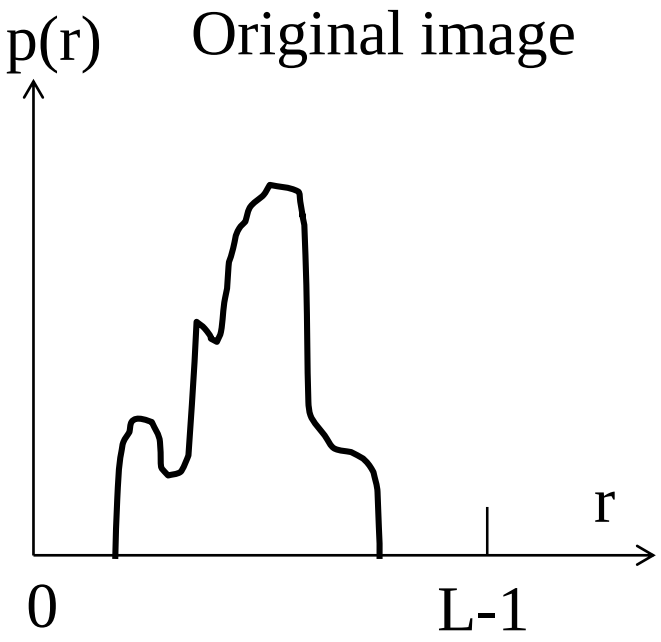
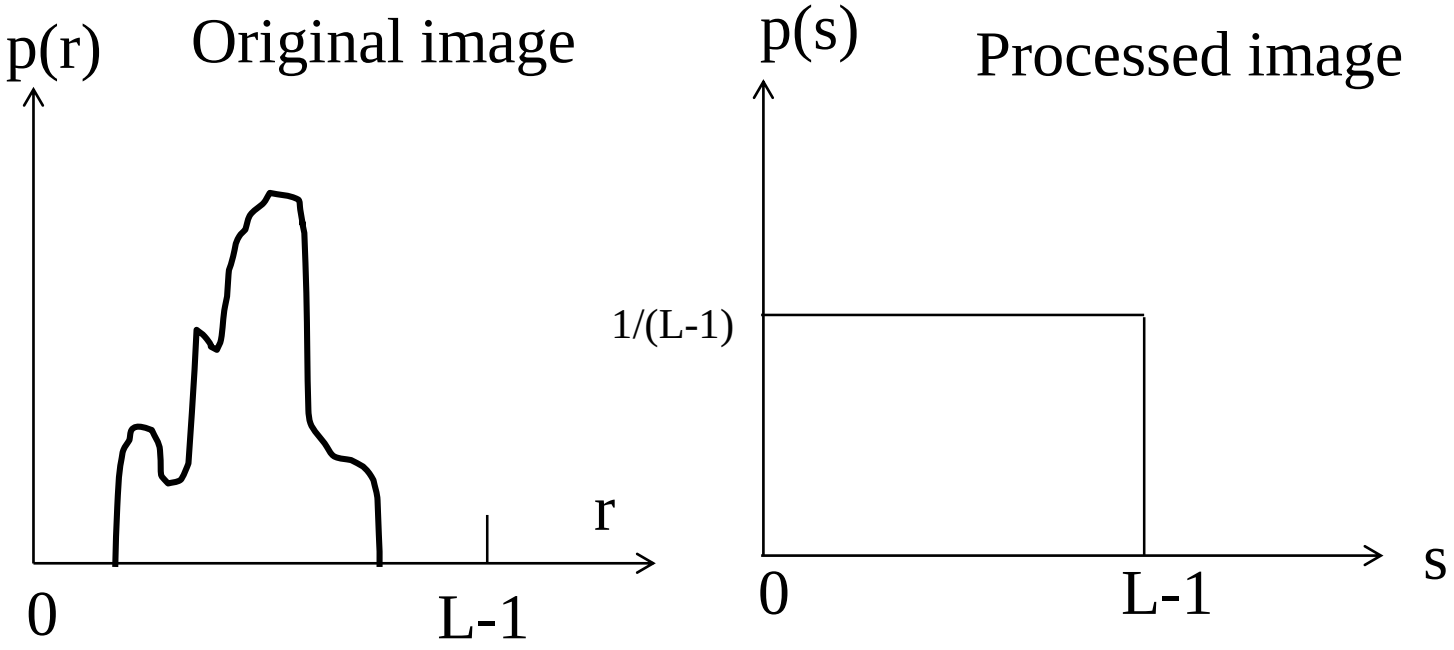
Image Enhancement (Histogram Processing)

Histogram Equalization



p(r) is probability of pixels for original image

Histogram Equalization



p(r) is probability of pixels for original image

p(s) is probability of pixels for equalized image

Histogram Equalization

- An image has N pixels
- The probability of occurrence of gray level r_k in an image is

$$p_r(r_k) = \frac{n_k}{N} \qquad k = 0,1,2,\ldots,L-1$$

  Where n_k is number of pixels with intensity r_k
- The transformation function is

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k} \frac{n_j}{n} \qquad k = 0,1,2,\ldots,L-1$$

- An output image is obtained by mapping each pixel with level r_k in the input image into a corresponding pixel with level s_k

Histogram Equalization

- 3-bit image of size 64x64 has intensity distribution table

r	nr	c	(L-1) x c/MN	s	ns
0	790				
1	1023				
2	850				
3	656				
4	329				
5	245				
6	122				
7	81				

Histogram Equalization

- 3-bit image of size 64x64 has intensity distribution table

r	nr	c	(L-1) x c/MN	s	ns
0	790	790			
1	1023	1813			
2	850				
3	656				
4	329				
5	245				
6	122				
7	81				

Histogram Equalization

- 3-bit image of size 64x64 has intensity distribution table

r	nr	c	(L-1) x c/MN	s	ns
0	790	790			
1	1023	1813			
2	850	2663			
3	656	3319			
4	329	3648			
5	245	3893			
6	122	4015			
7	81	4096			

Histogram Equalization

- 3-bit image of size 64x64 has intensity distribution table

r	nr	c	(L-1) x c/MN	s	ns
0	790	790	1.35		
1	1023	1813			
2	850	2663			
3	656	3319			
4	329	3648			
5	245	3893			
6	122	4015			
7	81	4096			

Histogram Equalization

- 3-bit image of size 64x64 has intensity distribution table

r	nr	c	(L-1) x c/MN	s	ns
0	790	790	1.35		
1	1023	1813	3.09		
2	850	2663	4.55		
3	656	3319	5.67		
4	329	3648	6.23		
5	245	3893	6.65		
6	122	4015	6.86		
7	81	4096	7		

