## RISC-V Bitmanip Extension

Document Version 0.92

Editor: Clifford Wolf Symbiotic GmbH clifford@symbioticeda.com November 8, 2019 Contributors to all versions of the spec in alphabetical order (please contact editors to suggest corrections): Jacob Bachmeyer, Allen Baum, Ari Ben, Alex Bradbury, Steven Braeger, Rogier Brussee, Michael Clark, Ken Dockser, Paul Donahue, Dennis Ferguson, Fabian Giesen, John Hauser, Robert Henry, Bruce Hoult, Po-wei Huang, Ben Marshall, Rex McCrary, Lee Moore, Jiří Moravec, Samuel Neves, Markus Oberhumer, Christopher Olson, Nils Pipenbrinck, Joseph Rahmeh, Xue Saw, Tommy Thorn, Avishai Tvila, Andrew Waterman, Thomas Wicki, and Clifford Wolf.

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## Chapter 1

## Introduction

This is the RISC-V Bitmanip Extension draft spec.

## 1.1 ISA Extension Proposal Design Criteria

Any proposed changes to the ISA should be evaluated according to the following criteria.

- Architecture Consistency: Decisions must be consistent with RISC-V philosophy. ISA changes should deviate as little as possible from existing RISC-V standards (such as instruction encodings), and should not re-implement features that are already found in the base specification or other extensions.
- Threshold Metric: The proposal should provide *significant* savings in terms of clocks or instructions. As a heuristic, any proposal should replace at least three instructions. An instruction that only replaces two may be considered, but only if the frequency of use is very high and/or the implementation very cheap.
- Data-Driven Value: Usage in real world applications, and corresponding benchmarks showing a performance increase, will contribute to the score of a proposal. A proposal will not be accepted on the merits of its *theoretical* value alone, unless it is used in the real world.
- Hardware Simplicity: Though instructions saved is the primary benefit, proposals that dramatically increase the hardware complexity and area, or are difficult to implement, should be penalized and given extra scrutiny. The final proposals should only be made if a test implementation can be produced.
- Compiler Support: ISA changes that can be natively detected by the compiler, or are already used as intrinsics, will score higher than instructions which do not fit that criteria.

## 1.2 B Extension Adoption Strategy

The overall goal of this extension is pervasive adoption by minimizing potential barriers and ensuring the instructions can be mapped to the largest number of ops, either direct or pseudo, that are supported by the most popular processors and compilers. By adding generic instructions and taking advantage of the RISC-V base instructions that already operate on bits, the minimal set of instructions need to be added while at the same time enabling a rich of operations.

The instructions cover the four major categories of bit manipulation: Count, Extract, Insert, Swap. The spec supports RV32, RV64, and RV128. "Clever" obscure and/or overly specific instructions are avoided in favor of more straightforward, fast, generic ones. Coordination with other emerging RISC-V ISA extensions groups is required to ensure our instruction sets are architecturally consistent.

### 1.3 Next steps

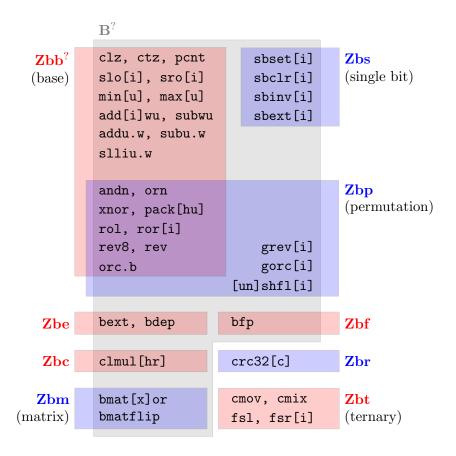
- Assign concrete instruction encodings so that we can start implementing the extension in processor cores and compilers.
- Add support for this extension to processor cores and compilers so we can run quantitative evaluations on the instructions.
- Create assembler snippets for common operations that do not map 1:1 to any instruction in this spec, but can be implemented easily using clever combinations of the instructions. Add support for those snippets to compilers.

## Chapter 2

# RISC-V Bitmanip Extension

In the proposals provided in this chapter, the C code examples are for illustration purposes only. They are not optimal implementations, but are intended to specify the desired functionality.

The final standard will likely define a range of Z-extensions for different bit manipulation instructions, with the "B" extension itself being a mix of instructions from those Z-extensions. It is unclear as of yet what this will look like exactly, but it will probably look something like this:



The main open questions are:

- Which of the "Zb\*" extensions should be included in "B"?
- There is some redundancy/overlap with add[i]wu, subwu, addu.w, subu.w, and slliu.w. Which instructions should stay?
- Which "Zbp" pseudo-ops should be included in "Zbb"?

These decisions will be informed in big part by evaluations of the cost and added value for the individual instructions.

For the purpose of tool-chain development "B" is currently everything.

For extensions that only implement certain pseudo-instructions (such as "Zbb" implements rev8 and rev, which are pseudo-instructions for grevi rd, rs1, -8 and grevi rd, rs1, -1 respectively, the same binary encoding is used for those instructions as are used on a core with full support for the grev[i] instruction.

Like in the base ISA, we add \*W instruction variants on RV64 with the semantic of the matching RV32 instruction. Those instructions ignore the upper 32 bit of their input and sign-extend their 32 bit output values. The \*W instruction is omitted when the 64-bit instruction produces the same result as the \*W instruction would when the 64-bit instruction is fed sign-extended 32 bit values.

## 2.1 Basic bit manipulation instructions

#### 2.1.1 Count Leading/Trailing Zeros (clz, ctz)

```
RISC-V Bitmanip ISA

RV32, RV64:
clz rd, rs
ctz rd, rs

RV64 only:
clzw rd, rs
ctzw rd, rs
```

The clz operation counts the number of 0 bits at the MSB end of the argument. That is, the number of 0 bits before the first 1 bit counting from the most significant bit. If the input is 0, the output is XLEN. If the input is -1, the output is 0.

The ctz operation counts the number of 0 bits at the LSB end of the argument. If the input is 0, the output is XLEN. If the input is -1, the output is 0.

```
uint_xlen_t clz(uint_xlen_t rs1)
{
    for (int count = 0; count < XLEN; count++)
        if ((rs1 << count) >> (XLEN - 1))
        return count;
    return XLEN;
}
uint_xlen_t ctz(uint_xlen_t rs1)
{
    for (int count = 0; count < XLEN; count++)
        if ((rs1 >> count) & 1)
        return count;
    return XLEN;
}
```

The expression XLEN-1-clz(x) evaluates to the index of the most significant set bit, also known as integer base-2 logarithm, or -1 if x is zero.

These instructions are commonly used for scanning bitmaps for set bits, for example in malloc(), in binary GCD, or in priority queues such as the sched\_find\_first\_bit() function used in the Linux kernel real-time scheduler.

Another common applications include normalization in fixed-point code and soft float libraries, null suppression in data compression.

#### 2.1.2 Count Bits Set (pcnt)

```
RV32, RV64:
pcnt rd, rs

RV64 only:
pcntw rd, rs
```

This instruction counts the number of 1 bits in a register. This operations is known as population count, popcount, sideways sum, bit summation, or Hamming weight. [22, 20]

```
uint_xlen_t pcnt(uint_xlen_t rs1)
{
   int count = 0;
   for (int index = 0; index < XLEN; index++)
        count += (rs1 >> index) & 1;
   return count;
}
```

#### 2.1.3 Logic-with-negate (andn, orn, xnor)

```
RISC-V Bitmanip ISA
andn rd, rs1, rs2
orn rd, rs1, rs2
xnor rd, rs1, rs2
```

This instructions implement AND, OR, and XOR with the 2nd arument inverted.

```
uint_xlen_t andn(uint_xlen_t rs1, uint_xlen_t rs2)
{
    return rs1 & ~rs2;
}
uint_xlen_t orn(uint_xlen_t rs1, uint_xlen_t rs2)
{
    return rs1 | ~rs2;
}
uint_xlen_t xnor(uint_xlen_t rs1, uint_xlen_t rs2)
{
    return rs1 ^ ~rs2;
}
```

This can use the existing inverter on rs2 in the ALU that's already there to implement subtract.

Among other things, those instructions allow implementing the "trailing bit manipulation" code patterns in two instructions each. For example, (x - 1) & x produces a mask from trailing zero bits in x.

#### 2.1.4 Pack two words in one register (pack, packu, packh)

```
RV32, RV64:

pack rd, rs1, rs2
packu rd, rs1, rs2
packh rd, rs1, rs2

RV64 only:
packw rd, rs1, rs2
packu rd, rs1, rs2
packu rd, rs1, rs2
```

The pack instruction packs the XLEN/2-bit lower halves of rs1 and rs2 into rd, with rs1 in the lower half and rs2 in the upper half.

```
uint_xlen_t pack(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t lower = (rs1 << XLEN/2) >> XLEN/2;
    uint_xlen_t upper = rs2 << XLEN/2;
    return lower | upper;
}</pre>
```

The packu instruction packs the upper halves of rs1 and rs2 into rd.

```
uint_xlen_t packu(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t lower = rs1 >> XLEN/2;
    uint_xlen_t upper = (rs2 >> XLEN/2) << XLEN/2;
    return lower | upper;
}</pre>
```

And the packh instruction packs the LSB bytes of rs1 and rs2 into the 16 LSB bits of rd, zero extending the rest of rd.

```
uint_xlen_t packh(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t lower = rs1 & 255;
    uint_xlen_t upper = (rs2 & 255) << 8;
    return lower | upper;
}</pre>
```

Applications include XLEN/2-bit funnel shifts, zero-extend XLEN/2 bit values, duplicate the lower XLEN/2 bits (e.g. for mask creation), loading unsigned 32 constants on RV64, and packing C structs that fit in a register and are therefore passed in a register according to the RISC-V calling convention.

```
; Constructing a 32-bit int from four bytes (RV32) packh a0, a0, a1 packh a1, a2, a3 pack a0, a0, a1 ; Load Oxffff0000ffff0000 on RV64 lui rd, Oxffff0 pack rd, rd, rd ; Same as FSLW on RV64 pack rd, rs1, rs3 rol rd, rd, rs2 addiw rd, rd, 0 ; Clear the upper half of rd pack rd, rd, zero
```

Paired with shfli/unshfli and the other bit permutation instructions, pack can interleave arbitrary power-of-two chunks of rs1 and rs2. For example, interleaving the bytes in the lower halves of rs1 and rs2:

```
pack rd, rs1, rs2
zip8 rd, rd
```

pack is most commonly used to zero-extend words <XLEN. For this purpose we define the following assembler pseudo-ops:

```
RV32:
 zext.b rd, rs
                 ->
                      andi rd, rs, 255
 zext.h rd, rs
                 ->
                      pack rd, rs, zero
RV64:
 zext.b rd, rs
                 ->
                    andi rd, rs, 255
                      packw rd, rs, zero
 zext.h rd, rs
               ->
 zext.w rd, rs
               ->
                      pack rd, rs, zero
RV128:.
 zext.b rd, rs
               ->
                      andi rd, rs, 255
 zext.h rd, rs
               ->
                      packw rd, rs, zero
 zext.w rd, rs ->
                    packd rd, rs, zero
 zext.d rd, rs ->
                      pack rd, rs, zero
```

#### 2.1.5 Min/max instructions (min, max, minu, maxu)

```
RV32, RV64:
min rd, rs1, rs2
max rd, rs1, rs2
minu rd, rs1, rs2
maxu rd, rs1, rs2
maxu rd, rs1, rs2
```

We define 4 R-type instructions min, max, minu, maxu with the following semantics:

```
uint_xlen_t min(uint_xlen_t rs1, uint_xlen_t rs2)
{
    return (int_xlen_t)rs1 < (int_xlen_t)rs2 ? rs1 : rs2;
}
uint_xlen_t max(uint_xlen_t rs1, uint_xlen_t rs2)
{
    return (int_xlen_t)rs1 > (int_xlen_t)rs2 ? rs1 : rs2;
}
uint_xlen_t minu(uint_xlen_t rs1, uint_xlen_t rs2)
{
    return rs1 < rs2 ? rs1 : rs2;
}</pre>
```

```
uint_xlen_t maxu(uint_xlen_t rs1, uint_xlen_t rs2)
{
    return rs1 > rs2 ? rs1 : rs2;
}
```

Code that performs saturated arithmetic on a word size < XLEN needs to perform min/max operations frequently. A simple way of performing those operations without branching can benefit those programs.

Some applications spend a lot of time on calculating the absolute values of signed integers. One example of that would be SAT solvers, due to the way CNF literals are commonly encoded [10]. With max (or minu) this is a two-instruction operation:

```
neg a1, a0
max a0, a0, a1
```

#### 2.1.6 Sign-extend instructions (sext.b, sext.h)

```
RISC-V Bitmanip ISA
RV32, RV64:
sext.b rd, rs
sext.h rd, rs
```

For sign-extending a byte or half-word we define two unary instructions:

```
uint_xlen_t sextb(uint_xlen_t x)
{
    return int_xlen_t(x << (XLEN-8)) >> (XLEN-8);
}
uint_xlen_t sexth(uint_xlen_t x)
{
    return int_xlen_t(x << (XLEN-16)) >> (XLEN-16);
}
```

Additionally, we define pseudo-instructions for zero extending bytes or half-words and for zero- and sign-extending words:

```
RV32:
                       andi rd, rs, 255
  zext.b rd, rs
                  ->
  zext.h rd, rs
                       pack rd, rs, zero
                  ->
RV64:
  zext.b rd, rs
                       andi rd, rs, 255
                  ->
  zext.h rd, rs
                  ->
                       packw rd, rs, zero
  zext.w rd, rs
                  ->
                       pack rd, rs, zero
                       addiw rd, rs, 0
  sext.w rd, rs
                  ->
```

Sign extending 8-bit and 16-bit values is needed when calling a function that accepts an 8-bit or 16-bit signed argument, because the RISC-V calling conventions dictates that such an argument

must be passed in sign-extended form.

#### 2.1.7 Single-bit instructions (sbset, sbclr, sbinv, sbext)

```
🗕 RISC-V Bitmanip ISA 🗕
RV32, RV64:
  sbset rd, rs1, rs2
  sbclr rd, rs1, rs2
  sbinv rd, rs1, rs2
  sbext rd, rs1, rs2
  sbseti rd, rs1, imm
  sbclri rd, rs1, imm
  sbinvi rd, rs1, imm
  sbexti rd, rs1, imm
RV64:
  sbsetw rd, rs1, rs2
  sbclrw rd, rs1, rs2
  sbinvw rd, rs1, rs2
  sbextw rd, rs1, rs2
  sbsetiw rd, rs1, imm
  sbclriw rd, rs1, imm
  sbinviw rd, rs1, imm
```

We define 4 single-bit instructions sbset (set), sbclr (clear), sbinv (invert), and sbext (extract), and their immediate-variants, with the following semantics:

```
uint_xlen_t sbset(uint_xlen_t rs1, uint_xlen_t rs2)
{
    int shamt = rs2 & (XLEN - 1);
    return rs1 | (uint_xlen_t(1) << shamt);
}
uint_xlen_t sbclr(uint_xlen_t rs1, uint_xlen_t rs2)
{
    int shamt = rs2 & (XLEN - 1);
    return rs1 & ~(uint_xlen_t(1) << shamt);
}
uint_xlen_t sbinv(uint_xlen_t rs1, uint_xlen_t rs2)
{
    int shamt = rs2 & (XLEN - 1);
    return rs1 ^ (uint_xlen_t(1) << shamt);
}</pre>
```

```
uint_xlen_t sbext(uint_xlen_t rs1, uint_xlen_t rs2)
{
   int shamt = rs2 & (XLEN - 1);
   return 1 & (rs1 >> shamt);
}
```

### 2.1.8 Shift Ones (Left/Right) (slo, sloi, sro, sroi)

```
RV32, RV64:
slo rd, rs1, rs2
sro rd, rs1, imm
sroi rd, rs1, imm

RV64 only:
slow rd, rs1, rs2
srow rd, rs1, rs2
sloiw rd, rs1, rs2
sloiw rd, rs1, imm
sroiw rd, rs1, imm
```

These instructions are similar to shift-logical operations from the base spec, except instead of shifting in zeros, they shift in ones.

```
uint_xlen_t slo(uint_xlen_t rs1, uint_xlen_t rs2)
{
    int shamt = rs2 & (XLEN - 1);
    return ~(~rs1 << shamt);
}
uint_xlen_t sro(uint_xlen_t rs1, uint_xlen_t rs2)
{
    int shamt = rs2 & (XLEN - 1);
    return ~(~rs1 >> shamt);
}
```

ISAs with flag registers often have a "Shift in Carry" or "Rotate through Carry" instruction. Arguably a "Shift Ones" is an equivalent on an ISA like RISC-V that avoids such flag registers.

The main application for the Shift Ones instruction is mask generation.

When implementing this circuit, the only change in the ALU over a standard logical shift is that the value shifted in is not zero, but is a 1-bit register value that has been forwarded from the high bit of the instruction decode. This creates the desired behavior on both logical zero-shifts and logical ones-shifts.

## 2.2 Bit permutation instructions

The following sections describe 3 types of bit permutation instructions: Rotate shift, generalized reverse, and generalized shuffle.

A bit permutation essentially is applying an invertible function to the bit addresses. (Bit addresses are 5 bit values on RV32 and 6 bit values on RV64.)

Rotate shift by k is simply addition (rol) or subtraction (ror) modulo XLEN.

$$i'_{\text{rot}} := i \pm k \mod XLEN$$

Generalized reverse with control argument k is simply XOR-ing the bit addresses with k:

$$i'_{\text{grev}} := i \oplus k$$

And generalized shuffle is performing a bit permutation on the bits of the bit addresses:

$$i'_{\text{shfl}} := \operatorname{perm}_k(i)$$

With the caveat that a single shf1/unshf1 instruction can only perferm a certain sub-set of bit address permutations, but a sequence of 4 shf1/unshf1 instructions can perform any of the 120 such permutations on RV32, and a sequence of 5 shf1/unshf1 instructions can perform any of the 720 such permutations on RV64.

Combining those three types of operations makes a vast number of bit permutations accessible within only a few instructions [24] (see Table 2.1).

