Existence and Uniqueness

Theorem 1 *Suppose that both $f(x,y)$ and its partial derivative $f_y(x,y)$ are continuous on some rectangle R in the xy-plane containing the point (x_0, y_0). Then for some open interval $I = (a,b)$ so that $a < x_0 < b$, the initial value problem*

$$\frac{dy}{dx} = f(x,y), \qquad y(x_0) = y_0$$

has one and only one solution on the interval I.

1. The theorem above does not guarantee the existence of solutions for the following problems. Why? Fix each problems so that the theorem guarantees a locally unique solution.

   (a) $y'(x) = \dfrac{1}{x-y}, \quad y(2) = 2$

   (b) $y'(x) = \sqrt{y}, \quad y(0) = -1$

   (c) $y'(x) = \ln x, \quad y(-1) = 3$

Differential Equation Solution Methods

2. Classify the following differential equations as being Separable, Exact, and/or Homogeneous.

Equation	Separable	Exact	Homogeneous
$x^3y' = -3yx^2$			
$xy' = \left[1 + \ln\left(\dfrac{y}{x}\right)\right] y$			
$y' + 2xy = \sin xy^3$			

3. Solve the following initial value problem.

$$xy' + 3y = 2x^5, \quad y(2) = 1$$

4. Find the general solution to the following differential equation.

$$xy' = \left[1 + \ln\left(\frac{y}{x}\right)\right] y$$

5. Solve the following initial value problem:

$$y' = y(10 - y), \quad y(0) = 13.$$