$$\underbrace{\bar{f}}_{L\times 1}(\underbrace{\bar{d}}_{N\times 1},\underbrace{\bar{p}}_{M\times 1}) = \bar{0}$$

$$\Gamma(\bar{d},\bar{p},\bar{\lambda}) = \underbrace{\phi(\bar{d})}_{\substack{\text{misfit + Regul dado +}\\ \text{Regul passo}}} + \underbrace{\Psi(\bar{p})}_{\substack{\text{Regul passo +}\\ \text{Regul parâmetro}}} + \bar{\lambda}^T \underbrace{\bar{f}(\bar{d},\bar{p})}_{\substack{\text{modelo}\\ \text{direto}}}$$

Universidade do Estado do Rio de Janeiro - Faculdade de Geologia

Geofísica 1 | Prática 8 - Magnetometria + Transformações

Nome: ______ Data:

1) Sobre a Amplitude da Derivada Total (TGA): (a) Como o TGA é afetado pelo erro nos dados e por que isso acontece? (b) O TGA concentra a anomalia sobre o corpo para todas as inclinações e declinações? Dê exemplos. Dicas: experimente inc=0 e dec=0; varie um parâmetro de cada vez.

$$\bar{U} = \begin{bmatrix} \bar{d} \\ \bar{\lambda} \\ \bar{p} \end{bmatrix} \qquad \min \Gamma(\bar{U}) \qquad \underline{\text{Newton}} \qquad \Gamma(\bar{U}) \Rightarrow \text{Taylor 2ª ordem em torno de } \bar{U}^0$$

$$\Gamma'(\bar{U}) \approx \Gamma(\bar{U}^0) + \underbrace{\bar{\nabla}_U \Gamma(\bar{U}^0)}_{\text{gradiente de }\Gamma}{}^T \Delta\bar{U} + \frac{1}{2}\Delta\bar{U}^T \underbrace{\bar{\bar{H}}_U \Gamma(\bar{U}^0)}_{\text{Hessiana de }\Gamma} \Delta\bar{U}$$

$$\min \Gamma \Rightarrow \bar{\nabla}_U \Gamma' = \bar{0} \Rightarrow \bar{\nabla}_U \Gamma(\bar{U}^0) + \bar{\bar{H}}_U \Gamma(\bar{U}^0)\Delta\bar{U} = \bar{0}$$

$$\boxed{\bar{\bar{H}}_U \Gamma(\bar{U}^0)\, \Delta\bar{U} = -\bar{\nabla}_U \Gamma(\bar{U}^0)}$$

2) Sobre a redução ao polo: (a) Como o erro nos dados influencia a redução ao polo? Compare com o TGA. (b) A redução ao polo é bem sucedida (capaz de concentrar a anomalia em cima do corpo) para qualquer valor de declinação? (c) Como a qualidade da redução ao polo varia com a inclinação? Como isso limita a aplicabilidade da redução ao polo?

<u>Quem é quem?</u>

$$\bar{\nabla}_U \Gamma = \begin{bmatrix} \bar{\nabla}_d \Gamma \\ \bar{\nabla}_\lambda \Gamma \\ \bar{\nabla}_p \Gamma \end{bmatrix}$$

$$\bar{\nabla}_\lambda \Gamma = \bar{f}(\bar{d},\bar{p})$$

$$\bar{\nabla}_d \Gamma = \bar{\nabla}_d \phi + \cancel{\bar{\nabla}_d \Psi}^{\,\bar{0}} + \bar{\nabla}_d \bar{\lambda}^T \bar{f}$$

$$\bar{\nabla}_p \Gamma = \cancel{\bar{\nabla}_p \phi}^{\,\bar{0}} + \bar{\nabla}_p \Psi + \bar{\nabla}_p \bar{\lambda}^T \bar{f}$$

$$\boxed{\begin{aligned} \bar{\nabla}_\lambda \Gamma &= \bar{f}^0 \\ \bar{\nabla}_d \Gamma &= \bar{\nabla}_d \phi + \bar{\bar{B}}^T \bar{\lambda} \\ \bar{\nabla}_p \Gamma &= \bar{\nabla}_p \Psi + \bar{\bar{A}}^T \bar{\lambda} \end{aligned}}$$

