# Morphological Image Processing

# Introduction

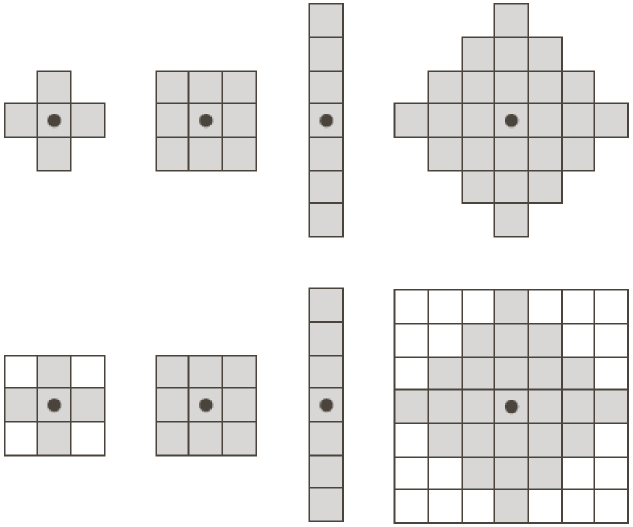
- *'Morphology'* denotes a branch of biology
 that deals with the form and structure of
animals and plants.
- For images, morphological operations change the shape of the object

# Basic Principle

- Extraction of geometrical information from an unknown image through transformations
- Use a well-defined, set known as Structuring Element (SE) for extraction
- Design of SEs, their shape and size, is crucial to the success of the morphological operations

# Structuring Element

- Small sets or subimages
- Used to probe an image to study region of interest
- Ex:Grey square (foreground) shows true ('1')
- Ex: White square (background) shows false ('0')



Some structuring elements

# Some Morphological Operations

- Erosion
- Dilation
- Opening
- Closing

# Erosion

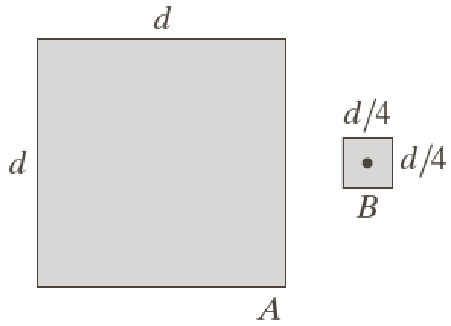
- Erosion is used  for shrinking element A by using element B
- Set B is a structuring element
  - Erosion for Sets A and B is defined by

$$A \ominus B = \{z | [(B)z \subseteq A\}$$

- Erosion of A by B is the set of all points, z such that B, translated by z, is contained in A
- That is eroded image contains center of structuring element after translation

