

# The Reference

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# 1 Data structures

## 1.1 Dsu

```
struct DSU {
    vector<int> ps;
    vector<int> size;
    DSU(int N) : ps(N + 1), size(N + 1, 1) { iota(ps.begin(), ps.end(), 0); }
    int find_set(int x) { return ps[x] == x ? x : ps[x] = find_set(ps[x]); }
    bool same_set(int x, int y) { return find_set(x) == find_set(y); }
    void union_set(int x, int y) {
        if (same_set(x, y)) return;

        int px = find_set(x);
        int py = find_set(y);

        if (size[px] < size[py]) swap(px, py);

        ps[py] = px;
        size[px] += size[py];
    }
};
```

## 1.2 Dsu (Python)

```
class DSU:
    def __init__(self, n):
        self.n = n
        self.p = [x for x in range(0, n + 1)]
        self.size = [0 for i in range(0, n + 1)]

    def find_set(self, x): # log n
        if self.p[x] == x:
            return x
        else:
            self.p[x] = self.find_set(self.p[x])
            return self.p[x]

    def same_set(self, x, y): # log n
        return bool(self.find_set(x) == self.find_set(y))

    def union_set(self, x, y): # log n
        px = self.find_set(x)
        py = self.find_set(y)

        if px == py:
            return

        size_x = self.size[px]
        size_y = self.size[py]

        if size_x > size_y:
            self.p[py] = self.p[px]
            self.size[px] += self.size[py]
        else:
            self.p[px] = self.p[py]
            self.size[py] += self.size[px]
```

## 1.3 Ordered Set Gnu Pbds

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename T>
// using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
// tree_order_statistics_node_update>;

// if you want to find the elements less or equal :p
using ordered_set = tree<T, null_type, less_equal<T>, rb_tree_tag,
                        tree_order_statistics_node_update>;
```

## 1.4 Segtree Rmaxq Rmaxu

```
template <typename T = ll>
struct SegTree {
    int N;
    T nu, nq;
    vector<T> st, lazy;
    SegTree(const vector<T> &xs)
        : N(len(xs)),
          nu(numeric_limits<T>::min()),
          nq(numeric_limits<T>::min()),
          st(4 * N + 1, nu),
          lazy(4 * N + 1, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    void update(int l, int r, T value) { update(1, 0, N - 1, l, r, value); }

    T query(int l, int r) { return query(1, 0, N - 1, l, r); }

    void update(int node, int nl, int nr, int ql, int qr, T v) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return;

        st[node] = max(st[node], v);
        if (ql <= nl and nr <= qr) {
            if (nl < nr) {
                lazy[left(node)] = max(lazy[left(node)], v);
                lazy[right(node)] = max(lazy[right(node)], v);
            }
            return;
        }
        update(left(node), nl, mid(nl, nr), ql, qr, v);
        update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

        st[node] = max(st[left(node)], st[right(node)]);
    }

    T query(int node, int nl, int nr, int ql, int qr) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return nq;

        if (ql <= nl and nr <= qr) return st[node];
```

```

    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return max(x, y);
}

void propagation(int node, int nl, int nr) {
    if (lazy[node] != nu) {
        st[node] = max(st[node], lazy[node]);

        if (nl < nr) {
            lazy[left(node)] = max(lazy[left(node)], lazy[node]);
            lazy[right(node)] = max(lazy[right(node)], lazy[node]);
        }

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

int main() {
    int n;
    cin >> n;
    vector<array<int, 3>> xs(n);
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < 3; ++j) {
            cin >> xs[i][j];
        }
    }
    vi aux(n, 0);
    SegTree<int> st(aux);
    for (int i = 0; i < n; ++i) {
        int a = min(i + xs[i][1], n);
        int b = min(i + xs[i][2], n);
        st.update(i, i, st.query(i, i) + xs[i][0]);
        int cur = st.query(i, i);
        st.update(a, b, cur);
    }

    cout << st.query(0, n) << '\n';
}

```

## 1.5 Segtree Rminq Pau

```

template <typename T = ll>
struct SegTree {
    int n;
    T nu, nq;
    vector<T> st;
    SegTree(const vector<T> &v)
        : n(len(v)), nu(0), nq(numeric_limits<T>::max()), st(n * 4 + 1, nu) {
        for (int i = 0; i < n; ++i) update(i, v[i]);
    }
    void update(int p, T v) { update(1, 0, n - 1, p, v); }

```

```

    T query(int l, int r) { return query(1, 0, n - 1, l, r); }

    void update(int node, int nl, int nr, int p, T v) {
        if (p < nl or p > nr) return;

        if (nl == nr) {
            st[node] = v;
            return;
        }

        update(left(node), nl, mid(nl, nr), p, v);
        update(right(node), mid(nl, nr) + 1, nr, p, v);

        st[node] = min(st[left(node)], st[right(node)]);
    }

    T query(int node, int nl, int nr, int ql, int qr) {
        if (ql <= nl and qr >= nr) return st[node];
        if (nl > qr or nr < ql) return nq;
        if (nl == nr) return st[node];

        return min(query(left(node), nl, mid(nl, nr), ql, qr),
                    query(right(node), mid(nl, nr) + 1, nr, ql, qr));
    }

    int left(int p) { return p << 1; }
    int right(int p) { return (p << 1) + 1; }
    int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

## 1.6 Segtree Rminq Rsu

```

template <typename t = ll>
struct SegTree {
    int n;
    t nu;
    t nq;
    vector<t> st, lazy;
    SegTree(const vector<t> &xs)
        : n(len(xs)),
          nu(0),
          nq(numeric_limits<t>::max()),
          st(4 * n, nu),
          lazy(4 * n, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }

    SegTree(int n) : n(n), st(4 * n, nu), lazy(4 * n, nu) {}

    void update(int l, int r, ll value) { update(1, 0, n - 1, l, r, value); }

    t query(int l, int r) { return query(1, 0, n - 1, l, r); }

    void update(int node, int nl, int nr, int ql, int qr, ll v) {
        propagation(node, nl, nr);

        if (ql > nr or qr < nl) return;

```

```

    if (ql <= nl and nr <= qr) {
        st[node] += (nr - nl + 1) * v;

        if (nl < nr) {
            lazy[left(node)] += v;
            lazy[right(node)] += v;
        }

        return;
    }

    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

    st[node] = min(st[left(node)], st[right(node)]);
}

t query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    t x = query(left(node), nl, mid(nl, nr), ql, qr);
    t y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return min(x, y);
}

void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
        st[node] += lazy[node];

        if (nl < nr) {
            lazy[left(node)] += lazy[node];
            lazy[right(node)] += lazy[node];
        }

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }
int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

## 1.7 Segtree Rsq Rsu

```

template <typename T = ll>
struct SegTree {
    int N;
    vector<T> st, lazy;
    T nu = 0;
    T nq = 0;
    SegTree(const vector<T> &xs) : N(len(xs)), st(4 * N, nu), lazy(4 * N, nu) {
        for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
    }
};

```

```

}

SegTree(int n) : N(n), st(4 * N, nu), lazy(4 * N, nu) {}

void update(int l, int r, ll value) { update(1, 0, N - 1, l, r, value); }

T query(int l, int r) { return query(1, 0, N - 1, l, r); }

void update(int node, int nl, int nr, int ql, int qr, ll v) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return;

    if (ql <= nl and nr <= qr) {
        st[node] += (nr - nl + 1) * v;

        if (nl < nr) {
            lazy[left(node)] += v;
            lazy[right(node)] += v;
        }

        return;
    }

    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);

    st[node] = st[left(node)] + st[right(node)];
}

T query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);

    if (ql > nr or qr < nl) return nq;

    if (ql <= nl and nr <= qr) return st[node];

    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);

    return x + y;
}

void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
        st[node] += (nr - nl + 1) * lazy[node];

        if (nl < nr) {
            lazy[left(node)] += lazy[node];
            lazy[right(node)] += lazy[node];
        }

        lazy[node] = nu;
    }
}

int left(int p) { return p << 1; }
int right(int p) { return (p << 1) + 1; }

```

```

    int mid(int l, int r) { return (r - l) / 2 + 1; }
};

```

## 1.8 Sparse Table Rminq

