Contents

1 Data structures

1.1 Disjoint Sparse Table

Answers queries of any monoid operation (i.e. has identity element and is associative)
Build: $O(N \log N)$, Query: $O(1)$

```
#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct DisjointSparseTable {
  using Operation = T (*)(T, T);

  vector<vector<T>> st;
  Operation f;
  T identity;

  static constexpr int log2_floor(unsigned long long i) noexcept {
    return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
  }

  // Lazy loading constructor. Needs to call build!
  DisjointSparseTable(Operation op, const T neutral = T())
    : st(), f(op), identity(neutral) {}

  DisjointSparseTable(vector<T> v) : DisjointSparseTable(v, F(min(a, b))) {}

  DisjointSparseTable(vector<T> v, Operation op, const T neutral = T())
    : st(), f(op), identity(neutral) {
    build(v);
  }

  void build(vector<T> v) {
    st.resize(log2_floor(v.size()) + 1,
              vector<T>(1ll << (log2_floor(v.size()) + 1)));
    v.resize(st[0].size(), identity);
    for (int level = 0; level < (int)st.size(); ++level) {
      for (int block = 0; block < (1 << level); ++block) {
        const auto l = block << (st.size() - level);
        const auto r = (block + 1) << (st.size() - level);
        const auto m = l + (r - l) / 2;

        st[level][m] = v[m];
        for (int i = m + 1; i < r; i++)
          st[level][i] = f(st[level][i - 1], v[i]);
        st[level][m - 1] = v[m - 1];
        for (int i = m - 2; i >= l; i--)
          st[level][i] = f(st[level][i + 1], v[i]);
      }
    }
  }

  T query(int l, int r) const {
    if (l > r) return identity;
    if (l == r) return st.back()[l];

    const auto k = log2_floor(l ^ r);
    const auto level = (int)st.size() - 1 - k;
    return f(st[level][l], st[level][r]);
  }
};
```

1.2 Dsu

```
struct DSU {
  vi ps;
  vi size;
  DSU(int N) : ps(N + 1), size(N + 1, 1) { iota(all(ps), 0); }
  int find_set(int x) { return ps[x] == x ? x : ps[x] = find_set(ps[x]); }
  bool same_set(int x, int y) { return find_set(x) == find_set(y); }
  void union_set(int x, int y) {
    if (same_set(x, y)) return;

    int px = find_set(x);
    int py = find_set(y);

    if (size[px] < size[py]) swap(px, py);

    ps[py] = px;
    size[px] += size[py];
  }
};
```

1.3 Ordered Set

If you need an ordered **multi**set you may add an id to each value. Using greater_equal, or less_equal is considered undefined behavior.

- **order_of_key (k)** : Number of items strictly smaller/greater than k .
- **find_by_order(k)** : K-th element in a set (counting from zero).

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set =
  tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
```

1.4 SegTree Point Update (dynamic function)

Answers queries of any monoid operation (i.e. has identity element and is associative)
Build: $O(N)$, Query: $O(\log N)$

```
#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct SegTree {
  using Operation = T (*)(T, T);

  int N;
  vector<T> ns;
  Operation operation;
  T identity;

  SegTree(int n, Operation op = F(a + b), T neutral = T())
    : N(n), ns(2 * N, neutral), operation(op), identity(neutral) {}

  SegTree(const vector<T> &v, Operation op = F(a + b), T neutral = T())
```

```
    : SegTree((int)v.size(), op, neutral) {
    copy(v.begin(), v.end(), ns.begin() + N);

    for (int i = N - 1; i > 0; --i) ns[i] = operation(ns[2 * i], ns[2 * i +
    1]);
  }

  T query(size_t i) const { return ns[i + N]; }

  T query(size_t l, size_t r) const {
    auto a = l + N, b = r + N;
    auto ans = identity;

    while (a <= b) {
      if (a & 1) ans = operation(ans, ns[a++]);
      if (not(b & 1)) ans = operation(ans, ns[b--]);

      a /= 2;
      b /= 2;
    }

    return ans;
  }

  void update(size_t i, T value) { update_set(i, operation(ns[i + N], value));
     }

  void update_set(size_t i, T value) {
    auto a = i + N;

    ns[a] = value;
    while (a >>= 1) ns[a] = operation(ns[2 * a], ns[2 * a + 1]);
  }
};
```

1.5 Segtree Range Max Query Range Max Update

```
template <typename T = ll>
struct SegTree {
  int N;
  T nu, nq;
  vector<T> st, lazy;
  SegTree(const vector<T> &xs)
    : N(len(xs)),
      nu(numeric_limits<T>::min()),
      nq(numeric_limits<T>::min()),
      st(4 * N + 1, nu),
      lazy(4 * N + 1, nu) {
    for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
  }

  void update(int l, int r, T value) { update(1, 0, N - 1, l, r, value); }

  T query(int l, int r) { return query(1, 0, N - 1, l, r); }

  void update(int node, int nl, int nr, int ql, int qr, T v) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return;

    st[node] = max(st[node], v);
    if (ql <= nl and nr <= qr) {
      if (nl < nr) {
        lazy[left(node)] = max(lazy[left(node)], v);
        lazy[right(node)] = max(lazy[right(node)], v);
      }
      return;
    }
    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

    st[node] = max(st[left(node)], st[right(node)]);
  }

  T query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return max(x, y);
  }

  void propagation(int node, int nl, int nr) {
    if (lazy[node] != nu) {
      st[node] = max(st[node], lazy[node]);

      if (nl < nr) {
        lazy[left(node)] = max(lazy[left(node)], lazy[node]);
        lazy[right(node)] = max(lazy[right(node)], lazy[node]);
      }

      lazy[node] = nu;
    }
  }

  int left(int p) { return p << 1; }
  int right(int p) { return (p << 1) + 1; }
  int mid(int l, int r) { return (r - l) / 2 + l; }
};
int main() {
  int n;
  cin >> n;
  vector<array<int, 3>> xs(n);
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < 3; ++j) {
      cin >> xs[i][j];
    }
  }
  vi aux(n, 0);
  SegTree<int> st(aux);
```

```cpp
  for (int i = 0; i < n; ++i) {
    int a = min(i + xs[i][1], n);
    int b = min(i + xs[i][2], n);
    st.update(i, i, st.query(i, i) + xs[i][0]);
    int cur = st.query(i, i);
    st.update(a, b, cur);
  }

  cout << st.query(0, n) << '\n';
}
```

1.6 SegTree Range Min Query Point Assign Update

```cpp
template <typename T = ll>
struct SegTree {
  int n;
  T nu, nq;
  vector<T> st;
  SegTree(const vector<T> &v)
    : n(len(v)), nu(0), nq(numeric_limits<T>::max()), st(n * 4 + 1, nu) {
    for (int i = 0; i < n; ++i) update(i, v[i]);
  }
  void update(int p, T v) { update(1, 0, n - 1, p, v); }
  T query(int l, int r) { return query(1, 0, n - 1, l, r); }

  void update(int node, int nl, int nr, int p, T v) {
    if (p < nl or p > nr) return;

    if (nl == nr) {
      st[node] = v;
      return;
    }

    update(left(node), nl, mid(nl, nr), p, v);
    update(right(node), mid(nl, nr) + 1, nr, p, v);

    st[node] = min(st[left(node)], st[right(node)]);
  }

  T query(int node, int nl, int nr, int ql, int qr) {
    if (ql <= nl and qr >= nr) return st[node];
    if (nl > qr or nr < ql) return nq;
    if (nl == nr) return st[node];

    return min(query(left(node), nl, mid(nl, nr), ql, qr),
               query(right(node), mid(nl, nr) + 1, nr, ql, qr));
  }

  int left(int p) { return p << 1; }
  int right(int p) { return (p << 1) + 1; }
  int mid(int l, int r) { return (r - l) / 2 + l; }
};
```

1.7 SegTree Range Min Query Range Sum Update

```cpp
template <typename t = ll>
struct SegTree {
  int n;
  t nu;
  t nq;
  vector<t> st, lazy;
  SegTree(const vector<t> &xs)
    : n(len(xs)),
      nu(0),
      nq(numeric_limits<t>::max()),
      st(4 * n, nu),
      lazy(4 * n, nu) {
    for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
  }

