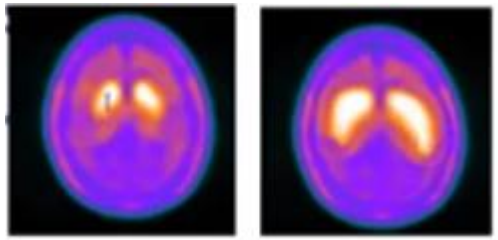
Image Registration

What is Image registration?

- Process of transforming different sets of images into one coordinate system
- After registration, images can be fused to extract information
- Images may be from multiple photographs, different sensors, different illuminations, different angles (view points) etc

Images from sources of different sensors



PET scan before and after few months

What is Image Registration?

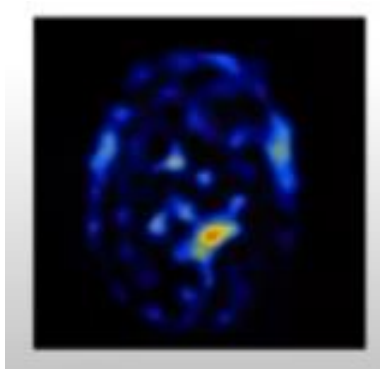
Different depths (far or close)

Camera is rotated

Camera is tilted

Fusion after Registration

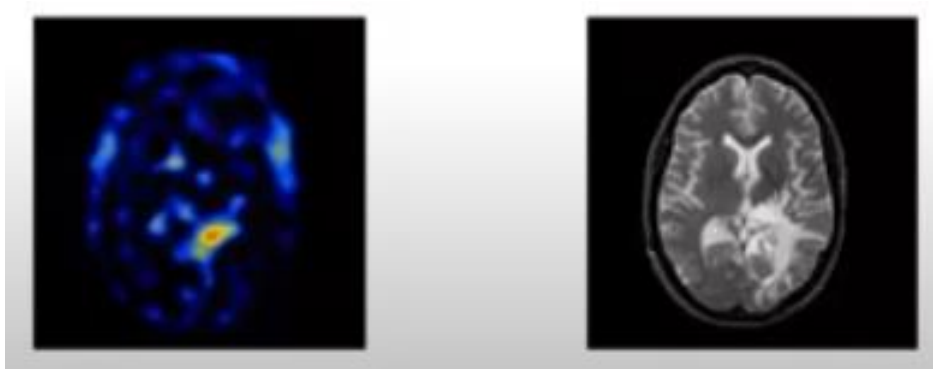
- Fusion of medical images for diagnosis
- PET+MRI, CT+PET, CT+MRI …



PET

Fusion after Registration

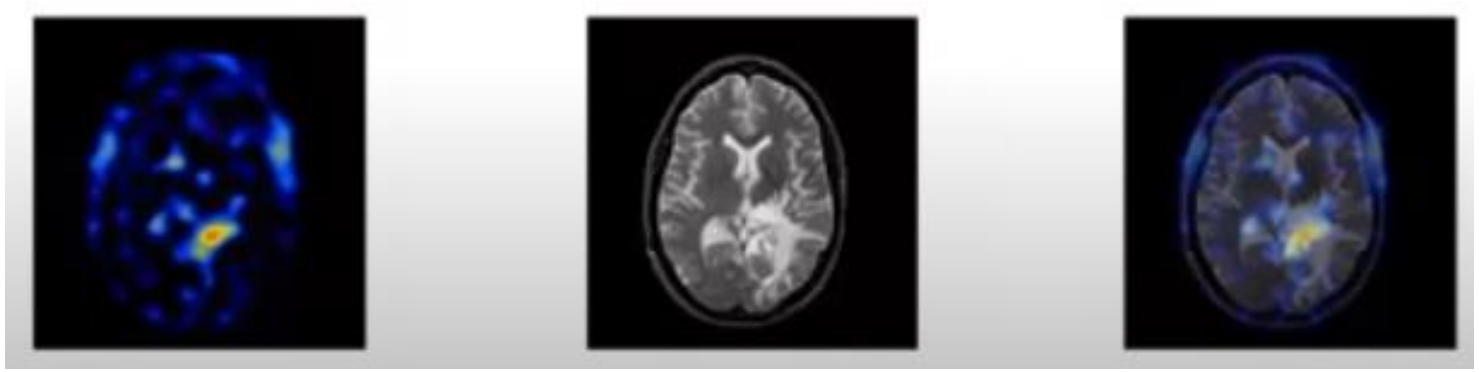
- Fusion of medical images for diagnosis
- PET+MRI, CT+PET, CT+MRI …



PET MRI

Fusion after Registration

- Fusion of medical images for diagnosis
- PET+MRI, CT+PET, CT+MRI …



PET MRI Image Fusion

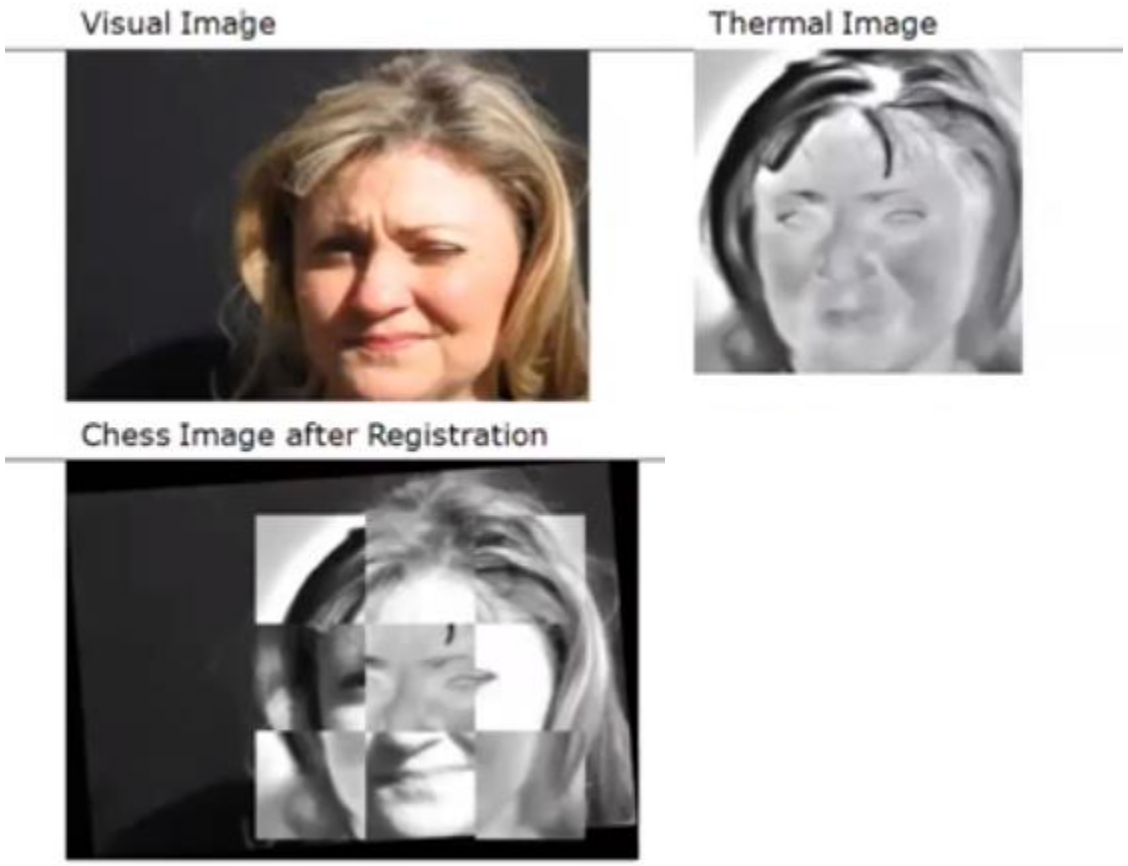
Tumor is clearly visible

Fusion after Registration

- Thermal image is captured when image is in dark

Fusion after Registration

- Thermal image is captured when image is in dark



Fusion after Registration

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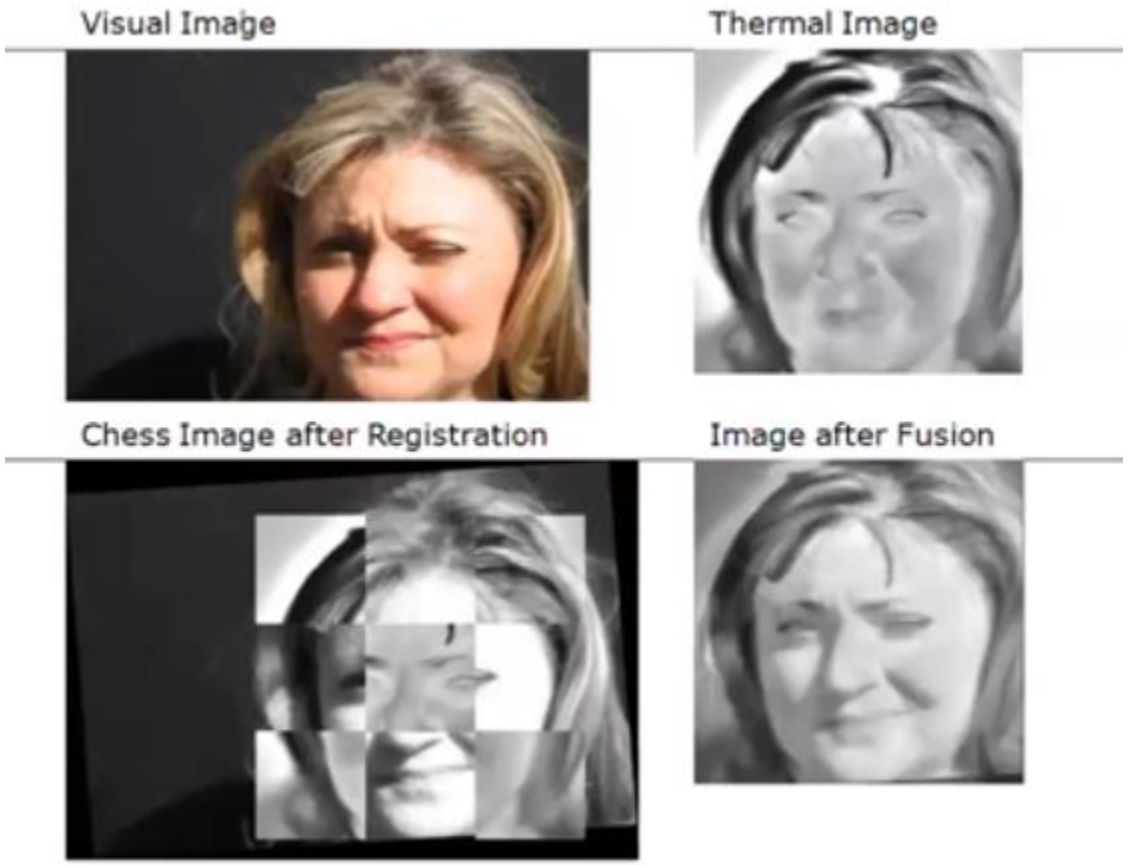


Panorama or mosaic using Registration

Satellite image 1

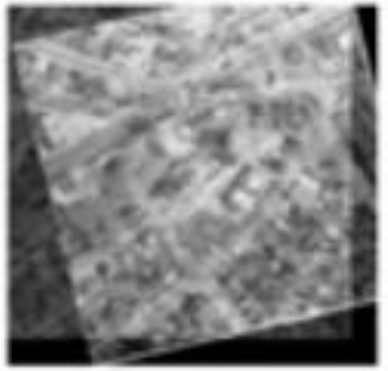
Satellite image 2

Panorama or mosaic using Registration

- Increases field of view

Satellite image 1

Satellite image 2



Panorama of Satellite image

How to do image registration?

