[p].sum;
        if (a <= l and r <= b) {
            seg[p].set_a = aa;
            seg[p].set_r = rr;
            prop(p, l, r);
            return seg[p].sum;
        }
        int m = (l + r) / 2;
        int tam_l = inter({l, m}, {a, b});
        return seg[p].sum =
            set(a, b, aa, rr, 2 * p, l, m) +
            set(a, b, aa + rr * tam_l, rr,
                2 * p + 1, m + 1, r);
    };
    void update_set(int l, int r, ll aa, ll rr) {
        set(l, r, aa, rr, 1, 0, n - 1);
    };
    ll add(int a, int b, ll aa, ll rr, int p, int l,
           int r) {
        prop(p, l, r);
        if (b < l or r < a) return seg[p].sum;
        if (a <= l and r <= b) {
            seg[p].add_a += aa;
            seg[p].add_r += rr;
            prop(p, l, r);
            return seg[p].sum;
        }
        int m = (l + r) / 2;
        int tam_l = inter({l, m}, {a, b});
        return seg[p].sum =
            add(a, b, aa, rr, 2 * p, l, m) +
            add(a, b, aa + rr * tam_l, rr,
                2 * p + 1, m + 1, r);
    };
    void update_add(int l, int r, ll aa, ll rr) {
        add(l, r, aa, rr, 1, 0, n - 1);
    };
    ll query(int a, int b, int p, int l, int r) {
        prop(p, l, r);
        if (b < l or r < a) return 0;
        if (a <= l and r <= b) return seg[p].sum;
        int m = (l + r) / 2;
        return query(a, b, 2 * p, l, m) +
            query(a, b, 2 * p + 1, m + 1, r);
    };
    ll query(int l, int r) {
        return query(l, r, 1, 0, n - 1);
    };
};
```

```
seg[p].set_a = aa;
seg[p].set_r = rr;
prop(p, l, r);
return seg[p].sum;
}
int m = (l + r) / 2;
int tam_l = inter({l, m}, {a, b});
return seg[p].sum =
    set(a, b, aa, rr, 2 * p, l, m) +
    set(a, b, aa + rr * tam_l, rr,
        2 * p + 1, m + 1, r);
}
void update_set(int l, int r, ll aa, ll rr) {
    set(l, r, aa, rr, 1, 0, n - 1);
}
ll add(int a, int b, ll aa, ll rr, int p, int l,
       int r) {
    prop(p, l, r);
    if (b < l or r < a) return seg[p].sum;
    if (a <= l and r <= b) {
        seg[p].add_a += aa;
        seg[p].add_r += rr;
        prop(p, l, r);
        return seg[p].sum;
    }
    int m = (l + r) / 2;
    int tam_l = inter({l, m}, {a, b});
    return seg[p].sum =
        add(a, b, aa, rr, 2 * p, l, m) +
        add(a, b, aa + rr * tam_l, rr,
            2 * p + 1, m + 1, r);
}
void update_add(int l, int r, ll aa, ll rr) {
    add(l, r, aa, rr, 1, 0, n - 1);
}
ll query(int a, int b, int p, int l, int r) {
    prop(p, l, r);
    if (b < l or r < a) return 0;
    if (a <= l and r <= b) return seg[p].sum;
    int m = (l + r) / 2;
    return query(a, b, 2 * p, l, m) +
        query(a, b, 2 * p + 1, m + 1, r);
}
ll query(int l, int r) {
    return query(l, r, 1, 0, n - 1);
}
};
```

#### 2.4.2 Increment update query min & max (bottom up)

```
using SegT = ll;
struct QueryT {
    SegT mx, mn;
    QueryT() {
        : mx(numeric_limits<SegT>::min()),
          mn(numeric_limits<SegT>::max()) {}
    };
    QueryT(SegT _v) : mx(_v), mn(_v) {}
};
inline QueryT combine(QueryT ln, QueryT rn,
                      pii lr1, pii lr2) {
    chmax(ln.mx, rn.mx);
    chmin(ln.mn, rn.mn);
    return ln;
}
```

```
using LazyT = SegT;
inline QueryT applyLazyInQuery(QueryT q, LazyT l,
                               pii lr) {
    if (q.mx == QueryT().mx) q.mx = SegT();
    if (q.mn == QueryT().mn) q.mn = SegT();
    q.mx += l, q.mn += l;
    return q;
}
inline LazyT applyLazyInLazy(LazyT a, LazyT b) {
    return a + b;
}
using UpdateT = SegT;
inline QueryT applyUpdateInQuery(QueryT q,
                                  UpdateT u,
                                  pii lr) {
    if (q.mx == QueryT().mx) q.mx = SegT();
    if (q.mn == QueryT().mn) q.mn = SegT();
    q.mx += u, q.mn += u;
    return q;
}
inline LazyT applyUpdateInLazy(LazyT l, UpdateT u,
                               pii lr) {
    return l + u;
}
template <typename Qt = QueryT,
          typename Lt = LazyT,
          typename Ut = UpdateT, auto C = combine,
          auto ALQ = applyLazyInQuery,
          auto ALL = applyLazyInLazy,
          auto AUQ = applyUpdateInQuery,
          auto AUL = applyUpdateInLazy>
struct LazySegmentTree {
    int n, h;
    vector<Qt> ts;
    vector<Lt> ds;
    vector<pii> lrs;
    LazySegmentTree(int _n) {
        : n(_n),
          h(sizeof(int) * 8 - __builtin_clz(n)),
          ts(n << 1),
          ds(n),
          lrs(n << 1) {
            rep(i, 0, n) lrs[i + n] = {i, i};
            rrep(i, n - 1, 0) {
                lrs[i] = {lrs[i << 1].first,
                          lrs[i << 1 | 1].second};
            }
        }
    };
    LazySegmentTree(const vector<Qt> &xs) {
        : LazySegmentTree(len(xs)) {
            copy(all(xs), ts.begin() + n);
            rep(i, 0, n) lrs[i + n] = {i, i};
            rrep(i, n - 1, 0) {
                ts[i] = C(ts[i << 1], ts[i << 1 | 1],
                          lrs[i << 1], lrs[i << 1 | 1]);
            }
        }
    };
    void set(int p, Qt v) {
        ts[p + n] = v;
        build(p + n);
    };
    void upd(int l, int r, Ut v) {
        l += n, r += n + 1;
        int l0 = l, r0 = r;
        for (; l < r; l >>= 1, r >>= 1) {
            applyUpdateInQuery(ts[l], v, {l, l});
            applyUpdateInQuery(ts[r], v, {r, r});
            l = l0, r = r0;
        }
    };
};
```

```

    if (l & 1) apply(l++, v);
    if (r & 1) apply(--r, v);
}
build(10), build(r0 - 1);
}
Qt qry(int l, int r) {
    l += n, r += n + 1;
    push(l), push(r - 1);
    Qt resl = Qt(), resr = Qt();
    pii lr1 = {l, l}, lr2 = {r, r};
    for (; l < r; l >= 1, r >= 1) {
        if (l & 1)
            resl = C(resl, ts[l], lr1, lrs[l]), l++;
        if (r & 1)
            r--, resr = C(ts[r], resr, lrs[r], lr2);
    }
    return C(resl, resr, lr1, lr2);
}
void build(int p) {
    while (p > 1) {
        p >= 1;
        ts[p] = ALQ(C(ts[p << 1], ts[p << 1 | 1],
            lrs[p << 1], lrs[p << 1 | 1]),
            ds[p], lrs[p]);
    }
}
void push(int p) {
    rrep(s, h, 0) {
        int i = p >> s;
        if (ds[i] != Lt()) {
            apply(i << 1, ds[i]),
            apply(i << 1 | 1, ds[i]);
            ds[i] = Lt();
        }
    }
}
inline void apply(int p, Ut v) {
    ts[p] = AUQ(ts[p], v, lrs[p]);
    if (p < n) ds[p] = AUL(ds[p], v, lrs[p]);
}
};

```

### 2.4.3 Increment update sum query (top down)

```

struct Lnode {
    ll v;
    bool assign;
    Lnode() : v(), assign() {} // Neutral element
    Lnode(ll _v, bool a = 0) : v(_v), assign(a){};
};
using Qnode = ll;
using Unode = Lnode;
struct LSegTree {
    int n, ql, qr;
    vector<Qnode> st;
    vector<Lnode> lz;
    /*-----*/
    Qnode merge(Qnode lv, Qnode rv, int nl,
        int nr) {
        return lv + rv;
    }
    void prop(int i, int l, int r) {
        if (lz[i].assign) {
            st[i] = lz[i].v * (r - l + 1);
            if (l != r) lz[tol(i)] = lz[tor(i)] = lz[i];

```

```

        } else {
            st[i] += lz[i].v * (r - l + 1);
            if (l != r)
                lz[tol(i)].v += lz[i].v,
                lz[tor(i)].v += lz[i].v;
        }
        lz[i] = Lnode();
    }
    void applyV(int i, Unode v) {
        if (v.assign) {
            lz[i] = v;
        } else {
            lz[i].v += v.v;
        }
    }
    /*-----*/
    LSegTree() {}
    LSegTree(int _n)
        : n(_n), st(_n << 2), lz(_n << 2) {}
    bool disjoint(int l, int r) {
        return qr < l or r < ql;
    }
    bool contains(int l, int r) {
        return ql <= l and r <= qr;
    }
    int tol(int i) { return i << 1; }
    int tor(int i) { return i << 1 | 1; }
    void build(vector<Qnode> &v) {
        build(v, 1, 0, n - 1);
    }
    void build(vector<Qnode> &v, int i, int l,
        int r) {
        if (l == r) {
            st[i] = v[l];
            return;
        }
        int m = midpoint(l, r);
        build(v, tol(i), l, m);
        build(v, tor(i), m + 1, r);
        st[i] = merge(st[tol(i)], st[tor(i)], l, r);
    }
    void upd(int l, int r, Unode v) {
        ql = l, qr = r;
        upd(1, 0, n - 1, v);
    }
    void upd(int i, int l, int r, Unode v) {
        prop(i, l, r);
        if (disjoint(l, r)) return;
        if (contains(l, r)) {
            applyV(i, v);
            prop(i, l, r);
            return;
        }
        int m = midpoint(l, r);
        upd(tol(i), l, m, v);
        upd(tor(i), m + 1, r, v);
        st[i] = merge(st[tol(i)], st[tor(i)], l, r);
    }
    Qnode qry(int l, int r) {
        ql = l, qr = r;
        return qry(1, 0, n - 1);
    }
    Qnode qry(int i, int l, int r) {
        prop(i, l, r);
        if (disjoint(l, r)) return Qnode();
        if (contains(l, r)) return st[i];
        int m = midpoint(l, r);
        return merge(qry(tol(i), l, m),

```

```

        qry(tor(i), m + 1, r), l, r);
    }
};

```

## 2.5 Bitree 2D

**Description:** Given a 2D array you can increment an arbitrary position, and also query the subsum of a subgrid

**Time:** Update and query in  $O(\log N^2)$

```

struct Bit2d {
    int n;
    vll2d bit;
    Bit2d(int ni) : n(ni), bit(n + 1, vll(n + 1)) {}
    Bit2d(int ni, vll2d &xs)
        : n(ni), bit(n + 1, vll(n + 1)) {
        for (int i = 1; i <= n; i++) {
            for (int j = 1; j <= n; j++) {
                update(i, j, xs[i][j]);
            }
        }
    }
    void update(int x, int y, ll val) {
        for (; x <= n; x += (x & (-x))) {
            for (int i = y; i <= n; i += (i & (-i))) {
                bit[x][i] += val;
            }
        }
    }
    ll sum(int x, int y) {
        ll ans = 0;
        for (int i = x; i; i -= (i & (-i))) {
            for (int j = y; j; j -= (j & (-j))) {
                ans += bit[i][j];
            }
        }
        return ans;
    }
    ll query(int x1, int y1, int x2, int y2) {
        return sum(x2, y2) - sum(x2, y1 - 1) -
            sum(x1 - 1, y2) + sum(x1 - 1, y1 - 1);
    }
};

```

## 2.6 Convex Hull Trick / Line Container

**Description:** Container where you can add lines of the form  $mx + b$ , and query the maximum value at point  $x$ .

**Usage:** `insert_line(m, b)` inserts the line  $m \cdot x + b$  in the container.

`eval(x)` find the highest value among all lines in the point  $x$ .

**Time:** Eval and insert in  $O(\log N)$

```

const ll LLINF = 1e18;
const ll is_query = -LLINF;
struct Line {
    ll m, b;
    mutable function<const Line *(>> succ;
    bool operator<(const Line &rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line *s = succ();
        if (!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};
struct Cht : public multiset<Line> { // maintain
    // max m*x+b

```

```

bool bad(iterator y) {
    auto z = next(y);
    if (y == begin()) {
        if (z == end()) return 0;
        return y->m == z->m && y->b <= z->b;
    }
    auto x = prev(y);
    if (z == end())
        return y->m == x->m && y->b <= x->b;
    return (ld)(x->b - y->b) * (z->m - y->m) >=
        (ld)(y->b - z->b) * (y->m - x->m);
}

void insert_line(
    ll m,
    ll b) { // min -> insert (-m,-b) -> -eval()
    auto y = insert({m, b});
    y->succ = [=] {
        return next(y) == end() ? 0 : &*next(y);
    };
    if (bad(y)) {
        erase(y);
        return;
    }
    while (next(y) != end() && bad(next(y)))
        erase(next(y));
    while (y != begin() && bad(prev(y)))
        erase(prev(y));
}

ll eval(ll x) {
    auto l = *lower_bound((Line){x, is_query});
    return l.m * x + l.b;
}
};

```

## 2.7 DSU / UFDS

**Usage:** You may uncomment the commented parts to find online which nodes belong to each set, it makes the *union\_set* method cost  $O(\log^2)$  instead  $O(A)$

```

struct DSU {
    vi ps, sz;
    // vector<unordered_set<int>> sts;
    DSU(int N)
        : ps(N + 1),
          sz(N, 1) /*, sts(N) */
    {
        iota(ps.begin(), ps.end(), 0);
        // for (int i = 0; i < N; i++)
        // sts[i].insert(i);
    }
    int find_set(int x) {
        return ps[x] == x ? x
            : ps[x] = find_set(ps[x]);
    }
    int size(int u) { return sz[find_set(u)]; }
    bool same_set(int x, int y) {
        return find_set(x) == find_set(y);
    }
    void union_set(int x, int y) {
        if (same_set(x, y)) return;
        int px = find_set(x);
        int py = find_set(y);
        if (sz[px] < sz[py]) swap(px, py);
        ps[py] = px;
    }
};

```

```

        sz[px] += sz[py];
        // sts[px].merge(sts[py]);
    }
};

```

## 2.8 Lichao Tree (dynamic)

**Description:** Lichao Tree that creates the nodes dynamically, allowing to query and update from range  $[MAXL, MAXR]$

**Usage:**

- *query(x)* : find the highest point among all lines in the structure
- *add(a, b)* : add a line of form  $y = ax + b$  in the structure
- *addSegment(a, b, l, r)* : add a line segment of form  $y = ax + b$  which covers from range  $[l, r]$

**Time:**  $O(\log N)$

```

template <typename T = ll, T MAXL = 0,
          T MAXR = 1'000'000'001>
struct LiChaoTree {
    static const T inf =
        -numeric_limits<T>::max() / 2;
    bool first_best(T a, T b) { return a > b; }
    T get_best(T a, T b) {
        return first_best(a, b) ? a : b;
    }
    struct line {
        T m, b;
        T operator()(T x) { return m * x + b; }
    };
    struct node {
        line li;
        node *left, *right;
        node(line _li = {0, inf})
            : li(_li),
              left(nullptr),
              right(nullptr) {}
        ~node() {
            delete left;
            delete right;
        }
    };
    node *root;
    LiChaoTree(line li = {0, inf})
        : root(new node(li)) {}
    ~LiChaoTree() { delete root; }
    T query(T x, node *cur, T l, T r) {
        if (cur == nullptr) return inf;
        if (x < l or x > r) return inf;
        T mid = midpoint(l, r);
        T ans = cur->li(x);
        ans = get_best(ans,
            query(x, cur->left, l, mid));
        ans = get_best(
            ans, query(x, cur->right, mid + 1, r));
        return ans;
    }
    T query(T x) {
        return query(x, root, MAXL, MAXR);
    }
    void add(line li, node *&cur, T l, T r) {
        if (cur == nullptr) {
            cur = new node(li);
            return;
        }
        T mid = midpoint(l, r);
        if (first_best(li(mid), cur->li(mid)))
            swap(li, cur->li);
        if (first_best(li(l), cur->li(l)))

```

```

            add(li, cur->left, l, mid);
            if (first_best(li(r), cur->li(r)))
                add(li, cur->right, mid + 1, r);
    }
    void add(T m, T b) {
        add({m, b}, root, MAXL, MAXR);
    }
    void addSegment(line li, node *&cur, T l, T r,
        T lseg, T rseg) {
        if (r < lseg || l > rseg) return;
        if (cur == nullptr) cur = new node;
        if (lseg <= l && r <= rseg) {
            add(li, cur, l, r);
            return;
        }
        T mid = midpoint(l, r);
        if (l != r) {
            addSegment(li, cur->left, l, mid, lseg,
                rseg);
            addSegment(li, cur->right, mid + 1, r, lseg,
                rseg);
        }
    }
    void addSegment(T a, T b, T l, T r) {
        addSegment({a, b}, root, MAXL, MAXR, l, r);
    }
};

```

## 2.9 Merge sort tree

**Description:** Like a segment tree but each node stores the ordered subsegment it represents.

**Usage:**

- *inrange(l, r, a, b)* : counts the number of positions  $i$ ,  $l \leq i \leq r$  such that  $a \leq x_i \leq b$ .

**Time:** Build  $O(N \log N^2)$ , *inrange*  $O(\log N^2)$

**Memory:**  $O(n \log N)$

```

template <class T>
struct MergeSortTree {
    int n;
    vector<vector<T>> st;
    MergeSortTree(vector<T> &xs)
        : n(len(xs)), st(n << 1) {
        rep(i, 0, n) st[i + n] = vector<T>({xs[i]});
        rrep(i, n - 1, 0) {
            st[i].resize(len(st[i << 1]) +
                len(st[i << 1 | 1]));
            merge(all(st[i << 1]), all(st[i << 1 | 1]),
                st[i].begin());
        }
    }
    int count(int i, T a, T b) {
        return upper_bound(all(st[i]), b) -
            lower_bound(all(st[i]), a);
    }
    int inrange(int l, int r, T a, T b) {
        int ans = 0;
        for (l += n, r += n + 1; l < r;
            l >>= 1, r >>= 1) {
            if (l & 1) ans += count(l++, a, b);
            if (r & 1) ans += count(--r, a, b);
        }
        return ans;
    }
};

```



## 2.10 Mex with update

**Description:** This DS allows you to maintain an array of elements, insert, and remove, and query the MEX at any time.

**Usage:**

- *Mex(mxsZ)*: Initialize the DS, *mxsz* must be the maximum number of elements that the structure may have.
- *add(x)*: just adds one copy of *x*.
- *rmv(x)*: just remove a copy of *x*.
- *operator()*: returns the MEX.

**Time:**

- *Mex(mxsZ)*:  $O(\log mxsz)$
- *add(x)*:  $O(\log mxsz)$
- *rmv(x)*:  $O(\log mxsz)$
- *operator()*:  $O(1)$

```
struct Mex {
    int mx_sz;
    vi hs;
    set<int> st;

    Mex(int _mx_sz) : mx_sz(_mx_sz), hs(mx_sz + 1) {
        auto it = st.begin();
        rep(i, 0, mx_sz + 1) it = st.insert(it, i);
    }

    void add(int x) {
        if (x > mx_sz) return;
        if (!hs[x]++) st.erase(x);
    }

    void rmv(int x) {
        if (x > mx_sz) return;
        if (!--hs[x]) st.erase(x);
    }

    int operator()() const { return *st.begin(); }

    /*
    Optional, you can just create with size
    len(xs) add N elements :D
    */
    Mex(const vi &xs, int _mx_sz = -1)
        : Mex(~_mx_sz ? _mx_sz : len(xs)) {
        for (auto xi : xs) add(xi);
    }
};
```

## 2.11 Orderd Set (GNU PBDS)

**Usage:** If you need an ordered **multi** set you may add an id to each value. Using greater\_equal, or less\_equal is considered undefined behavior.

- **order\_of\_key(k)**: Number of items strictly smaller/greater than *k*.
- **find\_by\_order(k)**: K-th element in a set (counting from zero).

**Time:** Both  $O(\log N)$

**Warning:** Is 2 or 3 times slower then a regular set/map.

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set =
    tree<T, null_type, less<T>, rb_tree_tag,
        tree_order_statistics_node_update>;
```

## 2.12 Prefix Sum 2D

**Description:** Given an 2D array with *N* lines and *M* columns, find the sum of the subarray that have the left upper corner at  $(x1, y1)$  and right bottom corner at  $(x2, y2)$ .