  SegTree(int n) : n(n), st(4 * n, nu), lazy(4 * n, nu) {}

  void update(int l, int r, ll value) { update(1, 0, n - 1, l, r, value); }

  t query(int l, int r) { return query(1, 0, n - 1, l, r); }

  void update(int node, int nl, int nr, int ql, int qr, ll v) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return;

    if (ql <= nl and nr <= qr) {
      st[node] += (nr - nl + 1) * v;

      if (nl < nr) {
        lazy[left(node)] += v;
        lazy[right(node)] += v;
      }

      return;
    }

    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

    st[node] = min(st[left(node)], st[right(node)]);
  }

  t query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    t x = query(left(node), nl, mid(nl, nr), ql, qr);
    t y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return min(x, y);
  }

  void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
      st[node] += lazy[node];

      if (nl < nr) {
```

```
        lazy[left(node)] += lazy[node];
        lazy[right(node)] += lazy[node];
      }

      lazy[node] = nu;
    }
  }

  int left(int p) { return p << 1; }
  int right(int p) { return (p << 1) + 1; }
  int mid(int l, int r) { return (r - l) / 2 + l; }
};
```

1.8 SegTree Range Sum Query Range Sum Update

```
template <typename T = ll>
struct SegTree {
  int N;
  vector<T> st, lazy;
  T nu = 0;
  T nq = 0;
  SegTree(const vector<T> &xs) : N(len(xs)), st(4 * N, nu), lazy(4 * N, nu) {
    for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
  }

  SegTree(int n) : N(n), st(4 * N, nu), lazy(4 * N, nu) {}

  void update(int l, int r, ll value) { update(1, 0, N - 1, l, r, value); }

  T query(int l, int r) { return query(1, 0, N - 1, l, r); }

  void update(int node, int nl, int nr, int ql, int qr, ll v) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return;

    if (ql <= nl and nr <= qr) {
      st[node] += (nr - nl + 1) * v;

      if (nl < nr) {
        lazy[left(node)] += v;
        lazy[right(node)] += v;
      }

      return;
    }

    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

    st[node] = st[left(node)] + st[right(node)];
  }

  T query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;
    if (ql <= nl and nr <= qr) return st[node];

    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return x + y;
  }

  void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
      st[node] += (nr - nl + 1) * lazy[node];

      if (nl < nr) {
        lazy[left(node)] += lazy[node];
        lazy[right(node)] += lazy[node];
      }

      lazy[node] = nu;
    }
  }

  int left(int p) { return p << 1; }
  int right(int p) { return (p << 1) + 1; }
  int mid(int l, int r) { return (r - l) / 2 + l; }
};
```

1.9 Sparse Table Range Min Query

Build: O(NlogN), Query: O(1)

```
int fastlog2(ll x) {
  ull i = x;
  return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
}
template <typename T>
class SparseTable {
 public:
  int N;
  int K;
  vector<vector<T>> st;
  SparseTable(vector<T> vs)
    : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector<T>(N + 1)) {
    copy(vs.begin(), vs.end(), st[0].begin());

    for (int i = 1; i <= K; ++i)
      for (int j = 0; j + (1 << i) <= N; ++j)
        st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
  }
  T RMQ(int l, int r) {  // [l, r], 0 indexed
    int i = fastlog2(r - l + 1);
    return min(st[i][l], st[i][r - (1 << i) + 1]);
  }
};
```

2 Dynamic programming

2.1 Edit Distance

$O(N * M)$

```
int edit_distance(const string &a, const string &b) {
  int n = a.size();
  int m = b.size();
  vector<vi> dp(n + 1, vi(m + 1, 0));

  int ADD = 1, DEL = 1, CHG = 1;
  for (int i = 0; i <= n; ++i) {
    dp[i][0] = i * DEL;
  }
  for (int i = 1; i <= m; ++i) {
    dp[0][i] = ADD * i;
  }

  for (int i = 1; i <= n; ++i) {
    for (int j = 1; j <= m; ++j) {
      int add = dp[i][j - 1] + ADD;
      int del = dp[i - 1][j] + DEL;
      int chg = dp[i - 1][j - 1] + (a[i - 1] == b[j - 1] ? 0 : 1) * CHG;
      dp[i][j] = min({add, del, chg});
    }
  }

  return dp[n][m];
}
```

2.2 Kadane

Find the maximum subarray sum in a given a rray.

```
int kadane(const vi &as) {
  vi s(len(as));
  s[0] = as[0];

  for (int i = 1; i < len(as); ++i) s[i] = max(as[i], s[i - 1] + as[i]);

  return *max_element(all(s));
}
```

2.3 Knapsack (value)

Finds the maximum points possible

```
const int MAXN{2010}, MAXM{2010};

ll st[MAXN][MAXM];

ll dp(int i, int m, int M, const vii &cs) {
  if (i < 0) return 0;

  if (st[i][m] != -1) return st[i][m];

  auto res = dp(i - 1, m, M, cs);

  auto [w, v] = cs[i];

  if (w <= m) res = max(res, dp(i - 1, m - w, M, cs) + v);

  st[i][m] = res;
  return res;
}

ll knapsack(int M, const vii &cs) {
  memset(st, -1, sizeof st);

  return dp((int)cs.size() - 1, M, M, cs);
}
```

2.4 Knapsack (elements)

Finds the maximum posisble points carry and which elements to achieve it

```
const int MAXN{2010}, MAXM{2010};
ll st[MAXN][MAXM];
char ps[MAXN][MAXM];

pair<ll, vi> knapsack(int M, const vii &cs) {
  int N = len(cs) - 1;

  for (int i = 0; i <= N; ++i) st[i][0] = 0;

  for (int m = 0; m <= M; ++m) st[0][m] = 0;

  for (int i = 1; i <= N; ++i) {
    for (int m = 1; m <= M; ++m) {
      st[i][m] = st[i - 1][m];
      ps[i][m] = 0;
      auto [w, v] = cs[i];

      if (w <= m and st[i - 1][m - w] + v > st[i][m]) {
        st[i][m] = st[i - 1][m - w] + v;
        ps[i][m] = 1;
      }
    }
  }

  int m = M;
  vi is;
  for (int i = N; i >= 1; --i) {
    if (ps[i][m]) {
      is.push_back(i);
      m -= cs[i].first;
    }
  }

  reverse(all(is));

  // max value, items
  return {st[N][M], is};
}
```

2.5 Longest Increasing Subsequence (LIS)

Finds the lenght of the longest subsequence in

$$O(n \log n)$$

.

```
int LIS(const vi& as) {
  const ll oo = 1e18;
  int n = len(as);
  vll lis(n + 1, oo);
  lis[0] = -oo;

  auto ans = 0;

  for (int i = 0; i < n; ++i) {
    auto it = lower_bound(all(lis), as[i]);
    auto pos = (int)(it - lis.begin());

    ans = max(ans, pos);
    lis[pos] = as[i];
  }

  return ans;
}
```

2.6 Money Sum (Bottom Up)

Find every possible sum using the given values only once.

```
set<int> money_sum(const vi &xs) {
  using vc = vector<char>;
  using vvc = vector<vc>;
  int _m = accumulate(all(xs), 0);
  int _n = xs.size();
  vvc _dp(_n + 1, vc(_m + 1, 0));
  set<int> _ans;
  _dp[0][xs[0]] = 1;
  for (int i = 1; i < _n; ++i) {
    for (int j = 0; j <= _m; ++j) {
      if (j == 0 or _dp[i - 1][j]) {
        _dp[i][j + xs[i]] = 1;
        _dp[i][j] = 1;
      }
    }
  }

  for (int i = 0; i < _n; ++i)
    for (int j = 0; j <= _m; ++j)
      if (_dp[i][j]) _ans.insert(j);
  return _ans;
}
```

2.7 Travelling Salesman Problem

```
using vi = vector<int>;
vector<vi> dist;
vector<vi> memo;
/* O ( N^2 * 2^N )*/
int tsp(int i, int mask, int N) {
  if (mask == (1 << N) - 1) return dist[i][0];
  if (memo[i][mask] != -1) return memo[i][mask];
  int ans = INT_MAX << 1;
  for (int j = 0; j < N; ++j) {
    if (mask & (1 << j)) continue;
    auto t = tsp(j, mask | (1 << j), N) + dist[i][j];
    ans = min(ans, t);
  }
  return memo[i][mask] = ans;
}
```

3 Geometry

3.1 Convex Hull

Given a set of points find the smallest convex polygon that contains all the given points.
Time: $O(N \log N)$
By default it removes the collinear points, set the boolean to true if you don't want that

