

上海交通大学

# 计算机视觉

教师: 赵旭

班级: AI4701

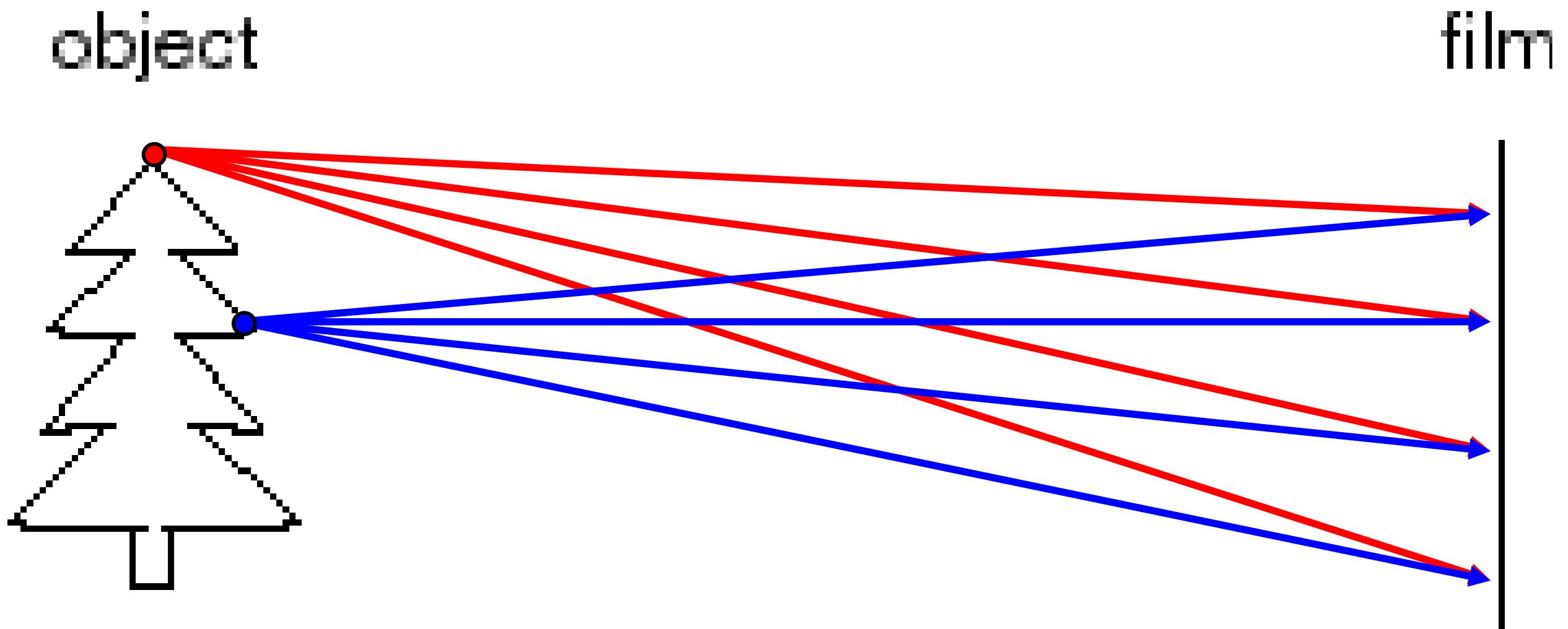
2024 春

# 4. 摄像机及其几何模型

# 主要内容

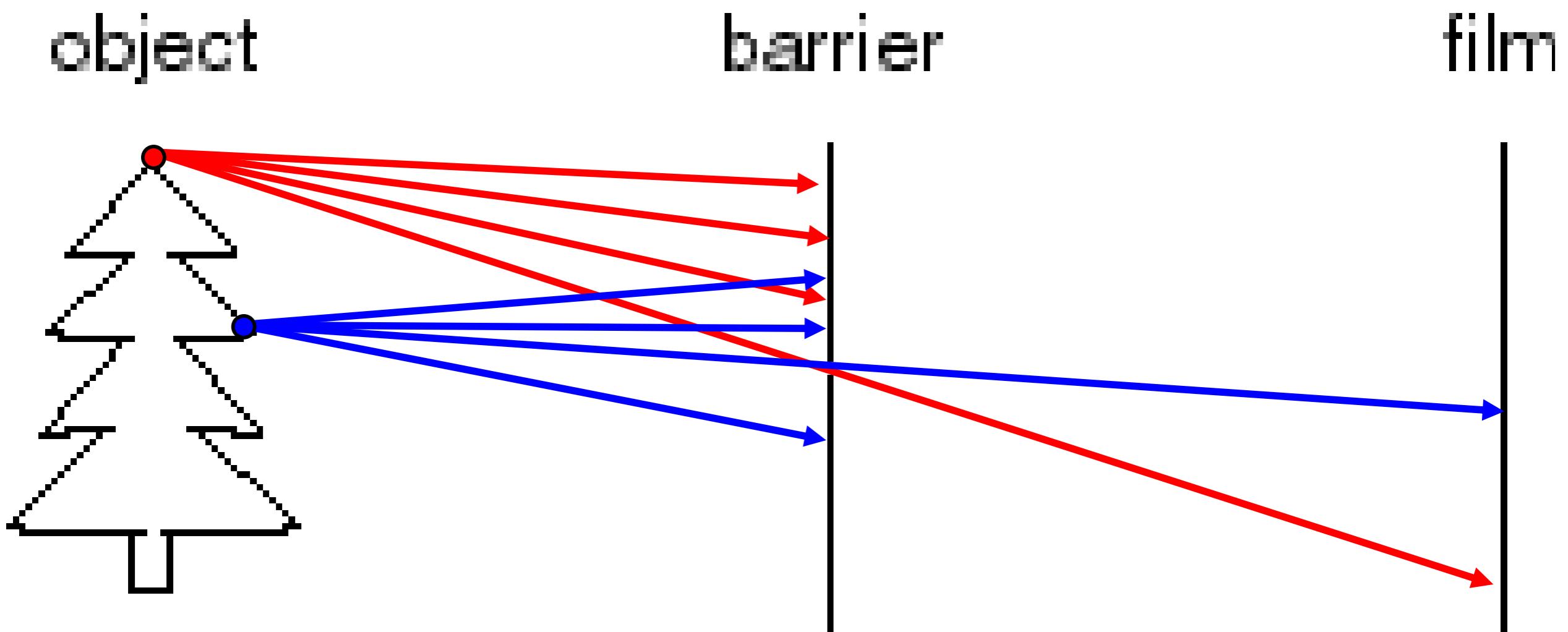
- ❖ 小孔相机模型
  - 暗房
  - 小孔几何模型
- ❖ 带镜头的摄像机
  - 景深
  - 视场
  - 镜头畸变
- ❖ 数字相机

