$$\bar f(\underset{N\times 1}{\bar d},\underset{M\times 1}{\bar p}) = \bar 0 \qquad \bar f: L\times 1$$

$$\Gamma(\bar d,\bar p,\bar\lambda) = \underbrace{\phi(\bar d)}_{\text{misfit + Regul dado + Regul passo}} + \underbrace{\Psi(\bar p)}_{\text{Regul passo + Regul parâmetro}} + \underbrace{\bar\lambda^T \bar f(\bar d,\bar p)}_{\text{modelo direto}}$$

Universidade do Estado do Rio de Janeiro - Faculdade de Geologia

Geofísica 1 | Prática 8 | Magnetometria - Transformações

Nome: ________________ Data:

1) Sobre a Amplitude da Derivada Total (TGA): (a) Como o TGA é afetado pelo erro nos dados e por que isso acontece? (b) O TGA concentra a anomalia sobre o corpo para todas as inclinações e declinações? Dê exemplos. Dicas: experimente inc=0 e dec=0; varie um parâmetro de cada vez.

$$\bar u = \begin{bmatrix} \bar d \\ \bar\lambda \\ \bar p \end{bmatrix} \qquad \min \Gamma(\bar u) \qquad \underline{\text{Newton}} \qquad \Gamma(\bar u) \Rightarrow \text{Taylor 2ª ordem em torno de } \bar u^0$$

$$\Gamma'(\bar u) \approx \Gamma(\bar u^0) + \underbrace{\bar\nabla_u \Gamma(\bar u^0)^T}_{\text{gradiente de }\Gamma} \Delta\bar u + \frac{1}{2}\Delta\bar u^T \underbrace{\bar{\bar H}_u \Gamma(\bar u^0)}_{\text{Hessiana de }\Gamma} \Delta\bar u$$

$$\min \Gamma \Rightarrow \bar\nabla_u \Gamma' = \bar 0 \Rightarrow \bar\nabla_u\Gamma(\bar u^0) + \bar{\bar H}_u\Gamma(\bar u^0)\Delta\bar u = \bar 0$$

$$\bar{\bar H}_u \Gamma(\bar u^0)\,\Delta\bar u = -\bar\nabla_u\Gamma(\bar u^0)$$

2) Sobre a redução ao polo: (a) Como o erro nos dados influencia a redução ao polo? Compare com o TGA. (b) A redução ao polo é bem sucedida (capaz de concentrar a anomalia em cima do corpo) para qualquer valor de declinação? (c) Como a qualidade da redução ao polo varia com a inclinação? Como isso limita a aplicabilidade da redução ao polo?

<u>Quem é quem?</u>

$$\bar\nabla_u\Gamma = \begin{bmatrix} \bar\nabla_d \Gamma \\ \bar\nabla_\lambda \Gamma \\ \bar\nabla_p\Gamma \end{bmatrix}$$

$$\boxed{\bar\nabla_\lambda\Gamma = \bar f(\bar d,\bar p)}$$

$$\bar\nabla_d\Gamma = \bar\nabla_d\phi + \cancel{\bar\nabla_d\Psi}^{\,\bar 0} + \bar\nabla_d\bar\lambda^T\bar f$$

$$\bar\nabla_p\Gamma = \cancel{\bar\nabla_p\phi}^{\,\bar 0} + \bar\nabla_p\Psi + \bar\nabla_p\bar\lambda^T\bar f$$

$$\bar\nabla_p\bar\lambda^T\bar f = \begin{bmatrix} \frac{\partial\bar\lambda^T\bar f}{\partial p_1} \\ \vdots \\ \frac{\partial\bar\lambda^T\bar f}{\partial p_M}\end{bmatrix} \longrightarrow \bar\lambda^T\frac{\partial\bar f}{\partial p_1} = \frac{\partial\bar f}{\partial p_1}^T\bar\lambda \Rightarrow \underbrace{\begin{bmatrix} \frac{\partial\bar f}{\partial p_1}^T \\ \frac{\partial\bar f}{\partial p_2}^T \\ \vdots \\ \frac{\partial\bar f}{\partial p_M}^T \end{bmatrix}}_{\bar{\bar A}^T}\bar\lambda$$