```
struct pt {
  double x, y;
  int id;
};

int orientation(pt a, pt b, pt c) {
  double v = a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y - b.y);
  if (v < 0) return -1;  // clockwise
  if (v > 0) return +1;  // counter-clockwise
  return 0;
}

bool cw(pt a, pt b, pt c, bool include_collinear) {
  int o = orientation(a, b, c);
  return o < 0 || (include_collinear && o == 0);
}
bool collinear(pt a, pt b, pt c) { return orientation(a, b, c) == 0; }

void convex_hull(vector<pt>& pts, bool include_collinear = false) {
  pt p0 = *min_element(all(pts), [](pt a, pt b) {
    return make_pair(a.y, a.x) < make_pair(b.y, b.x);
  });
  sort(all(pts), [&p0](const pt& a, const pt& b) {
    int o = orientation(p0, a, b);
    if (o == 0)
      return (p0.x - a.x) * (p0.x - a.x) + (p0.y - a.y) * (p0.y - a.y) <
             (p0.x - b.x) * (p0.x - b.x) + (p0.y - b.y) * (p0.y - b.y);
    return o < 0;
  });
  if (include_collinear) {
    int i = len(pts) - 1;
    while (i >= 0 && collinear(p0, pts[i], pts.back())) i--;
    reverse(pts.begin() + i + 1, pts.end());
  }

  vector<pt> st;
```

```
  for (int i = 0; i < len(pts); i++) {
    while (st.size() > 1 &&
           !cw(st[len(st) - 2], st.back(), pts[i], include_collinear))
      st.pop_back();
    st.push_back(pts[i]);
  }

  pts = st;
}
```

3.2 Determinant

```
#include "Point.cpp"

template <typename T>
T D(const Point<T> &P, const Point<T> &Q, const Point<T> &R) {
  return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
         (R.x * Q.y + R.y * P.x + Q.x * P.y);
}
```

3.3 Equals

```
template <typename T>
bool equals(T a, T b) {
  const double EPS{1e-9};
  if (is_floating_point<T>::value)
    return fabs(a - b) < EPS;
  else
    return a == b;
}
```

3.4 Line

```
#include <bits/stdc++.h>

#include "point-struct-and-utils.cpp"
using namespace std;

struct line {
  ld a, b, c;
};

// the answer is stored in the third parameter (pass by reference)
void pointsToLine(const point &p1, const point &p2, line &l) {
  if (fabs(p1.x - p2.x) < EPS)
    // vertical line
    l = {1.0, 0.0, -p1.x};
  // default values
  else
    l = {-(ld)(p1.y - p2.y) / (p1.x - p2.x), 1.0, -(ld)(l.a * p1.x) - p1.y};
}
```

3.5 Point Struct And Utils (2d)

```
#include <bits/stdc++.h>
using namespace std;
using ld = long double;

struct point {
  ld x, y;
  int id;
  point(ld x = 0.0, ld y = 0.0, int id = -1) : x(x), y(y), id(id) {}

  point& operator+=(const point& t) {
    x += t.x;
    y += t.y;
    return *this;
  }
  point& operator-=(const point& t) {
    x -= t.x;
    y -= t.y;
    return *this;
  }
  point& operator*=(ld t) {
    x *= t;
    y *= t;
    return *this;
  }
  point& operator/=(ld t) {
    x /= t;
    y /= t;
    return *this;
  }
  point operator+(const point& t) const { return point(*this) += t; }
  point operator-(const point& t) const { return point(*this) -= t; }
  point operator*(ld t) const { return point(*this) *= t; }
  point operator/(ld t) const { return point(*this) /= t; }
};

ld dot(point& a, point& b) { return a.x * b.x + a.y * b.y; }

ld norm(point& a) { return dot(a, a); }

ld abs(point a) { return sqrt(norm(a)); }

ld proj(point a, point b) { return dot(a, b) / abs(b); }

ld angle(point a, point b) { return acos(dot(a, b) / abs(a) / abs(b)); }

ld cross(point a, point b) { return a.x * b.y - a.y * b.x; }
```

3.6 Segment

```
#include "Line.cpp"
#include "Point.cpp"
#include "equals.cpp"

template <typename T>
struct segment {
  Point<T> A, B;

  bool contains(const Point<T> &P) const;

  Point<T> closest(const Point<T> &p) const;
```

```
};

template <typename T>
bool segment<T>::contains(const Point<T> &P) const {
  // verifica se P áest contido na reta
  double dAB = Point<T>::dist(A, B), dAP = Point<T>::dist(A, P),
         dPB = Point<T>::dist(P, B);

  return equals(dAP + dPB, dAB);
}

template <typename T>
Point<T> segment<T>::closest(const Point<T> &P) const {
  Line<T> R(A, B);
  auto Q = R.closest(P);

  if (this->contains(Q)) return Q;

  auto distA = Point<T>::dist(P, A);
  auto distB = Point<T>::dist(P, B);

  if (distA <= distB)
    return A;
  else
    return B;
}
```

4 Graphs

4.1 2 SAT

```
struct SAT2 {
  ll n;
  vll2d adj, adj_t;
  vc used;
  vll order, comp;
  vc assignment;
  bool solvable;
  SAT2(ll _n)
    : n(2 * _n),
      adj(n),
      adj_t(n),
      used(n),
      order(n),
      comp(n, -1),
      assignment(n / 2) {}
  void dfs1(int v) {
    used[v] = true;
    for (int u : adj[v]) {
      if (!used[u]) dfs1(u);
    }
    order.push_back(v);
  }

  void dfs2(int v, int cl) {
    comp[v] = cl;
    for (int u : adj_t[v]) {
      if (comp[u] == -1) dfs2(u, cl);
    }
  }

  bool solve_2SAT() {
    // find and label each SCC
    for (int i = 0; i < n; ++i) {
      if (!used[i]) dfs1(i);
    }
    reverse(all(order));
    ll j = 0;
    for (auto &v : order) {
      if (comp[v] == -1) dfs2(v, j++);
    }

    assignment.assign(n / 2, false);
    for (int i = 0; i < n; i += 2) {
      // x and !x belong to the same SCC
      if (comp[i] == comp[i + 1]) {
        solvable = false;
        return false;
      }

      assignment[i / 2] = comp[i] > comp[i + 1];
    }
    solvable = true;
    return true;
  }

  void add_disjunction(int a, bool na, int b, bool nb) {
    a = (2 * a) ^ na;
    b = (2 * b) ^ nb;
    int neg_a = a ^ 1;
    int neg_b = b ^ 1;
    adj[neg_a].push_back(b);
    adj[neg_b].push_back(a);
    adj_t[b].push_back(neg_a);
    adj_t[a].push_back(neg_b);
  }
};
```

4.2 SCC (struct)

Able to find the component of each node and the total of SCC in $O(V * E)$ and build the SCC graph ($O(V * E)$).

```
struct SCC {
  ll N;
  int totscc;
  vll2d adj, tadj;
  vll todo, comps, comp;
  vector<set<ll>> sccadj;
  vchar vis;
  SCC(ll _N)
    : N(_N), totscc(0), adj(_N), tadj(_N), comp(_N, -1), sccadj(_N), vis(_N)
    {}

  void add_edge(ll x, ll y) { adj[x].eb(y), tadj[y].eb(x); }
```

```
  void dfs(ll x) {
    vis[x] = 1;
    for (auto &y : adj[x])
      if (!vis[y]) dfs(y);
    todo.pb(x);
  }
  void dfs2(ll x, ll v) {
    comp[x] = v;
    for (auto &y : tadj[x])
      if (comp[y] == -1) dfs2(y, v);
  }
  void gen() {
    for (ll i = 0; i < N; ++i)
      if (!vis[i]) dfs(i);
    reverse(all(todo));
    for (auto &x : todo)
      if (comp[x] == -1) {
        dfs2(x, x);
        comps.pb(x);
        totscc++;
      }
  }

  void genSCCGraph() {
    for (ll i = 0; i < N; ++i) {
      for (auto &j : adj[i]) {
        if (comp[i] != comp[j]) {
          sccadj[comp[i]].insert(comp[j]);
        }
      }
    }
  }
};
```

4.3 Bellman Ford

Find shortest path from a single source to all other nodes. Can detect negative cycles.
Time: $O(V * E)$

```
bool bellman_ford(const vector<vector<pair<int, ll>>> &g, int s,
                  vector<ll> &dist) {
  int n = (int)g.size();
  dist.assign(n, LLONG_MAX);

  vector<int> count(n);
  vector<char> in_queue(n);
  queue<int> q;

  dist[s] = 0;
  q.push(s);
  in_queue[s] = true;

  while (not q.empty()) {
    int cur = q.front();
    q.pop();
    in_queue[cur] = false;

    for (auto [to, w] : g[cur]) {
      if (dist[cur] + w < dist[to]) {
        dist[to] = dist[cur] + w;
        if (not in_queue[to]) {
          q.push(to);
          in_queue[to] = true;
          count[to]++;
          if (count[to] > n) return false;
        }
      }
    }
  }

  return true;
}
```

4.4 Binary Lifting

$far[h][i]$ = the node that is 2^h distance from node i
Build : $O(N * \log N)$
sometimes is useful invert the order of loops