```

/*
    Sparse table implementation for rmq.
    build: O(NlogN)
    query: O(1)
*/
int fastlog2(ll x) {
    ull i = x;
    return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
}
template <typename T>
class SparseTable {
public:
    int N;
    int K;
    vector<vector<T>> st;
    SparseTable(vector<T> vs)
        : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector<T>(N + 1)) {
        copy(vs.begin(), vs.end(), st[0].begin());

        for (int i = 1; i <= K; ++i)
            for (int j = 0; j + (1 << i) <= N; ++j)
                st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
    }
    T RMQ(int l, int r) { // [l, r], 0 indexed
        int i = fastlog2(r - l + 1);
        return min(st[i][l], st[i][r - (1 << i) + 1]);
    }
};

```

## 2 Dynamic programming

### 2.1 Edit Distance

```

int edit_distance(const string &a, const string &b) {
    int n = a.size();
    int m = b.size();
    vector<vi> dp(n + 1, vi(m + 1, 0));

    int ADD = 1, DEL = 1, CHG = 1;
    for (int i = 0; i <= n; ++i) {
        dp[i][0] = i * DEL;
    }
    for (int i = 1; i <= m; ++i) {
        dp[0][i] = ADD * i;
    }

    for (int i = 1; i <= n; ++i) {
        for (int j = 1; j <= m; ++j) {
            int add = dp[i][j - 1] + ADD;
            int del = dp[i - 1][j] + DEL;
            int chg = dp[i - 1][j - 1] + (a[i - 1] == b[j - 1] ? 0 : 1) * CHG;
            dp[i][j] = min({add, del, chg});
        }
    }
}

```

```

    }
}

return dp[n][m];
}

```

### 2.2 Kadane

```

/*
    * Find the maximum sum subarray in a given array.
    * O(N)
    * */
int kadane(const vi &as) {
    vi s(len(as));
    s[0] = as[0];

    for (int i = 1; i < len(as); ++i) s[i] = max(as[i], s[i - 1] + as[i]);

    return *max_element(all(s));
}

```

### 2.3 Knapsack (value)

```

const int MAXN{2010}, MAXM{2010};

ll st[MAXN][MAXM];

ll dp(int i, int m, int M, const vii &cs) {
    if (i < 0) return 0;

    if (st[i][m] != -1) return st[i][m];

    auto res = dp(i - 1, m, M, cs);
    auto [w, v] = cs[i];

    if (w <= m) res = max(res, dp(i - 1, m - w, M, cs) + v);

    st[i][m] = res;
    return res;
}

ll knapsack(int M, const vii &cs) {
    memset(st, -1, sizeof st);

    return dp((int)cs.size() - 1, M, M, cs);
}

```

### 2.4 Knapsack With Elements

```

const int MAXN{2010}, MAXM{2010};
ll st[MAXN][MAXM];
char ps[MAXN][MAXM];

pair<ll, vi> knapsack(int M, const vii &cs) {
    int N = len(cs) - 1;

    for (int i = 0; i <= N; ++i) st[i][0] = 0;
}

```

```

for (int m = 0; m <= M; ++m) st[0][m] = 0;

for (int i = 1; i <= N; ++i) {
    for (int m = 1; m <= M; ++m) {
        st[i][m] = st[i - 1][m];
        ps[i][m] = 0;
        auto [w, v] = cs[i];

        if (w <= m and st[i - 1][m - w] + v > st[i][m]) {
            st[i][m] = st[i - 1][m - w] + v;
            ps[i][m] = 1;
        }
    }
}

int m = M;
vi is;
for (int i = N; i >= 1; --i) {
    if (ps[i][m]) {
        is.push_back(i);
        m -= cs[i].first;
    }
}

reverse(all(is));

// max value, items
return {st[N][M], is};
}

```

## 2.5 Longest Increasing Sequence

```

int LIS(int N, const vector<int> &as) {
    vector<int> lis(N + 1, oo);
    lis[0] = -oo;

    auto ans = 0;

    for (int i = 0; i < N; ++i) {
        auto it = lower_bound(lis.begin(), lis.end(), as[i]);
        auto pos = (int)(it - lis.begin());

        ans = max(ans, pos);
        lis[pos] = as[i];
    }

    return ans;
}

```

## 2.6 Money Sum Bottom Up

```

/*
    find every possible sum using
    the given values only once.
*/
set<int> money_sum(const vi &xs) {
    using vc = vector<char>;

```

```

    using vvc = vector<vc>;
    int _m = accumulate(all(xs), 0);
    int _n = xs.size();
    vvc _dp(_n + 1, vc(_m + 1, 0));
    set<int> _ans;
    _dp[0][xs[0]] = 1;
    for (int i = 1; i < _n; ++i) {
        for (int j = 0; j <= _m; ++j) {
            if (j == 0 or _dp[i - 1][j]) {
                _dp[i][j + xs[i]] = 1;
                _dp[i][j] = 1;
            }
        }
    }

    for (int i = 0; i < _n; ++i)
        for (int j = 0; j <= _m; ++j)
            if (_dp[i][j]) _ans.insert(j);
    return _ans;
}

```

## 2.7 Tsp

```

using vi = vector<int>;
vector<vi> dist;
vector<vi> memo;
/* 0 ( N^2 * 2^N )*/
int tsp(int i, int mask, int N) {
    if (mask == (1 << N) - 1) return dist[i][0];
    if (memo[i][mask] != -1) return memo[i][mask];
    int ans = INT_MAX << 1;
    for (int j = 0; j < N; ++j) {
        if (mask & (1 << j)) continue;
        auto t = tsp(j, mask | (1 << j), N) + dist[i][j];
        ans = min(ans, t);
    }
    return memo[i][mask] = ans;
}

```

## 3 Extras

### 3.1 Bigint

```

const int maxn = 1e2 + 14, lg = 15;
const int base = 1000000000;
const int base_digits = 9;
struct bigint {
    vector<int> a;
    int sign;

    int size() {
        if (a.empty()) return 0;
        int ans = (a.size() - 1) * base_digits;
        int ca = a.back();
        while (ca) ans++, ca /= 10;
        return ans;
    }
}

```

```

bigint operator^(const bigint &v) {
    bigint ans = 1, a = *this, b = v;
    while (!b.isZero()) {
        if (b % 2) ans *= a;
        a *= a, b /= 2;
    }
    return ans;
}

string to_string() {
    stringstream ss;
    ss << *this;
    string s;
    ss >> s;
    return s;
}

int sumof() {
    string s = to_string();
    int ans = 0;
    for (auto c : s) ans += c - '0';
    return ans;
}

/*</arpa>*/
bigint() : sign(1) {}

bigint(long long v) { *this = v; }

bigint(const string &s) { read(s); }

void operator=(const bigint &v) {
    sign = v.sign;
    a = v.a;
}

void operator=(long long v) {
    sign = 1;
    a.clear();
    if (v < 0) sign = -1, v = -v;
    for (; v > 0; v = v / base) a.push_back(v % base);
}

bigint operator+(const bigint &v) const {
    if (sign == v.sign) {
        bigint res = v;

        for (int i = 0, carry = 0; i < (int)max(a.size(), v.a.size()) || carry; ++i) {
            if (i == (int)res.a.size()) res.a.push_back(0);
            res.a[i] += carry + (i < (int)a.size() ? a[i] : 0);
            carry = res.a[i] >= base;
            if (carry) res.a[i] -= base;
        }
        return res;
    }
    return *this - (-v);
}

bigint operator-(const bigint &v) const {
    if (sign == v.sign) {

```

```

        if (abs() >= v.abs()) {
            bigint res = *this;
            for (int i = 0, carry = 0; i < (int)v.a.size() || carry; ++i) {
                res.a[i] -= carry + (i < (int)v.a.size() ? v.a[i] : 0);
                carry = res.a[i] < 0;
                if (carry) res.a[i] += base;
            }
            res.trim();
            return res;
        }
        return -(v - *this);
    }
}

return *this + (-v);
}

void operator*=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i) {
        if (i == (int)a.size()) a.push_back(0);
        long long cur = a[i] * (long long)v + carry;
        carry = (int)(cur / base);
        a[i] = (int)(cur % base);
        // asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) :
        // "A"(cur), "c"(base));
    }
    trim();
}

bigint operator*(int v) const {
    bigint res = *this;
    res *= v;
    return res;
}

