RADIOMETRY – CHAPTER 2

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RADIOMETRY DEFINED

the quantitative measurement of the energy content of optical radiation fields

- This includes how radiation is transmitted through the optical measurement system to the detector
- Traditionally, radiometry has focused on measurement of *incoherent* radiation fields

COHERENCE

- A monochromatic plane wave field $E(\vec{r}, t) = E_0 \mathrm{e}^{(\vec{k}\cdot\vec{r} - \omega t)}$ is *coherent*
  - The phase $\phi = \vec{k} \cdot \vec{r} - \omega t$ is fixed for all time and space
- A traditional laser is highly coherent
  - The autocorrelation $\langle E(x, t) E(x + \xi, t + \tau) \rangle$ is appreciable for "large" spatial coherence length ξ and temporal coherence time τ

THERMAL SOURCES ARE INCOHERENT

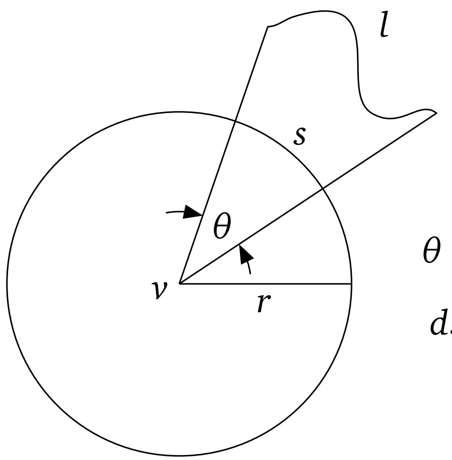
- Thermal radiation sources (sun, incandescent bulbs, blackbody, etc.) produce *incoherent* radiation
  - Myriad independent oscillators stochastically producing photons
  - Phase difference between any two plane waves is a random variable
  - Autocorrelation is essentially zero for any ξ and τ

ASSUMPTIONS

1. Radiation sources are incoherent
2. Radiation propagates via geometric optics
3. Energy of radiation fields is conserved when propagating through transparent media (vacuum)

We will relax #3 when we consider radiation propagation in participating media

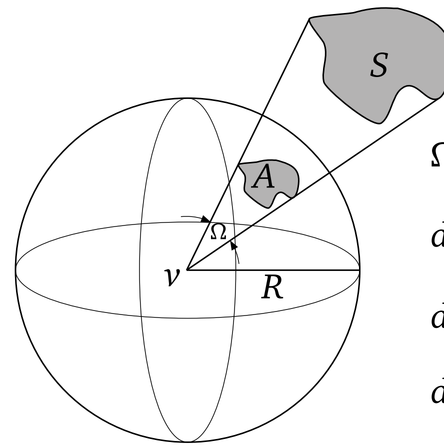
ANGLE AND SOLID ANGLE



$$\theta = \frac{s}{r} \quad [\theta] = \text{rad}$$

$$ds = r\,d\theta$$

Linear Angle



$$\Omega = \frac{A}{R^2} \quad [\Omega] = \text{sr}$$

$$dA = R^2 d\Omega$$

$$d\Omega = \frac{|d\vec{A} \cdot \hat{\Omega}|}{R^2}$$

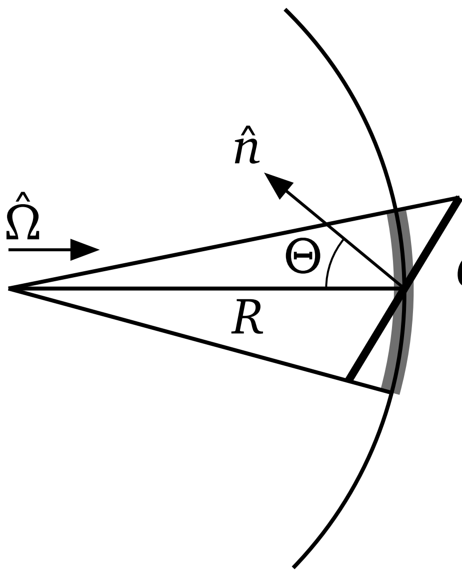
$$d\Omega = \sin(\theta)\,d\theta\,d\phi$$

Solid Angle

- Linear angle subtended by 2D object l at point v is $\theta = s/r$
- Solid angle subtended by 3D object S at point v is $\Omega = A/R^2$