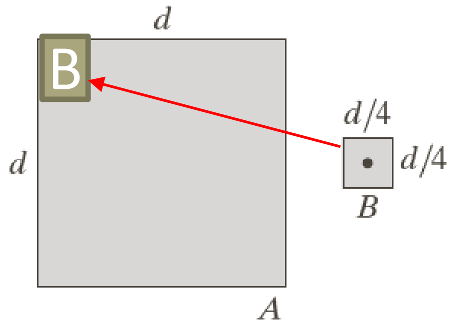
# Erosion of A by Structuring Element B

Image SE1



# Erosion of A by Structuring Element B

SE1 on image SE1

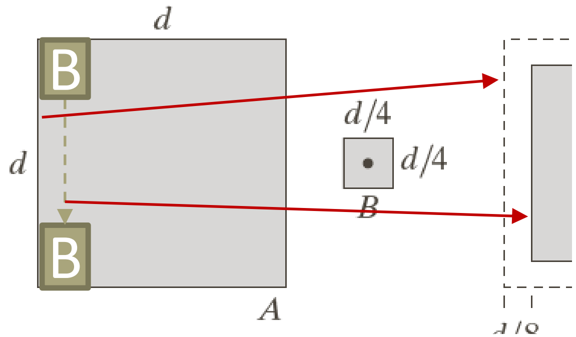



Eroded image with SE1

# Erosion of A by Structuring Element B

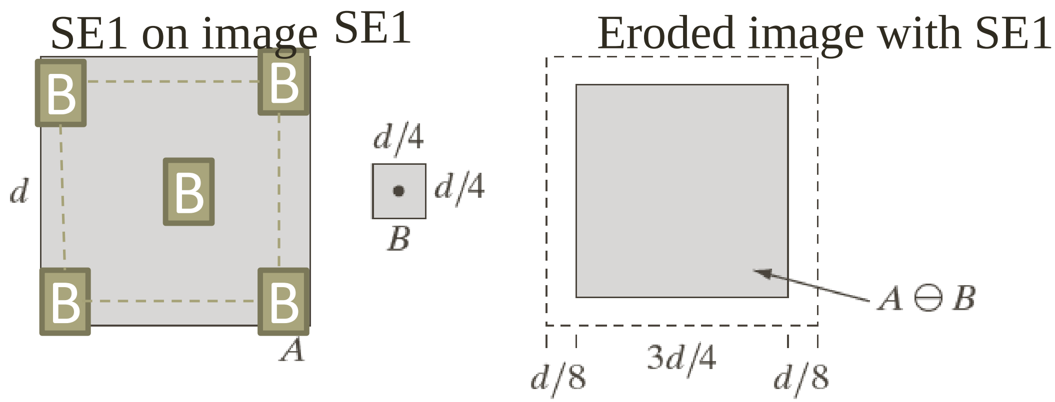
SE1 on image SE1

Eroded image with SE1



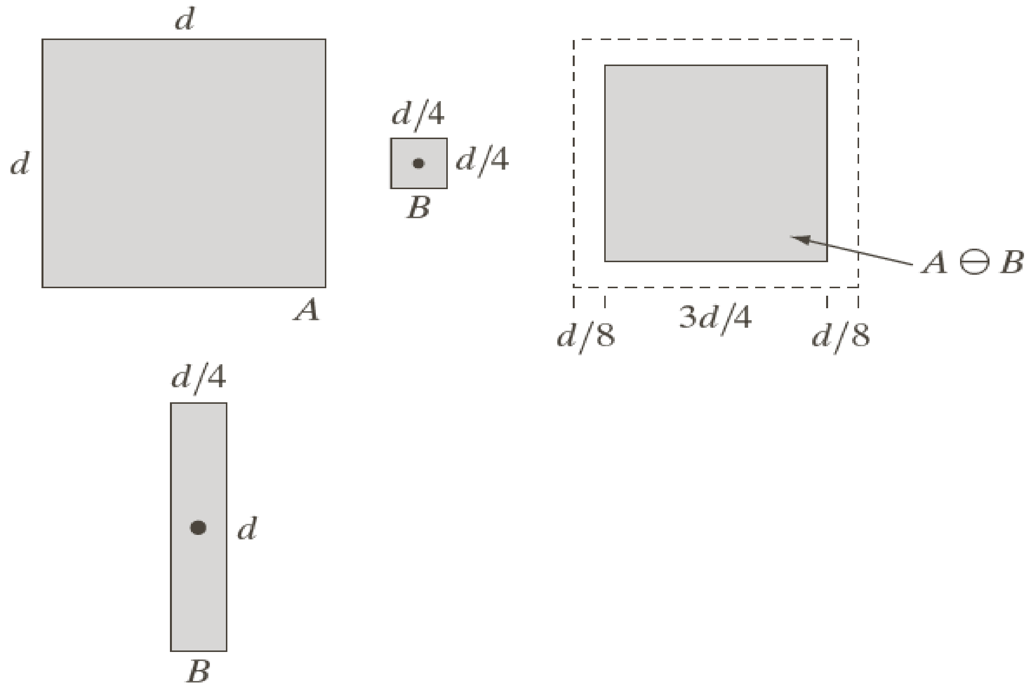

$A \ominus B$

# Erosion of A by Structuring Element B



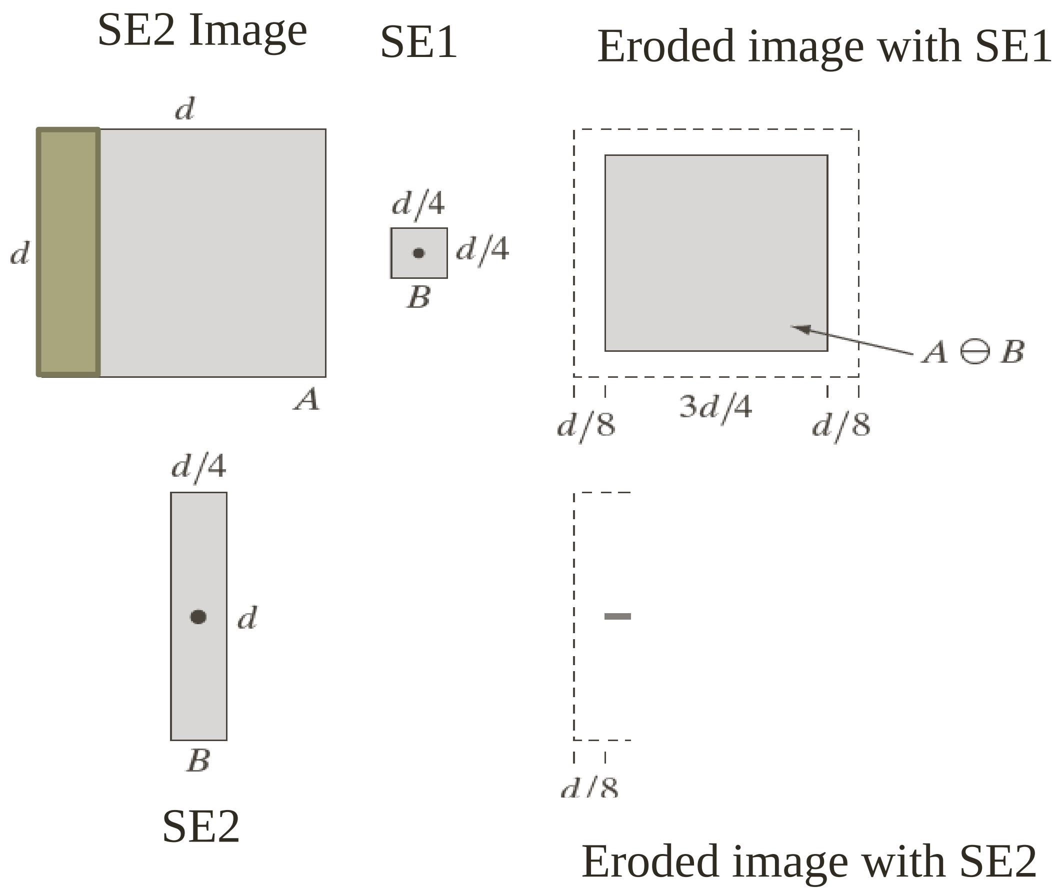
# Erosion of A by Structuring Element B

Image SE1 Eroded image with SE1




SE2

# Erosion of A by Structuring Element B

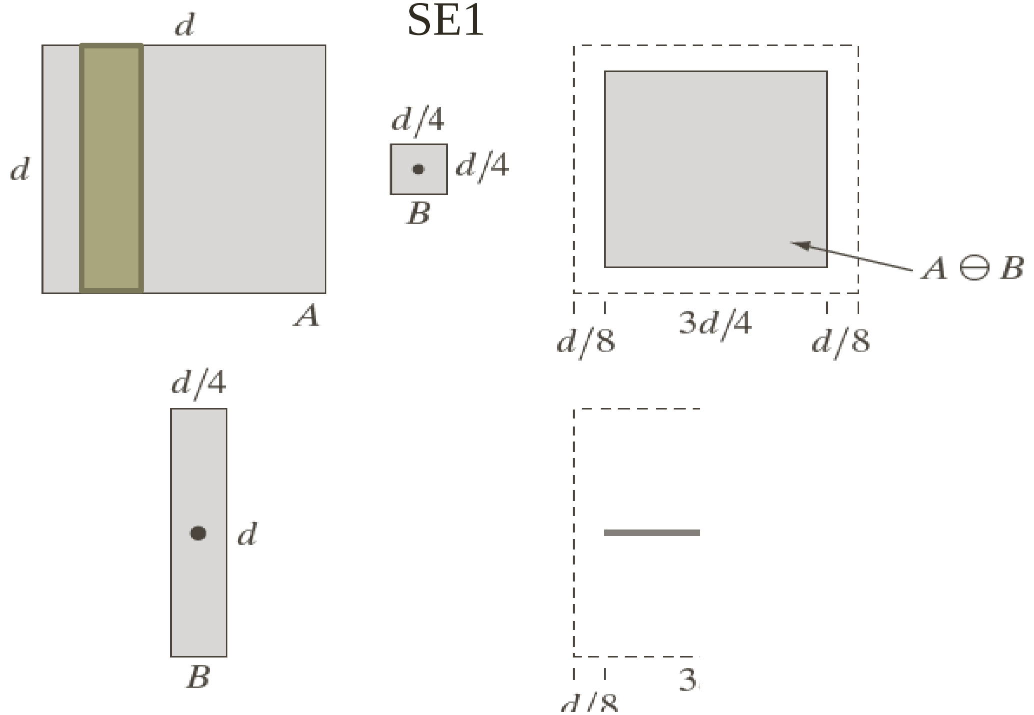


# Erosion of A by Structuring Element B

SE2 on Image

Eroded image with SE1

SE1

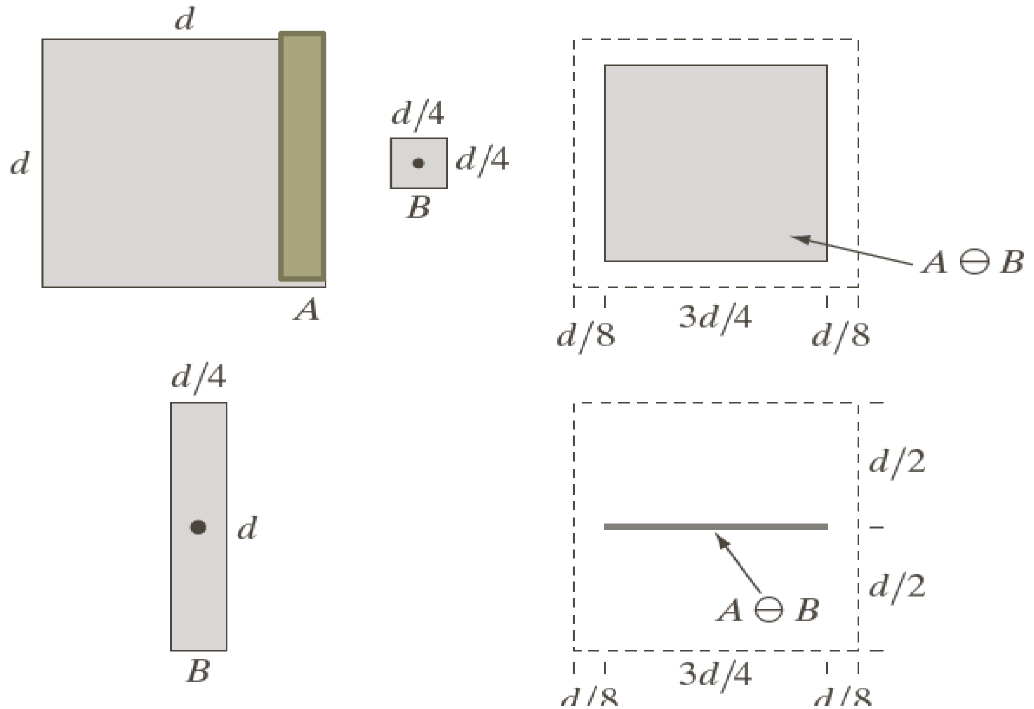



SE2

Eroded image with SE2

# Erosion of A by Structuring Element B

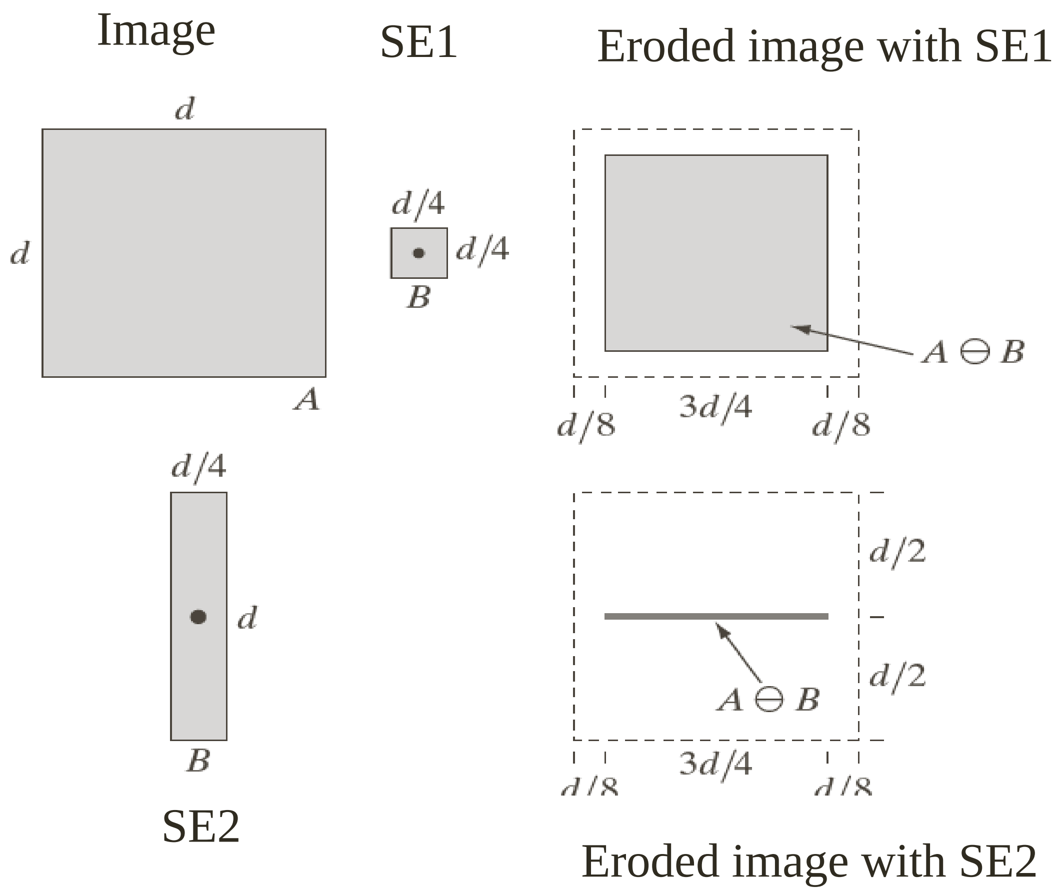
SE2 on Image SE1 Eroded image with SE1



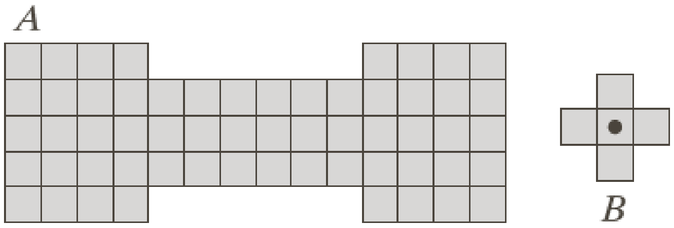

SE2

Eroded image with SE2

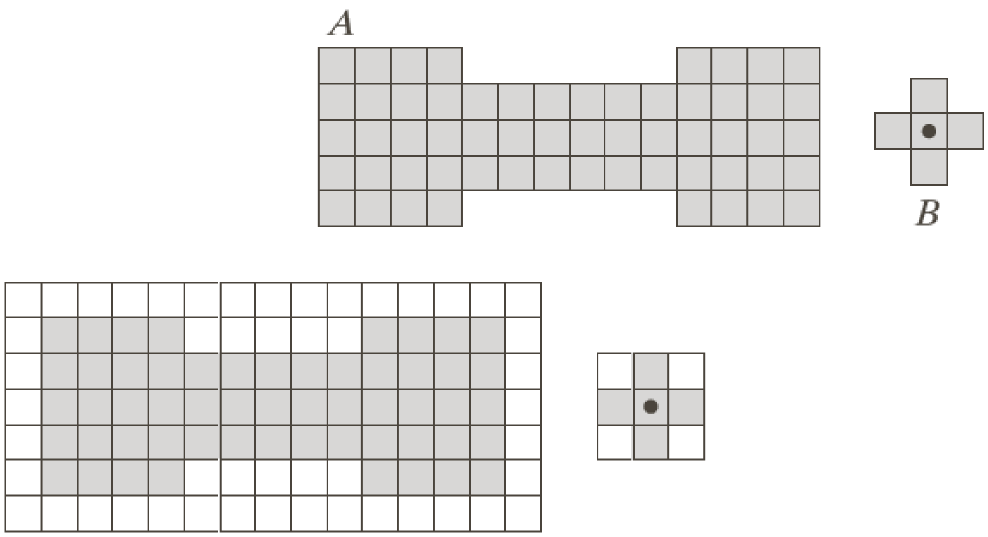
# Erosion of A by Structuring Element B



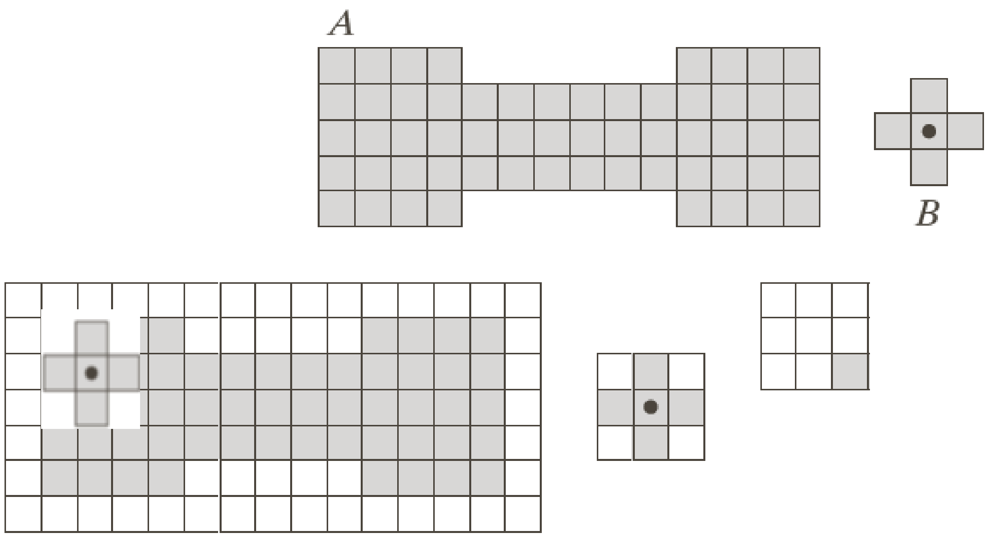
# Example: Erosion



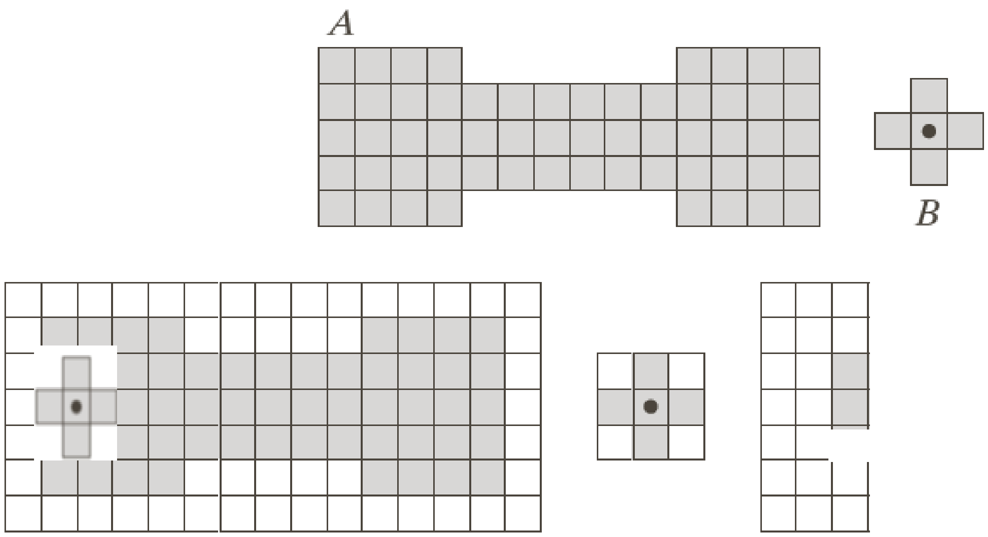
# Example: Erosion



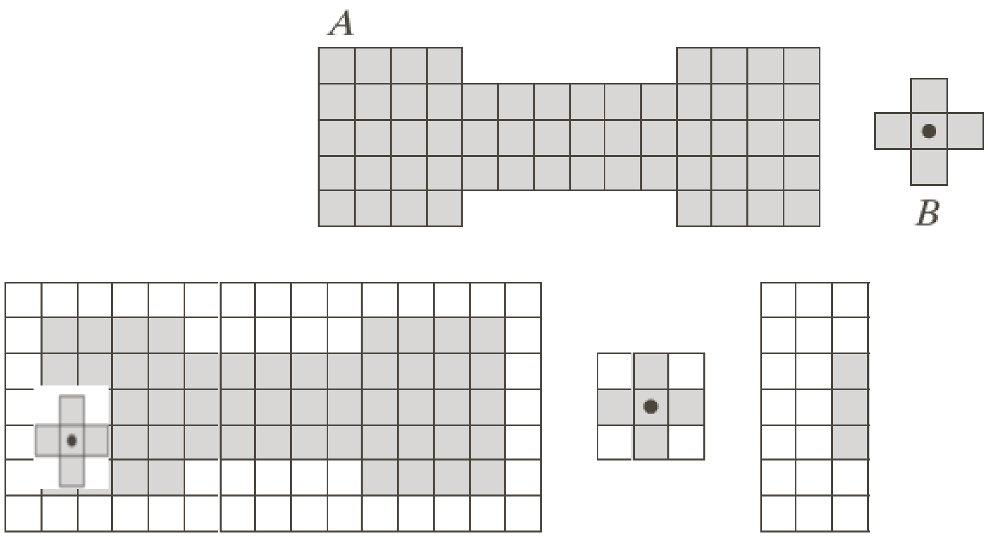
# Example: Erosion



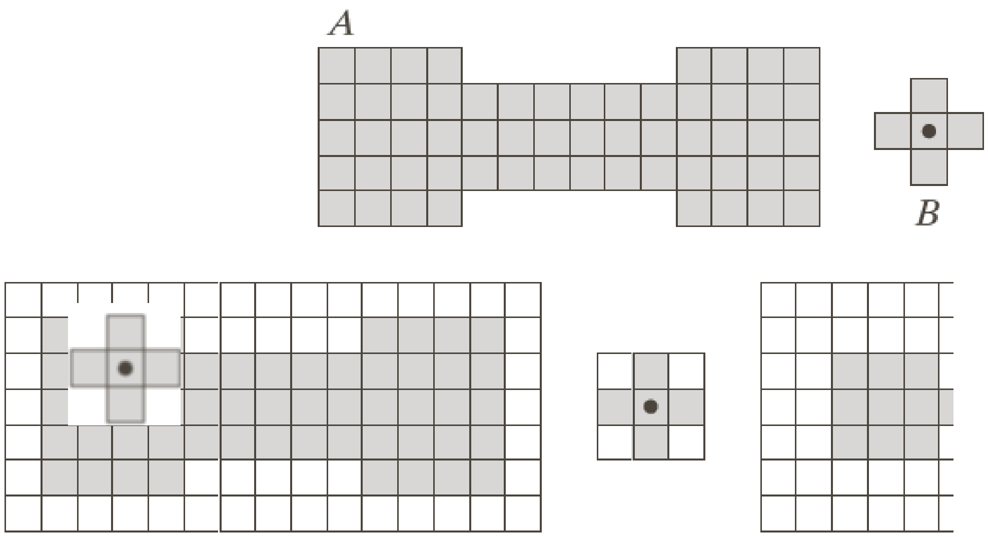
# Example: Erosion



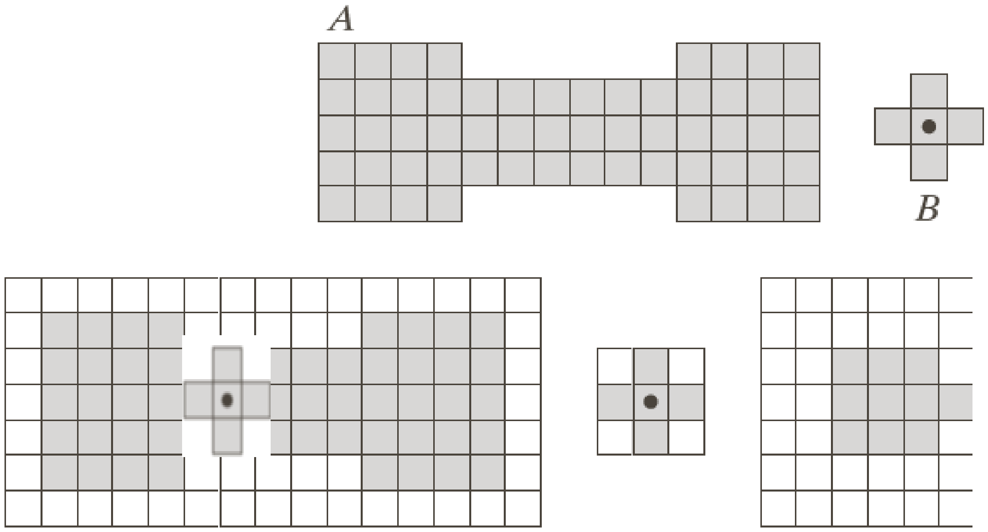
# Example: Erosion



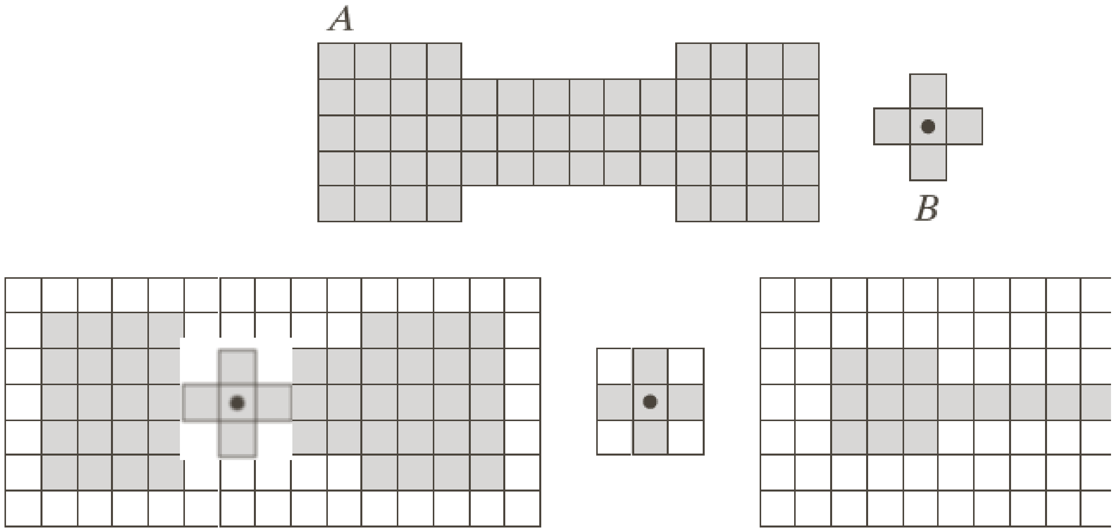
# Example: Erosion



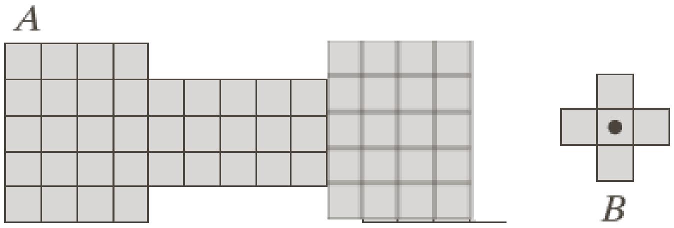
# Example: Erosion

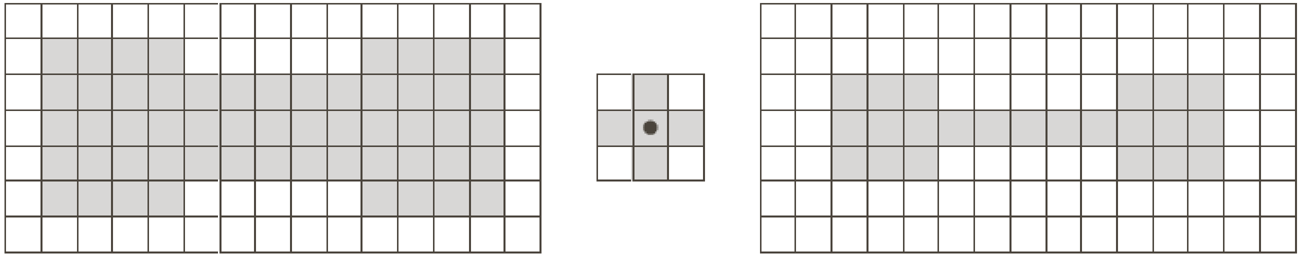


# Set and Structuring Element



# Set and Structuring Element



Pixels of SE and the eroded objects of image have same pixel intensity

# Erosion example

# Erosion with structuring elements

SE:11×11, white image

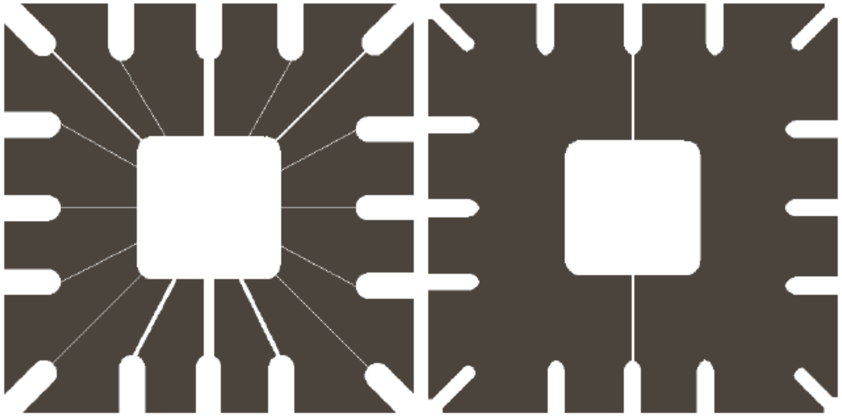
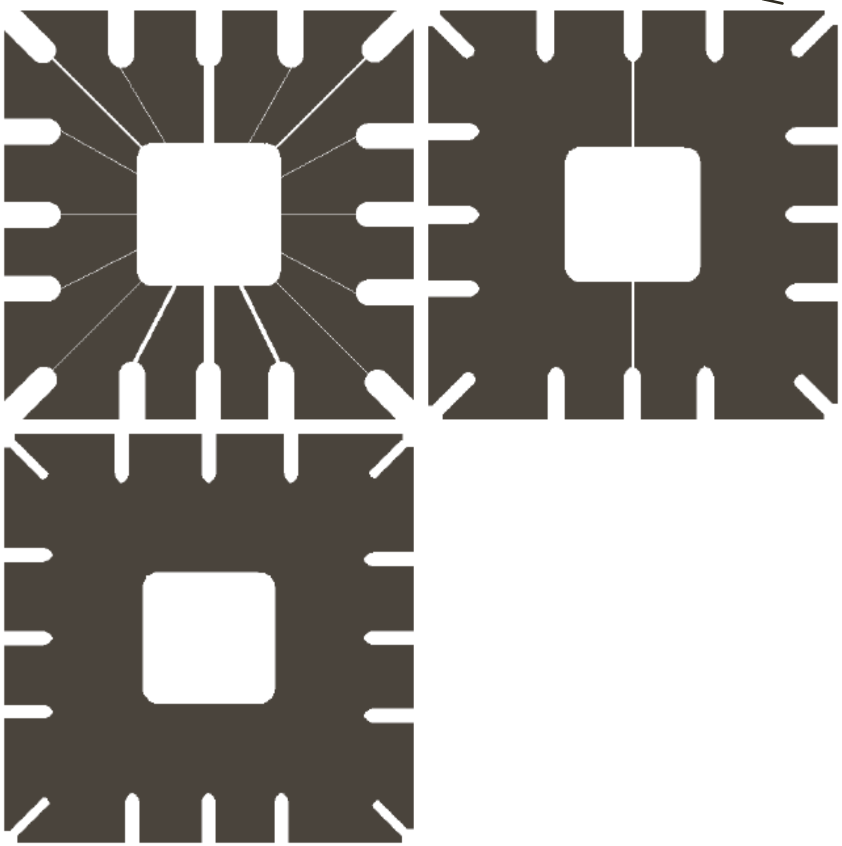


Image details smaller than the structuring element are removed

# Erosion with structuring elements

SE:11×11 white image

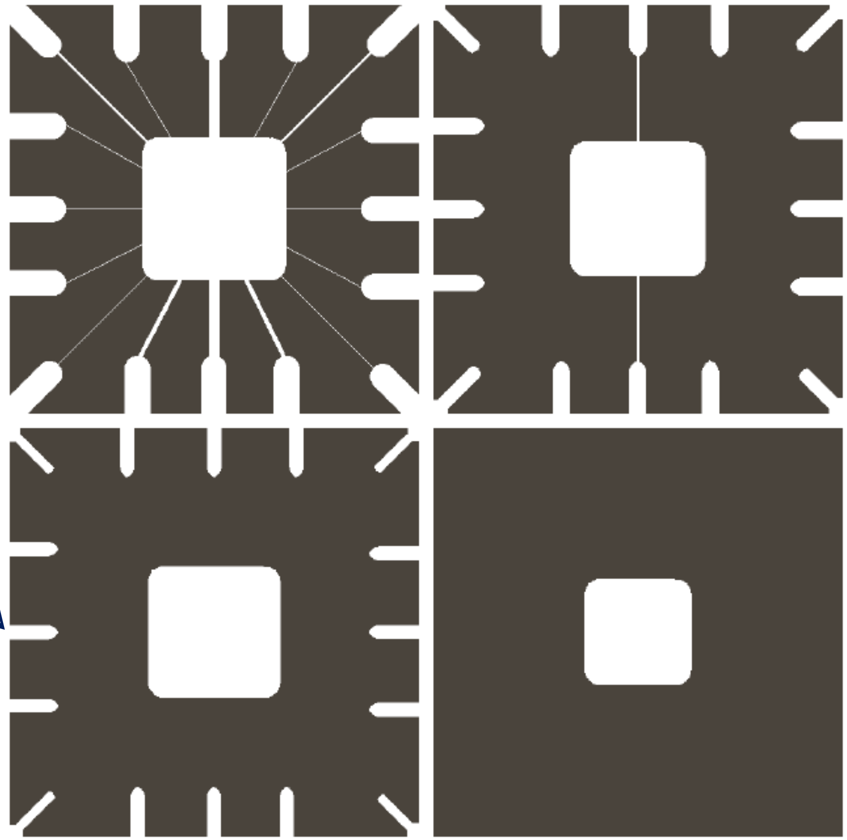


SE:15×15 white image

Image details smaller than the structuring element are removed

# Erosion with structuring elements

SE:11×11 white image



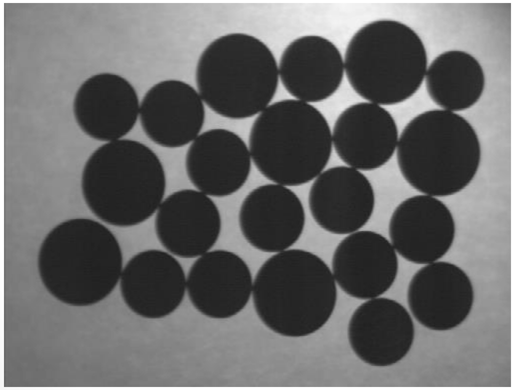
SE:15×15 white image

SE:45×45 white image

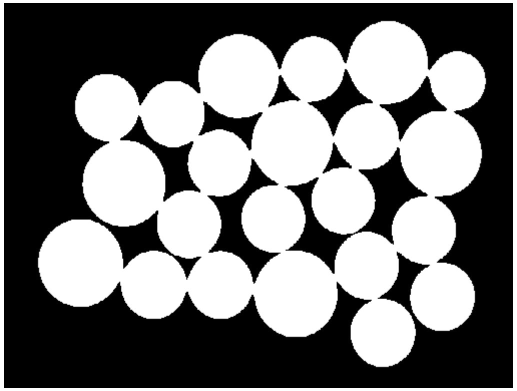
Image details smaller than the structuring element are removed

# Counting coins

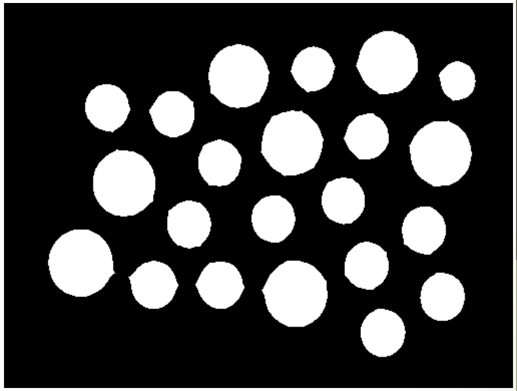
- Counting coins is difficult because they touch each other
- Solution: Binarization using thresholding and Erosion separates them
- Apply Structuring element of circular shape with size smaller than smallest coin



Gray Image



Binary Image



Eroded Binary Image

# Dilation

- Dilation is used for expanding an element A by using structuring element B

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$

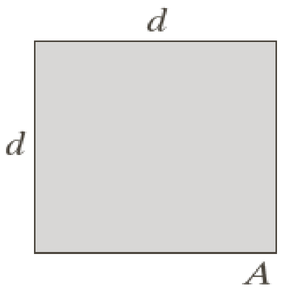
- Based on obtaining the reflection of B about its origin and shifting the reflection by z
- The dilation of A by B is the set of all displacements z, such that reflection of B and A overlap by at least one element

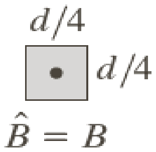
$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subset A\}$$

