CHAPTER 17

PROBLEM 17.1

It is known that 1500 revolutions are required for the 3000 kg flywheel to coast to rest from an angular velocity of 300 rpm. Knowing that the radius of gyration of the flywheel is 1 m determine the average magnitude of the couple due to kinetic friction in the bearings.

SOLUTION

Angular velocity:

$$\omega_0 = 300 \text{ rpm}\left(\frac{2\pi}{60}\right)$$
$$= 10\pi \text{ rad}$$
$$\omega_2 = 0$$

Moment of inertia:

$$\bar{I} = m\bar{k}^2 = (3000 \text{ kg})\,(1 \text{ m})^2$$
$$= 3000 \text{ kg}\cdot\text{m}^2$$

Kinetic energy:

$$T_1 = \frac{1}{2}\bar{I}\omega_0^2$$
$$= \frac{1}{2}(3000)(10\pi)^2$$
$$= 1.48044\times 10^6 \text{ N}\cdot\text{m}$$
$$T_2 = 0$$

Work:

$$U_{1\to 2} = -M\theta$$
$$= -M(1500 \text{ rev})(2\pi \text{ rad/rev})$$
$$= -9424.8M$$

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$
$$1.48044\times 10^6 - 9424.8M = 0$$

Average friction couple: $M = 157.08 \text{ N}\cdot\text{m}$ $\qquad M = 157.1 \text{ N}\cdot\text{m}$ ◀

PROBLEM 17.2

The rotor of an electric motor has an angular velocity of 3600 rpm when the load and power are cut off. The 50-kg rotor, which has a centroidal radius of gyration of 180 mm, then coasts to rest. Knowing that the kinetic friction of the rotor produces a couple of magnitude 3.5 N · m, determine the number of revolutions that the rotor executes before coming to rest.

SOLUTION

Angular velocity:

$$\omega_0 = 3600 \text{ rpm}\left(\frac{2\pi}{60}\right) = 120\pi \text{ rad/s}$$

$$\omega_2 = 0$$

Moment of inertia:

$$\bar{I} = m\bar{k}^2 = (50 \text{ kg})(0.180 \text{ m})^2 = 1.620 \text{ kg} \cdot \text{m}^2$$

Kinetic energy:

$$T_1 = \frac{1}{2}\bar{I}\omega_0^2 = \frac{1}{2}(1.620)(120\pi)^2 = 115.12 \text{ kJ},$$

$$T_2 = 0$$

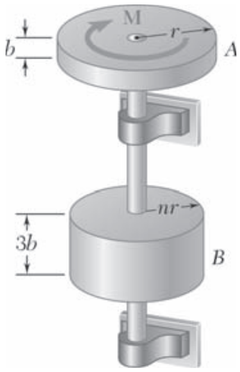
Work:

$$U_{1\to 2} = -M\theta = -(3.5 \text{ N} \cdot \text{m})\theta$$

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2: \quad 115.12 \text{ kJ} - (3.5 \text{ N} \cdot \text{m})\theta = 0$$

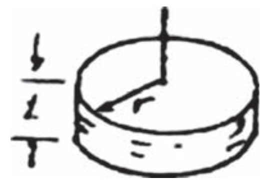
Rotation angle: $\theta = 32.891 \times 10^3 \text{ rad}$ $\qquad \theta = 5230 \text{ rev}$ ◄



PROBLEM 17.3

Two disks of the same material are attached to a shaft as shown. Disk A is of radius r and has a thickness b, while disk B is of radius nr and thickness $3b$. A couple **M** of constant magnitude is applied when the system is at rest and is removed after the system has executed 2 revolutions. Determine the value of n which results in the largest final speed for a point on the rim of disk B.

SOLUTION



For any disk: $$m = \rho(\pi r^2 t)$$

$$\bar{I} = \frac{1}{2}mr^2$$

$$= \frac{1}{2}\pi\rho t r^4$$

Moment of inertia.

Disk A: $$I_A = \frac{1}{2}\pi\,\rho\,b\,r^4$$

Disk B: $$I_B = \frac{1}{2}\pi\,\rho(3b)(nr)^4$$

$$= 3n^4\left[\frac{1}{2}\pi\rho b r^4\right]$$

$$= 3n^4 I_A$$

$$I_{\text{total}} = I_A + I_B$$

$$= (1+3n^4)I_A$$

Work-energy. $$T_1 = 0 \quad U_{1\to 2} = M\theta = M(4\pi\text{ rad})$$

$$T_2 = \frac{1}{2}I_{\text{total}}\,\omega_2^2$$

$$T_1 + U_{1\to 2} = T_2: \quad 0 + M(4\pi) = \frac{1}{2}(1+3n^4)I_A\omega_2^2$$

$$\omega_2^2 = \frac{8\pi M}{(1+3n^4)I_A}$$

For Point D on rim of disk B

$$v_D = (nr)\omega_2 \quad \text{or} \quad v_D^2 = n^2r^2\omega_2^2 = \frac{8\pi Mr^2}{I_A}\cdot\frac{n^2}{1+3n^4}$$

PROBLEM 17.3 (Continued)

Value of n for maximum final speed.

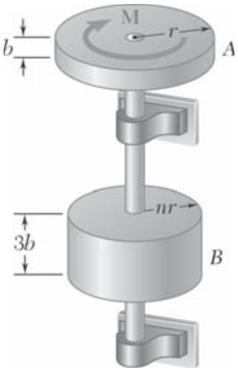
For maximum v_D: $\dfrac{d}{dn}\left(\dfrac{n^2}{1+3n^4}\right)=0$

$$\frac{1}{(1+3n^4)^2}[n^2(12n^3)-(1+3n^4)(2n)]=0$$

$$12n^5-2n-6n^5=0$$

$$2n(3n^4-1)=0$$

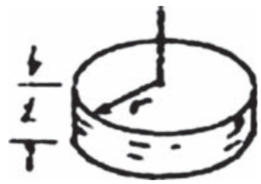
$n=0$ and $n=\left(\dfrac{1}{3}\right)^{0.25}=0.7598$ $\qquad n=0.760$ ◄



PROBLEM 17.4

Two disks of the same material are attached to a shaft as shown. Disk A has a mass of 15 kg and a radius $r = 125$ mm. Disk B is three times as thick as disk A. Knowing that a couple $\mathbf{M}$ of magnitude 20 N · m is to be applied to disk A when the system is at rest, determine the radius nr of disk B if the angular velocity of the system is to be 600 rpm after 4 revolutions.

SOLUTION



For any disk:

$$m = \rho(\pi r^2 t)$$

$$\bar{I} = \frac{1}{2}mr^2$$

$$= \frac{1}{2}\pi\rho t r^4$$

Moment of inertia.

Disk A:

$$I_A = \frac{1}{2}\pi\rho b r^4$$

Disk B:

$$I_B = \frac{1}{2}\pi\rho(3b)(nr)^4$$

$$= 3n^4\left[\frac{1}{2}\pi\rho b r^4\right]$$

$$= 3n^4 I_A$$

$$I_{\text{total}} = I_A + I_B = (1 + 3n^4)I_A \qquad (1)$$

Angular velocity:

$$\omega_1 = 0$$

$$\omega_2 = 600 \text{ rpm}$$

$$= 20\pi \text{ rad/s}$$

Rotation:

$$\theta = 4 \text{ rev} = 8\pi \text{ rad}$$

Kinetic energy:

$$T_1 = 0$$

$$T_2 = \frac{1}{2}I_{\text{total}}\,\omega_2^2$$

Work:

$$U_{1\to2} = M\theta$$

$$= (20 \text{ N}\cdot\text{m})(8\pi \text{ rad})$$

$$= 502.65 \text{ J}$$

PROBLEM 17.4 (Continued)

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + 502.65 = \frac{1}{2} I_{\text{total}} (20\pi)^2$$

$$I_{\text{total}} = 0.25465 \text{ kg} \cdot \text{m}^2$$

But,

$$I_A = \frac{1}{2} m_A r_A^2$$

$$= \frac{1}{2}(15 \text{ kg})(0.125 \text{ m})^2$$

$$= 0.117188 \text{ kg} \cdot \text{m}^2$$

From (1)

$$0.25465 = (1 + 3n^4)(0.117188)$$

$$n^4 = 0.390998$$

$$n = 0.79076$$

Radius of disk B: $r_B = nr_A = (0.79076)(125 \text{ mm})$ $\qquad r_B = 98.8 \text{ mm}$ ◀

PROBLEM 17.5

The flywheel of a punching machine has a mass of 300 kg and a radius of gyration of 600 mm. Each punching operation requires 2500 J of work. (*a*) Knowing that the speed of the flywheel is 300 rpm just before a punching, determine the speed immediately after the punching. (*b*) If a constant 25-N·m couple is applied to the shaft of the flywheel, determine the number of revolutions executed before the speed is again 300 rpm.

SOLUTION

Moment of inertia.

$$I = mk^2 = (300\text{ kg})(0.6\text{ m})^2 = 108\text{ kg}\cdot\text{m}^2$$

Kinetic energy. *Position 1*.

$$\omega_1 = 300\text{ rpm} = 10\pi\text{ rad/s}$$

$$T_1 = \frac{1}{2}I\omega_1^2 = \frac{1}{2}(108)(10\pi)^2 = 53.296\times10^3\text{ J}$$

Position 2.

$$T_2 = \frac{1}{2}I\omega_2^2 = 54\omega_2^2$$

Work.

$$U_{1\to2} = -2500\text{ J}$$

Principle of work and energy for punching.

$$T_1 + U_{1\to2} = T_2:\quad 53.296\times10^3 - 2500 = 54\omega_2^2$$

(*a*)

$$\omega_2^2 = 940.66$$

$$\omega_2 = 30.67\text{ rad/s}\qquad\qquad \omega_2 = 293\text{ rpm} \blacktriangleleft$$

Principle of work and energy for speed recovery.

$$T_2 + U_{2\to1} = T_1$$

$$U_{2\to1} = 2500\text{ J}$$

$$M = 25\text{ N}\cdot\text{m}$$

$$U_{2\to1} = M\theta\quad 2500 = 25\theta\quad \theta = 100\text{ rad}$$

(*b*)

$$\theta = 15.92\text{ rev} \blacktriangleleft$$

PROBLEM 17.6

The flywheel of a small punching machine rotates at 360 rpm. Each punching operation requires 2250 N·m of work and it is desired that the speed of the flywheel after each punching be not less than 95 percent of the original speed. (*a*) Determine the required moment of inertia of the flywheel. (*b*) If a constant 27 N·m couple is applied to the shaft of the flywheel, determine the number of revolutions that must occur between two successive punchings, knowing that the initial velocity is to be 360 rpm at the start of each punching.

SOLUTION

Angular speeds. $\omega_1 = 360 \text{ rpm} = 12\pi \text{ rad/s}$

$$\omega_2 = 0.95\,\omega_1$$

$$\omega_2 = 11.4\pi \text{ rad/s}$$

Work. $U_{1\to2} = 2250 \text{ N}\cdot\text{m}$

Principle of work and energy for punching

$$T_1 + U_{1\to2} = T_2$$

$$\frac{1}{2}I\omega_1^2 + U_{1\to2} = \frac{1}{2}I\omega_2^2$$

(*a*) Solving for I $\quad I = \dfrac{-2U_{1\to2}}{\omega_2^2 - \omega_1^2}$

$$= \frac{-(2)(-2250)}{(12\pi)^2 - (11.4\pi)^2} = 32.475 \text{ kg}\cdot\text{m}^2$$

$I = 32.5 \text{ kg}\cdot\text{m}^2$ ◀

Principle of work and energy for speed recovery.

$$T_2 + U_{2\to3} = T_3$$

But, $\quad T_3 = T_1$

$$U_{2\to3} = T_3 - T_2$$

$$= T_1 - T_2$$

$$= -U_{1\to2}$$

$$= 2250 \text{ N}\cdot\text{m}$$

Work, $\quad U_{2\to3} = M\theta$

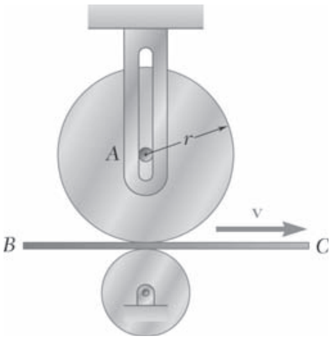
(*b*) $\quad \theta = \dfrac{U_{2\to3}}{M}$

$$= \frac{2250}{27}$$

$$= 83.33 \text{ rad}$$

$\theta = 13.26 \text{ rev}$ ◀

PROBLEM 17.7

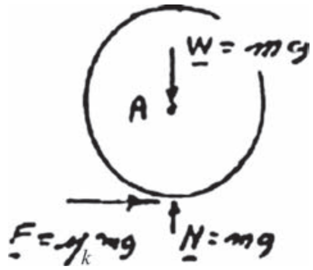


Disk A is of constant thickness and is at rest when it is placed in contact with belt BC, which moves with a constant velocity $\mathbf{v}$. Denoting by μ_k the coefficient of kinetic friction between the disk and the belt, derive an expression for the number of revolutions executed by the disk before it attains a constant angular velocity.

SOLUTION

Work of external friction force on disk A.

Only force doing work is F. Since its moment about A is $M = rF$, we have



$$\begin{aligned} U_{1\to 2} &= M\theta \\ &= rF\theta \\ &= r(\mu_k mg)\theta \end{aligned}$$

Kinetic energy of disk A.

Angular velocity becomes constant when $\omega_2 = \dfrac{v}{r}$

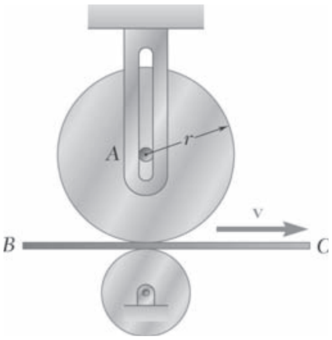
$$\begin{aligned} T_1 &= 0 \\ T_2 &= \frac{1}{2}\bar{I}\omega_2^2 \\ &= \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 \\ &= \frac{mv^2}{4} \end{aligned}$$

Principle of work and energy for disk A.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + r\mu_k mg\theta = \frac{mv^2}{4}$$

Angle change. $\theta = \dfrac{v^2}{4r\mu_k g}$ rad $\qquad \theta = \dfrac{v^2}{8\pi r\mu_k g}$ rev ◄

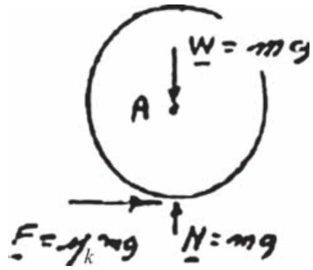
PROBLEM 17.8



Disk A, of weight 5 kg and radius $r = 150$ mm, is at rest when it is placed in contact with belt BC, which moves to the right with a constant speed $v = 12$ m/s. Knowing that $\mu_k = 0.20$ between the disk and the belt, determine the number of revolutions executed by the disk before it attains a constant angular velocity.

SOLUTION

Work of external friction force on disk A.



Only force doing work is F. Since its moment about A is $M = rF$, we have

$$\begin{aligned} U_{1\to 2} &= M\theta \\ &= rF\theta \\ &= r(\mu_k mg)\theta \end{aligned}$$

Kinetic energy of disk A.

Angular velocity becomes constant when $\omega_2 = \dfrac{v}{r}$

$$T_1 = 0$$

$$\begin{aligned} T_2 &= \frac{1}{2}\bar{I}\omega_2^2 \\ &= \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 \\ &= \frac{mv^2}{4} \end{aligned}$$

Principle of work and energy for disk A.

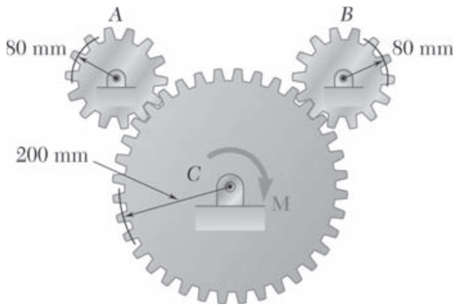
$$T_1 + U_{1-2} = T_2: \quad 0 + r\mu_k mg\theta = \frac{mv^2}{4}$$

Angle change $\quad \theta = \dfrac{v^2}{4r\mu_k g}$ rad $\qquad \theta = \dfrac{v^2}{8\pi r\mu_k g}$ rev

Data:

$$r = 0.15 \text{ m}$$
$$\mu_k = 0.20$$
$$v = 12 \text{ m/s}$$

$$\theta = \frac{(12 \text{ m/s})^2}{8\pi(0.15 \text{ m})(0.20)(9.81 \text{ m/s}^2)} \qquad \theta = 19.47 \text{ rev} \blacktriangleleft$$



PROBLEM 17.9

Each of the gears A and B has a mass of 2.4 kg and a radius of gyration of 60 mm, while gear C has a mass of 12 kg and a radius of gyration of 150 mm. A couple **M** of constant magnitude 10 N·m is applied to gear C. Determine (*a*) the number of revolutions of gear C required for its angular velocity to increase from 100 to 450 rpm, (*b*) the corresponding tangential force acting on gear A.

SOLUTION

Moments of inertia.

Gears A and B: $$I_A = I_B = mk^2 = (2.4)(0.06)^2 = 8.64\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Gear C: $$I_C = (12)(0.15)^2 = 270\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Kinematics. $$r_A\omega_A = r_B\omega_B = r_C\omega_C$$

$$\omega_A = \omega_B = \frac{200}{80}\omega_C = 2.5\omega_C$$

$$\theta_A = \theta_B = 2.5\theta_C$$

Kinetic energy. $$T = \frac{1}{2}I\omega^2:$$

Position 1. $$\omega_C = 100\ \text{rpm} = \frac{10}{3}\pi\ \text{rad/s}$$

$$\omega_A = \omega_B = 250\ \text{rpm} = \frac{25}{3}\pi\ \text{rad/s}$$

Gear A: $$(T_1)_A = \frac{1}{2}(8.64\times 10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\ \text{J}$$

Gear B: $$(T_1)_B = \frac{1}{2}(8.64\times 10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\ \text{J}$$

Gear C: $$(T_1)_C = \frac{1}{2}(270\times 10^{-3})\left(\frac{10\pi}{3}\right)^2 = 14.8044\ \text{J}$$

System: $$T_1 = (T_1)_A + (T_1)_B + (T_1)_C = 20.726\ \text{J}$$

Position 2. $$\omega_C = 450\ \text{rpm} = 15\pi\ \text{rad/s}$$

$$\omega_A = \omega_B = 37.5\pi\ \text{rad/s}$$

Gear A: $$(T_2)_A = \frac{1}{2}(8.64\times 10^{-3})(37.5\pi)^2 = 59.957\ \text{J}$$

PROBLEM 17.9 (Continued)

Gear B: $$(T_2)_B = \frac{1}{2}(8.64\times10^{-3})(37.5\pi)^2 = 59.957 \text{ J}$$

Gear C: $$(T_2)_C = \frac{1}{2}(270\times10^{-3})(15\pi)^2 = 299.789 \text{ J}$$

System: $$T_2 = (T_2)_A + (T_2)_B + (T_2)_C = 419.7 \text{ J}$$

Work of couple. $$U_{1\to2} = M\theta_C = 10\theta_C$$

Principle of work and energy for system.

$$T_1 + U_{1\to2} = T_2: \quad 20.726 + 10\theta_C = 419.7$$

$$\theta_C = 39.898 \text{ radians}$$

(a) Rotation of gear C. $\theta_C = 6.35 \text{ rev}$ ◀

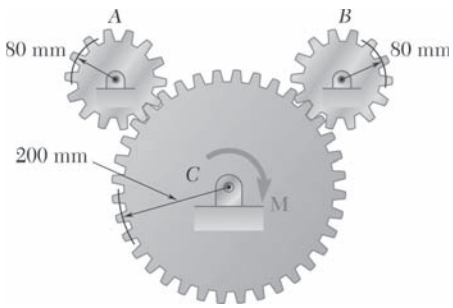
Rotation of gear A. $$\theta_A = (2.5)(39.898)$$
$$= 99.744 \text{ radians}$$

Principle of work and energy for gear A.

$$(T_1)_A + M_A\theta_A = (T_2)_A: \quad 2.9609 + M_A(99.744) = 59.957$$

$$M_A = 0.57142 \text{ N}\cdot\text{m}$$

(b) Tangential force on gear A. $$F_t = \frac{M_A}{r_A} = \frac{0.57142}{0.08}$$ $F_t = 7.14 \text{ N}$ ◀



PROBLEM 17.10

Solve Problem 17.9, assuming that the 10-N·m couple is applied to gear B.

PROBLEM 17.9 Each of the gears A and B has a mass of 2.4 kg and a radius of gyration of 60 mm, while gear C has a mass of 12 kg and a radius of gyration of 150 mm. A couple **M** of constant magnitude 10 N·m is applied to gear C. Determine (*a*) the number of revolutions of gear C required for its angular velocity to increase from 100 to 450 rpm, (*b*) the corresponding tangential force acting on gear A.

SOLUTION

Moments of inertia.

Gears A and B: $\quad I_A = I_B = mk^2 = (2.4)(0.06)^2 = 8.64\times 10^{-3}\,\text{kg}\cdot\text{m}^2$

Gear C: $\quad I_C = (12)(0.15)^2 = 270\times 10^{-3}\,\text{kg}\cdot\text{m}^2$

Kinematics. $\quad r_A\omega_A = r_B\omega_B = r_C\omega_C$

$$\omega_A = \omega_B = \frac{200}{80}\omega_C = 2.5\omega_C$$

$$\theta_A = \theta_B = 2.5\theta_C$$

Kinetic energy. $\quad T = \frac{1}{2}I\omega^2$:

Position 1. $\quad \omega_C = 100\text{ rpm} = \frac{10}{3}\pi\text{ rad/s}$

$$\omega_A = \omega_B = 250\text{ rpm} = \frac{25}{3}\pi\text{ rad/s}$$

Gear A: $\quad (T_1)_A = \frac{1}{2}(8.64\times 10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\text{ J}$

Gear B: $\quad (T_1)_B = \frac{1}{2}(8.64\times 10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\text{ J}$

Gear C: $\quad (T_1)_C = \frac{1}{2}(270\times 10^{-3})\left(\frac{10\pi}{3}\right)^2 = 14.8044\text{ J}$

System: $\quad T_1 = (T_1)_A + (T_1)_B + (T_1)_C = 20.726\text{ J}$

PROBLEM 17.10 (Continued)

Position 2. $\omega_C = 450 \text{ rpm} = 15\pi \text{ rad/s}$

$\omega_A = \omega_B = 37.5\pi \text{ rad/s}$

Gear *A*: $(T_2)_A = \frac{1}{2}(8.64\times 10^{-3})(37.5\pi)^2 = 59.957 \text{ J}$

Gear *B*: $(T_2)_B = \frac{1}{2}(8.64\times 10^{-3})(37.5\pi)^2 = 59.957 \text{ J}$

Gear *C*: $(T_2)_C = \frac{1}{2}(270\times 10^{-3})(15\pi)^2 = 299.789 \text{ J}$

System: $T_2 = (T_2)_A + (T_2)_B + (T_2)_C = 419.7 \text{ J}$

Work of couple. $U_{1\to 2} = M\theta_B = 10\theta_B$

Principle of work and energy for system.

$$T_1 + U_{1\to 2} = T_2: \quad 20.726 + 10\theta_B = 419.7$$

$$\theta_B = 39.898 \text{ radians}$$

(*a*) Rotation of gear *C*. $\theta_C = \frac{39.898}{2.5} = 15.959 \text{ radians}$ $\theta_C = 2.54 \text{ rev}$ ◄

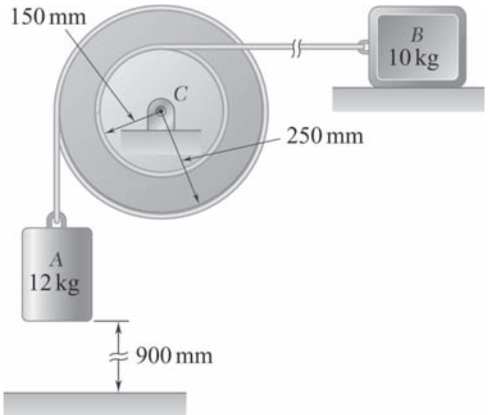
Rotation of gear *A*. $\theta_A = \theta_B = 39.898 \text{ radians}$

Principle of work and energy for gear *A*.

$$(T_1)_A + M_A\theta_A = (T_2)_A: \quad 2.9609 + M_A(39.898) = 59.957$$

$$M_A = 1.4285 \text{ N}\cdot\text{m}$$

(*b*) Tangential force on gear *A*. $F_t = \frac{M_A}{r_A} = \frac{1.4285}{0.08}$ $F_t = 17.86 \text{ N}$ ◄



PROBLEM 17.11

The double pulley shown weighs 15 kg and has a centroidal radius of gyration of 160 mm. Cylinder A and block B are attached to cords that are wrapped on the pulleys as shown. The coefficient of kinetic friction between block B and the surface is 0.25. Knowing that the system is released from rest in the position shown, determine (a) the velocity of cylinder A as it strikes the ground, (b) the total distance that block B moves before coming to rest.

SOLUTION

Let $v_A =$ speed of block $A, v_B =$ speed of block B, $\omega =$ angular speed of pulley.

Kinematics. $r_A = 0.25$ m, $r_B = 0.15$ m

$$v_A = r_A\omega = 0.25\,\omega$$
$$v_B = r_B\omega = 0.15\,\omega$$
$$s_A = r_A\theta = 0.25\,\theta$$
$$s_B = r_B\theta = 0.15\,\theta$$

(a) Cylinder A falls to ground. $s_A = 0.9$ m

$$s_B = (0.15)\left(\frac{0.9}{0.25}\right) = 0.54 \text{ m}$$

Work of weight A: $U_{1\to2} = (m_A g)\, s_A = (12.5)(9.81)(0.9)$

Normal contact force acting on block B: $N = m_B g = (10)(9.81) = 98.1$ N

Friction force on block B: $F_f = \mu_k N = (0.25)(98.1) = 24.525$ N

Work of friction force: $U_{1\to2} = -F_f s_B = -(24.525)(0.54) = -13.2435$ N·m

Total work: $U_{1\to2} = 110.3625 - 13.2435 = 97.119$ N·m

Kinetic energy: $T_1 = 0;\qquad T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}m_B v_B^2$

$$T_2 = \frac{1}{2}m_A r_A^2\omega^2 + \frac{1}{2}m_C k^2\omega^2 + \frac{1}{2}m_B r_B^2\omega^2$$
$$= \frac{1}{2}\Big[(12.5)\,(0.25)^2 + (15)\,(0.16)^2 + (10)\,(0.15)^2\Big]\omega^2$$
$$= 0.695125\omega^2$$

PROBLEM 17.11 (Continued)

Principle of work and energy.

$$T_1 + U_{1\to 2} = T: \quad 0 + 97.119 = 0.695125\omega^2$$

$$\omega = 11.820 \text{ rad/s}^2$$

Velocity of cylinder A: $v_A = (0.25)(11.820)$ $\mathbf{v}_A = 2.96 \text{ m/s} \downarrow$ ◀

(*b*) Block B comes to rest.

For block B and pulley C. $T_3 = \frac{1}{2}I\omega^2 + \frac{1}{2}m_B v_B^2; \qquad T_4 = 0:$

$$T_3 = \frac{1}{2}m_C k^2\omega^2 + \frac{1}{2}m_B r_B^2\omega^2$$

$$= \frac{1}{2}\left[(15)(0.16)^2 + (10)(0.15)^2\right](11.820)^2$$

$$= 42.5424 \text{ N}\cdot\text{m}$$

Work of friction force: $U_{3\to 4} = -F_f s'_B = -24.525\, s'_B$

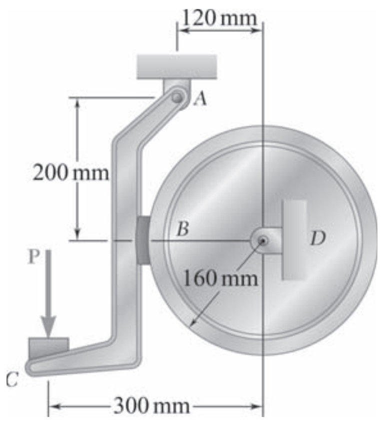
Principle of work and energy.

$$T_3 + U_{3\to 4} = T_4: \quad 42.5424 - 24.525 s'_B = 0$$

$$s'_B = 1.7347 \text{ m}$$

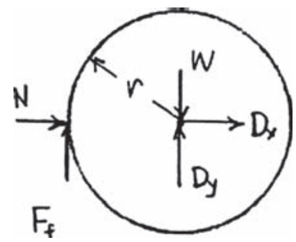
Total distance for block B. $d = s_B + s'_B: \quad d = 0.54 + 1.7347 = 2.2747 \text{ m}$ $d = 2.27 \text{ m}$ ◀

PROBLEM 17.12



The 160 mm-radius brake drum is attached to a larger flywheel that is not shown. The total mass moment of inertia of the flywheel and drum is 20 $\mathrm{kg \cdot m^2}$ and the coefficient of kinetic friction between the drum and the brake shoe is 0.35. Knowing that the initial angular velocity of the flywheel is 360 rpm counterclockwise, determine the vertical force **P** that must be applied to the pedal C if the system is to stop in 100 revolutions.

SOLUTION



Kinetic energy.

$$\omega_1 = 360 \text{ rpm} = 12\pi \text{ rad/s}$$

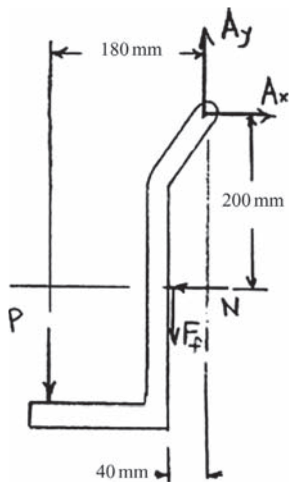
$$\omega_2 = 0$$

$$T_1 = \frac{1}{2} I \omega_1^2$$

$$= \frac{1}{2}(20)(12\pi)^2$$

$$= 14{,}212.23 \text{ N} \cdot \text{m}$$

$$T_2 = 0$$



Work.

$$\theta = (100)(2\pi) = 628.32 \text{ rad}$$

$$M_D = F_f r = F_f (0.16 \text{ m})$$

$$U_{1\to 2} = -M_D \theta = -F_f (0.16)(628.32)$$

$$= -100.531 F_f$$

Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 14{,}212.23 - 100.531 F_f = 0$$

$$F_f = 141.371 \text{ N}$$

Kinetic friction force.

$$F_f = \mu_k N$$

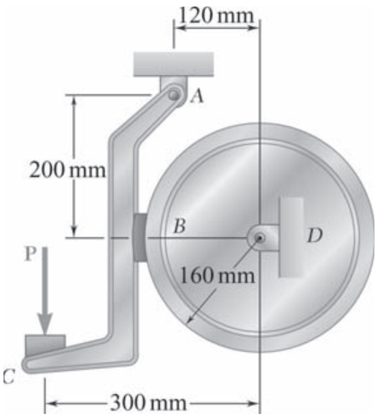
$$N = \frac{F_f}{\mu_k} = \frac{141.371}{0.35} = 403.918 \text{ N}$$

Statics. $\circlearrowleft + \Sigma M_A = 0: \quad (180 \text{ mm})P + (40 \text{ mm})F_f - (200 \text{ mm})N = 0$

$$180P + (40)(141.371) - (200)(403.918) = 0$$

$$P = 417.38 \text{ N}$$

$\mathbf{P} = 417 \text{ N} \downarrow$ ◀



PROBLEM 17.13

Solve Problem 17.12, assuming that the initial angular velocity of the flywheel is 360 rpm clockwise.

PROBLEM 17.12 The 160-mm-radius brake drum is attached to a larger flywheel that is not shown. The total mass moment of inertia of the flywheel and drum is 20 kg $\cdot$ m^2 and the coefficient of kinetic friction between the drum and the brake shoe is 0.35. Knowing that the initial angular velocity of the flywheel is 360 rpm counterclockwise, determine the vertical force **P** that must be applied to the pedal *C* if the system is to stop in 100 revolutions.

SOLUTION

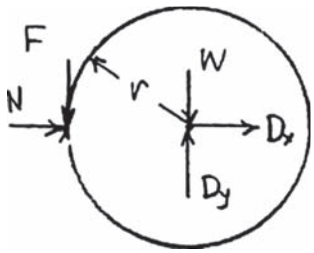
Kinetic energy.

$$\omega_1 = 360 \text{ rpm} = 12\pi \text{ rad/s}$$

$$\omega_2 = 0$$

$$T_1 = \frac{1}{2} I \omega_1^2 = \frac{1}{2}(20)(12\pi)^2 = 14{,}212.23 \text{ N} \cdot \text{m}$$

$$T_2 = 0$$

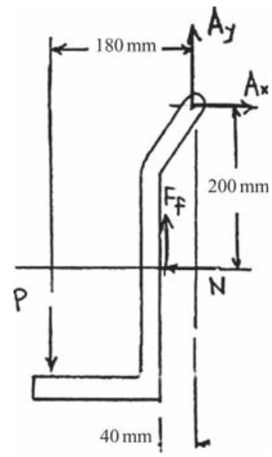


Work.

$$\theta = (100)(2\pi) = 628.32 \text{ rad}$$

$$M_D = F_f r = F_f(0.16)$$

$$U_{1\to2} = -M_D\theta = -F_f(0.16)(628.32) = -100.531 F_f$$



Principle of work and energy.

$$T_1 + U_{1\to2} = T_2: \quad 14{,}212.23 - 100.531 F_f = 0$$

$$F_f = 141.371 \text{ N}$$

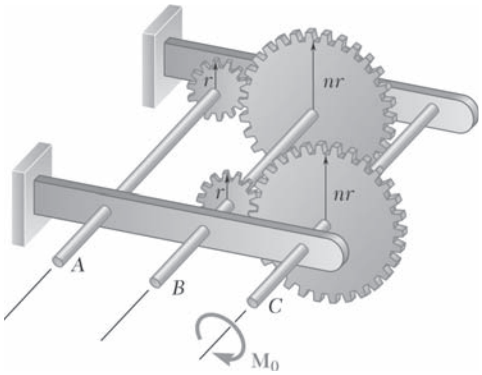
Kinetic friction. $\quad F_f = \mu_k N: \quad N = \dfrac{F_f}{\mu_k} = \dfrac{141.371}{0.35} = 403.918 \text{ N}$

Statics. $\quad +\Sigma M_A = 0: \quad (180 \text{ mm})P - (40 \text{ mm})F_f - (200 \text{ mm})N = 0$

$$180P - (40)(141.371) - (200)(403.918) = 0$$

$$P = 480.21 \text{ N} \qquad \mathbf{P} = 480 \text{ N} \downarrow \blacktriangleleft$$

PROBLEM 17.14



The gear train shown consists of four gears of the same thickness and of the same material; two gears are of radius r, and the other two are of radius nr. The system is at rest when the couple $\mathbf{M}_0$ is applied to shaft C. Denoting by I_0 the moment of inertia of a gear of radius r, determine the angular velocity of shaft A if the couple $\mathbf{M}_0$ is applied for one revolution of shaft C.

SOLUTION

Mass and moment of inertia:

For a disk of radius r and thickness t: $\quad m = \rho(\pi r^2)t = \rho\pi t r^2$

$$\bar{I}_0 = \frac{1}{2}mr^2 = \frac{1}{2}(\rho\pi + r^2)r^2 = \frac{1}{2}\rho\pi t r^4$$

For a disk of radius nr and thickness t, $\quad \bar{I} = \frac{1}{2}\rho\pi t(nr)^4 \quad \bar{I} = n^4\bar{I}_0$

Kinematics: If for shaft A we have $\omega_A \circlearrowright$

Then, for shaft B we have $\omega_B = \omega_A/n \circlearrowleft$

And, for shaft C we have $\omega_C = \omega_A/n^2 \circlearrowright$

Principle of work-energy:

Couple M_0 applied to shaft C for one revolution. $\theta = 2\pi$ radians, $T_1 = 0$,

$$U_{1-2} = M_0\theta = M_0(2\pi \text{ radians}) = 2\pi M_0$$

$$T_2 = \frac{1}{2}(\bar{I}_{\text{shaft } A})w_A^2 + \frac{1}{2}(\bar{I}_{\text{shaft } B})\omega_B^2 + \frac{1}{2}(\bar{I}_{\text{shaft } C})\omega_C^2$$

$$= \frac{1}{2}\bar{I}_0 w_A^2 + \frac{1}{2}(\bar{I}_0 + n^4\bar{I}_0)\left(\frac{\omega_A}{n}\right)^2 + \frac{1}{2}(n^4\bar{I}_0)\left(\frac{\omega_A}{n^2}\right)^2$$

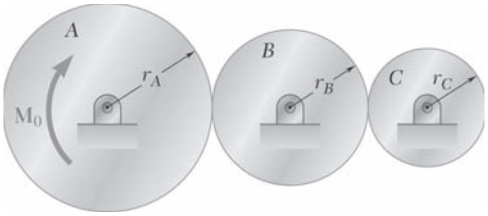
$$= \frac{1}{2}\bar{I}_0\omega_A^2\left(n^2 + 2 + \frac{1}{n^2}\right)$$

$$= \frac{1}{2}\bar{I}_0\omega_A^2\left(n + \frac{1}{n}\right)^2$$

$$T_1 + U_{1-2} = T_2: \quad 0 + 2\pi M_0 = \frac{1}{2}\bar{I}_0\omega_A^2\left(n + \frac{1}{n}\right)^2$$

Angular velocity. $\quad \omega_A^2 = \frac{4\pi M_0}{\bar{I}_0}\frac{1}{\left(n + \frac{1}{n}\right)^2} \qquad \omega_A = \frac{2n}{n^2+1}\sqrt{\frac{\pi M_0}{\bar{I}_0}}$ ◀

PROBLEM 17.15

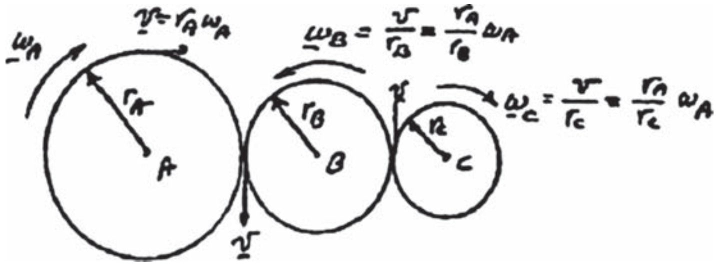


The three friction disks shown are made of the same material and have the same thickness. It is known that disk A weighs 6 kg and that the radii of the disks are $r_A = 200$ mm, $r_B = 150$ mm, and $r_C = 100$ mm. The system is at rest when a couple $\mathbf{M}_0$ of constant magnitude 7.5 N·m is applied to disk A. Assuming that no slipping occurs between disks, determine the number of revolutions required for disk A to reach an angular velocity of 150 rpm.

SOLUTION

Kinematics.

Denote velocity of perimeter by v.



Mass and moment of inertia of disks.

Denote mass density of material by ρ and thickness of disks by t.

Then mass of a disk is $\quad m = (\text{volume})\rho = (\pi r^2 t)\rho$

and $\quad \bar{I} = \frac{1}{2}mr^2 = \frac{\pi\rho t}{2}r^4$

Kinetic energy: $\quad T = \Sigma\frac{1}{2}\bar{I}\omega^2$

$$T = \frac{1}{2}\left(\frac{\pi\rho t}{2}\right)\left[r_A^4\omega_A^2 + r_B^4\omega_B^2 + r_C^4\omega_C^2\right]$$

$$= \frac{1}{2}\left(\frac{\pi\rho t}{2}\right)\left[r_A^4\omega_A^2 + r_B^4\left(\frac{r_A}{r_B}\right)^2\omega_A^2 + r_C^4\left(\frac{r_A}{r_C}\right)^2\omega_A^2\right]$$

$$T = \frac{1}{2}\left(\frac{\pi\rho t\omega_A^2}{2}\right)r_A^2\left[r_A^2 + r_B^2 + r_C^2\right] \qquad (1)$$

PROBLEM 17.15 (Continued)

Recall that $m_A = \pi r_A^2 t\rho$ and write

Eq. (1) as: $$T = \frac{1}{4}\left(\pi r_A^2 t\rho\right)\left(r_A^2 + r_B^2 + r_C^2\right)$$

$$T = \frac{1}{4}(m_A)r_A^2\left[1 + \left(\frac{r_B}{r_A}\right)^2 + \left(\frac{r_C}{r_A}\right)^2\right]\omega_A^2$$

Data: $$\omega_A = 150 \text{ rpm}\left(\frac{2\pi}{60}\right) = 5\pi \text{ rad/s}$$

$$m_A = 6 \text{ kg}, \quad r_A = 0.2 \text{ m}, \quad r_B = 0.15 \text{ m}, \quad r_C = 0.1 \text{ m}$$
$$M = 7.5 \text{ N}\cdot\text{m}$$

Work: $$U_{1-2} = M\theta = (7.5 \text{ N}\cdot\text{m})\theta$$

Principle of work and energy for system:

$$T_1 + U_{1\to 2} = T_2$$

$$0 + 7.5\theta = \frac{1}{4}(6)(0.2)^2\left[1 + \left(\frac{0.15}{0.2}\right)^2 + \left(\frac{0.1}{0.2}\right)^2\right](5\pi)^2$$

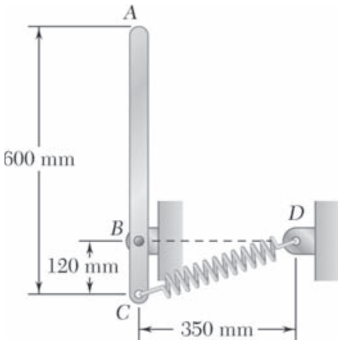
$$7.5\theta = 0.06\left[1 + \frac{9}{16} + \frac{1}{4}\right](25\pi)^2$$

$$7.5\theta = 26.833$$

$$\theta = 3.5777 \text{ rad}\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)$$

$$= 0.5694 \text{ rev}$$

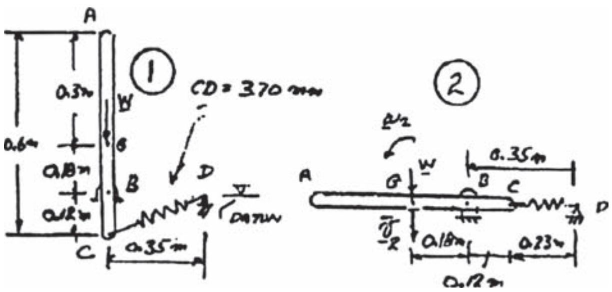
$\theta = 0.569 \text{ rev}$ ◄



PROBLEM 17.16

A slender 4-kg rod can rotate in a vertical plane about a pivot at B. A spring of constant $k = 400$ N/m and of unstretched length 150 mm is attached to the rod as shown. Knowing that the rod is released from rest in the position shown, determine its angular velocity after it has rotated through 90°.

SOLUTION



Position 1:

Spring: $$x_1 = CD - (\overbrace{150\text{ mm}}^{\text{Unstretched Length}}) = 370 - 150 = 220\text{ mm} = 0.22\text{ m}$$

$$V_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(400\text{ N/m})(0.22\text{ m})^2 = 9.68\text{ J}$$

Gravity: $$V_g = Wh = mgh = (4\text{ kg})(9.81\text{ m/s}^2)(0.18\text{ m}) = 7.063\text{ J}$$

$$V_1 = V_e + V_g = 9.68\text{ J} + 7.063\text{ J} = 16.743\text{ J}$$

Kinetic energy: $T_1 = 0$

Position 2:

Spring: $$x_2 = 230\text{ mm} - 150\text{ mm} = 80\text{ mm} = 0.08\text{ m}$$

$$V_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(400\text{ N/m})(0.08\text{ m})^2 = 1.28\text{ J}$$

Gravity: $V_g = Wh = 0$

$$V_2 = V_e + V_g$$
$$= 1.28\text{ J}$$

PROBLEM 17.16 (Continued)

Kinetic energy:

$$\bar{v}_2 = r\omega_2 = (0.18 \text{ m})\omega_2$$

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(4 \text{ kg})(0.6 \text{ m})^2 = 0.12 \text{ kg}\cdot\text{m}^2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\,\omega_2^2$$

$$= \frac{1}{2}(4 \text{ kg})(0.18\omega_2)^2 + \frac{1}{2}(0.12)\omega_2^2$$

$$T_2 = 0.1248\omega_2^2$$

Conservation of energy:

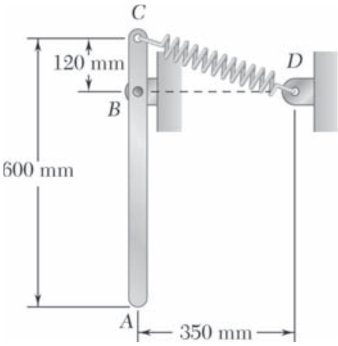
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 16.743 \text{ J} = 0.1248\omega_2^2 + 1.28 \text{ J}$$

$$\omega_2^2 = 123.9$$

$$\omega_2 = 11.131 \text{ rad/s}$$

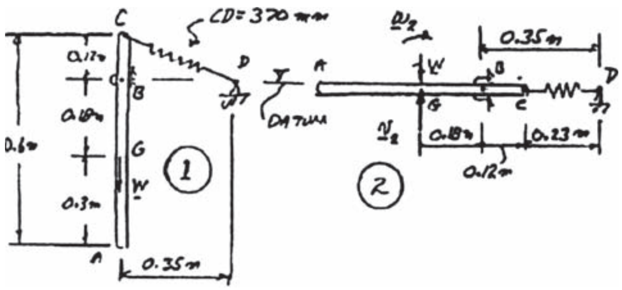
$\boldsymbol{\omega}_2 = 11.13 \text{ rad/s}$ ◀



PROBLEM 17.17

A slender 4-kg rod can rotate in a vertical plane about a pivot at B. A spring of constant $k = 400$ N/m and of unstretched length 150 mm is attached to the rod as shown. Knowing that the rod is released from rest in the position shown, determine its angular velocity after it has rotated through 90°.

SOLUTION



Position 1:

Spring: $$x_1 = CD - (\overbrace{150 \text{ mm}}^{\text{Unstretched Length}}) = 370 - 150 = 220 \text{ mm} = 0.22 \text{ m}$$

$$V_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(400 \text{ N/m})(0.22 \text{ m})^2 = 9.68 \text{ J}$$

Gravity: $$V_g = Wh = mgh = (4 \text{ kg})(9.81 \text{ m/s}^2)(-0.22 \text{ m}) = -7.063 \text{ J}$$

$$V_1 = V_e + V_g = 9.68 \text{ J} - 7.063 \text{ J} = 2.617 \text{ J}$$

Kinetic energy: $T_1 = 0$

Position 2:

Spring: $$x_2 = 230 \text{ mm} - 150 \text{ mm} = 80 \text{ mm} = 0.08 \text{ m}$$

$$V_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(400 \text{ N/m})(0.08 \text{ m})^2 = 1.28 \text{ J}$$

Gravity: $V_g = Wh = 0$

$$V_2 = V_e + V_g$$
$$= 1.28 \text{ J}$$

PROBLEM 17.17 (Continued)

Kinetic energy:

$$\bar{v}_2 = r\omega_2 = (0.18 \text{ m})\omega_2$$

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(4 \text{ kg})(0.6 \text{ m})^2 = 0.12 \text{ kg} \cdot \text{m}^2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}(4 \text{ kg})(0.18\omega_2)^2 + \frac{1}{2}(0.12)\omega_2^2$$

$$T_2 = 0.1248\omega_2^2$$

Conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2.617 \text{ J} = 0.1248\,\omega_2^2 + 1.28 \text{ J}$$

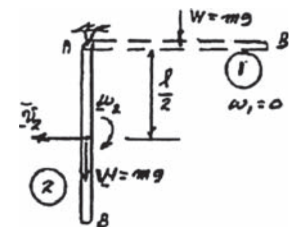
$$\omega_2^2 = 10.713$$

$$\omega_2 = 3.273 \text{ rad/s} \qquad \boldsymbol{\omega}_2 = 3.27 \text{ rad/s} \circlearrowright \blacktriangleleft$$

PROBLEM 17.18

A slender rod of length l and weight W is pivoted at one end as shown. It is released from rest in a horizontal position and swings freely. (*a*) Determine the angular velocity of the rod as it passes through a vertical position and determine the corresponding reaction at the pivot, (*b*) Solve Part *a* for $W = 10$ N and $l = 1$ m.

SOLUTION



Position 1:

$$v_1 = 0$$

$$\omega_1 = 0$$

$$T_1 = 0$$

$$\bar{v}_2 = \frac{l}{2}\omega_2$$

Position 2:

$$T_2 = \frac{1}{2}m\bar{r}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m\left(\frac{l}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega_2^2$$

$$T_2 = \frac{1}{6}ml^2\omega_2^2$$

Work:

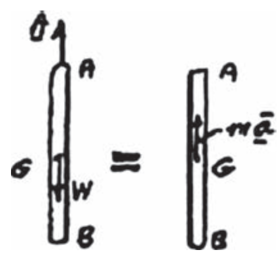
$$U_{1\to 2} = mg\frac{l}{2}$$

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$

$$0 + mg\frac{l}{2} = \frac{1}{6}ml^2\omega_2^2$$

(*a*) Expressions for angular velocity and reactions.



$$\omega_2^2 = \frac{3g}{l} \qquad \boldsymbol{\omega}_2 = \sqrt{\frac{3g}{l}} \circlearrowright \blacktriangleleft$$

$$\bar{a} = \frac{l}{2}\omega_2^2 = \frac{l}{2}\cdot\frac{3g}{l} = \frac{3}{2}g$$

$$+\uparrow \Sigma F = \Sigma(F)_{\text{eff}}: \quad A - W = m\bar{a}$$

$$A - mg = m\frac{3}{2}g$$

$$A = \frac{5}{2}mg \qquad \mathbf{A} = \frac{5}{2}W \uparrow \blacktriangleleft$$

PROBLEM 17.18 (Continued)

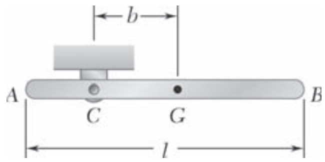
(*b*) Application of data:

$$W = 10\text{ N},\quad l = 1\text{ m}$$

$$\omega_2^2 = \frac{3g}{l} = \frac{3g}{1} = 29.43\text{ rad}^2/\text{s}^2 \qquad \boldsymbol{\omega}_2 = 5.42\text{ rad/s} \circlearrowright \blacktriangleleft$$

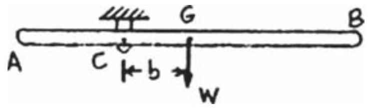
$$A = \frac{5}{2}W = \frac{5}{2}(10\text{ N}) = 25\text{ N} \qquad \mathbf{A} = 25.0\text{ N} \uparrow \blacktriangleleft$$

PROBLEM 17.19



A slender rod of length l is pivoted about a Point C located at a distance b from its center G. It is released from rest in a horizontal position and swings freely. Determine (a) the distance b for which the angular velocity of the rod as it passes through a vertical position is maximum, (b) the corresponding values of its angular velocity and of the reaction at C.

SOLUTION



Position 1. $\bar{v} = 0, \quad \omega = 0 \quad T_1 = 0$

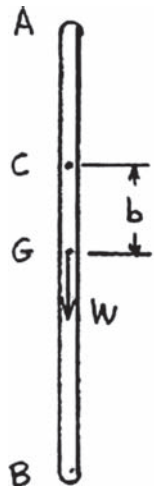
Elevation: $h = 0 \quad V_1 = mgh = 0$

Position 2. $\bar{v}_2 = b\omega_2$

$$I = \frac{1}{12}ml^2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m\left(b^2 + \frac{1}{12}l^2\right)\omega_2^2$$



Elevation: $h = -b \quad V_2 = -mgb$

Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{1}{2}m\left(b^2 + \frac{1}{12}l^2\right)\omega_2^2 - mgb$$

$$\omega_2^2 = \frac{2gb}{b^2 + \frac{1}{12}l^2}$$

(a) Value of b for maximum ω_2.

$$\frac{d}{db}\left(\frac{b}{b^2 + \frac{1}{12}l^2}\right) = \frac{\left(b^2 + \frac{1}{12}l^2\right) - b(2b)}{\left(b^2 + \frac{1}{12}l^2\right)^2} = 0 \qquad b^2 = \frac{1}{12}l^2 \qquad b = \frac{l}{\sqrt{12}} \blacktriangleleft$$

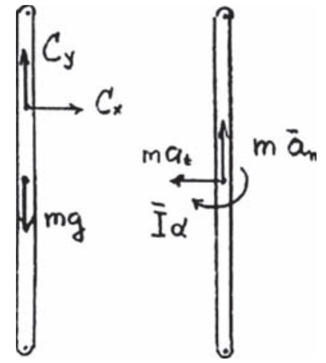
(b) Angular velocity.

$$\omega_2^2 = \frac{2g\frac{l}{\sqrt{12}}}{\frac{l^2}{12} + \frac{l^2}{12}}$$

$$= \sqrt{12}\frac{g}{l}$$

$$\omega_2 = 12^{1/4}\sqrt{\frac{g}{l}} \qquad \omega_2 = 1.861\sqrt{\frac{g}{l}} \blacktriangleleft$$

PROBLEM 17.19 (Continued)



Reaction at C.

$$a_n = b\omega_2^2$$

$$= \frac{l}{\sqrt{12}}\sqrt{12}\frac{g}{l}$$

$$= g$$

$$+\uparrow \Sigma F_y = ma_n: \quad C_y - mg = mg$$

$$C_y = 2mg$$

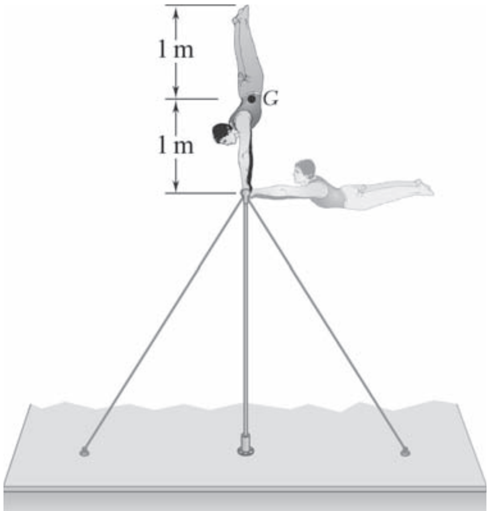
$$+\circlearrowleft \Sigma M_C = mba_t + \bar{I}\alpha: \quad 0 = (mb^2 + \bar{I})\alpha$$

$$\alpha = 0, \quad a_t = 0$$

$$\xrightarrow{+} \Sigma F_x = ma_t: \qquad C_x = -ma_t = 0$$

$\mathbf{C} = 2mg \uparrow$ ◀

PROBLEM 17.20



An 80-kg gymnast is executing a series of full-circle swings on the horizontal bar. In the position shown he has a small and negligible clockwise angular velocity and will maintain his body straight and rigid as he swings downward. Assuming that during the swing the centroidal radius of gyration of his body is 0.4 m determine his angular velocity and the force exerted on his hands after he has rotated through (*a*) 90°, (*b*) 180°.

SOLUTION

Position 1. (*Directly above the bar*).

Elevation: $h_1 = 1 \text{ m}$

Potential energy: $V_1 = mgh_1 = (80 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m}) = 784.8 \text{ N}\cdot\text{m}$

Speeds: $\omega_1 = 0, \;\; \bar{v}_1 = 0$

Kinetic energy: $T_1 = 0$

(*a*) *Position 2*. (*Body at level of bar after rotating* 90°).

Elevation: $h_2 = 0.$

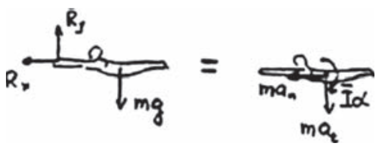
Potential energy: $V_2 = 0$

Speeds: $\bar{v}_2 = (1\text{m})\,\omega_2.$

Kinetic energy: $T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}mk^2\omega_2^2$

$$T_2 = \frac{1}{2}(80)\omega_2^2 + \frac{1}{2}(80)(0.4)^2\omega_2^2$$

$$= 46.4\omega_2^2$$



Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 784.8 = 46.4\omega_2^2$$

$$\omega_2^2 = 16.9138$$

$$\omega_2 = 4.11264 \text{ rad/s}$$

$\boldsymbol{\omega}_2 = 4.11 \text{ rad/s}$ ◄

PROBLEM 17.20 (Continued)

Kinematics: $\bar{a}_t = (1)(\alpha)$

$\bar{a}_n = (1)(\omega_2^2) = 16.9138 \text{ m/s}^2 \leftarrow$

$\circlearrowright^{+}\ \Sigma M_0 = \Sigma\left(M_0\right)_{\text{eff}}$: $(80)(9.81)(1) = (80)(1)(\alpha)(1) + (80)(0.4)^2(\alpha)$

$\alpha = 8.4569 \text{ rad/s}^2 \qquad \bar{a}_t = 8.4569 \text{ m/s}^2$

$\overset{+}{\longleftarrow}\ \Sigma F_x = ma_n$: $R_x = (80)(16.9138) = 1353.1 \text{ N}$

$+\!\uparrow \Sigma F_y = -ma_t$: $R_y - (80)(9.81) = -(80)(8.4569) \uparrow$

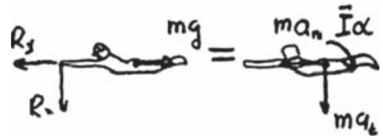
$R_y = 108.248 \text{ N} \uparrow$ $\qquad$ $\mathbf{R} = 1357 \text{ N} \measuredangle\ 4.57°$ ◀

(*b*) *Position 3*. (*Directly below bar after rotating* 180°).

Elevation: $h_3 = -1$ m.

Potential energy: $V_3 = Wh_3 = (80)(9.81)(-1) = -784.8 \text{ N}\cdot\text{m}$

Speeds: $\bar{v}_3 = (1)\omega_3$.



Kinetic energy: $T_3 = 46.4\omega_3^2$

Principle of conservation of energy.

$$T_1 + V_1 = T_3 + V_3: \quad 0 + 784.8 = 46.4\omega_3^2 - 784.8$$

$\omega_3^2 = 33.828 \text{ rad}^2/\text{s}^2$ $\qquad$ $\boldsymbol{\omega}_3 = 5.82 \text{ rad/s} \circlearrowright$ ◀

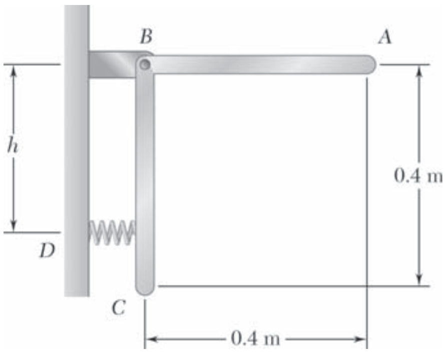
Kinematics: $a_n = (1)(33.828) = 33.828 \text{ m/s}^2 \uparrow$

From $\Sigma M_0 = \Sigma(M_0)_{\text{eff}}$ and $\Sigma F_x = 0$,

$\alpha = 0, \qquad a_t = 0 \qquad R_x = 0$

$+\!\uparrow \Sigma F_y = ma_n$: $R_y - (80)(9.81) = (80)(33.828)$

$R_y = 3491.04 \text{ N}$ $\qquad$ $\mathbf{R} = 3490 \text{ N} \uparrow$ ◀

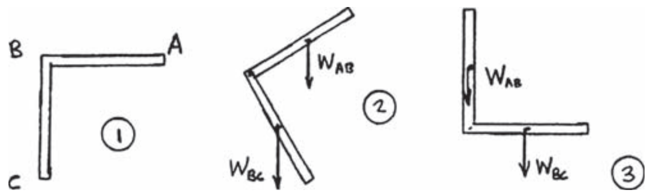


PROBLEM 17.21

Two identical slender rods AB and BC are welded together to form an L-shaped assembly. The assembly is pressed against a spring at D and released from the position shown. Knowing that the maximum angle of rotation of the assembly in its subsequent motion is 90° counterclockwise, determine the magnitude of the angular velocity of the assembly as it passes through the position where rod AB forms an angle of 30° with the horizontal.