```
const int maxlog = 20;
int far[maxlog + 1][n + 1];
int n;
for (int h = 1; h <= maxlog; h++) {
  for (int i = 1; i <= n; i++) {
    far[h][i] = far[h - 1][far[h - 1][i]];
  }
}
```

4.5 Check Bipartitie

$O(V)$

```
bool checkBipartite(const ll n, const vector<vll> &adj) {
  ll s = 0;
  queue<ll> q;
  q.push(s);
  vll color(n, INF);
  color[s] = 0;
  bool isBipartite = true;
  while (!q.empty() && isBipartite) {
    ll u = q.front();
    q.pop();
    for (auto &v : adj[u]) {
      if (color[v] == INF) {
        color[v] = 1 - color[u];
        q.push(v);
      } else if (color[v] == color[u]) {
        return false;
      }
    }
  }
  return true;
}
```

4.6 Dijkstra

Finds the minimum distance from s to every other node in

$$O(E * \log E)$$

time.

```
vll dijkstra(const vector<vector<pll>> &g, int n, int s) {
  priority_queue<pll, vector<pll>, greater<pll>> pq;
  vll dist(n + 1, oo);
  vector<char> vis(n + 1);
  pq.emplace(0, s);
  dist[s] = 0;
  while (!pq.empty()) {
    auto [d1, v] = pq.top();
    pq.pop();
    if (vis[v]) continue;
    vis[v] = true;

    for (auto [d2, u] : g[v]) {
      if (dist[u] > d1 + d2) {
        dist[u] = d1 + d2;
        pq.emplace(dist[u], u);
      }
    }
  }
  return dist;
}
```

4.7 Euler Path

Find a path that visits every edge exactly once.
Time: $O(E)$
graphs with sets are undirected, graphs with vectors are directed

```
// Directed Edges
vector<int> euler_cycle(vector<vector<int>> &g, int u) {
  vector<int> res;

  stack<int> st;
  st.push(u);
  while (!st.empty()) {
    auto cur = st.top();
    if (g[cur].empty()) {
      res.push_back(cur);
      st.pop();
    } else {
      auto next = g[cur].back();
      st.push(next);

      g[cur].pop_back();
    }
  }

  for (auto &x : g)
    if (!x.empty()) return {};

  return res;
}

// Directed Edges
vector<int> euler_path(vector<vector<int>> &g, int first) {
  {
    int n = (int)g.size();
    vector<int> in(n), out(n);
    for (int i = 0; i < n; i++)
      for (auto x : g[i]) in[x]++, out[i]++;

    int a = 0, b = 0, c = 0;
    for (int i = 0; i < n; i++)
      if (in[i] == out[i])
        c++;
      else if (in[i] - out[i] == 1)
        b++;
      else if (in[i] - out[i] == -1)
        a++;

    if (c != n - 2 or a != 1 or b != 1) return {};
  }

  auto res = euler_cycle(g, first);
  if (res.empty()) return res;

  reverse(all(res));
  return res;
}

// Undirected Edges
vector<int> euler_cycle(vector<set<int>> &g, int u) {
  vector<int> res;

  stack<int> st;
  st.push(u);
  while (!st.empty()) {
    auto cur = st.top();
    if (g[cur].empty()) {
      res.push_back(cur);
      st.pop();
    } else {
      auto next = *g[cur].begin();
      st.push(next);

      g[cur].erase(next);
      g[next].erase(cur);
    }
  }

  for (auto &x : g)
    if (!x.empty()) return {};

  return res;
}

// Undirected edges
vector<int> euler_path(vector<set<int>> &g, int first) {
  int n = (int)g.size();
```

```
  int v1 = -1, v2 = -1;
  {
    bool bad = false;
    for (int i = 0; i < n; i++)
      if (g[i].size() & 1) {
        if (v1 == -1)
          v1 = i;
        else if (v2 == -1)
          v2 = i;
        else
          bad = true;
      }

    if (bad or (v1 != -1 and v2 == -1)) return {};
  }

  if (v1 != -1) {
    // insert cycle
    g[v1].insert(v2);
    g[v2].insert(v1);
  }

  auto res = euler_cycle(g, first);
  if (res.empty()) return res;

  if (v1 != -1) {
    for (int i = 0; i + 1 < (int)res.size(); i++) {
      if ((res[i] == v1 and res[i + 1] == v2) ||
          (res[i] == v2 and res[i + 1] == v1)) {
        vector<int> res2;
        for (int j = i + 1; j < (int)res.size(); j++) res2.push_back(res[j]);
        for (int j = 1; j <= i; j++) res2.push_back(res[j]);
        res = res2;
        break;
      }
    }
  }

  reverse(all(res));
  return res;
}
```

4.8 Floyd Warshall

Simply finds the minimal distance for each node to every other node. $O(V^3)$

```
vector<vll> floyd_warshall(const vector<vll> &adj, ll n) {
  auto dist = adj;

  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
      for (int k = 0; k < n; ++k) {
        dist[j][k] = min(dist[j][k], dist[j][i] + dist[i][k]);
      }
    }
  }
  return dist;
}
```

4.9 Graph Cycle

Detects if a graph contains a cycle. If path parameter is not null, it will contain the cycle if one exists.
Time: $O(V + E)$

```
bool has_cycle(const vector<vector<int>> &g, int s, vector<char> &vis,
               vector<char> &in_path, vector<int> *path = nullptr) {
  vis[s] = in_path[s] = 1;
  if (path != nullptr) path->push_back(s);
  for (auto x : g[s]) {
    if (!vis[x] && has_cycle(g, x, vis, in_path, path))
      return true;
    else if (in_path[x]) {
      if (path != nullptr) path->push_back(x);
      return true;
    }
  }
  in_path[s] = 0;
  if (path != nullptr) path->pop_back();
  return false;
}
```

4.10 Kruskal

Find the minimum spanning tree of a graph.
Time: $O(E \log E)$
can be used to find the maximum spanning tree by changing the comparison operator in the sort

```
struct UFDS {
  vector<int> ps, sz;
  int components;

  UFDS(int n) : ps(n + 1), sz(n + 1, 1), components(n) { iota(all(ps), 0); }

  int find_set(int x) { return (x == ps[x] ? x : (ps[x] = find_set(ps[x]))); }

  bool same_set(int x, int y) { return find_set(x) == find_set(y); }

  void union_set(int x, int y) {
    x = find_set(x);
    y = find_set(y);

    if (x == y) return;

    if (sz[x] < sz[y]) swap(x, y);

    ps[y] = x;
    sz[x] += sz[y];

    components--;
  }
};

vector<tuple<ll, int, int>> kruskal(int n, vector<tuple<ll, int, int>> &edges)
    {
  UFDS ufds(n);
  vector<tuple<ll, int, int>> ans;
```

```
  sort(all(edges));
  for (auto [a, b, c] : edges) {
    if (ufds.same_set(b, c)) continue;

    ans.emplace_back(a, b, c);
    ufds.union_set(b, c);
  }

  return ans;
}
```

4.11 Lowest Common Ancestor

Given two nodes find the lowest common ancestor of both.
Build : $O(V)$, Query: $O(1)$

```
int fastlog2(ll x) {
  ull i = x;
  return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
}
template <typename T>
class SparseTable {
 public:
  int N;
  int K;
  vector<vector<T>> st;
  SparseTable(vector<T> vs)
    : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector<T>(N + 1)) {
    copy(vs.begin(), vs.end(), st[0].begin());

    for (int i = 1; i <= K; ++i)
      for (int j = 0; j + (1 << i) <= N; ++j)
        st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
  }
  SparseTable() {}
  T RMQ(int l, int r) {
    int i = fastlog2(r - l + 1);
    return min(st[i][l], st[i][r - (1 << i) + 1]);
  }
};
class LCA {
 public:
  int p;
  int n;
  vi first;
  vector<char> visited;
  vi vertices;
  vi height;
  SparseTable<int> st;

