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1 Data structures

1.1 Segtree Lazy (Atcoder)

```
struct Node {
 // need an empty constructor with the neutral node
  Node() : {}
};
struct Lazy {
 // need an empty constructor with the neutral lazy
 Lazy() : {}
};
// how to merge two nodes
Node op(Node a, Node b) {}
// how to apply the lazy into a node
Node mapping(Lazy a, Node b, int, int) {}
// how to merge two lazy
Lazy comp(Lazy a, Lazy b) {}
template <typename T, auto op, typename L, auto mapping, auto
   composition >
struct SegTreeLazy {
  static_assert(is_convertible_v < decltype(op), function < T(T, T)
                 "op must be a function T(T, T)");
  static assert(
    is_convertible_v < decltype(mapping), function < T(L, T, int,</pre>
   int)>>.
    "mapping must be a function T(L, T, int, int)");
  static_assert(is_convertible_v < decltype(composition),
   function < L(L, L) >>,
                "composition must be a function L(L, L)");
  int N, size, height;
  const T eT;
  const L eL;
  vector <T> d;
  vector < L > lz;
  SegTreeLazy(const T &eT_ = T(), const L &eL_ = L())
```

```
: SegTreeLazy(0, eT_, eL_) {}
explicit SegTreeLazy(int n, const T &eT_ = T(), const L &eL_
  : SegTreeLazy(vector<T>(n, eT_), eT_, eL_) {}
explicit SegTreeLazy(const vector<T> &v, const T &eT_ = T(),
                      const L &eL_ = L())
  : N(int(v.size())), eT(eT_), eL(eL_) {
  size = 1;
  height = 0;
  while (size < N) size <<= 1, height++;</pre>
  d = vector < T > (2 * size, eT);
  lz = vector < L > (size, eL);
 for (int i = 0; i < N; i++) d[size + i] = v[i];</pre>
 for (int i = size - 1; i >= 1; i--) {
    update(i);
 }
}
void set(int p, T x) {
 assert(0 <= p && p < N);
 p += size;
 for (int i = height; i >= 1; i--) push(p >> i);
  d[q] = x:
 for (int i = 1; i \le height; i++) update(p >> i);
T get(int p) {
  assert(0 <= p && p < N);
  p += size;
 for (int i = height; i >= 1; i--) push(p >> i);
  return d[p];
T query(int 1, int r) {
  assert(0 \le 1 \&\& 1 \le r \&\& r \le N);
 1 += size:
 r += size:
  for (int i = height; i >= 1; i--) {
   if (((1 >> i) << i) != 1) push(1 >> i);
   if ((((r + 1) >> i) << i) != (r + 1)) push(r >> i);
  }
  T \text{ sml} = eT, \text{ smr} = eT;
```

```
while (1 <= r) {
    if (1 \& 1) sml = op(sml, d[1++]);
    if (!(r \& 1)) smr = op(d[r--], smr);
    1 >>= 1:
    r >>= 1;
  }
  return op(sml, smr);
}
T query_all() { return d[1]; }
void update(int p, L f) {
  assert(0 <= p && p < N);
 p += size;
 for (int i = height; i >= 1; i--) push(p >> i);
 d[p] = mapping(f, d[p]);
  for (int i = 1; i <= height; i++) update(p >> i);
void update(int 1, int r, L f) {
  assert(0 <= 1 && 1 <= r && r < N);
 l += size;
 r += size;
  for (int i = height; i >= 1; i--) {
    if (((1 >> i) << i) != 1) push(1 >> i);
    if ((((r + 1) >> i) << i) != (r + 1)) push(r >> i);
  }
    int 12 = 1, r2 = r;
    while (1 <= r) {
      if (1 & 1) all_apply(l++, f);
     if (!(r & 1)) all_apply(r--, f);
      1 >>= 1;
      r >>= 1:
    }
    1 = 12;
    r = r2;
  }
  for (int i = 1; i <= height; i++) {</pre>
    if (((1 >> i) << i) != 1) update(1 >> i);
    if ((((r + 1) >> i) << i) != (r + 1)) update(r >> i);
```

```
}
  pair<int, int> node_range(int k) const {
    int remain = height;
    for (int kk = k; kk >>= 1; --remain)
    int fst = k << remain;</pre>
    int lst = min(fst + (1 << remain) - 1, size + N - 1);</pre>
    return {fst - size, lst - size};
  }
 private:
  void update(int k) { d[k] = op(d[2 * k], d[2 * k + 1]); }
  void all_apply(int k, L f) {
    auto [fst, lst] = node_range(k);
    d[k] = mapping(f, d[k], fst, lst);
    if (k < size) lz[k] = composition(f, lz[k]);</pre>
  void push(int k) {
    all_apply(2 * k, lz[k]);
    all_apply(2 * k + 1, lz[k]);
    lz[k] = eL;
  }
};
1.2 Bitree 2D
Given a 2d array allow you to sum val to the position (x,y) and find the sum of the rectangle with left top
corner (x1, y1) and right bottom corner (x2, y2)
Update and query 1 indexed!
Time: update O(logn^2), query O(logn^2)
struct Bit2d {
  int n;
  vll2d bit;
  Bit2d(int ni) : n(ni), bit(n + 1, vll(n + 1)) {}
  Bit2d(int ni, vll2d &xs) : n(ni), bit(n + 1, vll(n + 1)) {
    for (int i = 1; i <= n; i++) {
      for (int j = 1; j <= n; j++) {
        update(i, j, xs[i][j]);
      }
    }
  void update(int x, int y, ll val) {
    for (; x \le n; x += (x & (-x)))  {
      for (int i = y; i \le n; i += (i & (-i))) {
```

```
bit[x][i] += val;
      }
   }
 }
 11 sum(int x, int y) {
   11 \text{ ans} = 0:
    for (int i = x; i; i -= (i & (-i))) {
      for (int j = y; j; j -= (j & (-j))) {
        ans += bit[i][j];
      }
    }
    return ans;
 }
 ll query(int x1, int y1, int x2, int y2) {
    return sum(x2, y2) - sum(x2, y1 - 1) - sum(x1 - 1, y2) +
           sum(x1 - 1, y1 - 1);
 }
};
1.3 Bitree
template <typename T>
struct BITree {
 int N:
  vector <T> v;
  BITree(int n) : N(n), v(n + 1, 0) {}
  void update(int i, const T& x) {
   if (i == 0) return;
   for (; i <= N; i += i & -i) v[i] += x;
 }
 T range_sum(int i, int j) { return range_sum(j) - range_sum(i
    - 1); }
 T range_sum(int i) {
   T sum = 0;
   for (; i > 0; i -= i & -i) sum += v[i];
   return sum;
 }
```

};

1.4 Disjoint Sparse Table

```
Answers queries of any monoid operation (i.e. has identity element and is associative)
Build: O(N \log N), Query: O(1)
#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct DisjointSparseTable {
  using Operation = T (*)(T, T);
  vector < vector < T >> st;
  Operation f;
  T identity;
  static constexpr int log2_floor(unsigned long long i)
   noexcept {
    return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
  }
  // Lazy loading constructor. Needs to call build!
  DisjointSparseTable(Operation op, const T neutral = T())
    : st(), f(op), identity(neutral) {}
  DisjointSparseTable(vector<T> v) : DisjointSparseTable(v, F(
   min(a, b))) {}
  DisjointSparseTable(vector < T > v, Operation op, const T
   neutral = T())
   : st(), f(op), identity(neutral) {
    build(v):
  }
  void build(vector<T> v) {
    st.resize(log2_floor(v.size()) + 1,
               vector <T > (111 << (log2_floor(v.size()) + 1)));</pre>
    v.resize(st[0].size(), identity);
    for (int level = 0; level < (int)st.size(); ++level) {</pre>
      for (int block = 0; block < (1 << level); ++block) {</pre>
         const auto 1 = block << (st.size() - level);</pre>
         const auto r = (block + 1) << (st.size() - level);</pre>
        const auto m = 1 + (r - 1) / 2;
        st[level][m] = v[m];
        for (int i = m + 1; i < r; i++)</pre>
           st[level][i] = f(st[level][i - 1], v[i]);
        st[level][m - 1] = v[m - 1]:
```

```
for (int i = m - 2; i >= 1; i--)
          st[level][i] = f(st[level][i + 1], v[i]);
      }
    }
  }
  T query(int 1, int r) const {
    if (l > r) return identity;
    if (1 == r) return st.back()[1];
    const auto k = log2_floor(l ^ r);
    const auto level = (int)st.size() - 1 - k;
    return f(st[level][1], st[level][r]);
  }
};
    DSU/UFDS
Uncomment the lines to reover which element belong to each set.
Time: \approx O(1) for everything.
struct DSU {
  vi ps;
  vi size;
  // vector < unordered_set < int >> sts;
  DSU(int N) : ps(N + 1), size(N, 1), sts(N) {
    iota(all(ps), 0);
    // for (int i = 0; i < N; i++) sts[i].insert(i);
  int find_set(int x) { return ps[x] == x ? x : ps[x] =
   find_set(ps[x]); }
  bool same_set(int x, int y) { return find_set(x) == find_set(
   y); }
  void union_set(int x, int y) {
    if (same_set(x, y)) return;
    int px = find_set(x);
    int py = find_set(y);
    if (size[px] < size[py]) swap(px, py);</pre>
    ps[py] = px;
    size[px] += size[py];
    // sts[px].merge(sts[py]);
  }
```

};

1.6 Ordered Set

If you need an ordered **multi**set you may add an id to each value. Using greater_equal, or less_equal is considered undefined behavior.

- order of key (k): Number of items strictly smaller/greater than k.
- find by order(k): K-th element in a set (counting from zero).

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set =
   tree<T, null_type, less<T>, rb_tree_tag,
   tree_order_statistics_node_update>;
```

1.7 Prefix Sum 2D

Given an 2d array with n lines and m columns, find the sum of the subarray that have the left upper corner at (x1, y1) and right bottom corner at (x2, y2).

Time: build $O(n \cdot m)$, query O(1).

```
struct psum2d {
  v112d s;
  vll2d psum;
  psum2d(v112d &grid, int n, int m)
   : s(n + 1, vll(m + 1)), psum(n + 1, vll(m + 1)) {
   for (int i = 1; i <= n; i++)
      for (int j = 1; j \le m; j++) s[i][j] = s[i][j - 1] + grid
   [i][j];
   for (int i = 1; i <= n; i++)
      for (int j = 1; j <= m; j++) psum[i][j] = psum[i - 1][j]</pre>
   + s[i][i];
 }
 ll query(int x1, int y1, int x2, int y2) {
    ll ans = psum[x2][y2] + psum[x1 - 1][y1 - 1];
    ans -= psum[x2][y1 - 1] + psum[x1 - 1][y2];
    return ans:
 }
};
```

1.8 SegTree Range Sum Query Range PA sum/set Update

Makes arithmetic progression updates in range and sum queries.

```
Considering PA(A,R) = [A+R, A+2R, A+3R, ...]
  • update set(l, r, A, R): sets [l, r] to PA(A, R)
  • update add(l, r, A, R): sum PA(A, R) in [l, r]
  • query(l, r): sum in range [l, r]
0 indexed!
Time: build O(n), updates and queries O(\log n)
const 11 oo = 1e18;
struct SegTree {
  struct Data {
    ll sum:
    ll set_a, set_r, add_a, add_r;
    Data(): sum(0), set_a(oo), set_r(0), add_a(0), add_r(0) {}
  };
  int n;
  vector < Data > seg;
  SegTree(int n_{-}): n(n_{-}), seg(vector < Data > (4 * n)) {}
  void prop(int p, int l, int r) {
    int sz = r - 1 + 1;
    ll &sum = seg[p].sum, &set_a = seg[p].set_a, &set_r = seg[p
       &add_a = seg[p].add_a, &add_r = seg[p].add_r;
    if (set a != oo) {
      set_a += add_a, set_r += add_r;
      sum = set_a * sz + set_r * sz * (sz + 1) / 2;
      if (1 != r) {
        int m = (1 + r) / 2:
         seg[2 * p].set_a = set_a;
         seg[2 * p].set_r = set_r;
         seg[2 * p].add_a = seg[2 * p].add_r = 0;
         seg[2 * p + 1].set_a = set_a + set_r * (m - 1 + 1);
         seg[2 * p + 1].set_r = set_r;
         seg[2 * p + 1].add_a = seg[2 * p + 1].add_r = 0;
      set_a = oo, set_r = 0;
      add_a = add_r = 0;
    } else if (add_a or add_r) {
      sum += add_a * sz + add_r * sz * (sz + 1) / 2;
      if (1 != r) {
```

```
int m = (1 + r) / 2:
      seg[2 * p].add_a += add_a;
      seg[2 * p].add_r += add_r;
      seg[2 * p + 1].add_a += add_a + add_r * (m - 1 + 1);
      seg[2 * p + 1].add_r += add_r;
   }
    add_a = add_r = 0;
 }
}
int inter(pii a, pii b) {
 if (a.first > b.first) swap(a, b);
  return max(0, min(a.second, b.second) - b.first + 1);
11 set(int a, int b, ll aa, ll rr, int p, int l, int r) {
 prop(p, 1, r);
 if (b < l or r < a) return seg[p].sum;</pre>
 if (a <= 1 and r <= b) {
    seg[p].set_a = aa;
    seg[p].set_r = rr;
    prop(p, 1, r);
    return seg[p].sum;
 }
 int m = (1 + r) / 2;
 int tam_l = inter({1, m}, {a, b});
 return seg[p].sum = set(a, b, aa, rr, 2 * p, 1, m) +
                       set(a, b, aa + rr * tam_1, rr, 2 * p +
 1, m + 1, r);
void update_set(int 1, int r, 11 aa, 11 rr) {
  set(1, r, aa, rr, 1, 0, n - 1);
11 add(int a, int b, ll aa, ll rr, int p, int l, int r) {
 prop(p, 1, r);
 if (b < 1 or r < a) return seg[p].sum;
 if (a \le 1 \text{ and } r \le b) 
    seg[p].add_a += aa;
    seg[p].add_r += rr;
    prop(p, 1, r);
    return seg[p].sum;
 int m = (1 + r) / 2:
  int tam_l = inter({1, m}, {a, b});
```

```
return seg[p].sum = add(a, b, aa, rr, 2 * p, 1, m) +
                         add(a, b, aa + rr * tam_1, rr, 2 * p +
   1, m + 1, r);
  }
  void update_add(int 1, int r, ll aa, ll rr) {
    add(1, r, aa, rr, 1, 0, n - 1);
  ll query(int a, int b, int p, int l, int r) {
    prop(p, 1, r);
   if (b < 1 or r < a) return 0;
   if (a <= 1 and r <= b) return seg[p].sum;</pre>
   int m = (1 + r) / 2;
   return query(a, b, 2 * p, 1, m) + query(a, b, 2 * p + 1, m
   + 1, r);
  11 query(int 1, int r) { return query(1, r, 1, 0, n - 1); }
};
1.9 SegTree Point Update (dynamic function)
Answers queries of any monoid operation (i.e. has identity element and is associative)
Build: O(N), Query: O(\log N)
#define F(expr) [](auto a, auto b) { return expr; }
template <typename T>
struct SegTree {
  using Operation = T (*)(T, T);
  int N:
  vector <T> ns:
  Operation operation;
  T identity;
  SegTree(int n, Operation op = F(a + b), T neutral = T())
   : N(n), ns(2 * N, neutral), operation(op), identity(neutral
   ) {}
  SegTree(const vector<T> &v, Operation op = F(a + b), T
   neutral = T())
    : SegTree((int)v.size(), op, neutral) {
    copy(v.begin(), v.end(), ns.begin() + N);
    for (int i = N - 1; i > 0; --i) ns[i] = operation(ns[2 * i
   ], ns[2 * i + 1]);
  }
```

```
T query(size_t i) const { return ns[i + N]; }
  T query(size_t l, size_t r) const {
    auto a = 1 + N, b = r + N;
    auto ans = identity;
    // Non-associative operations needs to be processed
   backwards
    stack <T> st;
    while (a <= b) {</pre>
      if (a \& 1) ans = operation(ans, ns[a++]);
      if (not(b & 1)) st.push(ns[b--]);
      a >>= 1;
      b >>= 1;
    for (; !st.empty(); st.pop()) ans = operation(ans, st.top()
   );
    return ans;
  }
  void update(size_t i, T value) { update_set(i, operation(ns[i
    + N], value)); }
  void update_set(size_t i, T value) {
    auto a = i + N;
    ns[a] = value;
    while (a >>= 1) ns[a] = operation(ns[2 * a], ns[2 * a + 1])
  }
};
1.10 Segtree Range Max Query Point Max Assign Update
      (dynamic)
Answers range queries in ranges until 10<sup>9</sup> (maybe more)
Time: query and update O(n \cdot \log n)
struct node;
node *newNode();
struct node {
  node *left, *right;
  int lv. rv:
```

```
ll val;
  node() : left(NULL), right(NULL), val(-oo) {}
 inline void init(int 1, int r) {
   lv = 1:
   rv = r;
 }
  inline void extend() {
   if (!left) {
      int m = (lv + rv) / 2;
      left = newNode();
      right = newNode();
      left->init(lv, m);
      right -> init(m + 1, rv);
   }
 }
 11 query(int 1, int r) {
    if (r < lv || rv < l) {</pre>
      return 0;
    if (1 <= lv && rv <= r) {
      return val;
    extend();
    return max(left->query(l, r), right->query(l, r));
 }
  void update(int p, ll newVal) {
    if (lv == rv) {
      val = max(val, newVal);
      return;
   }
    extend():
    (p <= left->rv ? left : right)->update(p, newVal);
    val = max(left->val, right->val);
 }
};
const int BUFFSZ(1e7);
```

```
node *newNode() {
  static int bufSize = BUFFSZ;
  static node buf[(int)BUFFSZ];
  assert(bufSize):
 return &buf[--bufSize]:
}
struct SegTree {
  int n;
  node *root;
  SegTree(int _n) : n(_n) {
   root = newNode();
   root -> init(0, n);
 }
  11 query(int 1, int r) { return root->query(1, r); }
 void update(int p, ll v) { root->update(p, v); }
};
1.11 Segtree Range Max Query Range Max Update
template <typename T = 11>
struct SegTree {
  int N;
  T nu, nq;
  vector <T> st, lazy;
  SegTree(const vector<T> &xs)
   : N(len(xs)),
      nu(numeric_limits <T>::min()),
      nq(numeric_limits <T>::min()),
      st(4 * N + 1, nu),
      lazy(4 * N + 1, nu) {
    for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);</pre>
 }
  void update(int 1, int r, T value) { update(1, 0, N - 1, 1, r
   , value); }
  T query(int 1, int r) { return query(1, 0, N - 1, 1, r); }
  void update(int node, int nl, int nr, int ql, int qr, T v) {
    propagation(node, nl, nr);
    if (ql > nr or qr < nl) return;</pre>
    st[node] = max(st[node], v):
```