SOLUTION

Moment of inertia about B. $\quad I_B = \frac{1}{3}m_{AB}l^2 + \frac{1}{3}m_{BC}l^2$



Position 2. $\quad \theta = 30°$

$$V_2 = W_{AB}(h_{AB})_2 + W_{BC}(h_{BC})_2$$

$$= W_{AB}\frac{l}{2}\sin 30° + W_{BC}\left(-\frac{l}{2}\cos 30°\right)$$

$$T_2 = \frac{1}{2}I_B\omega_2^2 = \frac{1}{6}(m_{AB} + m_{BC})l^2\omega_2^2$$

Position 3. $\quad \theta = 90°$

$$V_3 = W_{AB}\frac{l}{2} \quad T_3 = 0$$

Conservation of energy.

$$T_2 + V_2 = T_3 + V_3:$$

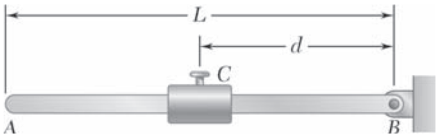
$$\frac{1}{6}(m_{AB} + m_{BC})l^2\omega_2^2 + W_{AB}\frac{l}{2}\sin 30° - W_{BC}\frac{l}{2}\cos 30° = 0 + W_{AB}\frac{l}{2}$$

$$\omega_2^2 = \frac{3}{l}\cdot\frac{W_{AB}(1-\sin 30°) + W_{BC}\cos 30°}{m_{AB} + m_{BC}}$$

$$= \frac{3}{2}\frac{g}{l}[1 - \sin 30° + \cos 30°]$$

$$= 2.049\frac{g}{l} = 2.049\frac{9.81}{0.4} = 50.25$$

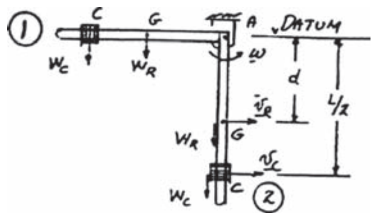
$\omega_2 = 7.09$ rad/s ◄



PROBLEM 17.22

A collar with a mass of 1 kg is rigidly attached at a distance $d = 300$ mm from the end of a uniform slender rod AB. The rod has a mass of 3 kg and is of length $L = 600$ mm. Knowing that the rod is released from rest in the position shown, determine the angular velocity of the rod after it has rotated through 90°.

SOLUTION



Kinematics.

Rod $\quad v_R = \dfrac{L}{2}\omega$

Collar $\quad v_C = d\omega$

Position 1. $\quad \omega = 0$

$$T_1 = 0 \quad V_1 = 0$$

Position 2.

$$T_2 = \frac{1}{2}m_R\bar{v}_R^2 + \frac{1}{2}\bar{I}_R\omega^2 + \frac{1}{2}m_C v_C^2$$

$$= \frac{1}{2}m_R\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_R L^2\right)\omega^2 + \frac{1}{2}m_C d^2\omega^2$$

$$= \frac{1}{6}m_R L^2\omega^2 + \frac{1}{2}m_C d^2\omega^2$$

$$V_2 = -W_C d - W_R\frac{L}{2}$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \left(\frac{1}{6}m_R L^2 + \frac{1}{2}m_C d^2\right)\omega^2 - W_C d - W_R\frac{L}{2}$$

$$\omega^2 = \frac{3(2W_C d + W_C L)}{3m_C d^2 + m_R L^2} = \frac{3g(2m_C d + m_R L)}{3m_C d^2 + m_R L^2} \qquad (1)$$

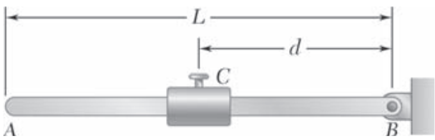
Data: $\quad m_C = 1$ kg, $\quad d = 0.3$ m, $\quad m_R = 3$ kg, $\quad L = 0.6$ m

From Eq. (1),

$$\omega^2 = 3(9.81)\left[\frac{(2)(1)(0.3) + 3(0.6)}{3(1)(0.3)^2 + 3(0.6)^2}\right]$$

$$= 52.32$$

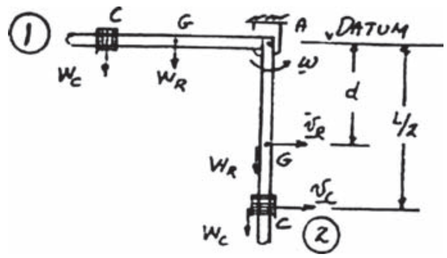
$\bar{\omega} = 7.23$ rad/s ↺ ◀



PROBLEM 17.23

A collar with a mass of 1 kg is rigidly attached to a slender rod AB of mass 3 kg and length L = 600 mm. The rod is released from rest in the position shown. Determine the distance d for which the angular velocity of the rod is maximum after it has rotated 90°.

SOLUTION



Kinematics.

<u>Rod</u> $$v_R = \frac{L}{2}\omega$$

<u>Collar</u> $$v_C = d\omega$$

Position 1. $$\omega = 0$$

$$T_1 = 0 \quad V_1 = 0$$

Position 2.

$$T_2 = \frac{1}{2}m_R\bar{v}_R^2 + \frac{1}{2}\bar{I}_R\omega^2 + \frac{1}{2}m_C v_C^2$$

$$= \frac{1}{2}m_R\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_R L^2\right)\omega^2 + \frac{1}{2}m_C d^2\omega^2$$

$$= \frac{1}{6}m_R L^2\omega^2 + \frac{1}{2}m_C d^2\omega^2$$

$$V_2 = -W_C d - W_R\frac{L}{2}$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \left(\frac{1}{6}m_R L^2 + \frac{1}{2}m_C d^2\right)\omega^2 - W_C d - W_R\frac{L}{2}$$

$$\omega^2 = \frac{3(2W_C d + W_C L)}{3m_C d^2 + m_R L^2} = \frac{3g(2m_C d + m_R L)}{3m_C d^2 + m_R L^2} \qquad (1)$$

Let $$x = \frac{d}{L}.$$

$$\omega^2 = \frac{3g}{L}\cdot\frac{2x + \frac{m_R}{m_C}}{3x^2 + \frac{m_R}{m_C}}$$

Data: $$m_C = 1 \text{ kg}, \quad m_R = 3 \text{ kg}$$

$$\frac{L\omega^2}{3g} = \frac{2x + 3}{3x^2 + 3}$$

PROBLEM 17.23 (Continued)

$L\omega^2/3g$ is maximum. Set its derivative with respect to x equal to zero.

$$\frac{d}{dx}\left(\frac{L\omega^2}{3g}\right) = \frac{(3x^2+3)(2)-(2x+3)(6x)}{(3x^2+3)^2} = 0$$

$$-6x^2 - 18x + 6 = 0$$

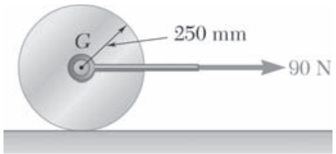
Solving the quadratic equation

$$x = -3.30 \quad \text{and} \quad x = 0.30278$$

$$\begin{aligned} d &= 0.30278L \\ &= (0.30278)(0.6) \\ &= 0.1817 \text{ m} \end{aligned}$$

$d = 181.7$ mm ◀

PROBLEM 17.24



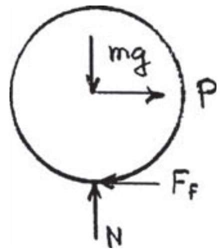
A 20-kg uniform cylindrical roller, initially at rest, is acted upon by a 90-N force as shown. Knowing that the body rolls without slipping, determine (*a*) the velocity of its center *G* after it has moved 1.5 m, (*b*) the friction force required to prevent slipping.

SOLUTION

Since the cylinder rolls without slipping, the point of contact with the ground is the instantaneous center.

Kinematics: $\bar{v} = r\omega$

Position 1. At rest. $T_1 = 0$



Position 2. $s = 1.5 \text{ m} \quad v_G = \bar{v} \quad \omega = \dfrac{v_G}{r}$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv_G^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v_G}{r}\right)^2$$

$$= \frac{3}{4}mv_G^2 = \frac{3}{4}(20)v_G^2 = 15v_G^2$$

Work: $U_{1\to2} = Ps = (90)(1.5) = 135 \text{ J.} \quad F_f$ does no work.

(*a*) Principle of work and energy.

$$T_1 + U_{1\to2} = T_2: \quad 0 + 135 = 15v_G^2$$

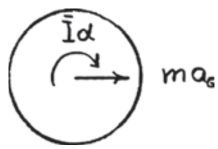
$$v_G^2 = 9 \qquad \mathbf{v}_G = 3.00 \text{ m/s} \rightarrow$$

(*b*) Since the forces are constant, $a_G = \bar{a} = \text{constant}$

$$a_G = \frac{v_G^2}{2s}$$

$$= \frac{9}{(2)(1.5)}$$

$$= 3 \text{ m/s}^2$$



$$\xrightarrow{+} \Sigma F_x = m\bar{a}: \quad P - F_f = m\bar{a}$$

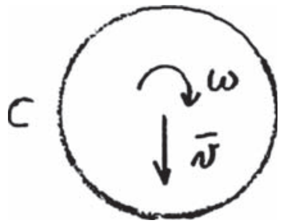
$$F_f = P - m\bar{a}$$

$$= 90 - (20)(3) \qquad \mathbf{F}_f = 30.0 \text{ N} \leftarrow \blacktriangleleft$$

PROBLEM 17.25

A rope is wrapped around a cylinder of radius r and mass m as shown. Knowing that the cylinder is released from rest, determine the velocity of the center of the cylinder after it has moved downward a distance s.

SOLUTION



Point C is the instantaneous center.

$$\bar{v} = r\omega \quad \omega = \frac{\bar{v}}{r}$$

Position 1. At rest. $T_1 = 0$

Position 2. Cylinder has fallen through distance s.

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\bar{v}}{r}\right)^2$$

$$= \frac{3}{4}m\bar{v}^2$$

Work. $U_{1\to 2} = mgs$

Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgs = \frac{3}{4}m\bar{v}^2$$

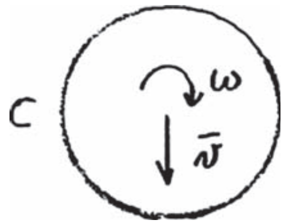
$$\bar{v}^2 = \frac{4gs}{3} \qquad \bar{\mathbf{v}} = \sqrt{\frac{4gs}{3}} \downarrow ◀$$

PROBLEM 17.26

Solve Problem 17.25, assuming that the cylinder is replaced by a thin-walled pipe of radius r and mass m.

PROBLEM 17.25 A rope is wrapped around a cylinder of radius r and mass m as shown. Knowing that the cylinder is released from rest, determine the velocity of the center of the cylinder after it has moved downward a distance s.

SOLUTION



Point C is the instantaneous center.

$$\bar{v} = r\omega \quad \omega = \frac{\bar{v}}{r}$$

Position 1. At rest. $\quad T_1 = 0$

Position 2. Cylinder has fallen through distance s.

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}(mr^2)\left(\frac{\bar{v}}{r}\right)^2$$

$$= m\bar{v}^2$$

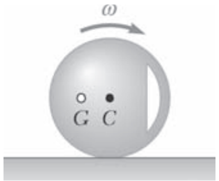
Work. $\quad U_{1\to 2} = mgs$

Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgs = m\bar{v}^2$$

$$\bar{v}^2 = gs$$

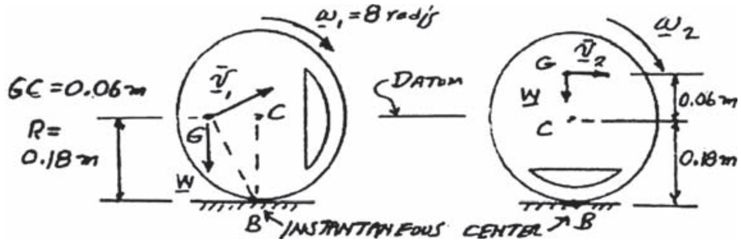
$\bar{\mathbf{v}} = \sqrt{gs} \downarrow$ ◀



PROBLEM 17.27

The mass center G of a 3-kg wheel of radius $R = 180$ mm is located at a distance $r = 60$ mm from its geometric center C. The centroidal radius of gyration of the wheel is $\bar{k} = 90$ mm. As the wheel rolls without sliding, its angular velocity is observed to vary. Knowing that $\omega = 8$ rad/s in the position shown, determine (*a*) the angular velocity of the wheel when the mass center G is directly above the geometric center C, (*b*) the reaction at the horizontal surface at the same instant.

SOLUTION



$$\bar{v}_1 = (BG)\omega_1$$
$$= \sqrt{(0.18)^2 + (0.06)^2}(8)$$
$$= 8\sqrt{0.036} \text{ m/s}$$
$$\bar{v}_2 = 0.24\omega_2$$
$$m = 3 \text{ kg}$$
$$\bar{k} = 0.09 \text{ m}$$

Position 1.

$$V_1 = 0$$
$$T_1 = \frac{1}{2}m\bar{v}_1^2 + \frac{1}{2}\bar{I}\omega_1^2$$
$$= \frac{1}{2}(3)(8\sqrt{0.036})^2 + \frac{1}{2}(3)(0.09)^2(8)^2$$
$$= 4.2336 \text{ J}$$

Position 2.

$$V_2 = Wh$$
$$= mgh$$
$$= (3)(9.81)(0.06)$$
$$= 1.7658 \text{ J}$$
$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}(3)(0.24\omega_2)^2 + \frac{1}{2}(3)(0.09)^2\omega_2^2$$
$$= 0.09855\omega_2^2$$

PROBLEM 17.27 (Continued)

(*a*) Conservation of energy.

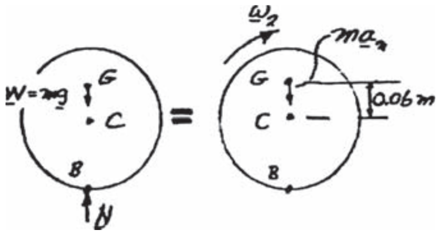
$$T_1 + V_1 = T_2 + V_2$$

$$4.2336 \text{ J} + 0 = 0.09855\omega_2^2 + 1.7658 \text{ J}$$

$$\omega_2^2 = 25.041$$

$$\omega_2 = 5.004 \text{ rad/s} \qquad \boldsymbol{\omega}_2 = 5.00 \text{ rad/s} \blacktriangleleft$$

(*b*) Reaction at *B*.



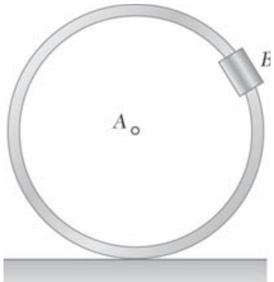
$$ma_n = m(CG)\omega_2^2$$

$$= (3 \text{ kg})(0.06 \text{ m})(5.00 \text{ rad/s})^2$$

$$= 4.5 \text{ N} \downarrow$$

$$+\uparrow \Sigma F_y = ma_y: \quad N - mg = -ma_n$$

$$N - (3)(9.81) = -4.5 \qquad \mathbf{N} = 24.9 \text{ N} \uparrow \blacktriangleleft$$



PROBLEM 17.28

A collar B, of mass m and of negligible dimension, is attached to the rim of a hoop of the same mass m and of radius r that rolls without sliding on a horizontal surface. Determine the angular velocity $\boldsymbol{\omega}_1$ of the hoop in terms of g and r when B is directly above the center A, knowing that the angular velocity of the hoop is $3\boldsymbol{\omega}_1$ when B is directly below A.

SOLUTION

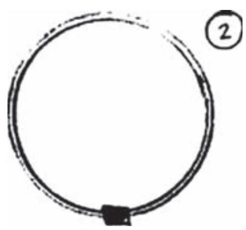
The point of contact with ground is the instantaneous center.

Position 1. Point B is at the top.

$$\omega = \omega_1 \quad v_B = 2r\omega_1 \quad v_A = r\omega_1$$

$$\begin{aligned} T_1 &= \frac{1}{2}mv_B^2 + \frac{1}{2}mv_A^2 + \frac{1}{2}\bar{I}\omega^2 \\ &= \frac{1}{2}m(2r\omega_1)^2 + \frac{1}{2}m(r\omega_1)^2 + \frac{1}{2}(mr^2)\omega_1^2 \\ &= 3mr^2\omega_1^2 \end{aligned}$$

Position 2. Point B is at the bottom.



$$\omega = \omega_2 \quad v_B = 0 \quad v_A = r\omega_2$$

$$\begin{aligned} T_2 &= \frac{1}{2}mv_B^2 + \frac{1}{2}mv_A^2 + \frac{1}{2}\bar{I}\omega_2^2 \\ &= 0 + \frac{1}{2}m(r\omega_2)^2 + \frac{1}{2}(mr^2)\omega_2^2 \\ &= mr^2\omega_2^2 \\ &= mr^2(3\omega_1)^2 \\ &= 9mr^2\omega_1^2 \end{aligned}$$

Work. $\quad U_{1\to 2} = mg(\Delta h) = 2mgr$

Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 3mr^2\omega_1^2 + 2mgr = 9m\omega_1^2$$

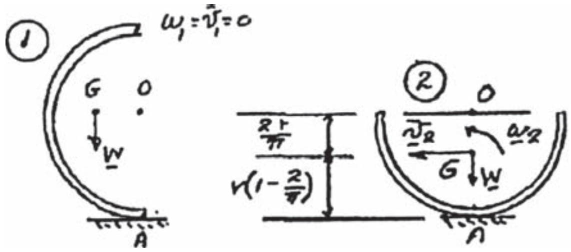
$$\omega_1^2 = \frac{g}{3r} \qquad \omega_1 = 0.577\sqrt{\frac{g}{r}} \blacktriangleleft$$

PROBLEM 17.29

A half section of pipe of mass m and radius r is released from rest in the position shown. Knowing that the pipe rolls without sliding, determine (*a*) its angular velocity after it has rolled through 90°, (*b*) the reaction at the horizontal surface at the same instant. [*Hint:* Note that $GO = 2r/\pi$ and that, by the parallel-axis theorem, $\bar{I} = mr^2 - m(GO)^2$.]

SOLUTION

Position 1. $\qquad \omega_1 = 0 \quad v_1 = 0 \quad T_1 = 0$



Position 2. Kinematics: $\qquad \bar{v}_2 = (AG)\omega_2 = r\left(1 - \frac{2}{\pi}\right)\omega_2$

Moment of inertia: $\qquad \bar{I} = mr^2 - m(0.6)^2 = mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1 - \frac{4}{\pi^2}\right)$

Kinetic energy:

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\,\omega_2^2$$

$$= \frac{1}{2}m\left(1 - \frac{2}{\pi}\right)^2 r^2\omega_2^2 + \frac{1}{2}mr^2\left(1 - \frac{4}{\pi^2}\right)\omega_2^2$$

$$= \frac{1}{2}mr^2\left[\left(1 - \frac{4}{\pi} + \frac{4}{\pi^2}\right) + \left(1 - \frac{4}{\pi^2}\right)\right]$$

$$= \frac{1}{2}mr^2\left(2 - \frac{4}{\pi}\right)$$

Work: $\qquad U_{1\to 2} = W(OG) = mg\frac{2r}{\pi} = \frac{2}{\pi}mgr$

Principle of work and energy: $\qquad T_1 + U_{1\to 2} = T_2$

$$0 + mg\frac{2r}{\pi} = \frac{1}{2}mr^2\left(2 - \frac{4}{\pi}\right)\omega_2^2$$

$$\omega_2^2 = \frac{2}{\pi\left(1 - \frac{2}{\pi}\right)} \cdot \frac{g}{r} = 1.7519\frac{g}{r}$$

PROBLEM 17.29 (Continued)

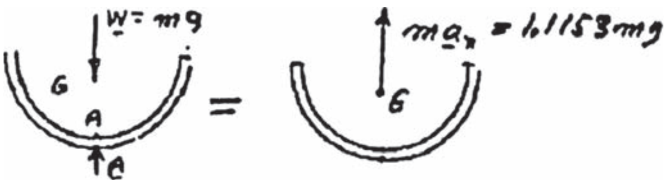
(a) Angular velocity. $\qquad \boldsymbol{\omega}_2 = 1.324\sqrt{\dfrac{g}{r}}$ ↺ ◀

(b) Reaction at A.

Kinematics: Since O moves horizontally, $(a_0)_y = 0$

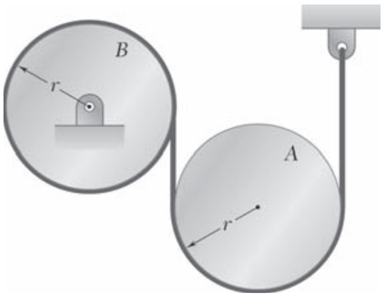
$$a_n = (0.6)\omega_2^2$$
$$= \frac{2r}{\pi}\left(1.7519\frac{g}{r}\right)$$
$$= 1.1153g \uparrow$$

Kinetics:



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A - mg = 1.1153mg$$

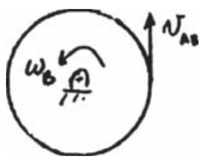
$\mathbf{A} = 2.12mg \uparrow$ ◀



PROBLEM 17.30

Two uniform cylinders, each of mass $m = 7$ kg and radius $r = 100$ mm are connected by a belt as shown. Knowing that the initial angular velocity of cylinder B is 30 rad/s counterclockwise, determine (*a*) the distance through which cylinder A will rise before the angular velocity of cylinder B is reduced to 5 rad/s, (*b*) the tension in the portion of belt connecting the two cylinders.

SOLUTION



Kinematics. $\quad v_{AB} = r\omega_B$

Point C is the instantaneous center of cylinder A.

$$\omega_A = \frac{v_{AB}}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

Moment of inertia. $\quad \bar{I} = \dfrac{1}{2}\dfrac{W}{g}r^2$

Kinetic energy.

Cyl B: $$\frac{1}{2}\bar{I}\omega_B^2 = \frac{1}{2}\left(\frac{1}{2}\frac{W}{g}r^2\right)\omega_B^2 = \frac{1}{4}\frac{W}{g}r^2\omega_B^2$$

Cyl A: $$\frac{1}{2}mv_A^2 + \frac{1}{2}\bar{I}\omega_A^2 = \frac{1}{2}\frac{W}{g}\left(\frac{1}{2}r\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}\frac{W}{g}r^2\right)\left(\frac{1}{2}\omega_B\right)^2$$

$$= \frac{3}{16}\frac{W}{g}r^2\omega_B^2$$

Total: $$T = \frac{7}{16}\frac{W}{g}r^2\omega_B^2$$

(*a*) *Distance h that cylinder A will rise*.
Conservation of energy for system.

$$T_1 + V_1 = T_2 + V_2: \quad \frac{7}{16}\frac{W}{g}r^2(\omega_B)_1^2 + 0 = \frac{7}{16}\frac{W}{g}r^2(\omega_B)_2^2 + Wh$$

$$h = \frac{7}{16}\frac{r^2}{g}[(\omega_B)_1^2 - (\omega_B)_2]^2$$

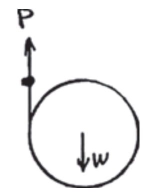
$$= \left(\frac{7}{16}\right)\frac{(0.1)^2}{(9.81)}(30^2 - 5^2)$$

$$= 0.3902 \text{ m}$$

$h = 0.390$ m ◀

PROBLEM 17.30 (Continued)

(*b*) *Tension in belt between the cylinders*.



When cylinder A moves up a distance h, the belt moves up a distance $2h$.

Work: $$U_{1\to 2} = P(2h) - Wh$$

Principle of work and energy for cylinder A.

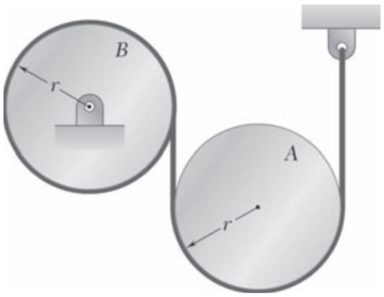
$$T_1 + U_{1\to 2} = T_2: \quad \frac{3}{16}\frac{W}{g}r^2(\omega_B)_1^2 + 2Ph - Wh = \frac{3}{16}\frac{W}{g}r^2(\omega_B)_2^2$$

$$P = \frac{1}{2}W - \frac{3}{32}\frac{Wr^2}{gh}\left[(\omega_B)_1^2 - (\omega_B)_2^2\right]$$

$$= \frac{1}{2}W - \frac{3}{14}W$$

$$= \frac{2}{7}W = \frac{2}{7}(7)(9.81)$$

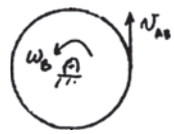
$P = 19.62$ N ◄



PROBLEM 17.31

Two uniform cylinders, each of mass $m = 7$ kg and radius $r = 100$ mm are connected by a belt as shown. If the system is released from rest, determine (*a*) the velocity of the center of cylinder *A* after it has moved through 1 m, (*b*) the tension in the portion of belt connecting the two cylinders.

SOLUTION



Kinematics. $$v_{AB} = r\omega_B$$

Point *C* is the instantaneous center of cylinder *A*.

$$\omega_A = \frac{v_{AB}}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

Moment of inertia. $$\bar{I} = \frac{1}{2}\frac{W}{g}r^2$$

Kinetic energy.

Cyl *B*: $$\frac{1}{2}\bar{I}\omega_B^2 = \frac{1}{2}\left(\frac{1}{2}\frac{W}{g}r^2\right)\omega_B^2 = \frac{1}{4}\frac{W}{g}r^2\omega_B^2$$

Cyl *A*: $$\frac{1}{2}mv_A^2 + \frac{1}{2}\bar{I}\omega_A^2 = \frac{1}{2}\frac{W}{g}\left(\frac{1}{2}r\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}\frac{W}{g}r^2\right)\left(\frac{1}{2}\omega_B\right)^2$$

$$= \frac{3}{16}\frac{W}{g}r^2\omega_B^2$$

Total: $$T = \frac{7}{16}\frac{W}{g}r^2\omega_B^2$$

Position 1. Rest $$T_1 = 0 \qquad V_1 = 0$$

Position 2. Cylinder has fallen through. $d = 1$ m.

$$T_2 = \frac{7}{16}\frac{W}{g}r^2\omega_B^2$$

$$V_2 = -Wd$$

PROBLEM 17.31 (Continued)

Principle of conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$0 + 0 = \frac{7}{16}\frac{W}{g}r^2\omega_B^2 - Wd$$

where $r = 0.1\text{ m}$

$$\omega_B^2 = \frac{16}{7}\frac{gd}{r^2}$$

$$= \frac{16}{7}\cdot\frac{(9.81)(1)}{(0.1)^2}$$

$$= 2242.29$$

$$\omega_B = 47.3528\text{ rad/s}$$

(*a*) Velocity of the center of cylinder *A*.

$$\bar{v}_A = \frac{1}{2}r\omega_B$$

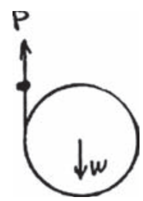
$$= \frac{1}{2}(0.1)(47.3528) \qquad \bar{\mathbf{v}}_A = 2.37\text{ m/s}\downarrow \blacktriangleleft$$

(*b*) Tension in belt between the cylinders.

When cylinder *A* moves down a distance *d*, the belt moves down a distance 2*d*.

Work: $U_{1\to 2} = \frac{P}{2}(2d) + Wd$

Principle of work and energy for cylinder *A*:

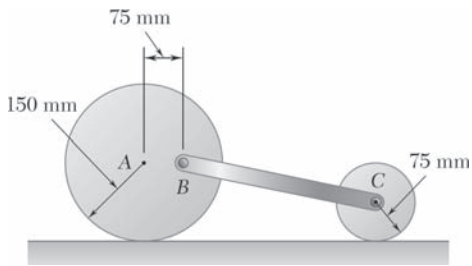


$$T_1 + U_{1\to 2} = T_2: \quad 0 - P(2d) + Wd = \frac{3}{16}\frac{W}{g}r^2\omega_B^2$$

$$P = \frac{1}{2}W - \frac{3}{32}\frac{Wr^2\omega_B^2}{g\,d}$$

$$P = \frac{1}{2}(7)(9.81) - \frac{3}{32}\frac{(7)(9.81)(0.1)^2(2242.29)}{(9.81)(1)}$$

$$= 34.335 - 14.715 = 19.620 \qquad P = 19.62\text{ N} \blacktriangleleft$$

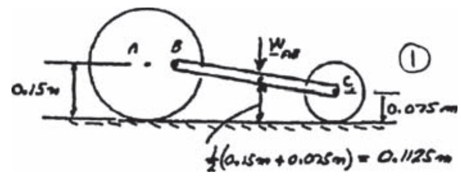


PROBLEM 17.32

The 5-kg rod *BC* is attached by pins to two uniform disks as shown. The mass of the 150-mm-radius disk is 6 kg and that of the 75-mm-radius disk is 1.5 kg. Knowing that the system is released from rest in the position shown, determine the velocity of the rod after disk *A* has rotated through 90°.

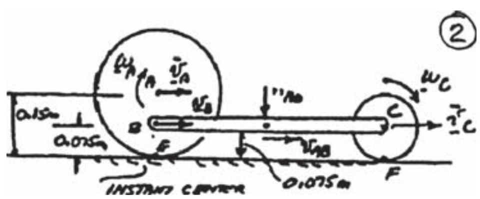
SOLUTION

Position 1. $T_1 = 0$



Position 2.

Kinematics.



$$v_B = v_{AB} \quad \omega_A = \frac{v_B}{BE} = \frac{v_{AB}}{0.075\text{ m}} \quad \bar{v}_A = 2v_B = 2v_{AB}$$

$$\bar{v}_C = v_{AB} \quad \omega_C = \frac{v_C}{CF} = \frac{v_{AB}}{0.075\text{ m}} \quad \omega_{AB} = 0$$

Kinetic energy.

$$T_2 = \frac{1}{2}m_A\bar{v}_A + \frac{1}{2}\bar{I}_A\omega_A^2 + \frac{1}{2}m_{AB}v_{AB}^2 + \frac{1}{2}m_B\bar{v}_B^2 + \frac{1}{2}\bar{I}_B\omega_B^2$$

$$= \frac{1}{2}\left[(2\text{ m/s})(2v_{AB})^2 + \frac{1}{2}(6\text{ kg})(0.15\text{ m})^2\left(\frac{v_{AB}}{0.075}\right)^2 + (5\text{ kg})v_{AB}^2\right.$$

$$\left. + (1.5\text{ kg})(v_{AB})^2 + \frac{1}{2}(1.5\text{ kg})(0.075\text{ m})\left(\frac{v_{AB}}{0.075}\right)^2\right]$$

$$= \frac{1}{2}\left[24 + 12 + 5 + 1.5 + 0.75\right]v_{AB}^2$$

$$T_2 = 21.625\ v_{AB}^2$$

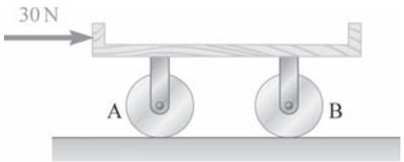
PROBLEM 17.32 (Continued)

Work:
$$U_{1\to 2} = W_{AB}(0.1125\text{ m} - 0.075\text{ m})$$
$$= (5\text{ kg})(9.81)(0.0375\text{ m})$$
$$U_{1\to 2} = 1.8394\text{ J}$$

Principle of work and energy:
$$T_1 + U_{1\to 2} = T_2$$
$$0 + 1.8394\text{ J} = 21.625\, v_{AB}^2$$
$$v_{AB}^2 = 0.08506$$
$$v_{AB} = 0.2916\text{ m}$$

Velocity of the rod. $\mathbf{v}_{AB} = 292\text{ mm/s} \rightarrow$ ◀

PROBLEM 17.33



The 9-kg cradle is supported as shown by two uniform disks that roll without sliding at all surfaces of contact. The mass of each disk is $m = 6$ kg and the radius of each disk is $r = 80$ mm. Knowing that the system is initially at rest, determine the velocity of the cradle after it has moved 250 mm.

SOLUTION

Moments of inertia. $\bar{I}_A = \bar{I}_B = \dfrac{Wr^2}{2g}$

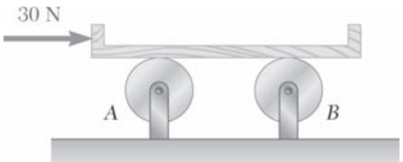
Kinematics. $\omega_A = \omega_B = \dfrac{v_C}{r} \quad v_A = v_B = v_C$

Kinetic energy. $T_1 = 0$

$$\begin{aligned}
T_2 &= \frac{1}{2}m_A v_A^2 + \frac{1}{2}\bar{I}_A\omega_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}\bar{I}_B\omega_B^2 + \frac{1}{2}m_C v_C^2 \\
&= \frac{1}{2}\left[m + \frac{1}{2}m + m + \frac{1}{2}m + m_C\right]v_C^2 \\
&= \frac{1}{2}(3m + m_C)v_C^2 \\
&= \frac{1}{2}[(3)(6) + 9]v_C^2 \\
&= 13.5v_C^2
\end{aligned}$$

Work. $U_{1\to 2} = Fs = (30\text{ N})(0.25\text{ m}) = 7.5\text{ J}$

Principle of work and energy. $T_1 + U_{1\to 2} = T_2: \quad 0 + 7.5 = 13.5v_C^2$ $\mathbf{v}_C = 0.745\text{ m/s} \rightarrow$ ◄



PROBLEM 17.34

The 9-kg cradle is supported as shown by two uniform disks that roll without sliding at all surfaces of contact. The mass of each disk is $m = 6$ kg and the radius of each disk is $r = 80$ mm. Knowing that the system is initially at rest, determine the velocity of the cradle after it has moved 250 mm.

SOLUTION

Moments of inertia. $\bar{I}_A = \bar{I}_B = \dfrac{Wr^2}{2g}$

Kinematics. $\omega_A = \omega_B = \dfrac{v_C}{r}$

$v_A = v_B = 0$

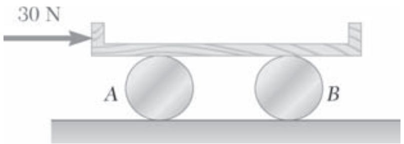
Kinetic energy. $T_1 = 0$

$$
\begin{aligned}
T_2 &= \frac{1}{2}m_A v_A^2 + \frac{1}{2}\bar{I}_A\omega_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}\bar{I}_B\omega_B^2 + \frac{1}{2}m_C v_C^2 \\
&= \frac{1}{2}\left[0 + \frac{1}{2}m + 0 + \frac{1}{2}m + m_C\right]v_C^2 \\
&= \frac{1}{2}(m + m_C)v_C^2 \\
&= \frac{1}{2}(6+9)v_C^2 \\
&= 7.5v_C^2
\end{aligned}
$$

Work. $U_{1\to 2} = Fs = (30 \text{ N})(0.25 \text{ m}) = 7.5 \text{ J}$

Principle of work and energy. $T_1 + U_{1\to 2} = T_2: \quad 0 + 7.5 = 7.5v_C^2$

$\mathbf{v}_C = 1.000 \text{ m/s} \rightarrow$ ◀



PROBLEM 17.35

The 9-kg cradle is supported as shown by two uniform disks that roll without sliding at all surfaces of contact. The mass of each disk is $m = 6$ kg and the radius of each disk is $r = 80$ mm. Knowing that the system is initially at rest, determine the velocity of the cradle after it has moved 250 mm.

SOLUTION

Moments of inertia. $\bar{I}_A = \bar{I}_B = \dfrac{Wr^2}{2g}$

Kinematics. $\omega_A = \omega_B = \dfrac{v_C}{2r} \quad v_A = v_B = \dfrac{1}{2}v_C$

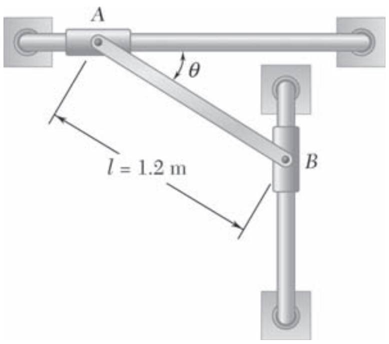
Kinetic energy. $T_1 = 0$

$$\begin{aligned}
T_2 &= \frac{1}{2}m_A v_A^2 + \frac{1}{2}\bar{I}_A\omega_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}\bar{I}_B\omega_B^2 + \frac{1}{2}m_C v_C^2 \\
&= \frac{1}{2}\left[\frac{1}{4}m + \frac{1}{8}m + \frac{1}{4}m + \frac{1}{8}m + m_C\right]v_C^2 \\
&= \frac{1}{2}(0.75m + m_C)v_C^2 \\
&= \frac{1}{2}[(0.75)(6) + 9]v_C^2 \\
&= 6.75v_C^2
\end{aligned}$$

Work. $U_{1\to 2} = Fs = (30\text{ N})(0.25\text{ m}) = 7.5\text{ J}$

Principle of work and energy. $T_1 + U_{1\to 2} = T_2: \quad 0 + 7.5 = 6.75v_C^2$

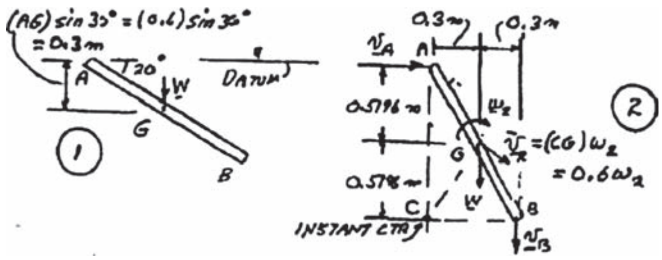
$\mathbf{v}_C = 1.054\text{ m/s} \rightarrow$ ◄



PROBLEM 17.36

The motion of the slender 10-kg rod AB is guided by collars of negligible mass that slide freely on the vertical and horizontal rods shown. Knowing that the bar is released from rest when $\theta = 30°$, determine the velocity of collars A and B when $\theta = 60°$.

SOLUTION



Position 1:

$$T_1 = 0$$
$$V_1 = W(0.3 \text{ m})$$
$$= -(10 \text{ kg})(9.8)(0.3)$$
$$= -29.43 \text{ J}$$

Position 2:

$$V_2 = -W(0.5196 \text{ m})$$
$$= -(10 \text{ kg})(9.81)(0.5196)$$
$$= -50.974 \text{ J}$$
$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}(10 \text{ kg})(0.6\omega_2)^2 + \frac{1}{2}\left(\frac{1}{2}(10 \text{ kg})(1.2 \text{ m})^2\omega_2^2\right)$$
$$= 2.4\omega_2^2$$

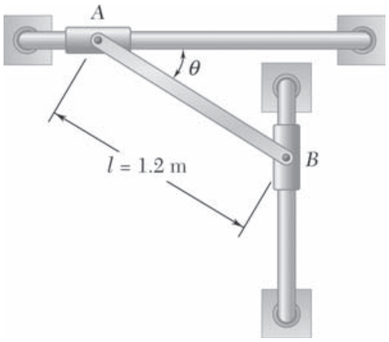
Principle of conservation of energy. $T_1 + V_1 = T_2 + V_2$

$$0 - 29.43 \text{ J} = 2.4\omega_2^2 - 50.974$$
$$\omega_2^2 = 8.9768 \qquad \omega_2 = 2.996 \text{ rad/s} \circlearrowleft$$

Velocity of collars when $\theta = 60°$

$v_A = (AC)\omega_2 = (2 \times 0.5196 \text{ m})(2.996 \text{ rad/s})$ $\qquad \mathbf{v}_A = 3.11 \text{ m/s} \rightarrow$ ◀

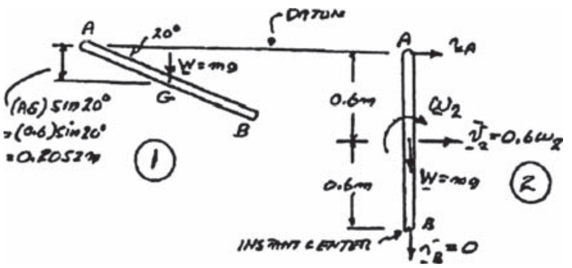
$v_B = (BC)\omega_2 = (2 \times 0.3 \text{ m})(2.996 \text{ rad/s})$ $\qquad \mathbf{v}_B = 1.798 \text{ m/s} \downarrow$ ◀



PROBLEM 17.37

The motion of the slender 10-kg rod AB is guided by collars of negligible mass that slide freely on the vertical and horizontal rods shown. Knowing that the bar is released from rest when $\theta = 20°$, determine the velocity of collars A and B when $\theta = 90°$.

SOLUTION



Position 1:

$$T_1 = 0$$

$$V_1 = -W(0.2052 \text{ m}) = -mg(0.2052)$$

Position 2:

$$V_2 = -W(0.6 \text{ m}) = -mg(0.6)$$

$$T_2 = \frac{1}{2}m\mathbf{v}_2^2 + \frac{1}{2}I\omega_2^2 = \frac{1}{2}m(0.6\omega_2)^2 + \frac{1}{2}\left(\frac{1}{12}m(1.2)^2\right)\omega_2^2$$

$$T_2 = 0.24m\omega_2^2$$

Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 - 0.2052mg = 0.24m\omega_2^2 - 0.6mg$$

$$\omega_2^2 = 1.645g = 1.645(9.81) = 16.137$$

$$\omega_2 = 4.017 \text{ rad/s}$$

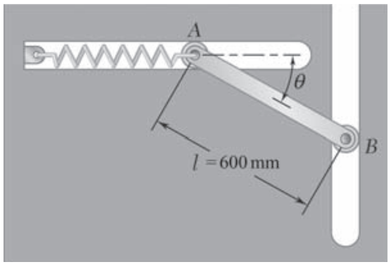
Velocity of collars when $\theta = 90°$

$$v_A = (AB)\omega_2 = (1.2 \text{ m})(4.017 \text{ rad/s})$$

$\mathbf{v}_A = 4.82 \text{ m/s} \rightarrow$ ◀

$$v_B = 0$$

$\mathbf{v}_B = 0$ ◀

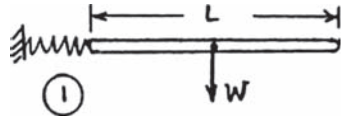


PROBLEM 17.38

The ends of a 4.5 kg rod AB are constrained to move along slots cut in a vertical plane as shown. A spring of constant $k = 600$ N/m is attached to end A in such a way that its tension is zero when $\theta = 0$. If the rod is released from rest when $\theta = 0$, determine the angular velocity of the rod and the velocity of end B when $\theta = 30°$.

SOLUTION

Moment of inertia. Rod. $\bar{I} = \frac{1}{12}mL^2$

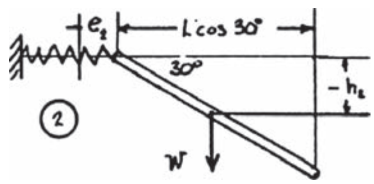


Position 1. $\theta_1 = 0 \qquad \bar{v}_1 = 0 \qquad \omega_1 = 0$

h_1 = elevation above slot. $\quad h_1 = 0$

e_1 = elongation of spring. $\quad e_1 = 0$

$$T_1 = \frac{1}{2}m\bar{v}_1^2 + \frac{1}{2}\bar{I}\omega_1^2 = 0$$



$$V_1 = \frac{1}{2}ke_1^2 + Wh_1 = 0$$

Position 2. $\theta = 30°$

$$e_2 + L\cos 30° = L$$

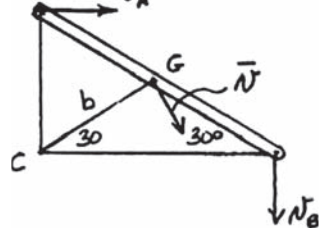
$$e_2 = L(1 - \cos 30°)$$

$$h_2 = -\frac{L}{2}\sin 30° = -\frac{1}{4}L$$

$$V_2 = \frac{1}{2}ke_2^2 + Wh_2 = \frac{1}{2}kL^2(1 - \cos 30°)^2 - \frac{1}{4}WL$$

Kinematics. Velocities at A and B are directed as shown. Point C is the instantaneous center of rotation. From geometry, $b = \frac{L}{2}$.

$$\bar{v} = b\omega = \frac{L}{2}\omega$$



$$v_B = (L\cos 30°)\omega$$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)$$

$$= \frac{1}{6}\frac{W}{g}L^2\omega^2$$

PROBLEM 17.38 (Continued)

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{1}{6}\frac{W}{g}L^2\omega^2 + \frac{1}{2}kL^2(1-\cos 30°)^2 - \frac{1}{4}WL$$

$$\omega^2 = \frac{3g}{L} - \frac{3\text{ kg}}{W}(1-\cos 30°)^2$$

Data:

$$W = (4.5)(9.81)\text{ N}$$

$$g = 9.81\text{ m/s}^2$$

$$L = 0.6\text{ m}$$

$$k = 600\text{ N/m}$$

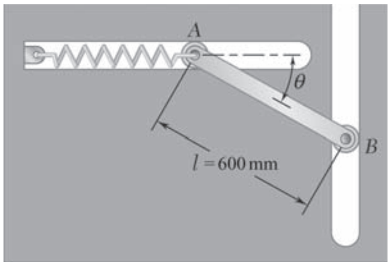
$$\omega^2 = \frac{(3)(9.81)}{(0.6)} - \frac{3(600)(9.81)}{(4.5)(9.81)}(1-\cos 30°)^2$$

$$= 41.8703$$

$\boldsymbol{\omega} = 6.47$ rad/s ◀

$$v_B = (0.6)(\cos 30°)(6.4707)$$

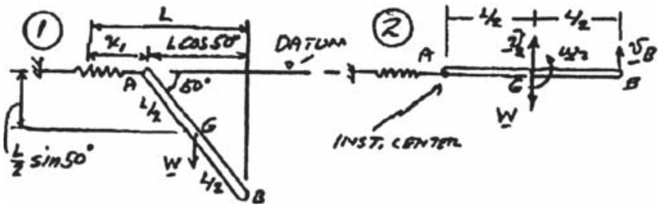
$\mathbf{v}_B = 3.36$ m/s ◀



PROBLEM 17.39

The ends of a 4.5 kg rod AB are constrained to move along slots cut in a vertical plate as shown. A spring of constant $k = 600$ N/m is attached to end A in such a way that its tension is zero when $\theta = 0$. If the rod is released from rest when $\theta = 50°$, determine the angular velocity of the rod and the velocity of end B when $\theta = 0$.

SOLUTION



$$\bar{v}_2 = \frac{L}{2}\omega_2$$

$$v_B = L\omega_2$$

$$x_1 = L - L\cos 50°$$
$$= 0.6\text{ m}(1-\cos 50°)$$
$$= 0.21433\text{ m}$$

Position 1.

$$V_1 = -W\frac{L}{2}\sin 50° + \frac{1}{2}kx_1^2$$

$$V_1 = -(4.5)(9.81)\left(\frac{0.6}{2}\right)\sin 50° + \frac{1}{2}(600)(0.21433)^2$$
$$= -10.1451 + 13.7812$$
$$= 3.6361\text{ N}\cdot\text{m}$$

$$T_1 = 0$$

Position 2.

$$V_2 = (V_g)_2 + (V_e)_2 = 0$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$$
$$= \frac{1}{6}mL^2\omega_2^2 = \frac{1}{6}(4.5\text{ kg})(0.6\text{ m})^2\omega_2^2 = 0.27\omega_2^2$$

PROBLEM 17.39 (Continued)

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$0 + 3.6361\ \text{N}\cdot\text{m} = 0.27\omega_2^2$$

$$\omega_2^2 = 13.467\ \text{rad}^2/\text{s}^2$$

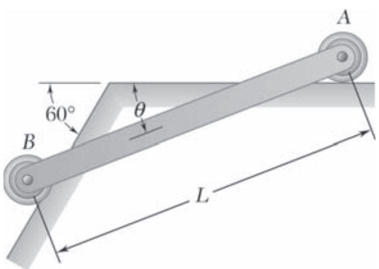
$$\omega_2 = 3.6697\ \text{rad/s}$$

$\omega_2 = 3.67\ \text{rad/s}$ ◄

Velocity of B: $v_B = L\omega_2 = (0.6\ \text{m})(3.6697\ \text{rad/s})$

$$= 2.2018\ \text{m/s}$$

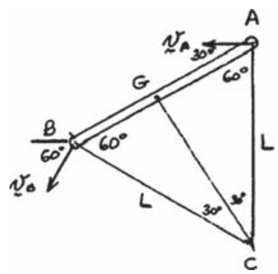
$\mathbf{v}_B = 2.20\ \text{m/s} \uparrow$ ◄



PROBLEM 17.40

The motion of the uniform rod AB is guided by small wheels of negligible mass that roll on the surface shown. If the rod is released from rest when $\theta = 0$, determine the velocities of A and B when $\theta = 30°$.

SOLUTION



Position 1. $\quad \theta = 0$

$$v_A = v_B = 0$$
$$\omega = 0$$
$$T_1 = 0$$
$$V_1 = 0$$

Position 2. $\quad \theta = 30°$

Kinematics. Locate the instantaneous center C. Triangle ABC is equilateral.

$$v_A = v_B = L\omega$$
$$v_G = L\omega \cos 30°$$

Moment of inertia. $\quad \bar{I} = \frac{1}{12} ml^2$

Kinetic energy. $\quad T_2 = \frac{1}{2} m v_G^2 + \frac{1}{2} \bar{I} \omega^2: \quad T_2 = \frac{1}{2} m (L\omega \cos 30°)^2 + \frac{1}{2}\left(\frac{1}{12} mL^2\right)\omega^2$

$$= \frac{5}{12} ml^2 \omega^2$$

Potential energy. $\quad V_2 = -mg\frac{L}{2}\sin 30° = -\frac{1}{4} mgL$

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{5}{12} mL^2\omega^2 - \frac{1}{4} mgL$$

$$\omega^2 = 0.6\frac{g}{L}$$

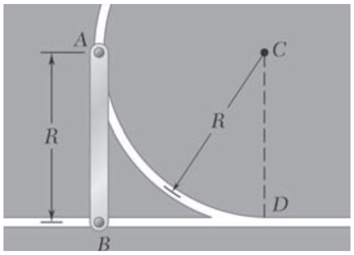
$$\omega = 0.775\sqrt{\frac{g}{L}}$$

$$v_A = 0.775\sqrt{gL}$$

$$v_B = 0.775\sqrt{gL}$$

$\mathbf{v}_A = 0.775\sqrt{gL} \leftarrow$ ◀

$\mathbf{v}_B = 0.775\sqrt{gL} \measuredangle 60°$ ◀



PROBLEM 17.41

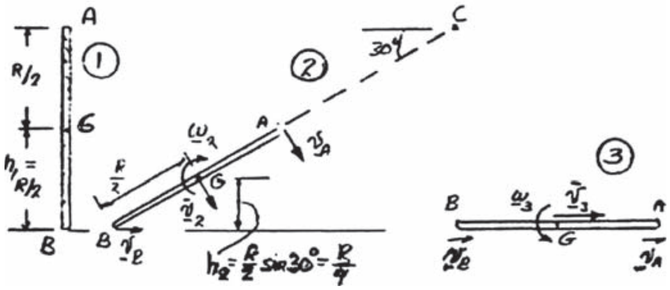
The motion of a slender rod of length R is guided by pins at A and B which slide freely in slots cut in a vertical plate as shown. If end B is moved slightly to the left and then released, determine the angular velocity of the rod and the velocity of its mass center (*a*) at the instant when the velocity of end B is zero, (*b*) as end B passes through Point D.

SOLUTION

The rod AB moves from *Position 1*, where it is nearly vertical, to *Position 2*, where $\mathbf{v}_B = 0$.

In *Position 2*, $\mathbf{v}_A$ is perpendicular to both CA and AB, so CAB is a straight line of length $2L$ and slope angle 30°.

In *Position 3* the end B passes through Point D.



Position 1: $T_1 = 0 \quad V_1 = \frac{v}{h_1} = mg\frac{R}{2}$

Position 2: Since instantaneous center is at B,

$$v_2 = \frac{1}{2}R\omega_2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}I\omega_2^2$$

$$= \frac{1}{2}m\left(\frac{1}{2}R\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mR^2\right)\omega_2^2$$

$$= \frac{1}{6}mR^2\omega_2^2$$

$$V_2 = Wh_2 = mg\frac{R}{4}$$

Position 3: $V_3 = 0$

Since both $\mathbf{v}_A$ and $\mathbf{v}_B$ are horizontal, $\omega_3 = 0$ (1)

$$T_3 = \frac{1}{2}m\bar{v}_2^2$$

PROBLEM 17.41 (Continued)

(*a*) From 1 to 2: Conservation of energy

$$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{1}{2} mgR = \frac{1}{6} mR^2 \omega_2^2 + \frac{1}{4} mgR$$

$$\omega_2^2 = \frac{3}{2} \frac{g}{R}$$

$$\omega_2 = \sqrt{\frac{3}{2} \frac{g}{R}} \qquad \boldsymbol{\omega}_2 = 1.225 \sqrt{\frac{g}{R}} \circlearrowright \blacktriangleleft$$

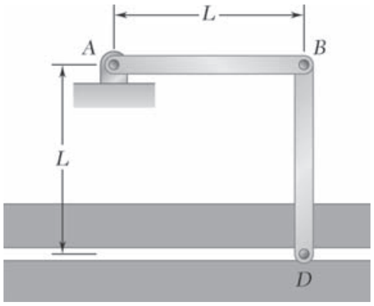
$$\bar{v}_2 = \frac{1}{2} R\omega_2 = \frac{1}{2} \sqrt{\frac{3}{2} gR} = \sqrt{\frac{3}{8} gR} \qquad \mathbf{v}_R = 0.612 \sqrt{gR} \searrow 60° \blacktriangleleft$$

(*b*) From 1 to 3: Conservation of energy

From Eq. (1) we have $\boldsymbol{\omega}_3 = 0 \blacktriangleleft$

$$T_1 + V_1 = T_3 + V_3: \quad 0 + \frac{1}{2} mgR = \frac{1}{2} m\bar{v}_3^2$$

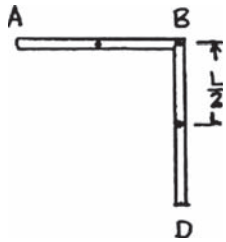
$$\bar{v}_3^2 = gR \qquad \mathbf{v}_3 = \sqrt{gR} \rightarrow \blacktriangleleft$$



PROBLEM 17.42

Two uniform rods, each of mass m and length L, are connected to form the linkage shown. End D of rod BD can slide freely in the horizontal slot, while end A of rod AB is supported by a pin and bracket. If end D is moved slightly to the left and then released, determine its velocity (a) when it is directly below A, (b) when rod AB is vertical.

SOLUTION



Moments of inertia. $\qquad \bar{I}_{AB} = \bar{I}_{BD} = \dfrac{1}{12}mL^2$

$$I_A = \frac{1}{3}mL^2$$

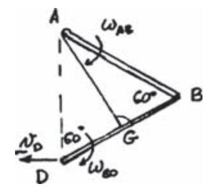
Position 1. At rest as shown. $\qquad T_1 = 0$

$$V_1 = mgh_{AB} + mgh_{BC}$$

$$= 0 + mg\left(-\frac{L}{2}\right)$$

$$= -\frac{1}{2}mgL$$

(a) *Position 2*. In *Position 2*, Point A is the instantaneous center of both AB and BD. Since Point B is common to both bars,



$$\omega_{AB} = \omega_{BD}$$

$$= \omega_2$$

$$v_D = l\omega_2$$

$$v_G = l\omega \cos 30°$$

Kinetic energy.

$$T_2 = \frac{1}{2}I_A\omega_2^2 + \frac{1}{2}mv_G^2 + \frac{1}{2}\bar{I}_{BD}\omega_2^2: \quad T_2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_2^2 + \frac{1}{2}m(l\omega_2 \cos 30°)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$$

$$= \frac{7}{12}mL^2\omega_2^2$$

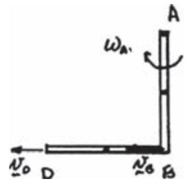
Potential energy. $\qquad V_2 = mgh_{AB} + mgh_{BD}$

$$V_2 = mg\left(-\frac{L}{2}\sin 30°\right) + mg\left(-\frac{3L}{2}\sin 30°\right)$$

$$= -mgL$$

PROBLEM 17.42 (Continued)

Conservation of energy.



$$T_1 + V_1 = T_2 + V_2: \quad 0 - \frac{1}{2}mgL = \frac{7}{12}mL^2\omega_2^2 - mgL$$

$$\omega_2^2 = \frac{6}{7}\frac{g}{L}$$

$$\omega_2 = 0.926\sqrt{\frac{g}{L}}$$

$$v_D = L\omega_2 = 0.926\sqrt{gL} \qquad \mathbf{v}_D = 0.926\sqrt{gL} \leftarrow \blacktriangleleft$$

(*b*) *Position 3*. In *Position 3*, bar *BD* is in translation.

$$v_D = v_B = L\omega_{AB} = L\omega_3$$

Kinetic energy.

$$T_3 = \frac{1}{2}I_A\omega_3^2 + \frac{1}{2}mv_B^2: \quad T_3 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_3^2 + \frac{1}{2}m(L\omega_3)^2 = \frac{2}{3}mL^2\omega_3^2$$

Potential energy.

$$V_3 = mg\left(-\frac{L}{2}\right) + mg(-L)$$

$$= -\frac{3}{2}mgL$$

Conservation of energy.

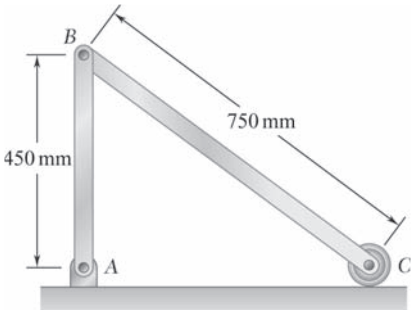
$$T_1 + V_1 = T_3 + V_3: \quad 0 - \frac{1}{2}mgL = \frac{2}{3}mL^2\omega_3^2 - \frac{3}{2}mgL$$

$$\omega_3^2 = \frac{3}{2}\frac{g}{L}$$

$$\omega_3 = 1.225\sqrt{\frac{g}{L}}$$

$$v_D = L\omega_3$$

$$= 1.225\sqrt{gL} \qquad \mathbf{v}_D = 1.225\sqrt{gL} \leftarrow \blacktriangleleft$$



PROBLEM 17.43

The uniform rods AB and BC have a mass of 1.2 kg and 2 kg, respectively, and the small wheel at C is of negligible weight. If the wheel is moved slightly to the right and then released, determine the velocity of pin B after rod AB has rotated through 90°.