# 设计一款照相机



- ❖ 想法1：直接在物体前面放置感光胶片
- ❖ 可以得到合理的图像吗？

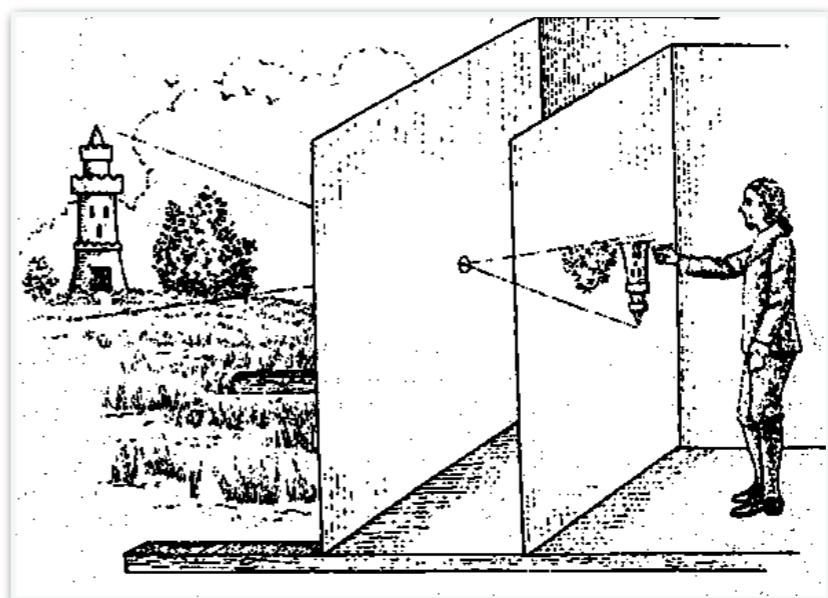
# 设计一款照相机



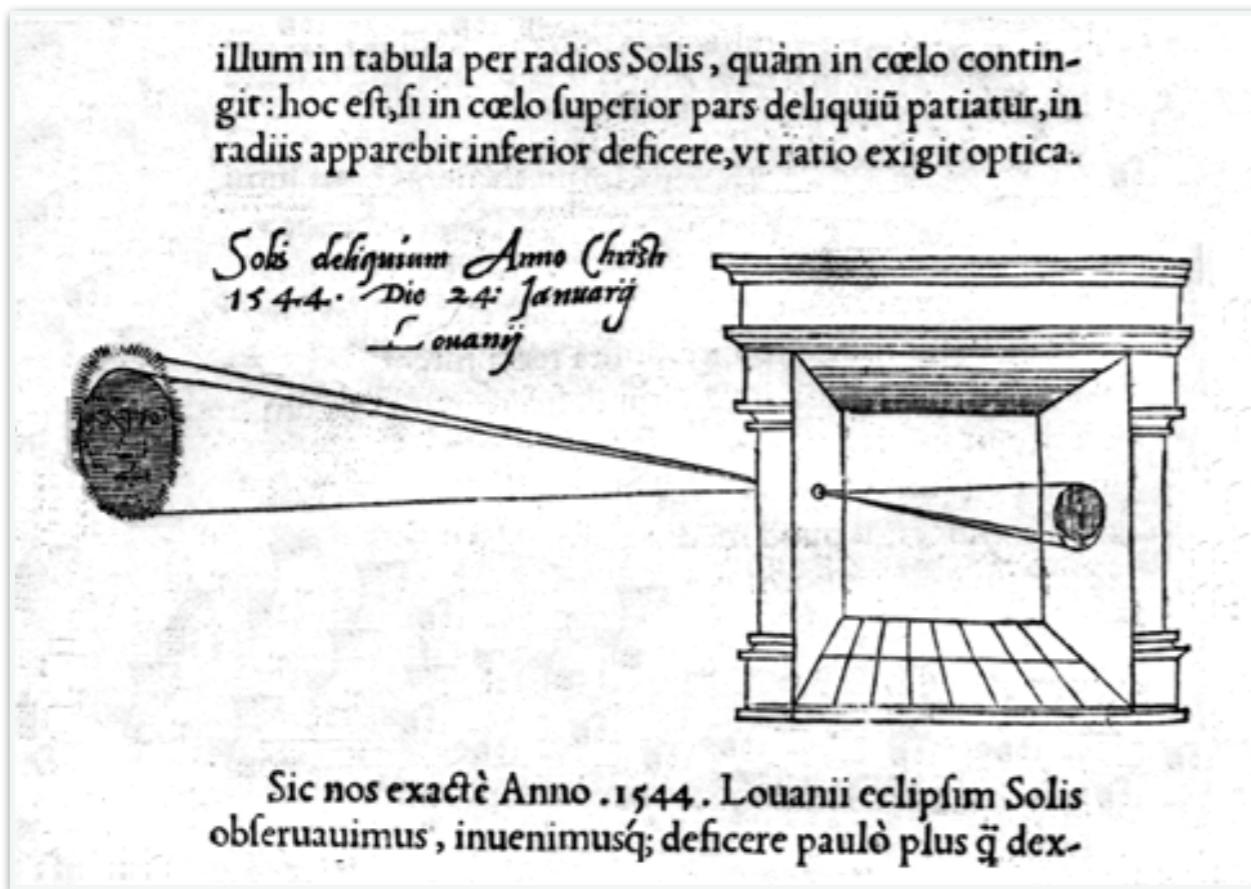
- ❖ 挡住大部分光线，只让一部分光到达胶片
  - ❖ 消除了模糊
  - ❖ 光线进入的通道：光圈

