# The Reference

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## Data structures

### 1.1 Dsu

```
struct DSU {
 vector < int > ps;
 vector < int > size;
 DSU(int N) : ps(N + 1), size(N + 1, 1) { iota(ps.begin(), ps.end(), 0); }
 int find_set(int x) { return ps[x] == x ? x : ps[x] = find_set(ps[x]); }
 bool same_set(int x, int y) { return find_set(x) == find_set(y); }
 void union_set(int x, int y) {
    if (same_set(x, y)) return;
   int px = find_set(x);
    int py = find_set(y);
    if (size[px] < size[py]) swap(px, py);</pre>
   ps[py] = px;
    size[px] += size[py];
};
     Dsu (Python)
class DSU:
    def init (self. n):
        self.n = n
        self.p = [x for x in range(0, n + 1)]
        self.size = [0 for i in range(0, n + 1)]
    def find_set(self, x): # log n
        if self.p[x] == x:
            return x
        else:
            self.p[x] = self.find_set(self.p[x])
            return self.p[x]
    def same_set(self, x, y): # log n
        return bool(self.find_set(x) == self.find_set(y))
    def union_set(self, x, y): # log n
        px = self.find set(x)
        py = self.find_set(y)
        if px == py:
            return
        size x = self.size[px]
        size_y = self.size[py]
        if size_x > size_y:
            self.p[pv] = self.p[px]
            self.size[px] += self.size[py]
            self.p[px] = self.p[py]
```

self.size[py] += self.size[px]

### 1.3 Ordered Set Gnu Pbds

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename T>
// using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
// tree_order_statistics_node_update>;
// if you want to find the elements less or equal :p
using ordered_set = tree<T, null_type, less_equal<T>, rb_tree_tag,
                        tree_order_statistics_node_update>;
1.4 Segtree Rmaxq Rmaxu
 int N:
```

```
template <typename T = 11>
struct SegTree {
 T nu, nq;
 vector <T> st, lazy;
 SegTree(const vector <T> &xs)
   : N(len(xs)),
     nu(numeric_limits <T>::min()),
     ng(numeric_limits <T>::min()),
     st(4 * N + 1, nu),
     lazy(4 * N + 1, nu) {
   for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);</pre>
 void update(int 1, int r, T value) { update(1, 0, N - 1, 1, r, value); }
 T query(int 1, int r) { return query(1, 0, N - 1, 1, r); }
 void update(int node, int nl, int nr, int ql, int qr, T v) {
   propagation(node, nl, nr);
   if (ql > nr or qr < nl) return;
   st[node] = max(st[node], v):
   if (ql <= nl and nr <= qr) {</pre>
     if (nl < nr) {
       lazy[left(node)] = max(lazy[left(node)], v);
        lazy[right(node)] = max(lazy[right(node)], v);
     return;
   update(left(node), nl, mid(nl, nr), ql, qr, v);
   update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);
    st[node] = max(st[left(node)], st[right(node)]);
 T query(int node, int nl, int nr, int ql, int qr) {
   propagation(node, nl, nr);
   if (ql > nr or qr < nl) return nq;
   if (ql <= nl and nr <= qr) return st[node];</pre>
```

```
T x = query(left(node), nl, mid(nl, nr), ql, qr);
   T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);
   return max(x, y);
 }
 void propagation(int node, int nl, int nr) {
   if (lazv[node] != nu) {
      st[node] = max(st[node], lazy[node]);
     if (nl < nr) {
       lazy[left(node)] = max(lazy[left(node)], lazy[node]);
       lazy[right(node)] = max(lazy[right(node)], lazy[node]);
     lazy[node] = nu;
   }
 }
 int left(int p) { return p << 1; }</pre>
 int right(int p) { return (p << 1) + 1; }</pre>
 int mid(int 1, int r) { return (r - 1) / 2 + 1; }
int main() {
 int n;
 cin >> n:
 vector < array < int , 3>> xs(n);
 for (int i = 0; i < n; ++i) {
   for (int j = 0; j < 3; ++j) {
      cin >> xs[i][i]:
 }
 vi aux(n, 0);
 SegTree < int > st(aux);
 for (int i = 0; i < n; ++i) {
   int a = min(i + xs[i][1], n);
   int b = min(i + xs[i][2], n);
   st.update(i, i, st.query(i, i) + xs[i][0]);
   int cur = st.query(i, i);
   st.update(a, b, cur);
 }
 cout << st.query(0, n) << '\n';
     Segtree Rming Pau
template <typename T = 11>
struct SegTree {
 int n;
 T nu, nq;
 vector <T> st:
 SegTree(const vector <T> &v)
   : n(len(v)), nu(0), nq(numeric_limits < T > :: max()), st(n * 4 + 1, nu) {
   for (int i = 0; i < n; ++i) update(i, v[i]);</pre>
```

void update(int p, T v) { update(1, 0, n - 1, p, v); }

```
T query(int 1, int r) { return query(1, 0, n - 1, 1, r); }
  void update(int node, int nl, int nr, int p, T v) {
    if (p < nl or p > nr) return;
    if (nl == nr) {
      st[node] = v;
      return;
    update(left(node), nl, mid(nl, nr), p, v);
    update(right(node), mid(nl, nr) + 1, nr, p, v);
    st[node] = min(st[left(node)], st[right(node)]);
  T query(int node, int nl, int nr, int ql, int qr) {
    if (ql <= nl and qr >= nr) return st[node];
    if (nl > gr or nr < gl) return ng;
    if (nl == nr) return st[node]:
    return min(query(left(node), nl, mid(nl, nr), ql, qr),
               query(right(node), mid(nl, nr) + 1, nr, ql, qr));
  }
  int left(int p) { return p << 1; }</pre>
  int right(int p) { return (p << 1) + 1; }</pre>
 int mid(int 1, int r) { return (r - 1) / 2 + 1; }
};
     Segtree Rming Rsu
template <typename t = 11>
struct SegTree {
  int n;
  t nu;
  t nq;
  vector < t> st, lazy;
  SegTree(const vector <t > &xs)
   : n(len(xs)),
      nu(0),
      nq(numeric_limits <t>::max()),
      st(4 * n, nu),
      lazv(4 * n. nu) {
    for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
  SegTree(int n): n(n), st(4 * n, nu), lazy(4 * n, nu) {}
  void update(int l. int r. ll value) { update(1, 0, n - 1, l, r, value); }
  t query(int 1, int r) { return query(1, 0, n - 1, 1, r); }
  void update(int node, int nl, int nr, int ql, int qr, ll v) {
    propagation(node, nl, nr);
    if (ql > nr or qr < nl) return;
```

```
if (gl <= nl and nr <= gr) {
      st[node] += (nr - nl + 1) * v;
     if (nl < nr) {
       lazy[left(node)] += v;
        lazy[right(node)] += v;
      return:
    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);
    st[node] = min(st[left(node)], st[right(node)]);
 }
 t query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);
   if (ql > nr or qr < nl) return nq;</pre>
    if (ql <= nl and nr <= qr) return st[node];</pre>
    t x = query(left(node), nl, mid(nl, nr), ql, qr);
    t y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);
   return min(x, y);
 }
  void propagation(int node, int nl, int nr) {
   if (lazy[node]) {
      st[node] += lazv[node];
     if (nl < nr) {
       lazy[left(node)] += lazy[node];
        lazy[right(node)] += lazy[node];
     lazv[node] = nu;
 }
 int left(int p) { return p << 1; }</pre>
 int right(int p) { return (p << 1) + 1; }</pre>
 int mid(int 1, int r) { return (r - 1) / 2 + 1; }
};
     Segtree Rsq Rsu
template <typename T = 11>
struct SegTree {
 int N:
 vector <T> st, lazy;
 T nu = 0:
 T nq = 0;
 SegTree(const vector<T> &xs) : N(len(xs)), st(4 * N, nu), lazy(4 * N, nu) {
   for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
```