$$\bar{\nabla}_p \bar{\lambda}^T \bar{f} = \begin{bmatrix} \dfrac{\partial \bar{\lambda}^T \bar{f}}{\partial p_1} \\ \vdots \\ \dfrac{\partial \bar{\lambda}^T \bar{f}}{\partial p_M} \end{bmatrix} \longrightarrow \bar{\lambda}^T \frac{\partial \bar{f}}{\partial p_1} = \frac{\partial \bar{f}}{\partial p_1}^T \bar{\lambda} \Rightarrow \underbrace{\begin{bmatrix} \dfrac{\partial \bar{f}}{\partial p_1}^T \\ \dfrac{\partial \bar{f}}{\partial p_2}^T \\ \vdots \\ \dfrac{\partial \bar{f}}{\partial p_M}^T \end{bmatrix}}_{\bar{\bar{A}}^T} \bar{\lambda}$$

$$\boxed{\begin{aligned} \bar{\nabla}_p \bar{\lambda}^T \bar{f} &= \bar{\bar{A}}^T \bar{\lambda} \\ \bar{\nabla}_d \bar{\lambda}^T \bar{f} &= \bar{\bar{B}}^T \bar{\lambda} \end{aligned}}$$

$$
\bar{\bar{H}}_U\Gamma = \left[\begin{array}{cccc|cccc|ccc}
\frac{\partial^2\Gamma}{\partial d_1\partial d_1} & \frac{\partial^2\Gamma}{\partial d_1\partial d_2} & \cdots & \frac{\partial^2\Gamma}{\partial d_1\partial d_N} & \frac{\partial^2\Gamma}{\partial d_1\partial p_1} & \frac{\partial^2\Gamma}{\partial d_1\partial p_2} & \cdots & \frac{\partial^2\Gamma}{\partial d_1\partial p_M} & \frac{\partial^2\Gamma}{\partial d_1\partial\lambda_1} & \cdots & \frac{\partial^2\Gamma}{\partial d_1\partial\lambda_L} \\
\frac{\partial^2\Gamma}{\partial d_2\partial d_1} & \frac{\partial^2\Gamma}{\partial d_2^2} & & \frac{\partial^2\Gamma}{\partial d_2\partial d_N} & \frac{\partial^2\Gamma}{\partial d_2\partial p_1} & & & \frac{\partial^2\Gamma}{\partial d_2\partial p_M} & \frac{\partial^2\Gamma}{\partial d_2\partial\lambda_1} & \cdots & \frac{\partial^2\Gamma}{\partial d_2\partial\lambda_L} \\
\frac{\partial^2\Gamma}{\partial d_N\partial d_1} & \frac{\partial^2\Gamma}{\partial d_N\partial d_2} & \cdots & \frac{\partial^2\Gamma}{\partial d_N^2} & \frac{\partial^2\Gamma}{\partial d_N\partial p_1} & & \cdots & \frac{\partial^2\Gamma}{\partial d_N\partial p_M} & \frac{\partial^2\Gamma}{\partial d_N\partial\lambda_1} & \cdots & \\
\hline
\frac{\partial^2\Gamma}{\partial p_1\partial d_1} & \frac{\partial^2\Gamma}{\partial p_1\partial d_2} & \cdots & \frac{\partial^2\Gamma}{\partial p_1\partial d_N} & \frac{\partial^2\Gamma}{\partial p_1^2} & & \cdots & \frac{\partial^2\Gamma}{\partial p_1\partial p_M} & \frac{\partial^2\Gamma}{\partial p_1\partial\lambda_1} & \cdots & \frac{\partial^2\Gamma}{\partial p_1\partial\lambda_L} \\
\vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial^2\Gamma}{\partial p_M\partial d_1} & \frac{\partial^2\Gamma}{\partial p_M\partial d_2} & \cdots & \frac{\partial^2\Gamma}{\partial p_M\partial d_N} & \frac{\partial^2\Gamma}{\partial p_M\partial p_1} & & \cdots & \frac{\partial^2\Gamma}{\partial p_M^2} & \frac{\partial^2\Gamma}{\partial p_M\partial\lambda_1} & \cdots & \frac{\partial^2\Gamma}{\partial p_M\partial\lambda_L} \\
\hline
\frac{\partial^2\Gamma}{\partial\lambda_1\partial d_1} & \frac{\partial^2\Gamma}{\partial\lambda_1\partial d_2} & \cdots & \frac{\partial^2\Gamma}{\partial\lambda_1\partial d_N} & \frac{\partial^2\Gamma}{\partial\lambda_1\partial p_1} & & \cdots & \frac{\partial^2\Gamma}{\partial\lambda_1\partial p_M} & \frac{\partial^2\Gamma}{\partial\lambda_1^2} & \cdots & \frac{\partial^2\Gamma}{\partial\lambda_1\partial\lambda_L} \\
\vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial^2\Gamma}{\partial\lambda_L\partial d_1} & & \cdots & \frac{\partial^2\Gamma}{\partial\lambda_L\partial d_N} & \frac{\partial^2\Gamma}{\partial\lambda_L\partial p_1} & & \cdots & \frac{\partial^2\Gamma}{\partial\lambda_L\partial p_M} & \frac{\partial^2\Gamma}{\partial\lambda_L\partial\lambda_1} & \cdots & \frac{\partial^2\Gamma}{\partial\lambda_L^2}
\end{array}\right]
$$