**Time:** Build  $O(N \cdot M)$ , Query  $O(1)$ .

```
template <typename T>
struct psum2d {
    vector<vector<T>>> s;
    vector<vector<T>>> psum;
    psum2d(vector<vector<T>>> &grid, int n, int m)
        : s(n + 1, vector<T>(m + 1)),
          psum(n + 1, vector<T>(m + 1)) {
        for (int i = 1; i <= n; i++)
            for (int j = 1; j <= m; j++) {
                s[i][j] =
                    s[i][j - 1] + grid[i - 1][j - 1];
                psum[i][j] = psum[i - 1][j] + s[i][j];
            }
    }

    T query(int x1, int y1, int x2, int y2) {
        T ans = psum[x2 + 1][y2 + 1] + psum[x1][y1];
        ans -= psum[x2 + 1][y1] + psum[x1][y2 + 1];
        return ans;
    }
};
```

## 2.13 Venice Set

**Description:** A container that you can insert *q* copies of element *e*, increment every element in the container in *x*, query which is the best element and it's quantity and also remove *k* copies of the greatest element.

**Time:**

- add elment  $O(\log N)$
- remove  $O(\log N)$
- update:  $O(1)$
- query  $O(1)$

```
template <typename T = ll>
struct VeniceSet {
    using T2 = pair<T, ll>;
    priority_queue<T2, vector<T2>, greater<T2>> pq;
    T acc;
    VeniceSet() : acc() {}

    void add_element(const T &e, const ll q) {
        pq.emplace(e - acc, q);
    }

    void update_all(const T &x) { acc += x; }

    T2 best() {
        auto ret = pq.top();
        ret.first += acc;
        return ret;
    }

    void pop() { pq.pop(); }

    void pop_k(int k) {
        auto [e, q] = pq.top();
        pq.pop();
        q -= k;
        if (q) pq.emplace(e, q);
    }
};
```

## 2.14 Venice Set (complete)

**Description:** A container which you can insert elements update all at once and also make a few queries

**Usage:**

- *add\_element(e, q)*: adds *q* copies of *e*, if no *q* is provided adds a single one

- *update\_all(x)*: increment every value by *x*
- *erase(e)*: removes every copy of *e*, and returns how much was removed.
- *count(e)*: returns the number of *e* in the container
- *high()/low()*: returns the highest/lowest element, and it's quantity
- *pop\_low(q)/pop\_high(q)*: removes *q* copies of the lowest/highest elements if no *q* is provided removes all copies of the lowest/highest element.

You may answer which is the *K* - *th* value and it's quantity using an *ordered\_set*.

Probably works with other operations

**Time:** Considering *N* the number of distinct numbers in the container

- *add\_element(e, q)*:  $O(\log(N))$
- *update\_all(x)*:  $O(1)$
- *erase(e)*:  $O(\log(N))$
- *count(e)*:  $O(\log(N))$
- *high()/low()*:  $O(1)$
- *pop\_low(q)/pop\_high(q)*: worst case is  $O(N \cdot \log(N))$  if you remove all elements and so on...

**Warning:** There is no error handling if you try to *pop* more elements than exists or related stuff

```
struct VeniceSet {
    set<pll> st;
    ll acc;
    VeniceSet() : acc() {}

    ll add_element(ll e, ll q = 1) {
        q += erase(e);
        e -= acc;
        st.emplace(e, q);
        return q;
    }

    void update_all(ll x) { acc += x; }

    ll erase(ll e) {
        e -= acc;
        auto it = st.lb({e, LLONG_MIN});
        if (it == end(st) || (*it).first != e)
            return 0;
        ll ret = (*it).second;
        st.erase(it);
        return ret;
    }

    ll count(ll x) {
        x -= acc;
        auto it = st.lb({x, LLONG_MIN});
        if (it == end(st) || (*it).first != x)
            return 0;
        return (*it).second;
    }

    pll high() { return *rbegin(st); }
    pll low() { return *begin(st); }

    void pop_high(ll q = -1) {
        if (q == -1) q = high().second;
        while (q) {
            auto [e, eq] = high();
            st.erase(prev(end(st)));
            if (eq > q) add_element(e, eq - q);
            q = max(0ll, q - eq);
        }
    }

    void pop_low(ll q = -1) {
        if (q == -1) q = low().second;
        while (q) {
            auto [e, eq] = low();
            st.erase(st.begin());
            if (eq > q) add_element(e, eq - q);
            q = max(0ll, q - eq);
        }
    }
};
```

```

    }
}
};

```

## 2.15 Wavelet tree

```

using ll = long long;
template <typename T>
struct WaveletTree {
    struct Node {
        T lo, hi;
        int left_child, right_child;
        vector<int> pcnt;
        vector<ll> psum;

        Node(int lo_, int hi_)
            : lo(lo_), hi(hi_),
              left_child(0), right_child(0),
              pcnt(), psum() {}
    };

    vector<Node> nodes;
    WaveletTree(vector<T> v) {
        nodes.reserve(2 * v.size());
        auto [mn, mx] = minmax_element(v.begin(), v.end());
        auto build = [&](auto &&self, Node &node,
                           auto from, auto to) {
            if (node.lo == node.hi or from >= to)
                return;
            auto mid = midpoint(node.lo, node.hi);
            auto f = [&mid](T x) { return x <= mid; };
            node.pcnt.reserve(to - from + 1);
            node.pcnt.push_back(0);
            node.psum.reserve(to - from + 1);
            node.psum.push_back(0);
            T left_upper = node.lo,
              right_lower = node.hi;
            for (auto it = from; it != to; it++) {
                auto value = f(*it);
                node.pcnt.push_back(node.pcnt.back() + value);
                node.psum.push_back(node.psum.back() + *it);

                if (value)
                    left_upper = max(left_upper, *it);
                else
                    right_lower = min(right_lower, *it);
            }
            auto pivot = stable_partition(from, to, f);
            node.left_child = make_node(node.lo, left_upper);
            self(self, nodes[node.left_child], from, pivot);
            node.right_child = make_node(right_lower, node.hi);
            self(self, nodes[node.right_child], pivot, to);
        };
        build(build, nodes[make_node(*mn, *mx)], v.begin(), v.end());
    }

    T kth_element(int L, int R, int K) const {
        auto f = [&](auto &&self, const Node &node,
                           int l, int r, int k) -> T {

```

```

            if (l > r) return 0;
            if (node.lo == node.hi) return node.lo;
            int lb = node.pcnt[l],
                rb = node.pcnt[r + 1],
                left_size = rb - lb;
            return (left_size > k
                    ? self(self, nodes[node.left_child],
                           lb, rb - 1, k)
                    : self(self, nodes[node.right_child],
                           l - lb, r - rb,
                           k - left_size));
        };
        return f(f, nodes[0], L, R, K);
    }

    pair<int, ll> count_and_sum_in_range(
        int L, int R, T a, T b) const {
        auto f = [&](auto &&self, const Node &node,
                           int l, int r) -> pair<int, ll> {
            if (l > r or node.lo > b or node.hi < a)
                return {0, 0};
            if (a <= node.lo and node.hi <= b)
                return {r - l + 1,
                        (node.lo == node.hi
                         ? (r - l + 1ll) * node.lo
                         : node.psum[r + 1] - node.psum[l])};

            int lb = node.pcnt[l],
                rb = node.pcnt[r + 1];
            auto [left_cnt, left_sum] = self(self, nodes[node.left_child], lb, rb - 1);
            auto [right_cnt, right_sum] = self(self, nodes[node.right_child], l - lb, r - rb);
            return {left_cnt + right_cnt, left_sum + right_sum};
        };
        return f(f, nodes[0], L, R);
    }

    inline int count_in_range(int L, int R, T a, T b) const {
        return count_and_sum_in_range(L, R, a, b).first;
    }

    inline ll sum_in_range(int L, int R, T a, T b) const {
        return count_and_sum_in_range(L, R, a, b).second;
    }

private:
    int make_node(T lo, T hi) {
        int id = (int)nodes.size();
        nodes.emplace_back(lo, hi);
        return id;
    }
};

```

## 3 Dynamic Programming

### 3.1 Binary Knapsack (bottom up)

**Description:** Given the points each element have, and it respective cost, computes the maximum points we can get if we can ignore/choose an

element, in such way that the sum of costs don't exceed the maximum cost allowed.  
**Time:**  $O(N * W)$   
**Warning:** The vectors  $VS$  and  $WS$  starts at one, so it need an empty value at index 0.

```

const int MAXN(1'000), MAXCOST(1'000 * 20);
ll dp[MAXN + 1][MAXCOST + 1];
bool ps[MAXN + 1][MAXCOST + 1];
pair<ll, vi> knapsack(const vll &points,
                     const vi &costs,
                     int maxCost) {
    int n = len(points) - 1; // ELEMENTS START AT INDEX 1 !
    for (int m = 0; m <= maxCost; m++) {
        dp[0][m] = 0;
    }
    for (int i = 1; i <= n; i++) {
        dp[i][0] = dp[i - 1][0] + (costs[i] == 0) * points[i];
        ps[i][0] = costs[i] == 0;
    }
    for (int i = 1; i <= n; i++) {
        for (int m = 1; m <= maxCost; m++) {
            dp[i][m] = dp[i - 1][m], ps[i][m] = 0;
            int w = costs[i];
            ll v = points[i];
            if (w <= m and dp[i - 1][m - w] + v > dp[i][m]) {
                dp[i][m] = dp[i - 1][m - w] + v,
                ps[i][m] = 1;
            }
        }
    }
    vi is;
    for (int i = n, m = maxCost; i >= 1; --i) {
        if (ps[i][m]) {
            is.emplace_back(i);
            m -= costs[i];
        }
    }
    return {dp[n][maxCost], is};
}

```

### 3.2 Edit Distance

**Time:**  $O(N * M)$

```

int edit_distance(const string &a,
                  const string &b) {
    int n = a.size();
    int m = b.size();
    vector<vi> dp(n + 1, vi(m + 1, 0));
    int ADD = 1, DEL = 1, CHG = 1;
    for (int i = 0; i <= n; ++i) {
        dp[i][0] = i * DEL;
    }
    for (int i = 1; i <= m; ++i) {
        dp[0][i] = ADD * i;
    }
    for (int i = 1; i <= n; ++i) {
        for (int j = 1; j <= m; ++j) {
            int add = dp[i][j - 1] + ADD;
            int del = dp[i - 1][j] + DEL;
            int chg =

```

```

        dp[i - 1][j - 1] +
        (a[i - 1] == b[j - 1] ? 0 : 1) * CHG;
    dp[i][j] = min({add, del, chg});
}
}
return dp[n][m];
}

```

### 3.3 Knapsack

**Description:** Finds the maximum score you can achieve, given that you have  $N$  items, each item has a *cost*, a *point* and a *quantity*, you can spent at most *maxcost* and buy each item the maximum quantity it has.

**Time:**  $O(n \cdot maxcost \cdot \log maxqtd)$

**Memory:**  $O(maxcost)$ .

```

11 knapsack(const vi &weight, const vll &value,
           const vi &qtd, int maxCost) {
    vi costs;
    vll values;
    for (int i = 0; i < len(weight); i++) {
        ll q = qtd[i];
        for (ll x = 1; x <= q; q -= x, x <= 1) {
            costs.eb(x * weight[i]);
            values.eb(x * value[i]);
        }
        if (q) {
            costs.eb(q * weight[i]);
            values.eb(q * value[i]);
        }
    }
    vll dp(maxCost + 1);
    for (int i = 0; i < len(values); i++) {
        for (int j = maxCost; j > 0; j--) {
            if (j >= costs[i])
                dp[j] = max(dp[j],
                           values[i] + dp[j - costs[i]]);
        }
    }
    return dp[maxCost];
}

```

### 3.4 Longest Increasing Subsequence

**Description:** Find the pair  $(sz, psx)$  where  $sz$  is the size of the longest subsequence and  $psx$  is a vector where  $psx_i$  tells the size of the longest increase subsequence that ends at position  $i$ . *getidx* just tells which indices could be in the longest increasing subsequence.

**Time:**  $O(n \log n)$

```

template <typename T>
pair<int, vi> lis(const vector<T> &xs, int n) {
    vector<T> dp(n + 1, numeric_limits<T>::max());
    dp[0] = numeric_limits<T>::min();
    int sz = 0;
    vi psx(n);
    rep(i, 0, n) {
        int pos =
            lower_bound(all(dp), xs[i]) - dp.begin();
        sz = max(sz, pos);
        dp[pos] = xs[i];
        psx[i] = pos;
    }
}

```

```

    return {sz, psx};
}
template <typename T>
vi get_idx(vector<T> xs) {
    int n = xs.size();
    auto [sz1, psx1] = lis(xs, n);
    transform(rall(xs), xs.begin(),
              [](T x) { return -x; });
    auto [sz2, psx2] = lis(xs, n);
    vi ans;
    rep(i, 0, n) {
        int l = psx1[i];
        int r = psx2[n - i - 1];
        if (l + r - 1 == sz1) ans.eb(i);
    }
    return ans;
}

```

### 3.5 Monery sum

**Description:** Find every possible sum using the given values only once.

```

set<int> money_sum(const vi &xs) {
    using vc = vector<char>;
    using vvc = vector<vc>;
    int _m = accumulate(all(xs), 0);
    int _n = xs.size();
    vvc _dp(_n + 1, vc(_m + 1, 0));
    set<int> _ans;
    _dp[0][xs[0]] = 1;
    for (int i = 1; i < _n; ++i) {
        for (int j = 0; j <= _m; ++j) {
            if (j == 0 or _dp[i - 1][j]) {
                _dp[i][j + xs[i]] = 1;
                _dp[i][j] = 1;
            }
        }
    }
    for (int i = 0; i < _n; ++i)
        for (int j = 0; j <= _m; ++j)
            if (_dp[i][j]) _ans.insert(j);
    return _ans;
}

```

### 3.6 SOS

```

const int LOG = 17;
int dp[(1<< LOG)];
for(int b = 0; b < LOG; b++){
    for(int mask = 0; mask < (1<< LOG); mask++){
        if(mask & (1<< b)){
            dp[mask] |= dp[mask ^ (1LL << b)];
        }
    }
}

```

### 3.7 Travelling Salesman Problem

**Time:**  $O(N^2 \cdot 2^N)$

**Memory:**  $O(N^2 \cdot 2^N)$

```

vll2d dist;
vll memo;
int tsp(int i, int mask, int N) {
    if (mask == (1 << N) - 1) return dist[i][0];
    if (memo[i][mask] != -1) return memo[i][mask];
    int ans = INT_MAX << 1;
    for (int j = 0; j < N; ++j) {
        if (mask & (1 << j)) continue;
        auto t =
            tsp(j, mask | (1 << j), N) + dist[i][j];
        ans = min(ans, t);
    }
    return memo[i][mask] = ans;
}

```

## 4 Extras

### 4.1 Binary to gray

```

string binToGray(string bin) {
    string gray(bin.size(), '0');
    int n = bin.size() - 1;
    gray[0] = bin[0];
    for (int i = 1; i <= n; i++) {
        gray[i] = '0' + (bin[i - 1] == '1') ^
                    (bin[i] == '1');
    }
    return gray;
}

```

### 4.2 Get permutation cycles

**Description:** Receives a permutation  $[0, n-1]$  and return a vector 2D with each cycle.

```

vll2d getPermutationCicles(const vll &ps) {
    ll n = len(ps);
    vector<char> visited(n);
    vector<vll> cicles;
    rep(i, 0, n) {
        if (visited[i]) continue;
        vll cicle;
        ll pos = i;
        while (!visited[pos]) {
            cicle.pb(pos);
            visited[pos] = true;
            pos = ps[pos];
        }
        cicles.push_back(vll(all(cicle)));
    }
    return cicles;
}

```

### 4.3 Max & Min Check

**Description:** Returns the min/max value in range [l, r] that satisfies the lambda function check, if there is no such value the 'nullopt' is returned.

**Usage:** check must be a function that receives an integer and return a boolean.

**Time:**  $O(\log r - l + 1)$

```
template <typename T>
optional<T> maxCheck(T l, T r, auto check) {
    optional<T> ret;
    while (l <= r) {
        T m = midpoint(l, r);
        if (check(m))
            ret ? chmax(ret, m) : ret = m, l = m + 1;
        else
            r = m - 1;
    }
    return ret;
}

template <typename T>
optional<T> minCheck(T l, T r, auto check) {
    optional<T> ret;
    while (l <= r) {
        T m = midpoint(l, r);
        if (check(m))
            ret ? chmin(ret, m) : ret = m, r = m - 1;
        else
            l = m + 1;
    }
    return ret;
}
```

### 4.4 Mo's algorithm

```
template <typename T, typename Tans>
struct Mo {
    struct Query {
        int l, r, idx, block;
        Query(int _l, int _r, int _idx, int _block)
            : l(_l), r(_r), idx(_idx), block(_block) {}
        bool operator<(const Query &q) const {
            if (block != q.block)
                return block < q.block;
            return (block & 1 ? (r < q.r) : (r > q.r));
        }
    };
    vector<T> vs;
    vector<Query> qs;
    const int block_size;
    Mo(const vector<T> &a)
        : vs(a), block_size((int)ceil(sqrt(a.size()))) {}
    void add_query(int l, int r) {
        qs.emplace_back(l, r, qs.size(), l / block_size);
    }
    auto solve() {
        // get answer return type
        vector<Tans> answers(qs.size());
        sort(all(qs));
        int cur_l = 0, cur_r = -1;
        for (auto q : qs) {
```

```
            while (cur_l > q.l) add(--cur_l);
            while (cur_r < q.r) add(++cur_r);
            while (cur_l < q.l) remove(cur_l++);
            while (cur_r > q.r) remove(cur_r--);
            answers[q.idx] = get_answer();
        }
        return answers;
    }
private:
    // add value at idx from data structure
    inline void add(int idx) {}
    // remove value at idx from data structure
    inline void remove(int idx) {}
    // extract current answer of the data structure
    inline Tans get_answer() {}
};
```

### 4.5 \_\_int128t stream

```
void print(__int128 x) {
    if (x < 0) {
        cout << '-';
        x = -x;
    }
    if (x > 9) print(x / 10);
    cout << (char)((x % 10) + '0');
}

__int128 read() {
    string s;
    cin >> s;
    __int128 x = 0;
    for (auto c : s) {
        if (c != '-') x += c - '0';
        x *= 10;
    }
    x /= 10;
    if (s[0] == '-') x = -x;
    return x;
}
```

## 5 Geometry

### 5.1 Check if a point belong to line segment

```
// Verifica se o ponto P pertence ao segmento de
// reta AB
const ld EPS = 1e-9;
template <typename T>
struct Point {
    T x, y;
    Point(T _x, T _y) : x(_x), y(_y) {}
};

template <typename T>
bool equals(const T a, const T b) {
    if (is_floating_point<T>) {
        return fabs1(a - b) <= EPS;
    }
    return a == b;
}

/*
Verify if the segment AB contains point P
*/
template <typename T>
```

```
bool contains(const Point<T> &A,
              const Point<T> &B,
              const Point<T> &P) {
    auto xmin = min(A.x, B.x);
    auto xmax = max(A.x, B.x);
    auto ymin = min(A.y, B.y);
    auto ymax = max(A.y, B.y);
    if (P.x < xmin || P.x > xmax || P.y < ymin ||
        P.y > ymax)
        return false;
    return equals((P.y - A.y) * (B.x - A.x),
                  (P.x - A.x) * (B.y - A.y));
}
```