- Establish correspondence between the images of the same scene

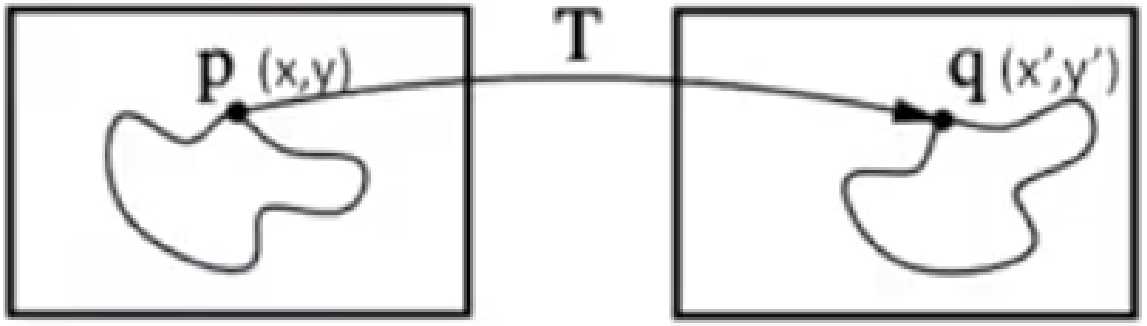


Image 1

Image 2

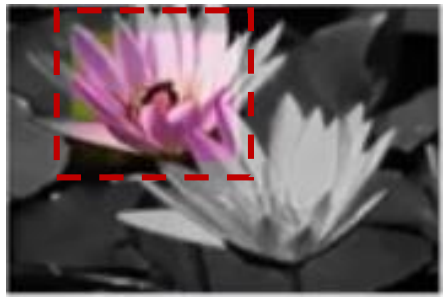
- Object is same
- Image 2 is displaced right, down and rotated version of image 1
- Interest point (key point), p is displaced to q
- Image2 is spatially transformed version of image 1
- Establishment of correspondence of two images is to find spatial transformation
- Then apply registration is to align the images

Template Matching using Correspondence

- Pixels of sample flower are matching with bigger image
- Take a pixel of sample image
- Find coordinates of the same pixel in bigger image
- Repeat the same for all the pixels of sample image

Template of flower

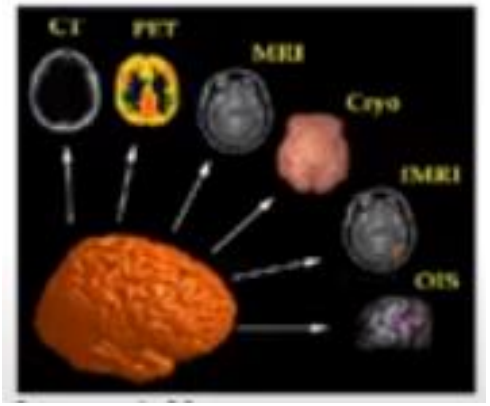
Match template



Types of Image Registration

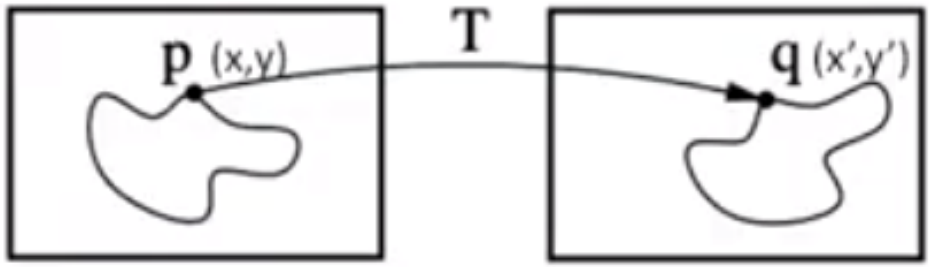
- Mono modal
  - Establishing correspondence among images captured using same sensor or camera

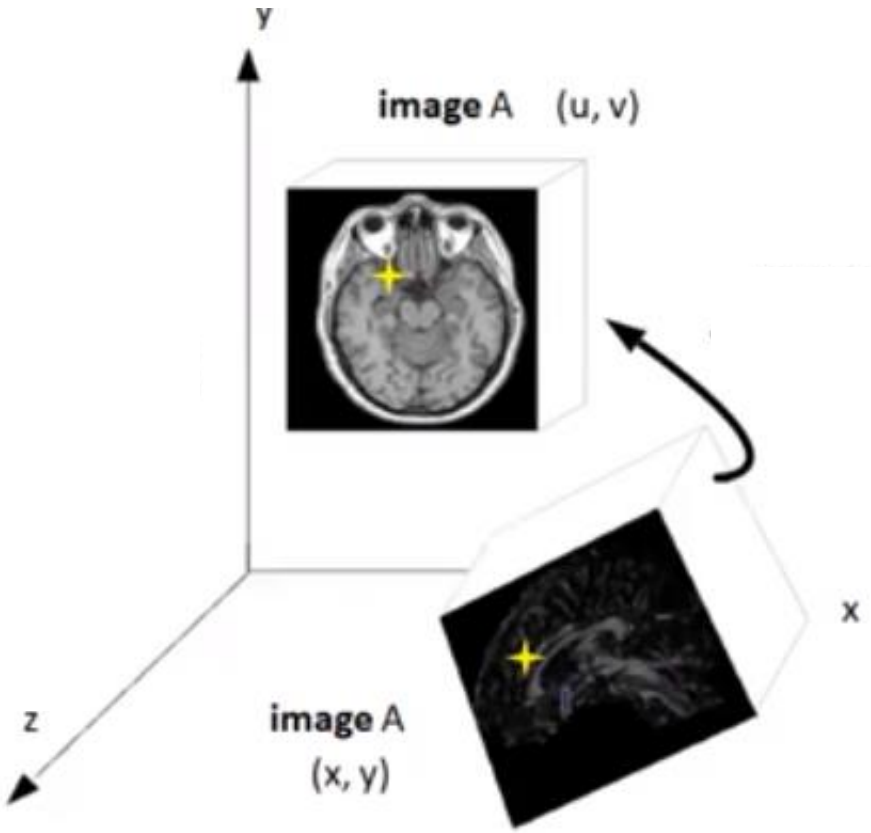
- Multi modal
  - Establishing correspondence among images captured using different sensor or camera



Geometric (Spatial) Transformation

- Widely used for transformation of coordinate system
- Each pixel location, (x, y) on one image is transformed to new location, (x', y') in another image



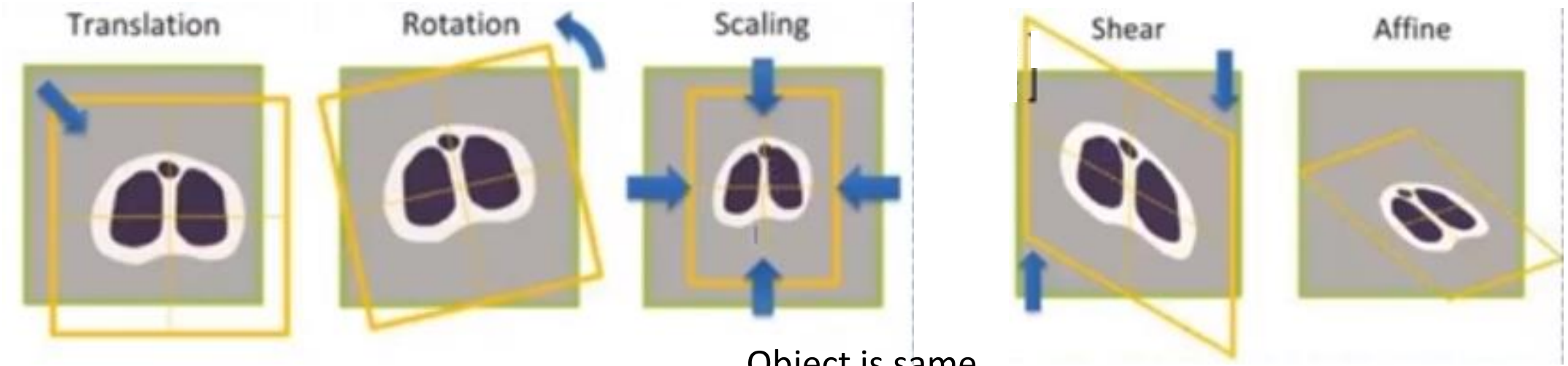


Geometric (Spatial) Transformation

- Two types
  - Linear
    - Translation
    - Rotation
    - Scaling
    - Shear
    - Affine
  - Non linear

Linear Geometrical Transformation

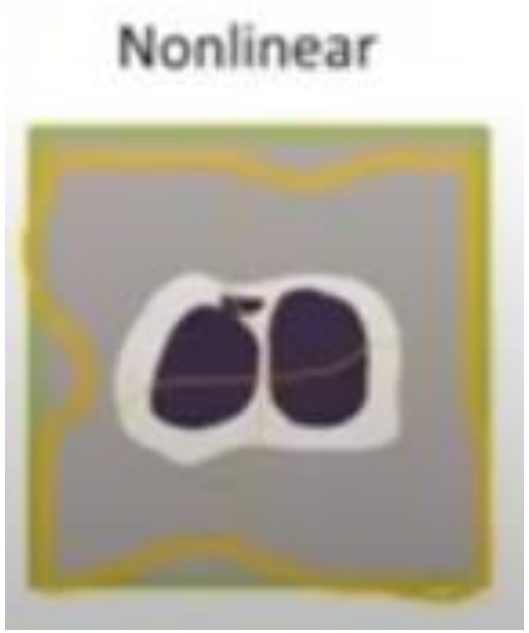
- Translation: Camera is displaced in x and y direction
- Rotation: Camera is rotated
- Scaling: Far or close view
- Shear: Camera is tilted
- Affine: Translation, rotation and scaling on the same image



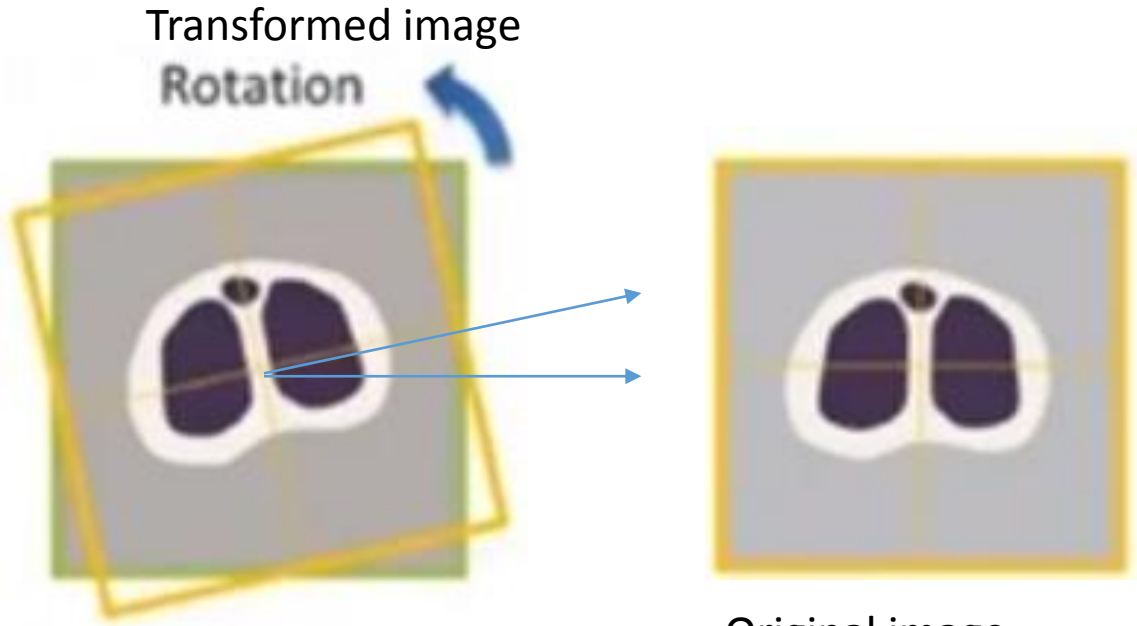
Object is same

Non linear Transformation

- Not easy to align or apply registration

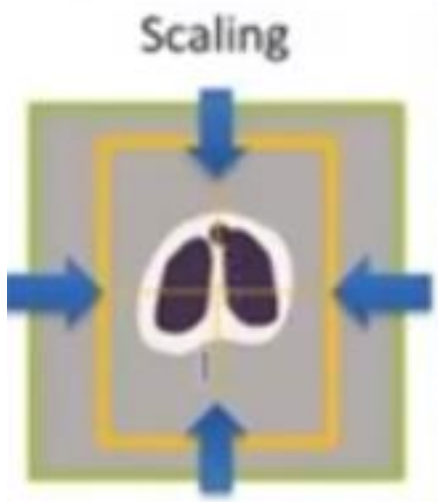


Rotation



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

θ is angle of rotation

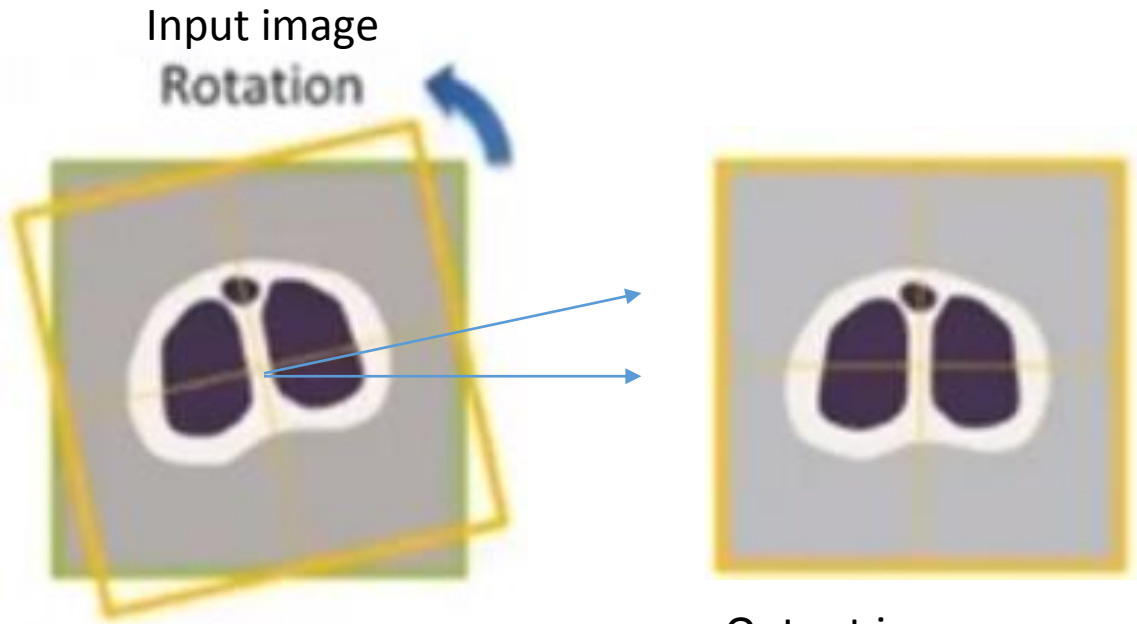


$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

S decides the size of object

(x', y') are pixel coordinates of transformed image (x, y) are pixel coordinates of original image