Histogram Equalization

- 3-bit image of size 64x64 has intensity distribution table

r	nr	c	(L-1) x c/MN = (7x c)/(64x64)	s	ns
0	790	790	1.35	1	
1	1023	1813	3.09	3	
2	850	2663	4.55	5	
3	656	3319	5.67	6	
4	329	3648	6.23	6	
5	245	3893	6.65	7	
6	122	4015	6.86	7	
7	81	4096	7	7	

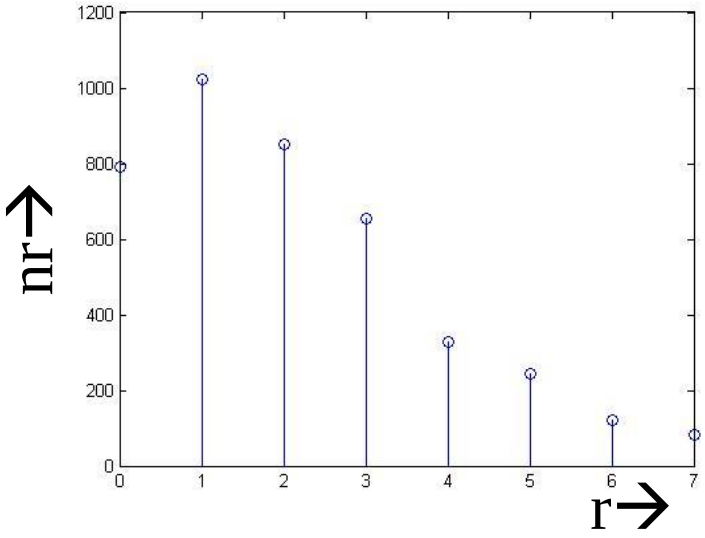
Histogram Equalization

- 3-bit image of size 64x64 has intensity distribution table
- M rows and N columns

r	nr	c	(L-1) x c/MN = (7x c)/(64x64)	s	ns
0	790	790	1.35	1	790
1	1023	1813	3.09	3	1023
2	850	2663	4.55	5	850
3	656	3319	5.67	6	985
4	329	3648	6.23	6	
5	245	3893	6.65	7	448
6	122	4015	6.86	7	
7	81	4096	7	7	

Histogram before and after equalization

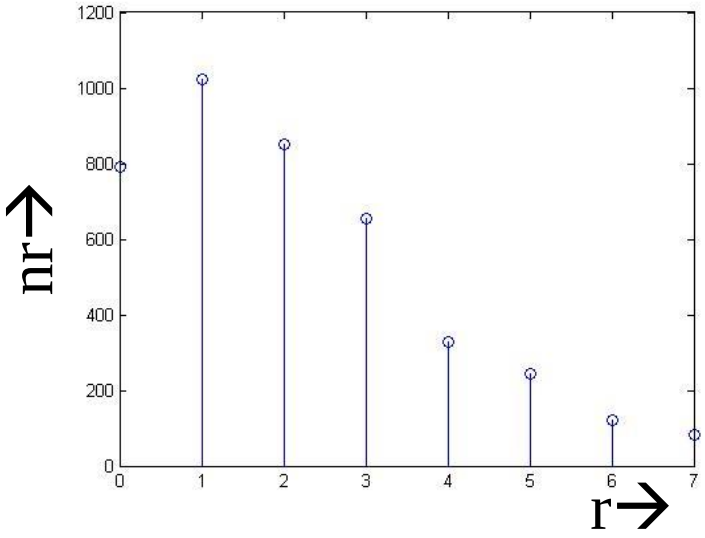
r	nr	s	ns
0	790	1	790
1	1023	3	1023
2	850	5	850
3	656	6	985
4	329	6	
5	245	7	448
6	122	7	
7	81	7	



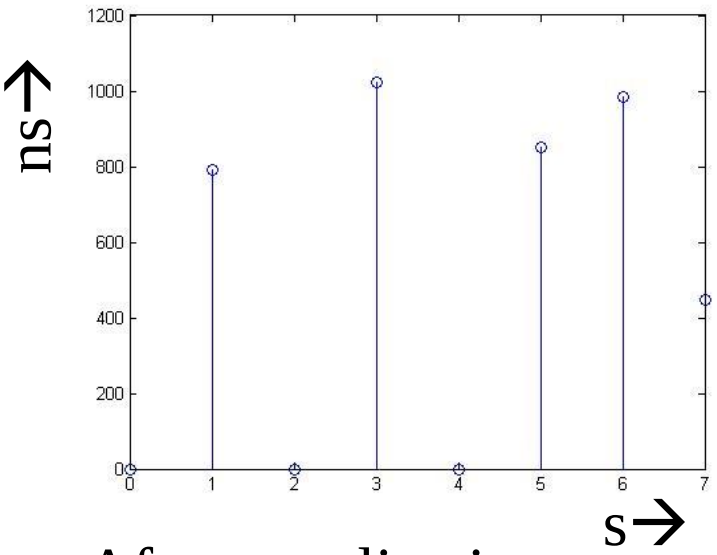
Before equalization

Histogram before and after equalization

r	nr	s	ns
0	790	1	790
1	1023	3	1023
2	850	5	850
3	656	6	985
4	329	6	
5	245	7	448
6	122	7	
7	81	7	



Before equalization



After equalization

Histogram Equalization

- Image matrix of an 3-bit image is given below. Improve contrast the image using histogram processing

$$A = \begin{bmatrix} 7 & 5 & 6 & 4 & 3 & 1 & 7 & 6 & 2 & 4 \\ 5 & 5 & 5 & 2 & 1 & 3 & 7 & 6 & 5 & 1 \\ 3 & 6 & 7 & 5 & 3 & 4 & 5 & 5 & 5 & 6 \\ 3 & 7 & 7 & 6 & 5 & 6 & 7 & 3 & 4 & 4 \\ 6 & 7 & 6 & 6 & 7 & 6 & 6 & 4 & 4 & 3 \\ 6 & 7 & 7 & 5 & 5 & 4 & 2 & 0 & 1 & 2 \\ 5 & 6 & 5 & 6 & 6 & 3 & 4 & 6 & 2 & 5 \\ 7 & 6 & 7 & 5 & 1 & 3 & 2 & 4 & 4 & 2 \\ 5 & 4 & 3 & 2 & 3 & 3 & 3 & 2 & 2 & 0 \\ 4 & 5 & 5 & 6 & 6 & 6 & 7 & 7 & 2 & 5 \end{bmatrix}$$

r	nr	c	(L-1) c/MN	s	ns
0					
1					
2					
3					
4					
5					
6					
7					

Histogram Equalization

- Image matrix of an 3-bit image is given below. Improve contrast the image using histogram processing