### 5.2 Check if point is inside triangle

```
struct point {
    int x, y;
    int id;
    point operator-(const point &o) const {
        return {x - o.x, y - o.y};
    }
    int operator^(const point &o) const {
        return x * o.y - y * o.x;
    }
};

/*
Verify the direction that the point
_e_ is in relation to the vector
formed by the points a->b
-1 = right
0 = collinear
1 = left
*/
int ccw(point a, point b, point e) {
    int tmp = (b - a) ^ (e - a);
    return (tmp > 0) - (tmp < 0);
}

/*
Verify if the point e
is inside the triangle formed by
the points t1, t2, t3
*/
bool inside_triangle(point t1, point t2, point t3,
                    point e) {
    int x = ccw(t1, t2, e);
    int y = ccw(t2, t3, e);
    int z = ccw(t3, t1, e);
    return !((x == 1 or y == 1 or z == 1) and
              (x == -1 or y == -1 or z == -1));
}
```

### 5.3 Convex hull

```
struct pt {
    double x, y;
    int id;
};

int orientation(pt a, pt b, pt c) {
    double v = a.x * (b.y - c.y) +
              b.x * (c.y - a.y) +
              c.x * (a.y - b.y);
```

```

    if (v < 0) return -1; // clockwise
    if (v > 0) return +1; // counter-clockwise
    return 0;
}

bool cw(pt a, pt b, pt c,
        bool include_collinear) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}

bool collinear(pt a, pt b, pt c) {
    return orientation(a, b, c) == 0;
}

void convex_hull(vector<pt> &pts,
                 bool include_collinear = false) {
    pt p0 = *min_element(all(pts), [](pt a, pt b) {
        return make_pair(a.y, a.x) <
               make_pair(b.y, b.x);
    });
    sort(all(pts), [&p0](const pt &a, const pt &b) {
        int o = orientation(p0, a, b);
        if (o == 0)
            return (p0.x - a.x) * (p0.x - a.x) +
                   (p0.y - a.y) * (p0.y - a.y) <
                   (p0.x - b.x) * (p0.x - b.x) +
                   (p0.y - b.y) * (p0.y - b.y);
        return o < 0;
    });
    if (include_collinear) {
        int i = len(pts) - 1;
        while (i >= 0 &&
               collinear(p0, pts[i], pts.back()))
            i--;
        reverse(pts.begin() + i + 1, pts.end());
    }
    vector<pt> st;
    for (int i = 0; i < len(pts); i++) {
        while (st.size() > 1 &&
               !cw(st[len(st) - 2], st.back(), pts[i],
                  include_collinear))
            st.pop_back();
        st.push_back(pts[i]);
    }
    pts = st;
}

```

## 5.4 Polygon lattice points

```

11 cross(ll x1, ll y1, ll x2, ll y2) {
    return x1 * y2 - x2 * y1;
}

11 polygonArea(vector<pll> &pts) {
    ll ats = 0;
    for (int i = 2; i < len(pts); i++)
        ats +=
            cross(pts[i].first - pts[0].first,
                  pts[i].second - pts[0].second,
                  pts[i - 1].first - pts[0].first,
                  pts[i - 1].second - pts[0].second);
    return abs(ats / 2ll);
}

11 boundary(vector<pll> &pts) {
    ll ats = pts.size();
    for (int i = 0; i < len(pts); i++) {
        ll deltax = (pts[i].first -

```

```

        pts[(i + 1) % pts.size()].first);
        ll deltax =
            (pts[i].second -
             pts[(i + 1) % pts.size()].second);
        ats += abs(__gcd(deltax, deltax)) - 1;
    }
    return ats;
}

pll latticePoints(vector<pll> &pts) {
    ll bounds = boundary(pts);
    ll area = polygonArea(pts);
    ll inside = area + 1ll - bounds / 2ll;
    return {inside, bounds};
}

```

## 5.5 Segment intersection

```

using ld = long double;

template <typename T = ld>
struct Point {
    T x, y;
    bool is_port;
};

template <typename T = ld>
bool operator==(const Point<T> &a,
                const Point<T> &b) {
    return a.x == b.x and a.y == b.y;
}

template <typename T = ld>
struct Segment {
    Point<T> p1, p2;
};

template <typename T>
int orientation(Point<T> p, Point<T> q,
                Point<T> r) {
    int val = (q.y - p.y) * (r.x - q.x) -
              (q.x - p.x) * (r.y - q.y);
    // TODO: if it's a float must use other way to
    // compare
    if (val == 0)
        return 0; // colinear
    else if (val > 0)
        return 1; // clockwise
    else
        return 2; // counterclockwise
}

template <typename T>
bool do_segment_intersect(Segment<T> s1,
                          Segment<T> s2) {
    int o1 = orientation(s1.p1, s1.p2, s2.p1);
    int o2 = orientation(s1.p1, s1.p2, s2.p2);
    int o3 = orientation(s2.p1, s2.p2, s1.p1);
    int o4 = orientation(s2.p1, s2.p2, s1.p2);
    return (o1 != o2 and o3 != o4) or
           (o1 == 0 and o3 == 0) or
           (o2 == 0 and o4 == 0);
}

```

## 6 Graphs

### 6.1 Heavy-Light Decomposition (point update)

#### 6.1.1 Maximum number on path

```

struct Node {
    ll value;
    Node()
        : value(numeric_limits<
                ll>::min()){}; // Neutral
    // element

    Node(ll v) : value(v){};
};

Node combine(Node l, Node r) {
    Node m;
    m.value = max(l.value, r.value);
    return m;
}

template <typename T = Node, auto F = combine>
struct SegTree {
    int n;
    vector<T> st;
    SegTree(int _n) : n(_n), st(n << 1) {}

    void set(int p, const T &k) {
        for (st[p += n] = k; p >= 1;)
            st[p] = F(st[p << 1], st[p << 1 | 1]);
    }

    T query(int l, int r) {
        T ans1, ansr;
        for (l += n, r += n + 1; l < r;
             l >= 1, r >= 1) {
            if (l & 1) ans1 = F(ans1, st[l++]);
            if (r & 1) ansr = F(st[--r], ansr);
        }
        return F(ans1, ansr);
    }
};

template <typename SegT = Node,
          auto SegOp = combine>
struct HeavyLightDecomposition {
    int n;
    vi ps, ds, sz, heavy, head, pos;
    SegTree<SegT, SegOp> seg;

    HeavyLightDecomposition(const vi2d &g,
                            const vector<SegT> &v,
                            int root = 0)
        : n(len(g)), seg(n) {
        ps = ds = sz = heavy = head = pos = vi(n, -1);
        auto dfs = [&](auto &&self, int u) -> void {
            sz[u] = 1;
            int mx = 0;
            for (auto x : g[u])
                if (x != ps[u]) {
                    ps[x] = u;
                    ds[x] = ds[u] + 1;
                    self(self, x);
                    sz[u] += sz[x];
                    if (sz[x] > mx)
                        mx = sz[x], heavy[u] = x;
                }
        };
        dfs(dfs, root);
        for (int i = 0, cur = 0; i < n; i++) {
            if (ps[i] == -1 or heavy[ps[i]] != i)

```

```

        for (int j = i; j != -1; j = heavy[j]) {
            head[j] = i;
            pos[j] = cur++;
        }
    }
    rep(i, 0, n) seg.set(pos[i], v[i]);
}
vector<pii> disjoint_ranges(int u, int v) {
    vector<pii> ret;
    for (; head[u] != head[v]; v = ps[head[v]]) {
        if (ds[head[u]] > ds[head[v]]) swap(u, v);
        ret.eb(pos[head[v]], pos[v]);
    }
    if (ds[u] > ds[v]) swap(u, v);
    ret.eb(pos[u], pos[v]);
    return ret;
}
SegT query_path(int u, int v) {
    SegT res;
    for (auto [l, r] : disjoint_ranges(u, v)) {
        res = SegOp(res, seg.query(l, r));
    }
    return res;
}
SegT query_subtree(int u) const {
    return seg.query(pos[u], pos[u] + sz[u] - 1);
}
void set(int u, SegT x) { seg.set(pos[u], x); }
};

```

## 6.2 2-SAT

**Description:** Calculates a valid assignment to boolean variables  $a, b, c, \dots$  to a 2-SAT problem, so that an expression of the type  $(a|b)\&\&(!a|c)\&\&(d|!b)\&\&\dots$  becomes true, or reports that it is unsatisfiable.

**Usage:** Negated variables are represented by bit-inversions ( $\bar{x}$ ).

Returns true iff it is solvable ts.values[0..N-1] holds the assigned values to the vars.

**Time:**  $O(N + E)$ , where  $N$  is the number of boolean variables, and  $E$  is the number of clauses.

```

struct TwoSat {
    int N;
    vector<vi> gr;
    vi values; // 0 = false, 1 = true
    TwoSat(int n = 0) : N(n), gr(2 * n) {}
    int addVar() { // (optional)
        gr.eb();
        gr.eb();
        return N++;
    }
    void either(int f, int j) {
        f = max(2 * f, -1 - 2 * f);
        j = max(2 * j, -1 - 2 * j);
        gr[f].pb(j ^ 1);
        gr[j].pb(f ^ 1);
    }
    void setValue(int x) { either(x, x); }
    void atMostOne(const vi &li) { // (optional)
        if (sz(li) <= 1) return;
        int cur = ~li[0];
        rep(i, 2, sz(li)) {
            int next = addVar();
            either(cur, ~li[i]);
        }
    }
};

```

```

        either(cur, next);
        either(~li[i], next);
        cur = ~next;
    }
    either(cur, ~li[1]);
}
vi val, comp, z;
int time = 0;
int dfs(int i) {
    int low = val[i] = ++time, x;
    z.pb(i);
    for (int e : gr[i])
        if (!comp[e])
            low = min(low, val[e] ? dfs(e));
    if (low == val[i]) do {
        x = z.back();
        z.pop_back();
        comp[x] = low;
        if (values[x >> 1] == -1)
            values[x >> 1] = x & 1;
    } while (x != i);
    return val[i] = low;
}
bool solve() {
    values.assign(N, -1);
    val.assign(2 * N, 0);
    comp = val;
    rep(i, 0, 2 * N) if (!comp[i]) dfs(i);
    rep(i, 0, N) if (comp[2 * i] ==
                    comp[2 * i + 1]) return 0;
    return 1;
}
};

```

## 6.3 BFS-01

**Description:** Similar to a Dijkstra given a weighted graph finds the distance from source  $s$  to every other node.

**Time:**  $O(V + E)$

**Warning:** Applicable only when the weight of the edges  $\in \{0, x\}$

```

vector<pair<ll, int>> adj[maxn];
ll dists[maxn];
int s, n;
void bfs_01() {
    fill(dists, dists + n, oo);
    dist[s] = 0;
    deque<int> q;
    q.emplace_back(s);
    while (not q.empty()) {
        auto u = q.front();
        q.pop_front();
        for (auto [v, w] : adj[u]) {
            if (dist[v] <= dist[u] + w) continue;
            dist[v] = dist[u] + w;
            w ? q.emplace_back(v) : q.emplace_front(v);
        }
    }
}

```

## 6.4 Bellman ford

**Description:** Find shortest path from a single source to all other nodes. Can detect negative cycles.

**Time:**  $O(V \cdot E)$

```

bool bellman_ford(
    const vector<vector<pair<int, ll>>> &g, int s,
    vector<ll> &dist) {
    int n = (int)g.size();
    dist.assign(n, LLONG_MAX);
    vector<int> count(n);
    vector<char> in_queue(n);
    queue<int> q;
    dist[s] = 0;
    q.push(s);
    in_queue[s] = true;
    while (not q.empty()) {
        int cur = q.front();
        q.pop();
        in_queue[cur] = false;
        for (auto [to, w] : g[cur]) {
            if (dist[cur] + w < dist[to]) {
                dist[to] = dist[cur] + w;
                if (not in_queue[to]) {
                    q.push(to);
                    in_queue[to] = true;
                    count[to]++;
                    if (count[to] > n) return false;
                }
            }
        }
    }
    return true;
}

```

## 6.5 Bellman-Ford (find negative cycle)

**Description:** Given a directed graph find a negative cycle by running  $n$  iterations, and if the last one produces a relaxation than there is a cycle.

**Time:**  $O(V \cdot E)$

```

const ll oo = 2500 * 1e9;
using graph = vector<vector<pair<int, ll>>>>;
vi negative_cycle(graph &g, int n) {
    vll d(n, oo);
    vi p(n, -1);
    int x = -1;
    d[0] = 0;
    for (int i = 0; i < n; i++) {
        x = -1;
        for (int u = 0; u < n; u++) {
            for (auto &[v, l] : g[u]) {
                if (d[u] + l < d[v]) {
                    d[v] = d[u] + l;
                    p[v] = u;
                    x = v;
                }
            }
        }
    }
    if (x == -1)
        return {};
    else {
        for (int i = 0; i < n; i++) x = p[x];
        vi cycle;
        for (int v = x;; v = p[v]) {
            cycle.eb(v);
            if (v == x and len(cycle) > 1) break;
        }
        reverse(all(cycle));
    }
}

```

```

        return cycle;
    }
}

```

## 6.6 Biconnected Components

**Description:** Build a vector of vectors, where the  $i$ -th vector correspond to the nodes of the  $i$ -th, biconnected component, a biconnected component is a subset of nodes and edges in which there is no cut point, also exist at least two distinct routes in vertex between any two vertex in the same biconnected component.

**Time:**  $O(N + M)$

```

const int maxn(5 '00' 000);
int tin[maxn], stck[maxn], bcc_cnt, n, top = 0, timer = 1;
vector<int> g[maxn], nodes[maxn];

int tarjan(int u, int p = -1) {
    int lowu = tin[u] = timer++;
    int son_cnt = 0;
    stck[++top] = u;
    for (auto v : g[u]) {
        if (!tin[v]) {
            son_cnt++;
            int lowx = tarjan(v, u);
            lowu = min(lowu, lowx);
            if (lowx >= tin[u]) {
                while (top != -1 && stck[top + 1] != v)
                    nodes[bcc_cnt].emplace_back(
                        stck[top--]);
                nodes[bcc_cnt++].emplace_back(u);
            }
        } else {
            lowu = min(lowu, tin[v]);
        }
    }
    if (p == -1 && son_cnt == 0) {
        nodes[bcc_cnt++].emplace_back(u);
    }
    return lowu;
}

void build_bccs() {
    timer = 1;
    top = -1;
    memset(tin, 0, sizeof(int) * n);
    for (int i = 0; i < n; i++) nodes[i] = {};
    bcc_cnt = 0;
    for (int u = 0; u < n; u++)
        if (!tin[u]) tarjan(u);
}

```

## 6.7 Binary Lifting/Jumping

**Description:** Given a function/successor graph answers queries of the form which is the node after  $k$  moves starting from  $u$ .

**Time:** Build  $O(N \cdot MAXLOG2)$ , Query  $O(MAXLOG2)$ .

```
const int MAXN(2e5), MAXLOG2(30);
int bl[MAXN][MAXLOG2 + 1];
int N;
int jump(int u, ll k) {
    for (int i = 0; i <= MAXLOG2; i++) {
        if (k & (1ll << i)) u = bl[u][i];
    }
    return u;
}
```

```

}
void build() {
    for (int i = 1; i <= MAXLOG2; i++) {
        for (int j = 0; j < N; j++) {
            bl[j][i] = bl[bl[j][i - 1]][i - 1];
        }
    }
}
}

```

## 6.8 Bipartite Graph

**Description:** Given a graph, find the 'left' and 'right' side if is a bipartite graph, if is not then a empty vi2d is returned

**Time:**  $O(N + M)$

```

vi2d bipartite_graph(vi2d &adj) {
    int n = len(adj);
    vi side(n, -1);
    vi2d ret(2);
    rep(u, 0, n) {
        if (side[u] == -1) {
            queue<int> q;
            q.emp(u);
            side[u] = 0;
            ret[0].eb(u);
            while (len(q)) {
                int u = q.front();
                q.pop();
                for (auto v : adj[u]) {
                    if (side[v] == -1) {
                        side[v] = side[u] ^ 1;
                        ret[side[v]].eb(v);
                        q.push(v);
                    } else if (side[u] == side[v])
                        return {};
                }
            }
        }
    }
    return ret;
}

```

## 6.9 Block-Cut tree

```

struct block_cut_tree {
    int n;
    vector<int> id, is_cutpoint, tin, low, stk;
    vector<vector<int>> comps, tree;
    block_cut_tree(vector<vector<int>> &g)
        : n(g.size()),
          id(n),
          is_cutpoint(n),
          tin(n),
          low(n) {
        // build comps
        for (int i = 0; i < n; i++) {
            if (!tin[i]) {
                int timer = 0;
                dfs(i, -1, timer, g);
            }
        }
        int node_id = 0;
        for (int u = 0; u < n; u++) {
            if (is_cutpoint[u]) {
                id[u] = node_id++;
                tree.push_back({});
            }
        }
    }
};

```

```

    }
    for (auto &comp : comps) {
        int node = node_id++;
        tree.push_back({});
        for (int u : comp) {
            if (!is_cutpoint[u]) {
                id[u] = node;
            } else {
                tree[node].emplace_back(id[u]);
                tree[id[u]].emplace_back(node);
            }
        }
    }
}

void dfs(int u, int p, int &timer,
         vector<vector<int>> &g) {
    tin[u] = low[u] = ++timer;
    stk.emplace_back(u);
    for (auto v : g[u]) {
        if (v == p) continue;
        if (!tin[v]) {
            dfs(v, u, timer, g);
            low[u] = min(low[u], low[v]);
            if (low[v] >= tin[u]) {
                is_cutpoint[u] =
                    (tin[u] > 1 or tin[v] > 2);
                comps.push_back({u});
                while (comps.back().back() != v) {
                    comps.back().emplace_back(stk.back());
                    stk.pop_back();
                }
            }
        } else
            low[u] = min(low[u], tin[v]);
    }
}
};

```

## 6.10 D'Escopo-Pape

**Description:** Is a single source shortest path that works faster than Dijkstra's algorithm and the Bellman-Ford algorithm in most cases, and will also work for negative edges. However not for negative cycles. There exists cases where it runs in exponential time.

**Usage:** Returns a pair containing two vectors, the first one with the distance from  $s$  to every other node, and another one with the ancestor of each node. note that the ancestor of  $s$  is  $-1$

```
using Edge = pair<ll, int>;
using Adj = vector<vector<Edge>>;

pair<vll, vi> desopo_pape(int s, int n,
                           const Adj &adj) {
    vll ds(n, LLONG_MAX), ps(n, -1);
    ds[s] = 0;
    vi ms(n, 2);
    deque<int> q;
    q.pb(s);
    while (len(q)) {
        int u = q.front();
        q.pop_front();
        ms[u] = 0;
        for (auto [w, v] : adj[u]) {
            if (chmin(ds[v], w + ds[u])) {
                ps[v] = u;
                if (ms[v] == 2)
```

```

        ms[v] = 1, q.pb(v);
    else if (ms[v] == 0)
        ms[v] = 1, q.pf(v);
    }
}
return {ds, ps};
}

```

## 6.11 Dijkstra

```

const int MAXN = 1'00'000;
const ll MAXW = 1'000'000ll;
constexpr ll OO = MAXW * MAXN + 1;
using Edge = pair<ll, int>; // { weight, node}
using Adj = vector<vector<Edge>>;
template <typename T>
using min_heap =
    priority_queue<T, vector<T>, greater<T>>;
pair<vll, vi> dijkstra(const Adj &g, int s) {
    int n = len(g);
    min_heap<Edge> pq;
    vll ds(n, OO);
    vi ps(n, -1);
    pq.emp(0, s);
    ds[s] = 0;
    while (len(pq)) {
        auto [du, u] = pq.top();
        pq.pop();
        if (ds[u] < du) continue;
        for (auto [w, v] : g[u]) {
            ll ndv = du + w;
            if (chmin(ds[v], ndv)) {
                ps[v] = u;
                pq.emp(ndv, v);
            }
        }
        return {ds, ps};
    }
    // optional !
    vi recover_path(int source, int ending,
        const vi &ps) {
        if (ps[ending] == -1) return {};
        int cur = ending;
        vi ans;
        while (cur != -1) {
            ans.eb(cur);
            cur = ps[cur];
        }
        reverse(all(ans));
        return ans;
    }
}

```

## 6.12 Dijkstra (K-shortest paths)

```

const ll oo = 1e9 * 1e5 + 1;
using adj = vector<vector<pll>>;
vector<priority_queue<ll>> dijkstra(
    const vector<vector<pll>> &g, int n, int s,
    int k) {
    priority_queue<pll, vector<pll>, greater<pll>>
        pq;
    vector<priority_queue<ll>> dist(n);

```

```

    dist[0].emplace(0);
    pq.emplace(0, s);
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();
        if (not dist[v].empty() and
            dist[v].top() < d1)
            continue;
        for (auto [d2, u] : g[v]) {
            if (len(dist[u]) < k) {
                pq.emplace(d2 + d1, u);
                dist[u].emplace(d2 + d1);
            } else {
                if (dist[u].top() > d1 + d2) {
                    dist[u].pop();
                    dist[u].emplace(d1 + d2);
                    pq.emplace(d2 + d1, u);
                }
            }
        }
    }
    return dist;
}

```

## 6.13 Extra Edges to Make Digraph Fully Strongly Connected

**Description:** Given a directed graph  $G$  find the necessary edges to add to make the graph a single strongly connected component.