void operator*=(long long v) {
    if (v < 0) sign = -sign, v = -v;
    if (v > base) {
        *this = *this * (v / base) * base + *this * (v % base);
        return;
    }
    for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i) {
        if (i == (int)a.size()) a.push_back(0);
        long long cur = a[i] * (long long)v + carry;
        carry = (int)(cur / base);
        a[i] = (int)(cur % base);
        // asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) :
        // "A"(cur), "c"(base));
    }
    trim();
}

bigint operator*(long long v) const {
    bigint res = *this;
    res *= v;
    return res;
}

friend pair<bigint, bigint> divmod(const bigint &a1, const bigint &b1) {

```

```

int norm = base / (b1.a.back() + 1);
bigint a = a1.abs() * norm;
bigint b = b1.abs() * norm;
bigint q, r;
q.a.resize(a.a.size());

for (int i = a.a.size() - 1; i >= 0; i--) {
    r *= base;
    r += a.a[i];
    int s1 = r.a.size() <= b.a.size() ? 0 : r.a[b.a.size()];
    int s2 = r.a.size() <= b.a.size() - 1 ? 0 : r.a[b.a.size() - 1];
    int d = ((long long)base * s1 + s2) / b.a.back();
    r -= b * d;
    while (r < 0) r += b, --d;
    q.a[i] = d;
}

q.sign = a1.sign * b1.sign;
r.sign = a1.sign;
q.trim();
r.trim();
return make_pair(q, r / norm);
}

bigint operator/(const bigint &v) const { return divmod(*this, v).first; }

bigint operator%(const bigint &v) const { return divmod(*this, v).second; }

void operator/=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = (int)a.size() - 1, rem = 0; i >= 0; --i) {
        long long cur = a[i] + rem * (long long)base;
        a[i] = (int)(cur / v);
        rem = (int)(cur % v);
    }
    trim();
}

bigint operator/(int v) const {
    bigint res = *this;
    res /= v;
    return res;
}

int operator%(int v) const {
    if (v < 0) v = -v;
    int m = 0;
    for (int i = a.size() - 1; i >= 0; --i)
        m = (a[i] + m * (long long)base) % v;
    return m * sign;
}

void operator+=(const bigint &v) { *this = *this + v; }
void operator-=(const bigint &v) { *this = *this - v; }
void operator*=(const bigint &v) { *this = *this * v; }
void operator/=(const bigint &v) { *this = *this / v; }

bool operator<(const bigint &v) const {

```

```

    if (sign != v.sign) return sign < v.sign;
    if (a.size() != v.a.size()) return a.size() * sign < v.a.size() * v.sign;
    for (int i = a.size() - 1; i >= 0; i--)
        if (a[i] != v.a[i]) return a[i] * sign < v.a[i] * v.sign;
    return false;
}

bool operator>(const bigint &v) const { return v < *this; }
bool operator<=(const bigint &v) const { return !(v < *this); }
bool operator>=(const bigint &v) const { return !(*this < v); }
bool operator==(const bigint &v) const {
    return !(*this < v) && !(v < *this);
}
bool operator!=(const bigint &v) const { return *this < v || v < *this; }

void trim() {
    while (!a.empty() && !a.back()) a.pop_back();
    if (a.empty()) sign = 1;
}

bool isZero() const { return a.empty() || (a.size() == 1 && !a[0]); }

bigint operator-() const {
    bigint res = *this;
    res.sign = -sign;
    return res;
}

bigint abs() const {
    bigint res = *this;
    res.sign *= res.sign;
    return res;
}

long long longValue() const {
    long long res = 0;
    for (int i = a.size() - 1; i >= 0; i--) res = res * base + a[i];
    return res * sign;
}

friend bigint gcd(const bigint &a, const bigint &b) {
    return b.isZero() ? a : gcd(b, a % b);
}

friend bigint lcm(const bigint &a, const bigint &b) {
    return a / gcd(a, b) * b;
}

void read(const string &s) {
    sign = 1;
    a.clear();
    int pos = 0;
    while (pos < (int)s.size() && (s[pos] == '-' || s[pos] == '+')) {
        if (s[pos] == '-') sign = -sign;
        ++pos;
    }
    for (int i = s.size() - 1; i >= pos; i -= base_digits) {
        int x = 0;
        for (int j = max(pos, i - base_digits + 1); j <= i; j++)

```



```

        x = x * 10 + s[j] - '0';
        a.push_back(x);
    }
    trim();
}

friend istream &operator>>(istream &stream, bigint &v) {
    string s;
    stream >> s;
    v.read(s);
    return stream;
}

friend ostream &operator<<(ostream &stream, const bigint &v) {
    if (v.sign == -1) stream << '-';
    stream << (v.a.empty() ? 0 : v.a.back());
    for (int i = (int)v.a.size() - 2; i >= 0; --i)
        stream << setw(base_digits) << setfill('0') << v.a[i];
    return stream;
}

static vector<int> convert_base(const vector<int> &a, int old_digits,
                               int new_digits) {
    vector<long long> p(max(old_digits, new_digits) + 1);
    p[0] = 1;
    for (int i = 1; i < (int)p.size(); i++) p[i] = p[i - 1] * 10;
    vector<int> res;
    long long cur = 0;
    int cur_digits = 0;
    for (int i = 0; i < (int)a.size(); i++) {
        cur += a[i] * p[cur_digits];
        cur_digits += old_digits;
        while (cur_digits >= new_digits) {
            res.push_back(int(cur % p[new_digits]));
            cur /= p[new_digits];
            cur_digits -= new_digits;
        }
    }
    res.push_back((int)cur);
    while (!res.empty() && !res.back()) res.pop_back();
    return res;
}

typedef vector<long long> vll;

static vll karatsubaMultiply(const vll &a, const vll &b) {
    int n = a.size();
    vll res(n + n);
    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++) res[i + j] += a[i] * b[j];
        return res;
    }

    int k = n >> 1;
    vll a1(a.begin(), a.begin() + k);
    vll a2(a.begin() + k, a.end());
    vll b1(b.begin(), b.begin() + k);

```

```

        vll b2(b.begin() + k, b.end());

    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);

    for (int i = 0; i < k; i++) a2[i] += a1[i];
    for (int i = 0; i < k; i++) b2[i] += b1[i];

    vll r = karatsubaMultiply(a2, b2);
    for (int i = 0; i < (int)a1b1.size(); i++) r[i] -= a1b1[i];
    for (int i = 0; i < (int)a2b2.size(); i++) r[i] -= a2b2[i];

    for (int i = 0; i < (int)r.size(); i++) res[i + k] += r[i];
    for (int i = 0; i < (int)a1b1.size(); i++) res[i] += a1b1[i];
    for (int i = 0; i < (int)a2b2.size(); i++) res[i + n] += a2b2[i];
    return res;
}

bigint operator*(const bigint &v) const {
    vector<int> a6 = convert_base(this->a, base_digits, 6);
    vector<int> b6 = convert_base(v.a, base_digits, 6);
    vll a(a6.begin(), a6.end());
    vll b(b6.begin(), b6.end());
    while (a.size() < b.size()) a.push_back(0);
    while (b.size() < a.size()) b.push_back(0);
    while (a.size() & (a.size() - 1)) a.push_back(0), b.push_back(0);
    vll c = karatsubaMultiply(a, b);
    bigint res;
    res.sign = sign * v.sign;
    for (int i = 0, carry = 0; i < (int)c.size(); i++) {
        long long cur = c[i] + carry;
        res.a.push_back((int)(cur % 1000000));
        carry = (int)(cur / 1000000);
    }
    res.a = convert_base(res.a, 6, base_digits);
    res.trim();
    return res;
}
};

```

## 3.2 Binary To Gray

```

string binToGray(string bin) {
    string gray(bin.size(), '0');
    int n = bin.size() - 1;
    gray[0] = bin[0];
    for (int i = 1; i <= n; i++) {
        gray[i] = '0' + (bin[i - 1] == '1') ^ (bin[i] == '1');
    }
    return gray;
}

```

## 3.3 Get Permutation Cicles

```

/*
 * receives a permutation [0, n-1]
 * returns a vector of cicles
 * for example: [ 1, 0, 3, 4, 2] -> [[0, 1], [2, 3, 4]]
 */

```