N	ROT-only	GREV-only	SHFL-only	ROT+GREV	ROT+GREV+SHFL
0	1	1	1	1	1
1	32	32	24	62	85
2			86	864	3030
3			119	4 640	78659
4			120	23312	2002167
5				92 192	50106844
6				294992	1234579963
7				703744	est. $30000000000$
8				1012856	est. $700000000000$
9				1046224	est. $15000000000000$
10				1048576	
11	_	_	_		

Table 2.1: Number of permutations reachable with N permutation instructions on RV32. "—" indicates that additional instructions don't increase the space of reachable permutations.

Sequences of ror, grev, and [un]shfl instructions can generate any arbitrary bit permutation. Often in surprising ways. For example, the following sequence swaps the two LSB bits of a0:

Instruction	State (XLEN=8)	Bit-Index Op
initial value	76543210	_
rori a0, a0, 2	$1\ 0\ 7\ 6\ 5\ 4\ 3\ 2$	i' := i - 2
unshfli a0, a0, -1	$1\ 7\ 5\ 3\ 0\ 6\ 4\ 2$	i' := ror(i)
roli a0, a0, 1	$7\ 5\ 3\ 0\ 6\ 4\ 2\ 1$	i' := i + 1
shfli a0, a0, -1	$7\ 6\ 5\ 4\ 3\ 2\ 0\ 1$	$i' := \operatorname{rol}(i)$

Table 2.2: Breakdown of the ror+[un]shfl sequence for swapping the two LSB bits of a word, using XLEN=8 for simplicity.

```
rori a0, a0, 2
unshfli a0, a0, -1
roli a0, a0, 1
shfli a0, a0, -1
```

The mechanics of this sequence is closely related to the fact that rol(ror(x-2)+1) is a function that maps 1 to 0 and 0 to 1 and every other number to itself. (With rol and ror denoting 1-bit rotate left and right shifts respectively.) See Table 2.2 for details.

The numbers in the right column of Table 2.1 might appear large, but they are tiny in comparison to the total number of 32-bit permutations  $(2.63 \cdot 10^{35})$ . Fortunately the [un]shfl instruction "explores" permutations that involve fields that have a size that's a power-of-two, and/or moves that are a power-of-two first, which means we get shorter sequences for permutations we more often care about in real-world applications.

For example, there are 24 ways of arranging the four bytes in a 32-bit word. ror, grev, and [un]shfl can perform any of those permutations in at most 3 instructions. See Table 4.1 for a list of those 24 sequences.

There are 40320 ways of arranging the eight bytes in a 64-bit word or the eight nibbles in a 32-bit word. ror, grev, and [un]shfl can perform any of those permutations in at most 9 instructions. [24]

Besides the more-or-less arbitrary permutations we get from combining ror, grev, and [un]shfl in long sequences, there are of course many cases where just one of these instructions does exactly what we need. For grev and [un]shfl we define rev\* and [un]zip\* pseudo instructions for the most common use-cases.

#### 2.2.1 Rotate (Left/Right) (rol, ror, rori)

```
RV32, RV64:
ror rd, rs1, rs2
rol rd, rs1, imm

RV64 only:
rorw rd, rs1, rs2
rolw rd, rs1, rs2
roriw rd, rs1, rs2
roriw rd, rs1, imm
```

These instructions are similar to shift-logical operations from the base spec, except they shift in the values from the opposite side of the register, in order. This is also called 'circular shift'.

```
uint_xlen_t rol(uint_xlen_t rs1, uint_xlen_t rs2)
{
    int shamt = rs2 & (XLEN - 1);
    return (rs1 << shamt) | (rs1 >> ((XLEN - shamt) & (XLEN - 1)));
}
uint_xlen_t ror(uint_xlen_t rs1, uint_xlen_t rs2)
{
    int shamt = rs2 & (XLEN - 1);
    return (rs1 >> shamt) | (rs1 << ((XLEN - shamt) & (XLEN - 1)));
}</pre>
```

### 2.2.2 Generalized Reverse (grev, grevi, rev)

```
RV32, RV64:
grev rd, rs1, rs2
grevi rd, rs1, imm

RV64 only:
grevw rd, rs1, rs2
greviw rd, rs1, imm
```

This instruction provides a single hardware instruction that can implement all of byte-order swap, bitwise reversal, short-order-swap, word-order-swap (RV64), nibble-order swap, bitwise reversal in a byte, etc, all from a single hardware instruction.

The Generalized Reverse (GREV) operation iteratively checks each bit i in the 2nd argument from i = 0 to  $log_2(XLEN) - 1$ , and if the corresponding bit is set, swaps each adjacent pair of  $2^i$  bits.

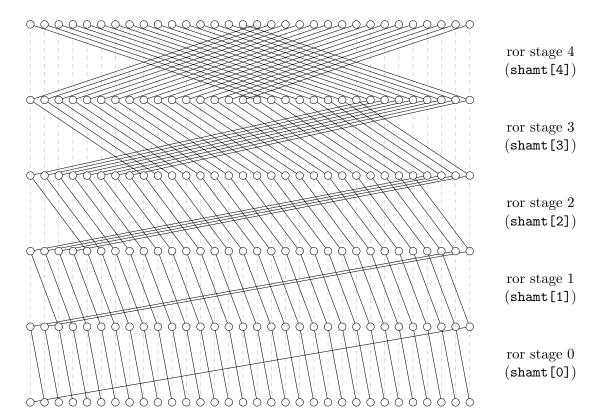


Figure 2.1: ror permutation network

```
uint32_t grev32(uint32_t rs1, uint32_t rs2)
{
    uint32_t x = rs1;
    int shamt = rs2 & 31;
    if (shamt & 1) x = ((x & 0x555555555) << 1) | ((x & 0xAAAAAAAA) >> 1);
    if (shamt & 2) x = ((x & 0x333333333) << 2) | ((x & 0xCCCCCCCC) >> 2);
    if (shamt & 4) x = ((x & 0x0F0F0F0F) << 4) | ((x & 0xF0F0F0F0) >> 4);
    if (shamt & 8) x = ((x & 0x00FF00FF) << 8) | ((x & 0xFF00FF00) >> 8);
    if (shamt & 16) x = ((x & 0x0000FFFF) << 16) | ((x & 0xFFFF0000) >> 16);
    return x;
}
```

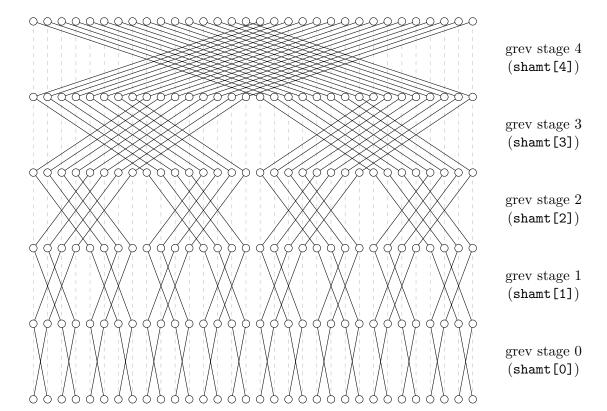


Figure 2.2: grev permutation network

```
uint64_t grev64(uint64_t rs1, uint64_t rs2)
   uint64_t x = rs1;
   int shamt = rs2 \& 63;
   ((x & OxAAAAAAAAAAAAAALL) >>
                                                  1);
   if (shamt & 2) x = ((x \& 0x333333333333333331L) <<
                                                  2) |
                     ((x & 0xCCCCCCCCCCCLL) >>
   if (shamt & 4) x = ((x & 0x0F0F0F0F0F0F0FLL) <<
                                                  4)
                      ((x & 0xF0F0F0F0F0F0F0F0LL) >>
                                                  4);
   if (shamt & 8) x = ((x & 0x00FF00FF00FF00FFLL) <<
                                                  8) I
                     ((x & 0xFF00FF00FF00LL) >> 8);
   if (shamt & 16) x = ((x & 0x0000FFFF0000FFFFLL) << 16) |
                      ((x & 0xFFFF0000FFFF0000LL) >> 16);
   if (shamt & 32) x = ((x & 0x00000000FFFFFFFLL) << 32) |
                     ((x & 0xFFFFFFF00000000LL) >> 32);
   return x;
}
```

The above pattern should be intuitive to understand in order to extend this definition in an obvious manner for RV128.

The grev operation can easily be implemented using a permutation network with  $log_2(XLEN)$ 

R	V32			RV64	
shamt	Instruction	shamt	Instruction	shamt	Instruction
0: 00000		0: 000000		32: 100000	rev32
1: 00001	rev.p	1: 000001	rev.p	33: 100001	
2: 00010	rev2.n	2: 000010	rev2.n	34: 100010	
3: 00011	rev.n	3: 000011	rev.n	35: 100011	_
4: 00100	rev4.b	4: 000100	rev4.b	36: 100100	
5: 00101	_	5: 000101	_	37: 100101	
6: 00110	rev2.b	6: 000110	rev2.b	38: 100110	
7: 00111	rev.b	7: 000111	rev.b	39: 100111	
8: 01000	rev8.h	8: 001000	rev8.h	40: 101000	
9: 01001	_	9: 001001	_	41: 101001	_
10: 01010	_	10: 001010	_	42: 101010	_
11: 01011	_	11: 001011		43: 101011	
12: 01100	rev4.h	12: 001100	rev4.h	44: 101100	
13: 01101	_	13: 001101		45: 101101	
14: 01110	rev2.h	14: 001110	rev2.h	46: 101110	
15: 01111	rev.h	15: 001111	rev.h	47: 101111	_
16: 10000	rev16	16: 010000	rev16.w	48: 110000	rev16
17: 10001	_	17: 010001	_	49: 110001	_
18: 10010	_	18: 010010	_	50: 110010	_
19: 10011	_	19: 010011	_	51: 110011	_
20: 10100	_	20: 010100	_	52: 110100	_
21: 10101	_	21: 010101	_	53: 110101	_
22: 10110	_	22: 010110	_	54: 110110	_
23: 10111	_	23: 010111	_	55: 110111	_
24: 11000	rev8	24: 011000	rev8.w	56: 111000	rev8
25: 11001	_	25: 011001	_	57: 111001	_
26: 11010	_	26: 011010	_	58: 111010	_
27: 11011	_	27: 011011	_	59: 111011	_
28: 11100	rev4	28: 011100	rev4.w	60: 111100	rev4
29: 11101	_	29: 011101	_	61: 111101	_
30: 11110	rev2	30: 011110	rev2.w	62: 111110	rev2
31: 11111	rev	31: 011111	rev.w	63: 111111	rev

Table 2.3: Pseudo-instructions for grevi instruction

stages. Figure 2.1 shows the permutation network for ror for reference. Figure 2.2 shows the permutation network for grev.

Pseudo-instructions are provided for the most common GREVI use-cases. Their names consist of a prefix and and optional suffix. Each prefix and suffix corresponds to a bit mask (see Table 2.4). The GREVI control word is obtained by AND-ing the two masks together.

In other words, the prefix controls the number of zero bits at the LSB end of the control word, and the suffix controls the number of zeros at the MSB end of the control word.

rev8 reverses the order of bytes in a word, thus performs endianness conversion. This is equivalent to the ARM REV instructions or BSWAP on x86. ARM also has instructions for swapping the bytes in 16-bit and 32-bit words, and reversing the bit order (see table 2.5).

Prefix	Mask	Suffix	Mask	
rev	111111		111111	
rev2	111110	. W	011111	(w = word)
rev4	111100	.h	001111	(h = half word)
rev8	111000	.b	000111	(b = byte)
rev16	110000	.n	000011	(n = nibble)
rev32	100000	.p	000001	(p = pair)

Table 2.4: Naming scheme for grevi pseudo-instructions. The prefix and suffix masks are ANDed to compute the immediate argument.

RISC-V	ARM	X86
rev	RBIT	_
rev8.h	REV16	_
rev8.w	REV32	_
rev8	REV	BSWAP

Table 2.5: Comparison of bit/byte reversal instructions

#### 2.2.3 Generalized Shuffle (shfl, unshfl, shfli, unshfli, zip, unzip)

```
RV32, RV64:
shfl rd, rs1, rs2
unshfl rd, rs1, imm
unshfli rd, rs1, imm
unshfli rd, rs1, imm

RV64 only:
shflw rd, rs1, rs2
unshflw rd, rs1, rs2
unshflw rd, rs1, rs2
```

Shuffle is the third bit permutation instruction in the RISC-V Bitmanip extension, after rotary shift and generalized reverse. It implements a generalization of the operation commonly known as perfect outer shuffle and its inverse (shuffle/unshuffle), also known as zip/unzip or interlace/uninterlace.

Bit permutations can be understood as reversible functions on bit indices (i.e. 5 bit functions on RV32 and 6 bit functions on RV64).

Operation	Corresponding function on bit indices
Rotate shift	Addition modulo XLEN
Generalized reverse	XOR with bitmask
Generalized shuffle	Bitpermutation

A generalized (un)shuffle operation has  $log_2(XLEN) - 1$  control bits, one for each pair of neighbouring bits in a bit index. When the bit is set, generalized shuffle will swap the two index bits.

The shf1 operation performs this swaps in MSB-to-LSB order (performing a rotate left shift on contiguous regions of set control bits), and the unshf1 operation performs the swaps in LSB-to-MSB order (performing a rotate right shift on contiguous regions of set control bits). Combining up to  $log_2(XLEN)$  of those shf1/unshf1 operations can implement any bitpermutation on the bit indices.

The most common type of shuffle/unshuffle operation is one on an immediate control value that only contains one contiguous region of set bits. We call those operations zip/unzip and provide pseudo-instructions for them. The naming scheme for those pseudo-instructions is similar to the naming scheme for the **grevi** pseudo-instructions (see Tables 2.3 and 2.4), except that the LSB bit of the masks in Table 2.4 is not used for zip/unzip.

Shuffle/unshuffle operations that only have individual bits set (not a contiguous region of two or more bits) are their own inverse.

shamt	inv	Bit index rotations	Pseudo-Instruction
0: 0000	0	no-op	_
0000	1	no-op	_
1: 0001	0	i[1] -> i[0]	zip.n, unzip.n
0001	1	equivalent to 0001 0	_
2: 0010	0	i[2] -> i[1]	zip2.b, unzip2.b
0010	1	$equivalent\ to\ 0010\ 0$	_
3: 0011	0	i[2] -> i[0]	zip.b
0011	1	i[2] <- i[0]	unzip.b
4: 0100	0	i[3] -> i[2]	zip4.h, unzip4.h
0100	1	$equivalent\ to\ 0100\ 0$	_
5: 0101	0	i[3] -> i[2], i[1] -> i[0]	_
0101	1	equivalent to 0101 0	_
6: 0110	0	i[3] -> i[1]	zip2.h
0110	1	i[3] <- i[1]	unzip2.h
7: 0111	0	i[3] -> i[0]	zip.h
0111	1	i[3] <- i[0]	unzip.h
8: 1000	0	i[4] -> i[3]	zip8, unzip8
1000	1	$equivalent\ to\ 1000\ 0$	_
9: 1001	0	i[4] -> i[3], i[1] -> i[0]	_
1001	1	$equivalent\ to\ 1001\ 0$	_
10: 1010	0	i[4] -> i[3], i[2] -> i[1]	
1010	1	$equivalent\ to\ 1010\ 0$	_
11: 1011	0	i[4] -> i[3], i[2] -> i[0]	_
1011	1	i[4] <- i[3], i[2] <- i[0]	_
12: 1100	0	i[4] -> i[2]	zip4
1100	1	i[4] <- i[2]	unzip4
13: 1101	0	i[4] -> i[2], i[1] -> i[0]	_
1101	1	i[4] <- i[2], i[1] <- i[0]	_
14: 1110	0	i[4] -> i[1]	zip2
1110	1	i[4] <- i[1]	unzip2
15: 1111	0	i[4] -> i[0]	zip
1111	1	i[4] <- i[0]	unzip

Table 2.6: RV32 modes and pseudo-instructions for shfli/unshfli instruction

Like GREV and rotate shift, the (un)shuffle instruction can be implemented using a short sequence

shamt	inv	Pseudo-Instruction	shamt	inv	Pseudo-Instruction
0: 00000	0		16: 10000	0	zip16, unzip16
00000	1	_	10000	1	_
1: 00001	0	zip.n, unzip.n	17: 10001	0	_
00001	1	_	10001	1	_
2: 00010	0	zip2.b, unzip2.b	18: 10010	0	_
00010	1	_	10010	1	_
3: 00011	0	zip.b	19: 10011	0	_
00011	1	unzip.b	10011	1	_
4: 00100	0	zip4.h, unzip4.h	20: 10100	0	_
00100	1	_	10100	1	_
5: 00101	0	_	21: 10101	0	_
00101	1	_	10101	1	_
6: 00110	0	zip2.h	22: 10110	0	_
00110	1	unzip2.h	10110	1	_
7: 00111	0	zip.h	23: 10111	0	_
00111	1	unzip.h	10111	1	_
8: 01000	0	zip8.w, unzip8.w	24: 11000	0	zip8
01000	1	_	11000	1	unzip8
9: 01001	0	_	25: 11001	0	_
01001	1	_	11001	1	_
10: 01010	0	_	26: 11010	0	_
01010	1	_	11010	1	_
11: 01011	0	_	27: 11011	0	_
01011	1	_	11011	1	_
12: 01100	0	zip4.w	28: 11100	0	zip4
01100	1	unzip4.w	11100	1	unzip4
13: 01101	0	_	29: 11101	0	_
01101	1	_	11101	1	_
14: 01110	0	zip2.w	30: 11110	0	zip2
01110	1	unzip2.w	11110	1	unzip2
15: 01111	0	zip.w	31: 11111	0	zip
01111	1	unzip.w	11111	1	unzip

Table 2.7: RV64 modes and pseudo-instructions for shfli/unshfli instruction

of elementary permutations, that are enabled or disabled by the shamt bits. But (un)shuffle has one stage fewer than GREV. Thus shfli+unshfli together require the same amount of encoding space as grevi.

```
uint32_t shuffle32_stage(uint32_t src, uint32_t maskL, uint32_t maskR, int N)
{
    uint32_t x = src & ~(maskL | maskR);
    x |= ((src << N) & maskL) | ((src >> N) & maskR);
    return x;
}
```