$$A = \begin{bmatrix} 7 & 5 & 6 & 4 & 3 & 1 & 7 & 6 & 2 & 4 \\ 5 & 5 & 5 & 2 & 1 & 3 & 7 & 6 & 5 & 1 \\ 3 & 6 & 7 & 5 & 3 & 4 & 5 & 5 & 5 & 6 \\ 3 & 7 & 7 & 6 & 5 & 6 & 7 & 3 & 4 & 4 \\ 6 & 7 & 6 & 6 & 7 & 6 & 6 & 4 & 4 & 3 \\ 6 & 7 & 7 & 5 & 5 & 4 & 2 & 0 & 1 & 2 \\ 5 & 6 & 5 & 6 & 6 & 3 & 4 & 6 & 2 & 5 \\ 7 & 6 & 7 & 5 & 1 & 3 & 2 & 4 & 4 & 2 \\ 5 & 4 & 3 & 2 & 3 & 3 & 3 & 2 & 2 & 0 \\ 4 & 5 & 5 & 6 & 6 & 6 & 7 & 7 & 2 & 5 \end{bmatrix}$$

r	nr	c	(L-1) c/MN	s	ns
0	2				
1	5				
2	11				
3	13				
4	13				
5	20				
6	21				
7	15				

Histogram Equalization

r	nr	c	(L-1) x c/MN	s	ns
0	2				
1	5				
2	11				
3	13				
4	13				
5	20				
6	21				
7	15				

Histogram Equalization

r	nr	c	(L-1) x c/MN	s	ns
0	2	2			
1	5	7			
2	11	18			
3	13	31			
4	13	44			
5	20	64			
6	21	85			
7	15	100			

Histogram Equalization

r	nr	c	(L-1) x c/MN	s	ns
0	2	2	0.14		
1	5	7	0.49		
2	11	18	1.26		
3	13	31	2.17		
4	13	44	3.08		
5	20	64	4.48		
6	21	85	5.95		
7	15	100	7		

Histogram Equalization

r	nr	c	(L-1) x c/MN	s	ns
0	2	2	0.14	0	
1	5	7	0.49	0	
2	11	18	1.26	1	
3	13	31	2.17	2	
4	13	44	3.08	3	
5	20	64	4.48	4	
6	21	85	5.95	6	
7	15	100	7	7	

Histogram Equalization

r	nr	c	(L-1) x c/MN	s	ns
0	2	2	0.14	0	7
1	5	7	0.49	0	
2	11	18	1.26	1	11
3	13	31	2.17	2	13
4	13	44	3.08	3	13
5	20	64	4.48	4	20
6	21	85	5.95	6	21
7	15	100	7	7	15

r	s
0	0

Histogram Equalization

r	nr	c	(L-1) x c/MN	s	ns
0	2	2	0.14	0	7
1	5	7	0.49	0	
2	11	18	1.26	1	11
3	13	31	2.17	2	13
4	13	44	3.08	3	13
5	20	64	4.48	4	20
6	21	85	5.95	6	21
7	15	100	7	7	15

r	s
0	0
1	0

Histogram Equalization

r	nr	c	(L-1) x c/MN	s	ns
0	2	2	0.14	0	7
1	5	7	0.49	0	
2	11	18	1.26	1	11
3	13	31	2.17	2	13
4	13	44	3.08	3	13
5	20	64	4.48	4	20
6	21	85	5.95	6	21
7	15	100	7	7	15

r	s
0	0
1	0
2	1
3	2
4	3
5	4
6	6
7	7

Histogram Equalization

Before equalization

$$A = \begin{bmatrix} 7 & 5 & 6 & 4 & 3 & 1 & 7 & 6 & 2 & 4 \\ 5 & 5 & 5 & 2 & 1 & 3 & 7 & 6 & 5 & 1 \\ 3 & 6 & 7 & 5 & 3 & 4 & 5 & 5 & 5 & 6 \\ 3 & 7 & 7 & 6 & 5 & 6 & 7 & 3 & 4 & 4 \\ 6 & 7 & 6 & 6 & 7 & 6 & 6 & 4 & 4 & 3 \\ 6 & 7 & 7 & 5 & 5 & 4 & 2 & 0 & 1 & 2 \\ 5 & 6 & 5 & 6 & 6 & 3 & 4 & 6 & 2 & 5 \\ 7 & 6 & 7 & 5 & 1 & 3 & 2 & 4 & 4 & 2 \\ 5 & 4 & 3 & 2 & 3 & 3 & 3 & 2 & 2 & 0 \\ 4 & 5 & 5 & 6 & 6 & 6 & 7 & 7 & 2 & 5 \end{bmatrix}$$

