

# Max-Subarray Sum Problem and Solution Algorithms

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November 17, 2019

**Abstract**—Maximum Subarray Sum is a well-known problem in the field of computer science. There are multiple number of solution algorithms with different complexities. In this paper, we demonstrated and compared 3 of these algorithms with quadratic, linear, and logarithmic complexities.

## I. THE PROBLEM

The Maximum Subarray Sum problem is the task of finding the contiguous subarray with largest sum in a given array of integers. Each number in the array could be positive, negative, or zero. For example: Given the array  $[-2, 1, -3, 4, -1, 2, 1, -5, 4]$  the solution would be  $[4, -1, 2, 1]$  with a sum of 6.

## II. SOLUTIONS

### A. Brute-Force Approach

This is the most intuitive solution to anyone. You basically traverse over the array and compare every possible combination of start and end index for the solution array.



The *start* index will be incremented by 1 everytime the *end* index reaches the end of the array. Then the *end* index will start from the element right next to the start element. At every iteration the sum between *start* and *end* indexes will be calculated. Hence, the maximum sum will be determined by computing every sum for the every possible sub-array. The complexity of this algorithm is  $\mathcal{O}(n^3)$ . With a little improvement we can convert this algorithm to be  $\mathcal{O}(n^2)$ . Instead of calculating the sum between two array indices at every iteration from scratch, we know that the current sum will be the (current element + previous sum). Consequently, eliminating the loop which is used for calculating the sum from the algorithm will reduce the time complexity.

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### Algorithm 1 Brute-Force

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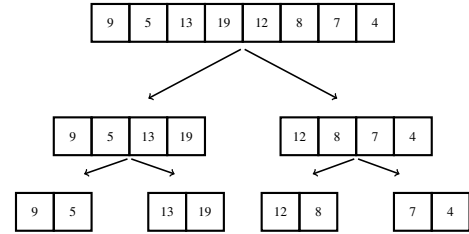
 $n \leftarrow \text{len}(\text{array})$ 
 $\text{max\_sum} \leftarrow 0$ 
for  $i \leftarrow 1$  to  $n$  do
  for  $j \leftarrow 1$  to  $n$  do
     $\text{sum} += \text{array}[j]$ 
    if  $\text{sum} > \text{max\_sum}$  then
       $\text{max\_sum} \leftarrow \text{sum}$ 
    end if
  end for
end for

```

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### B. Divide & Conquer Approach

Another solution is to divide the array into half recursively and computing the max subarray sum for each half and the sub array for crossing both halves. After calculating the summation for these 3 cases, we choose the largest one, thus we determine the maximum sub array.



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**Algorithm 2** Divide & Conquer
 

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function MAX_CROSSING_SUB_ARRAY(array, l, m, h)
  left_max_sum  $\leftarrow$   $-100000$ 
  sum_l  $\leftarrow$  0
  for i  $\leftarrow$  m downto l  $- 1$  do
    sum_l  $\leftarrow$  sum_l + array[i]
    if sum_l > left_max_sum then
      left_max_sum  $\leftarrow$  sum_l
    end if
  end for
  sum_r  $\leftarrow$  0
  right_max_sum  $\leftarrow$   $-100000$ 
  for j  $\leftarrow$  m + 1 to h + 1 do
    sum_r  $\leftarrow$  sum_r + array[j]
    if sum_r > right_max_sum then
      right_max_sum  $\leftarrow$  sum_r
    end if
  end for
  return left_max_sum + right_max_sum
end function

```

```

function MAX_SUB_ARRAY(array, l, h)
  m  $\leftarrow$   $((h + l) / 2)$ 
  if l = h then
    return array[l]
  end if
  return max(max_subarray(array, l, m),
    max_subarray(array, m + 1, h),
    max_crossing_subarray(array, l, m, h)
  )
end function

```

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