Rotation



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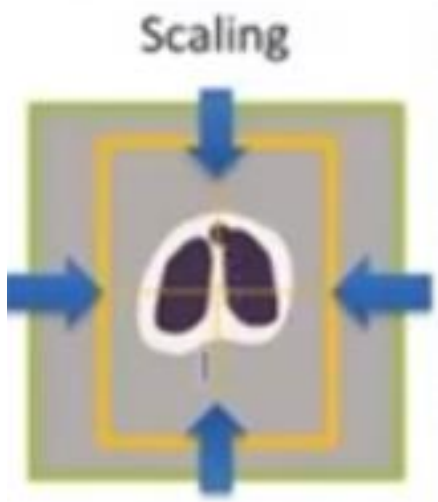
θ is angle of rotation

- If θ=90°, determine coordinates of original image

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos90 & -sin90 \\ sin90 & cos90 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

x‘= -y, y’ = x



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

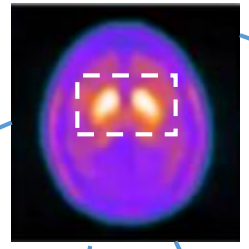
Assume s = ½, determine coordinates of input image

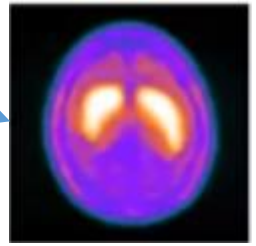
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

x‘= (½)x, y’ = (½)y

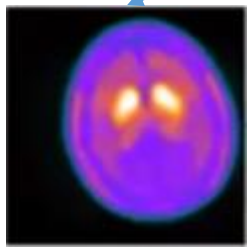
For x=10 and y = 10,x’= 5 and y’ = 5

Other Geometric Transformation

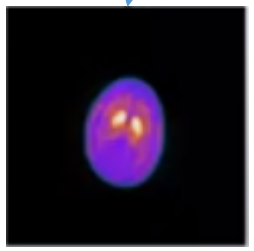




Deformation
Object is deformed



Translation and rotation
(rigid body transformation)

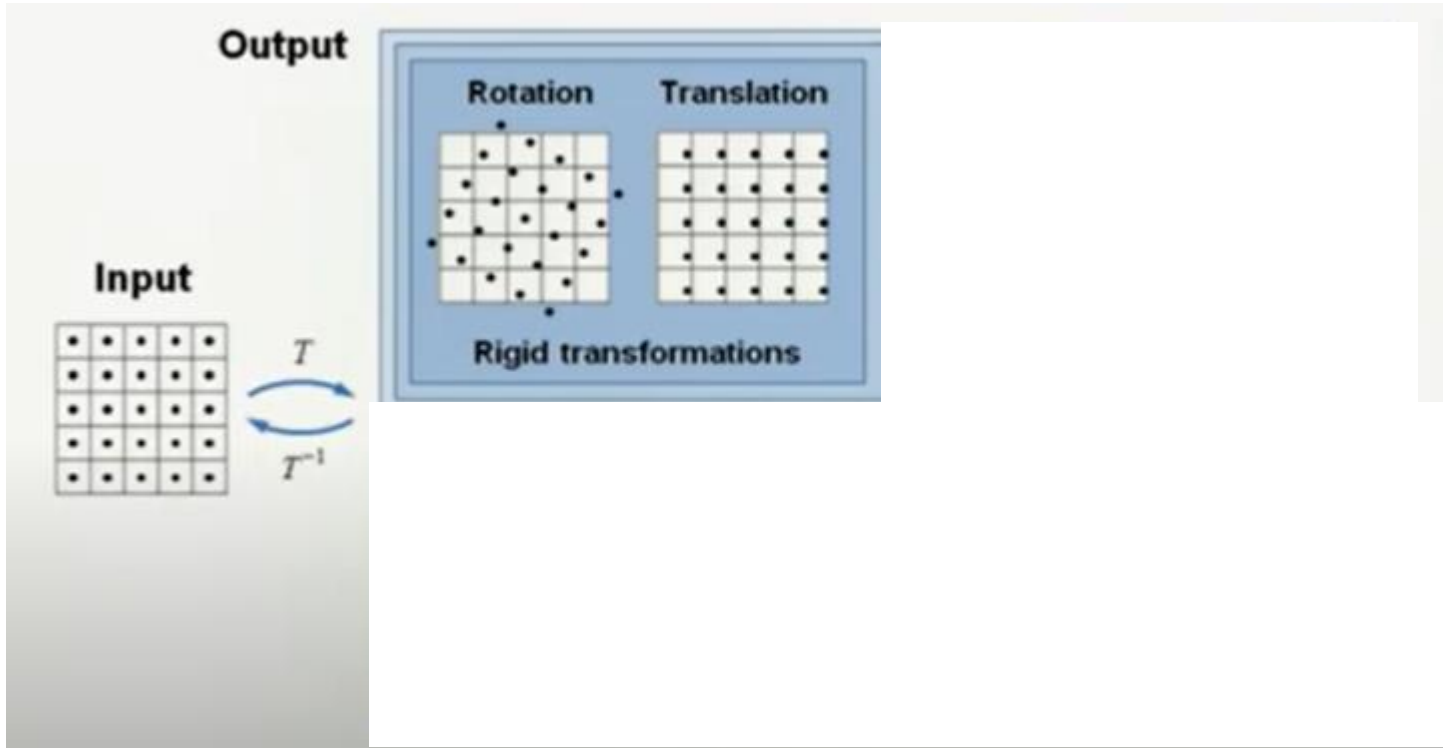


Translation, rotation and scaling (similarity transformation)

- Combination of all transformation (Affine)
- Difficult to correct this image to bring it normal

Geometric Transformation

- Can consider any or some of them for registration
- Rigid is less complex than affine



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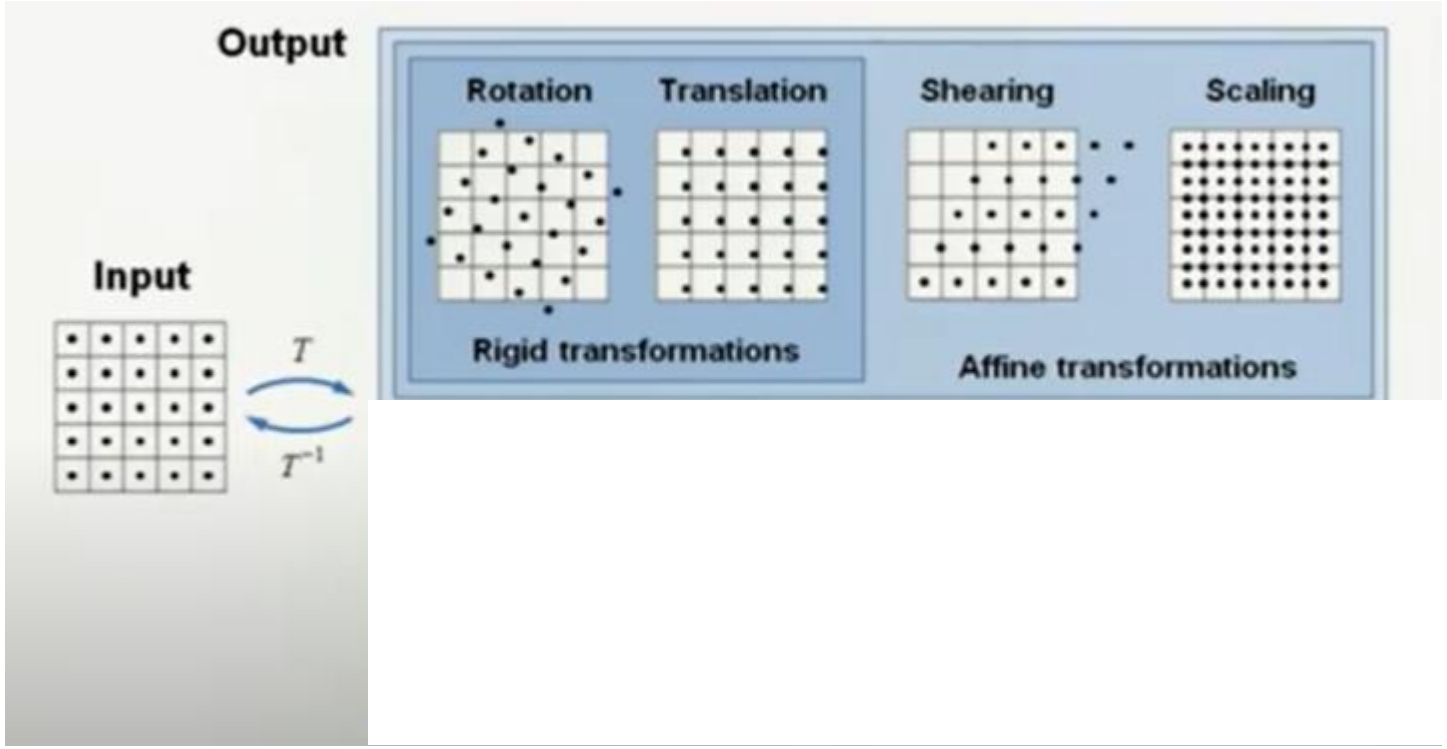
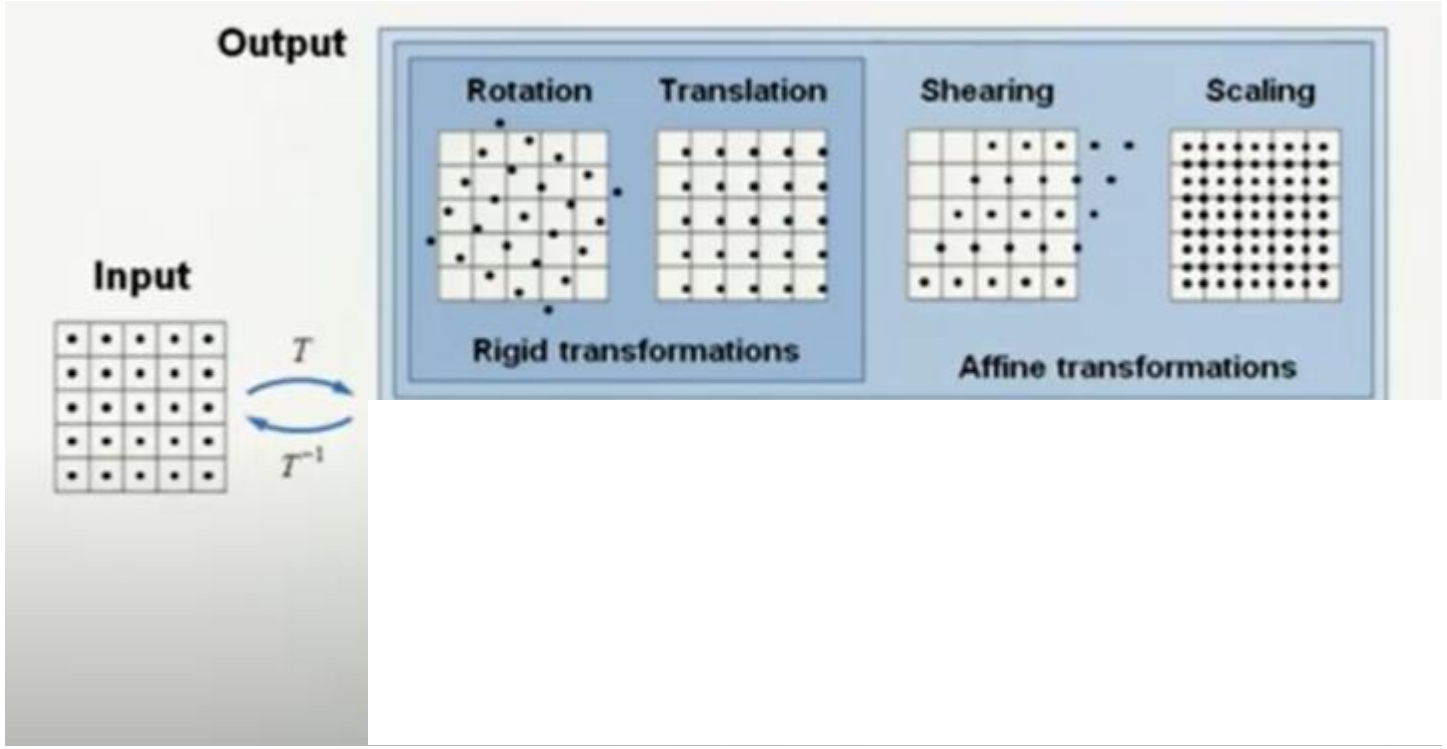