```

* */
vector<vll> getPermutationCicles(const vll &ps) {
    ll n = len(ps);
    vector<char> visited(n);
    vector<vll> cicles;
    for (int i = 0; i < n; ++i) {
        if (visited[i]) continue;

        vll cicle;
        ll pos = i;
        while (!visited[pos]) {
            cicle.pb(pos);
            visited[pos] = true;
            pos = ps[pos];
        }

        cicles.push_back(vll(all(cicle)));
    }
    return cicles;
}

```

## 4 Geometry

### 4.1 Point Template

```

const ld EPS = 1e-6;

typedef ld T;
bool eq(T a, T b) { return abs(a - b) <= EPS; }
struct point {
    T x, y;
    int id;
    point(T x = 0, T y = 0) : x(x), y(y) {}
    point operator+(const point &o) const { return {x + o.x, y + o.y}; }
    point operator-(const point &o) const { return {x - o.x, y - o.y}; }
    point operator*(T t) const { return {x * t, y * t}; }
    point operator/(T t) const { return {x / t, y / t}; }
    T operator*(const point &o) const {
        return x * o.x + y * o.y;
    } // dot product
    T operator^(const point &o) const {
        return x * o.y - y * o.x;
    } // cross product
};

ld dist(point a, point b) {
    point d = a - b;
    return sqrt(d * d);
}

```

## 5 Graphs

### 5.1 2 SAT (struct)

```

struct SAT2 {

```

```

    ll n;
    vll2d adj, adj_t;
    vc used;
    vll order, comp;
    vc assignment;
    bool solvable;
    SAT2(ll _n)
        : n(2 * _n),
          adj(n),
          adj_t(n),
          used(n),
          order(n),
          comp(n, -1),
          assignment(n / 2) {}
    void dfs1(int v) {
        used[v] = true;
        for (int u : adj[v]) {
            if (!used[u]) dfs1(u);
        }
        order.push_back(v);
    }

    void dfs2(int v, int c1) {
        comp[v] = c1;
        for (int u : adj_t[v]) {
            if (comp[u] == -1) dfs2(u, c1);
        }
    }

    bool solve_2SAT() {
        // find and label each SCC
        for (int i = 0; i < n; ++i) {
            if (!used[i]) dfs1(i);
        }
        reverse(all(order));
        ll j = 0;
        for (auto &v : order) {
            if (comp[v] == -1) dfs2(v, j++);
        }

        assignment.assign(n / 2, false);
        for (int i = 0; i < n; i += 2) {
            // x and !x belong to the same SCC
            if (comp[i] == comp[i + 1]) {
                solvable = false;
                return false;
            }

            assignment[i / 2] = comp[i] > comp[i + 1];
        }
        solvable = true;
        return true;
    }

    void add_disjunction(int a, bool na, int b, bool nb) {
        a = (2 * a) ^ na;
        b = (2 * b) ^ nb;
        int neg_a = a ^ 1;

```

```

    int neg_b = b ^ 1;
    adj[neg_a].push_back(b);
    adj[neg_b].push_back(a);
    adj_t[b].push_back(neg_a);
    adj_t[a].push_back(neg_b);
}
};

```

## 5.2 SCC (struct)

```

struct SCC {
    ll N;
    vll2d adj, tadj;
    vll todo, comps, comp;
    vector<set<ll>> sccadj;
    vchar vis;
    SCC(ll _N) : N(_N), adj(_N), tadj(_N), comp(_N, -1), sccadj(_N), vis(_N) {}

    void add_edge(ll x, ll y) { adj[x].eb(y), tadj[y].eb(x); }

    void dfs(ll x) {
        vis[x] = 1;
        for (auto &y : adj[x])
            if (!vis[y]) dfs(y);
        todo.pb(x);
    }
    void dfs2(ll x, ll v) {
        comp[x] = v;
        for (auto &y : tadj[x])
            if (comp[y] == -1) dfs2(y, v);
    }
    void gen() {
        for (ll i = 0; i < N; ++i)
            if (!vis[i]) dfs(i);
        reverse(all(todo));
        for (auto &x : todo)
            if (comp[x] == -1) {
                dfs2(x, x);
                comps.pb(x);
            }
    }

    void genSCCGraph() {
        for (ll i = 0; i < N; ++i) {
            for (auto &j : adj[i]) {
                if (comp[i] != comp[j]) {
                    sccadj[comp[i]].insert(comp[j]);
                }
            }
        }
    }
};

```

## 5.3 SCC Nodes (kosajaru)

```

/*
 * O(n+m)

```

```

* Returns a pair <a, b>
*     a: number of SCCs
*     b: vector of size n, where b[i] is the SCC id of node i
* */
void dfs(ll u, vchar &visited, const vll2d &g, vll &scc, bool buildScc, ll id,
        vll &sccid) {
    visited[u] = true;
    sccid[u] = id;
    for (auto &v : g[u])
        if (!visited[v]) dfs(v, visited, g, scc, buildScc, id, sccid);

    // if it's the first pass, add the node to the scc
    if (buildScc) scc.eb(u);
}

pair<ll, vll> kosajaru(vll2d &g) {
    ll n = len(g);
    vll scc;
    vchar vis(n);
    vll sccid(n);
    for (ll i = 0; i < n; i++)
        if (!vis[i]) dfs(i, vis, g, scc, true, 0, sccid);

    // build the transposed graph
    vll2d gt(n);
    for (int i = 0; i < n; ++i)
        for (auto &v : g[i]) gt[v].eb(i);

    // run the dfs on the previous scc order
    ll id = 1;
    vis.assign(n, false);
    for (ll i = len(scc) - 1; i >= 0; i--)
        if (!vis[scc[i]]) {
            dfs(scc[i], vis, gt, scc, false, id++, sccid);
        }
    return {id - 1, sccid};
}

```

## 5.4 Bellman Ford

```

bool bellman_ford(const vector<vector<pair<int, ll>>> &g, int s,
                 vector<ll> &dist) {
    int n = (int)g.size();
    dist.assign(n, LLONG_MAX);

    vector<int> count(n);
    vector<char> in_queue(n);
    queue<int> q;

    dist[s] = 0;
    q.push(s);
    in_queue[s] = true;

    while (not q.empty()) {
        int cur = q.front();
        q.pop();
        in_queue[cur] = false;

```

```

    for (auto [to, w] : g[cur]) {
        if (dist[cur] + w < dist[to]) {
            dist[to] = dist[cur] + w;
            if (not in_queue[to]) {
                q.push(to);
                in_queue[to] = true;
                count[to]++;
                if (count[to] > n) return false;
            }
        }
    }
}

return true;
}

```

## 5.5 Check Bipartite

```

// O(V)
bool checkBipartite(const ll n, const vector<vll> &adj) {
    ll s = 0;
    queue<ll> q;
    q.push(s);
    vll color(n, INF);
    color[s] = 0;
    bool isBipartite = true;
    while (!q.empty() && isBipartite) {
        ll u = q.front();
        q.pop();
        for (auto &v : adj[u]) {
            if (color[v] == INF) {
                color[v] = 1 - color[u];
                q.push(v);
            } else if (color[v] == color[u]) {
                return false;
            }
        }
    }
    return true;
}

```

## 5.6 Count SCC (kosajaru)

```

void dfs(ll u, vchar &visited, const vll2d &g, vll &scc, bool buildScc) {
    visited[u] = true;
    for (auto &v : g[u])
        if (!visited[v]) dfs(v, visited, g, scc, buildScc);

    // if it's the first pass, add the node to the scc
    if (buildScc) scc.eb(u);
}

ll kosajaru(vll2d &g) {
    ll n = len(g);
    vll scc;
    vchar vis(n);
    for (ll i = 0; i < n; i++)
        if (!vis[i]) dfs(i, vis, g, scc, true);
}

```

```

// build the transposed graph
vll2d gt(n);
for (int i = 0; i < n; ++i)
    for (auto &v : g[i]) gt[v].eb(i);

// run the dfs on the previous scc order
ll scccnt = 0;
vis.assign(n, false);
for (ll i = len(scc) - 1; i >= 0; i--)
    if (!vis[scc[i]]) dfs(scc[i], vis, gt, scc, false), scccnt++;
return scccnt;
}

```

## 5.7 Dijkstra