# Dilation

Image

SE1



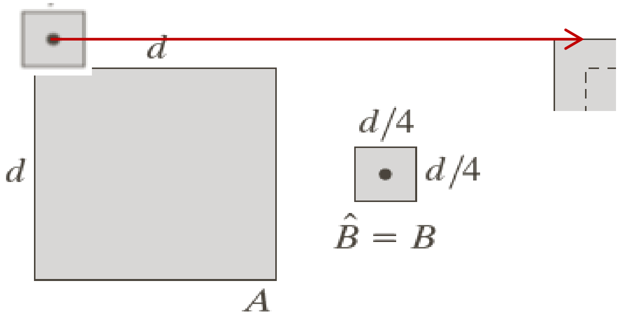



# Dilation

Image with SE1

Dilation of Image with SE1

SE1

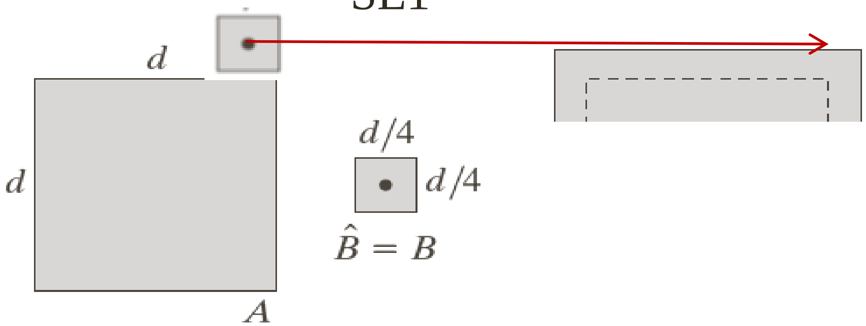


# Dilation

Image with SE1

Dilation of Image with SE1

SE1

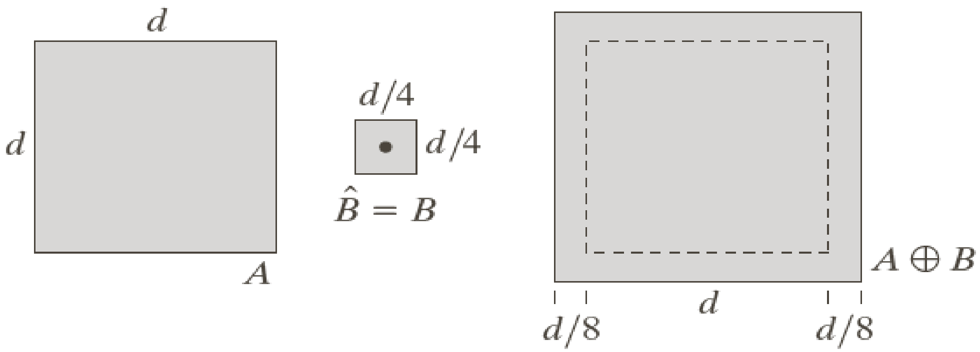


# Dilation

Image

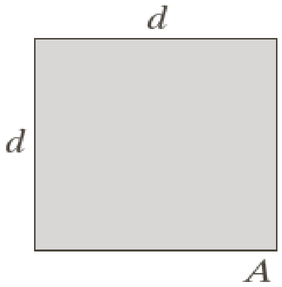
SE1

Dilation of Image with SE1

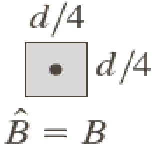


# Dilation

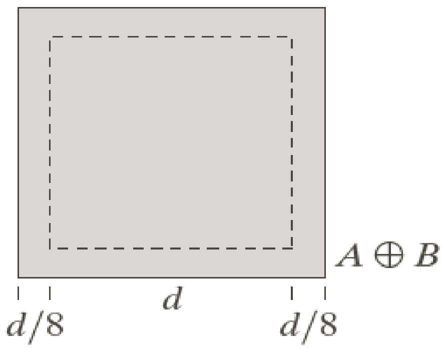
Image



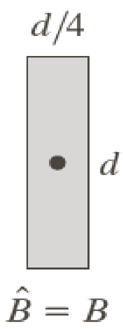

SE1




Dilation of Image with SE1

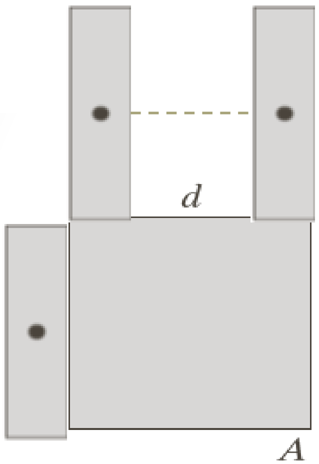



SE2

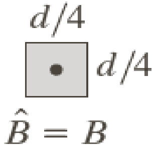


# Dilation

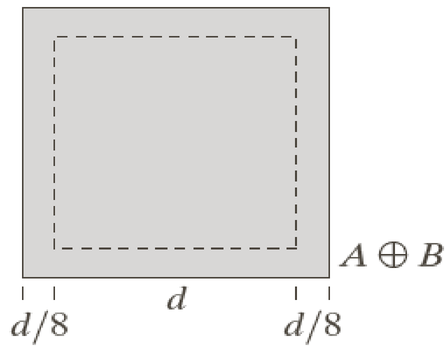
Image with SE2



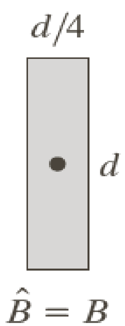

SE1

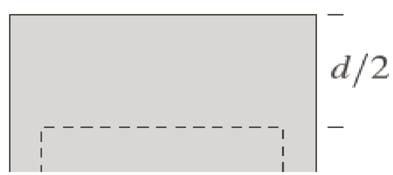



Dilation of Image with SE1




SE2




Dilation of Image with SE2

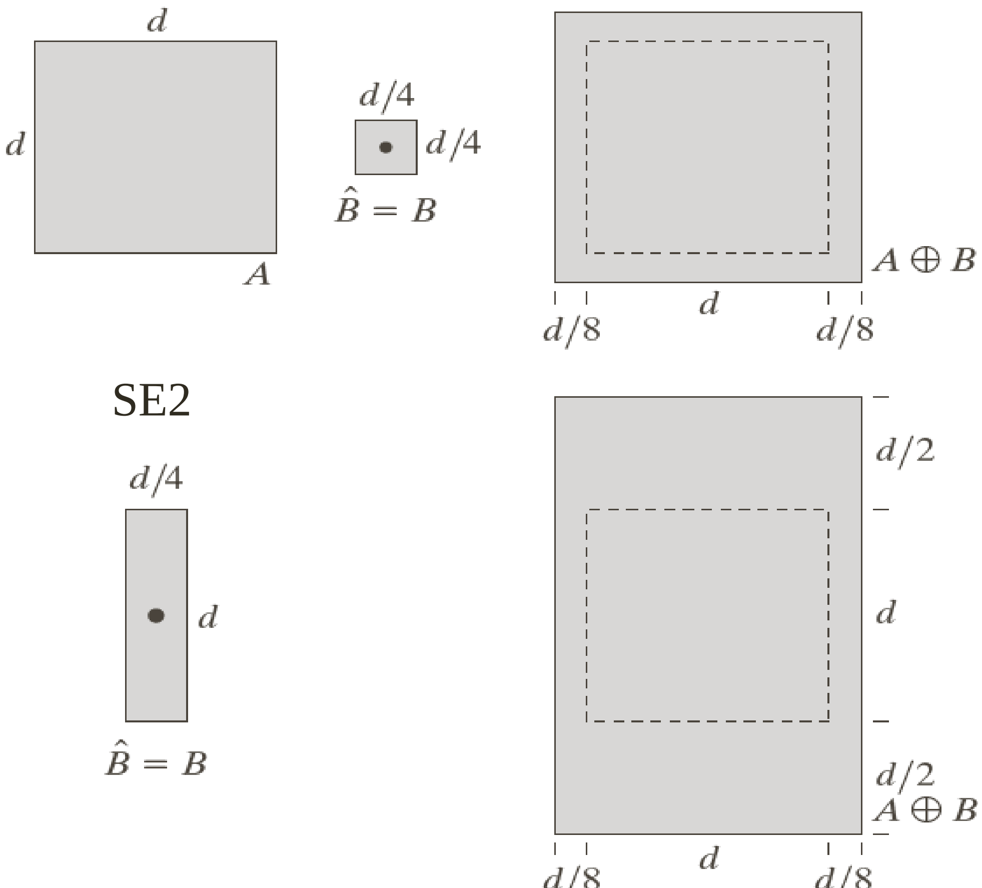
# Dilation

Image

SE1

Dilation of Image with SE1

SE2




Dilation of Image with SE2

# Example: Dilation

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

Structuring element

# Example: Dilation

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

Structuring element

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

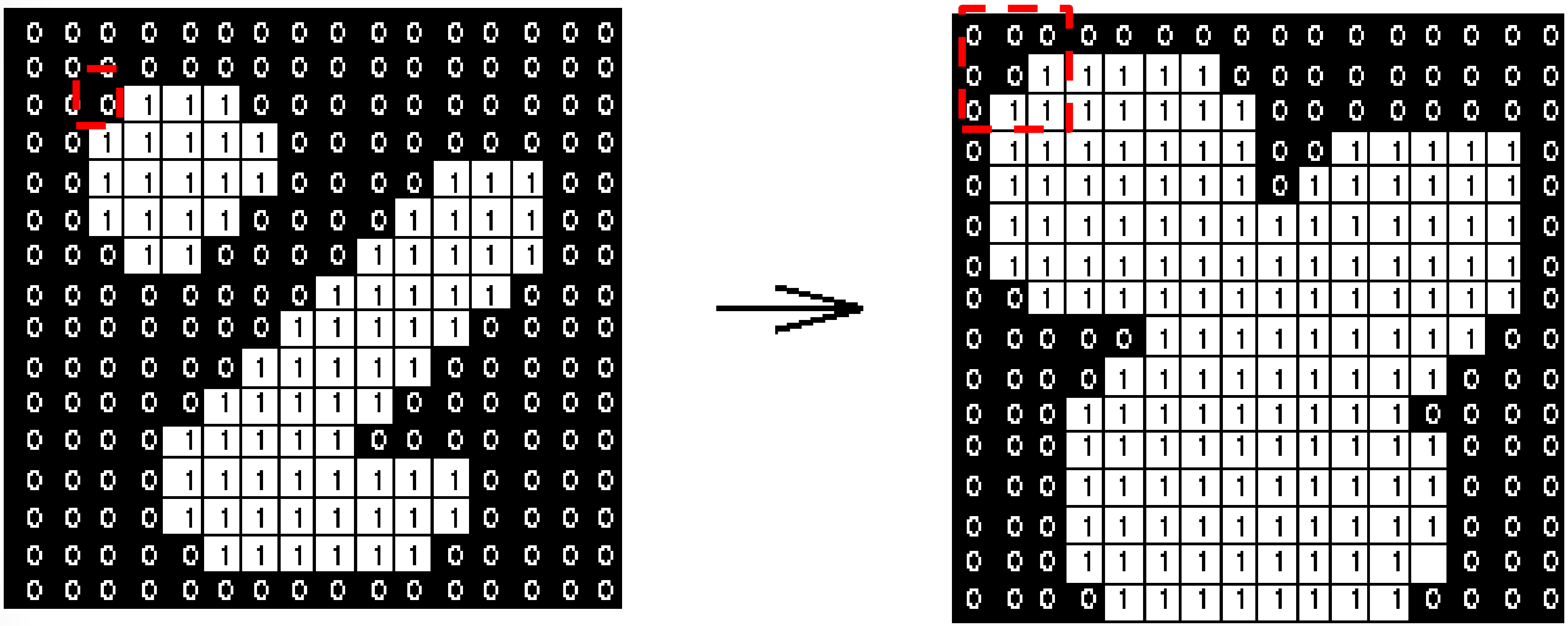
# Example: Dilation

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

Structuring element

# Example: Dilation

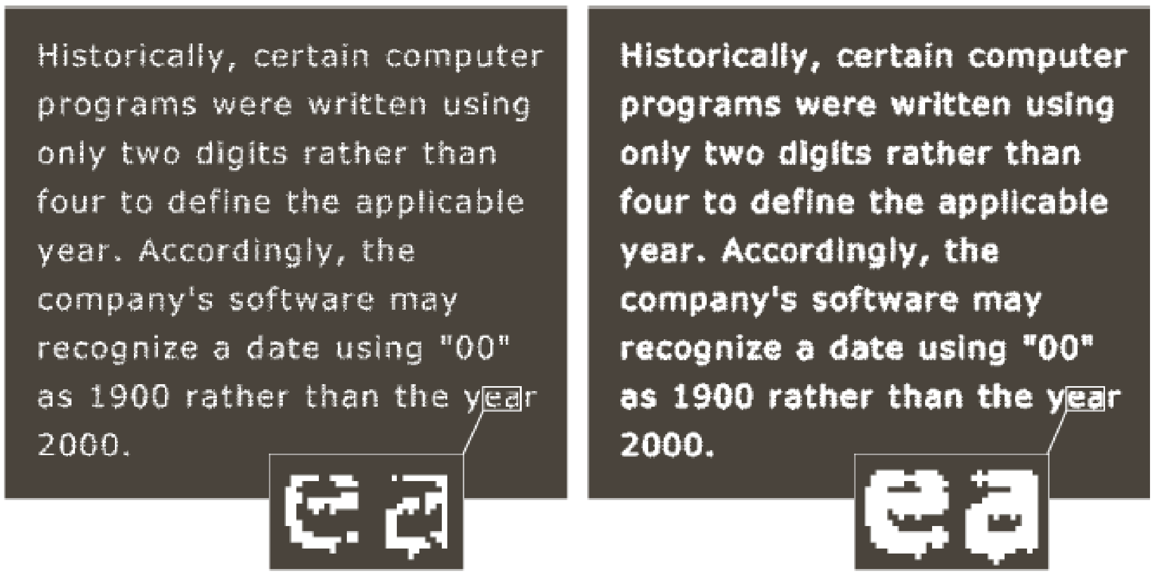


| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

Structuring element

append one white pixel to border pixels of object

# Dilation




| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

# Erosion and Dilation

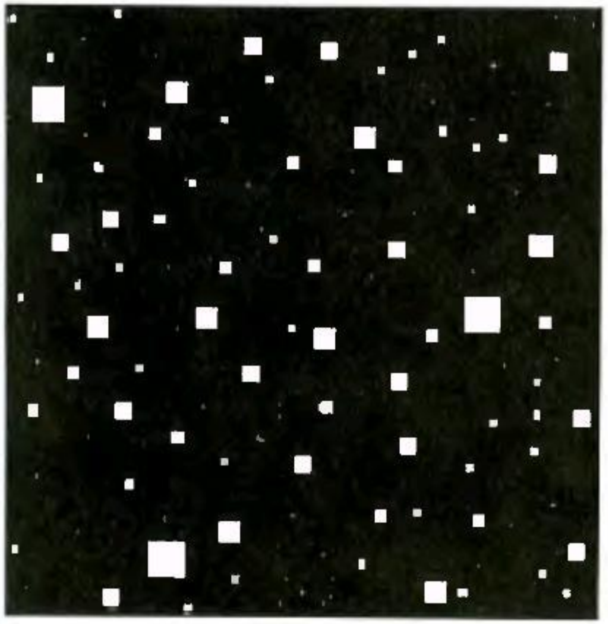


Image with squares of 1,3,5,7,9 and 15 pixels

# Erosion

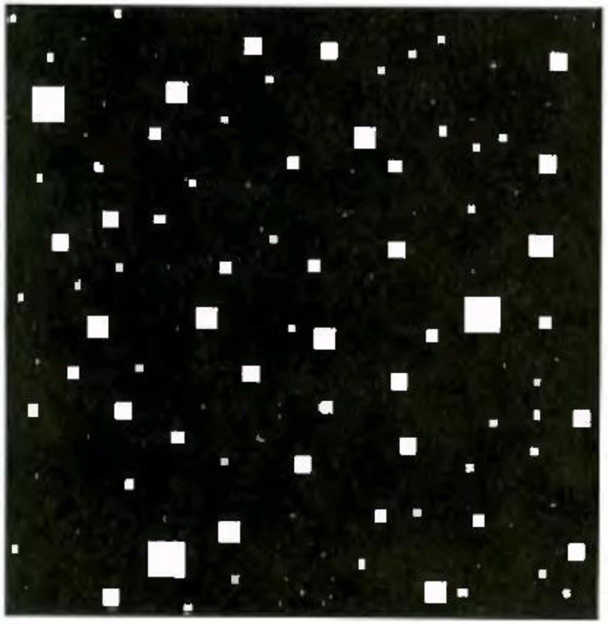


Image with squares of 1,3,5,7,9 and 15 pixels

Erosion with square SE of size 13 pixels

# Erosion and Dilation

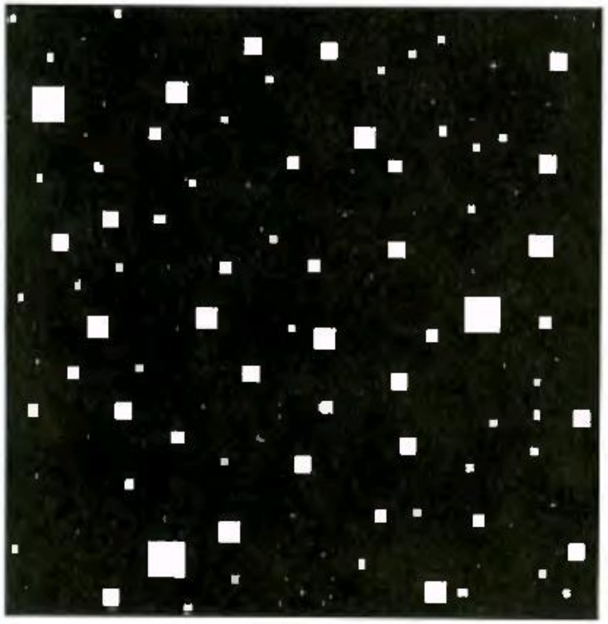
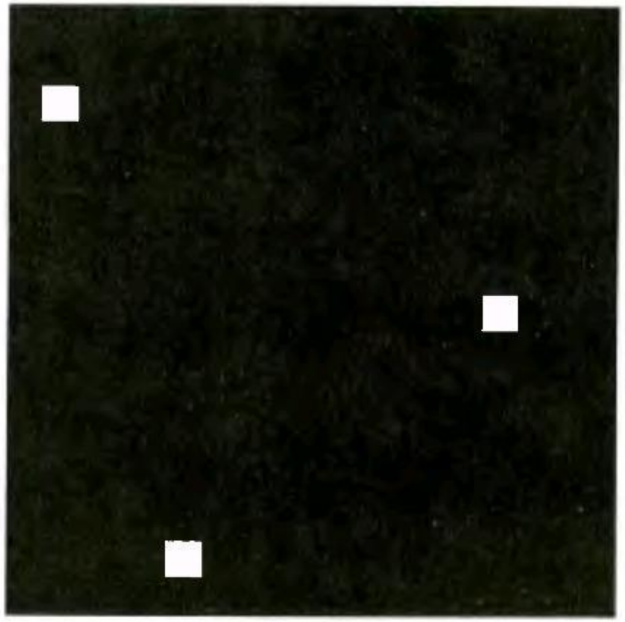


Image with squares of 1,3,5,7,9 and 15 pixels

Erosion with square SE of size 13 pixels



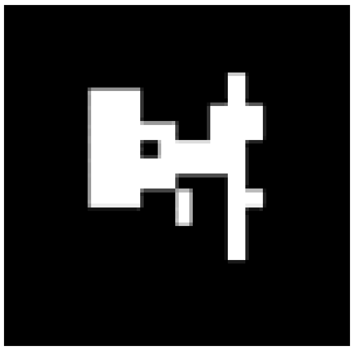
Dilation with square SE of size 13 pixels

# Duality of Erosion and dilation

$$(A\ e\ B)^c = (A^c\ d\ \hat{B})$$

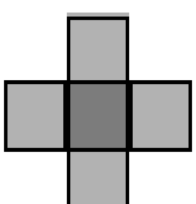
$$(A\ d\ B)^c = (A^c\ e\ \hat{B})$$

# Duality of dilation and erosion

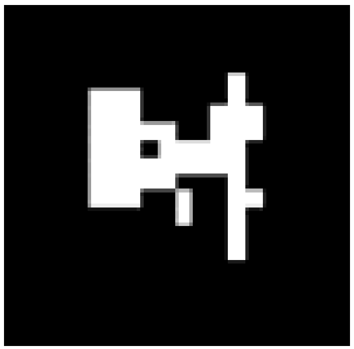


A

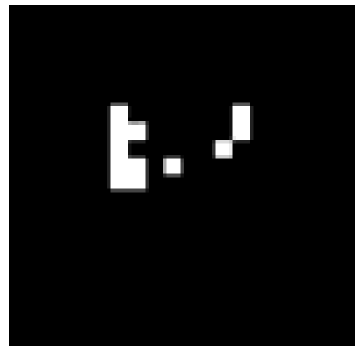
$B = \hat{B}$



# Duality of dilation and erosion

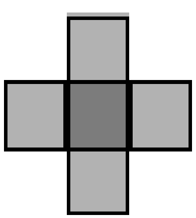


A

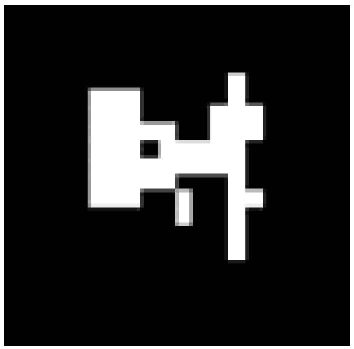


$A \ominus B$

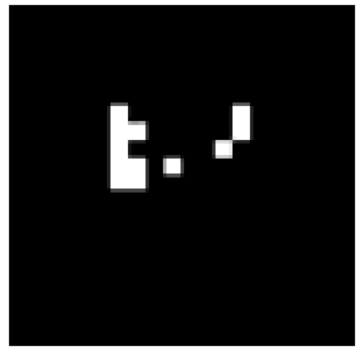
$$B = \hat{B}$$



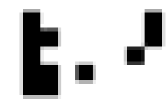
# Duality of dilation and erosion



A

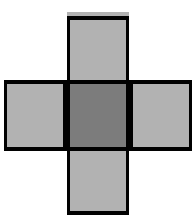


$A \ominus B$

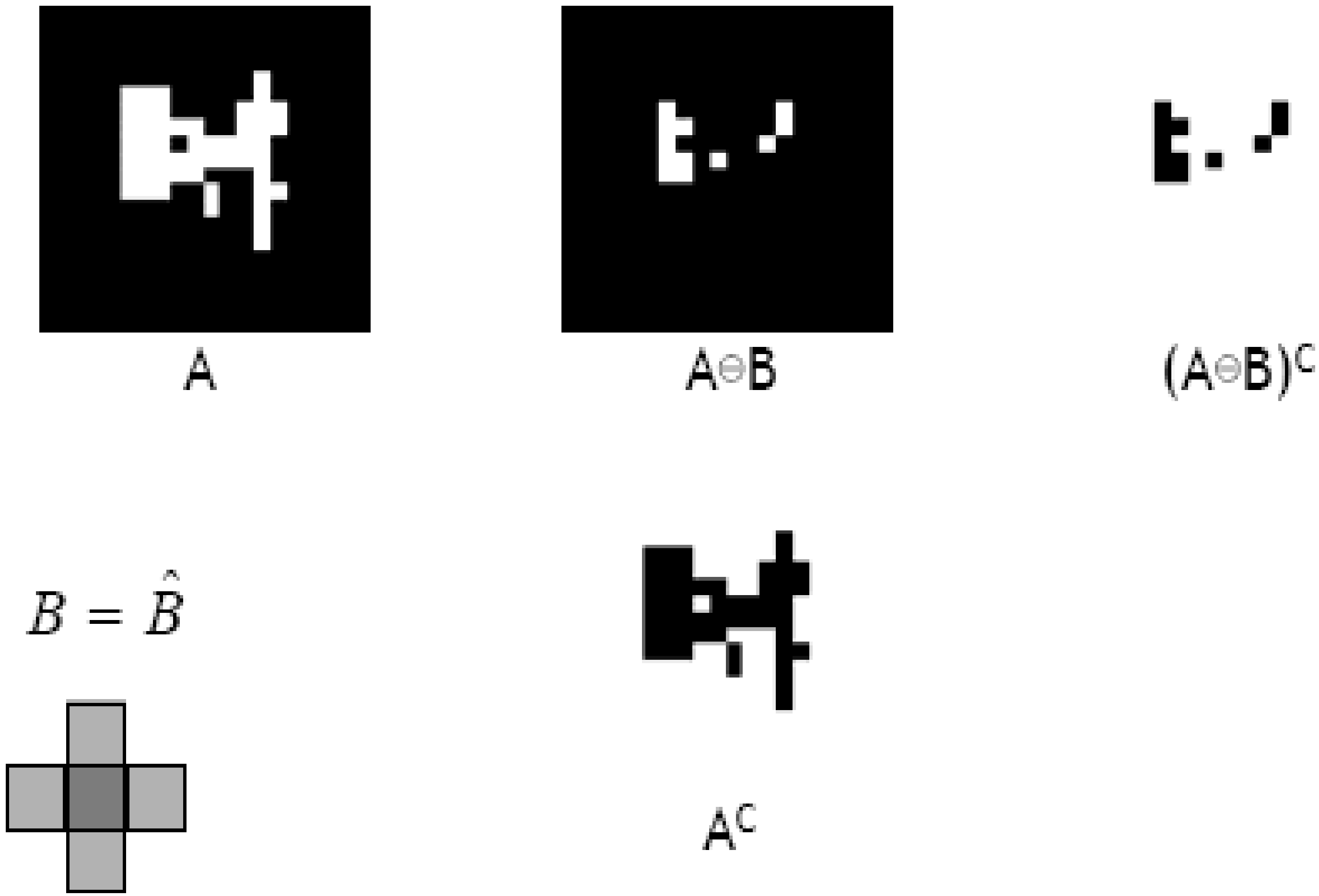


$(A \ominus B)^c$

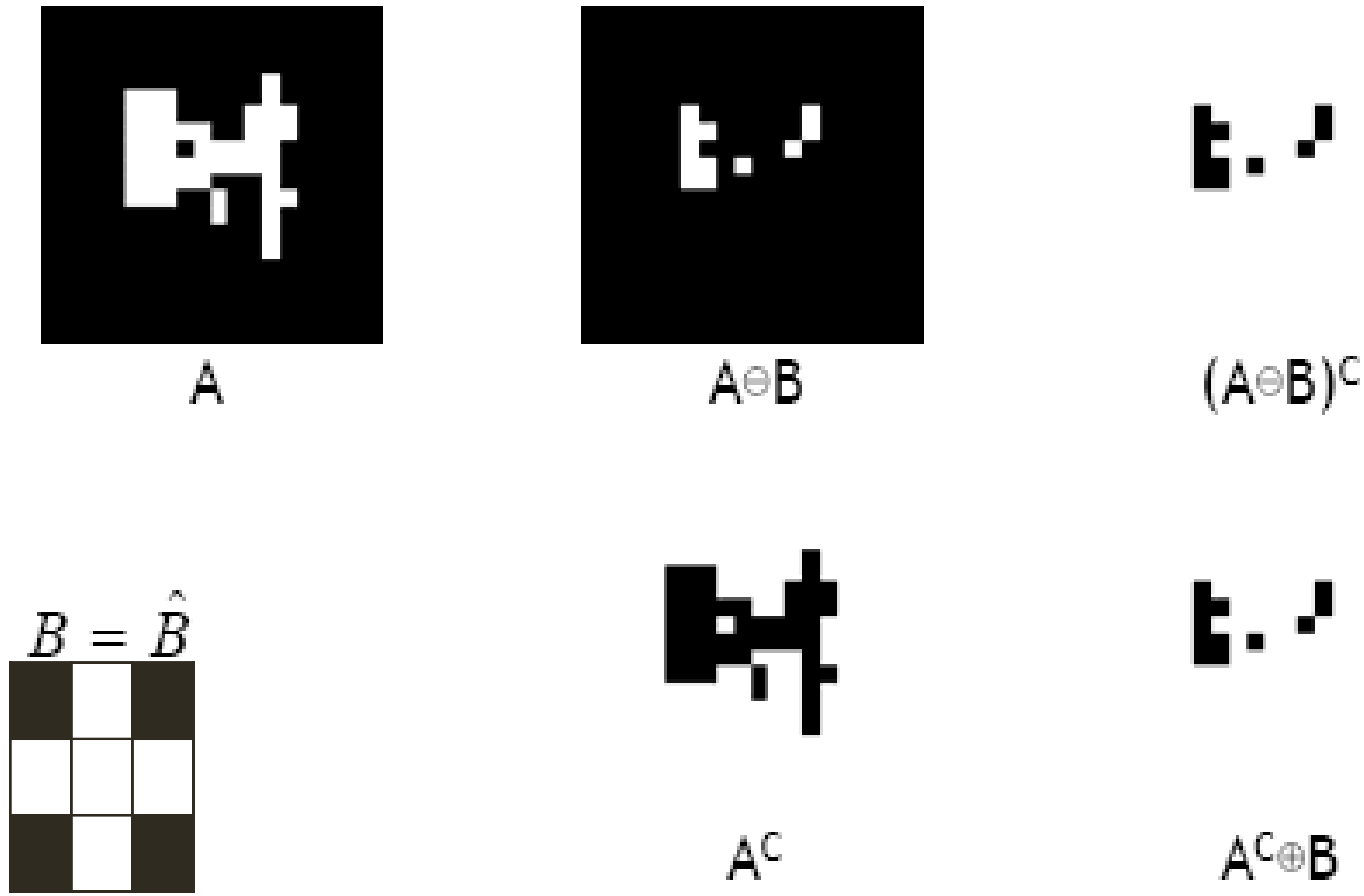
$B = \hat{B}$



# Duality of dilation and erosion



# Duality of dilation and erosion



# Opening

- erosion followed by dilation
- denoted by ∘

$$A \circ B = (A \ominus B) \oplus B$$

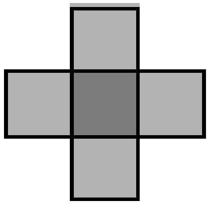
- eliminates protrusions
- breaks necks
- smoothens contour

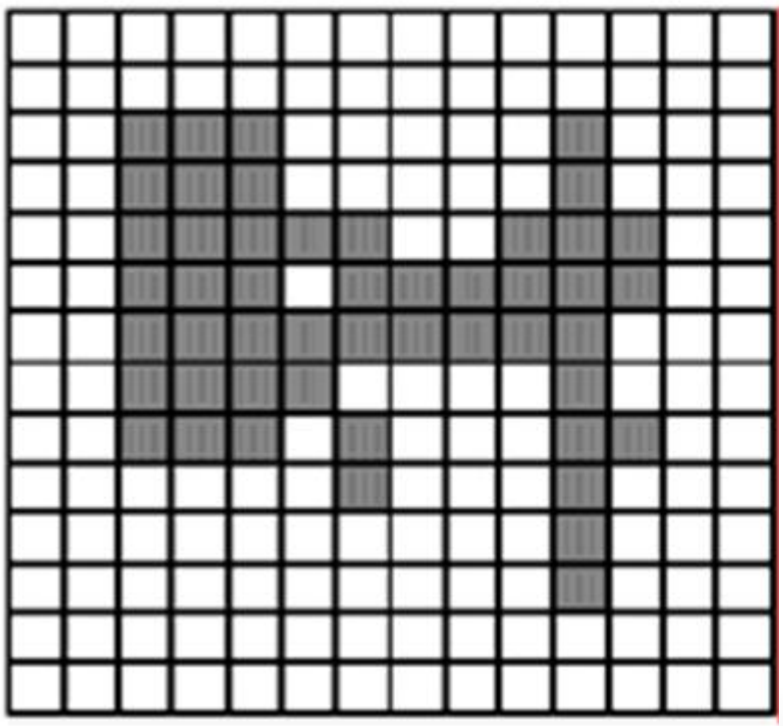
# Opening

- once an image is opened with a certain SE
- subsequent applications of the opening algorithm with the same SE will not cause any effect on the image.
- Mathematically,