```
}
SegTree(int n): N(n), st(4 * N, nu), lazy(4 * N, nu) {}
void update(int 1, int r, 11 value) { update(1, 0, N - 1, 1, r, value); }
T query(int 1, int r) { return query(1, 0, N - 1, 1, r); }
void update(int node, int nl, int nr, int ql, int qr, ll v) {
  propagation(node, nl, nr);
  if (ql > nr or qr < nl) return;</pre>
  if (ql <= nl and nr <= qr) {</pre>
    st[node] += (nr - nl + 1) * v:
    if (nl < nr) {
      lazv[left(node)] += v;
      lazv[right(node)] += v;
    return:
  7
  update(left(node), nl, mid(nl, nr), ql, qr, v);
  update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);
  st[node] = st[left(node)] + st[right(node)];
T query(int node, int nl, int nr, int ql, int qr) {
  propagation (node, nl, nr);
  if (ql > nr or qr < nl) return nq;</pre>
  if (ql <= nl and nr <= qr) return st[node];</pre>
  T x = query(left(node), nl, mid(nl, nr), ql, qr);
  T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);
  return x + y;
void propagation(int node, int nl, int nr) {
  if (lazy[node]) {
    st[node] += (nr - nl + 1) * lazy[node];
    if (nl < nr) {
      lazy[left(node)] += lazy[node];
      lazy[right(node)] += lazy[node];
    lazy[node] = nu;
}
int left(int p) { return p << 1; }</pre>
int right(int p) { return (p << 1) + 1; }</pre>
```

```
int mid(int 1, int r) { return (r - 1) / 2 + 1; }
};
     Sparse Table Rming
/*
        Sparse table implementation for rmq.
       build: O(NlogN)
       query: 0(1)
int fastlog2(11 x) {
 ull i = x;
 return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
template <typename T>
class SparseTable {
public:
 int N;
 int K:
 vector < vector < T >> st;
 SparseTable(vector<T> vs)
    : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector < T > (N + 1)) {
    copy(vs.begin(), vs.end(), st[0].begin());
   for (int i = 1; i <= K; ++i)
     for (int i = 0; i + (1 << i) <= N; ++i)
        st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
 T RMQ(int 1, int r) { // [1, r], 0 indexed
    int i = fastlog2(r - 1 + 1);
    return min(st[i][1], st[i][r - (1 << i) + 1]);</pre>
 }
};
    Dynamic programming
```

### Edit Distance

```
int edit_distance(const string &a, const string &b) {
 int n = a.size();
 int m = b.size():
 vector < vi > dp(n + 1, vi(m + 1, 0));
 int ADD = 1, DEL = 1, CHG = 1;
  for (int i = 0; i <= n; ++i) {</pre>
    dp[i][0] = i * DEL;
  for (int i = 1; i <= m; ++i) {</pre>
    dp[0][i] = ADD * i;
  for (int i = 1; i <= n; ++i) {
    for (int j = 1; j \le m; ++ j) {
      int add = dp[i][j - 1] + ADD;
      int del = dp[i - 1][j] + DEL;
      int chg = dp[i - 1][j - 1] + (a[i - 1] == b[j - 1]?0:1) * CHG;
      dp[i][j] = min({add, del, chg});
```

```
}
 return dp[n][m];
     Kadane
2.2
* Find the maximum sum subarray in a given array.
 * 0(N)
 * */
int kadane(const vi &as) {
 vi s(len(as));
 s[0] = as[0];
 for (int i = 1; i < len(as); ++i) s[i] = max(as[i], s[i - 1] + as[i]);
 return *max_element(all(s));
     Knapsack (value)
const int MAXN{2010}, MAXM{2010};
11 st[MAXN][MAXM];
11 dp(int i, int m, int M, const vii &cs) {
 if (i < 0) return 0:
  if (st[i][m] != -1) return st[i][m];
  auto res = dp(i - 1, m, M, cs);
  auto [w, v] = cs[i];
  if (w \le m) res = max(res, dp(i - 1, m - w, M, cs) + v);
  st[i][m] = res;
 return res:
11 knapsack(int M, const vii &cs) {
 memset(st, -1, sizeof st);
  return dp((int)cs.size() - 1, M, M, cs);
     Knapsack With Elements
const int MAXN{2010}, MAXM{2010};
11 st[MAXN][MAXM];
char ps[MAXN][MAXM];
pair<11, vi> knapsack(int M, const vii &cs) {
 int N = len(cs) - 1;
 for (int i = 0; i <= N; ++i) st[i][0] = 0;
```

```
for (int m = 0; m \le M; ++m) st[0][m] = 0;
 for (int i = 1: i <= N: ++i) {
   for (int m = 1; m <= M; ++m) {</pre>
      st[i][m] = st[i - 1][m];
     ps[i][m] = 0;
     auto [w, v] = cs[i];
     if (w \le m \text{ and } st[i - 1][m - w] + v > st[i][m]) {
       st[i][m] = st[i - 1][m - w] + v;
       ps[i][m] = 1;
     }
 int m = M:
 vi is:
 for (int i = N; i >= 1; --i) {
   if (ps[i][m]) {
     is.push_back(i);
     m -= cs[i].first:
 }
 reverse(all(is));
 // max value, items
 return {st[N][M], is};
    Longest Increasing Sequence
int LIS(int N, const vector < int > &as) {
 vector < int > lis(N + 1, oo);
 lis[0] = -oo;
 auto ans = 0;
 for (int i = 0; i < N; ++i) {
   auto it = lower_bound(lis.begin(), lis.end(), as[i]);
   auto pos = (int)(it - lis.begin());
   ans = max(ans, pos);
   lis[pos] = as[i];
 return ans;
     Money Sum Bottom Up
  find every possible sum using
  the given values only once.
```

set < int > money\_sum(const vi &xs) {

using vc = vector < char >;

```
using vvc = vector <vc>:
  int _m = accumulate(all(xs), 0);
  int _n = xs.size();
  vvc _dp(_n + 1, vc(_m + 1, 0));
  set < int > _ans;
  dp[0][xs[0]] = 1;
  for (int i = 1; i < _n; ++i) {
    for (int j = 0; j <= _m; ++j) {
      if (j == 0 or _dp[i - 1][j]) {
        _{dp[i][j + xs[i]] = 1;}
        _{dp[i][j]} = 1;
    }
  }
  for (int i = 0; i < _n; ++i)
    for (int j = 0; j <= _m; ++j)
      if (_dp[i][j]) _ans.insert(j);
  return _ans;
2.7 Tsp
using vi = vector<int>;
vector < vi > dist;
vector < vi> memo:
/* 0 ( N^2 * 2^N )*/
int tsp(int i, int mask, int N) {
  if (mask == (1 << N) - 1) return dist[i][0];</pre>
  if (memo[i][mask] != -1) return memo[i][mask];
  int ans = INT_MAX << 1;</pre>
  for (int j = 0; j < N; ++j) {
    if (mask & (1 << j)) continue;</pre>
    auto t = tsp(j, mask | (1 << j), N) + dist[i][j];</pre>
    ans = min(ans, t):
  return memo[i][mask] = ans;
     Extras
3.1 Bigint
const int maxn = 1e2 + 14, 1g = 15;
const int base = 1000000000;
const int base_digits = 9;
struct bigint {
  vector < int > a;
  int sign;
  int size() {
    if (a.empty()) return 0;
    int ans = (a.size() - 1) * base_digits;
    int ca = a.back();
    while (ca) ans++, ca \neq 10;
    return ans;
```

```
bigint operator^(const bigint &v) {
  bigint ans = 1, a = *this, b = v;
  while (!b.isZero()) {
   if (b % 2) ans *= a:
    a *= a, b /= 2;
  return ans;
}
string to_string() {
  stringstream ss;
  ss << *this;
  string s;
  ss >> s;
  return s:
int sumof() {
  string s = to_string();
  int ans = 0;
  for (auto c : s) ans += c - '0';
  return ans:
}
/*</arpa>*/
bigint() : sign(1) {}
bigint(long long v) { *this = v; }
bigint(const string &s) { read(s); }
void operator=(const bigint &v) {
  sign = v.sign;
  a = v.a:
void operator=(long long v) {
  sign = 1;
  a.clear():
 if (v < 0) sign = -1, v = -v;
 for (; v > 0; v = v / base) a.push_back(v \% base);
}
bigint operator+(const bigint &v) const {
  if (sign == v.sign) {
    bigint res = v;
    for (int i = 0, carry = 0; i < (int)max(a.size(), v.a.size()) || carry;</pre>
         ++i) {
      if (i == (int)res.a.size()) res.a.push_back(0);
      res.a[i] += carry + (i < (int)a.size() ? a[i] : 0);
      carry = res.a[i] >= base;
      if (carry) res.a[i] -= base;
    return res;
  return *this - (-v);
}
bigint operator - (const bigint &v) const {
  if (sign == v.sign) {
```