```

ll __inf = LLONG_MAX >> 5;
vll dijkstra(const vector<vector<pll>> &g, ll n) {
    priority_queue<pll, vector<pll>, greater<pll>> pq;
    vll dist(n, __inf);
    vector<char> vis(n);
    pq.emplace(0, 0);
    dist[0] = 0;
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();
        if (vis[v]) continue;
        vis[v] = true;

        for (auto [d2, u] : g[v]) {
            if (dist[u] > d1 + d2) {
                dist[u] = d1 + d2;
                pq.emplace(dist[u], u);
            }
        }
    }
    return dist;
}

```

## 5.8 Floyd Warshall

```

vector<vll> floyd_warshall(const vector<vll> &adj, ll n) {
    auto dist = adj;

    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            for (int k = 0; k < n; ++k) {
                dist[j][k] = min(dist[j][k], dist[j][i] + dist[i][k]);
            }
        }
    }
    return dist;
}

```

## 5.9 Kruskal (Python)

```

class DSU:

```

```

def __init__(self, n):
    self.n = n
    self.p = [x for x in range(0, n + 1)]
    self.size = [0 for i in range(0, n + 1)]

def find_set(self, x):
    if self.p[x] == x:
        return x
    else:
        self.p[x] = self.find_set(self.p[x])
        return self.p[x]

def same_set(self, x, y):
    return bool(self.find_set(x) == self.find_set(y))

def union_set(self, x, y):
    px = self.find_set(x)
    py = self.find_set(y)

    if px == py:
        return

    size_x = self.size[px]
    size_y = self.size[py]

    if size_x > size_y:
        self.p[py] = self.p[px]
        self.size[px] += self.size[py]
    else:
        self.p[px] = self.p[py]
        self.size[py] += self.size[px]

def kruskal(gv, n):
    """
    Receives the list of edges as a list of tuple in the form:
        d, u, v
        d: distance between u and v
    And also n as the total of vertices.
    """
    dsu = DSU(n)

    c = 0
    for e in gv:
        d, u, v = e
        if not dsu.same_set(u, v):
            c += d
            dsu.union_set(u, v)

    return c

```

## 5.10 Lowest Common Ancestor Sparse Table

```

int fastlog2(ll x) {
    ull i = x;
    return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
}
template <typename T>

```

```

class SparseTable {
public:
    int N;
    int K;
    vector<vector<T>> st;
    SparseTable(vector<T> vs)
        : N((int)vs.size()), K(fastlog2(N) + 1), st(K + 1, vector<T>(N + 1)) {
        copy(vs.begin(), vs.end(), st[0].begin());

        for (int i = 1; i <= K; ++i)
            for (int j = 0; j + (1 << i) <= N; ++j)
                st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
    }
    SparseTable() {}
    T RMQ(int l, int r) {
        int i = fastlog2(r - l + 1);
        return min(st[i][l], st[i][r - (1 << i) + 1]);
    }
};

class LCA {
public:
    int p;
    int n;
    vi first;
    vector<char> visited;
    vi vertices;
    vi height;
    SparseTable<int> st;

    LCA(const vector<vi> &g)
        : p(0), n((int)g.size()), first(n + 1), visited(n + 1, 0), height(n + 1) {
        build_dfs(g, 1, 1);
        st = SparseTable<int>(vertices);
    }

    void build_dfs(const vector<vi> &g, int u, int hi) {
        visited[u] = true;
        height[u] = hi;
        first[u] = vertices.size();
        vertices.push_back(u);
        for (auto uv : g[u]) {
            if (!visited[uv]) {
                build_dfs(g, uv, hi + 1);
                vertices.push_back(uv);
            }
        }
    }

    int lca(int a, int b) {
        int l = min(first[a], first[b]);
        int r = max(first[a], first[b]);
        return st.RMQ(l, r);
    }
};

```

## 5.11 Topological Sorting

```

/*

```

```

* O(V)
* assumes:
*     * vertices have index [0, n-1]
* if is a DAG:
*     * returns a topological sorting
* else:
*     * returns an empty vector
* */
enum class state { not_visited, processing, done };
bool dfs(const vector<vll> &adj, ll s, vector<state> &states, vll &order) {
    states[s] = state::processing;
    for (auto &v : adj[s]) {
        if (states[v] == state::not_visited) {
            if (not dfs(adj, v, states, order)) return false;
        } else if (states[v] == state::processing)
            return false;
    }
    states[s] = state::done;
    order.pb(s);
    return true;
}
vll topologicalSorting(const vector<vll> &adj) {
    ll n = len(adj);
    vll order;
    vector<state> states(n, state::not_visited);
    for (int i = 0; i < n; ++i) {
        if (states[i] == state::not_visited) {
            if (not dfs(adj, i, states, order)) return {};
        }
    }
    reverse(all(order));
    return order;
}

```

## 6 Math

### 6.1 Combinatorics With Repetitions

```

void combinations_with_repetition(int n, int k,
                                   function<void(const vector<int> &)> process)
{
    vector<int> v(k, 1);
    int pos = k - 1;

    while (true) {
        process(v);

        v[pos]++;

        while (pos > 0 and v[pos] > n) {
            --pos;
            v[pos]++;
        }

        if (pos == 0 and v[pos] > n) break;

        for (int i = pos + 1; i < k; ++i) v[i] = v[pos];
    }
}

```

```

        pos = k - 1;
    }
}

```

### 6.2 Count Divisors Memo

```

const ll mod = 1073741824;
const ll maxd = 100 * 100 * 100 + 1;
vector<ll> memo(maxd, -1);
ll countdivisors(ll x) {
    ll ox = x;
    ll ans = 1;
    for (ll i = 2; i <= x; ++i) {
        if (memo[x] != -1) {
            ans *= memo[x];
            break;
        }
        ll count = 0;
        while (x and x % i == 0) {
            x /= i;
            count++;
        }
        ans *= (count + 1);
    }
    memo[ox] = ans;
    return ans;
}

```

### 6.3 Euler Phi

```

const ll MAXN = 1e5;
vll list_primes(ll n) { // Nlog * log N
    vll ps;
    bitset<MAXN> sieve;
    sieve.set();
    sieve.reset(1);
    for (ll i = 2; i <= n; ++i) {
        if (sieve[i]) ps.push_back(i);
        for (ll j = i * 2; j <= n; j += i) {
            sieve.reset(j);
        }
    }
    return ps;
}

vector<pll> factorization(ll n, const vll &primes) {
    vector<pll> ans;
    for (auto &p : primes) {
        if (n == 1) break;
        ll cnt = 0;
        while (n % p == 0) {
            cnt++;
            n /= p;
        }
        if (cnt) ans.emplace_back(p, cnt);
    }
    return ans;
}

```

```

}

ll phi(ll n, vector<pll> factors) {
    if (n == 1) return 1;
    ll ans = n;

    for (auto [p, k] : factors) {
        ans /= p;
        ans *= (p - 1);
    }

    return ans;
}

```

## 6.4 Factorial Factorization

```

// O(logN) greater k that p^k | n
ll E(ll n, ll p) {
    ll k = 0, b = p;
    while (b <= n) {
        k += n / b;
        b *= p;
    }
    return k;
}

// list every prime until MAXN O(Nlog * log N)
const ll MAXN = 1e5;
vll list_primes(ll n) {
    vll ps;
    bitset<MAXN> sieve;
    sieve.set();
    sieve.reset(1);
    for (ll i = 2; i <= n; ++i) {
        if (sieve[i]) ps.push_back(i);
        for (ll j = i * 2; j <= n; j += i) sieve.reset(j);
    }
    return ps;
}

// O(pi(N)*logN)
map<ll, ll> factorial_factorization(ll n, const vll &primes) {
    map<ll, ll> fs;
    for (const auto &p : primes) {
        if (p > n) break;
        fs[p] = E(n, p);
    }
    return fs;
}

```

## 6.5 Factorial

```

const ll MAX = 18;
vll fv(MAX, -1);
ll factorial(ll n) {
    if (fv[n] != -1) return fv[n];
    if (n == 0) return 1;
    return n * factorial(n - 1);
}

```

```

}

```

## 6.6 Factorization With Primes

```

// Nlog * log N
const ll MAXN = 1e5;
vll list_primes(ll n) {
    vll ps;
    bitset<MAXN> sieve;
    sieve.set();
    sieve.reset(1);
    for (ll i = 2; i <= n; ++i) {
        if (sieve[i]) ps.push_back(i);
        for (ll j = i * 2; j <= n; j += i) sieve.reset(j);
    }
    return ps;
}