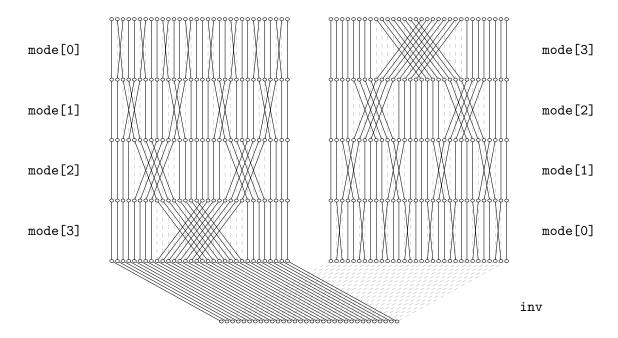


Figure 2.3: (un)shuffle permutation network without "flip" stages

```
uint32_t shf132(uint32_t rs1, uint32_t rs2)
{
    uint32_t x = rs1;
    int shamt = rs2 & 15;
    if (shamt & 8) x = shuffle32\_stage(x, 0x00ff0000, 0x0000ff00, 8);
    if (shamt & 4) x = shuffle32\_stage(x, 0x0f000f00, 0x00f000f0, 4);
    if (shamt & 2) x = shuffle32\_stage(x, 0x30303030, 0x0c0c0c0c, 2);
    if (shamt & 1) x = shuffle32\_stage(x, 0x44444444, 0x222222222, 1);
    return x;
}
uint32_t unshf132(uint32_t rs1, uint32_t rs2)
{
    uint32_t x = rs1;
    int shamt = rs2 \& 15;
    if (shamt & 1) x = shuffle32\_stage(x, 0x44444444, 0x222222222, 1);
    if (shamt & 2) x = shuffle32_stage(x, 0x30303030, 0x0c0c0c0c, 2);
    if (shamt & 4) x = shuffle32_stage(x, 0x0f000f00, 0x00f000f0, 4);
    if (shamt & 8) x = shuffle32\_stage(x, 0x00ff0000, 0x0000ff00, 8);
    return x;
}
```

Or for RV64:

```
uint64_t shuffle64_stage(uint64_t src, uint64_t maskL, uint64_t maskR, int N)
{
   uint64_t x = src & ~(maskL | maskR);
   x \mid = ((src \ll N) \& maskL) \mid ((src >> N) \& maskR);
   return x;
}
uint64_t shf164(uint64_t rs1, uint64_t rs2)
{
   uint64_t x = rs1;
   int shamt = rs2 \& 31;
   if (shamt & 16) x = shuffle64_stage(x, 0x0000ffff00000000LL,
                                       0x0000000ffff0000LL, 16);
   if (shamt & 8) x = shuffle64_stage(x, 0x00ff000000ff0000LL,
                                       0x0000ff000000ff00LL, 8);
   if (shamt & 4) x = shuffle64_stage(x, 0x0f000f000f000f00LL,
                                       0x00f000f000f000f0LL, 4);
   if (shamt & 2) x = shuffle64\_stage(x, 0x303030303030303030LL,
                                       0x0c0c0c0c0c0c0c0cLL, 2);
   0x222222222222LL, 1);
   return x;
}
uint64_t unshf164(uint64_t rs1, uint64_t rs2)
{
   uint64_t x = rs1;
   int shamt = rs2 \& 31;
   0x222222222222LL, 1);
   if (shamt & 2) x = shuffle64\_stage(x, 0x303030303030303030LL,
                                       0x0c0c0c0c0c0c0c0cLL, 2);
   if (shamt & 4) x = shuffle64\_stage(x, 0x0f000f000f000f000LL,
                                       0x00f000f000f0ULL, 4);
   if (shamt & 8) x = shuffle64\_stage(x, 0x00ff000000ff0000LL,
                                       0x0000ff000000ff00LL, 8);
   if (shamt & 16) x = shuffle64\_stage(x, 0x0000ffff00000000LL,
                                       0x0000000ffff0000LL, 16);
   return x;
}
```

The above pattern should be intuitive to understand in order to extend this definition in an obvious manner for RV128.

Alternatively (un)shuffle can be implemented in a single network with one more stage than GREV,

with the additional first and last stage executing a permutation that effectively reverses the order of the inner stages. However, since the inner stages only mux half of the bits in the word each, a hardware implementation using this additional "flip" stages might actually be more expensive than simply creating two networks.

```
uint32_t shuffle32_flip(uint32_t src)
{
    uint32_t x = src & 0x88224411;
    x = ((src << 6) & 0x22001100) | ((src >> 6) & 0x00880044);
    x = ((src \ll 9) \& 0x00440000) | ((src >> 9) \& 0x00002200);
    x = ((src \ll 15) \& 0x44110000) | ((src >> 15) \& 0x00008822);
    x = ((src \ll 21) \& 0x11000000) | ((src >> 21) \& 0x00000088);
    return x;
}
uint32_t unshf132alt(uint32_t rs1, uint32_t rs2)
{
    uint32_t shfl_mode = 0;
    if (rs2 & 1) shfl_mode |= 8;
    if (rs2 & 2) shfl_mode |= 4;
    if (rs2 & 4) shfl_mode |= 2;
    if (rs2 & 8) shfl_mode |= 1;
    uint32_t x = rs1;
    x = shuffle32_flip(x);
    x = shf132(x, shfl_mode);
    x = shuffle32_flip(x);
    return x;
}
```

Figure 2.4 shows the (un)shuffle permutation network with "flip" stages and Figure 2.3 shows the (un)shuffle permutation network without "flip" stages.

The **zip** instruction with the upper half of its input cleared performs the commonly needed "fanout" operation. (Equivalent to **bdep** with a 0x55555555 mask.) The **zip** instruction applied twice fans out the bits in the lower quarter of the input word by a spacing of 4 bits.

For example, the following code calculates the bitwise prefix sum of the bits in the lower byte of a 32 bit word on RV32:

```
andi a0, a0, 0xff
zip a0, a0
zip a0, a0
slli a1, a0, 4
c.add a0, a1
slli a1, a0, 8
c.add a0, a1
slli a1, a0, 16
c.add a0, a1
```

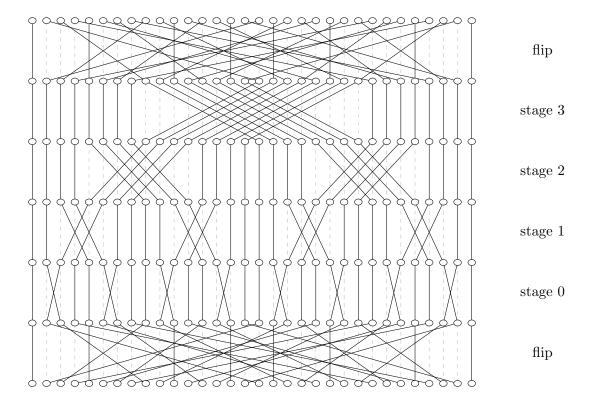


Figure 2.4: (un)shuffle permutation network with "flip" stages

The final prefix sum is stored in the 8 nibbles of the a0 output word.

Similarly, the following code stores the indices of the set bits in the LSB nibbles of the output word (with the LSB bit having index 1), with the unused MSB nibbles in the output set to zero:

```
andi a0, a0, 0xff
zip a0, a0
zip a0, a0
orc.n a0, a0
li a1, 0x87654321
and a1, a0, a1
bext a0, a1, a0
```

Other zip modes can be used to "fan-out" in blocks of 2, 4, 8, or 16 bit. zip can be combined with grevi to perform inner shuffles. For example on RV64:

```
li a0, 0x0000000012345678

zip4 t0, a0 ; <- 0x0102030405060708

rev4.b t1, t0 ; <- 0x1020304050607080

zip8 t2, a0 ; <- 0x00120034005600780

rev8.h t3, t2 ; <- 0x1200340056007800

zip16 t4, a0 ; <- 0x0000123400005678

rev16.w t5, t4 ; <- 0x1234000056780000
```

Another application for the zip instruction is generating Morton code [23].

The x86 PUNPCK [LH] \* MMX/SSE/AVX instructions perform similar operations as zip8 and zip16.

R	V32			RV64	
shamt	Instruction	shamt	Instruction	shamt	Instruction
0: 00000		0: 000000		32: 100000	orc32
1: 00001	orc.p	1: 000001	orc.p	33: 100001	_
2: 00010	orc2.n	2: 000010	orc2.n	34: 100010	_
3: 00011	orc.n	3: 000011	orc.n	35: 100011	_
4: 00100	orc4.b	4: 000100	orc4.b	36: 100100	_
5: 00101	_	5: 000101	_	37: 100101	_
6: 00110	orc2.b	6: 000110	orc2.b	38: 100110	_
7: 00111	orc.b	7: 000111	orc.b	39: 100111	_
8: 01000	orc8.h	8: 001000	orc8.h	40: 101000	
9: 01001	_	9: 001001	_	41: 101001	
10: 01010	_	10: 001010	_	42: 101010	
11: 01011	_	11: 001011		43: 101011	
12: 01100	orc4.h	12: 001100	orc4.h	44: 101100	
13: 01101	_	13: 001101		45: 101101	
14: 01110	orc2.h	14: 001110	orc2.h	46: 101110	
15: 01111	orc.h	15: 001111	orc.h	47: 101111	
16: 10000	orc16	16: 010000	orc16.w	48: 110000	orc16
17: 10001	_	17: 010001		49: 110001	
18: 10010	_	18: 010010		50: 110010	
19: 10011	_	19: 010011		51: 110011	
20: 10100	_	20: 010100		52: 110100	
21: 10101	_	21: 010101		53: 110101	
22: 10110	_	22: 010110		54: 110110	
23: 10111	_	23: 010111		55: 110111	
24: 11000	orc8	24: 011000	orc8.w	56: 111000	orc8
25: 11001	_	25: 011001	_	57: 111001	_
26: 11010	_	26: 011010	_	58: 111010	_
27: 11011	_	27: 011011	_	59: 111011	_
28: 11100	orc4	28: 011100	orc4.w	60: 111100	orc4
29: 11101	_	29: 011101		61: 111101	
30: 11110	orc2	30: 011110	orc2.w	62: 111110	orc2
31: 11111	orc	31: 011111	orc.w	63: 111111	orc

Table 2.8: Pseudo-instructions for gorci instruction

## 2.3 Generalized OR-Combine (gorc, gorci)

```
RV32, RV64:
gorc rd, rs1, rs2
gorci rd, rs1, imm

RV64 only:
gorcw rd, rs1, rs2
gorciw rd, rs1, imm
```

The GORC operation is similar to GREV, except that instead of swapping pairs of bits, GORC ORs them together, and writes the new value in both positions.

```
uint32_t gorc32(uint32_t rs1, uint32_t rs2)
{
   uint32_t x = rs1;
   int shamt = rs2 \& 31;
   if (shamt & 1) x = ((x & 0x555555555) << 1) | ((x & 0xAAAAAAAA) >> 
                                                                   1);
   if (shamt & 2) x |= ((x & 0x33333333) << 2) | ((x & 0xCCCCCCCC) >>
                                                                   2);
   if (shamt & 4) x |= ((x & 0x0F0F0F0F) << 4) | ((x & 0xF0F0F0F0) >>
                                                                   4);
   if (shamt & 8) x |= ((x & 0x00FF00FF) << 8) | ((x & 0xFF00FF00) >>
                                                                   8);
   if (shamt & 16) x |= ((x & 0x0000FFFF) << 16) | ((x & 0xFFFF0000) >> 16);
   return x;
}
uint64_t gorc64(uint64_t rs1, uint64_t rs2)
{
   uint64_t x = rs1;
   int shamt = rs2 \& 63;
   1) |
                      ((x & Oxaaaaaaaaaaaaaaall) >>
                                                   1);
   2) |
                      ((x & 0xCCCCCCCCCCCLL) >>
                                                   2);
   if (shamt & 4) x = ((x & 0x0F0F0F0F0F0F0F1L) <<
                                                   4) |
                      ((x & 0xF0F0F0F0F0F0F0F0LL) >>
                                                   4);
   if (shamt & 8) x = ((x & 0x00FF00FF00FF00FFLL) <<
                                                   8) |
                      ((x & 0xFF00FF00FF00LL) >>
                                                   8);
   if (shamt & 16) x |= ((x & 0x0000FFFF0000FFFFLL) << 16) |
                      ((x & 0xFFFF0000FFFF0000LL) >> 16);
   if (shamt & 32) x |= ((x & 0x00000000FFFFFFFLL) << 32) |
                      ((x & 0xFFFFFFF00000000LL) >> 32);
   return x;
}
```

GORC can be usefull for copying naturally aligned fields in a word, and testing such fields for being equal zero.

gorci pseudo-instructions follow the same naming scheme as grevi pseudo-instructions (see Tables 2.3 and 2.4), except the prefix orc is used instead of rev. See Table 2.8 for a full list of gorci pseudo-instructions.

An important use-case is strlen() and strcpy(), which can utilize orc.b for testing for zero bytes, and counting trailing non-zero bytes in a word.

## 2.4 Bit-Field Place (bfp)

```
RV32, RV64:
bfp rd, rs1, rs2

RV64 only:
bfpw rd, rs1, rs2
```

The bit field place (bfp) instruction places up to XLEN/2 LSB bits from rs2 into the value in rs1. The upper bits of rs2 control the length of the bit field and target position. The layout of rs2 is chosen in a way that makes it possible to construct rs2 easily using pack[h] instructions and/or andi/lui.

```
uint_xlen_t bfp(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t cfg = rs2 >> (XLEN/2);
    if ((cfg >> 30) == 2)
        cfg = cfg >> 16;
    int len = (cfg >> 8) & (XLEN/2-1);
    int off = cfg & (XLEN-1);
    len = len ? len : XLEN/2;
    uint_xlen_t mask = slo(0, len) << off;
    uint_xlen_t data = rs2 << off;
    return (data & mask) | (rs1 & ~mask);
}</pre>
```

The layout of the control word in rs2 is as follows for RV32. LEN=0 encodes for LEN=16.

And on RV64 (with LEN=0 encoding for LEN=32):

When SEL=10 then LEN and OFF are used, otherwise LEN' and OFF' are used.

Placing bits from a0 in a1, with results in t0 on RV32:

```
addi t0, zero, {length[3:0], offset[7:0]}
pack t0, a0, t0
bfp t0, a1, t0
```

```
And on RV64:
```

```
lui t0, zero, {3'b 100, length[4:0], offset[7:0], 4'b 0000}
pack t0, a0, t0
bfp t0, a1, t0

Or with a2=length and a3=offset:

packh t0, a3, a2
pack t0, a0, t0
bfp t0, a1, t0

Placing up to 16 constant bits in any contiguous region:
 lui t0, ...
addi t0, t0, ...
bfp[w] t0, a1, t0
```

Note that above sequences only modify one register (t0), which makes them fuse-able sequences.

## 2.5 Bit Extract/Deposit (bext, bdep)

```
RV32, RV64:
bext rd, rs1, rs2
bdep rd, rs1, rs2

RV64 only:
bextw rd, rs1, rs2
bdepw rd, rs1, rs2
```

This instructions implement the generic bit extract and bit deposit functions. This operation is also referred to as bit gather/scatter, bit pack/unpack, parallel extract/deposit, compress/expand, or right\_compress/right\_expand.

bext collects LSB justified bits to rd from rs1 using extract mask in rs2.

bdep writes LSB justified bits from rs1 to rd using deposit mask in rs2.

Implementations may choose to use smaller multi-cycle implementations of bext and bdep, or even emulate the instructions in software.

Even though multi-cycle bext and bdep often are not fast enough to outperform algorithms that use sequences of shifts and bit masks, dedicated instructions for those operations can still be of great advantage in cases where the mask argument is not constant.

For example, the following code efficiently calculates the index of the tenth set bit in a0 using bdep:

```
li a1, 0x00000200
bdep a0, a1, a0
ctz a0, a0
```

For cases with a constant mask an optimizing compiler would decide when to use **bext** or **bdep** based on the optimization profile for the concrete processor it is optimizing for. This is similar to the decision whether to use MUL or DIV with a constant, or to perform the same operation using a longer sequence of much simpler operations.

The bext and bdep instructions are equivalent to the x86 BMI2 instructions PEXT and PDEP. But there is much older prior art. For example, the soviet BESM-6 mainframe computer, designed and built in the 1960s, had APX/AUX instructions with almost the same semantics. [1] (The BESM-6 APX/AUX instructions packed/unpacked at the MSB end instead of the LSB end. Otherwise it is the same instruction.)

Efficient hardware implementations of bext and bdep are described in [13] and demonstrated in [25].

## 2.6 Carry-Less Multiply (clmul, clmulh, clmulr)

```
RV32, RV64:
clmul rd, rs1, rs2
clmulh rd, rs1, rs2
clmulr rd, rs1, rs2
RV64 only:
clmulw rd, rs1, rs2
clmulhw rd, rs1, rs2
clmulhw rd, rs1, rs2
clmulrw rd, rs1, rs2
```

Calculate the carry-less product [21] of the two arguments. clmul produces the lower half of the carry-less product and clmulh produces the upper half of the 2·XLEN carry-less product.

Carry-less multiplication is equivalent to multiplication in the polynomial ring over GF(2).

clmulr produces bits 2·XLEN-2:XLEN-1 of the 2·XLEN carry-less product. That means clmulh is equivalent to clmulr followed by a 1-bit right shift. (The MSB of a clmulh result is always zero.) Another equivalent definition of clmulr is clmulr(a,b) := rev(clmul(rev(a), rev(b))). (The "r" in clmulr means reversed.)

