

Review

1. Give examples of the following or explain why they don't exist.
 - (a) Values of a and b so that the differential equation $y'' + ay' + by = 12x^2e^x$ has particular solution $y_P = x^4e^x$.
 - (b) A first-order differential equation with solution $x^2y^2 + e^{xy} = k$.
 - (c) A stepsize for Euler's method that overestimates the solution of the initial value problem $y' = 2y$, $y(0) = 3$ at the point $x = 5$.

Power Series

2. Show Euler's formula,

$$e^{i\theta} = \cos \theta + i \sin \theta,$$

by using the Taylor series for e^x , $\cos x$, and $\sin x$.

3. The power series

$$1 - x + x^2 - x^3 + \cdots = \frac{1}{1 - (-x)}$$

can be thought of as a geometric series with multiplier $-x$.

(a) For what values of the multiplier x does the series converge?

(b) The derivative of $\ln(1 + x)$ is $\frac{1}{1 + x}$. Use the series above to derive a power series for $\ln(1 + x)$ by integrating the series term by term.

4. Determine a power series solution to the following linear initial value problem.

$$y' = (x - 1)^2 y, \quad y(1) = -1$$