(PROJECTED) SOLID ANGLE

Take care to account for the projected area of the source onto the sphere

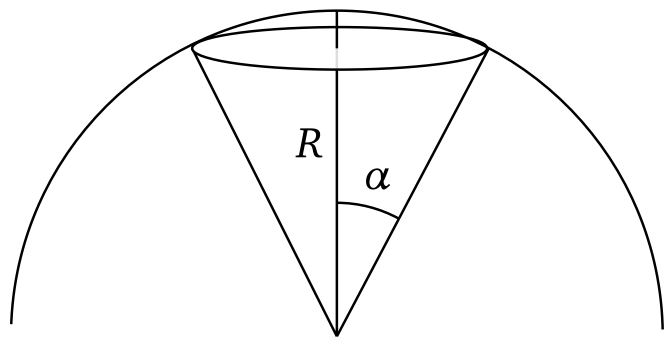


$$d\vec{A} = \hat{n}\, dA$$

$$\boxed{d\Omega = \frac{|d\vec{A} \cdot \hat{\Omega}|}{R^2} = \frac{dA \cos(\Theta)}{R^2}}$$

EXAMPLE

What is the solid angle subtended by a right circular cone of half-angle α?



$$A = \int_0^{2\pi} \int_0^{\alpha} R^2 \sin\theta \, d\theta \, d\phi$$

$$A = 2\pi R^2 (1 - \cos\alpha) = 4\pi R^2 \sin(\alpha/2)$$

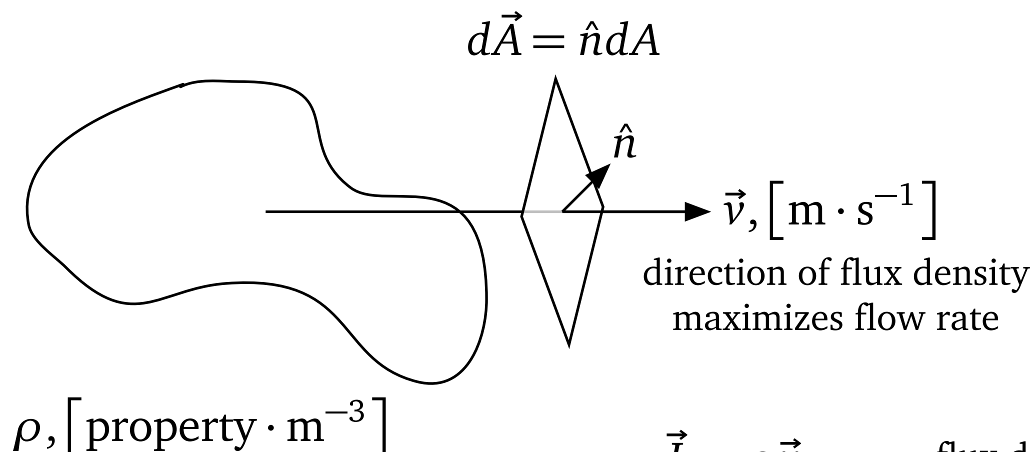
$$\Omega = 2\pi(1 - \cos\alpha)$$

NOMENCLATURE

symbol	term	units	notes
Q	radiant energy	$[\mathrm{J}]$	
$\Phi = \frac{dQ}{dt}$	radiant flux	$[\mathrm{W}]$	
$M = \frac{d\Phi}{dA}$	exitance	$\left[\mathrm{W}/\mathrm{m}^2\right]$	leaving surface
$E = \frac{d\Phi}{dA}$	irradi-ance	$\left[\mathrm{W}/\mathrm{m}^2\right]$	arriving at surface
$I = \frac{d\Phi}{d\Omega}$	intensity	$[\mathrm{W}/\mathrm{sr}]$	
$L = \frac{d^2\Phi}{dA_p d\Omega}$	radiance	$\left[\mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{sr})\right]$	$dA_p d\Omega = \mathrm{abs}(d\vec{A} \cdot d\vec{\Omega})$

FLUX DENSITY

- Useful concept in many transport phenomena
- Flow rate of a property density (mass, heat, charge, etc.) per unit area
- $\vec{J} = \rho\vec{v}$, $[(\text{property units})\cdot\text{m}^{-2}\cdot\text{s}^{-1}]$
- $\vec{J}(\vec{r})$ quantifies magnitude and direction of *maximal* flow rate at $\vec{r}$