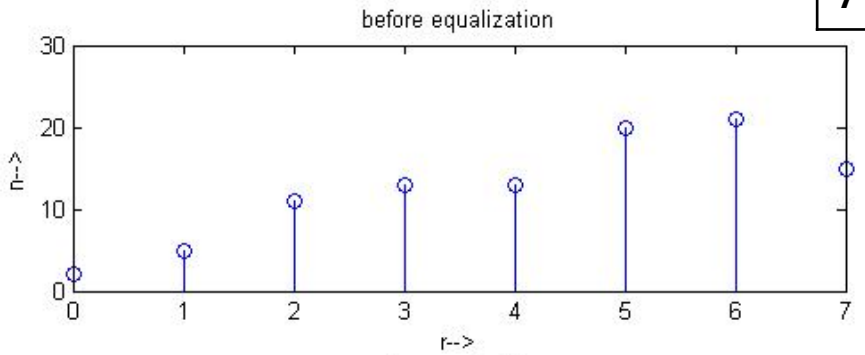
r	s
0	0
1	0
2	1
3	2
4	3
5	4
6	6
7	7

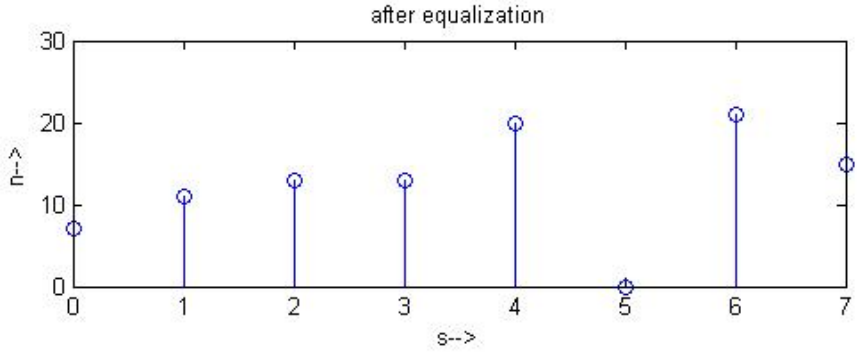
After equalization

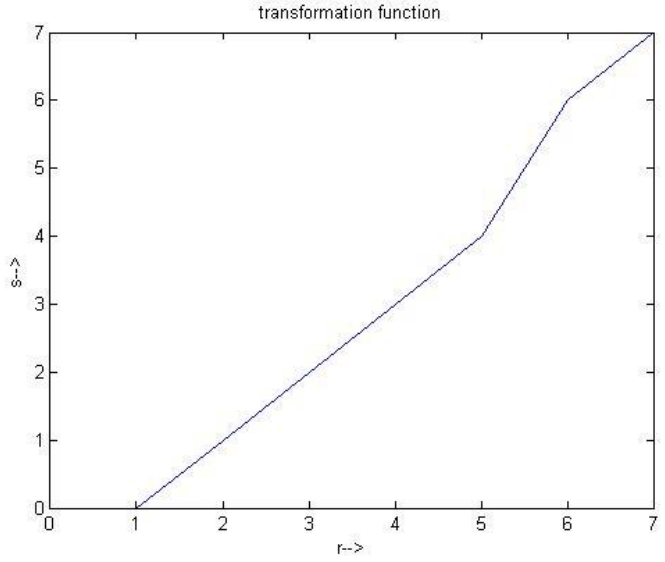
$$A = \begin{bmatrix} 7 & 4 & 6 & 3 & 2 & 0 & 7 & 6 & 1 & 3 \\ 4 & 4 & 4 & 1 & 0 & 2 & 7 & 6 & 4 & 0 \\ 2 & 6 & 7 & 4 & 2 & 3 & 4 & 4 & 4 & 6 \\ 2 & 7 & 7 & 6 & 4 & 6 & 7 & 2 & 3 & 3 \\ 6 & 7 & 6 & 6 & 7 & 6 & 6 & 3 & 3 & 2 \\ 6 & 7 & 7 & 4 & 4 & 3 & 1 & 0 & 0 & 1 \\ 4 & 6 & 4 & 6 & 6 & 2 & 3 & 6 & 2 & 4 \\ 7 & 6 & 7 & 4 & 0 & 2 & 1 & 3 & 3 & 1 \\ 4 & 3 & 2 & 1 & 2 & 2 & 2 & 1 & 1 & 0 \\ 3 & 4 & 4 & 6 & 6 & 6 & 7 & 7 & 1 & 4 \end{bmatrix}$$

HISTOGRAMS

r	nr	s	ns
0	2	0	7
1	5	0	
2	11	1	11
3	13	2	13
4	13	3	13
5	20	4	20
6	21	6	21
7	15	7	15







Histogram Matching/ Specification

- Similar to histogram equalization
- Does not try to make the histogram flat
- Histogram of a specified shape, say $p_z(z)$ is computed

Histogram Matching

- Given 3-bit image of size 64x64 has intensity distribution table determine transformation to match required intensity, z and corresponding number of pixels, n_z
  1. Compute histogram, p(r) of image and equalize it to generate,s

r	n_r	s_r	z	n_z		
0	790		0	0		
1	1023		1	0		
2	850		2	0		
3	656		3	614		
4	329		4	819		
5	245		5	1229		
6	122		6	819		
7	81		7	614		

Histogram Matching

1. Compute histogram equalization to generate, s

r	n_r	s_r	z	n_z		
0	790	1	0	0		
1	1023	3	1	0		
2	850	5	2	0		
3	656	6	3	614		
4	329	6	4	819		
5	245	7	5	1229		
6	122	7	6	819		
7	81	7	7	614		

Histogram Matching

2. Determine histogram equalization for specific requirement

r	n_r	s_r	z	n_z	c_z	
0	790	1	0	0		
1	1023	3	1	0		
2	850	5	2	0		
3	656	6	3	614		
4	329	6	4	819		
5	245	7	5	1229		
6	122	7	6	819		
7	81	7	7	614		

Histogram Matching

2. Determine histogram equalization, s_z

r	n_r	s_r	z	n_z	c_z	s_z = $c_z(L-1)/MN$ = $c_z(7)/4096$
0	790	1	0	0	0	
1	1023	3	1	0	0	
2	850	5	2	0	0	
3	656	6	3	614	614	
4	329	6	4	819	1433	
5	245	7	5	1229	2662	
6	122	7	6	819	3482	
7	81	7	7	614	4096	

Histogram Matching

2. Determine histogram equalization, s_z

r	n_r	s_r	z	n_z	c_z	s_z = $c_z(L-1)/MN$ = $c_z(7)/4096$
0	790	1	0	0	0	0
1	1023	3	1	0	0	0
2	850	5	2	0	0	0
3	656	6	3	614	614	1.05
4	329	6	4	819	1433	2.45
5	245	7	5	1229	2662	4.55
6	122	7	6	819	3482	5.95
7	81	7	7	614	4096	7

Histogram Matching

3. Determine histogram equalization, $\sim s_z$

r	n_r	s_r	z	n_z	s_z	$\sim s_z$
0	790	1	0	0	0	0
1	1023	3	1	0	0	0
2	850	5	2	0	0	0
3	656	6	3	614	1.05	1
4	329	6	4	819	2.45	2
5	245	7	5	1229	4.55	5
6	122	7	6	819	5.95	6
7	81	7	7	614	7	7

Histogram Matching

- For every value of s_r, use the stored value of $\sim s_z$ which is closest to s
- Choose corresponding value of z

r	n_r	s_r	z	n_z	s_z	$\sim s_z$
0	790	1	0	0	0	0
1	1023	3	1	0	0	0
2	850	5	2	0	0	0
3	656	6	3	614	1.05	1
4	329	6	4	819	2.45	2
5	245	7	5	1229	4.55	5
6	122	7	6	819	5.95	6
7	81	7	7	614	7	7

s to z mapping

s	z
1	3
3	
5	
6	
7	

Histogram Matching

- For every value of s_r, use the stored value of $\sim s_z$ which is closest to s
- Choose corresponding value of z

r	n_r	s_r	z	n_z	s_z	$\sim s_z$
0	790	1	0	0	0	0
1	1023	3	1	0	0	0
2	850	5	2	0	0	0
3	656	6	3	614	1.05	1
4	329	6	4	819	2.45	2
5	245	7	5	1229	4.55	5
6	122	7	6	819	5.95	6
7	81	7	7	614	7	7

s to z mapping

s	z
1	3
3	4
5	
6	
7	

Histogram Matching

- For every value of s_r, use the stored value of $\sim s_z$ which is closest to s
- Choose corresponding value of z