```
if (ql <= nl and nr <= qr) {</pre>
      if (nl < nr) {</pre>
        lazy[left(node)] = max(lazy[left(node)], v);
        lazy[right(node)] = max(lazy[right(node)], v);
      }
      return;
    update(left(node), nl, mid(nl, nr), ql, qr, v);
    update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);
    st[node] = max(st[left(node)], st[right(node)]);
  }
  T query(int node, int nl, int nr, int ql, int qr) {
    propagation(node, nl, nr);
    if (ql > nr or qr < nl) return nq;
    if (ql <= nl and nr <= qr) return st[node];</pre>
    T x = query(left(node), nl, mid(nl, nr), ql, qr);
    T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);
    return max(x, y);
  }
  void propagation(int node, int nl, int nr) {
    if (lazy[node] != nu) {
      st[node] = max(st[node], lazy[node]);
      if (nl < nr) {
        lazy[left(node)] = max(lazy[left(node)], lazy[node]);
        lazy[right(node)] = max(lazy[right(node)], lazy[node]);
      lazy[node] = nu;
    }
  }
  int left(int p) { return p << 1; }</pre>
  int right(int p) { return (p << 1) + 1; }</pre>
  int mid(int 1, int r) { return (r - 1) / 2 + 1; }
};
int main() {
  int n;
```

```
cin >> n:
vector < array < int , 3>> xs(n);
for (int i = 0; i < n; ++i) {</pre>
  for (int j = 0; j < 3; ++j) {
    cin >> xs[i][j];
  }
}
vi aux(n, 0);
SegTree < int > st(aux);
for (int i = 0; i < n; ++i) {
  int a = min(i + xs[i][1], n);
  int b = min(i + xs[i][2], n);
  st.update(i, i, st.query(i, i) + xs[i][0]);
  int cur = st.query(i, i);
  st.update(a, b, cur);
}
cout << st.query(0, n) << '\n';
```

1.12 SegTree Range Min Query Point Assign Update

```
template <typename T = 11>
struct SegTree {
  int n;
  T nu, nq;
  vector <T> st;
  SegTree(const vector <T> &v)
   : n(len(v)), nu(0), nq(numeric_limits < T > :: max()), st(n * 4
   + 1. nu) {
    for (int i = 0; i < n; ++i) update(i, v[i]);</pre>
  void update(int p, T v) { update(1, 0, n - 1, p, v); }
  T query(int 1, int r) { return query(1, 0, n - 1, 1, r); }
  void update(int node, int nl, int nr, int p, T v) {
    if (p < nl or p > nr) return;
    if (nl == nr) {
      st[node] = v;
      return;
    update(left(node), nl, mid(nl, nr), p, v);
    update(right(node), mid(nl, nr) + 1, nr, p, v);
```

```
st[node] = min(st[left(node)], st[right(node)]);
  }
 T query(int node, int nl, int nr, int ql, int qr) {
    if (ql <= nl and qr >= nr) return st[node];
    if (nl > qr or nr < ql) return nq;</pre>
    if (nl == nr) return st[node];
    return min(query(left(node), nl, mid(nl, nr), ql, qr),
                query(right(node), mid(nl, nr) + 1, nr, ql, qr))
  }
  int left(int p) { return p << 1; }</pre>
 int right(int p) { return (p << 1) + 1; }</pre>
  int mid(int 1, int r) { return (r - 1) / 2 + 1; }
};
1.13 Segtree Range Sum Query Point Sum Update (dynamic)
Answers range queries in ranges until 10<sup>9</sup> (maybe more)
Time: query and update O(n \cdot \log n)
struct node;
node *newNode();
struct node {
  node *left, *right;
  int lv, rv;
  ll val;
  node() : left(NULL), right(NULL), val(0) {}
  inline void init(int 1, int r) {
    lv = 1:
    rv = r;
  inline void extend() {
    if (!left) {
      int m = (rv - lv) / 2 + lv;
      left = newNode();
      right = newNode();
      left->init(lv, m);
      right -> init(m + 1, rv);
    }
```

```
}
  11 query(int 1, int r) {
    if (r < lv || rv < l) {
      return 0:
    }
    if (1 <= lv && rv <= r) {
      return val;
    extend();
    return left->query(l, r) + right->query(l, r);
  }
  void update(int p, ll newVal) {
    if (lv == rv) {
      val += newVal:
      return;
    }
    extend();
    (p <= left ->rv ? left : right) ->update(p, newVal);
    val = left->val + right->val;
};
const int BUFFSZ(1.3e7);
node *newNode() {
  static int bufSize = BUFFSZ;
  static node buf[(int)BUFFSZ];
  // assert(bufSize);
  return &buf[--bufSize];
struct SegTree {
  int n:
  node *root;
  SegTree(int _n) : n(_n) {
    root = newNode();
    root -> init(0, n);
  ll query(int 1, int r) { return root->query(1, r); }
  void update(int p, ll v) { root->update(p, v); }
};
```

1.14 SegTree Range Xor query Point Assign Update

```
template <typename T = 11>
struct SegTree {
  int n;
 T nu, nq;
  vector <T> st;
  SegTree(const vectorT> &v) : n(len(v)), nu(0), nq(0), st(n *
    4 + 1, nu) {
   for (int i = 0; i < n; ++i) update(i, v[i]);</pre>
 }
 void update(int p, T v) { update(1, 0, n - 1, p, v); }
 T query(int 1, int r) { return query(1, 0, n - 1, 1, r); }
  void update(int node, int nl, int nr, int p, T v) {
   if (p < nl or p > nr) return;
   if (nl == nr) {
      st[node] = v;
      return;
    }
    update(left(node), nl, mid(nl, nr), p, v);
    update(right(node), mid(nl, nr) + 1, nr, p, v);
    st[node] = st[left(node)] ^ st[right(node)];
 }
 T query(int node, int nl, int nr, int ql, int qr) {
   if (ql <= nl and qr >= nr) return st[node];
   if (nl > qr or nr < ql) return nq;</pre>
   if (nl == nr) return st[node];
    return query(left(node), nl, mid(nl, nr), ql, qr) ^
           query(right(node), mid(nl, nr) + 1, nr, ql, qr);
 }
 int left(int p) { return p << 1; }</pre>
 int right(int p) { return (p << 1) + 1; }</pre>
  int mid(int 1, int r) { return (r - 1) / 2 + 1; }
};
```

1.15 SegTree Range Min Query Range Sum Update

```
template <typename t = 11>
struct SegTree {
```

```
int n:
t nu;
t nq;
vector < t> st, lazy;
SegTree(const vector < t > &xs)
 : n(len(xs)),
    nu(0).
    nq(numeric_limits <t>::max()),
    st(4 * n, nu).
    lazv(4 * n, nu) {
  for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);</pre>
}
SegTree(int n): n(n), st(4 * n, nu), lazy(4 * n, nu) {}
void update(int 1, int r, 11 value) { update(1, 0, n - 1, 1,
 r, value); }
t query(int 1, int r) { return query(1, 0, n - 1, 1, r); }
void update(int node, int nl, int nr, int ql, int qr, ll v) {
  propagation(node, nl, nr);
  if (ql > nr or qr < nl) return;</pre>
  if (ql <= nl and nr <= qr) {</pre>
    st[node] += (nr - nl + 1) * v;
    if (nl < nr) {</pre>
      lazy[left(node)] += v;
      lazy[right(node)] += v;
    return;
  update(left(node), nl, mid(nl, nr), ql, qr, v);
  update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);
  st[node] = min(st[left(node)], st[right(node)]);
}
t query(int node, int nl, int nr, int ql, int qr) {
  propagation(node, nl, nr);
```

```
if (ql > nr or qr < nl) return nq;</pre>
    if (ql <= nl and nr <= qr) return st[node];</pre>
   t x = query(left(node), nl, mid(nl, nr), ql, qr);
   t y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);
    return min(x, y);
  void propagation(int node, int nl, int nr) {
    if (lazy[node]) {
      st[node] += lazy[node];
      if (nl < nr) {</pre>
        lazy[left(node)] += lazy[node];
        lazy[right(node)] += lazy[node];
      }
      lazy[node] = nu;
  }
  int left(int p) { return p << 1; }</pre>
  int right(int p) { return (p << 1) + 1; }</pre>
 int mid(int 1, int r) { return (r - 1) / 2 + 1; }
};
```

1.16 SegTree Range Sum Query Range Sum Update

```
template <typename T = 11>
struct SegTree {
   int N;
   T nu;
   T nq;
   vector<T> st, lazy;
   SegTree(const vector<T> &xs)
      : N(len(xs)), nu(0), nq(0), st(4 * N, nu), lazy(4 * N, nu)
   {
      for (int i = 0; i < len(xs); ++i) update(i, i, xs[i]);
   }

SegTree(int n) : N(n), nu(0), nq(0), st(4 * N, nu), lazy(4 * N, nu) {}
</pre>
```

```
void update(int 1, int r, 11 value) { update(1, 0, N - 1, 1,
 r, value); }
T query(int 1, int r) { return query(1, 0, N - 1, 1, r); }
void update(int node, int nl, int nr, int ql, int qr, ll v) {
  propagation(node, nl, nr);
  if (ql > nr or qr < nl) return;</pre>
  if (ql <= nl and nr <= qr) {</pre>
    st[node] += (nr - nl + 1) * v;
    if (nl < nr) {</pre>
      lazv[left(node)] += v;
      lazy[right(node)] += v;
    return;
  }
  update(left(node), nl, mid(nl, nr), ql, qr, v);
  update(right(node), mid(nl, nr) + 1, nr, ql, qr, v);
  st[node] = st[left(node)] + st[right(node)];
}
T query(int node, int nl, int nr, int ql, int qr) {
  propagation(node, nl, nr);
  if (ql > nr or qr < nl) return nq;</pre>
  if (ql <= nl and nr <= qr) return st[node];</pre>
  T x = query(left(node), nl, mid(nl, nr), ql, qr);
  T y = query(right(node), mid(nl, nr) + 1, nr, ql, qr);
  return x + y;
}
void propagation(int node, int nl, int nr) {
  if (lazy[node]) {
    st[node] += (nr - nl + 1) * lazy[node];
    if (nl < nr) {
```

```
lazy[right(node)] += lazy[node];
      lazy[node] = nu;
    }
  }
  int left(int p) { return p << 1; }</pre>
  int right(int p) { return (p << 1) + 1; }</pre>
  int mid(int 1, int r) { return (r - 1) / 2 + 1; }
};
1.17 Sparse Table
Answer the range query defined at the function op.
Build: O(NlogN), Query: O(1)
template <typename T>
struct SparseTable {
  vector <T> v;
  int n;
  static const int b = 30;
  vi mask, t;
  int op(int x, int y) { return v[x] < v[y] ? x : y; }</pre>
  int msb(int x) { return __builtin_clz(1) - __builtin_clz(x);
   }
  SparseTable() {}
  SparseTable(const vector<T>& v_) : v(v_), n(v.size()), mask(n
   ), t(n) {
    for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
      at = (at << 1) & ((1 << b) - 1):
      while (at and op(i, i - msb(at & -at)) == i) at ^= at & -
   at;
    }
    for (int i = 0; i < n / b; i++)
      t[i] = b * i + b - 1 - msb(mask[b * i + b - 1]);
    for (int j = 1; (1 << j) <= n / b; j++)
      for (int i = 0; i + (1 << j) <= n / b; i++)
        t[n / b * j + i] =
          op(t[n / b * (j - 1) + i], t[n / b * (j - 1) + i + (1)
    << (j - 1))]);
  int small(int r, int sz = b) { return r - msb(mask[r] & ((1
   << sz) - 1)); }
  T query(int 1, int r) {
```

lazy[left(node)] += lazy[node];

```
if (r - l + 1 <= b) return small(r, r - l + 1);
int ans = op(small(l + b - 1), small(r));
int x = l / b + 1, y = r / b - 1;
if (x <= y) {
   int j = msb(y - x + 1);
   ans = op(ans, op(t[n / b * j + x], t[n / b * j + y - (1 << j) + 1]));
}
return ans;
}
};</pre>
```

2 Dynamic programming

2.1 Binary Knapsack (bottom up)

Given N items, each with its own value V_i and weight W_i and a maximum knapsack weight W, compute the maximum value of the items that we can carry, if we can either ignore or take a particular item. Assume that $1 \le n \le 1000$, $1 \le S \le 10000$.

Time and space: O(N * W)

the vectors VS and WS starts at one, so it need an empty value at index 0.

```
const int MAXN(2010), MAXM(2010);
ll st[MAXN + 1][MAXM + 1];
char ps[MAXN + 1][MAXM + 1];
pair<11, vi> knapsack(int M, const vll &VS, const vi &WS) {
  memset(st, 0, sizeof(st));
  memset(ps, 0, sizeof(ps));
  int N = len(VS) - 1; // ELEMENTS START AT INDEX 1 !
  for (int i = 0; i \le N; ++i) st[i][0] = 0;
  for (int m = 0; m \le M; ++m) st[0][m] = 0;
  for (int i = 1; i <= N; ++i) {</pre>
    for (int m = 1; m <= M; ++m) {</pre>
      st[i][m] = st[i - 1][m];
      ps[i][m] = 0;
      int w = WS[i];
      11 v = VS[i];
      if (w \le m \text{ and } st[i - 1][m - w] + v > st[i][m]) {
        st[i][m] = st[i - 1][m - w] + v;
        ps[i][m] = 1;
      }
    }
```

```
}
  int m = M;
  vi is:
  for (int i = N; i >= 1; --i) {
    if (ps[i][m]) {
      is.emplace_back(i - 1);
      m -= WS[i];
    }
  }
  return {st[N][M], is};
    Binary Knapsack (top down)
Given N items, each with its own value V_i and weight W_i and a maximum knapsack weight W, compute
the maximum value of the items that we can carry, if we can either ignore or take a particular item.
Assume that 1 \le n \le 1000, 1 \le S \le 10000.
Time and space: O(N * W)
the bottom up version is 5 times faster!
const int MAXN(2000), MAXM(2000);
11 memo[MAXN][MAXM + 1];
char choosen[MAXN][MAXM + 1];
ll knapSack(int u, int w, vll &VS, vi &WS) {
  if (u < 0) return 0;
  if (memo[u][w] != -1) return memo[u][w];
  11 a = 0, b = 0;
  a = knapSack(u - 1, w, VS, WS);
  if (WS[u] \le w) b = knapSack(u - 1, w - WS[u], VS, WS) + VS[u]
  ];
  if (b > a) {
    choosen[u][w] = true;
  }
  return memo[u][w] = max(a, b);
pair<ll, vi> knapSack(int W, vll &VS, vi &WS) {
  memset(memo, -1, sizeof(memo));
  memset(choosen, 0, sizeof(choosen));
  int n = len(VS);
  ll v = knapSack(n - 1, W, VS, WS);
  11 cw = W;
  vi choosed;
  for (int i = n - 1; i >= 0; i--) {
```

if (choosen[i][cw]) {

```
cw -= WS[i]:
      choosed.emplace_back(i);
    }
 }
 return {v, choosed};
2.3 Digits
Finds the number of digits between 1 and x that don't have 4 or 13 as substring.
ll memo[19][10][2];
11 dp(int p, int d, bool l, const vi &digits) {
  if (p == len(digits)) return 0;
  if (memo[p][d][l] != -1) return memo[p][d][l];
  11 \text{ tot} = 0;
  ll k = l and d == digits[p] ? digits[p + 1] : 9;
  for (int i = 0; i <= k; i++) {
    if (i == 4) continue;
    if (d == 1 and i == 3) continue;
    tot += dp(p + 1, i, l and d == digits[p], digits);
  return memo[p][d][l] = tot;
vi get_digits(ll x) {
  vi digits;
  while (x) {
    digits.emplace_back(x % 10);
    x /= 10:
  }
  reverse(all(digits));
 return digits;
ll dp(ll x) {
  auto digits = get_digits(x);
  memset(memo, -1, sizeof memo);
  dbg(x, len(digits) - 1, digits);
  for (int i = 0; i \le 9; i++) {
```

```
memo[len(digits) - 1][i][0] = 1;
    memo[len(digits) - 1][i][1] = i <= digits.back();</pre>
  }
  11 \text{ tot} = 0:
  for (int i = 0; i <= digits[0]; i++) {</pre>
   if (i == 4) continue;
   tot += dp(0, i, i == digits[0], digits);
  return tot - 1;
2.4 Edit Distance
O(N*M)
int edit_distance(const string &a, const string &b) {
  int n = a.size();
  int m = b.size();
  vector < vi > dp(n + 1, vi(m + 1, 0));
  int ADD = 1, DEL = 1, CHG = 1;
  for (int i = 0; i <= n; ++i) {
    dp[i][0] = i * DEL;
  for (int i = 1; i <= m; ++i) {</pre>
    dp[0][i] = ADD * i;
  for (int i = 1; i <= n; ++i) {
    for (int j = 1; j <= m; ++j) {
      int add = dp[i][j - 1] + ADD;
      int del = dp[i - 1][j] + DEL;
      int chg = dp[i - 1][j - 1] + (a[i - 1] == b[j - 1]?0:
   1) * CHG;
      dp[i][j] = min({add, del, chg});
    }
  }
  return dp[n][m];
```

2.5 Kadane

Find the maximum subarray sum in a given a rray.

