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Exercise 8

TTK4130 Modeling and Simulation

Problem 1 (Implementation of friction models)

In this problem we will implement different *static* and *dynamic* friction models. To test the developed models, we will use a simple *testbench* as illustrated in Figure 1. The testbench is based on a model based on Newton's law of a box of mass $m = 1$kg sliding on a table due to application of an external force F_a (assumed to be a ramp function). A friction force F_f opposes motion. We will use the parameters

$F_c = 1$	Coulomb friction
$F_s = 1.2$	Stiction (static friction)
$F_v = 0.05$	Viscous friction
$v_s = 0.1$	Characteristic Stribeck velocity

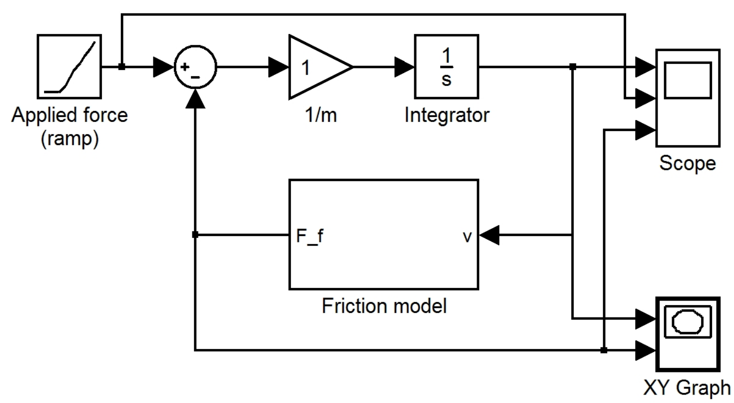

Figure 1: Testbench for friction models

(a) We start with static friction models, more specifically *Coulomb's model*, given by

$$F_f = F_c \,\mathrm{sign}(v), \quad v \neq 0 \tag{1}$$

where v is velocity. Implement this model in Simulink using a sign-block (do not use the built-in Coulomb friction block). Simulate the model over 10s, with ramp slope of 0.5 and 2.0. Use first a variable step solver. Explain what happens (hint: does the sign block include zero-crossing detection?). Choose the fixed-step Euler solver with sample time 0.01 instead – how does the model simulate now, for both values of the ramp?

The above (Coulomb) model has the disadvantage that it is not defined at $v = 0$. The Karnopp friction model (for Coulomb friction) remedies this by defining

$$F_f = \begin{cases} \mathrm{sat}(F_a, F_c), & v = 0 \\ F_c \,\mathrm{sign}(v), & v \neq 0 \end{cases}$$

where the sat-function is as defined in the book (the Simulink saturation function can be used for this saturation function by "hard-coding" the upper and lower limit at the second argument (F_c)). For this model to work properly, we must either use variable-step methods with event-detection to detect exactly when $v = 0$, or we have to use some kind of dead-zone around zero-velocity to treat the velocity as zero when it is small. Here, we will implement both of these approaches.

(b) Implement Karnopp's friction model in the setup in Figure 3, that is, fill in the two If Action Subsystems. Note that the if-block (by default) generate events when the if-clause changes. The Merge-block can be found under Signal Routing, and the If Action Subsystems under Ports & Subsystems.

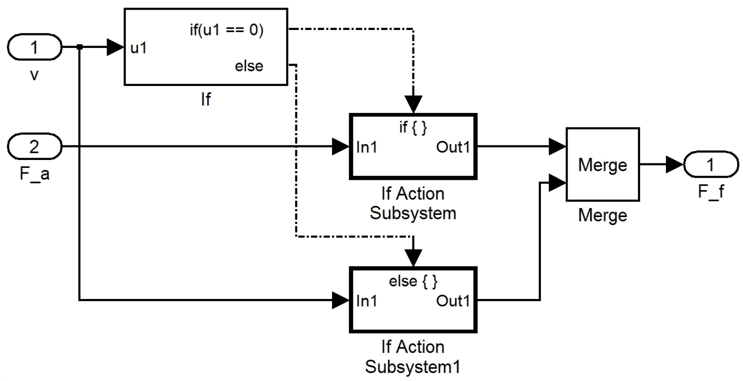

Figure 3: Setup for implementing Karnopp's friction model with event detection

Simulate using a variable step method (for example ode45) and comment. How does event-detection help? Note: It might be necessary to turn off event-detection for the sign-function inside the If Action Subsystem (why?). On the other hand, event-detection for the saturation function inside the other If Action Subsystem block should be turned on (why?).