**Time:**  $O(N + M)$

**Memory:**  $O(N)$

```

struct SCC {
    int n, num_sccs;
    vi2d adj;
    vi scc_id;
    SCC(int _n)
        : n(_n),
          num_sccs(0),
          adj(n),
          scc_id(n, -1) {}
    SCC(const vi2d &adj) : SCC(len(_adj)) {
        adj = _adj;
        find_sccs();
    }
    void add_edge(int u, int v) { adj[u].eb(v); }
    void find_sccs() {
        int timer = 1;
        vi tin(n), st;
        st.reserve(n);
        function<int(int)> dfs = [&](int u) -> int {
            int low = tin[u] = timer++;
            siz = len(st);
            st.eb(u);
            for (int v : adj[u])
                if (scc_id[v] < 0)
                    low = min(low, tin[v] ? tin[v] : dfs(v));
            if (tin[u] == low) {
                rep(i, siz, len(st)) scc_id[st[i]] =
                    num_sccs;
                st.resize(siz);
                num_sccs++;
            }
            return low;
        };
        for (int i = 0; i < n; i++)

```

```

            if (!tin[i]) dfs(i);
        }
    }
    vector<array<int, 2>> extra_edges(
        const vi2d &adj) {
        SCC scc(adj);
        auto scc_id = scc.scc_id;
        auto num_sccs = scc.num_sccs;
        if (num_sccs == 1) return {};
        int n = len(adj);
        vi2d scc_adj(num_sccs);
        vi zero_in(num_sccs, 1);
        rep(u, 0, n) {
            for (int v : adj[u]) {
                if (scc_id[u] == scc_id[v]) continue;
                scc_adj[scc_id[u]].eb(scc_id[v]);
                zero_in[scc_id[v]] = 0;
            }
        }
        int random_source =
            max_element(all(zero_in)) - zero_in.begin();
        vi vis(num_sccs);
        function<int(int)> dfs = [&](int u) {
            if (empty(scc_adj[u])) return u;
            for (int v : scc_adj[u])
                if (!vis[v]) {
                    vis[v] = 1;
                    int zero_out = dfs(v);
                    if (zero_out != -1) return zero_out;
                }
            return (int)-1;
        };
        vector<array<int, 2>> edges;
        vi in_unused;
        rep(i, 0, num_sccs) {
            if (zero_in[i]) {
                vis[i] = 1;
                int zero_out = dfs(i);
                if (zero_out != -1)
                    edges.push_back({zero_out, i});
                else
                    in_unused.push_back(i);
            }
        }
        rep(i, 1, len(edges)) {
            swap(edges[i][0], edges[i - 1][0]);
        }
        rep(i, 0, num_sccs) {
            if (scc_adj[i].empty() && !vis[i]) {
                if (!in_unused.empty()) {
                    edges.push_back({i, in_unused.back()});
                    in_unused.pop_back();
                } else {
                    edges.push_back({i, random_source});
                }
            }
        }
        for (int u : in_unused) edges.push_back({0, u});
        vi to_node(num_sccs);
        rep(i, 0, n) to_node[scc_id[i]] = i;
        for (auto &[u, v] : edges)
            u = to_node[u], v = to_node[v];
        return edges;
    }
}

```



## 6.14 Find Articulation/Cut Points

**Description:** Given an **undirected** graph find it's articulation points.  
**Time:**  $O(N + M)$   
**Warning:** A vertex  $u$  can be an articulation point if and only if has at least 2 adjacent vertex

```
const int MAXN(100);
int N;
vi2d G;
int timer;
int tin[MAXN], low[MAXN];
set<int> cpoints;

int dfs(int u, int p = -1) {
    int cnt = 0;
    low[u] = tin[u] = timer++;
    for (auto v : G[u]) {
        if (not tin[v]) {
            cnt++;
            dfs(v, u);
            if (low[v] >= tin[u]) cpoints.insert(u);
            low[u] = min(low[u], low[v]);
        } else if (v != p)
            low[u] = min(low[u], tin[v]);
    }
    return cnt;
}

void getCutPoints() {
    memset(low, 0, sizeof(low));
    memset(tin, 0, sizeof(tin));
    cpoints.clear();
    timer = 1;
    for (int i = 0; i < N; i++) {
        if (tin[i]) continue;
        int cnt = dfs(i);
        if (cnt == 1) cpoints.erase(i);
    }
}
```

## 6.15 Find Bridge-Tree components

**Usage:** *label2CC(u, p)* finds the 2-edge connected component of every node.  
**Time:**  $O(n + m)$

```
const int maxn(3 '00' 000);
int tin[maxn], compId[maxn], qtdComps;
vi g[maxn], stck;
int n;
int dfs(int u, int p = -1) {
    int low = tin[u] = len(stck);
    stck.emplace_back(u);
    bool multEdge = false;
    for (auto v : g[u]) {
        if (v == p and !multEdge) {
            multEdge = 1;
            continue;
        }
        low = min(low, tin[v] == -1 ? dfs(v, u) : tin[v]);
    }
    if (low == tin[u]) {
        for (int i = tin[u]; i < len(stck); i++)
            compId[stck[i]] = qtdComps;
        stck.resize(tin[u]);
        qtdComps++;
    }
}
```

```
    return low;
}

void label2CC() {
    memset(compId, -1, sizeof(int) * n);
    memset(tin, -1, sizeof(int) * n);
    stck.reserve(n);
    for (int i = 0; i < n; i++) {
        if (tin[i] == -1) dfs(i);
    }
}
```

## 6.16 Find Bridges

**Description:** Find every bridge in a **undirected** connected graph.  
**Warning:** Remember to read the graph as pair where the second is the id of the edge !

```
Time : $O(N + M)$ const int MAXN(10000),
      MAXM(100000);
int N, M, clk, tin[MAXN], low[MAXN],
    isBridge[MAXN];
vector<pii> G[MAXN];

void dfs(int u, int p = -1) {
    tin[u] = low[u] = clk++;
    for (auto [v, i] : G[u]) {
        if (v == p) continue;
        if (tin[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] > tin[u]) {
                isBridge[i] = 1;
            }
        }
    }
}

void findBridges() {
    fill(tin, tin + N, 0);
    fill(low, low + N, 0);
    fill(isBridge, isBridge + M, 0);
    clk = 1;
    for (int i = 0; i < N; i++) {
        if (!tin[i]) dfs(i);
    }
}
```

## 6.17 Find Centroid

**Description:** Given a tree (don't forget to make it 'undirected'), find it's centroids.  
@Time :  $O(V)$

```
void dfs(int u, int p, int n, vi2d &g, vi &sz,
        vi &centroid) {
    sz[u] = 1;
    bool iscentroid = true;
    for (auto v : g[u])
        if (v != p) {
            dfs(v, u, n, g, sz, centroid);
            if (sz[v] > n / 2) iscentroid = false;
            sz[u] += sz[v];
        }
    if (n - sz[u] > n / 2) iscentroid = false;
```

```
    if (iscentroid) centroid.eb(u);
}

vi getCentroid(vi2d &g, int n) {
    vi centroid;
    vi sz(n);
    dfs(0, -1, n, g, sz, centroid);
    return centroid;
}
```

## 6.18 Find bridges (online)

```
// O((n+m)*log(n))
struct BridgeFinder {
    // 2ecc = 2 edge conected component
    // cc = conected component
    vector<int> parent, dsu_2ecc, dsu_cc,
        dsu_cc_size;
    int bridges, lca_iteration;
    vector<int> last_visit;

    BridgeFinder(int n)
        : parent(n, -1),
          dsu_2ecc(n),
          dsu_cc(n),
          dsu_cc_size(n, 1),
          bridges(0),
          lca_iteration(0),
          last_visit(n) {
        for (int i = 0; i < n; i++) {
            dsu_2ecc[i] = i;
            dsu_cc[i] = i;
        }
    }

    int find_2ecc(int v) {
        if (v == -1) return -1;
        return dsu_2ecc[v] == v
            ? v
            : dsu_2ecc[v] =
                find_2ecc(dsu_2ecc[v]);
    }

    int find_cc(int v) {
        v = find_2ecc(v);
        return dsu_cc[v] == v
            ? v
            : dsu_cc[v] = find_cc(dsu_cc[v]);
    }

    void make_root(int v) {
        v = find_2ecc(v);
        int root = v;
        int child = -1;
        while (v != -1) {
            int p = find_2ecc(parent[v]);
            parent[v] = child;
            dsu_cc[v] = root;
            child = v;
            v = p;
        }
        dsu_cc_size[root] = dsu_cc_size[child];
    }

    void merge_path(int a, int b) {
        ++lca_iteration;
        vector<int> path_a, path_b;
        int lca = -1;
        while (lca == -1) {
            if (a != -1) {
                a = find_2ecc(a);
                path_a.push_back(a);
                if (last_visit[a] == lca_iteration) {
```

```

        lca = a;
        break;
    }
    last_visit[a] = lca_iteration;
    a = parent[a];
}
if (b != -1) {
    b = find_2ecc(b);
    path_b.push_back(b);
    if (last_visit[b] == lca_iteration) {
        lca = b;
        break;
    }
    last_visit[b] = lca_iteration;
    b = parent[b];
}
}
for (auto v : path_a) {
    dsu_2ecc[v] = lca;
    if (v == lca) break;
    --bridges;
}
for (auto v : path_b) {
    dsu_2ecc[v] = lca;
    if (v == lca) break;
    --bridges;
}
}
void add_edge(int a, int b) {
    a = find_2ecc(a);
    b = find_2ecc(b);
    if (a == b) return;
    int ca = find_cc(a);
    int cb = find_cc(b);
    if (ca != cb) {
        ++bridges;
        if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
            swap(a, b);
            swap(ca, cb);
        }
        make_root(a);
        parent[a] = dsu_cc[a] = b;
        dsu_cc_size[cb] += dsu_cc_size[a];
    } else {
        merge_path(a, b);
    }
}
};

```

## 6.19 Floyd Warshall

**Description:** Simply finds the minimal distance for each node to every other node.  $O(V^3)$

```

vector<vll> floyd_warshall(const vector<vll> &adj,
                           ll n) {
    auto dist = adj;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            for (int k = 0; k < n; ++k) {
                dist[j][k] = min(dist[j][k],
                                   dist[j][i] + dist[i][k]);
            }
        }
    }
    return dist;
}

```

## 6.20 Functional/Successor Graph

**Description:** Given a functional graph find the vertex after  $k$  moves starting at  $u$  and also the distance between  $u$  and  $v$ , if it's impossible to reach  $v$  starting at  $u$  returns -1.

**Time:** build  $O(N \cdot \text{MAXLOG2})$ , kth  $O(\text{MAXLOG2})$ , dist  $O(\text{MAXLOG2})$

```

const int MAXN(2'000'000), MAXLOG2(24);
int N;
vi2d succ(MAXN, vi(MAXLOG2 + 1));
vi dst(MAXN, 0);
int vis[MAXN];
void dfsbuild(int u) {
    if (vis[u]) return;
    vis[u] = 1;
    int v = succ[u][0];
    dfsbuild(v);
    dst[u] = dst[v] + 1;
}
void build() {
    for (int i = 0; i < N; i++) {
        if (not vis[i]) dfsbuild(i);
    }
    for (int k = 1; k <= MAXLOG2; k++) {
        for (int i = 0; i < N; i++) {
            succ[i][k] = succ[succ[i][k - 1]][k - 1];
        }
    }
}
int kth(int u, ll k) {
    if (k <= 0) return u;
    for (int i = 0; i <= MAXLOG2; i++)
        if ((1ll << i) & k) u = succ[u][i];
    return u;
}
int dist(int u, int v) {
    int cu = kth(u, dst[u]);
    if (kth(u, dst[u] - dst[v]) == v)
        return dst[u] - dst[v];
    else if (kth(cu, dst[cu] - dst[v]) == v)
        return dst[u] + (dst[cu] - dst[v]);
    else
        return -1;
}
}

```

## 6.21 Kruskal

**Description:** Find the minimum spanning tree of a graph.

**Time:**  $O(E \log E)$

```

struct UFDS {
    vector<int> ps, sz;
    int components;
    UFDS(int n)
        : ps(n + 1), sz(n + 1, 1), components(n) {
        iota(all(ps), 0);
    }
    int find_set(int x) {
        return (x == ps[x]
                ? x
                : (ps[x] = find_set(ps[x])));
    }
    bool same_set(int x, int y) {
        return find_set(x) == find_set(y);
    }
    void union_set(int x, int y) {

```

```

        x = find_set(x);
        y = find_set(y);
        if (x == y) return;
        if (sz[x] < sz[y]) swap(x, y);
        ps[y] = x;
        sz[x] += sz[y];
        components--;
    }
};
vector<tuple<ll, int, int>> kruskal(
    int n, vector<tuple<ll, int, int>> &edges) {
    UFDS ufds(n);
    vector<tuple<ll, int, int>> ans;
    sort(all(edges));
    for (auto [a, b, c] : edges) {
        if (ufds.same_set(b, c)) continue;
        ans.emplace_back(a, b, c);
        ufds.union_set(b, c);
    }
    return ans;
}

```

## 6.22 Lowest Common Ancestor

**Description:** Given two nodes of a tree find their lowest common ancestor, or their distance

```

template <typename T>
struct SparseTable {
    vector<T> v;
    int n;
    static const int b = 30;
    vi mask, t;
    int op(int x, int y) {
        return v[x] < v[y] ? x : y;
    }
    int msb(int x) {
        return __builtin_clz(1) - __builtin_clz(x);
    }
    SparseTable() {}
    SparseTable(const vector<T> &v_)
        : v(v_), n(v.size()), mask(n), t(n) {
        for (int i = 0, at = 0; i < n;
             mask[i++] = at | = 1) {
            at = (at << 1) & ((1 << b) - 1);
            while (at and op(i, i - msb(at & -at)) == i)
                at ^= at & -at;
        }
        for (int i = 0; i < n / b; i++)
            t[i] = b * i + b - 1 -
                msb(mask[b * i + b - 1]);
        for (int j = 1; (1 << j) <= n / b; j++)
            for (int i = 0; i + (1 << j) <= n / b; i++)
                t[n / b * j + i] =
                    op(t[n / b * (j - 1) + i],
                      t[n / b * (j - 1) + i +
                        (1 << (j - 1))]);
    }
    int small(int r, int sz = b) {
        return r - msb(mask[r] & ((1 << sz) - 1));
    }
}

```

```

T query(int l, int r) {
    if (r - l + 1 <= b)
        return small(r, r - l + 1);
    int ans = op(small(l + b - 1), small(r));
    int x = l / b + 1, y = r / b - 1;
    if (x <= y) {
        int j = msb(y - x + 1);
        ans = op(
            op(t[n / b * j + x],
               t[n / b * j + y - (1 << j) + 1]));
    }
    return ans;
};

struct LCA {
    SparseTable<int> st;
    int n;
    vi v, pos, dep;
    LCA(const vi2d &g, int root)
        : n(len(g)), pos(n) {
        dfs(root, 0, -1, g);
        st = SparseTable<int>(vector<int>(all(dep)));
    }
    void dfs(int i, int d, int p, const vi2d &g) {
        v.eb(len(dep)) = i, pos[i] = len(dep),
        dep.eb(d);
        for (auto j : g[i])
            if (j != p) {
                dfs(j, d + 1, i, g);
                v.eb(len(dep)) = i, dep.eb(d);
            }
    }
    int lca(int a, int b) {
        int l = min(pos[a], pos[b]);
        int r = max(pos[a], pos[b]);
        return v[st.query(l, r)];
    }
    int dist(int a, int b) {
        return dep[pos[a]] + dep[pos[b]] -
            2 * dep[pos[lca(a, b)]];
    }
};

```

## 6.23 Lowest Common Ancestor (Binary Lifting)

**Description:** Given a directed tree, finds the LCA between two nodes using binary lifting, and answer a few queries with it.

**Usage:**

- `lca`: returns the LCA between the two given nodes
- `on_path`: fids if `c` is in the path from `a` to `b`

**Time:** build  $O(N \cdot \text{MAXLOG}2)$ , all queries  $O(\text{MAXLOG}2)$

```

struct LCA {
    int n;
    const int maxlog;
    vector<vector<int>> up;
    vector<int> depth;
    LCA(const vector<vector<int>> &tree)
        : n(tree.size()),
          maxlog(ceil(log2(n))),
          up(n, vector<int>(maxlog + 1)),
          depth(n, -1) {
        for (int i = 0; i < n; i++) {
            if (depth[i] == -1) {
                depth[i] = 0;

```

```

                dfs(i, -1, tree);
            }
        }
    }
    void dfs(int u, int p,
             const vector<vector<int>> &tree) {
        if (p != -1) {
            depth[u] = depth[p] + 1;
            up[u][0] = p;
            for (int i = 1; i <= maxlog; i++) {
                up[u][i] = up[up[u][i - 1]][i - 1];
            }
        }
        for (int v : tree[u]) {
            if (v == p) continue;
            dfs(v, u, tree);
        }
    }
    int kth_jump(int u, int k) {
        for (int i = maxlog; i >= 0; i--) {
            if ((1 << i) & k) {
                u = up[u][i];
            }
        }
        return u;
    }
    int lca(int u, int v) {
        if (depth[u] < depth[v]) swap(u, v);
        int diff = depth[u] - depth[v];
        u = kth_jump(u, diff);
        if (u == v) return u;
        for (int i = maxlog; i >= 0; i--) {
            if (up[u][i] != up[v][i]) {
                u = up[u][i];
                v = up[v][i];
            }
        }
        return up[u][0];
    }
    bool on_path(int u, int v, int s) {
        int uv = lca(u, v), us = lca(u, s),
        vs = lca(v, s);
        return (uv == s or (us == uv and vs == s) or
            (vs == uv and us == s));
    }
    int dist(int u, int v) {
        return depth[u] + depth[v] -
            2 * depth[lca(u, v)];
    }
};

```

## 6.24 Maximum flow (Dinic)

**Description:** Finds the **maximum flow** in a graph network, given the **source** `s` and the **sink** `t`. Add edge from `a` to `b` with capcity `c`.

**Time:** In general  $O(E \cdot V^2)$ , if every capacity is 1, and every vertex has in degree equal 1 or out degree equal 1 then  $O(E \cdot \sqrt{V})$ ,

**Warning:** Suffle the edges list for every vertex may take you out of the worst case

```

struct Dinic {
    struct Edge {
        int to, rev;
        ll c, oc;
        ll flow() {
            return max(oc - c, 0LL);

```

```

        } // if you need flows
    };
    vi lvl, ptr, q;
    vector<vector<Edge>> adj;
    Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
    void addEdge(int a, int b, ll c, ll rcap = 0) {
        adj[a].pb({b, len(adj[b]), c, c});
        adj[b].pb({a, len(adj[a]) - 1, rcap, rcap});
    }
    ll dfs(int v, int t, ll f) {
        if (v == t || !f) return f;
        for (int &i = ptr[v]; i < len(adj[v]); i++) {
            Edge &e = adj[v][i];
            if (lvl[e.to] == lvl[v] + 1)
                if (ll p = dfs(e.to, t, min(f, e.c))) {
                    e.c -= p, adj[e.to][e.rev].c += p;
                    return p;
                }
        }
        return 0;
    }
    ll maxFlow(int s, int t) {
        ll flow = 0;
        q[0] = s;
        rep(L, 0, 31) {
            do { // 'int L=30' maybe faster for random
                // data
                lvl = ptr = vi(len(q));
                int qi = 0, qe = lvl[s] = 1;
                while (qi < qe && !lvl[t]) {
                    int v = q[qi++];
                    for (Edge e : adj[v])
                        if (!lvl[e.to] && e.c >> (30 - L))
                            q[qi++] = e.to,
                            lvl[e.to] = lvl[v] + 1;
                }
                while (ll p = dfs(s, t, LLONG_MAX))
                    flow += p;
            } while (lvl[t]);
        }
        return flow;
    }
    bool leftOfMinCut(int a) { return lvl[a] != 0; }
};

```

## 6.25 Minimum Cost Flow

**Description:** Given a network find the minimum cost to achieve a flow of at most `f`. Works with **directed** and **undirected** graphs

**Usage:**

- `add(u, v, c, w)`: adds an edge from `u` to `v` with capacity `c` and cost `w`.
- `flow(f)`: return a pair  $(\text{flow}, \text{cost})$  with the maximum flow until `f` with source at `s` and sink at `t`, with the minimum cost possible.