```
int kadane(const vi &as) {
  vi s(len(as));
  s[0] = as[0];
  for (int i = 1; i < len(as); ++i) s[i] = max(as[i], s[i-1]
   + as[i]):
  return *max_element(all(s));
2.6 Longest Increasing Subsequence (LIS)
Finds the length of the longest subsequence in
                               O(n \log n)
int LIS(const vi& as) {
  const ll oo = 1e18:
  int n = len(as);
  vll lis(n + 1, oo);
  lis[0] = -oo:
  auto ans = 0;
 for (int i = 0; i < n; ++i) {
    auto it = lower_bound(all(lis), as[i]);
    auto pos = (int)(it - lis.begin());
    ans = max(ans, pos);
    lis[pos] = as[i];
  return ans;
2.7 Money Sum (Bottom Up)
Find every possible sum using the given values only once.
```

```
set < int > money_sum(const vi &xs) {
  using vc = vector < char >;
  using vvc = vector < vc >;
  int _m = accumulate(all(xs), 0);
  int _n = xs.size();
  vvc _dp(_n + 1, vc(_m + 1, 0));
  set < int > _ans;
  _dp[0][xs[0]] = 1;
```

```
for (int i = 1; i < _n; ++i) {
    for (int j = 0; j <= _m; ++j) {</pre>
      if (j == 0 or _dp[i - 1][j]) {
        dp[i][j + xs[i]] = 1;
        _{dp[i][j]} = 1;
      }
    }
  }
  for (int i = 0; i < _n; ++i)</pre>
    for (int j = 0; j <= _m; ++j)</pre>
      if (_dp[i][j]) _ans.insert(j);
  return _ans;
   Travelling Salesman Problem
using vi = vector<int>;
vector < vi > dist;
vector < vi > memo;
/* 0 ( N^2 * 2^N )*/
int tsp(int i, int mask, int N) {
  if (mask == (1 << N) - 1) return dist[i][0];</pre>
  if (memo[i][mask] != -1) return memo[i][mask];
  int ans = INT_MAX << 1;</pre>
  for (int j = 0; j < N; ++ j) {
   if (mask & (1 << j)) continue;</pre>
   auto t = tsp(j, mask | (1 << j), N) + dist[i][j];</pre>
    ans = min(ans, t);
 }
  return memo[i][mask] = ans;
  Extras
3.1 Bigint
const int maxn = 1e2 + 14, 1g = 15;
const int base = 1000000000;
const int base_digits = 9;
struct bigint {
 vector<int> a;
 int sign;
```

int size() {

```
if (a.empty()) return 0;
  int ans = (a.size() - 1) * base_digits;
  int ca = a.back();
  while (ca) ans++, ca \neq 10;
  return ans:
bigint operator^(const bigint &v) {
  bigint ans = 1, a = *this, b = v;
  while (!b.isZero()) {
   if (b % 2) ans *= a;
    a *= a, b /= 2;
 }
  return ans;
string to_string() {
  stringstream ss;
  ss << *this;
  string s;
  ss >> s;
  return s;
int sumof() {
 string s = to_string();
 int ans = 0;
  for (auto c : s) ans += c - '0';
  return ans;
/*</arpa>*/
bigint() : sign(1) {}
bigint(long long v) { *this = v; }
bigint(const string &s) { read(s); }
void operator=(const bigint &v) {
 sign = v.sign;
 a = v.a;
}
void operator=(long long v) {
 sign = 1;
 a.clear():
 if (v < 0) sign = -1, v = -v;
  for (; v > 0; v = v / base) a.push_back(v % base);
```

```
bigint operator+(const bigint &v) const {
 if (sign == v.sign) {
    bigint res = v;
    for (int i = 0, carry = 0; i < (int)max(a.size(), v.a.
 size()) || carry;
         ++i) {
      if (i == (int)res.a.size()) res.a.push_back(0);
      res.a[i] += carry + (i < (int)a.size() ? a[i] : 0);
      carry = res.a[i] >= base;
      if (carry) res.a[i] -= base;
    }
    return res;
  return *this - (-v);
}
bigint operator-(const bigint &v) const {
  if (sign == v.sign) {
    if (abs() >= v.abs()) {
      bigint res = *this;
      for (int i = 0, carry = 0; i < (int)v.a.size() || carry</pre>
 ; ++i) {
        res.a[i] -= carry + (i < (int)v.a.size() ? v.a[i] :
 0);
        carry = res.a[i] < 0;
        if (carry) res.a[i] += base;
      res.trim();
      return res;
    return -(v - *this);
  return *this + (-v);
void operator*=(int v) {
  if (v < 0) sign = -sign, v = -v;
  for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i)</pre>
    if (i == (int)a.size()) a.push_back(0);
    long long cur = a[i] * (long long)v + carry;
    carry = (int)(cur / base);
    a[i] = (int)(cur \% base);
```

```
// asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) :
   // "A"(cur), "c"(base));
 }
 trim():
}
bigint operator*(int v) const {
 bigint res = *this;
 res *= v;
 return res;
void operator*=(long long v) {
 if (v < 0) sign = -sign, v = -v;
 if (v > base) {
    *this = *this * (v / base) * base + *this * (v % base);
   return;
 for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i)</pre>
    if (i == (int)a.size()) a.push_back(0);
   long long cur = a[i] * (long long)v + carry;
    carry = (int)(cur / base);
    a[i] = (int)(cur \% base);
   // asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) :
   // "A"(cur), "c"(base));
 trim();
bigint operator*(long long v) const {
 bigint res = *this;
 res *= v;
 return res;
}
friend pair < bigint, bigint > divmod(const bigint &a1, const
 bigint &b1) {
 int norm = base / (b1.a.back() + 1);
 bigint a = a1.abs() * norm;
 bigint b = b1.abs() * norm;
  bigint q, r;
 q.a.resize(a.a.size());
 for (int i = a.a.size() - 1; i >= 0; i--) {
```

```
r *= base:
    r += a.a[i];
    int s1 = r.a.size() <= b.a.size() ? 0 : r.a[b.a.size()];</pre>
    int s2 = r.a.size() <= b.a.size() - 1 ? 0 : r.a[b.a.size</pre>
 () - 1]:
    int d = ((long long)base * s1 + s2) / b.a.back();
    r \rightarrow b * d;
    while (r < 0) r += b, --d;
    q.a[i] = d;
 q.sign = a1.sign * b1.sign;
 r.sign = a1.sign;
 q.trim();
 r.trim();
 return make_pair(q, r / norm);
bigint operator/(const bigint &v) const { return divmod(*this
 , v).first; }
bigint operator%(const bigint &v) const { return divmod(*this
 , v).second; }
void operator/=(int v) {
  if (v < 0) sign = -sign, v = -v;
 for (int i = (int)a.size() - 1, rem = 0; i >= 0; --i) {
    long long cur = a[i] + rem * (long long)base;
    a[i] = (int)(cur / v);
    rem = (int)(cur \% v);
  trim();
bigint operator/(int v) const {
  bigint res = *this;
 res /= v:
  return res:
}
int operator%(int v) const {
 if (v < 0) v = -v;
 int m = 0:
 for (int i = a.size() - 1; i >= 0; --i)
    m = (a[i] + m * (long long)base) % v;
```

```
return m * sign;
}
void operator+=(const bigint &v) { *this = *this + v; }
void operator -= (const bigint &v) { *this = *this - v; }
void operator*=(const bigint &v) { *this = *this * v; }
void operator/=(const bigint &v) { *this = *this / v; }
bool operator < (const bigint &v) const {</pre>
  if (sign != v.sign) return sign < v.sign;</pre>
  if (a.size() != v.a.size()) return a.size() * sign < v.a.</pre>
 size() * v.sign;
  for (int i = a.size() - 1; i >= 0; i--)
    if (a[i] != v.a[i]) return a[i] * sign < v.a[i] * sign;</pre>
  return false;
}
bool operator>(const bigint &v) const { return v < *this; }</pre>
bool operator <= (const bigint &v) const { return !(v < *this);</pre>
  }
bool operator >= (const bigint &v) const { return !(*this < v);</pre>
bool operator == (const bigint &v) const {
  return !(*this < v) && !(v < *this);</pre>
bool operator!=(const bigint &v) const { return *this < v ||</pre>
 v < *this; }</pre>
void trim() {
  while (!a.empty() && !a.back()) a.pop_back();
  if (a.empty()) sign = 1;
bool isZero() const { return a.empty() || (a.size() == 1 && !
 a[0]); }
bigint operator -() const {
  bigint res = *this;
 res.sign = -sign;
  return res:
}
bigint abs() const {
  bigint res = *this;
  res.sign *= res.sign;
```

```
return res;
}
long longValue() const {
 long long res = 0;
 for (int i = a.size() - 1; i >= 0; i--) res = res * base +
 return res * sign;
friend bigint gcd(const bigint &a, const bigint &b) {
  return b.isZero() ? a : gcd(b, a % b);
friend bigint lcm(const bigint &a, const bigint &b) {
  return a / gcd(a, b) * b;
}
void read(const string &s) {
  sign = 1;
 a.clear();
 int pos = 0;
  while (pos < (int)s.size() && (s[pos] == '-' || s[pos] == '
    if (s[pos] == '-') sign = -sign;
    ++pos;
  for (int i = s.size() - 1; i >= pos; i -= base_digits) {
    int x = 0;
    for (int j = max(pos, i - base_digits + 1); j <= i; j++)</pre>
     x = x * 10 + s[j] - '0';
    a.push_back(x);
  trim();
friend istream &operator>>(istream &stream, bigint &v) {
  string s;
 stream >> s;
 v.read(s);
  return stream;
}
friend ostream &operator<<(ostream &stream, const bigint &v)</pre>
 if (v.sign == -1) stream << '-';</pre>
```

```
stream << (v.a.empty() ? 0 : v.a.back());
 for (int i = (int)v.a.size() - 2; i >= 0; --i)
    stream << setw(base_digits) << setfill('0') << v.a[i];</pre>
 return stream:
}
static vector < int > convert_base (const vector < int > &a, int
 old_digits,
                                 int new_digits) {
  vector < long long > p(max(old_digits, new_digits) + 1);
 for (int i = 1; i < (int)p.size(); i++) p[i] = p[i - 1] *
 10:
  vector<int> res;
 long long cur = 0;
 int cur_digits = 0;
  for (int i = 0; i < (int)a.size(); i++) {</pre>
    cur += a[i] * p[cur_digits];
    cur_digits += old_digits;
    while (cur_digits >= new_digits) {
      res.push_back(int(cur % p[new_digits]));
      cur /= p[new_digits];
      cur_digits -= new_digits;
   }
  }
  res.push_back((int)cur);
  while (!res.empty() && !res.back()) res.pop_back();
  return res;
}
typedef vector<long long> vll;
static vll karatsubaMultiply(const vll &a, const vll &b) {
 int n = a.size();
 vll res(n + n);
 if (n <= 32) {
    for (int i = 0; i < n; i++)</pre>
      for (int j = 0; j < n; j++) res[i + j] += a[i] * b[j];
    return res:
  int k = n \gg 1:
 vll a1(a.begin(), a.begin() + k);
  vll a2(a.begin() + k, a.end());
  vll b1(b.begin(), b.begin() + k);
```

```
vll b2(b.begin() + k, b.end());
    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);
    for (int i = 0; i < k; i++) a2[i] += a1[i];
    for (int i = 0; i < k; i++) b2[i] += b1[i];</pre>
    vll r = karatsubaMultiply(a2, b2);
    for (int i = 0; i < (int)a1b1.size(); i++) r[i] -= a1b1[i];
    for (int i = 0; i < (int)a2b2.size(); i++) r[i] -= a2b2[i];</pre>
    for (int i = 0; i < (int)r.size(); i++) res[i + k] += r[i];</pre>
   for (int i = 0; i < (int)a1b1.size(); i++) res[i] += a1b1[i</pre>
   for (int i = 0; i < (int)a2b2.size(); i++) res[i + n] +=
   a2b2[i];
    return res;
  bigint operator*(const bigint &v) const {
    vector < int > a6 = convert_base(this -> a, base_digits, 6);
    vector < int > b6 = convert_base(v.a, base_digits, 6);
    vll a(a6.begin(), a6.end());
    vll b(b6.begin(), b6.end());
    while (a.size() < b.size()) a.push_back(0);</pre>
    while (b.size() < a.size()) b.push_back(0);</pre>
    while (a.size() & (a.size() - 1)) a.push_back(0), b.
   push_back(0);
    vll c = karatsubaMultiply(a, b);
    bigint res;
    res.sign = sign * v.sign;
    for (int i = 0, carry = 0; i < (int)c.size(); i++) {</pre>
      long long cur = c[i] + carry;
      res.a.push_back((int)(cur % 1000000));
      carry = (int)(cur / 1000000);
    res.a = convert_base(res.a, 6, base_digits);
    res.trim():
    return res;
 }
}:
```

3.2 Binary To Gray

```
string binToGray(string bin) {
  string gray(bin.size(), '0');
  int n = bin.size() - 1;
  grav[0] = bin[0]:
 for (int i = 1; i <= n; i++) {
    gray[i] = '0' + (bin[i - 1] == '1') ^ (bin[i] == '1');
  return gray;
3.3 Get Permutation Cicles
* receives a permutation [0, n-1]
* returns a vector of cicles
* for example: [1, 0, 3, 4, 2] \rightarrow [[0, 1], [2, 3, 4]]
vector<vll> getPermutationCicles(const vll &ps) {
  ll n = len(ps):
  vector < char > visited(n):
  vector < vll > cicles;
  for (int i = 0; i < n; ++i) {</pre>
    if (visited[i]) continue;
    vll cicle;
    ll pos = i;
    while (!visited[pos]) {
      cicle.pb(pos);
      visited[pos] = true;
      pos = ps[pos];
    }
    cicles.push_back(vll(all(cicle)));
  return cicles;
3.4 Mo's Algorithm
template <typename T>
struct Mo {
  struct Query {
    int 1, r, idx, block;
    Query(int _1, int _r, int _idx, int _block)
      : l(_l), r(_r), idx(_idx), block(_block) {}
```

```
bool operator<(const Query &g) const {</pre>
      if (block != q.block) return block < q.block;</pre>
      return (block & 1 ? (r < q.r) : (r > q.r));
    }
  };
  vector <T> vs;
  vector < Query > qs;
  const int block_size;
  Mo(const vector <T > &a) : vs(a), block_size((int)ceil(sqrt(a.
   size()))) {}
  void add_query(int 1, int r) {
    qs.emplace_back(l, r, qs.size(), l / block_size);
  auto solve() {
   // get answer return type
   vector < 11 > answers(qs.size());
    sort(all(qs));
    int cur_1 = 0, cur_r = -1;
    for (auto q : qs) {
      while (cur_l > q.1) add(--cur_l);
      while (cur_r < q.r) add(++cur_r);</pre>
      while (cur_l < q.1) remove(cur_l++);</pre>
      while (cur_r > q.r) remove(cur_r--);
      answers[q.idx] = get_answer();
    return answers;
  }
 private:
 // add value at idx from data structure
  inline void add(int idx) {}
  // remove value at idx from data structure
  inline void remove(int idx) {}
  // extract current answer of the data structure
 inline auto get_answer() {}
};
```

3.5 Number Of Elements Greater Than K

```
template <typename T>
// Query is of the form {L, R, K}
vector<T> count_greater_k(const vector<T> &v,
                           const vector < tuple < int , int , T >> &q)
  struct Node {
   int pos, value, l, r;
  };
  int n = (int)v.size();
  int m = (int)q.size();
  vector < Node > a(n + m):
  for (int i = 0; i < n; i++) {</pre>
    a[i].pos = a[i].l = -1;
    a[i].r = i;
    a[i].value = v[i];
  }
  for (int j = 0; j < m; j++) {
    int i = j + n;
    auto [1, r, k] = q[j];
    a[i].pos = j;
    a[i].l = 1;
    a[i].r = r;
    a[i].value = k;
  sort(all(a), [](auto x, auto y) {
    if (x.value == y.value) return x.l > y.l;
    return x.value > y.value;
  }):
  vector < int > ans(m);
  BITree < int > bit(n + m);
  for (int i = 0; i < n + m; i++) {</pre>
    if (a[i].pos != -1) {
      ans[a[i].pos] = bit.range_sum(a[i].1, a[i].r);
      bit.update(a[i].r, 1);
  }
```

```
return ans;
```

4 Geometry

4.1 Convex Hull

Given a set of points find the smallest convex polygon that contains all the given points. Time: $O(N \log N)$

By default it removes the collinear points, set the boolean to true if you don't want that

```
struct pt {
  double x, y;
  int id:
};
int orientation(pt a, pt b, pt c) {
  double v = a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y)
    - b.v);
  if (v < 0) return -1; // clockwise
  if (v > 0) return +1; // counter-clockwise
  return 0;
}
bool cw(pt a, pt b, pt c, bool include_collinear) {
  int o = orientation(a, b, c);
  return o < 0 | | (include_collinear && o == 0);
bool collinear(pt a, pt b, pt c) { return orientation(a, b, c)
   == 0; }
void convex_hull(vector<pt>& pts, bool include_collinear =
   false) {
  pt p0 = *min_element(all(pts), [](pt a, pt b) {
    return make_pair(a.y, a.x) < make_pair(b.y, b.x);</pre>
  }):
  sort(all(pts), [&p0](const pt& a, const pt& b) {
    int o = orientation(p0, a, b);
    if (o == 0)
      return (p0.x - a.x) * (p0.x - a.x) + (p0.y - a.y) * (p0.y
    - a.y) <
             (p0.x - b.x) * (p0.x - b.x) + (p0.y - b.y) * (p0.y
    - b.y);
    return o < 0;
  });
  if (include_collinear) {
    int i = len(pts) - 1;
```

```
while (i >= 0 && collinear(p0, pts[i], pts.back())) i--;
    reverse(pts.begin() + i + 1, pts.end());
  vector <pt> st;
 for (int i = 0; i < len(pts); i++) {</pre>
    while (st.size() > 1 &&
           !cw(st[len(st) - 2], st.back(), pts[i],
   include_collinear))
      st.pop_back();
    st.push_back(pts[i]);
  pts = st;
4.2 Determinant
#include "Point.cpp"
template <typename T>
T D(const Point<T> &P, const Point<T> &Q, const Point<T> &R) {
 return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
         (R.x * Q.y + R.y * P.x + Q.x * P.y);
}
4.3 Equals
template <typename T>
bool equals(T a, T b) {
  const double EPS{1e-9};
 if (is_floating_point <T>::value)
    return fabs(a - b) < EPS;</pre>
  else
    return a == b;
}
4.4 Line
#include <bits/stdc++.h>
#include "point-struct-and-utils.cpp"
using namespace std;
struct line {
```