Geometric Transformation

- Can consider any or some of them for registration
- Rigid is less complex than affine
- Affine provides for flexibility in transformation
- For more flexibility and more degrees of freedom, non rigid transformation is useful



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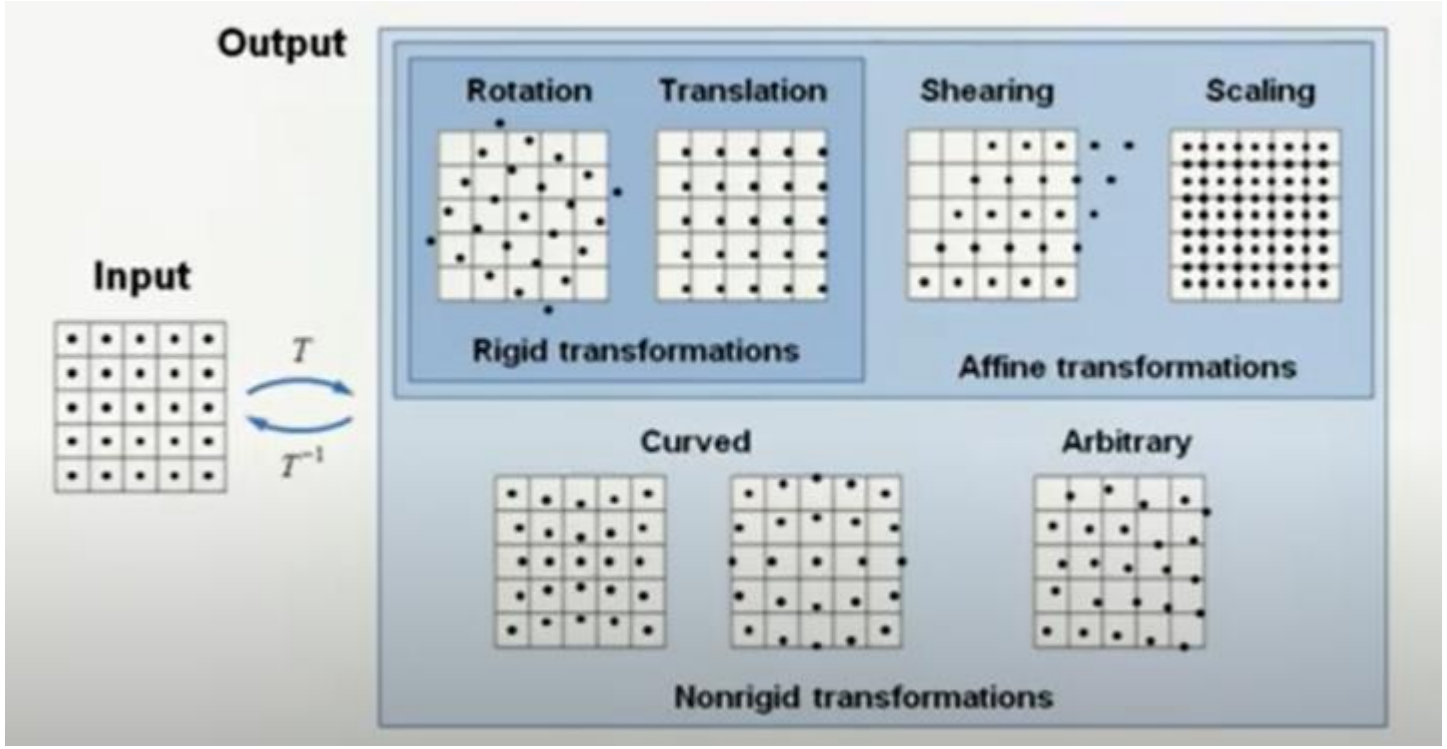
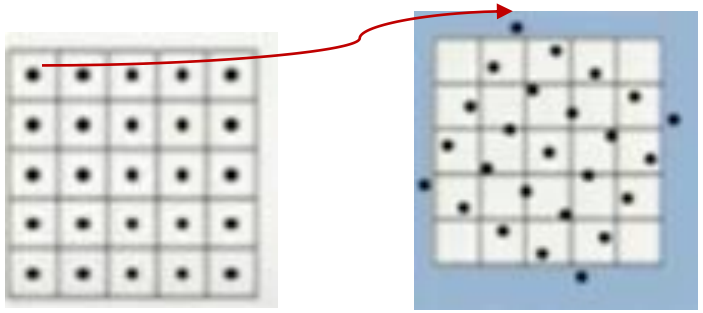


Process of Image Registration

- Various methods are available
- One of the methods is geometrical transformation
- Choose key (control) points in original image
- Find corresponding points in the image to be registered



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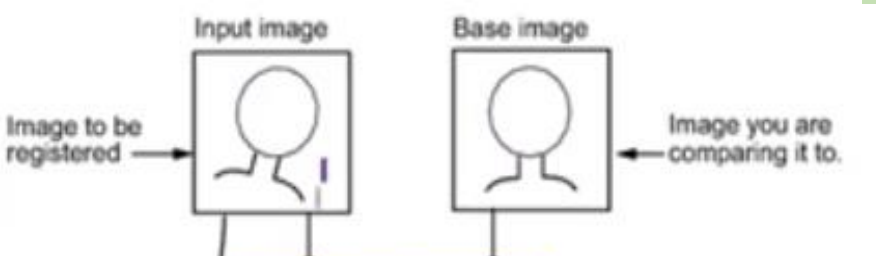

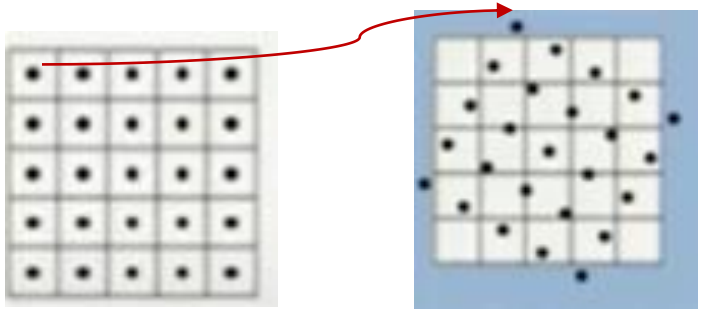

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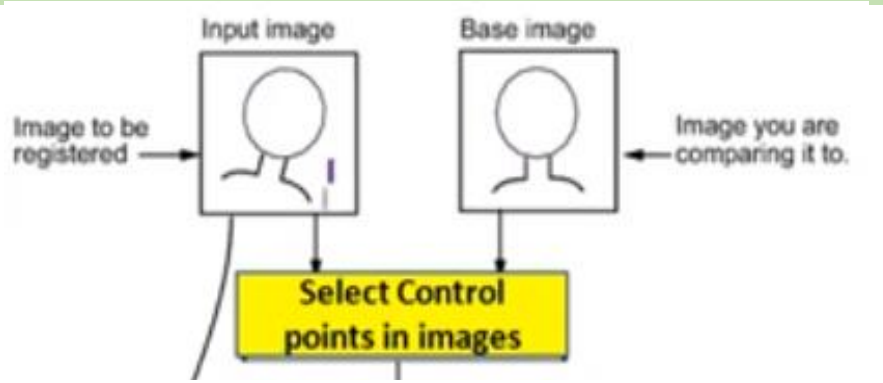

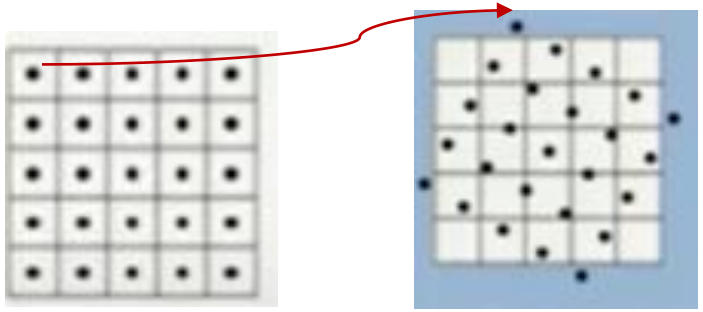

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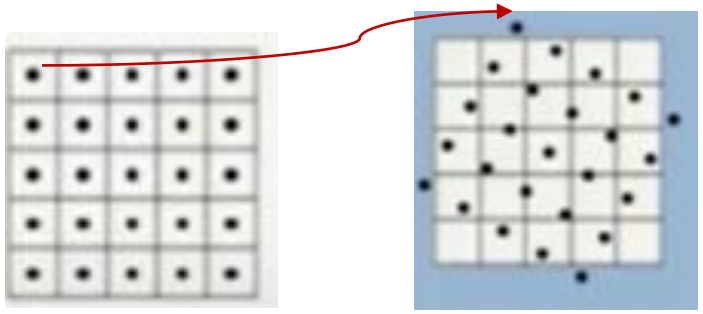

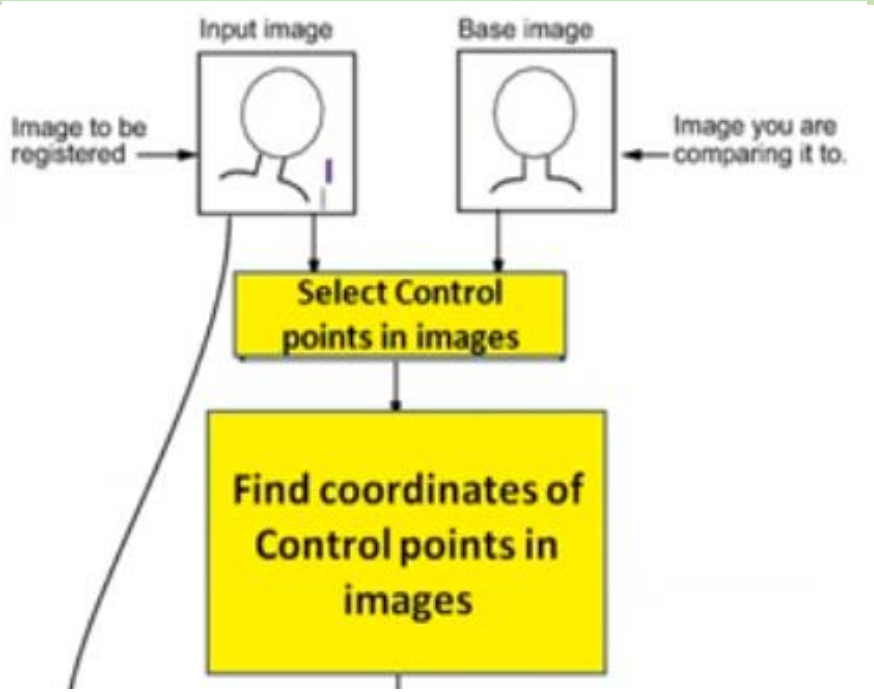

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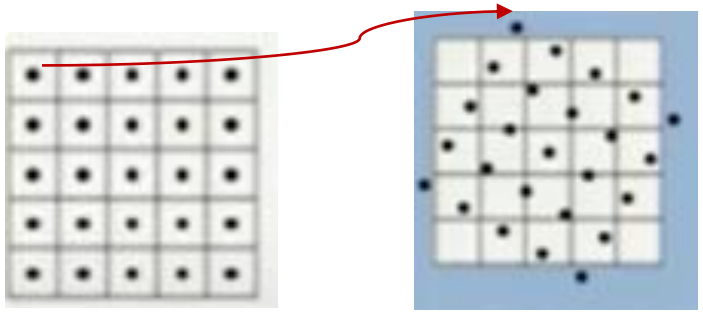