# 暗房

- ❖ 基本原理可以追溯到墨子 (470-390 BCE, 墨经) 和亚里士多德 (384-322 BCE)
- ❖ 辅助画家作画：Leonardo da Vinci (1452-1519)



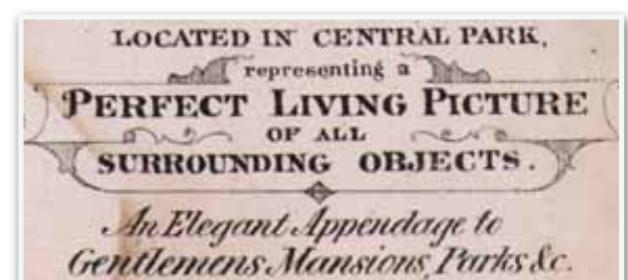
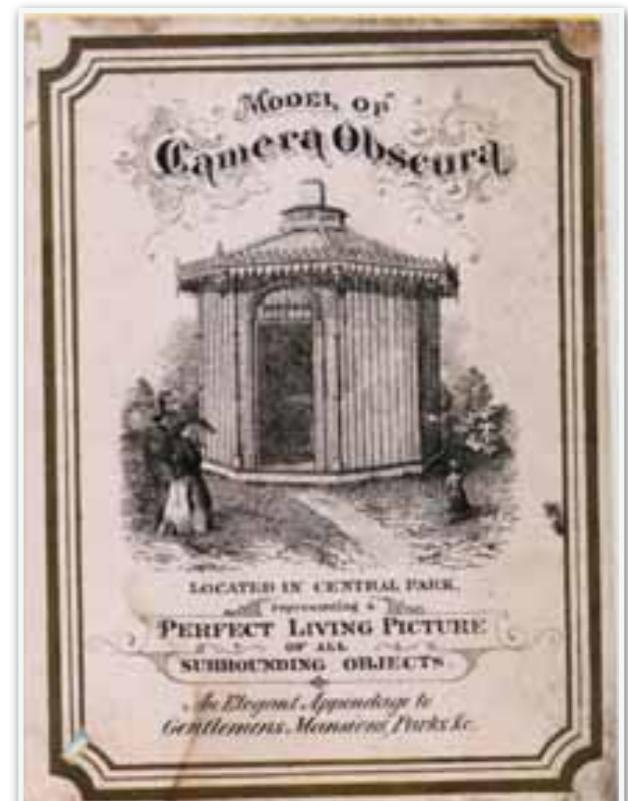
# 暗房



"Reinerus Gemma-Frisius, observed an eclipse of the sun at Louvain on January 24, 1544, and later he used this illustration of the event in his book De Radio Astronomica et Geometrica, 1545. It is thought to be the first published illustration of a camera obscura..."

Hammond, John H., The Camera Obscura, A Chronicle

# 暗房



19世纪晚期的景点

约1870s

# 第一张照片

目前存世的最古老的照片：需要曝光8小时以上



Joseph Niepce, 1826

第一张照片的照片

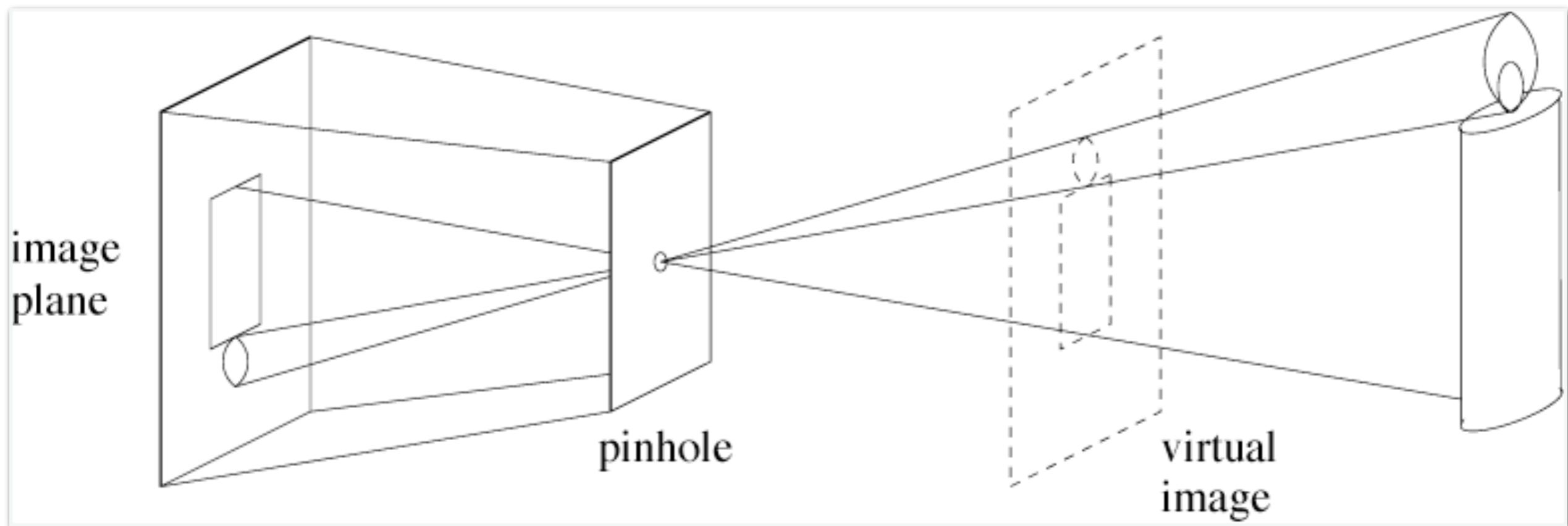


目前存放在 UT Austin

Niepce later teamed up with Daguerre, who eventually created Daguerrotypes (银板摄影术)