```
if (abs() >= v.abs()) {
      bigint res = *this;
      for (int i = 0, carry = 0; i < (int)v.a.size() || carry; ++i) {</pre>
        res.a[i] -= carry + (i < (int)v.a.size() ? v.a[i] : 0);
        carry = res.a[i] < 0;</pre>
        if (carry) res.a[i] += base;
      res.trim();
      return res:
    return -(v - *this);
  return *this + (-v);
void operator*=(int v) {
  if (v < 0) sign = -sign, v = -v;
 for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i) {
    if (i == (int)a.size()) a.push_back(0);
    long long cur = a[i] * (long long)v + carry;
    carry = (int)(cur / base);
    a[i] = (int)(cur % base);
   // asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) :
    // "A"(cur), "c"(base));
  trim();
bigint operator*(int v) const {
 bigint res = *this;
 res *= v:
  return res;
}
void operator*=(long long v) {
 if (v < 0) sign = -sign, v = -v;
 if (v > base) {
    *this = *this * (v / base) * base + *this * (v % base);
    return:
  for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i) {</pre>
    if (i == (int)a.size()) a.push_back(0);
    long long cur = a[i] * (long long)v + carry;
    carry = (int)(cur / base);
    a[i] = (int)(cur \% base);
    // asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) :
    // "A"(cur), "c"(base));
  trim():
bigint operator*(long long v) const {
 bigint res = *this;
 res *= v;
  return res;
friend pair < bigint, bigint > divmod(const bigint &a1, const bigint &b1) {
```

```
int norm = base / (b1.a.back() + 1):
  bigint a = a1.abs() * norm;
  bigint b = b1.abs() * norm;
  bigint q. r:
  q.a.resize(a.a.size());
  for (int i = a.a.size() - 1: i >= 0: i--) {
   r *= base:
   r += a.a[i]:
    int s1 = r.a.size() <= b.a.size() ? 0 : r.a[b.a.size()];
    int s2 = r.a.size() \le b.a.size() - 1 ? 0 : r.a[b.a.size() - 1]:
    int d = ((long long)base * s1 + s2) / b.a.back();
   r \rightarrow b * d;
    while (r < 0) r += b, --d:
    q.a[i] = d;
  q.sign = a1.sign * b1.sign;
  r.sign = a1.sign;
  q.trim();
 r.trim();
 return make_pair(q, r / norm);
bigint operator/(const bigint &v) const { return divmod(*this, v).first; }
bigint operator%(const bigint &v) const { return divmod(*this, v).second; }
void operator/=(int v) {
  if (v < 0) sign = -sign, v = -v;
  for (int i = (int)a.size() - 1, rem = 0; i \ge 0; --i) {
    long long cur = a[i] + rem * (long long)base;
   a[i] = (int)(cur / v):
    rem = (int)(cur % v);
  trim();
}
bigint operator/(int v) const {
  bigint res = *this;
 res /= v;
  return res;
int operator%(int v) const {
 if (v < 0) v = -v:
  int. m = 0:
  for (int i = a.size() - 1; i >= 0; --i)
   m = (a[i] + m * (long long)base) % v;
  return m * sign;
}
void operator+=(const bigint &v) { *this = *this + v; }
void operator -=(const bigint &v) { *this = *this - v; }
void operator*=(const bigint &v) { *this = *this * v; }
void operator/=(const bigint &v) { *this = *this / v; }
bool operator < (const bigint &v) const {</pre>
```

```
if (sign != v.sign) return sign < v.sign;</pre>
  if (a.size() != v.a.size()) return a.size() * sign < v.a.size() * v.sign;</pre>
  for (int i = a.size() - 1; i >= 0; i--)
    if (a[i] != v.a[i]) return a[i] * sign < v.a[i] * sign:
 return false:
}
bool operator>(const bigint &v) const { return v < *this; }</pre>
bool operator <= (const bigint &v) const { return !(v < *this); }</pre>
bool operator>=(const bigint &v) const { return !(*this < v); }</pre>
bool operator == (const bigint &v) const {
  return !(*this < v) && !(v < *this):
bool operator!=(const bigint &v) const { return *this < v || v < *this; }
void trim() {
  while (!a.empty() && !a.back()) a.pop_back();
 if (a.empty()) sign = 1;
bool isZero() const { return a.empty() || (a.size() == 1 && !a[0]); }
bigint operator -() const {
 bigint res = *this;
 res.sign = -sign;
  return res;
bigint abs() const {
 bigint res = *this;
  res.sign *= res.sign;
  return res;
long longValue() const {
 long long res = 0:
 for (int i = a.size() - 1; i >= 0; i--) res = res * base + a[i];
 return res * sign:
friend bigint gcd(const bigint &a, const bigint &b) {
  return b.isZero() ? a : gcd(b, a % b);
friend bigint lcm(const bigint &a, const bigint &b) {
  return a / gcd(a, b) * b;
void read(const string &s) {
 sign = 1:
  a.clear();
 int pos = 0;
  while (pos < (int)s.size() && (s[pos] == '-' || s[pos] == '+')) {
    if (s[pos] == '-') sign = -sign;
    ++pos:
 }
  for (int i = s.size() - 1; i >= pos; i -= base_digits) {
    for (int j = max(pos, i - base_digits + 1); j <= i; j++)</pre>
```

```
x = x * 10 + s[i] - '0':
   a.push_back(x);
  trim():
friend istream &operator>>(istream &stream, bigint &v) {
  string s;
 stream >> s:
 v.read(s);
 return stream;
friend ostream & operator << (ostream & stream, const bigint &v) {
  if (v.sign == -1) stream << '-':
  stream << (v.a.empty() ? 0 : v.a.back());
 for (int i = (int)v.a.size() - 2; i >= 0; --i)
    stream << setw(base_digits) << setfill('0') << v.a[i];</pre>
 return stream;
static vector < int > convert base (const vector < int > &a. int old digits.
                                 int new digits) {
  vector <long long > p(max(old_digits, new_digits) + 1);
  p[0] = 1:
  for (int i = 1; i < (int)p.size(); i++) p[i] = p[i - 1] * 10;
  vector<int> res:
  long long cur = 0;
  int cur_digits = 0;
  for (int i = 0; i < (int)a.size(); i++) {</pre>
   cur += a[i] * p[cur_digits];
    cur_digits += old_digits;
    while (cur digits >= new digits) {
     res.push_back(int(cur % p[new_digits]));
      cur /= p[new_digits];
      cur_digits -= new_digits;
  res.push back((int)cur);
  while (!res.empty() && !res.back()) res.pop_back();
  return res;
typedef vector<long long> vll;
static vll karatsubaMultiply(const vll &a, const vll &b) {
  int n = a.size();
  vll res(n + n);
  if (n <= 32) {
   for (int i = 0; i < n; i++)
     for (int j = 0; j < n; j++) res[i + j] += a[i] * b[j];
    return res:
  int k = n \gg 1:
  vll a1(a.begin(), a.begin() + k);
  vll a2(a.begin() + k, a.end());
  vll b1(b.begin(), b.begin() + k);
```

```
vll b2(b.begin() + k. b.end()):
    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);
    for (int i = 0; i < k; i++) a2[i] += a1[i];
    for (int i = 0; i < k; i++) b2[i] += b1[i];
    vll r = karatsubaMultiplv(a2, b2);
    for (int i = 0; i < (int)a1b1.size(); i++) r[i] -= a1b1[i];
    for (int i = 0; i < (int)a2b2.size(); i++) r[i] -= a2b2[i];</pre>
    for (int i = 0; i < (int)r.size(); i++) res[i + k] += r[i];
    for (int i = 0: i < (int)a1b1.size(): i++) res[i] += a1b1[i]:
    for (int i = 0: i < (int)a2b2.size(): i++) res[i + n] += a2b2[i]:
    return res:
  bigint operator*(const bigint &v) const {
    vector < int > a6 = convert_base(this -> a, base_digits, 6);
    vector < int > b6 = convert_base(v.a, base_digits, 6);
    vll a(a6.begin(), a6.end());
    vll b(b6.begin(), b6.end());
    while (a.size() < b.size()) a.push_back(0);</pre>
    while (b.size() < a.size()) b.push_back(0);</pre>
    while (a.size() & (a.size() - 1)) a.push_back(0), b.push_back(0);
    vll c = karatsubaMultiply(a, b);
    bigint res;
    res.sign = sign * v.sign;
    for (int i = 0, carry = 0; i < (int)c.size(); i++) {</pre>
      long long cur = c[i] + carry;
      res.a.push_back((int)(cur % 1000000));
      carrv = (int)(cur / 1000000):
    res.a = convert_base(res.a, 6, base_digits);
    res.trim():
    return res;
};
     Binary To Grav
string binToGray(string bin) {
  string gray(bin.size(), '0');
  int n = bin.size() - 1;
  grav[0] = bin[0];
  for (int i = 1: i <= n: i++) {
    grav[i] = '0' + (bin[i - 1] == '1') ^ (bin[i] == '1');
  return gray;
     Get Permutation Cicles
```