// O(pi(sqrt(n)))
map<ll, ll> factorization(ll n, const vll &primes) {
    map<ll, ll> ans;
    for (auto p : primes) {
        if (p * p > n) break;
        ll count = 0;
        for (; n % p == 0; count++, n /= p)
            ;
        if (count) ans[p] = count;
    }
    return ans;
}

```

## 6.7 Factorization

```

// O(sqrt(n))
map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;
        for (; n % i == 0; count++, n /= i)
            ;
        if (count) ans[i] = count;
    }
    if (n > 1) ans[n]++;
    return ans;
}

```

## 6.8 Fast Fourier Transform

```

template <bool invert = false>
void fft(vector<complex<double>>& xs) {
    int N = (int)xs.size();

    if (N == 1) return;

    vector<complex<double>> es(N / 2), os(N / 2);

    for (int i = 0; i < N / 2; ++i) es[i] = xs[2 * i];
}

```

```

for (int i = 0; i < N / 2; ++i) os[i] = xs[2 * i + 1];

fft<invert>(es);
fft<invert>(os);

auto signal = (invert ? 1 : -1);
auto theta = 2 * signal * acos(-1) / N;
complex<double> S{1}, S1{cos(theta), sin(theta)};

for (int i = 0; i < N / 2; ++i) {
    xs[i] = (es[i] + S * os[i]);
    xs[i] /= (invert ? 2 : 1);

    xs[i + N / 2] = (es[i] - S * os[i]);
    xs[i + N / 2] /= (invert ? 2 : 1);

    S *= S1;
}
}

```

## 6.9 Fast Exp

```

/*
Fast exponentiation algorithm,
compute a^n in O(log(n))
*/
ll fexp(ll a, int n) {
    if (n == 0) return 1;
    if (n == 1) return a;
    ll x = fexp(a, n / 2);
    return x * x * (n & 1 ? a : 1);
}

```

## 6.10 Gauss Elimination

```

template <size_t Dim>
struct GaussianElimination {
    vector<ll> basis;
    size_t size;

    GaussianElimination() : basis(Dim + 1), size(0) {}

    void insert(ll x) {
        for (ll i = Dim; i >= 0; i--) {
            if ((x & 1ll << i) == 0) continue;

            if (!basis[i]) {
                basis[i] = x;
                size++;
                break;
            }

            x ^= basis[i];
        }
    }

    void normalize() {

```

```

        for (ll i = Dim; i >= 0; i--)
            for (ll j = i - 1; j >= 0; j--)
                if (basis[i] & 1ll << j) basis[i] ^= basis[j];
    }

    bool check(ll x) {
        for (ll i = Dim; i >= 0; i--) {
            if ((x & 1ll << i) == 0) continue;

            if (!basis[i]) return false;

            x ^= basis[i];
        }

        return true;
    }

    auto operator[](ll k) { return at(k); }

    ll at(ll k) {
        ll ans = 0;
        ll total = 1ll << size;
        for (ll i = Dim; ~i; i--) {
            if (!basis[i]) continue;

            ll mid = total >> 1ll;
            if ((mid < k and (ans & 1ll << i) == 0) ||
                (k <= mid and (ans & 1ll << i)))
                ans ^= basis[i];

            if (mid < k) k -= mid;

            total >>= 1ll;
        }
        return ans;
    }

    ll at_normalized(ll k) {
        ll ans = 0;
        k--;
        for (size_t i = 0; i <= Dim; i++) {
            if (!basis[i]) continue;
            if (k & 1) ans ^= basis[i];
            k >>= 1;
        }
        return ans;
    }
};

```

## 6.11 Gcd Using Factorization

```

// O(sqrt(n))
map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;
        for (; n % i == 0; count++, n /= i)
            ;
    }
}

```



```

    if (count) ans[i] = count;
}
if (n > 1) ans[n]++;
return ans;
}

ll gcd_with_factorization(ll a, ll b) {
    map<ll, ll> fa = factorization(a);
    map<ll, ll> fb = factorization(b);
    ll ans = 1;
    for (auto fai : fa) {
        ll k = min(fai.second, fb[fai.first]);
        while (k--) ans *= fai.first;
    }
    return ans;
}

```

## 6.12 Gcd

```
ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
```

## 6.13 Integer Mod

```

const ll INF = 1e18;
const ll mod = 998244353;
template <ll MOD = mod>
struct Modular {
    ll value;
    static const ll MOD_value = MOD;

    Modular(ll v = 0) {
        value = v % MOD;
        if (value < 0) value += MOD;
    }
    Modular(ll a, ll b) : value(0) {
        *this += a;
        *this /= b;
    }

    Modular& operator+=(Modular const& b) {
        value += b.value;
        if (value >= MOD) value -= MOD;
        return *this;
    }
    Modular& operator--(Modular const& b) {
        value -= b.value;
        if (value < 0) value += MOD;
        return *this;
    }
    Modular& operator*(Modular const& b) {
        value = (ll)value * b.value % MOD;
        return *this;
    }

    friend Modular mexp(Modular a, ll e) {
        Modular res = 1;
        while (e) {
            if (e & 1) res *= a;

```

```

        a *= a;
        e >>= 1;
    }
    return res;
}
friend Modular inverse(Modular a) { return mexp(a, MOD - 2); }

Modular& operator/=(Modular const& b) { return *this *= inverse(b); }
friend Modular operator+(Modular a, Modular const b) { return a += b; }
Modular operator++(int) { return this->value = (this->value + 1) % MOD; }
Modular operator++() { return this->value = (this->value + 1) % MOD; }
friend Modular operator-(Modular a, Modular const b) { return a -= b; }
friend Modular operator-(Modular const a) { return 0 - a; }
Modular operator--(int) {
    return this->value = (this->value - 1 + MOD) % MOD;
}

Modular operator--() { return this->value = (this->value - 1 + MOD) % MOD; }
friend Modular operator*(Modular a, Modular const b) { return a *= b; }
friend Modular operator/(Modular a, Modular const b) { return a /= b; }
friend std::ostream& operator<<(std::ostream& os, Modular const& a) {
    return os << a.value;
}
friend bool operator==(Modular const& a, Modular const& b) {
    return a.value == b.value;
}
friend bool operator!=(Modular const& a, Modular const& b) {
    return a.value != b.value;
}
};

```

## 6.14 Is Prime

```

bool isprime(ll n) { // 0(sqrt(n))
    if (n < 2) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    for (ll i = 3; i * i < n; i += 2)
        if (n % i == 0) return false;
    return true;
}

```

## 6.15 Lcm Using Factorization

```

map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;
        for (; n % i == 0; count++, n /= i)
            ;
        if (count) ans[i] = count;
    }
    if (n > 1) ans[n]++;
    return ans;
}

ll lcm_with_factorization(ll a, ll b) {
    map<ll, ll> fa = factorization(a);

```

```

map<ll, ll> fb = factorization(b);
ll ans = 1;
for (auto fai : fa) {
    ll k = max(fai.second, fb[fai.first]);
    while (k--) ans *= fai.first;
}
return ans;
}

```

## 6.16 Lcm

```

ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
ll lcm(ll a, ll b) { return a / gcd(a, b) * b; }

```

## 6.17 Modular Inverse Using Phi

```

map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;
        for (; n % i == 0; count++, n /= i)
            ;
        if (count) ans[i] = count;
    }
    if (n > 1) ans[n]++;
    return ans;
}

```

```

ll phi(ll n) {
    if (n == 1) return 1;

    auto fs = factorization(n);
    auto res = n;

    for (auto [p, k] : fs) {
        res /= p;
        res *= (p - 1);
    }

    return res;
}

```

```

ll fexp(ll a, ll n, ll mod) {
    if (n == 0) return 1;
    if (n == 1) return a;
    ll x = fexp(a, n / 2, mod);
    return x * x * (n & 1 ? a : 1) % mod;
}

```

```

ll inv(ll a, ll mod) { return fexp(a, phi(mod) - 1, mod); }

```

## 6.18 N Choose K Count

```

/*
 * O(nm) time, O(m) space
 * equal to n choose k
 * */

```

```

ll binom(ll n, ll k) {
    if (k > n) return 0;
    vll dp(k + 1, 0);
    dp[0] = 1;
    for (ll i = 1; i <= n; i++)
        for (ll j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
    return dp[k];
}

```

## 6.19 Permutation Count