Slide from James Tompkin

# 小孔模型

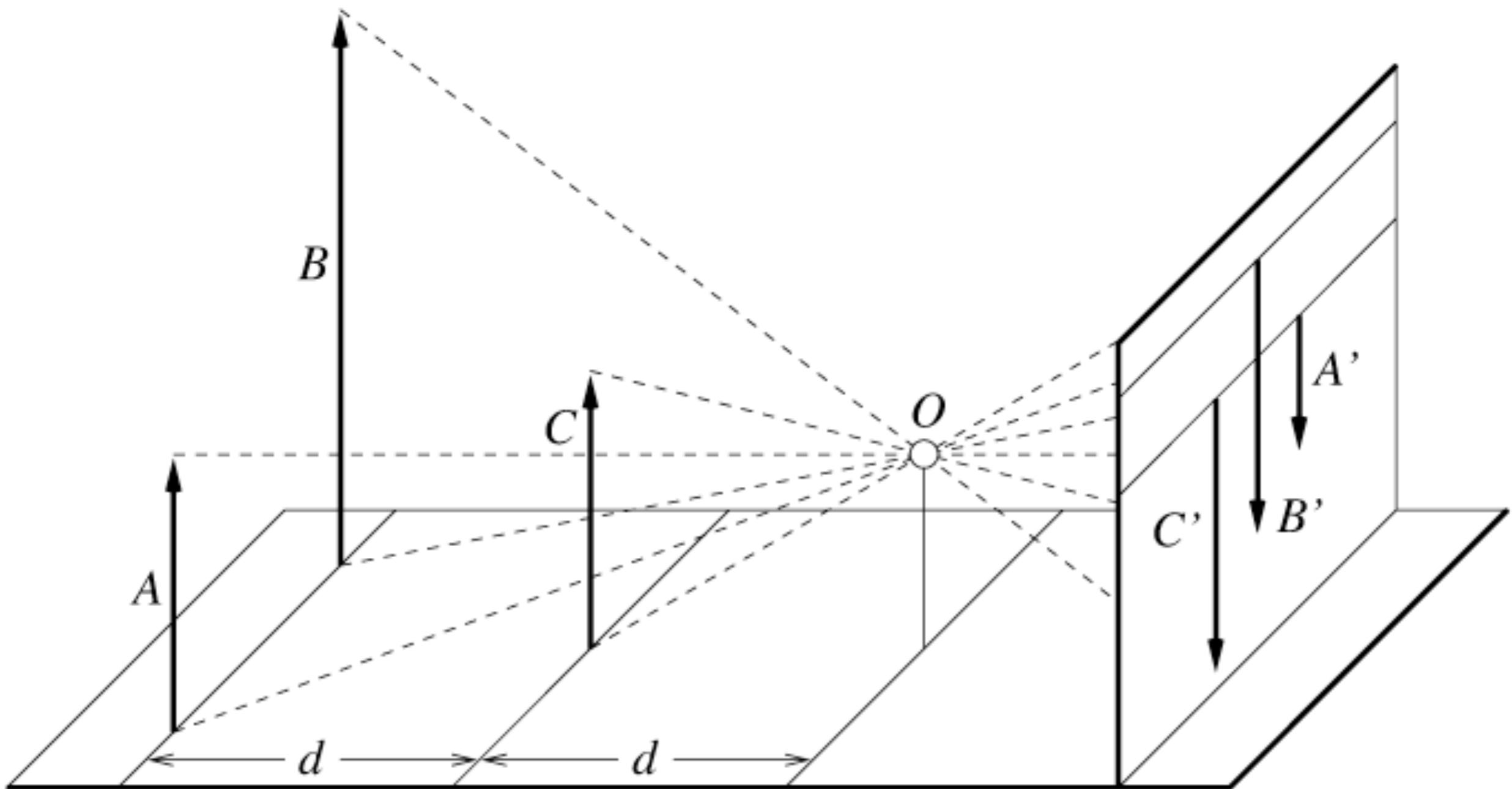


- ❖ 所有的光线只能通过一个唯一的小孔进入暗室，到达胶片
- ❖ 小孔（点）：投影中心
- ❖ 图像平面：在该平面上形成了图像

# 透视投影效应



# 效应1:近大远小

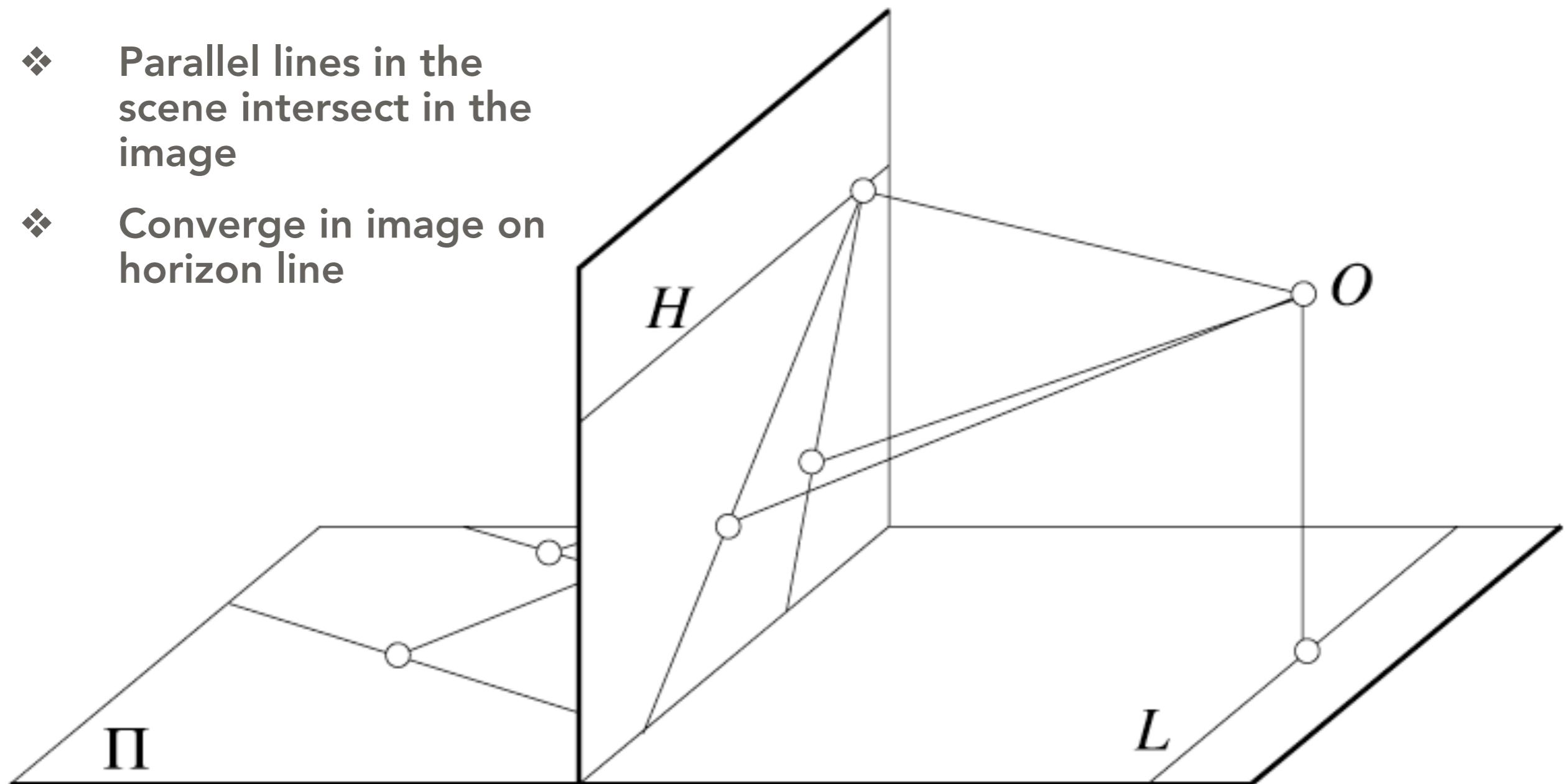