**Time:**  $O(N \cdot M + f \cdot m \log n)$

```

template <typename T>
struct MinCostFlow {
    struct Edge {
        int to;
        ll c, rc; // capcity, residual capacity
        T w; // cost
    };
    int n, s, t;
    vector<Edge> edges;
    vi2d g;

```

```

vector<T> dist;
vi pre;
MinCostFlow() {}
MinCostFlow(int n_, int _s, int _t)
: n(n_), s(_s), t(_t), g(n) {}

void addEdge(int u, int v, ll c, T w) {
    g[u].pb(len(edges));
    edges.eb(v, c, 0, w);
    g[v].pb(len(edges));
    edges.eb(u, 0, 0, -w);
}

// {flow, cost}
pair<ll, T> flow(ll flow_limit = LLONG_MAX) {
    ll flow = 0;
    T cost = 0;
    while (flow < flow_limit and dijkstra(s, t)) {
        ll aug = LLONG_MAX;
        for (int i = t; i != s;
            i = edges[pre[i] ^ 1].to) {
            aug = min({flow_limit - flow, aug,
                edges[pre[i]].c});
        }
        for (int i = t; i != s;
            i = edges[pre[i] ^ 1].to) {
            edges[pre[i]].c -= aug;
            edges[pre[i] ^ 1].c += aug;
            edges[pre[i]].rc += aug;
            edges[pre[i] ^ 1].rc -= aug;
        }
        flow += aug;
        cost += (T)aug * dist[t];
    }
    return {flow, cost};
}

// Needs to be called after flow method
vi2d paths() {
    vi2d p;
    for (;;) {
        int cur = s;
        auto &res = p.eb();
        res.pb(cur);
        while (cur != t) {
            bool found = false;
            for (auto i : g[cur]) {
                auto &[v, _, c, cost] = edges[i];
                if (c > 0) {
                    res.pb(cur = v);
                    found = true;
                    break;
                }
            }
            if (!found) break;
        }
        if (cur != t) {
            p.ppb();
            break;
        }
    }
    return p;
}

private:
bool bellman_ford(int s, int t) {
    dist.assign(n, numeric_limits<T>::max());
    pre.assign(n, -1);
    vc inq(n, false);

```

```

queue<int> q;
dist[s] = 0;
q.push(s);
while (len(q)) {
    int u = q.front();
    q.pop();
    inq[u] = false;
    for (int i : g[u]) {
        auto [v, c, w, _] = edges[i];
        auto new_dist = dist[u] + w;
        if (c > 0 and dist[v] > new_dist) {
            dist[v] = new_dist;
            pre[v] = i;
            if (not inq[v]) {
                inq[v] = true;
                q.push(v);
            }
        }
    }
}

return dist[t] != numeric_limits<T>::max();
}

bool dijkstra(int s, int t) {
    dist.assign(n, numeric_limits<T>::max());
    pre.assign(n, -1);
    dist[s] = 0;
    using PQ = pair<T, int>;
    pqmn<PQ> pq;
    pq.emp(0, s);
    while (len(pq)) {
        auto [cost, u] = pq.top();
        pq.pop();
        if (cost != dist[u]) continue;
        for (int i : g[u]) {
            auto [v, c, _, w] = edges[i];
            auto new_dist = dist[u] + w;
            if (c > 0 and dist[v] > new_dist) {
                dist[v] = new_dist;
                pre[v] = i;
                pq.emp(new_dist, v);
            }
        }
    }

return dist[t] != numeric_limits<T>::max();
};

```

## 6.26 Minimum Vertex Cover (already divided)

**Description:** Given a bipartite graph  $g$  with  $n$  vertices at left and  $m$  vertices at right, where  $g[i]$  are the possible right side matches of vertex  $i$  from left side, find a minimum vertex cover. The size is the same as the size of the maximum matching, and the complement is a maximum independent set.

```

vector<int> min_vertex_cover(
    vector<vector<int>> &g, int n, int m) {
    vector<int> match(m, -1), vis;
    auto find = [&](auto &&self, int j) -> bool {
        if (match[j] == -1) return 1;
        vis[j] = 1;
        int di = match[j];
        for (int e : g[di])
            if (!vis[e] and self(self, e)) {

```

```

                match[e] = di;
                return 1;
            }
        return 0;
    };
    for (int i = 0; i < (int)g.size(); i++) {
        vis.assign(match.size(), 0);
        for (int j : g[i]) {
            if (find(find, j)) {
                match[j] = i;
                break;
            }
        }
    }
    int res =
        (int)match.size() -
        (int)count(match.begin(), match.end(), -1);
    vector<char> lfound(n, true), seen(m);
    for (int it : match)
        if (it != -1) lfound[it] = false;
    vector<int> q, cover;
    for (int i = 0; i < n; i++)
        if (lfound[i]) q.push_back(i);
    while (!q.empty()) {
        int i = q.back();
        q.pop_back();
        lfound[i] = 1;
        for (int e : g[i])
            if (!seen[e] and match[e] != -1) {
                seen[e] = true;
                q.push_back(match[e]);
            }
    }
    for (int i = 0; i < n; i++)
        if (!lfound[i]) cover.push_back(i);
    for (int i = 0; i < m; i++)
        if (seen[i]) cover.push_back(n + i);
    assert((int)size(cover) == res);
    return cover;
}

```

## 6.27 Prim (MST)

**Description:** Given a graph with  $N$  vertex finds the minimum spanning tree, if there is no such three returns inf, it starts using the edges that connect with each  $s_i \in s$ , if none is provided than it starts with the edges of node 0.

**Time:**  $O(V \log E)$

```

const int MAXN(1'000'000);
int N;
vector<pair<ll, int>> G[MAXN];
ll prim(vi s = vi(1, 0)) {
    priority_queue<pair<ll, int>,
        vector<pair<ll, int>>,
        greater<pair<ll, int>>>
        pq;
    vector<char> ingraph(MAXN);
    int ingraphcnt(0);
    for (auto si : s) {
        ingraphcnt++;
        ingraph[si] = true;
        for (auto &[w, v] : G[si]) pq.emplace(w, v);
    }
}

```

```

ll mstcost = 0;
while (ingraphcnt < N and !pq.empty()) {
    ll w;
    int v;
    do {
        tie(w, v) = pq.top();
        pq.pop();
    } while (not pq.empty() and ingraph[v]);
    mstcost += w, ingraph[v] = true, ingraphcnt++;
    for (auto &[w2, v2] : G[v]) {
        pq.emplace(w2, v2);
    }
}
return ingraphcnt == N ? mstcost : oo;
}

```

## 6.28 Shortest Path With K-edges

**Description:** Given an adjacency matrix of a graph, and a number  $K$  computes the shortest path between all nodes that uses exactly  $K$  edges, so for  $0 \leq i, j \leq N - 1$  ans[i][j] = "the shortest path between  $i$  and  $j$  that uses exactly  $K$  edges, remember to initialize the adjacency matrix with  $\infty$ .  
**Time:**  $O(N^3 \cdot \log K)$

```

template <typename T>
vector<vector<T>> prod(vector<vector<T>> &a,
                     vector<vector<T>> &b) {
    const T _oo = numeric_limits<T>::max();
    int n = a.size();
    vector<vector<T>> c(n, vector<T>(n, _oo));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            for (int k = 0; k < n; k++)
                if (a[i][k] != _oo and b[k][j] != _oo)
                    c[i][j] =
                        min(c[i][j], a[i][k] + b[k][j]);
    return c;
}

template <typename T>
vector<vector<T>> shortest_with_k_moves(
    vector<vector<T>> adj, long long k) {
    if (k == 1) return adj;
    auto ans = adj;
    k--;
    while (k) {
        if (k & 1) ans = prod(ans, adj);
        k >>= 1;
        adj = prod(adj, adj);
    }
    return ans;
}

```

## 6.29 Strongly Connected Components (struct)

**Description:** Find the connected component for each edge (already in a topological order), some additional functions are also provided.  
**Time:** Build:  $O(V + E)$

```

struct SCC {
    int n, num_sccs;
    vi2d adj;
    vi scc_id;
    SCC(int _n)

```

```

        : n(_n),
        num_sccs(0),
        adj(n),
        scc_id(n, -1) {}
void add_edge(int u, int v) { adj[u].eb(v); }
void find_sccs() {
    int timer = 1;
    vi tin(n), st;
    st.reserve(n);
    function<int(int)> dfs = [&](int u) -> int {
        int low = tin[u] = timer++;
        siz = len(st);
        st.eb(u);
        for (int v : adj[u])
            if (scc_id[v] < 0)
                low = min(low, tin[v] ? tin[v] : dfs(v));
        if (tin[u] == low) {
            rep(i, siz, len(st)) scc_id[st[i]] =
                num_sccs;
            st.resize(siz);
            num_sccs++;
        }
        return low;
    };
    for (int i = 0; i < n; i++)
        if (!tin[i]) dfs(i);
}

vector<set<int>> build_gsc() {
    vector<set<int>> gsc;
    for (int i = 0; i < len(adj); ++i)
        for (auto j : adj[i])
            if (scc_id[i] != scc_id[j])
                gsc[scc_id[i]].emplace(scc_id[j]);
    return gsc;
}

vi2d per_comp() {
    vi2d ret(num_sccs);
    rep(i, 0, n) ret[scc_id[i]].eb(i);
    reverse(all(
        ret)); // already in topological order ;
    return ret;
}
}

```

## 6.30 Topological Sorting (Kahn)

**Description:** Finds the topological sorting in a **DAG**, if the given graph is not a **DAG** than an empty vector is returned, need to 'initialize' the INCNT as you build the graph.  
**Time:**  $O(V + E)$

```

const int MAXN(2 '00' 000);
int INCNT[MAXN];
vi2d GOUT(MAXN);
int N;

vi toposort() {
    vi order;
    queue<int> q;
    for (int i = 0; i < N; i++)
        if (!INCNT[i]) q.emplace(i);
    while (!q.empty()) {
        auto u = q.front();
        q.pop();
        order.emplace_back(u);
        for (auto v : GOUT[u]) {
            INCNT[v]--;

```

```

                if (INCNT[v] == 0) q.emplace(v);
            }
        }
        return len(order) == N ? order : vi();
    }
}

```

## 6.31 Topological Sorting (Tarjan)

**Description:** Finds a the topological order for the graph, if there is no such order it means the graph is cyclic, then it returns an empty vector  
**Time:**  $O(V + E)$

```

const int maxn(1 '00' 000);
int n, m;
vi g[maxn];
int not_found = 0, found = 1, processed = 2;
int state[maxn];

bool dfs(int u, vi &order) {
    if (state[u] == processed) return true;
    if (state[u] == found) return false;
    state[u] = found;
    for (auto v : g[u]) {
        if (not dfs(v, order)) return false;
    }
    state[u] = processed;
    order.emplace_back(u);
    return true;
}

vi topo_sort() {
    vi order;
    memset(state, 0, sizeof state);
    for (int u = 0; u < n; u++) {
        if (state[u] == not_found and
            not dfs(u, order))
            return {};
    }
    reverse(all(order));
    return order;
}

```

## 6.32 Tree Isomorphism (not rooted)

**Description:** Two trees are considered **isomorphic** if the hash given by *thash()* is the same.  
**Time:**  $O(V \cdot \log V)$

```

map<vi, int> mphash;

struct Tree {
    int n;
    vi2d g;
    vi sz, cs;
    Tree(int n_) : n(n_), g(n), sz(n) {}
    void add_edge(int u, int v) {
        g[u].emplace_back(v);
        g[v].emplace_back(u);
    }
    void dfs_centroid(int v, int p) {
        sz[v] = 1;
        bool cent = true;
        for (int u : g[v])
            if (u != p) {

```

```
        dfs_centroid(u, v);
        sz[v] += sz[u];
        cent &= not(sz[u] > n / 2);
    }
    if (cent and n - sz[v] <= n / 2)
        cs.push_back(v);
}

int fhash(int v, int p) {
    vi h;
    for (int u : g[v])
        if (u != p) h.push_back(fhash(u, v));
    sort(all(h));
    if (!mhash.count(h))
        mhash[h] = mhash.size();
    return mhash[h];
}

ll thash() {
    cs.clear();
    dfs_centroid(0, -1);
    if (cs.size() == 1) return fhash(cs[0], -1);
    ll h1 = fhash(cs[0], cs[1]);
    h2 = fhash(cs[1], cs[0]);
    return (min(h1, h2) << 3011) + max(h1, h2);
}

};
```

6.33 Tree Isomorphism (rooted)

**Description:** Given a rooted tree find the hash of each subtree, if two roots of two distinct trees have the same hash they are considered isomorphic  
**Time:** hash first time in  $O(\log N_v \cdot N_v)$  where  $(N_v)$  is the of the subtree of  $v$

```
map<vi, int> hasher;
int hs = 0;
struct RootedTreeIso {
    int n;
    vi2d adj;
    vi hashes;
    RootedTreeIso(int _n)
        : n(_n), adj(_n), hashes(_n, -1){};

    void add_edge(int u, int v) {
        adj[u].emplace_back(v);
        adj[v].emplace_back(u);
    }

    int hash(int u, int p = -1) {
        if (hashes[u] != -1) return hashes[u];
        vi children;
        for (auto v : adj[u])
            if (v != p)
                children.emplace_back(hash(v, u));
        sort(all(children));
        if (!hasher.count(children))
            hasher[children] = hs++;
        return hashes[u] = hasher[children];
    }
};
```

6.34 Tree diameter (DP)

```
const int MAXN(1'000'000);
int N;
vi G[MAXN];
```

```
int diameter, toLeaf[MAXN];
void calcDiameter(int u = 0, int p = -1) {
    int d1, d2;
    d1 = d2 = -1;
    for (auto v : G[u]) {
        if (v != p) {
            calcDiameter(v, u);
            d1 = max(d1, toLeaf[v]);
            tie(d1, d2) = minmax({d1, d2});
        }
    }
    toLeaf[u] = d2 + 1;
    diameter = max(diameter, d1 + d2 + 2);
}
```

7 Math

7.1 Arithmetic Progression Sum

Usage:

- $s$  : first term
- $d$  : common difference
- $n$  : number of terms

```
ll arithmeticProgressionSum(ll s, ll d, ll n) {
    return (s + (s + d * (n - 1))) * n / 2ll;
}
```

7.2 Binomial

**Time:**  $O(N \cdot K)$   
**Memory:**  $O(K)$

```
ll binom(ll n, ll k) {
    if (k > n) return 0;
    vll dp(k + 1, 0);
    dp[0] = 1;
    for (ll i = 1; i <= n; i++)
        for (ll j = k; j > 0; j--)
            dp[j] = dp[j] + dp[j - 1];
    return dp[k];
}
```

7.3 Binomial MOD

**Description:** find  $\binom{n}{k} \pmod{MOD}$   
**Time:**

- precompute: on first call it takes  $O(MAXNBIN)$  to precompute the factorials
- query:  $O(1)$ .

**Memory:**  $O(MAXNBIN)$   
**Warning:** Remember to set  $MAXNBIN$  properly !

```
const ll MOD = 998244353;
inline ll binom(ll n, ll k) {
    static const int BINMAX = 2'000'000;
    static vll FAC(BINMAX + 1), FINV(BINMAX + 1);
    static bool done = false;
    if (!done) {
        vll INV(BINMAX + 1);
        FAC[0] = FAC[1] = INV[1] = FINV[0] = FINV[1] = 1;
        for (int i = 2; i <= BINMAX; i++) {
            FAC[i] = FAC[i - 1] * i % MOD;
            INV[i] = MOD - MOD / i * INV[MOD % i] % MOD;
            FINV[i] = FINV[i - 1] * INV[i] % MOD;
        }
    }
    return FAC[n] * FINV[k] % MOD * FINV[n - k] % MOD;
}
```

```
    }
    done = true;
}

if (n < k || n < 0 || k < 0) return 0;
return FAC[n] * FINV[k] % MOD * FINV[n - k] % MOD;
}
```

7.4 Chinese Remainder Theorem

**Description:** Find the solution  $X$  to the  $N$  modular equations.

$$\begin{matrix} x \equiv a_1 \pmod{m_1} \\ \dots \\ x \equiv a_n \pmod{m_n} \end{matrix} \tag{1}$$

The  $m_i$  don't need to be coprime, if there is no solution then it returns -1.

```
tuple<ll, ll, ll> ext_gcd(ll a, ll b) {
    if (!a) return {b, 0, 1};
    auto [g, x, y] = ext_gcd(b % a, a);
    return {g, y - b / a * x, x};
}

template <typename T = ll>
struct crt {
    T a, m;

    crt() : a(0), m(1) {}
    crt(T a_, T m_) : a(a_), m(m_) {}
    crt operator*(crt C) {
        auto [g, x, y] = ext_gcd(m, C.m);
        if ((a - C.a) % g != 0) a = -1;
        if (a == -1 or C.a == -1) return crt(-1, 0);
        T lcm = m / g * C.m;
        T ans =
            a + (x * (C.a - a) / g % (C.m / g)) * m;
        return crt((ans % lcm + lcm) % lcm, lcm);
    }
};

template <typename T = ll>
struct Congruence {
    T a, m;
};

template <typename T = ll>
T chinese_remainder_theorem(
    const vector<Congruence<T>> &equations) {
    crt<T> ans;
    for (auto &[a_, m_] : equations) {
        ans = ans * crt<T>(a_, m_);
    }
    return ans.a;
}
```

7.5 Derangement / Matching Problem

**Description:** Computes the derangement of  $N$ , which is given by the formula:  $D_N = N! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^N \frac{1}{N!}\right)$   
**Time:**  $O(N)$

```
#warning Remember to call precompute !
const ll MOD = 1e9 + 7;
const int MAXN(1'000'000);
ll fats[MAXN + 1];
void precompute() {
    fats[0] = 1;
    for (ll i = 1; i <= MAXN; i++) {
        fats[i] = (fats[i - 1] * i) % MOD;
    }
}
ll fastpow(ll a, ll p, ll m) {
    ll ret = 1;
    while (p) {
        if (p & 1) ret = (ret * a) % MOD;
        p >>= 1;
        a = (a * a) % MOD;
    }
    return ret;
}
ll divmod(ll a, ll b) {
    return (a * fastpow(b, MOD - 2, MOD)) % MOD;
}
ll derangement(const ll n) {
    ll ans = fats[n];
    for (ll i = 1; i <= n; i++) {
        ll k = divmod(fats[n], fats[i]);
        if (i & 1) {
            ans = (ans - k + MOD) % MOD;
        } else {
            ans = (ans + k) % MOD;
        }
    }
    return ans;
}
```

## 7.6 Euler Phi $\varphi(N)$

**Description:** Computes the number of positive integers less than  $N$  that are coprimes with  $N$ , in  $O(\sqrt{N})$ .

```
int phi(int n) {
    if (n == 1) return 1;
    auto fs = factorization(
        n); // a vctor of pair or a map
    auto res = n;
    for (auto [p, k] : fs) {
        res /= p;
        res *= (p - 1);
    }
    return res;
}
```

## 7.7 Euler phi $\varphi(N)$ (in range)

**Description:** Computes the number of positive integers less than  $n$  that are coprimes with  $N$ , in the range  $[1, N]$ , in  $O(N \log N)$ .

```
const int MAX = 1e6;
vi range_phi(int n) {
    bitset<MAX> sieve;
    vi phi(n + 1);
    iota(phi.begin(), phi.end(), 0);
    sieve.set();
    for (int p = 2; p <= n; p += 2) phi[p] /= 2;
```

```
for (int p = 3; p <= n; p += 2) {
    if (sieve[p]) {
        for (int j = p; j <= n; j += p) {
            sieve[j] = false;
            phi[j] /= p;
            phi[j] *= (p - 1);
        }
    }
}
return phi;
}
```