```
ld a, b, c;
};
// the answer is stored in the third parameter (pass by
   reference)
void pointsToLine(const point &p1, const point &p2, line &1) {
 if (fabs(p1.x - p2.x) < EPS)
   // vertical line
   1 = \{1.0, 0.0, -p1.x\};
  // default values
  else
    1 = \{-(1d)(p1.y - p2.y) / (p1.x - p2.x), 1.0, -(1d)(1.a * p2.x)\}
   p1.x) - p1.y};
4.5 Point Struct And Utils (2d)
#include <bits/stdc++.h>
using namespace std;
using ld = long double;
struct point {
 ld x, y;
  int id;
  point (ld x = 0.0, ld y = 0.0, int id = -1) : x(x), y(y), id(
  id) {}
  point& operator+=(const point& t) {
    x += t.x;
   y += t.y;
    return *this;
  point& operator -= (const point& t) {
   x -= t.x:
   y = t.y;
    return *this;
  point& operator*=(ld t) {
   x *= t;
   y *= t;
    return *this;
  point& operator/=(ld t) {
   x /= t:
    y /= t;
```

```
return *this:
  point operator+(const point& t) const { return point(*this)
   += t: }
  point operator-(const point& t) const { return point(*this)
  point operator*(ld t) const { return point(*this) *= t; }
  point operator/(ld t) const { return point(*this) /= t; }
};
ld dot(point& a, point& b) { return a.x * b.x + a.y * b.y; }
ld norm(point& a) { return dot(a, a); }
ld abs(point a) { return sqrt(norm(a)); }
ld proj(point a, point b) { return dot(a, b) / abs(b); }
ld angle(point a, point b) { return acos(dot(a, b) / abs(a) /
   abs(b)): }
ld cross(point a, point b) { return a.x * b.y - a.y * b.x; }
4.6 Segment
#include "Line.cpp"
#include "Point.cpp"
#include "equals.cpp"
template <typename T>
struct segment {
  Point <T> A. B:
  bool contains(const Point<T> &P) const;
  Point <T> closest(const Point <T> &p) const;
};
template <typename T>
bool segment <T>::contains(const Point <T> &P) const {
 // verifica se P áest contido na reta
  double dAB = Point<T>::dist(A, B), dAP = Point<T>::dist(A, P)
         dPB = Point < T > :: dist(P, B);
```

```
return equals(dAP + dPB, dAB);
}
template <typename T>
Point <T > segment <T > :: closest(const Point <T > &P) const {
  Line \langle T \rangle R(A, B);
  auto Q = R.closest(P);
  if (this->contains(Q)) return Q;
  auto distA = Point<T>::dist(P, A);
  auto distB = Point<T>::dist(P, B);
  if (distA <= distB)</pre>
    return A;
  else
    return B;
5 Graphs
5.1 2 SAT
struct SAT2 {
  11 n;
  vll2d adj, adj_t;
  vc used;
  vll order, comp;
  vc assignment;
  bool solvable;
  SAT2(11 _n)
   : n(2 * _n),
      adj(n),
      adj_t(n),
      used(n),
      order(n),
      comp(n, -1),
      assignment(n / 2) {}
  void dfs1(int v) {
    used[v] = true;
    for (int u : adj[v]) {
      if (!used[u]) dfs1(u);
    }
    order.push_back(v);
  }
```

```
void dfs2(int v, int cl) {
    comp[v] = c1;
   for (int u : adj_t[v]) {
      if (comp[u] == -1) dfs2(u, cl);
   }
 }
  bool solve_2SAT() {
   // find and label each SCC
    for (int i = 0; i < n; ++i) {
      if (!used[i]) dfs1(i);
    }
    reverse(all(order));
    11 i = 0;
   for (auto &v : order) {
      if (comp[v] == -1) dfs2(v, j++);
    assignment.assign(n / 2, false);
    for (int i = 0; i < n; i += 2) {
     // x and !x belong to the same SCC
     if (comp[i] == comp[i + 1]) {
        solvable = false;
       return false;
      assignment[i / 2] = comp[i] > comp[i + 1];
    solvable = true;
    return true;
  void add_disjunction(int a, bool na, int b, bool nb) {
    a = (2 * a) ^ na;
    b = (2 * b) ^ nb;
    int neg_a = a ^1;
    int neg_b = b ^1;
    adj [neg_a].push_back(b);
    adj[neg_b].push_back(a);
    adj_t[b].push_back(neg_a);
    adj_t[a].push_back(neg_b);
 }
};
```

5.2 Cycle Distances

Time: O(N)

Given a vertex s finds the longest cycle that end's in s, note that the vector **dist** will contain the distance that each vertex u needs to reach s.

```
using adj = vector<vector<pair<int, 11>>>;
ll cycleDistances(int u, int n, int s, vc &vis, adj &g, vll &
   dist) {
 vis[u] = 1;
 for (auto [v, d] : g[u]) {
   if (v == s) {
      dist[u] = max(dist[u], d);
      continue;
   }
   if (vis[v] == 1) {
      continue;
   }
   if (vis[v] == 2) {
      dist[u] = max(dist[u], dist[v] + d);
   } else {
      11 d2 = cycleDistances(v, n, s, vis, g, dist);
      if (d2 != -oo) {
        dist[u] = max(dist[u], d2 + d);
     }
```

5.3 SCC (struct)

vis[u] = 2;

return dist[u];

}

Able to find the component of each node and the total of SCC in O(V*E) and build the SCC graph (O(V*E)).

```
struct SCC {
    ll N;
    int totscc;
    vll2d adj, tadj;
    vll todo, comps, comp;
    vector<set<ll>> sccadj;
    vchar vis;
    SCC(ll _N)
```

```
: N(_N), totscc(0), adj(_N), tadj(_N), comp(_N, -1), sccadj
   (N), vis(N)
  void add_edge(ll x, ll y) { adj[x].eb(y), tadj[y].eb(x); }
  void dfs(ll x) {
    vis[x] = 1:
    for (auto &y : adj[x])
      if (!vis[y]) dfs(y);
    todo.pb(x);
  void dfs2(ll x, ll v) {
    comp[x] = v;
    for (auto &y : tadj[x])
      if (comp[y] == -1) dfs2(y, v);
  }
  void gen() {
    for (11 i = 0; i < N; ++i)</pre>
      if (!vis[i]) dfs(i);
    reverse(all(todo));
    for (auto &x : todo)
      if (comp[x] == -1) {
        dfs2(x, x);
        comps.pb(x);
        totscc++;
      }
  }
  void genSCCGraph() {
    for (11 i = 0; i < N; ++i) {
      for (auto &j : adj[i]) {
        if (comp[i] != comp[j]) {
          sccadj[comp[i]].insert(comp[j]);
      }
  }
};
5.4 Array Cycle
struct ArrayCycle {
  vector < vector < int >> paths;
  vector < int > path_num, pos;
  vector < char > is_cycle;
```

```
ArrayCycle(const vector<int> &v) : path_num(v.size()), pos(v.
 size()) {
 paths.reserve(v.size());
  is_cycle.reserve(v.size());
  vector < char > vis(v.size());
  for (auto i : topological_order(v)) {
    if (vis[i]) continue;
    vector < int > path;
    int cur;
    for (cur = i; not vis[cur]; cur = v[cur]) {
      path.push_back(cur);
      vis[cur] = 1;
    }
      int cycle_start = 0;
      for (; cycle_start < (int)path.size() and path[</pre>
 cycle_start] != cur;
           cycle_start++)
        ;
      if (cycle_start > 0) {
        paths.emplace_back();
        for (int j = 0; j < cycle_start; j++) {</pre>
          paths.back().push_back(path[j]);
          pos[path[j]] = j;
          path_num[path[j]] = (int)paths.size() - 1;
        paths.back().push_back(cur);
        is_cycle.push_back(false);
      if (cycle_start < (int)path.size()) {</pre>
        paths.emplace_back();
        for (int j = cycle_start; j < (int)path.size(); j++)</pre>
 {
          paths.back().push_back(path[j]);
          pos[path[j]] = j - cycle_start;
          path_num[path[j]] = (int)paths.size() - 1;
        is_cycle.push_back(true);
```

```
}
const vector<int> &path(int cur) const { return paths[
 path_num[cur]]; }
int kth_pos(int cur, ll k) const {
  while (not is_cycle[path_num[cur]]) {
    auto &p = path(cur);
    int remain = (int)p.size() - pos[cur] - 1;
    if (k <= remain) return p[pos[cur] + k];</pre>
    cur = p.back();
    k -= remain;
  auto &p = path(cur);
  return p[(pos[cur] + k) % p.size()];
// {element, number_of_moves}
pair < int , int > go_to_cycle(int cur) const {
  int moves = 0:
  while (not is_cycle[path_num[cur]]) {
    auto &p = path(cur);
    moves += (int)p.size() - pos[cur] - 1;
    cur = p.back();
  return {cur, moves};
void topological_order(const vector<int> &g, vector<char> &
 vis,
                        vector < int > & order, int u) {
  vis[u] = true;
  if (not vis[g[u]]) topological_order(g, vis, order, g[u]);
  order.push_back(u);
}
vector < int > topological_order(const vector < int > &g) {
  vector < char > vis(g.size(), false);
  vector<int> order:
  for (auto i = 0; i < (int)g.size(); i++)</pre>
    if (not vis[i]) topological_order(g, vis, order, i);
  reverse(order.begin(), order.end());
```

```
return order;
};
```

5.5 Bellman-Ford (find negative cycle)

Given a directed graph find a negative cycle by running n iterations, and if the last one produces a relaxation than there is a cycle.

Time: $O(V \cdot E)$

```
const 11 oo = 2500 * 1e9;
using graph = vector < vector < pair < int , 11 >> >;
vi negative_cycle(graph &g, int n) {
 vll d(n, oo);
  vi p(n, -1);
 int x = -1;
  d[0] = 0:
 for (int i = 0; i < n; i++) {
   x = -1;
   for (int u = 0; u < n; u++) {
      for (auto &[v, 1] : g[u]) {
        if (d[u] + 1 < d[v]) {
          d[v] = d[u] + 1;
          p[v] = u;
          x = v;
      }
 if (x == -1)
    return {};
  else {
   for (int i = 0; i < n; i++) x = p[x];
   vi cycle;
    for (int v = x; v = p[v]) {
      cycle.eb(v);
      if (v == x and len(cycle) > 1) break;
    reverse(all(cycle));
    return cycle;
```

5.6 Bellman Ford

```
Find shortest path from a single source to all other nodes. Can detect negative cycles.
Time: O(V * E)
bool bellman_ford(const vector<vector<pair<int, ll>>> &g, int s
                     vector<ll> &dist) {
  int n = (int)g.size();
  dist.assign(n, LLONG_MAX);
  vector < int > count(n);
  vector < char > in_queue(n);
  queue < int > q;
  dist[s] = 0;
  q.push(s);
  in_queue[s] = true;
  while (not q.empty()) {
    int cur = q.front();
    q.pop();
    in_queue[cur] = false;
    for (auto [to, w] : g[cur]) {
       if (dist[cur] + w < dist[to]) {</pre>
         dist[to] = dist[cur] + w:
         if (not in_queue[to]) {
           q.push(to);
           in_queue[to] = true;
           count[to]++;
           if (count[to] > n) return false;
  return true:
5.7 BFS 01
Similar to a Dijkstra given a weighted graph finds the distance from source s to every other node (SSSP).
```

Similar to a Dijkstra given a weighted graph finds the distance from source s to every other node (SSSP) Applicable only when the weight of the edges $\in \{0, x\}$ Time: O(V + E)

```
vector < pair < ll, int >> adj[maxn];
ll dists[maxn];
```

```
int s, n;
void bfs_01() {
  fill(dists, dists + n, oo);
  dist[s] = 0;
  deque < int > q;
  q.emplace_back(s);
  while (not q.empty()) {
    auto u = q.front();
   q.pop_front();
    for (auto [v, w] : adj[u]) {
      if (dist[v] <= dist[u] + w) continue;</pre>
      dist[v] = dist[u] + w;
      w ? q.emplace_back(v) : q.emplace_front(v);
    }
 }
    Block Cut Tree
// O(n + m)
struct BlockCutTree {
  vector < vector < int >> blocks, tree;
  vector < vector < pair < int , int >>> block_edges;
  vector < int > articulation, pos;
  BlockCutTree(const vector<vector<int>> &g)
    : articulation(g.size()), pos(g.size()) {
    int t = 0:
    vector < int > id(g.size(), -1);
    stack<int> s1;
    stack<pair<int, int>> s2;
    for (int i = 0; i < (int)g.size(); i++)</pre>
      if (id[i] == -1) dfs(g, i, -1, t, id, s1, s2);
    tree.resize(blocks.size());
    for (int i = 0; i < (int)g.size(); i++)</pre>
      if (articulation[i]) pos[i] = (int)tree.size(), tree.
   emplace_back();
    for (int i = 0; i < (int)blocks.size(); i++) {</pre>
      for (auto j : blocks[i]) {
        if (not articulation[j])
```

```
pos[j] = i;
        else
          tree[i].push_back(pos[j]), tree[pos[j]].push_back(i);
      }
    }
  }
 private:
  int dfs(const vector < vector < int >> &g, int i, int p, int &t,
   vector < int > & id ,
          stack<int> &s1, stack<pair<int, int>> &s2) {
    int lo = id[i] = t++;
    s1.push(i);
    if (p != -1) s2.emplace(i, p);
    for (auto j : g[i])
      if (j != p and id[j] != -1) s2.emplace(i, j);
    for (auto j : g[i])
      if (j != p) {
        if (id[i] == -1) {
          int val = dfs(g, j, i, t, id, s1, s2);
          lo = min(lo, val);
          if (val >= id[i]) {
            articulation[i]++;
            blocks.emplace_back(1, i);
            for (; blocks.back().back() != j; s1.pop())
              blocks.back().push_back(s1.top());
            block_edges.emplace_back(1, s2.top());
            s2.pop();
            for (; block_edges.back().back() != make_pair(j, i)
   ; s2.pop())
              block_edges.back().push_back(s2.top());
        } else {
          lo = min(lo, id[j]);
        }
      }
    if (p == -1 and articulation[i]) --articulation[i];
    return lo:
  }
};
```

```
Check Bipartitie
O(V)
vi2d G;
int N, M;
bool check() {
  vi side (N, -1);
  queue < int > q;
  for (int st = 0; st < N; st++) {</pre>
    if (side[st] == -1) {
      q.emplace(st);
      side[st] = 0;
      while (not q.empty()) {
        int u = q.front();
        q.pop();
        for (auto v : G[u]) {
          if (side[v] == -1) {
            side[v] = side[u] ^ 1;
            q.push(v);
          } else if (side[u] == side[v])
            return false;
        }
      }
    }
  return true;
5.10 Dijkstra (k Shortest Paths)
const ll oo = 1e9 * 1e5 + 1;
using adj = vector<vector<pll>>;
vector<priority_queue<ll>> dijkstra(const vector<vector<pll>> &
   g, int n, int s,
                                      int k) {
  priority_queue < pll , vector < pll > , greater < pll >> pq;
  vector < priority_queue < ll >> dist(n);
  dist[0].emplace(0);
  pq.emplace(0, s);
  while (!pq.empty()) {
    auto [d1, v] = pq.top();
    pq.pop();
```

```
if (not dist[v].empty() and dist[v].top() < d1) continue;
    for (auto [d2, u] : g[v]) {
      if (len(dist[u]) < k) {</pre>
         pq.emplace(d2 + d1, u);
         dist[u].emplace(d2 + d1);
      } else {
        if (dist[u].top() > d1 + d2) {
           dist[u].pop();
           dist[u].emplace(d1 + d2);
           pq.emplace(d2 + d1, u);
      }
    }
  return dist;
5.11 Dijkstra
Finds the shortest path from s to every other node, and keep the 'parent' tracking.
Time: O(E \cdot \log V)
pair < vll, vi > dijkstra(const vector < vector < pll >> &g, int n, int
  priority_queue < pll , vector < pll > , greater < pll >> pq;
  vll dist(n, oo);
  vi p(n, -1);
  pq.emplace(0, s);
  dist[s] = 0;
  while (!pq.empty()) {
    auto [d1, v] = pq.top();
    pq.pop();
    if (dist[v] < d1) continue;</pre>
    for (auto [d2, u] : g[v]) {
      if (dist[u] > d1 + d2) {
         dist[u] = d1 + d2;
         p[u] = v;
         pq.emplace(dist[u], u);
    }
  return {dist, p};
```

5.12 Disjoint Edges Path (Maxflow)

```
Given a directed graph find's every path with disjoint edges that starts at s and ends at t
Time : O(E \cdot V^2)
struct DisjointPaths {
  int n;
  vi2d g, capacity;
  vector < vc > isedge;
  DisjointPaths(int _n) : n(_n), g(n), capacity(n, vi(n)),
   isedge(n, vc(n)) {}
  void add(int u, int v, int w = 1) {
    g[u].emplace_back(v);
    g[v].emplace_back(u);
    capacity[u][v] += w;
    isedge[u][v] = true;
  }
  // finds the new flow to insert
  int bfs(int s, int t, vi &parent) {
    fill(all(parent), -1);
    parent[s] = -2;
    queue <pair <int, int >> q;
    q.push({oo, s});
    while (!q.empty()) {
      auto [flow, cur] = q.front();
      q.pop();
      for (auto next : g[cur]) {
        if (parent[next] == -1 and capacity[cur][next]) {
          parent[next] = cur;
          ll new_flow = min(flow, capacity[cur][next]);
          if (next == t) return new_flow;
          q.push({new_flow, next});
        }
      }
    return 0;
  int maxflow(int s, int t) {
    int flow = 0;
```

```
vi parent(n);
  int new_flow;
  while ((new_flow = bfs(s, t, parent))) {
    flow += new_flow;
    int cur = t;
    while (cur != s) {
      int prev = parent[cur];
      capacity[prev][cur] -= new_flow;
      capacity[cur][prev] += new_flow;
      cur = prev;
   }
  }
  return flow;
}
// build the distinct routes based in the capacity set by
void dfs(int u, int t, vc2d &vis, vi &route, vi2d &routes) {
  route.eb(u);
 if (u == t) {
   routes.emplace_back(route);
    route.pop_back();
    return;
  }
  for (auto &v : g[u]) {
    if (capacity[u][v] == 0 and isedge[u][v] and not vis[u][v
 ]) {
      vis[u][v] = true;
      dfs(v, t, vis, route, routes);
      route.pop_back();
      return:
    }
  }
}
vi2d disjoint_paths(int s, int t) {
  int mf = maxflow(s, t);
  vi2d routes;
  vi route:
  vc2d vis(n, vc(n));
  for (int i = 0; i < mf; i++) dfs(s, t, vis, route, routes);</pre>
  return routes;
```