Unlike mulh[[s]u], we add a \*W variant of clmulh. This is because we expect some code to use 32-bit clmul intrisics, even on 64-bit architectures. For example in cases where data is processed in 32-bit chunks.

```
uint_xlen_t clmul(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t x = 0;
    for (int i = 0; i < XLEN; i++)
        if ((rs2 >> i) & 1)
            x ^= rs1 << i;
    return x;
}
uint_xlen_t clmulh(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t x = 0;
    for (int i = 1; i < XLEN; i++)
        if ((rs2 >> i) & 1)
            x = rs1 \gg (XLEN-i);
    return x;
}
```

```
uint_xlen_t clmulr(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t x = 0;
    for (int i = 0; i < XLEN; i++)
        if ((rs2 >> i) & 1)
            x ^= rs1 >> (XLEN-i-1);
    return x;
}
```

The classic applications for clmul are Cyclic Redundancy Check (CRC) [11, 26] and Galois/Counter Mode (GCM), but more applications exist, including the following examples.

There are obvious applications in hashing and pseudo random number generations. For example, it has been reported that hashes based on carry-less multiplications can outperform Google's CityHash [17].

clmul of a number with itself inserts zeroes between each input bit. This can be useful for generating Morton code [23].

clmul of a number with -1 calculates the prefix XOR operation. This can be useful for decoding gray codes.

Another application of XOR prefix sums calculated with clmul is branchless tracking of quoted strings in high-performance parsers. [16]

Carry-less multiply can also be used to implement Erasure code efficiently. [14]

SPARC introduced similar instructions (XMULX, XMULXHI) in SPARC T3 in 2010. [6]

TI C6000 introduced a similar instruction (XORMPY) in C64x+. [7]

## 2.7 CRC Instructions (crc32.[bhwd], crc32c.[bhwd])

```
RV32, RV64:
crc32.b rd, rs
crc32.h rd, rs
crc32.w rd, rs
crc32c.b rd, rs
crc32c.h rd, rs
crc32c.h rd, rs
crc32c.w rd, rs
crc32c.w rd, rs
crc32c.w rd, rs
```

Unary Cyclic Redundancy Check (CRC) instructions that interpret the bits of rs1 as a CRC32/CRC32C state and perform a polynomial reduction of that state shifted left by 8, 16,

32, or 64 bits.

The instructions return the new CRC32/CRC32C state.

The crc32.w/crc32c.w instructions are equivalent to executing crc32.h/crc32c.h twice, and crc32.h/crc32c.h instructions are equivalent to executing crc32.b/crc32c.b twice.

All 8 CRC instructions operate on bit-reflected data.

```
uint_xlen_t crc32(uint_xlen_t x, int nbits)
{
    for (int i = 0; i < nbits; i++)
        x = (x >> 1) ^ (0xEDB88320 & ^((x&1)-1));
    return x;
}
uint_xlen_t crc32c(uint_xlen_t x, int nbits)
{
    for (int i = 0; i < nbits; i++)</pre>
        x = (x >> 1) ^ (0x82F63B78 & ^ ((x&1)-1));
    return x;
}
uint_xlen_t crc32_b(uint_xlen_t rs1) { return crc32(rs1, 8); }
uint_xlen_t crc32_h(uint_xlen_t rs1) { return crc32(rs1, 16); }
uint_xlen_t crc32_w(uint_xlen_t rs1) { return crc32(rs1, 32); }
uint_xlen_t crc32c_b(uint_xlen_t rs1) { return crc32c(rs1, 8); }
uint_xlen_t crc32c_h(uint_xlen_t rs1) { return crc32c(rs1, 16); }
uint_xlen_t crc32c_w(uint_xlen_t rs1) { return crc32c(rs1, 32); }
#if XLEN > 32
uint_xlen_t crc32_d (uint_xlen_t rs1) { return crc32 (rs1, 64); }
uint_xlen_t crc32c_d(uint_xlen_t rs1) { return crc32c(rs1, 64); }
#endif
```

Payload data must be XOR'ed into the LSB end of the state before executing the CRC instruction. The following code demonstrates the use of crc32.b:

```
uint32_t crc32_demo(const uint8_t *p, int len)
{
   uint32_t x = 0xfffffffff;
   for (int i = 0; i < len; i++) {
      x = x ^ p[i];
      x = crc32_b(x);
   }
   return ~x;
}</pre>
```

In terms of binary polynomial arithmetic those instructions perform the operation

$$rd'(x) = (rs1'(x) \cdot x^N) \mod \{1, P'\}(x),$$

with  $N \in \{8, 16, 32, 64\}$ ,  $P = 0xEDB8_8320$  for CRC32 and  $P = 0x82F6_3B78$  for CRC32C, a'

denoting the XLEN bit reversal of a, and  $\{a,b\}$  denoting bit concatenation. Note that for example for CRC32  $\{1,P'\}=0$ x1\_04C1\_1DB7 on RV32 and  $\{1,P'\}=0$ x1\_04C1\_1DB7\_0000\_0000 on RV64.

These dedicated CRC instructions are meant for RISC-V implementations without fast multiplier and therefore without fast clmul[h]. For implementations with fast clmul[h] it is recommended to use the methods described in [11] and demonstrated in [26] that can process XLEN input bits using just one carry-less multiply for arbitrary CRC polynomials.

In applications where those methods are not applicable it is possible to emulate the dedicated CRC instructions using two carry-less multiplies that implement a Barrett reduction. The following example implements a replacement for crc32.w (RV32).

```
crc32_w:
    li t0, 0xF7011641
    li t1, 0xEDB88320
    clmul a0, a0, t0
    clmulr a0, a0, t1
    ret
```

# 2.8 Bit-Matrix Instructions (bmatxor, bmator, bmatflip, RV64 only)

```
RISC-V Bitmanip ISA

RV64 only:
bmator rd, rs1, rs2
bmatxor rd, rs1, rs2
bmatflip rd, rs
```

These are 64-bit-only instruction that are not available on RV32. On RV128 they ignore the upper half of operands and sign extend the results.

This instructions interpret a 64-bit value as 8x8 binary matrix.

bmatxor performs a matrix-matrix multiply with boolean AND as multiply operator and boolean XOR as addition operator.

bmator performs a matrix-matrix multiply with boolean AND as multiply operator and boolean OR as addition operator.

bmatflip is a unary operator that transposes the source matrix. It is equivalent to zip; zip; zip on RV64.

```
uint64_t bmatflip(uint64_t rs1)
{
   uint64_t x = rs1;
   x = shf164(x, 31);
   x = shf164(x, 31);
   x = shf164(x, 31);
   return x;
}
uint64_t bmatxor(uint64_t rs1, uint64_t rs2)
   // transpose of rs2
   uint64_t rs2t = bmatflip(rs2);
   uint8_t u[8]; // rows of rs1
   uint8_t v[8]; // cols of rs2
   for (int i = 0; i < 8; i++) {
        u[i] = rs1 >> (i*8);
        v[i] = rs2t >> (i*8);
    }
   uint64_t x = 0;
    for (int i = 0; i < 64; i++) {
        if (pcnt(u[i / 8] & v[i % 8]) & 1)
           x |= 1LL << i;
    }
   return x;
}
```

```
uint64_t bmator(uint64_t rs1, uint64_t rs2)
{
    // transpose of rs2
    uint64_t rs2t = bmatflip(rs2);
    uint8_t u[8]; // rows of rs1
    uint8_t v[8]; // cols of rs2
    for (int i = 0; i < 8; i++) {
        u[i] = rs1 >> (i*8);
        v[i] = rs2t >> (i*8);
    }
    uint64_t x = 0;
    for (int i = 0; i < 64; i++) {
        if ((u[i / 8] & v[i % 8]) != 0)
            x = 1LL \ll i;
    }
    return x;
}
```

Among other things, bmatxor/bmator can be used to perform arbitrary permutations of bits within each byte (permutation matrix as 2nd operand) or perform arbitrary permutations of bytes within a 64-bit word (permutation matrix as 1st operand).

There are similar instructions in Cray XMT [5]. The Cray X1 architecture even has a full 64x64 bit matrix multiply unit [4]. (See Section 4.6 for how to implement 64x64 bit matrix operations with bmat[x]or.)

The MMIX architecture has MOR and MXOR instructions with the same semantic. [15, p. 182f]

The x86 EVEX/VEX/SSE instruction GF2P8AFFINEQB is equivalent to bmatxor.

The bmm.8 instruction proposed in [12] is also equivalent to bmatxor.

## 2.9 Ternary Bit-Manipulation Instructions

## 2.9.1 Conditional Mix (cmix)

```
RV32, RV64:
cmix rd, rs2, rs1, rs3
```

(Note that the assembler syntax of cmix has the rs2 argument first to make assembler code more readable. But the reference C code code below uses the "architecturally correct" argument order rs1, rs2, rs3.)

The cmix rd, rs2, rs1, rs3 instruction selects bits from rs1 and rs3 based on the bits in the control word rs2.

```
uint_xlen_t cmix(uint_xlen_t rs1, uint_xlen_t rs2, uint_xlen_t rs3)
{
    return (rs1 & rs2) | (rs3 & ~rs2);
}
```

It replaces sequences like the following.

```
and rd, rs1, rs2
andn t0, rs3, rs2
or rd, rd, t0
```

Using cmix a single butterfly stage can be implemented in only two instructions. Thus, arbitrary bit-permutations can be implemented using only 18 instruction (32 bit) or 22 instructions (64 bits).

#### 2.9.2 Conditional Move (cmov)

```
RV32, RV64:
cmov rd, rs2, rs1, rs3
```

(Note that the assembler syntax of cmov has the rs2 argument first to make assembler code more readable. But the reference C code code below uses the "architecturally correct" argument order rs1, rs2, rs3.)

The cmov rd, rs2, rs1, rs3 instruction selects rs1 if the control word rs2 is non-zero, and rs3 if the control word is zero.

```
uint_xlen_t cmov(uint_xlen_t rs1, uint_xlen_t rs2, uint_xlen_t rs3)
{
    return rs2 ? rs1 : rs3;
}
```

The cmov instruction helps avoiding branches, which can lead to better performance, and helps with constant-time code as used in some cryptography applications.

#### 2.9.3 Funnel Shift (fsl, fsr, fsri)

```
RV32, RV64:
fsl rd, rs1, rs3, rs2
fsr rd, rs1, rs3, imm

RV64 only:
fslw rd, rs1, rs3, rs2
fsrw rd, rs1, rs3, rs2
fsriw rd, rs1, rs3, rs2
fsriw rd, rs1, rs3, imm
```

(Note that the assembler syntax for funnel shifts has the rs2 argument last to make assembler code more readable. But the reference C code code below uses the "architecturally correct" argument order rs1, rs2, rs3.)

The fsl rd, rs1, rs2 instruction creates a  $2 \cdot \text{XLEN}$  word by concatenating rs1 and rs3 (with rs1 in the MSB half), rotate-left-shifts that word by the amount indicated in the  $log_2(\text{XLEN})$ + 1 LSB bits in rs2, and then writes the MSB half of the result to rd.

The fsr rd, rs1, rs2 instruction creates a  $2 \cdot \text{XLEN}$  word by concatenating rs1 and rs3 (with rs1 in the LSB half), rotate-right-shifts that word by the amount indicated in the  $log_2(\text{XLEN}) + 1$  LSB bits in rs2, and then writes the LSB half of the result to rd.

```
uint_xlen_t fsl(uint_xlen_t rs1, uint_xlen_t rs2, uint_xlen_t rs3)
    int shamt = rs2 & (2*XLEN - 1);
    uint_xlen_t A = rs1, B = rs3;
    if (shamt >= XLEN) {
        shamt -= XLEN;
        A = rs3;
        B = rs1;
    return shamt ? (A << shamt) | (B >> (XLEN-shamt)) : A;
}
uint_xlen_t fsr(uint_xlen_t rs1, uint_xlen_t rs2, uint_xlen_t rs3)
{
    int shamt = rs2 & (2*XLEN - 1);
    uint_xlen_t A = rs1, B = rs3;
    if (shamt >= XLEN) {
        shamt -= XLEN;
        A = rs3;
        B = rs1;
    }
    return shamt ? (A >> shamt) | (B << (XLEN-shamt)) : A;
}
```

A shift unit capable of either fsl or fsr is capable of performing all the other shift functions, including the other funnel shift, with only minimal additional logic.

For any values of A, B, and C:

```
fsl(A, B, C) = fsr(A, -B, C)
```

And for any values x and  $0 \le \text{shamt} < \text{XLEN}$ :

```
sll(x, shamt) == fsl(x, shamt, 0)
srl(x, shamt) == fsr(x, shamt, 0)
sra(x, shamt) == fsr(x, shamt, sext_x)
slo(x, shamt) == fsl(x, shamt, ~0)
sro(x, shamt) == fsr(x, shamt, ~0)
ror(x, shamt) == fsr(x, shamt, x)
rol(x, shamt) == fsl(x, shamt, x)
```

Furthermore an RV64 implementation of either fsl or fsr is capable of performing the \*W versions of all shift operations with only a few gates of additional control logic.

On RV128 there is no fsri instruction. But there is fsriw and fsrid.

## 2.10 Unsigned address calculation instructions

Consider C code that's using unsigned 32-bit ints as array indices. For example:

```
char addiwu_demo(char *p, unsigned int i) {
  return p[i-1];
}
int slliuw_demo(int *p, unsigned int i, unsigned int j) {
  return p[i^j];
}
```

In both cases the expression within p[...] must overflow according to 32-bit arithmetic, then be zero-extended, and then this zero-extended result must be used in the address calculation.

The instructions below make sure that no explicit zext.w instruction is needed in those cases, to make sure there is no systematic performance penalty for code like shown above on RV64 compared to RV32.

#### 2.10.1 Add/sub with postfix zero-extend (addwu, subwu, addiwu)

```
RV64:
addwu rd, rs1, rs2
subwu rd, rs1, rs2
addiwu rd, rs1, imm
```

These instructions are identical to addw, subw, addiw, except that bits XLEN-1:32 of the result are cleared after the addition. I.e. these instructions zero-extend instead of sign-extend the 32-bit result.

```
uint_xlen_t addwu(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t result = rs1 + rs2;
    return (uint32_t)result;
}
uint_xlen_t subwu(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t result = rs1 - rs2;
    return (uint32_t)result;
}
```

The 12-bit immediate to addiwn is sign-extended, exactly like the immediate to addiw.

#### 2.10.2 Add/sub/shift with prefix zero-extend (addu.w, subu.w, slliu.w)

```
RISC-V Bitmanip ISA
addu.w rd, rs1, rs2
subu.w rd, rs1, rs2
slliu.w rd, rs1, imm
```

slliu.w is identical to slli, except that bits XLEN-1:32 of the rs1 argument are cleared before the shift.

addu.w and subu.w are identical to add and sub, except that bits XLEN-1:32 of the rs2 argument are cleared before the add/subtract.

```
uint_xlen_t slliuw(uint_xlen_t rs1, int imm)
{
    uint_xlen_t rs1u = (uint32_t)rs1;
    int shamt = imm & (XLEN - 1);
    return rs1u << shamt;
}
uint_xlen_t adduw(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t rs2u = (uint32_t)rs2;
    return rs1 + rs2u;
}
uint_xlen_t subuw(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t rs2u = (uint32_t)rs2;
    return rs1 - rs2u;
}</pre>
```

### 2.11 Opcode Encodings

This chapter contains proposed encodings for most of the instructions described in this document. **DO NOT IMPLEMENT THESE OPCODES YET.** We are trying to get official opcodes assigned and will update this chapter soon with the official opcodes.

The andn, orn, and xnor instruction are encoded the same way as and, or, and xor, but with op[30] set, mirroring the encoding scheme used for add and sub.

All shift instructions use funct3=001 for left shifts and funct3=101 for right shifts.

op[26]=1 selects funnel shifts. For funnel shifts op[30:29] is part if the 3rd operand and therefore unused for encoding the operation. For all other shift operations op[26]=0.

fsri is also encoded with op[26]=1, leaving a 6 bit immediate. The 7th bit, that is necessary to perform a 128 bit funnel shift on RV64, can be emulated by swapping rs1 and rs3.

There is no shfliw instruction. The slliu.w instruction occupies the encoding slot that would be occupied by shfliw.

On RV128 op [26] contains the MSB of the immediate for the shift instructions. Therefore there is no FSRI instruction on RV128. (But there is FSRIW/FSRID.)

		SLL	SRL	SRA		SLO	SRO		ROL	ROR		FSL	FSR
op[30]		0	0	1	-	0	0		1	1	1	-	_
op[29]		0	0	0		1	1		1	1		-	-
op[26]		0	0	0		0	0		0	0		1	1
funct3	3	001	101	101	1	001	101	1	001	101	Ι	001	101

Only an encoding for RORI exists, as ROLI can be implemented with RORI by negating the immediate. Unary functions are encoded in the spot that would correspond to ROLI, with the function encoded in the 5 LSB bits of the immediate.

The CRC instructions are encoded as unary instructions with op [24] set. The polynomial is selected via op [23], with op [23]=0 for CRC32 and op [23]=1 for CRC32C. The width is selected with op [22:20], using the same encoding as is used in funct3 for load/store operations.

cmix and cmov are encoded using the two remaining ternary operator encodings in funct3=001 and funct3=101. (There are two ternary operator encodings per minor opcode using the op[26]=1 scheme for marking ternary OPs.)

The single-bit instructions are also encoded within the shift opcodes, with op[27] set, and using op[30] and op[29] to select the operation:

	SBCLR	SBSET	SBINV	-	SBEXT	GORC	GREV
op[30]	1	0	1	1	1	0	1
op[29]	0	1	1		0	1	1
op[27]	1	1	1		1	1	1
funct3	001	001	001		101	101	101

There is no sbextiw instruction as it can be emulated trivially using sbexti. However, there is

sbsetiw, sbclriw, and sbinviw as changing bit 31 would change the sign extend. There are non-immediate \*W instructions of all single-bit instructions, including sbextw, because the number of used bits in rs2 is different in sbext and sbextw.

GORC and GREV are encoded in the two remaining slots in the single-bit instruction encoding space.

The remaining instructions are encoded within funct7=0000100 and funct7=0000101.

The funct7=0000101 block contains clmul[hr], min[u], and max[u].

The encoding of clmul, clmulr, clmulh is identical to the encoding of mulh, mulhsu, mulhu, except that op[27]=1.

The encoding of min[u]/max[u] uses funct3=100..111. The funct3 encoding matches op[31:29] of the AMO min/max functions.

