Python for Orbital Mechanics

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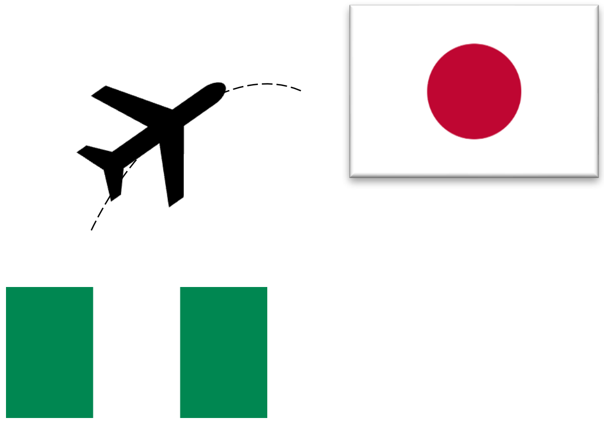
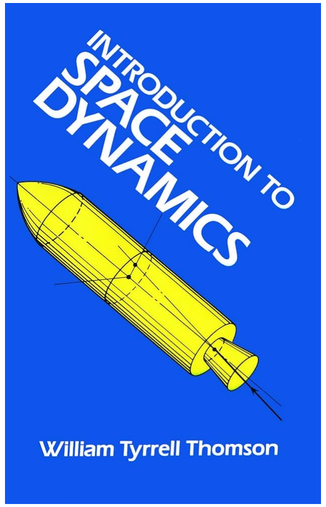
WHO & WHY…





CONTENT

WEEK 1:

- INTRO & 2-BODY PROBLEM

WEEK 2:

- MOVING IN AN ELLIPTICAL ORBIT

WEEK 3:

- COORDINATE SYSTEMS

WEEK 4:

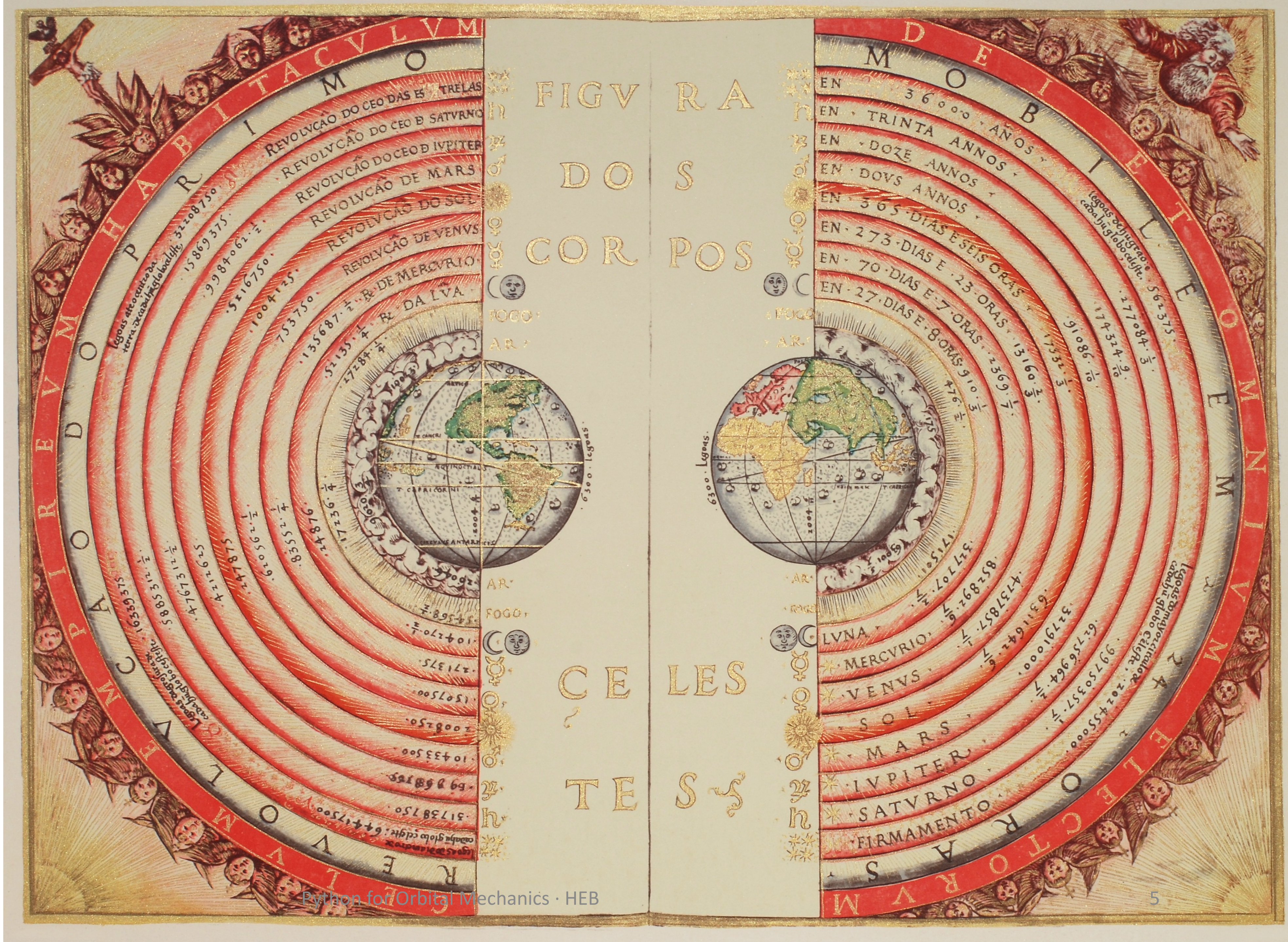
- MANOEUVRING THE SPACECRAFT - HOHMANN TRANSFER

WEEK 5:

- LAMBERT’S PROBLEM & PYTHON LIBRARIES FOR SPACE APPLICATIONS

INTRO &
2-BODY PROBLEM

GREEKS



FIGVRA
DOS
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CELESTES
REVOLVÇÃO DO CEO DAS ESTRELAS
REVOLVÇÃO DO CEO D SATVRNO
REVOLVÇÃO DO CEO D IVPITER
REVOLVÇÃO DE MARS
REVOLVÇÃO DO SOL
REVOLVÇÃO DE VENVS
R. DE MERCVRIO
R. DA LVA
EN 36000 AÑOS
EN TRINTA ANNOS
EN DOZE ANNOS
EN DOVS ANNOS
EN 365 DIAS E SEIS ORAS
EN 273 DIAS E 23 ORAS
EN 70 DIAS E 7 ORAS
EN 27 DIAS E 8 ORAS
FOGO
AR
LVNA
MERCVRIO
VENVS
SOL
MARS
IVPITER
SATVRNO
FIRMAMENTO