# 透视投影效应



# 效应2: 平行线相交

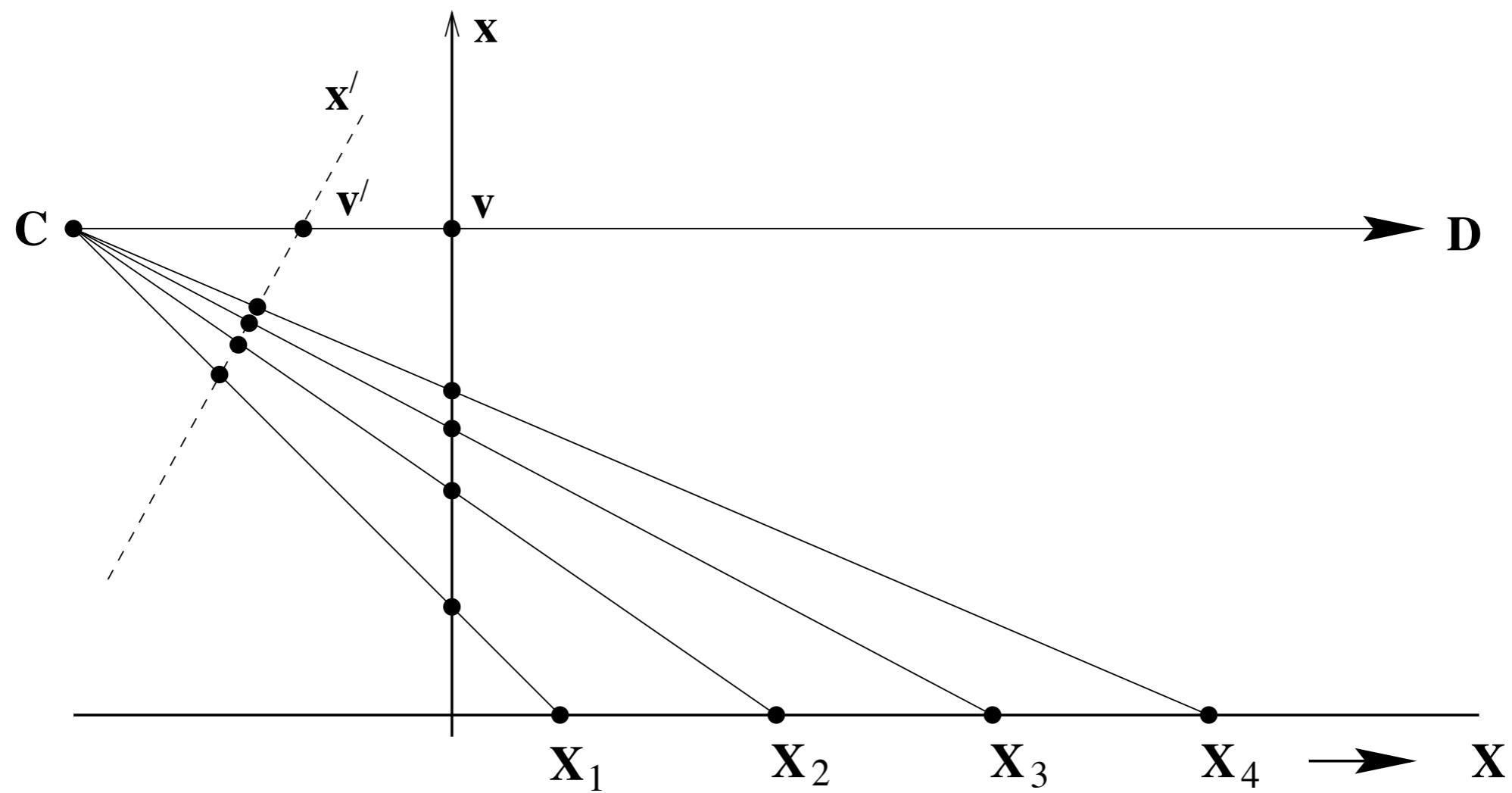
- ❖ Parallel lines in the scene intersect in the image
- ❖ Converge in image on horizon line

