STAT 502 - Homework 2

Due date: Thursday, October 21, 11:59PM. Submit your homework solutions to the course Canvas page. Total points: 20. **Late homework will not be accepted.**

1. **(5 Points)** The class notes mention that the two-sample t-test based on normal distribution theory and the randomization test that uses the same test statistic often yield similar p-values. Design and implement a simulation study to explore the truth of this statement.

   Set the seed as 123. Use `set.seed(123)`. First, simulate $n_A = 30$ samples from a normal distrtibution with mean 0 and variance 1 (a standard normal distribution) and, then simulate $n_B = 10$ samples from a normal distrtibution with mean 0 and variance 4. Your homework will be graded based on your output, so make sure that your sample can be reproduced!

   (a) **(1pt)** What is the observed two sample t-statistic?

   (b) **(1pt)** The null hypothesis we will test is $\mu_A = \mu_B$. Use Monte Carlo sampling (with replacement) to simulate 10,000 samples from the randomization null distribution. We will test the null hypothesis using a 0.05-level randomization test. Calculate the p-value of the randomization test based on your sample from the randomization null distribution and the observed t-statistic. What is the p-value of your test? Do you reject the null hypothesis?

   (c) **(3pt)** We now use the two-sided t-test to test the same null hypothesis at the 0.05-level.

      i. **(1pt)** What is the distribution of our test statistic under the null (specify df)?

      ii. **(1pt)**What is the critical value of this t-test?

      iii. **(1pt)** What is the p-value of this t-test? Do you reject the null hypothesis?

2. **(5 Points)** The simulated samples in Problem 1 came from normal distribution with unequal variances and unequal sample sizes. Use Welch's t-test on the data set from Problem 1 to test the null hypothesis of $\mu_A = \mu_B$ at a 0.05 level.

   (a) **(1pt)** What is the observed test statistic?

   (b) **(1pt)** What is the approximate distribution of Welch's test statistic under the null (specify df)?

   (c) **(1pt)** What is the critical value of this test (based on the approximate distribution of the test statistic under the null)?

   (d) **(1pt)** What is the p-value of this t-test? Do you reject the null hypothesis?

   (e) **(1pt)** Give a brief explanation of the differences you observe between the results when conducting a t-test in Problem 1 vs. Welch's modified t-test.

3. **(4 Points)** (Checking Normality using the `qqnorm() and qqline()` functions in R).

   (a) **(2pt)** Simulate 1000 observations of a random varible with:

      i. **(0.5pt)** A long tailed distribution.

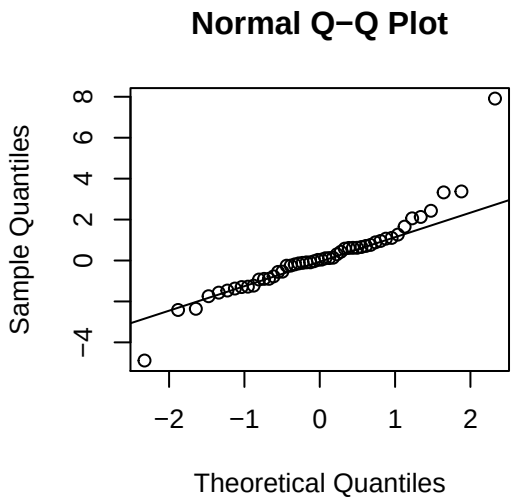
      ii. **(0.5pt)** A short tailed distribution.

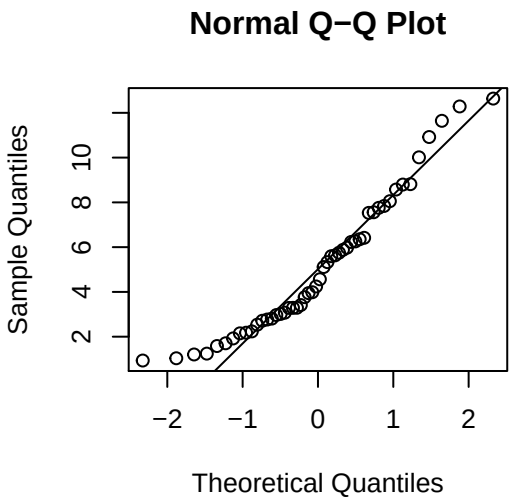
      iii. **(0.5pt)** A left skewed distribution.