$N+M+L \times N+M+L$

$$
\bar{\bar{H}}_U\Gamma = \begin{bmatrix}
\bar{\bar{H}}_d\Gamma_{N\times N} & \bar{\bar{H}}_{dp}\Gamma_{N\times M} & \bar{\bar{H}}_{d\lambda}\Gamma_{N\times L} \\
\bar{\bar{H}}_{pd}\Gamma_{M\times N} & \bar{\bar{H}}_{p}\Gamma_{M\times M} & \bar{\bar{H}}_{p\lambda}\Gamma_{M\times L} \\
\bar{\bar{H}}_{\lambda d}\Gamma_{L\times N} & \bar{\bar{H}}_{\lambda p}\Gamma_{L\times M} & \bar{\bar{H}}_{\lambda}\Gamma_{L\times L}
\end{bmatrix}
$$

Tá tudo trocado! é λ depois p!

$$
\bar{\bar{H}}_U = \begin{bmatrix}
\bar{\bar{H}}_d & \bar{\bar{H}}_{d\lambda} & \bar{\bar{H}}_{dp} \\
\bar{\bar{H}}_{d\lambda}^T & \bar{\bar{H}}_{\lambda} & \bar{\bar{H}}_{p\lambda}^T \\
\bar{\bar{H}}_{dp}^T & \bar{\bar{H}}_{p\lambda} & \bar{\bar{H}}_{p}
\end{bmatrix}
$$

$\bar{\bar{H}}_{d\lambda} = \bar{\bar{H}}_{\lambda d}^T$

$\bar{\bar{H}}_{p\lambda} = \bar{\bar{H}}_{\lambda p}^T$

$\bar{\bar{H}}_{dp} = \bar{\bar{H}}_{pd}^T$

$$
\bar{\bar{H}}_U = \begin{bmatrix}
\bar{\bar{H}}_d & \bar{\bar{H}}_{dp} & \bar{\bar{H}}_{d\lambda} \\
\bar{\bar{H}}_{dp}^T & \bar{\bar{H}}_{p} & \bar{\bar{H}}_{p\lambda} \\
\bar{\bar{H}}_{d\lambda}^T & \bar{\bar{H}}_{p\lambda}^T & \bar{\bar{H}}_{\lambda}
\end{bmatrix}
$$

Quem é quem? $\quad \bar{\bar{H}}_{\lambda}\Gamma = \bar{\bar{0}} \Rightarrow \Gamma$ é linear com $\bar{\lambda}$

$$\bar{\bar{H}}_{d\lambda}\Gamma = \bar{\nabla}_d\left[\bar{\nabla}_\lambda \Gamma\right]^T = \bar{\nabla}_d \bar{f}^T = \begin{bmatrix} \frac{\partial \bar{f}^T}{\partial d_1} \\ \vdots \\ \frac{\partial \bar{f}^T}{\partial d_N} \end{bmatrix} = \boxed{\bar{\bar{B}}^T}$$