r	n_r	s_r	z	n_z	s_z	$\sim s_z$
0	790	1	0	0	0	0
1	1023	3	1	0	0	0
2	850	5	2	0	0	0
3	656	6	3	614	1.05	1
4	329	6	4	819	2.45	2
5	245	7	5	1229	4.55	5
6	122	7	6	819	5.95	6
7	81	7	7	614	7	7

s to z mapping

s	z
1	3
3	4
5	5
6	6
7	7

Mapping for Histogram Matching

- For every value of r, select corresponding value of s
- For every value of s select corresponding value of z

r	n_r	s	z
0	790	1	
1	1023	3	
2	850	5	
3	656	6	
4	329	6	
5	245	7	
6	122	7	
7	81	7	

r to z mapping

s to z mapping

s	z
1	3
3	4
5	5
6	6
7	7

Mapping for Histogram Matching

- For every value of r, select corresponding value of s
- For every value of s select corresponding value of z

r	n_r	s	z
0	790	1	3
1	1023	3	4
2	850	5	5
3	656	6	6
4	329	6	6
5	245	7	7
6	122	7	7
7	81	7	7

r to z mapping

s to z mapping

s	z
1	3
3	4
5	5
6	6
7	7

Mapping for Histogram Matching

For each value of z, determine number of pixels

r	s	z	n_r	n_z
0	1	3	790	790
1	3	4		
2	5	5		
3	6	6		
4	6	6		
5	7	7		
6	7	7		
7	7	7		

r to z mapping and corresponding number of pixels

Mapping for Histogram Matching

For each value of z, determine number of pixels

r	s	z	n_r	n_z
0	1	3	790	790
1	3	4	1023	1023
2	5	5		
3	6	6		
4	6	6		
5	7	7		
6	7	7		
7	7	7		

r to z mapping and corresponding number of pixels

Mapping for Histogram Matching

For each value of z, determine number of pixels

r	s	z	n_r	n_z
0	1	3	790	790
1	3	4	1023	1023
2	5	5	850	850
3	6	6	656	985
4	6	6	329	
5	7	7		
6	7	7		
7	7	7		

r to z mapping and corresponding number of pixels

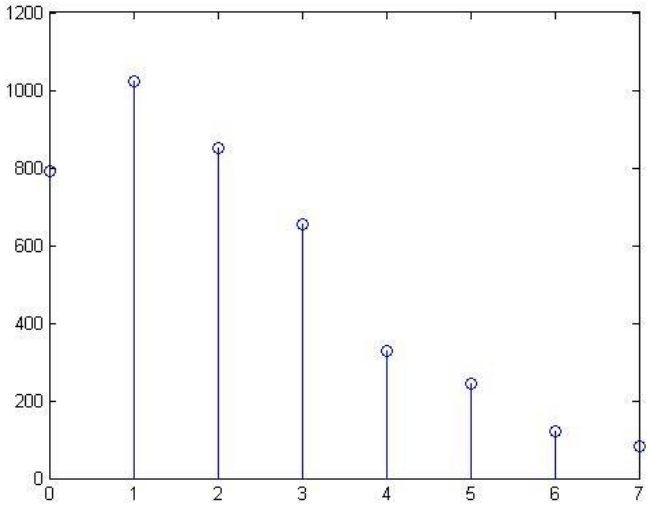
Mapping for Histogram Matching

For each value of z, determine number of pixels

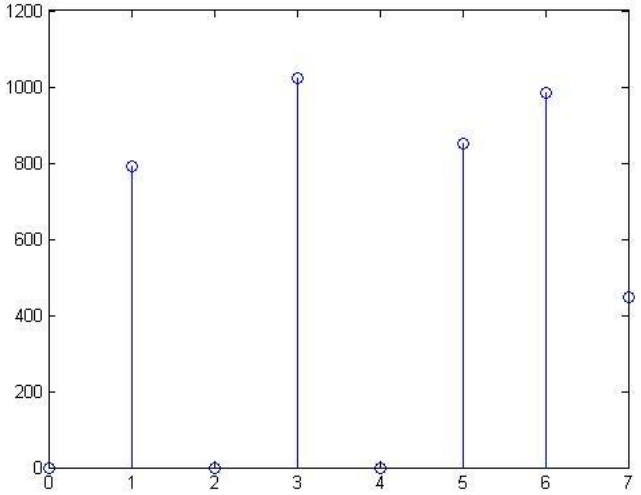
r	s	z	n_r	n_z
0	1	3	790	790
1	3	4	1023	1023
2	5	5	850	850
3	6	6	656	985
4	6	6	329	
5	7	7	245	448
6	7	7	122	
7	7	7	81	

