Chapter 12 Review

1. Given the points $A(1, 2, 3)$ and $B(5, 4, -2)$.

   (a) Find both the parametric and symmetric equations of the straight line connecting A and B.

   (b) Find the plane perpendicular to this line that goes through the point $(1, 2, 4)$.

   (c) Find a plane that this line does not intersect.

2. Give examples of the following, or explain why no such example exists.

   (a) An equation for a cylinder so that the point $(1, 2, 3)$ is on its surface.

   (b) A paraboloid that opens downwards in the z direction and intersects the xy-plane in the ellipse $4x^2 + 9y^2 = 36$.

   (c) Two parallel lines L_1 and L_2 in the parallel planes $x + 2y + 2z = 3$ and $x + 2y + 2z = 6$ so that the distance between L_1 and L_2 is 3.

   (d) Two skew lines L_1 and L_2 that sit in parallel planes $2x + 2y + z = 5$ and $2x + 2y + z = -4$ so that the minimum distance between L_1 and L_2 is 9.

3. Given the parameterization $\mathbf{r}(t) = \langle \cos t, \sin t, 3t \rangle$.

   (a) Find the unit tangent vector at time t.

   (b) Find an equation for the line tangent to the curve at $t = \pi$.

   (c) Calculate the curvature κ of the curve at $t = \pi$.

   (d) Where does this line intersect the plane $z = 0$.

4. For the following parameterization $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$

   (a) Find the velocity and acceleration vectors at time $t = 1$, $\mathbf{v}(1) = \mathbf{r}'(1)$ and $\mathbf{a}(1) = \mathbf{r}''(1)$.

   (b) Find the unit tangent vector $\mathbf{T}$ and the unit normal vector $\mathbf{N}$ at time $t = 1$. [Hint: you can find the normal direction by taking $\mathbf{v} \times \mathbf{a} \times \mathbf{v}$.]

   (c) Find the curvature κ at time $t = 1$.

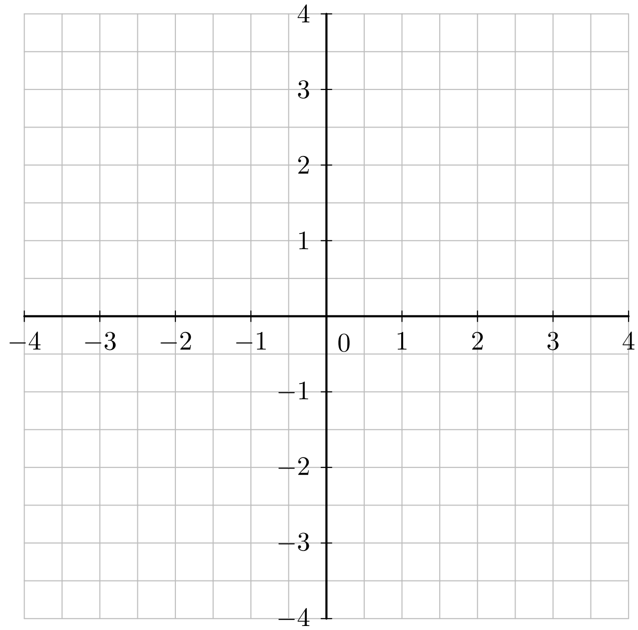
The Gradient

5. Given each of the following equations,

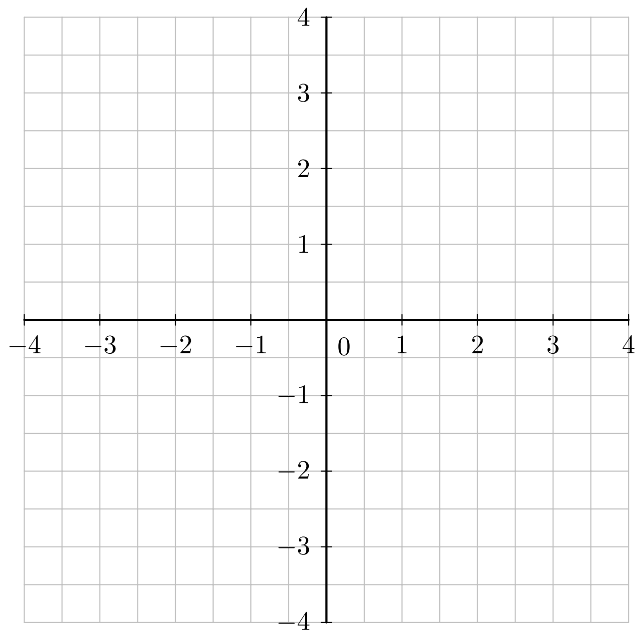
   (a) On the axes on the right, draw level curves of the function $f(x, y)$.

   (b) Find the gradient of f, $\langle f_x, f_y \rangle$, and draw some gradient vectors on the graph of the level curves. What is true about the gradient and level curves?

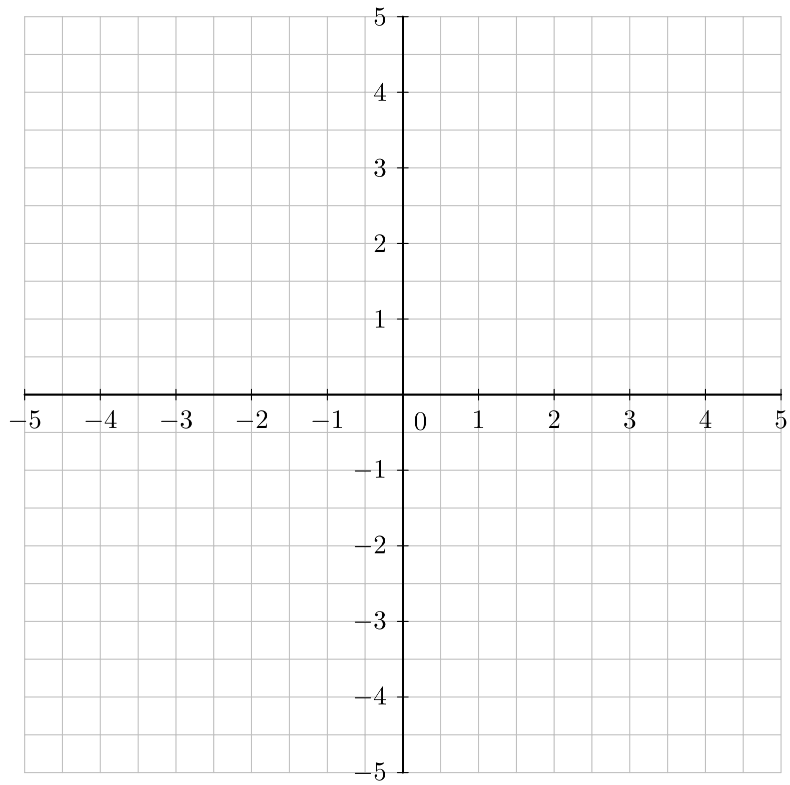
   - $f(x, y) = x + y + 3$



   - $f(x, y) = x^2 + y^2$



(c) Show on the graph below that the points where the level curves of $f(x, y) = x + y + 3$ are tangent to the curve $x^2 + y^2 = 8$ are $(2, 2)$ and $(-2, -2)$.



(d) Show that the gradient of $f(x, y) = x+y+3$ and the gradient of $g(x, y) = x^2+y^2$ are parallel (or antiparallel) at (2,2) and (-2,-2).

(e) Are these two points the maximum and minimum value of $f(x, y) = x + y + 3$ under the constraint $g(x, y) = x^2 + y^2 = 8$?