```
}
};
5.13 Euler Path (directed)
Given a directed graph finds a path that visits every edge exactly once.
Time: O(E)
vector < int > euler_cycle(vector < vector < int >> &g, int u) {
  vector<int> res;
  stack<int> st:
  st.push(u);
  while (!st.empty()) {
    auto cur = st.top();
    if (g[cur].empty()) {
      res.push_back(cur);
      st.pop();
    } else {
      auto next = g[cur].back();
      st.push(next);
      g[cur].pop_back();
  }
  for (auto &x : g)
    if (!x.empty()) return {};
  return res;
}
vector<int> euler_path(vector<vector<int>> &g, int first) {
  {
    int n = (int)g.size();
    vector < int > in(n), out(n);
    for (int i = 0; i < n; i++)</pre>
      for (auto x : g[i]) in[x]++, out[i]++;
    int a = 0, b = 0, c = 0;
    for (int i = 0; i < n; i++)</pre>
      if (in[i] == out[i])
         c++;
      else if (in[i] - out[i] == 1)
        b++;
```

```
else if (in[i] - out[i] == -1)
        a++:
    if (c != n - 2 or a != 1 or b != 1) return {}:
  }
  auto res = euler_cycle(g, first);
  if (res.empty()) return res;
  reverse(all(res));
  return res;
}
5.14 Euler Path (undirected)
Given a undirected graph finds a path that visits every edge exactly once.
Time: O(E)
vector < int > euler_cycle(vector < vector < int >> &g, int u) {
  vector<int> res;
  multiset < pair < int , int >> vis;
  stack<int> st;
  st.push(u);
  while (!st.empty()) {
    auto cur = st.top();
    while (!g[cur].empty()) {
      auto it = vis.find(make_pair(cur, g[cur].back()));
      if (it == vis.end()) break;
      g[cur].pop_back();
      vis.erase(it):
    }
    if (g[cur].empty()) {
      res.push_back(cur);
      st.pop();
    } else {
      auto next = g[cur].back();
      st.push(next);
      vis.emplace(next, cur);
      g[cur].pop_back();
  }
```

```
for (auto &x : g)
    if (!x.empty()) return {};
  return res:
}
vector < int > euler_path(vector < vector < int >> &g, int first) {
  int n = (int)g.size();
  int v1 = -1, v2 = -1;
    bool bad = false;
    for (int i = 0; i < n; i++)</pre>
      if (g[i].size() & 1) {
        if (v1 == -1)
          v1 = i;
        else if (v2 == -1)
          v2 = i:
        else
          bad = true;
      }
    if (bad or (v1 != -1 and v2 == -1)) return \{\};
  }
  if (v2 != -1) {
   // insert cycle
    g[v1].push_back(v2);
    g[v2].push_back(v1);
  auto res = euler_cycle(g, first);
  if (res.empty()) return res;
  if (v1 != -1) {
    for (int i = 0; i + 1 < (int)res.size(); i++) {
      if ((res[i] == v1 \text{ and } res[i + 1] == v2)
           (res[i] == v2 \text{ and } res[i + 1] == v1)) {
         vector<int> res2:
        for (int j = i + 1; j < (int)res.size(); j++) res2.
   push_back(res[j]);
        for (int j = 1; j <= i; j++) res2.push_back(res[j]);</pre>
        res = res2;
         break;
      }
```

```
}
reverse(all(res));
return res;
}
```

5.15 Find Articulation/Cut Points

Given an undirected graph find it's articulation points.

articulation point (or cut vertex): is defined as a vertex which, when removed along with associated edges, increases thee number of connected components in the graph.

A vertex u can be an articulation point if and only if has at least 2 adjascent vertex

Time: O(N+M)

```
const int MAXN(100);
int N;
vi2d G;
int timer;
int tin[MAXN], low[MAXN];
set < int > cpoints;
int dfs(int u, int p = -1) {
  int cnt = 0;
 low[u] = tin[u] = timer++;
 for (auto v : G[u]) {
   if (not tin[v]) {
      cnt++:
      dfs(v, u);
      if (low[v] >= tin[u]) cpoints.insert(u);
      low[u] = min(low[u], low[v]);
   } else if (v != p)
      low[u] = min(low[u], tin[v]);
 }
 return cnt;
void getCutPoints() {
  memset(low, 0, sizeof(low));
  memset(tin, 0, sizeof(tin));
  cpoints.clear();
  timer = 1:
  for (int i = 0; i < N; i++) {
```

```
if (tin[i]) continue;
    int cnt = dfs(i);
    if (cnt == 1) cpoints.erase(i);
 }
}
5.16 Find Bridges (online)
// O((n+m)*log(n))
struct BridgeFinder {
 // 2ecc = 2 edge conected component
 // cc = conected component
  vector<int> parent, dsu_2ecc, dsu_cc, dsu_cc_size;
  int bridges, lca_iteration;
  vector < int > last_visit;
  BridgeFinder(int n)
    : parent(n, -1),
      dsu_2ecc(n),
      dsu_cc(n),
      dsu_cc_size(n, 1),
      bridges(0),
      lca_iteration(0),
      last_visit(n) {
    for (int i = 0; i < n; i++) {</pre>
      dsu_2ecc[i] = i;
      dsu_cc[i] = i;
    }
  }
  int find 2ecc(int v) {
    if (v == -1) return -1:
    return dsu_2ecc[v] == v ? v : dsu_2ecc[v] = find_2ecc(
   dsu 2ecc[v]):
  }
  int find_cc(int v) {
   v = find_2ecc(v);
    return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(dsu_cc[v]);
  }
  void make_root(int v) {
   v = find_2ecc(v);
    int root = v;
    int child = -1;
```

```
while (v != -1) {
    int p = find_2ecc(parent[v]);
   parent[v] = child;
    dsu_cc[v] = root;
   child = v:
   v = p;
 }
  dsu_cc_size[root] = dsu_cc_size[child];
void merge_path(int a, int b) {
 ++lca_iteration;
 vector < int > path_a, path_b;
 int lca = -1;
 while (lca == -1) {
   if (a != -1) {
      a = find_2ecc(a);
      path_a.push_back(a);
      if (last_visit[a] == lca_iteration) {
       lca = a:
       break;
      last_visit[a] = lca_iteration;
      a = parent[a];
   if (b != -1) {
     b = find_2ecc(b);
      path_b.push_back(b);
      if (last_visit[b] == lca_iteration) {
       lca = b;
       break:
      last_visit[b] = lca_iteration;
      b = parent[b];
    }
 }
 for (auto v : path_a) {
    dsu_2ecc[v] = lca;
   if (v == lca) break;
    --bridges;
 }
 for (auto v : path_b) {
   dsu_2ecc[v] = lca;
    if (v == lca) break;
```

```
--bridges;
   }
 }
  void add_edge(int a, int b) {
    a = find_2ecc(a);
   b = find_2ecc(b);
    if (a == b) return;
    int ca = find_cc(a);
    int cb = find_cc(b);
    if (ca != cb) {
      ++bridges;
      if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
        swap(a, b);
        swap(ca, cb);
      make_root(a);
      parent[a] = dsu_cc[a] = b;
      dsu_cc_size[cb] += dsu_cc_size[a];
    } else {
      merge_path(a, b);
 }
};
```

5.17 Find Bridges

Find every bridge in a undirected connected graph.

bridge: A bridge is defined as an **edge** which, when removed, increases the number of connected components in the graph.

Time: O(N+M)

```
const int MAXN(50);
vi2d G(MAXN);
int tin[MAXN];
int low[MAXN];
char vis[MAXN];
int timer;
int N, M;
vector<pii> bridges;

void dfs(int u, int p = -1) {
  vis[u] = true;
  tin[u] = low[u] = timer++;
```

```
for (auto v : G[u]) {
    if (v == p) continue;
    if (vis[v]) {
      low[u] = min(low[u], tin[v]);
    } else {
      dfs(v, u);
      low[u] = min(low[u], low[v]);
      if (low[v] > tin[u]) {
        bridges.emplace_back(u, v);
    }
  }
void getBridges() {
  timer = 0;
  memset(vis, 0, sizeof(vis));
  memset(tin, -1, sizeof(tin));
  memset(low, -1, sizeof(low));
  bridges.clear();
  for (int i = 0; i < N; i++) {
    if (not vis[i]) dfs(i);
}
5.18 Find Centroid
Given a tree (don't forget to make it 'undirected'), find it's centroids.
Time: O(V)
void dfs(int u, int p, int n, vi2d &g, vi &sz, vi &centroid) {
  sz[u] = 1;
  bool iscentroid = true;
 for (auto v : g[u])
   if (v != p) {
      dfs(v, u, n, g, sz, centroid);
      if (sz[v] > n / 2) iscentroid = false;
      sz[u] += sz[v];
    }
 if (n - sz[u] > n / 2) iscentroid = false;
  if (iscentroid) centroid.eb(u);
```

}

```
vi getCentroid(vi2d &g, int n) {
  vi centroid;
  vi sz(n);
  dfs(0, -1, n, g, sz, centroid);
  return centroid:
}
5.19 Floyd Warshall
Simply finds the minimal distance for each node to every other node. O(V^3)
vector<vll> floyd_warshall(const vector<vll> &adj, ll n) {
  auto dist = adj;
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++ j) {
      for (int k = 0; k < n; ++k) {
         dist[j][k] = min(dist[j][k], dist[j][i] + dist[i][k]);
    }
  }
  return dist;
     Graph Cycle (directed)
Given a directed graph finds a cycle (or not).
Time : O(E)
bool dfs(int v, vi2d &adj, vc &visited, vi &parent, vc &color,
   int &cycle_start,
          int &cycle_end) {
  color[v] = 1:
  for (int u : adj[v]) {
    if (color[u] == 0) {
      parent[u] = v;
      if (dfs(u, adj, visited, parent, color, cycle_start,
   cycle_end))
        return true;
    } else if (color[u] == 1) {
      cycle_end = v;
      cycle_start = u;
      return true;
    }
  color[v] = 2;
  return false:
```

```
vi find_cycle(vi2d &g, int n) {
  vc visited(n):
  vi parent(n);
  vc color(n);
  int cycle_start, cycle_end;
  color.assign(n, 0);
  parent.assign(n, -1);
  cycle_start = -1;
  for (int v = 0; v < n; v++) {
    if (color[v] == 0 &&
        dfs(v, g, visited, parent, color, cycle_start,
   cvcle_end))
      break;
  }
  if (cycle_start == -1) {
    return {};
 } else {
    vector < int > cycle;
    cycle.push_back(cycle_start);
    for (int v = cycle_end; v != cycle_start; v = parent[v])
   cycle.push_back(v);
    cycle.push_back(cycle_start);
    reverse(cycle.begin(), cycle.end());
    return cycle;
}
5.21 Graph Cycle (undirected)
Detects if a graph contains a cycle. If path parameter is not null, it will contain the cycle if one exists.
Time: O(V+E)
void graph_cycles(const vector<vector<int>> &g, int u, int p,
   vector < int > &ps,
                    vector<int> &color, int &cn, vector<vector<</pre>
   int>> &cycles) {
  if (color[u] == 2) {
    return;
  if (color[u] == 1) {
    cn++:
```

}

```
int cur = p;
    cycles.emplace_back();
    auto &v = cycles.back();
    v.push_back(cur);
    while (cur != u) {
      cur = ps[cur];
      v.push_back(cur);
    }
    reverse(all(v));
    return;
  ps[u] = p;
  color[u] = 1;
  for (auto v : g[u]) {
   if (v != p) graph_cycles(g, v, u, ps, color, cn, cycles);
  color[u] = 2;
vector < vector < int >> graph_cycles(const vector < vector < int >> &g)
  vector<int> ps(g.size(), -1), color(g.size());
  int cn = 0:
  vector < vector < int >> cycles;
  for (int i = 0; i < (int)g.size(); i++)</pre>
    graph_cycles(g, i, -1, ps, color, cn, cycles);
 return cycles;
5.22 Heavy Light Decomposition
struct HeavyLightDecomposition {
  vector<int> parent, depth, size, heavy, head, pos;
  using SegT = int;
  static SegT op(SegT a, SegT b) { return max(a, b); }
  SegTree < SegT, op > seg;
  HeavyLightDecomposition(const vector < vector < int >> &g, const
   vector < int > &v,
                           int root = 0
    : parent(g.size()),
      depth(g.size()),
```

```
size(g.size()),
     heavy(g.size(), -1),
    head(g.size()),
    pos(g.size()),
     seg((int)g.size()) {
  dfs(g, root);
  int cur_pos = 0;
  decompose(g, root, root, cur_pos);
  for (int i = 0; i < (int)g.size(); i++) {</pre>
     seg.set(pos[i], v[i]);
  }
}
SegT query_path(int a, int b) const {
  int res = 0;
  for (; head[a] != head[b]; b = parent[head[b]]) {
    if (depth[head[a]] > depth[head[b]]) swap(a, b);
    res = op(res, seg.query(pos[head[b]], pos[b]));
  }
  if (depth[a] > depth[b]) swap(a, b);
  return op(res, seg.query(pos[a], pos[b]));
SegT query_subtree(int a) const {
  return seg.query(pos[a], pos[a] + size[a] - 1);
void set(int a, int x) { seg.set(pos[a], x); }
private:
void dfs(const vector<vector<int>> &g, int u) {
  size[u] = 1;
  int mx_child_size = 0;
  for (auto x : g[u])
    if (x != parent[u]) {
       parent[x] = u:
       depth[x] = depth[u] + 1;
      dfs(g, x);
       size[u] += size[x];
       if (size[x] > mx_child_size) mx_child_size = size[x],
  heavy[u] = x;
    }
}
```

```
void decompose(const vector < vector < int >> &g, int u, int h,
   int &cur_pos) {
    head[u] = h;
    pos[u] = cur_pos++;
    if (heavy[u] != -1) decompose(g, heavy[u], h, cur_pos);
    for (auto x : g[u])
      if (x != parent[u] and x != heavy[u]) {
        decompose(g, x, x, cur_pos);
  }
};
5.23
     Kruskal
Find the minimum spanning tree of a graph.
Time: O(E \log E)
can be used to find the maximum spanning tree by changing the comparison operator in the sort
struct UFDS {
  vector < int > ps, sz;
  int components;
  UFDS(int n): ps(n + 1), sz(n + 1, 1), components(n) { iota(
   all(ps), 0); }
  int find_set(int x) { return (x == ps[x] ? x : (ps[x] =
   find_set(ps[x])); }
  bool same_set(int x, int y) { return find_set(x) == find_set(
   y); }
  void union_set(int x, int y) {
    x = find set(x):
    y = find_set(y);
    if (x == y) return;
    if (sz[x] < sz[y]) swap(x, y);
    ps[y] = x;
    sz[x] += sz[y];
    components --;
};
```

```
vector<tuple<11, int, int>> kruskal(int n, vector<tuple<11, int</pre>
   , int >> &edges) {
  UFDS ufds(n);
  vector < tuple < ll, int, int >> ans;
  sort(all(edges));
  for (auto [a, b, c] : edges) {
    if (ufds.same_set(b, c)) continue;
    ans.emplace_back(a, b, c);
    ufds.union_set(b, c);
  }
  return ans;
5.24 Maximum Flow (Edmonds-Karp)
Finds the maximum flow in a graph network, given the source s and the sink t.
Time: O(V \cdot E^2)
struct maxflow {
  int n;
  vi2d g;
  vll2d cps;
  vi ps;
  vector < vector < char >> isedge;
  maxflow(int _n) : n(_n), g(n), cps(n, vll(n)), ps(n), isedge(
   n, vc(n)) {}
  void add(int u, int v, ll c, bool set = true) {
    g[u].emplace_back(v);
    g[v].emplace_back(u);
    cps[u][v] = cps[u][v] * (!set) + c;
    isedge[u][v] = true;
  11 bfs(int s, int t) {
    fill(all(ps), -1);
    ps[s] = -2;
    queue <pair <11, int >> q;
    q.emplace(oo, s);
    while (!q.empty()) {
      auto [flow, cur] = q.front();
```

```
q.pop();
      for (auto next : g[cur]) {
        if (ps[next] == -1 and cps[cur][next]) {
          ps[next] = cur;
          ll new_flow = min(flow, cps[cur][next]);
          if (next == t) return new_flow;
          q.emplace(new_flow, next);
      }
    }
    return 011;
  }
  11 flow(int s, int t) {
    11 flow = 0;
    11 new_flow;
    while ((new_flow = bfs(s, t))) {
      flow += new_flow;
      int cur = t;
      while (cur != s) {
        int prev = ps[cur];
        cps[prev][cur] -= new_flow;
        cps[cur][prev] += new_flow;
        cur = prev;
      }
    }
    return flow;
  vector<pii> get_used() {
    vector < pii > used;
    for (int i = 0; i < n; i++) {</pre>
      for (int j = 0; j < n; j++) {
        if (isedge[i][j] and cps[i][j] == 0) used.emplace_back(
   i, j);
      }
    }
    return used:
  }
};
```

5.25 Minimum Cost Flow

Given a network find the minimum cost to achieve a flow of at most f. Works with **directed** and **undirected** graphs

- add(u, v, w, c): adds an edge from u to v with capacity w and cost c.
- flow(s, t, f): return a pair (flow, cost) with the maximum flow until f with source at s and sink at t, with the minimum cost possible.

