## 1 Summary

This is a note for [1]. The implementation code for this paper can be found at <sup>1</sup>. The important components are as follows:

- 1. RNN language model
- 2. Morphological priors
- 3. Latent word embedding  $b_w$ .
- 4. Morpheme embedding  $u_m$ .
- 5. Variational distribution Q(b)

## 2 Latent Word Embedding and Morpheme Embedding

Each morpheme is segmented in unsupervised fashion according to Morfessor. For example,  $u_{-ism} = (-0.24, 5, -111)$ .

When inferring P(x), we will have to infer P(b) too since P(b) appears in the lower variational bound.

$$b_{w,i} \sim Bernouli(sigmoid(\sum_{m \in M_w} u_{m,i}))$$

i.e. for outcomes or the range of a probabilistic variable  $b_{w,i}$  is either 0 or 1,

$$P(b_{w,i}) = sigmoid(\sum_{m \in M_w} u_{m,i})^{b_{w,i}} (1 - sigmoid(\sum_{m \in M_w} u_{m,i}))^{1 - b_{w,i}}$$

So let's look into an example. Let  $M = perfection, -ism\ u_{perfection} = (0, -1.1, 1)$ 

 $u_{-ism} = (2, 5.1, 3)$ 

When w = perfectionism, then

 $b_{w,0} \sim Bernoulli(sigmoid(0+2)) \approx 0.88$ 

 $b_{w,1} \sim Bernoulli(sigmoid(-1.1 + 5.1)) \approx 0.98$ 

 $b_{w,2} \sim Bernoulli(sigmoid(1+3)) \approx 0.98$ 

So  $P(b_w = (1, 1, 1)) = 0.88 * 0.98 * 0.98 \approx 0.84$ .

## 3 Hidden state

The hidden state at time  $h_t$  (vector) is

$$h_t = sigmoid(\Theta h_{t-1} + b_{x_t})$$

where  $x_t$  is the word corresponding to the position t, and  $\Theta$  is the parameter for the recurrence function (recurrent weights<sup>2</sup>).

<sup>&</sup>lt;sup>1</sup>https://github.com/rguthrie3/MorphologicalPriorsForWordEmbeddings

<sup>2</sup>http://peterroelants.github.io/posts/rnn\_implementation\_part01/

4 What is going on inside  $D_{KL}(Q(b)||P(b))$ ?

$$\begin{split} D_{KL}(q(b_{w,i})||P(b_{w,i})) &= q(b_{w,i})\log(\frac{q(b_{w,i})}{P(b_{w,i})}) \\ &= q(b_{w,i})(\log(q(b_{w,i})) - \log(P(b_{w,i})) \\ &= q(b_{w,i})(\log(q(b_{w,i})) - b_{w,i}\log(sigmoid(\sum_{m \in M_w} u_{m,i})) - (1 - b_{w,i})\log(1 - sigmoid(\sum_{m \in M_w} u_{m,i}))) \\ q(b_{w,i}; \gamma_{w,i}) &= \gamma_{w,i}^{b_{w,i}}(1 - \gamma_{w,i})^{1 - b_{w,i}} \\ \log(q(b_{w,i}; \gamma_{w,i})) &= b_{w,i}\gamma_{w,i} + (1 - b_{w,i})(1 - \gamma_{w,i}) \end{split}$$

## References

[1] Parminder Bhatia, Robert Guthrie, and Jacob Eisenstein. Morphological priors for probabilistic neural word embeddings. In *Proceedings of the 2016 Conference on Empirical Methods in Natural Language Processing*, pages 490–500, Austin, Texas, November 2016. Association for Computational Linguistics.