$$(A \circ B) \circ B = A \circ B$$

# Erode

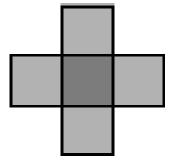
B= 

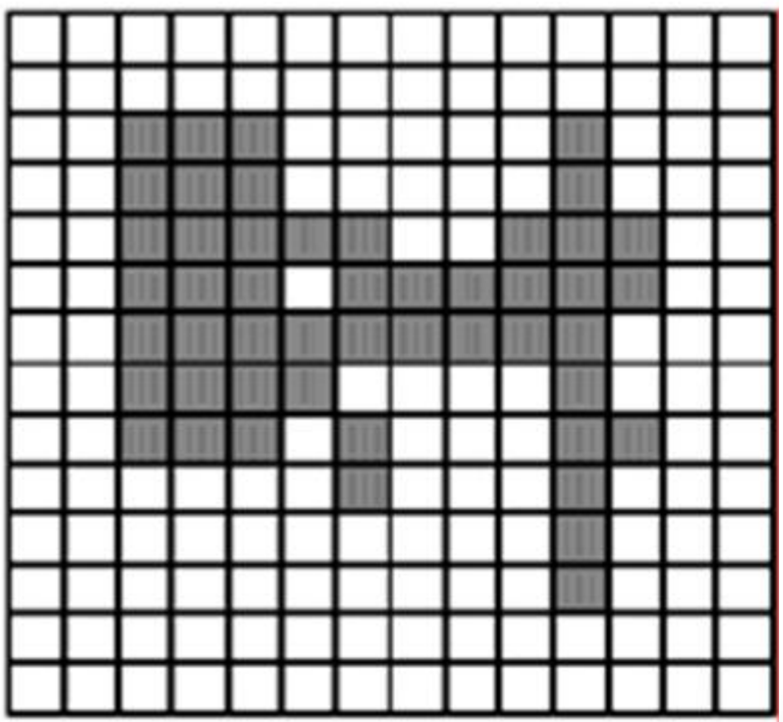


A

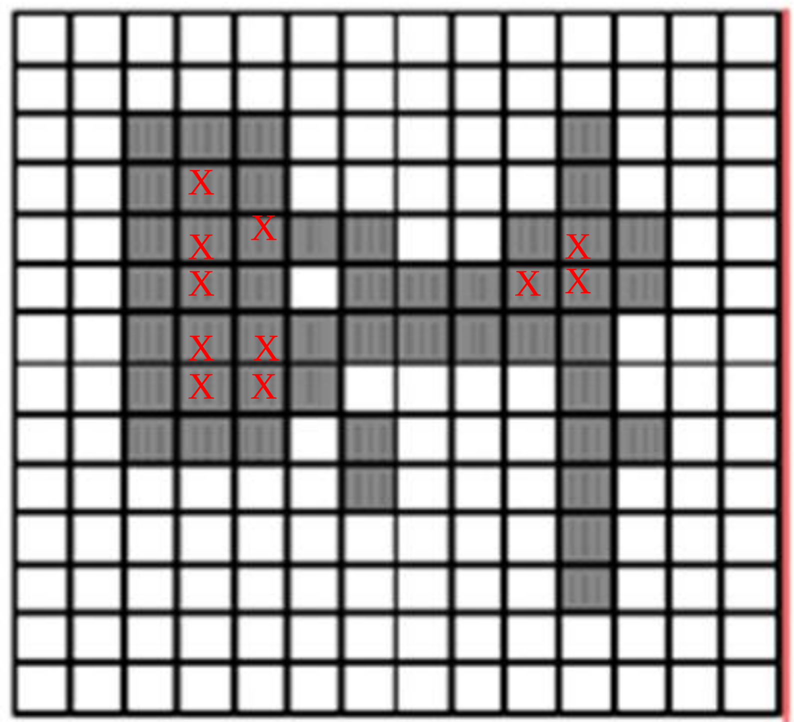
A e B

# Erode

B= 



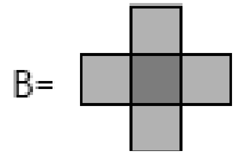
A

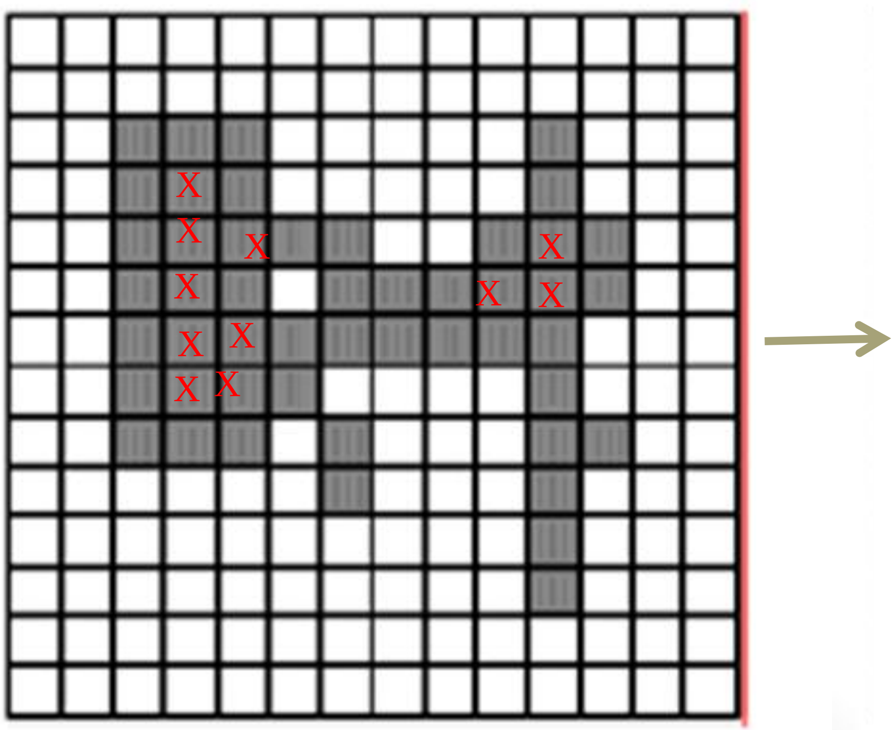


A e B

Red pixels are of eroded image

# Dilate Eroded Image



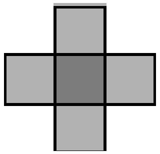




A e B

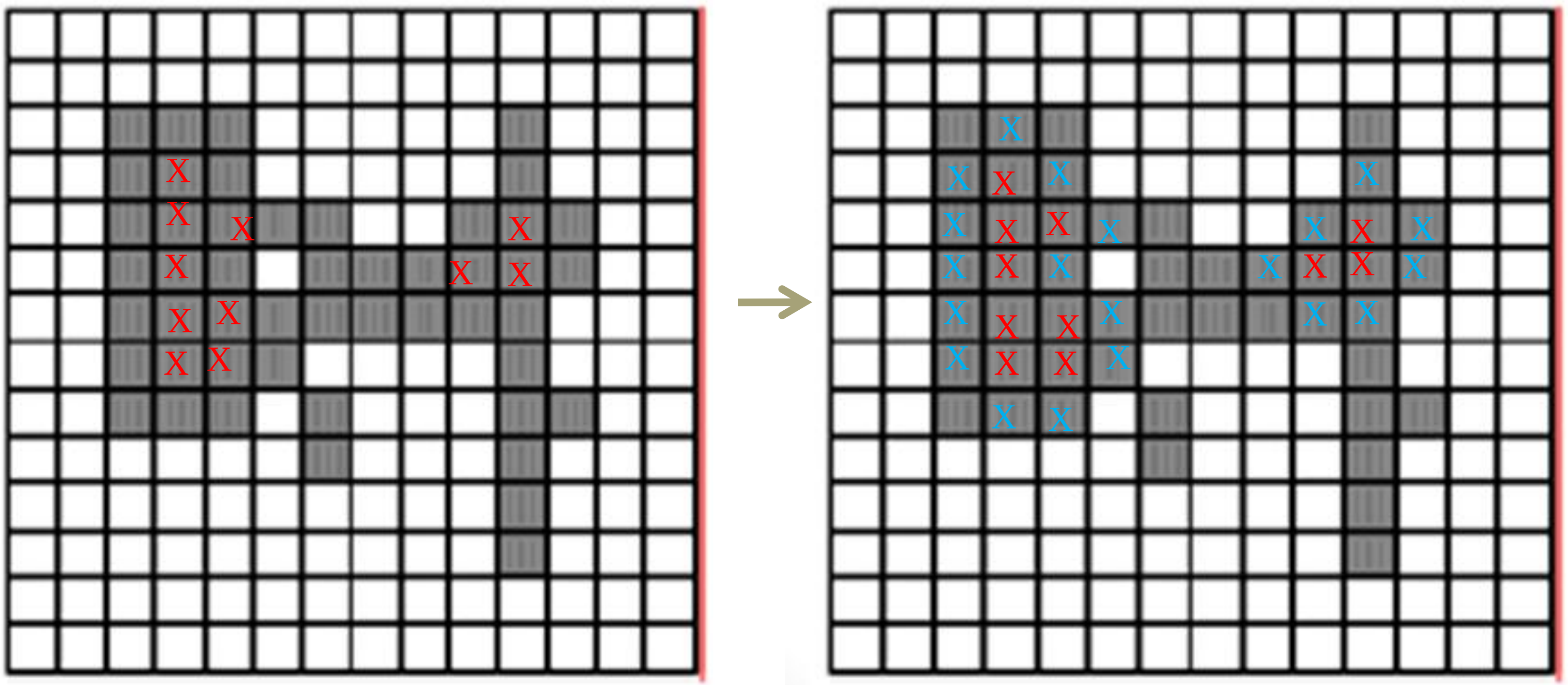
(A e B)d B

# Dilate Eroded Image

B=



A e B

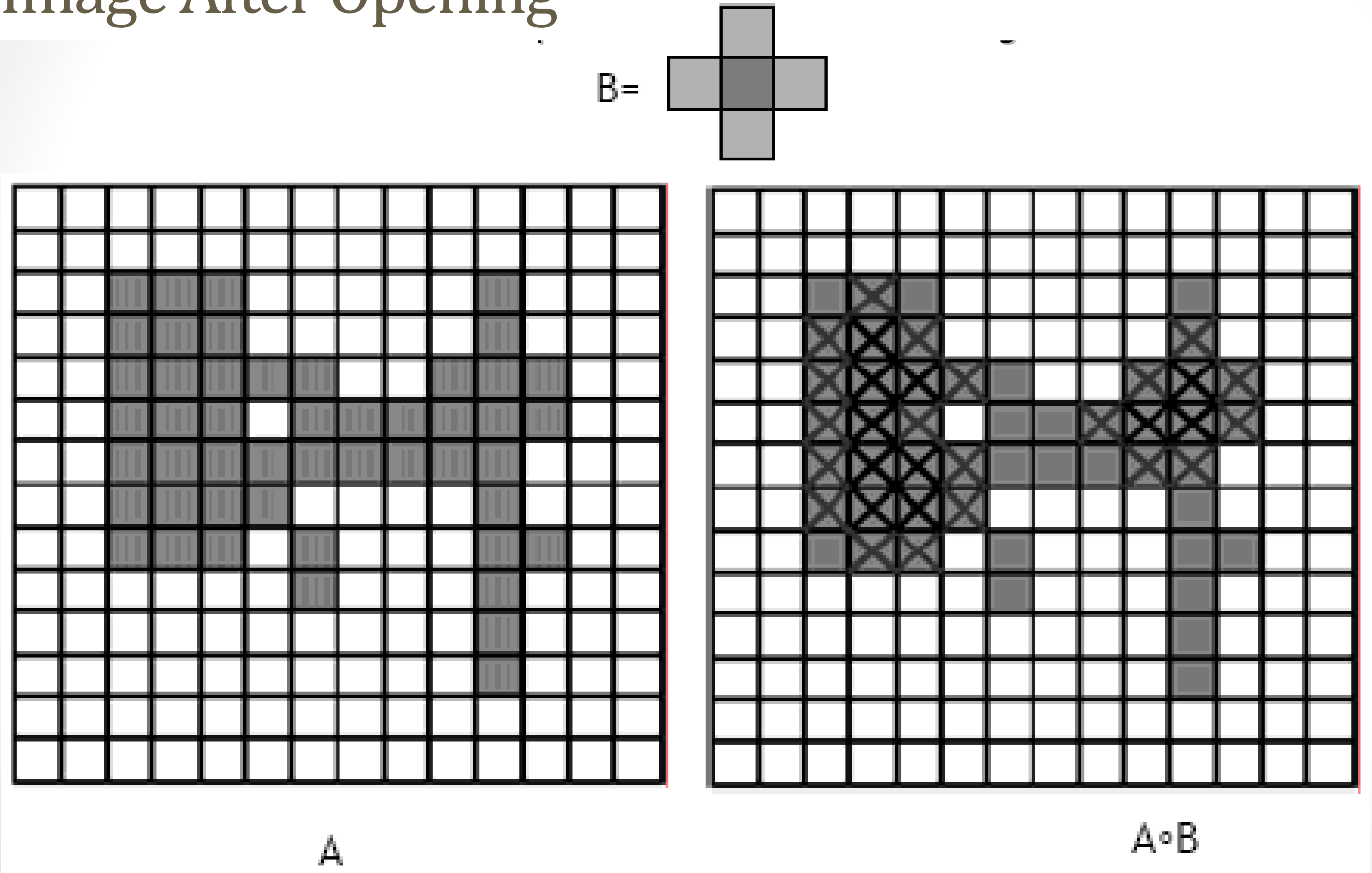


(A e B)d B

Opened image has  red and blue pixels

# Image After Opening

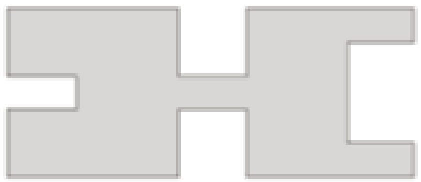
B=



# Opening

- Used to remove thin protrusions from objects
- To open up a gap between objects connected by a thin bridge without shrinking the objects
- This is because after erosion, dilation is used
- Also causes a smoothening of the object’s boundary

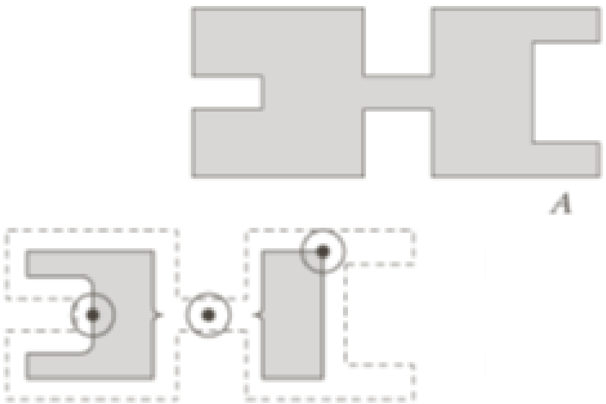
# Opening




Erosion

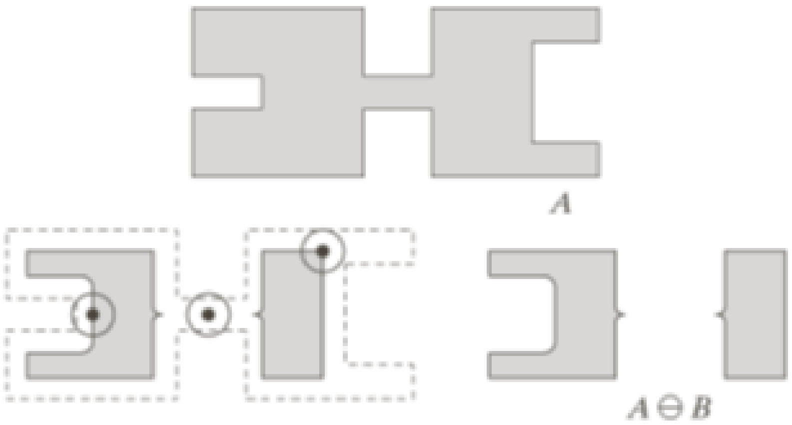
# Opening

Erosion



# Opening

Erosion



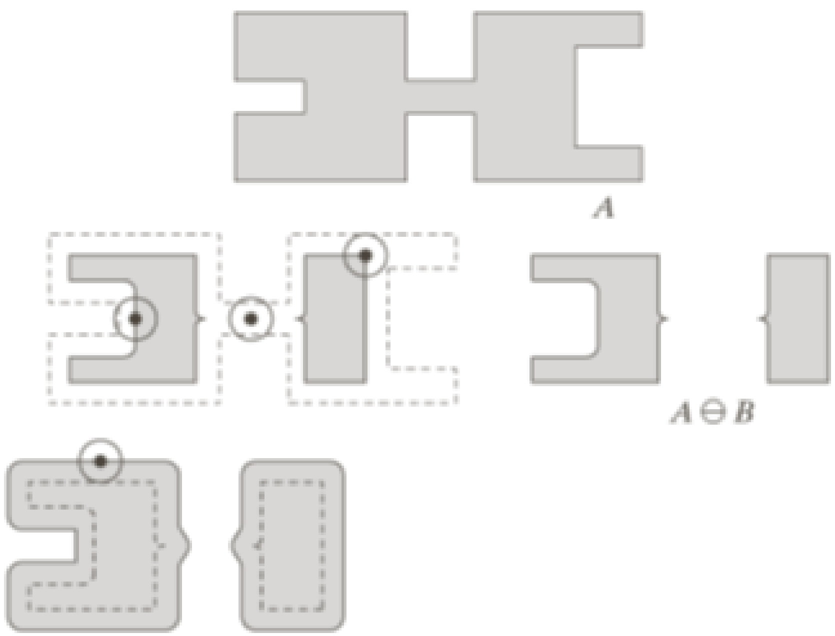

Eroded image

# Opening

Erosion

Eroded image

dilation of eroded image



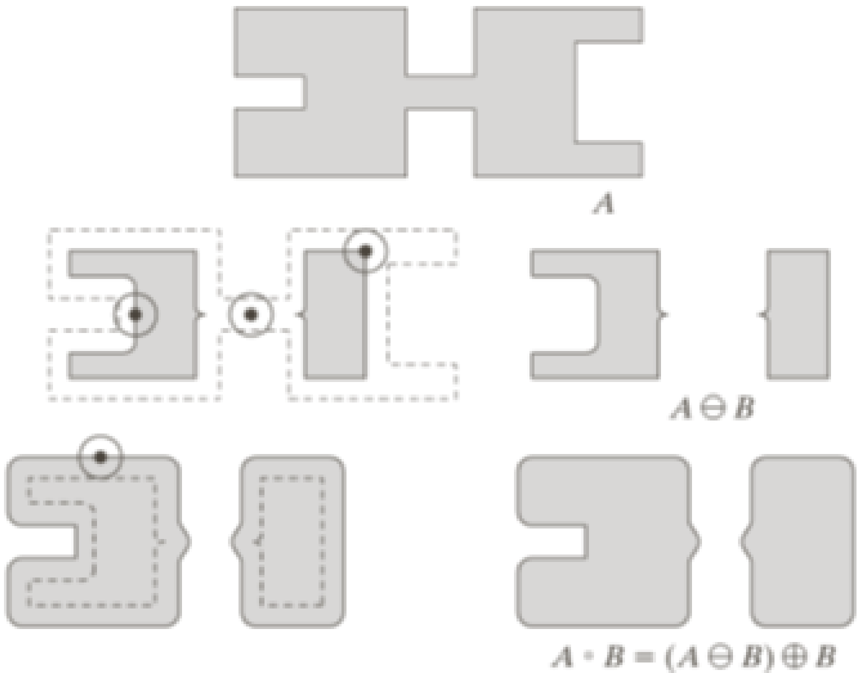
# Opening

Erosion

Eroded image

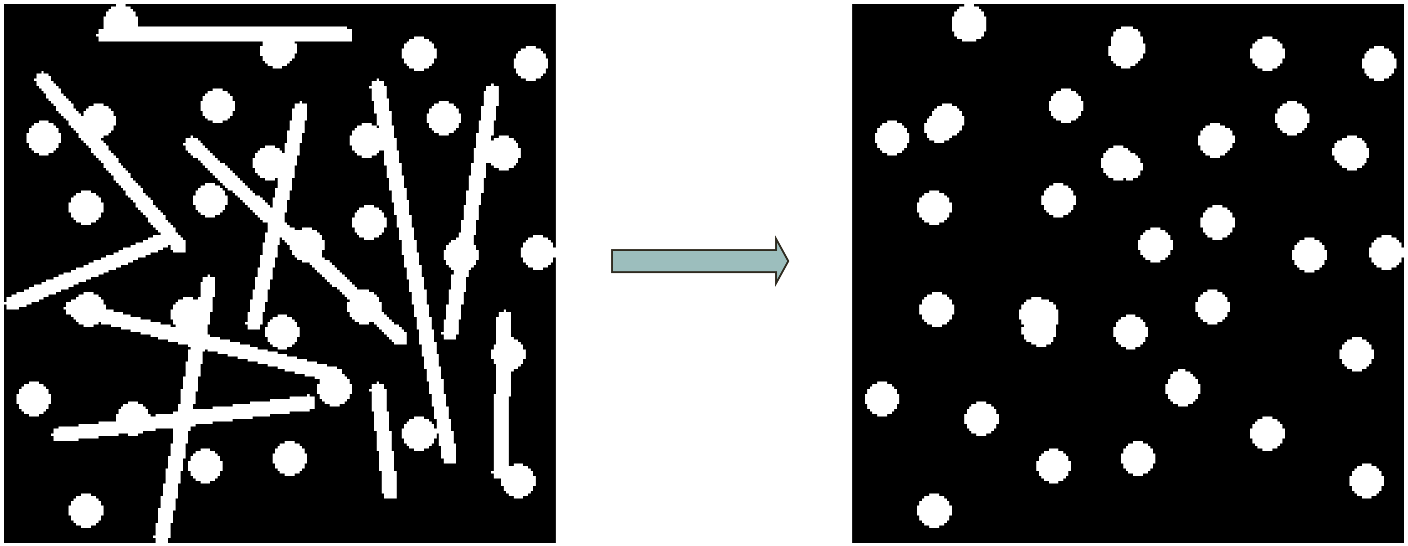
dilation of eroded image

Image after opening



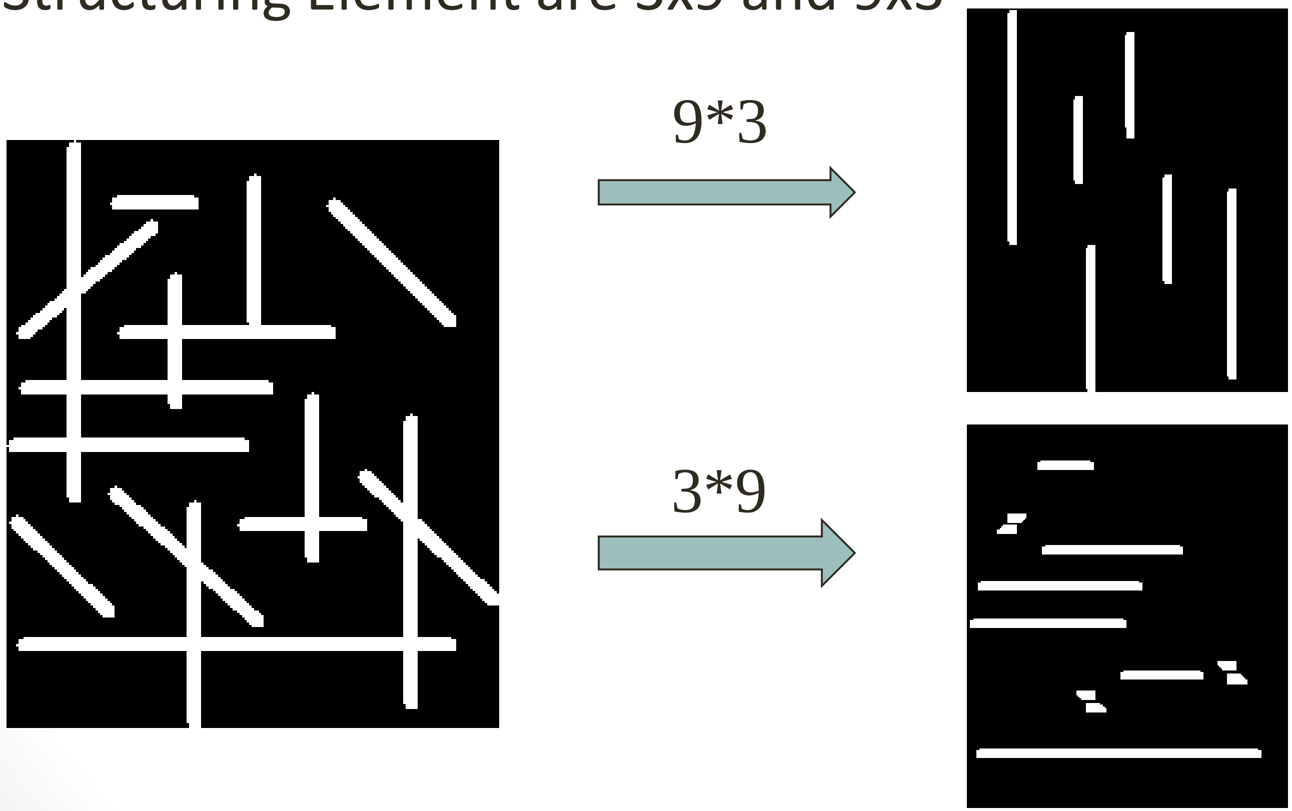
# opening

- Opening with a 11 pixel diameter disc
- Thickness of lines is less than 11 pixels



# opening

- Assume that image has vertical bars of size greater than 3x9 and 9x3
- Structuring Element are 3x9 and 9x3



# Closing

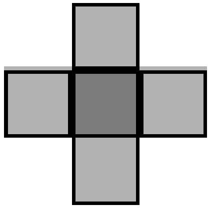
- dilation followed by erosion

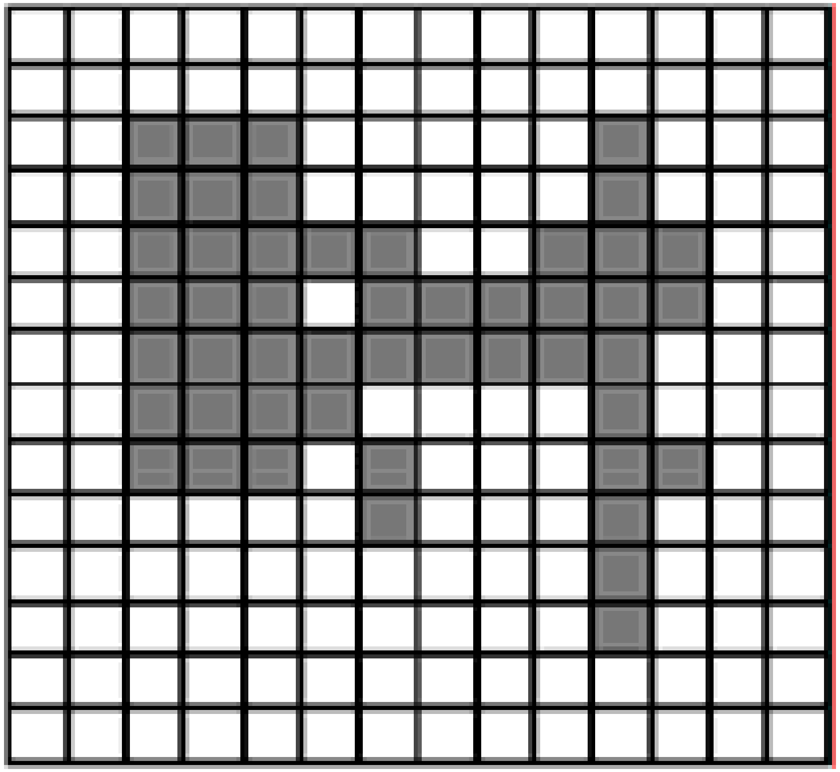
$$A \bullet B = (A \oplus B) \ominus B$$

- smooth contour
- fuse narrow breaks and long thin gulfs
- eliminate small holes
- fill gaps in the contour