$\vec{J} = \rho\vec{v}$ flux density

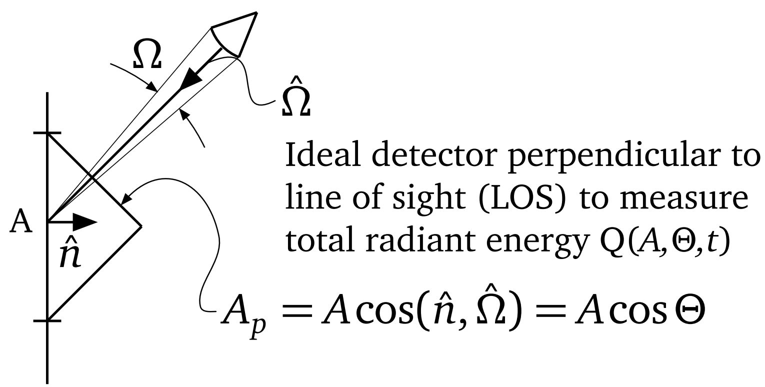
$d\Phi = \vec{J}\cdot d\vec{A}$ flux through dA

FLUX DENSITY AND FLUX

- Flux is the total (property) flow across a specified area per unit time
  - Flux density: $d\Phi = \vec{J} \cdot d\vec{A}$
  - Flux: $\Phi = \int_A \vec{J} \cdot d\vec{A}$
- Radiant flux for plane wave (here $\vec{S} = \vec{J}$)
  - $E(z,t) = E_0 \mathrm{e}^{(kz-\omega t)}\,\hat{e}_z$
  - $\langle u \rangle = \frac{1}{2}\epsilon|E_0|^2$
  - $\vec{S} = \vec{E} \times \vec{H} = \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}|E_0|^2\hat{e}_z,\ \left[\mathrm{W/m^2}\right]$
  - Factor out ϵ yields $\vec{S} = \left(\frac{1}{2}\epsilon|E_0|^2\right)\frac{1}{\sqrt{\epsilon\mu}}\hat{e}_z = u\vec{v}$ where $\vec{v} = \frac{c_0}{n}\hat{e}_z$

RADIANCE

- Non-negative scalar distribution function describing the radiation field
- Strictly, radiance (L) is defined in terms of its integral properties



- Radiance is the function who's integral over area and solid angle and time gives the total radiant energy

$$Q(\vec{r}) = \int_{t_1}^{t_2}\int_A\int_\Omega L(\vec{r},\hat{\Omega},t)d\hat{\Omega}\cdot d\vec{A}dt$$

RADIANCE (2)

Radiance of a field can be defined as the radiant energy $Q(A, \hat{\Omega}, \Delta t)$, as measured by an ideal detector of area A collimated so that it only receives radiation within a solid angle Ω, normalized by these extrinsic factors

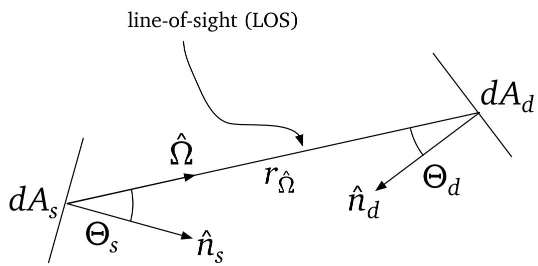
$$L = \lim_{A,\Omega,t \to 0} \frac{Q(A, \Omega, t)}{A \cos \Theta \, \Omega \, \Delta t}$$

Why radiance?

