Laplace Transforms

1. To solve the problem $y'' + 3y' + 2y = e^t, y(0) = 1, y'(0) = -1$.

   (a) First find the Laplace transform of the entire equation.

   (b) Then use the initial conditions and solve for $\mathcal{L}(y)$.

   (c) Then find the partial fractions of the right side of your equation and use the Laplace table to find y.

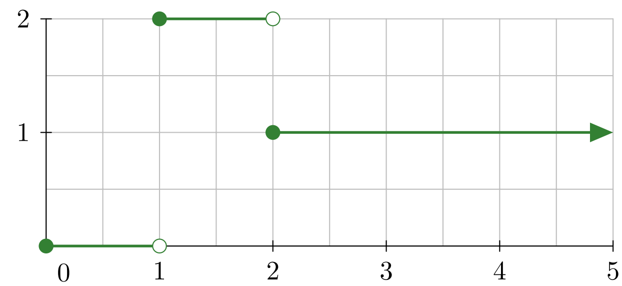
2. Solve the following differential equations using Laplace transforms.

   (a) $y'' + 3y' + 2y = 3\sin 2t, y(0) = 1, y'(0) = 0.$

   (b) $y'' + 3y' + 2y = 3u(t-2), y(0) = 0, y'(0) = 2.$

3. Solve the following initial value problem

$$y'' + 2y' + 5y = \begin{cases} 0 & 0 \le t < 1 \\ 2 & 1 \le t < 2 \\ 1 & 2 \le t \end{cases}, \quad y(0) = 1, y'(0) = -1.$$



(a) First show that the right hand side, which is shown on the graph above, can be written as $2u(t-1) - u(t-2)$.

(b) Then use Laplace transforms to solve the initial value problem.

Laplace Transform Table

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}, \quad s > 0$
e^{at}	$\frac{1}{s-a}, \quad s > a$
$t^n, n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
$\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
$\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
$\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \|a\|$
$\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \|a\|$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
$t^n e^{at}, n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$e^{ct}f(t)$	$F(s-c)$
$f(ct)$	$\frac{1}{s}F(\frac{s}{c}), \quad c > 0$
$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
$\delta(t-c)$	e^{-cs}
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^(n-1)(0)$
$(-t)^n f(t)$	$F^{(n)}(s)$