# Closing

B=

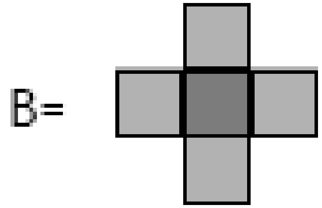




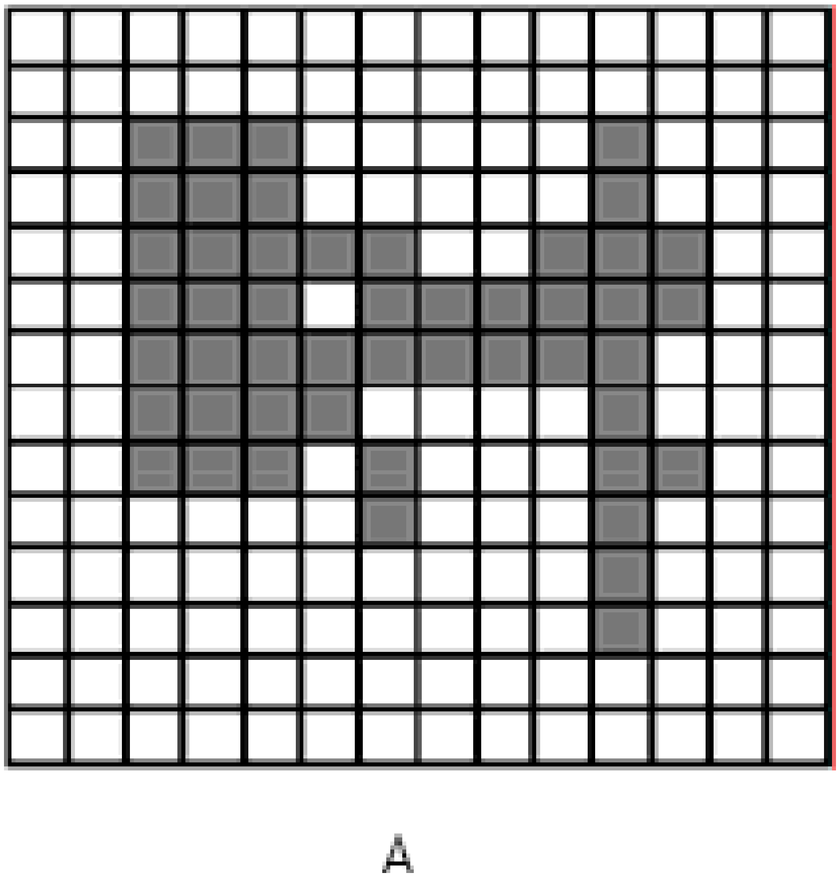
A

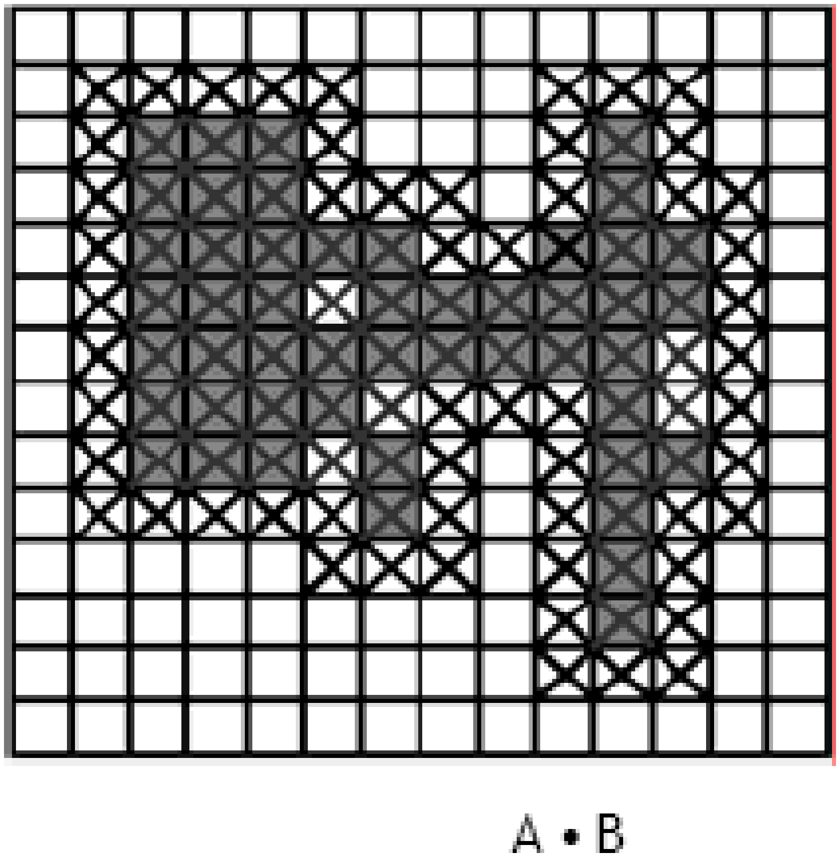
A • B

# Closing

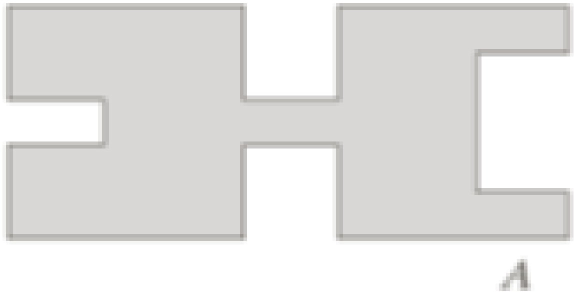



- Dilation

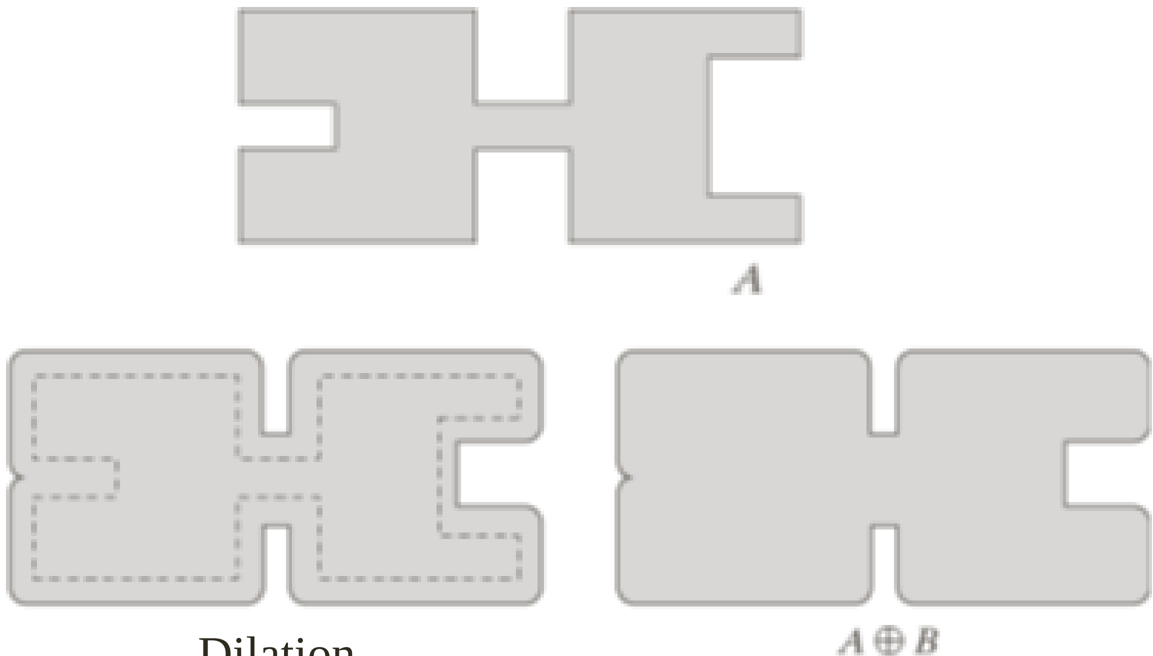


# Closing

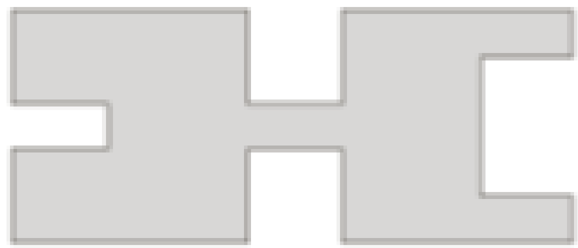


# Closing

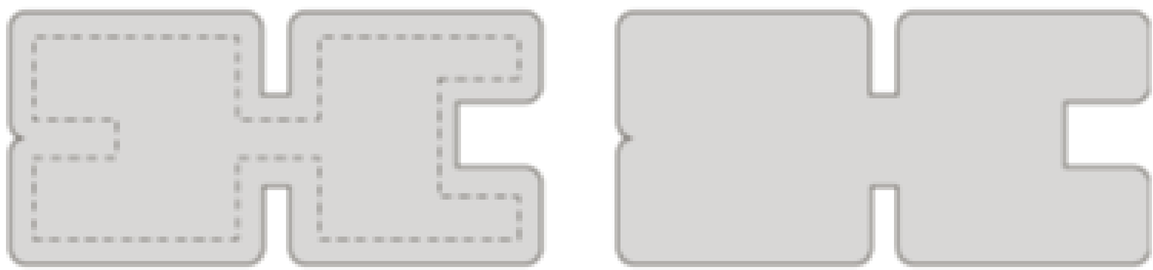



Dilation

# Closing

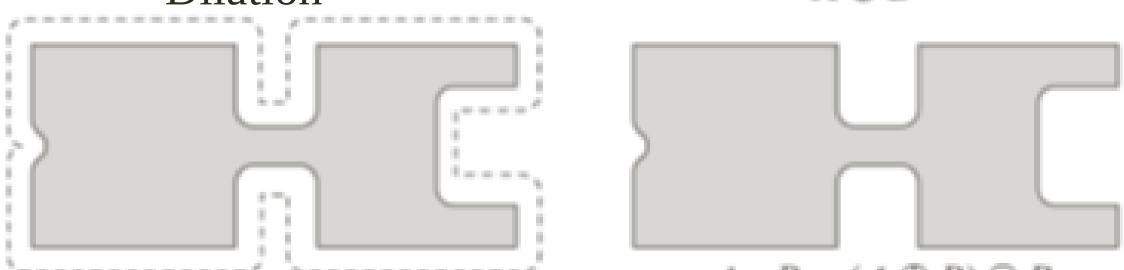


$A$



Dilation

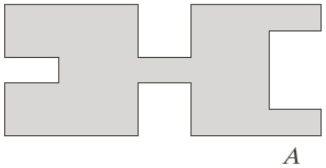
$A \oplus B$



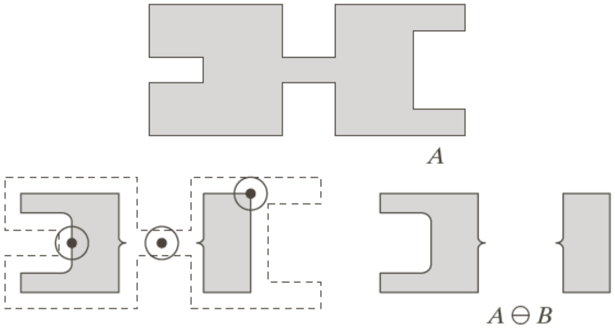
$A \cdot B = (A \oplus B) \ominus B$

Erosion of dilate image

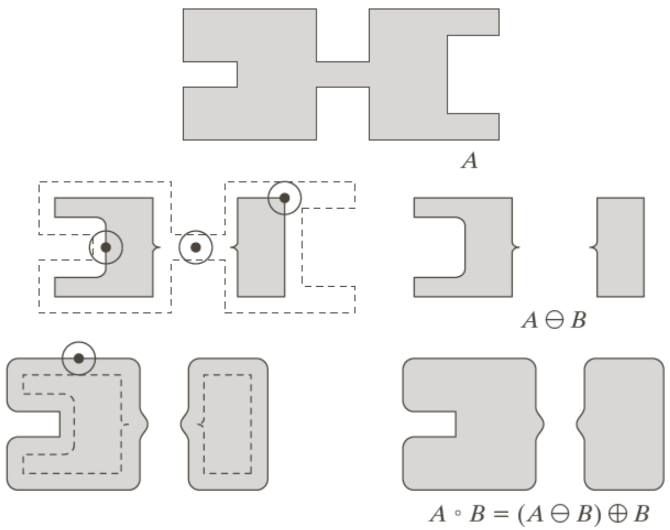
# Erosion, opening, dilation and closing



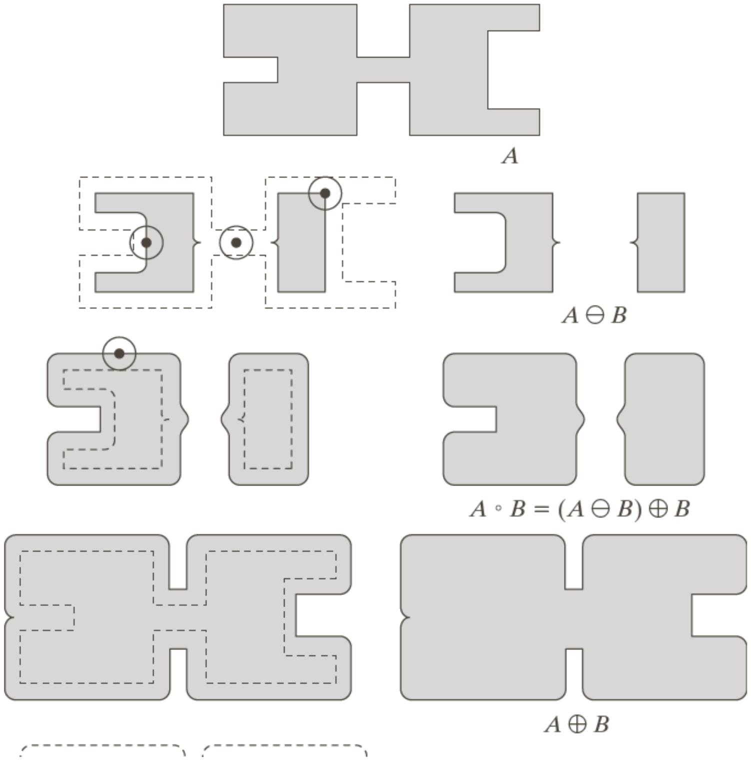
# Erosion, opening, dilation and closing



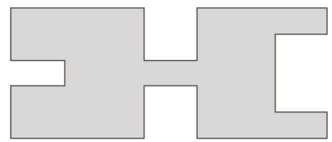
# Erosion, opening, dilation and closing



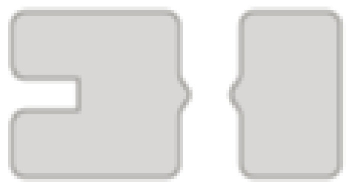
# Erosion, opening, dilation and closing



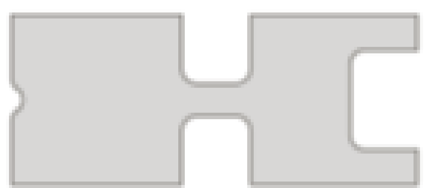
# Erosion, opening, dilation and closing



$A$



$A \circ B = (A \ominus B) \oplus B$



$A \bullet B = (A \oplus B) \ominus B$

# Erosion, opening, dilation and closing

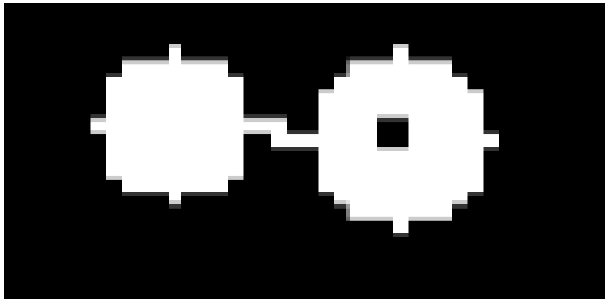
$$(A\ e\ B)^c = (A^c\ d\ \hat{B})$$

$$(A\ d\ B)^c = (A^c\ e\ \hat{B})$$

$$(A\ c\ B)^c = (A^c\ o\ \hat{B})$$

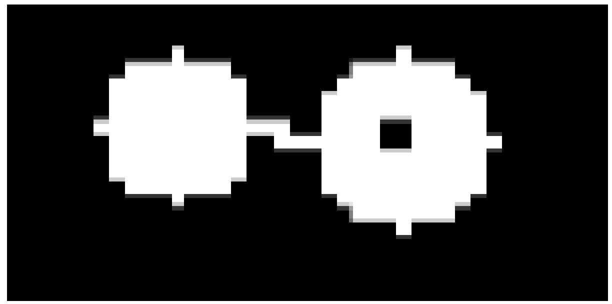
$$(A\ o\ B)^c = (A^c\ c\ \hat{B})$$

# Example: opening & closing



A

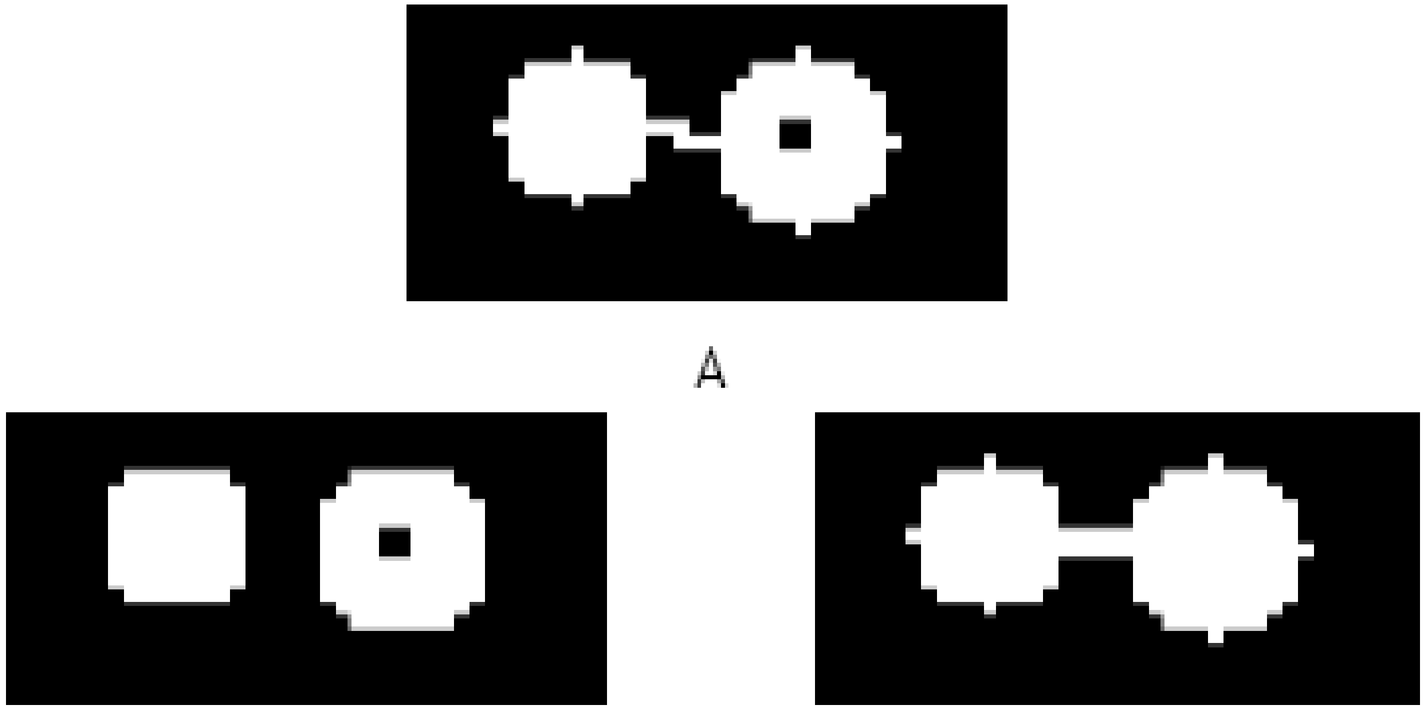
# Example: opening & closing



A

opening of A
→removal of small protrusions, thin connections, ...

# Example: opening & closing



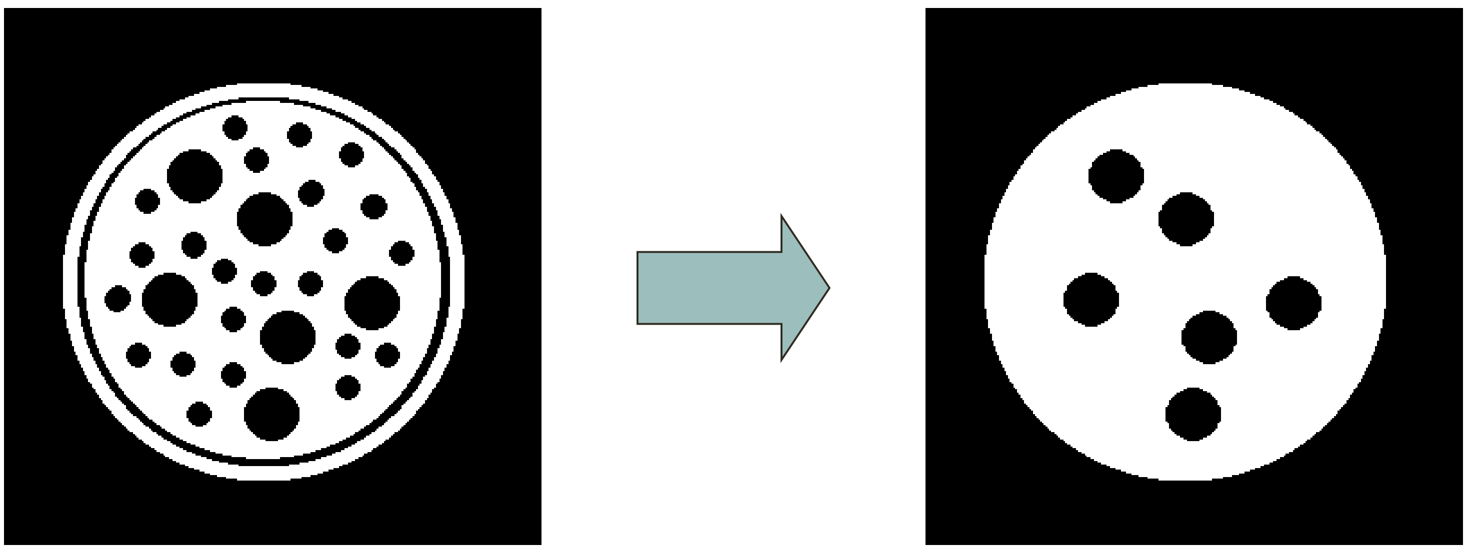
A

opening of A
→removal of small protrusions, thin connections, ...

closing of A
→ removal of holes

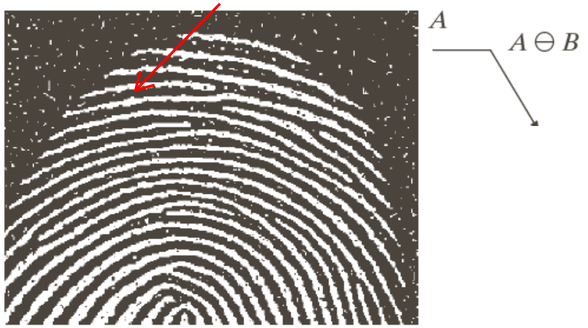
# Closing

- Closing operation with a 22 pixel disc (white)
- Closes small holes in the foreground