The remaining instructions are encoded within funct7=0000100. The shift-like shf1/unshf1 instructions uses the same funct3 values as the shift operations. bdep and bext are encoded in a way so that funct3[2] selects the "direction", similar to shift operations.

bmat[x] or use funct3=011 and funct3=111 in funct7=0000100.

pack occupies funct3=100 in funct7=0000100.

addwu and subwu are encoded like addw and subw, except that op [25] = 1 and op [27] = 1.

addu.w and subu.w are encoded like addw and subw, except that op [27] =1.

addiwu is encoded using funct3=100 (XOR) instead of funct3=000 in OP-32.

Finally, RV64 has \*W instructions for all bitmanip instructions, with the following exceptions:

andn, cmix, cmov, min[u], max[u] have no \*W variants because they already behave in the way a \*W instruction would when presented with sign-exteded 32-bit arguments.

bmatflip, bmatxor, bmator have no \*W variants because they are 64-bit only instructions.

crc32. [bhwd], crc32c. [bhwd] have no \*W variants because crc32[c].w is deemed sufficient.

There is no [un]shfliw, as a perfect outer shuffle always preserves the MSB bit, thus [un]shfli preserves proper sign extension when the upper bit in the control word is set. There's still [un]shflw that masks that upper control bit and sign-extends the output.

Relevant instruction encodings from the base ISA are included in the table below and are marked with a \*.

3 1 0 9 8 7	6 5 4 3 2 1 0		4 3 2	1 1 0 9 8 7	6 5 4 3 2 1 0	
funct	7   rs2	   rs1	f3	rd	   opcode	R-type
rs3	f2  rs2	rs1	f3	rd	opcode	R4-type
=======	imm 	rs1 	f3	rd	opcode   ========	I-type
0000000	   rs2	   rs1	111	rd	   0110011	AND*
0000000	rs2	rs1	110	rd	0110011	OR*
0000000	rs2	rs1	100	rd	0110011	XOR*
0100000	rs2	rs1	111	rd	0110011	ANDN
0100000	rs2	rs1	110	rd	0110011	ORN
0100000	rs2	rs1	100	rd	0110011	XNOR
0000000	rs2	   rs1	001	rd	   0110011	SLL*
0000000	rs2	rs1	101	rd	0110011	SRL*
0100000	rs2	rs1	101	rd	0110011	SRA*
0010000	rs2	rs1	001	rd	0110011	SLO
0010000	rs2	rs1	101	rd	0110011	SRO
0110000	rs2	rs1	001	rd	0110011	ROL
0110000	rs2	rs1	101	rd	0110011	ROR
0100100	   rs2	   rs1	001	rd	   0110011	SBCLR
0010100	rs2	rs1	001	rd	0110011	SBSET
0110100	rs2	rs1	001	rd	0110011	SBINV
0100100	rs2	rs1	101	rd	0110011	SBEXT
0010100	rs2	rs1	101	rd	0110011	GORC
0110100	rs2	rs1	101	rd	0110011	GREV
00000	   imm	   rs1	001	rd	   0010011	SLLI*
00000	imm	rs1	101	rd	0010011	SRLI*
01000	imm	rs1	101	rd	0010011	SRAI*
00100	imm	rs1	001	rd	0010011	SLOI
00100	imm	rs1	101		0010011	SROI
01100	imm	rs1	101	rd	0010011	RORI
01001	   imm	   rs1	001	rd	   0010011	SBCLRI
00101	imm	rs1	001		0010011	SBSETI
01101	imm	rs1	001		0010011	
01001	imm		101		0010011	
00101	imm	rs1	101	rd	0010011	GORCI
01101	imm	rs1	101	rd	0010011	GREVI
rs3	   11  rs2	   rs1	001	rd	   0110011	CMIX
rs3	11  rs2	rs1	101	rd	0110011	CMOV
rs3	10  rs2	rs1	001	rd	0110011	FSL
rs3	10  rs2	rs1	101	rd	0110011	FSR
rs3	1  imm	rs1	101	rd	0010011	FSRI

3  1 0 9 8 7 6	5 4 3 2 1 0	98765	4 3 2	1 1 0 9 8 7	6 5 4 3 2 1 0	
0110000	00000	========   rs1	001	 rd	======================================	CLZ
0110000	00001	rs1	001	rd	0010011	CTZ
0110000	00010	rs1	001	rd	0010011	PCNT
0110000	00011	rs1	001	_	0010011	BMATFLIP
0110000	00100	rs1	001	rd	0010011	SEXT.B
0110000	00101	rs1	001		0010011	SEXT.H
0110000	10000	   rs1	001	rd	   0010011	CRC32.B
0110000	10001	rs1	001	rd	0010011	CRC32.H
0110000	10010	rs1	001	rd	0010011	CRC32.W
0110000	10011	rs1	001	rd	0010011	CRC32.D
0110000	11000	rs1	001	rd	0010011	CRC32C.B
0110000	11001	rs1	001	rd	0010011	CRC32C.H
0110000	11010	rs1	001	rd	0010011	CRC32C.W
0110000	11011	rs1	001	rd	0010011	CRC32C.D
0000101	rs2	   rs1	001	rd	   0110011	CLMUL
0000101	rs2	rs1	010	rd	0110011	CLMULR
0000101	rs2	rs1	011	rd	0110011	CLMULH
0000101	rs2	rs1	100	rd	0110011	MIN
0000101	rs2	rs1	101	rd	0110011	MAX
0000101	rs2	rs1	110	rd	0110011	MINU
0000101	rs2	rs1 	111	rd	0110011   	MAXU
0000100	rs2	rs1	001	rd	0110011	SHFL
0000100	rs2	rs1	101	rd	0110011	UNSHFL
0100100	rs2	rs1	110	rd	0110011	BDEP
0000100	rs2	rs1	110	rd	0110011	BEXT
0000100	rs2	rs1	100	rd	0110011	PACK
0100100	rs2	rs1	100	rd	0110011	PACKU
0000100	rs2	rs1	011	rd	0110011	BMATOR
0100100	rs2	rs1	011	rd	0110011	BMATXOR
0000100	rs2	rs1	111	rd	0110011	PACKH
0100100	rs2	rs1 	111	rd	0110011   	BFP
000010	imm	rs1	001		0010011	SHFLI
000010	imm ========	rs1 =======	101	rd =======	0010011   =======	UNSHFLI
·	diate	rs1	000		O011011	ADDIW*
immed	diate	rs1	100		0011011	ADDIWU
00001	imm 	rs1 	001	rd	0011011   	SLLIU.W
0000000	rs2	rs1	000		0111011	ADDW*
0100000	rs2	rs1	000	rd	0111011	SUBW*
0000101	rs2	rs1	000		0111011	ADDWU
0100101	rs2	rs1	000	rd	0111011	SUBWU
0000100	rs2	rs1	000		0111011	ADDU.W
0100100	rs2 	rs1 	000	rd	0111011   	SUBU.W

3	2			1		I	
1 0 9 8 7 6 5	4 3 2 1 0	98765	4 3 2	1 0 9 8	7 6	5 4 3 2 1 0	
0010000	rs2	   rs1	   001	 rd		0111011	SLOW
0010000	rs2	rs1	101	rd	Ì	0111011	SROW
0110000	rs2	rs1	001	rd	Ì	0111011	ROLW
0110000	rs2	rs1	101	rd	Ī	0111011	RORW
0100100	rs2	   rs1	   001	rd		 0111011	SBCLRW
0010100	rs2	rs1	001	rd	i	0111011	SBSETW
0110100	rs2	rs1	001	rd	Ì	0111011	SBINVW
0100100	rs2	rs1	101	rd	Ì	0111011	SBEXTW
0010100	rs2	rs1	101	rd	I	0111011	GORCW
0110100	rs2	rs1	101	rd	Ī	0111011	GREVW
0010000	imm	   rs1	   001	rd	 I	   0011011	SLOIW
0010000	imm	rs1	101	rd	i	0011011	SROIW
0110000	imm	rs1	101	rd	i	0011011	RORIW
0100100	 imm	   rs1	   001	 rd	 I	 0011011	SBCLRIW
0010100	imm	rs1	001     001	rd	i	0011011	SBSETIW
0110100	imm	rs1	001     001	rd	i	0011011	SBINVIW
0010100	imm	rs1	101     101	rd	i	0011011	GORCIW
0110100	imm	rs1	101	rd	i	0011011	GREVIW
   rs3   10	 rs2	   rs1	   001	 rd	 I	 0111011	FSLW
rs3   10	rs2	rs1	101     101	rd	i	0111011	FSRW
rs3   10	imm	rs1	101	rd	i	0011011	FSRIW
	00000	   rs1	   001	 rd	 I	 0011011	CLZW
0110000	00001	rs1	001     001	rd	i	0011011	CTZW
0110000	00010	rs1	001	rd	i	0011011	PCNTW
	rs2	   rs1	   001	 rd	 I	 0111011	CLMULW
0000101	rs2	rs1	001     010	rd	i	0111011	CLMULRW
0000101	rs2		010	rd	i	0111011	CLMULHW
	 rs2	   rs1	   001	 rd	 I	 0111011	SHFLW
0000100	rs2	rs1	101	rd	i	0111011	UNSHFLW
0100100	rs2	rs1	101	rd	İ	0111011	BDEPW
0000100	rs2	rs1	110	rd	i	0111011	BEXTW
0000100	rs2		100	rd	i	0111011	PACKW
0100100	rs2	rs1	100     100	rd	i	0111011	PACKUW
0100100	rs2	rs1	111	rd	i	0111011	BFPW

### 2.12 Future compressed instructions

The RISC-V ISA has no dedicated instructions for bitwise inverse (not). Instead not is implemented as xori rd, rs, -1 and neg is implemented as sub rd, x0, rs.

In bitmanipulation code not is a very common operation. But there is no compressed encoding for those operation because there is no c.xori instruction.

On RV64 (and RV128) zext.w and zext.d (pack and packw) are commonly used to zero-extend unsigned values <XLEN.

It presumably would make sense for a future revision of the "C" extension to include compressed opcodes for those instructions.

An encoding with the constraint rd = rs would fit nicely in the reserved space in c.addi16sp/c.lui.

15 14	13 12	11 10	9	8	7	6	5	4	3	2	1	0	
011	nzimm[9]		2			nz	imn	n[4]6	3 8:7	[5]	(	01	C.ADDI16SP $(RES, nzimm=\theta)$
011	nzimm[17]	rd	<b>≠</b> {0.	$, 2$ }		r	nzim	m[1	6:12	2]	(	01	C.LUI (RES, nzimm= $\theta$ ; HINT, rd= $\theta$ )
011	0	00	rs	1'/r	$\mathrm{d}'$			0			(	01	C.NOT
011	0	01	rs	1'/r	$\mathrm{d}'$			0			(	01	C.NEG
011	0	10	rs	1'/r	$\mathrm{d}'$			0			(	01	C.ZEXT.W (RV64/128; RV32 RES)
011	0	11	rs	1'/r	$\mathrm{d}'$			0			(	01	C.ZEXT.D (RV128; RV32/64 RES)

The entire RVC encoding space is 15.585 bits wide, the remaining reserved encoding space in RVC is 11.155 bits wide, not including space that is only reserved on RV32/RV64. This means that above encoding would use 0.0065% of the RVC encoding space, or 1.4% of the remaining reserved RVC encoding space. Preliminary experiments have shown that NOT instructions alone make up approximately 1% of bitmanipulation code size. [27]

## 2.13 Macro-op fusion patterns

Some bitmanip operations have been left out of this spec because of lack of a (sensible) way to encode them in the 32-bit RISC-V encoding space. Instead we present fuse-able sequences of up to three instructions for those operations, so that high-end processors can implement them in a single fused macro-op, should they decide to do so.

For this document we only consider sequences fuse-able if they read at most two registers, only write one register, and contain no branch instructions.

#### 2.13.1 Fused \*-bfp sequences

The bfp instruction is most commonly used in sequences of one the the following forms.

```
For 32-bit (RV32 or *W instructions on RV64):

addi rd, zero, ...

pack[w] rd, rs2, rd

bfp[w] rd, rs1, rd

lui rd, ...

addi rd, rd, ...

bfp[w] rd, rs1, rd
```

And for 64-bit:

```
lui rd, ...
pack rd, rs2, rd
bfp rd, rs1, rd
```

#### 2.13.2 Load-immediate

For 32-bit code (RV32 or \*W instructions on RV64) we recommend to fuse the lui+addi pattern for loading a 32-bit constants:

```
lui rd, imm
addi[w] rd, rd, imm
```

For RV64 we also recommend to fuse the followings patterns for loading a 32-bit constant zero-extended into a 64-bit register:

```
lui rd, imm
addiwu rd, rd, imm
addiw rd, zero, v
pack rd, rd, zero
```

Further, for loading 64-bit consants in two macro-ops:

```
lui rd, imm
addiw rd, rd, imm
pack rd, rd, rs
```

#### 2.13.3 Fused shift sequences

Pairs of left and right shifts are common operations for extracting a bit field.

To extract the contiguous bit field starting at pos with length len from rs (with pos > 0, len > 0, and pos + len  $\le$  XLEN):

```
slli rd, rs, (XLEN-len-pos)
srli rd, rd, (XLEN-len)
```

Using srai instead of srli will sign-extend the extracted bit-field.

Similarly, placing a bit field with length len at the position pos:

```
slli rd, rs, (XLEN-len)
srli rd, rd, (XLEN-len-pos)
```

Note that the postfix right shift instruction can use a compressed encoding, yielding a 48-bit fused instruction if the left shift is a 32-bit instruction.

More generally, a processor might fuse all destructive shift operations following any other ALU operation.

We define the following assembler pseudo-ops for sr[la]i postfix fusion:

```
bfext rd, rs, len, pos  -> slli rd, rs, (XLEN-len-pos); srai rd, rd, (XLEN-len)
bfextu rd, rs, len, pos  -> slli rd, rs, (XLEN-len-pos); srli rd, rd, (XLEN-len)
bfmak rd, len, pos  -> sroi rd, zero, len; srli rd, rd, (XLEN-len-pos)
```

(The names bfext, bfextu, and bfmak are borrowed from m88k, that had dedicated instructions of those names (without bf-prefix) with equivalent semantics. [3, p. 3-28])

#### 2.14 C intrinsics via <rvintrin.h>

A C header file <rvintrin.h> is provided that contains assembler templates for directly creating assembler instructions from C code.

The header defines \_rv\_\*(...) functions that operate on the long data type, \_rv32\_\*(...) functions that operate on the int32\_t data type, and \_rv64\_\*(...) functions that operate on the int64\_t data type. The \_rv64\_\*(...) functions are only available on RV64. See table 2.9 for a complete list of intrinsics defined in <ruintin.h>.

Usage example:

```
#include <rvintrin.h>
int find_nth_set_bit(unsigned int value, int cnt) {
  return _rv32_ctz(_rv32_bdep(1 << cnt, value));
}</pre>
```

Defining RVINTRIN\_EMULATE before including <rvintrin.h> will define plain C functions that emulate the behavior of the RISC-V instructions. This is useful for testing software on non-RISC-V platforms.

	R	V32		RV64	
Instruction	_rv_*	_rv32_*	_rv_*	_rv32_*	_rv64_*
clz	<b>'</b>	<b>V</b>	~	<b>V</b>	<b>V</b>
ctz	<b>'</b>	<b>✓</b>	<b>'</b>	<b>✓</b>	<b>✓</b>
pcnt	<b>'</b>	<b>✓</b>	<b>/</b>	<b>✓</b>	✓
pack	<b>'</b>	<b>✓</b>	~	<b>✓</b>	<b>✓</b>
min	<b>'</b>	~	<b>'</b>	<b>✓</b>	<b>✓</b>
minu	<b>'</b>	<b>✓</b>	<b>'</b>	<b>✓</b>	<b>✓</b>
max	<b>'</b>	•	<b>'</b>	<b>✓</b>	~
maxu	~	<b>'</b>	~	<b>/</b>	<b>/</b>
sbset	~	<b>/</b>	~	<b>/</b>	<b>V</b>
sbclr	<b>'</b>	<b>/</b>	<b>'</b>	<b>/</b>	<b>V</b>
sbinv	<b>'</b>	<b>V</b>	<b>/</b>	<b>/</b>	<b>/</b>
sbext	<b>'</b>	V	<b>/</b>	<u> </u>	~
sll	~	V	<b>/</b>	V	<b>V</b>
srl	<b>V</b>	<b>V</b>	<b>V</b>	<b>V</b>	<b>V</b>
sra	\ \ \ \ \ \	<b>V</b>	\ \ \ \ \	<b>V</b>	<b>V</b>
slo					
sro	~	<b>V</b>	~	<b>V</b>	<b>V</b>
rol	1	<b>V</b>	1	~	<i>V</i>
ror	<i>V</i>	· ·	<i>V</i>		
grev			l		
gorc shfl	~	<b>V</b>	~	<b>V</b>	<b>V</b>
unshfl	\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	<b>V</b>	\ \ \	<b>V</b>	/
	<i>'</i>	· ·	<b>V</b>	· ·	
bfp bext	<i>'</i>	~	~	· ·	~
bext bdep	<b>'</b>	~	/	~	~
clmul	<b>/</b>	· ·	~	~	~
clmulh	<b>'</b>	~	/	1	/
clmulr		/	/	1	/
bmatflip			~		
bmatrrip			/		1
bmatxor					Ž
fsl			~		<u> </u>
fsr	/	/	\( \times \)	/	~
cmix	~		~		
cmov	/		<b>V V</b>		
crc32_b	\( \times \)		~		
crc32_h	/		/		
crc32_w	<b>/</b>		<b>/</b>		
crc32_d			\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \		
crc32c_b	~				
crc32c_h	<b>/</b>		\ \ \ \		
crc32c_w	<b>/</b>		<b>/</b>		
crc32c_d			<b>/</b>		
	I		1 -		

Table 2.9: C intrinsics defined in <rvintrin.h>

## Chapter 3

## Reference Implementations

### 3.1 Verilog reference implementations

We have implemented Verilog cores for all instructions proposed in this specification. These cores are permissively licensed under the ISC license and can be obtained from https://github.com/riscv/riscv-bitmanip/tree/master/verilog.

