??

Énergie

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MP2I

$$
\begin{aligned}
\text{PFD} : \left(m\frac{d\vec{v}}{dt} = \vec{F} \right) \cdot \vec{v} \\
m\frac{d}{dt}\left[\frac{1}{2}\|\vec{v}\|^2\right] = \mathscr{P}(\vec{F}) \\
\frac{d\mathscr{E}_c}{dt} = \mathscr{P}(\vec{F})
\end{aligned}
$$

force $\vec{F}$ motrice : $\mathscr{P}(\vec{F}) > 0 \iff \dfrac{d\mathscr{E}_c}{dt} > 0$

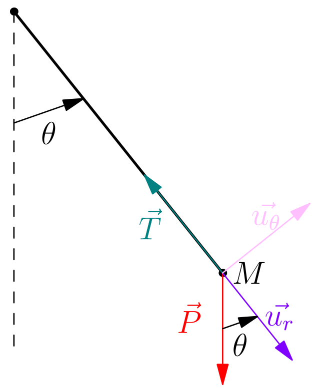
$$
\begin{aligned}
\int_{t_1}^{t_2} \frac{d\mathscr{E}_c}{dt}\, dt &= \int_{t_1}^{t_2} \mathscr{P}(\vec{F})\, dt \\
&= \int_{M_1 \to M_2} \delta W(\vec{F}) \\
&= W_{M_1 M_2}(\vec{F})
\end{aligned}
$$

Or,

$$
\int_{t_1}^{t_2} \frac{d\mathscr{E}_c}{dt}\, dt = [\mathscr{E}_c]_{t_1}^{t_2} = \mathscr{E}_c(t_2) - \mathscr{E}_c(t_1) = \Delta\mathscr{E}_c
$$

Donc,

$$
\Delta\mathscr{E}_c = W_{M_1 M_2}(\vec{F})
$$



$$
\begin{aligned}
\mathscr{P}(m\vec{g}) &= (mg\cos\theta\vec{u_r} - mg\sin\theta\vec{u_\theta}) \cdot \ell\dot{\theta}\vec{u_\theta} \\
&= -mg\ell\dot{\theta}\sin\theta
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{E}_c = \frac{1}{2}mv^2 = \frac{1}{2}m\ell^2\dot{\theta}^2 \\
\frac{d\mathscr{E}_c}{dt} = \mathscr{P}(m\vec{g}) + \underbrace{\mathscr{P}(\vec{T})}_{=0}
\end{aligned}
$$

Donc

$$
\begin{aligned}
\frac{1}{2}m\ell^2 \times 2\dot{\theta}\ddot{\theta} = -mg\ell\dot{\theta}\sin(\theta) \\
\dot{\theta} \neq 0 : \qquad \boxed{\ddot{\theta} + \frac{g}{\ell}\sin\theta = 0}
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{E}_c(M_2) - \mathscr{E}_c(M_1) &= W_{M_1M_2}(\vec{F}) \\
\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 &= W_{M_1M_2}(m\vec{g}) + \underbrace{W_{M_1M_2}(\vec{T})}_{=0} \\
W_{M_1M_2}(m\vec{g}) &= mg(z_1 - z_2) \\
v_2^2 &= v_1^2 + 2g(z_1 - z_2)
\end{aligned}
$$

$$
\frac{d\mathscr{E}_p}{dx}dx = -f dx \implies f = -\frac{d\mathscr{E}_p}{dx}
$$

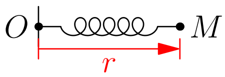
La force dérive l'énergie potentielle.

$$
\int_{M_0}^{M} d\mathscr{E}_p = -\int_{M_0} f \ dx
$$

On a $\begin{cases} M_0 : x_0 \\ M : x \end{cases}$

$$
\mathscr{E}_p(M) - \mathscr{E}_p(M_0) = -(x - x_0)
$$

$$
\vec{f} = -k\vec{u_r}
$$



$$
\delta W = \vec{f} \cdot \mathrm{d}\vec{OM} = \vec{f} \cdot (\mathrm{d}r \ \vec{u_r} + r\mathrm{d}\theta \ \vec{u_\theta} + \mathrm{d}z \ \vec{u_z}) = -kr \ \mathrm{d}r
$$

$$
\begin{aligned}
\mathrm{d}\mathscr{E}_p &= -\delta W = kr\mathrm{d}r \\
\int_{M_0}^{M} \mathrm{d}\mathscr{E}_p &= \mathscr{E}_p(M) - \mathscr{E}_p(M_0) \\
&= \int_{r_0}^{r} kr \ \mathrm{d}r = \frac{k}{2}(r^2 - r_0^2)
\end{aligned}
$$