(c) Implement Karnopp's model without relying on event-detection, by using dead-zone:

$$F_f = \begin{cases} \text{sat}(F_a, F_c), & |v| \leq \delta \\ F_c \,\text{sign}(v), & |v| > \delta \end{cases}$$

Choose $\delta = 0.5$. Simulate using a fixed-step solver, and comment. Test also using an initial velocity of $v(0) = -1$.

(d) Extend the model in (b) with sticking, Stribeck-effect (see 5.2.6 in book) and linear viscous friction,

$$F_f = \begin{cases} \text{sat}(F_a, F_s), & v = 0 \\ \left[F_c + (F_s - F_c)e^{-(v/v_s)^2}\right] \text{sign}(v) + F_v v, & v \neq 0 \end{cases}$$

(note slight error in book). Compare with Coulomb friction. Use ramp slope 0.5.

(e) Finally, we shall implement the LuGre dynamic friction model,

$$\dot{z} = (v - \sigma_0 \frac{|v|}{g(v)} z),$$
$$F_f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v,$$

where z may represent a small displacement in the stick-zone and σ_0 "spring-stiffness" of asperities. Set $\sigma_0 = 500$, $\sigma_1 = 0$ and $\sigma_2 = F_v$, and and implement $g(v)$ to get the same steady-state solution as in (d). Simulate and compare with solution in (d). Play around with the parameters σ_0 and σ_1, and comment.

Problem 2 (Transmission line)

A lossless transmission line is described by

$$\frac{\partial u}{\partial t} = -cz_0 \frac{\partial y}{\partial x} \tag{2a}$$

$$\frac{\partial y}{\partial t} = -\frac{c}{z_0} \frac{\partial u}{\partial x} \tag{2b}$$

where c is wave velocity (speed of sound) and z_0 is line (characteristic) impedance. The wave variables are

$$a = u + z_0 y,$$
$$b = u - z_0 y.$$

(a) We first assume that the equations describe a lossless hydraulic transmission line filled with water. What physical variables are u and y in this case?

The transmission line is a pipe with circular cross section with radius $R = 10\,\mathrm{cm}$. Water has speed of sound

$$c = \sqrt{\frac{\beta}{\rho}} = 1500\,\mathrm{m/s} \tag{3}$$

and density $\rho = 980\,\mathrm{kg/m^3}$. What is the bulk modulus of water, and what is (characteristic) line impedance for the transmission line? What are the units?

(b) We now assume the equations describe an electrical transmission line in the form of a high voltage line in air. The wave velocity is the speed of light $c = 3 \cdot 10^8\,\mathrm{m/s}$ and the characteristic impedance is $z_0 = 300\,\Omega$. What are the line inductance L and capacitance C per unit length? What physical variables are u and y in this case?

Problem 3 (Passivity of transmission line)

A lossless transmission line is described by (2). We use index 1 to denote input-side and index 2 to denote output-side. That is, u_1 is input and y_2 is the output/measurement. On the output side we have a load impedance $Z_2(s)$ such that

$$U_2(s) = Z_2(s)Y_2(s). \tag{4}$$

The corresponding wave variables fulfil

$$B_2(s) = H_2(s)A_2(s) \tag{5}$$

where

$$H_2(s) = \frac{G_2(s) - 1}{G_2(s) + 1} \tag{6}$$

and

$$G_2(s) := \frac{Z_2(s)}{z_0}. \tag{7}$$

See e.g. Chapter 1.6.8 in book.

A linear system with input u and output y is passive if the transfer function $Y(s)/U(s) = G(s)$ is positive real.

(a) Show that if $G_2(s)$ is positive real, then $H_2(s)$ is bounded real (hint: see book Chapter 2.4.11).

(b) Define $H_1(s)$ by

$$B_1(s) = H_1(s)A_1(s). \tag{8}$$

Show that if $H_2(s)$ is bounded real, then $H_1(s)$ is also bounded real. Use that

$$H_1(s) = \frac{B_1(s)}{A_1(s)} = e^{-2Ts}H_2(s).$$

(c) We have the load impedance $Z_1(s)$ such that

$$Z_1(s) = \frac{Y_1(s)}{U_1(s)}. \tag{9}$$

Further, we define the transfer function

$$G_1(s) = Z_1(s) z_0. \tag{10}$$

Show that

$$G_1(s) = \frac{1 - H_1(s)}{1 + H_1(s)} \tag{11}$$

and that if $H_1(s)$ is bounded real, then $G_1(s)$ is positive real. (Hint 1: Check how Eq. 6 is found. Hint 2: $2\,\mathrm{Re}\,[G_1(s)] = G_1(s) + G_1^*(s)$).

(d) Show that if $Z_2(s)$ is positive real, then the system with input u_1 and output y_1 is passive.