$$\boxed{\bar{\bar{H}}_{p\lambda}\Gamma = \bar{\nabla}_p\left[\bar{\nabla}_\lambda \Gamma\right]^T = \bar{\nabla}_p \bar{f}^T = \bar{\bar{A}}^T}$$

$$\bar{\bar{H}}_d\Gamma \Rightarrow H_{d_{ij}}\Gamma = \frac{\partial}{\partial d_i}\left[\frac{\partial}{\partial d_j}\left(\phi(\bar{d}) + \psi(\bar{p}) + \bar{\lambda}^T\bar{f}(\bar{d},\bar{p})\right)\right]$$

$$= \frac{\partial}{\partial d_i}\left[\frac{\partial \phi(\bar{d})}{\partial d_j} + 0 + \frac{\partial \bar{f}^T}{\partial d_j}\bar{\lambda}\right]$$

$$\bar{\bar{H}}_{d_{ij}}\Gamma = \frac{\partial^2\phi(\bar{d})}{\partial d_i \partial d_j} + \frac{\partial^2 \bar{f}^T}{\partial d_i \partial d_j}\bar{\lambda}$$

Desprezando derivadas segundas de $\bar{f}$ (Gauss-Newton)...

$$\boxed{\bar{\bar{H}}_d\Gamma = \bar{\bar{H}}_d\phi} \qquad \boxed{\bar{\bar{H}}_p\Gamma = \bar{\bar{H}}_p\psi}$$

$$H_{pd_{ij}}\Gamma = \frac{\partial}{\partial p_i}\left[\frac{\partial}{\partial d_j}\left(\phi + \psi + \bar{\lambda}^T\bar{f}\right)\right] = \frac{\partial}{\partial p_i}\left[\frac{\partial \phi}{\partial d_j} + 0 + \frac{\partial f^T}{\partial d_j}\lambda\right]$$

$$= \cancel{\frac{\partial}{\partial p_i}\frac{\partial \phi}{\partial d_j}}^{\,0} + \cancel{\frac{\partial^2 \bar{f}^T}{\partial p_i \partial d_j}\bar{\lambda}}^{\,0 \text{ (Gauss-Newton)}}$$

$$\boxed{\bar{\bar{H}}_{pd}\Gamma \approx \bar{\bar{0}}}$$

Juntando tudo: $\bar{\bar{H}}_u \Gamma \bar{\Delta}_u + \bar{\nabla}_u \Gamma = \bar{0}$

$$\cancel{\begin{bmatrix} \bar{\bar{H}}_d & \bar{\bar{0}} & \bar{\bar{B}}^T \\ \bar{\bar{0}} & \bar{\bar{H}} & \end{bmatrix}}$$

$$\left[\begin{array}{c|cc} \bar{\bar{H}}_d & \bar{\bar{B}}^T & \bar{\bar{0}} \\ \bar{\bar{B}} & \bar{\bar{0}} & \bar{\bar{A}} \\ \bar{\bar{0}} & \bar{\bar{A}}^T & \bar{\bar{H}}_p \end{array}\right] \begin{bmatrix} \bar{\Delta d} \\ \bar{\Delta \lambda} \\ \bar{\Delta}_p \end{bmatrix} + \begin{bmatrix} \bar{\nabla}_d \phi + \bar{\bar{B}}^T \bar{\lambda} \\ \bar{f}^0 \\ \bar{\nabla}_p \psi + \bar{\bar{A}}^T \bar{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{0} \\ \bar{0} \\ \bar{0} \end{bmatrix}$$

$$\begin{bmatrix} \bar{\bar{A}} & \bar{\bar{B}} \\ \bar{\bar{C}} & \bar{\bar{D}} \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + \begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix} = \begin{bmatrix} \bar{0} \\ \bar{0} \end{bmatrix} \Rightarrow (\bar{\bar{D}} - \bar{\bar{C}}\bar{\bar{A}}^{-1}\bar{\bar{B}})\bar{y} + \bar{v} - \bar{\bar{C}}\bar{\bar{A}}^{-1}\bar{u} = \bar{0}$$