  LCA(const vector<vi> &g)
    : p(0), n((int)g.size()), first(n + 1), visited(n + 1, 0), height(n + 1) {
    build_dfs(g, 1, 1);
    st = SparseTable<int>(vertices);
  }

  void build_dfs(const vector<vi> &g, int u, int hi) {
    visited[u] = true;
    height[u] = hi;
    first[u] = vertices.size();
    vertices.push_back(u);
    for (auto uv : g[u]) {
      if (!visited[uv]) {
        build_dfs(g, uv, hi + 1);
        vertices.push_back(u);
      }
    }
  }

  int lca(int a, int b) {
    int l = min(first[a], first[b]);
    int r = max(first[a], first[b]);
    return st.RMQ(l, r);
  }
};
```

4.12 Tree Maximum Distance

Returns the maximum distance from every node to any other node in the tree. $O(6V) = O(V)$

```
pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
  // O(V)
  // 0 indexed
  ll mostDistantNode = root;
  ll nodeDistance = 0;
  queue<pll> q;
  vector<char> vis(n);
  q.emplace(root, 0);
  vis[root] = true;
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
    if (dist > nodeDistance) {
      nodeDistance = dist;
      mostDistantNode = node;
    }
    for (auto u : adj[node]) {
      if (!vis[u]) {
        vis[u] = true;
        q.emplace(u, dist + 1);
      }
    }
  }
  return {mostDistantNode, nodeDistance};
}

ll twoNodesDist(const vector<vll> &adj, ll n, ll a, ll b) {
  queue<pll> q;
  vector<char> vis(n);
  q.emplace(a, 0);
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
    if (node == b) return dist;
    for (auto u : adj[node]) {
      if (!vis[u]) {
```

```
        vis[u] = true;
        q.emplace(u, dist + 1);
      }
    }
  }
  return -1;
}

tuple<ll, ll, ll> tree_diameter(const vector<vll> &adj, ll n) {
  // returns two points of the diameter and the diameter itself
  auto [node1, dist1] = mostDistantFrom(adj, n, 0);      // O(V)
  auto [node2, dist2] = mostDistantFrom(adj, n, node1);  // O(V)
  auto diameter = twoNodesDist(adj, n, node1, node2);    // O(V)
  return make_tuple(node1, node2, diameter);
}

vll everyDistanceFromNode(const vector<vll> &adj, ll n, ll root) {
  // Single Source Shortest Path, from a given root
  queue<pair<ll, ll>> q;
  vll ans(n, -1);
  ans[root] = 0;
  q.emplace(root, 0);
  while (!q.empty()) {
    auto [u, d] = q.front();
    q.pop();

    for (auto w : adj[u]) {
      if (ans[w] != -1) continue;
      ans[w] = d + 1;
      q.emplace(w, d + 1);
    }
  }
  return ans;
}

vll maxDistances(const vector<vll> &adj, ll n) {
  auto [node1, node2, diameter] = tree_diameter(adj, n);   // O(3V)
  auto distances1 = everyDistanceFromNode(adj, n, node1);  // O(V)
  auto distances2 = everyDistanceFromNode(adj, n, node2);  // O(V)
  vll ans(n);
  for (int i = 0; i < n; ++i)
    ans[i] = max(distances1[i], distances2[i]);  // O(V)
  return ans;
}
```

4.13 Small to Large

Answer queries of the form "How many vertices in the subtree of vertex v have property P?"
Build: $O(N)$, Query: $O(N \log N)$

```
struct SmallToLarge {
  vector<vector<int>> tree, vis_childs;
  vector<int> sizes, values, ans;
  set<int> cnt;

  SmallToLarge(vector<vector<int>> &&g, vector<int> &&v)
    : tree(g), vis_childs(g.size()), sizes(g.size()), values(v), ans(g.size())
     {
    update_sizes(0);
  }

  inline void add_value(int u) { cnt.insert(values[u]); }

  inline void remove_value(int u) { cnt.erase(values[u]); }

  inline void update_ans(int u) { ans[u] = (int)cnt.size(); }

  void dfs(int u, int p = -1, bool keep = true) {
    int mx = -1;
    for (auto x : tree[u]) {
      if (x == p) continue;

      if (mx == -1 or sizes[mx] < sizes[x]) mx = x;
    }

    for (auto x : tree[u]) {
      if (x != p and x != mx) dfs(x, u, false);
    }

    if (mx != -1) {
      dfs(mx, u, true);
      swap(vis_childs[u], vis_childs[mx]);
    }

    vis_childs[u].push_back(u);
    add_value(u);

    for (auto x : tree[u]) {
      if (x != p and x != mx) {
        for (auto y : vis_childs[x]) {
          add_value(y);
          vis_childs[u].push_back(y);
        }
      }
    }

    update_ans(u);

    if (!keep) {
      for (auto x : vis_childs[u]) remove_value(x);
    }
  }

  void update_sizes(int u, int p = -1) {
    sizes[u] = 1;
    for (auto x : tree[u]) {
      if (x != p) {
        update_sizes(x, u);
        sizes[u] += sizes[x];
      }
    }
  }
};
```

4.14 Topological Sorting

Assumes that :

- vertices index $[0, n-1]$
- is a DAG (else it returns an empty vector)

$O(V)$

```
enum class state { not_visited, processing, done };
bool dfs(const vector<vll> &adj, ll s, vector<state> &states, vll &order) {
  states[s] = state::processing;
  for (auto &v : adj[s]) {
    if (states[v] == state::not_visited) {
      if (not dfs(adj, v, states, order)) return false;
    } else if (states[v] == state::processing)
      return false;
  }
  states[s] = state::done;
  order.pb(s);
  return true;
}
vll topologicalSorting(const vector<vll> &adj) {
  ll n = len(adj);
  vll order;
  vector<state> states(n, state::not_visited);
  for (int i = 0; i < n; ++i) {
    if (states[i] == state::not_visited) {
      if (not dfs(adj, i, states, order)) return {};
    }
  }
  reverse(all(order));
  return order;
}
```

4.15 Tree Diameter

Finds the lenght of the diameter of the tree in $O(V)$, it's easy to recover the nodes at the point of the diameter .

```
pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
  // 0 indexed
  ll mostDistantNode = root;
  ll nodeDistance = 0;
  queue<pll> q;
  vector<char> vis(n);
  q.emplace(root, 0);
  vis[root] = true;
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
    if (dist > nodeDistance) {
      nodeDistance = dist;
      mostDistantNode = node;
    }
    for (auto u : adj[node]) {
      if (!vis[u]) {
        vis[u] = true;
        q.emplace(u, dist + 1);
      }
    }
  }
  return {mostDistantNode, nodeDistance};
}
ll twoNodesDist(const vector<vll> &adj, ll n, ll a, ll b) {
  // 0 indexed
  queue<pll> q;
  vector<char> vis(n);
  q.emplace(a, 0);
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
    if (node == b) {
      return dist;
    }
    for (auto u : adj[node]) {
      if (!vis[u]) {
        vis[u] = true;
        q.emplace(u, dist + 1);
      }
    }
  }
  return -1;
}
ll tree_diameter(const vector<vll> &adj, ll n) {
  // 0 indexed !!!
  auto [node1, dist1] = mostDistantFrom(adj, n, 0);      // O(V)
  auto [node2, dist2] = mostDistantFrom(adj, n, node1);  // O(V)
  auto diameter = twoNodesDist(adj, n, node1, node2);    // O(V)
  return diameter;
}
```

5 Math

5.1 GCD (with factorization)

$O(\sqrt{n})$ due to factorization.

```
ll gcd_with_factorization(ll a, ll b) {
  map<ll, ll> fa = factorization(a);
  map<ll, ll> fb = factorization(b);
  ll ans = 1;
  for (auto fai : fa) {
    ll k = min(fai.second, fb[fai.first]);
    while (k--) ans *= fai.first;
  }
  return ans;
}
```

5.2 GCD

```
ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
```

5.3 LCM (with factorization)

$O(\sqrt{n})$ due to factorization.

```
ll lcm_with_factorization(ll a, ll b) {
  map<ll, ll> fa = factorization(a);
  map<ll, ll> fb = factorization(b);
  ll ans = 1;
  for (auto fai : fa) {
    ll k = max(fai.second, fb[fai.first]);
    while (k--) ans *= fai.first;
  }
  return ans;
}
```

5.4 LCM

```
ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
ll lcm(ll a, ll b) { return a / gcd(a, b) * b; }
```

5.5 Arithmetic Progression Sum

- s : first term
- d : common difference
- n : number of terms

```
ll arithmeticProgressionSum(ll s, ll d, ll n) {
  return (s + (s + d * (n - 1))) * n / 2ll;
}
```

5.6 Binomial MOD

Precompute every factorial until $maxn$ ($O(maxn)$) allowing to answer the $\binom{n}{k}$ in $O(\log mod)$ time, due to the fastpow. Note that it needs $O(maxn)$ in memory.

