NTNU

Department of Engineering Cybernetics

# Examination paper for TTK4130 Modeling and Simulation

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**Examination time (from-to): 0900—1300**

**Permitted examination support material: Code A.**

**Other information:**

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**Number of pages enclosed:**

**Informasjon om trykking av eksamensoppgave**

**Originalen er:**

**1-sidig** ☐ **2-sidig X**

**sort/hvit** ☐ **farger X**

**skal ha flervalgskjema** ☐

**Checked by:**

Date Signature

Students will find the examination results in Studentweb. Please contact the department if you have questions about your results. The Examinations Office will not be able to answer this.

**NTNU – Trondheim**
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# Exam in TTK4130 Modeling and Simulation

Thursday, May 15 2019
09:00 – 13:00

Permitted aids (code A): All written and handwritten examination support materials are permitted.

Answers in English, Norwegian, or a mixture of the two are accepted.

Grades available: As specified by regulations.

## Problem 1 (11 %)

(2 %) (a) Explain briefly how the Modelica commands `replaceable` and `redeclare` are connected.

(6 %) (b) You have available the three models: `RedModel`, `WhiteModel`, and `BlueModel`. You like to implement a flexible model `Austria`, where the `RedModel`, the `WhiteModel` and another `RedModel` are connected and can be exchanged (Fig. 1). The models are connected by just one connector variable.

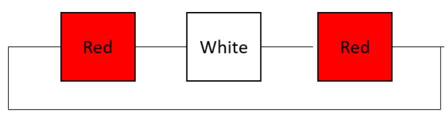


Figure 1: Model `Austria`.

Moreover, you like to set the parameter $p1$ in one of the `RedModel` to $p1 = 5$ and in the other to $p1 = 10$. Write down the Modelica code of the *flexible* model `Austria`.

(3 %) (c) A new model `Russia` should be implemented with the structure shown in Fig. 2. Write the Modelica code of the new model by using model `Austria`.

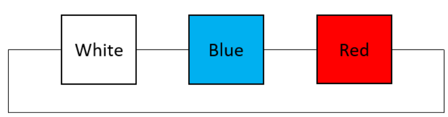


Figure 2: Model `Russia`.

## Problem 2 (10 %)

Consider the following differential-algebraic system with input $u$

$$\begin{aligned}
\dot{x}_1 &= x_3 - x_1 z, \\
\dot{x}_2 &= u - x_2 z, \\
\dot{x}_3 &= -x_1, \\
0 &= 1/2(1 - x_1^2 - x_2^2).
\end{aligned}$$

(4 %) (a) Derive mathematically the differential index of the system.

For the rest of the problem you can neglect the algebraic equation and assume $z = 1/2$

(6 %) (b) Is the ODE system passive for input $u$ and output $x_2$?

**Problem 3 (26 %)**

Consider the following Runge-Kutta method with the Butcher tableau

$$\begin{array}{c|cc} \gamma & \gamma & 0 \\ 1-\gamma & 1-2\gamma & \gamma \\ \hline & 1/2 & 1/2 \end{array} .$$

(5 %) (a) Write down the equations of the method. What are the limits for $\gamma$, and is the method explicit or implicit?

(7 %) (b) Find the stability function of the method.

(6 %) (c) Find the limits on $\gamma$ so the method is B-stable.
*Hint: Make appropriate assumptions.*

(4 %) (d) Find the limits on $\gamma$ for which the method is B- and L-stable.

(4 %) (e) Explain why the method can be seen as numerical efficient for $\gamma \neq 0$.

**Problem 4 (25 %)**

The following system consisting of a slender beam with mass $m_1$ and length $l$ and a disc with mass $m_2$ and radius $R$ should be modeled. The disc rolls without slipping on the ground (Fig. 3). The system is frictionless.

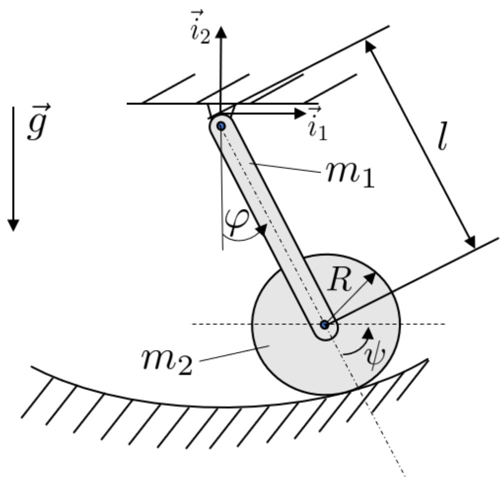


Figure 3: Slender beam with disc.

*Hint: The moment of inertia of a disc:* $I = 1/2 mR^2$.

(3 %) (a) Find a connection between $\varphi$ and $\psi$. How many degrees of freedom have the system?