SOLUTION

Moments of inertia.
$$m_{AB} = 1.2 \text{ kg}$$
$$m_{BC} = 2 \text{ kg}$$

Bar AB:
$$I_A = \frac{1}{3} m_{AB} L_{AB}^2 = \frac{1}{3}(1.2)(0.45)^2 = 0.081 \text{ kg} \cdot \text{m}^2$$

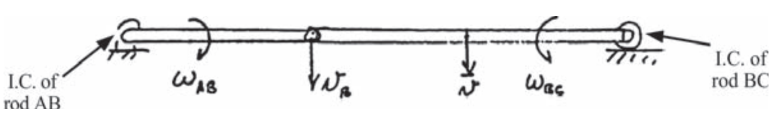
Bar BC:
$$\overline{I} = \frac{1}{12} m_{BC} L_{BC}^2 = \frac{1}{12}(2)(0.75)^2 = 0.09375 \text{ kg} \cdot \text{m}^2$$

Position 1. As shown with bar AB vertical. Point G is the midpoint of BC.

$$V_1 = W_{AB}\overline{h}_{AB} + W_{BC}\overline{h}_{BC}$$
$$= (1.2)(9.81)\left(\frac{0.45}{2}\right) + (2)(9.81)\left(\frac{0.45}{2}\right)$$
$$= 7.0632 \text{ N} \cdot \text{m}$$

Bar BC is at rest.
$$\omega_{BC} = 0$$
$$\overline{v} = v_G = v_B = v_C = 0$$
$$\omega_{AB} = \frac{v_B}{L_{AB}} = 0$$
$$T_1 = 0$$

Position 2. Bar AB is horizontal.



$$V_2 = 0$$

PROBLEM 17.43 (Continued)

Kinematics. $$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{v_B}{0.45}$$

$$\omega_{BC} = \frac{v_B}{L_{BC}} = \frac{v_B}{0.75}$$

$$\bar{v} = \frac{1}{2} v_B$$

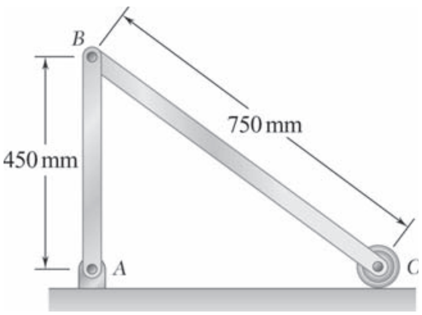
$$T_2 = \frac{1}{2} I_A \omega_{AB}^2 + \frac{1}{2} m_{BC} \bar{v}^2 + \frac{1}{2} \bar{I} \omega_{BC}^2$$

$$= \frac{1}{2}(0.081)\left(\frac{v_B}{0.45}\right)^2 + \frac{1}{2}(2)\left(\frac{1}{2} v_B\right)^2 + \frac{1}{2}(0.09375)\left(\frac{v_B}{0.75}\right)^2$$

$$= 0.53333 v_B^2$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2: \quad 0 + 7.0632 = 0.53333 v_B^2$

$v_B = 3.6392 \text{ m/s}$ $\mathbf{v}_B = 3.64 \text{ m/s} \downarrow$ ◄



PROBLEM 17.44

The uniform rods AB and BC have a mass 1.2 kg and 2 kg, respectively, and the small wheel at C is of negligible weight. Knowing that in the position shown the velocity of wheel C is 2 m/s to the right, determine the velocity of pin B after rod AB has rotated through 90°.

SOLUTION

Moments of inertia.

Bar AB: $$I_A = \frac{1}{3}m_{AB}L_{AB}^2 = \frac{1}{3}(1.2)(0.45)^2 = 0.081\ \text{kg}\cdot\text{m}^2$$

Bar BC: $$\bar{I} = \frac{1}{12}m_{BC}L_{BC^2} = \frac{1}{12}(2)(0.75)^2 = 0.09375\ \text{kg}\cdot\text{m}^2$$

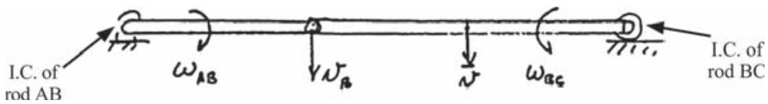
Position 1. As shown with bar AB vertical. Point G is the midpoint of BC.

$$V_1 = W_{AB}\bar{h}_{AB} + W_{BC}\bar{h}_{BC}$$
$$= (1.2)(9.81)\left(\frac{0.45}{2}\right) + (2)(9.81)\left(\frac{0.45}{2}\right)$$
$$= 7.0632\ \text{N}\cdot\text{m}$$

Kinematics, Bar BC is in translation. $$\omega_{BC} = 0$$

$$\bar{v} = v_G = v_B = v_C = 2\ \text{m/s}$$
$$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{2}{0.45} = \frac{40}{9}\ \text{rad/s}$$
$$T_1 = \frac{1}{2}I_A\omega_{AB}^2 + \frac{1}{2}m_{BC}\bar{v}^2 + \frac{1}{2}I\omega_{BC}^2$$
$$= \frac{1}{2}(0.081)\left(\frac{40}{9}\right)^2 + \frac{1}{2}(2)(2)^2 + 0$$
$$= 4.8\ \text{N}\cdot\text{m}$$

Position 2. Bar AB is horizontal.



$$V_2 = 0$$

PROBLEM 17.44 (Continued)

Kinematics.

$$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{v_B}{0.45}$$

$$\omega_{BC} = \frac{v_B}{L_{BC}} = \frac{v_B}{0.75}$$

$$\bar{v} = \frac{1}{2} v_B$$

$$T_2 = \frac{1}{2} I_A \omega_{AB}^2 + \frac{1}{2} m_{BC} \bar{v}^2 + \frac{1}{2} \bar{I} \omega_{BC}^2$$

$$= \frac{1}{2}(0.081)\left(\frac{v_B}{0.45}\right)^2 + \frac{1}{2}(2)\left(\frac{1}{2} v_B\right)^2 + \frac{1}{2}(0.09375)\left(\frac{v_B}{0.75}\right)^2$$

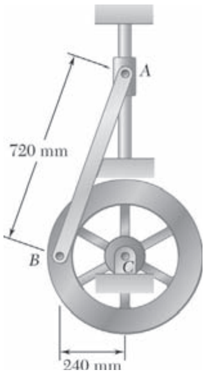
$$= 0.53333\, v_B^2$$

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 7.0632 + 4.8 = 0.53333 v_B^2 + 0$$

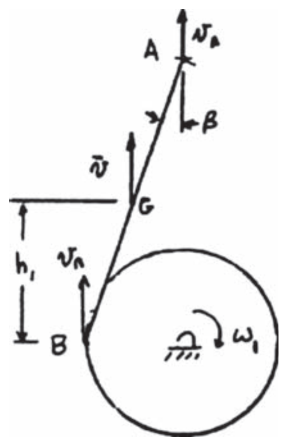
$$v_B = 4.7163 \text{ m/s} \qquad \mathbf{v}_B = 4.72 \text{ m/s} \downarrow \blacktriangleleft$$

PROBLEM 17.45



The 4-kg rod AB is attached to a collar of negligible mass at A and to a flywheel at B. The flywheel has a mass of 16 kg and a radius of gyration of 180 mm. Knowing that in the position shown the angular velocity of the flywheel is 60 rpm clockwise, determine the velocity of the flywheel when Point B is directly below C.

SOLUTION



Moments of inertia.

Rod AB:

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2$$
$$= \frac{1}{12}(4 \text{ kg})(0.72 \text{ m})^2$$
$$= 0.1728 \text{ kg} \cdot \text{m}^2$$

Flywheel:

$$I_C = m\bar{k}^2$$
$$= (16 \text{ kg})(0.18 \text{ m})^2$$
$$= 0.5184 \text{ kg} \cdot \text{m}^2$$

Position 1. As shown. $\boldsymbol{\omega} = \omega_1 \circlearrowright$

$$\sin\beta = \frac{0.12}{0.72} \qquad \beta = 19.471°$$
$$h_1 = \frac{1}{2}(0.72)\cos\beta = 0.33941 \text{ m}$$
$$V_1 = W_{AB} h_1$$
$$= (4)(9.81)(0.33941)$$
$$= 13.3185 \text{ J}$$

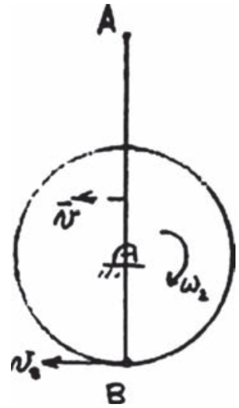
Kinematics. $v_B = r\omega_1 = 0.24\omega_1$

Bar AB is in translation. $\omega_{AB} = 0, \quad \bar{v} = v_B$

$$T_1 = \frac{1}{2} m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2} I_C \omega_1^2$$
$$= \frac{1}{2}(4)(0.24\omega_1)^2 + 0 + \frac{1}{2}(0.5184)\omega_1^2$$
$$= 0.3744\omega_1^2$$

PROBLEM 17.45 (Continued)

Position 2. Point *B* is directly below *C*.



$$h_2 = \frac{1}{2}L_{AB} - r$$
$$= \frac{1}{2}(0.72) - 0.24$$
$$= 0.12 \text{ m}$$
$$V_2 = W_{AB}h_2$$
$$= (4)(9.81)(0.12)$$
$$= 4.7088 \text{ J}$$

Kinematics.

$$v_B = r\omega_2 = 0.24\omega_2$$
$$\omega_{AB} = \frac{v_B}{0.72} = 0.33333\omega_2$$
$$\bar{v} = \frac{1}{2}v_B = 0.12\omega_2$$
$$T_2 = \frac{1}{2}m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}I_C\omega_2^2$$
$$= \frac{1}{2}(4)(0.12\omega_2)^2 + \frac{1}{2}(0.1728)(0.33333\omega_2)^2 + \frac{1}{2}(0.5184)\omega_2^2$$
$$= 0.2976\omega_2^2$$

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0.3744\omega_1^2 + 13.3185 = 0.2976\omega_2^2 + 4.7088 \qquad (1)$$

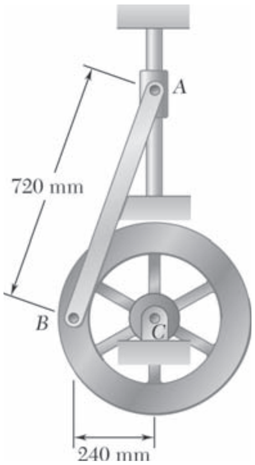
Angular speed data:

$$\omega_1 = 60 \text{ rpm} = 2\pi \text{ rad/s}$$

Solving Equation (1) for ω_2,

$$\omega_2 = 8.8655 \text{ rad/s}$$

$\boldsymbol{\omega}_2 = 84.7 \text{ rpm}$ ◄

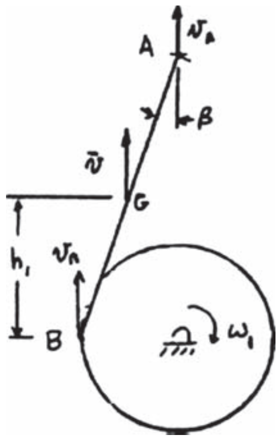


PROBLEM 17.46

If in Problem 17.45 the angular velocity of the flywheel is to be the same in the position shown and when Point B is directly above C, determine the required value of its angular velocity in the position shown.

SOLUTION

Moments of inertia.



Rod AB:

$$\bar{I}_{AB} = m_{AB}L_{AB}^2$$
$$= \frac{1}{12}4\ \text{kg}(0.72\ \text{m})^2$$
$$= 0.1728\ \text{kg}\cdot\text{m}^2$$

Flywheel:

$$I_C = mk^2$$
$$= 16\ \text{kg}(0.18\ \text{m})^2$$
$$= 0.5184\ \text{kg}\cdot\text{m}^2$$

Position 1. As shown.

$$\boldsymbol{\omega} = \omega_1 \circlearrowright$$

$$\sin\beta = \frac{0.12}{0.72}$$
$$\beta = 19.471°$$
$$h_1 = \frac{1}{2}0.72\cos\beta = 0.33941\ \text{m}$$
$$V_1 = W_{AB}h_1 = (4)(9.81)(0.33941)$$
$$= 13.3185$$

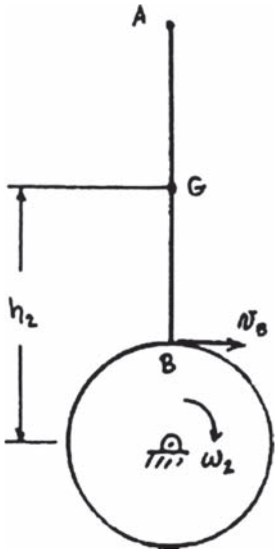
Kinematics.

$$v_B = r\omega_1 = 0.24\omega_1$$

PROBLEM 17.46 (Continued)

Bar AB is in translation. $\omega_{AB} = 0, \quad \bar{v} = v_B$

$$T_1 = \frac{1}{2} m_{AB} \bar{v}^2 + \frac{1}{2} \bar{I}_{AB} \omega_{AB}^2 + \frac{1}{2} I_C \omega_1^2$$
$$= \frac{1}{2}(4)(0.24\omega_1)^2 + 0 + \frac{1}{2}(0.55901)\omega_1^2$$
$$= 0.3744\omega_1^2$$



Position 2. Point B is directly above C.

$$h_2 = \frac{1}{2} L_{AB} + r$$
$$= \frac{1}{2}(0.72) + 0.24$$
$$= 0.6 \text{ m}$$
$$V_2 = W_{AB} h_2$$
$$= (4)(9.81)(0.6)$$
$$= 23.544 \text{ J}$$

Kinematics. $v_B = r\omega_2 = 0.24\omega_2$

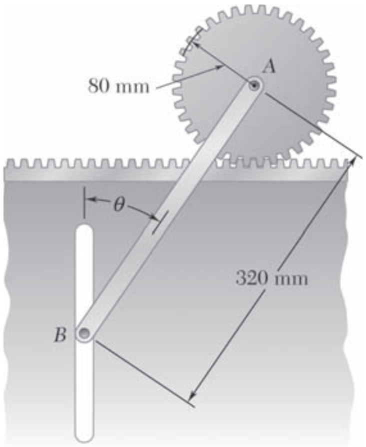
$$\omega_{AB} = \frac{v_B}{0.72} = 0.33333\omega_2$$
$$\bar{v} = \frac{1}{2} v_B = 0.12\omega_2$$
$$T_2 = \frac{1}{2} \frac{W_{AB}}{g} \bar{v}^2 + \frac{1}{2} \bar{I}_{AB} \omega_{AB}^2 + \frac{1}{2} I_C \omega_2^2$$
$$= \frac{1}{2} \frac{8}{32.2}(0.500\omega_2)^2 + \frac{1}{2}(0.186335)(0.33333\omega_2)^2$$
$$+ \frac{1}{2}(0.55901)\omega_2^2$$
$$= 0.320913\omega_2^2$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2: \quad 0.3744\omega_1^2 + 13.3135 = 0.2976\omega_2^2 + 23.544$ (1)

Angular speed data: $\omega_2 = \omega_1$

Then, $0.0760\omega_1^2 = +0.4105$

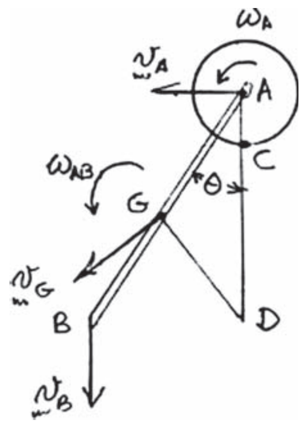
$\omega_1 = 11.602$ rad/s $\quad \boldsymbol{\omega}_1 = 110.8$ rpm ◄



PROBLEM 17.47

The 80-mm-radius gear shown has a mass of 5 kg and a centroidal radius of gyration of 60 mm. The 4-kg rod AB is attached to the center of the gear and to a pin at B that slides freely in a vertical slot. Knowing that the system is released from rest when $\theta = 60°$, determine the velocity of the center of the gear when $\theta = 20°$.

SOLUTION



Kinematics. $\mathbf{v}_A = v_A \leftarrow$

$\mathbf{v}_B = v_B \downarrow$

Point D is the instantaneous center of rod AB.

$$\omega_{AB} = \frac{v_A}{L\cos\theta}$$

$$v_B = (L\sin\theta)\omega_{AB} = v_A \tan\theta$$

$$v_G = \frac{L}{2}\omega_{AB} = \frac{v_A}{2\cos\theta}$$

Gear A effectively rolls with slipping, with Point C being the contact point.

$$v_C = 0$$

Angular velocity of gear $\omega_A = \dfrac{v_A}{r}.$

Potential energy: Use the level of the center of gear A as the datum.

$$V = -W_{AB}\left(\frac{L}{2}\cos\theta\right) = -\frac{1}{2}m_{AB}gL\cos\theta$$

Kinetic energy: $T = \dfrac{1}{2}m_A v_A^2 + \dfrac{1}{2}I_A\omega_A^2 + \dfrac{1}{2}m_{AB}v_\theta^2 + \dfrac{1}{2}\bar{I}_{AB}\omega_{AB}^2$

Masses and moments of inertia: $m_A = 5 \text{ kg}, \quad m_{AB} = 4 \text{ kg}$

$$I_A = m_A k^2 = (5)(0.060)^2 = 0.018 \text{ kg}\cdot\text{m}^2$$

$$\bar{I}_{AB} = \frac{1}{12}m_{AB}L^2 = \frac{1}{12}(4)(0.320)^2 = 0.03413 \text{ kg}\cdot\text{m}^2$$

PROBLEM 17.47 (Continued)

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

Position 1: $\theta = 60°$

$$v_A = 0 \quad T_1 = 0$$

$$V_1 = -\frac{1}{2}(4)(9.81)(0.320)\cos 60°$$

$$= -3.1392 \text{ J}$$

Position 2: $\theta = 20° \quad v_A = ?$

$$T_2 = \frac{1}{2}(5)v_A^2 + \frac{1}{2}(0.018)\left(\frac{v_A}{0.080}\right)^2 + \frac{1}{2}(4)\left(\frac{v_A}{2\cos 20°}\right)^2$$

$$+\frac{1}{2}(0.03413)\left(\frac{v_A}{0.320\cos 20°}\right)^2$$

$$= (2.5 + 1.40625 + 0.56624 + 0.18875)v_A^2$$

$$= 4.66124 v_A^2$$

$$V_2 = \frac{1}{2}(4)(9.81)(0.320)\cos 20°$$

$$= -5.8998 \text{ J}$$

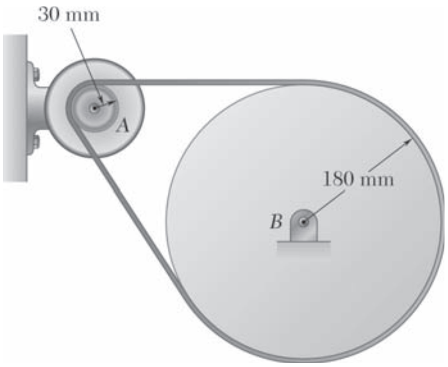
Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$0 - 3.1392 = 4.66124 v_A^2 - 5.8998$$

$$v_A^2 = 0.59225 \text{ m}^2/\text{s}^2$$

$$v_A = 0.770 \text{ m/s}$$

$\mathbf{v}_A = 0.770 \text{ m/s} \leftarrow$ ◀



PROBLEM 17.48

The motor shown rotates at a frequency of 22.5 Hz and runs a machine attached to the shaft at *B*. Knowing that the motor develops 3 kW, determine the magnitude of the couple exerted (*a*) by the motor on pulley *A*, (*b*) by the shaft on pulley *B*.

SOLUTION

$$\omega_A = 22.5\ \text{Hz}\left(\frac{2\pi\ \text{rad}}{\text{cycle}}\right)$$

$$= 45\pi\ \text{rad/s}$$

$$r_A\omega_A = r_B\omega_B: \quad (0.03\ \text{m})(45\pi\ \text{rad/s}) = (0.180\ \text{m})\omega_B$$

$$\omega_B = 7.5\pi\ \text{rad/s}$$

(*a*) Pulley *A*:

$$\text{Power} = M_A\omega_A$$

$$3000\ \text{W} = M_A(45\pi\ \text{rad/s})$$

$$M_A = 21.2\ \text{N}\cdot\text{m} \quad ◀$$

(*b*) Pulley *B*:

$$\text{Power} = M_B\omega_B$$

$$3000\ \text{W} = M_B(7.5\pi\ \text{rad/s})$$

$$M_B = 127.3\ \text{N}\cdot\text{m} \quad ◀$$

PROBLEM 17.49

Knowing that the maximum allowable couple that can be applied to a shaft is $2000\,\text{N}\cdot\text{m}$ determine the maximum power (m kW) that can be transmitted by the shaft at (*a*) 180 rpm, (*b*) 480 rpm.

SOLUTION

$$M = 2000\ \text{N}\cdot\text{m}$$

(*a*)

$$\omega = 180\ \text{rpm}\left(\frac{2\pi}{60}\right)$$
$$= 6\pi\ \text{rad/s}$$
$$\text{Power} = M\omega$$
$$= (2000\ \text{N}\cdot\text{m})(6\pi\ \text{rad/s}) = 37699.1\ \text{W}$$
$$= 37.699\ \text{kW}$$

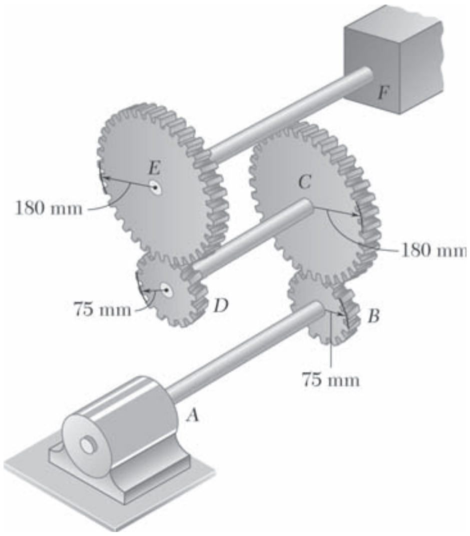
37.7 kW ◀

(*b*)

$$\omega = 480\ \text{rpm}\left(\frac{2\pi}{60}\right)$$
$$= 16\pi\ \text{rad/s}$$
$$\text{Power} = M\omega$$
$$= (2000\ \text{N}\cdot\text{m})(16\pi\ \text{rad/s}) = 100531\ \text{W}$$
$$= 100.531\ \text{kW}$$

100.5 kW ◀

PROBLEM 17.50



Three shafts and four gears are used to form a gear train which will transmit 7.5 kW from the motor at A to a machine tool at F. (Bearings for the shafts are omitted in the sketch.) Knowing that the frequency of the motor is 30 Hz, determine the magnitude of the couple which is applied to shaft (a) AB, (b) CD, (c) EF.

SOLUTION

Kinematics.

$$\omega_{AB} = 30 \text{ Hz} = 30(2\pi)\text{rad/s} = 60\pi \text{ rad/s}$$

Gears B and C.

$$r_B = 75 \text{ mm}$$
$$r_C = 180 \text{ mm}$$

$$r_B\omega_{AB} = r_C\omega_{CD}: \quad (75 \text{ mm})(60\pi \text{ rad/s}) = (180 \text{ mm})(\omega_{CD})$$

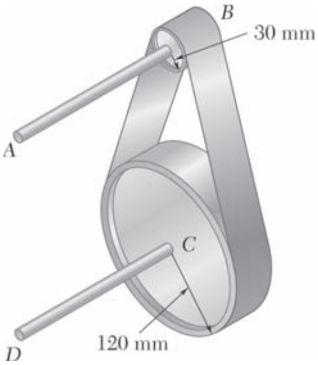
Gears D and E.

$$\omega_{CD} = 25\pi \text{ rad/s}$$
$$r_D = 75 \text{ mm}$$
$$r_E = 180 \text{ mm}$$
$$r_D\omega_{CD} = r_E\omega_{EF}: \quad (75 \text{ mm})(25\pi \text{ rad/s}) = (180 \text{ mm})(\omega_{EF})$$
$$\omega_{EF} = 10.4167\pi \text{ rad/s}$$
$$\text{Power} = 7.5 \text{ kW}$$

(a) <u>Shaft AB</u>. $\quad \text{Power} = M_{AB}\omega_{AB}: \quad 7500 \text{ W} = M_{AB}(60\pi \text{ rad/s}) \qquad M_{AB} = 39.8 \text{ N}\cdot\text{m}$ ◄

(b) <u>Shaft CD</u>. $\quad \text{Power} = M_{CD}\omega_{CD}: \quad 7500 \text{ W} = M_{CD}(25\pi \text{ rad/s}) \qquad M_{CD} = 95.5 \text{ N}\cdot\text{m}$ ◄

(c) <u>Shaft EF</u>. $\quad \text{Power} = M_{EF}\omega_{EF}: \quad 7500 \text{ W} = M_{EF}(10.4167\pi \text{ rad/s}) \qquad M_{EF} = 229 \text{ N}\cdot\text{m}$ ◄



PROBLEM 17.51

The shaft-disk-belt arrangement shown is used to transmit 2.4 kW from Point A to Point D. Knowing that the maximum allowable couples that can be applied to shafts AB and CD are $25\ \text{N}\cdot\text{m}$ and $80\ \text{N}\cdot\text{m}$, respectively, determine the required minimum speed of shaft AB.

SOLUTION

Power.

$$2.4\ \text{kW} = 2400\ \text{W}$$

$$M_{AB} < 25\ \text{N}\cdot\text{m}$$

$$P = M_{AB}\omega_{AB}$$

$$\min\omega_{AB} = \frac{P}{\max M_{AB}} = \frac{2400}{25} = 96\ \text{rad/s}$$

$$M_{CD} < 80\ \text{N}\cdot\text{m}$$

$$P = M_{CD}\omega_{CD}$$

$$\min\omega_{CD} = \frac{P}{\max M_{CD}} = \frac{2400}{80} = 30\ \text{rad/s}$$

Kinematics.

$$r_A\omega_{AB} = r_C\omega_{CD}$$

$$\min\omega_{AB} = \frac{r_C}{r_A}(\min\omega_{CD})$$

$$= \left(\frac{120}{30}\right)(30)$$

$$= 120\ \text{rad/s}$$

Choose the larger value for $\min\omega_{AB}$. $\quad \min\omega_{AB} = 120\ \text{rad/s} \quad\quad \min\omega_{AB} = 1146\ \text{rpm}$ ◄

PROBLEM 17.52

The rotor of an electric motor has a mass of 25 kg and a radius of gyration of 180 mm. It is observed that 4.2 min is required for the rotor to coast to rest from an angular velocity of 3600 rpm. Determine the average magnitude of the couple due to kinetic friction in the bearings of the motor.

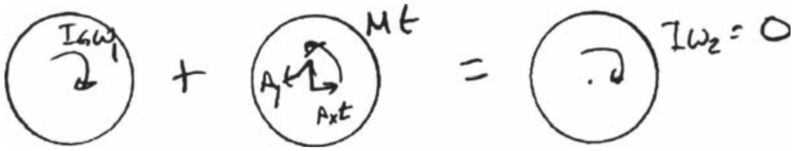
SOLUTION

Time. $t = 4.2 \text{ min} = 252 \text{ s}$

Moment of inertia.
$$I_G = m\bar{k}^2 = (25)(0.18 \text{ m})^2 = 0.81 \text{ kg}\cdot\text{m}^2$$

Angular velocities.
$$\omega_1 = 3600 \text{ rev/min} = 376.99 \text{ rad/s}$$
$$\omega_2 = 0$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about A. $I_G\omega_1 - Mt = 0$

Average magnitude of couple. $M = \dfrac{I_G\omega_1}{t} = \dfrac{(0.81 \text{ kg}\cdot\text{m}^2)(376.99 \text{ rad/s})}{252 \text{ s}}$ $M = 1.212 \text{ N}\cdot\text{m}$ ◀

PROBLEM 17.53

A 2000-kg flywheel with a radius of gyration of 700 mm is allowed to coast to rest from an angular velocity of 450 rpm. Knowing that kinetic friction produces a couple of magnitude 16 N·m determine the time required for the flywheel to coast to rest.

SOLUTION

Moment of inertia.

$$\bar{I} = m\bar{k}^2$$
$$= (2000 \text{ kg})(0.7 \text{ m})^2$$
$$= 980 \text{ kg}\cdot\text{m}^2$$

Angular velocities.

$$\omega_1 = 450 \text{ rpm}\left(\frac{2\pi}{60}\right)$$
$$= 47.125 \text{ rad/s}$$
$$M = 16 \text{ N}\cdot\text{m}$$

Principle of impulse and momentum.

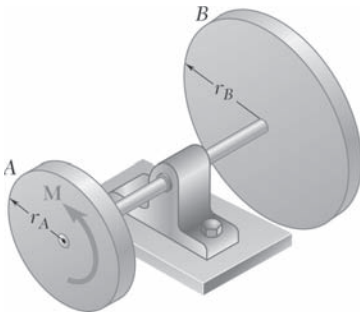
Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

Required time.

$\circlearrowright$ Moments about A:

$$\bar{I}\omega_1 - Mt = 0$$
$$t = \frac{I\omega_1}{M}$$
$$= \frac{(980 \text{ kg}\cdot\text{m}^2)(47.125 \text{ rad/s})}{16 \text{ N}\cdot\text{m}}$$
$$= 2886.4 \text{ s}$$
$$t = 2886.4 \text{ s}\left(\frac{\text{min}}{60 \text{ s}}\right)$$

$t = 48$ min 6 sec ◄



PROBLEM 17.54

Two disks of the same thickness and same material are attached to a shaft as shown. The 4-kg disk A has a radius $r_A = 100$ mm and disk B has a radius $r_B = 150$ mm. Knowing that a couple **M** of magnitude 2.5 N·m is applied to disk A when the system is at rest, determine the time required for the angular velocity of the system to reach 960 rpm.

SOLUTION

Mass of disk B.

$$m_B = \left(\frac{r_B}{r_A}\right)^2 m_A$$

$$= \left(\frac{0.15\text{ m}}{0.1\text{ m}}\right)^2 (4\text{ kg})$$

$$= 9\text{ kg}$$

Moment of inertia.

$$\bar{I} = \bar{I}_A + \bar{I}_B$$

$$= \frac{1}{2}(4\text{ kg})(0.1\text{ m})^2 + \frac{1}{2}(9\text{ kg})(0.15\text{ m})^2$$

$$= 0.12125\text{ kg}\cdot\text{m}^2$$

Angular velocity.

$$\omega_2 = 960\text{ rpm}\left(\frac{2\pi}{60}\right) = 100.53\text{ rad/s}$$

Moment.

$$M = 2.5\text{ N}\cdot\text{m}$$

Principle of impulse and momentum.

Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to2}$** = **Syst. Momenta$_2$**

$\circlearrowleft +$ Moments about C:

$$0 + Mt = \bar{I}\omega_2$$

Required time.

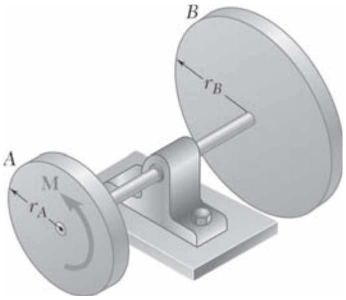
$$t = \frac{\bar{I}\omega_2}{M}$$

$$= \frac{(0.12125\text{ kg}\cdot\text{m}^2)(100.53\text{ rad/s})}{2.5\text{ N}\cdot\text{m}}$$

$$t = 4.8757\text{ s}$$

$t = 4.88$ s ◀

PROBLEM 17.55



Two disks of the same thickness and same material are attached to a shaft as shown. The 3-kg disk A has a radius $r_A = 100$ mm, and disk B has a radius $r_B = 125$ mm. Knowing that the angular velocity of the system is to be increased from 200 rpm to 800 rpm during a 3-s interval, determine the magnitude of the couple **M** that must be applied to disk A.

SOLUTION

Mass of disk B.

$$m_B = \left(\frac{r_B}{r_A}\right)^2 m_A$$

$$= \left(\frac{125 \text{ mm}}{100 \text{ mm}}\right)^2 3 \text{ kg}$$

$$= 4.6875 \text{ kg}$$

Moment of inertia.

$$\bar{I} = \bar{I}_A + \bar{I}_B$$

$$= \frac{1}{2}(3 \text{ kg})(0.1 \text{ m})^2 + \frac{1}{2}(4.6875 \text{ kg})(0.125 \text{ m})^2$$

$$= 0.05162 \text{ kg} \cdot \text{m}^2$$

Angular velocities.

$$\omega_1 = 200 \text{ rpm}\left(\frac{2\pi}{60}\right) = 20.944 \text{ rad/s}$$

$$\omega_2 = 800 \text{ rpm}\left(\frac{2\pi}{60}\right) = 83.776 \text{ rad/s}$$

Principle of impulse and momentum.

Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↺ Moments about B: $\bar{I}\omega_1 + Mt = \bar{I}\omega_2$

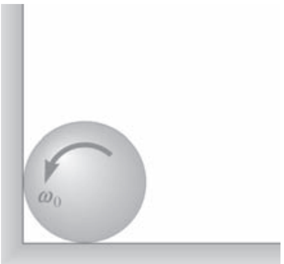
Couple M.

$$M = \frac{\bar{I}}{t}(\omega_2 - \omega_1)$$

$$= \frac{0.05162 \text{ kg} \cdot \text{m}^2}{3 \text{ s}}(83.776 \text{ rad/s} - 20.944 \text{ rad/s})$$

$M = 1.081 \text{ N} \cdot \text{m}$ ◀

PROBLEM 17.56



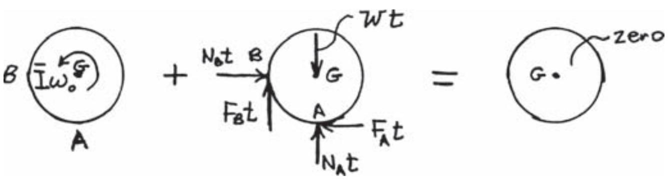
A cylinder of radius r and weight W with an initial counterclockwise angular velocity ω_0 is placed in the corner formed by the floor and a vertical wall. Denoting by μ_k the coefficient of kinetic friction between the cylinder and the wall and the floor derive an expression for the time required for the cylinder to come to rest.

SOLUTION

Since the cylinder's centre does not move, the center is the instantaneous center. Hence at both the contacts, the contact points move w.r.t. ground and wall and hence friction force at each contact = $\mu_k N$.

For the cylinder $\qquad \bar{I} = \frac{1}{2}mr^2, \quad W = mg$

Principle of impulse and momentum.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

Linear momentum $\xrightarrow{+}$: $\qquad 0 + N_B t - F_A t = 0$

$$N_B = F_A$$

Linear momentum $+\uparrow$: $\qquad 0 + N_A t + F_B t - Wt = 0$

$$N_A + F_B = N_A + \mu_k N_B$$

$$= N_A + \mu F_A + N_A + \mu_k^2 N_A = W$$

$$N_A = \frac{W}{1+\mu_k^2}$$

$$F_A = \mu_k N_A = \frac{\mu_k W}{1+\mu_k^2}$$

$$N_B = \frac{\mu_k W}{1+\mu_k^2}$$

$$F_B = \frac{\mu_k^2 W}{1+\mu_k^2}$$

$\circlearrowleft +$ Moments about G: $\qquad \bar{I}\omega_0 - F_A rt - F_B rt = 0$

$$t = \frac{\bar{I}\omega_0}{(F_A + F_B)r} = \frac{(1+\mu_k^2)\bar{I}\omega_0}{\mu_k(1+\mu_k)Wr} \qquad t = \frac{1+\mu_k^2}{2\mu_k(1+\mu_k)}\frac{r\omega_0}{g} \blacktriangleleft$$

PROBLEM 17.57

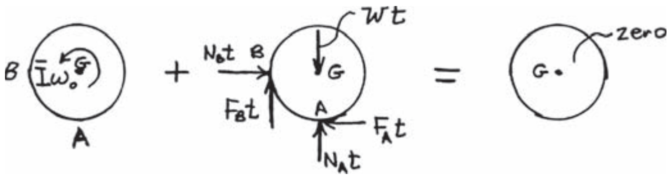
A 3-kg cylinder of radius $r = 125$ mm with an initial counterclockwise angular velocity $\omega_0 = 90$ rad/s is placed in the corner formed by the floor and a vertical wall. Knowing that the coefficient of kinetic friction is 0.10 between the cylinder and the wall and the floor, determine the time required for the cylinder to come to rest.

SOLUTION

Since the cylinder's centre does not move, the center is the instantaneous center. Hence at both the contacts, the contact points move w.r.t. ground and wall and hence friction force at each contact = $\mu_k N$.

For the cylinder $\qquad \bar{I} = \frac{1}{2}mr^2, \quad W = mg$

Principle of impulse and momentum.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

Linear momentum $\xrightarrow{+}$: $\quad 0 + N_Bt - F_At = 0$

$$N_B = F_A$$

Linear momentum $+\uparrow$: $\quad 0 + N_At + F_Bt - Wt = 0$

$$N_A + F_B = N_A + \mu_k N_B$$

$$= N_A + \mu F_A + N_A + \mu_k^2 N_A = W$$

$$N_A = \frac{W}{1+\mu_k^2}$$

$$F_A = \mu_k N_A = \frac{\mu_k W}{1+\mu_k^2}$$

$$N_B = \frac{\mu_k W}{1+\mu_k^2}$$

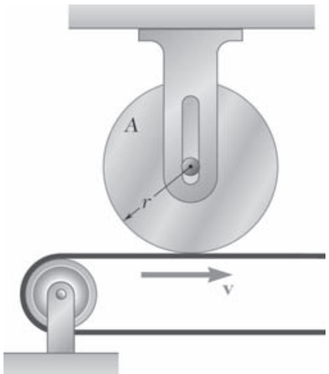
$$F_B = \frac{\mu_k^2 W}{1+\mu_k^2}$$

$\circlearrowleft +$ Moments about G: $\quad \bar{I}\omega_0 - F_Art - F_Brt = 0$

$$t = \frac{1+\mu_k^2}{2\mu_k(1+\mu_k)}\frac{r\omega_0}{g}$$

$$= \frac{1+(0.1)^2}{2(0.1)(1+0.1)}\frac{(0.125)(90)}{9.81}$$

$t = 5.26$ s ◀



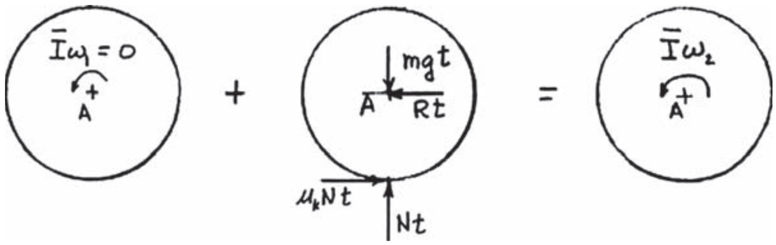
PROBLEM 17.58

A disk of constant thickness, initially at rest, is placed in contact with a belt that moves with a constant velocity **v**. Denoting by μ_k the coefficient of kinetic friction between the disk and the belt, derive an expression for the time required for the disk to reach a constant angular velocity.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{2}mr^2$

Final state of constant angular velocity. $\omega_2 = \frac{v}{r}$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

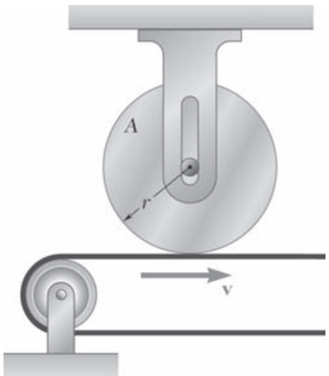
$+\uparrow y$ components: $0 + Nt - mgt = 0 \quad N = mg$

$\circlearrowleft +$ Moments about A: $0 + \mu_k Ntr = \bar{I}\omega_2$

$$t = \frac{\bar{I}\omega_2}{\mu_k mgr} = \frac{\frac{1}{2}mr^2\frac{v}{r}}{\mu_k mgr} = \frac{v}{2\mu_k g}$$

$$t = \frac{v}{2\mu_k g} \blacktriangleleft$$

PROBLEM 17.59

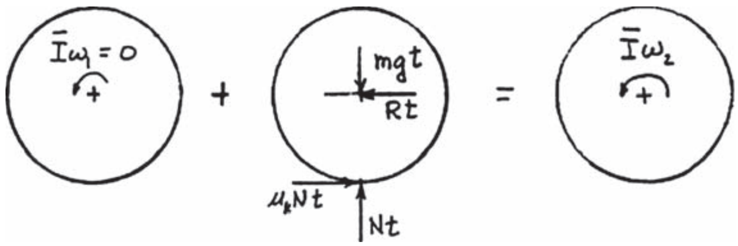


Disk A, of weight 2.5 kg and radius $r = 100$ mm is at rest when it is placed in contact with a belt which moves at a constant speed $v = 15$ m/s. Knowing that $\mu_k = 0.20$ between the disk and the belt, determine the time required for the disk to reach a constant angular velocity.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{2}mr^2$

Final state of constant angular velocity. $\omega_2 = \frac{v}{r}$



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

$+\uparrow y$ components: $0 + Nt - mgt = 0 \quad N = mg$

$+\circlearrowleft$ Moments about A: $0 + \mu_k Ntr = \bar{I}\omega_2$

$$t = \frac{\bar{I}\omega_2}{\mu_k mgr} = \frac{\frac{1}{2}mr^2 \frac{v}{r}}{\mu_k mgr} = \frac{v}{2\mu_k g}$$

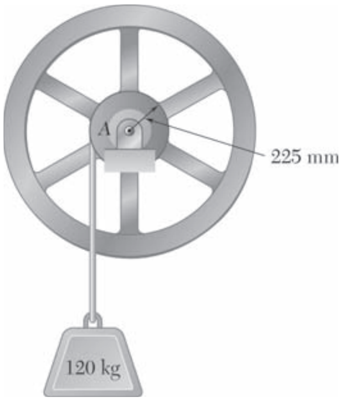
$$t = \frac{v}{2\mu_k g}$$

Data: $v = 15$ m/s

$\mu_k = 0.20$

$$t = \frac{15}{(2)(0.20)(9.81)}$$

$t = 3.82$ s ◄

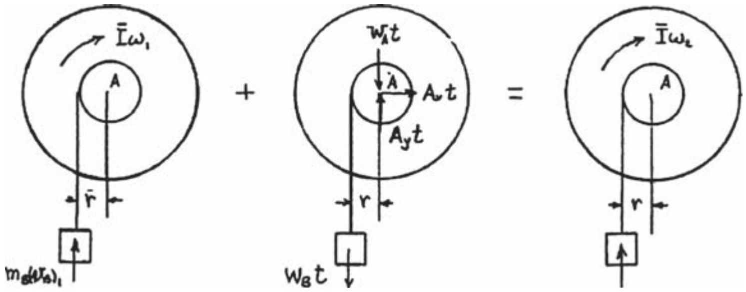


PROBLEM 17.60

The 350-kg flywheel of a small hoisting engine has a radius of gyration of 600 mm. If the power is cut off when the angular velocity of the flywheel is 100 rpm clockwise, determine the time required for the system to come to rest.

SOLUTION

Kinematics. $v_B = r\omega$



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

Moments about A: $\bar{I}_A\omega_1 + m_B(v_B)_1 r - W_B t r = \bar{I}_A\omega_2 + m_B(v_B)_2 r$

$$(m_A k^2 + m_B r^2)\omega_1 - m_B g t r = (m_A k^2 + m_B r^2)\omega_2$$

$$t = \frac{(m_A k^2 + m_B r^2)(\omega_1 - \omega_2)}{m_B r g}$$

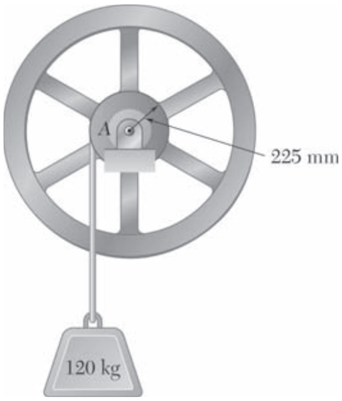
Data:

$$m_A = 350 \text{ kg}$$
$$k = 600 \text{ mm} = 0.6 \text{ m}$$
$$m_B = 120 \text{ kg}$$
$$r = 225 \text{ mm} = 0.225 \text{ m}$$
$$\omega_1 = 100 \text{ rpm} = 10.472 \text{ rad/s}$$
$$\omega_2 = 0$$

$$t = \frac{[(350)(0.6)^2 + (120)(0.225)^2](10.472 - 0)}{(120)(0.225)(9.81)}$$

$t = 5.22$ s ◀

PROBLEM 17.61

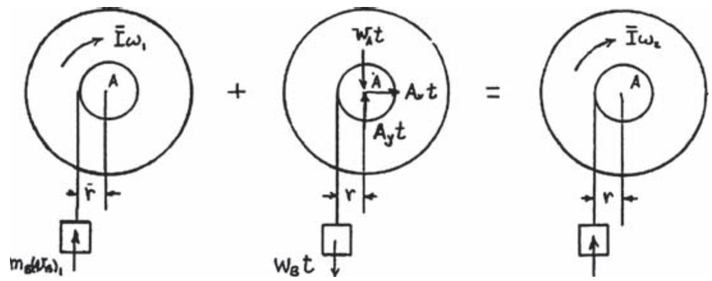


In Problem 17.60, determine the time required for the angular velocity of the flywheel to be reduced to 40 rpm clockwise.

PROBLEM 17.60 The 350-kg flywheel of a small hoisting engine has a radius of gyration of 600 mm. If the power is cut off when the angular velocity of the flywheel is 100 rpm clockwise, determine the time required for the system to come to rest.

SOLUTION

Kinematics. $v_B = r\omega$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowright$+ Moments about A: $\bar{I}_A\omega_1 + m_B(v_B)_1 r - W_B t r = \bar{I}_A\omega_2 + m_B(v_B)_2 r$

$$(m_A k^2 + m_B r^2)\omega_1 - m_B g t r = (m_A k^2 + m_B r^2)\omega_2$$

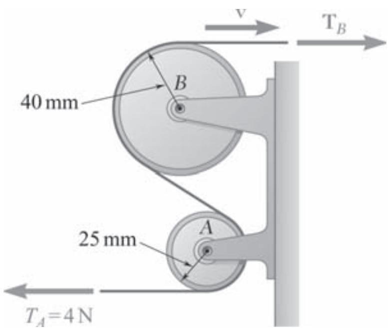
$$t = \frac{(m_A k^2 + m_B r^2)(\omega_1 - \omega_2)}{m_B r g}$$

Data:

$$m_A = 350 \text{ kg}$$
$$k = 600 \text{ mm} = 0.6 \text{ m}$$
$$m_B = 120 \text{ kg}$$
$$r = 225 \text{ mm} = 0.225 \text{ m}$$
$$\omega_1 = 100 \text{ rpm} = 10.472 \text{ rad/s}$$
$$\omega_2 = 40 \text{ rpm} = 4.189 \text{ rad/s}$$

$$t = \frac{[(350)(0.6)^2 + (120)(0.225)^2](10.472 - 4.189)}{(120)(0.225)(9.81)}$$

$t = 3.13 \text{ s}$ ◄



PROBLEM 17.62

A tape moves over the two drums shown. Drum A weighs 0.6 kg and has a radius of gyration of 20 mm, while drum B weighs 1.75 kg and has a radius of gyration of 30 mm. In the lower portion of the tape the tension is constant and equal to $T_A = 4\text{ N}$. Knowing that the tape is initially at rest, determine (a) the required constant tension T_B if the velocity of the tape is to be $v = 3\text{ m/s}$ after 0.24 s, (b) the corresponding tension in the portion of tape between the drums.

SOLUTION

Kinematics. Drums A and B rotate about fixed axes. Let v be the tape velocity in m/s.

$$v = r_A\omega_A = 0.025\,\omega_A \qquad \omega_A = 40\,v$$

$$v = r_B\omega_B = 0.04\,\omega_B \qquad \omega_B = 25\,v$$

Moments of inertia. $\bar{I}_A = m_A\bar{k}_A^2 = (0.6\text{ kg})(0.02\text{ m})^2 = 2.4\times10^{-4}\text{ kg}\cdot\text{m}^2$

$$\bar{I}_B = m_B\bar{k}_B^2 = (1.75\text{ kg})(0.03\text{ m})^2 = 15.75\times10^{-4}\text{ kg}\cdot\text{m}^2$$

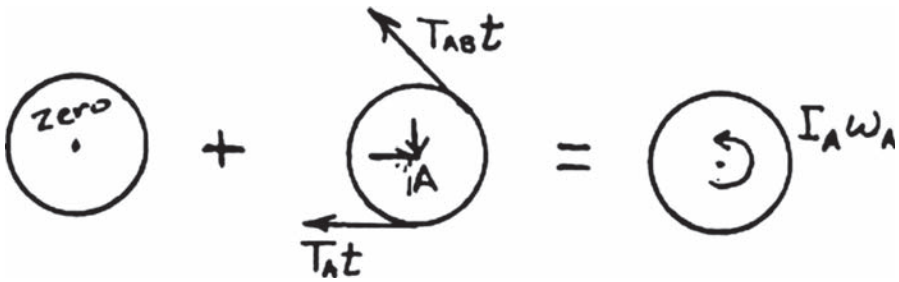
State 1. $t = 0$ $\quad v = 0$ $\quad \omega_A = \omega_B = 0$

State 2. $t = 0.24$ s, $\quad v = 3$ m/s

$$\omega_A = (40)(3) = 120\text{ rad/s}$$

$$\omega_B = (25)(3) = 75\text{ rad/s}$$

Drum A.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

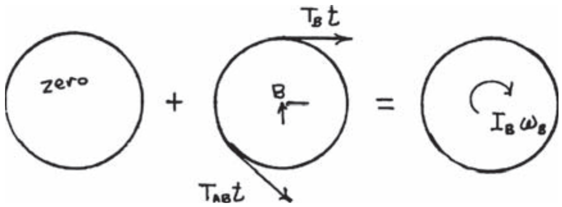
PROBLEM 17.62 (Continued)

$\circlearrowleft$+ Moments about A: $$0 + r_A T_{AB} t - r_A T_A t = I_A \omega_A$$

$$0 + (0.025)(T_{AB} t) - (0.025)(4)(0.24) = (2.4 \times 10^{-4})(120)$$

$$T_{AB} t = 2.112 \text{ N} \cdot \text{s}$$

Drum B.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+$\circlearrowright$ Moments about B: $$0 + r_B T_B t - r_B T_{AB} t = I_B \omega_B$$

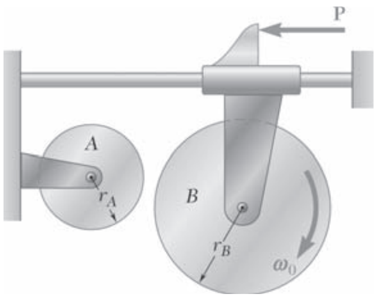
$$0 + (0.04)(T_B t) - (0.04)(2.112) = (15.75 \times 10^{-4})(75)$$

$$T_B t = 5.065125 \text{ N} \cdot \text{s}$$

(*a*) $$T_B = \frac{T_B t}{t} = \frac{5.065125}{0.24}$$ $T_B = 21.1 \text{ N}$ ◀

(*b*) $$T_{AB} = \frac{T_{AB} t}{t} = \frac{2.112}{0.24}$$ $T_{AB} = 8.80 \text{ N}$ ◀

PROBLEM 17.63



Disk B has an initial angular velocity ω_0 when it is brought into contact with disk A which is at rest. Show that the final angular velocity of disk B depends only on $\boldsymbol{\omega}_0$ and the ratio of the masses m_A and m_B of the two disks.

SOLUTION

Let Points A and B be the centers of the two disks and Point C be the contact point between the two disks.

Let ω_A and ω_B be the final angular velocities of disks A and B, respectively, and let v_C be the final velocity at C common to both disks.

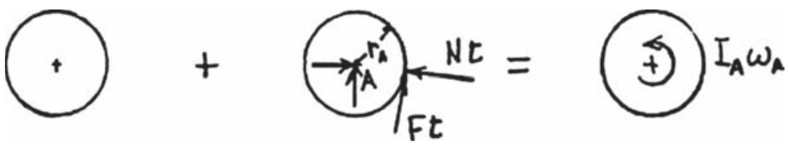
Kinematics: No slipping $\qquad v_C = r_A\omega_A = r_B\omega_B$

Moments of inertia. Assume that both disks are uniform cylinders.

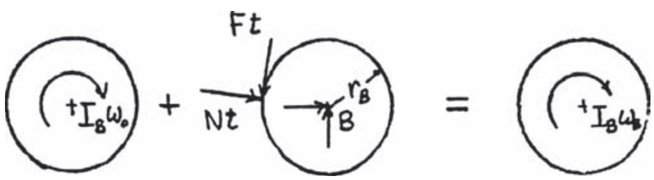
$$I_A = \frac{1}{2}m_A r_A^2 \qquad I_B = \frac{1}{2}m_B r_B^2$$

Principle of impulse and momentum.

Disk A



Disk B



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Disk A: $\circlearrowleft$ Moments about A: $\qquad 0 + r_A Ft = I_A\omega_A$

PROBLEM 17.63 (Continued)

$$Ft = \frac{I_A \omega_A}{r_A} = \frac{1}{2}\frac{m_A r_A^2 v_C}{r_A^2}$$

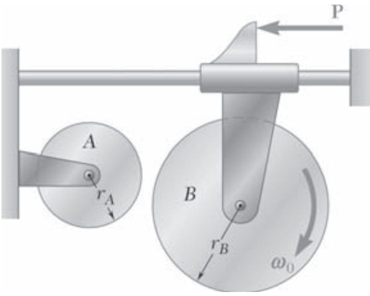
$$= \frac{1}{2} m_A v_C$$

$$= \frac{1}{2} m_A r_B \omega_B$$

Disk B: ↻ Moments about B: $I_B \omega_0 - r_B Ft = I_B \omega_B$

$$\frac{1}{2} m_B r_B^2 \omega_0 - r_B\left(\frac{1}{2} m_A r_B \omega_B\right) = \frac{1}{2} m r_B^2 \omega_B$$

$$\omega_B = \frac{\omega_0}{1 + \frac{m_A}{m_B}}$$ ◀



PROBLEM 17.64

The 4 kg disk A has a radius $r_A = 150$ mm and is initially at rest. The 5 kg disk B has a radius $r_B = 200$ mm and an angular velocity ω_0 of 900 rpm when it is brought into contact with disk A. Neglecting friction in the bearings, determine (*a*) the final angular velocity of each disk, (*b*) the total impulse of the friction force exerted on disk A.

SOLUTION

Let Points A and B be the centers of the two disks and Point C be the contact point between the two disks.

Let ω_A and ω_B be the final angular velocities of disks A and B, respectively, and let v_C be the final velocity at C common to both disks.

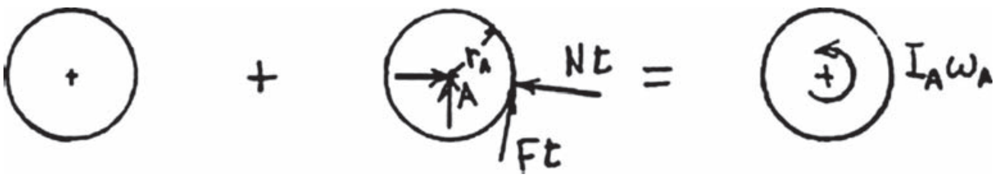
Kinematics: No slipping $\qquad v_C = r_A\omega_A = r_B\omega_B$

Moments of inertia. Assume that both disks are uniform cylinders.

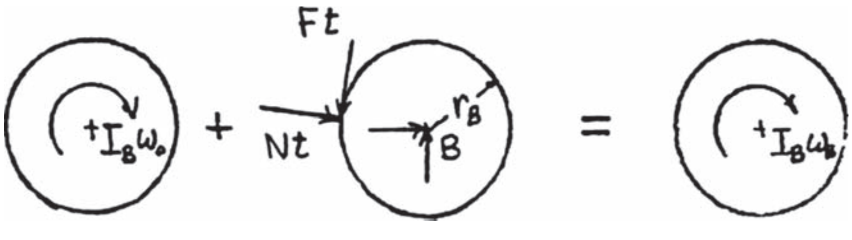
$$I_A = \frac{1}{2}m_A r_A^2 \qquad I_B = \frac{1}{2}m_B r_B^2$$

Principle of impulse and momentum.

Disk A



Disk B



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.64 (Continued)

Disk A: $\circlearrowleft$ Moments about A:

$$0 + r_A Ft = I_A \omega_A$$

$$Ft = \frac{I_A \omega_A}{r_A}$$

$$= \frac{1}{2}\frac{m_A r_A^2 v_C}{r_A^2} = \frac{1}{2} m_A v_C$$

$$= \frac{1}{2} m_A r_B \omega_B$$

Disk B: $\circlearrowright$ Moments about B:

$$I_B \omega_0 - r_B Ft = I_B \omega_B$$

$$\frac{1}{2} m_B r_B^2 \omega_0 - r_B \left(\frac{1}{2} m_A r_B \omega_B\right) = \frac{1}{2} m r_B^2 \omega_B$$

$$\omega_B = \frac{\omega_0}{1 + \frac{m_A}{m_B}}$$

Data:

$$m_A = 4\,\text{kg}$$

$$\frac{m_A}{m_B} = \frac{4}{5} = 0.8$$

$$r_B = 0.2\ \text{m}$$

$$\frac{r_B}{r_A} = \frac{200\ \text{mm}}{150\ \text{mm}} = \frac{4}{3}$$

$$\omega_0 = 900\ \text{rpm} = 30\pi\ \text{rad/s}$$

(a)

$$\omega_B = \frac{\omega_0}{1 + 0.8}$$

$$= \frac{30\pi}{1.8}$$

$$= 52.3599\ \text{rad/s}$$

$$\omega_A = \frac{r_B}{r_A}\omega_B$$

$$= \left(\frac{4}{3}\right)(52.3599)$$

$$= 69.8132\ \text{rad/s}$$

$\boldsymbol{\omega}_A = 667$ rpm $\circlearrowleft$ ◀

$\boldsymbol{\omega}_B = 500$ rpm $\circlearrowright$ ◀

(b)

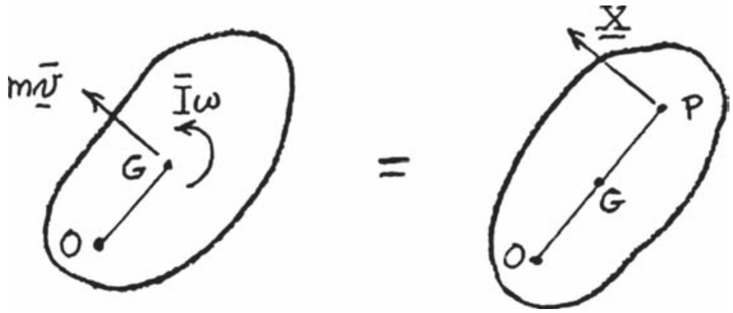
$$Ft = \left(\frac{1}{2}\right)(4)(0.2)(52.3599)$$

$\mathbf{F}t = 20.9\ \text{N}\cdot\text{s}$ $\uparrow$ ◀

PROBLEM 17.65

Show that the system of momenta for a rigid slab in plane motion reduces to a single vector, and express the distance from the mass center G to the line of action of this vector in terms of the centroidal radius of gyration $\bar{k}$ of the slab, the magnitude $\bar{v}$ of the velocity of G, and the angular velocity ω.

SOLUTION

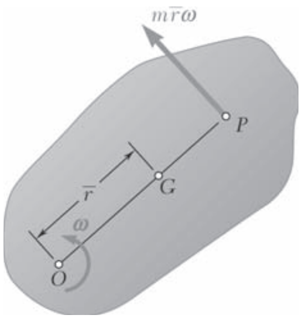


Syst. Momenta = **Single Vector**

↖ Components parallel to $m\bar{\mathbf{v}}$: $\quad m\bar{\mathbf{v}} = \mathbf{X}$ $\qquad \mathbf{X} = m\bar{\mathbf{v}}$ ◀

↺ Moments about G: $\quad \bar{I}\omega = (m\bar{v})d$

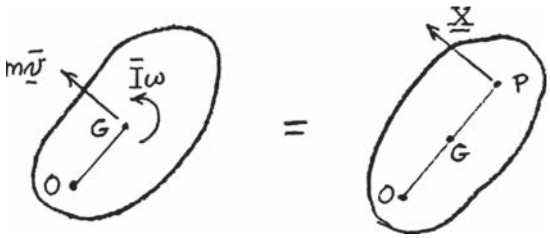
$$d = \frac{\bar{I}\omega}{m\bar{v}} = \frac{m\bar{k}^2\omega}{m\bar{v}} \qquad d = \frac{\bar{k}^2\omega}{\bar{v}} \blacktriangleleft$$



PROBLEM 17.66

Show that, when a rigid slab rotates about a fixed axis through O perpendicular to the slab, the system of the momenta of its particles is equivalent to a single vector of magnitude $m\bar{r}\omega$, perpendicular to the line OG, and applied to a Point P on this line, called the *center of percussion*, at a distance $GP = \bar{k}^2/\bar{r}$ from the mass center of the slab.