```
const ll maxn = 1e6;
vll fats(maxn + 1, -1);
const ll mod = 1e9 + 7;
void precompute() {
  fats[0] = 1;
  for (ll i = 1; i <= maxn; i++) {
    fats[i] = (fats[i - 1] * i) % mod;
  }
}

ll fpow(ll a, ll n, ll mod = LLONG_MAX) {
  if (n == 0) return 1;
  if (n == 1) return a;
  ll x = fpow(a, n / 2, mod) % mod;
  return ((x * x) % mod * (n & 1 ? a : 1ll)) % mod;
}

ll binommod(ll n, ll k) {
  ll upper = fats[n];
  ll lower = (fats[k] * fats[n - k]) % mod;
  return (upper * fpow(lower, mod - 2, mod)) % mod;
}
```

5.7 Binomial

O(nm) time, O(m) space
Equal to n choose k

```
ll binom(ll n, ll k) {
  if (k > n) return 0;
  vll dp(k + 1, 0);
  dp[0] = 1;
  for (ll i = 1; i <= n; i++)
    for (ll j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
  return dp[k];
}
```

5.8 Euler phi $\varphi(n)$ (in range)

Computes the number of positive integers less than n that are coprimes with n, in the range $[1, n]$, in $O(N \log N)$.

```
const int MAX = 1e6;
vi range_phi(int n) {
  bitset<MAX> sieve;
  vi phi(n + 1);

  iota(phi.begin(), phi.end(), 0);
  sieve.set();

  for (int p = 2; p <= n; p += 2) phi[p] /= 2;
  for (int p = 3; p <= n; p += 2) {
    if (sieve[p]) {
      for (int j = p; j <= n; j += p) {
        sieve[j] = false;
        phi[j] /= p;
        phi[j] *= (p - 1);
      }
    }
  }

  return phi;
}
```

5.9 Euler phi $\varphi(n)$

Computes the number of positive integers less than n that are coprimes with n, in $O(\sqrt{N})$.

```
int phi(int n) {
  if (n == 1) return 1;

  auto fs = factorization(n);  // a vctor of pair or a map
  auto res = n;

  for (auto [p, k] : fs) {
    res /= p;
    res *= (p - 1);
  }

  return res;
}
```

5.10 Factorial Factorization

Computes the factorization of $n!$ in $\pi(N) * \log n$

```
// O(logN)
ll E(ll n, ll p) {
  ll k = 0, b = p;
  while (b <= n) {
    k += n / b;
    b *= p;
  }
  return k;
}

// O(pi(N)*logN)
map<ll, ll> factorial_factorization(ll n, const vll &primes) {
  map<ll, ll> fs;
  for (const auto &p : primes) {
    if (p > n) break;
    fs[p] = E(n, p);
  }
  return fs;
}
```

5.11 Factorial

```
const ll MAX = 18;
vll fv(MAX, -1);
ll factorial(ll n) {
  if (fv[n] != -1) return fv[n];
  if (n == 0) return 1;
  return n * factorial(n - 1);
}
```

5.12 Factorization (Pollard Rho)

Factorizes a number into its prime factors in $O(n^{(\frac{1}{4})} * \log(n))$.

```
ll mul(ll a, ll b, ll m) {
  ll ret = a * b - (ll)((ld)1 / m * a * b + 0.5) * m;
  return ret < 0 ? ret + m : ret;
}

ll pow(ll a, ll b, ll m) {
  ll ans = 1;
  for (; b > 0; b /= 2ll, a = mul(a, a, m)) {
    if (b % 2ll == 1) ans = mul(ans, a, m);
  }
  return ans;
}

bool prime(ll n) {
  if (n < 2) return 0;
  if (n <= 3) return 1;
  if (n % 2 == 0) return 0;

  ll r = __builtin_ctzll(n - 1), d = n >> r;
  for (int a : {2, 325, 9375, 28178, 450775, 9780504, 795265022}) {
    ll x = pow(a, d, n);
    if (x == 1 or x == n - 1 or a % n == 0) continue;

    for (int j = 0; j < r - 1; j++) {
      x = mul(x, x, n);
      if (x == n - 1) break;
    }
    if (x != n - 1) return 0;
  }
  return 1;
}

ll rho(ll n) {
  if (n == 1 or prime(n)) return n;
  auto f = [n](ll x) { return mul(x, x, n) + 1; };

  ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
  while (t % 40 != 0 or gcd(prd, n) == 1) {
    if (x == y) x = ++x0, y = f(x);
    q = mul(prd, abs(x - y), n);
    if (q != 0) prd = q;
    x = f(x), y = f(f(y)), t++;
  }
  return gcd(prd, n);
}

vll fact(ll n) {
  if (n == 1) return {};
  if (prime(n)) return {n};
  ll d = rho(n);
  vll l = fact(d), r = fact(n / d);
  l.insert(l.end(), r.begin(), r.end());
  return l;
}
```

5.13 Factorization

Computes the factorization of n in $O(\sqrt{n})$.

```
map<ll, ll> factorization(ll n) {
  map<ll, ll> ans;
  for (ll i = 2; i * i <= n; i++) {
    ll count = 0;
    for (; n % i == 0; count++, n /= i)
      ;
    if (count) ans[i] = count;
  }
  if (n > 1) ans[n]++;
  return ans;
}
```

5.14 Fast Fourrier Transform

```
template <bool invert = false>
void fft(vector<complex<double>>& xs) {
  int N = (int)xs.size();
```

```
  if (N == 1) return;

  vector<complex<double>> es(N / 2), os(N / 2);

  for (int i = 0; i < N / 2; ++i) es[i] = xs[2 * i];

  for (int i = 0; i < N / 2; ++i) os[i] = xs[2 * i + 1];

  fft<invert>(es);
  fft<invert>(os);

  auto signal = (invert ? 1 : -1);
  auto theta = 2 * signal * acos(-1) / N;
  complex<double> S{1}, S1{cos(theta), sin(theta)};

  for (int i = 0; i < N / 2; ++i) {
    xs[i] = (es[i] + S * os[i]);
    xs[i] /= (invert ? 2 : 1);

    xs[i + N / 2] = (es[i] - S * os[i]);
    xs[i + N / 2] /= (invert ? 2 : 1);

    S *= S1;
  }
}
```

5.15 Fast pow

Computes a^n in $O(\log N)$.

```
ll fpow(ll a, int n, ll mod = LLONG_MAX) {
  if (n == 0) return 1;
  if (n == 1) return a;
  ll x = fpow(a, n / 2, mod) % mod;
  return ((x * x) % mod * (n & 1 ? a : 1ll)) % mod;
}
```

5.16 Gauss Elimination

```
template <size_t Dim>
struct GaussianElimination {
  vector<ll> basis;
  size_t size;

  GaussianElimination() : basis(Dim + 1), size(0) {}

  void insert(ll x) {
    for (ll i = Dim; i >= 0; i--) {
      if ((x & 1ll << i) == 0) continue;

      if (!basis[i]) {
        basis[i] = x;
        size++;
        break;
      }

      x ^= basis[i];
    }
  }

  void normalize() {
    for (ll i = Dim; i >= 0; i--)
      for (ll j = i - 1; j >= 0; j--)
        if (basis[i] & 1ll << j) basis[i] ^= basis[j];
  }

  bool check(ll x) {
    for (ll i = Dim; i >= 0; i--) {
      if ((x & 1ll << i) == 0) continue;

      if (!basis[i]) return false;

      x ^= basis[i];
    }

    return true;
  }

  auto operator[](ll k) { return at(k); }

  ll at(ll k) {
    ll ans = 0;
    ll total = 1ll << size;
    for (ll i = Dim; ~i; i--) {
      if (!basis[i]) continue;

      ll mid = total >> 1ll;
      if ((mid < k and (ans & 1ll << i) == 0) ||
          (k <= mid and (ans & 1ll << i)))
        ans ^= basis[i];

      if (mid < k) k -= mid;

      total >>= 1ll;
    }
    return ans;
  }

  ll at_normalized(ll k) {
    ll ans = 0;
    k--;
    for (size_t i = 0; i <= Dim; i++) {
      if (!basis[i]) continue;
      if (k & 1) ans ^= basis[i];
      k >>= 1;
    }
    return ans;
  }
};
```

5.17 Integer Mod

```
const ll INF = 1e18;
const ll mod = 998244353;
```

```cpp
template <ll MOD = mod>
struct Modular {
  ll value;
  static const ll MOD_value = MOD;