```

const ll MAX = 18;
vll fv(MAX, -1);
ll factorial(ll n) {
    if (fv[n] != -1) return fv[n];
    if (n == 0) return 1;
    return n * factorial(n - 1);
}

template <typename T>
ll permutation_count(vector<T> xs) {
    map<T, ll> h;
    for (auto xi : xs) h[xi]++;
    ll ans = factorial((ll)xs.size());
    dbg(ans);
    for (auto [v, cnt] : h) {
        dbg(cnt);
        ans /= cnt;
    }

    return ans;
}

```

## 6.20 Polynomial

```

using polynomial = vector<ll>;
int degree(const polynomial &xs) { return xs.size() - 1; }
ll horner_evaluate(const polynomial &xs, ll x) {
    ll ans = 0;
    ll n = degree(xs);
    for (int i = n; i >= 0; --i) {
        ans *= x;
        ans += xs[i];
    }
    return ans;
}

polynomial operator+(const polynomial &a, const polynomial &b) {
    int n = degree(a);
    int m = degree(b);
    polynomial r(max(n, m) + 1, 0);

    for (int i = 0; i <= n; ++i) r[i] += a[i];
    for (int j = 0; j <= m; ++j) r[j] += b[j];
    while (!r.empty() and r.back() == 0) r.pop_back();
    if (r.empty()) r.push_back(0);
    return r;
}

polynomial operator*(const polynomial &p, const polynomial &q) {

```

```

int n = degree(p);
int m = degree(q);
polynomial r(n + m + 1, 0);
for (int i = 0; i <= n; ++i)
    for (int j = 0; j <= m; ++j) r[i + j] += (p[i] * q[j]);
return r;
}

```

## 6.21 Power Sum

```

// calculates  $K^0 + K^1 + \dots + K^n$ 
ll fastpow(ll a, int n) {
    if (n == 1) return a;
    ll x = fastpow(a, n / 2);
    return x * x * (n & 1 ? a : 1);
}
ll powersum(ll n, ll k) { return (fastpow(n, k + 1) - 1) / (n - 1); }

```

## 6.22 Sieve List Primes

```

// list every prime until MAXN
const ll MAXN = 1e5;
vll list_primes(ll n) { // Nlog * log N
    vll ps;
    bitset<MAXN> sieve;
    sieve.set();
    sieve.reset(1);
    for (ll i = 2; i <= n; ++i) {
        if (sieve[i]) ps.push_back(i);
        for (ll j = i * 2; j <= n; j += i) {
            sieve.reset(j);
        }
    }
    return ps;
}

```

# 7 Searching

## 7.1 Ternary Search Recursive

```

const double eps = 1e-6;

// IT MUST BE AN UNIMODAL FUNCTION
double f(int x) { return x * x + 2 * x + 4; }

double ternary_search(double l, double r) {
    if (fabs(f(l) - f(r)) < eps) return f((l + (r - l) / 2.0));

    auto third = (r - l) / 3.0;
    auto m1 = l + third;
    auto m2 = r - third;

    // change the signal to find the maximum point.
    return m1 < m2 ? ternary_search(m1, r) : ternary_search(l, m2);
}

```

# 8 Strings

## 8.1 Hash Range Query

```

struct Hash {
    const ll P = 31;
    int n;
    string s;
    vll h, hi, p;
    Hash() {}
    Hash(string s) : s(s), n(s.size()), h(n), hi(n), p(n) {
        for (int i = 0; i < n; i++) p[i] = (i ? P * p[i - 1] : 1) % MOD;
        for (int i = 0; i < n; i++) h[i] = (s[i] + (i ? h[i - 1] : 0) * P) % MOD;
        for (int i = n - 1; i >= 0; i--)
            hi[i] = (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % MOD;
    }
    ll query(int l, int r) {
        ll hash = (h[r] - (l ? h[l - 1] * p[r - l + 1] % MOD : 0));
        return hash < 0 ? hash + MOD : hash;
    }
    ll query_inv(int l, int r) {
        ll hash = (hi[l] - (r + 1 < n ? hi[r + 1] * p[r - l + 1] % MOD : 0));
        return hash < 0 ? hash + MOD : hash;
    }
};

```

## 8.2 Longest Palindrome

```

string longest_palindrome(const string &s) {
    int n = (int)s.size();
    vector<array<int, 2>> dp(n);

    pii odd(0, -1), even(0, -1);
    pii ans;
    for (int i = 0; i < n; i++) {
        int k = 0;
        if (i > odd.second)
            k = 1;
        else
            k = min(dp[odd.first + odd.second - i][0], odd.second - i + 1);
        while (i - k >= 0 and i + k < n and s[i - k] == s[i + k]) k++;
        dp[i][0] = k--;
        if (i + k > odd.second) odd = {i - k, i + k};
        if (2 * dp[i][0] - 1 > ans.second) ans = {i - k, 2 * dp[i][0] - 1};

        k = 0;
        if (i <= even.second)
            k = min(dp[even.first + even.second - i + 1][1], even.second - i + 1);
        while (i - k - 1 >= 0 and i + k < n and s[i - k - 1] == s[i + k]) k++;
        dp[i][1] = k--;
        if (i + k > even.second) even = {i - k - 1, i + k};
        if (2 * dp[i][1] > ans.second) ans = {i - k - 1, 2 * dp[i][1]};
    }
    return s.substr(ans.first, ans.second);
}

```

## 8.3 Rabin Karp

```

size_t rabin_karp(const string &s, const string &p) {
    if (s.size() < p.size()) return 0;

    auto n = s.size(), m = p.size();
    const ll p1 = 31, p2 = 29, q1 = 1e9 + 7, q2 = 1e9 + 9;
    const ll p1_1 = fpow(p1, q1 - 2, q1), p1_2 = fpow(p1, m - 1, q1);
    const ll p2_1 = fpow(p2, q2 - 2, q2), p2_2 = fpow(p2, m - 1, q2);

    pair<ll, ll> hs, hp;
    for (int i = (int)m - 1; ~i; --i) {
        hs.first = (hs.first * p1) % q1;
        hs.first = (hs.first + (s[i] - 'a' + 1)) % q1;
        hs.second = (hs.second * p2) % q2;
        hs.second = (hs.second + (s[i] - 'a' + 1)) % q2;

        hp.first = (hp.first * p1) % q1;
        hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
        hp.second = (hp.second * p2) % q2;
        hp.second = (hp.second + (p[i] - 'a' + 1)) % q2;
    }

    size_t occ = 0;
    for (size_t i = 0; i < n - m; i++) {
        occ += (hs == hp);

        int fi = s[i] - 'a' + 1;
        int fm = s[i + m] - 'a' + 1;

        hs.first = (hs.first - fi + q1) % q1;
        hs.first = (hs.first * p1_1) % q1;
        hs.first = (hs.first + fm * p1_2) % q1;
        hs.second = (hs.second - fi + q2) % q2;
        hs.second = (hs.second * p2_1) % q2;
        hs.second = (hs.second + fm * p2_2) % q2;
    }
    occ += hs == hp;

    return occ;
}

```

## 8.4 String Psum

```

struct strPsum {
    ll n;
    ll k;
    vector<vll> psum;
    strPsum(const string &s) : n(s.size()), k(100), psum(k, vll(n + 1)) {
        for (ll i = 1; i <= n; ++i) {
            for (ll j = 0; j < k; ++j) {
                psum[j][i] = psum[j][i - 1];
            }
            psum[s[i - 1]][i]++;
        }
    }

    ll qtd(ll l, ll r, char c) { // [0,n-1]
        return psum[c][r + 1] - psum[c][l];
    }
}

```

```

}

```

## 8.5 Trie Naive

```

// time: O(n^2) memory: O(n^2)
using Node = map<char, int>;
using vi = vector<int>;
using Trie = vector<Node>;

Trie build(const string &s) {
    int n = (int)s.size();
    Trie trie(1);
    string suffix;

    for (int i = n - 1; i >= 0; --i) {
        suffix = s.substr(i) + '#';

        int v = 0; // root
        for (auto c : suffix) {
            if (c == '#') { // marks the poistion of an occurence
                trie[v][c] = i;
                break;
            }
            if (trie[v][c])
                v = trie[v][c];
            else {
                trie.push_back({});
                trie[v][c] = trie.size() - 1;
                v = trie.size() - 1;
            }
        }
    }
    return trie;
}

vi search(Trie &trie, string s) {
    int p = 0;
    vi occ;
    for (auto &c : s) {
        p = trie[p][c];
        if (!p) return occ;
    }

    queue<int> q;
    q.push(0);
    while (!q.empty()) {
        auto cur = q.front();
        q.pop();
        for (auto [c, v] : trie[cur]) {
            if (c == '#')
                occ.push_back(v);
            else
                q.push(v);
        }
    }
    return occ;
}

```