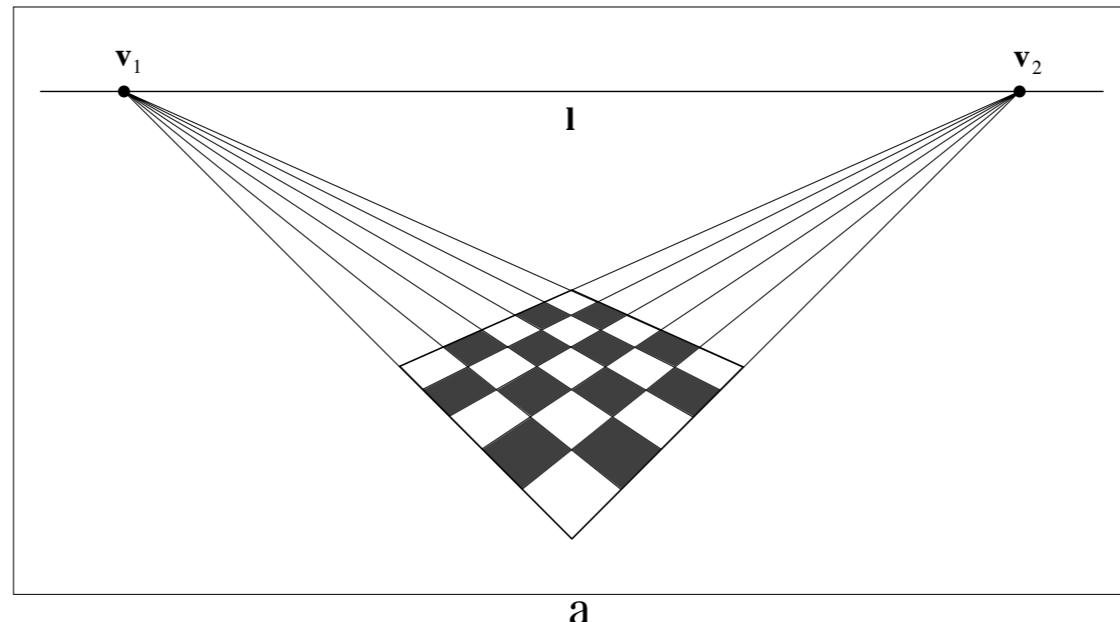


# 消影点 (Vanishing points)

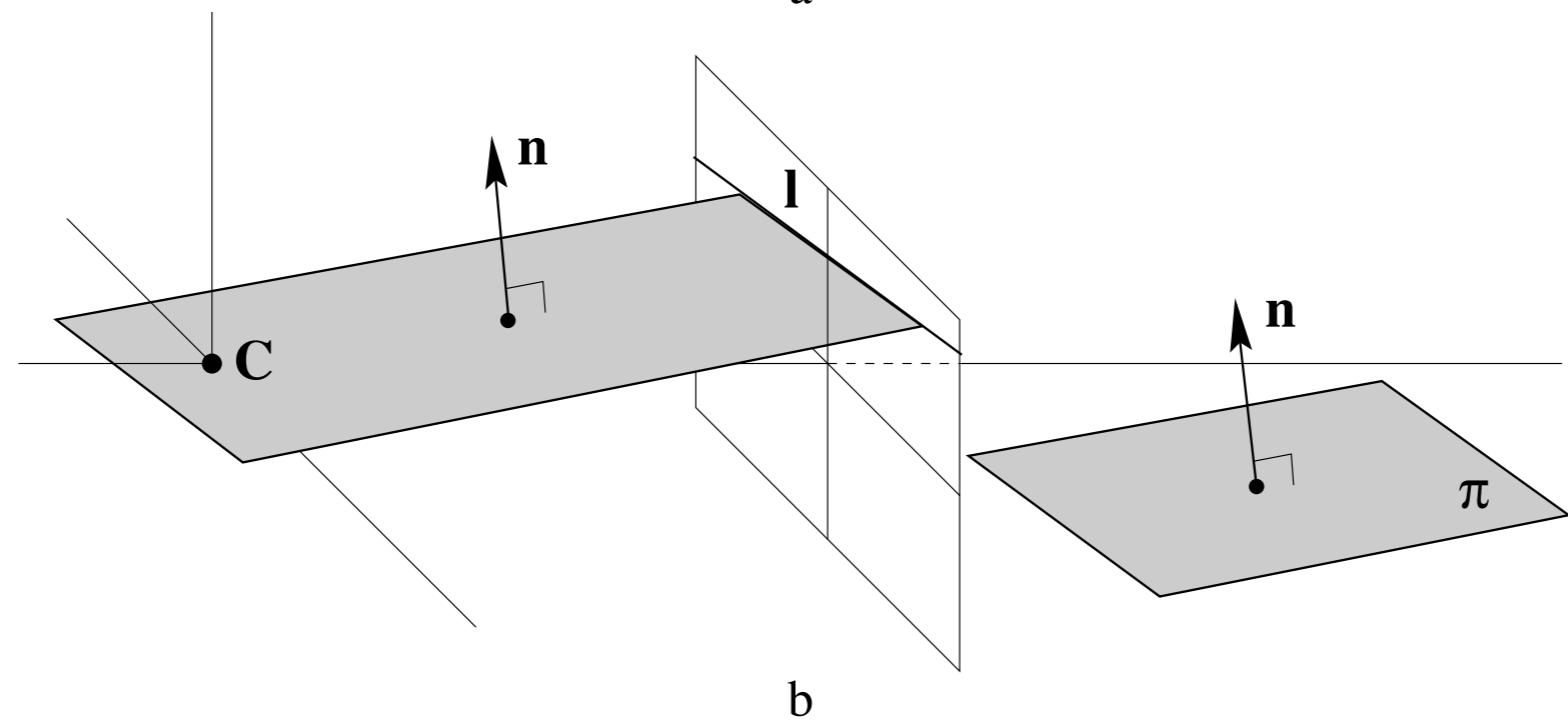
- ❖ 空间中的每个方向都存在对应的消影点
  - ❖ 所有同向的直线通过投影后交于对该方向对应的消影点
  - ❖ 退化情况：与图像平面平行的方向
- ❖ 位于相同平面上的具有不同方向的所有直线，对应的消影点位于同一条直线上
  - ❖ 该直线就是该平面对应的消影线
- ❖ 判别“假图像”