```
* receives a permutation [0, n-1]
* returns a vector of cicles
* for example: [1, 0, 3, 4, 2] \rightarrow [[0, 1], [2, 3, 4]]
```

```
* */
vector<vll> getPermutationCicles(const vll &ps) {
    ll n = len(ps);
    vector<char> visited(n);
    vector<vll> cicles;
    for (int i = 0; i < n; ++i) {
        if (visited[i]) continue;

        vll cicle;
        ll pos = i;
        while (!visited[pos]) {
            cicle.pb(pos);
            visited[pos] = true;
            pos = ps[pos];
        }

        cicles.push_back(vll(all(cicle)));
    }
    return cicles;
}</pre>
```

# 4 Geometry

## 4.1 Point Template

```
const ld EPS = 1e-6;
typedef ld T;
bool eq(T a, T b) { return abs(a - b) <= EPS; }</pre>
struct point {
 T x, v;
 int id:
 point(T x = 0, T y = 0) : x(x), y(y) {}
 point operator+(const point &o) const { return {x + o.x, y + o.y}; }
 point operator-(const point &o) const { return {x - o.x, y - o.y}; }
 point operator*(T t) const { return {x * t, y * t}; }
 point operator/(T t) const { return {x / t, y / t}; }
 T operator*(const point &o) const {
   return x * o.x + y * o.y;
 } // dot product
 T operator^(const point &o) const {
   return x * o.y - y * o.x;
 } // cross product
ld dist(point a, point b) {
 point d = a - b;
 return sqrt(d * d);
```

# 5 Graphs

# 5.1 2 SAT (struct)

```
struct SAT2 {
```

```
11 n:
vll2d adj, adj_t;
vc used;
vll order, comp;
vc assignment;
bool solvable;
SAT2(11 _n)
 : n(2 * _n),
    adi(n).
    adj_t(n),
    used(n),
    order(n),
    comp(n, -1),
    assignment(n / 2) {}
void dfs1(int v) {
  used[v] = true;
 for (int u : adj[v]) {
    if (!used[u]) dfs1(u);
  order.push_back(v);
}
void dfs2(int v, int cl) {
  comp[v] = c1;
 for (int u : adj_t[v]) {
    if (comp[u] == -1) dfs2(u, cl);
}
bool solve_2SAT() {
  // find and label each SCC
  for (int i = 0; i < n; ++i) {
    if (!used[i]) dfs1(i);
  reverse(all(order));
  11 j = 0;
  for (auto &v : order) {
    if (comp[v] == -1) dfs2(v, j++);
  assignment.assign(n / 2, false);
  for (int i = 0; i < n; i += 2) {
   // x and !x belong to the same SCC
    if (comp[i] == comp[i + 1]) {
      solvable = false;
      return false:
    assignment[i / 2] = comp[i] > comp[i + 1];
  solvable = true;
  return true;
void add_disjunction(int a, bool na, int b, bool nb) {
 a = (2 * a) ^ na;
 b = (2 * b) ^n b:
  int neg_a = a ^ 1;
```

```
int neg_b = b ^ 1;
    adj[neg_a].push_back(b);
    adj[neg_b].push_back(a);
    adj_t[b].push_back(neg_a);
    adj_t[a].push_back(neg_b);
 }
};
     SCC (struct)
struct SCC {
 11 N;
 vll2d adj, tadj;
 vll todo, comps, comp;
 vector<set<ll>> sccadj;
 SCC(11 _N) : N(_N), adj(_N), tadj(_N), comp(_N, -1), sccadj(_N), vis(_N) {}
 void add_edge(ll x, ll y) { adj[x].eb(y), tadj[y].eb(x); }
  void dfs(ll x) {
    vis[x] = 1;
   for (auto &y : adj[x])
     if (!vis[y]) dfs(y);
    todo.pb(x);
 void dfs2(11 x, 11 v) {
    comp[x] = v;
   for (auto &y : tadj[x])
      if (comp[y] == -1) dfs2(y, v);
 void gen() {
   for (11 i = 0; i < N; ++i)</pre>
     if (!vis[i]) dfs(i);
    reverse(all(todo));
    for (auto &x : todo)
     if (comp[x] == -1) {
        dfs2(x, x);
        comps.pb(x);
 }
 void genSCCGraph() {
   for (11 i = 0; i < N; ++i) {</pre>
     for (auto &j : adj[i]) {
        if (comp[i] != comp[j]) {
          sccadj[comp[i]].insert(comp[j]);
     }
};
     SCC Nodes (kosajaru)
* O(n+m)
```

```
* Returns a pair <a. b>
        a: number of SCCs
        b: vector of size n, where b[i] is the SCC id of node i
void dfs(ll u, vchar &visited, const vll2d &g, vll &scc, bool buildScc, ll id,
         vll &sccid) {
 visited[u] = true:
  sccid[u] = id;
 for (auto &v : g[u])
   if (!visited[v]) dfs(v, visited, g, scc, buildScc, id, sccid);
 // if it's the first pass, add the node to the scc
 if (buildScc) scc.eb(u);
pair <11, vll > kosajaru(vll2d &g) {
 ll n = len(g);
  vll scc;
  vchar vis(n);
  vll sccid(n):
 for (11 i = 0; i < n; i++)
   if (!vis[i]) dfs(i, vis, g, scc, true, 0, sccid);
  // build the transposed graph
  v112d gt(n);
 for (int i = 0; i < n; ++i)
   for (auto &v : g[i]) gt[v].eb(i);
  // run the dfs on the previous scc order
  ll id = 1:
  vis.assign(n, false);
 for (ll i = len(scc) - 1; i \ge 0; i--)
   if (!vis[scc[i]]) {
      dfs(scc[i], vis, gt, scc, false, id++, sccid);
 return {id - 1, sccid};
5.4 Bellman Ford
bool bellman_ford(const vector < vector < pair < int, 11>>> &g, int s,
                  vector<ll> &dist) {
 int n = (int)g.size();
  dist.assign(n, LLONG_MAX);
  vector < int > count(n);
  vector < char > in_queue(n);
  queue < int > q;
  dist[s] = 0:
  q.push(s);
  in_queue[s] = true;
  while (not q.empty()) {
   int cur = q.front();
    q.pop();
    in_queue[cur] = false;
```

```
for (auto [to, w] : g[cur]) {
     if (dist[cur] + w < dist[to]) {</pre>
        dist[to] = dist[cur] + w;
        if (not in_queue[to]) {
         q.push(to);
          in_queue[to] = true;
          count[to]++:
          if (count[to] > n) return false;
     }
  return true:
     Check Bipartite
bool checkBipartite(const 11 n, const vector < v11 > & adj) {
 11 s = 0:
 queue < 11 > q;
 q.push(s);
 vll color(n, INF);
 color[s] = 0;
 bool isBipartite = true;
 while (!q.empty() && isBipartite) {
   11 u = q.front();
   q.pop();
   for (auto &v : adj[u]) {
     if (color[v] == INF) {
        color[v] = 1 - color[u];
        q.push(v);
     } else if (color[v] == color[u]) {
        return false;
   }
 }
 return true;
     Count SCC (kosajaru)
void dfs(ll u, vchar &visited, const vll2d &g, vll &scc, bool buildScc) {
 visited[u] = true:
 for (auto &v : g[u])
   if (!visited[v]) dfs(v, visited, g, scc, buildScc);
 // if it's the first pass, add the node to the scc
 if (buildScc) scc.eb(u):
ll kosajaru(v112d &g) {
 ll n = len(g);
 vll scc:
 vchar vis(n);
 for (11 i = 0; i < n; i++)
   if (!vis[i]) dfs(i, vis, g, scc, true);
```