For evaluation purposes we synthesized these cores for RV32 and RV64 to the following mockup ASIC cell library:

Cell	Gate Count	Cell	Gate Count
NOT	0.5	AOI3	1.5
NAND	1	OAI3	1.5
NOR	1	AOI4	2
XOR	3	OAI4	2
XNOR	3	NMUX	2.5
DFF	4	MUX	3

For comparison we also synthesized the rocket-chip MulDiv cores obtained using the following rocket-chip configurations:

```
class MulDivConfig64 extends Config(
   new WithFastMulDiv ++
   new DefaultConfig
)

class MulDivConfig32 extends Config(
   new WithRV32 ++
   new WithFastMulDiv ++
   new DefaultConfig
)
```

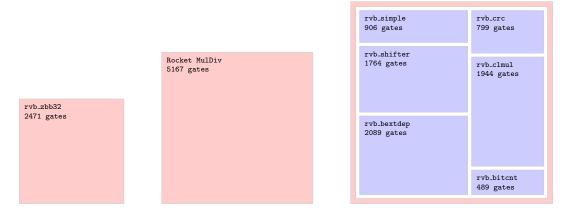


Figure 3.1: Area of 32-bit Rocket MulDiv core (center) compared to a complete implementation of all 32-bit instructions proposed in this specification (right), and the 32-bit "Zbb" extension (left).

The following table lists the verilog reference cores and the instructions they implement:

Module	Instructions
rvb_bextdep	bext bdep grev gorc shfl unshfl
rvb_clmul	clmul clmulr clmulh
$rvb\_shifter$	sll srl sra slo sro rol ror fsl fsr slliu.w
	sbset sbclr sbinv sbext bfp
$rvb\_bmatxor$	bmatxor bmator
${\tt rvb\_simple}$	min max minu maxu andn orn
	xnor pack cmix cmov addiwu addwu
	subwu adduw subuw
${\tt rvb\_bitcnt}$	clz ctz pcnt bmatflip
rvb_crc	crc32.[bhwd] crc32c.[bhwd]
$rvb\_full$	All of the above

On RV64 these cores also implement all \*W instruction variants of the above instructions.

Note that rvb\_shifter also implements the base ISA sll, srl, and sra instructions. Thus it can replace an existing implementation of the base ISA shift instructions.

Fig. 3.1 shows the area comparison for RV32 and fig. 3.2 shows the comparison for RV64. The area of the red frame surrounding the blue rvb\_\* modules accurately represents the added area by the rvb\_full wrapper module.

Regarding timing we evaluate the longest paths for rvb\_full and rocket-chip MulDiv, measured in gate delays:

	RV32	RV64
rvb_full	30	57
MulDiv	43	68

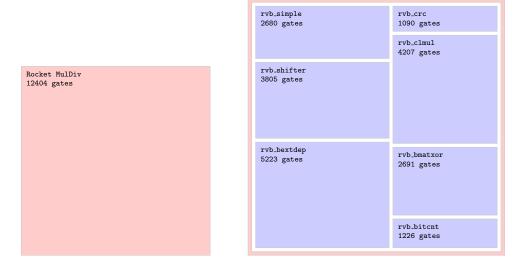


Figure 3.2: Area of 64-bit Rocket MulDiv core (left) compared to a complete implementation of all 64-bit instructions proposed in this specification (right).

All rvb\_\* reference cores provide single-cycle implementations of their functions, with the exception of rvb\_clmul which requires 4 cycles for a 32-bit carry-less multiply and 8 cycles for a 64-bit carry-less multiply, and rvb\_crc which requires 1 cycle for each payload byte.

## 3.2 Fast C reference implementations

GCC has intrinsics for the bit counting instructions clz, ctz, and pcnt. So a performance-sensitive application (such as an emulator) should probably just use those:

```
uint32_t fast_clz32(uint32_t rs1)
{
    if (rs1 == 0)
        return 32;
    assert(sizeof(int) == 4);
    return __builtin_clz(rs1);
}
uint64_t fast_clz64(uint64_t rs1)
{
    if (rs1 == 0)
        return 64;
    assert(sizeof(long long) == 8);
    return __builtin_clzll(rs1);
}
```

```
uint32_t fast_ctz32(uint32_t rs1)
  {
      if (rs1 == 0)
          return 32;
      assert(sizeof(int) == 4);
      return __builtin_ctz(rs1);
  }
 uint64_t fast_ctz64(uint64_t rs1)
  {
      if (rs1 == 0)
          return 64;
      assert(sizeof(long long) == 8);
      return __builtin_ctzll(rs1);
  }
 uint32_t fast_pcnt32(uint32_t rs1)
      assert(sizeof(int) == 4);
      return __builtin_popcount(rs1);
  }
 uint64_t fast_pcnt64(uint64_t rs1)
      assert(sizeof(long long) == 8);
      return __builtin_popcountll(rs1);
  }
For processors with BMI2 support GCC has intrinsics for bit extract and bit deposit instructions
(compile with -mbmi2 and include <x86intrin.h>):
 uint32_t fast_bext32(uint32_t rs1, uint32_t rs2)
  {
      return _pext_u32(rs1, rs2);
  }
 uint64_t fast_bext64(uint64_t rs1, uint64_t rs2)
  {
      return _pext_u64(rs1, rs2);
  }
 uint32_t fast_bdep32(uint32_t rs1, uint32_t rs2)
  {
      return _pdep_u32(rs1, rs2);
  }
 uint64_t fast_bdep64(uint64_t rs1, uint64_t rs2)
  {
      return _pdep_u64(rs1, rs2);
  }
```

For other processors we need to provide our own implementations. The following implementation

is a good compromise between code complexity and runtime:

```
uint_xlen_t fast_bext(uint_xlen_t rs1, uint_xlen_t rs2)
{
    uint_xlen_t c = 0, i = 0, mask = rs2;
    while (mask) {
        uint_xlen_t b = mask & ~((mask | (mask-1)) + 1);
        c |= (rs1 & b) >> (fast_ctz(b) - i);
        i += fast_pcnt(b);
        mask -= b;
    }
    return c;
}
uint_xlen_t fast_bdep(uint_xlen_t rs1, uint_xlen_t rs2)
    uint_xlen_t c = 0, i = 0, mask = rs2;
    while (mask) {
        uint_xlen_t b = mask & ~((mask | (mask-1)) + 1);
        c |= (rs1 << (fast_ctz(b) - i)) & b;</pre>
        i += fast_pcnt(b);
        mask -= b;
    }
    return c;
}
```

For the other Bitmanip instructions the C reference functions given in Chapter 2 are already reasonably efficient.

## 3.3 Bit permutation instructions as bit-index operations

For programs that synthesize sequences of bit permutation instructions it can be useful to describe bit permutation instructions in terms of bit-index operations.

Expressed as bit-index operation, ror is just subtraction and grev is just XOR:

```
uint_xlen_t ror(uint_xlen_t a, uint_xlen_t b)
{
  uint_xlen_t ret = 0;
  for (int i = 0; i < XLEN; i++) {
    int j = (i - b) & (XLEN-1);
    ret |= ((a >> i) & 1) << j;
  }
  return ret;
}</pre>
```

```
uint_xlen_t grev(uint_xlen_t a, uint_xlen_t b)
    uint_xlen_t ret = 0;
    for (int i = 0; i < XLEN; i++) {
      int j = (i \hat{b}) & (XLEN-1);
      ret |= ((a >> i) & 1) << j;
    return ret;
  }
The following unperm() function calculates the new position of bit i after unshfl by k.
  int unperm(int k, int i)
  {
    return ((k + (i & k & ~(k<<1))) & ~k) |
           (i \& (k | (k << 1))) | ((i >> 1) \& k);
  }
This allows us to write shfl and unshfl in the following way.
 uint_xlen_t shfl(uint_xlen_t a, uint_xlen_t b)
    uint_xlen_t ret = 0;
    for (int i = 0; i < XLEN; i++) {
      int j = unperm(b & (XLEN/2-1), i);
     ret |= ((a >> j) & 1) << i;
    return ret;
  }
 uint_xlen_t unshfl(uint_xlen_t a, uint_xlen_t b)
    uint_xlen_t ret = 0;
    for (int i = 0; i < XLEN; i++) {
      int j = unperm(b & (XLEN/2-1), i);
      ret |= ((a >> i) & 1) << j;
    return ret;
  }
```

## Chapter 4

## **Example Applications**

This chapter contains a collection of short code snippets and algorithms using the Bitmanip extension. It also contains some examples of bit manipulation code that doesn't require any extension beyond the base ISA.

## 4.1 Basic Bitmanipulation

#### 4.1.1 Loading constants

On RV32 any arbitrary constant can be loaded using LUI+ADDI.

```
lui a0, ((v - (v << 20 >> 20)) >> 12) & 0xffffff addi a0, a0, <math>(v << 20 >> 20)
```

(Assuming signed 32-bit arithmetic for the expression ( $v \ll 20 \gg 20$ ).)

Using the following sequence on RV64 will yield the 32-bit constant in sign-extended form.

```
lui a0, ((v - (v << 52 >> 52)) >> 12) & 0xffffff addiw a0, a0, (v << 52 >> 52)
```

(addiw is needed instead of addi to handle the cases correctly that have bits 11-30 of the constant set to one.)

Using addiwn instead of addiwn produces a zero-extended version of the same constant, iff any of the bits 11-31 of the constant is zero.

```
lui a0, ((v - (v << 52 >> 52)) >> 12) \& 0xffffff addiwu a0, a0, <math>(v << 52 >> 52)
```

In the remaining cases with bits 11-31 all set, addi+pack can be used to produce the constant:

```
addi a0, zero, v pack a0, a0, zero
```

64-bit constants of the form  $R \times 1S \times 0T \times 1$  (with R + S + T = 64) can be constructed using sloi and rori:

```
sloi a0, zero, R+T
rori a0, a0, R
```

Likewise, constructing  $R \times 0S \times 1T \times 0$ :

```
sloi a0, zero, S
slli a0, a0, T
```

Any constant that is just a repeating 16-bit pattern can be constructed in two instructions with lui and orc16. For example, constructing OxABCD\_ABCD\_ABCD\_ABCD:

```
lui a0, 0x0BCDA
orc16 a0, a0
```

Finally, any arbitrary 64-bit constant can be created using the following 5-instruction pattern and one spill register:

```
lui a0, ((v - (v << 52 >> 52)) >> 12) & Oxffffff addiw a0, a0, (v << 52 >> 52)
lui a1, ((v - (v << 20 >> 20)) >> 44) & Oxffffff addiw a1, a1, (v << 20 >> 52)
pack a0, a0, a1
```

#### 4.1.2 Bitfield extract

Extracting a bit field of length len at position pos can be done using two shift operations.

```
slli a0, a0, (XLEN-len-pos)
srli a0, a0, (XLEN-len)
```

Or using srai for a signed bit-field.

```
slli a0, a0, (XLEN-len-pos)
srai a0, a0, (XLEN-len)
```

#### 4.1.3 Packing bit-fields

There are different ways of packing bit-fields with the help of RISC-V BitManip instructions.

For example, packing a 16-bit RGB value in 5:6:5 format, from the bytes in a0, a1, and a2, using pack[h] and bext:

```
li t0, 0x00f8fcf8

packh a0, a0, a1

pack a0, a0, a2

bext a0, a0, t0
```

Or using funnel shifts (assuming a2 is already in zero-extended form and/or the upper bits of the return value do not matter):

```
srli a2, a2, 3
slli a1, a1, XLEN-8
fsli a1, a2, a1, 6
slli a0, a0, XLEN-8
fsli a0, a1, a0, 5
```

Using only base-ISA instructions, at least 7 instructions are needed to pack a 5:6:5 RGB value (assuming a0 is alredy in zero-extended form):

```
andi a2, a2, 0xf8
slli a2, a2, 9
andi a1, a1, 0xfc
slli a1, a1, 3
srli a0, a0, 3
or a1, a1, a2
or a0, a0, a1
```

Another example for packing bit fields is generating IEEE floats using only integer instructions (aka "soft float"). For example, generating a 32-bit float in the range  $[-1.0\cdots+1.0)$  from a signed 16-bit integer:

#### short2float:

```
neg a1, a0
max a1, a1, a0
clz a2, a1
srli a0, a0, 31
sll a3, a1, a2
srli a3, a3, 15
neg a2, a2
addi a2, a2, 143
packh a0, a2, a0
pack a0, a3, a0
slli a4, a4, 7
orc a1, a1
and a0, a0, a1
ret
```

Or using funnel shifts:

```
short2float:
neg a1, a0
max a1, a1, a0
clz a2, a1
sll a3, a1, a2
slli a3, a3, 1
neg a2, a2
addi a2, a2, 143
fsri a3, a3, a2, 8
srli a0, a0, 31
fsri a0, a3, a0, 1
cmov a0, a1, a0, zero
ret
```

#### 4.1.4 Parity check

The parity of a word (xor of all bits) is the LSB of the population count.

```
pcnt a0, a0 andi a0, a0, 1
```

#### 4.1.5 Average of two integers

The following four instructions calculate the average of the unsigned integers in a0 and a1, with compensation for overflow:

```
and a2, a0, a1
xor a0, a0, a1
srli a0, a0, 1
add a0, a0, a2
```

And likewise the average of two signed integers:

```
and a2, a0, a1
xor a0, a0, a1
srai a0, a0, 1
add a0, a0, a2
```

With fsri the unsigned case can be accomplished in just three instructions:

```
add a0, a0, a1 sltu a1, a0, a1 fsri a0, a1, 1
```

#### 4.1.6 Detecting integer overflow

Overflow in unsigned addition can be detected in two instructions:

```
add a0, a1, a2
bltu a0, a1, overflow
```

For signed addition, if the sign of one operand is known, for example because it is constant:

```
addi a0, a1, +imm
blt a0, a1, overflow
addi a0, a1, -imm
bgt a0, a1, overflow
```

And for signed addition in the general case:

```
add a0, a1, a2
slti a3, a1, 0
slt a4, a0, a2
bne a3, a4, overflow
```

And finally, generating the carry flag for an addition:

```
add a0, a1, a2 sltu a3, a0, a1
```

Thus, adding a0, a1, and a2 with results in a0 and carry-out in a1:

```
add a0, a0, a1
sltu a1, a0, a1
add a0, a0, a2
sltu a2, a0, a2
or a1, a1, a2
```

#### 4.1.7 Fuse-able sequences for logic operations

RISC-V has dedicated instructions for branching on equal/not-equal. But C code such as the following would require set-equal and set-not-equal instructions, similar to slt.

```
int is_equal = (a == b);
int is_noteq = (c != d);
```

Those can be implemented using the following fuse-able sequences:

```
sub rd, rs1, rs2
sltui rd, rd, 1

sub rd, rs1, rs2
sltu rd, zero, rd

Likewise for logic OR:
  int logic_or = (c || d);
  or rd, rs1, rs2
  sltu rd, zero, rd

And for logic AND, if rd != rs2:
  int logic_and = (c && d);
  orc rd, rs1
  and rd, rd, rs2
  sltu rd, zero, rd
```

#### 4.1.8 Rotate shift of bytes and half-words

Rotate right shift of the byte in a0 by the shift amount in a1, assuming a0 is stored in zero-extended form:

```
orc8 a0, a0
ror a0, a1
andi a0, a0, 255
```

And rotate right shift of the 16-bit half-word in a0 by the shift amount in a1, assuming a0 is stored in zero-extended form:

```
orc16 a0, a0
ror a0, a1
pack[w] a0, a0, zero
```

#### 4.1.9 Rank and select

Rank and select are fundamental operations in succinct data structures [19].

select(a0, a1) returns the position of the a1th set bit in a0. It can be implemented efficiently using bdep and ctz:

```
select:
    sbset a1, zero, a1
    bdep a0, a1, a0
    ctz a0, a0
    ret

rank(a0, a1) returns the number of set bits in a0 up to and including position a1.

rank:
    not a1, a1
    sll a0, a1
    pcnt a0, a0
    ret
```

#### 4.1.10 OR/AND/XOR-reduce in byte vectors

OR-ing the bytes in a register and returning the resulting byte is easy with GORC:

```
gorci a0, a0, -8
andi a0, 255
```

AND-ing can accomplished by applying De Morgan's laws:

```
not a0, a0
gorci a0, a0, -8
not a0, a0
andi a0, 255
```

XOR-ing can be accomplished with CLMUL (see also section 4.7).

```
andi a1, zero, 0x80 gorci a1, a1, -8 clmulr a0, a0, a1 andi a0, 255
```

Where the first two instructions (andi+gorci) just create the constant 0x8080..8080.

Finally, on RV64, XOR-ing the bytes in a register can also be accomplished with BMATXOR:

```
andi a1, zero, 0xff
bmatxor a0, a1, a0
```

#### 4.1.11 Counting trailing non-zero bytes

Counting the trailing (LSB-end) non-zero bytes in a word is a helpful operation in optimized implementations of strlen() and strcpy():

```
int count_trailing_nonzero_bytes(long x)
{
   return _rv_ctz(~_rv_orc_b(x)) >> 3;
}
```

#### 4.1.12 Finding bytes of certain values

Finding zero bytes is a useful operations for strchr() and memchr():

```
bool check_zero_bytes(long x)
{
  return ~_rv_orc_b(x) != 0;
}
```

To find other bytes we simply XOR the value with a mask of the byte value we are looking for:

```
bool check_byte(long x, unsigned char c)
{
  return ~_rv_orc_b(x ^ _rv_orc8(c)) != 0;
}
```

These schemes can easily be extended with ctz and pcnt to perform operations such as counting the number of bytes of a certain value within a word, or finding the position of the first such byte.

#### 4.1.13 Fill right of most significant set bit

The "fill right" or "fold right" operation is a pattern commonly used in bit manipulation code. [8]

The straight-forward RV64 implementation requires 12 instructions:

With clz it can be implemented in only 4 instructions. Notice the handling of the case where x=0 using sltiu+addi.

