Norwegian University of Science and Technology
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Exercise 2
LP and KKT Conditions

Problem 1 (30 %) The Mean Value Theorem

a Based on the Example A.2 (page 629) in the textbook, show that there exists one or more $\alpha \in (0, 1)$, given $x = [0,\ 0]^\top$ and $p = [2,\ 1]^\top$.

b $f(x) = x^{\frac{1}{2}}$ is a continuous function. Explain why it is not Lipschitz continuous at $x = 0$. (See page 624 in the textbook for an explanation of Lipschitz continuity.)

Problem 2 (25 %) LP and KKT-conditions (Exam August 2000)

The following linear program is in standard form:

$$\min_x c^\top x \qquad \text{s.t.} \qquad Ax = b, \quad x \geq 0 \tag{1}$$

with $c \in \mathbb{R}^n$, $x \in \mathbb{R}^n$, and $b \in \mathbb{R}^m$. Derive the KKT conditions for (1).

Problem 3 (45 %) Linear Programming

In a plant three products R, S, and T are made in two process stages A and B. To make a product the following time in each process stage is required:

- 1 tonne of R: 3 hours in stage A plus 2 hours in stage B.
- 1 tonne of S: 2 hours in stage A and 2 hours in stage B.
- 1 tonne of T: 1 hour in stage A and 3 hours in stage B.

During one year, stage A has 7200 hours and stage B has 6000 hours available production time. The rest of the time is needed for maintenance. It is *required* that the available production time should be *fully utilized* in both stages.[1]

The profit from the sale of the products is:

- R: 100 NOK per tonne.
- S: 75 NOK per tonne.
- T: 55 NOK per tonne.

We wish to maximize the yearly profit.

a Formulate this as an LP problem.

[1] This requirement is important when formulating the LP in part a).

b Which basic feasible points exist?

c Find the solution by checking the KKT conditions at all the feasible points found in **b)**.

d Formulate the dual problem for the LP in **a)**.

e Show that the optimal objective function value for the LP in **a)** equals the optimal objective function value for the dual problem in **d)** by showing that $c^\top x^* = b^\top \lambda^*$.

f If you can make either stage A or stage B more available (i.e., more production hours available because of more efficient maintenance), which of the production stages A or B would you choose to improve? Why? Check your answer by first increasing the capacity of A by 1 hour (i.e., to 7201 hours), and then by increasing B by 1 hour.