$$\boxed{\begin{aligned} \bar\nabla_p\bar\lambda^T\bar f &= \bar{\bar A}^T\bar\lambda \\ \bar\nabla_d\bar\lambda^T\bar f &= \bar{\bar B}^T\bar\lambda \end{aligned}}$$

$$\boxed{\begin{aligned} \bar\nabla_\lambda\Gamma &= \bar f^0 \\ \bar\nabla_d\Gamma &= \bar\nabla_d\phi + \bar{\bar B}^T\bar\lambda \\ \bar\nabla_p\Gamma &= \bar\nabla_p\Psi + \bar{\bar A}^T\bar\lambda \end{aligned}}$$

$$
\bar{\bar{H}}_U \Gamma = \left[\begin{array}{cccc|ccc|ccc}
\frac{\partial^2\Gamma}{\partial d_1 \partial d_1} & \frac{\partial^2\Gamma}{\partial d_1 \partial d_2} & \cdots & \frac{\partial^2\Gamma}{\partial d_1 \partial d_N} & \frac{\partial^2\Gamma}{\partial d_1 \partial p_1} & \frac{\partial^2\Gamma}{\partial d_1 \partial p_2} \cdots & \frac{\partial^2\Gamma}{\partial d_1 \partial p_M} & \frac{\partial^2\Gamma}{\partial d_1 \partial \lambda_1} & \cdots & \frac{\partial^2\Gamma}{\partial d_1 \partial \lambda_L} \\
\frac{\partial^2\Gamma}{\partial d_2 \partial d_1} & \frac{\partial^2\Gamma}{\partial d_2^2} & & \frac{\partial^2\Gamma}{\partial d_2 \partial d_N} & \frac{\partial^2\Gamma}{\partial d_2 \partial p_1} & & \frac{\partial^2\Gamma}{\partial d_2 \partial p_M} & \frac{\partial^2\Gamma}{\partial d_2 \partial \lambda_1} & \cdots & \frac{\partial^2\Gamma}{\partial d_2 \partial \lambda_L} \\
\frac{\partial^2\Gamma}{\partial d_N \partial d_1} & \frac{\partial^2\Gamma}{\partial d_N \partial d_2} & \cdots & \frac{\partial^2\Gamma}{\partial d_N^2} & \frac{\partial^2\Gamma}{\partial d_N \partial p_1} & \cdots & \frac{\partial^2\Gamma}{\partial d_N \partial p_M} & \frac{\partial^2\Gamma}{\partial d_N \partial \lambda_1} & \cdots & \\
\hline
\frac{\partial^2\Gamma}{\partial p_1 \partial d_1} & \frac{\partial^2\Gamma}{\partial p_1 \partial d_2} & \cdots & \frac{\partial^2\Gamma}{\partial p_1 \partial d_N} & \frac{\partial^2\Gamma}{\partial p_1^2} & \cdots & \frac{\partial^2\Gamma}{\partial p_1 \partial p_M} & \frac{\partial^2\Gamma}{\partial p_1 \partial \lambda_1} & \cdots & \frac{\partial^2\Gamma}{\partial p_1 \partial \lambda_L} \\
\vdots & & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial^2\Gamma}{\partial p_M \partial d_1} & \frac{\partial^2\Gamma}{\partial p_M \partial d_2} & \cdots & \frac{\partial^2\Gamma}{\partial p_M \partial d_N} & \frac{\partial^2\Gamma}{\partial p_M \partial p_1} & \cdots & \frac{\partial^2\Gamma}{\partial p_M^2} & \frac{\partial^2\Gamma}{\partial p_M \partial \lambda_1} & \cdots & \frac{\partial^2\Gamma}{\partial p_M \partial \lambda_L} \\
\hline
\frac{\partial^2\Gamma}{\partial \lambda_1 \partial d_1} & \frac{\partial^2\Gamma}{\partial \lambda_1 \partial d_2} & \cdots & \frac{\partial^2\Gamma}{\partial \lambda_1 \partial d_N} & \frac{\partial^2\Gamma}{\partial \lambda_1 \partial p_1} & \cdots & \frac{\partial^2\Gamma}{\partial \lambda_1 \partial p_M} & \frac{\partial^2\Gamma}{\partial \lambda_1^2} & \cdots & \frac{\partial^2\Gamma}{\partial \lambda_1 \partial \lambda_L} \\
\vdots & & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial^2\Gamma}{\partial \lambda_L \partial d_1} & & \cdots & \frac{\partial^2\Gamma}{\partial \lambda_L \partial d_N} & \frac{\partial^2\Gamma}{\partial \lambda_L \partial p_1} & \cdots & \frac{\partial^2\Gamma}{\partial \lambda_L \partial p_M} & \frac{\partial^2\Gamma}{\partial \lambda_L \partial \lambda_1} & \cdots & \frac{\partial^2\Gamma}{\partial \lambda_L^2}
\end{array}\right]
$$

