Is your robot motion jerky?

Generalizing Trajectory Retiming to Quadratic Objective Functions

Gerry Chen, Frank Dellaert, and Seth Hutchinson
Georgia Institute of Technology

Introduction

<u>Trajectory Retiming</u>:
compute a feasible speed profile to execute a path

<u>Example Applications</u>:
Decoupled approach to Motion Planning

Path Planning	Trajectory Retiming
Compute a collision-free, kinematically feasible path	*Compute dynamically feasible timestamps*

Predefined path (e.g. painting, machining)

Problem Formulation

Path: $\boldsymbol{q}(s): [0, 1] \to \mathbb{R}^n$
Parameterization: $s(t): [0, T] \to [0, 1]$

$$\begin{aligned} s^*(t) = \ & \underset{s(t)}{\arg\min} \quad C(s) \longleftarrow \text{objectives} \\ & \text{subject to} \\ & \boldsymbol{A}(s)\ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T \boldsymbol{B}(s)\dot{\boldsymbol{q}} + \boldsymbol{f}(s) \in \mathscr{C}(s), \\ & \boldsymbol{A}^v(s)\dot{\boldsymbol{q}} + \boldsymbol{f}^v(s) \in \mathscr{C}^v(s) \end{aligned}$$

Dynamics & state/control limits

Related Works

<u>Time-Optimal Path Parameterization (TOPP)</u>
Minimize trajectory duration ($C(s) \coloneqq T$)

<u>The Problem</u>
Bang-bang solution saturates control limits
- No margin for closed-loop controller
- Cannot handle secondary objectives

Approach

Instead of minimizing time, let's minimize quadratic objectives!

$$\begin{aligned} s^*(t) = \ & \underset{s(t)}{\arg\min} \quad \|SpeedObjective\|^2 + \|ControlEffort\|^2 + \dots \\ & \text{subject to} \quad Dynamics \\ & \qquad\qquad\quad Control/State\ Limits \end{aligned}$$

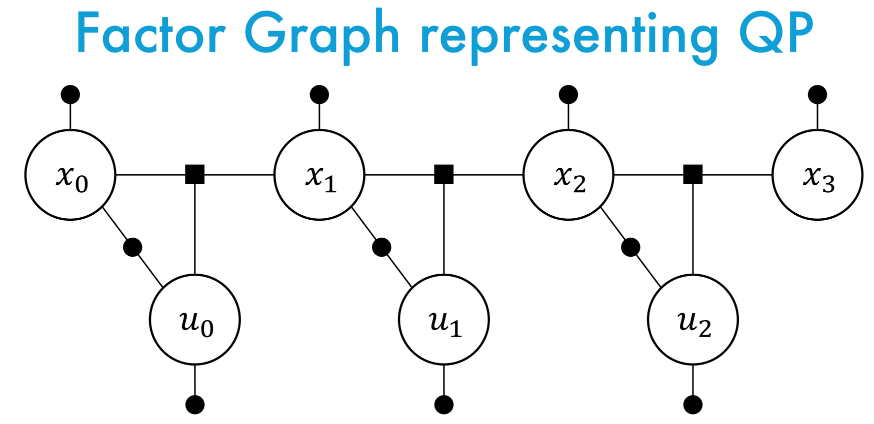
With a general quadratic objective function, we can balance multiple objectives such as max speed (min time), match target speed, max control margin, min control effort, etc.

We call this "QOPP": <u>Q</u>uadratic <u>O</u>bjective <u>P</u>ath <u>P</u>arameterization

Sparse QP

$$\begin{aligned} \boldsymbol{x}^*, \boldsymbol{u}^* = \ & \underset{u_0, \dots, x_N}{\arg\min} \quad \sum_{k=0}^{N} Q_k x_k^2 + R_k x_k u_k + N_k u_k^2 \\ & \text{subject to} \\ & \boldsymbol{a}_k u_k + \boldsymbol{b}_k x_k + \boldsymbol{c}_k \in \mathscr{C}_k, \quad k = 0, \dots, N, \\ & x_{k+1} - x_k - 2u_k \Delta_s = 0, \quad k = 0, \dots, N-1, \\ & x_k > 0, \quad k = 0, \dots, N \end{aligned}$$

Factor Graph representing QP



Animated elimination procedure

<u>How to solve QOPP</u>

1. **Transcribe into sparse QP** using standard TOPP parameterization & discretization
2. **Solve sparse QP** using factor graph variable elimination[2]