  Modular(ll v = 0) {
    value = v % MOD;
    if (value < 0) value += MOD;
  }
  Modular(ll a, ll b) : value(0) {
    *this += a;
    *this /= b;
  }

  Modular& operator+=(Modular const& b) {
    value += b.value;
    if (value >= MOD) value -= MOD;
    return *this;
  }
  Modular& operator-=(Modular const& b) {
    value -= b.value;
    if (value < 0) value += MOD;
    return *this;
  }
  Modular& operator*=(Modular const& b) {
    value = (ll)value * b.value % MOD;
    return *this;
  }

  friend Modular mexp(Modular a, ll e) {
    Modular res = 1;
    while (e) {
      if (e & 1) res *= a;
      a *= a;
      e >>= 1;
    }
    return res;
  }
  friend Modular inverse(Modular a) { return mexp(a, MOD - 2); }

  Modular& operator/=(Modular const& b) { return *this *= inverse(b); }
  friend Modular operator+(Modular a, Modular const b) { return a += b; }
  Modular operator++(int) { return this->value = (this->value + 1) % MOD; }
  Modular operator++() { return this->value = (this->value + 1) % MOD; }
  friend Modular operator-(Modular a, Modular const b) { return a -= b; }
  friend Modular operator-(Modular const a) { return 0 - a; }
  Modular operator--(int) {
    return this->value = (this->value - 1 + MOD) % MOD;
  }

  Modular operator--() { return this->value = (this->value - 1 + MOD) % MOD; }
  friend Modular operator*(Modular a, Modular const b) { return a *= b; }
  friend Modular operator/(Modular a, Modular const b) { return a /= b; }
  friend std::ostream& operator<<(std::ostream& os, Modular const& a) {
    return os << a.value;
  }
  friend bool operator==(Modular const& a, Modular const& b) {
    return a.value == b.value;
  }
  friend bool operator!=(Modular const& a, Modular const& b) {
    return a.value != b.value;
  }
};
```

5.18 Is prime

$O(\sqrt{N})$

```cpp
bool isprime(ll n) {
  if (n < 2) return false;
  if (n == 2) return true;
  if (n % 2 == 0) return false;
  for (ll i = 3; i * i < n; i += 2)
    if (n % i == 0) return false;
  return true;
}
```

5.19 Number of Divisors $\tau(n)$

Find the total of divisors of N in $O(\sqrt{N})$

```cpp
ll number_of_divisors(ll n) {
  ll res = 0;

  for (ll d = 1; d * d <= n; ++d) {
    if (n % d == 0) res += (d == n / d ? 1 : 2);
  }

  return res;
}
```

5.20 Power Sum

Calculates $K^0 + K^1 + ... + K^n$

```cpp
ll powersum(ll n, ll k) { return (fastpow(n, k + 1) - 1) / (n - 1); }
```

5.21 Sieve list primes

List every prime until MAXN, $O(N \log N)$ in time and $O(MAXN)$ in memory.

```cpp
const ll MAXN = 1e5;
vll list_primes(ll n) {
  vll ps;
  bitset<MAXN> sieve;
  sieve.set();
  sieve.reset(1);
  for (ll i = 2; i <= n; ++i) {
    if (sieve[i]) ps.push_back(i);
    for (ll j = i * 2; j <= n; j += i) {
      sieve.reset(j);
    }
  }
  return ps;
}
```

5.22 Sum of Divisors $\sigma(n)$

Computes the sum of each divisor of n in $O(\sqrt{n})$.

```
ll sum_of_divisors(long long n) {
  ll res = 0;

  for (ll d = 1; d * d <= n; ++d) {
    if (n % d == 0) {
      ll k = n / d;

      res += (d == k ? d : d + k);
    }
  }

  return res;
}
```

6 Searching

6.1 Ternary Search Recursive

```
const double eps = 1e-6;

// IT MUST BE AN UNIMODAL FUNCTION
double f(int x) { return x * x + 2 * x + 4; }

double ternary_search(double l, double r) {
  if (fabs(f(l) - f(r)) < eps) return f((l + (r - l) / 2.0));

  auto third = (r - l) / 3.0;
  auto m1 = l + third;
  auto m2 = r - third;

  // change the signal to find the maximum point.
  return m1 < m2 ? ternary_search(m1, r) : ternary_search(l, m2);
}
```

7 Strings

7.1 Hash Range Query

```
struct Hash {
  const ll P = 31;
  const ll mod = 1e9 + 7;
  string s;
  int n;
  vll h, hi, p;
  Hash() {}
  Hash(string s) : s(s), n(s.size()), h(n), hi(n), p(n) {
    for (int i = 0; i < n; i++) p[i] = (i ? P * p[i - 1] : 1) % mod;
    for (int i = 0; i < n; i++) h[i] = (s[i] + (i ? h[i - 1] : 0) * P) % mod;
    for (int i = n - 1; i >= 0; i--)
      hi[i] = (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % mod;
  }
  ll query(int l, int r) {
    ll hash = (h[r] - (l ? h[l - 1] * p[r - l + 1] % mod : 0));
    return hash < 0 ? hash + mod : hash;
  }
  ll query_inv(int l, int r) {
    ll hash = (hi[l] - (r + 1 < n ? hi[r + 1] * p[r - l + 1] % mod : 0));
    return hash < 0 ? hash + mod : hash;
  }
};
```

7.2 Longest Palindrome

```
string longest_palindrome(const string &s) {
  int n = (int)s.size();
  vector<array<int, 2>> dp(n);

  pii odd(0, -1), even(0, -1);
  pii ans;
  for (int i = 0; i < n; i++) {
    int k = 0;
    if (i > odd.second)
      k = 1;
    else
      k = min(dp[odd.first + odd.second - i][0], odd.second - i + 1);
    while (i - k >= 0 and i + k < n and s[i - k] == s[i + k]) k++;
    dp[i][0] = k--;
    if (i + k > odd.second) odd = {i - k, i + k};
    if (2 * dp[i][0] - 1 > ans.second) ans = {i - k, 2 * dp[i][0] - 1};

    k = 0;
    if (i <= even.second)
      k = min(dp[even.first + even.second - i + 1][1], even.second - i + 1);
    while (i - k - 1 >= 0 and i + k < n and s[i - k - 1] == s[i + k]) k++;
    dp[i][1] = k--;
    if (i + k > even.second) even = {i - k - 1, i + k};
    if (2 * dp[i][1] > ans.second) ans = {i - k - 1, 2 * dp[i][1]};
  }
  return s.substr(ans.first, ans.second);
}
```

7.3 Rabin Karp

```
size_t rabin_karp(const string &s, const string &p) {
  if (s.size() < p.size()) return 0;

  auto n = s.size(), m = p.size();
  const ll p1 = 31, p2 = 29, q1 = 1e9 + 7, q2 = 1e9 + 9;
  const ll p1_1 = fpow(p1, q1 - 2, q1), p1_2 = fpow(p1, m - 1, q1);
  const ll p2_1 = fpow(p2, q2 - 2, q2), p2_2 = fpow(p2, m - 1, q2);

  pair<ll, ll> hs, hp;
  for (int i = (int)m - 1; ~i; --i) {
    hs.first = (hs.first * p1) % q1;
    hs.first = (hs.first + (s[i] - 'a' + 1)) % q1;
    hs.second = (hs.second * p2) % q2;
    hs.second = (hs.second + (s[i] - 'a' + 1)) % q2;

    hp.first = (hp.first * p1) % q1;
    hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
```

```
    hp.second = (hp.second * p2) % q2;
    hp.second = (hp.second + (p[i] - 'a' + 1)) % q2;
  }

  size_t occ = 0;
  for (size_t i = 0; i < n - m; i++) {
    occ += (hs == hp);

    int fi = s[i] - 'a' + 1;
    int fm = s[i + m] - 'a' + 1;

    hs.first = (hs.first - fi + q1) % q1;
    hs.first = (hs.first * p1_1) % q1;
    hs.first = (hs.first + fm * p1_2) % q1;
    hs.second = (hs.second - fi + q2) % q2;
    hs.second = (hs.second * p2_1) % q2;
    hs.second = (hs.second + fm * p2_2) % q2;
  }
  occ += hs == hp;

  return occ;
}
```

7.4 String Psum

```
struct strPsum {
  ll n;
  ll k;
  vector<vll> psum;
  strPsum(const string &s) : n(s.size()), k(100), psum(k, vll(n + 1)) {
    for (ll i = 1; i <= n; ++i) {
      for (ll j = 0; j < k; ++j) {
        psum[j][i] = psum[j][i - 1];
      }
      psum[s[i - 1]][i]++;
    }
  }

  ll qtd(ll l, ll r, char c) {  // [0,n-1]
    return psum[c][r + 1] - psum[c][l];
  }
}
```

7.5 Suffix Automaton (complete)

