

## Heat Equation

1. Solve the following heat equation.

$$u_t = u_{xx}, \quad 0 < x < 10, \quad 0 < t$$

$$u(0, t) = 20, \quad u(10, t) = 40$$

$$u(x, 0) = 60, \quad 0 < x < 10$$

## Wave Equation

2. Solve the wave equation with Dirichlet boundary conditions

$$\begin{aligned}u_{tt} &= c^2 u_{xx}, & 0 < x < L, & \quad t > 0 \\u(0, t) &= u(L, t) = 0, & t > 0 \\u(x, 0) &= L(1 - x), & u_t(x, 0) = 0, & \quad 0 < x < L\end{aligned}$$

- (a) We can either look for all solutions of the form  $u(x, t) = X(x)T(t)$ , or recognize the boundary conditions as Dirichlet boundary conditions, and look for solutions of the form

$$u(x, t) = \sum_n c_n(t) \phi_n(x).$$

What are the functions  $\phi_n(x)$ ? What eigenvalue problem do they solve?

- (b) What differential equation must  $c_n(t)$  solve?

- (c) Find  $c_n(t)$ .

- (d) How do we match up the initial conditions  $u(x, 0) = L(1 - x)$  and  $u_t(x, 0) = 0$ ?

1. Solve the following wave equation.

$$u_{tt} = 4u_{xx}, \quad 0 < x < 10, \quad 0 < t$$

$$u_x(0, t) = 0, \quad u_x(10, t) = 0$$

$$u(x, 0) = 40 - 2x, \quad 0 < x < 10$$

$$u_t(x, 0) = 2x - 40, \quad 0 < x < 10$$