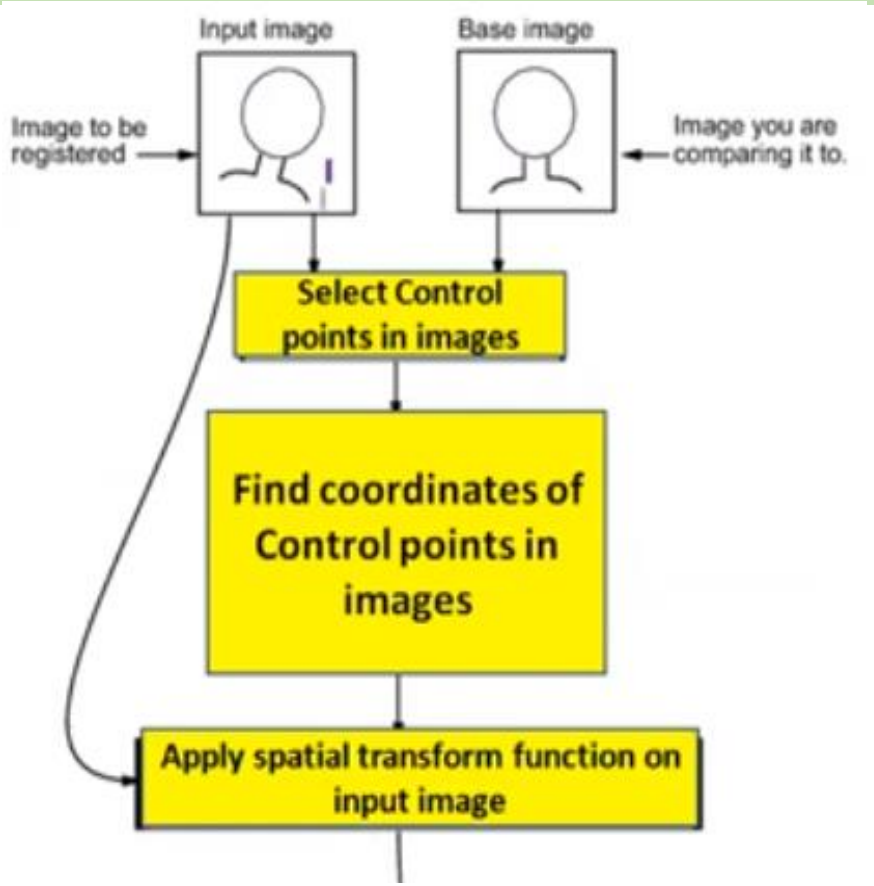

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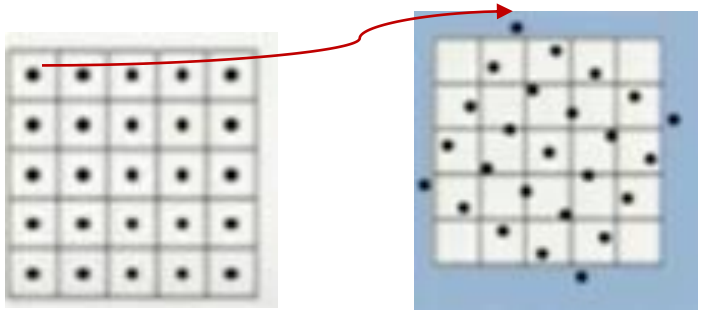
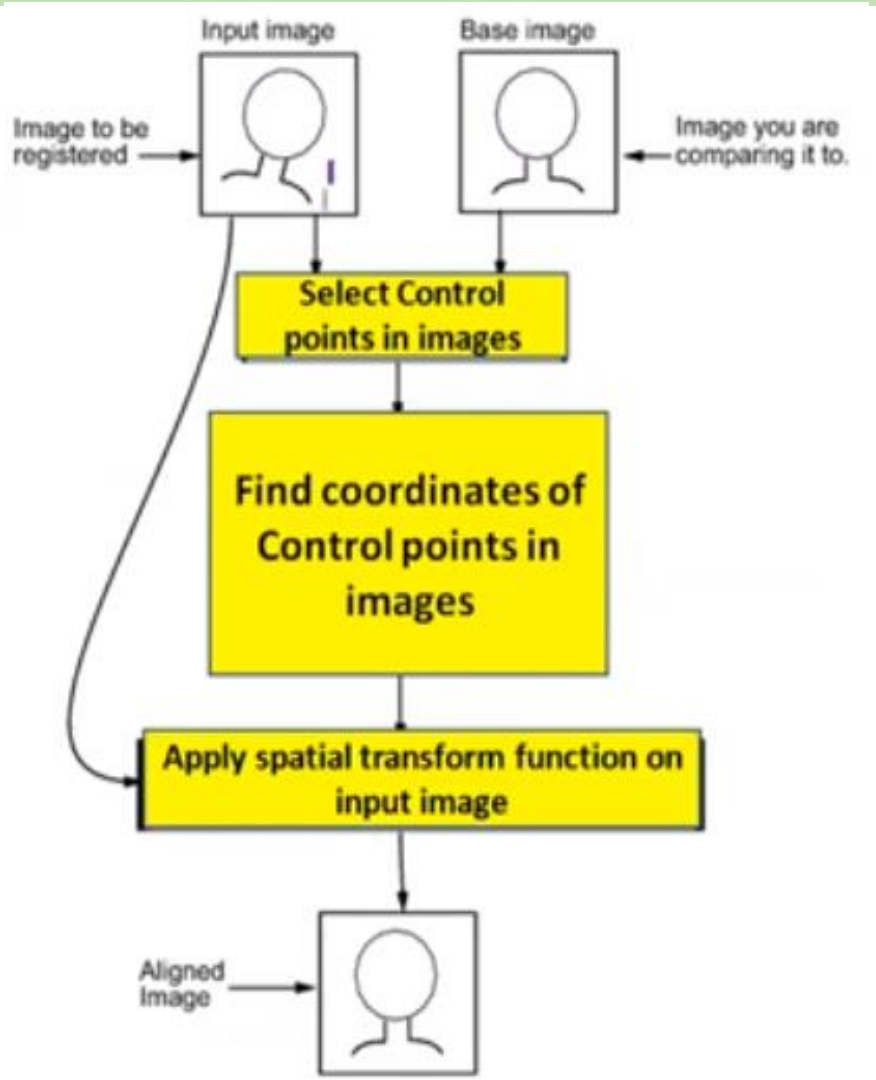
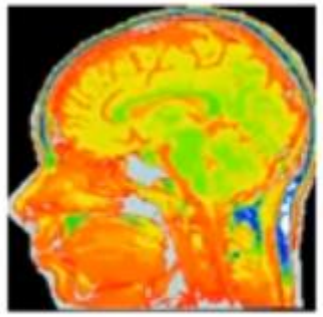




Medical Image Registration

- Alignment is registration
- Fusion, matching or superimposition is application
- Selection of key point and corresponding point in other image is critical
- Ex: Alignment of images to identify location of tumor should be precise



MR

Brain is visible
Tumor is not

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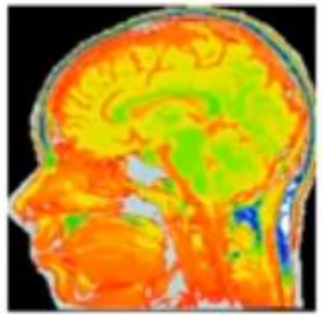

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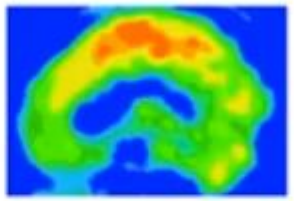
+



SPECT

- Tumor is visible
- Brain is not visible
- Therefore location of tumor in brain can not be identified

=

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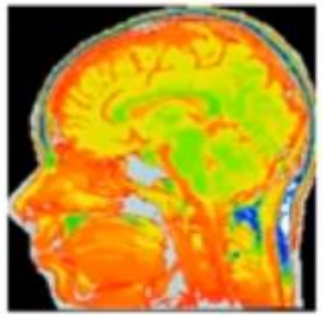

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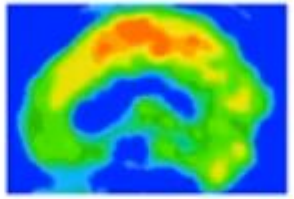

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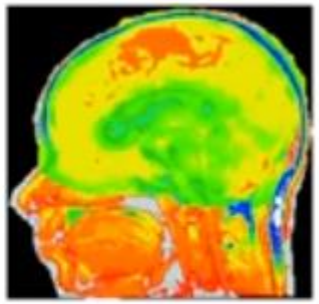


registered

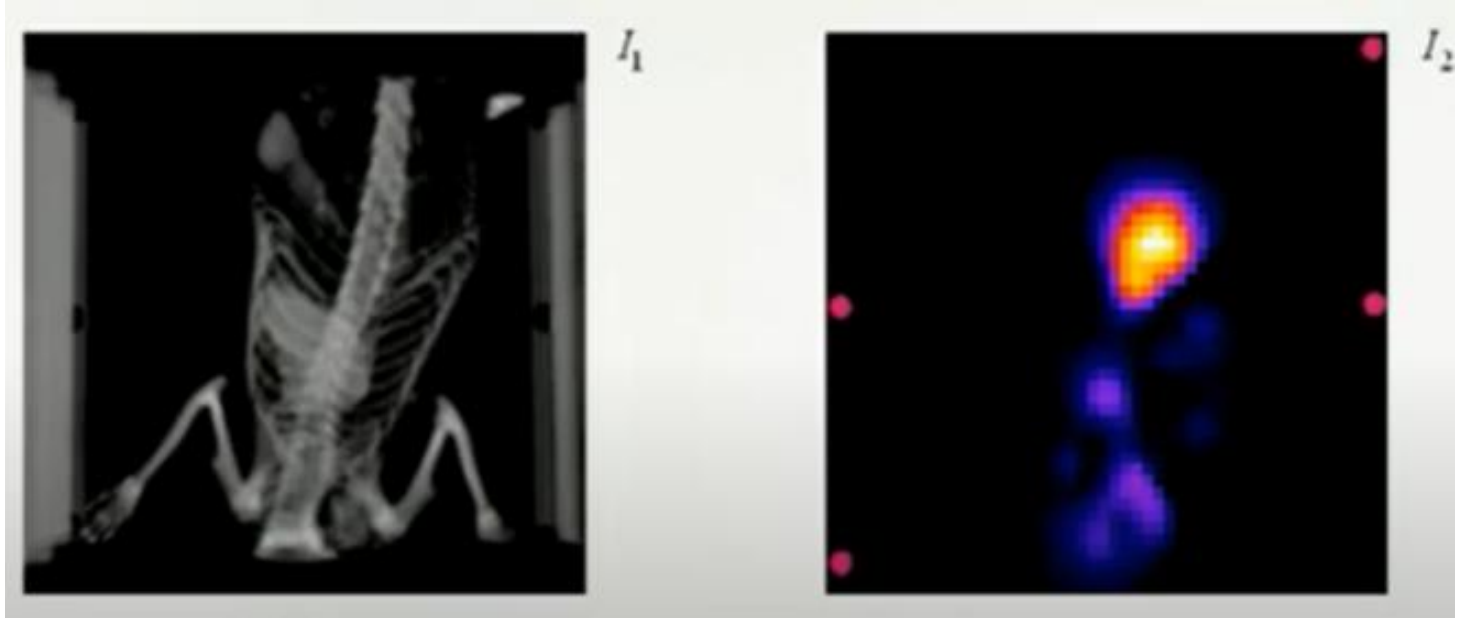
Shows location of tumor in brain

Applications of Image Registration

- Diagnosis
  - Combining images from different modalities
- Studying disease progression
  - Monitoring changes in shape, size, position, intensity in a specified time duration
- Image guided surgery
  - Make surgical plan based on pre operative images
- Patient comparison
  - Relate patient report with standard images

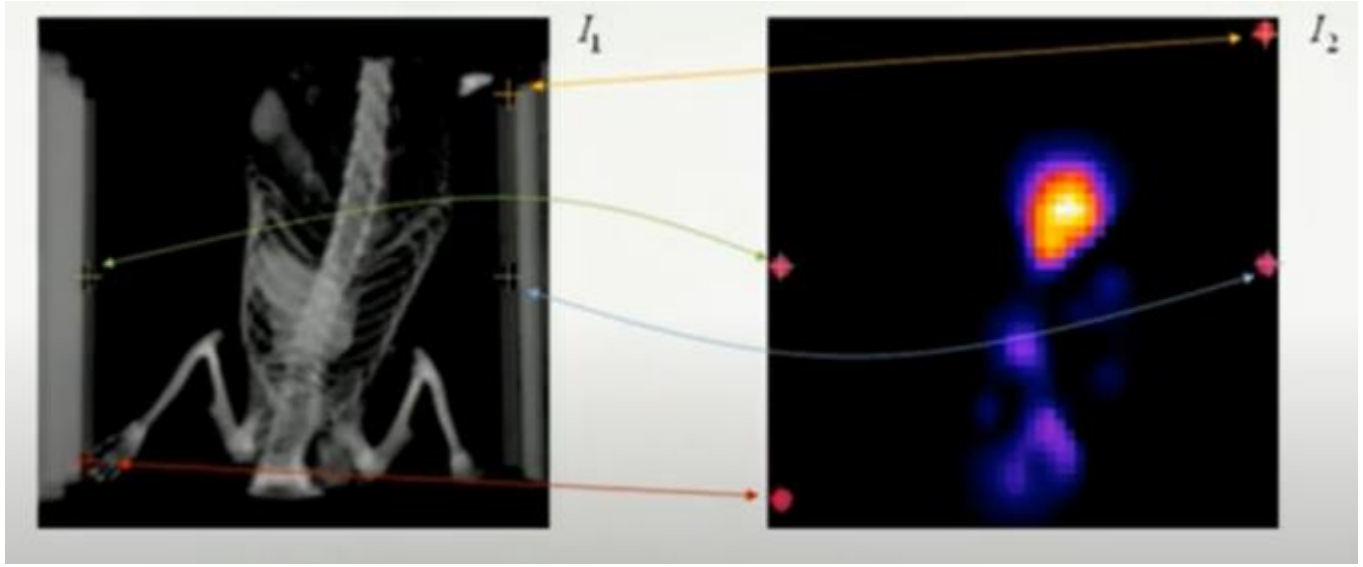
Extrinsic Information for Registration

- If images are very different from each other in terms of intensity distributions
- It is difficult to check whether these images are aligned or not
- Get key points which exist in both images (extrinsic information)
- Determine the correspondence between two points



