Python for Orbital Mechanics

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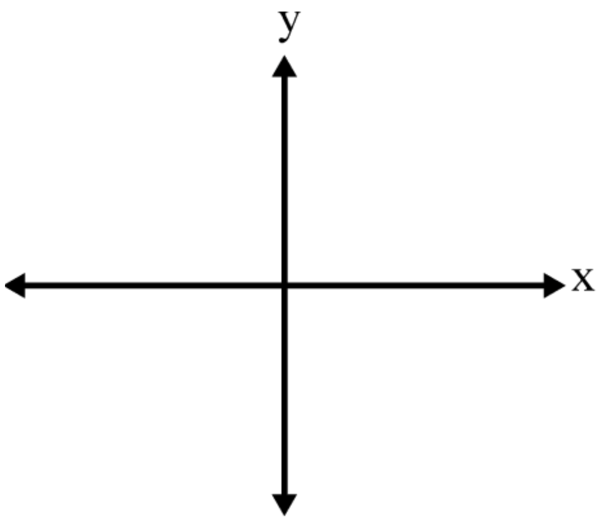
COORDINATE SYSTEMS

WHAT IS A COORDINATE SYSTEM?

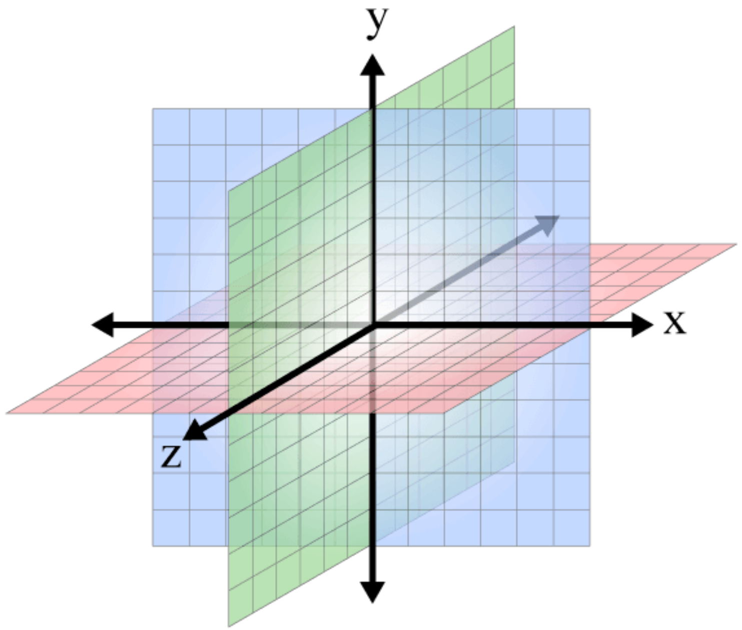
A coordinate system is a method for identifying the location of a point.

In geometry, a coordinate system is a system that uses one or more numbers, or coordinates, to uniquely determine the position of the points or other geometric elements on a manifold such as Euclidean space.