- Characterizes the (scalar) radiation field, independent of source size, detector size, viewing distance and orientation, integration time
- It is an invariant quantity in a *non-participating* medium

RADIANCE (3)

Differential flux - radiance relationship for *source* ($\boldsymbol{s}$) and *detector* ($\boldsymbol{d}$)



$$\Theta_s = \angle(\hat{n}_s, \hat{\Omega})$$

$$d\vec{\Omega}_d = \frac{\hat{\Omega} \cdot d\vec{A}_d}{r^2_{\hat{\Omega}}} \hat{\Omega}$$

$$\Theta_d = \angle(\hat{n}_d, \hat{\Omega})$$

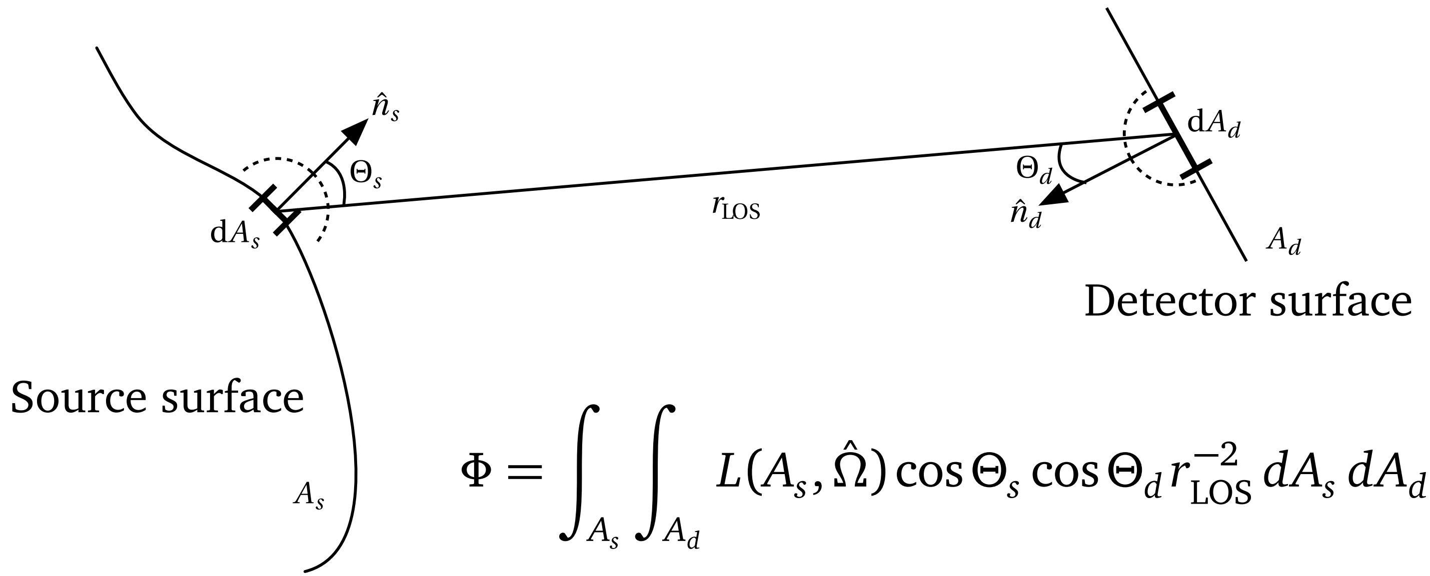
$$d\hat{\Omega}^{\perp}_d = \frac{dA_d |\cos\Theta_d|}{r^2_{\hat{\Omega}}}$$

$$\boxed{\begin{aligned} d^2\Phi &= L(A_s, \hat{\Omega}) d\vec{A}_s \cdot d\vec{\Omega}_d \\ &= L(A_s, \hat{\Omega}) \cos\Theta_s dA_s d\Omega^{\perp}_d \\ &= L(A_s, \hat{\Omega}) \cos\Theta_s \, dA_s \, r^{-2}_{\hat{\Omega}} \cos\Theta_d \, dA_d \\ &= L(A_s, \hat{\Omega}) d\Omega^{\perp}_s \cos\Theta_d dA_d \end{aligned}}$$

- Here, radiance is a property of the source and line-of-sight direction
- Technically, radiance is a property of the EM field

RADIANCE (4)

Total flux - radiance relationship for *source* (s) and *detector* (d)



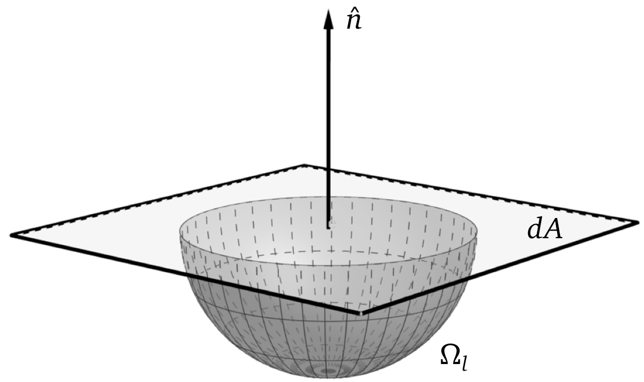
$$\Phi = \int_{A_s} \int_{A_d} L(A_s, \hat{\Omega}) \cos\Theta_s \cos\Theta_d \, r_{\mathrm{LOS}}^{-2} \, dA_s \, dA_d$$