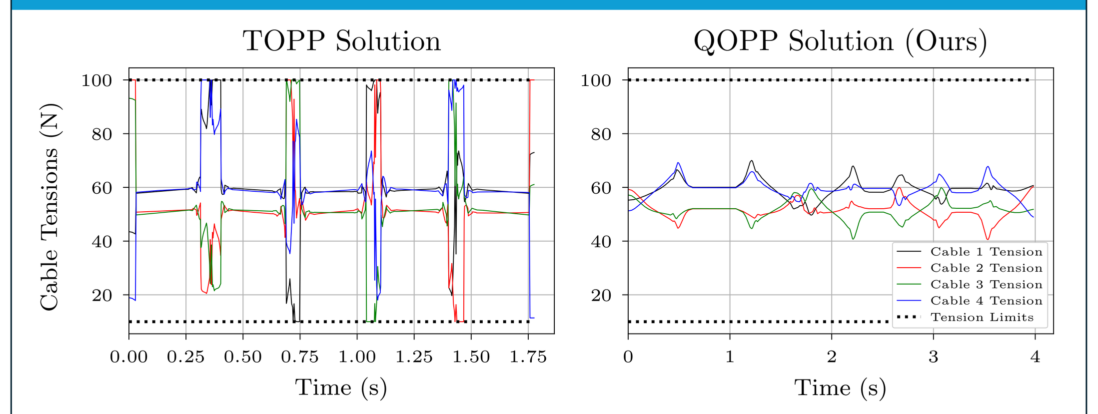
The factor graph depicts the sparsity pattern of the problem.

Performing elimination: During elimination, we only ever need to do 2 types of operations:
- Eliminate u_k: Re-write equality constraint & substitute
- Eliminate x_k: Solve 2-var parametric piecewise QP

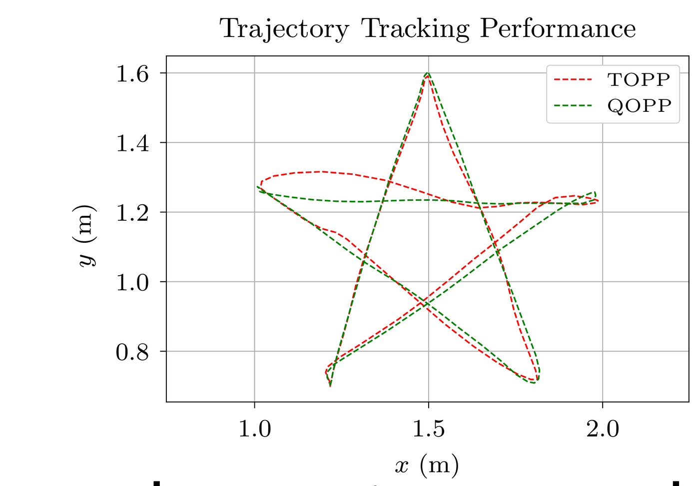
Thanks to the special structure we leverage with factor graphs,

we can solve QOPP in $\mathcal{O}(n)$ time!

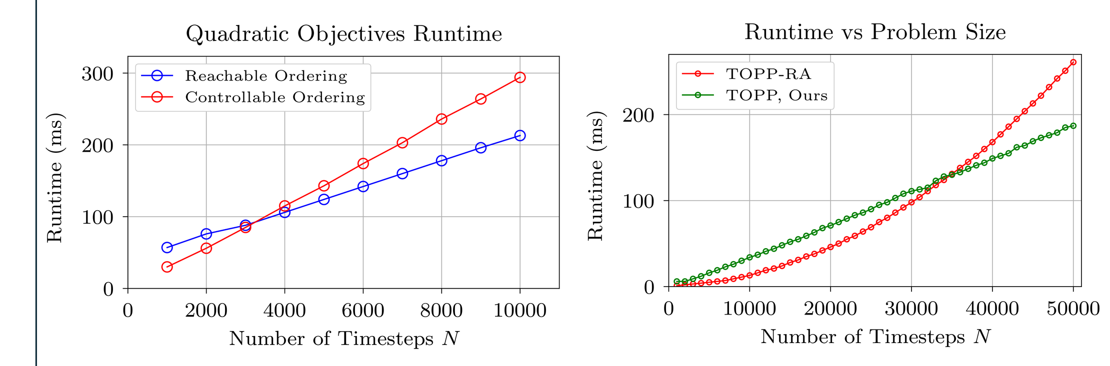
Results



TOPP constantly hits control limits. Meanwhile, QOPP allows us to trade-off speed for control margin.



As a result, executing on a <u>real robot</u>, TOPP has poor tracking performance (QOPP is still good).



QOPP is as-fast or faster than TOPP (C++)
Runtime is $\mathcal{O}(n)$ w.r.t. trajectory length

Conclusions

By balancing multiple objectives & constraints, QOPP achieves **better performance** in practice and can be solved as-fast or **faster than TOPP**.

Select References

[TOPP-RA]: H. Pham and Q.-C. Pham, "A New Approach to Time-Optimal Path Parameterization Based on Reachability Analysis," TRO, 2018.
[2]: S. Yang, G. Chen, Y. Zhang, H. Choset, and F. Dellaert, "Equality Constrained Linear Optimal Control With Factor Graphs," ICRA, 2021.