$N+M+L \times N+M+L$

$$
\bar{\bar{H}}_U \Gamma = \begin{bmatrix}
\bar{\bar{H}}_d \Gamma_{N\times N} & \bar{\bar{H}}_{dp} \Gamma_{N\times M} & \bar{\bar{H}}_{d\lambda} \Gamma_{N\times L} \\
\bar{\bar{H}}_{pd} \Gamma_{M\times N} & \bar{\bar{H}}_{p} \Gamma_{M\times M} & \bar{\bar{H}}_{p\lambda} \Gamma_{M\times L} \\
\bar{\bar{H}}_{\lambda d} \Gamma_{L\times N} & \bar{\bar{H}}_{\lambda p} \Gamma_{L\times M} & \bar{\bar{H}}_{\lambda} \Gamma_{L\times L}
\end{bmatrix}
$$

Tá tudo trocado! é λ depois p!

$$
\bar{\bar{H}}_U = \begin{bmatrix}
\bar{\bar{H}}_d & \bar{\bar{H}}_{d\lambda} & \bar{\bar{H}}_{dp} \\
\bar{\bar{H}}_{d\lambda}^T & \bar{\bar{H}}_{\lambda} & \bar{\bar{H}}_{p\lambda}^T \\
\bar{\bar{H}}_{dp}^T & \bar{\bar{H}}_{p\lambda} & \bar{\bar{H}}_{p}
\end{bmatrix}
$$

$\bar{\bar{H}}_{d\lambda} = \bar{\bar{H}}_{\lambda d}^T$

$\bar{\bar{H}}_{p\lambda} = \bar{\bar{H}}_{\lambda p}^T$

$\bar{\bar{H}}_{dp} = \bar{\bar{H}}_{pd}^T$

$$
\bar{\bar{H}}_U = \begin{bmatrix}
\bar{\bar{H}}_d & \bar{\bar{H}}_{dp} & \bar{\bar{H}}_{d\lambda} \\
\bar{\bar{H}}_{dp}^T & \bar{\bar{H}}_{p} & \bar{\bar{H}}_{p\lambda} \\
\bar{\bar{H}}_{d\lambda}^T & \bar{\bar{H}}_{p\lambda}^T & \bar{\bar{H}}_{\lambda}
\end{bmatrix}
$$