BRAHE

KEPLER

KEPLER'S 1ST LAW

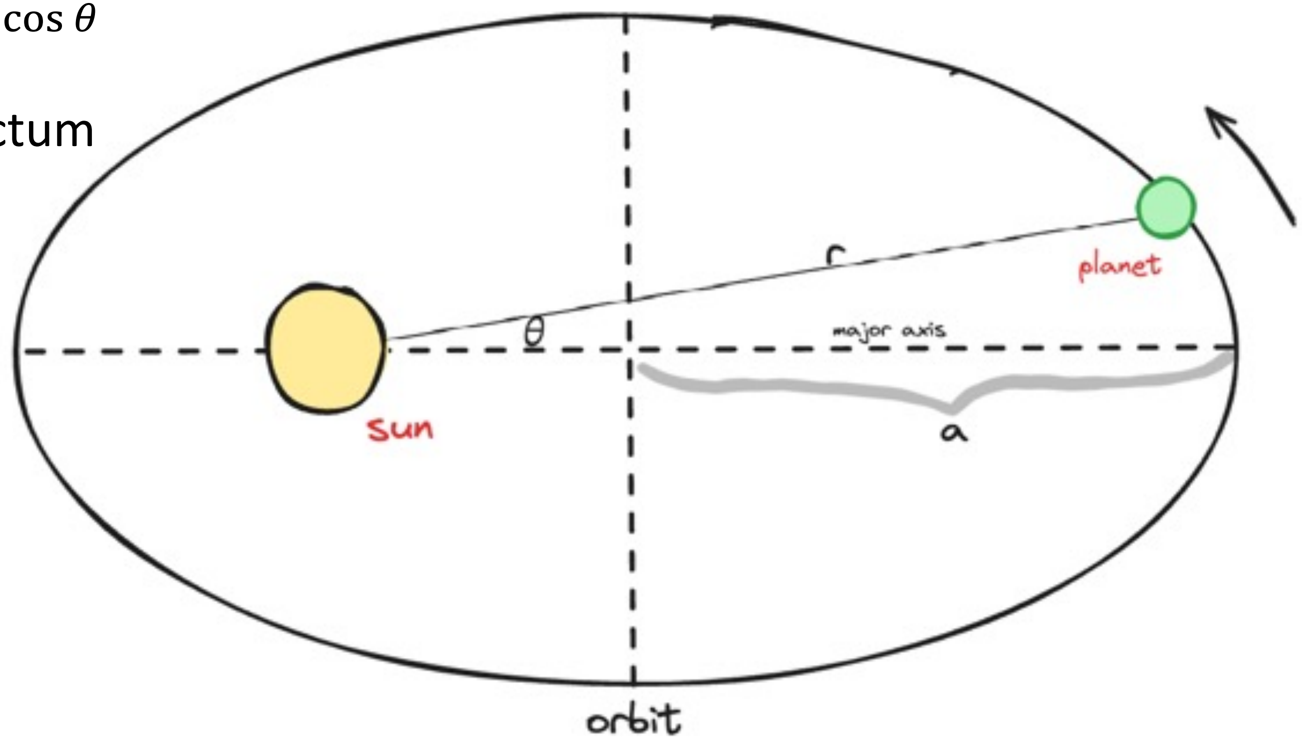
Law of orbits : planets orbit the sun elliptically

The distance from the Sun to the planet, $\mathrm{r} = \frac{p}{1+\varepsilon\cos\theta}$

Where $p = a(1-\varepsilon^2)$ is called the semi-latus rectum

For $0 \le \varepsilon \le 1$ is the eccentricity

a – semimajor axis



KEPLER'S 2ND LAW

Law of areas: a line connecting a planet to the sun covers an equal area over equal periods of time