r to z mapping and corresponding number of pixels

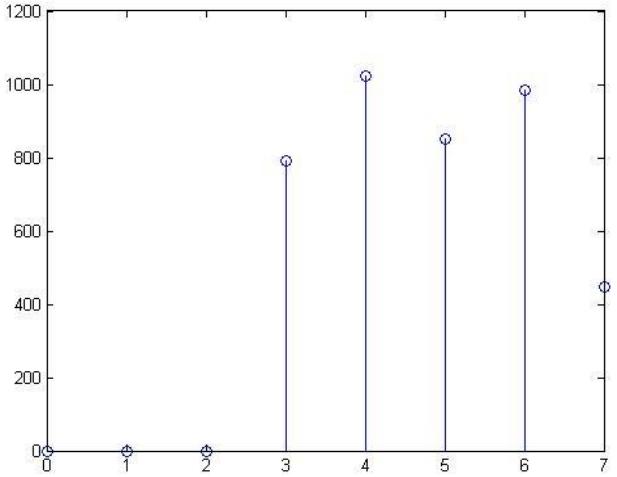
Histogram before and after matching



Before equalization

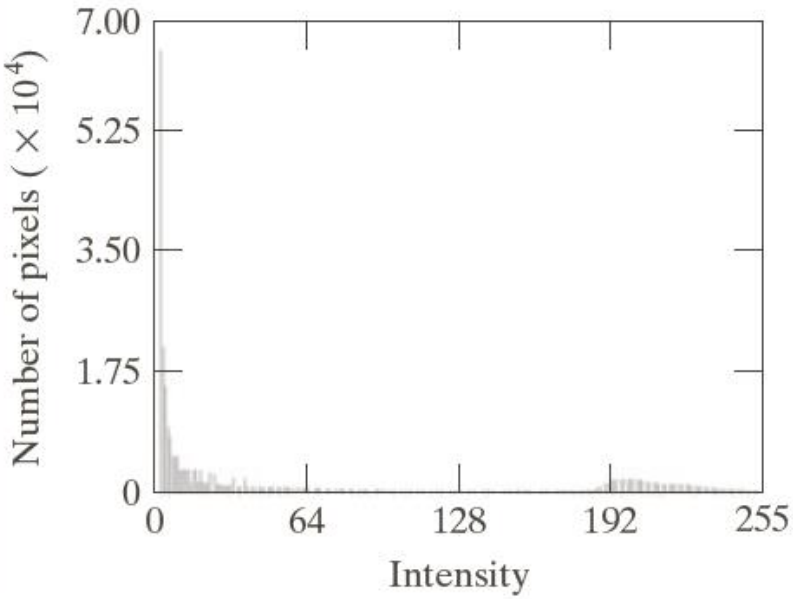


After equalization



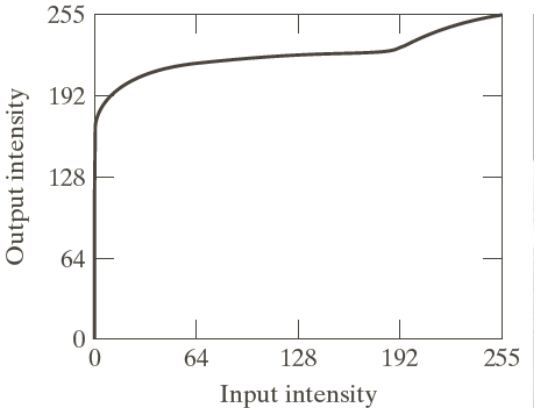
After matching

IMAGE OF MARS

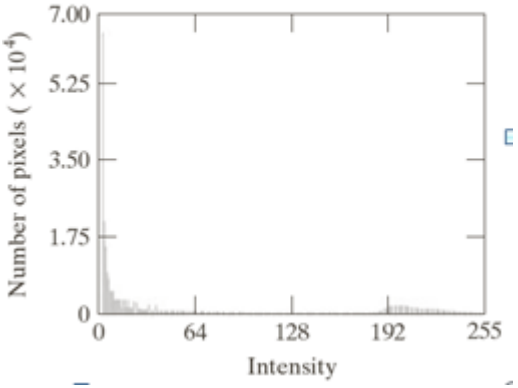


HISTOGRAM EQUALIZATION

Transfer Function

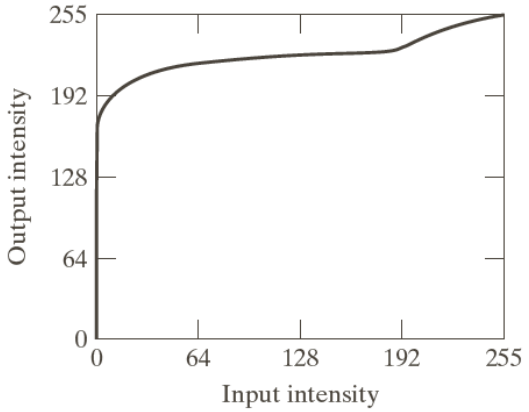


Histogram of original image

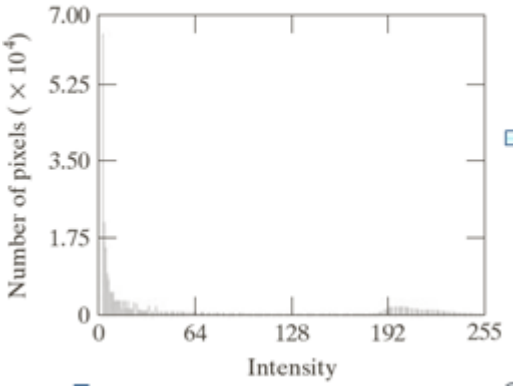


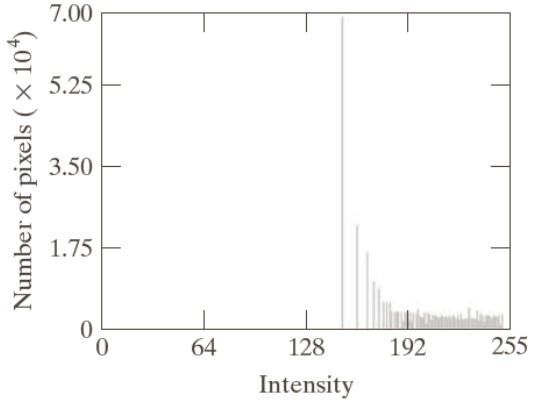
HISTOGRAM EQUALIZATION

Transfer Function



Histogram of original image

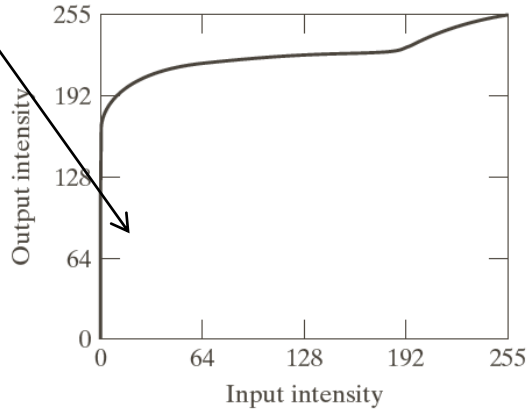




Histogram of equalized Image

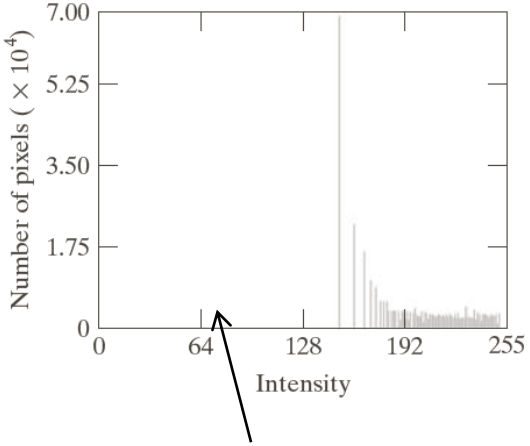
HISTOGRAM EQUALIZATION

Transfer Function



Sometimes equalization is not effective

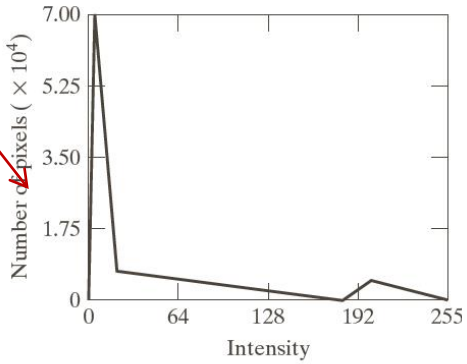
Equalized Image



Histogram of equalized Image

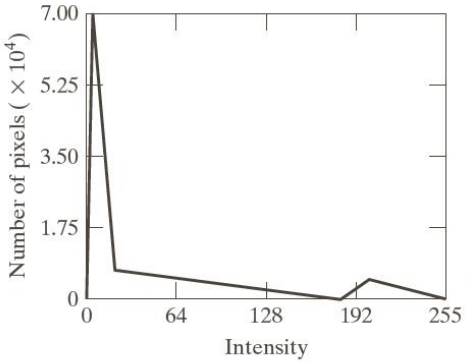
HISTOGRAM MATCHING

Required Histogram

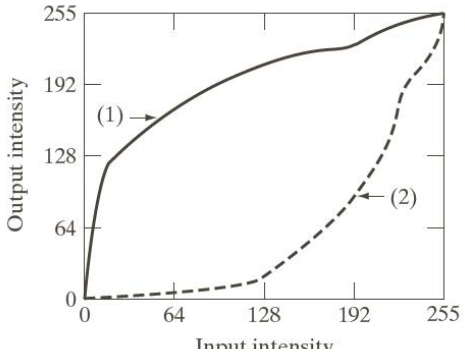


HISTOGRAM MATCHING

Required Histogram

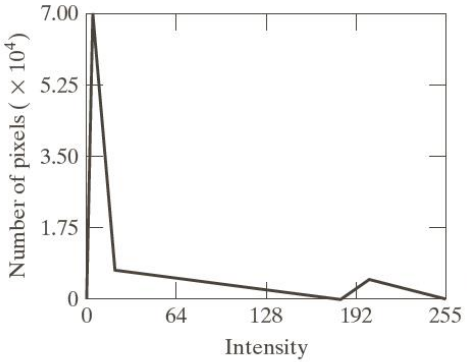