Quem é quem? $\quad \bar{\bar{H}}_{\lambda}\Gamma = \bar{\bar{0}} \Rightarrow \Gamma$ é linear com $\bar{\lambda}$

$$\bar{\bar{H}}_{d\lambda}\Gamma = \bar{\nabla}_d\left[\bar{\nabla}_\lambda \Gamma\right]^T = \bar{\nabla}_d \bar{f}^T = \begin{bmatrix} \dfrac{\partial \bar{f}^T}{\partial d_1} \\ \vdots \\ \dfrac{\partial \bar{f}^T}{\partial d_N} \end{bmatrix} = \boxed{\bar{\bar{B}}^T}$$

$$\bar{\bar{H}}_{p\lambda}\Gamma = \bar{\nabla}_p\left[\bar{\nabla}_\lambda \Gamma\right]^T = \bar{\nabla}_p \bar{f}^T = \boxed{\bar{\bar{A}}^T}$$

$$\bar{\bar{H}}_d\Gamma \Rightarrow H_{d_{ij}}\Gamma = \frac{\partial}{\partial d_i}\left[\frac{\partial}{\partial d_j}\left(\phi(\bar{d}) + \psi(\bar{p}) + \bar{\lambda}^T\bar{f}(\bar{d},\bar{p})\right)\right]$$

$$= \frac{\partial}{\partial d_i}\left[\frac{\partial \phi(\bar{d})}{\partial d_j} + 0 + \frac{\partial \bar{f}^T}{\partial d_j}\bar{\lambda}\right]$$

$$\bar{\bar{H}}_{d_{ij}}\Gamma = \frac{\partial^2 \phi(\bar{d})}{\partial d_i \partial d_j} + \frac{\partial^2 \bar{f}^T}{\partial d_i \partial d_j}\bar{\lambda}$$

Desprezando derivadas segundas de $\bar{f}$ (Gauss-Newton)...

$$\boxed{\bar{\bar{H}}_d\Gamma = \bar{\bar{H}}_d\phi} \qquad \boxed{\bar{\bar{H}}_p\Gamma = \bar{\bar{H}}_p\psi}$$

$$H_{pd_{ij}}\Gamma = \frac{\partial}{\partial p_i}\left[\frac{\partial}{\partial d_j}\left(\phi + \psi + \bar{\lambda}^T\bar{f}\right)\right] = \frac{\partial}{\partial p_i}\left[\frac{\partial \phi}{\partial d_j} + 0 + \frac{\partial f^T}{\partial d_j}\lambda\right]$$

$$= \cancel{\frac{\partial}{\partial p_i}\frac{\partial \phi}{\partial d_j}}^{\,0} + \cancel{\frac{\partial^2 \bar{f}^T}{\partial p_i \partial d_j}\bar{\lambda}} \rightarrow 0 \text{ (Gauss-Newton)}$$

$$\boxed{\bar{\bar{H}}_{pd}\Gamma \approx \bar{\bar{0}}}$$

Juntando tudo: $\bar{\bar{H}}_u \Gamma \bar{\Delta}_u + \bar{\nabla}_u \Gamma = \bar{0}$

$$\cancel{\begin{bmatrix} \bar{\bar{H}}_d & \bar{\bar{0}} & \bar{\bar{B}}^T \\ \bar{\bar{0}} & \bar{\bar{H}} & \end{bmatrix}}$$

$$\left[\begin{array}{c|cc} \bar{\bar{H}}_d & \bar{\bar{B}}^T & \bar{\bar{0}} \\ \bar{\bar{B}} & \bar{\bar{0}} & \bar{\bar{A}} \\ \bar{\bar{0}} & \bar{\bar{A}}^T & \bar{\bar{H}}_p \end{array}\right] \begin{bmatrix} \bar{\Delta}_d \\ \bar{\Delta}_\lambda \\ \bar{\Delta}_p \end{bmatrix} + \begin{bmatrix} \bar{\nabla}_d \phi + \bar{\bar{B}}^T \bar{\lambda} \\ \bar{f}^0 \\ \bar{\nabla}_p \psi + \bar{\bar{A}}^T \bar{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{0} \\ \bar{0} \\ \bar{0} \end{bmatrix}$$