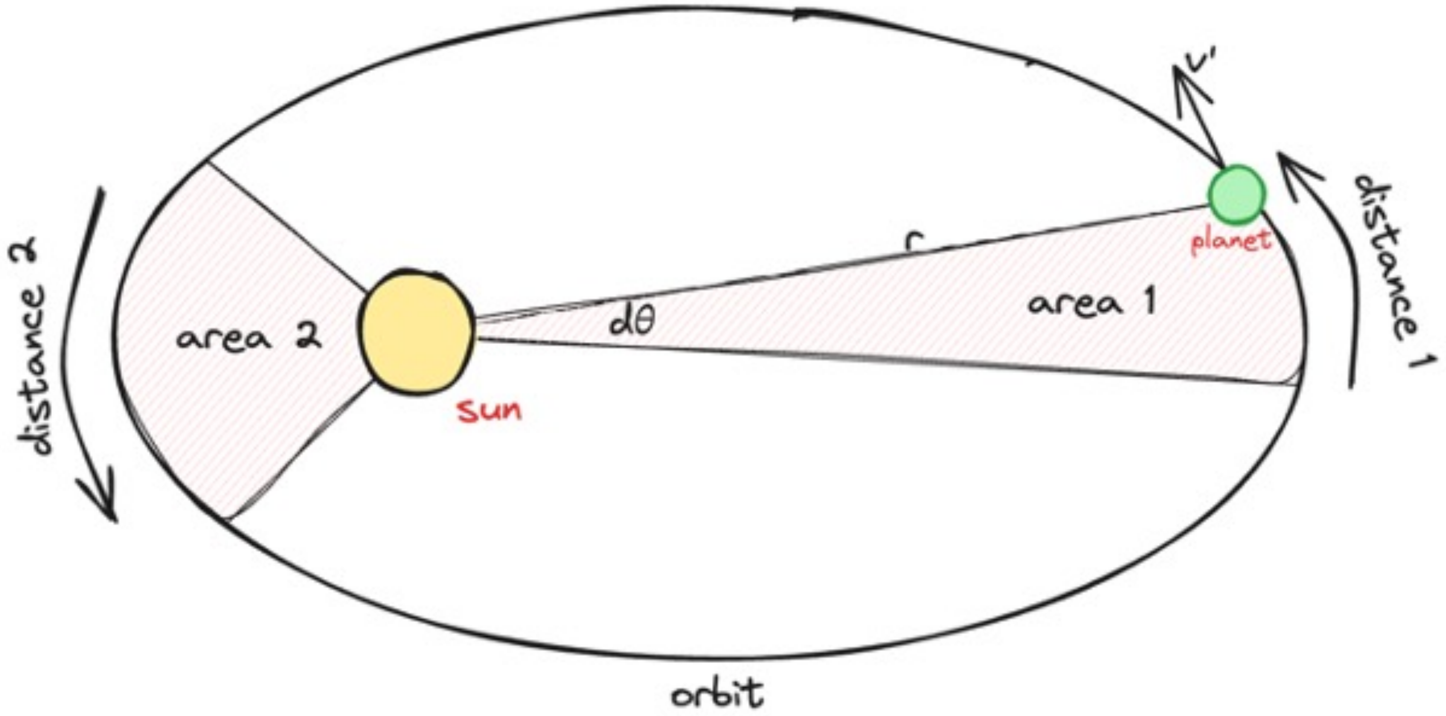
$$\frac{dA}{dt} = \frac{1}{2} r v_\theta$$