$$\left(\begin{bmatrix} \bar{\bar{0}} & \bar{\bar{A}} \\ \bar{\bar{A}}^T & \bar{\bar{H}}_p \end{bmatrix} - \begin{bmatrix} \bar{\bar{B}} \\ \bar{\bar{0}} \end{bmatrix} \bar{\bar{H}}_d^{-1} \begin{bmatrix} \bar{\bar{B}}^T & \bar{\bar{0}} \end{bmatrix} \right) \begin{bmatrix} \bar{\Delta\lambda} \\ \bar{\Delta}_p \end{bmatrix} + \begin{bmatrix} \bar{f}^0 \\ \bar{\nabla}_p \psi + \bar{\bar{A}}^T \bar{\lambda} \end{bmatrix} - \begin{bmatrix} \bar{\bar{B}} \\ \bar{\bar{0}} \end{bmatrix} \bar{\bar{H}}_d^{-1} (\bar{\nabla}_d \phi + \bar{\bar{B}}^T \bar{\lambda})$$

$$\left(\begin{bmatrix} \bar{\bar{0}} & \bar{\bar{A}} \\ \bar{\bar{A}}^T & \bar{\bar{H}}_p \end{bmatrix} - \begin{bmatrix} \bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\bar{B}}^T & \bar{\bar{0}} \\ \bar{\bar{0}} & \bar{\bar{0}} \end{bmatrix} \right) \begin{bmatrix} \bar{\Delta\lambda} \\ \bar{\Delta}_p \end{bmatrix} + \begin{bmatrix} \bar{f}^0 - B H_d^{-1} (\bar{\nabla}_d \phi + \bar{\bar{B}}^T \bar{\lambda}) \\ \bar{\nabla}_p \psi + \bar{\bar{A}}^T \bar{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{0} \\ \bar{0} \end{bmatrix}$$

$$\left[\begin{array}{c|c} -\bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\bar{B}}^T & \bar{\bar{A}} \\ \hline \bar{\bar{A}}^T & \bar{\bar{H}}_p \end{array}\right] \begin{bmatrix} \bar{\Delta\lambda} \\ \bar{\Delta p} \end{bmatrix} + \begin{bmatrix} \bar{f}^0 - \bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\nabla}_d \phi - \bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\bar{B}}^T \bar{\lambda} \\ \bar{\nabla}_p \psi + \bar{\bar{A}}^T \bar{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{0} \\ \bar{0} \end{bmatrix}$$

$$\Big(\bar{\bar{H}}_p + \bar{\bar{A}}^T \underbrace{(\bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\bar{B}}^T)}_{\bar{\bar{P}}}{}^{-1} \bar{\bar{A}} \Big) \bar{\Delta}_p + \bar{\nabla}_p \psi + \bar{\bar{A}}^T \bar{\lambda} + \bar{\bar{A}}^T (\bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\bar{B}}^T)^{-1} \Big[f^0 - \bar{\bar{B}}\bar{\bar{H}}_d^{-1} (\bar{\nabla}_d \phi + \bar{\bar{B}}^T \bar{\lambda}) \Big] = \bar{0}$$

$$(\bar{\bar{A}}^T \bar{\bar{P}} \bar{\bar{A}} + \bar{\bar{H}}_p) \bar{\Delta}_p = -\bar{\nabla}_p \psi - \bar{\bar{A}}^T \bar{\lambda} - \bar{\bar{A}}^T \bar{\bar{P}} \Big[\bar{f}^0 - \bar{\bar{B}}\bar{\bar{H}}_d^{-1} \bar{\nabla}_d \phi - \bar{\bar{P}}^{-1} \lambda \Big]$$

$$(A^T P A + H_p) \Delta_p = -\bar{\nabla}_p \psi - \cancel{A^T \lambda} - A^T P f^0 + A^T P B H_d^{-1} \bar{\nabla}_d \phi + \cancel{A^T \overbrace{P P^{-1}}^{I} \lambda}$$

$$\underline{\underline{(\bar{\bar{A}}^T \bar{\bar{P}} \bar{\bar{A}} + \bar{\bar{H}}_p) \bar{\Delta p} = \bar{\bar{A}}^T \bar{\bar{P}} (\bar{\bar{B}}\bar{\bar{H}}_d^{-1} \bar{\nabla}_d \phi - \bar{f}^0) - \bar{\nabla}_p \psi}}$$