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1) $R = \{A1, A2, A3, A4, A5\}$
   Functional dependencies:
   - $A2 \ A3 \rightarrow A4$
   - $A1 \ A3 \rightarrow A5$

a. BCNF: If for every non-trivial functional dependency is a superkey.
   3NF: If it is in BCNF and there are no transitive dependencies.

   - $A2 \ A3 \rightarrow A4$: $A2 \ A3$ is not a superkey because $A1$ is missing.
   - $A1 \ A3 \rightarrow A5$: $A1 \ A3$ is not a superkey because $A2$ is missing.

   Since neither functional dependency has a superkey on the left-hand side, the relation R does not satisfy BCNF.

   - $A2 \ A3 \rightarrow A4$: There are no transitive dependencies involved.
   - $A1 \ A3 \rightarrow A5$: There are no transitive dependencies involved.

   Since there are no transitive dependencies but it is not BCNF. So relation R is not 3NF.

b. Using BCNF Decomposition Algorithm

   $F = \{(A1 \ A3 \rightarrow A5), (A2 \ A3 \rightarrow A4)\}$

   - Step 1 : $A1 \ A3 \rightarrow A5$ is not BCNF.
     Split it.
     $R_1 = \{A1, A3, A5\}$
     $R_2 = \{A1, A2, A3, A4\}$

- Step 2: R_1 is BCNF.
  Store it.
- Step 3: R_2 is not BCNF.
  Split it.
    $R_3 = \{ A_2, A_3, A_4 \}$
    $R_4 = \{ A_1, A_2, A_3 \}$
- Step 4: R_3 is BCNF.
  Store it.
- Step 5: R_4 is BCNF
  Store it.

Finally, we have
  $R_1 = \{ A_1, A_3, A_5 \}$
  $R_2 = \{ A_2, A_3, A_4 \}$
  $R_3 = \{ A_1, A_2, A_3 \}$

$F_1: A_1 A_3 \rightarrow A_5$
$F_2: A_2 A_3 \rightarrow A_4$

c.

R1: • $A_1 A_3$ is candidate key
• A_5 is dependent

R2: • $A_2 A_3$ is candidate key
• A_4 is dependent

R3: no functional dependency.
  Satisfy BCNF.

d. There is no needed functional dependencies.

• $A_1 A_3 \rightarrow A_5$: Yes, preserved. R_1 includes items.
• $A_2 A_3 \rightarrow A_4$: Yes, preserved. R_2 includes items.