```
struct state {
  int len, link, cnt, firstpos;
  // this can be optimized using a vector with the alphabet size
  map<char, int> next;
  vi inv_link;
};
struct SuffixAutomaton {
  vector<state> st;
  int sz = 0;
  int last;
  vc cloned;

  SuffixAutomaton(const string &s, int maxlen)
    : st(maxlen * 2), cloned(maxlen * 2) {
    st[0].len = 0;
    st[0].link = -1;
    sz++;
    last = 0;
    for (auto &c : s) add_char(c);

    // precompute for count occurences
    for (int i = 1; i < sz; i++) {
      st[i].cnt = !cloned[i];
    }
    vector<pair<state, int>> aux;
    for (int i = 0; i < sz; i++) {
      aux.push_back({st[i], i});
    }

    sort(all(aux), [](const pair<state, int> &a, const pair<state, int> &b) {
      return a.fst.len > b.fst.len;
    });

    for (auto &[stt, id] : aux) {
      if (stt.link != -1) {
        st[stt.link].cnt += st[id].cnt;
      }
    }

    // for find every occurende position
    for (int v = 1; v < sz; v++) {
      st[st[v].link].inv_link.push_back(v);
    }
  }

  void add_char(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    st[cur].firstpos = st[cur].len - 1;
    int p = last;
    // follow the suffix link until find a transition to c
    while (p != -1 and !st[p].next.count(c)) {
      st[p].next[c] = cur;
      p = st[p].link;
    }
    // there was no transition to c so create and leave
    if (p == -1) {
      st[cur].link = 0;
      last = cur;
      return;
    }

    int q = st[p].next[c];
    if (st[p].len + 1 == st[q].len) {
      st[cur].link = q;
    } else {
      int clone = sz++;
      cloned[clone] = true;
      st[clone].len = st[p].len + 1;
      st[clone].next = st[q].next;
      st[clone].link = st[q].link;
```

```
    st[clone].firstpos = st[q].firstpos;
    while (p != -1 and st[p].next[c] == q) {
      st[p].next[c] = clone;
      p = st[p].link;
    }
    st[q].link = st[cur].link = clone;
  }
  last = cur;
}

bool checkOccurrence(const string &t) {  // O(len(t))
  int cur = 0;
  for (auto &c : t) {
    if (!st[cur].next.count(c)) return false;
    cur = st[cur].next[c];
  }
  return true;
}
ll totalSubstrings() {  // distinct, O(len(s))
  ll tot = 0;
  for (int i = 1; i < sz; i++) {
    tot += st[i].len - st[st[i].link].len;
  }
  return tot;
}

// count occurences of a given string t
int countOccurences(const string &t) {
  int cur = 0;
  for (auto &c : t) {
    if (!st[cur].next.count(c)) return 0;
    cur = st[cur].next[c];
  }
  return st[cur].cnt;
}

// find the first index where t appears a substring O(len(t))
int firstOccurence(const string &t) {
  int cur = 0;
  for (auto c : t) {
    if (!st[cur].next.count(c)) return -1;
    cur = st[cur].next[c];
  }
  return st[cur].firstpos - len(t) + 1;
}

vi everyOccurence(const string &t) {
  int cur = 0;
  for (auto c : t) {
    if (!st[cur].next.count(c)) return {};
    cur = st[cur].next[c];
  }
  vi ans;
  getEveryOccurence(cur, len(t), ans);
  return ans;
}

void getEveryOccurence(int v, int P_length, vi &ans) {
    if (!cloned[v]) ans.pb(st[v].firstpos - P_length + 1);
    for (int u : st[v].inv_link) getEveryOccurence(u, P_length, ans);
  }
};
```

7.6 Z-function get occurence positions

$O(len(s) + len(p))$

```
vi getOccPos(string &s, string &p) {
  // Z-function
  char delim = '#';
  string t{p + delim + s};
  vi zs(len(t));

  int l = 0, r = 0;
  for (int i = 1; i < len(t); i++) {
    if (i <= r) zs[i] = min(zs[i - l], r - i + 1);
    while (zs[i] + i < len(t) and t[zs[i]] == t[i + zs[i]]) zs[i]++;
    if (r < i + zs[i] - 1) l = i, r = i + zs[i] - 1;
  }

  // Iterate over the results of Z-function to get ranges
  vi ans;
  int start = len(p) + 1 + 1 - 1;
  for (int i = start; i < len(zs); i++) {
    if (zs[i] == len(p)) {
      int l = i - start;
      ans.emplace_back(l);
    }
  }
  return ans;
}
```

8 Settings and macros

8.1 short-macro.cpp

```
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define fastio                        \
  ios_base::sync_with_stdio(false); \
  cin.tie(0);                       \
  cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using pii = pair<int, int>;
#define all(a) a.begin(), a.end()


void run() {}
int32_t main(void) {
  fastio;
  int t;
  t = 1;
  // cin >> t;
```

```cpp
    while (t--) run();
}
```

8.2 .vimrc

```
set ts=4 sw=4 sta nu rnu sc cindent
set bg=dark ruler clipboard=unnamed,unnamedplus, timeoutlen=100
colorscheme default

nnoremap <C-j> :botright belowright term bash <CR>
syntax on
```

8.3 degug.cpp

```cpp
#include <bits/stdc++.h>
using namespace std;
/******** Debug Code *******/
template <typename T>
concept Printable = requires(T t) {
    { std::cout << t } -> std::same_as<std::ostream &>;
};
template <Printable T>
void __print(const T &x) {
    cerr << x;
}
template <size_t T>
void __print(const bitset<T> &x) {
    cerr << x;
}
template <typename A, typename B>
void __print(const pair<A, B> &p);
template <typename... A>
void __print(const tuple<A...> &t);
template <typename T>
void __print(stack<T> s);
template <typename T>
void __print(queue<T> q);
template <typename T, typename... U>
void __print(priority_queue<T, U...> q);
template <typename A>
void __print(const A &x) {
    bool first = true;
    cerr << '{';
    for (const auto &i : x) {
        cerr << (first ? "" : ","), __print(i);
        first = false;
    }
    cerr << '}';
}
template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '(';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << ')';
}
template <typename... A>
void __print(const tuple<A...> &t) {
    bool first = true;
    cerr << '(';
    apply(
        [&first](const auto &...args) {
            ((cerr << (first ? "" : ","), __print(args), first = false), ...);
        },
        t);
    cerr << ')';
}
template <typename T>
void __print(stack<T> s) {
    vector<T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    }
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
}
template <typename T>
void __print(queue<T> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.front();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
template <typename T, typename... U>
void __print(priority_queue<T, U...> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.top();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
void _print() { cerr << "]\n"; }
template <typename Head, typename... Tail>
void _print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";
    _print(T...);
}

#define dbg(x...)                    \
    cerr << "[" << #x << "] = ["; \
    _print(x)
```

8.4 .bashrc

```bash
cpp() {
  echo ">> COMPILING <<" 1>&2
  g++ -std=c++17 \
```

```
        -O2 \
        -g \
        -g3 \
        -Wextra \
        -Wshadow \
        -Wformat=2 \
        -Wconversion \
        -fsanitize=address,undefined \
        -fno-sanitize-recover \
        -Wfatal-errors  \
        $1

    if [ $? -ne 0 ]; then
        echo ">> FAILED <<" 1>&2
        return 1
    fi
    echo ">> DONE << " 1>&2
    time  ./a.out ${@:2}
}

prepare() {
    for i in {a..z}
    do
        cp macro.cpp $i.cpp
        touch $i.py
    done

    for i in {1..10}
    do
        touch in${i}
        touch out${i}
        touch ans${i}
    done
}
```

8.5 macro.cpp

```cpp
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define fastio                          \
  ios_base::sync_with_stdio(false); \
  cin.tie(0);                       \
  cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using ld = long double;
using vll = vector<ll>;
using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
using vii = vector<pii>;
using vc = vector<char>;
#define all(a) a.begin(), a.end()
#define snd second
#define fst first
#define pb(___x) push_back(___x)
#define mp(___a, ___b) make_pair(___a, ___b)
#define eb(___x) emplace_back(___x)

const ll oo = 1e18;

void run() {}
int32_t main(void) {
  fastio;
  int t;
  t = 1;
  // cin >> t;
  while (t--) run();
}
```