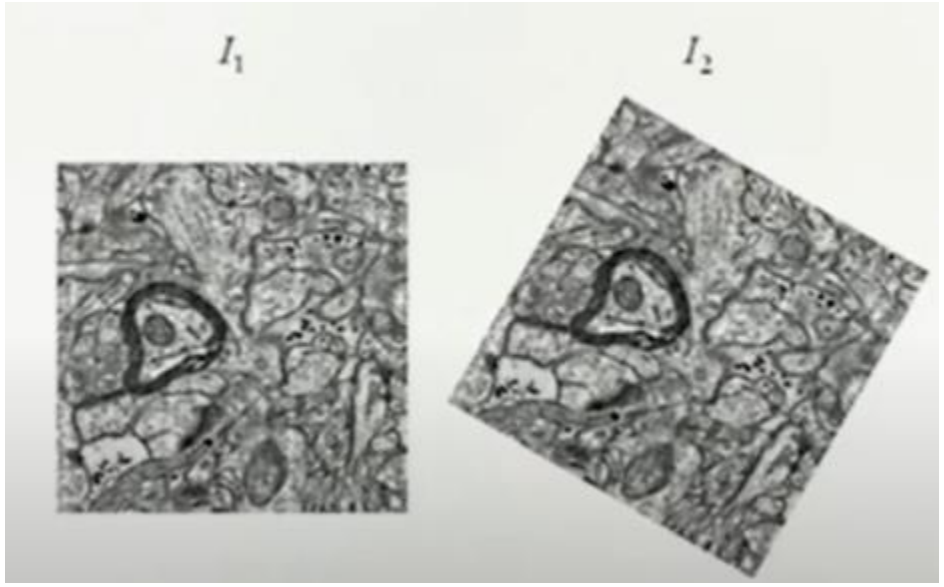
Extrinsic Information for Registration

- Check which point connects to which point in the second image
- Sufficient key points can be located, define geometrical transformations between them
- Landmark (key) points can be user defined or automatic like SIFT or URF features



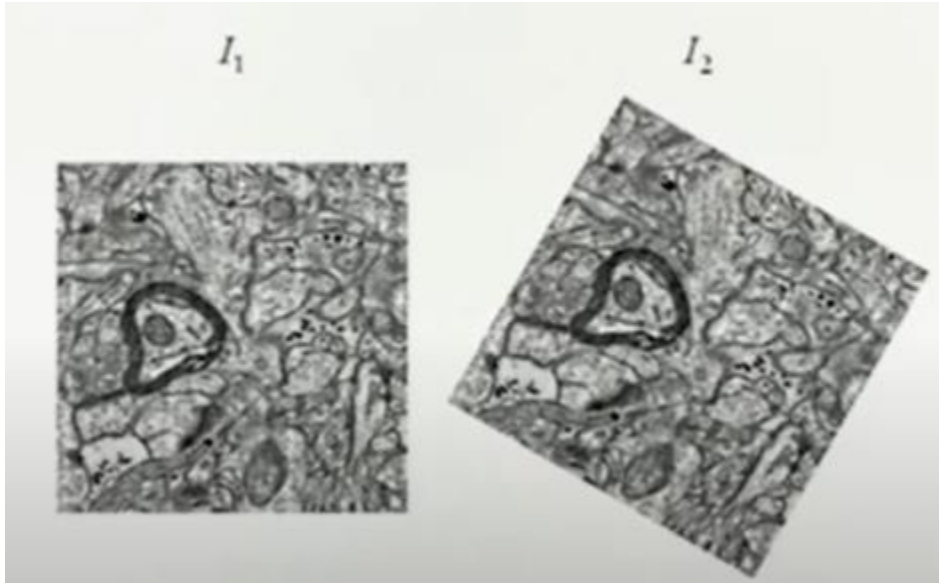
Intrinsic Information for Registration

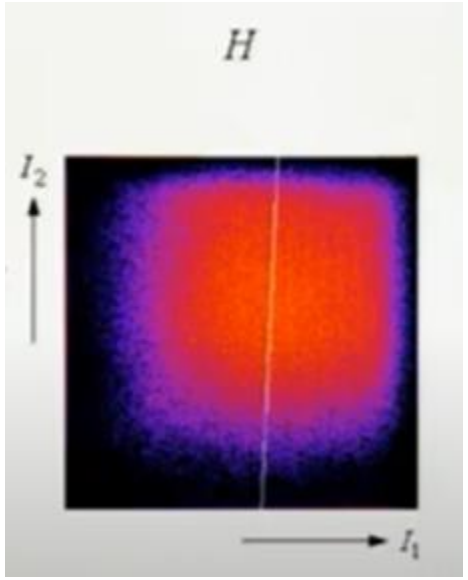
- If images are alike, intrinsic information be used
- Ex: Image is rotated version of original image
- Construct joint histogram



Joint Histogram of Images

- Joint histogram for two images I_1 and I_2
- Plot the intensity '*a*' of each pixel in image I_1 against the intensity '*b*' at the corresponding location in image I_2
- Rows and columns of matrix are a and b
- Value of each histogram at location H(*a, b*) i.e. row 'a' and column 'b' of H matrix correspond to the number of pixels with intensity '*a*' in I_1 and intensity '*b*' in image I_2
- Color map shows red color for large value
- Black color shows lowest value

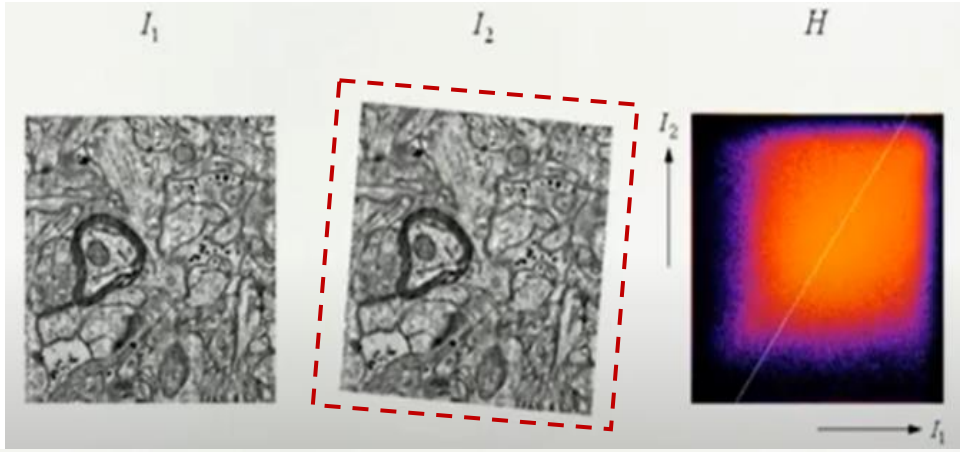


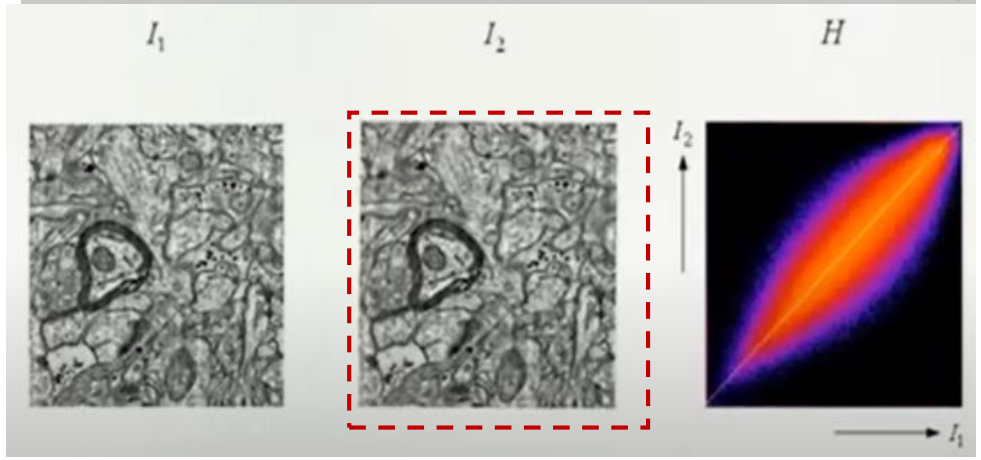


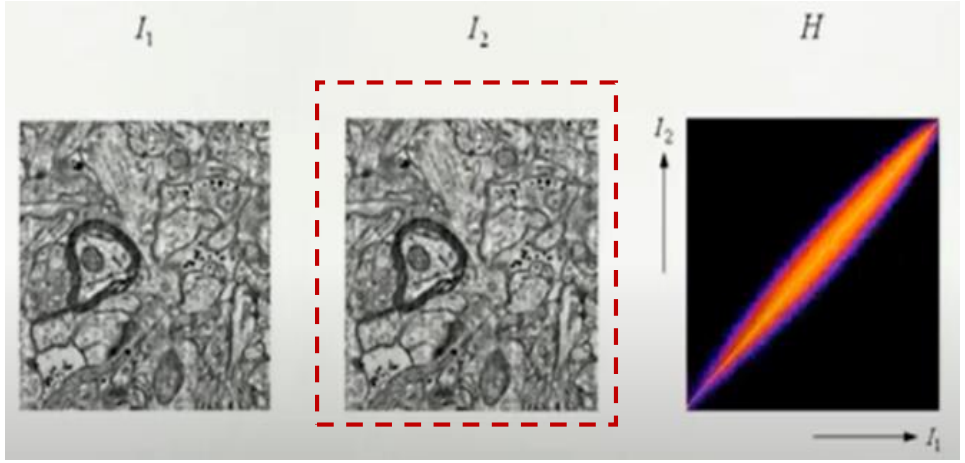
Histogram is dispersed

Joint Histogram of Images

- Joint histogram changes with the rotation in image
- Quantify similarity based on joint histogram
- Similarity measure is uses to quantify similarity







Similarity Measures

- Normalized Cross correlation (NCC)
- Sum of Intensity Squares (SIS)
- Mutual Image Information (MII)

Challenge is to find best transformation

Normalized Cross correlation (NCC)

- Is independent of brightness of images

  because mean of I1 and I2 are deducted from each pixel in numerator
- Denominator is used for normalization
- Therefore NCC does not change even if image is multiplied by a constant
- Transformation parameters for alignment should be selected such that NCC should be maximum
- NCC is useful if images have similar intensity distribution

$$NCC = \frac{\sum[I_1(x,y) - \bar{I_1}]\,[I_2(T(x,y)) - \bar{I_2}]}{\sqrt{\sum[I_1(x,y) - \bar{I_1}]^2 \sum[I_2(T(x,y)) - \bar{I_2}]^2}}$$

NCC for template matching

- Template is moved on each part of image
- Similarity of template and overlapped part of image is calculated