```
Time : O(N \cdot M + f \cdot m \log n)
template <typename T>
struct mcmf {
  struct edge {
   int to, rev, flow, cap;
    bool res; // if it's a reverse edge
    T cost; // cost per unity of flow
    edge(): to(0), rev(0), flow(0), cap(0), cost(0), res(false)
    edge(int to_, int rev_, int flow_, int cap_, T cost_, bool
   res )
      : to(to_), rev(rev_), flow(flow_), cap(cap_), res(res_),
   cost(cost ) {}
 };
  vector < vector < edge >> g;
  vector < int > par_idx, par;
  T inf;
  vector <T> dist:
  mcmf(int n) : g(n), par_idx(n), par(n), inf(numeric_limits<T</pre>
   >::max() / 3) {}
  void add(int u, int v, int w, T cost) {
    edge a = edge(v, g[v].size(), 0, w, cost, false);
    edge b = edge(u, g[u].size(), 0, 0, -cost, true);
    g[u].push_back(a);
    g[v].push_back(b);
  vector <T> spfa(int s) { // don't code it if there isn't
   negative cycles
    deque < int > q;
    vector < bool > is_inside(g.size(), 0);
    dist = vector <T>(g.size(), inf);
    dist[s] = 0:
```

```
q.push_back(s);
  is_inside[s] = true;
  while (!q.empty()) {
    int v = q.front();
    q.pop_front();
    is_inside[v] = false;
    for (int i = 0; i < g[v].size(); i++) {</pre>
      auto [to, rev, flow, cap, res, cost] = g[v][i];
      if (flow < cap and dist[v] + cost < dist[to]) {</pre>
        dist[to] = dist[v] + cost;
        if (is_inside[to]) continue;
        if (!q.empty() and dist[to] > dist[q.front()])
          q.push_back(to);
        else
          q.push_front(to);
        is_inside[to] = true;
      }
    }
  return dist;
bool dijkstra(int s, int t, vector<T>& pot) {
  priority_queue < pair < T, int > , vector < pair < T, int > > , greater
 <>> q;
  dist = vector<T>(g.size(), inf);
  dist[s] = 0;
  q.emplace(0, s);
  while (q.size()) {
    auto [d, v] = q.top();
    q.pop();
    if (dist[v] < d) continue;</pre>
    for (int i = 0; i < g[v].size(); i++) {</pre>
      auto [to, rev, flow, cap, res, cost] = g[v][i];
      cost += pot[v] - pot[to];
      if (flow < cap and dist[v] + cost < dist[to]) {</pre>
        dist[to] = dist[v] + cost;
        q.emplace(dist[to], to);
        par_idx[to] = i, par[to] = v;
      }
    }
  return dist[t] < inf;</pre>
```

```
}
  pair < int , T > min_cost_flow(int s, int t, int flow = inf) {
    vector <T> pot(g.size(), 0);
    pot = spfa(s); // comment this line if there isn't
   negative cycles
    int f = 0;
    T ret = 0;
    while (f < flow and dijkstra(s, t, pot)) {</pre>
      for (int i = 0; i < g.size(); i++)</pre>
        if (dist[i] < inf) pot[i] += dist[i];</pre>
      int mn_flow = flow - f, u = t;
      while (u != s) {
        mn_flow =
          min(mn_flow, g[par[u]][par_idx[u]].cap - g[par[u]][
   par_idx[u]].flow);
        u = par[u];
      ret += pot[t] * mn_flow;
      u = t:
      while (u != s) {
        g[par[u]][par_idx[u]].flow += mn_flow;
        g[u][g[par[u]][par_idx[u]].rev].flow -= mn_flow;
        u = par[u];
      f += mn_flow;
    return make_pair(f, ret);
};
```

5.26 Minimum Cut (unweighted)

After build the **direct/undirected** graph find the minimum of edges needed to be removed to make the sink t unreachable from the source s. Time: $O(V \cdot E^2)$

```
struct Mincut {
  int n;
  vi2d g;
```

```
vii edges;
vll2d capacity;
vi ps, vis;
Mincut(int _n) : n(_n), g(n), capacity(n, vll(n)), ps(n), vis
 (n) {}
void add(int u, int v, ll c = 1, bool directed = false, bool
 set = false) {
  edges.emplace_back(u, v);
 g[u].emplace_back(v);
  if (not set)
    capacity[u][v] += c;
    capacity[u][v] = c;
  if (not directed) {
    g[v].emplace_back(u);
    if (not set)
      capacity[v][u] += c;
      capacity[v][u] = c;
  }
}
11 bfs(int s, int t) {
 fill(all(ps), -1);
  ps[s] = -2;
  queue < pair < ll, int >> q;
  q.push({oo, s});
  while (!q.empty()) {
    auto [flow, cur] = q.front();
    q.pop();
    for (auto next : g[cur]) {
      if (ps[next] == -1 and capacity[cur][next]) {
        ps[next] = cur;
        11 new_flow = min(flow, capacity[cur][next]);
        if (next == t) return new_flow;
        q.push({new_flow, next});
      }
    }
```

```
}
  return 011;
}
ll maxflow(int s, int t) {
  11 \text{ flow} = 0;
  ll new_flow;
  while ((new_flow = bfs(s, t))) {
    flow += new_flow;
    int cur = t;
    while (cur != s) {
      int prev = ps[cur];
      capacity[prev][cur] -= new_flow;
      capacity[cur][prev] += new_flow;
      cur = prev;
    }
  }
  return flow;
void dfs(int u) {
  vis[u] = true;
 for (auto v : g[u]) {
    if (capacity[u][v] > 0 and not vis[v]) {
      dfs(v);
    }
  }
vii mincut(int s, int t) {
  maxflow(s, t);
  fill(all(vis), 0);
  dfs(s);
  vii removed:
  for (auto &[u, v] : edges) {
    if ((vis[u] and not vis[v]) or (vis[v] and not vis[u]))
      removed.emplace_back(u, v);
  }
  return removed:
```

```
};
```

5.27 Sum every node distance

Given a **tree**, for each node i find the sum of distance from i to every other node. don't forget to set the tree as undirected, that's needed to choose an arbitrary root Time: O(N)

```
void getRoot(int u, int p, vi2d &g, vll &d, vll &cnt) {
 for (int i = 0; i < len(g[u]); i++) {</pre>
    int v = g[u][i];
   if (v == p) continue;
    getRoot(v, u, g, d, cnt);
   d[u] += d[v] + cnt[v];
    cnt[u] += cnt[v];
 }
}
void dfs(int u, int p, vi2d &g, vll &cnt, vll &ansd, int n) {
 for (int i = 0; i < len(g[u]); i++) {
   int v = g[u][i];
   if (v == p) continue;
    ansd[v] = ansd[u] - cnt[v] + (n - cnt[v]);
    dfs(v, u, g, cnt, ansd, n);
 }
vll fromToAll(vi2d &g, int n) {
 vll d(n):
 vll cnt(n, 1);
  getRoot(0, -1, g, d, cnt);
 vll ansdist(n);
  ansdist[0] = d[0]:
 dfs(0, -1, g, cnt, ansdist, n);
  return ansdist;
```

5.28 Topological Sorting

Assumes that:

- vertices index [0, n-1]
- is a DAG (else it returns an empty vector)

```
O(V)
```

```
enum class state { not_visited, processing, done };
bool dfs(const vector < vll > & adj, ll s, vector < state > & states,
   vll &order) {
  states[s] = state::processing;
  for (auto &v : adj[s]) {
    if (states[v] == state::not_visited) {
      if (not dfs(adj, v, states, order)) return false;
    } else if (states[v] == state::processing)
      return false:
  states[s] = state::done;
  order.pb(s);
  return true;
vll topologicalSorting(const vector<vll> &adj) {
  ll n = len(adi);
  vll order;
  vector < state > states(n, state::not_visited);
  for (int i = 0; i < n; ++i) {
    if (states[i] == state::not_visited) {
      if (not dfs(adj, i, states, order)) return {};
    }
  reverse(all(order));
  return order;
}
  \mathbf{Math}
6.1 GCD (with factorization)
O(\sqrt{n}) due to factorization.
ll gcd_with_factorization(ll a, ll b) {
  map<11, 11> fa = factorization(a);
  map<11, 11> fb = factorization(b);
  ll ans = 1:
  for (auto fai : fa) {
    11 k = min(fai.second, fb[fai.first]);
    while (k--) ans *= fai.first;
  }
  return ans;
}
```

```
6.2 GCD
```

void precompute() {

for (ll i = 1; i <= maxn; i++) {</pre>

fats[i] = (fats[i - 1] * i) % MOD;

fats[0] = 1;

}

}

```
11 gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
6.3 LCM (with factorization)
O(\sqrt{n}) due to factorization.
ll lcm_with_factorization(ll a, ll b) {
  map<ll, ll> fa = factorization(a);
  map<11, 11> fb = factorization(b);
  11 \text{ ans} = 1;
  for (auto fai : fa) {
    11 k = max(fai.second, fb[fai.first]);
    while (k--) ans *= fai.first;
  }
  return ans;
6.4 LCM
11 gcd(11 a, 11 b) { return b ? gcd(b, a % b) : a; }
11 lcm(ll a, ll b) { return a / gcd(a, b) * b; }
6.5 Arithmetic Progression Sum
   \bullet s: first term
  \bullet d: common difference
  \bullet n: number of terms
ll arithmeticProgressionSum(ll s, ll d, ll n) {
  return (s + (s + d * (n - 1))) * n / 211;
}
    Binomial MOD
Precompute every factorial until maxn (O(maxn)) allowing to answer the \binom{n}{k} in O(\log mod) time, due to
the fastpow. Note that it needs O(maxn) in memory.
const 11 MOD = 1e9 + 7;
const 11 maxn = 2 * 1e6;
vll fats(maxn + 1, -1);
```

```
ll fpow(ll a, ll n, ll mod = LLONG_MAX) {
  if (n == 011) return 111;
  if (n == 111) return a:
 11 x = fpow(a, n / 211, mod) \% mod;
 return ((x * x) % mod * (n & 111 ? a : 111)) % mod;
ll binommod(ll n, ll k) {
  11 upper = fats[n];
  ll lower = (fats[k] * fats[n - k]) % MOD;
 return (upper * fpow(lower, MOD - 211, MOD)) % MOD;
6.7
   Binomial
O(nm) time, O(m) space
Equal to n choose k
ll binom(ll n, ll k) {
  if (k > n) return 0;
  vll dp(k + 1, 0);
  dp[0] = 1;
 for (ll i = 1; i <= n; i++)
    for (11 j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
 return dp[k];
6.8 Euler phi \varphi(n) (in range)
Computes the number of positive integers less than n that are coprimes with n, in the range [1, n], in
O(N \log N).
const int MAX = 1e6;
vi range_phi(int n) {
  bitset < MAX > sieve;
  vi phi(n + 1);
  iota(phi.begin(), phi.end(), 0);
  sieve.set();
  for (int p = 2; p <= n; p += 2) phi[p] /= 2;</pre>
 for (int p = 3; p <= n; p += 2) {
    if (sieve[p]) {
      for (int j = p; j <= n; j += p) {
         sieve[j] = false;
        phi[j] /= p;
```

```
phi[j] *= (p - 1);
      }
    }
  }
  return phi;
   Euler phi \varphi(n)
Computes the number of positive integers less than n that are coprimes with n, in O(\sqrt{N}).
int phi(int n) {
  if (n == 1) return 1;
  auto fs = factorization(n); // a vctor of pair or a map
  auto res = n;
  for (auto [p, k] : fs) {
   res /= p;
   res *= (p - 1);
  return res;
}
6.10 Factorial Factorization
Computes the factorization of n! in \pi(N) * \log n
// O(logN)
ll E(ll n, ll p) {
  11 k = 0, b = p;
  while (b <= n) {</pre>
   k += n / b;
    b *= p;
  return k;
// O(pi(N)*logN)
map<11, 11> factorial_factorization(11 n, const v11 &primes) {
  map<11, 11> fs;
  for (const auto &p : primes) {
    if (p > n) break;
    fs[p] = E(n, p);
```

```
return fs;
6.11 Factorial
const ll MAX = 18;
vll fv(MAX, -1);
11 factorial(ll n) {
  if (fv[n] != -1) return fv[n];
 if (n == 0) return 1;
  return n * factorial(n - 1);
}
6.12 Factorization (Pollard Rho)
Factorizes a number into its prime factors in O(n^{(\frac{1}{4})} * \log(n)).
11 mul(ll a, ll b, ll m) {
  11 \text{ ret} = a * b - (11)((1d)1 / m * a * b + 0.5) * m;
  return ret < 0 ? ret + m : ret;</pre>
}
ll pow(ll a, ll b, ll m) {
  ll ans = 1;
 for (; b > 0; b /= 211, a = mul(a, a, m)) {
    if (b % 211 == 1) ans = mul(ans, a, m);
  return ans;
bool prime(ll n) {
  if (n < 2) return 0;
  if (n <= 3) return 1;
  if (n % 2 == 0) return 0;
  ll r = \_builtin\_ctzll(n - 1), d = n >> r;
  for (int a: {2, 325, 9375, 28178, 450775, 9780504,
   795265022}) {
    11 x = pow(a, d, n);
    if (x == 1 \text{ or } x == n - 1 \text{ or a } \% n == 0) continue;
    for (int j = 0; j < r - 1; j++) {
     x = mul(x, x, n);
      if (x == n - 1) break:
    }
```

```
if (x != n - 1) return 0;
  }
  return 1;
ll rho(ll n) {
  if (n == 1 or prime(n)) return n;
  auto f = [n](11 x) \{ return mul(x, x, n) + 1; \};
  11 x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
  while (t \% 40 != 0 or gcd(prd, n) == 1) {
   if (x == y) x = ++x0, y = f(x);
   q = mul(prd, abs(x - y), n);
   if (q != 0) prd = q;
    x = f(x), y = f(f(y)), t++;
  return gcd(prd, n);
vll fact(ll n) {
  if (n == 1) return {};
  if (prime(n)) return {n};
  11 d = rho(n);
  vll l = fact(d), r = fact(n / d);
  1.insert(1.end(), r.begin(), r.end());
  return 1;
6.13 Factorization
Computes the factorization of n in O(\sqrt{n}).
map<ll, ll> factorization(ll n) {
  map<ll, ll> ans;
  for (11 i = 2; i * i <= n; i++) {
   11 count = 0;
    for (; n % i == 0; count++, n /= i)
    if (count) ans[i] = count;
  if (n > 1) ans [n] ++;
  return ans;
```

6.14 Fast Fourrier Transform

```
template <bool invert = false>
void fft(vector < complex < double >> & xs) {
  int N = (int)xs.size();
  if (N == 1) return:
  vector < complex < double >> es (N / 2), os (N / 2);
  for (int i = 0; i < N / 2; ++i) es[i] = xs[2 * i];
  for (int i = 0; i < N / 2; ++i) os[i] = xs[2 * i + 1];
  fft < invert > (es);
  fft < invert > (os);
  auto signal = (invert ? 1 : -1);
  auto theta = 2 * signal * acos(-1) / N;
  complex <double > S{1}, S1{cos(theta), sin(theta)};
  for (int i = 0; i < N / 2; ++i) {
    xs[i] = (es[i] + S * os[i]);
    xs[i] /= (invert ? 2 : 1);
    xs[i + N / 2] = (es[i] - S * os[i]);
    xs[i + N / 2] /= (invert ? 2 : 1);
    S *= S1:
 }
6.15 Fast pow
Computes a^n in O(\log N).
11 fpow(ll a, int n, ll mod = LLONG_MAX) {
 if (n == 0) return 1;
 if (n == 1) return a;
 11 x = fpow(a, n / 2, mod) \% mod;
  return ((x * x) % mod * (n & 1 ? a : 111)) % mod;
}
6.16 Gauss Elimination
template <size_t Dim>
struct GaussianElimination {
  vector < ll> basis:
```

```
size_t size;
GaussianElimination() : basis(Dim + 1), size(0) {}
void insert(ll x) {
  for (ll i = Dim; i >= 0; i--) {
    if ((x & 111 << i) == 0) continue;</pre>
    if (!basis[i]) {
      basis[i] = x;
      size++;
      break;
    }
    x ^= basis[i];
}
void normalize() {
  for (11 i = Dim; i >= 0; i--)
    for (11 j = i - 1; j \ge 0; j - -)
      if (basis[i] & 111 << j) basis[i] ^= basis[j];</pre>
}
bool check(ll x) {
  for (ll i = Dim; i >= 0; i--) {
    if ((x & 111 << i) == 0) continue;</pre>
    if (!basis[i]) return false;
    x ^= basis[i];
  return true;
}
auto operator[](11 k) { return at(k); }
ll at(ll k) {
  11 \text{ ans} = 0;
  11 total = 111 << size;</pre>
  for (ll i = Dim; ~i; i--) {
    if (!basis[i]) continue;
    11 mid = total >> 111;
```

```
if ((mid < k and (ans & 111 << i) == 0) ||</pre>
          (k <= mid and (ans & 111 << i)))
        ans ^= basis[i];
      if (mid < k) k = mid;
      total >>= 111;
    }
    return ans;
  }
  ll at_normalized(ll k) {
    11 \text{ ans} = 0;
    k--;
    for (size_t i = 0; i <= Dim; i++) {</pre>
     if (!basis[i]) continue;
     if (k & 1) ans ^= basis[i];
      k >>= 1;
    return ans;
};
6.17 Integer Mod
const ll INF = 1e18;
const 11 mod = 998244353;
template <11 MOD = mod>
struct Modular {
  ll value;
  static const 11 MOD_value = MOD;
  Modular(11 v = 0)  {
    value = v % MOD;
    if (value < 0) value += MOD;</pre>
  Modular(ll a, ll b) : value(0) {
    *this += a;
    *this /= b;
  Modular& operator+=(Modular const& b) {
    value += b.value;
    if (value >= MOD) value -= MOD;
    return *this;
```

```
}
Modular& operator -= (Modular const& b) {
  value -= b.value;
 if (value < 0) value += MOD;</pre>
  return *this:
}
Modular& operator*=(Modular const& b) {
  value = (11)value * b.value % MOD;
 return *this;
}
friend Modular mexp(Modular a, ll e) {
  Modular res = 1;
  while (e) {
   if (e & 1) res *= a;
    a *= a;
    e >>= 1;
  return res;
friend Modular inverse(Modular a) { return mexp(a, MOD - 2);
}
Modular& operator/=(Modular const& b) { return *this *=
 inverse(b); }
friend Modular operator+(Modular a, Modular const b) { return
  a += b; }
Modular operator++(int) { return this->value = (this->value +
  1) % MOD; }
Modular operator++() { return this->value = (this->value + 1)
friend Modular operator-(Modular a, Modular const b) { return
  a -= b; }
friend Modular operator - (Modular const a) { return 0 - a; }
Modular operator -- (int) {
  return this->value = (this->value - 1 + MOD) % MOD;
}
Modular operator -- () { return this -> value = (this -> value - 1
+ MOD) % MOD; }
friend Modular operator*(Modular a, Modular const b) { return
  a *= b: }
friend Modular operator/(Modular a, Modular const b) { return
friend std::ostream& operator << (std::ostream& os, Modular</pre>
```

```
const& a) {
    return os << a.value;</pre>
  friend bool operator == (Modular const& a, Modular const& b) {
    return a.value == b.value;
  friend bool operator!=(Modular const& a, Modular const& b) {
    return a.value != b.value;
  }
};
6.18 Is prime
O(\sqrt{N})
bool isprime(ll n) {
  if (n < 2) return false;
  if (n == 2) return true;
  if (n \% 2 == 0) return false;
  for (11 i = 3; i * i < n; i += 2)
    if (n % i == 0) return false;
  return true;
6.19 Number Of Divisors (sieve)
11 divisors(ll n) {
  ll ans = 1;
  for (auto p : primes) {
    if (p * p * p > n) break;
    int count = 1;
    while (n % p == 0) {
      n /= p;
      count++;
    }
    ans *= count;
  if (is_prime[n])
    ans *= 2;
  else if (is_prime_square[n])
    ans *= 3;
  else if (n != 1)
    ans *= 4;
```

```
return ans;
6.20 Number of Divisors \tau(n)
Find the total of divisors of N in O(\sqrt{N})
ll number_of_divisors(ll n) {
  11 \text{ res} = 0;
  for (11 d = 1: d * d \le n: ++d) {
    if (n % d == 0) res += (d == n / d ? 1 : 2);
  }
  return res;
}
6.21 Power Sum
Calculates K^0 + K^1 + ... + K^n
ll powersum(ll n, ll k) { return (fastpow(n, k + 1) - 1) / (n -
    1); }
6.22 Sieve list primes
List every prime until MAXN, O(N \log N) in time and O(MAXN) in memory.
const ll MAXN = 1e5;
vll list_primes(ll n) {
  vll ps;
  bitset < MAXN > sieve;
  sieve.set();
  sieve.reset(1);
  for (ll i = 2; i <= n; ++i) {
    if (sieve[i]) ps.push_back(i);
    for (11 j = i * 2; j <= n; j += i) {
       sieve.reset(j);
    }
  }
  return ps;
```

6.23 Sum of Divisors $\sigma(n)$

Computes the sum of each divisor of n in $O(\sqrt{n})$.