```

11 distinct_substr(const Trie &trie) {
    11 cnt = 0;
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
        auto u = q.front();
        q.pop();

        for (auto [c, v] : trie[u]) {
            if (c != '#') {
                cnt++;
                q.push(v);
            }
        }
    }
    return cnt;
}

```

## 9 Trees

### 9.1 Binary Lifting

```

/*
 * far[h][i] = the node that 2^h far from node i
 * sometimes is useful invert the order of loops
 * time : O(nlogn)
 * */
const int maxlog = 20;
int far[maxlog + 1][n + 1];
int n;
for (int h = 1; h <= maxlog; h++) {
    for (int i = 1; i <= n; i++) {
        far[h][i] = far[h - 1][far[h - 1][i]];
    }
}

```

### 9.2 Maximum Distances

```

/*
 * Returns the maximum distance from every node to any other node in the tree.
 * */
p11 mostDistantFrom(const vector<v11> &adj, 11 n, 11 root) {
    // 0 indexed
    11 mostDistantNode = root;
    11 nodeDistance = 0;
    queue<p11> q;
    vector<char> vis(n);
    q.emplace(root, 0);
    vis[root] = true;
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (dist > nodeDistance) {
            nodeDistance = dist;
            mostDistantNode = node;
        }
        for (auto u : adj[node]) {

```

```

            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return {mostDistantNode, nodeDistance};
}

```

```

11 twoNodesDist(const vector<v11> &adj, 11 n, 11 a, 11 b) {
    queue<p11> q;
    vector<char> vis(n);
    q.emplace(a, 0);
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (node == b) return dist;
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return -1;
}

```

```

tuple<11, 11, 11> tree_diameter(const vector<v11> &adj, 11 n) {
    // returns two points of the diameter and the diameter itself
    auto [node1, dist1] = mostDistantFrom(adj, n, 0);
    auto [node2, dist2] = mostDistantFrom(adj, n, node1);
    auto diameter = twoNodesDist(adj, n, node1, node2);
    return make_tuple(node1, node2, diameter);
}

```

```

v11 everyDistanceFromNode(const vector<v11> &adj, 11 n, 11 root) {
    // Single Source Shortest Path, from a given root
    queue<pair<11, 11>> q;
    v11 ans(n, -1);
    ans[root] = 0;
    q.emplace(root, 0);
    while (!q.empty()) {
        auto [u, d] = q.front();
        q.pop();

        for (auto w : adj[u]) {
            if (ans[w] != -1) continue;
            ans[w] = d + 1;
            q.emplace(w, d + 1);
        }
    }
    return ans;
}

```

```

v11 maxDistances(const vector<v11> &adj, 11 n) {
    auto [node1, node2, diameter] = tree_diameter(adj, n);
    auto distances1 = everyDistanceFromNode(adj, n, node1);
    auto distances2 = everyDistanceFromNode(adj, n, node2);
}

```

```

vll ans(n);
for (int i = 0; i < n; ++i) ans[i] = max(distances1[i], distances2[i]);
return ans;
}

```

## 9.3 Tree Diameter

```

pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
    // 0 indexed
    ll mostDistantNode = root;
    ll nodeDistance = 0;
    queue<pll> q;
    vector<char> vis(n);
    q.emplace(root, 0);
    vis[root] = true;
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (dist > nodeDistance) {
            nodeDistance = dist;
            mostDistantNode = node;
        }
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return {mostDistantNode, nodeDistance};
}

ll twoNodesDist(const vector<vll> &adj, ll n, ll a, ll b) {
    // 0 indexed
    queue<pll> q;
    vector<char> vis(n);
    q.emplace(a, 0);
    while (!q.empty()) {
        auto [node, dist] = q.front();
        q.pop();
        if (node == b) {
            return dist;
        }
        for (auto u : adj[node]) {
            if (!vis[u]) {
                vis[u] = true;
                q.emplace(u, dist + 1);
            }
        }
    }
    return -1;
}

ll tree_diameter(const vector<vll> &adj, ll n) {
    // 0 indexed !!!
    auto [node1, dist1] = mostDistantFrom(adj, n, 0);
    auto [node2, dist2] = mostDistantFrom(adj, n, node1);
    auto diameter = twoNodesDist(adj, n, node1, node2);
    return diameter;
}

```

## 10 Settings and macros

### 10.1 .vimrc

```

set ts=4 sw=4 sta nu rnu sc cindent
set bg=dark ruler clipboard=unnamed,unnamedplus, timeoutlen=100
colorscheme default

```

```

nnoremap <C-j> :botright belowright term bash <CR>
syntax on

```

### 10.2 degug.cpp

```

#include <bits/stdc++.h>
using namespace std;
/***** Debug Code *****/
template <typename T>
concept Printable = requires(T t) {
    { std::cout << t } -> std::same_as<std::ostream &>;
};
template <Printable T>
void __print(const T &x) {
    cerr << x;
}
template <size_t T>
void __print(const bitset<T> &x) {
    cerr << x;
}
template <typename A, typename B>
void __print(const pair<A, B> &p);
template <typename... A>
void __print(const tuple<A...> &t);
template <typename T>
void __print(stack<T> s);
template <typename T>
void __print(queue<T> q);
template <typename T, typename... U>
void __print(priority_queue<T, U...> q);
template <typename A>
void __print(const A &x) {
    bool first = true;
    cerr << '{';
    for (const auto &i : x) {
        cerr << (first ? "" : ","), __print(i);
        first = false;
    }
    cerr << '}';
}
template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '(';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << ')';
}
template <typename... A>
void __print(const tuple<A...> &t) {

```

```

bool first = true;
cerr << '(';
apply(
    [&first](const auto &...args) {
        ((cerr << (first ? "" : ","), __print(args), first = false), ...);
    },
    t);
cerr << ')';
}
template <typename T>
void __print(stack<T> s) {
    vector<T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    }
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
}
template <typename T>
void __print(queue<T> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.front();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
template <typename T, typename... U>
void __print(priority_queue<T, U...> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.top();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
void _print() { cerr << "]\n"; }
template <typename Head, typename... Tail>
void _print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";
    _print(T...);
}

#define dbg(x...) \
    cerr << "[" << #x << "]" = ["; \
    _print(x)

```

## 10.3 .bashrc

```

cpp() {
    echo ">> COMPILING <<" 1>&2
    g++ -std=c++17 \
        -O2 \

```

```

-g \
-g3 \
-Wextra \
-Wshadow \
-Wformat=2 \
-Wconversion \
-fsanitize=address,undefined \
-fno-sanitize-recover \
-Wfatal-errors \
-DDEBUG $1 \

if [ $? -ne 0 ]; then
    echo ">> FAILED <<" 1>&2
    return 1
fi
echo ">> DONE << " 1>&2
time ./a.out ${@:2}
}

prepare() {
    for i in {a..z}
    do
        cp macro.cpp $i.cpp
        touch $i.py
    done

    for i in {1..10}
    do
        touch in${i}
        touch out${i}
        touch ans${i}
    done
}

```

## 10.4 macro.cpp

```

#include <bits/stdc++.h>
using namespace std;
#ifdef DEBUG
#include "debug.cpp"
#else
#define dbg(...) 666
#endif
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(false); \
    cin.tie(0); \
    cout.tie(0);
#define len(__x) (ll) __x.size()
using ll = long long;
using vll = vector<ll>;
using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
using vii = vector<pii>;
using vc = vector<char>;

```

```
#define all(a) a.begin(), a.end()
#define snd second
#define fst first
#define pb(____x) push_back(____x)
#define mp(____a, ____b) make_pair(____a, ____b)
#define eb(____x) emplace_back(____x)

const ll INF = 1e18;

void run() {
```

```
}
int32_t main(void) {
    fastio;
    int t;
    t = 1;
    // cin >> t;
    while (t--) run();
}
```