$L = m r v_\theta$

is the angular momentum

Area 1 = Area 2

Time to travel distance 1 = Time to travel distance 2



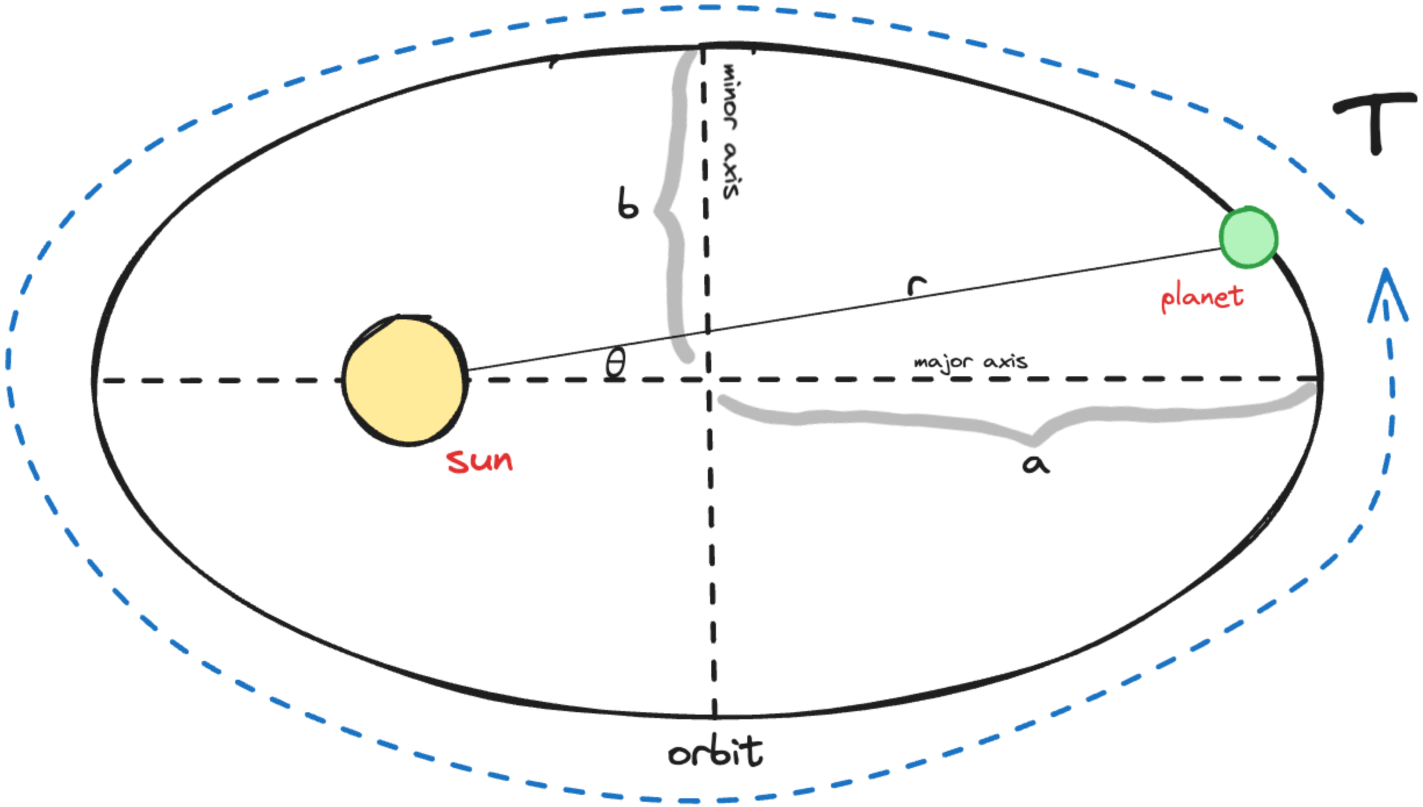
KEPLER'S 3RD LAW

Law of periods: establishes relationship between planet's orbital period and its distance from the sun

$T^2 \propto a^3$

$b^2 = a^2(1 - \varepsilon^2)$

T - time to complete orbit
a – semimajor axis
b – semiminor axis



NEWTON

NEWTON’S LAWS

2nd law of motion
“The net force on a body is equal to the body's acceleration multiplied by its mass.”

$$\hat{\boldsymbol{F}} = m\,\hat{\boldsymbol{a}}$$

Universal gravitation
“Every particle attracts every other particle in the universe with force directly proportional to the product of the masses and inversely proportional to the square of the distance between them.”

$$\mathrm{F} = G\,\frac{m_1 m_2}{r^2}$$

Gravitational constant of our universe, G = 6.67430 * 10^{-11}
Rewrite Kepler’s 3rd law as

$T^2 \ \alpha\ a^3 \quad \rightarrow \quad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$

T is in seconds
a is in astronomical units (AU) 1AU = 1.496e+11 metres
r is orbital radius in metres