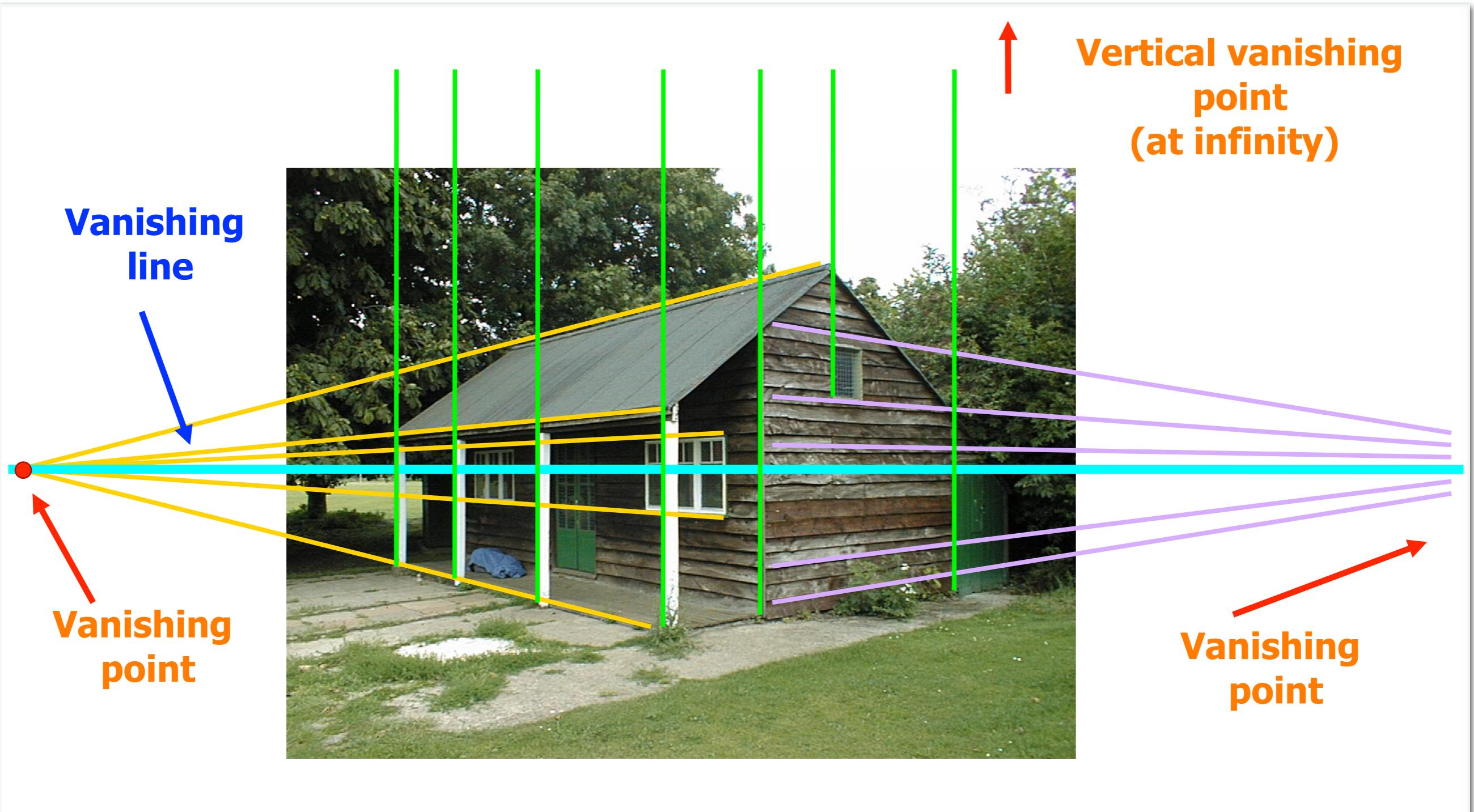
a



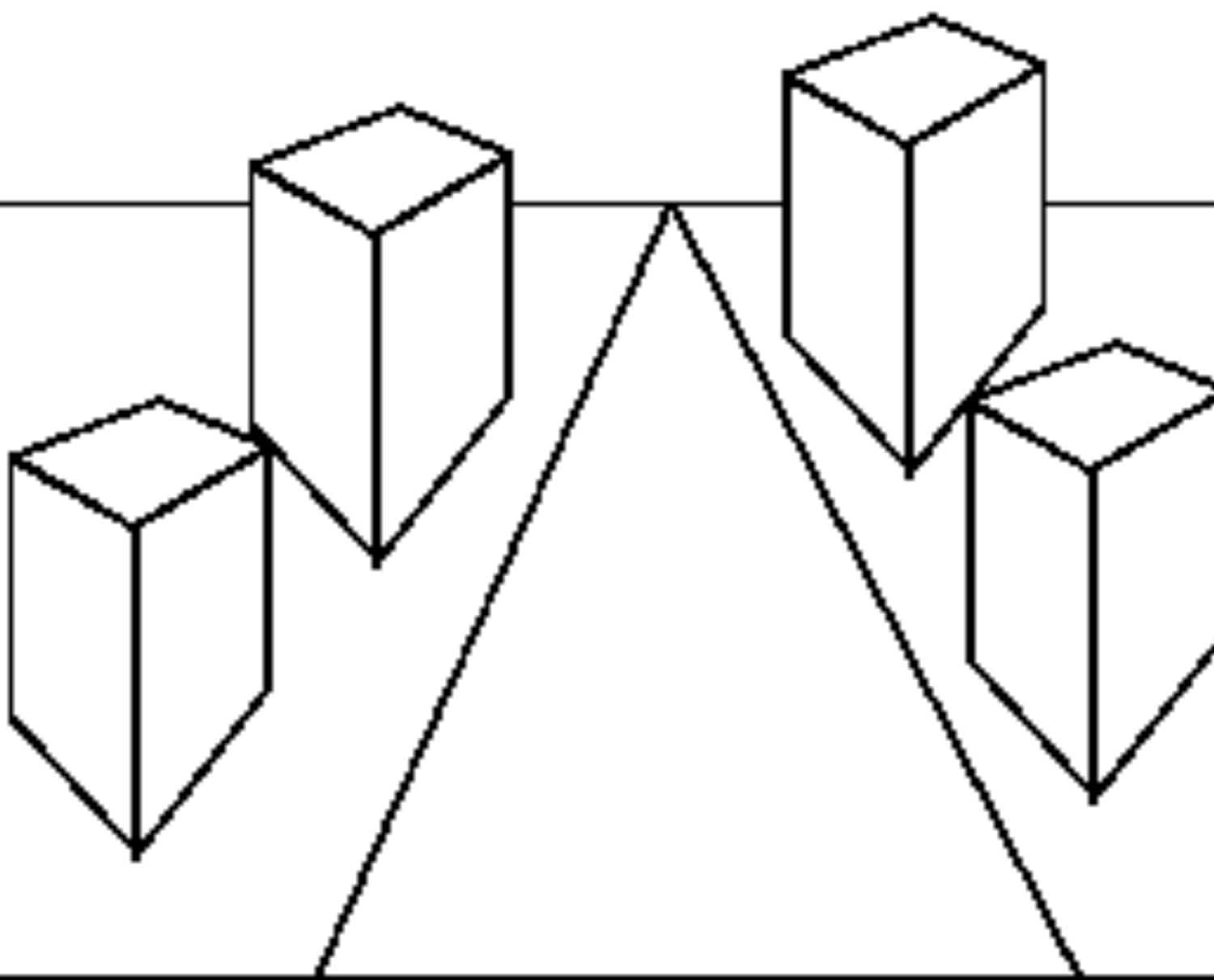
b



**Forsyth & Ponce**



Slide from Efros, Photo from Criminisi



Forsyth and Ponce

# 透视投影的几何属性

- ❖ 多对一：一条光线经过的空间各点映射为同一个图像点
- ❖ 点投影为点：
  - ❖ 图像平面上的点的投影未定义