SOLUTION



Kinematics. Point O is fixed. $\bar{v} = \bar{r}\omega$

System momenta.

Components parallel to $m\bar{\mathbf{v}}$: $X = m\bar{v} = m\bar{r}\omega$ $\quad X = m\bar{r}\omega$ ◀

Moments about G: $(GP)X = \bar{I}\omega$

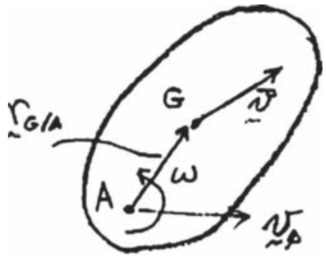
$$(GP)mr\omega = m\bar{k}^2\omega \qquad (GP) = \frac{\bar{k}^2}{\bar{r}}$$ ◀

PROBLEM 17.67

Show that the sum $\mathbf{H}_A$ of the moments about a Point A of the momenta of the particles of a rigid slab in plane motion is equal to $I_A\boldsymbol{\omega}$, where $\boldsymbol{\omega}$ is the angular velocity of the slab at the instant considered and I_A the moment of inertia of the slab about A, if and only if one of the following conditions is satisfied: (*a*) A is the mass center of the slab, (*b*) A is the instantaneous center of rotation, (*c*) the velocity of A is directed along a line joining Point A and the mass center G.

SOLUTION

Kinematics.

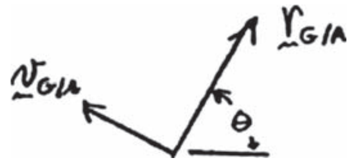


Let $\quad \omega = \omega\mathbf{k}$

and $\quad \mathbf{r}_{G/A} = r_{G/A} \measuredangle\, \theta$

Then, $\quad \mathbf{v}_{G/A} = \omega \times \mathbf{r}_{G/A} = (\omega r_{G/A}) \measuredangle\, \beta$

Where $\quad \beta = \theta + 90°$

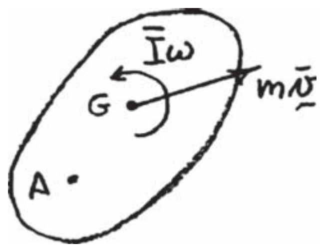


Also $\quad \bar{\mathbf{v}} = \mathbf{v}_A + \mathbf{v}_{G/A}$

Define $\quad \mathbf{h} = \mathbf{r}_{G/A} \times \mathbf{v}_{G/A}$

$$\mathbf{h} = (r_{G/A})(v_{G/A})\mathbf{k} = (r_{G/A})^2\omega\mathbf{k} = (r_{G/A})^2\omega$$

System momenta. Moments about A:



$$\begin{aligned}
\mathbf{H}_A &= \mathbf{r}_{G/A} \times m\bar{\mathbf{v}} + \bar{I}\omega \\
&= \mathbf{r}_{G/A} \times m(\mathbf{v}_A + \mathbf{v}_{G/A}) + \bar{I}\omega \\
&= \mathbf{r}_{G/A} \times m\mathbf{v}_A + m\mathbf{r}_{G/A} \times \mathbf{v}_{G/A} + \bar{I}\omega \\
&= \mathbf{r}_{G/A} \times m\mathbf{v}_A + m\mathbf{h} + \bar{I}\omega \\
&= \mathbf{r}_{G/A} \times m\mathbf{v}_A + mr_{G/A}^2\omega + \bar{I}\omega \\
&= \mathbf{r}_{G/A} \times m\mathbf{v}_A + (mr_{G/A}^2 + \bar{I})\omega
\end{aligned}$$

The first term on the right hand side is equal to zero if

(*a*) $\quad \mathbf{r}_{G/A} = 0 \quad$ (A is the mass center)

or (*b*) $\quad \mathbf{v}_A = 0 \quad$ (A is the instantaneous center of rotation)

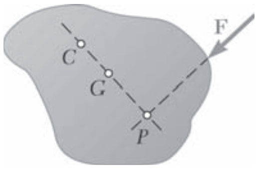
or (*c*) $\quad \mathbf{r}_{G/A}$ is perpendicular to $\mathbf{v}_A$.

In the second term, $\quad mr_{G/A}^2 + \bar{I} = I_A$

by the parallel axis theorem. Thus, $\quad \mathbf{H}_A = I_A\omega$

when one or more of the conditions (*a*), (*b*) or (*c*) is satisfied.

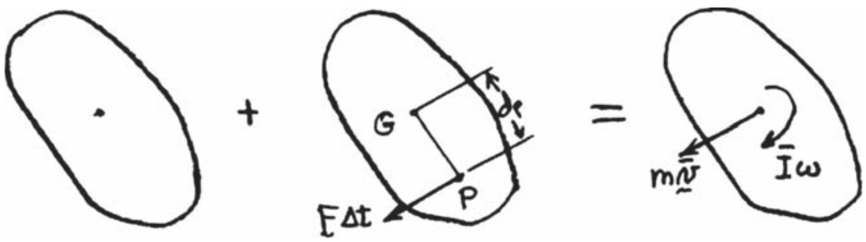
PROBLEM 17.68



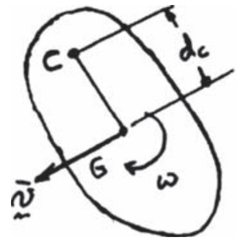
Consider a rigid slab initially at rest and subjected to an impulsive force **F** contained in the plane of the slab. We define the *center of percussion P* as the point of intersection of the line of action of **F** with the perpendicular drawn from G. (a) Show that the instantaneous center of rotation C of the slab is located on line GP at a distance $GC = \bar{k}^2/GP$ on the opposite side of G. (b) Show that if the center of percussion were located at C the instantaneous center of rotation would be located at P.

SOLUTION

(a) Locate the instantaneous center C corresponding to center of percussion P. Let $d_P = GP$.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$



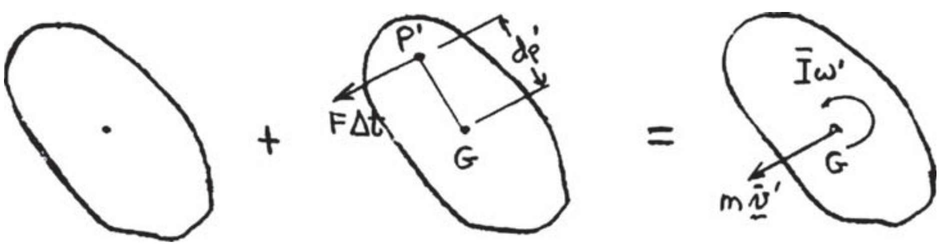
Components parallel to $\mathbf{F}\Delta t$: $0 + F\Delta t = m\bar{v}$

Moments about G: $0 + d_P(F\Delta t) = \bar{I}\omega$

Eliminate $F\Delta t$ to obtain
$$\frac{\bar{v}}{\omega} = \frac{\bar{I}}{md_P} = \frac{\bar{k}^2}{d_P}$$

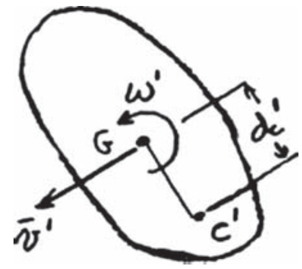
Kinematics. Locate Point C. $GC = d_C = \dfrac{\bar{v}}{\omega} = \dfrac{\bar{k}^2}{d_P}$ $\qquad GC = \dfrac{\bar{k}^2}{GP}$ ◀

(b) Place the center of percussion at $P' = C$. Locate the corresponding instantaneous center C'. Let
$$d_{P'} = GP' = GC = d_C.$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

PROBLEM 17.68 (Continued)



Components parallel to $\mathbf{F}\Delta t$: $0 + F\Delta t = m\bar{v}'$

Moments about G: $0 + d_{P'}(F\Delta t) = \bar{I}\omega'$

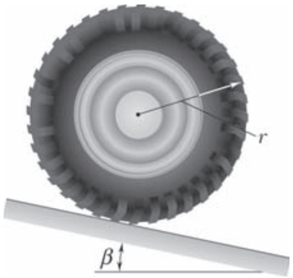
Eliminate $F\Delta t$ to obtain $\dfrac{\bar{v}'}{\omega'} = \dfrac{\bar{I}}{md_{P'}} = \dfrac{\bar{k}^2}{d_{P'}}$

Kinematics. Locate Point C'. $GC' = d_{C'} = \dfrac{\bar{v}'}{\omega'} = \dfrac{\bar{k}^2}{d_{P'}} = \dfrac{\bar{k}^2}{d_C}$

Using $d_C = d_{P'} = \dfrac{\bar{k}^2}{d_P}$ gives $d_{C'} = d_P$ or $GC' = GP$ ◀

Thus Point C' coincides with Point P.

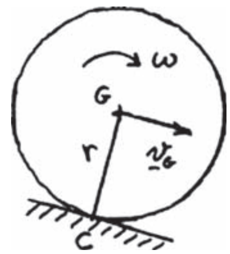
PROBLEM 17.69



A wheel of radius r and centroidal radius of gyration $\bar{k}$ is released from rest on the incline shown at time $t = 0$. Assuming that the wheel rolls without sliding, determine (*a*) the velocity of its center at time t, (*b*) the coefficient of static friction required to prevent slipping.

SOLUTION

Kinematics. Rolling motion. Instantaneous center at C.

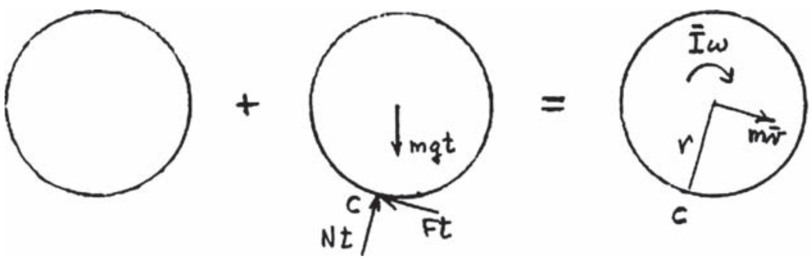


$$\bar{v} = v_G = r\omega$$

$$\omega = \frac{\bar{v}}{r}$$

Moment of inertia. $\quad \bar{I} = m\bar{k}^2$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

↻ moments about C: $\quad 0 + (mgt)r\sin\beta = m\bar{v}r + \bar{I}\,\omega$

$$(mgr\sin\beta)t = mr\bar{v} + \frac{m\bar{k}^2\bar{v}}{r}$$

(*a*) *Velocity of Point G*. $\quad \bar{\mathbf{v}} = \dfrac{r^2 gt\sin\beta}{r^2 + \bar{k}^2} \searrow \beta$ ◀

$+\searrow$ components parallel to incline:

$$0 + mgt\sin\beta - Ft = m\bar{v}$$

$$Ft = mgt\sin\beta - \frac{mr^2 gt\sin\beta}{r^2 + k^2}$$

$$= \frac{\bar{k}^2 mgt\sin\beta}{r^2 + k^2}$$

PROBLEM 17.69 (Continued)

$+\nearrow$ components normal to incline:

$$0 + Nt - mgt\cos\beta = 0$$

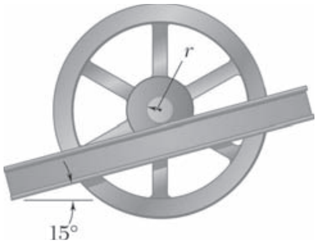
$$Nt = mgt\cos\beta$$

(*b*) *Required coefficient of static friction*.

$$\mu_s \geq \frac{F}{N}$$

$$= \frac{Ft}{Nt}$$

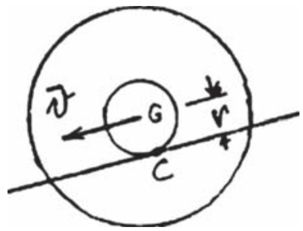
$$= \frac{\bar{k}^2 mgt\sin\beta}{(r^2 + \bar{k}^2)mgt\cos\beta} \qquad \mu_s \geq \frac{\bar{k}^2\tan\beta}{r^2 + \bar{k}^2} \blacktriangleleft$$



PROBLEM 17.70

A flywheel is rigidly attached to a 40 mm-radius shaft that rolls without sliding along parallel rails. Knowing that after being released from rest the system attains a speed of 150 mm/s in 30 s, determine the centroidal radius of gyration of the system.

SOLUTION



Kinematics. Rolling motion. Instantaneous center at C.

$$\bar{v} = v_G = r\omega$$

Moment of inertia. $\bar{I} = m\bar{k}^2$

Kinetics.

Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

↺ Moments about C: $0 + (mgt)r\sin\beta = m\bar{v}r + \bar{I}\omega$

$$mgtr\sin\beta = m\left(r + \frac{\bar{k}^2}{r}\right)\bar{v}$$

Solving for $\bar{k}^2$, $\bar{k}^2 = r^2\left(\frac{gt\sin\beta}{\bar{v}} - 1\right)$

Data: $r = 40 \text{ mm} = 0.04 \text{ m}$

$$g = 9.81 \text{ m/s}^2$$

$$t = 30 \text{ s}$$

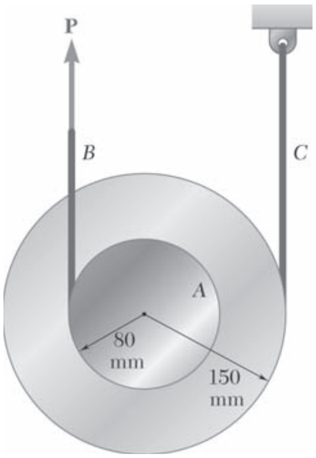
$$\bar{v} = 150 \text{ mm/s} = 0.15 \text{ m/s}$$

$$\bar{k}^2 = (0.04)^2\left[\frac{(9.81)(30)\sin 15^\circ}{0.15} - 1\right]$$

$$= 0.81088 \text{ m}^2$$

$$\bar{k}^2 = 0.90049 \text{ m}$$

$\bar{k} = 900$ mm ◀



PROBLEM 17.71

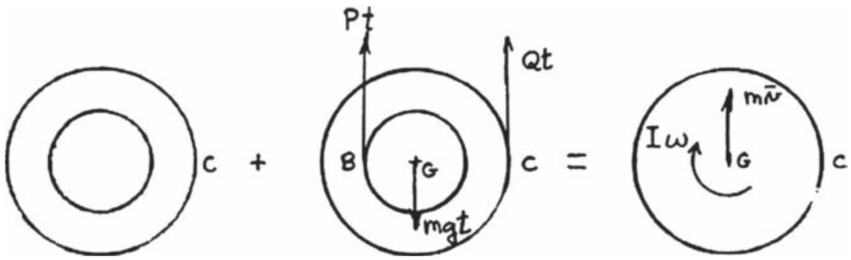
The double pulley shown has a mass of 3 kg and a radius of gyration of 100 mm. Knowing that when the pulley is at rest, a force **P** of magnitude 24 N is applied to cord B, determine (a) the velocity of the center of the pulley after 1.5 s, (b) the tension in cord C.

SOLUTION

For the double pulley,

$$r_C = 0.150 \text{ m}$$
$$r_B = 0.080 \text{ m}$$
$$k = 0.100 \text{ m}$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics. Point C is the instantaneous center. $\bar{v} = r_C\omega$

$\circlearrowleft$ Moments about C: $0 + Pt(r_C + r_B) - mgtr_C = \bar{I}\omega + m\bar{v}r_C$

$$= mk^2\omega + m(r_C\omega)r_C$$

$$\omega = \frac{Pt(r_C + r_B) - mgtr_C}{m\left(k^2 + r_C^2\right)}$$

$$= \frac{(24)(1.5)(0.230) - (3)(9.81)(1.5)(0.150)}{3(0.100^2 + 0.150^2)}$$

$$= 17.0077 \text{ rad/s}$$

PROBLEM 17.71 (Continued)

(*a*) $\bar{v} = (0.150)(17.0077) = 2.55115$ m/s $\qquad \bar{\mathbf{v}} = 2.55$ m/s $\uparrow$ ◀

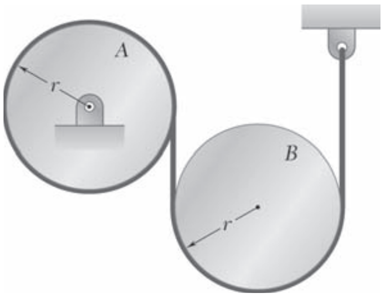
$+\uparrow$ Linear components: $\quad 0 + Pt - mgt + Qt = m\bar{v}$

$$Q = \frac{m\bar{v}}{t} + mg - P$$

$$= \frac{(3)(2.55115)}{1.5} + (3)(9.81) - 24$$

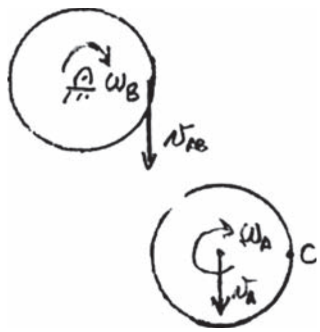
(*b*) *Tension in cord C.* $\qquad Q = 10.53$ N ◀

PROBLEM 17.72



Two uniform cylinders, each of mass $m = 7$ kg and radius $r = 100$ mm are connected by a belt as shown. If the system is released from rest when $t = 0$, determine (a) the velocity of the center of cylinder B at $t = 3$ s, (b) the tension in the portion of belt connecting the two cylinders.

SOLUTION



Kinematics. $\quad v_{AB} = r\omega_B$

Point C is the instantaneous center of cylinder A.

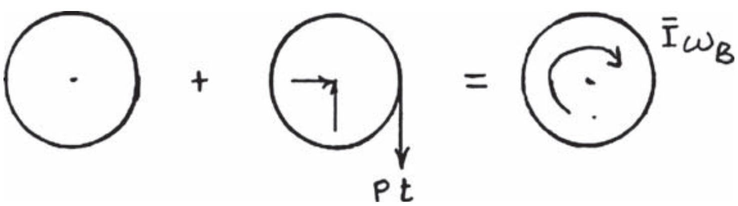
$$\omega_A = \frac{v_{AB}}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

Moment of inertia. $\quad \bar{I} = \frac{1}{2}mr^2$

(a) Velocity of the center of A.

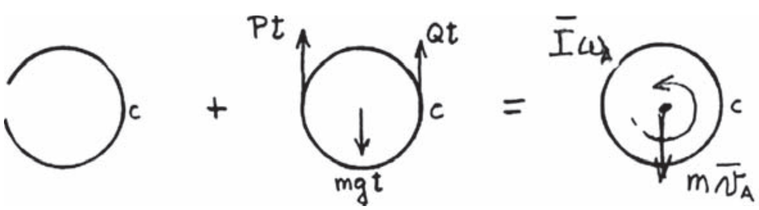
Cyl. B:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowright$ + Moments about B: $\quad 0 + Ptr = \bar{I}\omega_B \qquad (1)$

Cyl. A:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.72 (Continued)

Moments about C: $$0-2Ptr+mgtr=m\bar{v}_A r+\bar{I}\omega_A$$

$$0-2\bar{I}\omega_B-mgtr=m\left(\frac{1}{2}r\omega_B\right)r+\bar{I}\left(\frac{1}{2}\omega_B\right)$$

$$\left(\frac{5}{2}\bar{I}+\frac{1}{2}mr^2\right)\omega_B=mgrt$$

$$\left(\frac{5}{2}\frac{mr^2}{2}+\frac{1}{2}mr^2\right)\omega_B=mgrt$$

$$\frac{7}{4}r\omega_B=gt$$

$$\omega_B=\frac{4}{7}\frac{gt}{r} \qquad (2)$$

$$\bar{v}_A=\frac{1}{2}r\omega_B=\frac{2}{7}gt=\frac{2}{7}(9.81)(3)$$

$$=8.4086\text{ m/s}$$

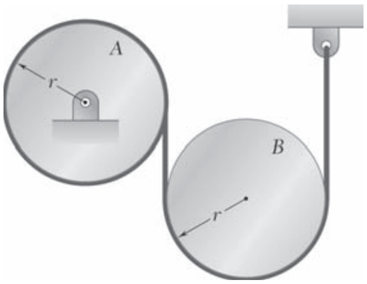
$\bar{\mathbf{v}}_A=8.41\text{ m/s}\downarrow$ ◀

(*b*) Tension in the belt.

From (1) and (2) $$Ptr=\bar{I}\left(\frac{4}{7}\frac{gt}{r}\right)$$

$$P=\frac{4}{7}\frac{\bar{I}g}{r^2}=\frac{4}{7}mg=\frac{4}{7}(7)(9.81)=39.24\text{ N}$$

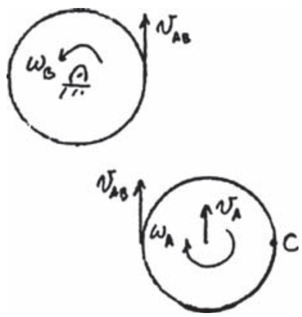
$P=39.2\text{ N}$ ◀



PROBLEM 17.73

Two uniform cylinders, each of mass $m = 7$ kg and radius $r = 100$ mm are connected by a belt as shown. Knowing that at the instant shown the angular velocity of cylinder A is 30 rad/s counterclockwise, determine (*a*) the time required for the angular velocity of cylinder A to be reduced to 5 rad/s, (*b*) the tension in the portion of belt connecting the two cylinders.

SOLUTION



Kinematics. $\qquad v_{AB} = r\omega_B$

Point C is the instantaneous center of cylinder A.

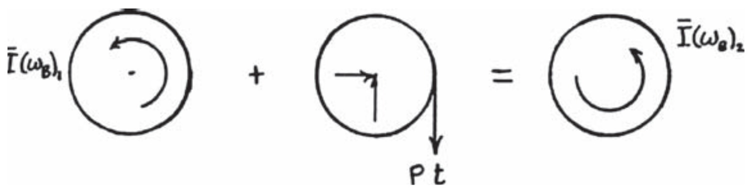
$$\omega_A = \frac{v_{AB}}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

Moment of inertia. $\qquad \bar{I} = \frac{1}{2}\frac{W}{g}r^2$

(*a*) *Required time*.

Cyl. *B*:



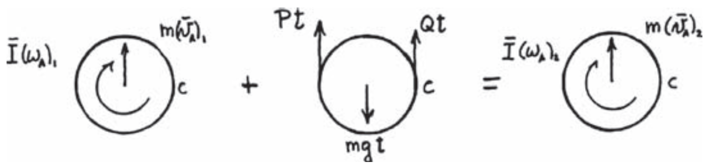
Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

Moments about B: $\qquad \bar{I}(\omega_B)_1 - Ptr = \bar{I}(\omega_B)_2$

$$Ptr = \bar{I}[(\omega_B)_1 - (\omega_B)_2]$$

$$= \frac{1}{2}mr^2[(\omega_B)_1 - (\omega_B)_2] \qquad (1)$$

Cyl. *A*:



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

PROBLEM 17.73 (Continued)

↻ Moments about C: $$\bar{I}(\omega_A)_1 + m(v_A)_1 r + 2Ptr - mgtr = \bar{I}(\omega_A)_2 + m(v_A)_2 r$$

$$\frac{1}{2}mr^2[(\omega_A)_1 - (\omega_A)_2 + mr[(\omega_A)_1 - (\omega_A)_2]r + 2Ptr - mgtr = 0$$

$$\frac{3}{2}mr^2\left[\left(\frac{1}{2}\omega_B\right)_1 - \frac{1}{2}(\omega_B)_2\right] + 2\left\{\frac{1}{2}mr^2[(\omega_B)_1 - (\omega_B)_2]\right\} - mgtr = 0$$

$$\frac{7}{4}mr^2[(\omega_B)_1 - (\omega_B)_2] - mgtr = 0$$

$$t = \frac{7r[(\omega_B)_1 - (\omega_B)_2]}{4g} \qquad (2)$$

Data: $m = 7$ kg

$r = 0.1$ m

From Equation (2), $$t = \frac{(7)(0.1)(30-5)}{(4)(9.81)} = 0.56612$$

$t = 0.446$ s ◀

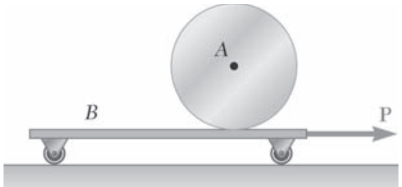
(*b*) *Tension in belt between cylinders*.

From Equation (1) and (2) $$P = \frac{1}{2}\cdot m \cdot \frac{4g}{7}$$

$$= \frac{1}{2}(7)\frac{(4)(9.81)}{7} = 19.62 \text{ N}$$

$P = 19.62$ N ◀

PROBLEM 17.74

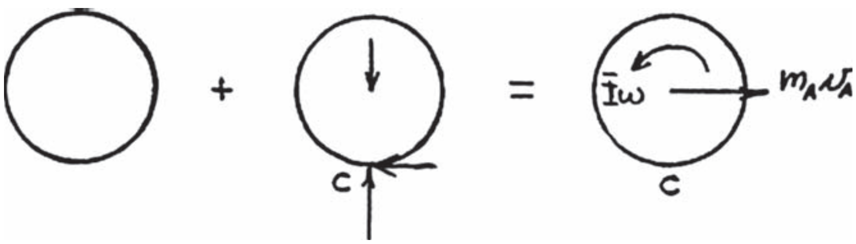


A 240-mm-radius cylinder of mass 8 kg rests on a 3-kg carriage. The system is at rest when a force **P** of magnitude 10 N is applied as shown for 1.2 s. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine the resulting velocity of (*a*) the carriage, (*b*) the center of the cylinder.

SOLUTION

Moment of inertia.
$$\bar{I} = \frac{1}{2} m_A r^2$$
$$= \frac{1}{2}(8\text{ kg})(0.24\text{ m})^2$$
$$= 0.2304\text{ kg}\cdot\text{m}^2$$

Cylinder alone:

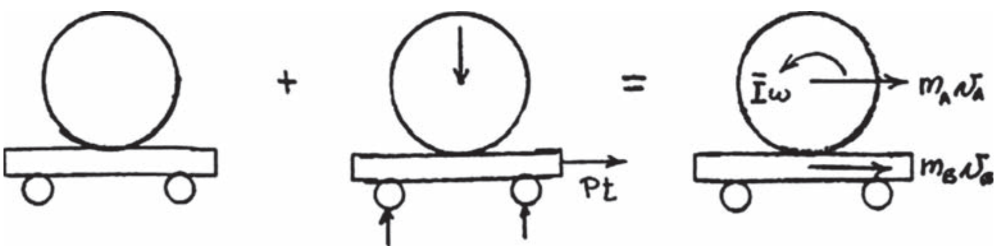


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

↺ Moments about *C*: $0 + 0 = \bar{I}\omega - m_A v_A r$

or $0 = 0.2304\omega - (8)(0.24)v_A$ (1)

Cylinder and carriage:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ Horizontal components: $0 + Pt = m_A v_A + m_B v_B$

or $0 + (10)(1.2) = 8v_A + 3v_B$ (2)

PROBLEM 17.74 (Continued)

Kinematics.

$$v_A = v_B - r\omega$$

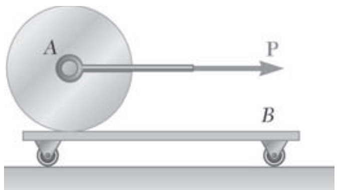
$$v_A = v_B - 0.24\omega \qquad (3)$$

Solving Equations (1), (2) and (3) simultaneously gives $\boldsymbol{\omega} = 5.68 \text{ rad/s}$ ↺

(*a*) Velocity of the carriage. $\mathbf{v}_B = 2.12 \text{ m/s} \rightarrow$ ◀

(*b*) Velocity of the center of the cylinder. $\mathbf{v}_A = 0.706 \text{ m/s} \rightarrow$ ◀

PROBLEM 17.75

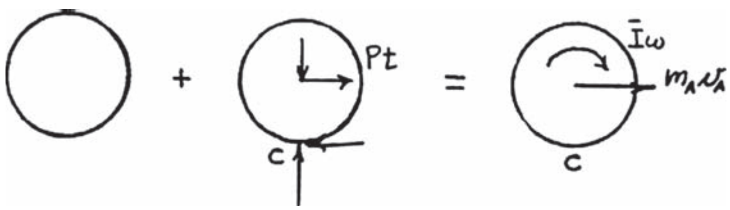


A 240-mm-radius cylinder of mass 8 kg rests on a 3-kg carriage. The system is at rest when a force **P** of magnitude 10 N is applied as shown for 1.2 s. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine the resulting velocity of (*a*) the carriage, (*b*) the center of the cylinder.

SOLUTION

Moment of inertia.
$$\bar{I} = \frac{1}{2}m_A r^2$$
$$= \frac{1}{2}(8\text{ kg})(0.24\text{ m})^2$$
$$= 0.2304\text{ kg}\cdot\text{m}^2$$

Cylinder alone:

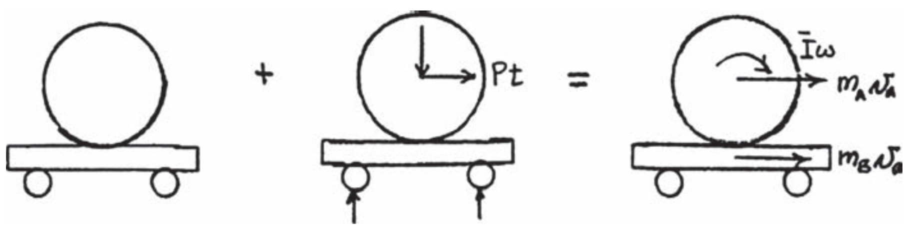


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about C:
$$0 + Ptr = \bar{I}\omega + m_A v_A r$$

or
$$0 + (10)(1.2)(0.24) = 0.2304\omega + (8)(0.24)v_A \qquad (1)$$

Cylinder and carriage:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ Horizontal components:
$$0 + Pt = m_A v_A + m_B v_B$$

or
$$0 + (10)(1.2) = 8v_A + 3v_B \qquad (2)$$

PROBLEM 17.75 (Continued)

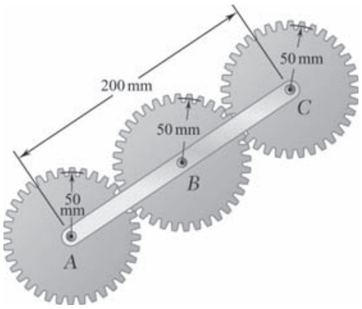
Kinematics.
$$v_A = v_B + r\omega$$
$$v_A = v_B + 0.24\omega \qquad (3)$$

Solving Equations (1), (2) and (3) simultaneously gives $\boldsymbol{\omega} = 2.21 \text{ rad/s}$ ↻

(*a*) Velocity of the carriage. $v_B = 0.706 \text{ m/s} \rightarrow$ ◀

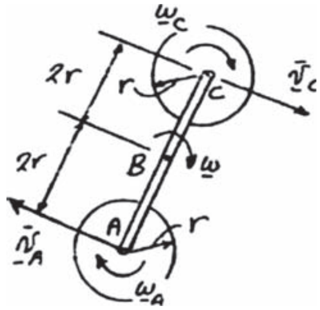
(*b*) Velocity of the center of the cylinder. $\mathbf{v}_A = 0.235 \text{ m/s} \rightarrow$ ◀

PROBLEM 17.76



In the gear arrangement shown, gears A and C are attached to rod ABC, which is free to rotate about B, while the inner gear B is fixed. Knowing that the system is at rest, determine the magnitude of the couple **M** which must be applied to rod ABC, if 2.5 s later the angular velocity of the rod is to be 240 rpm clockwise. Gears A and C have a mass of 1.25 kg each and may be considered as disks of radius 50 mm; rod ABC weighs 2 kg.

SOLUTION



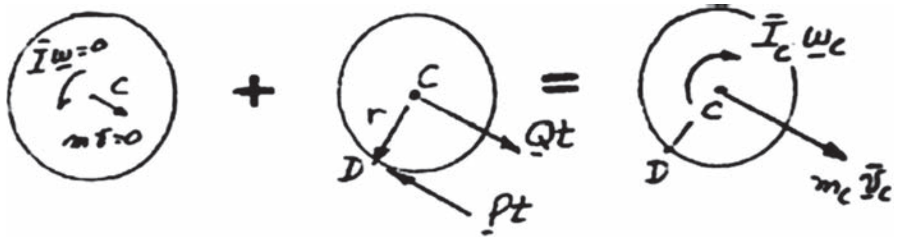
Kinematics of motion

Let $\omega_{ABC} = \omega$ $\qquad\qquad$ $\bar{v}_A = \bar{v}_C = (BC)\omega = 2r\omega$

Since gears A and C roll on the fixed gear B,

$$\omega_A = \omega_C = \frac{v_C}{r} = \frac{2r\omega}{r} = 2\omega$$

Principle of impulse and momentum.

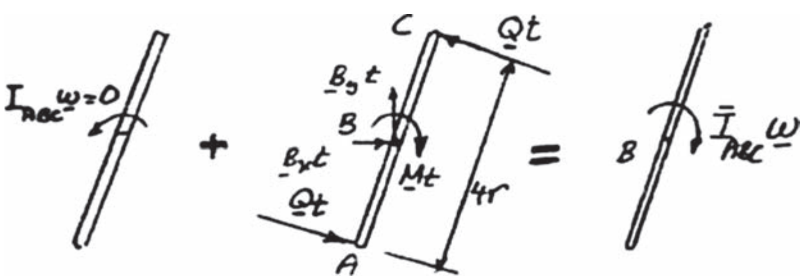


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowleft +$ Moments about D: $\qquad 0 + (Qt)r = m_C\bar{v}_C r + \bar{I}_C w_C$

$$(Qt)r = m_C(2r\omega)r + \frac{1}{2}m_C r^2(2\omega)$$

$$Qt = 3m_C rw \qquad (1)$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.76 (Continued)

Moments about B:

$$Mt - Qt(4r) = \bar{I}_{ABC}\omega$$

$$Mt - 4(Qt)r = \frac{1}{12}m_{ABC}(4r)^2\omega$$

$$Mt - 4(Qt)r = \frac{4}{3}m_{ABC}r^2\omega \qquad (2)$$

Substitute for (Qt) from (1) into (2):

$$Mt - 4(3m_C r\omega)r = \frac{4}{3}m_{ABC}r^2\omega$$

$$Mt = \frac{4}{3}r^2\omega(m_{ABC} + 9m_C) \qquad (3)$$

Couple M.

Data:

$$t = 2.5 \text{ s}$$

$$r = 50 \text{ mm} = 0.05 \text{ m}$$

$$m_{ABC} = 2 \text{ kg}$$

$$m_C = 1.25 \text{ kg}$$

$$\omega = 240 \text{ rpm}$$

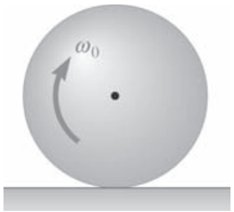
$$= 8\pi \text{ rad/s}$$

Eq. (3):

$$M(2.5 \text{ s}) = \frac{4}{3}(0.05)^2 (8\pi)\big(2 + 9(1.25)\big)$$

$$2.5 \text{ M} = 1.1100$$

$$M = 0.444 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 0.444 \text{ N}\cdot\text{m} \blacktriangleleft$$

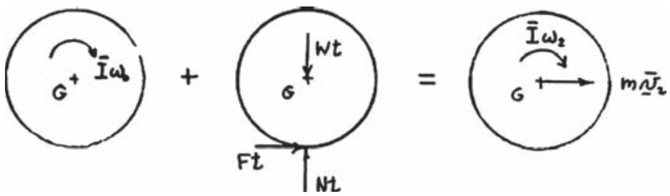


PROBLEM 17.77

A sphere of radius r and mass m is placed on a horizontal floor with no linear velocity but with a clockwise angular velocity $\boldsymbol{\omega}_0$. Denoting by μ_k the coefficient of kinetic friction between the sphere and the floor, determine (*a*) the time t_1 at which the sphere will start rolling without sliding, (*b*) the linear and angular velocities of the sphere at time t_1.

SOLUTION

Moment of inertia. Solid sphere. $\bar{I} = \frac{2}{5}mr^2$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

$+\uparrow$ y components: $Nt_1 - Wt_1 = 0 \qquad N = W = mg$ (1)

$\xrightarrow{+}$ x components: $Ft_1 = mv_2$ (2)

$\circlearrowright +$ Moments about G: $\bar{I}\omega_0 - Ft_1 r = \bar{I}\omega_2$ (3)

Since $F = \mu_k N = \mu_k mg$, Equation (2) gives

$$\mu_k mgt_1 = mv_2$$

or $$v_2 = \mu_k g t_1 \qquad (4)$$

Using the value for $\bar{I}$ in Equation (3),

$$\frac{2}{5}mr^2\omega_0 - \mu_k mgt_1 r = \frac{2}{5}mr^2\omega_2$$

or $$\omega_2 = \omega_0 - \frac{5}{2}\frac{\mu_k g t_1}{r} \qquad (5)$$

(*a*) *Time t_1 at which sliding stops.*

From kinematics, $$v_2 = r\omega$$

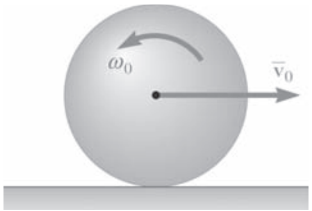
$$\mu_k g t_1 = r\omega_0 - \frac{5}{2}\mu_k g t_1 \qquad t_1 = \frac{2}{7}\frac{r\omega_0}{\mu_k g} \blacktriangleleft$$

PROBLEM 17.77 (Continued)

(b) *Linear and angular velocities.*

From Equation (4), $v_2 = \mu_k g \dfrac{2}{7} \dfrac{r\omega_0}{\mu_k g}$ $\qquad \mathbf{v}_2 = \dfrac{2}{7} r\omega_0 \rightarrow$ ◀

From Equation (6), $\omega_2 = \dfrac{v_2}{r} = \dfrac{2}{7}\omega_0$ $\qquad \boldsymbol{\omega}_2 = \dfrac{2}{7}\omega_0$ ↻ ◀

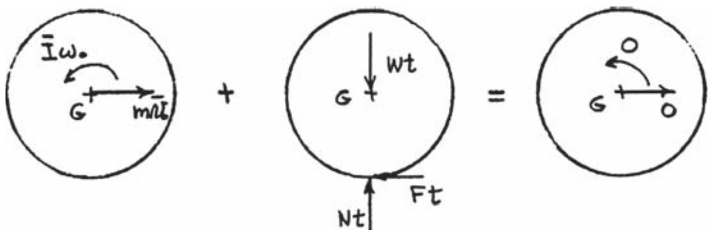


PROBLEM 17.78

A sphere of radius r and mass m is projected along a rough horizontal surface with the initial velocities shown. If the final velocity of the sphere is to be zero, express (a) the required magnitude of $\boldsymbol{\omega}_0$ in terms of v_0 and r, (b) the time required for the sphere to come to rest in terms of v_0 and coefficient of kinetic friction μ_k.

SOLUTION

Moment of inertia. Solid sphere. $\bar{I} = \dfrac{2}{5}mr^2$



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

$+\uparrow$ y components: $Nt - Wt = 0 \qquad N = W = mg$ (1)

$\xrightarrow{+}$ x components: $m\bar{v}_0 - Ft = 0 \qquad Ft = m\bar{v}_0$ (2)

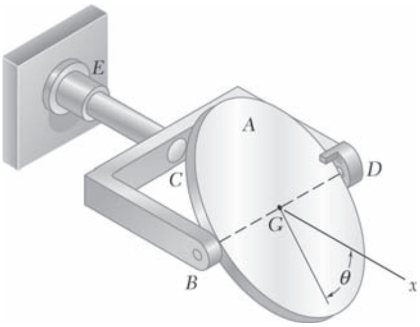
$\circlearrowleft +$ Moments about G: $\bar{I}\omega_0 - Ftr = 0$ (3)

$$\frac{2}{5}mr^2\omega_0 - m\bar{v}_0 r = 0$$

(a) *Solving for* ω_0, $\omega_0 = \dfrac{5}{2}\dfrac{\bar{v}_0}{r}$ ◀

(b) *Time to come to rest*.

From Equation (2), $t = \dfrac{mv_0}{F} = \dfrac{m\bar{v}_0}{\mu_k mg}$ $\qquad t = \dfrac{\bar{v}_0}{\mu_k g}$ ◀



PROBLEM 17.79

A 1.25-kg disk of radius 100 mm is attached to the yoke BCD by means of short shafts fitted in bearings at B and D. The 0.75 kg yoke has a radius of gyration of 75 mm about the x axis. Initially the assembly is rotating at 120 rpm with the disk in the plane of the yoke ($\theta = 0$). If the disk is slightly disturbed and rotates with respect to the yoke until $\theta = 90°$, where it is stopped by a small bar at D, determine the final angular velocity of the assembly.

SOLUTION

Moment of inertia of yoke: $$I_C = mk_C^2 = (0.75\,\text{kg})(0.075\,\text{m})^2 = 4.21875 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Moment of inertia of disk about x axis: $$\theta = 0: \quad I_A = \frac{1}{4}mr^2$$

$$= \frac{1}{4}(1.25)(0.1)^2 = 3.125 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$\theta = 90°: \quad I_A = \frac{1}{2}mr^2$$

$$= \frac{1}{2}(1.25)(0.1)^2 = 6.25 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Total moment of inertia about the x axis:

$$\theta = 0: \quad (I_x)_1 = I_C + I_A$$

$$= 7.34375 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$\theta = 90°: \quad (I_x)_2 = I_C + I_A$$

$$= 10.46875 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Angular momentum about the x axis:

$$\theta = 0: \quad H_1 = (I_x)_1\omega_1$$

$$= 7.34375 \times 10^{-3}\ \omega_1$$

$$\theta = 90°: \quad H_2 = (I_x)_2\omega_2$$

$$= 10.46875 \times 10^{-3}\omega_2$$

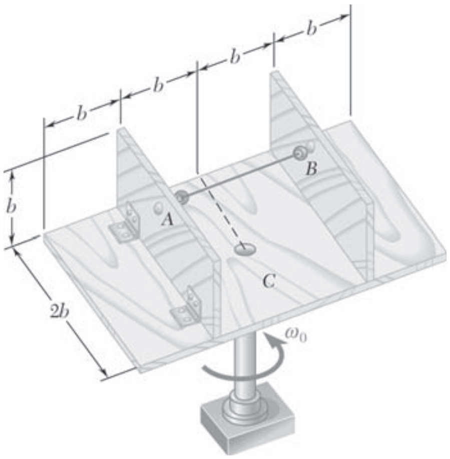
Conservation of angular momentum.

$$H_1 = H_2: \quad 7.34375 \times 10^{-3}\,\omega_1 = 10.46875 \times 10^{-3}\,\omega_2$$

$$\omega_2 = 0.7015\omega_1 = (0.7015)(120\,\text{rpm})$$

$\omega_2 = 84.2\ \text{rpm}$ ◀

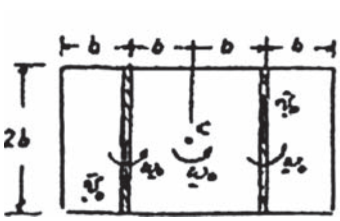
PROBLEM 17.80



Two panels A and B are attached with hinges to a rectangular plate and held by a wire as shown. The plate and the panels are made of the same material and have the same thickness. The entire assembly is rotating with an angular velocity $\boldsymbol{\omega}_0$ when the wire breaks. Determine the angular velocity of the assembly after the panels have come to rest against the plate.

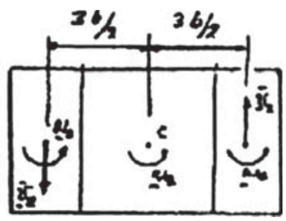
SOLUTION

Geometry and kinematics:



Panels in up position

$$\bar{v}_0 = b\omega_0$$



Panels in down position

$$\bar{v}_2 = \frac{3}{2} b\,\omega_0$$

Let ρ = mass density, t = thickness

Plate:

$$m_{\text{plate}} = \rho t(2b)(4b) = 8\rho t b^2$$

$$\bar{I}_{\text{plate}} = \frac{1}{12}(8\rho t b^2)[(2b)^2 + (4b)^2]$$

$$= \frac{160}{12}\rho t b^4$$

$$= \frac{40}{3}\rho t b^4$$

Each panel:

$$m_{\text{panel}} = \rho t(b)(2b) = 2\rho t b^2$$

<u>Panel in up position</u>

$$(\bar{I}_{\text{panel}})_0 = \frac{1}{12}(2\rho t\, b^2)(2b)^2$$

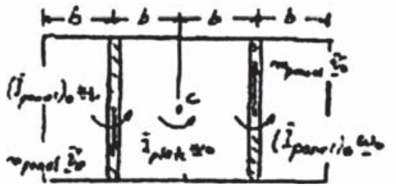
$$= \frac{8}{12}\rho\, t\, b^4 = \frac{2}{3}\rho t\, b^4$$

PROBLEM 17.80 (Continued)

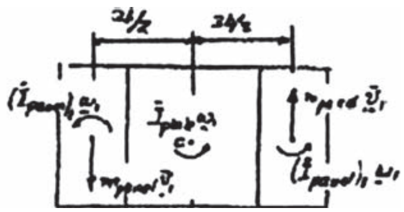
Panel in down position

$$(\bar{I}_{\text{panel}})_1 = \frac{1}{12}(2\rho t\, b^2)[b^2 + (2b)^2]$$
$$= \frac{10}{12}\rho t\, b^4$$
$$= \frac{5}{6}\rho t\, b^4$$

Conservation of angular momentum about the vertical spindle.



Initial momenta



Final momenta

$\circlearrowleft +$ Moments about C:

$$\bar{I}_{\text{plate}}\omega_0 + 2[(\bar{I}_{\text{panel}})_0\,\omega_0 + m_{\text{panel}}v_0(b)] = \bar{I}_{\text{plate}}\omega_1 + 2\left[(\bar{I}_{\text{panel}})_1\omega_1 + m_{\text{panel}}v_1\left(\frac{3b}{2}\right)\right]$$

$$\frac{40}{3}\rho t b^4\omega_0 + 2\left[\frac{2}{3}\rho t b^4\omega_0 + (2\rho t b^2)(b\omega_0)b\right] = \frac{40}{3}\rho t b^4\omega_0 + 2\left[\frac{5}{6}\rho t b^4\omega_1 + 2\rho t b^2\left(\frac{3}{2}b\omega_0\right)\left(\frac{3}{2}b\right)\right]$$

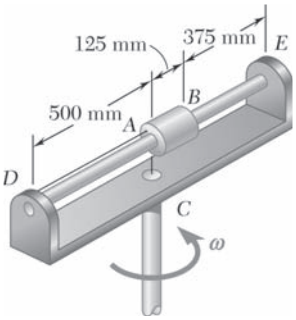
$$\left[\frac{40}{3} + \frac{4}{3} + 4\right]\rho t\, b^4\omega_0 = \left[\frac{40}{3} + \frac{10}{6} + 9\right]\rho t b^4\omega_1$$

$$\frac{56}{3}\omega_0 = 24\omega_1$$

$$\omega_1 = \frac{56}{(3)(24)}$$

$$\omega_1 = \frac{7}{9}\omega_2 \blacktriangleleft$$

PROBLEM 17.81



A 1.6-kg tube *AB* can slide freely on rod *DE* which in turn can rotate freely in a horizontal plane. Initially the assembly is rotating with an angular velocity $\omega = 5$ rad/s and the tube is held in position by a cord. The moment of inertia of the rod and bracket about the vertical axis of rotation is $0.30\ \text{kg}\cdot\text{m}^2$ and the centroidal moment of inertia of the tube about a vertical axis is $0.0025\ \text{kg}\cdot\text{m}^2$. If the cord suddenly breaks, determine (*a*) the angular velocity of the assembly after the tube has moved to end *E*, (*b*) the energy lost during the plastic impact at *E*.

SOLUTION

Let Point *C* be the intersection of axle *C* and rod *DE*. Let Point *G* be the mass center of tube *AB*.

Masses and moments of inertia about vertical axes.

$$m_{AB} = 1.6 \text{ kg}$$

$$\bar{I}_{AB} = 0.0025 \text{ kg}\cdot\text{m}^2$$

$$\bar{I}_{DCE} = 0.30 \text{ kg}\cdot\text{m}^2$$

State 1.

$$(r_{G/A})_1 = \frac{1}{2}(125)$$

$$= 62.5 \text{ mm}$$

$$\omega_1 = 5 \text{ rad/s}$$

State 2.

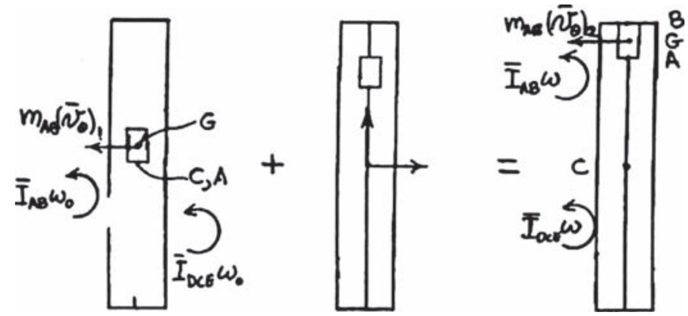
$$(r_{G/A})_2 = 500 - 62.5$$

$$= 437.5 \text{ mm}$$

$$\omega = \omega_2$$

Kinematics.

$$(v_G)_\theta = \bar{v}_\theta = r_{G/C}\omega$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.81 (Continued)

Moments about C:

$$\bar{I}_{AB}\omega_1 + \bar{I}_{DCE}\omega_1 + m_{AB}(\bar{v}_\theta)_1(r_{G/C})_1 + 0 = \bar{I}_{AB}\omega_2 + \bar{I}_{DCE}\omega_2 + m_{AB}(\bar{v}_\theta)_2(r_{G/C})_2$$

$$\left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/G})_1^2\right]\omega_1 = \left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/C})_2^2\right]\omega_2$$

$$[0.0025 + 0.30 + (1.6)(0.0625)^2](5) = [0.0025 + 0.30 + (1.6)(0.4375)^2]\omega_2$$

$$(0.30875)(5) = 0.60875\omega_2$$

$$\omega_2 = 2.5359 \text{ rad/s}$$

(a) *Angular velocity after the plastic impact.* 2.54 rad/s ◄

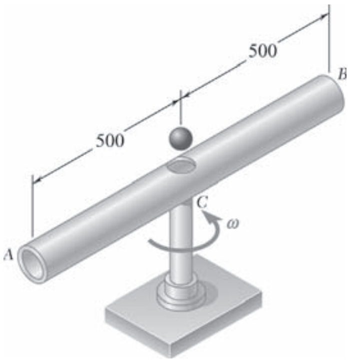
Kinetic energy. $T = \frac{1}{2}\bar{I}_{AB}\omega^2 + \frac{1}{2}\bar{I}_{DCE}\omega^2 + \frac{1}{2}m_{AB}\bar{v}^2$

$$T_1 = \frac{1}{2}(0.0025)(5)^2 + \frac{1}{2}(0.30)(5)^2 + \frac{1}{2}(1.6)(0.0625)^2(5)^2$$
$$= 3.859375 \text{ J}$$

$$T_2 = \frac{1}{2}(0.0025)(2.5359)^2 + \frac{1}{2}(0.30)(2.5359)^2 + \frac{1}{2}(1.6)(0.4375)^2(2.5359)^2$$
$$= 1.9573 \text{ J}$$

(b) *Energy lost.* $T_1 - T_2 = 1.902$ J ◄

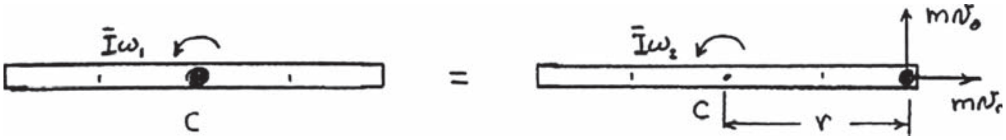
PROBLEM 17.82



Two 0.4-kg balls are to be put successively into the center C of the slender 2-kg tube AB. Knowing that when the first ball is put into the tube the initial angular velocity of the tube is 8 rad/s and neglecting the effect of friction, determine the angular velocity of the tube just after (a) the first ball has left the tube, (b) the second ball has left the tube.

SOLUTION

Conservation of angular momentum about C.



$$\bar{I}\omega_1 = \bar{I}\omega_2 + mrv_\theta$$
$$= \bar{I}\omega_2 + mr(r\omega_2)$$
$$\omega_2 = \frac{\bar{I}}{\bar{I} + mr^2}\omega_1$$
$$= C\omega_1 \qquad (1)$$

Data:

$$\bar{I} = \frac{1}{12} m_{\text{tube}} L^2$$
$$= \frac{1}{12}(2)(1)^2$$
$$= \frac{1}{6}\ \text{kg}\cdot\text{m}^2$$

(one ball) $m = 0.4$ kg

$$r = 500\ \text{mm} = 0.5\ \text{m}$$
$$\bar{I} + mr^2 = \frac{1}{6} + (0.4)(0.5)^2$$
$$= \frac{4}{15}\ \text{kg}\cdot\text{m}^2$$
$$C = \frac{\left(\frac{1}{6}\right)}{\left(\frac{4}{15}\right)} = 0.625$$

PROBLEM 17.82 (Continued)

(*a*) *First ball moves through the tube*.

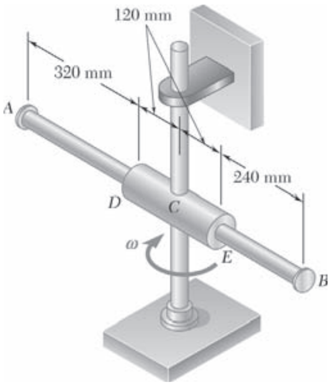
$$\omega_1 = 8 \text{ rad/s}$$

By Equation (1), $\omega_2 = (0.625)(8)$ $\omega_2 = 5.00 \text{ rad/s}$ ◄

(*b*) *Second ball moves through the tube*.

$$\omega_1 = 5.00 \text{ rad/s}$$

By Equation (1), $\omega_2 = (0.625)(5.00)$ $\omega_2 = 3.13 \text{ rad/s}$ ◄

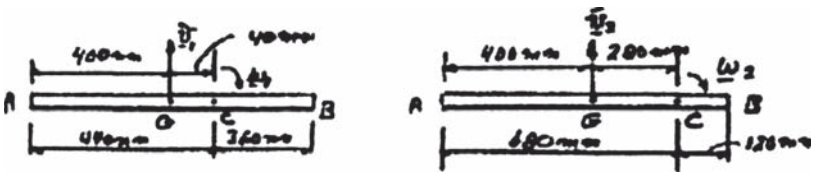


PROBLEM 17.83

A 3-kg rod of length 800 mm can slide freely in the 240-mm cylinder *DE*, which in turn can rotate freely in a horizontal plane. In the position shown the assembly is rotating with an angular velocity of magnitude $\omega = 40$ rad/s and end *B* of the rod is moving toward the cylinder at a speed of 75 mm/s relative to the cylinder. Knowing that the centroidal mass moment of inertia of the cylinder about a vertical axis is $0.025\ \text{kg}\cdot\text{m}^2$ and neglecting the effect of friction, determine the angular velocity of the assembly as end *B* of the rod strikes end *E* of the cylinder.

SOLUTION

Kinematics and geometry.

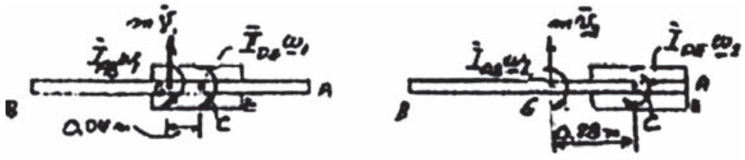


$\bar{v}_1 = (0.04\ \text{m})\omega_1 = (0.4\ \text{m})(40\ \text{rad/s})$ $\qquad\qquad$ $\bar{v}_2 = (0.28\ \text{m})\omega_2$

$\bar{v}_1 = 1.6\ \text{m/s}$

Initial position $\qquad\qquad$ Final position

Conservation of angular momentum about *C*.



$\circlearrowleft +$ Moments about *C*: $\qquad \bar{I}_{AB} = \dfrac{1}{12}(3\ \text{kg})(0.8\ \text{m})^2 = 0.16\ \text{kg}\cdot\text{m}^2$

$$\bar{I}_{AB}\omega_1 + m\bar{v}_1(0.04\ \text{m}) + \bar{I}_{DE}\omega_1 = \bar{I}_{AB}\omega_2 + m\bar{v}_2(0.028\ \text{m}) + \bar{I}_{DE}\omega_2$$

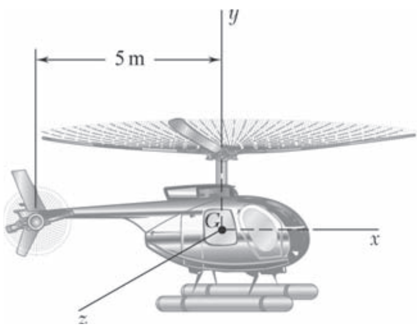
$$(0.16\ \text{kg}\cdot\text{m}^2)(40\ \text{rad/s}) + (8\ \text{kg})(1.6\ \text{m/s})\ (0.04\ \text{m}) + (0.025\ \text{kg}\cdot\text{m}^2)(40\ \text{rad/s})$$

$$= (0.16\ \text{kg}\cdot\text{m}^2)\omega_2 + (3\ \text{kg})(0.28\omega_2)(0.28) + (0.025\ \text{kg}\cdot\text{m}^2)\omega_2$$

$$(6.4 + 0.192 + 1.00) = (0.16 + 0.2352 + 0.025)\omega_2$$

$$7.592 = 0.4202\omega_2; \quad \omega_2 = 18.068\ \text{rad/s}$$

Angular velocity. $\qquad \omega_2 = 18.07\ \text{rad/s}$ ◄



PROBLEM 17.84

In the helicopter shown, a vertical tail propeller is used to prevent rotation of the cab as the speed of the main blades is changed. Assuming that the tail propeller is not operating, determine the final angular velocity of the cab after the speed of the main blades has been changed from 180 to 240 rpm. (The speed of the main blades is measured relative to the cab, and the cab has a centroidal moment of inertia of 1000 $\text{kg}\cdot\text{m}^2$. Each of the four main blades is assumed to be a slender 4.2-m rod of mass 25 kg.)

SOLUTION

Let $\mathbf{\Omega}$ be the angular velocity of the cab and ω be the angular velocity of the blades relative to the cab. The absolute angular velocity of the blades is $\Omega + \omega$.

$$\omega_1 = 180 \text{ rpm} = 6\pi \text{ rad/s}$$
$$\omega_2 = 240 \text{ rpm} = 8\pi \text{ rad/s}$$

Moments of inertia.

Cab: $I_C = 1000 \text{ kg}\cdot\text{m}^2$

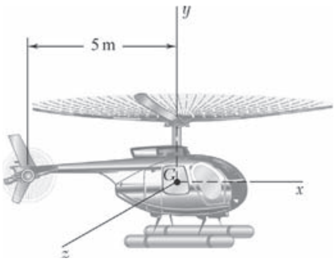
Blades:
$$I_B = 4\left(\frac{1}{3}mL^2\right)$$
$$= (4)\left(\frac{1}{3}\right)(25)(4.2)^2$$
$$= 588 \text{ kg}\cdot\text{m}^2$$

Assume $\Omega_1 = 0$.