2-BODY PROBLEM

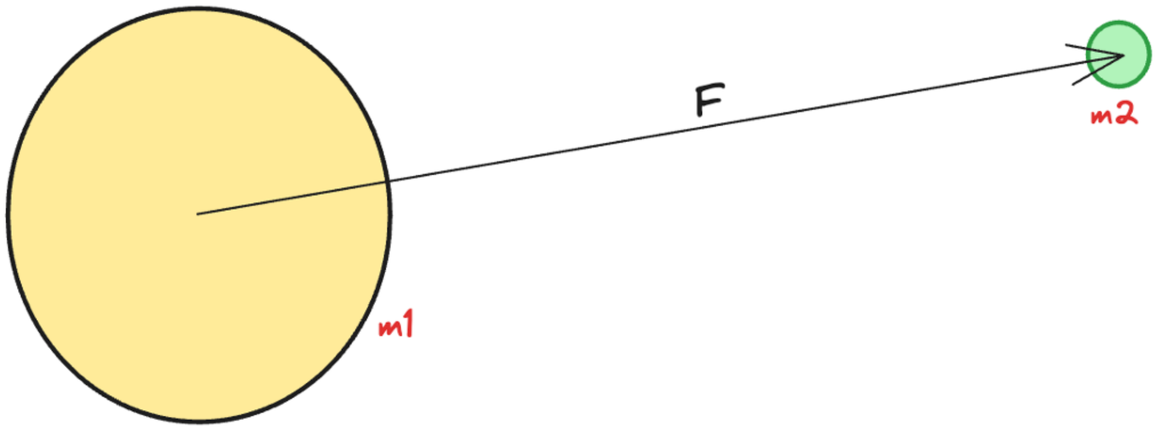
Assumptions

>> The universe only contains one very large body and one small body

>> Large body is spherical and mass is evenly distributed, centre of mass is at geometric centre

>> Large body influences small body gravitationally with force coming from its centre (small body does not influence large body)

>> Reference frame is centred at geometric centre of the large body



$$F = m_2 a_2 = G\frac{m_1 m_2}{r^2}, \quad a_2 = G\frac{m_1}{r^2} = \frac{\mu}{r^2}$$

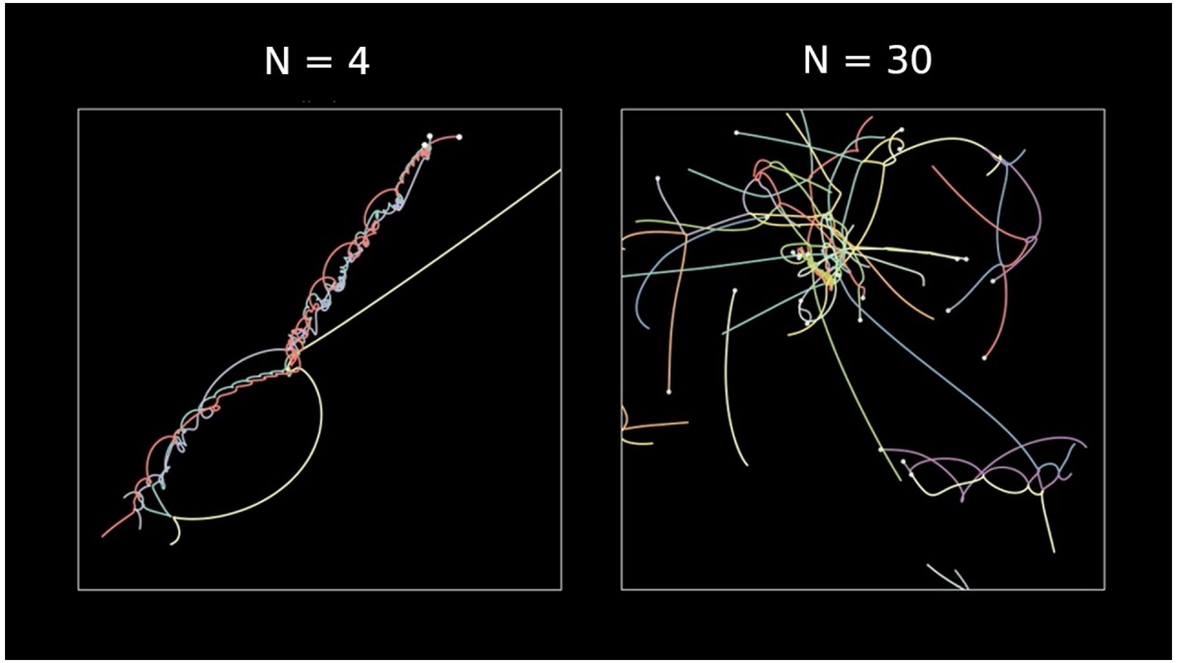
*every body has it's own mu constant value

N-BODY PROBLEM

Given the quasi-steady orbital properties (instantaneous position, velocity and time) of a group of celestial bodies, predict their interactive forces; and consequently, predict their true orbital motions for all future times.

>> The n-body problem is the problem of predicting the individual motions of a group of celestial objects interacting with each other gravitationally.