Transform ation function

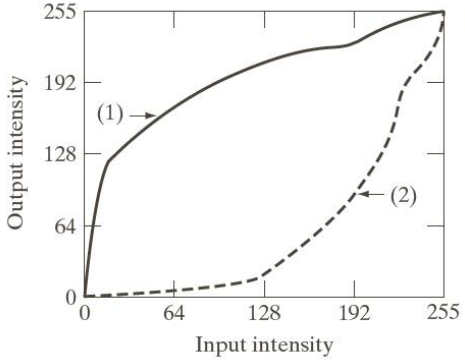


HISTOGRAM MATCHING

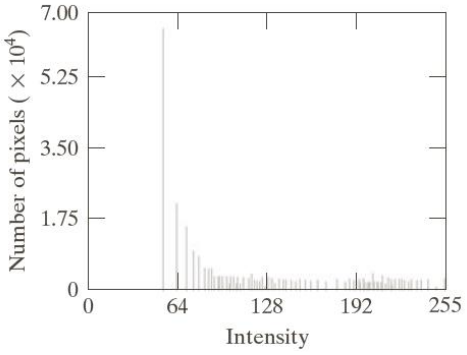
Required Histogram



Transform ation function

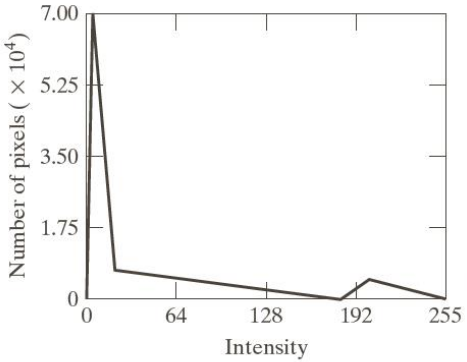


Histogram of matched image



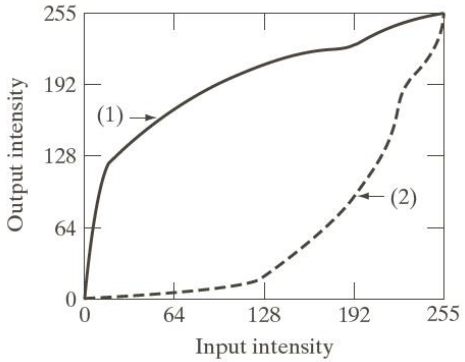
HISTOGRAM MATCHING

Required Histogram

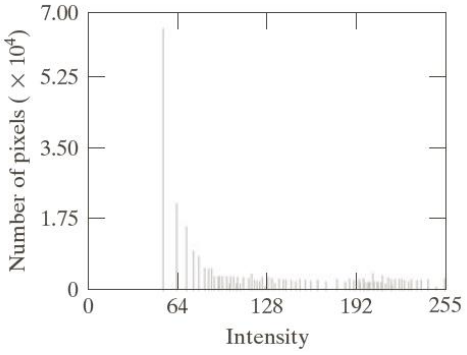


Enhanced Image

Transformation function

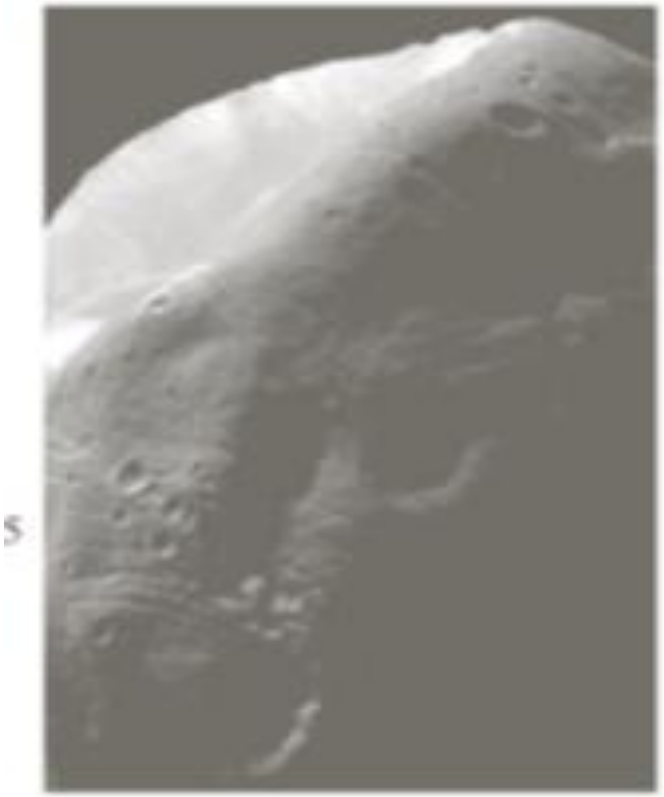


Histogram of matched image



HISTOGRAM EQUALIZED AND MATCHED IMAGES

equalized



matched

Global & Local Enhancement

- Sometimes it is necessary to enhance details over small area of an image
- Number of pixels in small area has negligible influence on the computation for entire image
- Global histogram processing
  - intensity distribution of entire image
  - Suitable for overall enhancement
  - Pixels are modified by a transformation function based on the gray-level content of an entire image
- Local histogram processing
  - Transformation function is derived for neighborhood of each pixel