$xy - axis$



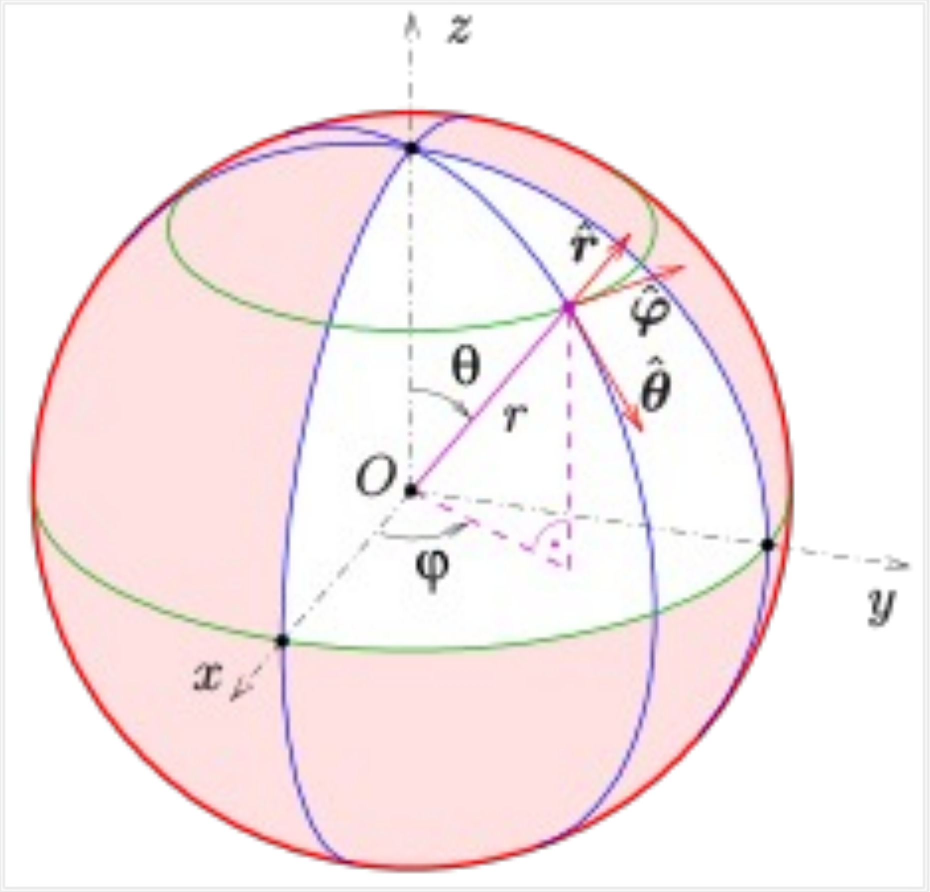
$xyz - axis$



DIFFERENT COORD SYSTEMS COMMONLY USED

SPHERICAL COORDINATES $(\mathbf{r}, \boldsymbol{\theta}, \boldsymbol{\varphi})$

r: the distance of the radial line r connecting the point to the fixed point of origin (which is located on a fixed zenith direction axis);
***θ*:** the polar or inclination angle θ of the radial line r (latitude)
***φ*:** the azimuthal angle φ of the radial line r (longitude)



The equations for converting from rectangular to spherical

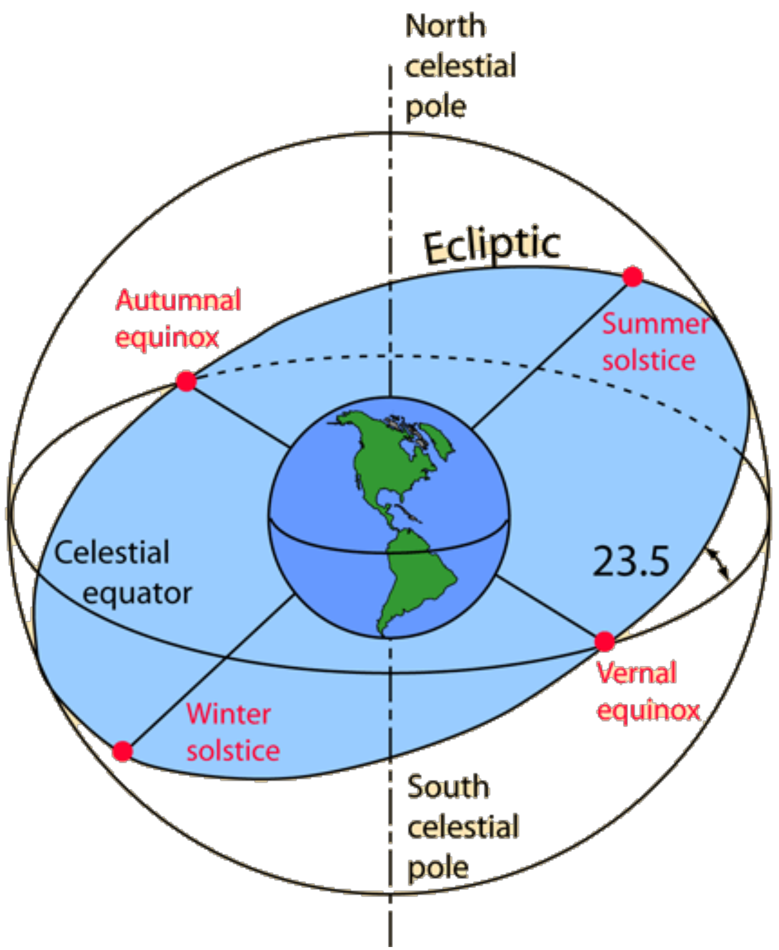
$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \tan^{-1}\frac{y}{x}$$
$$\varphi = \cos^{-1}\left(\frac{z}{\rho}\right)$$