# Example: Opening and closing

White and black dots due to noise

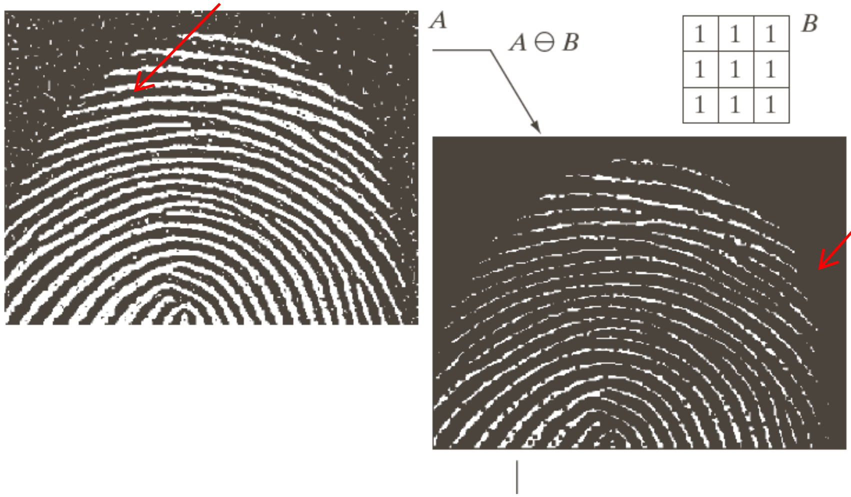



| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

B

# Example: Opening and closing

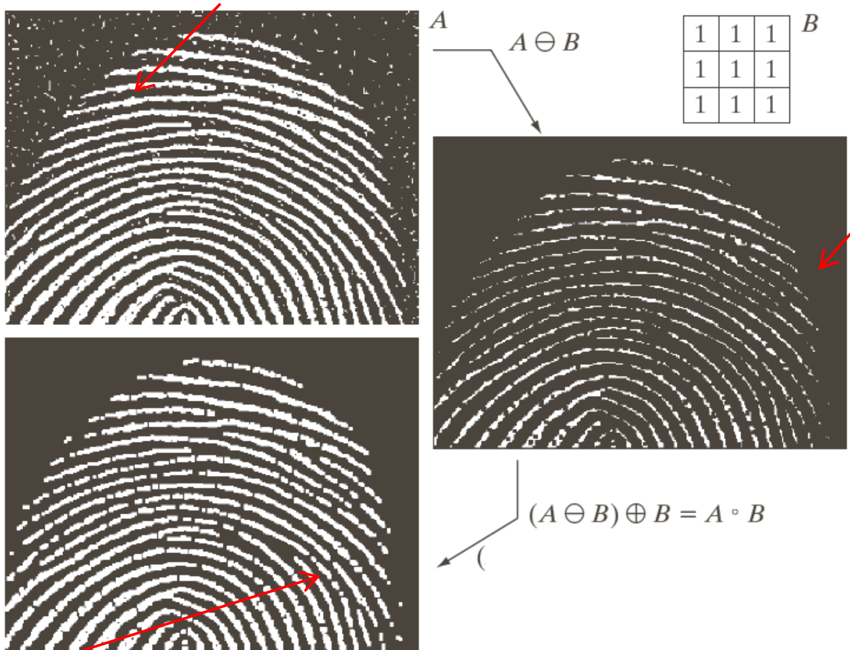
White and black dots due to noise




Background noise (white dots) removed
Black dots enhanced

# Example: Opening and closing

White and black dots due to noise

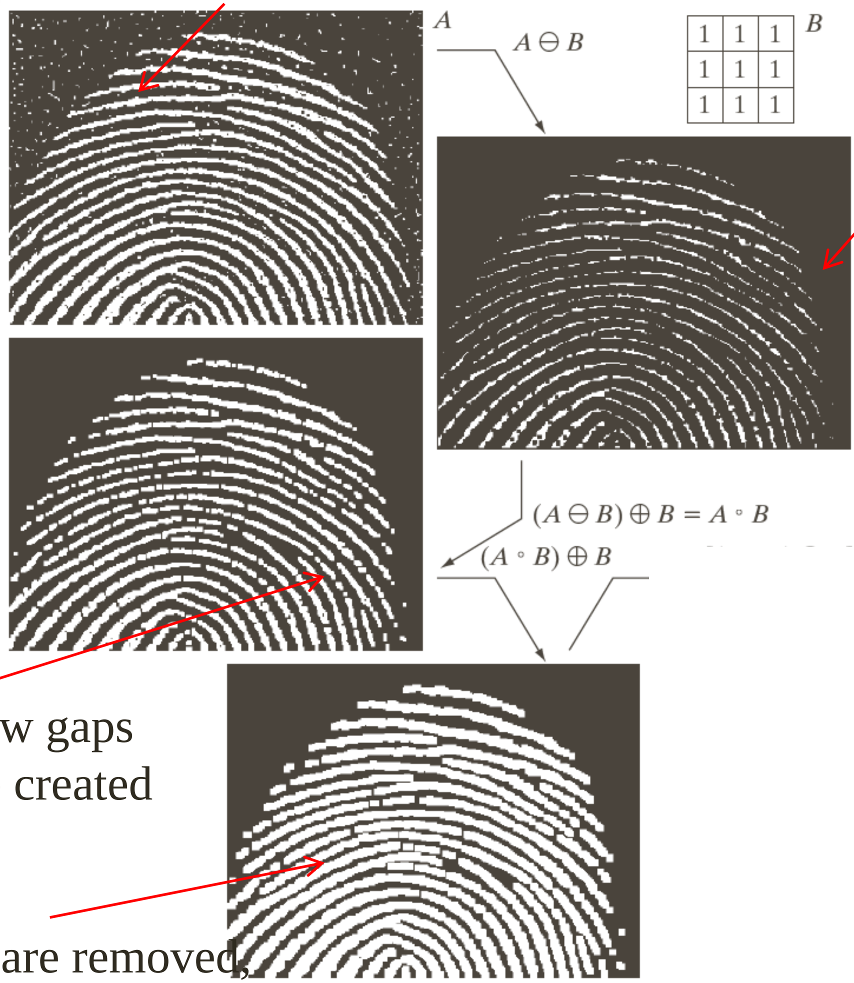



Background noise (white dots) removed
Black dots enhanced

New gaps are created

# Example: Opening and closing

White and black dots due to noise




Background noise (white dots) removed

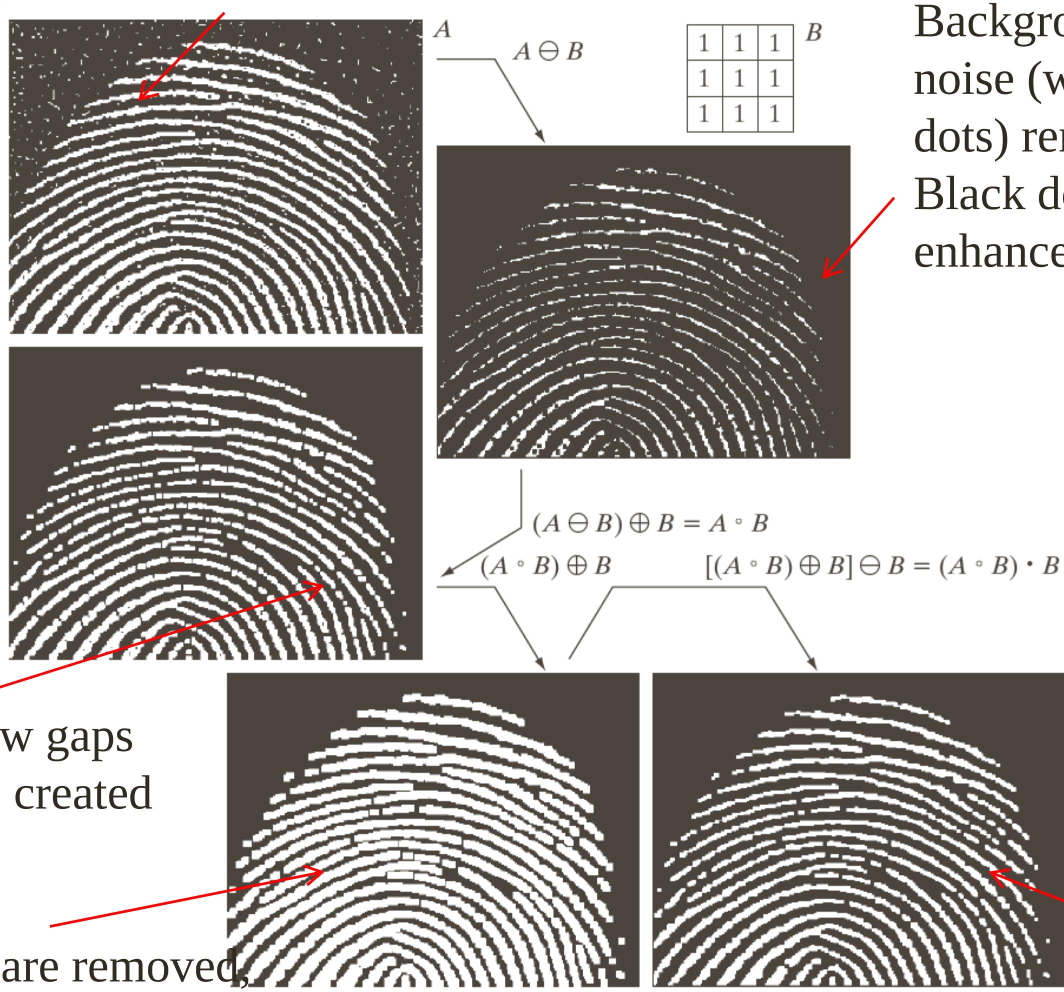
Black dots enhanced

New gaps are created

Gaps are removed, ridges thickened

# Example: Opening and closing

White and black dots due to noise




Background noise (white dots) removed

Black dots enhanced

New gaps are created

Gaps are removed, ridges thickened

Ridges thinned

# The Hit-or-Miss Transformation

- A basic morphological tool for existence of object
- Finds location of a object, **X** in a larger image, **A**

# The Hit-or-Miss Transformation

- Uses two structuring elements (B1 and B2)
- Mathematically, the HoM transform of image A by the structuring element set B (B = (B1,B2)),

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

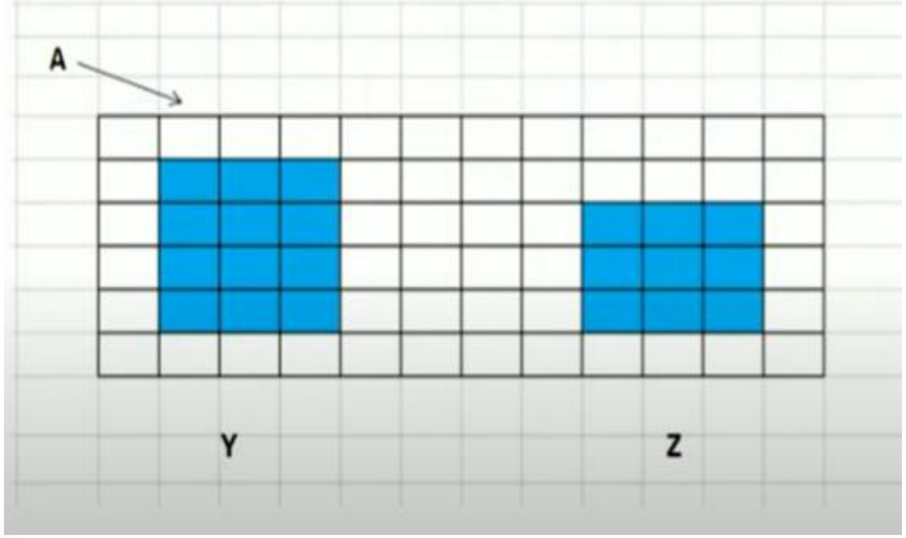
# The Hit-or-Miss Transformation

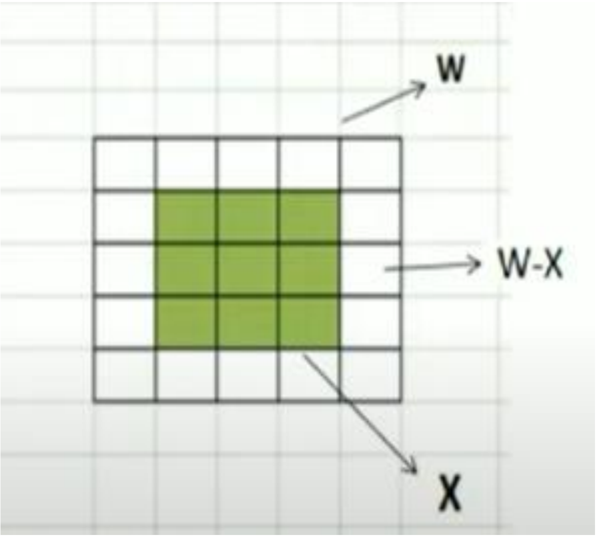
- Let X be enclosed by a small window, **W**
- Structuring Element, $B_1$= **X**
- The local background of X with respect to W is defined as $B_2$=(**W - X**)
- Apply erosion operator on **A** by $B_1$
- Apply erosion operator on the complement of **A** by the local background **(W – X)**
- Find intersection of outputs of the above two operations
- Intersection is precisely the location of object

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

# Example: Hit or Miss Transform

- Find whether object of size 3×3 exist (hit) or not (miss)
- Consider SE, X of size, 3×3
- Blue and green are representative colors

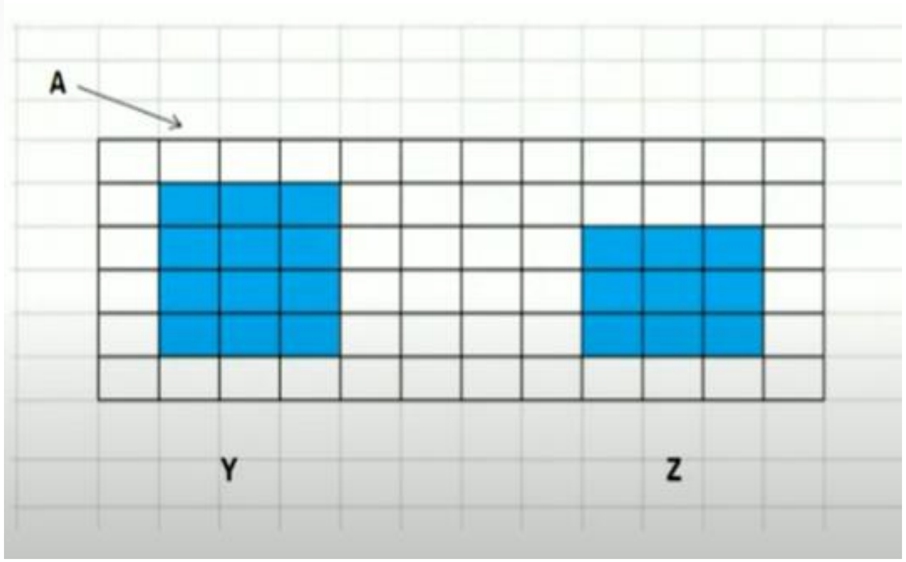


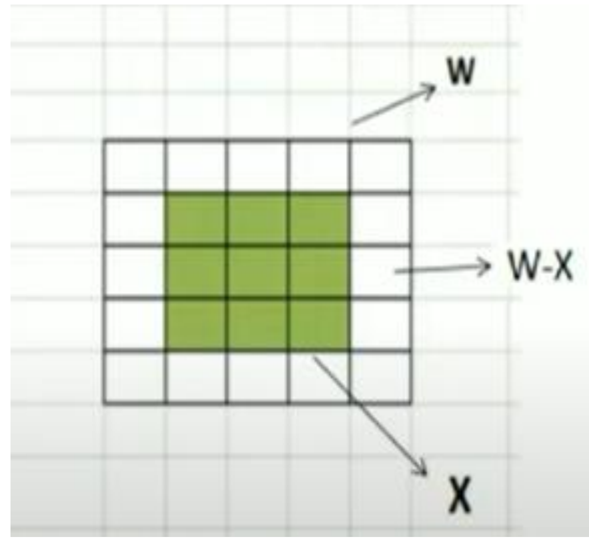




Image, A

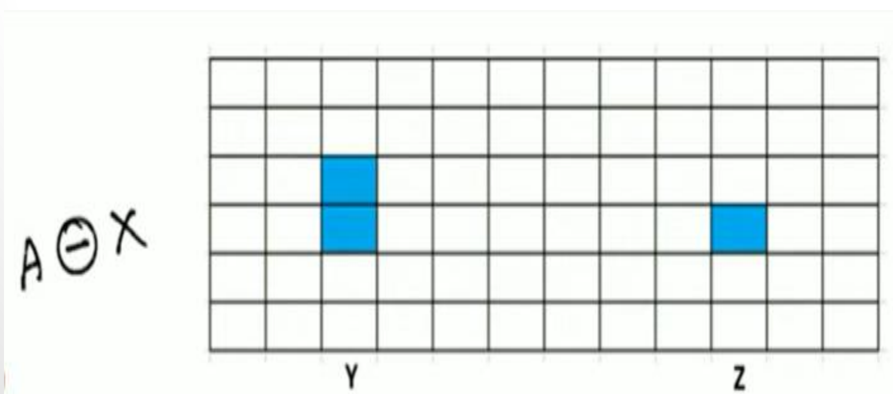
- W is window size larger than X
- Boundary of window is W-X

# Example: Hit or Miss Transform

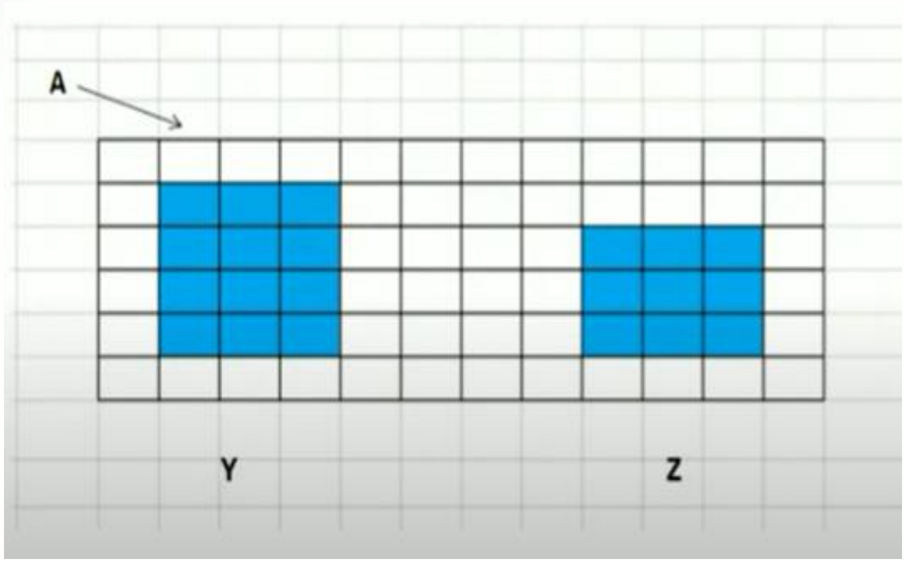


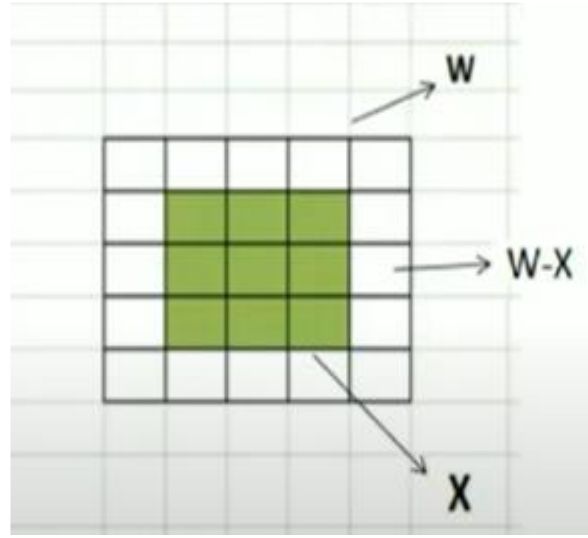



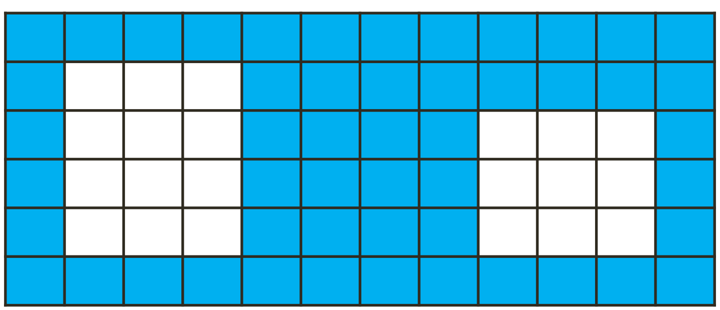

Image, A



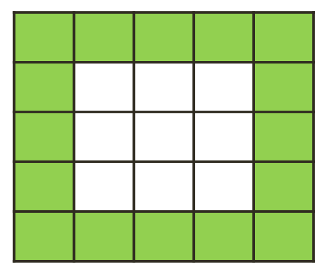
# Example: Hit or Miss Transform



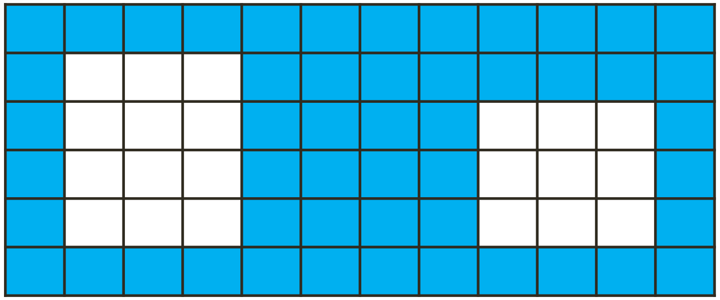



$A^c$

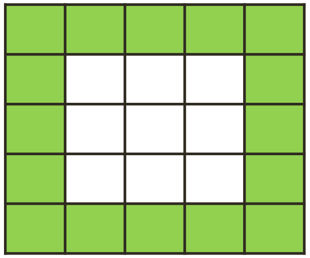


W-X

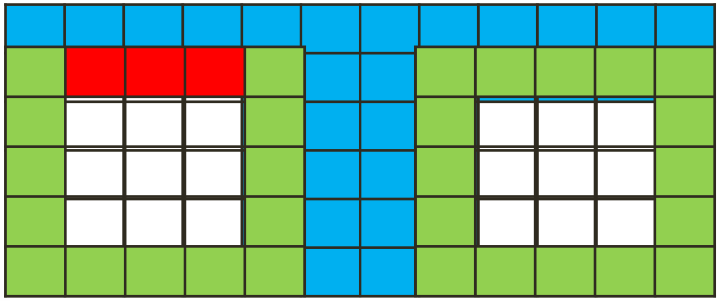
# Example: Hit or Miss Transform

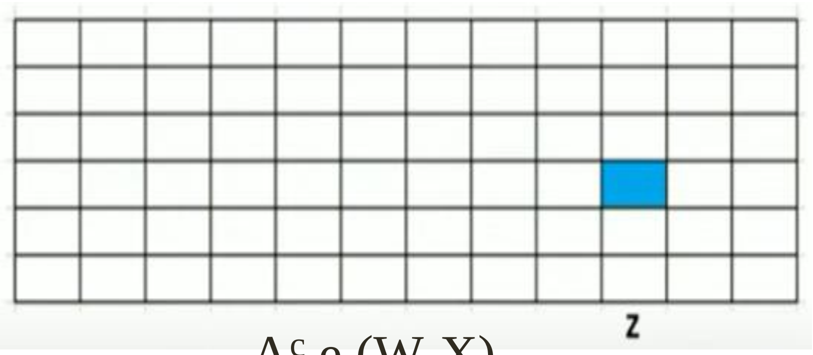


$A^c$



W-X

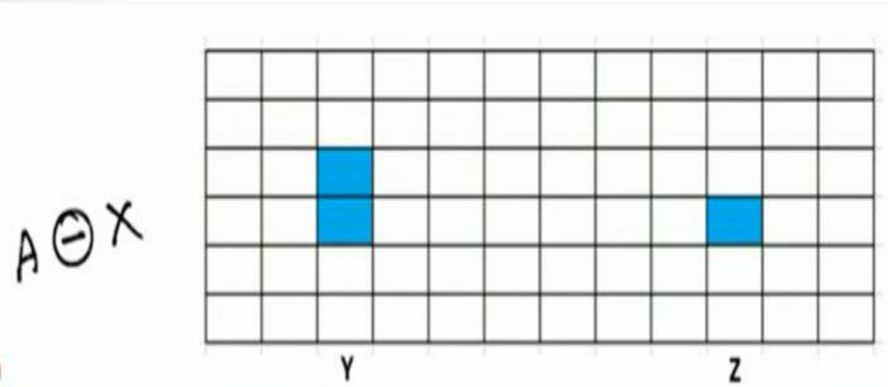


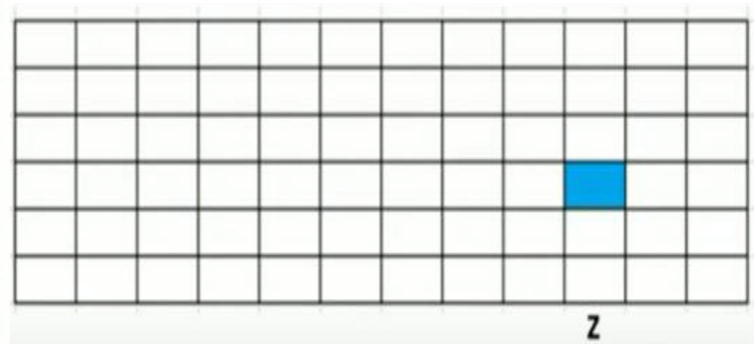



$A^c$ e (W-X)

For first region, red cells of $A^c$ and W-X do not match, it is a miss
For second Region, $A^c$ and W-X match, it is a hit

# Example: Hit or Miss Transform

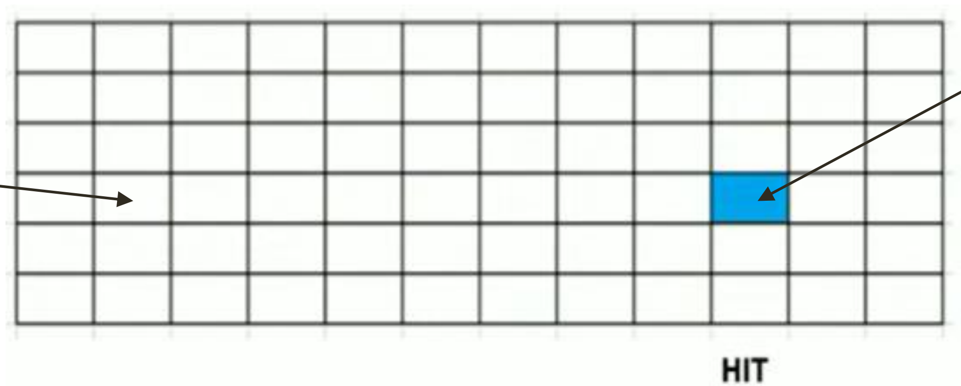



$A^c$ e (W-X)

At this location object, X exists

Miss




A e X ∩ $A^c$ e (W-X)

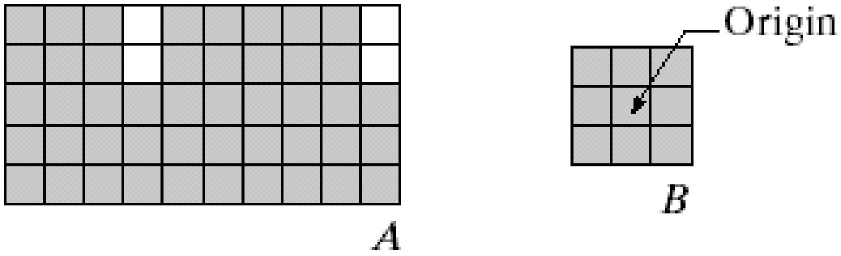
# Boundary extraction

$$\beta(A) = A - (A \ominus B)$$

# Boundary extraction

$$\beta(A) = A - (A \ominus B)$$

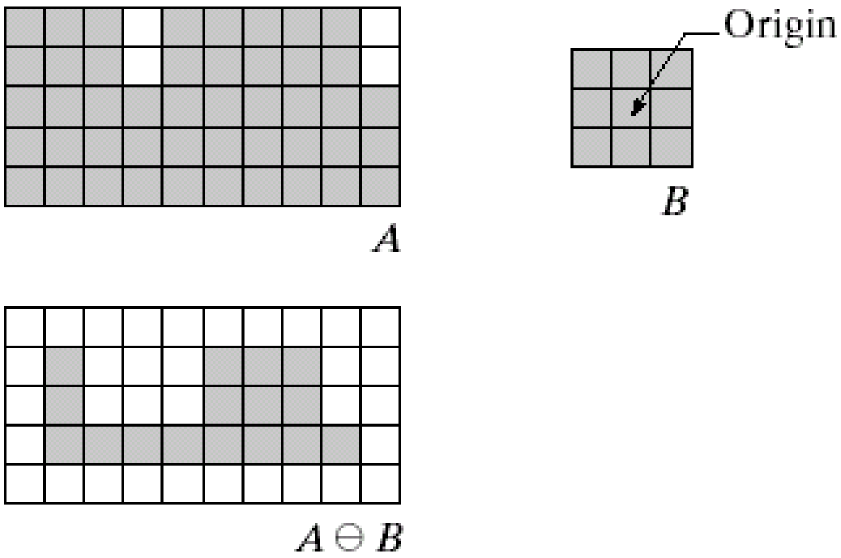
Foreground pixel(dark) is 1 and background (white) is 0