$$\begin{bmatrix} \bar{\bar{A}} & \bar{\bar{B}} \\ \bar{\bar{C}} & \bar{\bar{D}} \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + \begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix} = \begin{bmatrix} \bar{0} \\ \bar{0} \end{bmatrix} \Rightarrow (\bar{\bar{D}} - \bar{\bar{C}}\bar{\bar{A}}^{-1}\bar{\bar{B}})\bar{y} + \bar{v} - \bar{\bar{C}}\bar{\bar{A}}^{-1}\bar{u} = \bar{0}$$

$$\left(\begin{bmatrix} \bar{\bar{0}} & \bar{\bar{A}} \\ \bar{\bar{A}}^T & \bar{\bar{H}}_p \end{bmatrix} - \begin{bmatrix} \bar{\bar{B}} \\ \bar{\bar{0}} \end{bmatrix} \bar{\bar{H}}_d^{-1} \begin{bmatrix} \bar{\bar{B}}^T & \bar{\bar{0}} \end{bmatrix} \right) \begin{bmatrix} \bar{\Delta}_\lambda \\ \bar{\Delta}_p \end{bmatrix} + \begin{bmatrix} \bar{f}^0 \\ \bar{\nabla}_p \psi + \bar{\bar{A}}^T \bar{\lambda} \end{bmatrix} - \begin{bmatrix} \bar{\bar{B}} \\ \bar{\bar{0}} \end{bmatrix} \bar{\bar{H}}_d^{-1} (\bar{\nabla}_d \phi + \bar{\bar{B}}^T \bar{\lambda})$$

$$\left(\begin{bmatrix} \bar{\bar{0}} & \bar{\bar{A}} \\ \bar{\bar{A}}^T & \bar{\bar{H}}_p \end{bmatrix} - \begin{bmatrix} \bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\bar{B}}^T & \bar{\bar{0}} \\ \bar{\bar{0}} & \bar{\bar{0}} \end{bmatrix} \right) \begin{bmatrix} \bar{\Delta}_\lambda \\ \bar{\Delta}_p \end{bmatrix} + \begin{bmatrix} \bar{f}^0 - \bar{\bar{B}}\bar{\bar{H}}_d^{-1}(\bar{\nabla}_d \phi + \bar{\bar{B}}^T \bar{\lambda}) \\ \bar{\nabla}_p \psi + \bar{\bar{A}}^T \bar{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{0} \\ \bar{0} \end{bmatrix}$$

$$\left[\begin{array}{c|c} -\bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\bar{B}}^T & \bar{\bar{A}} \\ \hline \bar{\bar{A}}^T & \bar{\bar{H}}_p \end{array}\right] \begin{bmatrix} \bar{\Delta}_\lambda \\ \bar{\Delta}_p \end{bmatrix} + \begin{bmatrix} \bar{f}^0 - \bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\nabla}_d \phi - \bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\bar{B}}^T \bar{\lambda} \\ \bar{\nabla}_p \psi + \bar{\bar{A}}^T \bar{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{0} \\ \bar{0} \end{bmatrix}$$

$$\Big(\bar{\bar{H}}_p + \bar{\bar{A}}^T \underbrace{(\bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\bar{B}}^T)^{-1}}_{\bar{\bar{P}}} \bar{\bar{A}} \Big) \bar{\Delta}_p + \bar{\nabla}_p \psi + \bar{\bar{A}}^T \bar{\lambda} + \bar{\bar{A}}^T (\bar{\bar{B}}\bar{\bar{H}}_d^{-1}\bar{\bar{B}}^T)^{-1} \left[\bar{f}^0 - \bar{\bar{B}}\bar{\bar{H}}_d^{-1} (\bar{\nabla}_d \phi + \bar{\bar{B}}^T \bar{\lambda}) \right] = \bar{0}$$

$$(\bar{\bar{A}}^T \bar{\bar{P}} \bar{\bar{A}} + \bar{\bar{H}}_p) \bar{\Delta}_p = -\bar{\nabla}_p \psi - \bar{\bar{A}}^T \bar{\lambda} - \bar{\bar{A}}^T \bar{\bar{P}} \left[\bar{f}^0 - \bar{\bar{B}}\bar{\bar{H}}_d^{-1} \bar{\nabla}_d \phi - \bar{\bar{P}}^{-1} \lambda \right]$$