```
11 sum_of_divisors(long long n) {
    ll res = 0;

    for (ll d = 1; d * d <= n; ++d) {
        if (n % d == 0) {
            ll k = n / d;

            res += (d == k ? d : d + k);
        }
        return res;
}</pre>
```

7 Problems

7.1 Hanoi Tower

Let T_n be the total of moves to solve a hanoi tower, we know that $T_n >= 2 \cdot T_{n-1} + 1$, for n > 0, and $T_0 = 0$. By induction it's easy to see that $T_n = 2^n - 1$, for n > 0.

The following algorithm finds the necessary steps to solve the game for 3 stacks and n disks.

```
void move(int a, int b) { cout << a << ' ' ' << b << endl; }
void solve(int n, int s, int e) {
  if (n == 0) return;
  if (n == 1) {
    move(s, e);
    return;
  }
  solve(n - 1, s, 6 - s - e);
  move(s, e);
  solve(n - 1, 6 - s - e, e);
}</pre>
```

8 Searching

8.1 Meet in the middle

Answers the query how many subsets of the vector xs have sum equal x. Time: $O(N \cdot 2^{\frac{N}{2}})$

```
vll get_subset_sums(int 1, int r, vll &a) {
  int len = r - l + 1;
  vll res;

for (int i = 0; i < (1 << len); i++) {</pre>
```

```
11 sum = 0;
    for (int j = 0; j < len; j++) {
      if (i & (1 << j)) {</pre>
        sum += a[1 + j];
      }
   }
    res.push_back(sum);
  }
  return res;
};
ll count(vll &xs, ll x) {
  int n = len(xs);
  vll left = get_subset_sums(0, n / 2 - 1, xs);
  vll right = get_subset_sums(n / 2, n - 1, xs);
  sort(all(left));
  sort(all(right));
  11 \text{ ans} = 0;
  for (ll i : left) {
    auto start_index =
      lower_bound(right.begin(), right.end(), x - i) - right.
   begin();
    auto end index =
      upper_bound(right.begin(), right.end(), x - i) - right.
   begin();
    ans += end_index - start_index;
  return ans;
   Ternary Search Recursive
const double eps = 1e-6;
// IT MUST BE AN UNIMODAL FUNCTION
double f(int x) { return x * x + 2 * x + 4; }
double ternary_search(double 1, double r) {
  if (fabs(f(1) - f(r)) < eps) return f((1 + (r - 1) / 2.0));
  auto third = (r - 1) / 3.0;
  auto m1 = 1 + third;
  auto m2 = r - third;
  // change the signal to find the maximum point.
```

```
return m1 < m2 ? ternary_search(m1, r) : ternary_search(1, m2</pre>
   );
}
   Strings
9.1 Count Distinct Anagrams
const 11 MOD = 1e9 + 7;
const int maxn = 1e6;
vll fs(maxn + 1);
void precompute() {
 fs[0] = 1;
 for (ll i = 1; i <= maxn; i++) {
    fs[i] = (fs[i - 1] * i) % MOD;
}
ll fpow(ll a, int n, ll mod = LLONG_MAX) {
  if (n == 0) return 1;
  if (n == 1) return a:
 11 x = fpow(a, n / 2, mod) \% mod;
 return ((x * x) % mod * (n & 1 ? a : 111)) % mod;
}
11 distinctAnagrams(const string &s) {
  precompute();
 vi hist('z' - 'a' + 1, 0);
 for (auto &c : s) hist[c - 'a']++;
 ll ans = fs[len(s)];
 for (auto &q : hist) {
    ans = (ans * fpow(fs[q], MOD - 2, MOD)) % MOD;
  return ans;
9.2 Double Hash Range Query
const 11 MOD = 1000027957;
const int MOD2 = 1000015187;
struct Hash {
  const 11 P = 31;
  int n;
  string s;
  vll h, h2, hi, hi2, p, p2;
```

```
Hash() {}
  Hash(string _s) : s(_s), n(len(_s)), h(n), h2(n), hi(n), hi2(
   n), p(n), p2(n) {
   for (int i = 0; i < n; i++) p[i] = (i ? P * p[i - 1] : 1) %
    MOD:
    for (int i = 0; i < n; i++) p2[i] = (i ? P * p2[i - 1] : 1)
   for (int i = 0; i < n; i++) h[i] = (s[i] + (i ? h[i - 1] :
   0) * P) % MOD;
    for (int i = 0; i < n; i++) h2[i] = (s[i] + (i ? h2[i - 1]
   : 0) * P) % MOD2;
    for (int i = n - 1; i >= 0; i--)
      hi[i] = (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) % MOD;
    for (int i = n - 1; i >= 0; i--)
      hi2[i] = (s[i] + (i + 1 < n ? hi2[i + 1] : 0) * P) % MOD2
   ;
  }
  pii query(int 1, int r) {
    ll hash = (h[r] - (1 ? h[1 - 1] * p[r - 1 + 1] % MOD : 0));
    11 \text{ hash2} = (h2[r] - (1 ? h2[1 - 1] * p2[r - 1 + 1] % MOD2 :
    0));
    return \{(hash < 0 ? hash + MOD : hash), (hash2 < 0 ? hash2
   + MOD2 : hash2)};
  }
  pii query_inv(int 1, int r) {
   ll hash = (hi[1] - (r + 1 < n ? hi[r + 1] * p[r - 1 + 1] %
   MOD : O));
   11 \text{ hash2} = (\text{hi2}[1] - (r + 1 < n ? \text{hi2}[r + 1] * p2[r - 1 + 1])
   1] % MOD2 : 0));
   return {(hash < 0 ? hash + MOD : hash), (hash2 < 0 ? hash2</pre>
   + MOD2 : hash2)};
  }
};
9.3 Hash Range Query
struct Hash {
  const 11 P = 31;
  const ll mod = 1e9 + 7;
  string s;
  int n;
  vll h, hi, p;
  Hash() {}
```

 $Hash(string s) : s(s), n(s.size()), h(n), hi(n), p(n) {$

```
for (int i = 0; i < n; i++) p[i] = (i ? P * p[i - 1] : 1) %
    for (int i = 0; i < n; i++) h[i] = (s[i] + (i ? h[i - 1] :
   0) * P) % mod:
    for (int i = n - 1; i >= 0; i--)
      hi[i] = (s[i] + (i + 1 < n ? hi[i + 1] : 0) * P) \% mod;
  }
  11 query(int 1, int r) {
    ll hash = (h[r] - (1 ? h[1 - 1] * p[r - 1 + 1] % mod : 0));
    return hash < 0 ? hash + mod : hash;</pre>
  11 query_inv(int 1, int r) {
    ll hash = (hi[1] - (r + 1 < n ? hi[r + 1] * p[r - 1 + 1] %
   mod : 0));
    return hash < 0 ? hash + mod : hash;</pre>
 }
};
9.4 K-th digit in digit string
Find the k-th digit in a digit string, only works for 1 \le k \le 10^{18}!
Time: precompute O(1), query O(1)
using vull = vector<ull>;
vull pow10;
vector < array < ull, 4>> memo;
void precompute(int maxpow = 18) {
  ull qtd = 1;
  ull start = 1;
  ull end = 9;
  ull curlenght = 9;
  ull startstr = 1;
  ull endstr = 9:
 for (ull i = 0, j = 111; (int)i < maxpow; i++, j *= 1011)
   pow10.eb(j);
  for (ull i = 0; i < maxpow - 1ull; i++) {</pre>
    memo.push_back({start, end, startstr, endstr});
    start = end + 111;
    end = end + (911 * pow10[qtd]);
    curlenght = end - start + 1ull;
    qtd++;
    startstr = endstr + 1ull;
    endstr = (endstr + 1ull) + (curlenght)*qtd - 1ull;
```

```
}
char kthDigit(ull k) {
  int qtd = 1;
  for (auto [s, e, ss, es] : memo) {
    if (k >= ss and k <= es) {
      ull pos = k - ss;
      ull index = pos / qtd;
      ull nmr = s + index;
      int i = k - ss - qtd * index;

      return ((nmr / pow10[qtd - i - 1]) % 10) + '0';
    }
    qtd++;
}
return 'X';
}</pre>
```

9.5 Longest Palindrome Substring (Manacher)

Finds the longest palindrome substring, manacher returns a vector where the i-th position is how much is possible to grow the string to the left and the right of i and keep it a palindrome. Time: O(N)

```
vi manacher(string s) {
  string t2;
 for (auto c : s) t2 += string("#") + c;
  t2 = t2 + '#';
  int n = t2.size();
  t2 = "$" + t2 + "^";
  vi p(n + 2);
 int 1 = 1, r = 1;
 for (int i = 1; i <= n; i++) {
   p[i] = max(0, min(r - i, p[l + (r - i)]));
    while (t2[i - p[i]] == t2[i + p[i]]) {
      p[i]++;
   }
   if (i + p[i] > r) {
      l = i - p[i], r = i + p[i];
   }
    p[i]--;
 }
 return vi(begin(p) + 1, end(p) - 1);
string longest_palindrome(const string &s) {
```

```
vi xs = manacher(s):
  string s2;
  for (auto c : s) s2 += string("#") + c;
  s2 = s2 + '#':
  int mpos = 0;
  for (int i = 0; i < len(xs); i++) {</pre>
    if (xs[i] > xs[mpos]) {
      mpos = i;
    }
  }
  string ans;
  int k = xs[mpos];
  for (int i = mpos - k; i <= mpos + k; i++) {</pre>
    if (s2[i] != '#') {
      ans += s2[i];
    }
  }
  return ans;
void run() {
  string s;
  cin >> s;
  auto ans = longest_palindrome(s);
  cout << ans << endl;</pre>
9.6 Longest Palindrome
string longest_palindrome(const string &s) {
  int n = (int)s.size();
  vector < array < int, 2>> dp(n);
  pii odd(0, -1), even(0, -1);
  pii ans;
  for (int i = 0; i < n; i++) {
    int k = 0;
    if (i > odd.second)
      k = 1;
    else
      k = min(dp[odd.first + odd.second - i][0], odd.second - i
    + 1):
```

```
while (i - k \ge 0 \text{ and } i + k \le n \text{ and } s[i - k] == s[i + k]) k
   ++;
    dp[i][0] = k--;
    if (i + k > odd.second) odd = \{i - k, i + k\};
    if (2 * dp[i][0] - 1 > ans.second) ans = \{i - k, 2 * dp[i]\}
   ][0] - 1};
    k = 0;
    if (i <= even.second)</pre>
      k = min(dp[even.first + even.second - i + 1][1], even.
   second -i+1);
    while (i - k - 1) = 0 and i + k < n and s[i - k - 1] == s[i
    + k]) k++;
    dp[i][1] = k--;
    if (i + k > even.second) even = \{i - k - 1, i + k\};
   if (2 * dp[i][1] > ans.second) ans = \{i - k - 1, 2 * dp[i]\}
   ][1]};
  }
  return s.substr(ans.first, ans.second);
9.7 Rabin Karp
size_t rabin_karp(const string &s, const string &p) {
  if (s.size() < p.size()) return 0;</pre>
  auto n = s.size(), m = p.size();
  const 11 p1 = 31, p2 = 29, q1 = 1e9 + 7, q2 = 1e9 + 9;
  const ll p1_1 = fpow(p1, q1 - 2, q1), p1_2 = fpow(p1, m - 1,
   q1);
  const ll p2_1 = fpow(p2, q2 - 2, q2), p2_2 = fpow(p2, m - 1,
   q2);
  pair < ll, ll > hs, hp;
  for (int i = (int)m - 1; ~i; --i) {
    hs.first = (hs.first * p1) % q1;
    hs.first = (hs.first + (s[i] - 'a' + 1)) \% q1;
    hs.second = (hs.second * p2) % q2;
    hs.second = (hs.second + (s[i] - 'a' + 1)) \% q2;
    hp.first = (hp.first * p1) % q1;
    hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
    hp.second = (hp.second * p2) % q2;
```

hp.second = (hp.second + (p[i] - a' + 1)) % q2;

}

```
size_t occ = 0;
 for (size_t i = 0; i < n - m; i++) {</pre>
    occ += (hs == hp);
    int fi = s[i] - 'a' + 1;
    int fm = s[i + m] - 'a' + 1;
    hs.first = (hs.first - fi + q1) \% q1;
    hs.first = (hs.first * p1_1) % q1;
   hs.first = (hs.first + fm * p1_2) % q1;
    hs.second = (hs.second - fi + q2) \% q2;
    hs.second = (hs.second * p2_1) % q2;
    hs.second = (hs.second + fm * p2_2) \% q2;
  occ += hs == hp;
 return occ;
9.8 String Psum
struct strPsum {
 ll n;
 11 k;
  vector < vll > psum;
  strPsum(const string &s): n(s.size()), k(100), psum(k, vll(n
    + 1)) {
   for (ll i = 1; i <= n; ++i) {
      for (11 j = 0; j < k; ++j) {
        psum[j][i] = psum[j][i - 1];
      psum[s[i - 1]][i]++;
 }
 ll qtd(ll l, ll r, char c) { // [0,n-1]
    return psum[c][r + 1] - psum[c][1];
 }
    Suffix Automaton (complete)
struct state {
  int len, link, cnt, firstpos;
```

```
// this can be optimized using a vector with the alphabet
   size
  map < char , int > next;
  vi inv_link;
}:
struct SuffixAutomaton {
  vector<state> st:
  int sz = 0;
  int last;
  vc cloned;
  SuffixAutomaton(const string &s, int maxlen)
    : st(maxlen * 2), cloned(maxlen * 2) {
    st[0].len = 0;
    st[0].link = -1;
    sz++;
    last = 0;
    for (auto &c : s) add_char(c);
    // precompute for count occurences
    for (int i = 1; i < sz; i++) {</pre>
      st[i].cnt = !cloned[i];
    vector < pair < state , int >> aux;
    for (int i = 0; i < sz; i++) {</pre>
      aux.push_back({st[i], i});
    sort(all(aux), [](const pair<state, int> &a, const pair<</pre>
   state, int> &b) {
      return a.fst.len > b.fst.len;
    });
    for (auto &[stt, id] : aux) {
      if (stt.link != -1) {
        st[stt.link].cnt += st[id].cnt;
      }
    }
    // for find every occurende position
    for (int v = 1; v < sz; v++) {
      st[st[v].link].inv_link.push_back(v);
    }
  }
```

```
void add char(char c) {
 int cur = sz++;
 st[cur].len = st[last].len + 1;
  st[cur].firstpos = st[cur].len - 1;
 int p = last;
 // follow the suffix link until find a transition to c
 while (p != -1 and !st[p].next.count(c)) {
    st[p].next[c] = cur;
   p = st[p].link;
 // there was no transition to c so create and leave
 if (p == -1) {
    st[cur].link = 0;
   last = cur;
    return;
 }
 int q = st[p].next[c];
 if (st[p].len + 1 == st[q].len) {
    st[cur].link = q;
 } else {
   int clone = sz++;
    cloned[clone] = true;
    st[clone].len = st[p].len + 1;
    st[clone].next = st[q].next;
    st[clone].link = st[q].link;
    st[clone].firstpos = st[q].firstpos;
    while (p != -1 and st[p].next[c] == q) {
      st[p].next[c] = clone;
     p = st[p].link;
    st[q].link = st[cur].link = clone;
 last = cur;
bool checkOccurrence(const string &t) { // O(len(t))
 int cur = 0:
 for (auto &c : t) {
   if (!st[cur].next.count(c)) return false;
    cur = st[cur].next[c];
 return true;
11 totalSubstrings() { // distinct, O(len(s))
```