```
// build the transposed graph
  vll2d gt(n);
 for (int i = 0; i < n; ++i)</pre>
   for (auto &v : g[i]) gt[v].eb(i);
 // run the dfs on the previous scc order
 11 \ \text{scccnt} = 0;
  vis.assign(n, false);
 for (ll i = len(scc) - 1; i >= 0; i--)
    if (!vis[scc[i]]) dfs(scc[i], vis, gt, scc, false), scccnt++;
 return scccnt;
     Diikstra
11 __inf = LLONG_MAX >> 5;
vll dijkstra(const vector<vector<pll>>> &g, ll n) {
  priority_queue < pll , vector < pll > , greater < pll >> pq;
  vll dist(n, __inf);
  vector < char > vis(n):
  pq.emplace(0, 0);
  dist[0] = 0;
  while (!pq.empty()) {
   auto [d1, v] = pq.top();
    pq.pop();
    if (vis[v]) continue;
    vis[v] = true;
    for (auto [d2, u] : g[v]) {
      if (dist[u] > d1 + d2) {
        dist[u] = d1 + d2:
        pq.emplace(dist[u], u);
    }
 }
  return dist:
     Floyd Warshall
vector < vll > floyd_warshall(const vector < vll > & adj, ll n) {
  auto dist = adi:
 for (int i = 0; i < n; ++i) {
   for (int j = 0; j < n; ++ j) {
     for (int k = 0; k < n; ++k) {
        dist[j][k] = min(dist[j][k], dist[j][i] + dist[i][k]);
    }
 return dist;
     Kruskal (Python)
```

class DSU:

```
def init (self. n):
        self.n = n
        self.p = [x for x in range(0, n + 1)]
        self.size = [0 for i in range(0, n + 1)]
    def find_set(self, x):
        if self.p[x] == x:
           return x
        else:
            self.p[x] = self.find_set(self.p[x])
           return self.p[x]
    def same_set(self, x, y):
       return bool(self.find_set(x) == self.find_set(y))
    def union_set(self, x, y):
        px = self.find_set(x)
       py = self.find_set(y)
        if px == py:
           return
        size_x = self.size[px]
        size_y = self.size[py]
       if size_x > size_y:
            self.p[py] = self.p[px]
            self.size[px] += self.size[py]
        else:
            self.p[px] = self.p[py]
            self.size[py] += self.size[px]
def kruskal(gv, n):
    Receives te list of edges as a list of tuple in the form:
       d, u, v
       d: distance between u and v
    And also n as the total of verties.
    dsu = DSU(n)
    c = 0
    for e in gv:
       d, u, v = e
       if not dsu.same_set(u, v):
           dsu.union_set(u, v)
    return c
5.10 Lowest Common Ancestor Sparse Table
```

```
int fastlog2(11 x) {
 ull i = x:
 return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
template <typename T>
```

```
class SparseTable {
 public:
  int N;
  int K:
  vector < vector < T >> st;
  SparseTable(vector<T> vs)
   : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector < T > (N + 1)) {
    copy(vs.begin(), vs.end(), st[0].begin());
    for (int i = 1; i <= K; ++i)
      for (int j = 0; j + (1 << i) <= N; ++j)
        st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
  SparseTable() {}
  T RMO(int 1, int r) {
    int i = fastlog2(r - l + 1);
    return min(st[i][1], st[i][r - (1 << i) + 1]);</pre>
};
class LCA {
 public:
 int p;
  int n:
  vi first;
  vector < char > visited;
  vi vertices;
  vi height;
  SparseTable < int > st;
  LCA(const vector <vi> &g)
    : p(0), n((int)g.size()), first(n + 1), visited(n + 1, 0), height(n + 1) {
    build_dfs(g, 1, 1);
    st = SparseTable < int > (vertices);
  }
  void build_dfs(const vector < vi> &g, int u, int hi) {
    visited[u] = true;
    height[u] = hi;
    first[u] = vertices.size();
    vertices.push_back(u);
    for (auto uv : g[u]) {
      if (!visited[uv]) {
        build_dfs(g, uv, hi + 1);
        vertices.push_back(u);
    }
  7
  int lca(int a, int b) {
   int l = min(first[a], first[b]);
    int r = max(first[a], first[b]);
    return st.RMQ(1, r);
};
       Topological Sorting
5.11
```

/\*

```
* 0(V)
 * assumes:
        * vertices have index [0, n-1]
 * if is a DAG:
       * returns a topological sorting
 * else:
      * returns an empty vector
enum class state { not_visited, processing, done };
bool dfs(const vector <vll> &adj, ll s, vector <state> &states, vll &order) {
  states[s] = state::processing;
 for (auto &v : adj[s]) {
   if (states[v] == state::not_visited) {
     if (not dfs(adj, v, states, order)) return false;
   } else if (states[v] == state::processing)
      return false;
 states[s] = state::done;
 order.pb(s);
 return true:
vll topologicalSorting(const vector<vll> &adj) {
 ll n = len(adj);
 vll order;
 vector < state > states(n, state::not_visited);
 for (int i = 0; i < n; ++i) {</pre>
   if (states[i] == state::not_visited) {
     if (not dfs(adj, i, states, order)) return {};
   }
 }
 reverse(all(order));
 return order;
```

# 6 Math

# 6.1 Combinatorics With Repetitions

```
pos = k - 1;
 }
      Count Divisors Memo
const 11 mod = 1073741824;
const ll maxd = 100 * 100 * 100 + 1;
vector < 11 > memo(maxd, -1);
11 countdivisors(11 x) {
 11 ox = x:
 ll ans = 1;
 for (11 i = 2; i <= x; ++i) {
   if (memo[x] != -1) {
      ans *= memo[x];
      break;
    11 count = 0;
    while (x \text{ and } x \% i == 0) {
    x /= i;
      count++;
    ans *= (count + 1);
  memo[ox] = ans:
  return ans;
     Euler Phi
const ll MAXN = 1e5;
vll list_primes(ll n) { // Nlog * log N
 vll ps;
 bitset < MAXN > sieve;
  sieve.set();
  sieve.reset(1):
 for (11 i = 2; i <= n; ++i) {
   if (sieve[i]) ps.push_back(i);
   for (11 j = i * 2; j <= n; j += i) {
      sieve.reset(j);
 }
 return ps;
vector<pll> factorization(ll n, const vll &primes) {
 vector < pll > ans;
 for (auto &p : primes) {
   if (n == 1) break:
   11 cnt = 0;
   while (n \% p == 0) {
      cnt++:
      n /= p;
    if (cnt) ans.emplace_back(p, cnt);
 return ans;
```

```
11 phi(ll n, vector<pll> factors) {
 if (n == 1) return 1:
 11 \text{ ans } = n;
 for (auto [p, k] : factors) {
   ans /= p;
   ans *= (p - 1);
 return ans;
6.4 Factorial Factorization
// O(logN) greater k that p^k | n
11 E(11 n, 11 p) {
 11 k = 0, b = p;
 while (b \le n) {
  k += n / b;
   b *= p;
 return k;
// lsit every prime until MAXN O(Nlog * log N)
const ll MAXN = 1e5:
vll list_primes(ll n) {
 vll ps;
 bitset < MAXN > sieve;
 sieve.set();
 sieve.reset(1):
 for (11 i = 2; i <= n; ++i) {
   if (sieve[i]) ps.push_back(i);
   for (11 j = i * 2; j <= n; j += i) sieve.reset(j);</pre>
 }
 return ps;
// O(pi(N)*logN)
map<11, 11> factorial_factorization(11 n, const v11 &primes) {
 map < 11, 11 > fs;
 for (const auto &p : primes) {
   if (p > n) break;
   fs[p] = E(n, p);
 return fs;
6.5 Factorial
const 11 MAX = 18:
vll fv(MAX, -1);
11 factorial(ll n) {
 if (fv[n] != -1) return fv[n];
 if (n == 0) return 1;
 return n * factorial(n - 1);
```

# } 6.6 Factorization With Primes // Nlog \* log N const ll MAXN = 1e5; vll list\_primes(ll n) { vll ps; bitset < MAXN > sieve; sieve.set(); sieve.reset(1); for (11 i = 2; i <= n; ++i) { if (sieve[i]) ps.push\_back(i); for (11 j = i \* 2; j <= n; j += i) sieve.reset(j);</pre> } return ps; // O(pi(sqrt(n))) map<11, 11> factorization(11 n, const v11 &primes) { map < 11, 11 > ans; for (auto p : primes) { if (p \* p > n) break; 11 count = 0;for (; n % p == 0; count++, n /= p) if (count) ans[p] = count; return ans; Factorization // O(sqrt(n)) map<ll, ll> factorization(ll n) { map<11, 11> ans; for (ll i = 2; i \* i <= n; i++) { 11 count = 0: for (; n % i == 0; count++, n /= i) if (count) ans[i] = count; if (n > 1) ans[n]++; return ans: Fast Fourrier Transform template <bool invert = false> void fft(vector<complex<double>>& xs) { int N = (int)xs.size(); if (N == 1) return; vector < complex < double >> es(N / 2), os(N / 2);

for (int i = 0; i < N / 2; ++i) es[i] = xs[2 \* i];