The equations for converting from spherical to rectangular

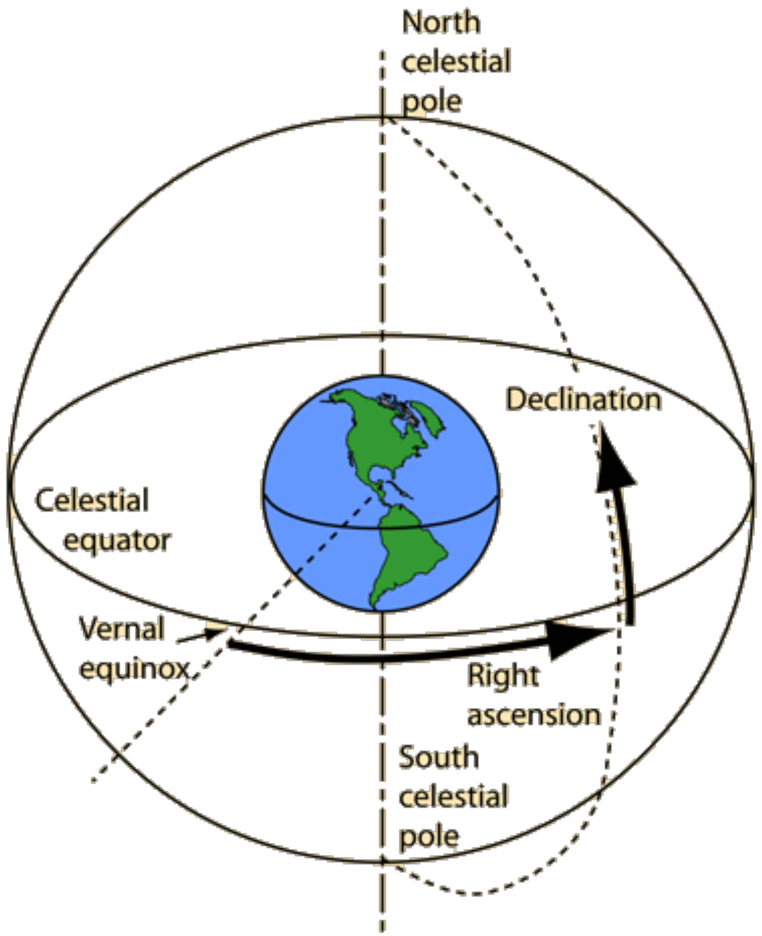
$$x = r\sin\theta\cos\varphi$$
$$y = r\sin\theta\sin\varphi$$
$$z = r\cos\theta$$