```
11 \text{ tot} = 0;
   for (int i = 1; i < sz; i++) {
      tot += st[i].len - st[st[i].link].len;
   }
   return tot;
 }
 // count occurences of a given string t
  int countOccurences(const string &t) {
    int cur = 0;
   for (auto &c : t) {
      if (!st[cur].next.count(c)) return 0;
      cur = st[cur].next[c];
   }
    return st[cur].cnt;
 // find the first index where t appears a substring O(len(t))
 int firstOccurence(const string &t) {
    int cur = 0;
    for (auto c : t) {
      if (!st[cur].next.count(c)) return -1;
      cur = st[cur].next[c];
    return st[cur].firstpos - len(t) + 1;
 }
 vi everyOccurence(const string &t) {
    int cur = 0;
   for (auto c : t) {
      if (!st[cur].next.count(c)) return {};
      cur = st[cur].next[c];
    vi ans;
    getEveryOccurence(cur, len(t), ans);
    return ans;
 }
 void getEveryOccurence(int v, int P_length, vi &ans) {
   if (!cloned[v]) ans.pb(st[v].firstpos - P_length + 1);
    for (int u : st[v].inv_link) getEveryOccurence(u, P_length,
    ans):
 }
};
```

9.10 Z-function get occurrence positions

```
O(len(s) + len(p))
vi getOccPos(string &s, string &p) {
 // Z-function
  char delim = '#':
  string t{p + delim + s};
  vi zs(len(t));
  int 1 = 0, r = 0;
 for (int i = 1; i < len(t); i++) {</pre>
    if (i <= r) zs[i] = min(zs[i - 1], r - i + 1);</pre>
    while (zs[i] + i < len(t)) and t[zs[i]] == t[i + zs[i]]) zs[
   if (r < i + zs[i] - 1) l = i, r = i + zs[i] - 1;
  }
  // Iterate over the results of Z-function to get ranges
  vi ans;
  int start = len(p) + 1 + 1 - 1;
 for (int i = start; i < len(zs); i++) {</pre>
    if (zs[i] == len(p)) {
      int l = i - start;
      ans.emplace_back(1);
    }
  return ans;
```

10 Trees

10.1 Binary Lifting (struct)

```
struct BinaryLifting {
  vector<int> far, level, parent;

BinaryLifting(const vector<vector<int>> &g, int root = 0)
  : far(g.size(), -1), level(g.size()), parent(g.size()) {
  level[root] = 1;
  vector<int> q{root};
  q.reserve(g.size());
  for (int u = 0; u < (int)q.size(); u++) {
    for (auto x : g[u])
    if (x != parent[u]) {
      parent[x] = u;
    }
}</pre>
```

```
level[x] = level[u] + 1;
        int p1 = u;
        int p2 = far[u];
        if (p2 > -1 \text{ and } far[p2] > -1 \text{ and}
             level[p1] - level[p2] == level[p2] - level[far[p2
 11)
           far[x] = far[p2];
         else
           far[x] = p1;
        q.push_back(x);
  }
}
int kth_parent(int node, int k) const {
  while (node \geq 0 and k \geq 0) {
    if (far[node] > -1 and level[node] - k <= level[far[node</pre>
 11) {
      k -= level[node] - level[far[node]];
      node = far[node];
    } else {
      k--;
      node = parent[node];
    }
  }
  return node;
int lca(int u, int v) const {
  if (level[u] < level[v]) swap(u, v);</pre>
  while (level[v] < level[u]) {</pre>
    if (far[u] > -1 and level[v] <= level[far[u]])</pre>
      u = far[u]:
    else
      u = parent[u];
  }
  if (u == v) return u:
  while (parent[u] != parent[v]) {
```

```
if (far[u] > -1 and far[v] > -1 and far[u] != far[v]) {
        u = far[u];
        v = far[v];
      } else {
        u = parent[u];
        v = parent[v];
    }
    return parent[u];
};
10.2 Binary Lifting
/*
* far[h][i] = the node that 2^h far from node i
* sometimes is useful invert the order of loops
* time : O(nlogn)
* */
const int maxlog = 20;
int far[maxlog + 1][n + 1];
int n;
for (int h = 1; h <= maxlog; h++) {</pre>
 for (int i = 1; i <= n; i++) {
    far[h][i] = far[h - 1][far[h - 1][i]];
}
10.3 Isomorphism
Two trees are considered isomorphic if the hash given by thash() is the same.
Time: O(V \cdot \log V)
map < vector < int > , int > mphash;
struct Tree {
  int n;
  vi2d g;
  vi sz, cs;
  Tree(int n_) : n(n_), g(n), sz(n) {}
  void add_edge(int u, int v) {
    g[u].emplace_back(v);
    g[v].emplace_back(u);
  }
```

```
void dfs_centroid(int v, int p) {
    sz[v] = 1;
    bool cent = true:
    for (int u : g[v])
      if (u != p) {
         dfs_centroid(u, v);
         sz[v] += sz[u];
         cent &= not(sz[u] > n / 2);
    if (cent and n - sz[v] <= n / 2) cs.push_back(v);</pre>
  }
  int fhash(int v, int p) {
    vi h;
    for (int u : g[v])
      if (u != p) h.push_back(fhash(u, v));
    sort(all(h)):
    if (!mphash.count(h)) mphash[h] = mphash.size();
    return mphash[h];
  }
  11 thash() {
    cs.clear();
    dfs_centroid(0, -1);
    if (cs.size() == 1) return fhash(cs[0], -1);
    ll h1 = fhash(cs[0], cs[1]), h2 = fhash(cs[1], cs[0]);
    return (min(h1, h2) << 3011) + max(h1, h2);</pre>
  }
};
10.4 Lowest Common Ancestor
Given two nodes of a tree find their lowest common ancestor, or their distance
Build : O(V), Queries: O(1)
0 indexed!
template <typename T>
struct SparseTable {
  vector <T> v;
  int n;
  static const int b = 30;
  vi mask, t;
```

int op(int x, int y) { return $v[x] < v[y] ? x : y; }$

```
int msb(int x) { return __builtin_clz(1) - __builtin_clz(x);
  }
  SparseTable() {}
  SparseTable(const vector<T>& v_) : v(v_), n(v.size()), mask(n
  ), t(n) {
   for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
      at = (at << 1) & ((1 << b) - 1);
      while (at and op(i, i - msb(at & -at)) == i) at ^= at & -
   at;
   }
   for (int i = 0; i < n / b; i++)
      t[i] = b * i + b - 1 - msb(mask[b * i + b - 1]);
   for (int j = 1; (1 << j) <= n / b; j++)
      for (int i = 0; i + (1 << j) <= n / b; i++)
        t[n / b * j + i] =
          op(t[n / b * (j - 1) + i], t[n / b * (j - 1) + i + (1)
    << (i - 1))]);
  int small(int r, int sz = b) { return r - msb(mask[r] & ((1
   << sz) - 1)); }
 T query(int 1, int r) {
    if (r - l + 1 \le b) return small(r, r - l + 1);
    int ans = op(small(1 + b - 1), small(r));
   int x = 1 / b + 1, y = r / b - 1;
   if (x <= y) {
     int j = msb(y - x + 1);
      ans = op(ans, op(t[n / b * j + x], t[n / b * j + y - (1)
   << j) + 1]));
   }
    return ans;
};
struct LCA {
  SparseTable < int > st;
 int n;
 vi v, pos, dep;
 LCA(const vi2d& g, int root) : n(len(g)), pos(n) {
   dfs(root, 0, -1, g);
    st = SparseTable < int > (vector < int > (all(dep)));
 }
 void dfs(int i, int d, int p, const vi2d& g) {
   v.eb(len(dep)) = i, pos[i] = len(dep), dep.eb(d);
```

```
for (auto j : g[i])
      if (j != p) {
        dfs(j, d + 1, i, g);
        v.eb(len(dep)) = i, dep.eb(d);
      }
  }
  int lca(int a, int b) {
    int 1 = min(pos[a], pos[b]);
   int r = max(pos[a], pos[b]);
    return v[st.query(1, r)];
  }
  int dist(int a, int b) {
    return dep[pos[a]] + dep[pos[b]] - 2 * dep[pos[lca(a, b)]];
  }
};
10.5 Tree Maximum Distance
Returns the maximum distance from every node to any other node in the tree. O(6V) = O(V)
pll mostDistantFrom(const vector<vll> &adj, ll n, ll root) {
 // O(V)
  // 0 indexed
  ll mostDistantNode = root;
  11 nodeDistance = 0;
  queue <pll> q;
  vector < char > vis(n);
  q.emplace(root, 0);
  vis[root] = true;
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
    if (dist > nodeDistance) {
      nodeDistance = dist:
      mostDistantNode = node;
    for (auto u : adj[node]) {
      if (!vis[u]) {
        vis[u] = true;
        q.emplace(u, dist + 1);
      }
    }
```

return {mostDistantNode, nodeDistance};

```
11 twoNodesDist(const vector < vll > &adj, ll n, ll a, ll b) {
  queue <pll> q;
  vector < char > vis(n):
  q.emplace(a, 0);
  while (!q.empty()) {
    auto [node, dist] = q.front();
    q.pop();
   if (node == b) return dist;
   for (auto u : adj[node]) {
      if (!vis[u]) {
        vis[u] = true;
        q.emplace(u, dist + 1);
    }
 }
 return -1;
tuple < 11, 11, 11> tree_diameter(const vector < v11> & adj, 11 n) {
 // returns two points of the diameter and the diameter itself
  auto [node1, dist1] = mostDistantFrom(adj, n, 0);
                                                           // O(V
  auto [node2, dist2] = mostDistantFrom(adj, n, node1); // O(V
  auto diameter = twoNodesDist(adj, n, node1, node2);
                                                          // O(V
 return make_tuple(node1, node2, diameter);
vll everyDistanceFromNode(const vector < vll > & adj, ll n, ll root
 // Single Source Shortest Path, from a given root
  queue <pair <11, 11>> q;
  vll ans(n, -1);
  ans[root] = 0;
  q.emplace(root, 0);
 while (!q.empty()) {
   auto [u, d] = q.front();
    q.pop();
   for (auto w : adj[u]) {
      if (ans[w] != -1) continue;
      ans[w] = d + 1;
      q.emplace(w, d + 1);
```

```
}
  return ans;
}
vll maxDistances(const vector < vll > & adj, ll n) {
  auto [node1, node2, diameter] = tree_diameter(adj, n);
   (3V)
  auto distances1 = everyDistanceFromNode(adj, n, node1); // 0
  auto distances2 = everyDistanceFromNode(adj, n, node2); // 0
  vll ans(n);
  for (int i = 0; i < n; ++i)</pre>
    ans[i] = \max(\text{distances1[i]}, \text{distances2[i]}); // O(V)
  return ans;
10.6 Small to Large
Answer queries of the form "How many vertices in the subtree of vertex v have property P?"
* this implementation answers how many distinct values[i] are in the subtree starting at u.
Build: O(N), Query: O(N \log N)
struct SmallToLarge {
  int n;
  vi2d tree, vis_childs;
  vi sizes, values, ans;
  set < int > cnt:
  SmallToLarge(vi2d &g, vi &v)
    : tree(g), vis_childs(len(g)), sizes(len(g)), values(v),
   ans(len(g)) {
    get_size(0);
    dfs(0):
  }
  inline void add_value(int u) { cnt.insert(values[u]); }
  inline void remove_value(int u) { cnt.erase(values[u]); }
  inline void update_ans(int u) { ans[u] = len(cnt); }
  void dfs(int u, int p = -1, bool keep = true) {
    int mx = -1;
```

for (auto x : tree[u]) {

```
if (x == p) continue;
      if (mx == -1 \text{ or sizes}[mx] < \text{sizes}[x]) mx = x;
    }
    for (auto x : tree[u]) {
      if (x != p and x != mx) dfs(x, u, false);
    }
    if (mx != -1) {
      dfs(mx, u, true);
      swap(vis_childs[u], vis_childs[mx]);
    }
    vis_childs[u].push_back(u);
    add_value(u);
    for (auto x : tree[u]) {
      if (x != p and x != mx) {
        for (auto y : vis_childs[x]) {
          add_value(y);
          vis_childs[u].push_back(y);
        }
      }
    }
    update_ans(u);
    if (!keep) {
      for (auto x : vis_childs[u]) remove_value(x);
  void get_size(int u, int p = -1) {
    sizes[u] = 1;
    for (auto x : tree[u])
      if (x != p) {
        get_size(x, u);
        sizes[u] += sizes[x];
  }
};
```

10.7 Tree Diameter

```
const int MAXN(2,00,000);
int N;
vi2d G(MAXN);
int toleaf[MAXN], maxdist[MAXN];
void dfs(int u, int p = -1) {
  int ds1, ds2;
  ds1 = ds2 = -1;
  for (auto v : G[u]) {
   if (v == p) continue;
   if (ds1 > ds2) swap(ds1, ds2);
    dfs(v, u);
    ds1 = max(ds1, toleaf[v]);
  toleaf[u] = max(ds1, ds2) + 1;
  maxdist[u] = 2 + ds1 + ds2;
int diameter(int root, int n) {
  dfs(root);
  int d = 0;
  for (int u = 0; u < n; ++u) d = max(d, maxdist[u]);
  return d;
10.8 Tree Flatten
void tree_flatten(const vector<vector<int>> &g, int u, int p,
   vector < int > & pre ,
                   vector<int> &pos, int &idx) {
  ++idx;
  pre.push_back(u);
  for (auto x : g[u])
    if (x != p) tree_flatten(g, x, u, pre, pos, idx);
  pos[u] = idx;
pair < vector < int >, vector < int >> tree_flatten(const vector < vector</pre>
   <int>> &g,
```

```
int root = 0) {
  vector<int> first(g.size()), last(g.size()), pre;
  int timer = -1;
 tree_flatten(g, root, -1, pre, last, timer);
 for (int i = 0; i < (int)g.size(); i++) first[pre[i]] = i;</pre>
  return {first, last};
    Settings and macros
11.1 short-macro.cpp
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define fastio
 ios_base::sync_with_stdio(false); \
cin.tie(0);
 cout.tie(0);
#define len(__x) (int) __x.size()
using ll = long long;
using pii = pair<int, int>;
#define all(a) a.begin(), a.end()
void run() {}
int32_t main(void) {
 fastio;
 int t;
 t = 1;
 // cin >> t;
  while (t--) run();
11.2 debug.cpp
#include <bits/stdc++.h>
using namespace std;
/****** Debug Code ******/
template <typename T>
concept Printable = requires(T t) {
    { std::cout << t } -> std::same_as<std::ostream &>;
};
template <Printable T>
void __print(const T &x) {
    cerr << x;
```

```
template <size_t T>
void __print(const bitset<T> &x) {
    cerr << x;
template <typename A, typename B>
void __print(const pair<A, B> &p);
template <typename... A>
void __print(const tuple <A...> &t);
template <typename T>
void __print(stack<T> s);
template <typename T>
void __print(queue < T > q);
template <typename T, typename... U>
void __print(priority_queue < T, U... > q);
template <typename A>
void __print(const A &x) {
    bool first = true:
    cerr << '{';
    for (const auto &i : x) {
        cerr << (first ? "" : ","), __print(i);</pre>
        first = false;
    cerr << '}';
template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '(';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << ')';
template <typename... A>
void __print(const tuple<A...> &t) {
    bool first = true;
    cerr << '(':
    apply(
        [&first](const auto &...args) {
            ((cerr << (first ? "" : ","), __print(args), first
   = false), ...);
        },
        t):
    cerr << ')':
}
```

```
template <typename T>
void __print(stack<T> s) {
    vector <T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    }
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
template <typename T>
void __print(queue < T > q) {
    vector <T> debugVector;
    while (!q.empty()) {
        T t = q.front();
        debugVector.push_back(t);
        q.pop();
    __print(debugVector);
template <typename T, typename... U>
void __print(priority_queue < T, U... > q) {
    vector <T> debugVector;
    while (!q.empty()) {
        T t = q.top();
        debugVector.push_back(t);
        q.pop();
    __print(debugVector);
void _print() { cerr << "]\n"; }</pre>
template <typename Head, typename... Tail>
void _print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";</pre>
    _print(T...);
}
#define dbg(x...)
    cerr << "[" << #x << "] = ["; \
    _print(x)
11.3 .vimrc
```

```
set ts=4 sw=4 sta nu rnu sc cindent
set bg=dark ruler clipboard=unnamed,unnamedplus, timeoutlen=100
colorscheme default
nnoremap <C-j>:botright belowright term bash <CR>
syntax on
11.4 .bashrc
cpp() {
 g++ -std=c++20 -fsanitize=address,undefined -Wall $1 && time
   ./a.out
}
cpp() {
  echo ">> COMPILING <<" 1>&2
  g++-std=c++17
      -02 \
      -g \
      -g3 \
      -Wextra \
      -Wshadow \
      -Wformat=2 \
      -Wconversion \
      -fsanitize=address,undefined \
      -fno-sanitize-recover \
      -Wfatal-errors \
      $1
  if [ $? -ne 0 ]; then
      echo ">> FAILED <<" 1>&2
      return 1
  fi
  echo ">> DONE << " 1>&2
  time ./a.out ${0:2}
prepare() {
    cp debug.cpp ./
    for i in {a..z}
        cp macro.cpp $i.cpp
        touch $i.py
    done
```

```
for i in {1..10}
    do
        touch in${i}
        touch out${i}
        touch ans${i}
    done
}
11.5 macro.cpp
#include <bits/stdc++.h>
using namespace std;
#ifdef LOCAL
#include "debug.cpp"
#else
#define dbg(...) 42
#endif
#define endl '\n'
#define fastio
ios_base::sync_with_stdio(false); \
 cin.tie(0);
cout.tie(0);
#define len(__x) (int)__x.size()
using ll = long long;
using ull = unsigned long long;
using ld = long double;
using vll = vector<ll>;
using pll = pair<11, 11>;
using v112d = vector < v11 >;
using vi = vector<int>;
using vi2d = vector < vi>;
using pii = pair<int, int>;
using vii = vector<pii>;
using vc = vector < char >;
#define all(a) a.begin(), a.end()
#define pb(___x) push_back(___x)
#define mp(__a, __b) make_pair(__a, __b)
#define eb(___x) emplace_back(___x)
// vector<string> dir({"LU", "U", "RU", "R", "RD", "D", "LD", "
// int dx[] = \{-1, -1, -1, 0, 1, 1, 1, 0\};
// int dy[] = \{-1, 0, 1, 1, 1, 0, -1, -1\};
vector<string> dir({"U", "R", "D", "L"});
int dx[] = \{-1, 0, 1, 0\};
```

```
int dy[] = {0, 1, 0, -1};
const ll oo = 1e18;
auto solve() {}
int32_t main(void) {
#ifndef LOCAL
fastio;
#endif
```

```
int t;
t = 1;
// cin >> t;
for (int i = 1; i <= t; i++) {
    solve();
}
</pre>
```