```
for (int i = 0; i < N / 2; ++i) os[i] = xs[2 * i + 1];
 fft < invert > (es):
 fft < invert > (os);
 auto signal = (invert ? 1 : -1);
 auto theta = 2 * signal * acos(-1) / N;
  complex <double > S{1}, S1{cos(theta), sin(theta)};
  for (int i = 0; i < N / 2; ++i) {
   xs[i] = (es[i] + S * os[i]);
   xs[i] /= (invert ? 2 : 1);
   xs[i + N / 2] = (es[i] - S * os[i]):
   xs[i + N / 2] /= (invert ? 2 : 1);
   S *= S1;
 }
     Fast Exp
 Fast exponentiation algorithm,
 compute a^n in O(log(n))
ll fexp(ll a, int n) {
 if (n == 0) return 1;
 if (n == 1) return a;
 ll x = fexp(a, n / 2);
 return x * x * (n & 1 ? a : 1);
6.10 Gauss Elimination
template <size t Dim>
struct GaussianElimination {
 vector <11> basis:
 size_t size;
 GaussianElimination() : basis(Dim + 1), size(0) {}
 void insert(ll x) {
   for (11 i = Dim; i >= 0; i--) {
     if ((x & 111 << i) == 0) continue;
     if (!basis[i]) {
       basis[i] = x;
       size++:
       break;
     }
     x ^= basis[i];
 }
 void normalize() {
```

```
for (11 i = Dim: i >= 0: i--)
      for (11 j = i - 1; j \ge 0; j--)
        if (basis[i] & 111 << j) basis[i] ^= basis[j];</pre>
  }
  bool check(ll x) {
    for (11 i = Dim; i >= 0; i--) {
      if ((x & 111 << i) == 0) continue;
      if (!basis[i]) return false;
      x ^= basis[i];
    return true:
  }
  auto operator[](11 k) { return at(k); }
  11 at(11 k) {
    11 \text{ ans} = 0;
    11 total = 111 << size:</pre>
    for (11 i = Dim; ~i; i--) {
      if (!basis[i]) continue;
      11 mid = total >> 111;
      if ((mid < k and (ans & 111 << i) == 0) ||
          (k <= mid and (ans & 111 << i)))
        ans ^= basis[i];
      if (mid < k) k -= mid:
      total >>= 111:
    }
    return ans;
  11 at_normalized(ll k) {
    11 \text{ ans} = 0:
    k--:
    for (size_t i = 0; i <= Dim; i++) {</pre>
      if (!basis[i]) continue;
      if (k & 1) ans ^= basis[i];
      k >>= 1:
    return ans:
};
6.11 Gcd Using Factorization
// O(sqrt(n))
map<ll, ll> factorization(ll n) {
 map<11, 11> ans;
 for (ll i = 2; i * i <= n; i++) {
    11 count = 0;
    for (; n % i == 0; count++, n /= i)
```

```
if (count) ans[i] = count:
 if (n > 1) ans [n]++;
 return ans:
ll gcd_with_factorization(ll a, ll b) {
 map<11, 11> fa = factorization(a);
 map<11, 11> fb = factorization(b);
 11 \text{ ans} = 1;
 for (auto fai : fa) {
   11 k = min(fai.second, fb[fai.first]);
   while (k--) ans *= fai.first;
 return ans:
6.12 Gcd
11 gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
6.13 Integer Mod
const 11 INF = 1e18;
const 11 mod = 998244353;
template <11 MOD = mod>
struct Modular {
 ll value:
 static const 11 MOD_value = MOD;
 Modular(11 v = 0) {
   value = v % MOD;
   if (value < 0) value += MOD;</pre>
 Modular(ll a, ll b) : value(0) {
   *this += a:
   *this /= b:
 Modular& operator+=(Modular const& b) {
    value += b.value;
   if (value >= MOD) value -= MOD:
   return *this;
  Modular& operator -= (Modular const& b) {
    value -= b.value;
   if (value < 0) value += MOD;</pre>
   return *this;
  Modular& operator *= (Modular const& b) {
    value = (11)value * b.value % MOD;
   return *this;
 friend Modular mexp(Modular a, ll e) {
    Modular res = 1;
    while (e) {
     if (e & 1) res *= a;
```

```
a *= a:
      e >>= 1;
    return res:
  friend Modular inverse (Modular a) { return mexp(a, MOD - 2); }
  Modular& operator/=(Modular const& b) { return *this *= inverse(b); }
  friend Modular operator+(Modular a. Modular const b) { return a += b; }
  Modular operator++(int) { return this->value = (this->value + 1) % MOD; }
  Modular operator++() { return this->value = (this->value + 1) % MOD; }
  friend Modular operator - (Modular a, Modular const b) { return a -= b; }
  friend Modular operator - (Modular const a) { return 0 - a; }
  Modular operator -- (int) {
    return this->value = (this->value - 1 + MOD) % MOD:
  }
  Modular operator -- () { return this -> value = (this -> value - 1 + MOD) % MOD; }
  friend Modular operator*(Modular a, Modular const b) { return a *= b; }
  friend Modular operator/(Modular a, Modular const b) { return a /= b; }
  friend std::ostream& operator<<(std::ostream& os, Modular const& a) {</pre>
    return os << a.value:
  friend bool operator == (Modular const& a, Modular const& b) {
    return a.value == b.value:
 friend bool operator!=(Modular const& a, Modular const& b) {
    return a.value != b.value:
};
6.14 Is Prime
bool isprime(ll n) { // O(sqrt(n))
  if (n < 2) return false;
  if (n == 2) return true:
  if (n % 2 == 0) return false;
  for (11 i = 3; i * i < n; i += 2)
    if (n % i == 0) return false:
  return true;
6.15 Lcm Using Factorization
map<ll, ll> factorization(ll n) {
  map<11, 11> ans;
 for (11 i = 2; i * i <= n; i++) {
   11 count = 0;
    for (; n % i == 0; count++, n /= i)
    if (count) ans[i] = count;
  if (n > 1) ans[n]++;
  return ans;
ll lcm_with_factorization(ll a, ll b) {
  map<ll, ll> fa = factorization(a);
```

```
map<ll. 11> fb = factorization(b):
  ll ans = 1;
  for (auto fai : fa) {
   11 k = max(fai.second, fb[fai.first]);
    while (k--) ans *= fai.first;
 }
  return ans;
6.16 Lcm
11 gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
11 lcm(ll a, ll b) { return a / gcd(a, b) * b; }
6.17 Modular Inverse Using Phi
map<ll, ll> factorization(ll n) {
  map < 11, 11 > ans;
  for (11 i = 2; i * i <= n; i++) {
    11 count = 0:
    for (; n % i == 0; count++, n /= i)
    if (count) ans[i] = count;
  if (n > 1) ans[n]++;
  return ans;
11 phi(11 n) {
  if (n == 1) return 1;
  auto fs = factorization(n);
  auto res = n:
  for (auto [p, k] : fs) {
   res /= p;
   res *= (p - 1);
  return res;
11 fexp(ll a, ll n, ll mod) {
 if (n == 0) return 1:
 if (n == 1) return a;
 11 x = fexp(a, n / 2, mod):
 return x * x * (n & 1 ? a : 1) % mod;
}
11 inv(11 a, 11 mod) { return fexp(a, phi(mod) - 1, mod); }
6.18 N Choose K Count
 * O(nm) time, O(m) space
* equal to n choose k
```

```
ll binom(ll n. ll k) {
 if (k > n) return 0;
  vll dp(k + 1, 0);
  dp[0] = 1;
 for (ll i = 1; i <= n; i++)
   for (ll j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
 return dp[k];
6.19 Permutation Count
const 11 MAX = 18:
vll fv(MAX, -1);
ll factorial(ll n) {
 if (fv[n] != -1) return fv[n]:
 if (n == 0) return 1;
  return n * factorial(n - 1);
template <typename T>
11 permutation_count(vector <T> xs) {
  map < T, 11 > h;
  for (auto xi : xs) h[xi]++;
  ll ans = factorial((ll)xs.size());
  dbg(ans);
 for (auto [v, cnt] : h) {
    dbg(cnt);
    ans /= cnt;
  }
  return ans;
6.20 Polynomial
using polynomial = vector<11>;
int degree(const polynomial &xs) { return xs.size() - 1; }
ll horner_evaluate(const polynomial &xs, ll x) {
 11 \text{ ans} = 0:
 11 n = degree(xs);
 for (int i = n; i >= 0; --i) {
   ans *= x:
    ans += xs[i];
  return ans;
polynomial operator+(const polynomial &a, const polynomial &b) {
 int n = degree(a);
  int m = degree(b);
  polynomial r(max(n, m) + 1, 0);
  for (int i = 0; i <= n; ++i) r[i] += a[i];
  for (int j = 0; j <= m; ++j) r[j] += b[j];
  while (!r.empty() and r.back() == 0) r.pop_back();
 if (r.empty()) r.push_back(0);
  return r;
polynomial operator*(const polynomial &p, const polynomial &q) {
```