FUNDAMENTAL PLANES

ECLIPTIC

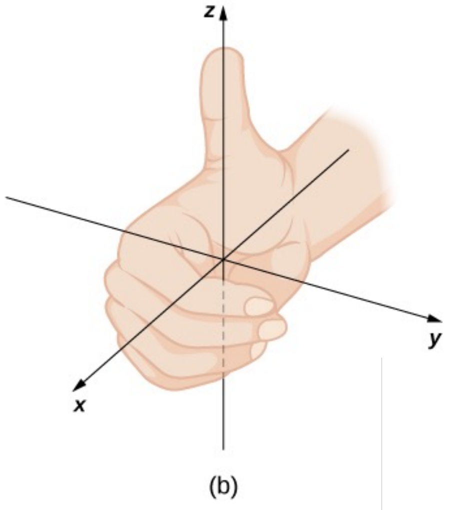


EQUATORIAL

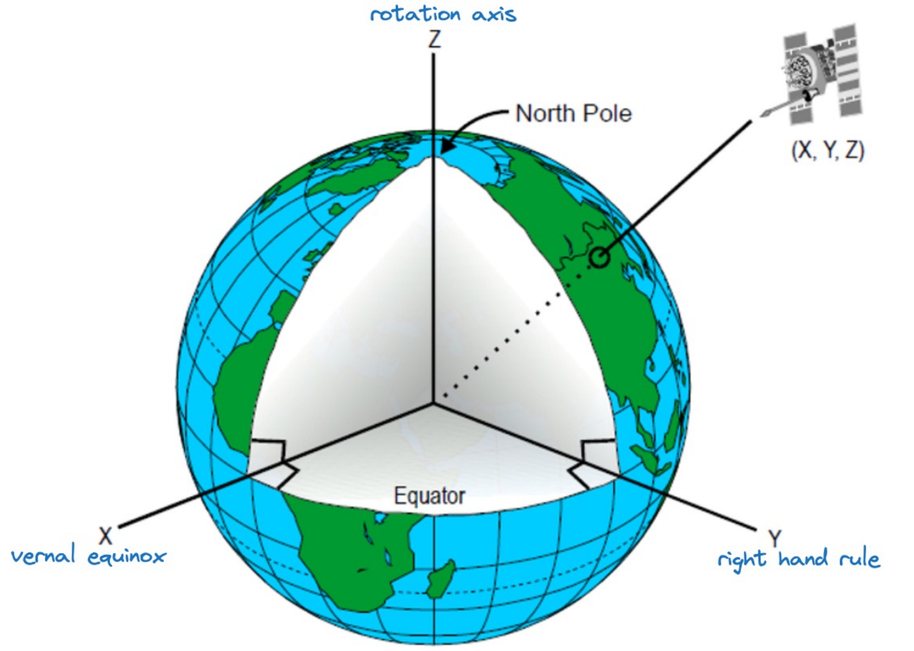


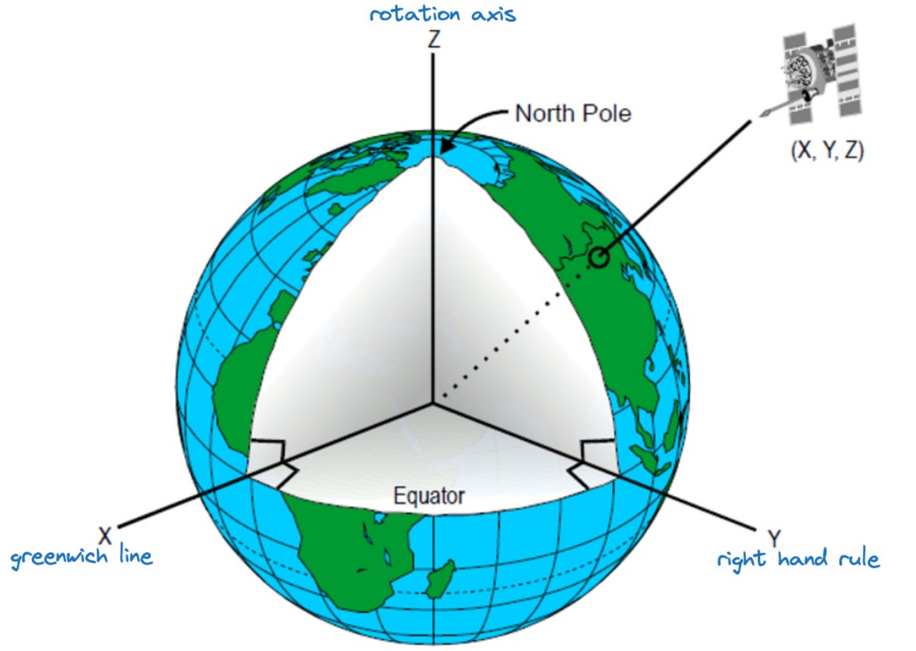
GEOCENTRIC FRAMES OF REFERENCE

Earth Centric Inertial: inertial, not rotating, with respect to the stars; useful to describe motion of celestial bodies and spacecraft.



Earth Centric Earth Fixed: not inertial, accelerated, rotating with respect to the stars; useful to describe motion of objects on Earth surface.