## 7.8 FFT convolution and exponentiation

```
const ld PI = acos(-1);
/* change the ld to doulbe may increase
 * performance =D */
struct num {
    ld a{0.0}, b{0.0};
    num() {}
    num(ld na) : a{na} {}
    num(ld na, ld nb) : a{na}, b{nb} {}
    const num operator+(const num &c) const {
        return num(a + c.a, b + c.b);
    }
    const num operator-(const num &c) const {
        return num(a - c.a, b - c.b);
    }
    const num operator*(const num &c) const {
        return num(a * c.a - b * c.b,
            a * c.b + b * c.a);
    }
    const num operator/(const ll &c) const {
        return num(a / c, b / c);
    }
};

void fft(vector<num> &a, bool invert) {
    int n = len(a);
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1) j ^= bit;
        j ^= bit;
        if (i < j) swap(a[i], a[j]);
    }
    for (int sz = 2; sz <= n; sz <= 1) {
        ld ang = 2 * PI / sz * (invert ? -1 : 1);
        num wsz(cos(ang), sin(ang));
        for (int i = 0; i < n; i += sz) {
            num w(1);
            rep(j, 0, sz / 2) {
                num u = a[i + j],
                    v = a[i + j + sz / 2] * w;
                a[i + j] = u + v;
                a[i + j + sz / 2] = u - v;
                w = w * wsz;
            }
        }
    }
    if (invert)
        for (num &x : a) x = x / n;
}

vi conv(vi const a, vi const b) {
    vector<num> fa(all(a));
    vector<num> fb(all(b));
    int n = 1;
```

```
while (n < len(a) + len(b)) n <= 1;
fa.resize(n);
fb.resize(n);
fft(fa, false);
fft(fb, false);
rep(i, 0, n) fa[i] = fa[i] * fb[i];
fft(fa, true);
vi result(n);
rep(i, 0, n) result[i] = round(fa[i].a);
while (len(result) and result.back() == 0)
    result.pop_back();
/* Uncomment this line if you want a boolean
 * convolution*/
// for (auto &xi : result) xi = min(xi, 1ll);
return result;
}

vll poly_exp(vll &ps, int k) {
    vll ret(len(ps));
    auto base = ps;
    ret[0] = 1;
    while (k) {
        if (k & 1) ret = conv(ret, base);
        k >>= 1;
        base = conv(base, base);
    }
    return ret;
}
```

## 7.9 Factorial Factorization

**Description:** Computes the factorization of  $N!$  in  $\varphi(N) * \log N$   
**Time:**  $O(\varphi(N) \cdot \log N)$

```
ll E(ll n, ll p) {
    ll k = 0, b = p;
    while (b <= n) {
        k += n / b;
        b *= p;
    }
    return k;
}

map<ll, ll> factorial_factorization(
    ll n, const vll &primes) {
    map<ll, ll> fs;
    for (const auto &p : primes) {
        if (p > n) break;
        fs[p] = E(n, p);
    }
    return fs;
}
```

## 7.10 Factorization

**Description:** Computes the factorization of  $N$ .  
**Time:**  $O(\sqrt{n})$ .

```
map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;
        for (; n % i == 0; count++, n /= i)
            ;
        if (count) ans[i] = count;
    }
    if (n > 1) ans[n]++;
    return ans;
}
```

7.11 Factorization (Pollard’s Rho)

**Description:** Factorizes a number into its prime factors.  
**Time:**  $O(N^{\frac{1}{4}} * \log(N))$ .

```
ll mul(ll a, ll b, ll m) {
    ll ret =
        a * b - (ll)((ld)1 / m * a * b + 0.5) * m;
    return ret < 0 ? ret + m : ret;
}

ll pow(ll a, ll b, ll m) {
    ll ans = 1;
    for (; b > 0; b /= 2ll, a = mul(a, a, m)) {
        if (b % 2ll == 1) ans = mul(ans, a, m);
    }
    return ans;
}

bool prime(ll n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;
    ll r = __builtin_ctzll(n - 1), d = n >> r;
    for (int a : {2, 325, 9375, 28178, 450775,
        9780504, 795265022}) {
        ll x = pow(a, d, n);
        if (x == 1 or x == n - 1 or a % n == 0)
            continue;
        for (int j = 0; j < r - 1; j++) {
            x = mul(x, x, n);
            if (x == n - 1) break;
        }
        if (x != n - 1) return 0;
    }
    return 1;
}

ll rho(ll n) {
    if (n == 1 or prime(n)) return n;
    auto f = [n](ll x) { return mul(x, x, n) + 1; };
    ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
    while (t % 40 != 0 or gcd(prd, n) == 1) {
        if (x == y) x = ++x0, y = f(x);
        q = mul(prd, abs(x - y), n);
        if (q != 0) prd = q;
        x = f(x), y = f(f(y)), t++;
    }
    return gcd(prd, n);
}

vector<ll> fact(ll n) {
    if (n == 1) return {};
    if (prime(n)) return {n};
    ll d = rho(n);
    vector<ll> l = fact(d), r = fact(n / d);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}
```

7.12 Fast Pow

**Description:** Computes  $a^b \pmod m$   
**Time:**  $O(\log B)$ .

```
ll fpow(ll a, ll b, ll m) {
    ll ret = 1;
    while (b) {
        if (b & 1) ret = (ret * a) % m;
        b >>= 1;
    }
```

```
        a = (a * a) % m;
    }
    return ret;
}
```

7.13 Find diophantine equation solution

**Description:** Given  $a, b, c$  finds the solution to the equation  $ax + by = c$ , the result will be stored in the reference variables  $x0$  and  $y0$ .  
**Time:**  $O(\log \min(a, b))$

```
template <typename T>
tuple<T, T, T> ext_gcd(T a, T b) {
    if (b == 0) return {a, 1, 0};
    auto [d, x1, y1] = ext_gcd(b, a % b);
    return {d, y1, x1 - y1 * (a / b)};
}

template <typename T>
tuple<bool, T, T> find_any_solution(T a, T b,
                                    T c) {
    assert(a != 0 or b != 0);
    #warning Be careful with overflow, use __int128 if
        needed !
    auto [d, x0, y0] =
        ext_gcd(a < 0 ? -a : a, b < 0 ? -b : b);
    if (c % d) return {false, 0, 0};
    x0 *= c / d;
    y0 *= c / d;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return {true, x0, y0};
}
```

7.14 Find multiplicatinve inverse

```
ll inv(ll a, ll m) {
    return a > 1ll ? m - inv(m % a, a) * m / a
        : 1ll;
}
```

7.15 GCD

```
ll gcd(ll a, ll b) {
    return b ? gcd(b, a % b) : a;
}
```

7.16 Gauss XOR elimination / XOR-SAT

**Description:** Execute gaussian elimination with xor over the system  $Ax = b$  in. The add method must receive a bitset indicating which variables are present in the equation, and the solution of the equation.  
**Time:**  $O(\frac{nm^2}{64})$

```
const int MAXXI = 2009;
using Equation = bitset<MAXXI>;
struct GaussXor {
    vector<char> B;
    vector<Equation> A;
    void add(const Equation &ai, bool bi) {
```

```
        A.push_back(ai);
        B.push_back(bi);
    }
    pair<bool, Equation> solution() {
        int cnt = 0, n = A.size();
        Equation vis;
        vis.set();
        Equation x;
        for (int j = MAXXI - 1, i; j >= 0; j--) {
            for (i = cnt; i < n; i++) {
                if (A[i][j]) break;
            }
            if (i == n) continue;
            swap(A[i], A[cnt]), swap(B[i], B[cnt]);
            i = cnt++;
            vis[j] = 0;
            for (int k = 0; k < n; k++) {
                if (i == k || !A[k][j]) continue;
                A[k] ^= A[i];
                B[k] ^= B[i];
            }
        }
        x = vis;
        for (int i = 0; i < n; i++) {
            int acum = 0;
            for (int j = 0; j < MAXXI; j++) {
                if (!A[i][j]) continue;
                if (!vis[j]) {
                    vis[j] = 1;
                    x[j] = acum ^ B[i];
                }
                acum ^= x[j];
            }
            if (acum != B[i])
                return {false, Equation()};
        }
        return {true, x};
    }
};
```

7.17 Integer partition

**Description:** Find the total of ways to partition a given number  $N$  in such way that none of the parts is greater than  $K$ .  
**Time:**  $O(N \cdot \min(N, K))$   
**Memory:**  $O(N)$   
**Warning:** Remember to memset everything to  $-1$  before using it

```
const ll MOD = 1000000007;
const int MAXN(100);
ll memo[MAXN + 1];
ll dp(ll n, ll k = oo) {
    if (n == 0) return 1;
    ll &ans = memo[n];
    if (ans != -1) return ans;
    ans = 0;
    for (int i = 1; i <= min(n, k); i++) {
        ans = (ans + dp(n - i, k)) % MOD;
    }
    return ans;
}
```



## 7.18 LCM

```
ll gcd(ll a, ll b) {
    return b ? gcd(b, a % b) : a;
}
ll lcm(ll a, ll b) { return a / gcd(a, b) * b; }
```

## 7.19 Linear Recurrence

**Description:** Find the  $n$ -th term of a linear recurrence, given the recurrence  $rec$  and the first  $K$  values of the recurrence, remember that  $first\_k[i]$  is the value of  $f(i)$ , considering 0-indexing.

**Time:**  $O(K^3 \log N)$

```
template <typename T>
vector<vector<T>> prod(vector<vector<T>> &a,
                      vector<vector<T>> &b,
                      const ll mod) {

    int n = a.size();
    vector<vector<T>> c(n, vector<T>(n));
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            for (int k = 0; k < n; k++) {
                c[i][j] = (c[i][j] +
                           ((a[i][k] * b[k][j]) % mod)) %
                           mod;
            }
        }
    }
    return c;
}

template <typename T>
vector<vector<T>> fpow(vector<vector<T>> &xs,
                     ll p, ll mod) {
    vector<vector<T>> ans(xs.size(),
                        vector<T>(xs.size()));
    for (int i = 0; i < (int)xs.size(); i++)
        ans[i][i] = 1;
    for (auto b = xs; p; p >>= 1, b = prod(b, b, mod))
        if (p & 1) ans = prod(ans, b, mod);
    return ans;
}

ll linear_req(vector<vector<ll>> rec,
              vector<ll> first_k, ll n, ll mod) {
    int k = first_k.size();
    if (n < k) {
        return first_k[n];
    }
    ll n2 = n - k + 1;
    rec = fpow(rec, n2, mod);
    ll ret = 0;
    for (int i = 0; i < k; i++) {
        ret = (ret +
              (rec.back()[i] * first_k[i]) % mod) %
              mod;
    }
    return ret;
}
```

## 7.20 List N elements choose K

**Description:** Process every possible combination of  $K$  elements from  $N$

elements, those index marked as 1 in the index vector says which elements are chosen at that moment.

**Time:**  $O(\binom{N}{K} \cdot O(process))$

```
void process(vi &index) {
    for (int i = 0; i < len(index); i++) {
        if (index[i])
            cout << i << " \n"[i == len(index) - 1];
    }
}

void n_choose_k(int n, in k) {
    vi index(n);
    fill(index.end() - k, index.end(), 1);
    do {
        process(index);
    } while (next_permutation(all(index)));
}
```

## 7.21 List primes (Sieve of Eratosthenes)

```
const ll MAXN = 2e5;
vll list_primes(ll n = MAXN) {
    vll ps;
    bitset<MAXN + 1> sieve;
    sieve.set();
    sieve.reset(1);
    for (ll i = 2; i <= n; ++i) {
        if (sieve[i]) ps.push_back(i);
        for (ll j = i * 2; j <= n; j += i) {
            sieve.reset(j);
        }
    }
    return ps;
}
```

## 7.22 Matrix exponentiation

```
const ll MOD = 1'000'000'007;
template <typename T>
vector<vector<T>> prod(vector<vector<T>> &a,
                      vector<vector<T>> &b) {
    int n = len(a);
    vector<vector<T>> c(n, vector<T>(n));
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            for (int k = 0; k < n; k++) {
                c[i][j] = (c[i][j] +
                           ((a[i][k] * b[k][j]) % MOD)) %
                           MOD;
            }
        }
    }
    return c;
}

template <typename T>
vector<vector<T>> fpow(vector<vector<T>> &xs,
                     ll p) {
    vector<vector<T>> ans(len(xs),
                        vector<T>(len(xs)));
    for (int i = 0; i < len(xs); i++) ans[i][i] = 1;
    auto b = xs;
    while (p) {
        if (p & 1) ans = prod(ans, b);
    }
```

```
    p >>= 1;
    b = prod(b, b);
}
return ans;
}
```

## 7.23 NTT integer convolution and exponentiation

**Time:**

- Convolution  $O(N \cdot \log N)$ ,
- Exponentiation:  $O(\log K \cdot N \cdot \log N)$

```
template <int _mod>
struct mint {
    ll expo(ll b, ll e) {
        ll ret = 1;
        while (e) {
            if (e % 2) ret = ret * b % _mod;
            e /= 2, b = b * b % _mod;
        }
        return ret;
    }
    ll inv(ll b) { return expo(b, _mod - 2); }
    using m = mint;
    ll v;
    mint() : v(0) {}
    mint(ll v_) {
        if (v_ >= _mod or v_ <= -_mod) v_ %= _mod;
        if (v_ < 0) v_ += _mod;
        v = v_;
    }
    m &operator+=(const m &a) {
        v += a.v;
        if (v >= _mod) v -= _mod;
        return *this;
    }
    m &operator-=(const m &a) {
        v -= a.v;
        if (v < 0) v += _mod;
        return *this;
    }
    m &operator*=(const m &a) {
        v = v * ll(a.v) % _mod;
        return *this;
    }
    m &operator/=(const m &a) {
        v = v * inv(a.v) % _mod;
        return *this;
    }
    m operator-() { return m(-v); }
    m &operator^=(ll e) {
        if (e < 0) {
            v = inv(v);
            e = -e;
        }
        v = expo(v, e);
        // possivel otimizacao:
        // cuidado com 0^0
        // v = expo(v, e%(p-1));
        return *this;
    }
    bool operator==(const m &a) { return v == a.v; }
    bool operator!=(const m &a) { return v != a.v; }

    friend istream &operator>>(istream &in, m &a) {
        ll val;
        in >> val;
        a = m(val);
        return in;
    }
    friend ostream &operator<<(ostream &out, m a) {
```

```

    return out << a.v;
}
friend m operator+(m a, m b) { return a += b; }
friend m operator-(m a, m b) { return a -= b; }
friend m operator*(m a, m b) { return a *= b; }
friend m operator/(m a, m b) { return a /= b; }
friend m operator^(m a, ll e) { return a ^= e; }
};

const ll MOD1 = 998244353;
const ll MOD2 = 754974721;
const ll MOD3 = 167772161;

template <int _mod>
void ntt(vector<mint<_mod>> &a, bool rev) {
    int n = len(a);
    auto b = a;
    assert(!(n & (n - 1)));
    mint<_mod> g = 1;
    while ((g ^ (_mod / 2)) == 1) g += 1;
    if (rev) g = 1 / g;
    for (int step = n / 2; step; step /= 2) {
        mint<_mod> w = g ^ (_mod / (n / step)),
            wn = 1;
        for (int i = 0; i < n / 2; i += step) {
            for (int j = 0; j < step; j++) {
                auto u = a[2 * i + j],
                    v = wn * a[2 * i + j + step];
                b[i + j] = u + v;
                b[i + n / 2 + j] = u - v;
            }
            wn = wn * w;
        }
        swap(a, b);
    }
    if (rev) {
        auto n1 = mint<_mod>(1) / n;
        for (auto &x : a) x *= n1;
    }
}

template <ll _mod>
vector<mint<_mod>> convolution(
    const vector<mint<_mod>> &a,
    const vector<mint<_mod>> &b) {
    vector<mint<_mod>> l(all(a)), r(all(b));
    int N = len(l) + len(r) - 1, n = 1;
    while (n <= N) n *= 2;
    l.resize(n), r.resize(n);
    ntt(l, false), ntt(r, false);
    for (int i = 0; i < n; i++) l[i] *= r[i];
    ntt(l, true);
    l.resize(N);
    // Uncomment for a boolean convolution :)
    /*
    for (auto &li : l) {
        li.v = min(li.v, 1ll);
    }
    */
    return l;
}

template <ll _mod>
vector<mint<_mod>> poly_exp(
    vector<mint<_mod>> &ps, int k) {
    vector<mint<_mod>> ret(len(ps));
    auto base = ps;
    ret[0] = 1;
    while (k) {
        if (k & 1) ret = convolution(ret, base);
        k >>= 1;
    }
}

```

```

    base = convolution(base, base);
}
return ret;
}

```

## 7.24 NTT integer convolution and exponentiation (2 mods) modules)

**Description:** Computes the convolution between the two polynomials and.  
**Time:**  $O(N \log N)$   
**Warning:** This is pure magic !

```

template <int _mod>
struct mint {
    ll expo(ll b, ll e) {
        ll ret = 1;
        while (e) {
            if (e % 2) ret = ret * b % _mod;
            e /= 2, b = b * b % _mod;
        }
        return ret;
    }
    ll inv(ll b) { return expo(b, _mod - 2); }
    using m = mint;
    ll v;
    mint() : v(0) {}
    mint(ll v_) {
        if (v_ >= _mod or v_ <= -_mod) v_ %= _mod;
        if (v_ < 0) v_ += _mod;
        v = v_;
    }
    m &operator+=(const m &a) {
        v += a.v;
        if (v >= _mod) v -= _mod;
        return *this;
    }
    m &operator-=(const m &a) {
        v -= a.v;
        if (v < 0) v += _mod;
        return *this;
    }
    m &operator*=(const m &a) {
        v = v * ll(a.v) % _mod;
        return *this;
    }
    m &operator/=(const m &a) {
        v = v * inv(a.v) % _mod;
        return *this;
    }
    m operator-() { return m(-v); }
    m &operator^=(ll e) {
        if (e < 0) {
            v = inv(v);
            e = -e;
        }
        v = expo(v, e);
        // possivel otimizacao:
        // cuidado com 0^0
        // v = expo(v, e%(p-1));
        return *this;
    }
    bool operator==(const m &a) { return v == a.v; }
    bool operator!=(const m &a) { return v != a.v; }
    friend istream &operator>>(istream &in, m &a) {
        ll val;
        in >> val;
        a = m(val);
        return in;
    }
}

```

```

friend ostream &operator<<(ostream &out, m a) {
    return out << a.v;
}
friend m operator+(m a, m b) { return a += b; }
friend m operator-(m a, m b) { return a -= b; }
friend m operator*(m a, m b) { return a *= b; }
friend m operator/(m a, m b) { return a /= b; }
friend m operator^(m a, ll e) { return a ^= e; }
};

const ll MOD1 = 998244353;
const ll MOD2 = 754974721;
const ll MOD3 = 167772161;

template <int _mod>
void ntt(vector<mint<_mod>> &a, bool rev) {
    int n = len(a);
    auto b = a;
    assert(!(n & (n - 1)));
    mint<_mod> g = 1;
    while ((g ^ (_mod / 2)) == 1) g += 1;
    if (rev) g = 1 / g;
    for (int step = n / 2; step; step /= 2) {
        mint<_mod> w = g ^ (_mod / (n / step)),
            wn = 1;
        for (int i = 0; i < n / 2; i += step) {
            for (int j = 0; j < step; j++) {
                auto u = a[2 * i + j],
                    v = wn * a[2 * i + j + step];
                b[i + j] = u + v;
                b[i + n / 2 + j] = u - v;
            }
            wn = wn * w;
        }
        swap(a, b);
    }
    if (rev) {
        auto n1 = mint<_mod>(1) / n;
        for (auto &x : a) x *= n1;
    }
}

tuple<ll, ll, ll> ext_gcd(ll a, ll b) {
    if (!a) return {b, 0, 1};
    auto [g, x, y] = ext_gcd(b % a, a);
    return {g, y - b / a * x, x};
}

template <typename T = ll>
struct crt {
    T a, m;
    crt() : a(0), m(1) {}
    crt(T a_, T m_) : a(a_), m(m_) {}
    crt operator*(crt C) {
        auto [g, x, y] = ext_gcd(m, C.m);
        if ((a - C.a) % g != 0) a = -1;
        if (a == -1 or C.a == -1) return crt(-1, 0);
        T lcm = m / g * C.m;
        T ans =
            a + (x * (C.a - a) / g % (C.m / g)) * m;
        return crt((ans % lcm + lcm) % lcm, lcm);
    }
};

template <typename T = ll>
struct Congruence {
    T a, m;
};

template <typename T = ll>
T chinese_remainder_theorem(
    const vector<Congruence<T>> &equations) {
}

```

```

    crt<T> ans;
    for (auto &[a_, m_] : equations) {
        ans = ans * crt<T>(a_, m_);
    }
    return ans.a;
}

#define int long long
template <ll m1, ll m2>
vll merge_two_mods(const vector<mint<m1>> &a,
                   const vector<mint<m2>> &b) {
    int n = len(a);
    vll ans(n);
    for (int i = 0; i < n; i++) {
        auto cur = crt<ll>();
        auto ai = a[i].v;
        auto bi = b[i].v;
        cur = cur * crt<ll>(ai, m1);
        cur = cur * crt<ll>(bi, m2);
        ans[i] = cur.a;
    }
    return ans;
}

vll convolution_2mods(const vll &a,
                    const vll &b) {
    vector<mint<MOD1>> l(all(a)), r(all(b));
    int N = len(l) + len(r) - 1, n = 1;
    while (n <= N) n *= 2;
    l.resize(n), r.resize(n);
    ntt(l, false), ntt(r, false);
    for (int i = 0; i < n; i++) l[i] *= r[i];
    ntt(l, true);
    l.resize(N);
    vector<mint<MOD2>> l2(all(a)), r2(all(b));
    l2.resize(n), r2.resize(n);
    ntt(l2, false), ntt(r2, false);
    rep(i, 0, n) l2[i] *= r2[i];
    ntt(l2, true);
    l2.resize(N);
    return merge_two_mods(l, l2);
}

vll poly_exp(const vll &xs, ll k) {
    vll ret(len(xs));
    ret[0] = 1;
    auto base = xs;
    while (k) {
        if (k & 1) ret = convolution_2mods(ret, base);
        k >>= 1;
        base = convolution_2mods(base, base);
    }
    return ret;
}

```

## 7.25 Polyominoes

**Usage:** *buildPolyominoes*(*x*) creates every polyomino until size *x*, and put it in *polyominoes*[*x*], access *polyomino.v* to find the vector of pairs representing the coordinates of each piece, considering that the polyomino was 'rooted' in coordinate (0,0).