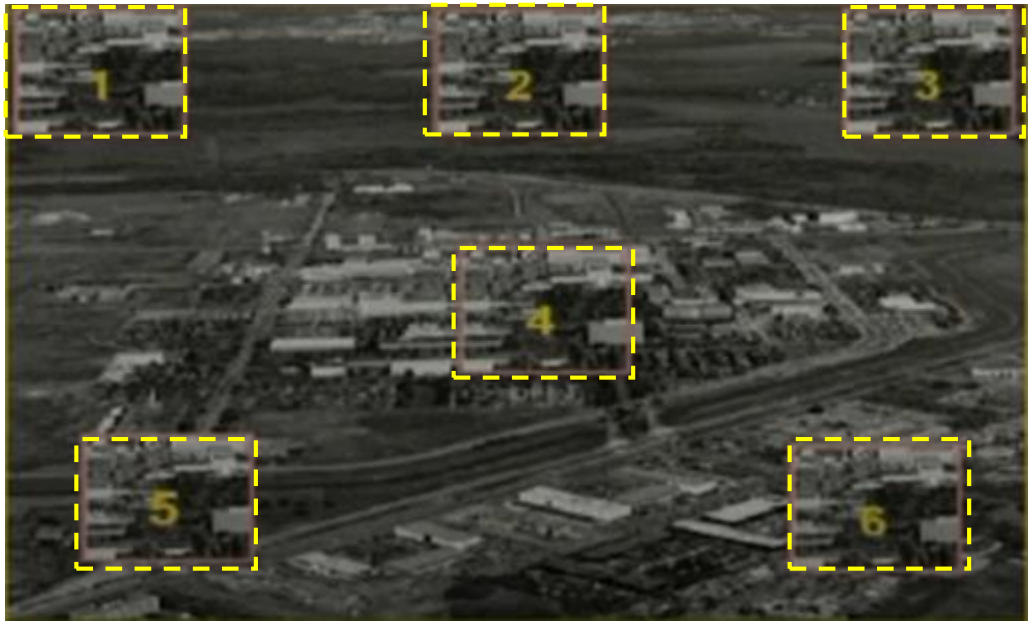
Template

Image

NCC for template matching

- Template is moved on each part of image
- Similarity of template and overlapped part of image is calculated

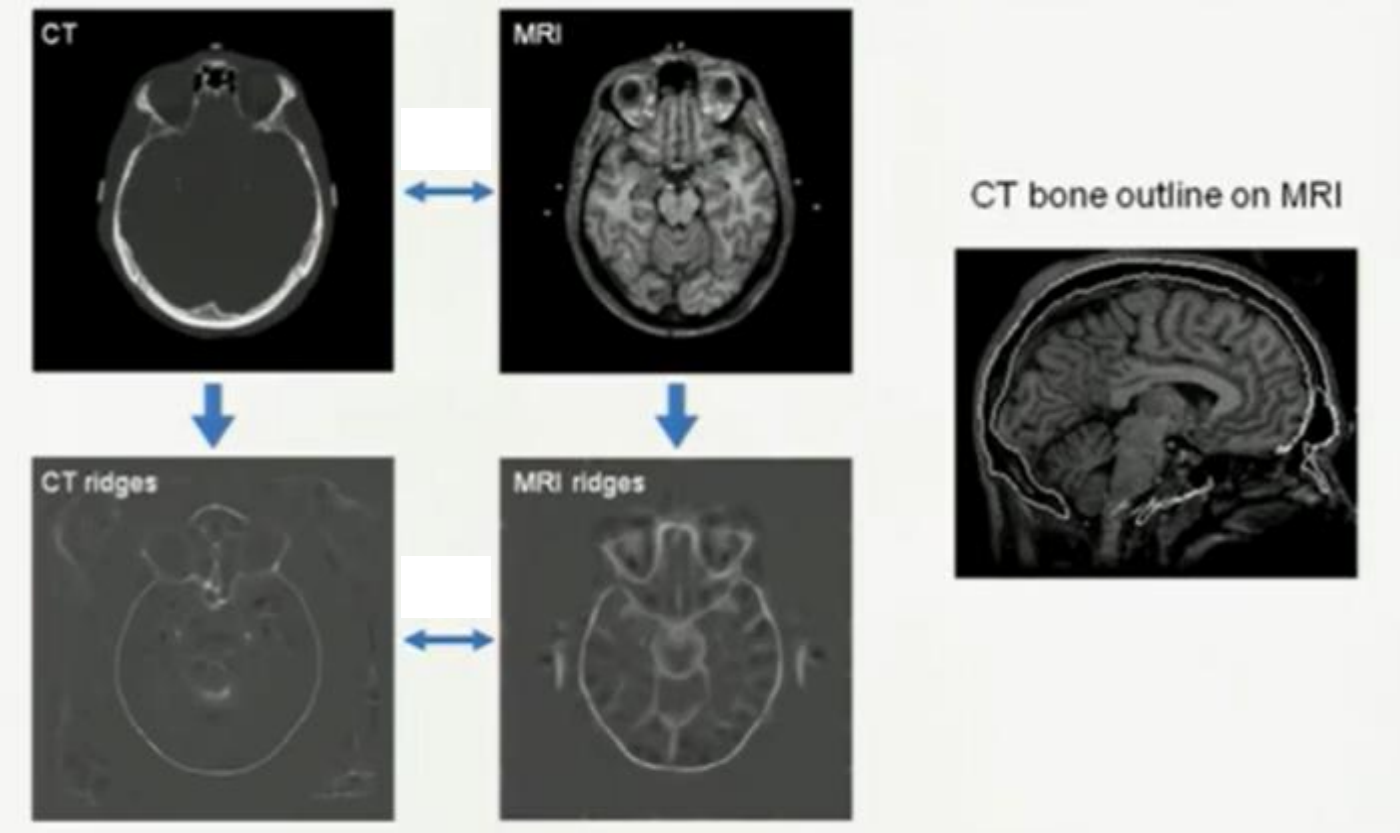
Location	Similarity Measure
1	431
2	462
3	436
4	635
5	417
6	511



Location 4 of image matches with template

NCC for Image Registration

- CT and MRI images have different intensity distribution
- NCC is not effective
- Apply ridge filter, edge detecting filter to detect edges (pre-process)
- Then apply image registration



Similarity measures

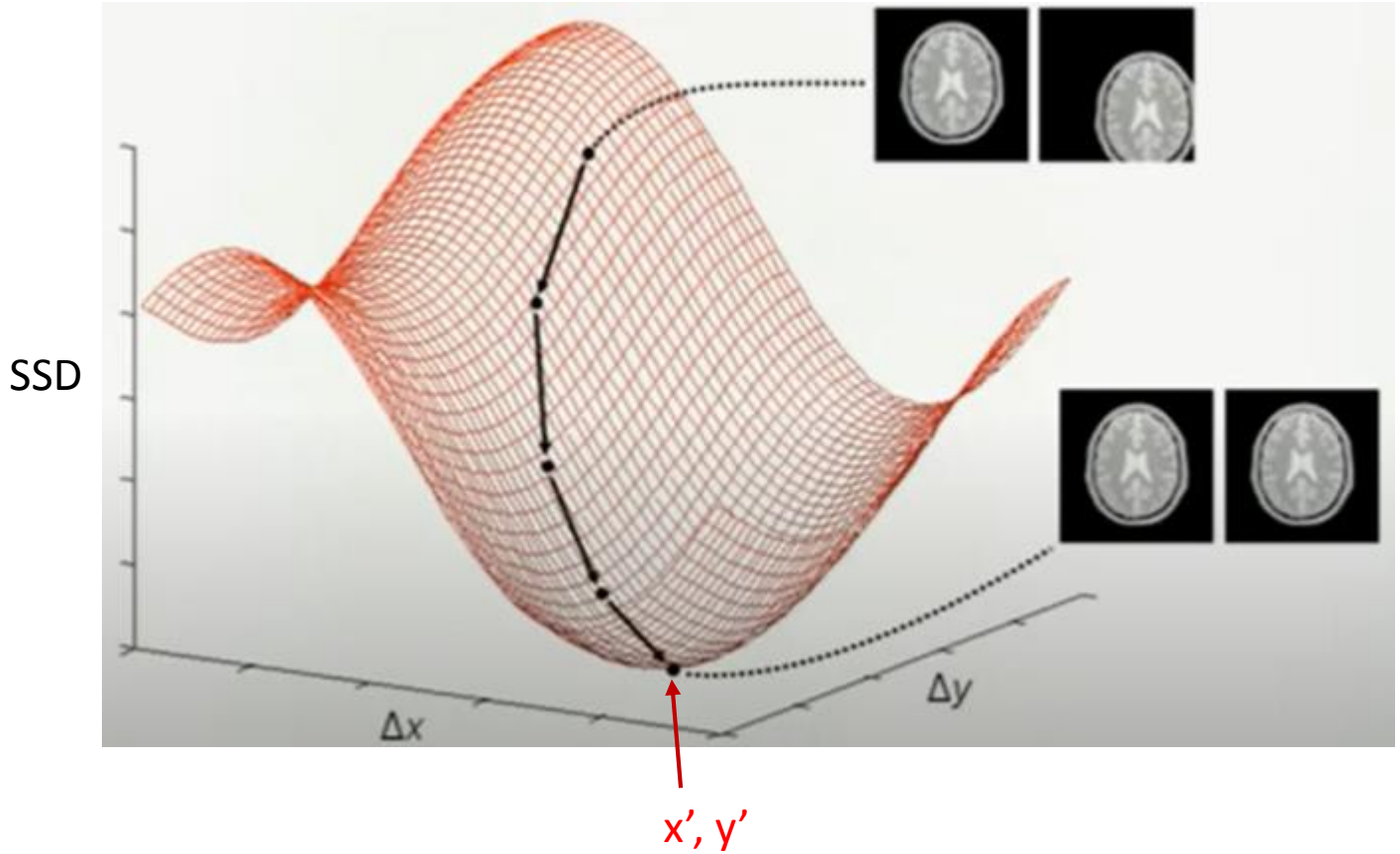
Sum of Intensity Squares (SIS) or Sum of Square Distance (SSD)

- Transformation parameters should be selected such that SID is minimum

$$SID = \sum [I_1(x, y) - I_2(T(x, y))]^2$$

Sum of Intensity Squares (SIS) for determining transformation

- At (x’, y’), SSD is minimum at best translation (shifting)
- If rotation is added , graph has three dimensions



Mutual Image Information (MII)

- Uses entropy of each image and joint entropy of both images
- Transformation parameters should be selected such that MII is maximum

$$MII = E(I_1) + E\big(T(I_2)\big) - E(I_1,\mathsf{T}(I_2))$$

Where,

$$E(I) = \sum P(i) log\left(\frac{1}{P(i)}\right)$$

Uses normalized histogram of image

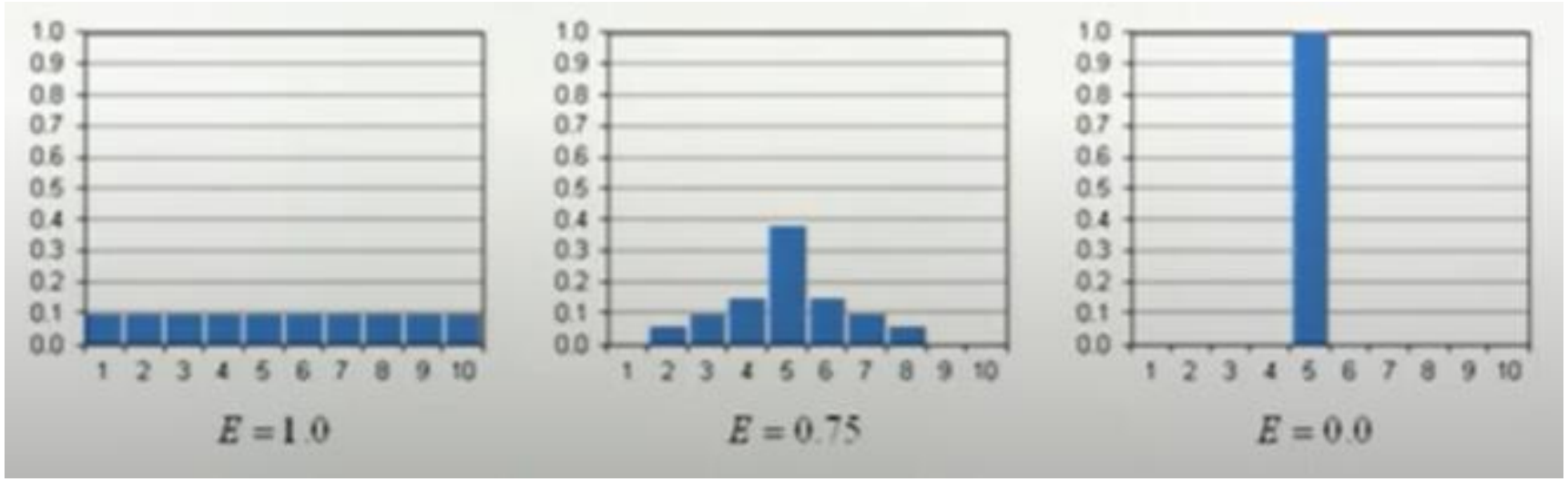
$$E(I_1, I_2) = \sum P(i,j) log\left(\frac{1}{P(i,j)}\right)$$

Uses normalized joint histogram

Entropy

- Entropy is a measure of uncertainty
- For better matching, entropy of individual images should be highest
- Joint entropy should be minimum (based on 2D joint histogram)

Normalized histogram of an image



Maximum uncertainty as probability of pixels for all intensities is same

Less uncertainty as probability of pixels for some intensities is more than others