# Boundary extraction

$$\beta(A) = A - (A \ominus B)$$

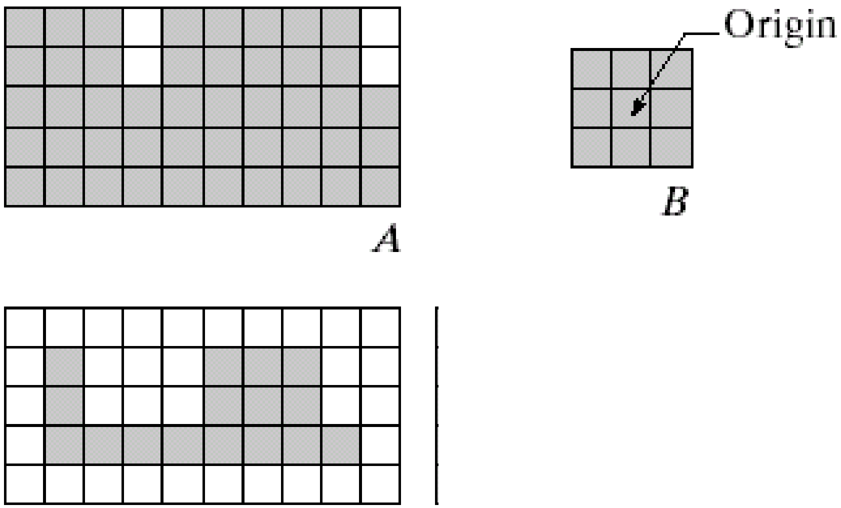
Foreground pixel(dark) is 1 and background (white) is 0



# Boundary extraction

$$\beta(A) = A - (A \ominus B)$$

Foreground pixel(dark) is 1 and background (white) is 0



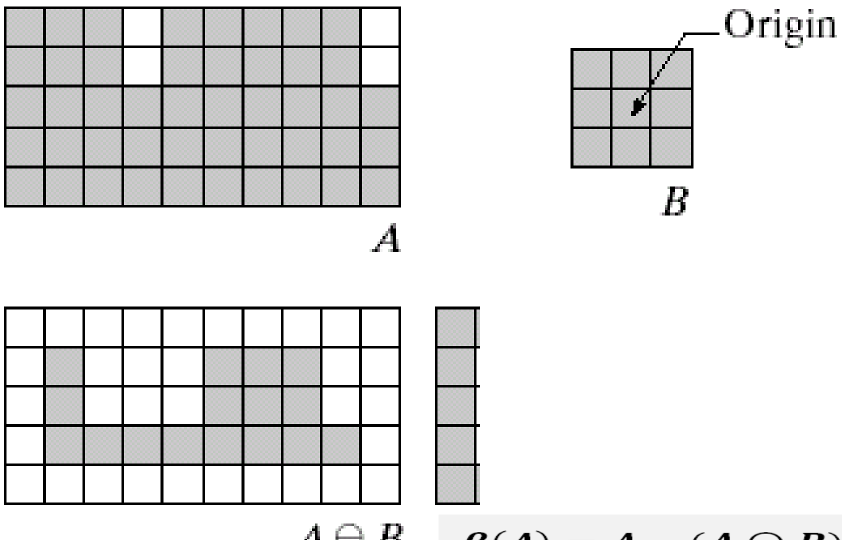



$$\beta(A) = A - (A \ominus B)$$

# Boundary extraction

$$\beta(A) = A - (A \ominus B)$$

Foreground pixel(dark) is 1 and background (white) is 0

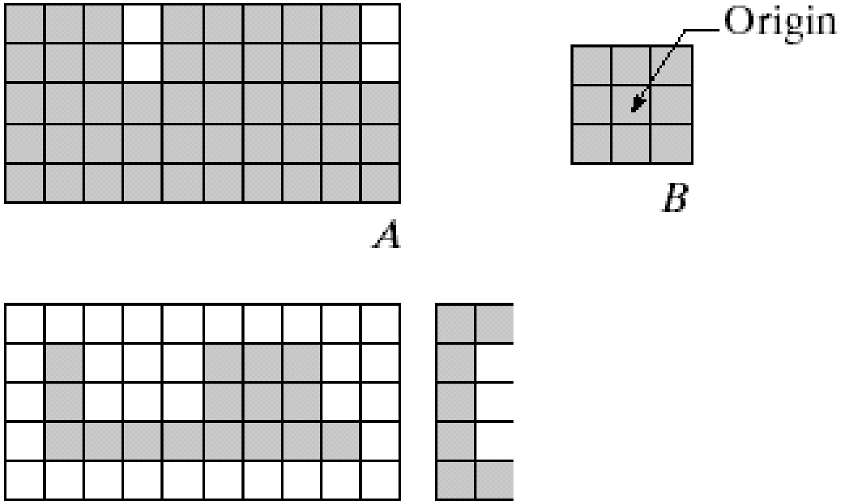



$$\beta(A) = A - (A \ominus B)$$

# Boundary extraction

$$\boldsymbol{\beta}(\boldsymbol{A}) = \boldsymbol{A} - (\boldsymbol{A} \ominus \boldsymbol{B})$$

Foreground pixel(dark) is 1 and background (white) is 0

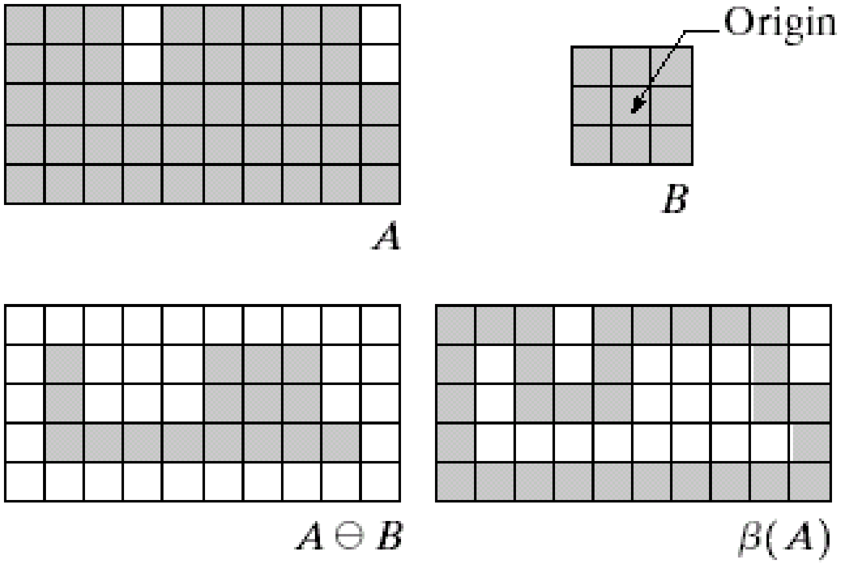



$$\boldsymbol{\beta}(\boldsymbol{A}) = \boldsymbol{A} - (\boldsymbol{A} \ominus \boldsymbol{B})$$

# Boundary extraction

$$\beta(A) = A - (A \ominus B)$$

Foreground pixel(dark) is 1 and background (white) is 0



# Boundary extraction

Foreground pixel is 1 (white)
background pixel is 0 (black)

Original image

3x3 structuring element is used

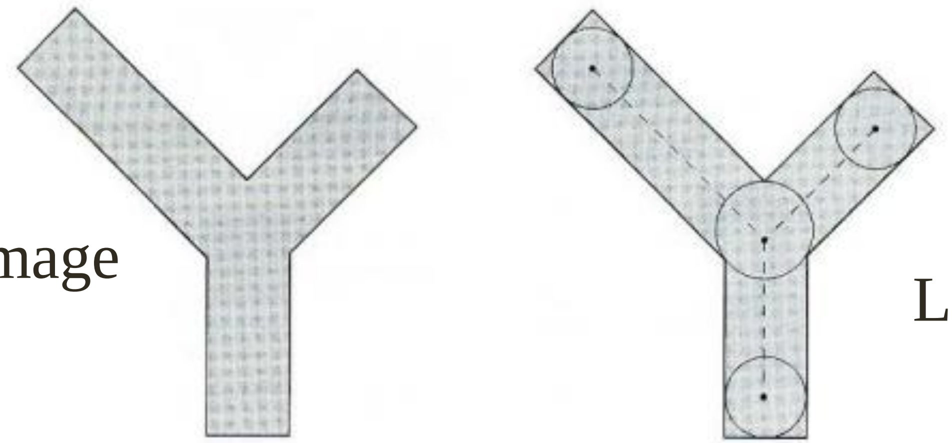
# Boundary extraction

Foreground pixel is 1 (white)
background pixel is 0 (black)

Original image    Boundary in image

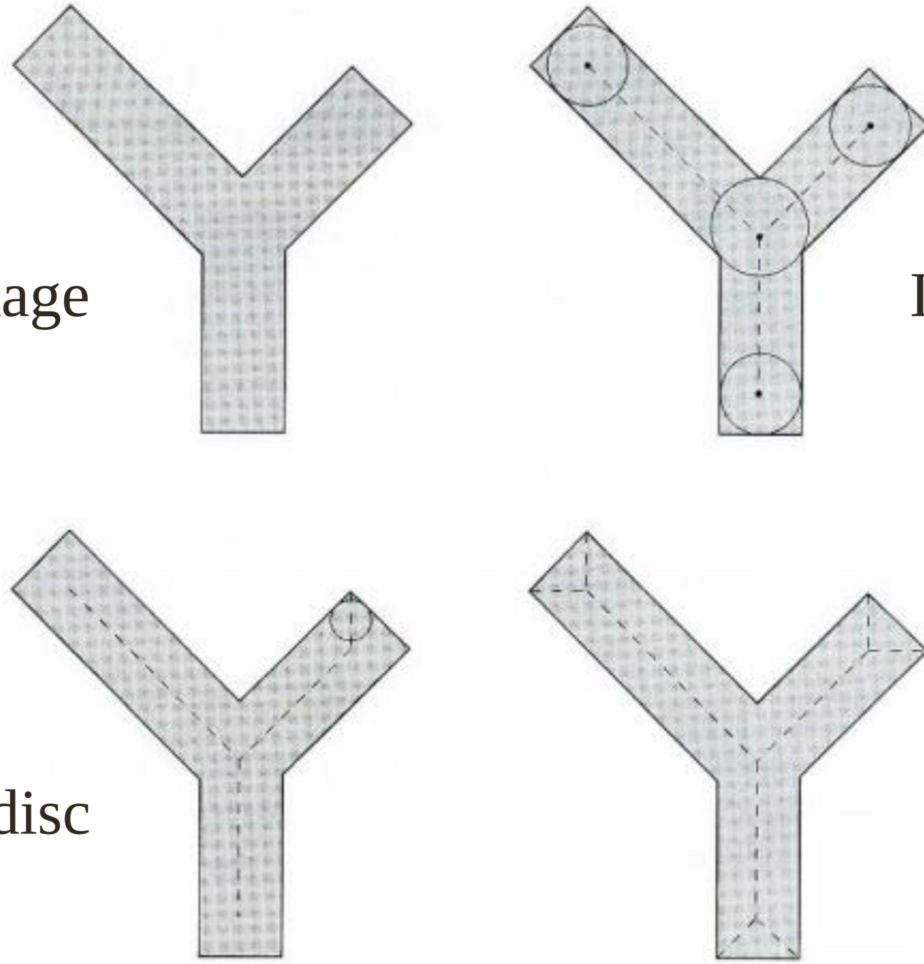
3x3 structuring element is used

# Skeleton (intuition)




- Largest disk centered in the image and is contained in A
- Dotted line is skeleton of image

# Skeleton



Original image

Large disc

Smaller disc

Skeleton

# Skeleton

- The skeleton of A is

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

- Where B is structuring element
- And $S_k(A)$ is skeleton subset
- And kB indicates k successive erosions of A

$$(A \ominus kB) = (\ldots((A \ominus B) \ominus B) \ominus \ldots) \ominus B$$

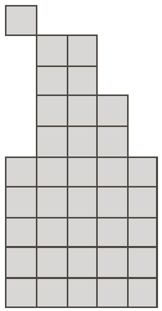
# Skeleton

- K is the last iterative step before A erodes to an empty set

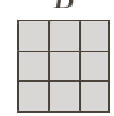
$$K = \max\{k \mid (A \ominus kB) \neq \emptyset\}$$

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

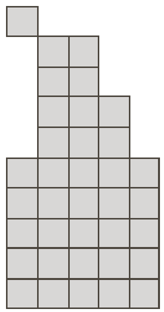
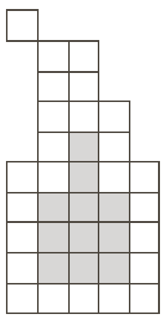
- S(A) can be obtained as the union of skeleton subsets $S_k(A)$

| $k$ | $A \ominus kB$ |
| --- | --- |
| 0 |  |

$B$

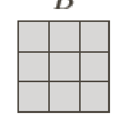


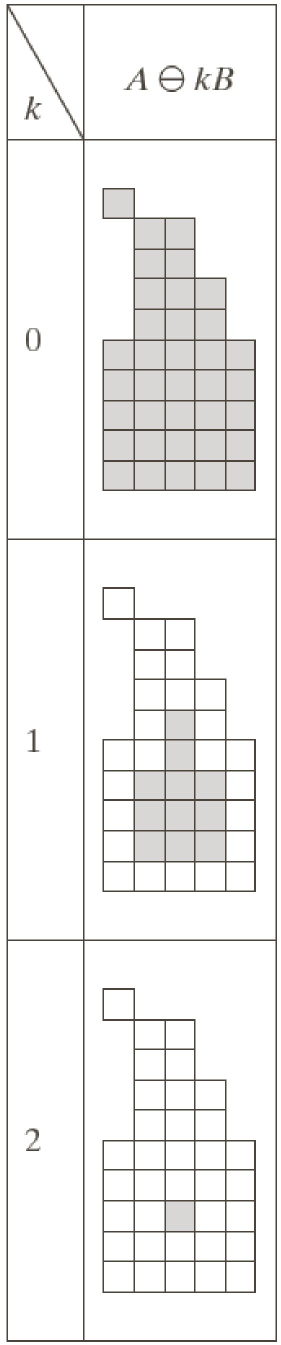
$$S_k(A) = \boxed{(A \ominus kB)} - (A \ominus kB) \circ B$$

| $k$ | $A \ominus kB$ |
|---|---|
| 0 |  |
| 1 |  |

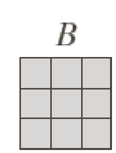
$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

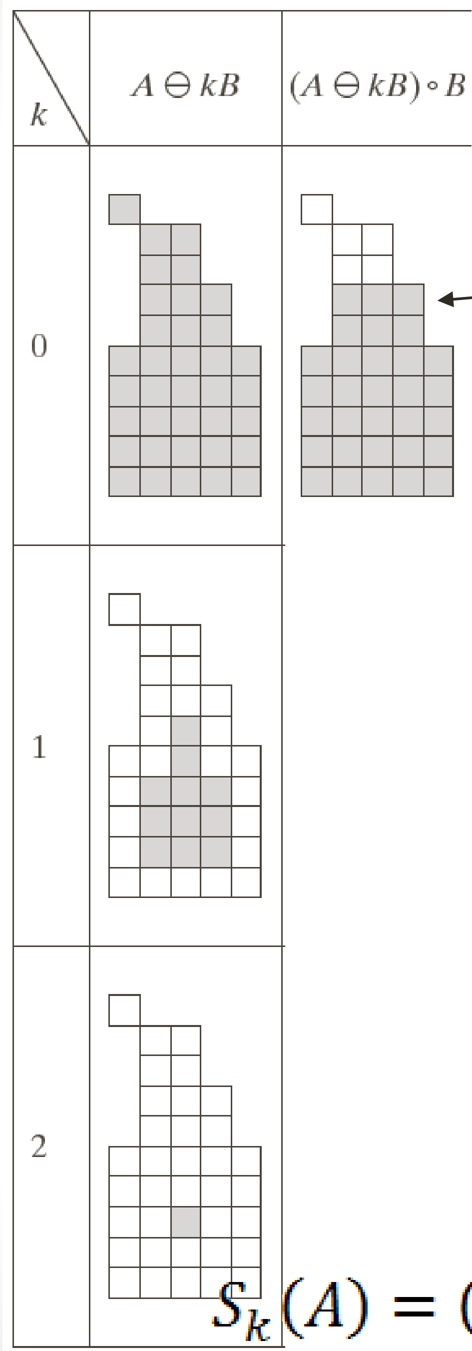
$B$





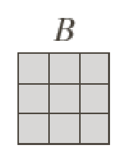

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

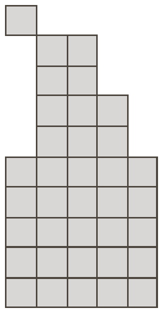
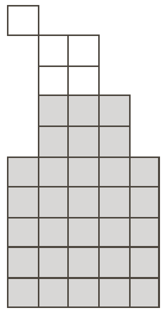
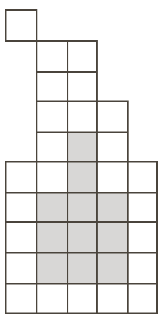
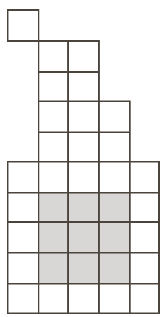
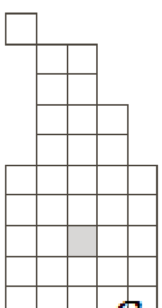





Opening removes parts of image which can not contain B

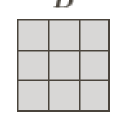
$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

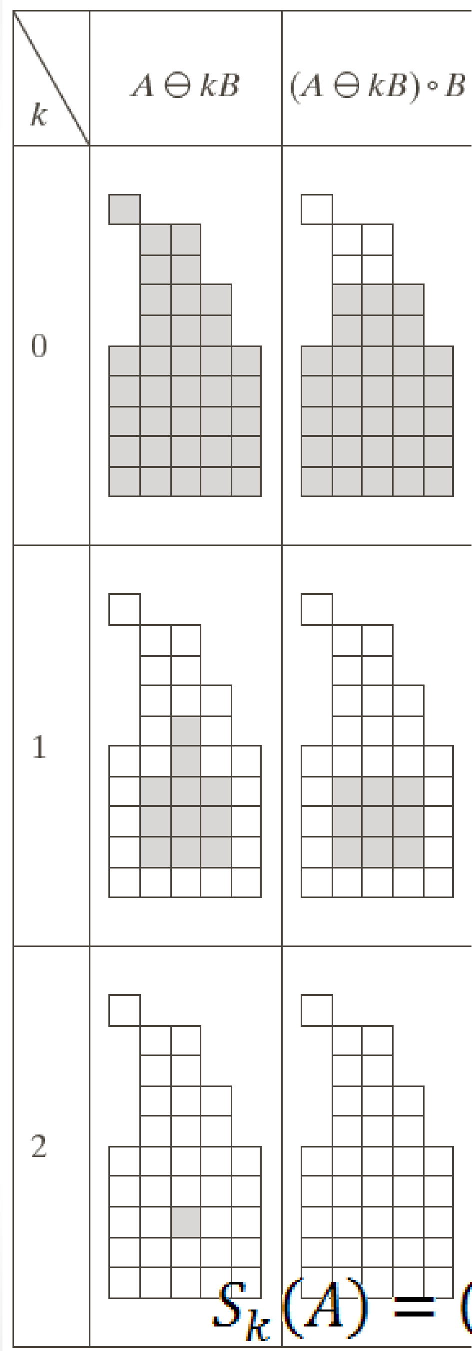


| $k$ | $A \ominus kB$ | $(A \ominus kB) \circ B$ |
| --- | --- | --- |
| 0 |  |  |
| 1 |  |  |
| 2 |  | |

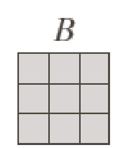
$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