```
int n = degree(p);
int m = degree(q);
polynomial r(n + m + 1, 0);
for (int i = 0; i <= n; ++i)
   for (int j = 0; j <= m; ++j) r[i + j] += (p[i] * q[j]);
return r;</pre>
```

## 6.21 Power Sum

```
// calculates K^0 + K^1 ... + K^n
ll fastpow(ll a, int n) {
   if (n == 1) return a;
   ll x = fastpow(a, n / 2);
   return x * x * (n & 1 ? a : 1);
}
ll powersum(ll n, ll k) { return (fastpow(n, k + 1) - 1) / (n - 1); }
```

## 6.22 Sieve List Primes

```
// lsit every prime until MAXN
const ll MAXN = 1e5;
vll list_primes(ll n) { // Nlog * log N
  vll ps;
  bitset<MAXN> sieve;
  sieve.set();
  sieve.reset(1);
  for (ll i = 2; i <= n; ++i) {
    if (sieve[i]) ps.push_back(i);
    for (ll j = i * 2; j <= n; j += i) {
       sieve.reset(j);
    }
  }
  return ps;
}</pre>
```

# 7 Searching

# 7.1 Ternary Search Recursive

```
const double eps = 1e-6;

// IT MUST BE AN UNIMODAL FUNCTION
double f(int x) { return x * x + 2 * x + 4; }

double ternary_search(double 1, double r) {
   if (fabs(f(1) - f(r)) < eps) return f((1 + (r - 1) / 2.0));

   auto third = (r - 1) / 3.0;
   auto m1 = 1 + third;
   auto m2 = r - third;

   // change the signal to find the maximum point.
   return m1 < m2 ? ternary_search(m1, r) : ternary_search(1, m2);
}</pre>
```

# 8 Strings

# 8.1 Hash Range Query

```
struct Hash {
  const 11 P = 31;
  int n;
  string s;
  vll h, hi, p;
  Hash() {}
  Hash(string s) : s(s), n(s.size()), h(n), hi(n), p(n) {
    for (int i = 0; i < n; i++) p[i] = (i ? P * p[i - 1] : 1) % MOD;
    for (int i = 0; i < n; i++) h[i] = (s[i] + (i ? h[i - 1] : 0) * P) % MOD;
    for (int i = n - 1; i >= 0; i--)
      hi[i] = (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % MOD;
  11 query(int 1, int r) {
    ll hash = (h[r] - (1 ? h[1 - 1] * p[r - 1 + 1] % MOD : 0));
    return hash < 0 ? hash + MOD : hash:
  11 query_inv(int 1, int r) {
    ll hash = (hi[1] - (r + 1 < n ? hi[r + 1] * p[r - 1 + 1] % MOD : 0));
    return hash < 0 ? hash + MOD : hash;
};
```

## 8.2 Longest Palindrome

```
string longest_palindrome(const string &s) {
 int n = (int)s.size();
 vector < array < int , 2>> dp(n);
 pii odd(0, -1), even(0, -1);
 pii ans;
 for (int i = 0: i < n: i++) {
   int k = 0;
   if (i > odd.second)
     k = 1:
     k = min(dp[odd.first + odd.second - i][0], odd.second - i + 1);
    while (i - k \ge 0 \text{ and } i + k < n \text{ and } s[i - k] == s[i + k]) k++;
    dp[i][0] = k--:
   if (i + k > odd.second) odd = \{i - k, i + k\};
   if (2 * dp[i][0] - 1 > ans.second) ans = \{i - k, 2 * dp[i][0] - 1\};
   k = 0:
    if (i <= even.second)</pre>
     k = min(dp[even.first + even.second - i + 1][1], even.second - i + 1);
    while (i - k - 1) = 0 and i + k < n and s[i - k - 1] == s[i + k]) k++;
    dp[i][1] = k--;
   if (i + k > even.second) even = \{i - k - 1, i + k\};
   if (2 * dp[i][1] > ans.second) ans = \{i - k - 1, 2 * dp[i][1]\};
 return s.substr(ans.first, ans.second);
```

# 8.3 Rabin Karp

```
size_t rabin_karp(const string &s, const string &p) {
 if (s.size() < p.size()) return 0;</pre>
 auto n = s.size(). m = p.size();
 const 11 p1 = 31, p2 = 29, q1 = 1e9 + 7, q2 = 1e9 + 9;
 const ll p1_1 = fpow(p1, q1 - 2, q1), p1_2 = fpow(p1, m - 1, q1);
 const 11 p2_1 = fpow(p2, q2 - 2, q2), p2_2 = fpow(p2, m - 1, q2);
 pair <11, 11> hs. hp:
 for (int i = (int)m - 1; ~i; --i) {
   hs.first = (hs.first * p1) % q1;
   hs.first = (hs.first + (s[i] - 'a' + 1)) % q1;
   hs.second = (hs.second * p2) % q2;
   hs.second = (hs.second + (s[i] - 'a' + 1)) % q2:
    hp.first = (hp.first * p1) % q1;
   hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
   hp.second = (hp.second * p2) % q2;
   hp.second = (hp.second + (p[i] - 'a' + 1)) \% q2;
 size t occ = 0:
 for (size t i = 0: i < n - m: i++) {</pre>
    occ += (hs == hp);
   int fi = s[i] - 'a' + 1;
    int fm = s[i + m] - a' + 1;
   hs.first = (hs.first - fi + q1) % q1;
   hs.first = (hs.first * p1_1) % q1;
   hs.first = (hs.first + fm * p1 2) % a1:
   hs.second = (hs.second - fi + q2) \% q2;
   hs.second = (hs.second * p2_1) % q2;
   hs.second = (hs.second + fm * p2_2) % q2;
 occ += hs == hp;
 return occ;
8.4 String Psum
struct strPsum {
 11 n:
 vector < vll> psum;
 strPsum(const string \&s) : n(s.size()), k(100), psum(k, vll(n + 1)) {
   for (ll i = 1; i <= n; ++i) {
      for (11 j = 0; j < k; ++j) {
        psum[i][i] = psum[i][i - 1]:
      psum[s[i - 1]][i]++;
 }
 ll qtd(ll l, ll r, char c) { // [0,n-1]}
   return psum[c][r + 1] - psum[c][1];
```

## 8.5 Trie Naive

```
// time: O(n^2) memory: O(n^2)
using Node = map < char, int >;
using vi = vector<int>;
using Trie = vector < Node >;
Trie build(const string &s) {
 int n = (int)s.size();
 Trie trie(1):
  string suffix;
 for (int i = n - 1; i \ge 0; --i) {
    suffix = s.substr(i) + '#':
   int v = 0: // root
   for (auto c : suffix) {
     if (c == '#') { // makrs the poistion of an occurence
        trie[v][c] = i:
        break:
      if (trie[v][c])
       v = trie[v][c];
      else {
        trie.push_back({});
        trie[v][c] = trie.size() - 1:
       v = trie.size() - 1;
   }
 }
 return trie;
vi search(Trie &trie, string s) {
 int p = 0:
 vi occ;
 for (auto &c : s) {
   p = trie[p][c];
   if (!p) return occ;
  queue < int > q:
  q.push(0);
  while (!q.empty()) {
   auto cur = q.front();
   q.pop();
   for (auto [c, v] : trie[cur]) {
     if (c == '#')
        occ.push_back(v);
      else
        q.push(v);
  return occ;
```

```
11 distinct_substr(const Trie &trie) {
    11 cnt = 0;
    queue <int > q;
    q.push(0);
    while (!q.empty()) {
        auto u = q.front();
        q.pop();

        for (auto [c, v] : trie[u]) {
            if (c != '#') {
                cnt++;
                 q.push(v);
            }
        }
    }
    return cnt;
}
```

## 8.6 Z Function Get Occurrence Positions