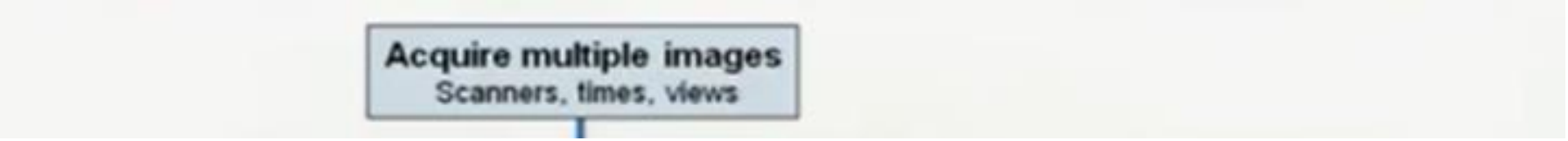
Least uncertainty as probability is nonzero for only one intensity. All pixels have same intensity

Mutual Information in overlapping regions

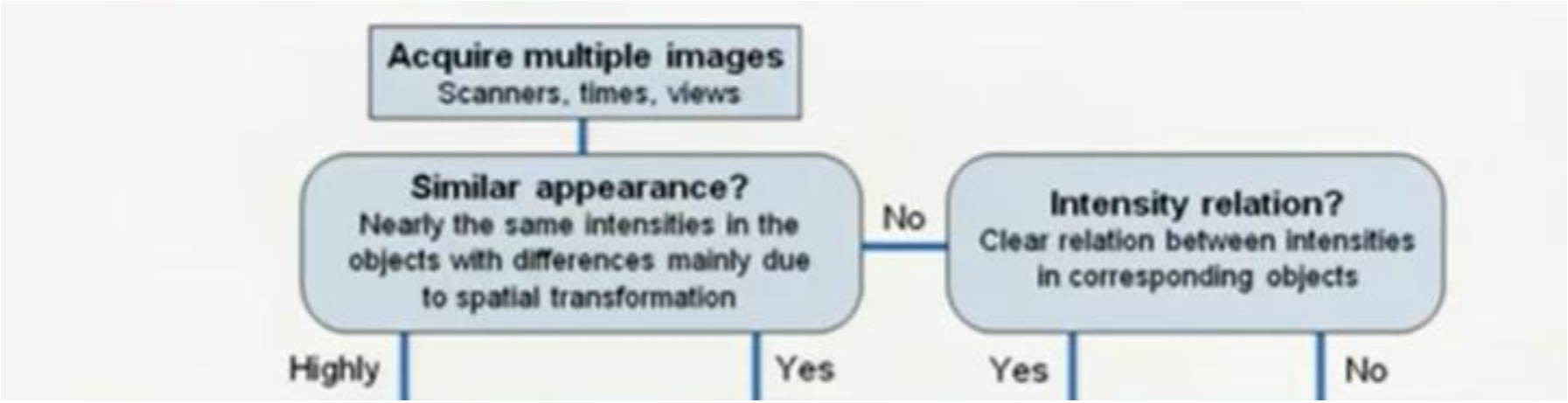
$$MI = E(I_1) + E\big(T(I_2)\big) - E(I_1,\mathsf{T}(I_2)) \qquad E(I) = \sum P(i) log\left(\frac{1}{P(i)}\right)$$

- Transformation is used to match two images
- Some of the pixels match
- Generally few pixels remain which do not match in two images
- Transformation should be such that MI is maximum
- $E(I_1)$ and $E(I_2)$ should be maximum and $E(I_1,T(I_2))$ should be minimum
- MI is Based on intensity distribution not on actual intensity
- Therefore useful for multi modal registration

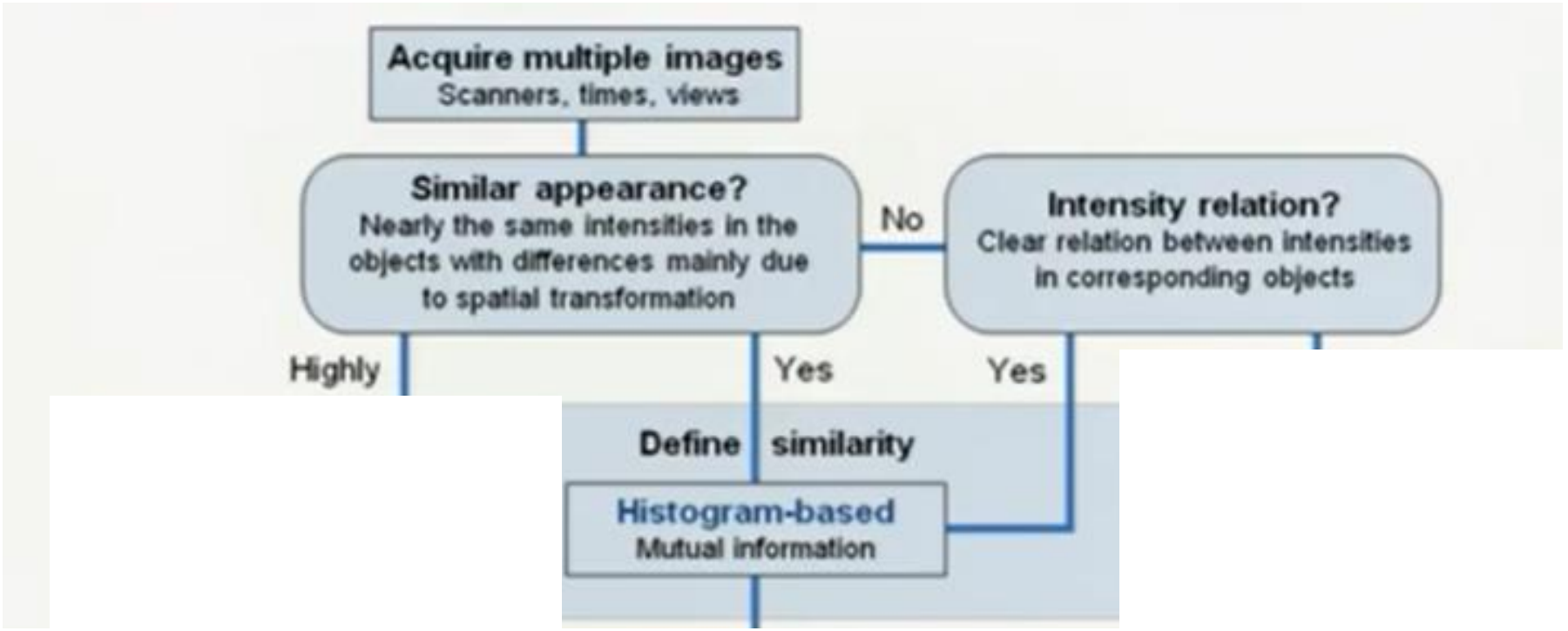
Registration using Similarity Measures



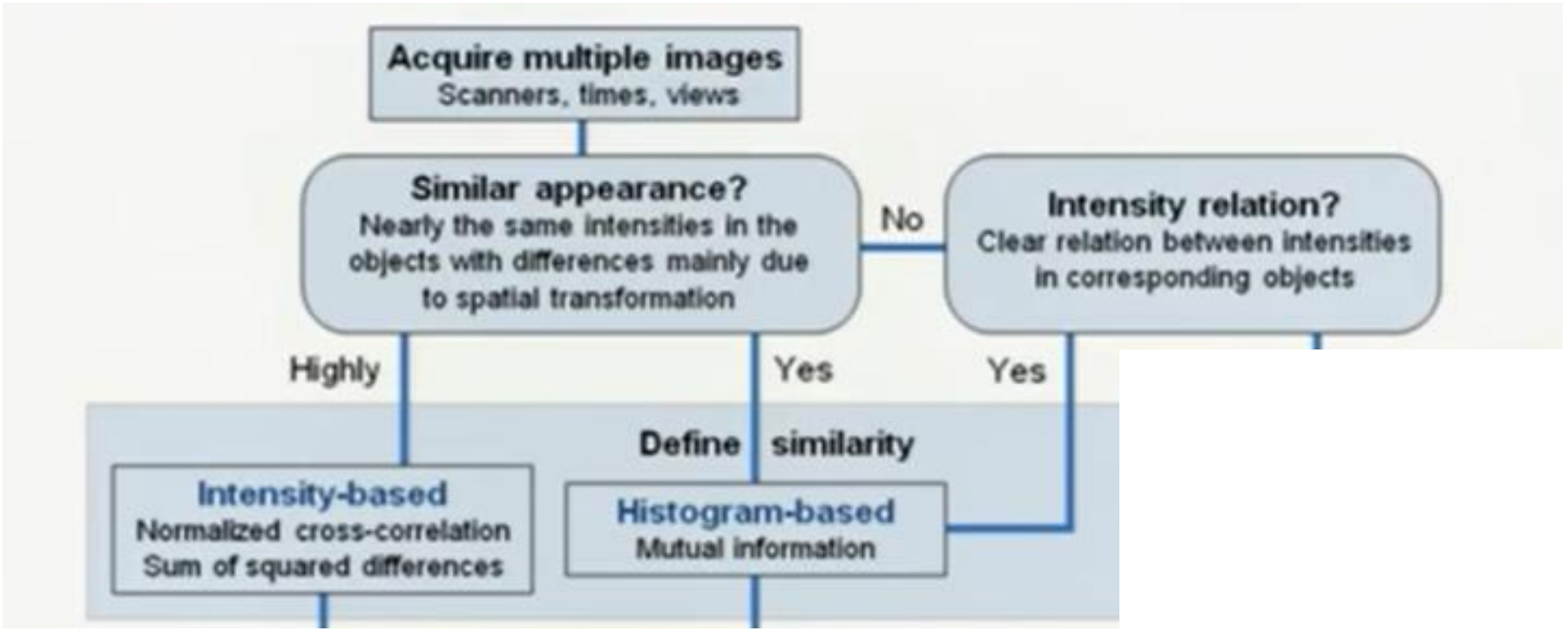
Registration using Similarity Measures



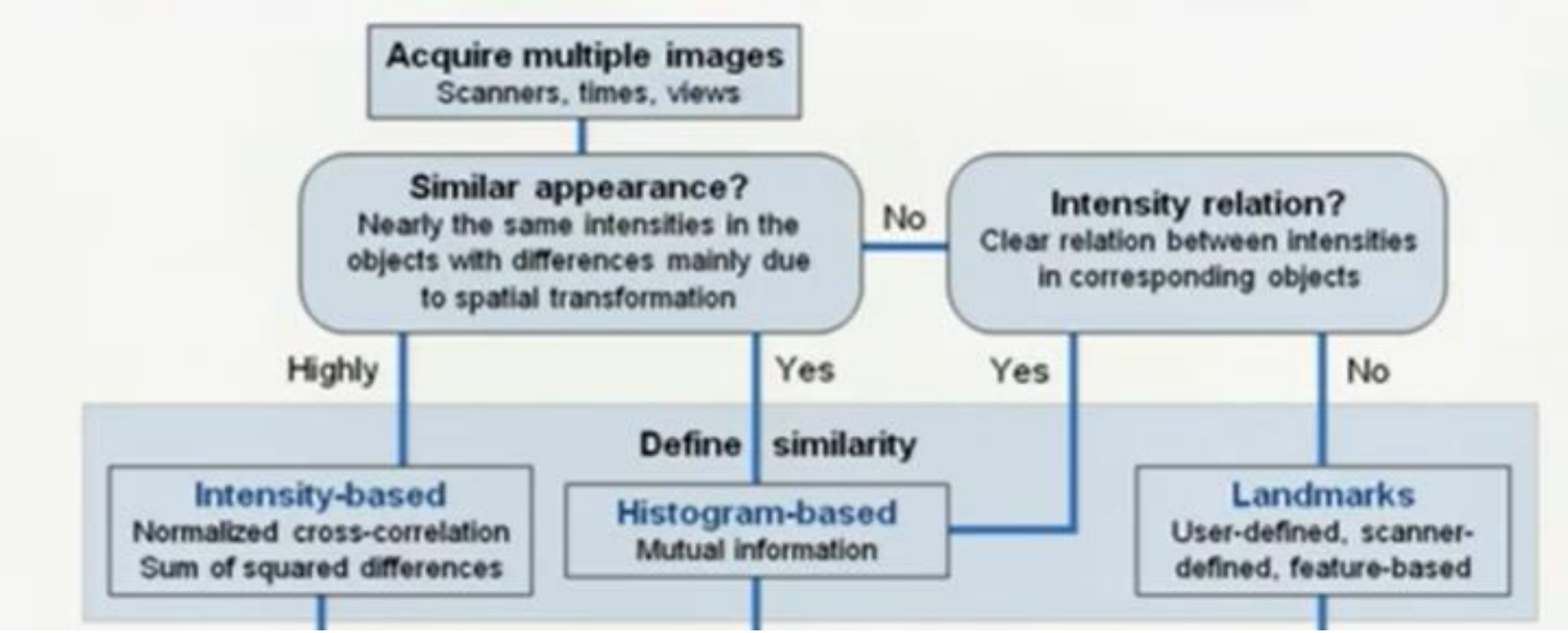
Registration using Similarity Measures



Registration using Similarity Measures



Registration using Similarity Measures



Registration using Similarity Measures