- Later, we'll learn how to relate source radiance to its material properties ($\tilde{N}(\nu)$, T) and its radiation environment (sun, sky, etc.)

IRRADIANCE / EXITANCE

- Radiant power per unit area
- Units: $\left[\mathrm{W}/\mathrm{m}^2\right]$
- Irradiance (E): *arriving*
- Exitance (M): *leaving*

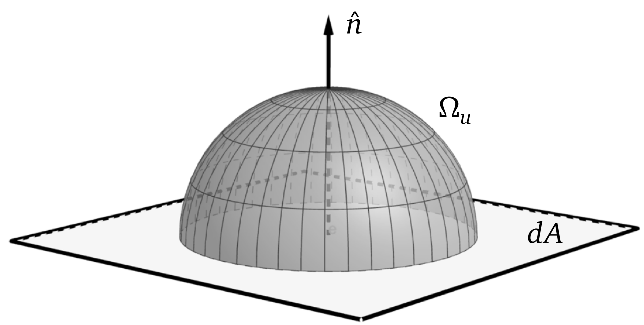
For an elemental area with specified unit normal:



Irradiance

$\hat{n} \cdot \hat{\Omega} < 0, \forall \hat{\Omega}$

$$E = \frac{d\Phi}{dA} = \int_{\Omega_l} L(\hat{\Omega}) \cos(\hat{n}, \hat{\Omega})\, d\Omega$$



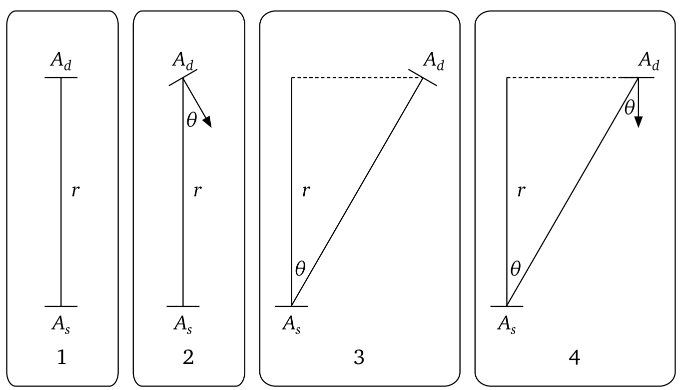
Exitance

$\hat{n} \cdot \hat{\Omega} > 0, \forall \hat{\Omega}$

$$M = \frac{d\Phi}{dA} = \int_{\Omega_u} L(\hat{\Omega}) \cos(\hat{n}, \hat{\Omega})\, d\Omega$$

"COS TO THE FOURTH"

Problem: consider a small, uniform Lambertian source A_s and a detector A_d separated by r where $r^2 \gg A_s, A_d$. What is the radiant flux at the detector in each case?:



$$\Phi = \int_{A_s}\int_{\Omega_d} L(A_s, \hat{\Omega}) \cos\Theta_s dA_s d\Omega_d^{\perp} \simeq L \cos\Theta_s A_s \Omega_d^{\perp}$$

1 $\quad \Theta_s = 0, \Omega_d^{\perp} \simeq \frac{A_d}{r^2} \Rightarrow \Phi_1 = LA_s\frac{A_d}{r^2}$

2 $\quad \Theta_s = 0, \Omega_d^{\perp} \simeq \frac{A_d \cos\theta}{r^2} \Rightarrow \Phi_2 = LA_s\frac{A_d \cos\theta}{r^2} = \Phi_1 \cos\theta$

3 $\quad \Theta_s = \theta, \Omega_d^{\perp} \simeq \frac{A_d}{(r/\cos\theta)^2} \Rightarrow \Phi_3 = LA_s \cos\theta \frac{A_d}{(r/\cos\theta)^2} = \Phi_1 \cos^3\theta$