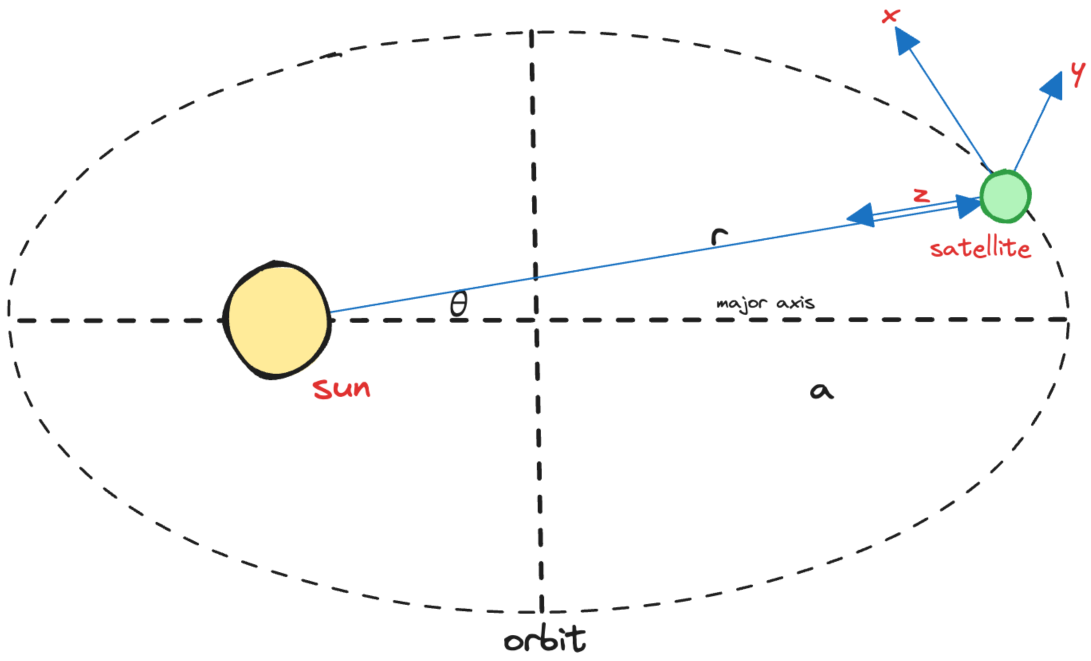
OTHER FRAMES OF REFERENCE

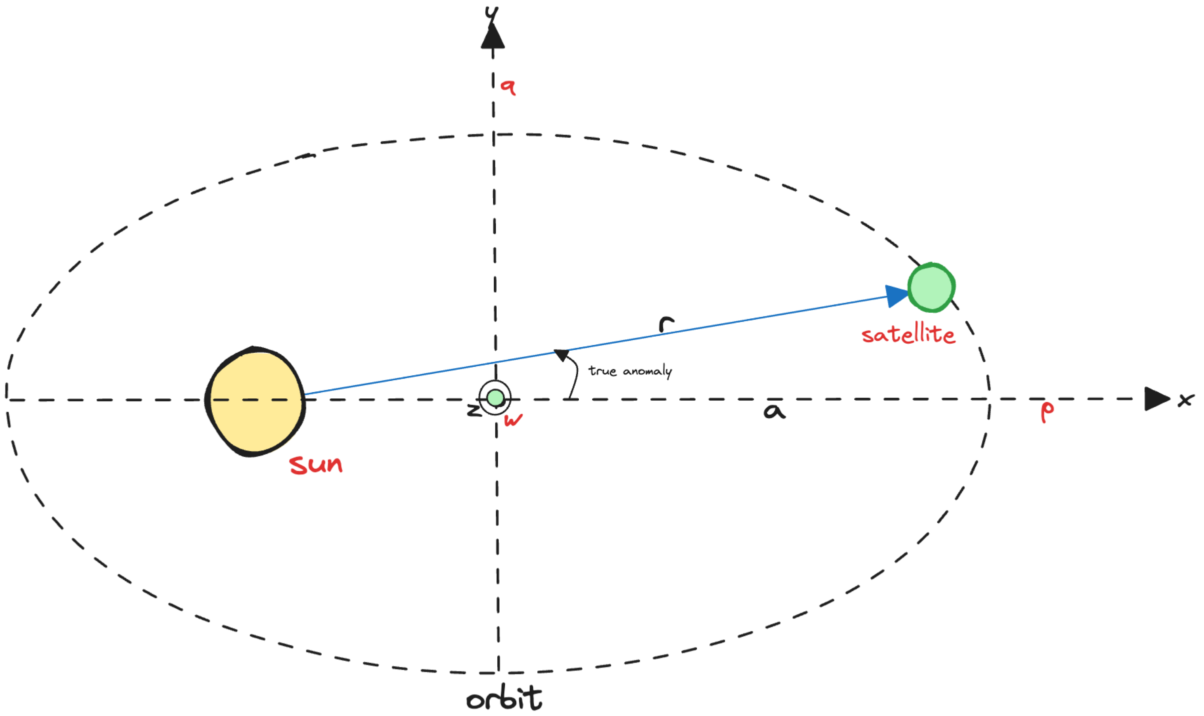
SATELLITE NORMAL

x: the velocity vector
y: the angular momentum (cross product of x & z)
z: the (nadir) negative of the position vector from the source of orbit

PERIFOCAL COORDINATES $(\mathrm{r}, \theta, \varphi)$

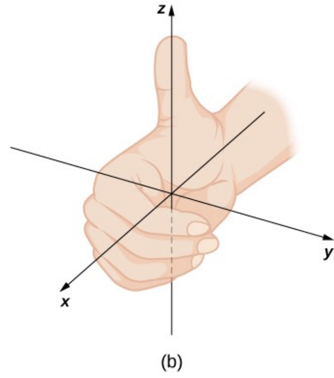
$\hat{p}$: pointing along the x axis
$\hat{q}$: the pointing along the x axis
$\hat{w}$: pointing along the x axis

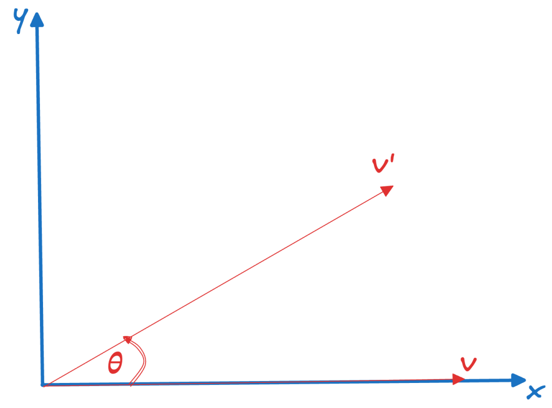




FRAME CONVERSIONS

Position vector in one frame can be converted to another frame using a rotation matrix. Bear in mind that vector rotations are different to coordinate rotations.