Conservation of angular momentum about shaft.

$$I_B(\omega_1 + \Omega_1) + I_C\Omega_1 = I_B(\omega_2 + \Omega_2) + I_C\Omega_2$$
$$\Omega_2 = -\frac{I_B(\omega_2 - \omega_1)}{I_C + I_B}$$
$$= -\frac{(588)(8\pi - 6\pi)}{(588 + 1000)}$$
$$= -2.3265 \text{ rad/s}$$

$\Omega_2 = -22.2 \text{ rpm}$ ◄



PROBLEM 17.85

Assuming that the tail propeller in Problem 17.84 is operating and that the angular velocity of the cab remains zero, determine the final horizontal velocity of the cab when the speed of the main blades is changed from 180 to 240 rpm. The cab weighs 625 kg and is initially at rest. Also determine the force exerted by the tail propeller if the change in speed takes place uniformly in 12 s.

SOLUTION

Let $\mathbf{\Omega}$ be the angular velocity of the cab and ω be the angular velocity of the blades relative to the cab. The absolute angular velocity of the blades is $\Omega + \omega$.

$$\omega_1 = 180 \text{ rpm} = 6\pi \text{ rad/s}$$
$$\omega_2 = 240 \text{ rpm} = 8\pi \text{ rad/s}$$

Moments of inertia.

Cab: $\quad I_C = 1000 \text{ kg}\cdot\text{m}^2$

Blades: $\quad I_B = 4\left(\frac{1}{3}mL^2\right) = (4)\left(\frac{1}{3}\right)(25)(4.2)^2$

$$= 588 \text{ kg}\cdot\text{m}^2$$

The cab does not rotate. $\quad \Omega_1 = \Omega_2 = 0$

$$\textbf{Syst. Momenta}_1 + \textbf{Syst. Ext. Imp.}_{1\to 2} = \textbf{Syst. Momenta}_2$$

Moments about shaft: $\quad I_B(\omega_1 + \Omega_1) + I_C\Omega_1 + Frt = I_B(\omega_2 + \Omega_2) + I_C\Omega_2$

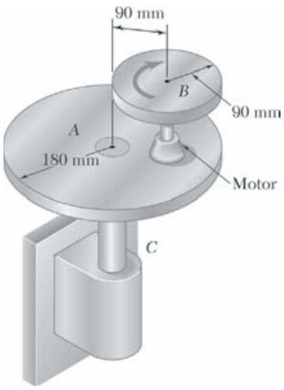
$$Frt = I_B(\omega_2 - \omega_1)$$
$$= (588)(8\pi - 6\pi)$$
$$= 3694.51 \text{ N}\cdot\text{m}\cdot\text{s}$$
$$Ft = \frac{Frt}{r} = \frac{3694.51}{5} = 738.903 \text{ N}\cdot\text{s}$$

Linear components: $\quad mv_1 + Ft = mv_2$

$$v_2 - v_1 = \frac{Ft}{m} = \frac{738.903}{625 + (4)(25)}$$
$$= 1.0192 \text{ m/s}$$

(*a*) Assume $v_1 = 0$. $\quad v_2 = 1.019 \text{ m/s}$ ◄

(*b*) Force. $\quad F = \frac{Ft}{t} = \frac{738.903}{12} \quad F = 61.6 \text{ N}$ ◄

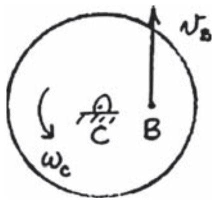


PROBLEM 17.86

The 4-kg disk B is attached to the shaft of a motor mounted on plate A, which can rotate freely about the vertical shaft C. The motor-plate-shaft unit has a moment of inertia of $0.20 \text{ kg}\cdot\text{m}^2$ with respect to the axis of the shaft. If the motor is started when the system is at rest, determine the angular velocities of the disk and of the plate after the motor has attained its normal operating speed of 360 rpm.

SOLUTION

Moments of inertia. motor-plate-shaft:

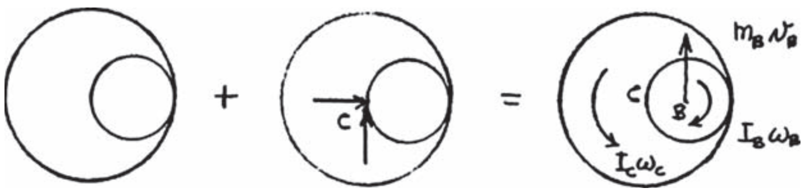


$$I_C = 0.20 \text{ kg}\cdot\text{m}^2$$

Disk:
$$I_B = \frac{1}{2} m_B r_B^2$$
$$= \frac{1}{2}(4 \text{ kg})(0.090 \text{ m})^2$$
$$= 0.0162 \text{ kg}\cdot\text{m}^2$$

Kinematics. $\quad v_B = r_{B/C}\omega_C = 0.090\omega_C$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about C: $\quad 0 + 0 = I_C\omega_C + m_B v_B - I_B\omega_B$

$$I_C\omega_C + m_B(r_{B/C}\omega_C)r_{B/C} - I_B\omega_B = 0$$
$$(I_C + m_B r_{B/C}^2)\omega_C = I_B\omega_B$$
$$[0.20 + (4)(0.090)^2]\omega_C = 0.0162\omega_B \qquad \omega_C = 0.069707$$

PROBLEM 17.86 (Continued)

Angular velocity of motor.

$$\omega_M = \omega_B + \omega_C$$
$$= 1.06971\omega_B$$

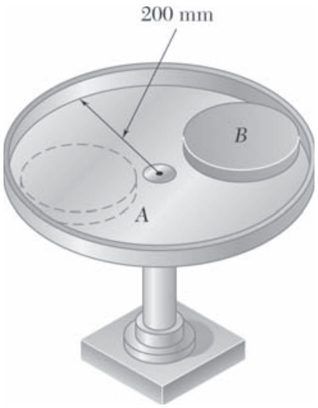
$$\omega_M = 360 \text{ rpm}$$

$$\omega_B = \frac{\omega_M}{1.06971}$$
$$= \frac{360 \text{ rpm}}{1.06971}$$
$$= 336.54 \text{ rpm}$$

$\omega_B = 337 \text{ rpm}$ ◀

$$\omega_C = (0.069707)(336.54 \text{ rpm})$$

$\omega_C = 32.5 \text{ rpm}$ ◀



PROBLEM 17.87

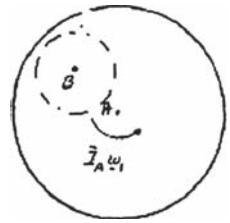
The circular platform A is fitted with a rim of 200-mm inner radius and can rotate freely about the vertical shaft. It is known that the platform-rim unit has a mass of 5 kg and a radius of gyration of 175 mm with respect to the shaft. At a time when the platform is rotating with an angular velocity of 50 rpm, a 3-kg disk B of radius 80 mm is placed on the platform with no velocity. Knowing that disk B then slides until it comes to rest relative to the platform against the rim, determine the final angular velocity of the platform.

SOLUTION

Moments of inertia.

$$\bar{I}_A = m_A k^2 = (5\text{ kg})(0.175\text{ m})^2 = 0.153125\text{ kg}\cdot\text{m}^2$$

$$\bar{I}_B = \frac{1}{2}m_B r_B^2 = \frac{1}{2}(3\text{ kg})(0.08\text{ m})^2 = 9.6\times10^{-3}\text{ kg}\cdot\text{m}^2$$

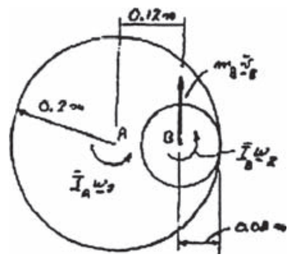


Syst. Momenta$_1$

State 1 Disk B is at rest.

State 2 Disk B moves with platform A.

Kinematics. In State 2, $\bar{v}_B = (0.12\text{ m})\omega_2$



Syst. Momenta$_2$

Principle of conservation of angular momentum.

Moments about D:

$$\bar{I}_A\omega_1 = \bar{I}_A\omega_2 + \bar{I}_B\omega_2 + m_B\bar{v}_B(0.12\text{ m})$$

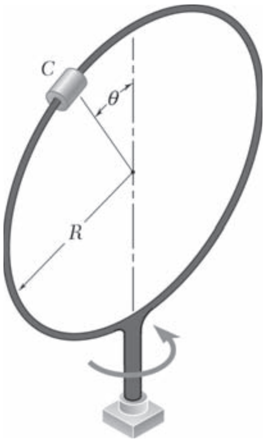
$$(0.153125\text{ kg}\cdot\text{m}^2)\omega_1 = (0.153125\text{ kg}\cdot\text{m}^2)\omega_2 + (9.6\times10^{-3}\text{ kg}\cdot\text{m}^2)\omega_2 + (3\text{ kg})(0.12\text{ m})^2\omega_2$$

$$0.153125\omega_2 = 0.20593\omega_1$$

$$\omega_2 = 0.7436\omega_1 = 0.7436(50\text{ rpm})$$

Final angular velocity $\omega_2 = 37.2\text{ rpm}$ ◀

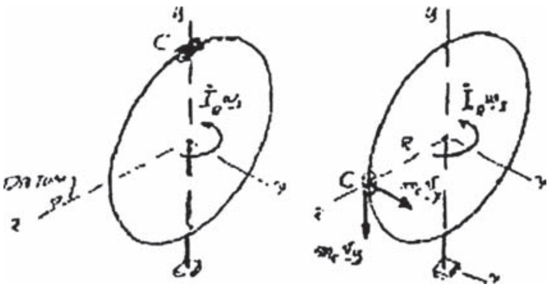
PROBLEM 17.88



A small 2-kg collar C can slide freely on a thin ring of mass 3 kg and radius 250 mm. The ring is welded to a short vertical shaft, which can rotate freely in a fixed bearing. Initially the ring has an angular velocity of 35 rad/s and the collar is at the top of the ring $(\theta = 0)$ when it is given a slight nudge. Neglecting the effect of friction, determine (*a*) the angular velocity of the ring as the collar passes through the position $\theta = 90°$, (*b*) the corresponding velocity of the collar relative to the ring.

SOLUTION

Moment of inertia of ring. $\qquad \bar{I}_R = \frac{1}{2}m_R R^2$



Position 1 $\qquad\qquad$ Position 2

Position 1. $\qquad \theta = 0.$

$$v_C = 0$$

Position 2. $\qquad \theta = 90°$

$$(v_C)_y = v_y = R\omega_2$$

Conservation of angular momentum about y axis for system.

$$\bar{I}_R\omega_1 = \bar{I}_R\omega_2 + m_C v_y R$$

$$\frac{1}{2}m_R R^2\omega_1 = \frac{1}{2}m_R R^2\omega_2 + m_C R^2\omega_2$$

$$m_R R^2\omega_1 = (m_R + 2m_C)R^2\omega_2$$

$$\omega_2 = \frac{m_R}{m_R + 2m_C}\omega_1 \qquad (1)$$

PROBLEM 17.88 (Continued)

Potential energy. Datum is the center of the ring.

$$V_1 = m_C gR \quad V_2 = 0$$

Kinetic energy:

$$T_1 = \frac{1}{2}\bar{I}_R\omega_1^2 = \frac{1}{2}\left(\frac{1}{2}m_R R^2\right)\omega_1^2$$

$$= \frac{1}{4}m_R R^2\omega_1^2$$

$$T_2 = \frac{1}{2}\bar{I}_R\omega_2^2 + \frac{1}{2}m_C\left(v_x^2 + v_y^2\right)$$

$$= \frac{1}{4}m_R R^2\omega_2^2 + \frac{1}{2}m_C R^2 v_2^2 + \frac{1}{2}m_C v_y^2$$

Principle of conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{4}m_R R^2\omega_1^2 + m_C gR = \left(\frac{1}{4}m_R + \frac{1}{2}m_C\right)R^2\omega_2^2 + \frac{1}{2}m_C v_y^2 \qquad (2)$$

Data:

$$m_C = 2 \text{ kg}$$
$$m_R = 3 \text{ kg}$$
$$R = 0.25 \text{ m}$$
$$\omega_1 = 35 \text{ rad/s}$$

(*a*) Angular velocity.

From Eq. (1), $$\omega_2 = \frac{3 \text{ kg}}{3 \text{ kg} + 2(2 \text{ kg})}(35 \text{ rad/s})$$ $\omega_2 = 15.00$ rad/s ◀

(*b*) Velocity of collar relative to ring.

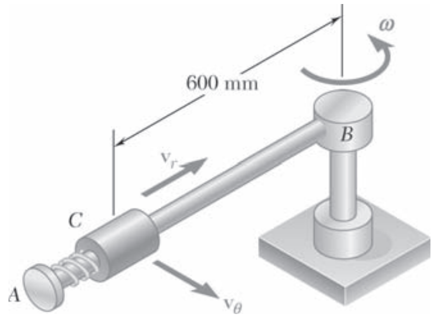
From Eq. (2), $$\frac{1}{4}(3 \text{ kg})(0.25 \text{ m})^2(35 \text{ rad/s})^2 + (2 \text{ kg})(9.81 \text{ m/s}^2)(0.25 \text{ m})$$

$$= \left[\frac{1}{4}(3 \text{ kg}) + \frac{1}{2}(2 \text{ kg})\right](0.25 \text{ m})^2(15 \text{ rad/s})^2 + \frac{1}{2}(2 \text{ kg})v_y^2$$

$$57.422 + 4.905 = 24.609 + v_y^2$$

$$v_y^2 = 37.716$$ $v_y = 6.14$ m/s ◀

PROBLEM 17.89



Collar C has a mass of 8 kg and can slide freely on rod AB, which in turn can rotate freely in a horizontal plane. The assembly is rotating with an angular velocity $\boldsymbol{\omega}$ of 1.5 rad/s when a spring located between A and C is released, projecting the collar along the rod with an initial relative speed $v_r = 1.5$ m/s. Knowing that the combined mass moment of inertia about B of the rod and spring is $1.2\ \text{kg}\cdot\text{m}^2$, determine (*a*) the minimum distance between the collar and Point B in the ensuing motion, (*b*) the corresponding angular velocity of the assembly.

SOLUTION

Kinematics. $\quad v_\theta = r\omega$

Moments of inertia.

$$I = I_B + m_C r^2$$
$$= 1.2 + 8r^2$$

Conservation of angular momentum.

$$(H_B)_1 = (H_B)_2: \quad I_B\omega_1 + m_C(v_\theta)_1 r_1 = I_B\omega_2 + m_C(v_\theta)_2 r_2$$

$$(I_B + m_C r_1^2)\omega_1 = (I_B + m_C r_2^2)\omega_2 \quad \text{or} \quad I_1\omega_1 = I_2\omega_2$$

$$\omega_2 = \frac{I_1\omega_1}{I_2} \qquad (1)$$

Potential energy. $\quad V_1 = 0 \quad V_2 = 0$

Kinetic energy.

$$T = \frac{1}{2}m_C v_r^2 + \frac{1}{2}m_C v_\theta^2 + \frac{1}{2}I_B\omega^2$$
$$= \frac{1}{2}m_C v_r^2 + \frac{1}{2}I\omega^2$$

Position 1. Just after spring is released.

$$r = r_1$$
$$v_r = (v_r)_1$$
$$\omega = \omega_1$$
$$I = I_1$$

Position 2. Distance r is minimum.

$$r = r_2$$
$$(v_r) = (v_r) = 0$$
$$\omega = \omega_2$$
$$I = I_2$$

PROBLEM 17.89 (Continued)

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad \frac{1}{2} m_C (v_r)_1^2 + \frac{1}{2} I_1 \omega_1^2 + 0 = 0 + \frac{1}{2} I_2 \omega_2^2$$

$$m_C (v_r)_1^2 = I_2 \left(\frac{I_1 \omega_1}{I_2} \right)^2 - I_1 \omega_1^2$$

$$= I_1 \left(\frac{I_1}{I_2} - 1 \right) \omega_1^2 \qquad (2)$$

Data.

$$r_1 = 600 \text{ mm} = 0.6 \text{ m}$$
$$\omega_1 = 15 \text{ rad/s}$$
$$(v_r)_1 = 1.5 \text{ m}$$
$$I_1 = 1.2 + (8)(0.6)^2$$
$$= 4.08 \text{ kg} \cdot \text{m}^2$$

Equation (2):

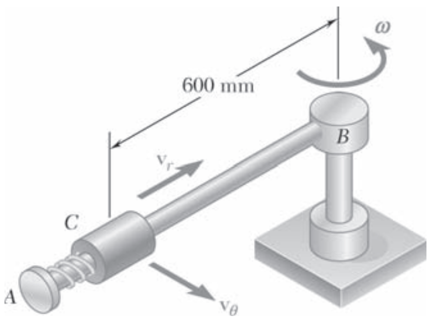
$$(8)(1.5)^2 = 4.08 \left(\frac{I_1}{I_2} - 1 \right) (1.5)^2$$

$$\frac{I_1}{I_2} = 2.9608$$

$$I_2 = \frac{4.08}{2.9608} = 1.378 = 1.2 + 8 r_2^2$$

(*a*) $\quad r_2 = 0.14917 \text{ m} \qquad r_2 = 149.2 \text{ mm}$ ◄

(*b*) Equation (1): $\quad \omega_2 = \frac{4.08}{1.378}(1.5) \qquad \omega_2 = 4.44 \text{ rad/s}$ ◄



PROBLEM 17.90

In Problem 17.89, determine the required magnitude of the initial relative speed v_r if during the ensuing motion the minimum distance between collar C and Point B is to be 300 mm.

SOLUTION

Kinematics. $$v_\theta = r\omega$$

Moments of inertia. $$I = I_B + m_C r^2$$
$$= 1.2 + 8r^2$$

Conservation of angular momentum.

$$(H_B)_1 = (H_B)_2: \quad I_B\omega_1 + m_C(v_\theta)_1 r_1 = I_B\omega_2 + m_C(v_\theta)_2 r_2$$
$$(I_B + m_C r_1^2)\omega_1 = (I_B + m_C r_2^2)\omega_2 \quad \text{or} \quad I_1\omega_1 = I_2\omega_2$$
$$\omega_2 = \frac{I_1\omega_1}{I_2} \qquad (1)$$

Potential energy. $$V_1 = 0 \quad V_2 = 0$$

Kinetic energy. $$T = \frac{1}{2}m_C v_r^2 + \frac{1}{2}m_C v_\theta^2 + \frac{1}{2}I_B\omega^2$$
$$= \frac{1}{2}m_C v_r^2 + \frac{1}{2}I\omega^2$$

Position 1. Just after spring is released.
$$r = r_1$$
$$v_r = (v_r)_1$$
$$\omega = \omega_1$$
$$I = I_1$$

Position 2. Distance r is minimum.
$$r = r_2$$
$$(v_r) = (v_r) = 0$$
$$\omega = \omega_2$$
$$I = I_2$$

PROBLEM 17.90 (Continued)

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad \frac{1}{2} m_C (v_r)_1^2 + \frac{1}{2} I_1 \omega_1^2 + 0 = 0 + \frac{1}{2} I_2 \omega_2^2$$

$$m_C (v_r)_1^2 = I_2 \left(\frac{I_1 \omega_1}{I_2} \right)^2 - I_1 \omega_1^2$$

$$= I_1 \left(\frac{I_1}{I_2} - 1 \right) \omega_1^2 \qquad (2)$$

Data.

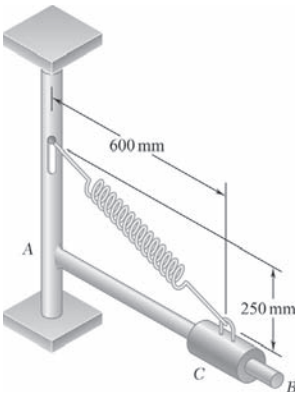
$$r_1 = 600 \text{ mm} = 0.6 \text{ m}$$
$$\omega_1 = 1.5 \text{ rad/s}$$
$$r_2 = 300 \text{ mm} = 0.3 \text{ m}$$
$$I_1 = 1.2 + (8)(0.6)^2 = 4.08 \text{ kg} \cdot \text{m}^2$$
$$I_2 = 1.2 + (8)(0.3)^2 = 1.92 \text{ kg} \cdot \text{m}^2$$

Equation (2):

$$8(v_r)_1^2 = 4.08 \left(\frac{4.08}{1.92} - 1 \right) (1.5)^2$$

$$(v_r)_1^2 = 1.29094 \text{ m}^2/\text{s}^2 \qquad (v_r)_1 = 1.136 \text{ m/s} \blacktriangleleft$$

PROBLEM 17.91

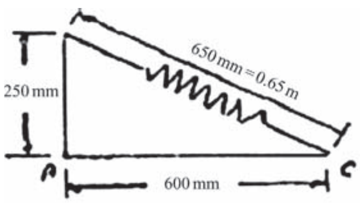


A 3 kg collar C is attached to a spring and can slide on rod AB, which in turn can rotate in a horizontal plane. The mass moment of inertia of rod AB with respect to end A is 0.5 kg$\cdot$m^2. The spring has a constant $k = 3000$ N/m and an undeformed length of 250 mm. At the instant shown the velocity of the collar relative to the rod is zero and the assembly is rotating with an angular velocity of 12 rad/s. Neglecting the effect of friction, determine (a) the angular velocity of the assembly as the collar passes through a point located 180 mm from end A of the rod, (b) the corresponding velocity of the collar relative to the rod.

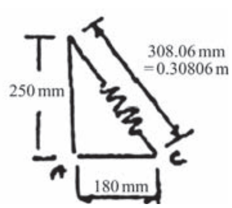
SOLUTION

Potential energy of spring: undeformed length = 250 mm = 0.25 m

Position 1:



Position 2:



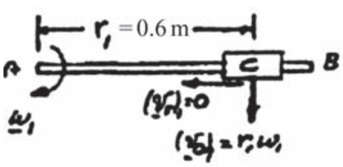
$$\Delta = 0.65\text{ m} - 0.25\text{ m} = 0.4\text{ m}$$

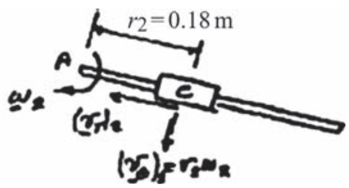
$$V_1 = \frac{1}{2}k\Delta^2 = \frac{1}{2}(3000)(0.4)^2$$
$$= 240\text{ N}\cdot\text{m}$$

$$\Delta = 0.30806 - 0.25 = 0.05806\text{ m}$$

$$V_2 = \frac{1}{2}k\Delta^2 = \frac{1}{2}(3000)(0.05806)^2$$
$$= 5.0564\text{ N}\cdot\text{m}$$

Kinematics:





Kinetics: Since moments of all forces about shift at A are zero, $(\mathbf{H}_A)_1 = (\mathbf{H}_A)_2$

$$I_R\omega_1 + m_C(v_0)r_1 = I_R\omega_C + m_C(v_0)_2 r_2$$
$$\left(I_R + m_C r_1^2\right)\omega_1 = \left(I_R + m_C r_2^3\right)\omega_2$$

PROBLEM 17.91 (Continued)

Data: $I_R = 0.5 \text{ kg}\cdot\text{m}^2, \quad m_C = 3 \text{ kg}$

$$r_1 = 0.6 \text{ m}, \quad r_2 = 0.18 \text{ m}, \quad \omega_1 = 12 \text{ rad/s}$$

$$\left[0.5 + (3)(0.6)^2\right](12 \text{ rad/s}) = \left[0.5 + (3)(0.18)^2\right]\omega_2$$

$$18.96 = 0.5972\omega_2; \quad \omega_2 = 31.748 \text{ rad/s}$$

(*a*) Angular velocity. $\omega_2 = 31.7 \text{ rad/s}$ ◀

Kinetic energy.

$$T_1 = \frac{1}{2}I_A\omega_1^2 + \frac{1}{2}m_C(v_D)_1^2 + \frac{1}{2}m_C(v_R)_1^2$$

$$= \frac{1}{2}(0.5)(12)^2 + \frac{1}{2}(3)(0.6)^2(12)^2 + 0$$

$$= 36 + 77.76$$

$$T_1 = 113.76 \text{ N}\cdot\text{m}$$

$$T_2 = \frac{1}{2}I_R\omega_2^2 + \frac{1}{2}m(v_B)_2^2 + \frac{1}{2}m_2(v_R)_2^2$$

$$= \frac{1}{2}(0.5)(31.748)^2$$

$$+ \frac{1}{2}(3)(0.18)^2(31.748)^2 + \frac{1}{2}(3)(v_R)_2^2$$

$$T_2 = 300.9695 + 1.5\,(v_r)_2^2$$

Principle of conservation of energy: $T_1 + V_1 = T_2 + V_2$

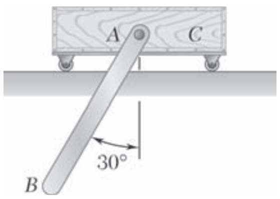
Recall: $V_1 = 240 \text{ N}\cdot\text{m} \quad \text{and} \quad V_2 = 5.0564 \text{ N}\cdot\text{m}$

$$113.76 + 240 = 300.9695 + 1.5(v_r)_2^2 + 5.0564$$

$$47.7341 = 1.5\,{v_{r2}}^2$$

(*b*) Velocity of collar relative to rod. $(v_r)_2 = 5.64 \text{ m/s}$ ◀

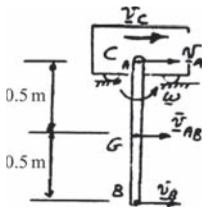
PROBLEM 17.92



A uniform rod AB, of mass 7.5 kg and length 1 m is attached to the 12.5-kg cart C. Knowing that the system is released from rest in the position shown and neglecting friction, determine (*a*) the velocity of Point B as rod AB passes through a vertical position (*b*) the corresponding velocity of cart C.

SOLUTION

Kinematics $\qquad \mathbf{v}_C = \mathbf{v}_A$



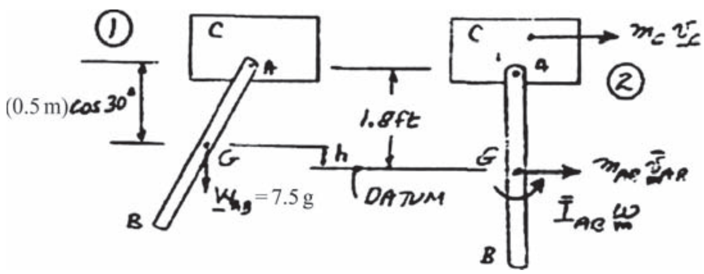
$$\mathbf{v}_C \rightarrow = \overline{\mathbf{v}}_{AB} \rightarrow + (0.5 \text{ m})\omega \leftarrow$$

$$v_C = v_{AB} - 0.5\omega \qquad (1)$$

$$AB = 1 \text{ m}$$

Mass. $\qquad m_C = 12.5 \text{ kg}, \quad m_{AB} = 7.5 \text{ kg}$

Kinetics



Linear momentum

$$\xrightarrow{+} 0 = m_C v_C + m_{AB}\overline{v}_{AB}$$

$$\overline{v}_{AB} = -\frac{m_C}{m_{AB}} v_C = -\frac{(12.5 \text{ kg})}{(7.5 \text{ kg})} v_C, \quad \overline{v}_{AB} = -\frac{5}{3} v_C \qquad (2)$$

Substitute into Eq. (1): $\qquad v_C = -\frac{5}{3} v_C - 0.5\omega$

$$\frac{8}{3} v_C = -0.5\omega \qquad v_C = -0.1875\omega \quad (3)$$

Substitute into Eq. (2): $\qquad \overline{v}_{AB} = -\frac{5}{3}(-0.1875\omega)$

$$\overline{v}_{AB} = 0.3125\omega \qquad (4)$$

PROBLEM 17.92 (Continued)

Kinetic and potential energies.

$$T_1 = 0$$

$$V_1 = W_{AB}b = (7.5)(9.81)(0.5)(1-\cos 30°)$$

$$= 4.9286 \text{ N} \cdot \text{m}$$

$$V_2 = 0$$

$$T_2 = \frac{1}{2}m_C v_C^2 + \frac{1}{2}m_{AB}\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}_{AB}\omega^2$$

$$= \frac{1}{2}(12.5)(-0.1875\omega)^2 + \frac{1}{2}(7.5)(0.3125\omega)^2 + \frac{1}{2}\left[\frac{1}{12}(7.5)(1)^2\right]\omega^2$$

$$= \omega^2(0.21973 + 0.36621 + 0.3125)$$

$$= 0.89844\omega^2$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$0 + 4.9286 = 0.89844\omega^2$$

$$\omega^2 = 5.4857 \quad \omega = 2.3422 \text{ rad/s}$$

(*b*) Velocity of *C*: Eq. (3) $v_C = -0.1875\omega$ $\quad v_C = 0.439 \text{ m/s} \leftarrow$ ◀

(*a*) Velocity of *B*: $\mathbf{v}_B = v_C + (1)\omega \rightarrow = (0.43916 \text{ m/s} \leftarrow) + [(1)(2.3422)] \rightarrow]$

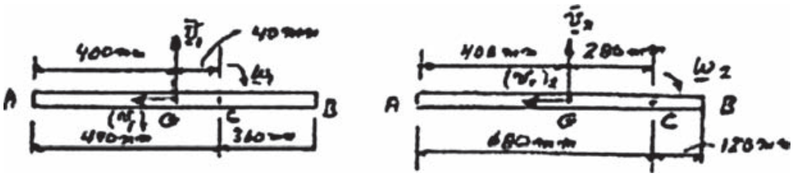
$v_B = 0.43916 \leftarrow + 2.3422 \rightarrow$ $\quad \mathbf{v}_B = 1.903 \text{ m/s} \rightarrow$ ◀

PROBLEM 17.93

In Problem 17.83, determine the velocity of rod *AB* relative to cylinder *DE* as end *B* of the rod strikes the end *E* of the cylinder.

SOLUTION

Kinematics and geometry.

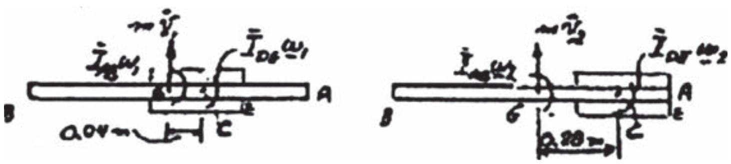


$\bar{v}_1 = (0.04 \text{ m})\omega_1 = (0.4 \text{ m})(40 \text{ rad/s})$ $\qquad$ $\bar{v}_2 = (0.28 \text{ m})\omega_2$

$\bar{v}_1 = 1.6 \text{ m/s}$

Initial position $\qquad$ Final position

Conservation of angular momentum about *C*.



↺ Moments about *C*: $\qquad \bar{I}_{AB} = \frac{1}{12}(3 \text{ kg})(0.9 \text{ m})^2 = 0.16 \text{ kg}\cdot\text{m}^2$

$$\bar{I}_{AB}\omega_1 + m\bar{v}_1(0.04 \text{ m}) + I_{DE}\omega_1 = I_{AB}\omega_2 + m\bar{v}_2(0.28 \text{ m}) + I_{DE}\omega_2$$

$$(0.16 \text{ kg}\cdot\text{m}^2)(40 \text{ rad/s}) + (8 \text{ kg})(1.6 \text{ m/s})\ (0.04 \text{ m}) + (0.025 \text{ kg}\cdot\text{m}^2)(40 \text{ rad/s})$$

$$= (0.16 \text{ kg}\cdot\text{m}^2)\omega_2 + (3 \text{ kg})(0.28\omega_2)(0.28) + (0.025 \text{ kg}\cdot\text{m}^2)\omega_2$$

$$(6.4 + 0.192 + 1.00) = (0.16 + 0.2352 + 0.025)\omega_2$$

$$7.592 = 0.4202\omega_2; \quad \omega_2 = 18.068 \text{ rad/s:} \quad \omega_2 = 18.07 \text{ rad/s}$$

Conservation of energy $\qquad (v_r) = 0.075 \text{ m/s}$

$$V_1 = V_2 = 0$$

$$T_1 = \frac{1}{2}\bar{I}_{DE}\omega_1^2 + \frac{1}{2}\bar{I}_{AB}\omega_1^2 + \frac{1}{2}m_{AB}\bar{v}_1^2 + \frac{1}{2}m_{AB}(v_r)_1^2$$

$$= \frac{1}{2}(0.025 \text{ kg}\cdot\text{m}^2)(40 \text{ rad/s})^2 + \frac{1}{2}(0.16 \text{ kg}\cdot\text{m}^2)(40 \text{ rad/s})^2$$

$$+ \frac{1}{2}(3 \text{ kg})(1.6 \text{ m/s})^2 + \frac{1}{3}(3 \text{ kg})(0.075 \text{ m/s})^2$$

PROBLEM 17.93 (Continued)

$$T_1 = 20\text{ J} + 120\text{ J} + 3.84\text{ J} + 0.008\text{ J} = 151.85\text{ J}$$

$$\bar{v}_2 = (0.28\text{ m})\omega_2 + (0.28\text{ m})(18.068\text{ rad/s}) = 5.059\text{ m/s}$$

$$T_2 = \frac{1}{2}\bar{I}_{DE}\omega_2^2 + \frac{1}{2}\bar{I}_{AB}\omega_2^2 + \frac{1}{2}m_{AB}v_2^2 + \frac{1}{2}m_{AB}(v_r)_2^2$$

$$= \frac{1}{2}(0.025\text{ kg}\cdot\text{m}^2)(18.068\text{ rad/s})^2$$

$$+\frac{1}{2}(0.16\text{ kg}\cdot\text{m}^2)(18.068\text{ rad/s})^2$$

$$= \frac{1}{2}(3\text{ kg})(5.059\text{ m/s})^2 + \frac{1}{2}(3\text{ kg})(v_r)_2^2$$

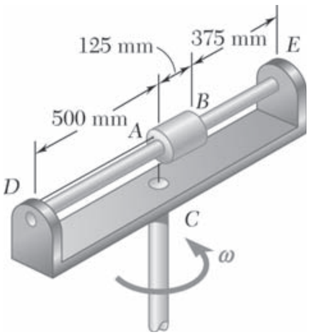
$$T_2 = 4.081\text{ J} + 26.116\text{ J} + 38.391\text{ J} + 1.5(v_r)_2^2$$

$$T_2 = 68.587\text{ J} + 1.5(v_r)_2^2$$

$$T_1 + V_1 = T_2 + V_2: \quad 151.85\text{ J} + 0 = 68.587\text{ J} + 1.5(v_r)_2^2$$

$$83.263 = 1.5(v_r)_2^2$$

Velocity of rod relative to cylinder. $(v_r)_2 = 7.45\text{ m/s}$ ◄



PROBLEM 17.94

In Problem 17.81 determine the velocity of the tube relative to the rod as the tube strikes end E of the assembly.

PROBLEM 17.81 A 1.6-kg tube AB can slide freely on rod DE which in turn can rotate freely in a horizontal plane. Initially the assembly is rotating with an angular velocity $\omega = 5$ rad/s and the tube is held in position by a cord. The moment of inertia of the rod and bracket about the vertical axis of rotation is $0.30\ \text{kg}\cdot\text{m}^2$ and the centroidal moment of inertia of the tube about a vertical axis is $0.0025\ \text{kg}\cdot\text{m}^2$. If the cord suddenly breaks, determine (a) the angular velocity of the assembly after the tube has moved to end E, (b) the energy lost during the plastic impact at E.

SOLUTION

Let Point C be the intersection of axle C and rod DE. Let Point G be the mass center of tube AB.

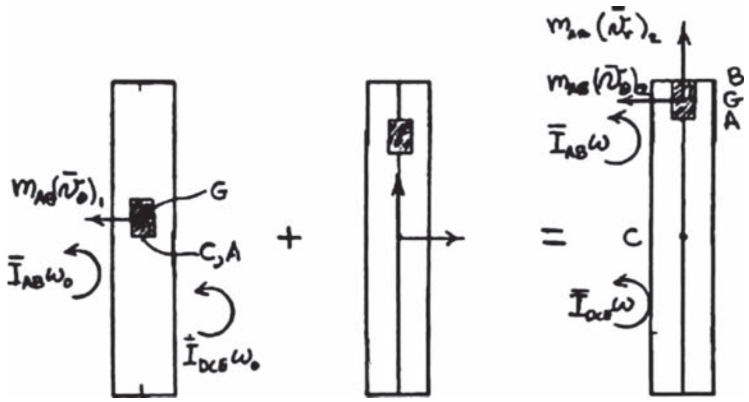
Masses and moments of inertia about vertical axes.

$$m_{AB} = 1.6\ \text{kg}, \qquad \bar{I}_{AB} = 0.0025\ \text{kg}\cdot\text{m}^2, \qquad \bar{I}_{DCE} = 0.30\ \text{kg}\cdot\text{m}^2$$

State 1. $\quad (r_{G/A})_1 = \frac{1}{2}(125) = 62.5\ \text{mm}, \qquad \omega_1 = 5\ \text{rad/s}, \quad (v_r)_1 = 0$

State 2. $\quad (r_{G/A})_2 = 500 - 62.5 = 437.5\ \text{mm}, \qquad \omega = \omega_2, \qquad v_r = (v_r)_2 = 0$

Kinematics. $\quad (v_G)_\theta = \bar{v}_\theta = r_{G/C}\,\omega$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.94 (Continued)

Moments about C:

$$\bar{I}_{AB}\omega_1 + \bar{I}_{DCE}\omega_1 + m_{AB}(\bar{v}_\theta)_1(r_{G/C})_1 + 0 = \bar{I}_{AB}\omega_2 + \bar{I}_{DCE}\omega_2 + m_{AB}(\bar{v}_\theta)_2(r_{G/C})_2$$

$$\left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/C})_1^2\right]\omega_1 = \left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/C})_2^2\right]\omega_2$$

$$[0.0025 + 0.30 + (1.6)(0.0625)^2](5) = [0.0025 + 0.30 + (1.6)(0.4375)^2]\omega_2$$

$$(0.30875)(5) = 0.60875\omega_2 \qquad \omega_2 = 2.5359 \text{ rad/s}$$

Kinetic energy.

$$T = \frac{1}{2}\bar{I}_{AB}\omega^2 + \frac{1}{2}\bar{I}_{DCE}\omega^2 + \frac{1}{2}m_{AB}\bar{v}^2$$

$$= \frac{1}{2}\bar{I}_{AB}\omega^2 + \frac{1}{2}\bar{I}_{DCE}\omega^2 + \frac{1}{2}m_{AB}\left(r_{G/C}^2\omega^2 + \bar{v}_r^2\right)$$

$$T_1 = \frac{1}{2}(0.0025)(5)^2 + \frac{1}{2}(0.3)(5)^2 + \frac{1}{2}(1.6)(0.0625)^2 + 0 = 3.859375 \text{ J}$$

$$T_2 = \frac{1}{2}(0.0025)(2.5359)^2 + \frac{1}{2}(0.30)(2.5359)^2$$

$$+ \frac{1}{2}(1.6)(0.0625)^2(2.5359)^2 + \frac{1}{2}(1.6)(\bar{v}_r)_2^2$$

$$= 1.95737 + 0.8(\bar{v}_r)_2^2$$

Work. The work of the bearing reactions at C is zero. Since the sliding contact between the rod and the tube is frictionless, the work of the contact force is zero.

$$U_{1\to 2} = 0$$

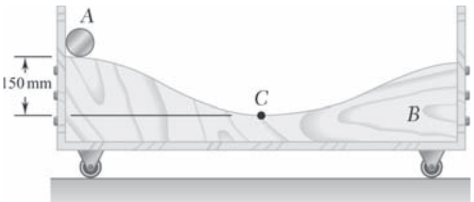
Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2$$

$$3.859375 + 0 = 1.95737 + 0.8(\bar{v}_r)_2^2$$

Velocity of the tube relative to the rod. $(\bar{v}_r)_2 = 1.542 \text{ m/s}$ ◄

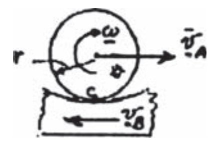
PROBLEM 17.95



The 3-kg steel cylinder A and the 5-kg wooden cart B are at rest in the position shown when the cylinder is given a slight nudge, causing it to roll without sliding along the top surface of the cart. Neglecting friction between the cart and the ground, determine the velocity of the cart as the cylinder passes through the lowest point of the surface at C.

SOLUTION

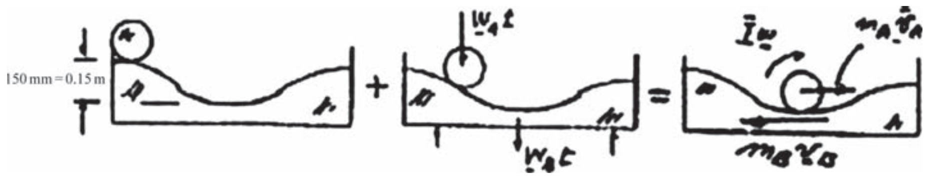
Kinematics (when cylinder is passing C)



$$\overset{+}{\longleftarrow}\quad v_B = v_C = r\omega - \bar{v}_A$$

$$\omega = \frac{\bar{v}_A + v_B}{r}$$

Principle of impulse and momentum.



Syst. of Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

$\overset{+}{\longrightarrow}$ x components: $\quad m_A\bar{v}_A - m_B v_B = 0$

$$3\bar{v}_A = 5v_B; \quad v_B = 0.6\,\bar{v}_A$$

Work: $\quad U_{1\to2} = W_A(0.15\text{ m}) = (3\text{ kg})(9.81\text{ m/s}^2)(0.15\text{ m}) = 4.4145\text{ N}\cdot\text{m}; \quad T_1 = 0$

Kinetic energy: $\quad T_2 = \frac{1}{2}m_A\bar{v}_A^2 + \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m_B v_3^2$

$$v_B = 0.6\bar{v}_A; \quad \omega = \frac{\bar{v}_A + v_B}{r} = \frac{v_A + 0.6v_A}{r} = \frac{1.6v_A}{r}$$

$$T_2 = \frac{1}{2}(3\text{ kg})\left(\bar{v}_A^2\right) + \frac{1}{2}\left(\frac{3\text{ kg}\,r^2}{2}\right)\left(\frac{1.6v_A}{r}\right)^2 + \frac{1}{2}(5\text{ kg})(0.6v_A)^2$$

$$= 1.5\bar{v}_A^2 + 1.92\bar{v}_A^2 + 0.9\bar{v}_A^2 = 4.32\bar{v}_A^2$$

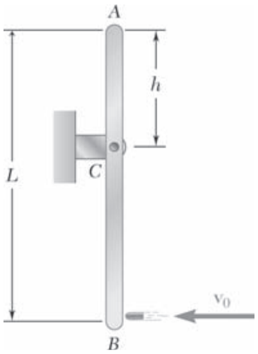
Principle of work and energy: $\quad T_1 + U_{1\to2} = T_2$

$$0 + 4.4145 = 4.32\bar{v}_A^2$$

$$\bar{v}_A^2 = 1.021875 \quad \bar{\mathbf{v}}_A = 1.01088\text{ m/s} \rightarrow$$

$$v_B = 0.6\bar{v}_A = 0.6(1.01088) \qquad \mathbf{v}_B = 0.607\text{ m/s} \leftarrow \blacktriangleleft$$

PROBLEM 17.96



A bullet weighing 40 gm is fired with a horizontal velocity of 550 m/s into the lower end of a slender 7.5-kg bar of length $L = 800$ mm. Knowing that $h = 300$ mm and that the bar is initially at rest, determine (*a*) the angular velocity of the bar immediately after the bullet becomes embedded, (*b*) the impulsive reaction at *C*, assuming that the bullet becomes embedded in 0.001 s.

SOLUTION

Bar: $L = 800 \text{ mm} = 0.8 \text{ m} \quad m = 7.5 \text{ kg}$

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(7.5)(0.8)^2 = 0.4 \text{ kg} \cdot \text{m}^2$$

Bullet: $m_0 = 40 \text{ g} = 0.04 \text{ kg}$

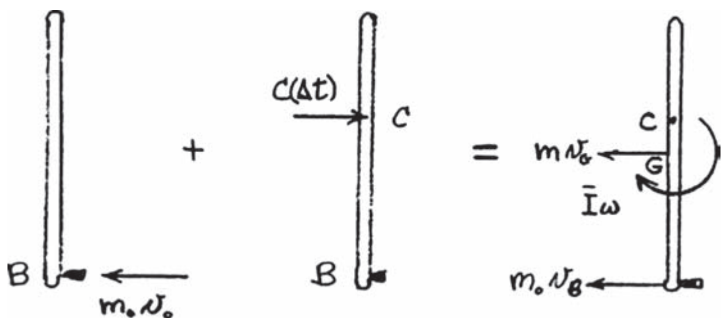
Support location: $h = 300 \text{ mm} = 0.3 \text{ m}$

Kinematics.

$$v_B = (L - h)\omega = (0.8 - 0.3)\omega = 0.5\omega$$

$$v_G = \left(\frac{L}{2} - h\right)\omega = (0.4 - 0.3)\omega = 0.1\omega$$

Kinetics.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

Moments about *C*:

$$m_0 v_0 (L - h) = m_0 v_B (L - h) + m v_G \left(\frac{L}{2} - h\right) + \bar{I}\omega$$

$$(0.04)(550)(0.5) = (0.04)(0.5\omega)(0.5) + (7.5)(0.1\omega)(0.1) + (0.4\omega)$$

PROBLEM 17.96 (Continued)

(*a*) $$11 = 0.01\omega + 0.075\omega + 0.4\omega = 0.485\omega \qquad \text{or} \qquad \omega = 22.6804$$

$\boldsymbol{\omega} = 22.7 \text{ rad/s} \circlearrowright$ ◄

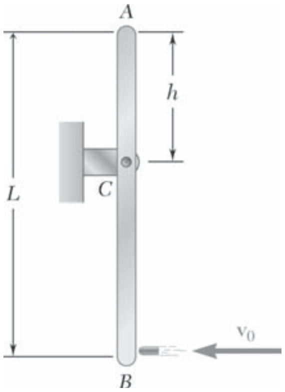
$$v_B = (0.5)(22.6804) = 11.3402 \text{ m/s}$$
$$v_G = (0.1)(22.6804) = 2.26804 \text{ m/s}$$

$\xrightarrow{+}$ Horizontal components:

$$-m_0 v_0 + C(\Delta t) = -m_0 v_B - m v_G: \quad C(\Delta t) = m_0(v_0 - v_B) - m v_0$$
$$C(\Delta t) = (0.04)(550 - 11.3402) - (7.5)(2.26804)$$
$$= 4.53609 \text{ N} \cdot \text{s}$$

(*b*) $$C = \frac{C\Delta t}{\Delta t} = \frac{4.53609}{0.001}$$

$\mathbf{C} = 4540 \text{ N} \rightarrow$ ◄



PROBLEM 17.97

In Problem 17.96, determine (*a*) the required distance *h* if the impulsive reaction at *C* is to be zero, (*b*) the corresponding angular velocity of the bar immediately after the bullet becomes embedded.

SOLUTION

Bar: $L = 800 \text{ mm} = 0.8 \text{ m} \quad m = 0.75 \text{ kg}$

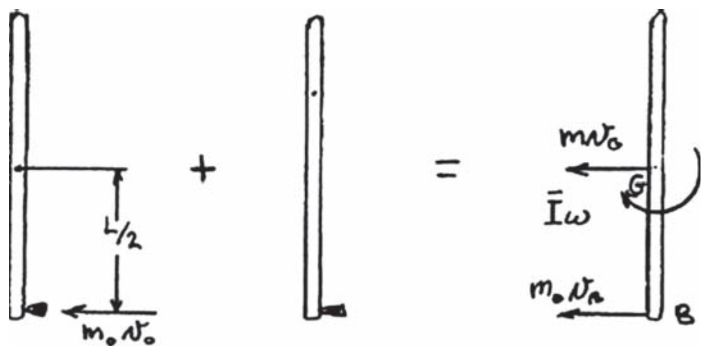
$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(7.5)(0.8)^2 = 0.4 \text{ kg}\cdot\text{m}^2$$

Bullet: $m_0 = 0.04 \text{ kg}$

Kinematics. $v_B = (L-h)\omega = (0.8-h)\omega$

$$v_G = \left(\frac{L}{2} - h\right)\omega = (0.4-h)\omega$$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

moment about *B*: $0 + 0 = \bar{I}\omega - mv_G\left(\frac{L}{2}\right)$

$$0 + 0 = 0.4\omega - (7.5)(0.4-h)\omega(0.4)$$

Divide by ω $\quad 0 = 0.4 - 3(0.4 - h)$

PROBLEM 17.97 (Continued)

(*a*) $$h = \frac{4}{15}\text{ m} = 0.26667\text{ m}$$ $h = 267$ mm ◀

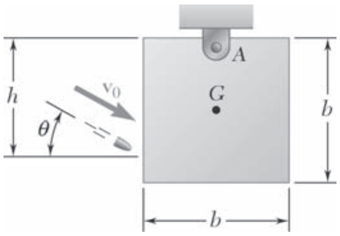
$$v_B = \left(0.8 - \frac{4}{15}\right)\omega = \frac{8}{15}\omega$$

$$v_G = \left(0.4 - \frac{4}{15}\right)\omega = \frac{2}{15}\omega$$

$\xleftarrow{+}$ Horizontal components: $m_0 v_0 + 0 = m v_G + m_0 v_B$

$$(0.04)(550) + 0 = (7.5)\left(\frac{2}{15}\omega\right) + (0.04)\left(\frac{8}{15}\omega\right)$$

(*b*) $$\omega = 21.541$$ $\omega = 21.5$ rad/s ◀



PROBLEM 17.98

A 45-g bullet is fired with a velocity of 400 m/s at $\theta = 30°$ into a 9-kg square panel of side $b = 200$ mm. Knowing that $h = 150$ mm and that the panel is initially at rest, determine (*a*) the velocity of the center of the panel immediately after the bullet becomes embedded, (*b*) the impulsive reaction at *A*, assuming that the bullet becomes embedded in 2 ms.

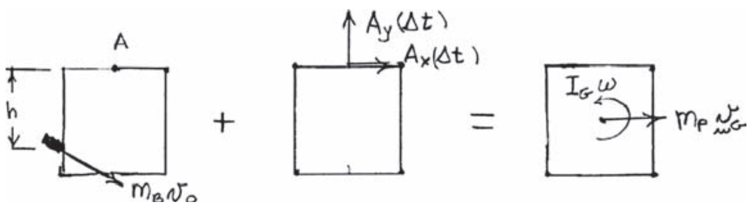
SOLUTION

$$m_B = 0.045 \text{ kg} \quad m_P = 9 \text{ kg} \quad I_G = \frac{1}{6} m_P b^2 = \frac{1}{6}(9)(0.200)^2 = 0.06 \text{ kg} \cdot \text{m}^2$$

Kinematics. After impact, the plate is rotating about the fixed Point *A* with angular velocity $\boldsymbol{\omega} = \omega \circlearrowleft$.

$$\mathbf{v}_G = \frac{b}{2}\omega \rightarrow .$$

Principle of impulse and momentum. To simplify the analysis, neglect the mass of the bullet after impact.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

(*a*) $\stackrel{+}{\circlearrowleft}$ Moments about *A*:

$$(m_B v_0 \cos 30°)h + m_B v_0 \sin 30° \left(\frac{b}{2}\right) + 0 = I_G \omega + m_P v_G \frac{b}{2}$$

$$m_B v_0 \left(h \cos 30° + \frac{b}{2} \sin 30°\right) = \left(I_G + \frac{1}{4} m_P b^2\right)\omega$$

$$(0.045)(400)(0.150 \cos 30° + 0.100 \sin 30°)$$

$$= \left[0.06 + \frac{1}{4}(9)(0.2)^2\right]\omega = 0.15\omega$$

$$\omega = 21.588 \text{ rad/s}$$

$$v_B = (0.100)(21.556) = 2.1556 \text{ m/s} \qquad \mathbf{v}_G = 2.16 \text{ m/s} \rightarrow ◀$$

PROBLEM 17.98 (Continued)

(*b*) $\xrightarrow{+}$ Linear momentum: $m_B v_0 \cos 30° + A_x(\Delta t) = m_P v_G$

$$(0.045)(400 \cos 30°) + A_x(0.002) = (9)(2.1556)$$

$\mathbf{A}_X = 1920\text{ N}$ $\mathbf{A}_x = 1920\text{ N} \rightarrow$

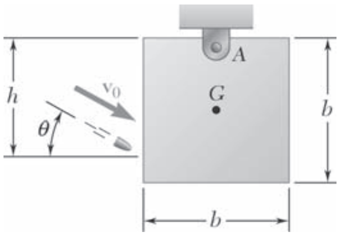
Linear momentum: $-m_B v_0 \sin 30° + A_y(\Delta t) = 0$

$$-(0.045)(400)\sin 30° + A_y(0.002) = 0$$

$A_y = 4500\text{ N}$ $\mathbf{A}_y = 4500\text{ N} \uparrow$

` $A = 4892\text{ N} = 4.892\text{ kW}$ $\tan\beta = \dfrac{4500}{1920}$ $\beta = 66.9°$

$\mathbf{A} = 4.87\text{ kN}$ ⦨ $66.9°$ ◀



PROBLEM 17.99

A 45-g bullet is fired with a velocity of 400 m/s at $\theta = 5°$ into a 9-kg square panel of side $b = 200$ mm. Knowing that $h = 150$ mm and that the panel is initially at rest, determine (*a*) the required distance h if the horizontal component of the impulsive reaction at A is to be zero, (*b*) the corresponding velocity of the center of the panel immediately after the bullet becomes embedded.

SOLUTION

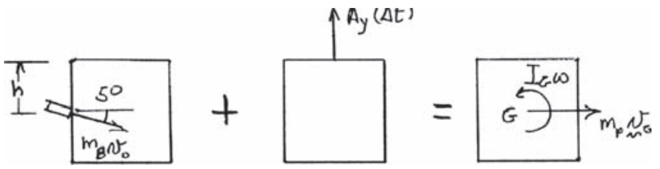
$$m_B = 0.045 \text{ kg} \quad m_P = 9 \text{ kg} \quad I_G = \frac{1}{6}m_P b^2 = \frac{1}{6}(9)(0.200)^2 = 0.06 \text{ kg}\cdot\text{m}^2$$

Kinematics. After impact, the plate is rotating about the fixed Point A with angular velocity $\omega = \omega \circlearrowleft$.

$$\mathbf{v}_G = \frac{b}{2}\omega \rightarrow .$$

Principle of impulse and momentum. To simplify the analysis, neglect the mass of the bullet after impact.

Also $$A_X(\Delta t) = 0.$$



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

$\xrightarrow{+}$ Linear momentum: $$m_B v_0 \cos 5° + 0 = m_P v_G = m_P\left(\frac{b}{2}\omega\right)$$

$$(0.045)(400\cos 5°) = (9)(0.100)\omega \quad \omega = 19.9239 \text{ rad/s}$$

$$v_G = (0.100)(19.9239) = 1.99239 \text{ m/s} \qquad (1)$$

$\circlearrowleft$+ Moments about A: $$(m_B v_0 \cos 5°)h + (m_B v_0 \sin 5°)\frac{b}{2} = I_G\omega + m_P v_G \frac{b}{2}$$

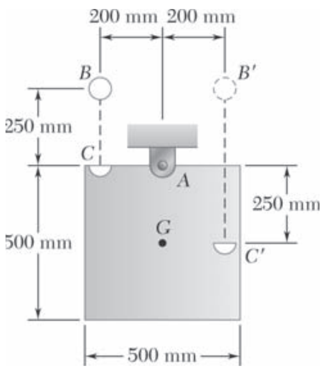
$$m_B v_0\left(b\cos 5° + \frac{b}{2}\sin 5°\right) = \left(I_G + \frac{1}{4}m_P b^2\right)\omega$$

$$(0.045)(400)(h\cos 5° + 0.100\sin 5°) = \left[0.06 + \frac{1}{4}(9)(0.100)^2\right](19.9239)$$

$$17.9315h + 1.5688 = 2.9886$$

(*a*) $$h = 0.07918 \text{ m}$$ $h = 79.2$ mm ◀

(*b*) From Eq. (1), $\mathbf{v}_G = 1.992$ m/s $\rightarrow$ ◀



PROBLEM 17.100

A 8-kg wooden panel is suspended from a pin support at *A* and is initially at rest. A 2-kg metal sphere is released from rest at *B* and falls into a hemispherical cup *C* attached to the panel at a point located on its top edge. Assuming that the impact is perfectly plastic, determine the velocity of the mass center *G* of the panel immediately after the impact.

SOLUTION

Mass and moment of inertia

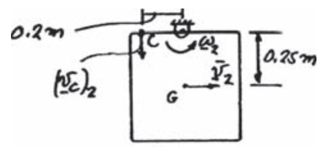
$$m_s = 2 \text{ kg} \quad m_P = 8 \text{ kg}$$

$$\bar{I} = \frac{1}{6} m_P (0.5 \text{ m})^2 = \frac{1}{6}(8)(0.5)^2 = 0.3333 \text{ kg} \cdot \text{m}^2$$

Velocity of sphere at *C*. $\quad (v_C)_1 = \sqrt{2gy} = \sqrt{2(9.81 \text{ m/s}^2)(0.25 \text{ m})} = 2.2147 \text{ m/s}$

Impact analysis.

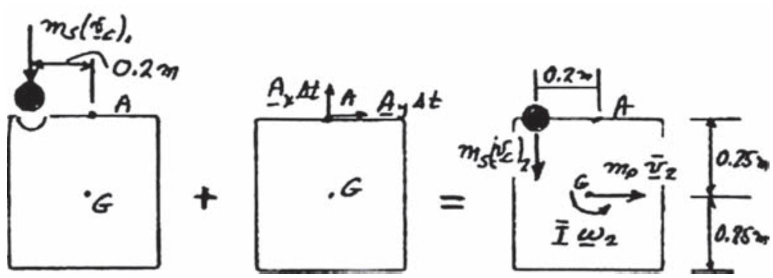
Kinematics $\quad$ Immediately after impact in terms of ω_2



$$\bar{v}_2 = 0.25\omega_2$$

$$(v_C)_2 = 0.2\omega_2$$

Principle of impulse and momentum.



Syst. Momenta$_1$ $\quad + \quad$ **Syst. Ext. Imp.**$_{1\to 2}$ $\quad = \quad$ **Syst. Momenta**$_2$

$\circlearrowleft +$ Moments about A:

$$m_s(v_C)_1(0.2\text{ m}) + 0 = m_s(v_C)_2(0.2\text{ m}) + \bar{I}\omega_2 + m_P\bar{v}_2(0.25\text{ m})$$

$$(2\text{ kg})(2.2147\text{ m/s})(0.2\text{ m}) = (2\text{ kg})(0.2\omega_2)(0.2\text{ m}) + 0.33333\omega_2 + (8\text{ kg})(0.25\text{ m})^2\omega_2$$

$$0.88589 = (0.08 + 0.33333 + 0.500)\omega_2$$

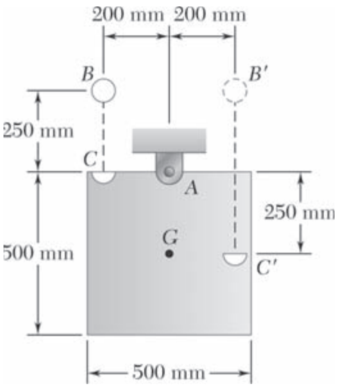
$$\omega_2 = 0.96995\text{ rad/s} \quad \boldsymbol{\omega}_2 = 0.96995\text{ rad/s} \circlearrowleft$$

Velocity of the mass center

$$\bar{v}_2 = (0.25\text{ m})\omega_2 = (0.25\text{ m})(0.96995\text{ rad/s})$$

$$\bar{v}_2 = 0.24249\text{ m/s} \qquad \bar{\mathbf{v}}_2 = 242\text{ mm/s} \rightarrow \blacktriangleleft$$

PROBLEM 17.101



An 8-kg wooden panel is suspended from a pin support at *A* and is initially at rest. A 2-kg metal sphere is released from rest at *B′* and falls into a hemispherical cup *C′* attached to the panel at the same level as the mass center *G*. Assuming that the impact is perfectly plastic, determine the velocity of the mass center *G* of the panel immediately after the impact.