*If you were not able to find a connection between $\varphi$ and $\psi$ use $\varphi = \psi$ [which is not necessarily the correct answer to (a)]*

(6 %) (b) Find the position, velocity and acceleration vector of the center of mass of both body parts.

(13 %) (c) Find the equation of motion of the system as a function of the given variables (Fig. 3) with the Newton-Euler approach.
*Caution: Solutions using the Lagrange approach will give no points in this task.*
*Hint 1: Given variables: $\vec{g}, m_1, m_2, l, R, \psi, \varphi$, including their derivatives.*
*Hint 2: The equation of motion does not necessary contain all given variables.*

*If you were not able to find the EoM use in the following $\ddot{\varphi}l^2(2m_1 + m_2) + {}^1\!/_3 lg(3m_1 + {}^1\!/_2 m_2)\sin\varphi = 0$ [which is not necessarily the correct answer to (c)]*

(3 %) (d) Find the period of oscillation.
*Hint: Make appropriate assumptions.*

**Problem 5 (12 %)**

Consider a thin plate (Fig. 6) of height $h$ with a constant density $\rho$ and the mass $m = {}^\pi\!/_4 abh\rho$. The outer contour of the cross-section can be described by $z^2 = b^2 - b^2 y^2/a^2$, where $a \gg h$ and $b \gg h$.

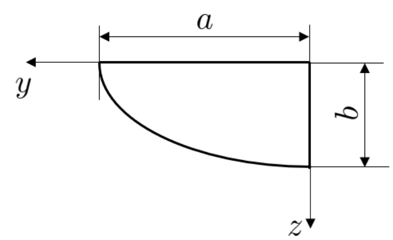

Figure 6: Thin plate

The following may be useful

$$\int \sqrt{X}dx = \frac{1}{2}(x\sqrt{X} + c^2 \arcsin\frac{x}{c})$$
$$\int x\sqrt{X}dx = -\frac{1}{3}\sqrt{X^3}$$
$$\int x^2\sqrt{X}dx = -\frac{x}{4}\sqrt{X^3} + \frac{c^2}{8}(x\sqrt{X} + c^2 \arcsin\frac{x}{c}),$$

with $X = (c^2 - x^2)$.

(7 %) (a) Find the moment of inertia $I_{y,o}$ of the plate for the given coordinate system as a function of mass $m$.

*If you were not able to find $I_{y,o}$ use in the following $I_{y,o} = {}^1\!/_6 a^2b^2m$ [which is not necessarily the correct answer to (a)]*

(5 %) (b) Find the moment of inertia $I_{y,c}$ around the center of mass of the plate as a function of mass $m$.

**Problem 6 (16 %)**

In an open wind channel of diameter $D$ the air flow around a wind turbine with diameter $d$ is investigated (Fig. 7). The air flow has a constant density $\rho$. The upstream wind velocity is $u_\infty$, the following velocity field over the cross-section of the channel is measured at a specific downstream position $x = \text{const}$.

$$u_e(r,\phi) = \begin{cases} u_\infty \left((1-c) - c\cos(\pi r/R)\right), & 0 \le r \le R \\ u_\infty, \quad R < r \le D/2 \end{cases}$$

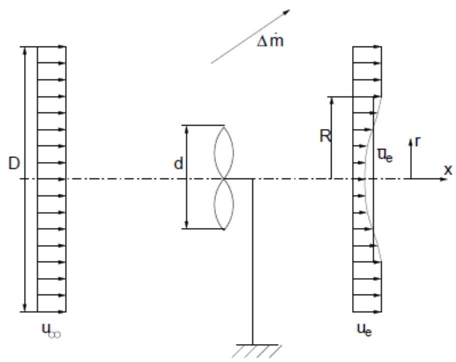


Figure 7: Open wind channel with wind turbine

Assume that the wind channel is in steady-state, $d \ll D$, $d/2 < R < D/2$ and $0 < c < 1$. Moreover, friction forces can be neglected.
The following may be useful

$$\int x\cos(ax)dx = \frac{\cos(ax)}{a^2} + \frac{x\sin(ax)}{a}.$$

(7 %) (a) Determine the displaced mass flow $\Delta\dot{m}$ in the wind channel.
*Hint: As usually for these problems you first have to define the control volume.*

*If you were not able to find the displaced mass flow continue with* $\Delta\dot{m} = \rho u_\infty c\left(1 - \frac{2}{\pi}\right)$ *[which is not necessarily the correct answer to (a)]*

(3 %) (b) The velocity profile $u_e(r,\phi)$ should be replaced with an piecewise constant velocity $u_e(r,\phi) = \bar{u}_e,\ 0 \le r \le R$ and $u_e(r,\phi) = u_\infty,\ R < r \le D/2$. Determine the average velocity $\bar{u}_e$ such that the displaced mass flow $\Delta\dot{m}$ stays the same as in a).

For the following task assume the average velocity $\bar{u}_e$ is known.

(6 %) (c) Determine the force $F$ produced from the wind turbine using the piecewise velocity profile.