Alternatively, a Trailing Bit Manipulation (TBM) code pattern can be used together with rev to implement this function in 4 instructions:

Finally, there is another implementation in 4 instructions using BMATOR, if we do not count the extra instructions for loading utility matrices.

```
uint64_t rfill_bmat(uint64_t x)
 uint64_t m0, m1, m2, t;
 m0 = 0xFF7F3F1F0F070301LL; // LD
                            // SLLI, BMATFLIP
 m1 = bmatflip(m0 << 8);
 m2 = -1LL;
                             // ADDI
                           // BMATOR
 t = bmator(x, m0);
 x = bmator(x, m2);
                            // BMATOR
                           // BMATOR
 x = bmator(m1, x);
 x \mid = t;
                             // OR
 return x;
}
```

#### 4.1.14 Round to next power of two

One common application of rfill() is rounding up to the next power of two:

```
uint64_t round_pow2(uint64_t x)
{
  return rfill(x-1)+1;
}
```

This can also be implemented in just 4 instructions, if we don't care about the case where the above code overflows because x is already larger than the largest power-of-two representable in an  $uint64_t$ .

Note that this code handles  $0 \to 0$  and  $1 \to 1$  correctly, i.e. equivialent to rfill(x-1)+1.

#### 4.2 Packed vectors

#### 4.2.1 Packing bytes

The following RV32 code packs the lower 8 bits from a0, a1, a2, a3 into a 32-bit word returned in a0, ignoring other bits in the input values.

```
packh a0, a0, a1
packh a1, a2, a3
pack a0, a0, a1
```

And the following RV64 code packs 8 bytes into a register.

```
packh a0, a0, a1
packh a1, a2, a3
packh a2, a4, a5
packh a3, a6, a7
packw a0, a0, a1
packw a1, a2, a3
pack a0, a0, a1
```

#### 4.2.2 Permuting bytes

There are 24 ways of arranging the four bytes in a 32-bit word. ror, grev, and [un]shfl can perform any of those permutations in at most 3 instructions. Table 4.1 lists those sequences. [24]

#### 4.2.3 Widening and narrowing

The [un]zip instructions can help with widening and narrowing packed vectors. For example, narrowing the bytes in two words into a single word with the values in nibbles with values from a0 in LSB half and values from a1 in MSB half:

```
unzip4 a0, a0
unzip4 a1, a1
pack a0, a0, a1
```

And widening the nibbles from a0 into bytes in a1 (MSB half) and a0 (LSB half), with zero extension:

```
srli a1, a0, XLEN/2
pack a0, a0, zero
zip4 a1, a1
zip4 a0, a0
```

And finally the same widening operation with sign extension:

Bytes	Instructions
ABCD	initial byte order
A B D C	ROR(24),SHFL(8),ROR(8)
A C B D	SHFL(8)
A C D B	ROR(8),GREV(8),SHFL(8)
ADBC	ROR(16),SHFL(8),ROR(24)
A D C B	ROR(8),GREV(8)
BACD	ROR(8),SHFL(8),ROR(24)
B A D C	GREV(8)
B C A D	ROR(16),SHFL(8),ROR(8)
B C D A	ROR(24)
BDAC	GREV(8),SHFL(8)
В D С А	ROR(24),SHFL(8)
CABD	ROR(8),GREV(24),SHFL(8)
C A D B	ROR(16),SHFL(8)
C B A D	ROR(8),GREV(24)
C B D A	SHFL(8),ROR(24)
CDAB	ROR(16)
CDBA	ROR(8),SHFL(8),ROR(8)
DABC	ROR(8)
DACB	SHFL(8),ROR(8)
DBAC	ROR(8),SHFL(8)
DBCA	GREV(24),SHFL(8)
D C A B	ROR(24),SHFL(8),ROR(24)
D C B A	GREV(24)

Table 4.1: Instruction sequences for arbitrary permutations of bytes in a 32-bit word.

```
addi t0, zero, 8 orc4 t0, t0 and t0, t0, a0 orc.n t0, t0 srli t1, t0, XLEN/2 srli a1, a0, XLEN/2 pack a1, a1, t1 pack a0, a0, t0 zip4 a1, a1 zip4 a0, a0
```

### 4.2.4 Shifting packed vector elements

Using zip we can re-arrange the bits in a packed vector of N elements so that a shift by k of each byte becomes a shift of Nk of the entire new vector. So we zip, shift, and then unzip to shuffle everything back. The number of zip and unzip is  $\log 2(N)$ . This works for all kinds of shift operations. For example, rotating a vector of bytes on RV32 in 6 instructions:

```
zip a0, a0
zip a0, a0
slli a1, a1, 2
ror a0, a0, a1
unzip a0, a0
unzip a0, a0
```

Because zip; zip; zip is equal to unzip; unzip on RV32, and equal to unzip; unzip; unzip on RV64, we need never more than 2 [un]zip on RV32, or 3 [un]zip on RV64.

#### 4.2.5 Adding packed vectors

The following six instructions will add the elements of the two vectors passed in a0 and a1, and return the vector of sums in a0.

This expects a mask in a2 that marks the MSB bit of each vector element. For a vector of bytes this mask would be 0x8080...80 (which can be obtained in two instructions via orc8(0x80)).

```
xor a3, a0, a1
and a3, a3, a2
andn a0, a0, a2
andn a1, a1, a2
add a0, a0, a1
xor a0, a0, a3
```

#### 4.3 Funnel shifts

A funnel shift takes two XLEN registers, concatenates them to a  $2 \times XLEN$  word, shifts that by a certain amount, then returns the lower half of the result for a right shift and the upper half of the result for a left shift.

The fsl, fsr, and fsri instructions perform funnel shifts.

#### 4.3.1 Bigint shift

A common application for funnel shifts is shift operations in bigint libraries.

For example, the following functions implement rotate-shift operations for bigints made from n XLEN words.

```
void bigint_rol(uint_xlen_t data[], int n, int shamt)
  if (n <= 0)
    return;
  uint_xlen_t buffer = data[n-1];
  for (int i = n-1; i > 0; i--)
    data[i] = fsl(data[i], shamt, data[i-1]);
  data[0] = fsl(data[0], shamt, buffer);
}
void bigint_ror(uint_xlen_t data[], int n, int shamt)
  if (n <= 0)
    return;
  uint_xlen_t buffer = data[0];
  for (int i = 0; i < n-1; i++)
    data[i] = fsr(data[i], shamt, data[i+1]);
  data[n-1] = fsr(data[n-1], shamt, buffer);
}
```

These version only works for shift-amounts <XLEN. But functions supporting other kinds of shift operations, or shifts  $\geq$ XLEN can easily be built with fsl and fsr.

### 4.3.2 Parsing bit-streams of 27-bit words

The following function parses n 27-bit words from a packed array of XLEN words:

```
void parse_27bit(uint_xlen_t *idata, uint_xlen_t *odata, int n)
  uint_xlen_t lower = 0, upper = 0;
  int reserve = 0;
  while (n--) {
    if (reserve < 27) {
      uint_xlen_t buf = *(idata++);
      lower |= sll(buf, reserve);
      upper = reserve ? srl(buf, -reserve) : 0;
      reserve += XLEN;
    }
    *(odata++) = lower & ((1 << 27)-1);
    lower = fsr(lower, 27, upper);
    upper = srl(upper, 27);
    reserve -= 27;
  }
}
```

And here the same thing in RISC-V assembler:

```
parse_27bit:
 li t1, 0
                        ; lower
 li t2, 0
                        ; upper
 li t3, 0
                        ; reserve
 li t4, 27
                        ; shamt
                       ; mask
 slo t5, zero, t4
 beqz a2, endloop
                        ; while (n--)
loop:
 addi a2, a2, -1
 bge t3, t4, output ; if (reserve < 27)
 lw t6, 0(a0)
                              ; buf = *(idata++)
 addi a0, a0, 4
 sll t7, t6, t3
                               ; lower |= sll(buf, reserve)
 or t1, t1, t7
 sub t7, zero, t3
                               ; upper = reserve ? srl(buf, -reserve) : 0
 srl t7, t6, t7
 cmov t2, t3, t7, zero
 addi t3, t3, 32
                               ; reserve += XLEN;
output:
 and t6, t1, t5
                           ; *(odata++) = lower & ((1 << 27)-1)
 sw t6, 0(a1)
 addi a1, a1, 4
 fsr t1, t1, t2, t4
                         ; lower = fsr(lower, 27, upper)
 srl t2, t2, t4
                         ; upper = srl(upper, 27)
 sub t3, t3, t4
                          ; reserve -= 27
 bnez a2, loop
                      ; while (n--)
endloop:
 ret
```

A loop iteration without fetch is 9 instructions long, and a loop iteration with fetch is 17 instructions long.

Without ternary operators that would be 13 instructions and 22 instructions, i.e. assuming one cycle per instruction, that function would be about 30% slower without ternary instructions.

### 4.3.3 Parsing bit-streams of 6-bit words

The following code accepts three 64-bit words in t0, t1, and t2 containing a bit-stream of 32 6-bit words, and outputs these 32 values in the bytes of t0, t1, t2, and t3.

That's 7 instructions without the two instructions for constructing the mask in a0.

Without funnel shift this operation requires 11 instructions:

Sign-extending the 6-bit values in the bytes of t0, t1, t2, and t3 with bit-matrix multiply:

```
li a0, 0x80c0e01008040201
```

```
bmator t0, t0, a0
bmator t1, t1, a0
bmator t2, t2, a0
bmator t3, t3, a0
```

Or without bit-matrix multiply:

```
addi a0, zer0, 0x60
orc8 a0, a0  // a0 = 0x60606060606060

add t0, t0, a0
add t1, t1, a0
add t2, t2, a0
add t3, t3, a0
xor t0, t0, a0
xor t1, t1, a0
xor t2, t2, a0
xor t3, t3, a0
```

### 4.3.4 Fixed-point multiply

A fixed-point multiply is simply an integer multiply, followed by a right shift. If the entire dynamic range of XLEN bits should be useable for the factors, then the product before shift must be 2\*XLEN wide. Therefore mul+mulh is needed for the multiplication, and funnel shift instructions can help with the final right shift. For fixed-point numbers with N fraction bits:

```
mul_fracN:
   mulh a2, a0, a1
   mul a0, a0, a1
   fsri a0, a0, a2, N
   ret
```

# 4.4 Arbitrary bit permutations

This section lists code snippets for computing arbitrary bit permutations that are defined by data (as opposed to bit permutations that are known at compile time and can likely be compiled into shift-and-mask operations and/or a few instances of bext/bdep).

### 4.4.1 Using butterfly operations

The following macro performs a stage-N butterfly operation on the word in a0 using the mask in a1.

```
grevi a2, a0, (1 << N) cmix a0, a1, a2, a0
```

The bitmask in a1 must be preformatted correctly for the selected butterfly stage. A butterfly operation only has a XLEN/2 wide control word. The following macros format the mask assuming those XLEN/2 bits in the lower half of a1 on entry:

```
bfly_msk_0:
```

```
pack a1, a1, a1
zip a1, a1

bfly_msk_1:
  pack a1, a1, a1
zip2 a1, a1

bfly_msk_2:
  pack a1, a1, a1
zip4 a1, a1
```

A sequence of  $2 \cdot log_2(XLEN) - 1$  butterfly operations can perform any arbitrary bit permutation (Beneš network):

```
butterfly(LOG2_XLEN-1)
butterfly(LOG2_XLEN-2)
...
butterfly(0)
...
butterfly(LOG2_XLEN-2)
butterfly(LOG2_XLEN-1)
```

Many permutations arising from real-world applications can be implemented using shorter sequences. For example, any sheep-and-goats operation (SAG, see section 4.4.4) with either the sheep or the goats bit reversed can be implemented in  $log_2(XLEN)$  butterfly operations.

Reversing a permutation implemented using butterfly operations is as simple as reversing the order of butterfly operations.

### 4.4.2 Using omega-flip networks

The omega operation is a stage-0 butterfly preceded by a zip operation:

```
zip a0, a0
grevi a2, a0, 1
cmix a0, a1, a2, a0
```

The flip operation is a stage-0 butterfly followed by an unzip operation:

```
grevi a2, a0, 1
cmix a0, a1, a2, a0
unzip a0, a0
```

A sequence of  $log_2(XLEN)$  omega operations followed by  $log_2(XLEN)$  flip operations can implement any arbitrary 32 bit permutation.

As for butterfly networks, permutations arising from real-world applications can often be implemented using a shorter sequence.

### 4.4.3 Using baseline networks

Another way of implementing arbitrary 32 bit permutations is using a baseline network followed by an inverse baseline network.

A baseline network is a sequence of  $log_2(XLEN)$  butterfly(0) operations interleaved with unzip operations. For example, a 32-bit baseline network:

```
butterfly(0)
unzip
butterfly(0)
unzip.h
butterfly(0)
unzip.b
butterfly(0)
unzip.n
butterfly(0)
```

An inverse baseline network is a sequence of  $log_2(XLEN)$  butterfly(0) operations interleaved with zip operations. The order is opposite to the order in a baseline network. For example, a 32-bit inverse baseline network:

```
butterfly(0)
zip.n
butterfly(0)
zip.b
butterfly(0)
zip.h
butterfly(0)
zip
butterfly(0)
```

A baseline network followed by an inverse baseline network can implement any arbitrary bit permutation.

### 4.4.4 Using sheep-and-goats

The Sheep-and-goats (SAG) operation is a common operation for bit permutations. It moves all the bits selected by a mask (goats) to the LSB end of the word and all the remaining bits (sheep)

to the MSB end of the word, without changing the order of sheep or goats.

The SAG operation can easily be performed using bext (data in a0 and mask in a1):

```
bext a2, a0, a1
not a1, a1
bext a0, a0, a1
pcnt a1, a1
ror a0, a0, a1
or a0, a0, a2
```

Any arbitrary bit permutation can be implemented in  $log_2(XLEN)$  SAG operations.

The Hacker's Delight describes an optimized standard C implementation of the SAG operation. Their algorithm takes 254 instructions (for 32 bit) or 340 instructions (for 64 bit) on their reference RISC instruction set. [9, p. 152f, 162f]

### 4.4.5 Using bit-matrix multiply

bat[x]or performs a permutation of bits within each byte when used with a permutation matrix in rs2, and performs a permutation of bytes when used with a permutation matrix in rs1.

### 4.5 Mirroring and rotating bitboards

Bitboards are 64-bit bitmasks that are used to represent part of the game state in chess engines (and other board game AIs). The bits in the bitmask correspond to squares on a  $8 \times 8$  chess board:

```
56 57 58 59 60 61 62 63 48 49 50 51 52 53 54 55 40 41 42 43 44 45 46 47 32 33 34 35 36 37 38 39 24 25 26 27 28 29 30 31 16 17 18 19 20 21 22 23 8 9 10 11 12 13 14 15 0 1 2 3 4 5 6 7
```

Many bitboard operations are simple straight-forward operations such as bitwise-AND, but mirroring and rotating bitboards can take up to 20 instructions on x86.

### 4.5.1 Mirroring bitboards

Flipping horizontally or vertically can easily done with grevi:

#### Flip horizontal: 63 62 61 60 59 58 57 56 RISC-V Bitmanip: 55 54 53 52 51 50 49 48 rev.b 47 46 45 44 43 42 41 40 39 38 37 36 35 34 33 32 31 30 29 28 27 26 25 24 x86: 23 22 21 20 19 18 17 16 13 operations 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0 Flip vertical: 0 1 2 3 4 5 6 7 RISC-V Bitmanip: 8 9 10 11 12 13 14 15 rev8 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 x86: 40 41 42 43 44 45 46 47 bswap 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63

Rotating by 180 (flip horizontal and vertical):

# Rotate 180: 7 6 5 4 3 2 1 0 RISC-V Bitmanip: 15 14 13 12 11 10 9 8 rev 23 22 21 20 19 18 17 16 31 30 29 28 27 26 25 24 39 38 37 36 35 34 33 32 x86: 47 46 45 44 43 42 41 40 14 operations 55 54 53 52 51 50 49 48 63 62 61 60 59 58 57 56

### 4.5.2 Rotating bitboards

Using zip a bitboard can be transposed easily:

### Transpose:

```
7 15 23 31 39 47 55 63 RISC-V Bitmanip:
6 14 22 30 38 46 54 62 zip, zip, zip
5 13 21 29 37 45 53 61
4 12 20 28 36 44 52 60
3 11 19 27 35 43 51 59 x86:
2 10 18 26 34 42 50 58 18 operations
1 9 17 25 33 41 49 57
0 8 16 24 32 40 48 56
```

A rotation is simply the composition of a flip operation and a transpose operation. This takes 19 operations on x86 [2]. With Bitmanip the rotate operation only takes 4 operations:

rotate\_bitboard:

rev8 a0, a0 zip a0, a0 zip a0, a0 zip a0, a0

### 4.5.3 Explanation

The bit indices for a 64-bit word are 6 bits wide. Let i[5:0] be the index of a bit in the input, and let i'[5:0] be the index of the same bit after the permutation.

As an example, a rotate left shift by N can be expressed using this notation as  $i'[5:0] = i[5:0] + N \pmod{64}$ .

The GREV operation with shamt N is  $i'[5:0] = i[5:0] \times N$ .

And a SHFL operation corresponds to a rotate left shift by one position of any contiguous region of i[5:0]. For example, zip is a left rotate shift of the entire bit index:

$$i'[5:0] = \{i[4:0], i[5]\}$$

And zip4 performs a left rotate shift on bits 5:2:

$$i'[5:0] = \{i[4:2], i[5], i[1:0]\}$$

In a bitboard, i[2:0] corresponds to the X coordinate of a board position, and i[5:3] corresponds to the Y coordinate.

Therefore flipping the board horizontally is the same as negating bits i[2:0], which is the operation performed by grevi rd, rs, 7 (rev.b).

Likewise flipping the board vertically is done by grevi rd, rs, 56 (rev8).

Finally, transposing corresponds by swapping the lower and upper half of i[5:0], or rotate shifting i[5:0] by 3 positions. This can easily done by rotate shifting the entire i[5:0] by one bit position (zip) three times.