SOLUTION

Mass and moment of inertia.

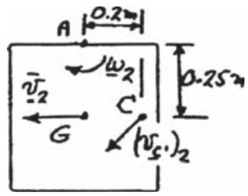
$$m_S = 2 \text{ kg}$$
$$m_P = 8 \text{ kg}$$
$$\bar{I} = \frac{1}{6} m_P (0.5 \text{ m})^2$$
$$= \frac{1}{6}(8)(0.5)^2$$
$$= 0.3333 \text{ kg} \cdot \text{m}^2$$

Velocity of sphere at *C′*.

$$(v_{C'})_1 = \sqrt{2gy}$$
$$= \sqrt{2(9.81 \text{ m/s}^2)(0.5 \text{ m})}$$
$$= 3.1321 \text{ m/s}$$

Impact analysis.

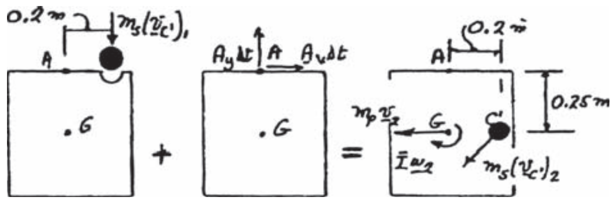
Kinematics Immediately after impact in terms of ω_2.



$$AC' = \sqrt{(0.2)^2 + (0.25)^2} = 0.32016 \text{ m}$$
$$(\mathbf{v}_{C'})_2 = AC'\omega_2$$
$$= 0.32016\omega_2$$

$$\bar{\mathbf{v}}_2 = 0.25\,\omega_2 \quad \theta \quad \text{(perpendicular to } AC.)$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1-2}$ = **Syst. Momenta**$_2$

PROBLEM 17.101 (Continued)

$\circlearrowleft +$ Moments about A:

$$m_S(v_{C'})_1(0.2\text{ m}) + 0 = m_S(v_{C'})_2(AC') + \bar{I}\omega_2 + m_P\bar{v}_2(0.25\text{ m})$$

$$(2\text{ kg})(3.1321\text{ m/s})(0.2\text{ m}) = (2\text{ kg})(0.32016\omega_2)(0.32016\text{ m})$$

$$+0.3333\omega_2 + (8\text{ kg})(0.25\text{ m})^2\omega_2$$

$$1.25284 = (0.2050 + 0.3333 + 0.500)\omega_2$$

$$\omega_2 = 1.2066\text{ rad/s}$$

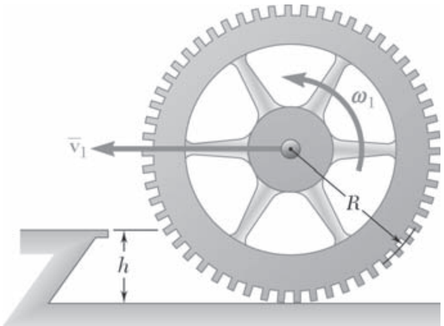
Velocity of the mass center.

$$\bar{v}_2 = (0.25\text{ m})\omega_2$$

$$= (0.25\text{ m})(1.2066\text{ rad/s})$$

$$= 0.3016\text{ m/s}$$

$\bar{\mathbf{v}}_2 = 302\text{ mm/s} \leftarrow$ ◀



PROBLEM 17.102

The gear shown has a radius $R = 150$ mm and a radius of gyration $\bar{k} = 125$ mm. The gear is rolling without sliding with a velocity $\bar{\mathbf{v}}_1$ of magnitude 3 m/s when it strikes a step of height $h = 75$ mm. Because the edge of the step engages the gear teeth, no slipping occurs between the gear and the step. Assuming perfectly plastic impact, determine the angular velocity of the gear immediately after the impact.

SOLUTION

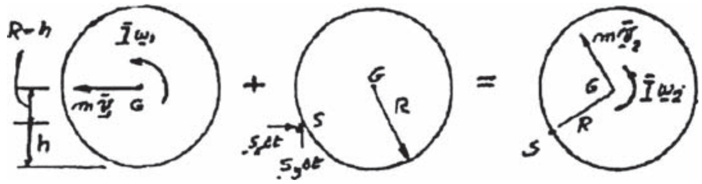
Kinematics. Just before impact, the contact point with the rack is the instantaneous center of rotation of the gear.

$$\bar{\mathbf{v}}_1 = R\omega_1 \leftarrow$$

Just after impact, Point S is the instantaneous center of rotation

$$\mathbf{v}_2 = R\omega_2 \measuredangle\, \theta \quad \text{(perpendicular to } GS\text{)}$$

Principle of impulse and momentum.



$\circlearrowleft$+ Moments about S: $\quad m\bar{v}_2(R-h) + \bar{I}\omega_1 = m\bar{v}_2 R + \bar{I}\omega_2$

$$m(R\omega_1)(R-h) + m\bar{k}^2\omega_1 = m(R\omega_2)R + m\bar{k}^2\omega_2$$

$$[R(R-h) + \bar{k}^2]\omega_1 = (R^2 + \bar{k}^2)\omega_2$$

$$\omega_2 = \frac{R^2 + \bar{k}^2 - Rh}{R^2 + k^2}\omega_1 \qquad \omega_2 = \left[1 - \frac{Rh}{R^2 + k^2}\right]\omega_1 \tag{1}$$

Data: $\quad R = 150$ mm, $\bar{k} = 125$ mm, $v_1 = 3$ m/s, $h = 75$ mm

$$\omega_1 = \frac{v_1}{R} = \frac{3 \text{ m/s}}{0.150 \text{ m}} = 20 \text{ rad/s}$$

Angular velocity.

From (1), $\quad \omega_2 = \left[1 - \frac{(150)(75)}{(150^2 + 125^2)}\right](20 \text{ rad/s}) = 0.7049(20) \qquad \boldsymbol{\omega}_2 = 14.10 \text{ rad/s} \circlearrowleft$ ◀

PROBLEM 17.103

A uniform slender rod AB of mass m is at rest on a frictionless horizontal surface when hook C engages a small pin at A. Knowing that the hook is pulled upward with a constant velocity $\mathbf{v}_0$, determine the impulse exerted on the rod (*a*) at A, (*b*) at B. Assume that the velocity of the hook is unchanged and that the impact is perfectly plastic.

SOLUTION

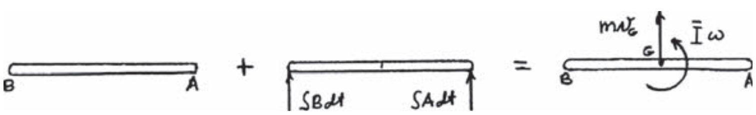
At the given instant, $\qquad v_B = 0.$

Moment of inertia. $\qquad \bar{I} = \frac{1}{12} mL^2$

Kinematics. (Rotation about B). $\qquad \omega = \frac{v_0}{L}$

$$v_G = \frac{L}{2}\omega = \frac{1}{2}v_0$$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

↺ Moments about B: $\qquad 0 + L\int A\,dt = mv_G \frac{L}{2} + \bar{I}\omega$

(*a*) $$\int A\,dt = \frac{mv_G}{2} + \frac{mL\omega}{12} = \frac{mv_0}{4} + \frac{mv_0}{12} = \frac{mv_0}{3}$$

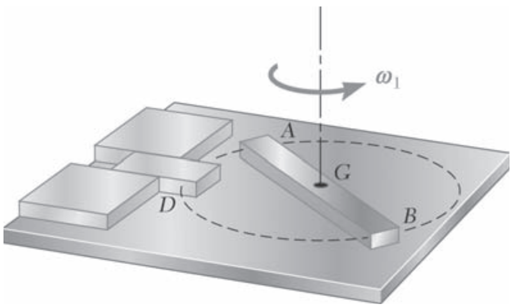
$$\int \mathbf{A}\,dt = \frac{mv_0}{3} \uparrow \blacktriangleleft$$

↻ Moments about A: $\qquad 0 + L\int B\,dt = mv_G \frac{L}{2} - \bar{I}\omega$

(*b*) $$\int B\,dt = \frac{mv_G}{2} - \frac{mL\omega}{12} = \frac{mv_0}{4} - \frac{mv_0}{12} = \frac{mv_0}{6}$$

$$\int \mathbf{B}\,dt = \frac{mv_0}{6} \uparrow \blacktriangleleft$$

PROBLEM 17.104



A uniform slender bar of length L and mass m is supported by a frictionless horizontal table. Initially the bar is spinning about its mass center G with a constant angular velocity ω_1. Suddenly latch D is moved to the right and is struck by end A of the bar. Assuming that the impact of A and D is perfectly plastic, determine the angular velocity of the bar and the velocity of its mass center immediately after the impact.

SOLUTION

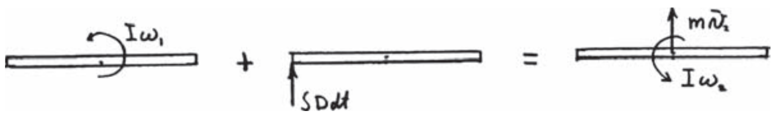
Moment of inertia. $\quad \bar{I} = \frac{1}{12} mL^2$

Before impact. $\quad (v_A)_1 = \frac{L}{2}\omega_1 \downarrow$

Impact condition. $\quad (v_A)_2 = -e(v_A)_1 = \frac{1}{2} eL\omega_1 \uparrow$

Kinematics after impact. $\quad \bar{v}_2 = (v_A)_2 + \frac{L}{2}\omega_2 = \frac{1}{2}eL\omega_1 + \frac{1}{2}L\omega_2$

Principle of impulse-momentum at impact.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowleft$ Moments about D: $\quad I\omega_1 + 0 = I\omega_2 + m\bar{v}_2 \frac{L}{2}$

$$I\omega_1 = I\omega_2 + m\left(\frac{1}{2}eL\omega_1 + \frac{1}{2}L\omega_2\right)\frac{L}{2}$$

$$\frac{1}{12}mL^2\omega_1 = \frac{1}{12}mL^2\omega_2 + \frac{1}{4}mL^2 e\omega_1 + \frac{1}{4}mL^2\omega_2$$

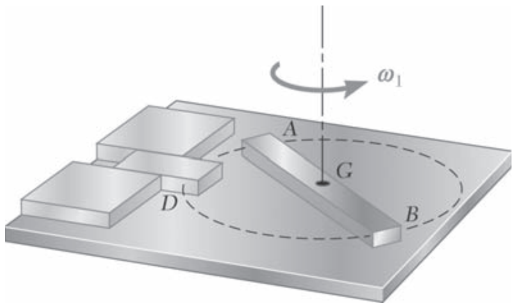
$$\omega_2 = \frac{1}{4}(1-3e)\omega_1$$

$$\bar{v}_2 = \frac{1}{2}Le\omega_1 + \frac{1}{2}L\left(\frac{1}{4}\right)(1-3e)\omega_1 = \frac{1}{8}(1+e)\omega_1 L$$

For perfectly plastic impact, $\quad e = 0 \qquad \boldsymbol{\omega}_2 = \frac{1}{4}\omega_1 \circlearrowleft$ ◀

$\bar{\mathbf{v}}_2 = \frac{1}{8}L\omega_1 \uparrow$ ◀

PROBLEM 17.105



Solve Problem 17.104, assuming that the impact of A and D is perfectly elastic.

PROBLEM 17.104 A uniform slender bar of length L and mass m is supported by a frictionless horizontal table. Initially the bar is spinning about its mass center G with a constant angular velocity ω_1. Suddenly latch D is moved to the right and is struck by end A of the bar. Assuming that the impact of A and D is perfectly plastic, determine the angular velocity of the bar and the velocity of its mass center immediately after the impact.

SOLUTION

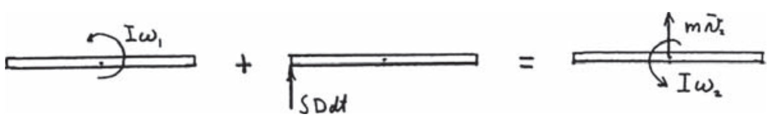
Moment of inertia. $\quad \bar{I} = \frac{1}{12}mL^2$

Before impact. $\quad (v_A)_1 = \frac{L}{2}\omega_1 \downarrow$

Impact condition. $\quad (v_A)_2 = -e(v_A)_1 = \frac{1}{2}eL\omega_1 \uparrow$

Kinematics after impact. $\quad \bar{v}_2 = (v_A)_2 + \frac{L}{2}\omega_2 = \frac{1}{2}eL\omega_1 + \frac{1}{2}L\omega_2$

Principle of impulse-momentum at impact.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about D:

$$I\omega_1 + 0 = I\omega_2 + m\bar{v}_2\frac{L}{2}$$

$$I\omega_1 = I\omega_2 + m\left(\frac{1}{2}eL\omega_1 + \frac{1}{2}L\omega_2\right)\frac{L}{2}$$

$$\frac{1}{12}mL^2\omega_1 = \frac{1}{12}mL^2\omega_2 + \frac{1}{4}mL^2e\omega_1 + \frac{1}{4}mL^2\omega_2$$

$$\omega_2 = \frac{1}{4}(1-3e)\omega_1$$

$$\bar{v}_2 = \frac{1}{2}Le\omega_1 + \frac{1}{2}L\left(\frac{1}{4}\right)(1-3e)\omega_1 = \frac{1}{8}(1+e)\omega_1 L$$

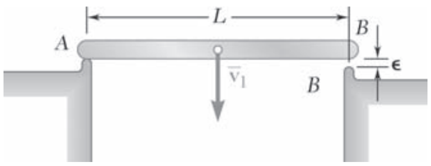
PROBLEM 17.105 (Continued)

For perfectly elastic impact, $e = 1$

$$\omega_2 = \frac{1}{4}(1-3)\omega_1 = -\frac{1}{2}\omega_1 \qquad \boldsymbol{\omega}_2 = \frac{1}{2}\omega_1 \circlearrowright \blacktriangleleft$$

$$\bar{v}_2 = \frac{1}{2}L\omega_1 + \frac{1}{2}L\left(-\frac{1}{2}\omega_1\right) = \frac{1}{4}L\omega_1 \qquad \bar{\mathbf{v}}_2 = \frac{1}{4}L\omega_1 \uparrow \blacktriangleleft$$

PROBLEM 17.106

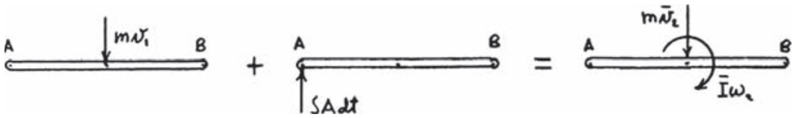


A uniform slender rod of length L is dropped onto rigid supports at A and B. Since support B is slightly lower than support A, the rod strikes A with a velocity $\bar{\mathbf{v}}_1$ before it strikes B. Assuming perfectly elastic impact at both A and B, determine the angular velocity of the rod and the velocity of its mass center immediately after the rod (*a*) strikes support A, (*b*) strikes support B, (*c*) again strikes support A.

SOLUTION

Moment of inertia. $\bar{I} = \dfrac{1}{12} mL^2$

(*a*) *First Impact at A*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Condition of impact: $e = 1$: $(\mathbf{v}_A)_2 = v_1 \uparrow$

Kinematics: $\bar{v}_2 = \dfrac{L}{2}\omega - (v_A)_2 = \dfrac{L}{2}\omega - v_1$

$\circlearrowright$ Moments about A:

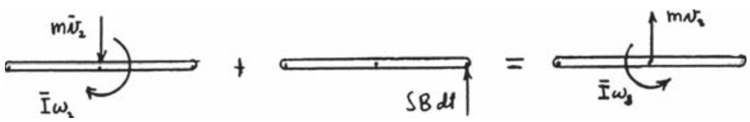
$$mv_1 \frac{L}{2} + 0 = m\bar{v}_2 \frac{L}{2} + \bar{I}\omega_2$$

$$= m\left(\frac{L}{2}\omega - v_1\right)\frac{L}{2} + \left(\frac{1}{12}mL^2\right)\omega_2 \qquad \boldsymbol{\omega}_2 = \frac{3v_1}{L} \circlearrowright \blacktriangleleft$$

$$\bar{v}_2 = \frac{L}{2}\left(\frac{3v_1}{L}\right) - v_1 = \frac{1}{2}v_1 \qquad \bar{\mathbf{v}}_2 = \frac{1}{2}v_1 \downarrow \blacktriangleleft$$

$$(v_B)_2 = L\omega - (v_A)_2 = 3v_1 - v_1 = 2v_1 \downarrow$$

(*b*) *Impact at B*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Condition of impact. $e = 1$: $(\mathbf{v}_B)_3 = 2v_1 \uparrow$

Kinematics: $\bar{v}_3 = (v_B)_2 - \dfrac{L}{2}\omega = 2v_1 - \dfrac{L}{2}\omega$

PROBLEM 17.106 (Continued)

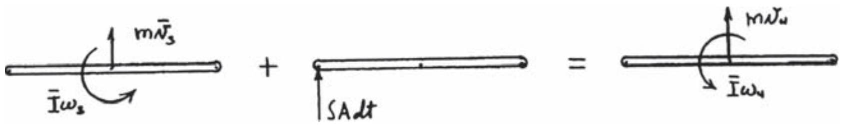
Moments about B: $-m\bar{v}_2 \dfrac{L}{2} + \bar{I}\omega_2 + 0 = m\bar{v}_3 \dfrac{L}{2} - \bar{I}\omega_3$

$$-m\left(\frac{1}{2}v_1\right)\frac{L}{2} + \left(\frac{1}{12}mL^2\right)\left(\frac{3v_1}{L}\right) + 0 = m\left(2v_1 - \frac{L}{2}\omega_3\right)\frac{L}{2} - \left(\frac{1}{12}mL^2\right)\omega_3 \qquad \boldsymbol{\omega}_3 = \frac{3v_1}{L} \circlearrowleft \blacktriangleleft$$

$$\bar{v}_3 = 2v_1 - \frac{L}{2}\left(\frac{3v_1}{L}\right) = \frac{1}{2}v_1 \qquad \bar{\mathbf{v}}_3 = \frac{1}{2}v_1 \uparrow \blacktriangleleft$$

$$(v_A)_3 = L\omega - (v_B)_3 = 3v_1 - 2v_1 = v_1 \downarrow$$

(*c*) *Second Impact at A*.



Syst. Momenta$_3$ + **Syst. Ext. Imp.**$_{3\to 4}$ = **Syst. Momenta**$_4$

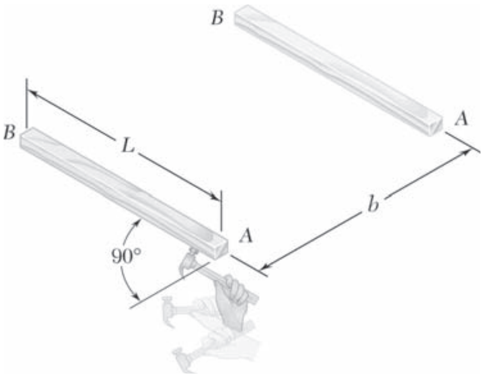
Condition of impact. $e = 1$: $(\mathbf{v}_A)_4 = v_1 \uparrow$

Kinematics: $\bar{v}_4 = (v_A)_4 + \dfrac{L}{2}\omega_4 = v_1 + \dfrac{L}{2}\omega_4$

Moments about A: $m\bar{v}_3 \dfrac{L}{2} + \bar{I}\omega_3 + 0 = m\bar{v}_4 \dfrac{L}{2} + \bar{I}\omega_4$

$$m\left(\frac{1}{2}v_1\right)\frac{L}{2} + \left(\frac{1}{12}mL^2\right)\left(\frac{3v_1}{L}\right) + 0 = m\left(v_1 + \frac{L}{2}\omega_4\right)\frac{L}{2} + \left(\frac{1}{12}mL^2\right)\omega_4 \qquad \boldsymbol{\omega}_4 = 0 \blacktriangleleft$$

$$\bar{v}_4 = v_1 + 0 \qquad \bar{\mathbf{v}}_4 = v_1 \uparrow \blacktriangleleft$$



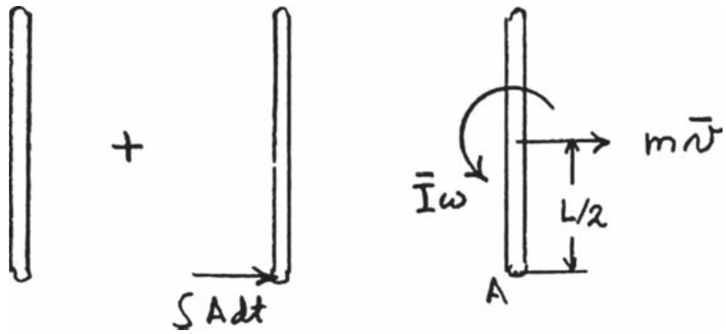
PROBLEM 17.107

A uniform slender rod AB is at rest on a frictionless horizontal table when end A of the rod is struck by a hammer which delivers an impulse that is perpendicular to the rod. In the subsequent motion, determine the distance b through which the rod will move each time it completes a full revolution.

SOLUTION

Moment of inertia. $\quad \bar{I} = \frac{1}{12} mL^2$

Kinetics.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

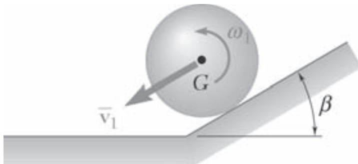
↺ Moments about A: $\quad 0 + 0 + \bar{I}\omega + m\bar{v}\frac{L}{2}$

$$\bar{v} = \frac{2\bar{I}\omega}{mL} = \frac{2\frac{1}{12}mL^2\omega}{mL} = \frac{1}{6}\omega L$$

Motion after impact. $\quad \theta = \omega t \qquad t = \frac{\theta}{\omega} = \frac{2\pi}{\omega}$

$$b = \bar{v}t = \left(\frac{1}{6}\omega L\right)\frac{2\pi}{\omega}$$

$$b = \frac{\pi}{3}L \blacktriangleleft$$



PROBLEM 17.108

A uniform sphere of radius r rolls down the incline shown without slipping. It hits a horizontal surface and, after slipping for a while, it starts rolling again. Assuming that the sphere does not bounce as it hits the horizontal surface, determine its angular velocity and the velocity of its mass center after it has resumed rolling.

SOLUTION

Moment of inertia. Solid sphere. $\bar{I} = \frac{2}{5}mr^2$

Kinematics.

Before

After

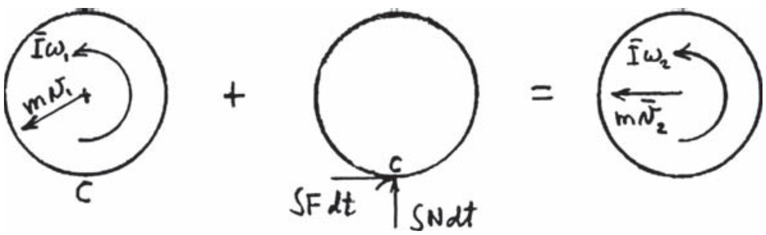
Before impact (rolling).

$$v_1 = r\omega_1$$

After slipping has stopped.

$$\bar{v}_2 = r\omega_2$$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

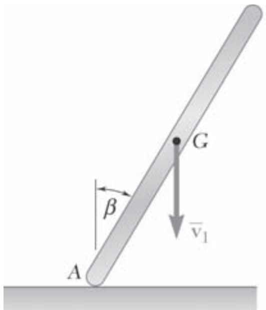
Moments about C:

$$\bar{I}\omega_1 + mv_1 r\cos\beta + 0 = \bar{I}\omega_2 + m\bar{v}_2 r$$

$$\bar{I}\omega_1 + mr^2\omega_1\cos\beta = \bar{I}\omega_2 + mr^2\omega_2$$

$$\omega_2 = \frac{\bar{I} + mr^2\cos\beta}{\bar{I} + mr^2}\omega_1 = \frac{\frac{2}{5}mr^2 + mr^2\cos\beta}{\frac{2}{5}mr^2 + mr^2}\omega_1 \qquad \boldsymbol{\omega}_2 = \frac{1}{7}(2 + 5\cos\beta)\omega_1 \circlearrowleft \blacktriangleleft$$

$$\bar{v}_2 = r\omega_2 = \frac{2 + 5\cos\beta}{7}r\omega_1 \qquad \bar{\mathbf{v}}_2 = \frac{1}{7}(2 + 5\cos\beta)\bar{v}_1 \leftarrow \blacktriangleleft$$



PROBLEM 17.109

The slender rod AB of length L forms an angle β with the vertical as it strikes the frictionless surface shown with a vertical velocity $\bar{\mathbf{v}}_1$ and no angular velocity. Assuming that the impact is perfectly elastic, derive an expression for the angular velocity of the rod immediately after the impact.

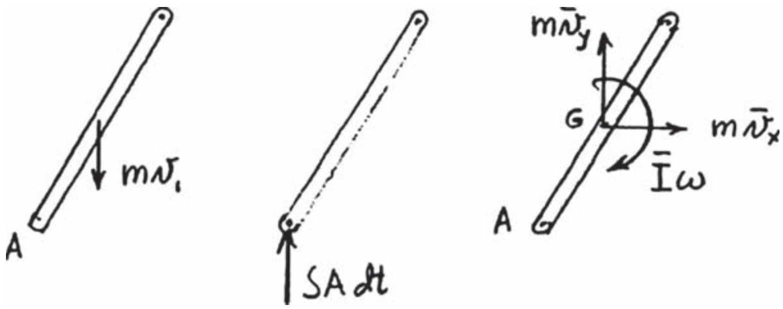
SOLUTION

Moment of inertia. $\quad \bar{I} = \dfrac{1}{12}mL^2$

Perfectly elastic impact. $\quad e = 1 \qquad [(v_A)_y]_2 = -e[(v_A)_y]_1 = ev_1 \uparrow$

$$\mathbf{v}_A = (v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} = (v_A)_x\mathbf{i} + v_1\mathbf{j}$$

Kinetics.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

$\xrightarrow{+}$ horizontal components: $\quad 0 + 0 = m\bar{v}_x \qquad m\bar{v}_x = 0$

Kinematics. $\quad \mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A} \qquad [\bar{v}_y \uparrow] = [v_1 \uparrow] + [(v_A)_x \rightarrow] + \left[\dfrac{L}{2}\omega \searrow \beta\right]$

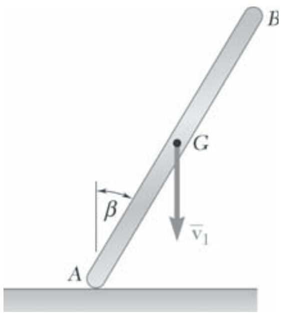
Velocity components $\uparrow$: $\quad \bar{v}_y = v_1 - \dfrac{L}{2}\omega\sin\beta$

$\circlearrowright$ Moments about A: $\quad mv_1\dfrac{L}{2}\sin\beta + 0 = -m\bar{v}_y\dfrac{L}{2}\sin\beta + \bar{I}\omega$

$$mv_1\frac{L}{2}\sin\beta = m\left(\frac{L}{2}\omega\sin\beta - v_1\right)\frac{L}{2}\sin\beta + \frac{1}{12}mL^2\omega$$

$$\left(\frac{1}{12}mL^2 + \frac{1}{4}mL^2\sin^2\beta\right)\omega = mv_1L\sin\beta \qquad \omega = \frac{12\sin\beta}{1+3\sin^2\beta}\frac{v_1}{L} \blacktriangleleft$$

PROBLEM 17.110



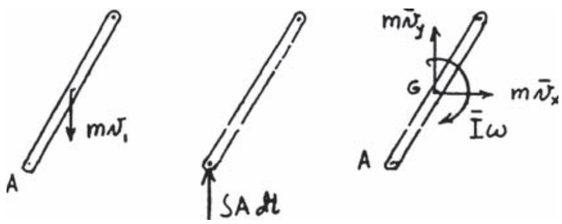
Solve Problem 17.109, assuming that the impact between rod *AB* and the frictionless surface is perfectly plastic.

SOLUTION

Moment of inertia.
$$\overline{I} = \frac{1}{12} mL^2$$

Perfectly plastic impact.
$$e = 0 \qquad [(v_A)_y]_2 = -e(v_A)_{y1} = 0$$
$$\mathbf{v}_A = (v_A)_x \mathbf{i} + (v_A)_y \mathbf{j} = (v_A)_x \mathbf{i}$$

Kinetics.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

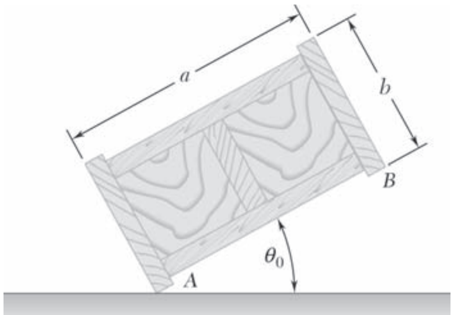
$\xrightarrow{+}$ horizontal components:
$$0 + 0 = m\overline{v}_x \qquad m\overline{v}_x = 0$$

Kinematics.
$$\mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A} \qquad [\overline{v}_y \uparrow] = [(v_A)_x \rightarrow] + + \left[\frac{L}{2}\omega \searrow \beta\right]$$

Velocity components $\uparrow$:
$$\overline{v}_y = -\frac{L}{2}\omega \sin\beta$$

$\circlearrowright$ Moments about *A*:
$$mv_1 \frac{L}{2}\sin\beta + 0 = -m\overline{v}_y \frac{L}{2}\sin\beta + \overline{I}\omega$$
$$mv_1 \frac{L}{2}\sin\beta = m\left(\frac{L}{2}\omega \sin\beta\right)\frac{L}{2}\sin\beta + \frac{1}{12}mL^2\omega$$
$$\left(\frac{1}{12}mL^2 + \frac{1}{4}mL^2\sin^2\beta\right)\omega = \frac{1}{2}mv_1 L\sin\beta$$

$$\omega = \frac{6\sin\beta}{1+3\sin^2\beta}\frac{v_1}{L} \blacktriangleleft$$

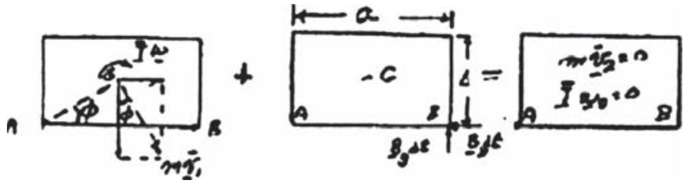


PROBLEM 17.111

A uniformly loaded rectangular crate is released from rest in the position shown. Assuming that the floor is sufficiently rough to prevent slipping and that the impact at B is perfectly plastic, determine the smallest value of the ratio a/b for which corner A will remain in contact with the floor.

SOLUTION

We consider the limiting case when the crate is just ready to rotate about B. At that instant the velocities must be zero and the reaction at corner A must be zero. Use the principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowright$+ Moments about B:

$$\bar{I}\omega_1 + (m\bar{v}_1)_2 \frac{b}{2} - (m\bar{v}_1)_y \frac{a}{2} + 0 = 0 \tag{1}$$

Note:
$$\sin\phi = \frac{b}{\sqrt{a^2+b^2}}, \quad \cos\phi = \frac{a}{\sqrt{a^2+b^2}}$$

$$\bar{v}_1 = (AG)v_1 = \frac{1}{2}\sqrt{a^2+b^2}\,\omega_1$$

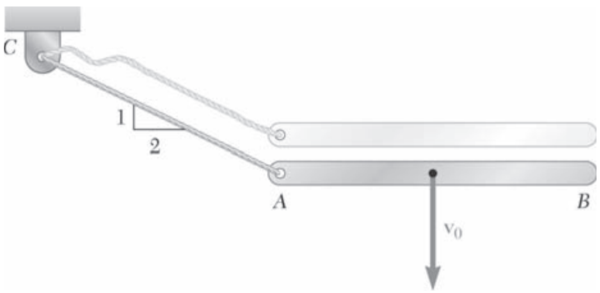
Thus:
$$(m\bar{v}_1)_x = (m\bar{v}_1)\sin\phi = \frac{m}{2}\sqrt{a^2+b^2}\,\omega_1 \frac{b}{\sqrt{a^2+b^2}} = \frac{1}{2}mb\omega_1$$

Also,
$$(m\bar{v}_1)_y = (m\bar{v}_1)\cos\phi = \frac{1}{2}ma\omega$$

$$\bar{I} = \frac{1}{12}m(a^2+b^2)$$

From Eq. (1)
$$\frac{1}{12}m(a^2+b^2)\omega_1 + \frac{1}{2}(mb\omega_1)\frac{b}{2} - \frac{1}{2}(ma\omega_1)\frac{a}{2} = 0$$

$$\frac{1}{3}mb^2\omega_1 - \frac{1}{6}ma^2v_1 = 0 \qquad \frac{a^2}{b^2} = 2 \quad \frac{a}{b} = \sqrt{2} \quad \frac{a}{b} = 1.414 \blacktriangleleft$$

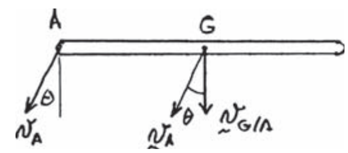


PROBLEM 17.112

A uniform slender rod AB of length L is falling freely with a velocity $\mathbf{v}_0$ when cord AC suddenly becomes taut. Assuming that the impact is perfectly plastic, determine the angular velocity of the rod and the velocity of its mass center immediately after the cord becomes taut.

SOLUTION

Immediately after impact



$$\tan\theta = \frac{1}{2} \quad \theta = 26.565°$$

Due to constraint of inextensible cord,

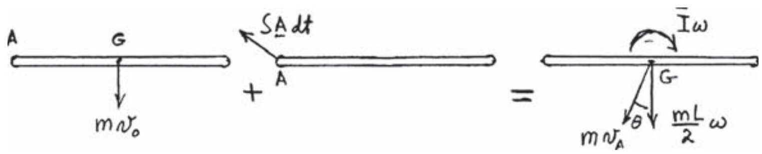
$$\mathbf{v}_A = v_A \swarrow \theta$$

Kinematics:

$$\mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A}$$

$$= v_A \swarrow \theta + \frac{L}{2}\omega \downarrow \qquad (1)$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

Components $\swarrow \theta$:

$$mv_0\cos\theta + 0 = mv_A + \frac{mL}{2}\omega\cos\theta$$

$$v_A + \left(\frac{1}{2}\cos\theta\right)(L\omega) = v_0\cos\theta \qquad (2)$$

$\circlearrowleft$ Moments about A:

$$mv_0\frac{L}{2} + 0 = (mv_A\cos\theta)\frac{L}{2} + \left(\frac{mL}{2}\omega\right)\left(\frac{L}{2}\right) + \bar{I}\omega$$

$$\left(\frac{L}{2}\cos\theta\right)v_A + \left(\frac{1}{4}L^2 + \frac{1}{12}L^2\right)\omega = \frac{L}{2}v_0 \qquad (3)$$

Solving Eqs. (2) and (3) simultaneously (with $\theta = 26.565°$)

$$v_A = 0.55902v_0 \quad L\omega = 0.75v_0$$

Angular velocity after impact. $\boldsymbol{\omega} = 0.750\dfrac{v_0}{L} \circlearrowright$ ◄

PROBLEM 17.112 (Continued)

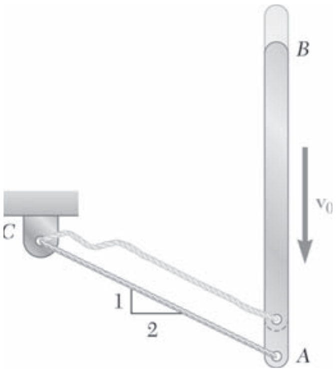
Velocity of mass center.

$$\mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A}$$

$$\mathbf{v}_G = 0.55902v_0 \;\; \theta + \frac{1}{2}(0.75v_0)\downarrow$$

$$= 0.25v_0 \leftarrow + 0.5v_0 \downarrow + 0.375v_0 \downarrow$$

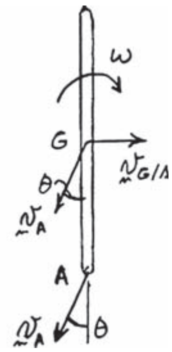
$$= 0.25v_0 \leftarrow + 0.875v_0 \downarrow \qquad \mathbf{v}_B = 0.910v_0 \;\; 74.1^\circ \blacktriangleleft$$



PROBLEM 17.113

A uniform slender rod AB of length L is falling freely with a velocity $\mathbf{v}_0$ when cord AC suddenly becomes taut. Assuming that the impact is perfectly plastic, determine the angular velocity of the rod and the velocity of its mass center immediately after the cord becomes taut.

SOLUTION



Immediately after impact,

$$\tan\theta = \frac{1}{2} \qquad \theta = 26.565°$$

Due to constraint of inextensible cord,

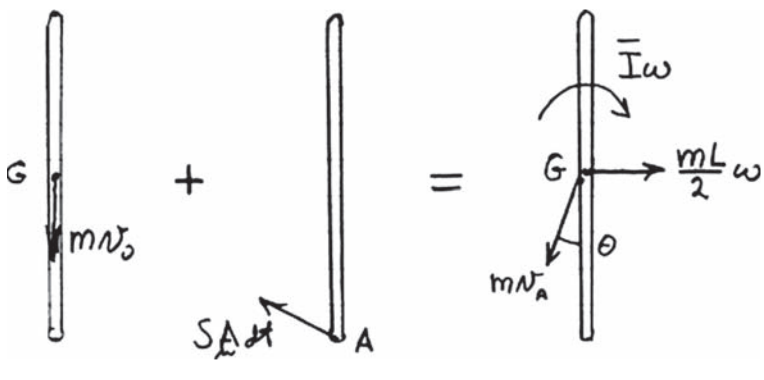
$$\mathbf{v}_A = v_A \measuredangle\, \theta$$

Kinematics.

$$\mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A}$$

$$= v_A \measuredangle\, \theta + \frac{L}{2}\omega \rightarrow$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Components $\measuredangle\, \theta$:

$$mv_0 \cos\theta + 0 = mv_A - \frac{mL}{2}\omega \sin\theta$$

$$v_A - \left(\frac{1}{2}\sin\theta\right)(L\omega) = v_0 \cos\theta \qquad (2)$$

PROBLEM 17.113 (Continued)

Moments about A:

$$0+0=-(mv_A\sin\theta)\frac{L}{2}+\left(\frac{mL}{2}\omega\right)\left(\frac{L}{2}\right)+\bar{I}\omega$$

$$-\left(\frac{L}{2}\sin\theta\right)v_A+\left(\frac{1}{4}L^2+\frac{1}{12}L^2\right)\omega=0 \qquad (3)$$

Solving Eqs. (2) and (3) simultaneously (with $\theta = 26.565°$),

$$v_A = 1.05527v_0 \quad L\omega = 0.70588v_0$$

Angular velocity after impact. $\boldsymbol{\omega} = 0.706\frac{v_0}{L}$ ◄

Velocity of mass center.

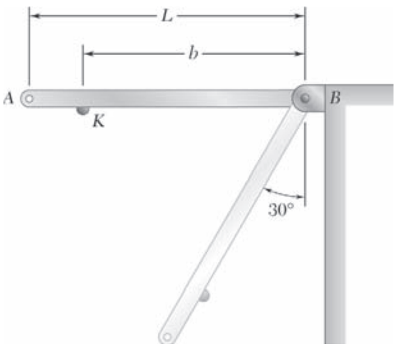
$$\mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A}$$

$$\mathbf{v}_G = 1.055227v_0 \ \angle\ \theta + \frac{1}{2}(0.70588)v_0 \rightarrow$$

$$= 0.47059v_0 \leftarrow + 0.94118v_0 \downarrow + 0.35294v_0 \rightarrow$$

$$= 0.11765v_0 \leftarrow + 0.94118v_0 \downarrow \qquad \mathbf{v}_G = 0.949v_0 \ \angle\ 82.9°$$ ◄

PROBLEM 17.114

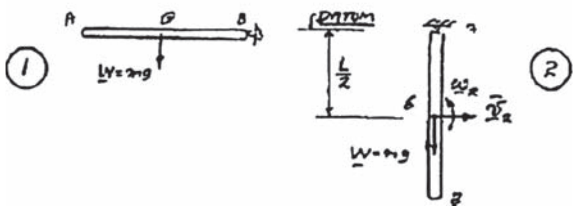


A slender rod of length L and mass m is released from rest in the position shown. It is observed that after the rod strikes the vertical surface it rebounds to form an angle of 30° with the vertical. (*a*) Determine the coefficient of restitution between knob K and the surface. (*b*) Show that the same rebound can be expected for any position of knob K.

SOLUTION

For analysis of the downward swing of the rod before impact and for the upward swing after impact use the principle of conservation of energy.

Before impact.



$$V_1 = 0$$

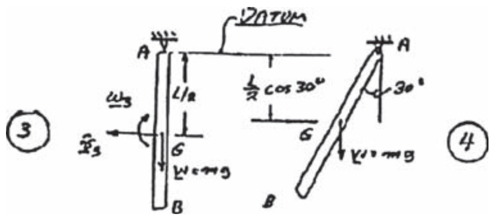
$$V_2 = -W\frac{L}{2} = -mg\frac{L}{2}$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\bar{v}_2^2 = \frac{1}{2}\left(\frac{1}{12}mL^2\right)v_2^2 + \frac{1}{2}m\left(\frac{1}{2}\omega_2\right)^2 = \frac{1}{6}mL^2\omega_2^2$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 = \frac{1}{6}mL^2\omega_2^2 = -mg\frac{L}{2}; \quad \omega_2^2 = 3\frac{g}{L} \qquad \omega_2 = 1.73205\sqrt{\frac{g}{L}}$$

After impact.



$$V_3 = -W\frac{L}{2} = -mg\frac{L}{2}$$

PROBLEM 17.114 (Continued)

$$V_4 = -W\frac{L}{2}\cos 30°$$

$$T_3 = \frac{1}{2}\bar{I}\omega_3^2 + \frac{1}{2}m\bar{v}_3^2$$

$$= \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_3^2 + \frac{1}{2}m\left(\frac{1}{2}\omega_3\right)^2$$

$$= \frac{1}{6}mL^2\omega_3^2$$

$$T_4 = 0$$

$$T_3 + V_3 = T_4 + V_4: \quad \frac{1}{6}mL^2\omega_3^2 - m\theta\frac{L}{2} = 0 - mg\frac{L}{2}\cos 30°$$

$$\omega_3^2 = (1-\cos 30°)\frac{g}{L} \qquad \boldsymbol{\omega}_3 = 0.63397\sqrt{\frac{g}{L}} \circlearrowright$$

Analysis of impact.

Let r be the distance BK.

Before impact, $\quad (\mathbf{v}_k)_3 = b\omega_2 \rightarrow\, = 1.73205b\sqrt{\frac{g}{L}} \rightarrow$

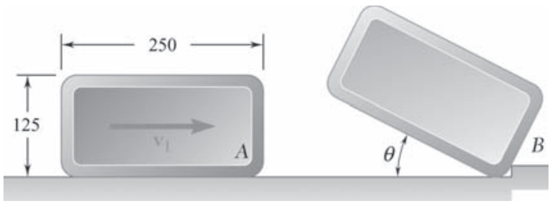
After impact, $\quad (\mathbf{v}_k)_4 = b\omega_3 \leftarrow\, = 0.63397b\sqrt{\frac{g}{L}} \leftarrow$

Coefficient of restitution. $\quad e = \frac{|(v_k)_{4n}|}{|(v_k)_{3n}|}$

$$e = \frac{0.63397}{1.73205} \qquad e = 0.366 \blacktriangleleft$$

(b) Clearly the answer is independent of b.

PROBLEM 17.115



The uniform rectangular block shown is moving along a frictionless surface with a velocity $\overline{\mathbf{v}}_1$ when it strikes a small obstruction at B. Assuming that the impact between corner A and obstruction B is perfectly plastic, determine the magnitude of the velocity $\overline{\mathbf{v}}_1$ for which the maximum angle θ through which the block will rotate is 30°.

SOLUTION

Let m be the mass of the block.

Dimensions: $a = 250 \text{ mm} = 0.25 \text{ m}$

$b = 125 \text{ mm} = 0.125 \text{ m}$

Moment of inertia about the mass center.

$$\bar{I} = \frac{1}{12}m(a^2 + b^2)$$

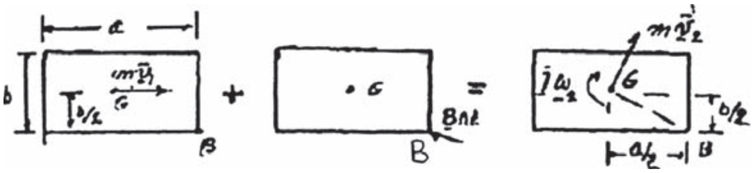
Let d be one half the diagonal. $d = \frac{1}{2}\sqrt{a^2 + b^2} = \frac{\sqrt{5}}{16} \text{ m} = 0.13975 \text{ m}$

Kinematics. Before impact $\bar{v}_1 = v_1 \rightarrow, \quad \boldsymbol{\omega}_1 = 0$

After impact, the block is rotating about corner at B.

$$\omega_2 = \omega_2 \circlearrowright \quad v_2 = d\omega_2 \nearrow$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\rightarrow 2}$ = **Syst. Momenta**$_2$

$\circlearrowright +$ Moments about B:

$$\frac{mv_1 b}{2} + 0 = \bar{I}\omega_2 + mdv_2$$

$$\frac{1}{2}mv_1 b = \frac{1}{12}m(a^2 + b^2)\omega_2 + md^2\omega_2$$

$$= \frac{1}{3}m(a^2 + b^2)\omega_2$$

PROBLEM 17.115 (Continued)

Angular velocity after impact $\qquad \boldsymbol{\omega}_2 = \dfrac{3v_1 b}{2(a^2+b^2)} \circlearrowright \qquad (1)$

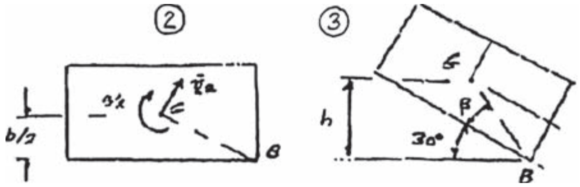
The motion after impact is a rotation about corner B.

Position 2 (immediately after impact). $\qquad \bar{v}_2 = d\omega_2$

Position 3 $(\theta = 30°)$. $\qquad \beta = \tan^{-1}\dfrac{b}{a} = \tan^{-1} 0.5 = 26.565°$

$$h = d\sin(\beta + 30°) = 0.13975 \sin 56.565° = 0.11663 \text{ m}$$

$$\omega_3 = 0 \qquad \bar{v}_3 = 0$$



Potential energy: $\qquad V_2 = \dfrac{mgb}{2} \quad V_3 = mgh$

Kinetic energy: $\qquad T_2 = \dfrac{1}{2}\bar{I}\omega_2^2 + \dfrac{1}{2}m\bar{v}_2^2 = \dfrac{1}{2}(\bar{I} + md^2)\omega_2^2$

$$= \frac{1}{6}m(a^2+b^2)\omega_2^2 \quad T_3 = 0$$

Principle of conservation of energy:

$$T_2 + V_3 = T_3 + V_3$$

$$\frac{1}{6}m(a^2+b^2)\omega_2^2 + \frac{mgb}{2} = 0 + mgh$$

$$\omega_2^2 = \frac{3g(2h-b)}{(a^2+b^2)} = \frac{(3)(9.81)(0.23325-0.125)}{(0.25)^2+(0.125)^2}$$

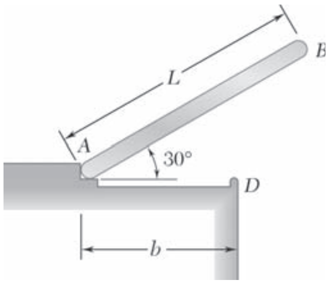
$$= 40.7794 \text{ (rad/s)}^2 \qquad \boldsymbol{\omega}_2 = 6.3859 \text{ rad/s}$$

Magnitude of initial velocity.

Solving Eq. (1) for v_1 $\qquad v_1 = \dfrac{2(a^2+b^2)\omega_2}{3b}$

$$v_1 = \frac{(2)\left[(0.25)^2 + (0.125)^2\right](6.3859)}{(3)(0.125)} \qquad v_1 = 2.66 \text{ m/s} \blacktriangleleft$$

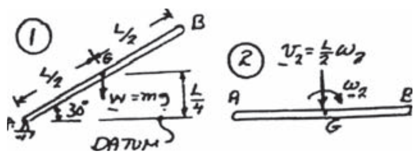
PROBLEM 17.116



A slender rod of mass m and length L is released from rest in the position shown and hits edge D. Assuming perfectly plastic impact at D, determine for $b = 0.6L$, (a) the angular velocity of the rod immediately after the impact, (b) the maximum angle through which the rod will rotate after the impact.

SOLUTION

For analysis of the falling motion before impact use the principle of conservation of energy.



Position 1: $\quad T_1 = 0, \quad V_1 = mg\dfrac{L}{4}$

Position 2: $\quad V_2 = 0$

$$T_2 = \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$$

$$T_2 = \frac{1}{6}mL^2\omega_2^2$$

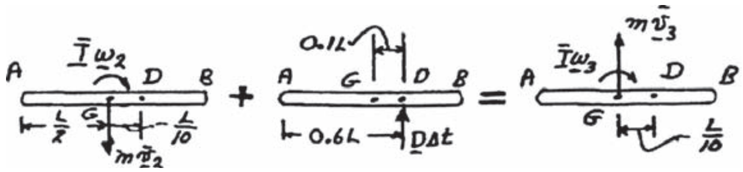
$$T_1 + V_1 = T_2 + V_2: \quad 0 + mg\frac{L}{4} = \frac{1}{6}mL^2\omega_2^2 \qquad \omega_2 = \sqrt{\frac{3g}{2L}}$$

Analysis of impact. Kinematics

Before impact, rotation is about Point A. $\qquad \bar{v}_2 = \dfrac{L}{2}\omega_2$

After impact, rotation is about Point D. $\qquad \bar{v}_3 = \dfrac{L}{10}\omega_3$

Principle of impulse-momentum.



Syst. Momenta$_2$ + **Syst. Ext. Imp.**$_{2\to3}$ = **Syst. Momenta**$_3$

Moments about D:

$$\bar{I}\omega_2 - m\bar{v}_2\left(\frac{L}{10}\right) = \bar{I}\omega_3 + m\bar{v}_3\left(\frac{L}{10}\right) \tag{1}$$

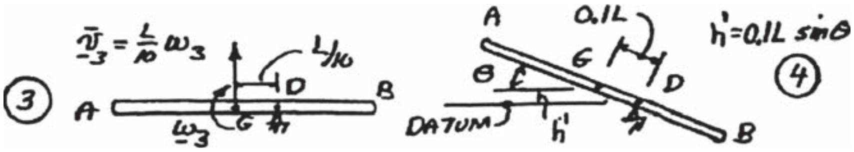
$$\frac{1}{12}mL^2\omega_2 - m\left(\frac{L}{2}\omega_2\right)\frac{L}{10} = \frac{1}{12}mL^2\omega_3 + m\left(\frac{L}{10}\omega_3\right)\frac{L}{10}$$

$$\left(\frac{1}{12} - \frac{1}{20}\right)\omega_2 = \left(\frac{1}{12} + \frac{1}{100}\right)\omega_3$$

PROBLEM 17.116 (Continued)

(*a*) Angular velocity. $\omega_3 = \frac{5}{14}\omega_2 = \frac{5}{14}\sqrt{\frac{3g}{2L}}$ $\boldsymbol{\omega}_3 = 0.437\sqrt{\frac{g}{L}}$ ◀

For analysis of the rotation about Point *D* after the impact use the principle of conservation of energy.



Position 3. (Just after impact)

$$\bar{v}_3 = \frac{L}{10}\omega_3 \quad V_3 = 0$$

$$T_3 = \frac{1}{2}m\left(\frac{L}{10}\omega_3\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_3^2 = \frac{14}{300}mL^2\omega_3^2$$

$$= \frac{14}{300}mL^2\left(\frac{5}{14}\sqrt{\frac{2g}{2L}}\right)^2 = \frac{mgL}{112}$$

Position 4. $\theta =$ maximum rotation angle.

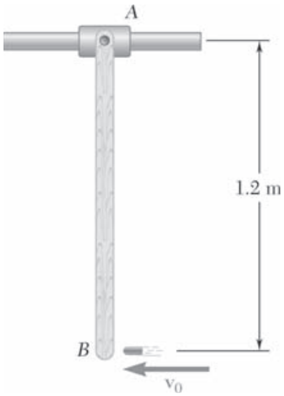
$$h' = \frac{L}{10}\sin\theta$$

$$V_4 = mgh' = \frac{mgL}{10}\sin\theta$$

$$\bar{v}_4 = 0, \quad \omega_4 = 0, \quad T_4 = 0$$

$$T_3 + V_3 = T_4 + V_4; \quad \frac{mgL}{112} + 0 = 0 + \frac{mgL}{10}\sin\theta$$

(*b*) Maximum rotation angle. $\sin\theta = \frac{10}{112}$ $\theta = 5.12°$ ◀



PROBLEM 17.117

A 30-g bullet is fired with a horizontal velocity of 350 m/s into the 8-kg wooden beam *AB*. The beam is suspended from a collar of negligible mass that can slide along a horizontal rod. Neglecting friction between the collar and the rod, determine the maximum angle of rotation of the beam during its subsequent motion.

SOLUTION

Mass of bullet. $m' = 30g = 0.03$ kg

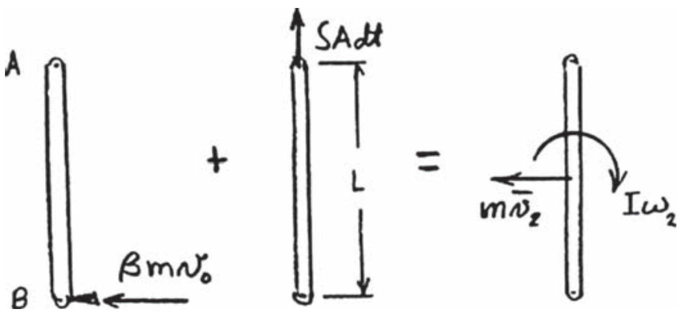
Mass of beam *AB*. $m = 8$ kg

Mass ratio. $\beta = \dfrac{m'}{m} = 0.00375 \quad m' = \beta m$

Since β is so small, the mass of the bullet will be neglected in comparison with that of the beam in determining the motion after the impact.

Moment of inertia. $\bar{I} = \dfrac{1}{12} mL^2$

Impact kinetics.



Syst. Momenta$_2$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

$\xleftarrow{+}$ linear components: $-\beta m v_0 + 0 = m\bar{v}_2 \qquad \bar{v}_2 = \beta v_0$

Moments about *B*: $0 + 0 = \bar{I}\omega - m\bar{v}_2 \dfrac{L}{2}$

$$\omega = \frac{m\bar{v}_2 L}{2\bar{I}} = \frac{12 m\beta v_0 L}{2mL^2}$$

$$\omega = \frac{6\beta v_0}{L}$$

PROBLEM 17.117 (Continued)

Motion during rising. *Position 2*. Just after the impact.

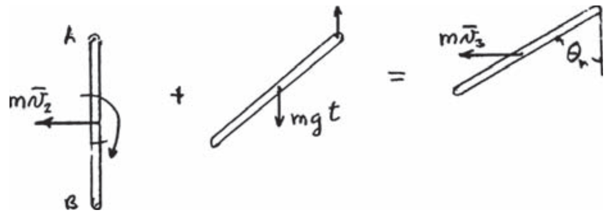
$$V_2 = -mg\frac{L}{2} \quad \text{(datum at level } A\text{)}$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m(\beta v_0)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\left(\frac{6\beta v_0}{L}\right)^2$$

$$= 2\beta^2 m v_0^2$$

Position 3. $\omega = 0, \quad \theta = \theta_m.$



Syst. Momenta$_2$ + **Syst. Ext. Imp.**$_{2\to 3}$ = **Syst. Momenta**$_3$

$$V_3 = -mg\frac{L}{2}\cos\theta_m$$

$$T_3 = \frac{1}{2}m\bar{v}_3^2$$

$\xleftarrow{+}$ linear components: $m\bar{v}_2 + 0 = m\bar{v}_3 \qquad \bar{v}_3 = \bar{v}_2 = \beta v_0$

Conservation of energy.

$$T_2 + V_2 = T_3 + V_3: \quad 2\beta^2 m v_0^2 - mg\frac{L}{2} = \frac{1}{2}m(\beta v_0)^2 - mg\frac{L}{2}\cos\theta_m$$

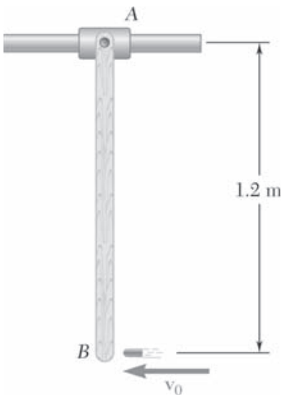
$$\frac{3\beta^2 v_0^2}{gL} = 1 - \cos\theta_m$$

$$\cos\theta_m = 1 - \frac{3\beta^2 v_0^2}{gL}$$

$$= 1 - \frac{(3)(0.00375)^2(350)^2}{(9.81)(1.2)}$$

$$= 0.56099$$

$\theta_m = 55.9°$ ◀



PROBLEM 17.118

For the beam of Problem 17.117, determine the velocity of the 30-g bullet for which the maximum angle of rotation of the beam will be 90°.

PROBLEM 17.117 A 30-g bullet is fired with a horizontal velocity of 350 m/s into the 8-kg wooden beam *AB*. The beam is suspended from a collar of negligible weight that can slide along a horizontal rod. Neglecting friction between the collar and the rod, determine the maximum angle of rotation of the beam during its subsequent motion.

SOLUTION

Mass of bullet. $m' = 30g = 0.03 \text{ kg}$

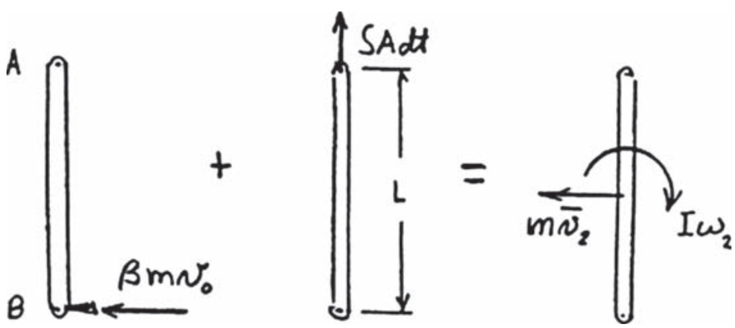
Mass of beam *AB*. $m = 8 \text{ kg}$

Mass ratio. $\beta = \dfrac{m'}{m} = 0.00375 \quad m' = \beta m$

Since β is so small, the mass of the bullet will be neglected in comparison with that of the beam in determining the motion after the impact.

Moment of inertia. $\bar{I} = \dfrac{1}{12} mL^2$

Impact Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xleftarrow{+}$ linear components: $-\beta m v_0 + 0 = m\bar{v}_2 \quad \bar{v}_2 = \beta v_0$

$\circlearrowright$ Moments about *B*: $0 + 0 = \bar{I}\omega - m\bar{v}_2 \dfrac{L}{2}$

$$\omega = \frac{m\bar{v}_2 L}{2\bar{I}} = \frac{12 m \beta v_0 L}{2mL^2}$$

$$\omega = \frac{6\beta v_0}{L}$$

PROBLEM 17.118 (Continued)

Motion during rising. *Position 2*. Just after the impact.