4 $\quad \Theta_s = \theta, \Omega_d^{\perp} \simeq \frac{A_d \cos\theta}{(r/\cos\theta)^2} \Rightarrow \Phi_3 = LA_s \cos\theta \frac{A_d \cos\theta}{(r\cos\theta)^2} = \Phi_1 \cos^4\theta$

INTENSITY

- **Caution:** *Intensity* is an abused term – when you encounter it in the literature, be sure you know what the units are!
- Radiant power per unit solid angle; units: $[\mathbf{W/sr}]$

$$I = \frac{d\Phi}{d\Omega} = \int_{A_s} L(A_s, \hat{\Omega}) \cos\Theta_s \, dA_s$$

- Note that $\hat{\Omega}$ is *constant* in the integral
- Intensity of common source from two distinct vantage points can differ
- Useful for point sources and *distant* extended objects subtending a *small* solid angle at the detector

INTENSITY AND IRRADIANCE

- Consider the irradiance (*at a point*) of a *point source*
  - There is only *one* direction $\hat{\Omega}$ since we're connecting a source *point* with a detector *point*
  - $d^2\Phi = L(A_s, \hat{\Omega}) \cos\Theta_s dA_s \cos\Theta_d dA_d r_{\hat{\Omega}}^{-2}$
  - $d\Phi = \left(\int_{A_s} L(A_s, \hat{\Omega}) \cos\Theta_s dA_s\right) \cos\Theta_d dA_d r_{\hat{\Omega}}^{-2} = I(\hat{\Omega}) \cos\Theta_d dA_d r_{\hat{\Omega}}^{-2}$
  - Thus, $E = \frac{d\Phi}{dA_d} = I(\hat{\Omega}) \cos\Theta_d / r_{\hat{\Omega}}^2$
- What about irradiance from a resolvable, *extended* source?
  - An intensity can be computed for any source, but it is not useful here
  - Now we're connecting *multiple source points* with a detector point, *i.e.* must account for $\hat{\Omega} = \hat{\Omega}(A_s, A_d)$ and $r_{\hat{\Omega}} = r_{\hat{\Omega}}(A_s, A_d)$ when summing over all elemental source contributions to the detector point.

SPECTRAL QUANTITIES

- The electromagnetic field can contain different amounts of energy at different wavelengths / frequencies
- Our previous radiometric terms implicitly *integrated* over all frequencies
- Now have spectral dependence to our distribution functions, *e.g.*
  - $L_\nu = \frac{\Phi(\Delta\Omega, \Delta A, \Delta\nu)}{\Delta\Omega \Delta A \cos\Theta \Delta\nu}, \; L_\lambda = \frac{\Phi(\Delta\Omega, \Delta A, \Delta\lambda)}{\Delta\Omega \Delta A \cos\Theta \Delta\lambda}$
- The spectral distributions are not equivalent functions!
  - $L_\nu d\nu = L_\lambda d\lambda$ and $L = \int_{\Delta\nu} L_\nu d\nu = \int_{\Delta\lambda} L_\lambda d\lambda$
  - $L_\nu \neq L_\lambda$
  - Must account for the differential relationship between ν and λ:
    $\lambda\nu = c \implies d\nu = -c\lambda^2 d\lambda$

SPECTRAL QUANTITIES (2)

- SI units
  - $L_\nu \ \left[\mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{sr} \cdot \mathrm{Hz})\right]$
  - $L_\lambda \ \left[\mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{sr} \cdot \mathrm{m})\right] \left(= \left[\mathrm{W}/(\mathrm{m}^3 \cdot \mathrm{sr})\right] \text{ NO!}\right)$
- Wavenumber: $\tilde{\nu} = \frac{1}{\lambda} \left[\mathrm{cm}^{-1}\right]$
  - $L_{\tilde{\nu}} \ \left[\mathrm{W}/(\mathrm{cm}^2 \cdot \mathrm{sr} \cdot \mathrm{cm}^{-1})\right] (= [\mathrm{W}/(\mathrm{cm} \cdot \mathrm{sr})] \text{ NO!})$
- Flick: $L_\lambda \ \left[\mathrm{W}/(\mathrm{cm}^2 \cdot \mathrm{sr} \cdot \mu\mathrm{m})\right]$