### 4.5.4 Rotating Bitcubes

Let's define a bitcube as a  $4 \times 4 \times 4$  cube with x = i[1:0], y = i[3:2], and z = i[5:4]. Using the same methods as described above we can easily rotate a bitcube by 90° around the X-, Y-, and Z-axis:

```
rotate_x: rotate_y: rotate_z:
rev16 a0, a0 rev.n a0, a0 rev4.h
zip4 a0, a0 zip a0, a0 zip.h a0, a0
zip4 a0, a0 zip4 a0, a0
zip4 a0, a0
zip4 a0, a0
```

### 4.6 Manipulating 64x64 Bit Matrices

The bmat[x]or and bmatflip instructions operate on 8x8 bit matrices stored in single 64-bit registers, where each byte of such a 64-bit value represents one row (column) of a 8x8 bit matrix.

Let's assume we have a 64x64 bit matrix in memory, stored as one row (column) per 64-bit value. In order to use bmat[x]or and bmatflip on such a matrix, we must first convert it into a 8x8 block matrix of 64 individual 8x8 matrices, each stored in a 64-bit value. The following function performs this transformation for a single row (column) of the block matrix in 40 instructions.

```
void conv8x8(const uint64_t x[8], uint64_t y[8])
 uint64_t x0_x1_31_00 = _rv64_pack (x[0], x[1]);
 uint64_t x2_x3_31_00 = rv64_pack (x[2], x[3]);
 uint64_t x4_x5_31_00 = rv64_pack (x[4], x[5]);
 uint64_t x6_x7_31_00 = rv64_pack (x[6], x[7]);
 uint64_t x0_x1_63_32 = _rv64_packu(x[0], x[1]);
 uint64_t x2_x3_63_32 = rv64_packu(x[2], x[3]);
 uint64_t x4_x5_63_32 = rv64_packu(x[4], x[5]);
 uint64_t x6_x7_63_32 = _rv64_packu(x[6], x[7]);
 uint64_t x0_x1_31_00_z = _rv64_unzip16(x0_x1_31_00);
 uint64_t x2_x3_31_00_z = _rv64_unzip16(x2_x3_31_00);
 uint64_t x4_x5_31_00_z = _rv64_unzip16(x4_x5_31_00);
 uint64_t x6_x7_31_00_z = _rv64_unzip16(x6_x7_31_00);
 uint64_t x0_x1_63_32_z = _rv64_unzip16(x0_x1_63_32);
 uint64_t x2_x3_63_32_z = rv64_unzip16(x2_x3_63_32);
 uint64_t x4_x5_63_32_z = rv64_unzip16(x4_x5_63_32);
 uint64_t x6_x7_63_32_z = rv64_unzip16(x6_x7_63_32);
 uint64_t x0_x1_x2_x3_15_00 = rv64_pack (x0_x1_31_00_z, x2_x3_31_00_z);
 uint64_t x4_x5_x6_x7_15_00 = rv64_pack (x4_x5_31_00_z, x6_x7_31_00_z);
 uint64_t x0_x1_x2_x3_31_16 = rv64_packu(x0_x1_31_00_z, x2_x3_31_00_z);
 uint64_t x4_x5_x6_x7_31_16 = rv64_packu(x4_x5_31_00_z, x6_x7_31_00_z);
 uint64_t x0_x1_x2_x3_47_32 = rv64_pack (x0_x1_63_32_z, x2_x3_63_32_z);
 uint64_t x4_x5_x6_x7_47_32 = rv64_pack (x4_x5_63_32_z, x6_x7_63_32_z);
 uint64_t x0_x1_x2_x3_63_48 = rv64_packu(x0_x1_63_32_z, x2_x3_63_32_z);
 uint64_t x4_x5_x6_x7_63_48 = rv64_packu(x4_x5_63_32_z, x6_x7_63_32_z);
```

```
uint64_t x0_x1_x2_x3_15_00_z = _rv64_unzip8(x0_x1_x2_x3_15_00);
uint64_t x4_x5_x6_x7_15_00_z = rv64_unzip8(x4_x5_x6_x7_15_00);
uint64_t x0_x1_x2_x3_31_16_z = _rv64_unzip8(x0_x1_x2_x3_31_16);
uint64_t x4_x5_x6_x7_31_16_z = rv64_unzip8(x4_x5_x6_x7_31_16);
uint64_t x0_x1_x2_x3_47_32_z = rv64_unzip8(x0_x1_x2_x3_47_32);
uint64_t x4_x5_x6_x7_47_32_z = rv64_unzip8(x4_x5_x6_x7_47_32);
uint64_t x0_x1_x2_x3_63_48_z = _rv64_unzip8(x0_x1_x2_x3_63_48);
uint64_t x4_x5_x6_x7_63_48_z = rv64_unzip8(x4_x5_x6_x7_63_48);
y[0] = rv64_{pack} (x0_x1_x2_x3_15_00_z, x4_x5_x6_x7_15_00_z);
y[1] = rv64_packu(x0_x1_x2_x3_15_00_z, x4_x5_x6_x7_15_00_z);
y[2] = rv64_pack (x0_x1_x2_x3_31_16_z, x4_x5_x6_x7_31_16_z);
y[3] = rv64_packu(x0_x1_x2_x3_31_16_z, x4_x5_x6_x7_31_16_z);
y[4] = rv64_pack (x0_x1_x2_x3_47_32_z, x4_x5_x6_x7_47_32_z);
y[5] = rv64_packu(x0_x1_x2_x3_47_32_z, x4_x5_x6_x7_47_32_z);
y[6] = rv64_pack (x0_x1_x2_x3_63_48_z, x4_x5_x6_x7_63_48_z);
y[7] = rv64_packu(x0_x1_x2_x3_63_48_z, x4_x5_x6_x7_63_48_z);
```

Each of the 5 blocks in this function only consumes the eight outputs of the previous block. Therefore 16 registers are sufficient to run this function in registers only without the need to spill any data on the stack.

Note that this function is its own inverse. Therefore the same function can be used for the convertion from block matrix form back to row (column) major form.

A bit 64x64 bit matrix in block matrix form can easily be transposed by running bmatflip (or zip; zip; zip) on the blocks of the matrix and then renaming the individual 64-bit variables.

To multiply 64x64 bit matrices in block matrix form, the matrix-matrix-product is decomposed in the obvious way in  $8 \times 8 \times 8 = 512$  bmat [x] or instructions and  $7 \times 8 \times 8 = 448$  [x] or instructions.

# 4.7 Inverting Xorshift RNGs

Xorshift RNGs are a class of fast RNGs for different bit widths. There are 648 Xorshift RNGs for 32 bits, but this is the one that the author of the original Xorshift RNG paper recommends. [18, p. 4]

```
uint32_t xorshift32(uint32_t x)
{
    x ^= x << 13;
    x ^= x >> 17;
    x ^= x << 5;
    return x;
}</pre>
```

This function of course has been designed and selected so it's efficient, even without special bit-manipulation instructions. So let's look at the inverse instead. First, the naïve form of inverting this function:

```
uint32_t xorshift32_inv(uint32_t x)
{
   uint32_t t;
   t = x ^ (x << 5);
   t = x ^ (t << 5);
   x = x ^ (t << 5);
   x = x ^ (t << 5);
   x = x ^ (t << 13);
   x = x ^ (t << 13);
   return x;
}</pre>
```

This translates to 18 RISC-V instructions, not including the function call overhead.

Obviously the C expression x  $\hat{}$  (x >> 17) is already its own inverse (because  $17 \ge XLEN/2$ ) and therefore already has an effecient inverse. But the two other blocks can easily be implemented using a single clmul instruction each:

```
uint32_t xorshift32_inv(uint32_t x)
{
    x = clmul(x, 0x42108421);
    x = x ^ (x >> 17);
    x = clmul(x, 0x04002001);
    return x;
}
```

This are 8 RISC-V instructions, including 4 instructions for loading the constants, but not including the function call overhead.

An optimizing compiler could easily generate the clmul instructions and the magic constants from the C code for the naïve implementation. (0x04002001 = (1 << 2\*13) | (1 << 13) | 1 and <math>0x42108421 = (1 << 6\*5) | (1 << 5\*5) | ... | (1 << 5) | 1)

The obvious remaining question is "if clmul(x, 0x42108421) is the inverse of x ^ (x << 5), what's the inverse of x ^ (x >> 5)?" It's clmulr(x, 0x84210842), where 0x84210842 is the bit-reversal of 0x42108421.

A special case of xorshift is  $x ^ (x >> 1)$ , which is a gray encoder. The corresponding gray decoder is clmulr(x, 0xffffffff).

# 4.8 Cyclic redundency checks (CRC)

There are special instructions for performing CRCs using the two most widespread 32-bit CRC polynomials, CRC-32 and CRC-32C.

CRCs with other polynomials can be computed efficiently using CLMUL. The following examples

assert(len >= 2); a0 = rev8(p[0]); a1 = rev8(p[1]);

are using CRC32Q. The easiest way of implementing CRC32Q with clmul is using a Barrett reduction. On RV32: uint32\_t crc32q\_simple(const uint32\_t \*data, int length) { uint32\_t P = 0x814141AB; // CRC polynomial (implicit x^32) uint32\_t mu = 0xFEFF7F62; // x^64 divided by CRC polynomial uint32\_t mu1 = 0xFF7FBFB1; // "mu" with leading 1, shifted right by 1 bit  $uint32_t crc = 0;$ for (int i = 0; i < length; i++) { crc ^= rev8(data[i]); crc = clmulr(crc, mu1); crc = clmul(crc, P); } return crc; The following python code calculates the value of mu for a given CRC polynomial: def polydiv(dividend, divisor): quotient = 0while dividend.bit\_length() >= divisor.bit\_length(): i = dividend.bit\_length() - divisor.bit\_length() dividend = dividend ^ (divisor << i)</pre> quotient |= 1 << i return quotient P = 0x1814141ABprint("0x%X" % (polydiv(1<<64, P))) # prints 0x1FEFF7F62</pre> A more efficient method would be the following, which processes 64-bit at a time (RV64): uint32\_t crc32q\_fast(const uint64\_t \*p, int len) uint64\_t P = 0x1814141ABLL; // CRC polynomial uint64\_t k1 = 0xA1FA6BECLL; // remainder of x^128 divided by CRC polynomial uint64\_t k2 = 0x9BE9878FLL; // remainder of x^96 divided by CRC polynomial uint64\_t k3 = 0xB1EFC5F6LL; // remainder of x^64 divided by CRC polynomial uint64\_t mu = 0x1FEFF7F62LL; // x^64 divided by CRC polynomial uint64\_t a0, a1, a2, t1, t2;

```
// Main loop: Reduce to 2x 64 bits
 for (const uint64_t *t0 = p+2; t0 != p+len; t0++)
 {
   a2 = rev8(*t0);
   t1 = clmulh(a0, k1);
   t2 = clmul(a0, k1);
   a0 = a1 ^t1;
   a1 = a2 ^t2;
 }
 // Reduce to 64 bit, add 32 bit zero padding
 t1 = clmulh(a0, k2);
 t2 = clmul(a0, k2);
 a0 = (a1 >> 32) ^ t1;
 a1 = (a1 << 32) ^ t2;
 t2 = clmul(a0, k3);
 a1 = a1 ^t2;
 // Barrett Reduction
 t1 = clmul(a1 >> 32, mu);
 t2 = clmul(t1 >> 32, P);
 a0 = a1 ^t2;
 return a0;
}
```

The main idea is to transform an array of arbitrary length to an array with the same CRC that's only two 64-bit elements long. (That's the "Main loop" portion of above code.)

Then we further reduce it to just 64-bit. And then we use a Barrett reduction to get the final 32-bit result.

The following python code can be used to calculate the "magic constants" k1, k2, and k3:

```
def polymod(dividend, divisor):
    quotient = 0
    while dividend.bit_length() >= divisor.bit_length():
        i = dividend.bit_length() - divisor.bit_length()
        dividend = dividend ^ (divisor << i)
        quotient |= 1 << i
        return dividend

print("0x%X" % (polymod(1<<128, P))) # prints 0xA1FA6BEC
print("0x%X" % (polymod(1<< 96, P))) # prints 0x9BE9878F
print("0x%X" % (polymod(1<< 64, P))) # prints 0xB1EFC5F6</pre>
```

The above example code is taken from [26]. A more detailed description of the algorithms employed can be found in [11].

### 4.9 Decoding RISC-V Immediates

The following code snippets decode and sign-extend the immediate from RISC-V S-type, B-type, J-type, and CJ-type instructions. They are nice "nothing up my sleeve"-examples for real-world bit permutations.

31	27	26	25	24	20	19	15	14	12	11	7	6	0	
	imm[11:5	5]								imm	[4:0]			S-type
	imm[12 10:5]									imm[4	4:1 11]			B-type
imm[20 10:1 11						9:12]								J-type

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	
				iı	$\overline{\mathrm{nm}}$	[11	4 9:	8 10	0 6 '	7 3:1	1 5					CJ-type

```
decode_s:
                                           // variant 1 (with RISC-V Bitmanip)
  li t0, 0xfe000f80
                                           decode_cj:
  bext a0, a0, t0
                                             li t0, 0x28800001
  c.slli a0, 20
                                             li t1, 0x000016b8
  c.srai a0, 20
                                             li t2, 0xb4e00000
  ret
                                             li t3, 0x4b000000
                                             bext a1, a0, t1
decode_b:
                                             bdep a1, a1, t2
  li t0, 0xeaa800aa
                                             rori a0, a0, 11
  rori a0, a0, 8
                                             bext a0, a0, t0
  grevi a0, a0, 8
                                             bdep a0, a0, t3
  shfli a0, a0, 7
                                             c.or a0, a1
  bext a0, a0, t0
                                             c.srai a0, 20
  c.slli a0, 20
                                             ret
  c.srai a0, 19
  ret
decode_j:
  li t0, 0x800003ff
  li t1, 0x800ff000
  bext a1, a0, t1
  c.slli a1, 23
  rori a0, a0, 21
  bext a0, a0, t0
  c.slli a0, 12
  c.or a0, a1
  c.srai a0, 11
  ret
```

```
// variant 3 (with RISC-V Zbp only)
// variant 2 (without RISC-V Bitmanip)
decode_cj:
                                           decode_cj:
  srli a5, a0, 2
                                             shfli
                                                     a0, a0, 15
  srli a4, a0, 7
                                                     a0, a0, 28
                                             rori
  c.andi a4, 16
                                                     a0, a0, 2
                                             shfli
  slli a3, a0, 3
                                                     a0, a0, 14
                                             shfli
  c.andi a5, 14
                                             rori
                                                     a0, a0, 26
  c.add a5, a4
                                             shfli
                                                     a0, a0, 8
  andi a3, a3, 32
                                             rori
                                                     a0, a0, 10
  srli a4, a0, 1
                                             unshfli a0, a0, 12
                                                     a0, a0, 18
  c.add a5, a3
                                             rori
  andi a4, a4, 64
                                             unshfli a0, a0, 14
  slli a2, a0, 1
                                             rori
                                                     a0, a0, 28
  c.add a5, a4
                                             shfli
                                                     a0, a0, 6
  andi a2, a2, 128
                                             rori
                                                     a0, a0, 28
  srli a3, a0, 1
                                             unshfli a0, a0, 15
  slli a4, a0, 19
                                             slli
                                                     a0, a0, 21
  c.add a5, a2
                                                     a0, a0, 20
                                             srai
  andi a3, a3, 768
                                             ret
  c.slli a0, 2
  c.add a5, a3
  andi a0, a0, 1024
  c.srai a4, 31
  c.add a5, a0
  slli a0, a4, 11
  c.add a0, a5
  ret
```

# Change History

Date	Rev	Changes
2017-07-17	0.10	Initial Draft
2017-11-02	0.11	Remove roli, assembler can convert it to use a rori
		Remove bitwise subset and replace with andc
		Doc source text same base for study and spec.
		Fix typos
2017-11-30	0.32	Jump rev number to be on par with associated Study
		Move pdep/pext into spec draft and called it scatter-gather
2018-04-07	0.33	Move to github, throw out study, convert from .md to .tex
		Fix typos and fix some reference C implementations
		Rename bgat/bsca to bext/bdep
		Remove post-add immediate from clz
		Clean up encoding tables and code sections
2018-04-20	0.34	Add GREV, CTZ, and compressed instructions
		Restructure document: Move discussions to extra sections
		Add FAQ, add analysis of used encoding space
		Add Pseudo-Ops, Macros, Algorithms
		Add Generalized Bit Permutations (shuffle)
2018-05-12	0.35	Replace shuffle with generalized zip (gzip)
		Add additional XBitfield ISA Extension
		Add figures and tables, Clean up document
		Extend discussion and evaluation chapters
		Add Verilog reference implementations
		Add fast C reference implementations

Date	Rev	Changes						
2018-10-05	0.36	XBitfield is now a proper extension proposal						
		Add bswaps. [hwd] instructions						
		Add cmix, cmov, fsl, fsr						
		Rename gzip to shfl/unshfl						
		Add min, max, minu, maxu						
		Add clri, maki, join						
		Add cseln, cselz, mvnez, mveqz						
		Add clmul, clmulh, bmatxor, bmator, bmatflip						
		Remove bswaps.[hwd], clri, maki, join						
		Remove cseln, cselz, mvnez, mveqz						
2019-06-10	0.90	Add dedicated CRC instructions						
		Add proposed opcode encodings						
		Rename from XBitmanip to RISC-V Bitmanip						
		Remove chapter on bfxp[c] instruction						
		Refactor proposal into one big chapter						
		Remove c.brev and c.neg instructions						
		$\operatorname{Add}$ fsri, pack, addiwu, slliu.w						
		$\operatorname{Add}$ addwu, subwu, addu.w, subu.w						
		Rename andc to andn, Add orn and xnor						
		Add sbset[i], sbclr[i], sbinv[i], sbext[i]						
		New naming scheme for grevi pseudo-ops						
		Add clmulr instruction (reversed clmul)						
		Jump to Rev 0.90 to indicate spec matureness						
2019-08-29	0.91	Change encodings of bmatxor and grev[i][w]						
		Add gorc[i][w] and bfp[w] instructions						
2019-11-08	0.92	Add packh and packu[w] instructions						
		Add sext.b and sext.h instructions						
		Change encoding and behavior of bfp[w]						
		Change encoding of bdep[w]						

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