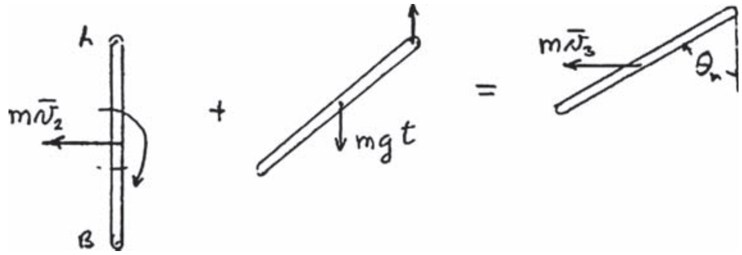
$$V_2 = -mg\frac{L}{2} \quad \text{(datum at level } A\text{)}$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m(\beta v_0)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\left(\frac{6\beta v_0}{L}\right)^2$$

$$= 2\beta^2 mv_0^2$$

Position 3. $\omega = 0, \quad \theta = \theta_m.$



Syst. Momenta$_2$ + Syst. Ext. Imp.$_{2\to3}$ = Syst. Momenta$_3$

$$V_3 = -mg\frac{L}{2}\cos\theta_m$$

$$T_3 = \frac{1}{2}m\bar{v}_3^2$$

$\xleftarrow{+}$ linear components: $m\bar{v}_2 + 0 = m\bar{v}_3 \quad \bar{v}_3 = \bar{v}_2 = \beta v_0$

Conservation of energy.

$$T_2 + V_2 = T_3 + V_3: \quad 2\beta^2 mv_0^2 - mg\frac{L}{2} = \frac{1}{2}m(\beta v_0)^2 - mg\frac{L}{2}\cos\theta_m$$

$$\beta v_0 = \sqrt{\frac{1}{3}gL(1-\cos\theta_m)}$$

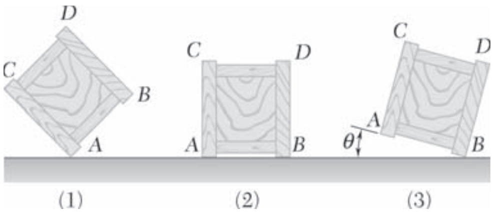
$$= \sqrt{\left(\frac{1}{3}\right)(9.81)(1.2)(1-\cos 90°)}$$

$$= 1.98091 \text{ m/s}$$

$$v_0 = \frac{1.98091}{0.00375}$$

$v_0 = 528$ m/s ◄

PROBLEM 17.119

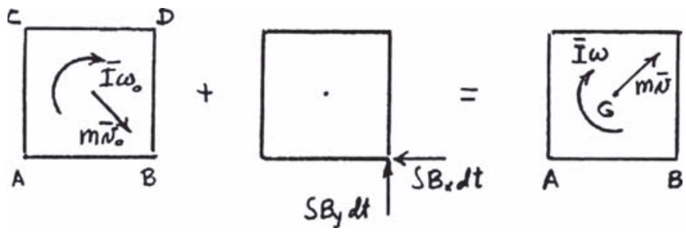


A uniformly loaded square crate is released from rest with its corner D directly above A; it rotates about A until its corner B strikes the floor, and then rotates about B. The floor is sufficiently rough to prevent slipping and the impact at B is perfectly plastic. Denoting by $\boldsymbol{\omega}_0$ the angular velocity of the crate immediately before B strikes the floor, determine (a) the angular velocity of the crate immediately after B strikes the floor, (b) the fraction of the kinetic energy of the crate lost during the impact, (c) the angle θ through which the crate will rotate after B strikes the floor.

SOLUTION

Let m be the mass of the crate and c be the length of an edge.

Moment of inertia $\qquad \bar{I} = \frac{1}{12}m(c^2 + c^2) = \frac{1}{6}mc^2$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

Kinematics:

$$\bar{v}_0 = r_{G/A}\omega_0 = \frac{1}{2}\sqrt{2}c\omega_0$$

$$\bar{v} = r_{G/B}\omega = \frac{1}{2}\sqrt{2}c\omega$$

Moments about B:

$$\bar{I}\omega_0 + 0 = \bar{I}\omega + r_{G/B}m\bar{v}$$

$$\frac{1}{6}mc^2\omega_0 + 0 = \frac{1}{6}mc^2\omega + \left(\frac{1}{2}\sqrt{2}c\right)m\left(\frac{1}{2}\sqrt{2}c\omega\right) = \frac{2}{3}mc^2\omega$$

(a) Solving for ω, $\qquad \boldsymbol{\omega} = \frac{1}{4}\omega_0 \circlearrowright$ ◀

Kinetic Energy.

Before impact:

$$T_1 = \frac{1}{2}\bar{I}\omega_0^2 + \frac{1}{2}m\bar{v}_0^2$$

$$= \frac{1}{2}\left(\frac{1}{6}mc^2\right)\omega_0^2 + \frac{1}{2}m\left(\frac{1}{2}\sqrt{2}c\omega_0\right)^2$$

$$= \frac{1}{3}mc^2\omega_0^2$$

PROBLEM 17.119 (Continued)

After impact: $$T_2 = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}\left(\frac{1}{6}mc^2\right)\omega^2 + \frac{1}{2}m\left(\frac{1}{2}\sqrt{2}c\omega\right)^2$$

$$= \frac{1}{3}mc^2\omega^2 = \frac{1}{3}mc^2\left(\frac{1}{4}\omega_0\right)^2 = \frac{1}{48}mc^2\omega_0$$

(*b*) Fraction of energy lost: $$\frac{T_1 - T_2}{T_1} = \frac{\frac{1}{3} - \frac{1}{48}}{\frac{1}{3}} = 1 - \frac{1}{16} \qquad \frac{15}{16} \blacktriangleleft$$

Conservation of energy during falling. $$T_0 + V_0 = T_1 + V_1 \qquad (1)$$

Conservation of energy during rising. $$T_3 + V_3 = T_2 + V_2 \qquad (2)$$

Conditions: $$T_0 = 0, \quad T_3 = 0 \quad T_2 = \frac{1}{16}T_1$$

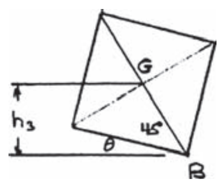
$$V_0 = mg\left(\frac{1}{2}\sqrt{2}c\right) \quad V_1 = V_2 = mg\left(\frac{1}{2}c\right) \quad V_3 = mgh_3$$

From Equation (1), $$T_1 = V_0 - V_1 = \frac{1}{2}\left(\sqrt{2} - 1\right)mgc$$

From Equation (2), $$T_2 = V_3 - V_2 = mgh_3 - \frac{1}{2}mgc$$

$$\frac{h_3 - \frac{1}{2}c}{\frac{1}{2}\sqrt{2} - 1} = \frac{1}{16} \quad h_3 = \left[\frac{1}{2} + \frac{1}{16}\left(\sqrt{2} - 1\right)\right]c$$

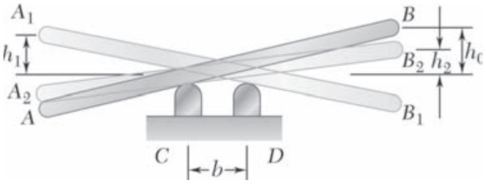
(*c*) From geometry, $$h_3 = \frac{1}{2}\sqrt{2}c\,\sin(\theta + 45°)$$



Equating the two expressions for h_3,

$$\sin(45° + \theta) = \frac{\frac{1}{2} + \frac{1}{16}\left(\sqrt{2} - 1\right)}{\frac{1}{2}\sqrt{2}}$$

$$45° + \theta = 46.503° \qquad \theta = 1.50° \blacktriangleleft$$



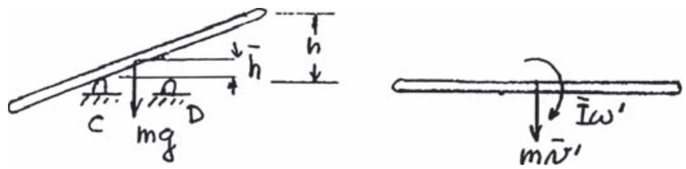
PROBLEM 17.120

A uniform slender rod AB of length $L = 800$ mm is placed with its center equidistant from two supports that are located at a distance $b = 200$ mm from each other. End B of the rod is raised a distance $h_0 = 100$ mm and released; the rod then rocks on the supports as shown. Assuming that the impact at each support is perfectly plastic and that no slipping occurs between the rod and the supports, determine (a) the height h_1 reached by end A after the first impact, (b) the height h_2 reached by end B after the second impact.

SOLUTION

Moment of inertia.
$$\bar{I} = \frac{1}{12}mL^2$$

Let θ be the angle of inclination of the bar and $\bar{h}$ the elevation of the center of gravity.



Position 0 Position′

Position 0.
$$V_0 = mg\bar{h}_0 = mg\frac{bh_0}{L+b} \quad T_0 = 0$$

Position′.
$$\bar{v}' = \frac{1}{2}b\omega' \quad V' = 0$$

$$T' = \frac{1}{2}m(\bar{v}')^2 + \frac{1}{2}\bar{I}(\omega')^2 = \frac{1}{2}m\left(\frac{1}{2}b\omega'\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)(\omega')^2$$
$$= \frac{1}{24}m(L^2 + 3b^2)(\omega')^2$$

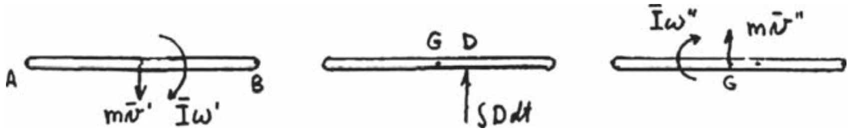
Conservation of energy.
$$T_0 + V_0 = T' + V': \quad 0 + \frac{mgbh_0}{L+b} = \frac{1}{24}m(L^2 + 3b^2)(\omega')^2$$

$$(\omega')^2 = \frac{24gbh_0}{(L+b)(L^2+3b^2)} = C_1h_0 \qquad (1)$$

where
$$C_1 = \frac{24gb}{(L+b)(L^2+3b^2)} \qquad (2)$$

PROBLEM 17.120 (Continued)

Impact at D.



Syst. Momenta′ + Syst. Ext. Imp.′→″ = Syst. Momenta″

Kinematics. $\quad \bar{v}' = \dfrac{1}{2}b\omega' \quad \bar{v}'' = \dfrac{1}{2}b\omega''$

↻ Moments about D: $\quad m\bar{v}'\dfrac{b}{2} - \bar{I}\omega' + 0 = m\bar{v}''\dfrac{b}{2} + \bar{I}\omega''$

$$-m\left(\frac{1}{2}b\omega'\right)\frac{b}{2} + \frac{1}{12}mL^2\omega' + 0 = m\left(\frac{1}{2}b\omega''\right)\frac{b}{2} + \frac{1}{12}mL^2\omega''$$

$$\omega'' = \frac{L^2 - 3b^2}{L^2 + 3b^2}\omega' = C_2\omega' \tag{3}$$

where
$$C_2 = \frac{L^2 - 3b^2}{L^2 + 3b^2} \tag{4}$$

Position 1 Maximum elevation of end A.

Conservation of energy. $\quad T_1 + V_1 = T'' + V'$

Result $\quad (\omega'')^2 = C_1 h_1$

$$h_1 = \frac{(\omega'')^2}{C_1} = \frac{C_2^2(\omega')^2}{C_1} = \frac{C_2^2 C_1}{C_1}h_1 = C_2^2 h_1$$

Data: $\quad L = 800 \text{ mm} = 0.8 \text{ m}, \quad b = 200 \text{ mm} = 0.2 \text{ m}$

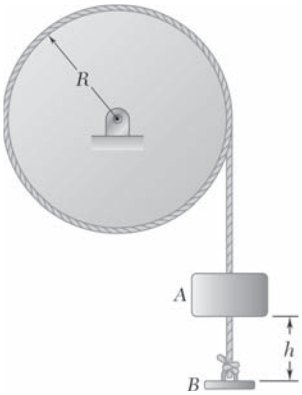
$$C_2 = \frac{(0.8)^2 - 3(0.2)^2}{(0.8)^2 + 3(0.2)^2} = \frac{13}{19} = 0.68421$$

$$C_2^2 = \frac{169}{361} = 0.468144$$

(a) $\quad h_1 = 0.468144h_0 = (0.468144)(100 \text{ mm}) = 46.8144 \text{ mm} \qquad h_1 = 46.8 \text{ mm}$ ◄

(b) $\quad h_2 = 0.468144h_1 = (0.468144)(46.8144) = 21.916 \text{ mm} \qquad h_2 = 21.9 \text{ mm}$ ◄

PROBLEM 17.121



A small plate B is attached to a cord that is wrapped around a uniform 4 kg disk of radius $R = 200$ mm. A 1.5 kg collar A is released from rest and falls through a distance $h = 300$ mm before hitting plate B. Assuming that the impact is perfectly plastic and neglecting the weight of the plate, determine immediately after the impact (*a*) the velocity of the collar, (*b*) the angular velocity of the disk.

SOLUTION

The collar A falls a distance h. From the principle of conservation of energy.

$$v_1 = \sqrt{2gh}$$

Impact analysis. *Kinematics*. Plastic impact. $e = 0$
Collar A and plate B move together. The cord is inextensible.

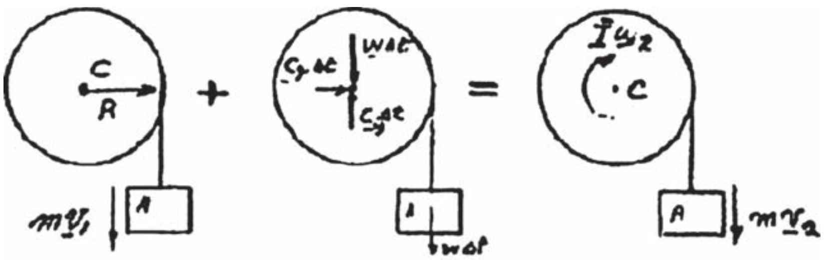
$$\bar{v}_2 = R\omega \quad \text{or} \quad \omega_2 = \frac{\bar{v}_2}{R}$$

Let m = mass of collar A and M = mass of disk

Moment of inertia of disk: $\bar{I} = \frac{1}{2}MR^2$

Principle of impulse and momentum.

$$\bar{I}\omega_1 = 0$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

PROBLEM 17.121 (Continued)

$\circlearrowright$ Moments about C:

$$m v_1 R = \bar{I}\omega_2 + m v_2 R \qquad (1)$$

$$m v_1 R = \frac{1}{2} M R^2 \left(\frac{v_2}{R}\right) + m v_2 R$$

$$m v_1 = \frac{1}{2} M v_2 + m v_2$$

$$v_2 = \frac{2m}{2m + M} v_1$$

Data:

$$m = 1.5 \text{ kg}$$

$$M = 4 \text{ kg}$$

$$h = 300 \text{ mm} = 0.3 \text{ m}$$

$$R = 200 \text{ mm} = 0.2 \text{ m}$$

$$v_1 = \sqrt{2(9.81)(0.3)}$$

$$= 2.4261 \text{ m/s}$$

(*a*) Velocity of *A*.

$$v_2 = \frac{(2)(1.5)}{[(2)(1.5) + 4]} v_1 = \frac{3}{7} v_1$$

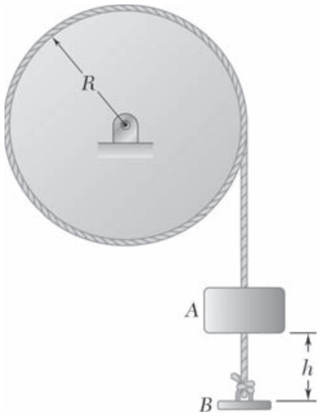
$$v_2 = \frac{3}{7}(2.4261) = 1.03976 \text{ m/s}$$

$$\mathbf{v}_2 = 1.040 \text{ m/s} \downarrow \blacktriangleleft$$

(*b*) Angular velocity.

$$\omega_2 = \frac{1.03976}{0.2} = 5.1988 \text{ rad/s}$$

$$\omega_2 = 5.20 \text{ rad/s} \circlearrowright \blacktriangleleft$$



PROBLEM 17.122

Solve Problem 17.121, assuming that the coefficient of restitution between A and B is 0.8.

PROBLEM 17.121 A small plate B is attached to a cord that is wrapped around a uniform 4-kg disk of radius 200 mm. A 1.5 kg collar A is released from rest and falls through a distance $h = 300\,\text{mm}$ before hitting plate B. Assuming that the impact is perfectly plastic and neglecting the weight of the plate, determine immediately after the impact (a) the velocity of the collar, (b) the angular velocity of the disk.

SOLUTION

$$m_D = 4\ \text{kg}$$

$$R = 0.2\ \text{m}$$

$$I_D = \frac{1}{2}m_D R^2 = \frac{1}{2}(4)(0.2)^2 = 0.08\ \text{kg}\cdot\text{m}^2$$

$$m_A = 1.5\ \text{kg}$$

$$h = 300\ \text{mm} = 0.3\ \text{m}$$

Collar A falls through distance h. Use conservation of energy.

$$T_1 = 0$$

$$V_1 = W_A h$$

$$T_2 = \frac{1}{2}m_A v_A^2$$

$$V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + W_A h = \frac{1}{2}m_A v_A^2 + 0$$

$$v_A^2 = \frac{2m_A h}{W_A} = 2gh$$

$$= (2)(9.81)(0.3)$$

$$= 5.886\ \text{m}^2/\text{s}^2$$

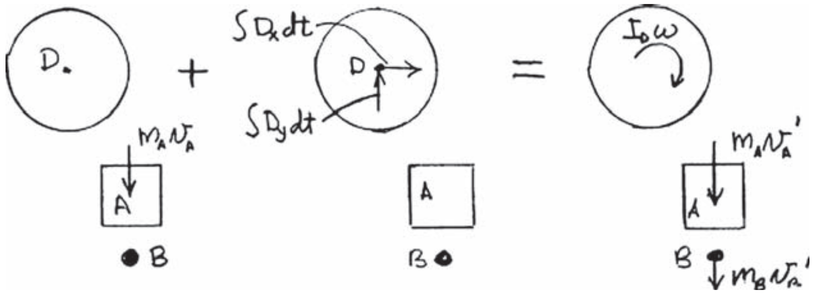
$$\mathbf{v}_A = 2.4261\ \text{m/s} \downarrow$$

Impact. Neglect the mass of plate B. Neglect the effect of weight during the duration of the impact.

PROBLEM 17.122 (Continued)

Kinematics. $\omega' = \omega$ ↻ $\mathbf{v}'_B = R\omega \downarrow = 0.2\omega' \downarrow$

Conservation of momentum.



↻+ Moments about D: $m_A v_A R + 0 = m_A v'_A R + I_D \omega' + m_B v'_B R$

$$(1.5)(2.4261)(0.2) = (1.5)(0.75)v'_A + (0.08\omega') + 0$$

$$1.125 v'_A + 0.08\omega' = 0.72783 \tag{1}$$

Coefficient of restitution. $v'_B - v'_A = e(v_A - v_B)$

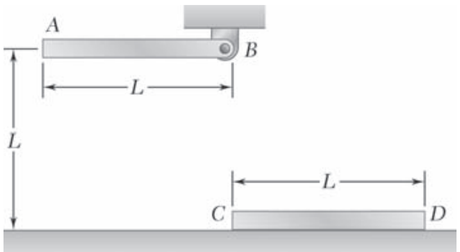
$$0.2\omega' - v'_A = 0.8(2.4261 - 0) \tag{2}$$

$$-v'_A + 0.2\omega' = 1.94088$$

Solving Eqs. (1) and (2) simultaneously

(*a*) Velocity of *A*. $v'_A = -0.0318$ m/s $\mathbf{v}'_A = 0.0318 \text{ m/s} \uparrow$ ◀

(*b*) Angular velocity. $\omega' = 9.545$ rad/s $\boldsymbol{\omega}' = 9.55 \text{ rad/s}$ ↻ ◀



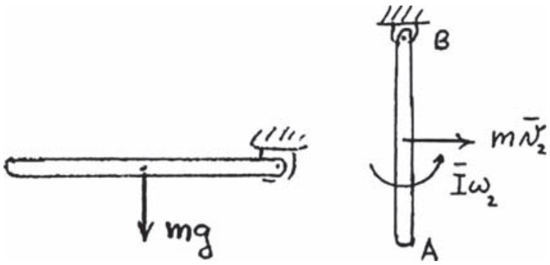
PROBLEM 17.123

A slender rod AB is released from rest in the position shown. It swings down to a vertical position and strikes a second and identical rod CD which is resting on a frictionless surface. Assuming that the coefficient of restitution between the rods is 0.5, determine the velocity of rod CD immediately after the impact.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{12} mL^2$ for each rod.

Rod AB swings to vertical position.



Position 1 Position 2

Position 1. $V_1 = 0 \qquad T_1 = 0$

Position 2. $V_2 = -mg\frac{L}{2}$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$$

$$= \frac{1}{6}mL^2\omega_2^2$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{1}{6}mL^2\omega_2^2 - mg\frac{L}{2}$

$$\boldsymbol{\omega}_2 = \sqrt{\frac{3g}{L}} \circlearrowleft$$

$$\bar{\mathbf{v}}_2 = \frac{L}{2}\sqrt{\frac{3g}{L}} \rightarrow$$

$$(v_B)_2 = L\omega = \sqrt{3gL}$$

PROBLEM 17.123 (Continued)

Impact condition:

$$(v_C)_3 - (v_B)_3 = e(v_B)_2$$

$$(v_C)_3 - L\omega_3 = e\sqrt{3gL}$$

$$(v_C)_3 = L\omega_3 + e\sqrt{3gL}$$

Principle of impulse-momentum at impact.

Syst. Momenta$_2$ + **Syst. Ext. Imp.**$_{2\to3}$ = **Syst. Momenta**$_3$

↺ Moments about B:

$$m\bar{v}_2\frac{L}{2} + \bar{I}\,\omega_2 + 0 = m\bar{v}_3\frac{L}{2} + \bar{I}\,\omega_3 + m(v_C)_3 L$$

$$m\left(\frac{L}{2}\sqrt{\frac{3g}{L}}\right)\frac{L}{2} + \frac{1}{12}mL^2\sqrt{\frac{3g}{L}} + 0 = m\left(\frac{L}{2}\omega_3\right)\frac{L}{2} + \frac{1}{12}mL^2\omega_3 + m\left(L\omega_3 + e\sqrt{3gL}\right)L$$

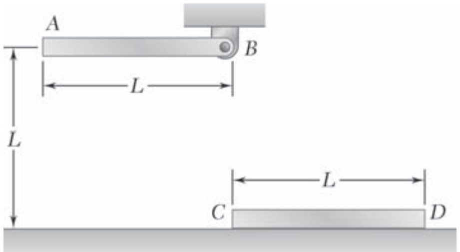
$$\omega_3 = \left(\frac{1}{4} - \frac{3}{4}e\right)\sqrt{\frac{3g}{L}}$$

$$(v_C)_3 = \left(\frac{1}{4} - \frac{3}{4}e\right)\sqrt{3gL} + e\sqrt{3gL}$$

$$(v_C)_3 = \frac{1}{4}(1+e)\sqrt{3gL}$$

For $e = 0.5$ $\quad (v_C)_3 = \frac{1}{4}(1+0.5)\sqrt{3gL}$ $\quad (\mathbf{v}_C)_3 = 0.650\sqrt{gL} \rightarrow$ ◀

PROBLEM 17.124



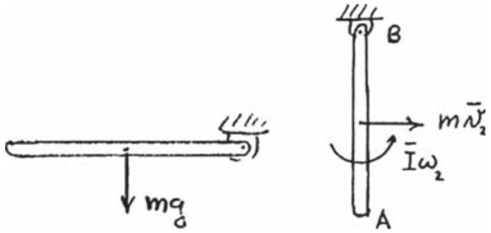
Solve Problem 17.123, assuming that the impact between the rods is perfectly elastic.

PROBLEM 17.123 A slender rod *AB* is released from rest in the position shown. It swings down to a vertical position and strikes a second and identical rod *CD* which is resting on a frictionless surface. Assuming that the coefficient of restitution between the rods is 0.5, determine the velocity of rod *CD* immediately after the impact.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{12}mL^2$ for each rod.

Rod *AB* swings to vertical position.



Position 1 Position 2

Position 1 $V_1 = 0 \qquad T_1 = 0$

Position 2 $V_2 = -mg\frac{L}{2}$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$$

$$= \frac{1}{6}mL^2\omega_2^2$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{1}{6}mL^2\omega_2^2 - mg\frac{L}{2}$

$$\boldsymbol{\omega}_2 = \sqrt{\frac{3g}{L}} \circlearrowleft$$

$$\bar{\mathbf{v}}_2 = \frac{L}{2}\sqrt{\frac{3g}{L}} \rightarrow$$

$$(v_B)_2 = L\omega = \sqrt{3gL}$$

PROBLEM 17.124 (Continued)

Impact condition:

$$-(v_C)_3 - (v_B)_3 = e(v_B)_2$$

$$(v_C)_3 - L\omega_3 = e\sqrt{3gL}$$

$$(v_C)_3 = L\omega_3 + e\sqrt{3gL}$$

Principle of impulse-momentum at impact.

Syst. Momenta$_2$ + **Syst. Ext. Imp.**$_{2\to3}$ = **Syst. Momenta**$_3$

Moments about B:

$$m\bar{v}_2\frac{L}{2} + \bar{I}\,\omega_2 + 0 = m\bar{v}_3\frac{L}{2} + \bar{I}\,\omega_3 + m(v_C)_3 L$$

$$m\left(\frac{L}{2}\sqrt{\frac{3g}{L}}\right)\frac{L}{2} + \frac{1}{12}mL^2\sqrt{\frac{3g}{L}} + 0 = m\left(\frac{L}{2}\omega_3\right)\frac{L}{2} + \frac{1}{12}mL^2\omega_3 + m\left(L\omega_3 + e\sqrt{3gL}\right)L$$

$$\omega_3 = \left(\frac{1}{4} - \frac{3}{4}e\right)\sqrt{\frac{3g}{L}}$$

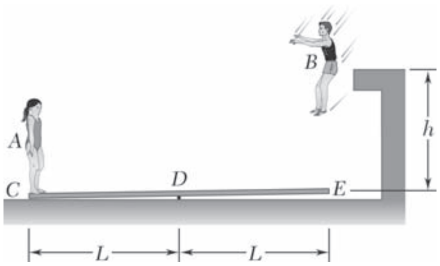
$$(v_C)_3 = \left(\frac{1}{4} - \frac{3}{4}e\right)\sqrt{3gL} + e\sqrt{3gL}$$

$$(v_C)_3 = \frac{1}{4}(1+e)\sqrt{3gL}$$

For perfectly elastic impact, $e = 1$.

$$(v_C)_3 = \frac{1}{4}(1+1)\sqrt{3gL} \qquad (\mathbf{v}_C)_3 = 0.866\sqrt{gL} \rightarrow \blacktriangleleft$$

PROBLEM 17.125



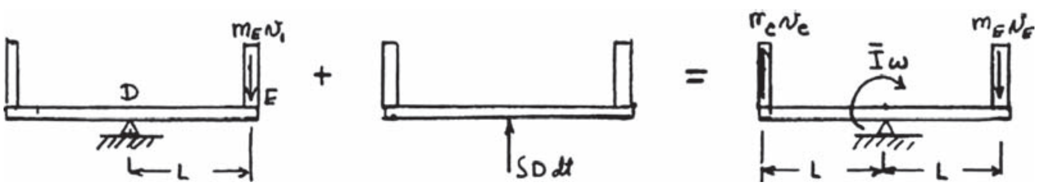
The plank *CDE* has a mass of 15 kg and rests on a small pivot at *D*. The 55-kg gymnast *A* is standing on the plank at *C* when the 70-kg gymnast *B* jumps from a height of 2.5 m and strikes the plank at *E*. Assuming perfectly plastic impact and that gymnast *A* is standing absolutely straight, determine the height to which gymnast *A* will rise.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{12} m_P (2L)^2 = \frac{1}{3} m_P L^2$

Velocity of jumper at *E*. $(v)_1 = \sqrt{2gh_1}$ (1)

Principle of impulse-momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics: $v_C = L\omega \quad v_D = L\omega$

↺ Moments about *D*:

$$m_E v_1 L + 0 = m_E v_E L + m_C v_C L + \bar{I}\omega$$

$$= m_E L^2 \omega + m_C L^2 \omega + \frac{1}{3} m_P L^2 \omega$$

$$\omega = \frac{m_E}{m_E + m_C + \frac{1}{3} m_P} \frac{v_1}{L}$$

$$v_C = L\omega = \frac{m_E v_1}{m_E + m_C + \frac{1}{3} m_P} \quad (2)$$

Gymnast (flier) rising. $h_C = \frac{v_C^2}{2g}$ (3)

Data:

$$m_E = m_B = 70 \text{ kg}$$
$$m_C = m_A = 55 \text{ kg}$$
$$m_P = 15 \text{ kg}$$
$$h_1 = 2.5 \text{ m}$$

PROBLEM 17.125 (Continued)

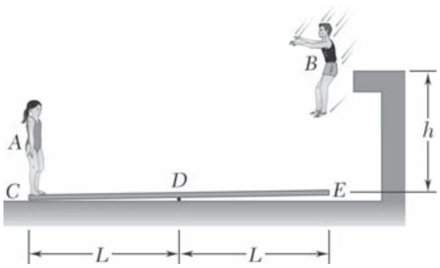
From Equation (1)
$$v_1 = \sqrt{(2)(9.81)(2.5)}$$
$$= 7.0036 \text{ m/s}$$

From Equation (2)
$$v_C = \frac{(70)(7.0036)}{70 + 55 + 5}$$
$$= 3.7712 \text{ m/s}$$

From Equation (3)
$$h_2 = \frac{(3.7712)^2}{(2)(9.81)}$$
$$= 0.725 \text{ m}$$

$h_2 = 725 \text{ mm}$ ◄

PROBLEM 17.126



Solve Problem 17.125, assuming that the gymnasts change places so that gymnast A jumps onto the plank while gymnast B stands at C.

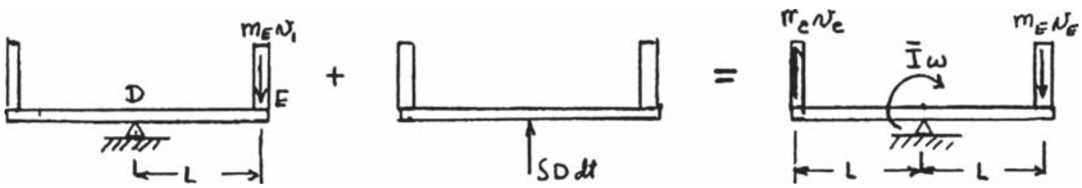
PROBLEM 17.125 The plank CDE has a mass of 15 kg and rests on a small pivot at D. The 55-kg gymnast A is standing on the plank at C when the 70-kg gymnast B jumps from a height of 2.5 m and strikes the plank at E. Assuming perfectly plastic impact and that gymnast A is standing absolutely straight, determine the height to which gymnast A will rise.

SOLUTION

Moment of inertia. $$\bar{I} = \frac{1}{12} m_P (2L)^2 = \frac{1}{3} m_P L^2$$

Velocity of jumper at E. $$(v)_1 = \sqrt{2gh_1} \qquad (1)$$

Principle of impulse-momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics: $$v_C = L\omega \quad v_D = L\omega$$

↺ Moments about D:

$$m_E v_1 L + 0 = m_E v_E L + m_C v_C L + \bar{I}\omega$$

$$= m_E L^2 \omega + m_C L^2 \omega + \frac{1}{3} m_P L^2 \omega$$

$$\omega = \frac{m_E}{m_E + m_C + \frac{1}{3} m_P} \frac{v_1}{L}$$

$$v_C = L\omega = \frac{m_E v_1}{m_E + m_C + \frac{1}{3} m_P} \qquad (2)$$

Gymnast (flier) rising. $$h_C = \frac{v_C^2}{2g} \qquad (3)$$

PROBLEM 17.126 (Continued)

Data:

$$m_E = m_A = 55 \text{ kg}$$
$$m_C = m_B = 70 \text{ kg}$$
$$m_P = 15 \text{ kg}$$
$$h_1 = 2.5 \text{ m}$$

From Equation (1)

$$v_1 = \sqrt{(2)(9.81)(2.5)}$$
$$= 7.0036 \text{ m/s}$$

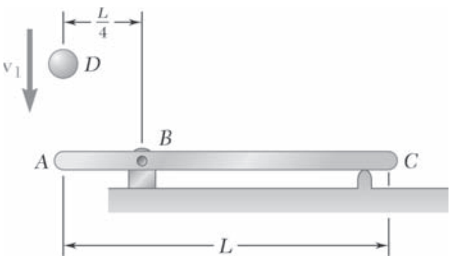
From Equation (2)

$$v_C = \frac{(55)(7.0036)}{55+70+5}$$
$$= 2.9631 \text{ m/s}$$

From Equation (3)

$$h_2 = \frac{(2.9631)^2}{(2)(9.81)}$$
$$= 0.447 \text{ m}$$

$h_2 = 447$ mm ◄



PROBLEM 17.127

Member ABC has a mass of 2.4 kg and is attached to a pin support at B. An 800-g sphere D strikes the end of member ABC with a vertical velocity $\mathbf{v}_1$ of 3 m/s. Knowing that $L = 750$ mm and that the coefficient of restitution between the sphere and member ABC is 0.5, determine immediately after the impact (a) the angular velocity of member ABC, (b) the velocity of the sphere.

SOLUTION

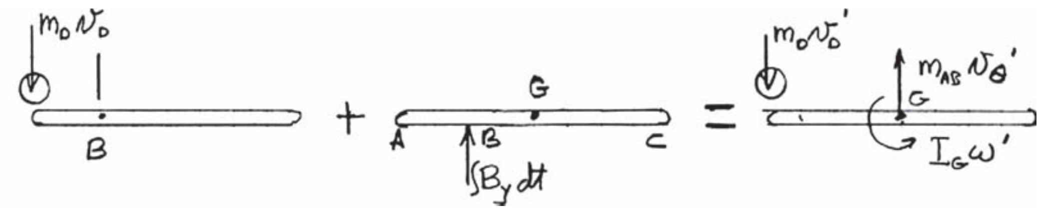
$$m_D = 0.800 \text{ kg}$$
$$L = 0.750 \text{ m}$$
$$\frac{1}{4}L = 0.1875 \text{ m}$$
$$m_{AC} = 2.4 \text{ kg}$$

Let Point G be the mass center of member ABC.

$$\begin{aligned} I_G &= \frac{1}{12} m_{AC} L^2 \\ &= \frac{1}{12}(2.4)(0.750)^2 \\ &= 0.1125 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

Kinematics after impact. $\quad \omega' = \omega' \circlearrowleft, \quad \mathbf{v}'_G = \frac{L}{4}\omega' \uparrow, \quad \mathbf{v}'_A = \frac{L}{4}\omega' \downarrow$

Conservation of momentum.



$\circlearrowleft +$ Moments about B: $\quad m_D v_D \frac{L}{2} + 0 = m_D v'_D \frac{L}{2} + I_G \omega' + m_{AC} v'_G \frac{L}{4}$

$$m_D v_D \frac{L}{4} = m_D v'_D \frac{L}{4} + \left[I_G + m_{AD}\left(\frac{L}{4}\right)^2 \right]\omega'$$

$$(0.800)(3)(0.1875) = (0.800)(0.1875)v'_D + [0.1125 + (2.4)(0.1875)^2]\omega'$$

$$0.45 = 0.15 v'_D + 0.196875\omega' \qquad (1)$$

PROBLEM 17.127 (Continued)

Coefficient of restitution.

$$v'_D - v'_A = v'_D - \frac{L}{4}\omega'$$

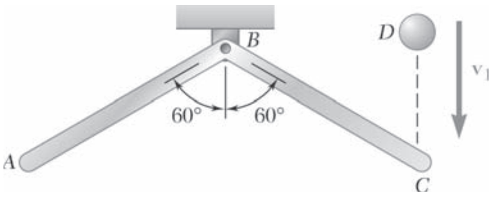
$$= -e(v_D - v_A)$$

$$v'_D - 0.1875\omega' = -(0.5)(3-0) \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously.

(*a*) Angular velocity. $\omega' = 3$ $\boldsymbol{\omega}' = 3.00 \text{ rad/s} \circlearrowleft$ ◄

(*b*) Velocity of *D*. $v'_D = -0.9375$ $v'_D = 0.938 \text{ m/s} \uparrow$ ◄



PROBLEM 17.128

Member ABC has a mass of 2.4 kg and is attached to a pin support at B. An 800-g sphere D strikes the end of member ABC with a vertical velocity $\mathbf{v}_1$ of 3 m/s. Knowing that $L = 750$ mm and that the coefficient of restitution between the sphere and member ABC is 0.5, determine immediately after the impact (a) the angular velocity of member ABC, (b) the velocity of the sphere.

SOLUTION

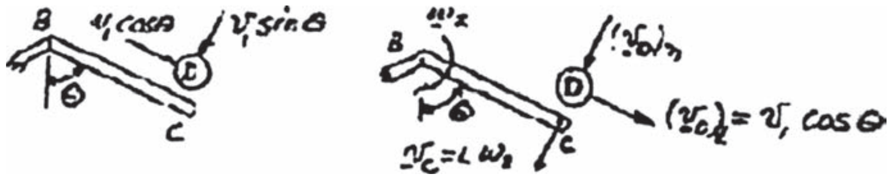
Let M be the mass of member ABC and $\bar{I}$ its moment of inertia about B.

$$M = 2.4 \text{ kg} \quad \bar{I} = \frac{1}{12}M(2L)^2$$

where $\quad L = 750 \text{ mm} = 0.75 \text{ m}$

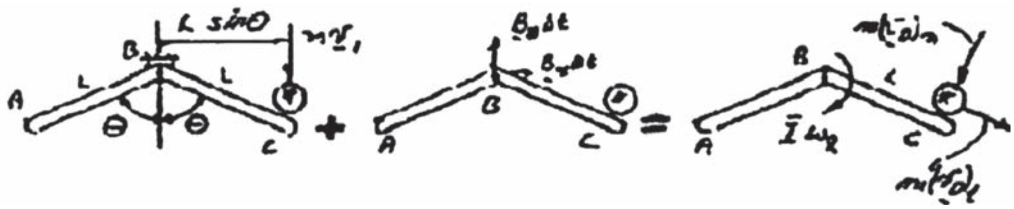
Let m be the mass of sphere D. $\quad m = 800 \text{ g} = 0.8 \text{ kg}$

Impact kinematics and coefficient of restitution.



$$(v_1 \sin\theta)e = L\omega_2 - (v_D)_n: \quad (v_D)_n = L\omega_2 - (v_1 \sin\theta)e \tag{1}$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

Moments about E: $\quad mv_1 L\sin\theta = \bar{I}\omega_2 + m(v_D)_n L$

$$mv_1 L\sin\theta = \frac{1}{12}M(2L)^2\omega_2 + m[L\omega_2 - (v_1\sin\theta)e]L$$

$$mv_1 \sin\theta = \frac{1}{3}ML\omega_2 - mL\omega_2 - m(v_1\sin\theta)e$$

$$m(1+e)\frac{v_1}{L}\sin\theta = \left(\frac{1}{3}M + m\right)\omega_2$$

PROBLEM 17.128 (Continued)

(*a*) Angular velocity.

$$\omega_2 = \frac{(3)(1+e)mv_1\sin\theta}{M+3m}$$

$$\omega_2 = \frac{(3)(1.5)(0.8)(3)\sin 60^\circ}{(2.4+2.4)(0.75)}$$

$$= 2.5981$$

$\omega_2 = 2.60$ rad/s ◀

(*b*) Velocity of *D*.

From Eq. (1),

$$(v_D)_n = (0.75)(2.5981) - (3\sin 60^\circ)(0.5)$$

$$= 0.64976 \text{ m/s}$$

$$(v_D)_t = v_1\cos 60^\circ$$

$$= 3\cos 60^\circ$$

$$= 1.5 \text{ m/s}$$

$$(\mathbf{v}_D)_n = 0.64976 \text{ m/s} \measuredangle 30^\circ$$

$$(\mathbf{v}_D)_t = 1.5 \text{ m/s} \measuredangle 30^\circ$$

$$v_D = \sqrt{(0.64976)^2 + (1.5)^2}$$

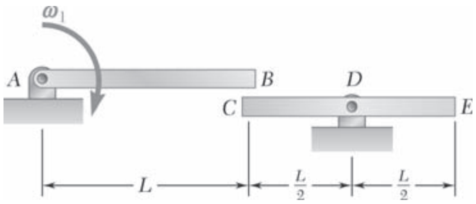
$$= 1.63468 \text{ m/s}$$

$$\tan\theta = \frac{0.64976}{1.5}$$

$$\theta = 23.4^\circ$$

$$\theta = 30^\circ = 53.4^\circ$$

$\mathbf{v}_D = 1.635$ m/s $\measuredangle$ 53.4° ◀



PROBLEM 17.129

A slender rod *CDE* of length *L* and mass *m* is attached to a pin support at its midpoint *D*. A second and identical rod *AB* is rotating about a pin support at *A* with an angular velocity ω_1 when its end *B* strikes end *C* of rod *CDE*. Denoting by *e* the coefficient of restitution between the rods, determine the angular velocity of each rod immediately after the impact.

SOLUTION

Rod *AB*.

Kinematics.

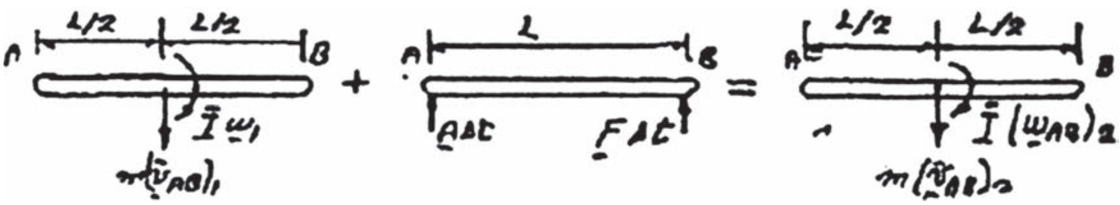
$$\boldsymbol{\omega}_1 = \omega_1 \circlearrowright$$

$$\boldsymbol{\omega}_2 = \omega_2 \circlearrowright$$

$$(\bar{\mathbf{v}}_{AB})_1 = \frac{L}{2}\omega_1 \downarrow$$

$$(\bar{v}_{AB})_2 = \frac{L}{2}\omega_2 \downarrow$$

<u>Principle of impulse and momentum.</u>



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowright$ Moments about *A*: $\bar{I}\omega_1 + m(\bar{v}_{AB})_1\frac{L}{2} - (F\Delta t)L = \bar{I}(\omega_{AB})_2 + m(\bar{v}_{AB})_2\frac{L}{2}$

$$\frac{1}{12}mL^2\omega_1 + m\left(\omega_1\frac{L}{2}\right)\frac{L}{2} - (F\Delta t)L = \frac{1}{12}mL^2(\omega_{AB})_2 + m\frac{L}{2}(\omega_{AB})_2\frac{L}{2}$$

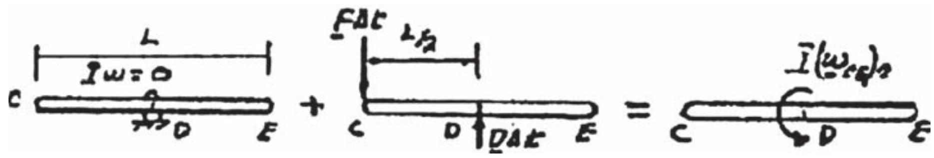
$$\frac{1}{3}mL^2\omega_1 - (F\Delta t)L = \frac{1}{3}mL^2(\omega_{AB})_2$$

$$F\Delta t = \frac{1}{3}mL^2[\omega_1 - (\omega_{AB})_2] \qquad (1)$$

PROBLEM 17.129 (Continued)

Rod *CE*.

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+ Moments about *D*:

$$(F\Delta t)\frac{L}{2} = \bar{I}(\omega_{CE})_2$$

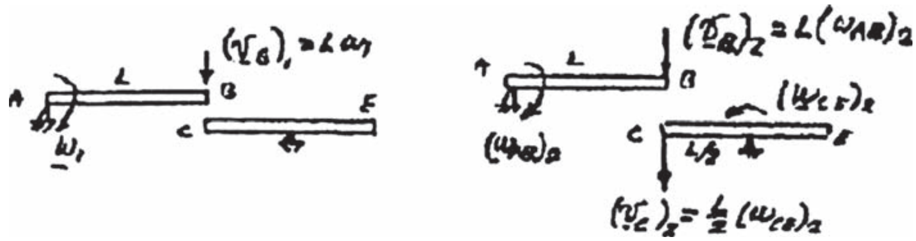
$$(F\Delta t)\frac{L}{2} = \frac{1}{12}mL^2(\omega_{CE})_2$$

Substitute for $(F\Delta t)$ from (1)

$$\frac{1}{3}mL^2[\omega_1 - (\omega_{AB})_2]\frac{L}{2} = \frac{1}{12}mL^2(\omega_{CE})_2$$

$$\omega_1 - (\omega_{AB})_2 = \frac{1}{2}(\omega_{CE})_2 \qquad (2)$$

Condition of impact. $e =$ coefficient of restitution.



$$(v_B)_1 e = (v_C)_2 - (v_B)_2$$

$$L\omega_1 e = \frac{L}{2}(\omega_{CE})_2 - L(\omega_{AB})_2$$

$$(\omega_{AB})_2 = \frac{1}{2}(\omega_{CE})_2 - \omega_1 e \qquad (3)$$

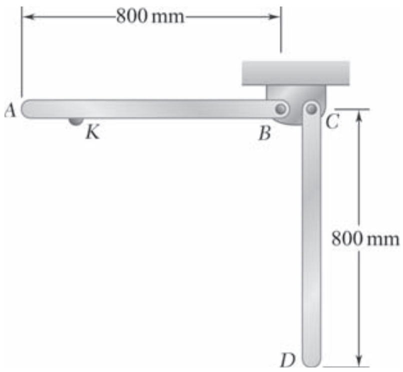
From Eq. (2)

$$\omega_1 - \left[\frac{1}{2}(\omega_{CE})_2 - \omega_1 e\right] = \frac{1}{2}(\omega_{CE})_2$$

$$\omega_1(1+e) = (\omega_{CE})_2 \qquad (\omega_{CE})_2 = \omega_1(1+e) \circlearrowleft \blacktriangleleft$$

From Eq. (3)

$$(\omega_{AB})_2 - \frac{1}{2}\omega_1(1+e) - \omega_1 e = \frac{1}{2}\omega_1 + \frac{1}{2}\omega_1 e - \omega_1 e \qquad (\omega_{AB})_2 = \frac{1}{2}\omega_1(1-e) \circlearrowright \blacktriangleleft$$



PROBLEM 17.130

The 2.5-kg slender rod AB is released from rest in the position shown and swings to a vertical position where it strikes the 1.5-kg slender rod CD. Knowing that the coefficient of restitution between the knob K attached to rod AB and rod CD is 0.8, determine the maximum angle θ_m through which rod CD will rotate after the impact.

SOLUTION

Let $$\beta = \frac{m_{CD}}{m_{AB}} = \frac{1.5 \text{ kg}}{2.5 \text{ kg}} = 0.6$$

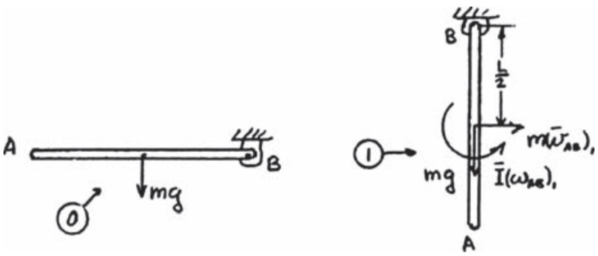
Let $$m = m_{AB}.$$

Then $$m_{CD} = \beta m.$$

Moments of inertia. $$\bar{I}_{AB} = \bar{I} = \frac{1}{12} mL^2$$

$$\bar{I}_{CD} = \frac{1}{12} \beta m L^2$$

Rod AB falls to vertical position.



Position 0. $$V_0 = 0 \qquad T_0 = 0$$

Position 1. $$V_1 = -mg\frac{L}{2}$$

$$(v_{AB})_1 = \frac{L}{2}(\omega_{AB})_1$$

$$T_1 = \frac{1}{2} m(\bar{v}_{AB})_1^2 + \frac{1}{2}\bar{I}(\omega_{AB})_1^2$$

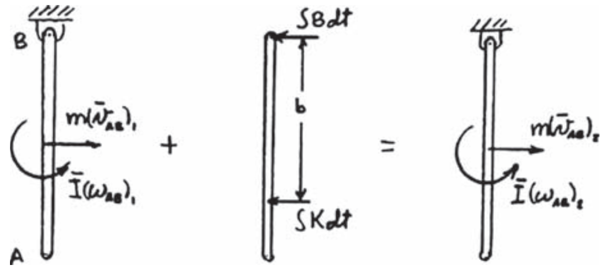
$$= \frac{1}{6} mL^2 (\omega_{AB})_1^2$$

PROBLEM 17.130 (Continued)

Conservation of energy. $T_0 + V_0 = T_1 + V_1: \quad 0 + 0 = \frac{1}{6} mL^2 (\omega_{AB})_1^2 - \frac{1}{2} mgL$

$$(\omega_{AB})_1^2 = \frac{3g}{L} \tag{1}$$

Impact.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

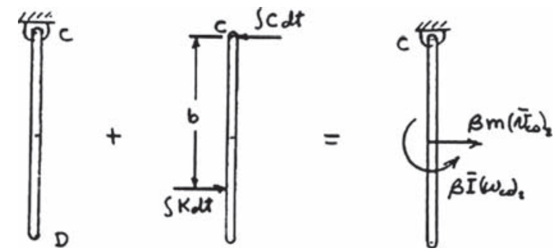
Kinematics

$$(v_{AB})_1 = \frac{L}{2}(\omega_{AB})_1$$

$$(v_{AB})_2 = \frac{L}{2}(\omega_{AB})_2$$

Moments about B: $m(v_{AB})_1 \frac{L}{2} + \bar{I}(\omega_{AB})_1 - b\int Kdt = m(v_{AB})_2 \left(\frac{L}{2}\right) + \bar{I}(\omega_{AB})_2$

$$\frac{1}{3} mL^2 (\omega_{AB})_1 - b\int Kdt = \frac{1}{3} mL^2 (\omega_{AB})_2 \tag{2}$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics

$$(\bar{v}_{CD})_2 = \frac{L}{2}(\omega_{CD})_2$$

Moments about C: $0 + b\int Kdt = \beta m(\bar{v}_{CD})_2 \frac{L}{2} + \beta\bar{I}(\omega_{CD})_2$

$$b\int Kdt = \frac{1}{3}\beta mL^2 (\omega_{CD})_2 \tag{3}$$

PROBLEM 17.130 (Continued)

Add Equations (1) and (2) to eliminate $b\int K dt$.

$$\frac{1}{3}mL^2(\omega_{AB})_1 = \frac{1}{3}mL^2(\omega_{AB})_2 + \frac{1}{3}\beta mL^2(\omega_{CD})_2$$

$$\beta(\omega_{CD})_2 + (\omega_{AB})_2 = (\omega_{AB})_1 \qquad (4)$$

Condition of impact. $\qquad b(\omega_{CD})_2 - b(\omega_{AB})_2 = eb(\omega_{AB})_1$

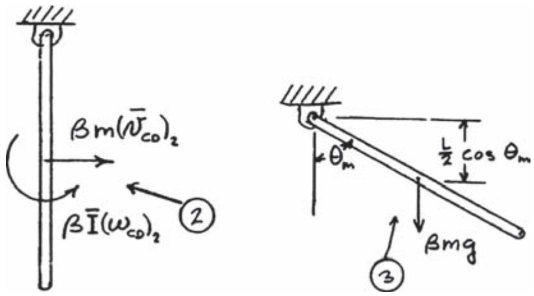
$$(\omega_{CD})_2 - (\omega_{AB})_2 = e(\omega_{AB})_1 \qquad (5)$$

Add Equations (4) and (5) to eliminate $(\omega_{AB})_2$.

$$(1+\beta)(\omega_{CD})_2 = (1+e)(\omega_{AB})_1$$

$$(\omega_{CD})_2 = \left(\frac{1+e}{1+\beta}\right)(\omega_{AB})_1 \qquad (6)$$

Rod CD rises to maximum height.



Position 2. $\qquad V_2 = -\beta mg\frac{L}{2}$

$$T_2 = \frac{1}{2}\beta m(\bar{v}_{CD})_2^2 + \frac{1}{2}\beta\bar{I}(\omega_{CD})_2^2$$

$$= \frac{1}{6}\beta mL^2(\omega_{CD})_2^2$$

Position 3. $\qquad V_3 = -\beta mg\frac{L}{2}\cos\theta_m$

$$T_3 = 0$$

PROBLEM 17.130 (Continued)

Conservation of energy.

$$T_2 + V_2 = T_3 + V_3: \quad \frac{1}{6}\beta m L^2 (\omega_{CD})_2^2 - \frac{1}{2}\beta m g L = 0 - \frac{1}{2}\beta m g L \cos\theta_m$$

$$1 - \cos\theta_m = \frac{L}{3g}(\omega_{CD})_2^2$$

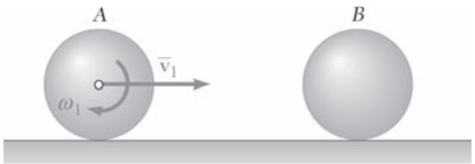
$$= \frac{L}{3g}\left(\frac{1+e}{1+\beta}\right)^2 (\omega_{AB})_1^2$$

$$= \frac{L}{3g}\left(\frac{1+e}{1+\beta}\right)^2 \frac{3g}{L}$$

$$\cos\theta_m = 1 - \left(\frac{1+e}{1+\beta}\right)^2$$

$$= 1 - \left(\frac{1+0.8}{1+0.6}\right)^2$$

$\theta_m = 105.4°$ ◀



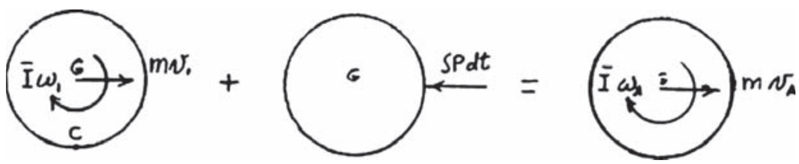
PROBLEM 17.131

Sphere A of mass m and radius r rolls without slipping with a velocity $\overline{\mathbf{v}}_1$ on a horizontal surface when it hits squarely an identical sphere B that is at rest. Denoting by μ_k the coefficient of kinetic friction between the spheres and the surface, neglecting friction between the spheres, and assuming perfectly elastic impact, determine (a) the linear and angular velocities of each sphere immediately after the impact, (b) the velocity of each sphere after it has started rolling uniformly.

SOLUTION

Moment of inertia. $$\bar{I} = \frac{2}{5}mr^2$$

Analysis of impact. Sphere A.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics: Rolling without slipping in Position 1.

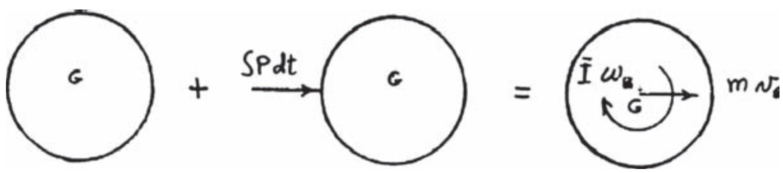
$$\omega_A = \frac{v_1}{r}$$

Moments about G: $$\bar{I}\omega_1 + 0 = \bar{I}\omega_A$$

$$\omega_A = \omega_1 = \frac{v_1}{r}$$

$\xrightarrow{+}$ Linear components: $$mv_1 - \int P dt = mv_A \qquad (1)$$

Analysis of impact. *Sphere B*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ Linear components: $$0 + \int P dt = mv_B \qquad (2)$$

PROBLEM 17.131 (Continued)

Add Equations (1) and (2) to eliminate $\int P dt$.

$$mv_1 = mv_A + mv_B \quad \text{or} \quad v_B + v_A = v_1 \tag{3}$$

Condition of impact. $e = 1$.

$$v_B - v_A = ev_1 = v_1 \tag{4}$$

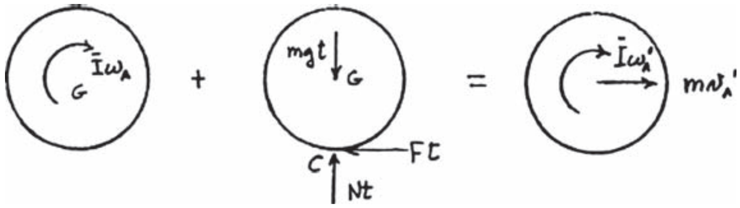
Solving Equations (4) and (5) simultaneously,

$$v_A = 0, \qquad v_B = v_1$$

Moments about G: $\quad 0 + 0 = \bar{I}\omega_B \quad \omega_B = 0$

(*a*) *Velocities after impact.* $\quad \mathbf{v}_A = 0; \quad \boldsymbol{\omega}_A = \dfrac{v_1}{r}$; $\mathbf{v}_B = v_1 \rightarrow; \quad \boldsymbol{\omega}_B = 0$ ◀

Motion after Impact. *Sphere A*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Condition of rolling without slipping: $\quad v_A' = \omega_A' r$

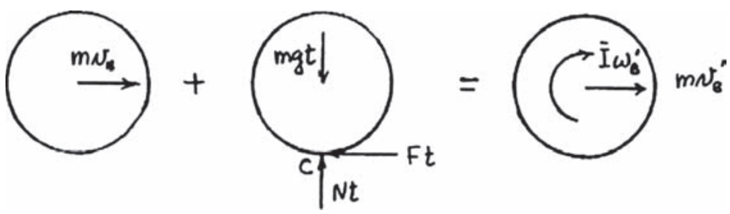
Moments about C: $\quad \bar{I}\omega_A + 0 + \bar{I}\omega_A' + mv_A' r$

$$\left(\frac{2}{5}mr^2\right)\left(\frac{v_1}{r}\right) + 0 = \left(\frac{2}{5}mr^2\right)\omega_A' + m(r\omega_A')r$$

$$\omega_A' = \frac{2}{7}\frac{v_1}{r}$$

$$v_A' = \frac{2}{7}v_1$$

Motion after impact. *Sphere B*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.131 (Continued)

Condition of rolling without slipping: $v'_B = r\omega'_B$

$\curvearrowright$ Moments about C:

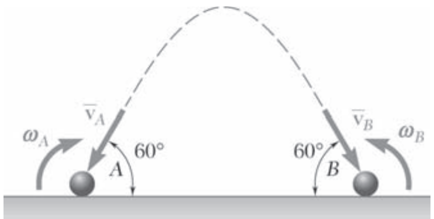
$$mv_B r + 0 = \bar{I}\,\omega'_B + mv'_B r$$

$$mv_1 r + 0 = \left(\frac{2}{5}mr^2\right)\omega'_B + m(r\omega'_B)r$$

$$\omega'_B = \frac{5}{7}\frac{v_1}{r}$$

$$v'_B = \frac{5}{7}v_1$$

(*b*) *Final Rolling Velocities*. $\mathbf{v}'_A = \frac{2}{7}v_1 \rightarrow; \quad \mathbf{v}'_B = \frac{5}{7}v_1 \rightarrow$ ◀



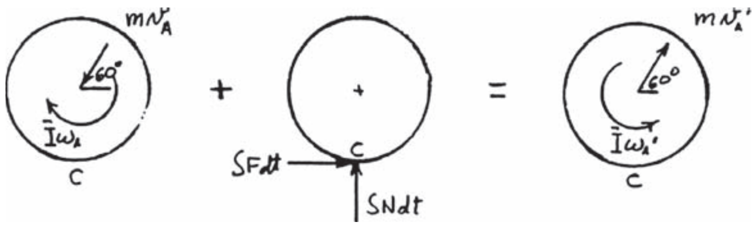
PROBLEM 17.132

A small rubber ball of radius r is thrown against a rough floor with a velocity $\bar{\mathbf{v}}_A$ of magnitude v_0 and a backspin $\boldsymbol{\omega}_A$ of magnitude ω_0. It is observed that the ball bounces from A to B, then from B to A, then from A to B, etc. Assuming perfectly elastic impact, determine the required magnitude ω_0 of the backspin in terms of $\bar{v}_0$ and r.