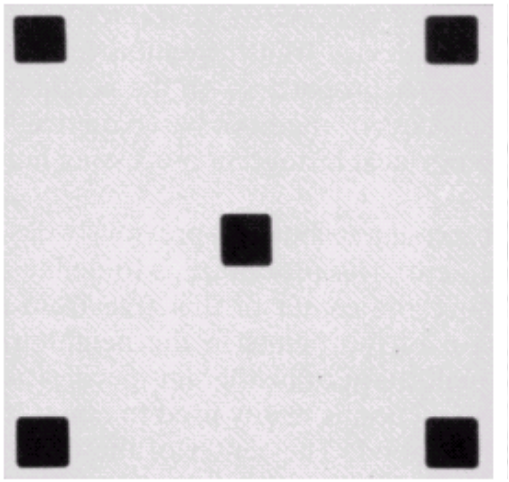
Local Histogram Equalization

- Image matrix of an 3-bit image is given below. Improve contrast the image using histogram processing
- Apply 5x5 histogram equalization

$$A = \begin{bmatrix} 7 & 5 & 6 & 4 & 3 & 1 & 7 & 6 & 2 & 4 \\ 5 & 5 & 5 & 2 & 1 & 3 & 7 & 6 & 5 & 1 \\ 3 & 6 & 7 & 5 & 3 & 4 & 5 & 5 & 5 & 6 \\ 3 & 7 & 7 & 6 & 5 & 6 & 7 & 3 & 4 & 4 \\ 6 & 7 & 6 & 6 & 7 & 6 & 6 & 4 & 4 & 3 \\ 6 & 7 & 7 & 5 & 5 & 4 & 2 & 0 & 1 & 2 \\ 5 & 6 & 5 & 6 & 6 & 3 & 4 & 6 & 2 & 5 \\ 7 & 6 & 7 & 5 & 1 & 3 & 2 & 4 & 4 & 2 \\ 5 & 4 & 3 & 2 & 3 & 3 & 3 & 2 & 2 & 0 \\ 4 & 5 & 5 & 6 & 6 & 6 & 7 & 7 & 2 & 5 \end{bmatrix}$$

Apply equalization to all 4 subparts of image separately

Local Enhancement

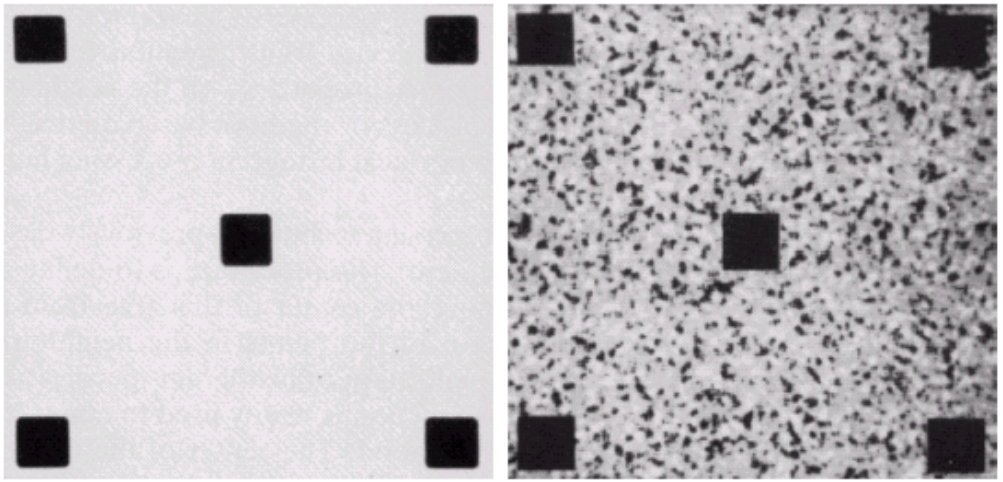


a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

original

Local Enhancement

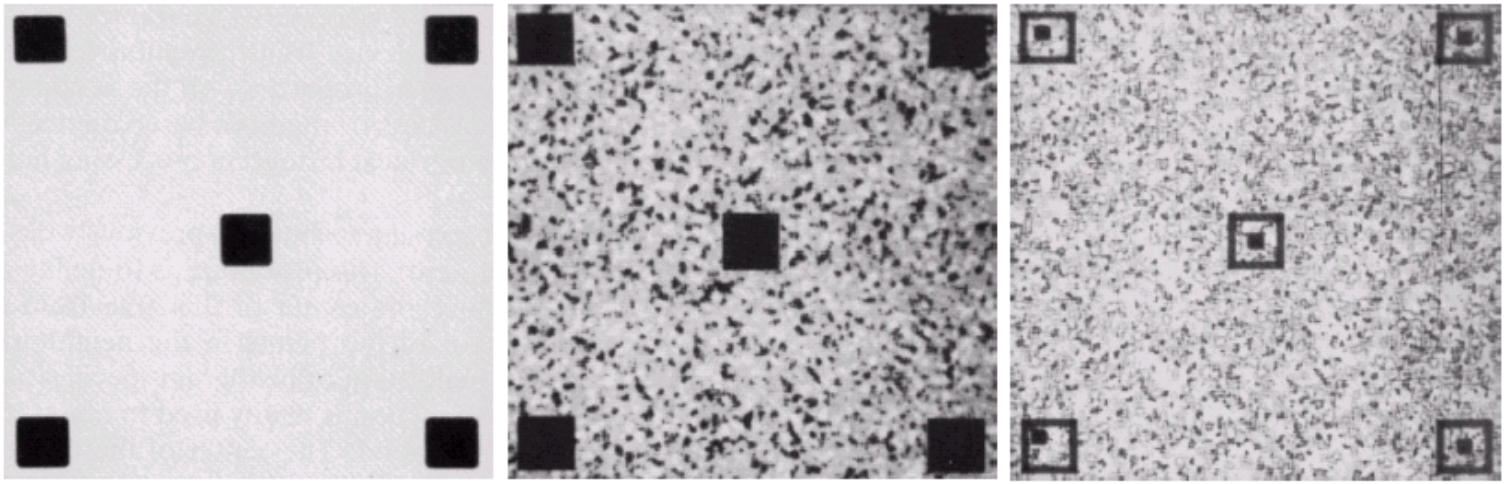


a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

original global

Local Enhancement



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

original global local