Limits

1. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^3}{x^5+y^5}$.

2. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^3}{x^2+y^2}$.

Vector Functions

3. For each of the following functions, first find the dimensions of the domain and the range. [For example, $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle : \mathbb{R} \to \mathbb{R}^3$.] Then find the partial derivatives of each component with respect to t, u, and/or v.

   (a) $\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle = \langle u, v, 9 - u^2 - v^2 \rangle$

   (b) $\mathbf{r}(t, u, v) = \langle x(t, u, v), y(t, u, v) \rangle = \langle \sin(t - 2v), \sqrt{u + 3v} \rangle$

4. Given the plane $x + 2y + 3z = 0$, find a parameterization $\mathbf{r}(x, y) : \mathbb{R}^2 \to \mathbb{R}^3$ for this plane.

The Chain Rule

5. *Related Rates:* Gasoline is pouring into a tank in the shape of a cone of radius 3 feet and height 4 feet. When the depth of the gasoline is 2 feet, the depth is increasing at 0.2 ft/sec. How fast is the volume of the gasoline changing at that instant?

6. Let $z = f(x, y)$ and let $x = r\cos\theta$ and $y = r\sin\theta$.

   (a) Find $\dfrac{\partial z}{\partial r}$.

   (b) Find $\dfrac{\partial^2 z}{\partial r^2}$.

7. Find a function $z = f(x, y)$ with $x = r\cos\theta$ and $y = r\sin\theta$ so that

$$\frac{\partial^2 z}{\partial r \partial \theta} = -2xr\sin\theta\cos\theta - x^2\sin\theta + 2yr\cos\theta\sin\theta + y^2\cos\theta.$$