$$(A^T P A + H_p) \Delta_p = -\bar{\nabla}_p \psi - \cancel{A^T \lambda} - A^T P f^0 + A^T P B H_d^{-1} \bar{\nabla}_d \phi + \cancel{A^T \overbrace{P P^{-1}}^{I} \lambda}$$

$$\boxed{(\bar{\bar{A}}^T \bar{\bar{P}} \bar{\bar{A}} + \bar{\bar{H}}_p) \bar{\Delta}_p = \bar{\bar{A}}^T \bar{\bar{P}} (\bar{\bar{B}}\bar{\bar{H}}_d^{-1} \bar{\nabla}_d \phi - \bar{f}^0) - \bar{\nabla}_p \psi}$$

Calculando $\overline{\Delta d}$: Volta no sistema 2x2 com $\Delta\lambda$ e Δp

$$-\bar{\bar{P}}^{-1}\overline{\Delta\lambda} + \bar{\bar{A}}\overline{\Delta p} + \bar{f}^{0} - \bar{\bar{B}}\bar{\bar{H}}_d^{-1}\left(\overline{\nabla_d\phi} + \bar{\bar{B}}^{T}\bar{\lambda}\right) = \bar{0}$$

$$\bar{\bar{P}}^{-1}\overline{\Delta\lambda} = \bar{\bar{A}}\overline{\Delta p} + \bar{f}^{0} - \bar{\bar{B}}\bar{\bar{H}}_d^{-1}\overline{\nabla_d\phi} - \bar{\bar{P}}^{-1}\bar{\lambda}$$

$$\overline{\Delta\lambda} = \bar{\bar{P}}\left(\bar{\bar{A}}\overline{\Delta p} + \bar{f}^{0} - \bar{\bar{B}}\bar{\bar{H}}_d^{-1}\overline{\nabla_d\phi}\right) - \overset{\bar{\bar{I}}}{\cancel{\bar{\bar{P}}\bar{\bar{P}}^{-1}}}\bar{\lambda}$$

Voltando na 1ª equação (sistema normal)

$$\bar{\bar{H}}_d\overline{\Delta d} + \bar{\bar{B}}^{T}\overline{\Delta\lambda} + \overline{\nabla_d\phi} + \bar{\bar{B}}^{T}\bar{\lambda} = \bar{0}$$

$$\bar{\bar{H}}_d\overline{\Delta d} + \overline{\nabla_d\phi} + \cancel{\bar{\bar{B}}^{T}\bar{\lambda}} + \bar{\bar{B}}^{T}\bar{\bar{P}}\left(\bar{\bar{A}}\overline{\Delta p} + \bar{f}^{0} - \bar{\bar{B}}\bar{\bar{H}}_d^{-1}\overline{\nabla_d\phi}\right) - \cancel{\bar{\bar{B}}^{T}\bar{\lambda}} = \bar{0}$$

$$\bar{\bar{H}}_d\overline{\Delta d} + \bar{\bar{B}}^{T}\bar{\bar{P}}\left(\bar{\bar{A}}\overline{\Delta p} + \bar{f}^{0}\right) + \left(\bar{\bar{I}} - \bar{\bar{B}}^{T}\bar{\bar{P}}\bar{\bar{B}}\bar{\bar{H}}_d^{-1}\right)\overline{\nabla_d\phi} = \bar{0}$$

$$\overline{\Delta d} = -\bar{\bar{H}}_d^{-1}\left[\bar{\bar{B}}^{T}\bar{\bar{P}}\left(\bar{\bar{A}}\overline{\Delta p} + \bar{f}^{0}\right) + \left(\bar{\bar{I}} - \bar{\bar{B}}^{T}\bar{\bar{P}}\bar{\bar{B}}\bar{\bar{H}}_d^{-1}\right)\overline{\nabla_d\phi}\right]$$