**Warning:** note that when accessing *polyominoes*[*x*] only the first *x* coordinates are valid.

```

const int MAXP = 10;
using pii = pair<int, int>;
// This implementation considers the rotations as
// distinct
//      0, 10, 10+9, 10+9+8...

```

```

int pos[11] = {0, 10, 19, 27, 34, 40,
               45, 49, 52, 54, 55};
struct Polyominoes {
    pii v[MAXP];
    ll id;
    int n;
    Polyominoes() {
        n = 1;
        v[0] = {0, 0};
        normalize();
    }
    pii &operator[](int i) { return v[i]; }
    bool add(int a, int b) {
        for (int i = 0; i < n; i++)
            if (v[i].first == a and v[i].second == b)
                return false;
        v[n++] = pii(a, b);
        normalize();
        return true;
    }
    void normalize() {
        int mxn = 100, mny = 100;
        for (int i = 0; i < n; i++)
            mxn = min(mxn, v[i].first),
            mny = min(mny, v[i].second);
        id = 0;
        for (int i = 0; i < n; i++) {
            v[i].first -= mxn, v[i].second -= mny;
            id |= (1LL << (pos[v[i].first] +
                           v[i].second));
        }
    }
};
vector<Polyominoes> polyominoes[MAXP + 1];
void buildPolyominoes(int mxN = 10) {
    vector<pair<int, int>> dt(
        {{1, 0}, {-1, 0}, {0, -1}, {0, 1}});
    for (int i = 0; i <= mxN; i++)
        polyominoes[i].clear();
    Polyominoes init;
    queue<Polyominoes> q;
    unordered_set<int64_t> used;
    q.push(init);
    used.insert(init.id);
    while (!q.empty()) {
        Polyominoes u = q.front();
        q.pop();
        polyominoes[u.n].push_back(u);
        if (u.n == mxN) continue;
        for (int i = 0; i < u.n; i++) {
            for (auto [dx, dy] : dt) {
                Polyominoes to = u;
                bool ok = to.add(to[i].first + dx,
                                to[i].second + dy);
                if (ok and !used.count(to.id)) {
                    q.push(to);
                    used.insert(to.id);
                }
            }
        }
    }
}

```

## 8 Primitives

### 8.1 Bigint

```

const int maxn = 1e2 + 14, lg = 15;
const int base = 1000000000;
const int base_digits = 9;
struct bigint {
    vi a;
    int sign;
    int size() {
        if (a.empty()) return 0;
        int ans = (a.size() - 1) * base_digits;
        int ca = a.back();
        while (ca) ans++, ca /= 10;
        return ans;
    }
    bigint operator^(const bigint &v) {
        bigint ans = 1, a = *this, b = v;
        while (!b.isZero()) {
            if (b % 2) ans *= a;
            a *= a, b /= 2;
        }
        return ans;
    }
    string to_string() {
        stringstream ss;
        ss << *this;
        string s;
        ss >> s;
        return s;
    }
    int sumof() {
        string s = to_string();
        int ans = 0;
        for (auto c : s) ans += c - '0';
        return ans;
    }
    /*</arpa>*/
    bigint() : sign(1) {}
    bigint(long long v) { *this = v; }
    bigint(const string &s) { read(s); }
    void operator=(const bigint &v) {
        sign = v.sign;
        a = v.a;
    }
    void operator=(long long v) {
        sign = 1;
        a.clear();
        if (v < 0) sign = -1, v = -v;
        for (; v > 0; v = v / base)
            a.push_back(v % base);
    }
    bigint operator+(const bigint &v) const {
        if (sign == v.sign) {
            bigint res = v;
            for (int i = 0, carry = 0;
                 i < (int)max(a.size(), v.a.size()) ||
                 carry;
                 ++i) {
                if (i == (int)res.a.size())
                    res.a.push_back(0);
                res.a[i] +=
                    carry +
                    (i < (int)a.size() ? a[i] : 0);
                carry = res.a[i] >= base;
                if (carry) res.a[i] -= base;
            }
            return res;
        }
        return *this - (-v);
    }
}

```

```

bigint operator-(const bigint &v) const {
    if (sign == v.sign) {
        if (abs() >= v.abs()) {
            bigint res = *this;
            for (int i = 0, carry = 0;
                 i < (int)v.a.size() || carry; ++i) {
                res.a[i] -=
                    carry +
                    (i < (int)v.a.size() ? v.a[i] : 0);
                carry = res.a[i] < 0;
                if (carry) res.a[i] += base;
            }
            res.trim();
            return res;
        }
        return -(v - *this);
    }
    return *this + (-v);
}

void operator*=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = 0, carry = 0;
         i < (int)a.size() || carry; ++i) {
        if (i == (int)a.size()) a.push_back(0);
        long long cur = a[i] * (long long)v + carry;
        carry = (int)(cur / base);
        a[i] = (int)(cur % base);
        // asm("divl %%ecx" : "=a"(carry),
        //      "=d"(a[i]) : "A"(cur), "c"(base));
    }
    trim();
}

bigint operator*(int v) const {
    bigint res = *this;
    res *= v;
    return res;
}

void operator*=(long long v) {
    if (v < 0) sign = -sign, v = -v;
    if (v > base) {
        *this = *this * (v / base) * base +
            *this * (v % base);
        return;
    }
    for (int i = 0, carry = 0;
         i < (int)a.size() || carry; ++i) {
        if (i == (int)a.size()) a.push_back(0);
        long long cur = a[i] * (long long)v + carry;
        carry = (int)(cur / base);
        a[i] = (int)(cur % base);
        // asm("divl %%ecx" : "=a"(carry),
        //      "=d"(a[i]) : "A"(cur), "c"(base));
    }
    trim();
}

bigint operator*(long long v) const {
    bigint res = *this;
    res *= v;
    return res;
}

friend pair<bigint, bigint> divmod(
    const bigint &a1, const bigint &b1) {
    int norm = base / (b1.a.back() + 1);
    bigint a = a1.abs() * norm;
    bigint b = b1.abs() * norm;
    bigint q, r;
    q.a.resize(a.a.size());

```

```

    for (int i = a.a.size() - 1; i >= 0; i--) {
        r *= base;
        r += a.a[i];
        int s1 = r.a.size() <= b.a.size()
            ? 0
            : r.a[b.a.size()];
        int s2 = r.a.size() <= b.a.size() - 1
            ? 0
            : r.a[b.a.size() - 1];
        int d = ((long long)base * s1 + s2) /
            b.a.back();
        r -= b * d;
        while (r < 0) r += b, --d;
        q.a[i] = d;
    }
    q.sign = a1.sign * b1.sign;
    r.sign = a1.sign;
    q.trim();
    r.trim();
    return make_pair(q, r / norm);
}

bigint operator/(const bigint &v) const {
    return divmod(*this, v).first;
}

bigint operator%(const bigint &v) const {
    return divmod(*this, v).second;
}

void operator/=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = (int)a.size() - 1, rem = 0;
         i >= 0; --i) {
        long long cur =
            a[i] + rem * (long long)base;
        a[i] = (int)(cur / v);
        rem = (int)(cur % v);
    }
    trim();
}

bigint operator/(int v) const {
    bigint res = *this;
    res /= v;
    return res;
}

int operator%(int v) const {
    if (v < 0) v = -v;
    int m = 0;
    for (int i = a.size() - 1; i >= 0; --i)
        m = (a[i] + m * (long long)base) % v;
    return m * sign;
}

void operator+=(const bigint &v) {
    *this = *this + v;
}

void operator-=(const bigint &v) {
    *this = *this - v;
}

void operator*=(const bigint &v) {
    *this = *this * v;
}

void operator/=(const bigint &v) {
    *this = *this / v;
}

bool operator<(const bigint &v) const {
    if (sign != v.sign) return sign < v.sign;
    if (a.size() != v.a.size())
        return a.size() * sign <

```

```

        v.a.size() * v.sign;
    for (int i = a.size() - 1; i >= 0; i--)
        if (a[i] != v.a[i])
            return a[i] * sign < v.a[i] * sign;
    return false;
}

bool operator>(const bigint &v) const {
    return v < *this;
}

bool operator<=(const bigint &v) const {
    return !(v < *this);
}

bool operator>=(const bigint &v) const {
    return !(*this < v);
}

bool operator==(const bigint &v) const {
    return !(*this < v) && !(v < *this);
}

bool operator!=(const bigint &v) const {
    return *this < v || v < *this;
}

void trim() {
    while (!a.empty() && !a.back()) a.pop_back();
    if (a.empty()) sign = 1;
}

bool isZero() const {
    return a.empty() || (a.size() == 1 && !a[0]);
}

bigint operator-() const {
    bigint res = *this;
    res.sign = -sign;
    return res;
}

bigint abs() const {
    bigint res = *this;
    res.sign *= res.sign;
    return res;
}

long long longValue() const {
    long long res = 0;
    for (int i = a.size() - 1; i >= 0; i--)
        res = res * base + a[i];
    return res * sign;
}

friend bigint gcd(const bigint &a,
                  const bigint &b) {
    return b.isZero() ? a : gcd(b, a % b);
}

friend bigint lcm(const bigint &a,
                  const bigint &b) {
    return a / gcd(a, b) * b;
}

void read(const string &s) {
    sign = 1;
    a.clear();
    int pos = 0;
    while (pos < (int)s.size() &&
           (s[pos] == '-' || s[pos] == '+')) {
        if (s[pos] == '-') sign = -sign;
        ++pos;
    }
    for (int i = s.size() - 1; i >= pos;
         i -= base_digits) {
        int x = 0;
        for (int j = max(pos, i - base_digits + 1);
             j <= i; j++)

```

```

        x = x * 10 + s[j] - '0';
    a.push_back(x);
}
trim();
}
friend istream &operator>>(istream &stream,
                           bigint &v) {
    string s;
    stream >> s;
    v.read(s);
    return stream;
}
friend ostream &operator<<(ostream &stream,
                           const bigint &v) {
    if (v.sign == -1) stream << '-';
    stream << (v.a.empty() ? 0 : v.a.back());
    for (int i = (int)v.a.size() - 2; i >= 0; --i)
        stream << setw(base_digits) << setfill('0')
                << v.a[i];
    return stream;
}
static vector<int> convert_base(
    const vector<int> &a, int old_digits,
    int new_digits) {
    vector<long long> p(
        max(old_digits, new_digits) + 1);
    p[0] = 1;
    for (int i = 1; i < (int)p.size(); i++)
        p[i] = p[i - 1] * 10;
    vector<int> res;
    long long cur = 0;
    int cur_digits = 0;
    for (int i = 0; i < (int)a.size(); i++) {
        cur += a[i] * p[cur_digits];
        cur_digits += old_digits;
        while (cur_digits >= new_digits) {
            res.push_back((int)(cur % p[new_digits]));
            cur /= p[new_digits];
            cur_digits -= new_digits;
        }
    }
    res.push_back((int)cur);
    while (!res.empty() && !res.back())
        res.pop_back();
    return res;
}
typedef vector<long long> vll;
static vll karatsubaMultiply(const vll &a,
                             const vll &b) {
    int n = a.size();
    vll res(n + n);
    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                res[i + j] += a[i] * b[j];
        return res;
    }
    int k = n >> 1;
    vll a1(a.begin(), a.begin() + k);
    vll a2(a.begin() + k, a.end());
    vll b1(b.begin(), b.begin() + k);
    vll b2(b.begin() + k, b.end());
    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);
    for (int i = 0; i < k; i++) a2[i] += a1[i];
    for (int i = 0; i < k; i++) b2[i] += b1[i];

```

```

    vll r = karatsubaMultiply(a2, b2);
    for (int i = 0; i < (int)a1b1.size(); i++)
        r[i] -= a1b1[i];
    for (int i = 0; i < (int)a2b2.size(); i++)
        r[i] -= a2b2[i];
    for (int i = 0; i < (int)r.size(); i++)
        res[i + k] += r[i];
    for (int i = 0; i < (int)a1b1.size(); i++)
        res[i] += a1b1[i];
    for (int i = 0; i < (int)a2b2.size(); i++)
        res[i + n] += a2b2[i];
    return res;
}
bigint operator*(const bigint &v) const {
    vector<int> a6 =
        convert_base(this->a, base_digits, 6);
    vector<int> b6 =
        convert_base(v.a, base_digits, 6);
    vll a(a6.begin(), a6.end());
    vll b(b6.begin(), b6.end());
    while (a.size() < b.size()) a.push_back(0);
    while (b.size() < a.size()) b.push_back(0);
    while (a.size() & (a.size() - 1))
        a.push_back(0), b.push_back(0);
    vll c = karatsubaMultiply(a, b);
    bigint res;
    res.sign = sign * v.sign;
    for (int i = 0, carry = 0; i < (int)c.size();
         i++) {
        long long cur = c[i] + carry;
        res.a.push_back((int)(cur % 1000000));
        carry = (int)(cur / 1000000);
    }
    res.a = convert_base(res.a, 6, base_digits);
    res.trim();
    return res;
}
};

```

## 8.2 Integer Mod

```

const ll MOD = 1'000'000'000 + 7;
template <ll _mod = MOD>
struct mint {
    ll value;
    static const ll MOD_value = _mod;
    mint(ll v = 0) {
        value = v % _mod;
        if (value < 0) value += _mod;
    }
    mint(ll a, ll b) : value(0) {
        *this += a;
        *this /= b;
    }
    mint &operator+=(mint const &b) {
        value += b.value;
        if (value >= _mod) value -= _mod;
        return *this;
    }
    mint &operator-=(mint const &b) {
        value -= b.value;
        if (value < 0) value += _mod;
        return *this;
    }
    mint &operator*=(mint const &b) {
        value = (ll)value * b.value % _mod;

```

```

        return *this;
    }
    friend mint mexp(mint a, ll e) {
        mint res = 1;
        while (e) {
            if (e & 1) res *= a;
            a *= a;
            e >>= 1;
        }
        return res;
    }
    friend mint inverse(mint a) {
        return mexp(a, _mod - 2);
    }
    mint &operator/=(mint const &b) {
        return *this *= inverse(b);
    }
    friend mint operator+(mint a, mint const b) {
        return a += b;
    }
    mint operator++(int) {
        return this->value = (this->value + 1) % _mod;
    }
    mint operator++() {
        return this->value = (this->value + 1) % _mod;
    }
    friend mint operator-(mint a, mint const b) {
        return a -= b;
    }
    friend mint operator-(mint const a) {
        return 0 - a;
    }
    mint operator--(int) {
        return this->value =
            (this->value - 1 + _mod) % _mod;
    }
    mint operator--() {
        return this->value =
            (this->value - 1 + _mod) % _mod;
    }
    friend mint operator*(mint a, mint const b) {
        return a *= b;
    }
    friend mint operator/(mint a, mint const b) {
        return a /= b;
    }
    friend std::ostream &operator<<(
        std::ostream &os, mint const &a) {
        return os << a.value;
    }
    friend bool operator==(mint const &a,
                           mint const &b) {
        return a.value == b.value;
    }
    friend bool operator!=(mint const &a,
                           mint const &b) {
        return a.value != b.value;
    }
};

```

## 8.3 Matrix

```

template <typename T>
struct Matrix {
    vector<vector<T>> d;
    Matrix() : Matrix(0) {}
    Matrix(int n) : Matrix(n, n) {}
    Matrix(int n, int m)

```

```

    : Matrix(
        vector<vector<T>>(n, vector<T>(m))) {}
Matrix(const vector<vector<T>> &v) : d(v) {}
constexpr int n() const {
    return (int)d.size();
}
constexpr int m() const {
    return n() ? (int)d[0].size() : 0;
}
void rotate() { *this = rotated(); }
Matrix<T> rotated() const {
    Matrix<T> res(m(), n());
    for (int i = 0; i < m(); i++) {
        for (int j = 0; j < n(); j++) {
            res[i][j] = d[n() - j - 1][i];
        }
    }
    return res;
}
Matrix<T> pow(int power) const {
    assert(n() == m());
    auto res = Matrix<T>::identity(n());
    auto b = *this;
    while (power) {
        if (power & 1) res *= b;
        b *= b;
        power >>= 1;
    }
    return res;
}
Matrix<T> submatrix(int start_i, int start_j,
                    int rows = INT_MAX,
                    int cols = INT_MAX) const {
    rows = min(rows, n() - start_i);
    cols = min(cols, m() - start_j);
    if (rows <= 0 or cols <= 0) return {};
    Matrix<T> res(rows, cols);
    for (int i = 0; i < rows; i++)
        for (int j = 0; j < cols; j++)
            res[i][j] = d[i + start_i][j + start_j];
    return res;
}
Matrix<T> translated(int x, int y) const {
    Matrix<T> res(n(), m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            if (i + x < 0 or i + x >= n() or
                j + y < 0 or j + y >= m())
                continue;
            res[i + x][j + y] = d[i][j];
        }
    }
    return res;
}
static Matrix<T> identity(int n) {
    Matrix<T> res(n);
    for (int i = 0; i < n; i++) res[i][i] = 1;
    return res;
}
vector<T> &operator[](int i) { return d[i]; }
const vector<T> &operator[](int i) const {
    return d[i];
}
Matrix<T> &operator+=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x += value;
    }
}

```

```

    return *this;
}
Matrix<T> operator+(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x + value;
    }
    return res;
}
Matrix<T> &operator-=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x -= value;
    }
    return *this;
}
Matrix<T> operator-(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x - value;
    }
    return res;
}
Matrix<T> operator*=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x *= value;
    }
    return *this;
}
Matrix<T> operator*(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x * value;
    }
    return res;
}
Matrix<T> &operator/=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x /= value;
    }
    return *this;
}
Matrix<T> operator/(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x / value;
    }
    return res;
}
Matrix<T> &operator+=(const Matrix<T> &o) {
    assert(n() == o.n() and m() == o.m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            d[i][j] += o[i][j];
        }
    }
    return *this;
}
Matrix<T> operator+(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());
    auto res = *this;
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            res[i][j] = res[i][j] + o[i][j];
        }
    }
    return res;
}
Matrix<T> operator-(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            d[i][j] -= o[i][j];
        }
    }
}