>> The three-body problem is a special case of the *n*-body problem. Unlike two-body problems, no general closed-form solution exists



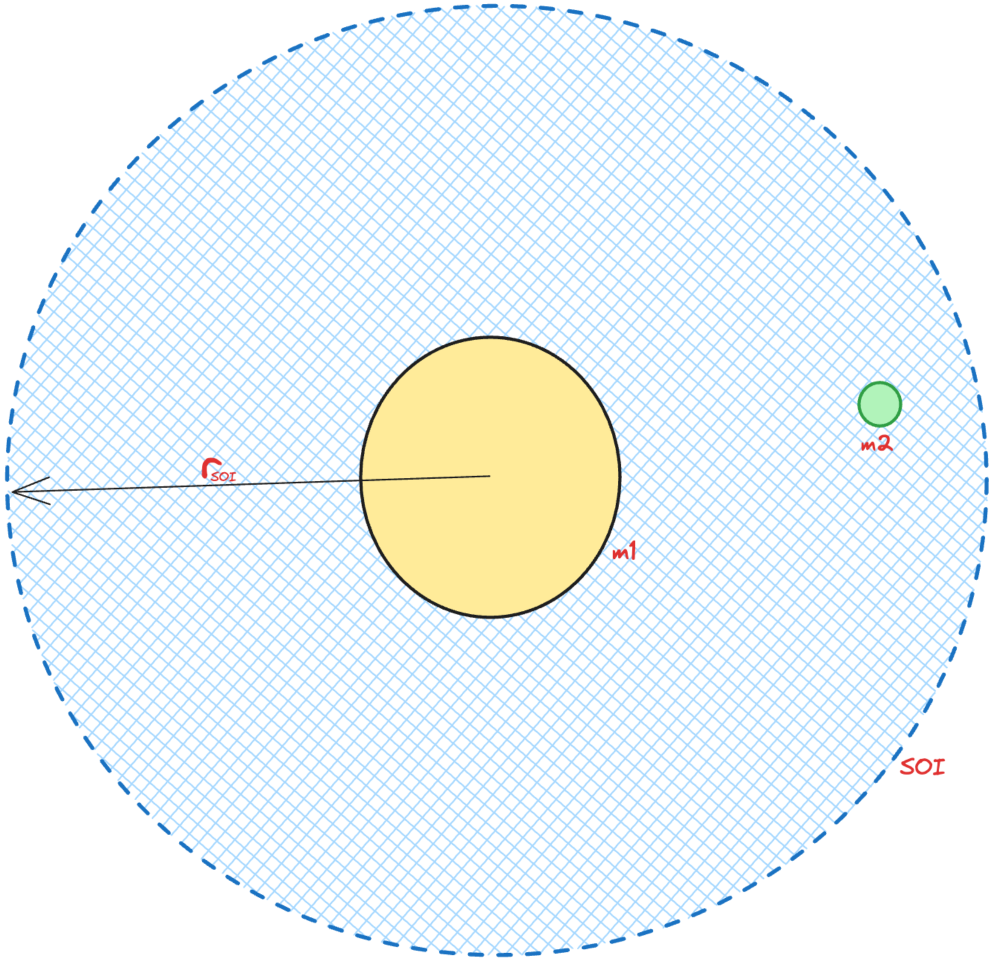
SPHERE OF INFLUENCE

Conceptually, we can imagine the sphere of influence as a boundary, inside of which the planet's gravitational pull dominates the pull of the Sun on a much smaller mass. This allows us to estimate a specific radius beyond which the focus of the trajectory switches from the planet to the Sun. Therefore, the velocity that the spacecraft has when it leaves the sphere of influence of a planet is the velocity that it starts with on its heliocentric trajectory.

The general equation describing the radius of the sphere r_{SOI} of a planet

$$r_{SOI} \approx a \left(\frac{m}{M}\right)^{2/5}$$

where
a is the semimajor axis of the smaller object's (usually a planet's) orbit around the larger body (usually the Sun).
m and M are the masses of the smaller and the larger object (usually a planet and the Sun), respectively.



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