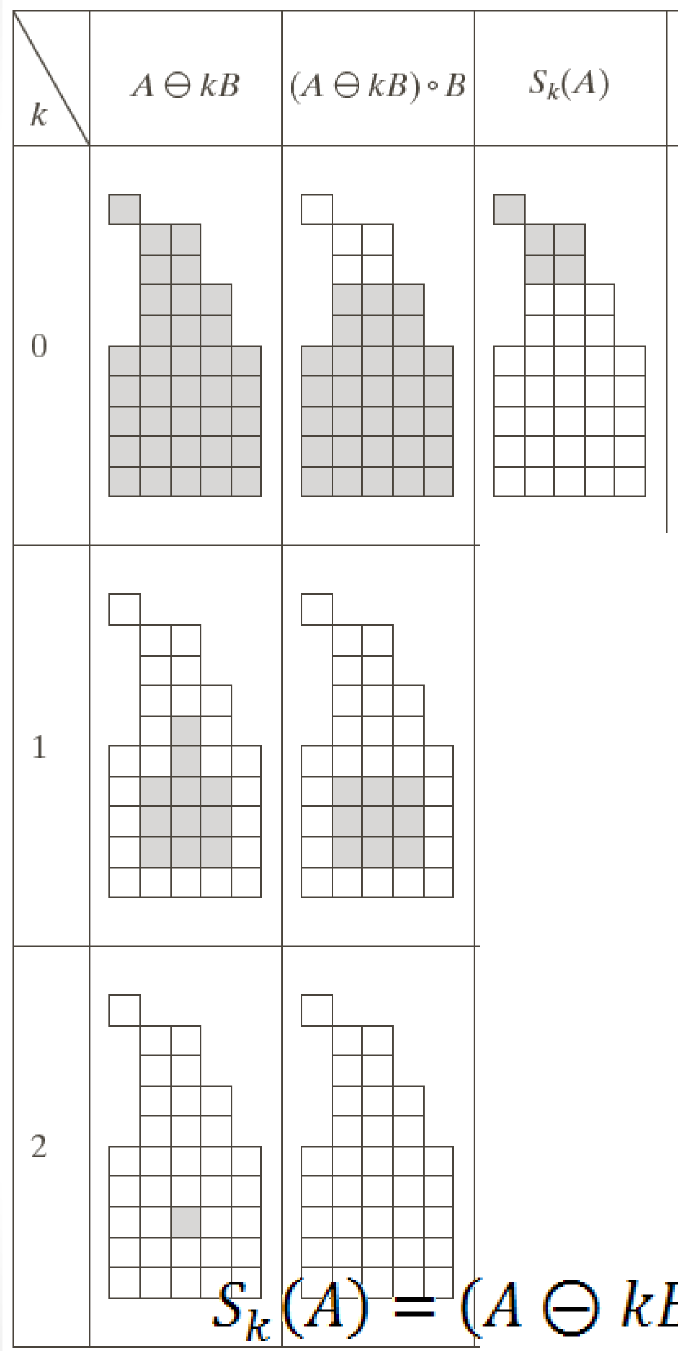
$B$



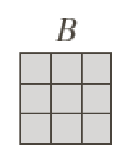


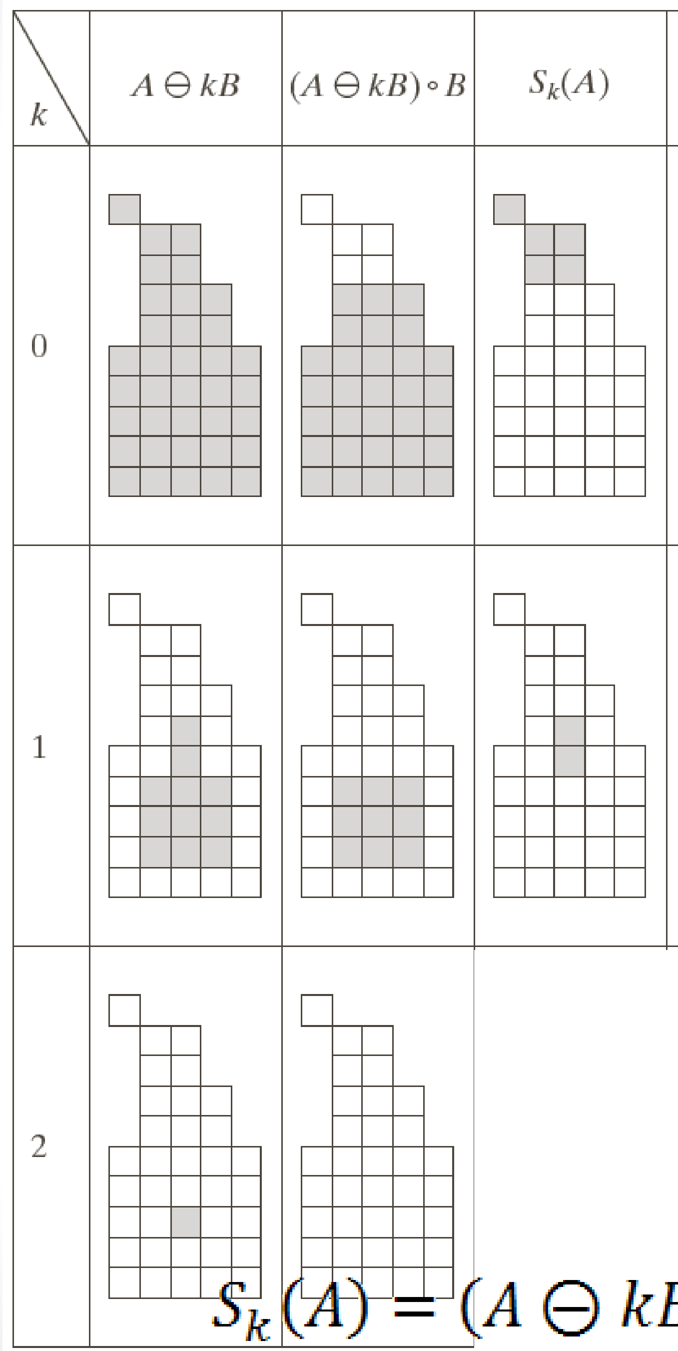

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$





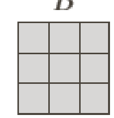

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

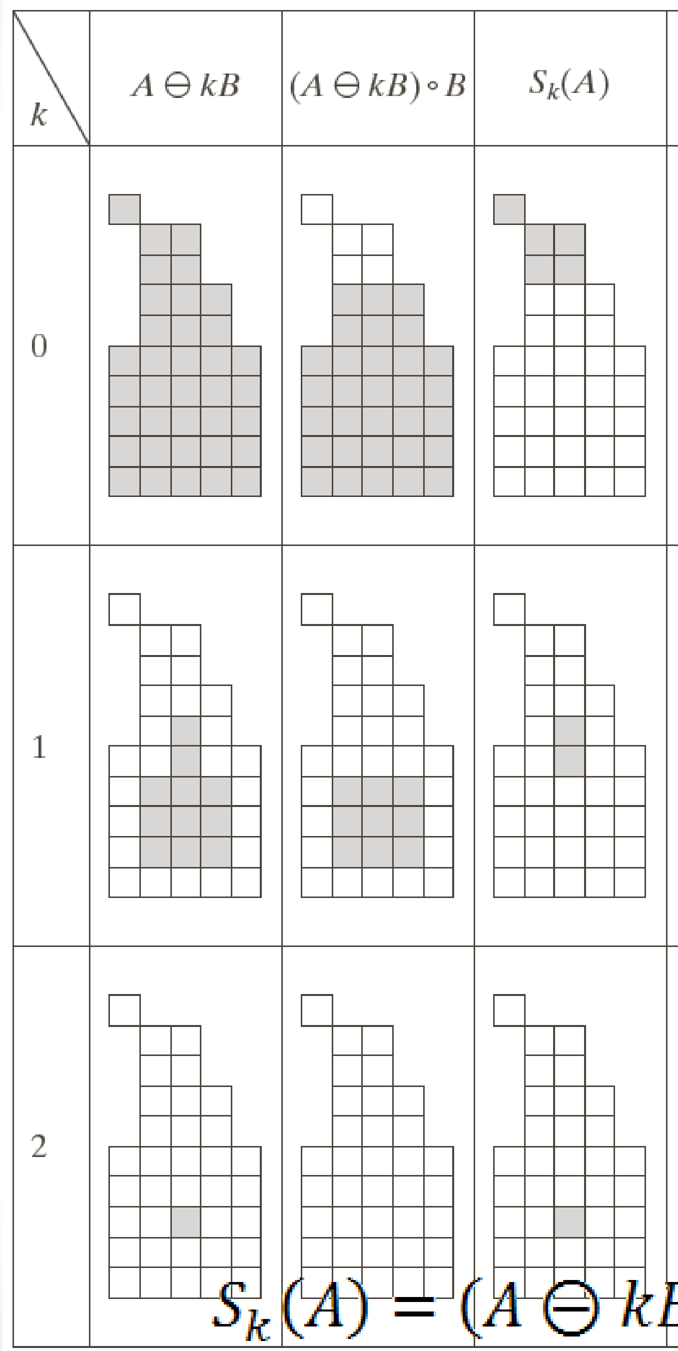




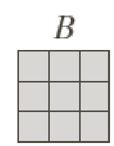

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

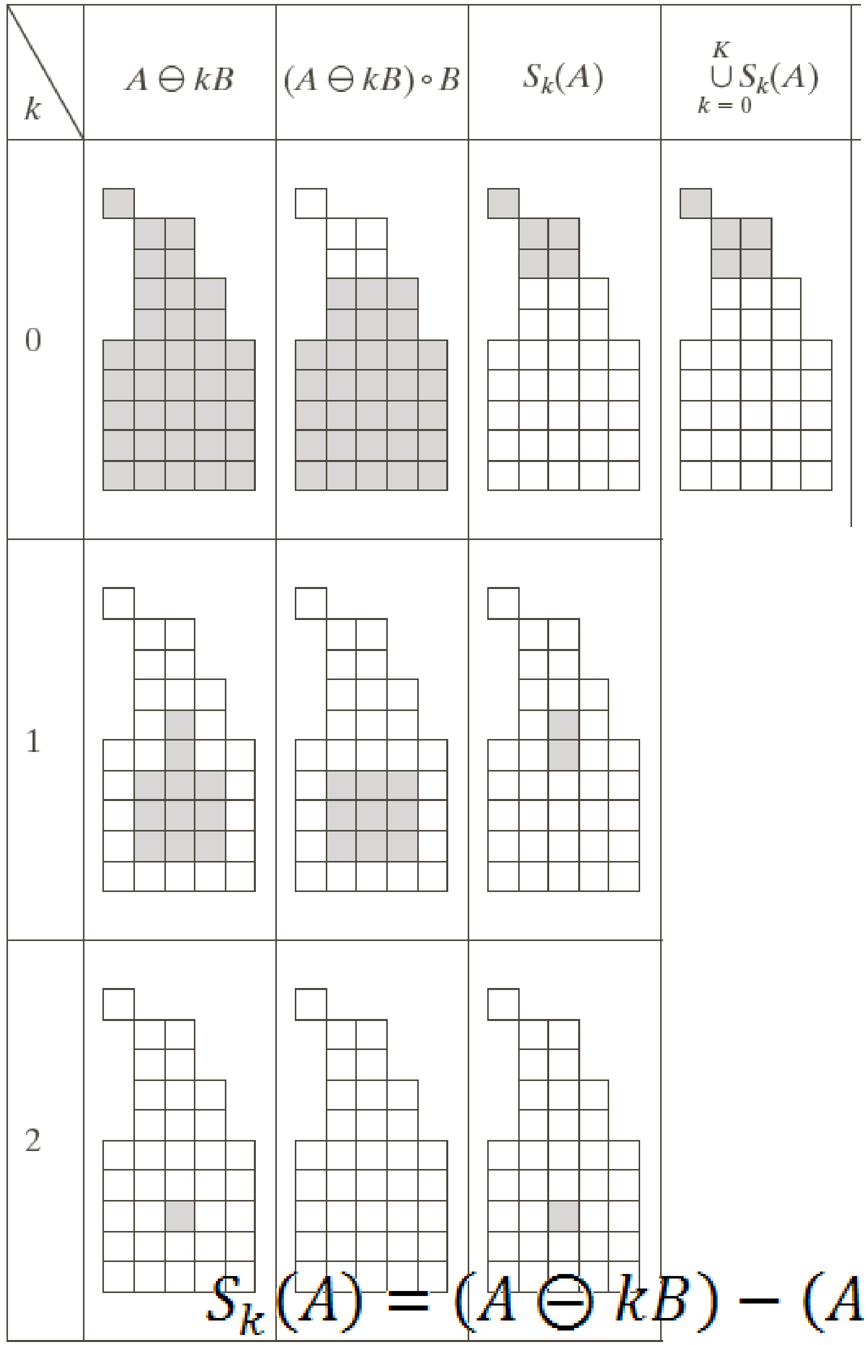
B





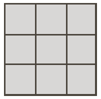

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

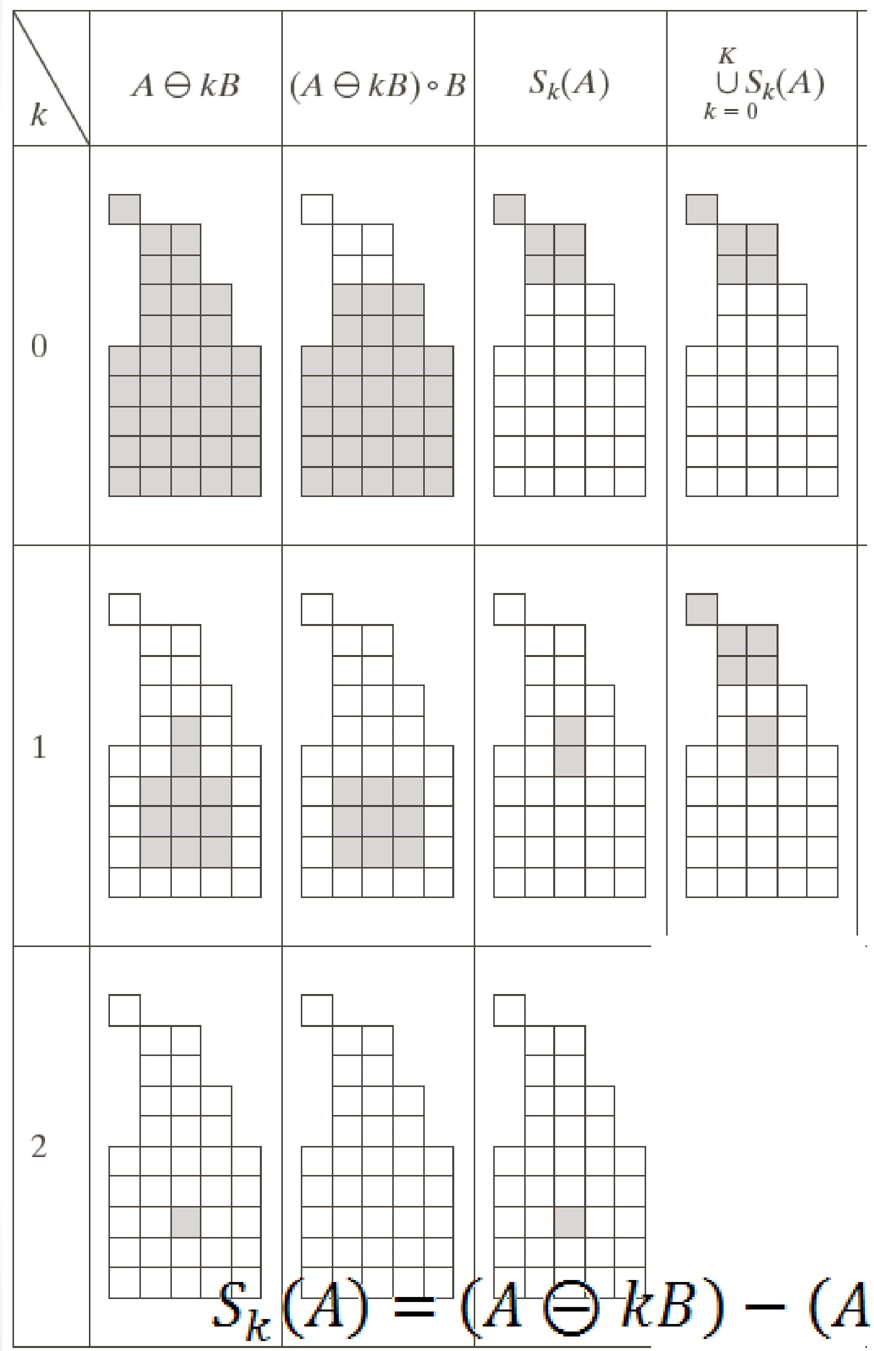





$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

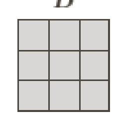
B

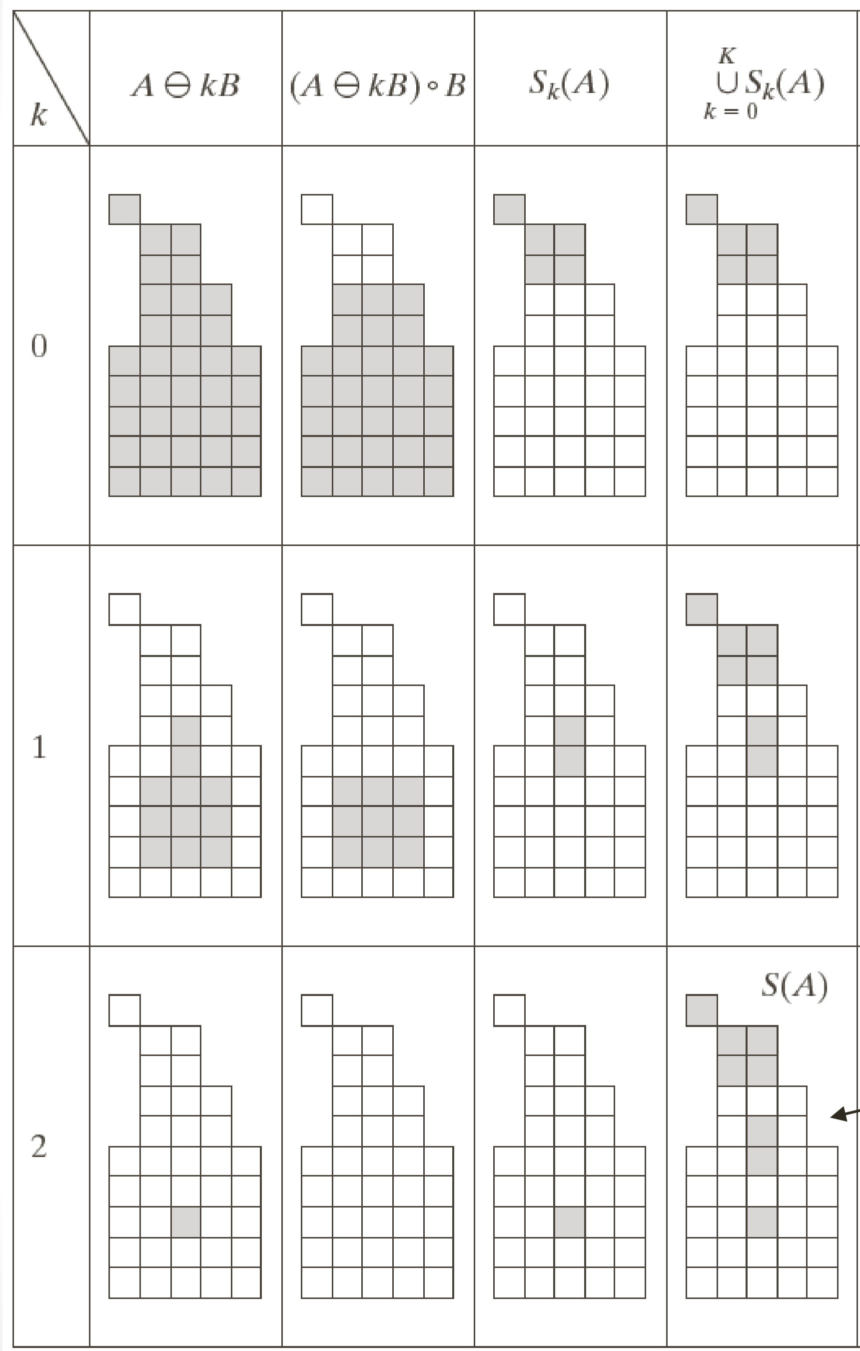




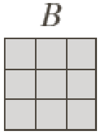

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

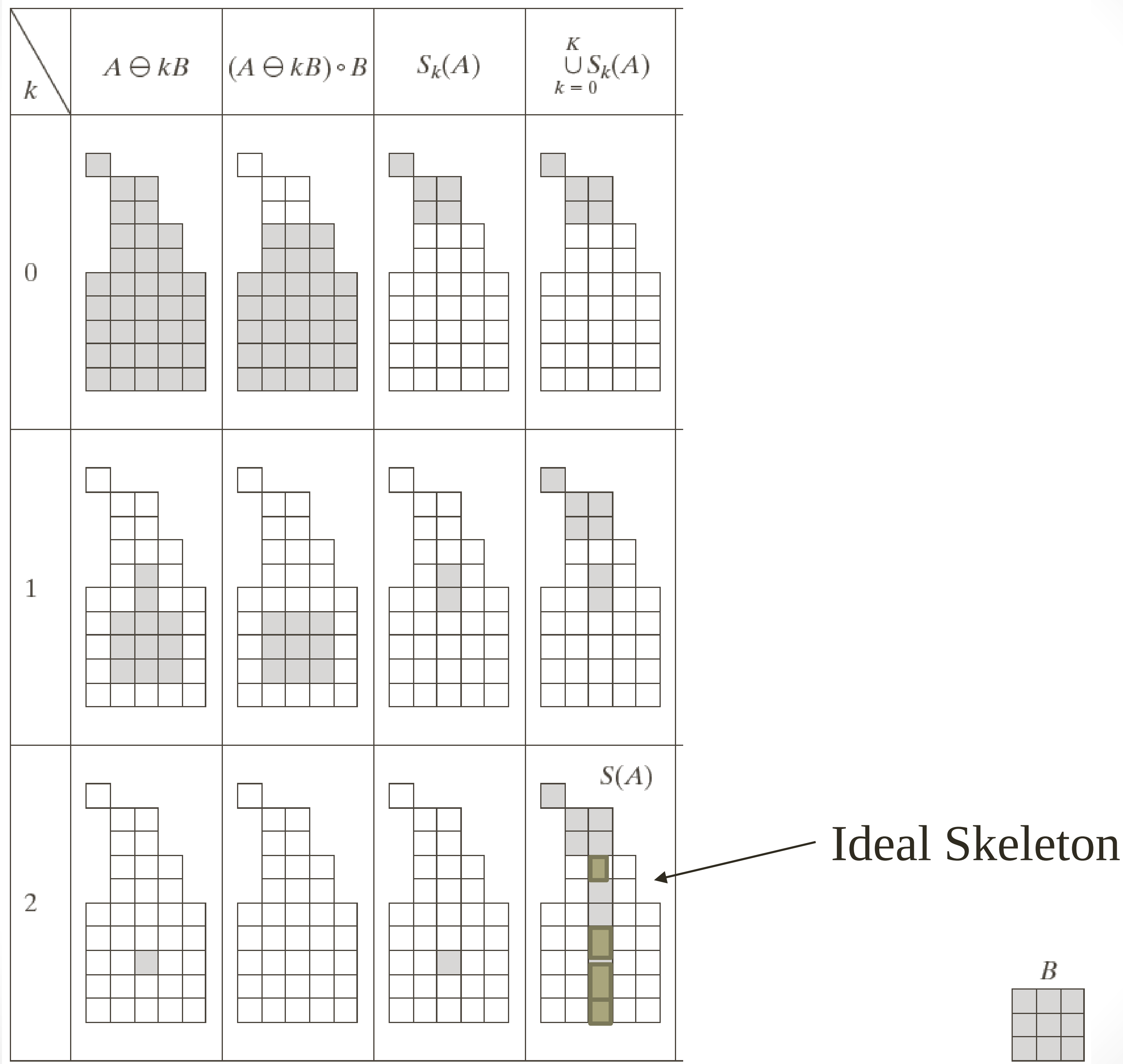
$B$






Skeleton





k
A ⊖ kB
(A ⊖ kB) ∘ B
S_k(A)
K
∪ S_k(A)
k = 0
0
1
2
S(A)
Ideal Skeleton
B

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

- Foreground is 1 and background is 0
- After thinning the number of zeroes increase

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

Structuring Elements (SE)

| | | |
|---|---|---|
| 0 | 0 | 0 |
| X | 1 | X |
| 1 | 1 | 1 |

$B_1$

| | | |
|---|---|---|
| X | 0 | 0 |
| 1 | 1 | 0 |
| 1 | 1 | X |

$B_2$

| | | |
|---|---|---|
| 1 | X | 0 |
| 1 | 1 | 0 |
| 1 | X | 0 |

$B_3$

- X is don't care
- Eight SEs are used
- $B_1$ is rotated clockwise to generate $B_2$
- Same process is repeated for remaining structuring elements

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

| | | |
|---|---|---|
| 0 | 0 | 0 |
| X | 1 | X |
| 1 | 1 | 1 |

$B_1$

| | | |
|---|---|---|
| X | 0 | 0 |
| 1 | 1 | 0 |
| 1 | 1 | X |

$B_2$

| | | |
|---|---|---|
| 1 | X | 0 |
| 1 | 1 | 0 |
| 1 | X | 0 |

$B_3$

- Thin(A,$B_1$) = A-(A$\circledast B_1$), where $\circledast$ is Hit or Miss Transform
- Overlap $B_1$ on each part of image
- If overlapped portion and B match then replace center of overlapped portion by 0 else don’t change the center

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

| | | |
|---|---|---|
| 0 | 0 | 0 |
| X | 1 | X |
| 1 | 1 | 1 |

$B_1$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

- Apply A- ($A \circledast B_1$) multiple times to generate $A_{B1}$

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

| | | |
|---|---|---|
| 0 | 0 | 0 |
| X | 1 | X |
| 1 | 1 | 1 |

$B_1$

- Apply A- (A $\circledast$ $B_1$) multiple times to generate $A_{B1}$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B1}$

| | | |
|---|---|---|
| X | 0 | 0 |
| 1 | 1 | 0 |
| 1 | 1 | X |

$B_2$

- Apply $A_{B1}$- ($A_{B1} \circledast B_2$) multiple times to generate $A_{B2}$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B2}$

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B2}$

| | | |
|---|---|---|
| 1 | X | 0 |
| 1 | 1 | 0 |
| 1 | X | 0 |

$B_3$

- Apply $A_{B2}$- ($A_{B2} \circledast B_3$) multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B3}$

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B3}$

| | | |
|---|---|---|
| 1 | X | 0 |
| 1 | 1 | 0 |
| 1 | X | 0 |

$B_3$

| | | |
|---|---|---|
| 1 | 1 | X |
| 1 | 1 | 0 |
| X | 0 | 0 |

$B_4$

- Apply $A_{B3} - (A_{B3} \circledast B_4)$ multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B4} = A_{B3}$ as B4 does not match with any part of image

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B4}$

| | | |
|---|---|---|
| 1 | 1 | X |
| 1 | 1 | 0 |
| X | 0 | 0 |

$B_4$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| X | 1 | X |
| 0 | 0 | 0 |

$B_5$

- Apply $A_{B4} - (A_{B4} \circledast B_5)$ multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B5} = A_{B4}$ as B5 does not match with any part of image

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B5}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| X | 1 | X |
| 0 | 0 | 0 |

$B_5$

| | | |
|---|---|---|
| X | 1 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | X |

$B_6$

| | | |
|---|---|---|
| 0 | X | 1 |
| 0 | 1 | 1 |
| 0 | X | 1 |

$B_7$

| | | |
|---|---|---|
| 0 | 0 | X |
| 0 | 1 | 1 |
| X | 1 | 1 |

$B_8$

- Apply $A_{B5}$- ($A_{B5} \circledast B_6$) multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B6} = A_{B5}$ as B6 does not match with any part of image

- Apply $A_{B6}$- ($A_{B6} \circledast B_6$) multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B7} = A_{B6}$ as B7 does not match with any part of image

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B7}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| X | 1 | X |
| 0 | 0 | 0 |

$B_5$

| | | |
|---|---|---|
| X | 1 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | X |

$B_6$

| | | |
|---|---|---|
| 0 | X | 1 |
| 0 | 1 | 1 |
| 0 | X | 1 |

$B_7$

| | | |
|---|---|---|
| 0 | 0 | X |
| 0 | 1 | 1 |
| X | 1 | 1 |

$B_8$

- Apply $A_{B7}$- ($A_{B7} \circledast B_8$) multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B8}$

# Image Thinning

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Thinned Image, $A_{B8}$

# Image Thickening

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

- Foreground is 1
- Background is 0
- After thickening number of ones increase

# Image Thickening

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

| | | |
|---|---|---|
| 0 | 0 | 0 |
| X | 0 | X |
| 1 | 1 | 1 |

$B_1$

| | | |
|---|---|---|
| X | 0 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | X |

$B_2$

| | | |
|---|---|---|
| 1 | X | 0 |
| 1 | 0 | 0 |
| 1 | X | 0 |

$B_3$

- X is don't care
- $B_1$ is rotated clockwise to generate $B_2$
- Same process repeated for remaining 8 structuring elements

# Image Thickening

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

| | | |
|---|---|---|
| 0 | 0 | 0 |
| X | 0 | X |
| 1 | 1 | 1 |

$B_1$

| | | |
|---|---|---|
| X | 0 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | X |

$B_2$

| | | |
|---|---|---|
| 1 | X | 0 |
| 1 | 0 | 0 |
| 1 | X | 0 |

$B_3$

- Thin(A,$B_1$) = AU(A $\circledast$ $B_1$), where $\circledast$ is Hit or Miss Transform
- Overlap $B_1$ on each part of image
- If overlapped portion and B match then replace center of overlapped portion by 1 else don’t change the center

# Image Thickening

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

| | | |
|---|---|---|
| 0 | 0 | 0 |
| X | 0 | X |
| 1 | 1 | 1 |

$B_1$

| | | |
|---|---|---|
| X | 0 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | X |

$B_2$

Apply $A \cup (A \circledast B_1)$ multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

No change in A, $A_{B1} = A$

Apply $A_{B1} \cup (A \circledast B_2)$ multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

No change $A_{B2} = A_{B1}$

# Image Thickening

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B2}$

| | | |
|---|---|---|
| 1 | X | 0 |
| 1 | 0 | 0 |
| 1 | X | 0 |

$B_3$

| | | |
|---|---|---|
| 1 | 1 | X |
| 1 | 0 | 0 |
| X | 0 | 0 |

$B_4$

Apply $A_{B2} \cup (A_{B2} \circledast B_3)$ multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B3}$

Apply $A_{B3} \cup (A_{B3} \circledast B_4)$ multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

No change, $A_{B4} = A_{B3}$

# Image Thickening

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B4}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| X | 0 | X |
| 0 | 0 | 0 |

$B_5$

| | | |
|---|---|---|
| X | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | X |

$B_6$

Apply $A_{B4} \cup (A_{B4} \circledast B_5)$ multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

No change, $A_{B5} = A_{B4}$

Apply $A_{B5} \cup (A_{B5} \circledast B_6)$ multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

No change, $A_{B6} = A_{B5}$

# Image Thickening

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, $A_{B6}$

| | | |
|---|---|---|
| 0 | X | 1 |
| 0 | 0 | 1 |
| 0 | X | 1 |

$B_7$

| | | |
|---|---|---|
| 0 | 0 | X |
| 0 | 0 | 1 |
| X | 1 | 1 |

$B_8$

Apply $A_{B6} \cup (A_{B6} \circledast B_7)$ multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

No change, $A_{B7} = A_{B6}$

Apply $A_{B7} \cup (A_{B7} \circledast B_8)$ multiple times

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

No change, $A_{B8} = A_{B7}$

# Image Thickening

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Image, A

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |

Thickened Image, $A_{B8}$