SOLUTION

Moment of inertia. $\bar{I} = \frac{2}{5}mr^2$ Ball is assumed to be a solid sphere.

Impact at A.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

For the velocity of the ball to be reversed on each impact,

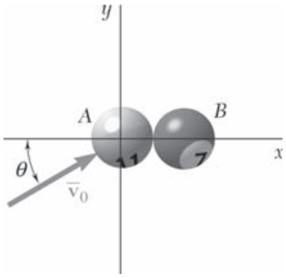
$$v_A' = v_A = v_0$$
$$\omega_A' = \omega_A = \omega_0$$

This is consistent with the assumption of perfectly elastic impact.

Moments about C: $mv_A r\cos 60° - \bar{I}\omega_A + 0 = \bar{I}\omega_A' - mv_A' r\cos 60°$

$$mv_0 r\cos 60° - \frac{2}{5}mr^2\omega_0 + 0 = \frac{2}{5}mr^2\omega_0 - mv_0 r\cos 60°$$

$$\frac{2}{5}r\omega_0 = v_0\cos 60° \qquad \omega_0 = \frac{5}{4}\frac{v_0}{r} \blacktriangleleft$$



PROBLEM 17.133

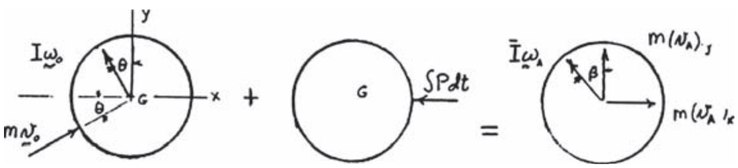
In a game of pool, ball A is rolling without slipping with a velocity $\overline{\mathbf{v}}_0$ as it hits obliquely ball B, which is at rest. Denoting by r the radius of each ball and by μ_k the coefficient of kinetic friction between the balls, and assuming perfectly elastic impact, determine (*a*) the linear and angular velocity of each ball immediately after the impact, (*b*) the velocity of ball B after it has started rolling uniformly.

SOLUTION

Moment of inertia. $\qquad \bar{I} = \frac{2}{5}mr^2$

(*a*) Impact analysis.

Ball A:



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

Kinematics of rolling: $\qquad \omega_0 = \frac{v_0}{r}$

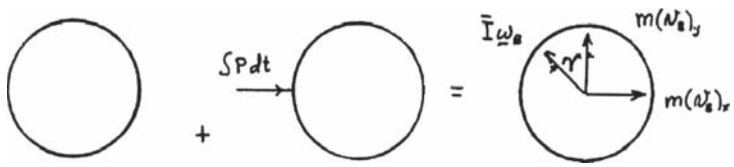
$\xrightarrow{+}$ Linear components: $\qquad mv_0 \cos\theta - \int P dt = m(v_A)_x \qquad (1)$

$+\uparrow$ Linear components: $\qquad mv_0 \sin\theta + 0 = m(v_A)_y \qquad (2)$

Moments about y axis: $\qquad \bar{I}\omega_0 \cos\theta + 0 = \bar{I}\omega_A \cos\beta \qquad (3)$

Moments about x axis: $\qquad -\bar{I}\omega_0 \sin\theta + 0 = -\bar{I}\omega_A \sin\beta \qquad (4)$

Ball B:



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

$\xrightarrow{+}$ Linear components: $\qquad 0 + \int P dt = m(v_B)_x \qquad (5)$

$+\uparrow$ Linear components: $\qquad 0 + 0 = m(v_B)_y \qquad (6)$

PROBLEM 17.133 (Continued)

Moments about y axis: $$0 + 0 = \bar{I}\omega_B \cos\gamma \qquad (7)$$

Moments about x axis: $$0 + 0 = \bar{I}\omega_B \sin\gamma \qquad (8)$$

Adding Equations (1) and (5) to eliminate $\int P dt$,

$$mv_0 \cos\theta + 0 = m(v_A)_x + m(v_B)_x$$

or $$(v_B)_x + (v_A)_x = v_0 \cos\theta \qquad (9)$$

Condition of impact. $$e = 1 \quad (v_B)_x - (v_A)_x = ev_0 \cos\theta = v_0 \cos\theta \qquad (10)$$

Solving Equations (9) and (10) simultaneously,

$$(v_A)_x = 0, \quad (v_B)_x = v_0 \cos\theta$$

From Equations (2) and (6), $$(v_A)_y = v_0 \sin\theta, \quad (v_B)_y = 0 \qquad \mathbf{v}_A = (v_0 \sin\theta)\mathbf{j} \blacktriangleleft$$

$$\mathbf{v}_B = (v_0 \cos\theta)\mathbf{i} \blacktriangleleft$$

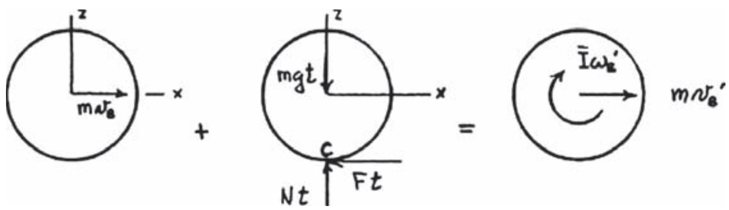
From Equations (3) and (4) simultaneously,

$$\beta = \theta, \quad \omega_A = \omega_0 = \frac{v_0}{r} \qquad \boldsymbol{\omega}_A = \frac{v_0}{r}(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j}) \blacktriangleleft$$

From Equations (7) and (8) simultaneously,

$$\omega_B = 0 \qquad \boldsymbol{\omega}_B = 0 \blacktriangleleft$$

(*b*) *Subsequent motion of ball B.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

Kinematics of rolling without slipping. $$v_B' = r\omega_B'$$

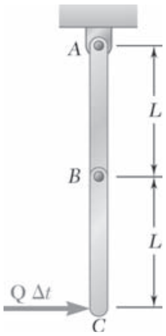
Moments about C:

$$mv_B r + 0 = \bar{I}\omega_B' + mv_B' r$$

$$= \frac{2}{5}mr^2\omega_B' + m(r\omega_B')r$$

$$\omega_B' = \frac{5}{7}\frac{v_B'}{r} = \frac{5}{7}\frac{v_1 \cos\theta}{r}$$

$$v_B' = \frac{5}{7}v_1 \cos\theta \qquad \mathbf{v}_B' = \frac{5}{7}(v_0 \cos\theta)\mathbf{i} \blacktriangleleft$$



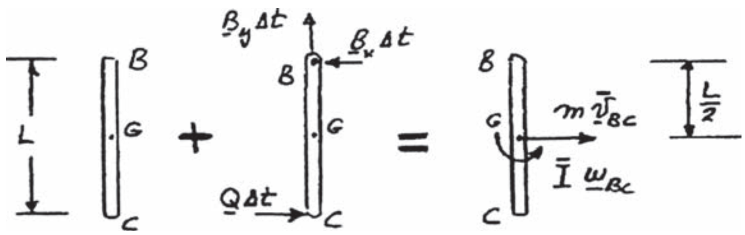
PROBLEM 17.134

Each of the bars AB and BC is of length $L = 400$ mm and mass $m = 1$ kg. Determine the angular velocity of each bar immediately after the impulse $Q\Delta t = (1.5\ \text{N}\cdot\text{s})\mathbf{i}$ is applied at C.

SOLUTION

Principle of impulse and momentum.

Bar BC:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowleft$+ Moments about B:

$$0 + (Q\Delta t)L = \bar{I}\,\omega_{BC} + m\bar{v}_{BC}\frac{L}{2}$$

$$(Q\Delta t)L = \frac{1}{12}mL^2\omega_{BC} + m\left(L\omega_{AB} + \frac{L}{2}\omega_{BC}\right)\frac{L}{2}$$

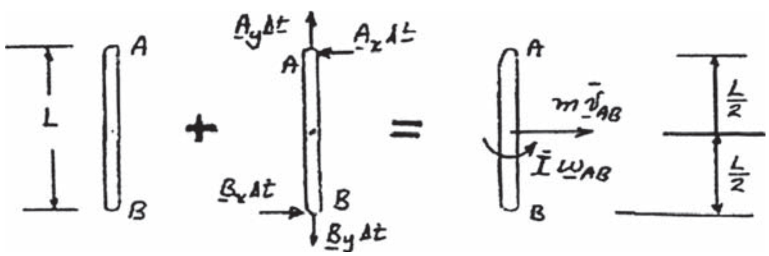
$$Q\Delta t = \frac{1}{2}mL\omega_{AB} + \frac{1}{3}mL\omega_{BC} \qquad (1)$$

$\xrightarrow{+}$ x components:

$$Q\Delta t - B_x\Delta t = m\bar{v}_{BC}$$

$$Q\Delta t - B_x\Delta t = m\left(L\omega_{AB} + \frac{1}{2}\omega_{BC}\right) \qquad (2)$$

Bar AB:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.134 (Continued)

$\circlearrowleft +$ Moments about A: $$0+(B_x\Delta t)L=\bar{I}\,\omega_{AB}+m\bar{v}_{AB}\frac{L}{2}$$

$$(B_x\Delta t)L=\frac{1}{12}mL^2\omega_{AB}+m\left(\frac{1}{2}\omega_{AB}\right)\frac{L}{2}$$

$$B_x\Delta t=\frac{1}{3}mL\omega_{AB} \qquad (3)$$

Add Eqs. (2) and (3): $$Q\Delta t=\frac{4}{3}mL\omega_{AB}+\frac{1}{2}mL\omega_{BC} \qquad (4)$$

Subtract Eq. (1) from Eq. (4): $$0=\frac{5}{6}mL\omega_{AB}+\frac{1}{6}mL\omega_{BC}$$

$$\omega_{BC}=-5\omega_{AB} \qquad (5)$$

Substitute for ω_{BC} in Eq. (1): $$Q\Delta t=\frac{1}{2}mL\omega_{AB}+\frac{1}{3}mL(-5\omega_{AB})$$

$$=-\frac{7}{6}mL\omega_{AB}$$

$$\omega_{AB}=-\frac{6}{7}\frac{Q\Delta t}{mL} \qquad (6)$$

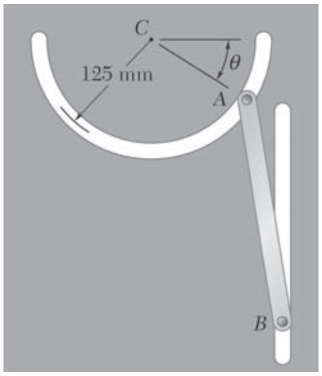
Substituting into Eq. (5): $$\omega_{BC}=-5\left(-\frac{6}{7}\frac{Q\Delta t}{mL}\right)$$

$$\omega_{BC}=\frac{30}{7}\frac{Q\Delta t}{mL} \qquad (7)$$

Given data: $$L=400\text{ mm}=0.4\text{ m}$$
$$Q\Delta t=1.5\text{ N}\cdot\text{s}$$
$$m=1\text{ kg}$$

Angular velocity of bar AB. $$\omega_{AB}=-\frac{6}{7}\frac{Q\Delta t}{mL}=-\frac{6}{7}\frac{(1.5)}{(1)(0.4)}$$ $\boldsymbol{\omega}_{AB}=3.21\text{ rad/s} \circlearrowright$ ◀

Angular velocity of bar BC. $$\omega_{BC}=\frac{30}{7}\frac{Q\Delta t}{mL}=\frac{(30)(1.5)}{(7)(1)(0.4)}$$ $\boldsymbol{\omega}_{BC}=16.07\text{ rad/s} \circlearrowleft$ ◀



PROBLEM 17.135

The motion of the slender 250-mm rod AB is guided by pins at A and B that slide freely in slots cut in a vertical plate as shown. Knowing that the rod has a mass of 2 kg and is released from rest when $\theta = 0$, determine the reactions at A and B when $\theta = 90°$.

SOLUTION

Let Point G be the mass center of rod AB.

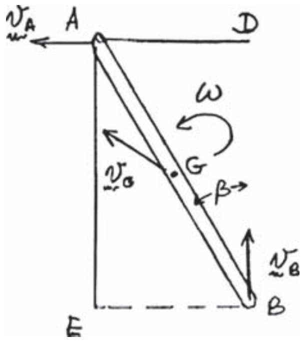
$$m = 2 \text{ kg}$$

$$L = 0.025 \text{ m}$$

$$I_G = \frac{1}{12}mL^2 = 0.0104667 \text{ kg}\cdot\text{m}^2$$

Kinematics.

$$\theta = 90°$$



$$\overline{AD} = R = 0.125 \text{ m}$$

$$AB = L = 0.25 \text{ m}$$

$$\sin\beta = \frac{R}{L} = \frac{1}{2} \quad \beta = 30°$$

$$\overline{AG} = \frac{L}{2} = 0.125 \text{ m}$$

$$\overline{BG} = 0.125 \text{ m}$$

Point E is the instantaneous center of rotation of bar AB.

$$v_G = \frac{L}{2}\omega = 0.125\ \omega$$

$$v_A = (L\cos 30°)\omega = 0.21651\omega$$

$$v_B = (L\sin 30°)\omega = 0.125\omega$$

Use principle of conservation of energy to obtain the velocities when $\theta = 90°$:

Use level A as the datum for potential energy.

Position 1.

$$\theta = 0 \quad T_1 = 0$$

$$V_1 = -mg\frac{L}{2}$$

$$= -(2)(9.81)(0.125)$$

$$= -2.4525 \text{ J}$$

PROBLEM 17.135 (Continued)

Position 2. $\theta = 90°$

$$T_2 = \frac{1}{2}I_G\omega^2 + \frac{1}{2}mv_G^2$$
$$= \frac{1}{2}(0.0104667)\omega^2 + \frac{1}{2}(0.125\omega)^2$$
$$= 0.0208583\omega^2$$
$$V_2 = -mg\left(R + \frac{L}{2}\cos\beta\right)$$
$$= -(2)(9.81)(0.125 + 0.125\cos 30°)$$
$$= -4.5764 \text{ J}$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 - 2.4525 = 0.0208583\omega^2 - 4.5764$$
$$\omega^2 = 101.826 \text{ rad}^2/\text{s}^2$$
$$\omega = 10.091 \text{ rad/s}$$
$$v_A = (0.21651)(10.091)$$
$$= 2.1848 \text{ m/s}$$
$$v_G = (0.125)(10.091)$$
$$= 1.2614 \text{ m/s}$$

More kinematics: For Point A moving in the curved slot,

$$\mathbf{a}_A = (a_C)_x\mathbf{i} + \frac{v_A^2}{R}\mathbf{j}$$
$$= (a_C)_x\mathbf{i} + \frac{(2.1847)^2}{0.125}\mathbf{j}$$
$$= (a_C)_x\mathbf{i} + 38.1833\mathbf{j}$$

For the rod AB, $\boldsymbol{\alpha} = \alpha\mathbf{k}, \quad \mathbf{v}_B = v_B\mathbf{j}$

$$\mathbf{r}_{A/B} = -L\sin 30°\mathbf{i} + L\cos 30°\mathbf{j}$$
$$= -0.125\mathbf{i} + 0.21651\mathbf{j}$$
$$\mathbf{r}_{G/B} = \frac{1}{2}\mathbf{r}_{A/B}$$
$$= -0.0625\mathbf{i} + 0.108253\mathbf{j}$$
$$\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} + \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B}$$
$$= a_B\mathbf{j} + \alpha\mathbf{k} \times (-0.125\mathbf{i} + 0.21651\mathbf{j})$$
$$-(10.091)^2(-0.125\mathbf{i} + 0.21651\mathbf{j})$$
$$= a_B\mathbf{j} - 0.125\alpha\mathbf{j} - 0.21651\alpha\mathbf{i} + 12.7285\mathbf{i} - 22.0468\mathbf{j}$$

PROBLEM 17.135 (Continued)

Matching vertical components of $\mathbf{a}_A$

$$38.1833 = a_B - 0.125\alpha - 22.0468$$

$$a_B = 0.125\alpha + 60.2301$$

$$\mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B}$$

$$= \mathbf{a}_B + \alpha\mathbf{k} \times \mathbf{r}_{G/B} - \omega^2 r_{G/B}$$

$$= (0.125\alpha + 60.2324)\mathbf{j} + \alpha\mathbf{k} \times (-0.0625\mathbf{i} + 0.108253\mathbf{j})$$

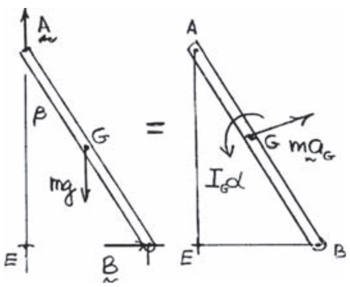
$$-(10.091)^2(-0.0625\mathbf{i} + 0.108253\mathbf{j})$$

$$= 0.125\alpha\mathbf{j} + 60.2301\mathbf{j} - 0.0625\alpha\mathbf{j} - 0.108253\alpha\mathbf{i}$$

$$+6.3643\mathbf{i} - 11.0232\mathbf{j}$$

$$\mathbf{a}_G = (-0.108253\alpha + 6.3643)\mathbf{i} + (0.0625\alpha + 49.2069)\mathbf{j}$$

Kinetics: Use rod AB as a free body.



$$\circlearrowleft^{+}\ \Sigma\mathbf{M}_E = \Sigma(M_E)_{\text{eff}}:$$

$$-mg\frac{L}{2}\sin\beta\mathbf{k} = I_G\boldsymbol{\alpha} + \mathbf{r}_{G/E} \times \mathbf{a}_G - (2)(9.81)(0.125)\sin 30°\mathbf{k}$$

$$= 0.0104667\boldsymbol{\alpha} + (0.0625\mathbf{i} + 0.108253\mathbf{j})(m\mathbf{a}_G)$$

$$-1.22625 = 0.0104667\alpha + 0.03125\alpha + 4.7730$$

$$0.0417167\alpha = -5.999R$$

$$\alpha = -143.808 \text{ rad/s}^2$$

$$a_G = (-21.933 \text{ m/s}^2)\mathbf{i} + (40.2189 \text{ m/s}^2)\mathbf{j}$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}} = m(a_G)_x: \quad -B = (2)(-21.932) = 43.864 \text{ N}$ $\qquad \mathbf{B} = 43.9 \text{ N} \rightarrow$ ◀

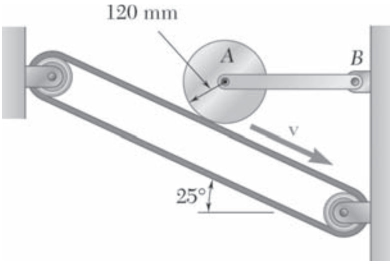
$+\!\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}} = m(a_G)_y: \quad A - mg = (2)(40.4289) = 80.4378$

$A = (2)(9.81) + 80.4378 = 100.058$ $\qquad \mathbf{A} = 100.1 \text{ N}\uparrow$ ◀

Check by considering

$$\circlearrowright^{+}\ \Sigma M_G = \Sigma M_{G\text{ eff}}: \circlearrowright\ \Sigma M_G = (0.0625)A - 0.108253B = 1.5052 \text{ N}\cdot\text{m} \circlearrowright$$

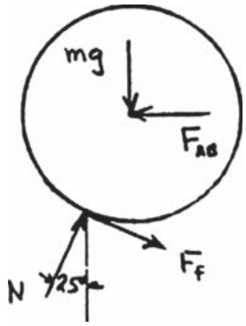
$$\circlearrowright\ \Sigma(M_G)_{\text{eff}} = I_G(-\alpha) = (0.0104667)(143.808) = 1.5052 \text{ N}\cdot\text{m} \circlearrowright$$



PROBLEM 17.136

A uniform disk of constant thickness and initially at rest is placed in contact with the belt shown, which moves at a constant speed $v = 25$ m/s. Knowing that the coefficient of kinetic friction between the disk and the belt is 0.15, determine (*a*) the number of revolutions executed by the disk before it reaches a constant angular velocity, (*b*) the time required for the disk to reach that constant angular velocity.

SOLUTION



Kinetic friction. $\quad F_f = \mu_k N = 0.15$ N

$$+\uparrow \Sigma F_y = N\cos 25° - F_f \sin 25° - mg = 0$$

$$(\cos 25° - \mu_k \sin 25°)N = mg$$

$$N = \frac{mg}{\cos 25° - 0.15\sin 25°} = 1.18636mg$$

$$F_f = (0.15)(1.18636)mg = 0.177954mg$$

Final angular velocity. $\quad \omega_2 = \dfrac{v}{r}$

Moment of inertia. $\quad \bar{I} = \dfrac{1}{2}mr^2$

(*a*) *Principle of work and energy.*

$$T_1 + W_{1\to 2} = T_2: \quad T_1 = 0$$

$$W_{1\to 2} = F_f r\theta = 0.177954mgr\theta$$

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}mv^2$$

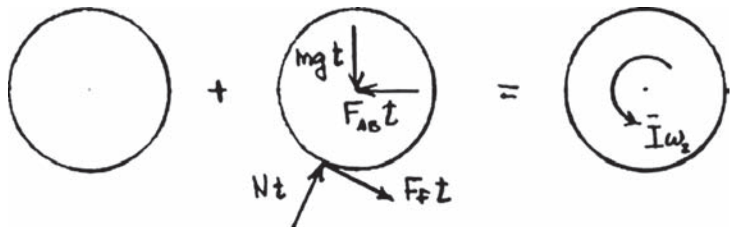
$$0 + 0.177954mgr\theta = \frac{1}{4}mv^2$$

$$\theta = 1.40486\frac{v^2}{gr} = \frac{(1.40486)(25)^2}{(9.81)(0.120)} = 745.87 \text{ radians}$$

$\theta = 118.7$ rev ◄

PROBLEM 17.136 (Continued)

(*b*) *Principle of impulse-momentum.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about A: $0 + F_f tr = \bar{I}\omega_2$

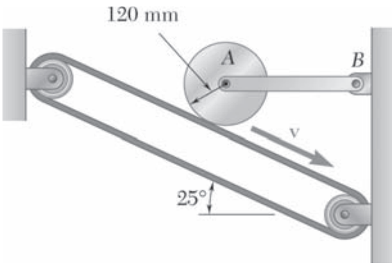
$$t = \frac{\bar{I}\omega_2}{F_f r}$$

$$= \frac{\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)}{0.177954mgr}$$

$$= 2.8097\frac{v}{g}$$

$$= \frac{(2.8097)(25)}{9.81}$$

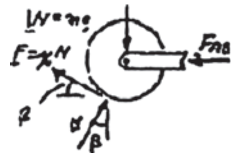
$t = 7.16$ s ◀



PROBLEM 17.137

Solve Problem 17.136, assuming that the direction of motion of the belt is reversed.

SOLUTION



While slipping occurs:

$$+\uparrow \Sigma F_y = 0: \quad N\cos\beta + \mu_k N \sin\beta - mg = 0$$

$$N = \frac{mg}{\cos\beta + \mu_k \sin\beta} \qquad (1)$$

For cylinder slipping occurs until $\omega = \dfrac{v}{r}$

$$M_F = Fr = \text{Moment of } F \text{ about } A.$$

Work:

$$U_{1\to 2} = M_F\theta \times Fr\theta$$
$$= \mu_k N r \theta$$

Kinetic energy:

$$T_1 = 0: \quad T_2 = \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2$$

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$

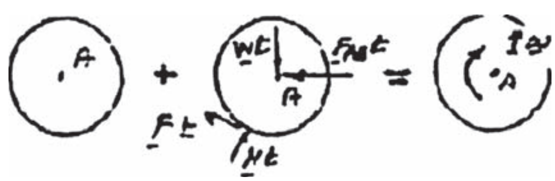
$$0 + \mu_k N r\theta = \frac{1}{4}mr^2$$

$$\theta = \frac{1}{4}\frac{mv^2}{\mu_k r}\cdot\frac{1}{N}$$

$$= \frac{1}{4}\frac{mv^2}{\mu_k r}\cdot\frac{\cos\beta + \mu_k \sin\theta}{mg}$$

$$\theta = \frac{1}{4}\cdot\frac{v^2}{\mu_k r g}(\cos\beta + \mu_k \sin\theta) \qquad (2)$$

<u>Principle of impulse-momentum</u>



PROBLEM 17.137 (Continued)

$\circlearrowleft^{+}$ Moments about A:

$$Ftr = \bar{I}\omega$$

$$\mu_k Ntr = \frac{1}{2}mv^2\left(\frac{v}{r}\right)$$

Substituting for N:

$$\mu_k\left(\frac{mg}{\cos\beta + \mu_k \sin\alpha}\right)tr = \frac{1}{2}mrv$$

$$t = \frac{1}{2}\cdot\frac{v}{\mu_k g}(\cos\beta + \mu_k \sin\beta) \tag{3}$$

Data:

$$\mu_k = 0.15$$
$$\beta = 25°$$
$$v = 25 \text{ m/s}$$
$$r = 0.12 \text{ m}$$

(*a*) From Eq. (2),

$$\theta = \frac{(25 \text{ m/s})^2}{4(0.15)(0.12 \text{ m})(9.81 \text{ m/s}^2)}\left[\cos 25° + (0.15)\sin 25°\right]$$

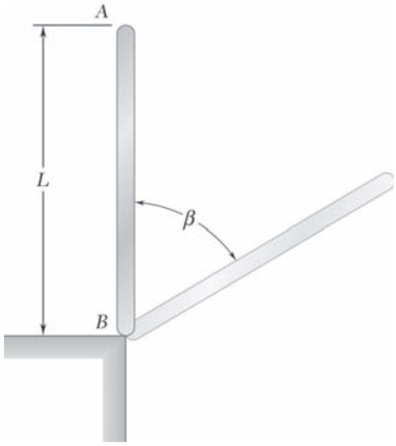
$$\theta = 858.05 \text{ rad}$$

$\theta = 136.6$ revolutions ◀

(*b*) From Eq. (3),

$$t = \frac{25 \text{ m/s}}{2(0.15)(9.81 \text{ m/s}^2)}[\cos 25° + (0.15)\sin 25°]$$

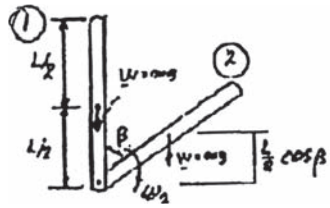
$t = 3.24$ s ◀



PROBLEM 17.138

A uniform slender rod is placed at corner B and is given a slight clockwise motion. Assuming that the corner is sharp and becomes slightly embedded in the end of the rod, so that the coefficient of static friction at B is very large, determine (*a*) the angle β through which the rod will have rotated when it loses contact with the corner, (*b*) the corresponding velocity of end A.

SOLUTION



Position 1.

$$T_1 = 0$$

$$V_1 = mgh_1 = \frac{mgL}{2}$$

Position 2.

$$V_2 = mgh_2 = \frac{mgL\cos\beta}{2}$$

$$T_2 = \frac{1}{2}I\omega_2^2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_2^2$$

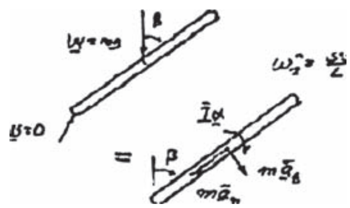
Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{mgL}{2} = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_2^2 + \frac{mgL\cos\beta}{2}$$

$$\omega_2^2 = \frac{3g}{L}(1-\cos\beta) \qquad (1)$$

Normal acceleration of mass center.



$$a_n = \frac{L}{2}\omega_2^2 = \frac{3}{2}g(1-\cos\beta)$$

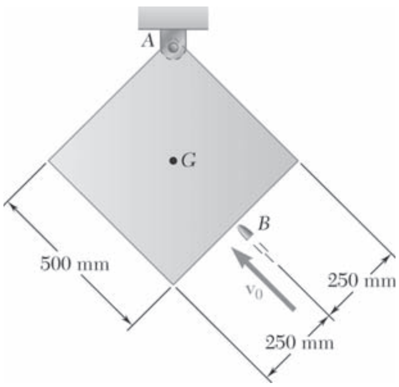
$$+\swarrow \Sigma F = +\Sigma F_{\text{eff}} = ma_n$$

$$mg\cos\beta = \frac{3}{2}mg(1-\cos\beta)$$

(*a*) Angle β. $\quad \dfrac{5}{2}\cos\beta = \dfrac{3}{2} \quad \cos\beta = 0.6$ $\quad \beta = 53.1° ◀$

From (1) $\quad \omega_2^2 = \dfrac{3g}{L}(1-0.6) = 1.2\dfrac{g}{L} \quad \omega_2 = 1.09545\sqrt{\dfrac{g}{L}}$

(*b*) Velocity of end A $\quad v_A = t\omega_2$ $\quad \mathbf{v}_A = 1.095\sqrt{gL}\ \searrow\ 53.1° ◀$



PROBLEM 17.139

A 35-g bullet B is fired with a velocity of 400 m/s into the side of a 3-kg square panel suspended as shown from a pin at A. Knowing that the panel is initially at rest, determine the components of the reaction at A after the panel has rotated 90°.

SOLUTION

Masses and moment of inertia.

$$m_B = 0.035 \text{ kg}$$

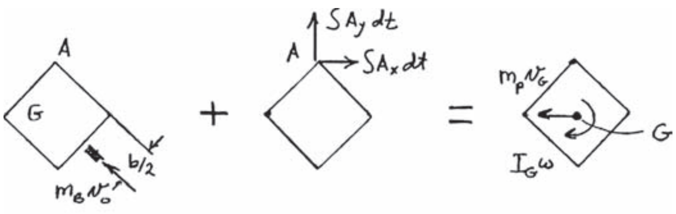
$$m_P = 3 \text{ kg}$$

$$b = 500 \text{ mm} = 0.5 \text{ m}$$

$$I_G = \frac{1}{6} m_P b^2 = \frac{1}{6}(3)(0.5)^2 = 0.125 \text{ kg} \cdot \text{m}^2$$

Note: The mass of the bullet is neglected in comparison with that of the plate after impact.

Analysis of impact: Use principle of impulse and momentum.



Kinematics: After impact the plate rotates about the pin at A.

$$v_G = \frac{b}{\sqrt{2}}\omega = \frac{0.5}{\sqrt{2}}\omega$$

+↻ Moments about A:

$$m_G v_0 \frac{b}{2} + 0 = I_G \omega + m_p v_G \frac{b}{\sqrt{2}}$$

$$\frac{1}{2} m_G v_0 b = (I_G + \frac{1}{2} m_P b^2)\omega$$

$$\frac{1}{2}(0.035)(400)(0.5) = \left[0.125 + \frac{1}{2}(3)(0.5)^2\right]\omega$$

$$\omega = 7 \text{ rad/s}$$

$$v_G = \frac{0.5}{\sqrt{2}}(7) = 2.4749 \text{ m/s}$$

PROBLEM 17.139 (Continued)

Corresponding kinetic energy. $T_1 = \frac{1}{2} I_G \omega^2 + \frac{1}{2} m_P v_G^2$

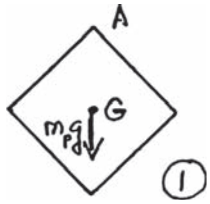
$$T_1 = \frac{1}{2}(0.125)(7)^2 + \frac{1}{2}(3)(2.4749)^2$$
$$= 12.25 \text{ J}$$

Plate rotates through 45°.

Position 1: $\theta = 0°$

Use Point A as the datum for potential energy

$$V_1 = -m_P g \frac{b}{\sqrt{2}}$$
$$= -(3)(9.81)\frac{0.5}{\sqrt{2}}$$
$$= -10.4051 \text{ J}$$



Position 2: $\theta = 90°$

$V_2 = 0$ since G is at level A.

$$T_2 = \frac{1}{2} I_G \omega_2^2 + \frac{1}{2} m_P (v_G)_2^2$$
$$= \frac{1}{2}(0.125)\omega_2^2 + \frac{1}{2}(3)\left(\frac{0.5}{\sqrt{2}}\omega_2\right)^2$$
$$= 0.25\omega_2^2$$

Principle of conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$
$$12.25 \text{ J} - 10.4051 \text{ J} = 0.25\omega_2^2 + 0$$
$$\omega_2^2 = 7.3796 \text{ (rad/s}^2)$$
$$\omega_2 = 2.7165 \text{ rad/s}$$

Analysis at 90° rotation. $\boldsymbol{\alpha} = \alpha$ ⟲

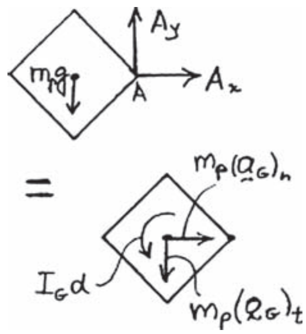
Kinematics: $(a_G)_t = \frac{b}{\sqrt{2}}\alpha = \frac{0.5}{\sqrt{2}}\alpha$ $\qquad (\mathbf{a}_G)_t = 0.35355\alpha \downarrow$

$$(a_G)_n = \frac{b}{\sqrt{2}}\omega^2$$
$$= \frac{(0.5)(7.3796)}{\sqrt{2}}$$

$(\mathbf{a}_G)_n = 2.6091 \text{ m/s}^2 \rightarrow$

PROBLEM 17.139 (Continued)

Kinematics: Use the free body diagram of the plate.



$$\circlearrowleft^{+} \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad m_P g \frac{b}{\sqrt{2}} = I_G \alpha + m_P (a_G)_t \frac{b}{\sqrt{2}}$$

$$= \left(I_G + \frac{1}{2} m_P b^2 \right) \alpha$$

$$\frac{(3)(9.81)(0.5)}{\sqrt{2}} = \left[0.125 + \frac{1}{2}(3)(0.5)^2 \right] \alpha$$

$$\boldsymbol{\alpha} = 20.810 \text{ rad/s}^2 \circlearrowleft$$

$$(\mathbf{a}_G)_t = 7.3574 \text{ m/s}^2 \downarrow$$

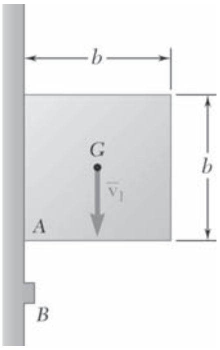
$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A_x = m_P (a_G)_n = (3)(2.6091)$$

$$\mathbf{A}_x = 7.83 \text{ N} \rightarrow \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A_y - m_P g = -m_P (a_G)_y$$

$$A_y - (3)(9.81) = (3)(-7.3574)$$

$$\mathbf{A}_y = 7.35 \text{ N} \uparrow \blacktriangleleft$$



PROBLEM 17.140

A square block of mass m is falling with a velocity $\overline{\mathbf{v}}_1$ when it strikes a small obstruction at B. Assuming that the impact between corner A and the obstruction B is perfectly plastic, determine immediately after the impact (*a*) the angular velocity of the block (*b*) the velocity of its mass center G.

SOLUTION

Moment of inertia. $\qquad \overline{I} = \dfrac{1}{6}mb^2$

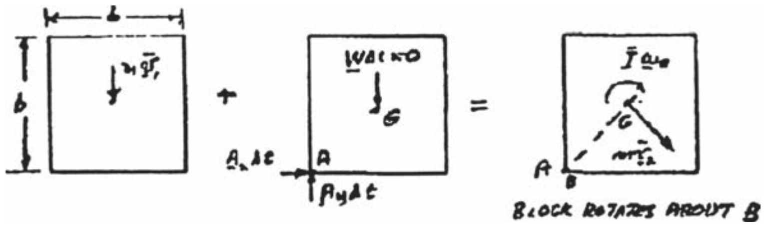
Kinematics. Before impact, block is translating.

$$\overline{\mathbf{v}}_1 = v_1 \downarrow \quad \omega_1 = 0$$

After impact, block is rotating about edge A.

$$\mathbf{v}_2 = \frac{b}{\sqrt{2}}\omega_2 \searrow 45^\circ$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowright$+ Moments about A.

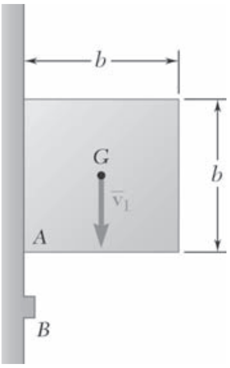
$$m\overline{v}_1\frac{b}{2} = \overline{I}\omega_2 + m\overline{v}_2(AG)$$

$$= \frac{1}{6}mb^2\omega_2 + m\left(\frac{b}{\sqrt{2}}\omega_2\right)\left(\frac{b}{\sqrt{2}}\right)$$

$$= \frac{2}{3}mb^2\omega_2$$

(*a*) Angular velocity. $\qquad \omega_2 = \dfrac{3}{4}\dfrac{v_1}{b} \qquad \boldsymbol{\omega}_2 = 0.750\dfrac{v_1}{b} \circlearrowright$ ◀

(*b*) Velocity of the mass center. $\qquad \overline{v}_2 = \dfrac{b}{\sqrt{2}}\omega_2 = \dfrac{3}{4\sqrt{2}}v_1 \qquad \overline{\mathbf{v}}_2 = 0.530v_1 \searrow 45^\circ$ ◀

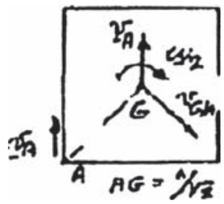


PROBLEM 17.141

Solve Problem 17.140, assuming that the impact between corner A and the obstruction B is perfectly elastic.

SOLUTION

Moments of inertia. $\quad \bar{I} = \frac{1}{6}mb^2$



Kinematics. Before impact, block is translating.

$$\mathbf{v}_1 = v_1 \downarrow \quad \omega_1 = 0$$

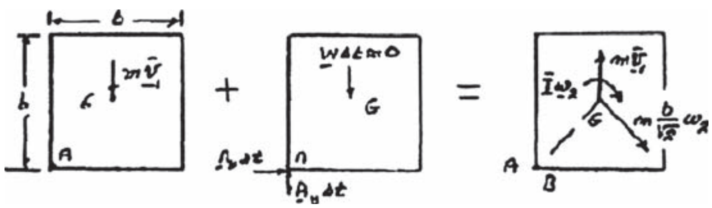
After impact with $e = 1$:

$$\mathbf{v}_A = v_1 \uparrow$$

$$\bar{\mathbf{v}}_2 = \mathbf{v}_A + \mathbf{v}_{G/A}$$

$$= [v_1 \uparrow] + \left[\frac{b}{\sqrt{2}}\omega_2 \searrow 45°\right] \qquad (1)$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowleft$+ Moments about A:

$$m\bar{v}_1\frac{b}{2} = \bar{I}\omega_1 - m\bar{v}_1\frac{b}{2} + m\frac{b}{\sqrt{2}}\omega_2\frac{b}{\sqrt{2}}$$

$$\bar{I} = \frac{1}{6}mb^2 \quad (Ag) = \frac{b}{\sqrt{2}}$$

$$m\bar{v}_1\frac{b}{2} = \frac{1}{b}mb^2\omega_2 - m\bar{v}_1\frac{b}{2} + m\left(\frac{b}{\sqrt{2}}\right)^2\omega_2$$

$$m\bar{v}_1 b = \frac{2}{3}b^2\omega_2$$

PROBLEM 17.141 (Continued)

(*a*) Angular velocity. $\omega_2 = \dfrac{3}{2}\dfrac{v_1}{b}$ $\qquad \omega_2 = 1.500\dfrac{v}{b}$ ◀

(*b*) Velocity of the mass center.

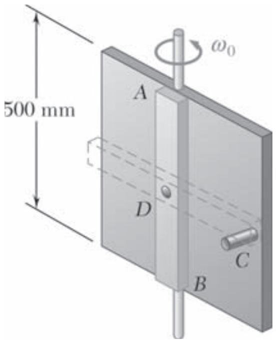
From Eq. (1),

$$\bar{v}_2 = [\bar{v}_1 \uparrow] + \left[\frac{b}{\sqrt{2}} \cdot \frac{3}{2}\frac{\bar{v}_1}{b} \searrow 45^\circ\right]$$

$$= [\bar{v}_1 \uparrow] + \left[\frac{3}{2\sqrt{2}}\bar{v}_1 \sin 45^\circ \downarrow\right] + \left[\frac{3}{2\sqrt{2}}\bar{v}_1 \cos 45^\circ \rightarrow\right]$$

$$= [\bar{v}_1 \uparrow] + \left[\frac{3}{4}\bar{v}_1 \downarrow\right] + \left[\frac{3}{4}\bar{v} \rightarrow\right] = \left[\frac{1}{4}v_1 \uparrow\right]\left[\frac{3}{4}v_1 \rightarrow\right]$$

$$\bar{\mathbf{v}}_2 = 0.791\bar{v}_1 \measuredangle 18.4^\circ \blacktriangleleft$$



PROBLEM 17.142

A 3-kg bar AB is attached by a pin at D to a 4-kg square plate, which can rotate freely about a vertical axis. Knowing that the angular velocity of the plate is 120 rpm when the bar is vertical, determine (*a*) the angular velocity of the plate after the bar has swung into a horizontal position and has come to rest against pin C, (*b*) the energy lost during the plastic impact at C.

SOLUTION

Moments of inertia about the vertical centroidal axis.

Square plate. $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(4)(0.500)^2 = 0.083333 \text{ kg}\cdot\text{m}^2$

Bar AB vertical. $\bar{I} =$ approximately zero

Bar AB horizontal. $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(3)(0.500)^2 = 0.0625 \text{ kg}\cdot\text{m}^2$

Position 1. Bar AB is vertical. $I_1 = 0.083333 \text{ kg}\cdot\text{m}^2$

Angular velocity. $\omega_1 = 120 \text{ rpm} = 4\pi \text{ rad/s}$

Angular momentum about the vertical axis.

$$(H_O)_1 = I_1\omega_1 = (0.083333)(4\pi) = 1.04720 \text{ kg}\cdot\text{m}^2/\text{s}$$

Kinetic energy. $T_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(0.083333)(4\pi)^2 = 6.5797 \text{ J}$

Position 2. Bar AB is horizontal. $I_2 = 0.145833 \text{ kg}\cdot\text{m}^2$

$$(H_O)_2 = I_2\omega_2 = 0.145833\omega_2$$

Conservation of angular momentum. $(H_O)_1 = (H_O)_2$:

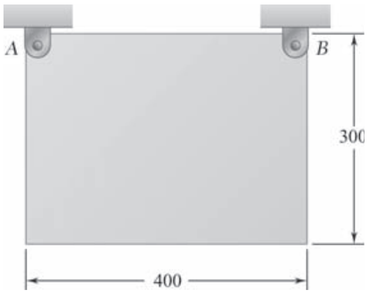
$$1.04720 = 0.145833\omega_2 \quad \omega_2 = 7.1808 \text{ rad/s}$$

(*a*) Final angular velocity. $\omega_2 = 68.6 \text{ rpm}$ ◀

(*b*) Loss of energy.

$$T_1 - T_2 = T_1 - \frac{1}{2}I_2\omega_2^2 = 6.5797 - \frac{1}{2}(0.145833)(0.71808)^2$$

$T_1 - T_2 = 2.82 \text{ J}$ ◀



PROBLEM 17.143

A 300×400 mm-rectangular plate is suspended by pins at *A* and *B*. The pin at *B* is removed and the plate swings freely about pin *A*. Determine (*a*) the angular velocity of the plate after it has rotated through 90°, (*b*) the maximum angular velocity attained by the plate as it swings freely.

SOLUTION

Let *m* be the mass of the plate.

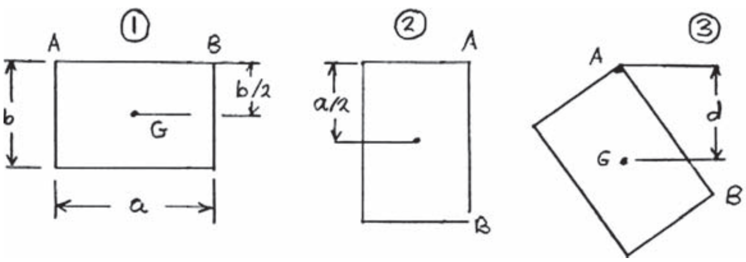
Dimensions: $a = 0.4\text{ m} \quad b = 0.3\text{ m}$

Moment of inertia about *A* $\quad I_A = \frac{1}{3}m(a^2 + b^2)$

Position 1. Initial position. $\quad \omega_1 = 0$

Position 2. Plate has rotated about *A* through 90°.

Position 3. Mass center is directly below pivot *A*.



Potential energy. Use level *A* as datum.

$$V_1 = -\frac{mab}{2} \quad V_2 = -\frac{mga}{2} \quad V_3 = -mgd$$

where $\quad d = \frac{1}{2}\sqrt{a^2 + b^2} = 0.25\text{ m}$

Kinetic energy. $\quad T_1 = 0 \quad T_2 = \frac{1}{2}I_A\omega_2^2 \quad T_3 = \frac{1}{2}I_A\omega_3^2$

PROBLEM 17.143 (Continued)

(*a*) 90° rotation. Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 - \frac{mgb}{2} = \frac{1}{2} \cdot \frac{1}{3} m(a^2 + b^2)\omega_2^2 + \frac{mga}{2}$$

$$\omega_2^2 = \frac{3g(a-b)}{a^2 + b^2} = \frac{(3)(9.81)(0.4 - 0.3)}{\left[(0.4)^2 + (0.3)^2\right]} = 11.772 (\text{rad/s})^2$$

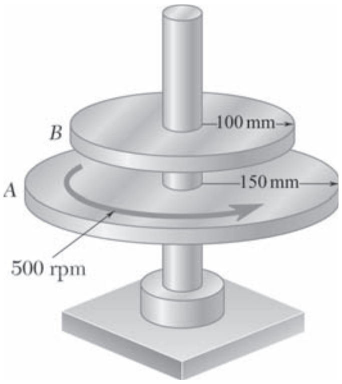
$\boldsymbol{\omega}_2 = 3.43 \text{ rad/s}$ ◄

(*b*) ω is maximum. Conservation of energy.

$$T_1 + V_1 = T_3 + V_3: \quad 0 - \frac{mgb}{2} = \frac{1}{2} \cdot \frac{1}{3} m(a^2 + b^2)\omega_3^2 - mgd$$

$$\omega_3^2 = \frac{g(6d + 3b)}{a^2 + b^2} = \frac{(9.81)(1.5 - 0.9)}{\left[(0.4)^2 + (0.3)^2\right]} = 23.544 \ (\text{rad/s})^2$$

$\boldsymbol{\omega}_3 = 4.85 \text{ rad/s}$ ◄



PROBLEM 17.144

Disks A and B are made of the same material and are of the same thickness; they can rotate freely about the vertical shaft. Disk B is at rest when it is dropped onto disk A, which is rotating with an angular velocity of 500 rpm. Knowing that disk A has a mass of 10 kg, determine (a) the final angular velocity of the disks, (b) the change is kinetic energy of the system.

SOLUTION

Disk A: $m_A = 10 \text{ kg} \quad r_A = 150 \text{ mm} = 0.15 \text{ m}$

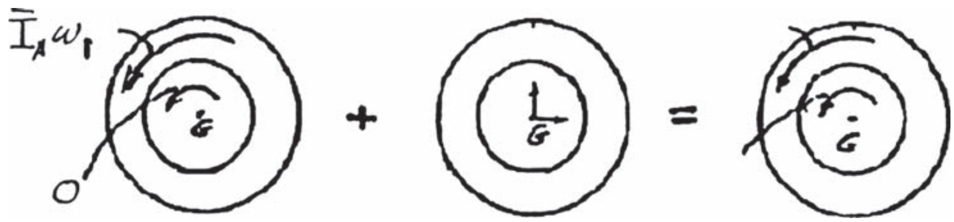
$$\bar{I}_A = \frac{1}{2}m_A r_A^2 = \frac{1}{2}(10)(0.15)^2 = \frac{9}{80} \text{ kg}\cdot\text{m}^2$$

Disk B: $r_B = 100 \text{ mm} = 0.1 \text{ m}$

$$m_B = m_A\left(\frac{r_B}{r_A}\right)^2 = (10)\left(\frac{100}{150}\right)^2 = \frac{40}{9} \text{ kg}$$

$$\bar{I}_B = \frac{1}{2}m_B r_B^2 = \frac{1}{2}\left(\frac{40}{9}\right)(0.1)^2 = \frac{1}{45} \text{ kg}\cdot\text{m}^2$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowleft +$ Moments about B: $\bar{I}_A\omega_1 + 0 + 0 = \bar{I}_A\omega_2 + \bar{I}_B\omega_2$

$$\omega_2 = \frac{\bar{I}_A}{\bar{I}_A + \bar{I}_B}\omega_1 = \frac{81}{97}\omega_1 = 0.83505\omega_1$$

Initial angular velocity of disk A: $\omega_1 = 500 \text{ rpm} = 52.36 \text{ rad/s}$

PROBLEM 17.144 (Continued)

(*a*) Final angular velocity of system: $\omega_2 = (0.83505)(52.36)$

$\omega_2 = 43.723 \text{ rad/s}$ $\qquad \omega_2 = 418 \text{ rpm}$ ◀

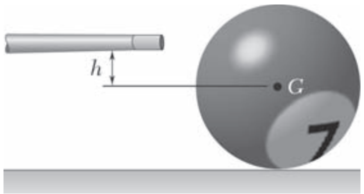
Initial kinetic energy: $T_1 = \frac{1}{2}\bar{I}_A \omega_1^2$

$$T_1 = \frac{1}{2}\left(\frac{9}{80}\right)(52.36)^2 = 154.213 \text{ N}\cdot\text{m}$$

Final kinetic energy: $T_2 = \frac{1}{2}(\bar{I}_A + \bar{I}_B)\omega_2^2$

$$T_2 = \frac{1}{2}\left(\frac{9}{80} + \frac{1}{45}\right)(43.723)^2 = 128.774 \text{ N}\cdot\text{m}$$

(*b*) Change in energy: $T_2 - T_1 = -25.439 \text{ N}\cdot\text{m}$ $\qquad \Delta T = -25.4 \text{ N}\cdot\text{m}$ ◀



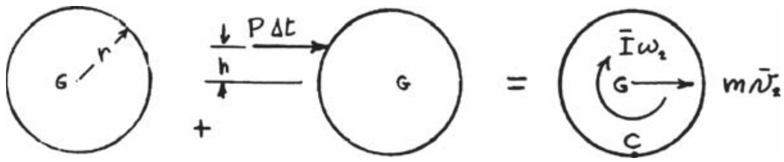
PROBLEM 17.145

At what height h above its center G should a billiard ball of radius r be struck horizontally by a cue if the ball is to start rolling without sliding?

SOLUTION

Moment of inertia. $\bar{I} = \frac{2}{5}mr^2$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics. Rolling without sliding. Point C is the instantaneous center of rotation.

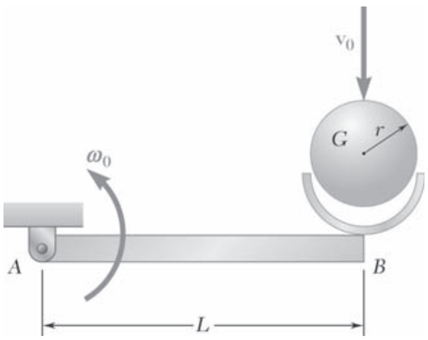
$\xrightarrow{+}$ Linear components: $0 + P\Delta t = m\bar{v}_2$

$= mr\omega_2$

$\circlearrowright$ Moments about G: $0 + hP\Delta t = \bar{I}\omega_2$

$$0 + h(mr\omega_2) = \left(\frac{2}{5}mr^2\right)\omega_2$$

$h = \frac{2}{5}r$ ◀



PROBLEM 17.146

A large 1.5 kg sphere with a radius $r = 100$ mm is thrown into a light basket at the end of a thin, uniform rod weighing 1 kg and length $L = 250$ mm as shown. Immediately before the impact the angular velocity of the rod is 3 rad/s counterclockwise and the velocity of the sphere is 0.5 m/s down. Assume the sphere sticks in the basket. Determine after the impact (*a*) the angular velocity of the bar and sphere, (*b*) the components of the reactions at *A*.

SOLUTION

Let Point *G* be the mass center of the sphere and Point *C* be that of the rod *AB*.

Rod *AB*: $m_{AB} = 1$ kg

$$I_{AB} = \frac{1}{12} m_{AB} L^2 = \frac{1}{12}(1)(0.25)^2$$

$$= \frac{1}{192}\ \text{kg}\cdot\text{m}^2$$

$$= 5.2083 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Sphere: $m_S = 1.5$ kg

$$I_G = \frac{2}{5} m_S r^2 = \frac{2}{5}(1.5)(0.1)^2 = \frac{3}{500} = 6 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

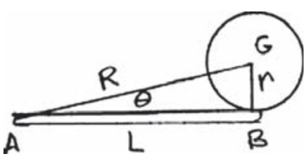
Impact. Before impact, bar *AB* is rotating about *A* with angular velocity $\boldsymbol{\omega}_0 = \omega_0 \circlearrowleft$ $(\omega_0 = 3\ \text{rad/s})$ and the sphere is falling with velocity $\mathbf{v}_0 = v_0 \downarrow (v_0 = 0.5\ \text{m/s})$. After impact, the rod and the sphere move together, rotating about *A* with angular velocity $\boldsymbol{\omega} = \omega \circlearrowright$.

Geometry. $R = \sqrt{L^2 + r^2} = \sqrt{(0.25)^2 + (0.1)^2} = 0.26926$ m

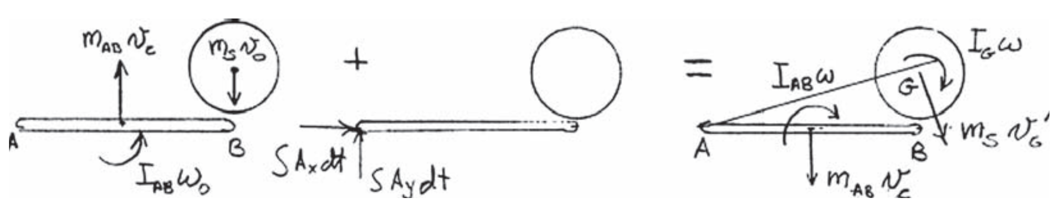
$$\tan\theta = \frac{r}{L} = \frac{0.1}{0.25} \qquad \theta = 21.8^\circ$$

Kinematics: Before impact, $v_C = \dfrac{L}{2}\omega_0 = (0.125)(3) = 0.375\ \text{m/s}\uparrow$

After impact, $\mathbf{v}_C = \dfrac{L}{2}\omega' \downarrow, \quad \mathbf{v}_G = R\omega' \measuredangle \theta$



Principle of impulse and momentum. Neglect mass of the rod and sphere over the duration of the impact.



PROBLEM 17.146 (Continued)

(*a*) $\circlearrowright$ Moments about *A*:

$$m_S v_0 L - I_{AB}\omega_0 - m_{AB} v_C \frac{L}{2} + 0 = I_G\omega' + m_S v'_G R + I_{AB}\omega' + m_{AB} v_C \frac{L}{2}$$

or
$$m_S v_0 L - I_{AB}\omega_0 - m_{AB} v_C \frac{L}{2} = \left(I_G + m_S R^2 + I_{AB} + \frac{1}{4} m_{AB} L^2\right)\omega' \qquad (1)$$

$$(1.5)(0.5)(0.25) - \left(\frac{1}{192}\right)(3) - (1)(0.375)(0.125)$$
$$= \left[\frac{3}{500} + (1.5)(0.26926)^2 + \frac{1}{192} + \left(\frac{1}{4}\right)(1)(0.25)^2\right]$$

$$0.125 = 0.13558\omega' \quad \omega' = 0.92193 \text{ rad/s}$$

$\boldsymbol{\omega}' = 0.922$ rad/s $\circlearrowright$ ◀

Normal accelerations at *C* and *G*.

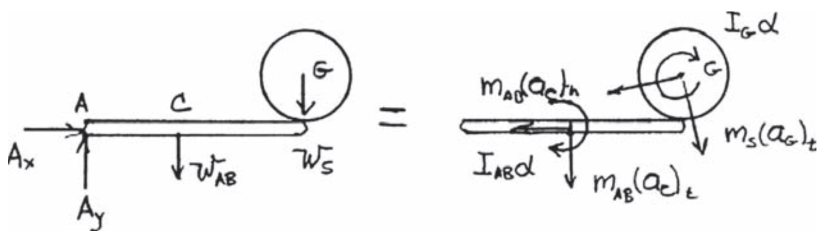
$$(\mathbf{a}_C)_n = \frac{L}{2}(\omega')^2 = (0.125)(0.92193)^2 = 0.10624 \text{ m/s}^2 \leftarrow$$

$$(a_G)_n = R(\omega')^2 = (0.26926)(0.92193)^2 = 0.22886 \text{ m/s}^2 \measuredangle\ 21.8°$$

Tangential accelerations at *C* and *G*. $\boldsymbol{\alpha} = \alpha \circlearrowright$

$$(a_C)_t = \frac{L}{2}\alpha = 0.125\,\alpha \downarrow \quad (a_G)_t = R\alpha = 0.26926\,\alpha \measuredangle\ 21.8°$$

(*b*) Kinetics. Use bar *AB* and the sphere as a free body.



$$\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}:$$

$$W_{AB}\frac{L}{2} + W_S L = I_{AB}\alpha + \frac{L}{2} m_{AB}(a_C)_t + I_G\alpha + m_S(a_G)_t R$$
$$= \left(I_{AB} + \frac{1}{4} m_{AB} L^2 + I_G + m_S R^2\right)\alpha$$

$$(1)(9.81)(0.125) + (1.5)(9.81)(0.25) = \left[\frac{1}{192} + \left(\frac{1}{4}\right)(1)(0.25)^2 + \frac{3}{500} + (1.5)(0.26926)^2\right]\alpha$$

$$4.905 = 0.13558\,\alpha$$

$\boldsymbol{\alpha} = 36.1766$ rad/s^2 $\circlearrowright$

PROBLEM 17.146 (Continued)

$$(\mathbf{a}_C)_t = (0.125)(36.1766) = 4.5221 \text{ m/s}^2 \downarrow, \quad (\mathbf{a}_G)_t = (0.26926)(36.1766) = 9.7409 \text{ m/s}^2 \searrow 21.8°$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}:$

$$A_x = -m_{AB}(a_C)_n - m_S(a_G)_n \cos 21.8° + m_S(a_G)_t \sin 21.8°$$

$$A_x = -(1)(0.10624) - (1.5)(0.22886)\cos 21.8° + (1.5)(9.7409)\sin 21.8°$$

$$A_x = 5.2137 \text{ N} \qquad \mathbf{A}_x = 5.21 \text{ N} \rightarrow \blacktriangleleft$$

$+\!\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A_y - W_{AB} - W_S = -m_{AB}(a_C)_t - m_S(a_G)_t \cos 21.8° - m_S(a_G)_n \sin 21.8°$

$$A_y - (1)(9.81) - (1.5)(9.81) = -(1)(4.5221) - (1.5)(9.7409)\cos 21.8° - (1.5)(0.22886)\sin 21.8°$$

$$A_y = 6.309 \text{ N} \qquad \mathbf{A}_y = 6.31 \text{ N} \uparrow \blacktriangleleft$$