```
* ans[i] = a position where p matchs
* with s perfectly starting
* */
vi getOccPos(string &s, string &p) {
 // Z-function
 char delim = '#';
 string t{p + delim + s};
 vi zs(len(t));
 int 1 = 0, r = 0;
 for (int i = 1: i < len(t): i++) {
   if (i <= r) zs[i] = min(zs[i - 1], r - i + 1);</pre>
   while (zs[i] + i < len(t)) and t[zs[i]] == t[i + zs[i]]) zs[i]++;
   if (r < i + zs[i] - 1) l = i, r = i + zs[i] - 1:
 // Iterate over the results of Z-function to get ranges
 vi ans:
 int start = len(p) + 1 + 1 - 1;
 for (int i = start; i < len(zs); i++) {</pre>
   if (zs[i] == len(p)) {
     int l = i - start:
      ans.emplace_back(1);
 }
 return ans;
```

# 9 Trees

# 9.1 Binary Lifting

```
/*
 * far[h][i] = the node that 2^h far from node i
 * sometimes is useful invert the order of loops
 * time : O(nlogn)
```

```
* */
const int maxlog = 20;
int far[maxlog + 1][n + 1];
int n:
for (int h = 1; h <= maxlog; h++) {</pre>
 for (int i = 1; i <= n; i++) {
    far[h][i] = far[h - 1][far[h - 1][i]];
 }
      Maximum Distances
* Returns the maximum distance from every node to any other node in the tree.
pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
 // O indexed
 11 mostDistantNode = root;
 11 nodeDistance = 0;
  queue <pll> q;
  vector < char > vis(n):
  q.emplace(root, 0);
  vis[root] = true;
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
    if (dist > nodeDistance) {
      nodeDistance = dist:
      mostDistantNode = node;
   for (auto u : adj[node]) {
      if (!vis[u]) {
        vis[u] = true;
        q.emplace(u, dist + 1);
    }
  return {mostDistantNode, nodeDistance};
ll twoNodesDist(const vector < vll > & adj, ll n, ll a, ll b) {
 queue <pll> q;
 vector < char > vis(n);
  q.emplace(a, 0);
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
   if (node == b) return dist;
   for (auto u : adj[node]) {
     if (!vis[u]) {
        vis[u] = true;
        q.emplace(u, dist + 1);
   }
  return -1;
```

```
tuple < 11, 11, 11> tree_diameter(const vector < v11> & adj, 11 n) {
 // returns two points of the diameter and the diameter itself
 auto [node1, dist1] = mostDistantFrom(adj, n, 0);
 auto [node2. dist2] = mostDistantFrom(adj. n. node1);
 auto diameter = twoNodesDist(adj, n, node1, node2);
 return make_tuple(node1, node2, diameter);
vll everyDistanceFromNode(const vector<vll> &adi. ll n. ll root) {
 // Single Source Shortest Path, from a given root
 queue <pair <11, 11>> q;
 vll ans(n, -1);
 ans[root] = 0;
 q.emplace(root, 0);
 while (!a.emptv()) {
   auto [u, d] = q.front();
   q.pop();
   for (auto w : adj[u]) {
     if (ans[w] != -1) continue:
     ans[w] = d + 1;
     q.emplace(w, d + 1);
 return ans;
vll maxDistances(const vector < vll > & adj, ll n) {
 auto [node1, node2, diameter] = tree_diameter(adj, n);
 auto distances1 = everyDistanceFromNode(adj, n, node1);
 auto distances2 = evervDistanceFromNode(adi. n. node2);
 for (int i = 0: i < n: ++i) ans[i] = max(distances1[i]. distances2[i]):
 return ans;
     Tree Diameter
pll mostDistantFrom(const vector < vll > & adi. 11 n. 11 root) {
 // O indexed
 11 mostDistantNode = root;
 11 nodeDistance = 0:
 queue <pll> q;
 vector < char > vis(n):
 q.emplace(root, 0);
 vis[root] = true;
 while (!q.empty()) {
   auto [node, dist] = q.front();
   q.pop();
   if (dist > nodeDistance) {
     nodeDistance = dist;
     mostDistantNode = node:
   for (auto u : adj[node]) {
     if (!vis[u]) {
       vis[u] = true;
       q.emplace(u, dist + 1);
```

```
return {mostDistantNode, nodeDistance};
11 twoNodesDist(const vector < vll> &adj, ll n, ll a, ll b) {
 // 0 indexed
  aueue <pl1> a:
  vector < char > vis(n);
  q.emplace(a, 0);
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
    if (node == b) {
     return dist:
   for (auto u : adj[node]) {
     if (!vis[u]) {
        vis[u] = true:
        q.emplace(u, dist + 1);
   }
  return -1:
ll tree_diameter(const vector < vll > & adj, ll n) {
 // 0 indexed !!!
  auto [node1, dist1] = mostDistantFrom(adj, n, 0);
  auto [node2, dist2] = mostDistantFrom(adj, n, node1);
  auto diameter = twoNodesDist(adj, n, node1, node2);
  return diameter:
      Settings and macros
10.1 .vimrc
set ts=4 sw=4 sta nu rnu sc cindent
set bg=dark ruler clipboard=unnamed,unnamedplus, timeoutlen=100
colorscheme default
nnoremap <C-j>:botright belowright term bash <CR>
syntax on
10.2
     degug.cpp
#include <bits/stdc++.h>
using namespace std;
/****** Debug Code ******/
template <tvpename T>
concept Printable = requires(T t) {
    { std::cout << t } -> std::same_as<std::ostream &>;
template <Printable T>
void __print(const T &x) {
    cerr << x;
```

template <size\_t T>

```
void __print(const bitset<T> &x) {
    cerr << x;
}
template <typename A, typename B>
void __print(const pair<A, B> &p);
                                                                                   }
template <typename... A>
void __print(const tuple<A...> &t);
template <typename T>
void __print(stack<T> s);
template <typename T>
void __print(queue < T > q);
template <typename T, typename... U>
void __print(priority_queue < T, U... > q);
template <typename A>
void __print(const A &x) {
    bool first = true;
    cerr << '{';
    for (const auto &i : x) {
        cerr << (first ? "" : ","), __print(i);</pre>
        first = false;
    }
    cerr << '}':
}
template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '(';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << ')':
template <typename... A>
void __print(const tuple < A... > &t) {
    bool first = true;
    cerr << '(';
    apply(
        [&first](const auto &...args) {
            ((cerr << (first ? "" : ","), __print(args), first = false), ...);
        },
        t);
    cerr << ')';
template <typename T>
void __print(stack<T> s) {
    vector <T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
template <typename T>
void __print(queue < T > q) {
    vector <T> debugVector;
    while (!q.empty()) {
        T t = q.front();
```

```
debugVector.push_back(t);
        q.pop();
   }
    __print(debugVector);
template <typename T, typename... U>
void __print(priority_queue < T, U... > q) {
    vector <T> debugVector;
    while (!q.empty()) {
       T t = q.top();
        debugVector.push_back(t);
        q.pop();
    __print(debugVector);
void _print() { cerr << "]\n"; }</pre>
template <typename Head, typename... Tail>
void _print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";
    _print(T...);
#define dbg(x...)
    cerr << "[" << #x << "] = [": \
    _print(x)
10.3 .bashrc
cpp() {
 echo ">> COMPILING <<" 1>&2
 g++ -std=c++17 \
      -02 \
      -g \
      -g3 \
      -Wextra \
      -Wshadow \
      -Wformat=2 \
      -Wconversion \
      -fsanitize=address,undefined \
      -fno-sanitize-recover \
      -Wfatal-errors \
      -DDEBUG $1 \
  if [ $? -ne 0 ]; then
      echo ">> FAILED <<" 1>&2
      return 1
  echo ">> DONE << " 1>&2
 time ./a.out ${@:2}
prepare() {
   for i in {a..z}
        cp macro.cpp $i.cpp
        touch $i.py
    done
```

```
for i in {1..10}
        touch in${i}
        touch out${i}
        touch ans${i}
    done
}
10.4 macro.cpp
#include <bits/stdc++.h>
using namespace std;
#ifdef DEBUG
#include "debug.cpp"
#else
#define dbg(...) 666
#endif
#define endl '\n'
#define fastio
   ios_base::sync_with_stdio(false); \
    cin.tie(0);
    cout.tie(0);
#define len(__x) (ll) __x.size()
```

using ll = long long; using vll = vector<ll>;

```
using pll = pair<11, 11>;
using v112d = vector < v11 >;
using vi = vector<int>;
using vi2d = vector < vi>;
using pii = pair < int , int >;
using vii = vector<pii>;
using vc = vector < char >;
#define all(a) a.begin(), a.end()
#define snd second
#define fst first
#define pb(___x) push_back(___x)
#define mp(__a, __b) make_pair(__a, __b)
#define eb(___x) emplace_back(___x)
const ll INF = 1e18:
void run() {
int32_t main(void) {
   fastio;
   int t:
   t = 1:
   // cin >> t;
   while (t--) run();
```