```

```

    }
}
return *this;
}
Matrix<T> operator-(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());
    auto res = *this;
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            res[i][j] = res[i][j] - o[i][j];
        }
    }
    return res;
}
Matrix<T> &operator*=(const Matrix<T> &o) {
    *this = *this * o;
    return *this;
}
Matrix<T> operator*(const Matrix<T> &o) const {
    assert(m() == o.n());
    Matrix<T> res(n(), o.m());
    for (int i = 0; i < res.n(); i++) {
        for (int j = 0; j < res.m(); j++) {
            auto &x = res[i][j];
            for (int k = 0; k < m(); k++) {
                x += (d[i][k] * o[k][j]);
            }
        }
    }
    return res;
}
friend istream &operator>>(istream &is,
                           Matrix<T> &mat) {
    for (auto &row : mat)
        for (auto &x : row) is >> x;
    return is;
}
friend ostream &operator<<(
    ostream &os, const Matrix<T> &mat) {
    bool frow = 1;
    for (auto &row : mat) {
        if (not frow) os << '\n';
        bool first = 1;
        for (auto &x : row) {
            if (not first) os << ' ';
            os << x;
            first = 0;
        }
        frow = 0;
    }
    return os;
}
auto begin() { return d.begin(); }
auto end() { return d.end(); }
auto rbegin() { return d.rbegin(); }
auto rend() { return d.rend(); }
auto begin() const { return d.begin(); }
auto end() const { return d.end(); }
auto rbegin() const { return d.rbegin(); }
auto rend() const { return d.rend(); }
};

```

## 9 Strings

### 9.1 Count distinct anagrams

```

const ll MOD = 1e9 + 7;
const int maxn = 1e6;
vll fs(maxn + 1);
void precompute() {
    fs[0] = 1;
    for (ll i = 1; i <= maxn; i++) {
        fs[i] = (fs[i - 1] * i) % MOD;
    }
}
ll fpow(ll a, int n, ll mod = LLONG_MAX) {
    if (n == 0) return 1;
    if (n == 1) return a;
    ll x = fpow(a, n / 2, mod) % mod;
    return ((x * x) % mod * (n & 1 ? a : 1ll)) % mod;
}
ll distinctAnagrams(const string &s) {
    precompute();
    vi hist('z' - 'a' + 1, 0);
    for (auto &c : s) hist[c - 'a']++;
    ll ans = fs[len(s)];
    for (auto &q : hist) {
        ans = (ans * fpow(fs[q], MOD - 2, MOD)) % MOD;
    }
    return ans;
}

```

## 9.2 Double hash range query

```

using ll = long long;
using vll = vector<ll>;
using pll = pair<ll, ll>;
const int MAXN(1'000'000);
const ll MOD = 1000027957;
const ll MOD2 = 1000015187;
const ll P = 31;
ll p[MAXN + 1], p2[MAXN + 1];
void precompute() {
    p[0] = p2[0] = 1;
    for (int i = 1; i <= MAXN; i++)
        p[i] = (P * p[i - 1]) % MOD,
        p2[i] = (P * p2[i - 1]) % MOD2;
}
struct Hash {
    int n;
    vll h, h2, hi, hi2;
    Hash() {}
    Hash(const string &s)
        : n(s.size()), h(n), h2(n), hi(n), hi2(n) {
        h[0] = h2[0] = s[0];
        for (int i = 1; i < n; i++)
            h[i] = (s[i] + h[i - 1] * P) % MOD,
            h2[i] = (s[i] + h2[i - 1] * P) % MOD2;
        hi[n - 1] = hi2[n - 1] = s[n - 1];
        for (int i = n - 2; i >= 0; i--)
            hi[i] = (s[i] + hi[i + 1] * P) % MOD,
            hi2[i] = (s[i] + hi2[i + 1] * P) % MOD2;
    }
    pll query(int l, int r) {
        ll hash =
            (h[r] -
             (l ? h[l - 1] * p[r - l + 1] % MOD : 0));
        ll hash2 =
            (h2[r] -
             (l ? h2[l - 1] * p2[r - l + 1] % MOD2
              : 0));
    }
}

```

```

        return {(hash < 0 ? hash + MOD : hash),
                 (hash2 < 0 ? hash2 + MOD2 : hash2)};
    }
    pll query_inv(int l, int r) {
        ll hash = (hi[l] -
                   (r + 1 < n ? hi[r + 1] *
                    p[r - l + 1] % MOD
                    : 0));
        ll hash2 =
            (hi2[l] -
             (r + 1 < n
              ? hi2[r + 1] * p2[r - l + 1] % MOD2
              : 0));
        return {(hash < 0 ? hash + MOD : hash),
                 (hash2 < 0 ? hash2 + MOD2 : hash2)};
    }
};

```

## 9.3 Hash range query

```

const ll P = 31;
const ll MOD = 1e9 + 9;
const int MAXN(1e6);
ll ppow[MAXN + 1];
void pre_calc() {
    ppow[0] = 1;
    for (int i = 1; i <= MAXN; i++)
        ppow[i] = (ppow[i - 1] * P) % MOD;
}
struct Hash {
    int n;
    vll h, hi;
    Hash(const string &s)
        : n(s.size()), h(n), hi(n) {
        h[0] = s[0];
        hi[n - 1] = s[n - 1];
        for (int i = 1; i < n; i++) {
            h[i] = (s[i] + h[i - 1] * P) % MOD;
            hi[n - i - 1] =
                (s[n - i - 1] + hi[n - i - 1] * P) % MOD;
        }
    }
    ll qry(int l, int r) {
        ll hash =
            (h[r] -
             (l ? h[l - 1] * ppow[r - l + 1] % MOD
              : 0));
        return hash < 0 ? hash + MOD : hash;
    }
    ll qry_inv(int l, int r) {
        ll hash =
            (hi[l] -
             (r + 1 < n
              ? hi[r + 1] * ppow[r - l + 1] % MOD
              : 0));
        return hash < 0 ? hash + MOD : hash;
    }
};

```

## 9.4 Hash unsigned long long $2^{64} - 1$

**Description:** Arithmetic mod  $2^{64} - 1$ . 2x slower than mod  $2^{64}$  and more code, but works on evil test data (e.g. Thue-Morse, where ABBA... and BAAB... of length  $2^{10}$  hash the same mod  $2^{64}$ ).

"typedef ull H;" instead if you think test data is random.

```

typedef uint64_t ull;
struct H {
    ull x;
    H(ull x = 0) : x(x) {}
    H operator+(H o) {
        return x + o.x + (x + o.x < x);
    }
    H operator-(H o) { return *this + ~o.x; }
    H operator*(H o) {
        auto m = (__uint128_t)x * o.x;
        return H((ull)m) + (ull)(m >> 64);
    }
    ull get() const { return x + !~x; }
    bool operator==(H o) const {
        return get() == o.get();
    }
    bool operator<(H o) const {
        return get() < o.get();
    }
};
static const H C =
    (long long)1e11 +
    3; // (order ~ 3e9; random also ok)
struct Hash {
    int n;
    vector<H> ha, pw;
    Hash(string &str)
        : n(str.size()),
          ha((int)str.size() + 1),
          pw(ha) {
        pw[0] = 1;
        for (int i = 0; i < (int)str.size(); i++)
            ha[i + 1] = ha[i] * C + str[i],
            pw[i + 1] = pw[i] * C;
    }
    H query(int a, int b) { // hash [a, b]
        b++;
        return ha[b] - ha[a] * pw[b - a];
    }
};
vector<H> getHashes(string &str, int length) {
    if ((int)str.size() < length) return {};
    H h = 0, pw = 1;
    for (int i = 0; i < length; i++)
        h = h * C + str[i], pw = pw * C;
    vector<H> ret = {h};
    for (int i = length; i < (int)str.size(); i++)
        ret.push_back(h = h * C + str[i] -
                      pw * str[i - length]);
    return ret;
}
H hashString(string &s) {
    H h{};
    for (char c : s) h = h * C + c;
    return h;
}

```

## 9.5 K-th digit in digit string

**Description:** Find the k-th digit in a *digit string*, only works for  $1 \leq k \leq 10^{18}$  !

**Time:** precompute  $O(1)$ , query  $O(1)$

```

using ull = vector<ull>;
ull pow10;
vector<array<ull, 4>> memo;
void precompute(int maxpow = 18) {
    ull qtd = 1;
    ull start = 1;
    ull end = 9;
    ull curlenght = 9;
    ull startstr = 1;
    ull endstr = 9;
    for (ull i = 0, j = 111; (int)i < maxpow;
        i++, j *= 1011)
        pow10.eb(j);
    for (ull i = 0; i < maxpow - 1ull; i++) {
        memo.push_back(
            {start, end, startstr, endstr});
        start = end + 111;
        end = end + (911 * pow10[qtd]);
        curlenght = end - start + 1ull;
        qtd++;
        startstr = endstr + 1ull;
        endstr =
            (endstr + 1ull) + (curlenght)*qtd - 1ull;
    }
    char kthDigit(ull k) {
        int qtd = 1;
        for (auto [s, e, ss, es] : memo) {
            if (k >= ss and k <= es) {
                ull pos = k - ss;
                ull index = pos / qtd;
                ull nmr = s + index;
                int i = k - ss - qtd * index;
                return ((nmr / pow10[qtd - i - 1]) % 10) +
                    '0';
            }
            qtd++;
        }
        return 'X';
    }
}

```

## 9.6 Longest Palindrome Substring (Manacher)

**Description:** Finds the longest palindrome substring, manacher returns a vector where the  $i$ -th position is how much is possible to grow the string to the left and the right of  $i$  and keep it a palindrome.

**Time:**  $O(N)$

```

vi manacher(const string &s) {
    int n = len(s) - 2;
    vi p(n + 2);
    int l = 1, r = 1;
    for (int i = 1; i <= n; i++) {
        p[i] = max(0, min(r - i, p[l + (r - i)]));
        while (s[i - p[i]] == s[i + p[i]]) p[i]++;
        if (i + p[i] > r) l = i - p[i], r = i + p[i];
        p[i]--;
    }
    return p;
}
string longest_palindrome(const string &s) {
    string t("$#");
    for (auto c : s)
        t.push_back(c), t.push_back('#');
    t.push_back('~');
    vi xs = manacher(t);
}

```

```

int mpos = max_element(all(xs)) - xs.begin();
string p;
for (int k = xs[mpos], i = mpos - k;
    i <= mpos + k; i++)
    if (t[i] != '#') p.push_back(t[i]);
return p;
}

```

## 9.7 Longest palindrome

```

string longest_palindrome(const string &s) {
    int n = (int)s.size();
    vector<array<int, 2>> dp(n);
    pii odd(0, -1), even(0, -1);
    pii ans;
    for (int i = 0; i < n; i++) {
        int k = 0;
        if (i > odd.second)
            k = 1;
        else
            k = min(dp[odd.first + odd.second - i][0],
                odd.second - i + 1);
        while (i - k >= 0 and i + k < n and
            s[i - k] == s[i + k])
            k++;
        dp[i][0] = k--;
        if (i + k > odd.second) odd = {i - k, i + k};
        if (2 * dp[i][0] - 1 > ans.second)
            ans = {i - k, 2 * dp[i][0] - 1};
        k = 0;
        if (i <= even.second)
            k = min(
                dp[even.first + even.second - i + 1][1],
                even.second - i + 1);
        while (i - k - 1 >= 0 and i + k < n and
            s[i - k - 1] == s[i + k])
            k++;
        dp[i][1] = k--;
        if (i + k > even.second)
            even = {i - k - 1, i + k};
        if (2 * dp[i][1] > ans.second)
            ans = {i - k - 1, 2 * dp[i][1]};
    }
    return s.substr(ans.first, ans.second);
}

```

## 9.8 Rabin-Karp

```

size_t rabin_karp(const string &s,
    const string &p) {
    if (s.size() < p.size()) return 0;
    auto n = s.size(), m = p.size();
    const ll p1 = 31, p2 = 29, q1 = 1e9 + 7,
        q2 = 1e9 + 9;
    const ll p1_1 = fpow(p1, q1 - 2, q1),
        p1_2 = fpow(p1, m - 1, q1);
    const ll p2_1 = fpow(p2, q2 - 2, q2),
        p2_2 = fpow(p2, m - 1, q2);
    pair<ll, ll> hs, hp;
    for (int i = (int)m - 1; ~i; --i) {
        hs.first = (hs.first * p1) % q1;
        hs.first = (hs.first + s[i] - 'a' + 1) % q1;
        hs.second = (hs.second * p2) % q2;
    }
}

```

```

hs.second =
    (hs.second + (s[i] - 'a' + 1)) % q2;
hp.first = (hp.first * p1) % q1;
hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
hp.second = (hp.second * p2) % q2;
hp.second =
    (hp.second + (p[i] - 'a' + 1)) % q2;
}
size_t occ = 0;
for (size_t i = 0; i < n - m; i++) {
    occ += (hs == hp);
    int fi = s[i] - 'a' + 1;
    int fm = s[i + m] - 'a' + 1;
    hs.first = (hs.first - fi + q1) % q1;
    hs.first = (hs.first * p1_1) % q1;
    hs.first = (hs.first + fm * p1_2) % q1;
    hs.second = (hs.second - fi + q2) % q2;
    hs.second = (hs.second * p2_1) % q2;
    hs.second = (hs.second + fm * p2_2) % q2;
}
occ += hs == hp;
return occ;
}

```

## 9.9 Suffix array

```

vector<int> sort_cyclic_shifts(string const &s) {
    int n = s.size();
    const int alphabet = 128;
    vector<int> p(n), c(n),
        cnt(max(alphabet, n), 0);
    for (int i = 0; i < n; i++) cnt[s[i]]++;
    for (int i = 1; i < alphabet; i++)
        cnt[i] += cnt[i - 1];
    for (int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i - 1]]) classes++;
        c[p[i]] = classes - 1;
    }
    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0) pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for (int i = 0; i < n; i++) cnt[c[pn[i]]]++;
        for (int i = 1; i < classes; i++)
            cnt[i] += cnt[i - 1];
        for (int i = n - 1; i >= 0; i--)
            p[--cnt[c[pn[i]]]] = pn[i];
        cn[p[0]] = 0;
        classes = 1;
        for (int i = 1; i < n; i++) {
            pair<int, int> cur = {
                c[p[i]], c[(p[i] + (1 << h)) % n]};
            pair<int, int> prev = {
                c[p[i - 1]],
                c[(p[i - 1] + (1 << h)) % n]};
            if (cur != prev) ++classes;
            cn[p[i]] = classes - 1;
        }
    }
}

```



```

    }
    c.swap(cn);
}
return p;
}
vector<int> suffix_array(string s) {
    s += "$";
    vector<int> p = sort_cyclic_shifts(s);
    p.erase(p.begin());
    return p;
}

```

## 9.10 Suffix automaton

```

struct state {
    int len, link, cnt, firstpos;
    // this can be optimized using a vector with the
    // alphabet size
    map<char, int> next;
    vi inv_link;
};

struct SuffixAutomaton {
    vector<state> st;
    int sz = 0;
    int last;
    vc cloned;

    SuffixAutomaton(const string &s, int maxlen)
        : st(maxlen * 2), cloned(maxlen * 2) {
        st[0].len = 0;
        st[0].link = -1;
        sz++;
        last = 0;
        for (auto &c : s) add_char(c);
        // precompute for count occurrences
        for (int i = 1; i < sz; i++) {
            st[i].cnt = !cloned[i];
        }
        vector<pair<state, int>> aux;
        for (int i = 0; i < sz; i++) {
            aux.push_back({st[i], i});
        }
        sort(all(aux), [](const pair<state, int> &a,
                           const pair<state, int> &b) {
            return a.fst.len > b.fst.len;
        });
        for (auto &[stt, id] : aux) {
            if (stt.link != -1) {
                st[stt.link].cnt += st[id].cnt;
            }
        }
        // for find every occurende position
        for (int v = 1; v < sz; v++) {
            st[st[v].link].inv_link.push_back(v);
        }
    }

    void add_char(char c) {
        int cur = sz++;
        st[cur].len = st[last].len + 1;
        st[cur].firstpos = st[cur].len - 1;
        int p = last;
        // follow the suffix link until find a
        // transition to c
        while (p != -1 and !st[p].next.count(c)) {
            st[p].next[c] = cur;
            p = st[p].link;
        }
    }
}

```

```

// there was no transition to c so create and
// leave
if (p == -1) {
    st[cur].link = 0;
    last = cur;
    return;
}
int q = st[p].next[c];
if (st[p].len + 1 == st[q].len) {
    st[cur].link = q;
} else {
    int clone = sz++;
    cloned[clone] = true;
    st[clone].len = st[p].len + 1;
    st[clone].next = st[q].next;
    st[clone].link = st[q].link;
    st[clone].firstpos = st[q].firstpos;
    while (p != -1 and st[p].next[c] == q) {
        st[p].next[c] = clone;
        p = st[p].link;
    }
    st[q].link = st[cur].link = clone;
}
last = cur;
}

bool checkOccurrence(
    const string &t) { // O(len(t))
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return false;
        cur = st[cur].next[c];
    }
    return true;
}

ll totalSubstrings() { // distinct, O(len(s))
    ll tot = 0;
    for (int i = 1; i < sz; i++) {
        tot += st[i].len - st[st[i].link].len;
    }
    return tot;
}

// count occurences of a given string t
int countOccurrences(const string &t) {
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return 0;
        cur = st[cur].next[c];
    }
    return st[cur].cnt;
}

// find the first index where t appears a
// substring O(len(t))
int firstOccurrence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return -1;
        cur = st[cur].next[c];
    }
    return st[cur].firstpos - len(t) + 1;
}

vi everyOccurrence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return {};
        cur = st[cur].next[c];
    }
    vi ans;
    getEveryOccurrence(cur, len(t), ans);
    return ans;
}

```

```

}

void getEveryOccurrence(int v, int P_length,
                        vi &ans) {
    if (!cloned[v])
        ans.pb(st[v].firstpos - P_length + 1);
    for (int u : st[v].inv_link)
        getEveryOccurrence(u, P_length, ans);
}
};

```

## 9.11 Trie

Description:

- build with the size of the alphabet ( $\sigma$ ) and the first char ( $norm$ )
- $insert(s)$  insert the string in the trie  $O(|s| * \sigma)$
- $erase(s)$  remove the string from the trie  $O(|s|)$
- $find(s)$  return the last node from the string  $s$ , 0 if not found  $O(|s|)$

```

struct trie {
    vi2d to;
    vi end, pref;
    int sigma;
    char norm;

    trie(int sigma_ = 26, char norm_ = 'a')
        : sigma(sigma_), norm(norm_) {
        to = {vector<int>(sigma)};
        end = {0}, pref = {0};
    }

    int next(int node, char key) {
        return to[node][key - norm];
    }

    void insert(const string &s) {
        int x = 0;
        for (auto c : s) {
            int &nxt = to[x][c - norm];
            if (!nxt) {
                nxt = len(to);
                to.push_back(vi(sigma));
                end.emplace_back(0), pref.emplace_back(0);
            }
            x = nxt, pref[x]++;
        }
        end[x]++, pref[0]++;
    }

    void erase(const string &s) {
        int x = 0;
        for (char c : s) {
            int &nxt = to[x][c - norm];
            x = nxt, pref[x]--;
            if (!pref[x]) nxt = 0;
        }
        end[x]--, pref[0]--;
    }

    int find(const string &s) {
        int x = 0;
        for (auto c : s) {
            x = to[x][c - norm];
            if (!x) return 0;
        }
        return x;
    }
};

```

## 9.12 Z-function get occurrence positions

Time:  $O(\text{len}(s) + \text{len}(p))$

```
vi getOccPos(string &s, string &p) {  
    // Z-function  
    char delim = '#';  
    string t{p + delim + s};  
    vi zs(len(t));  
    int l = 0, r = 0;  
    for (int i = 1; i < len(t); i++) {
```

```
        if (i <= r) zs[i] = min(zs[i - l], r - i + 1);  
        while (zs[i] + i < len(t) and  
               t[zs[i]] == t[i + zs[i]])  
            zs[i]++;  
        if (r < i + zs[i] - 1)  
            l = i, r = i + zs[i] - 1;  
    }  
    // Iterate over the results of Z-function to get  
    // ranges  
    vi ans;
```

```
    int start = len(p) + 1 + 1 - 1;  
    for (int i = start; i < len(zs); i++) {  
        if (zs[i] == len(p)) {  
            int l = i - start;  
            ans.emplace_back(l);  
        }  
    }  
    return ans;  
}
```