The Chain Rule

1. *Related Rates:* Gasoline is pouring into a tank in the shape of a cone (inverted cone? point at the bottom) of radius 3 feet and height 4 feet. When the depth of the gasoline is 2 feet, the depth is increasing at 0.2 ft/sec. How fast is the volume of the gasoline changing at that instant?

   (a) First write out the equation of the volume of a cone. Which variables are changing with time?

   (b) Differentiate implicitly with respect to time.

   (c) Solve for the rate of change of the volume.

2. If $z = x^2y^2$ and $x = u - v$ and $y = u + v$.

   (a) Find $\frac{\partial x}{\partial u}$ and $\frac{\partial y}{\partial u}$.

   (b) Write down $\frac{\partial z}{\partial u}$ in terms of $\frac{\partial x}{\partial u}$ and $\frac{\partial y}{\partial u}$.

   (c) Solve for $\frac{\partial z}{\partial u}$ in terms of u and v.

   (d) Find $\dfrac{\partial^2 z}{\partial u \partial v}$.

3. Let $z = f(x, y)$ and let $x = r\cos\theta$ and $y = r\sin\theta$.

   (a) Find $\dfrac{\partial z}{\partial r}$.

   (b) Find $\dfrac{\partial^2 z}{\partial r^2}$.

4. Find a function $z = f(x, y)$ with $x = r\cos\theta$ and $y = r\sin\theta$ so that

$$\frac{\partial^2 z}{\partial r \partial \theta} = -2xr\sin\theta\cos\theta - x^2\sin\theta + 2yr\cos\theta\sin\theta + y^2\cos\theta.$$

5. Given the function

$$g(t, x, y) = y \cos(xt),$$

with $y(x) = e^x$ and $x(t) = \sin t$.

(a) Find $\dfrac{\partial g}{\partial y}$.

(b) Find $\dfrac{\partial g}{\partial x}$.

(c) Find $\dfrac{\partial g}{\partial t}$.