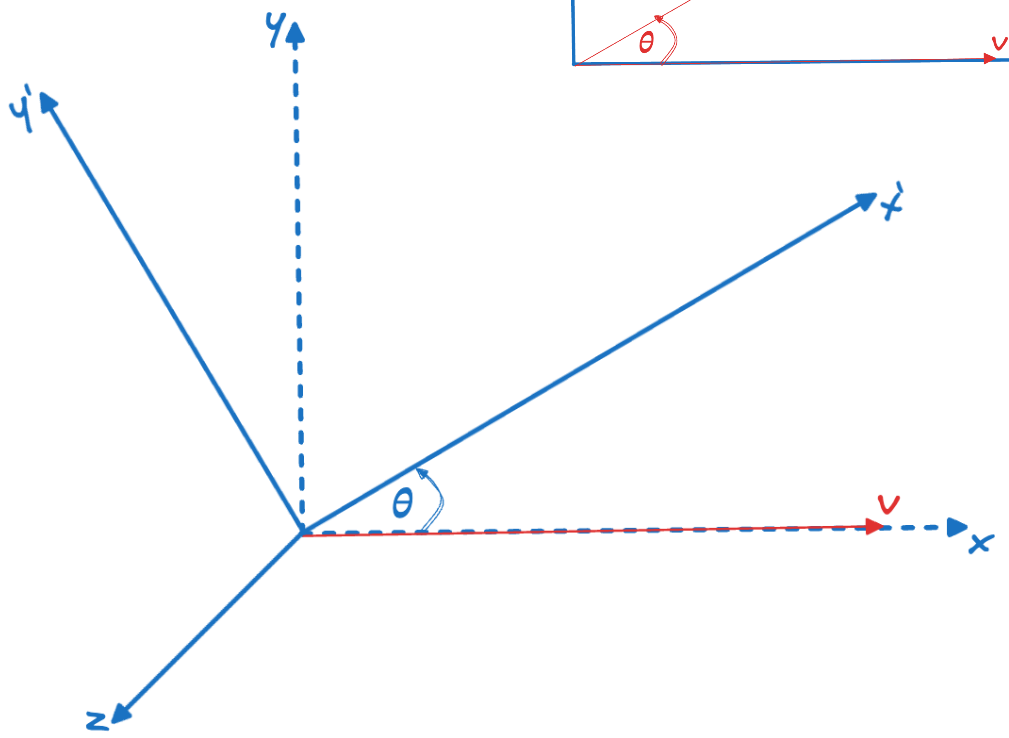




$$\begin{bmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{bmatrix} = \begin{bmatrix}cos\theta & sin\theta & 0\\ -sin\theta & cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} \rightarrow R_z$$

$$\begin{bmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & cos\theta & sin\theta\\ 0 & -sin\theta & cos\theta\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} \rightarrow R_x$$

$$\begin{bmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{bmatrix} = \begin{bmatrix}cos\theta & 0 & sin\theta\\ 0 & 1 & 0\\ -sin\theta & 0 & cos\theta\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} \rightarrow R_y$$



READING TLES

An example TLE for the International Space Station:

```
ISS (ZARYA)
1 25544U 98067A   08264.51782528 -.00002182  00000-0 -11606-4 0  2927
2 25544  51.6416 247.4627 0006703 130.5360 325.0288 15.72125391563537
```

LINE 1

Field	Columns	Content	Example
1	01	Line number	1
2	03–07	Satellite catalog number	25544
3	08	Classification (U: unclassified, C: classified, S: secret) [12]	U
4	10–11	International Designator (last two digits of launch year)	98
5	12–14	International Designator (launch number of the year)	067
6	15–17	International Designator (piece of the launch)	A
7	19–20	Epoch year (last two digits of year)	08
8	21–32	Epoch (day of the year and fractional portion of the day)	264.51782528
9	34–43	First derivative of mean motion; the ballistic coefficient [13]	-.00002182
10	45–52	Second derivative of mean motion (decimal point assumed) [13]	00000-0
11	54–61	*B**, the drag term, or radiation pressure coefficient (decimal point assumed) [13]	-11606-4
12	63–63	Ephemeris type (always zero; only used in undistributed TLE data) [14]	0
13	65–68	Element set number. Incremented when a new TLE is generated for this object.[13]	292
14	69	Checksum (modulo 10)	7

LINE 2

Field	Columns	Content	Example
1	01	Line number	2
2	03–07	Satellite Catalog number	25544
3	09–16	Inclination (degrees)	51.6416
4	18–25	Right ascension of the ascending node (degrees)	247.4627
5	27–33	Eccentricity (decimal point assumed)	0006703
6	35–42	Argument of perigee (degrees)	130.5360
7	44–51	Mean anomaly (degrees)	325.0288
8	53–63	Mean motion (revolutions per day)	15.72125391
9	64–68	Revolution number at epoch (revolutions)	56353
10	69	Checksum (modulo 10)	7

REFERENCES

https://www.youtube.com/watch?v=DbYapFLJsPA
https://www.youtube.com/watch?v=uuKzfwQu6eQ
https://www.youtube.com/watch?v=OOSQm8R9AsY&t=1832s
https://www.youtube.com/watch?v=qEdHZP_jRIE
https://en.wikipedia.org/wiki/Coordinate_system
https://en.wikipedia.org/wiki/Spherical_coordinate_system
https://en.wikipedia.org/wiki/Astronomical_coordinate_systems
http://hyperphysics.phy-astr.gsu.edu/hbase/eclip.html
https://en.wikipedia.org/wiki/Earth-centered_inertial
https://en.wikibooks.org/wiki/Astrodynamics/Coordinate_Systems
https://en.wikipedia.org/wiki/Earth-centered,_Earth-fixed_coordinate_system
https://orbital-mechanics.space/classical-orbital-elements/perifocal-frame.html
https://medium.com/@zackfizell10/reference-frames-every-aerospace-engineer-should-know-db10638b6d7a
https://en.wikipedia.org/wiki/Two-line_element_set