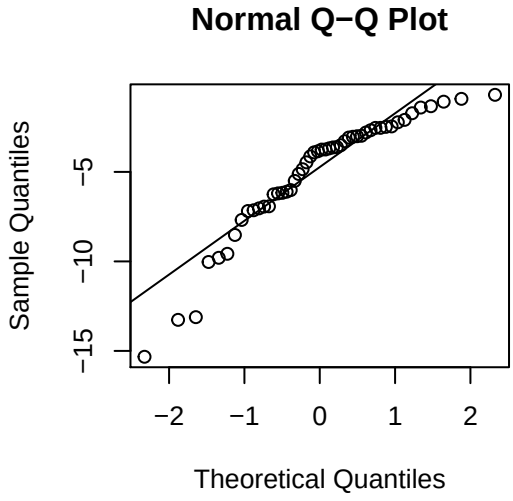
      iv. **(0.5pt)** A right skewed distribution.

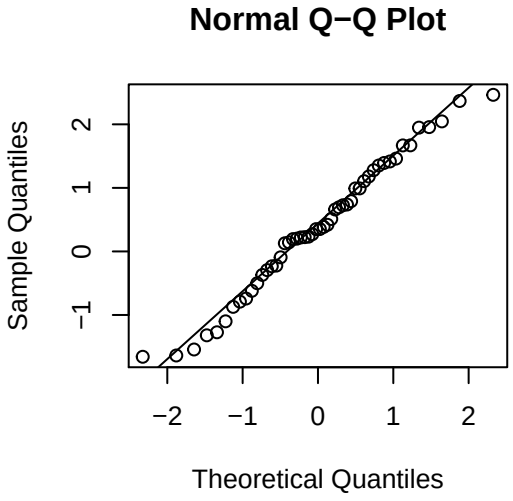
Indicate which distribution you sampled from and provide a plot of a normality check based on your sample.

(b) **(2pt, as above.)** Check the normality of the samples x_1, x_2, x_3, x_4 using the following 4 plots. What violations of normality (if any) do you notice in each plot?









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4. **(4 Points)** Wiebe and Bortolotti (2002) examined color in the tail feathers of northern flickers. Some of the birds had one "odd" feather that was different in color or length from the rest of the tail feathers, presumably because it was regrown after being lost. They measured the yellowness of one odd feather on each of 16 birds and compared it with the yellowness of one typical feather from the same bird. There are two nominal variables, type of feather (typical or odd) and the individual bird, and one measurement variable, yellowness.

Bird	Typical feather	Odd Feather
A	-0.255	-0.324
B	-0.213	-0.185
C	-0.19	-0.299
D	-0.185	-0.144
E	-0.045	-0.027
F	-0.025	-0.039
G	-0.015	-0.264
H	0.003	-0.077
I	0.015	-0.017
J	0.02	-0.169
K	0.023	-0.096
L	0.04	-0.33
M	0.04	-0.346
N	0.05	-0.191
O	0.055	-0.128
P	0.058	-0.182

We can conduct a paired t-test, to asses the null hypothesis of "There is no difference in the yellowness of the feathers" (or "$\mu = 0$") for this sample of northern flickers.

Suppose that the true difference in yellowness of a typical feather and an odd feather is 0.05, that the sample variance is a good estimate for the population variance.

(a) **(1pt)** What is the test statistic for this paired t-test and what distribution does it follow under the null?

(b) **(1pt)** What is the value of the non-centrality parameter for the t-distribution under H_1: $\mu = 0.05$?

(c) **(1pt)** What is the power of the two sided t-test performed at a 0.05-level under H_1: $\mu = 0.05$?

(d) **(1pt)** What sample size is required so that the the two sided t-test performed at a 0.05-level would have 90% power under H_1: $\mu = 0.05$?

5. **(2pt)** As part of a larger experiment, Dale (1992) looked at six samples of Exercise 2.4 a wetland soil undergoing a simulated snowmelt. Three were randomly selected for treatment with a neutral pH snowmelt; the other three got a reduced pH snowmelt. The observed response was the number of Copepoda removed from each microcosm during the first 14 days of snowmelt.

In the reduced pH group, there are three observed values: 256, 159, and 149. In the neutral pH group, there are three observed values: 54, 123, 248.

Using randomization methods, test the null hypothesis that the two treatments have equal average numbers of Copepoda versus a two-sided alternative.