- ❖ 线投影为线：
  - ❖ 退化情况：经过投影中心（焦点）的线被投影为点
- ❖ 平面投影为平面：
  - ❖ 退化情况：经过焦点的平面投影为线
- ❖ 距离和角度不再保持

# 透视变形

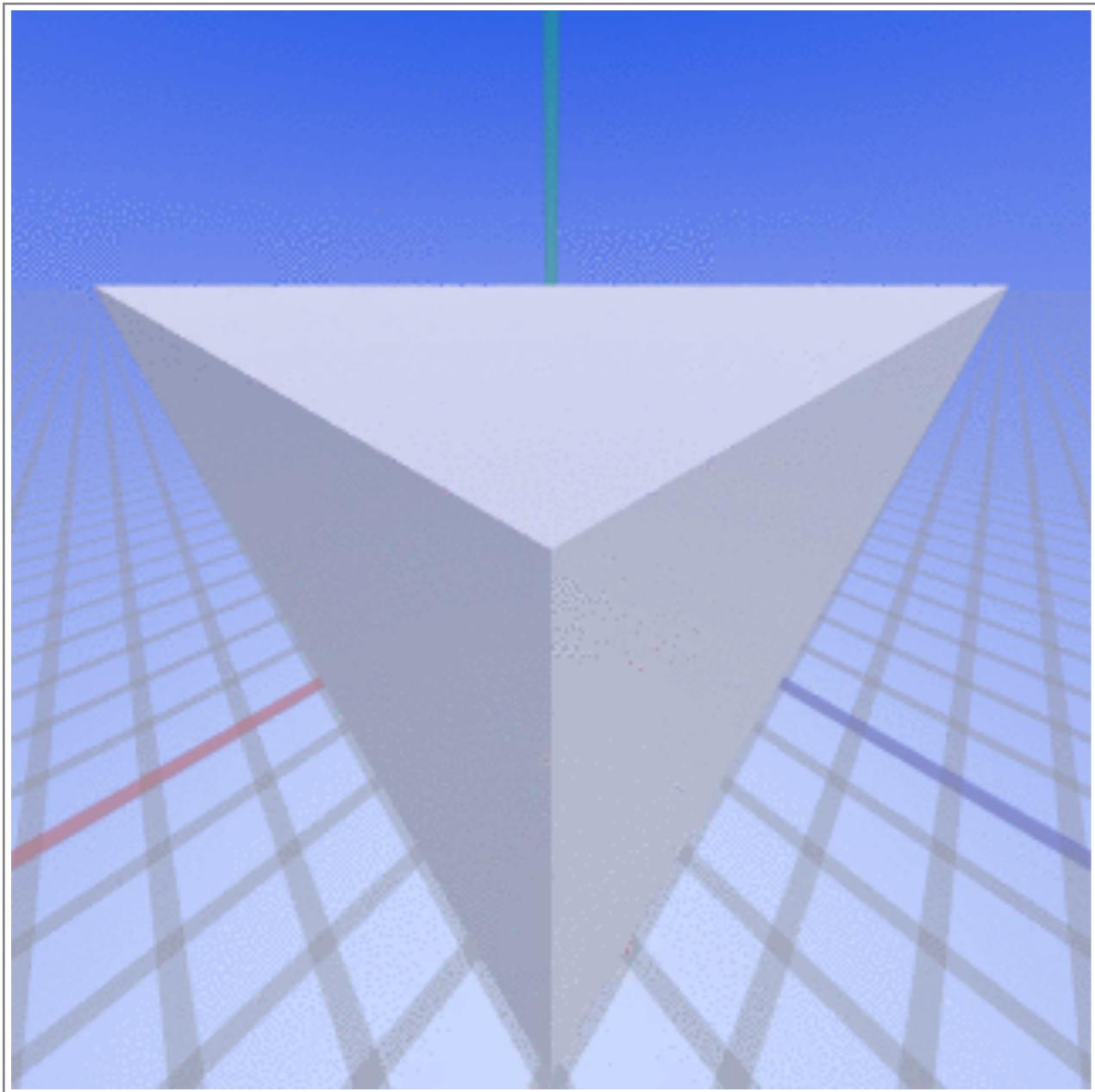


Image copyright 2007 [SharkD](#), licensed CC-BY-SA 3.0



Source: F. Durand

# 透视投影与艺术



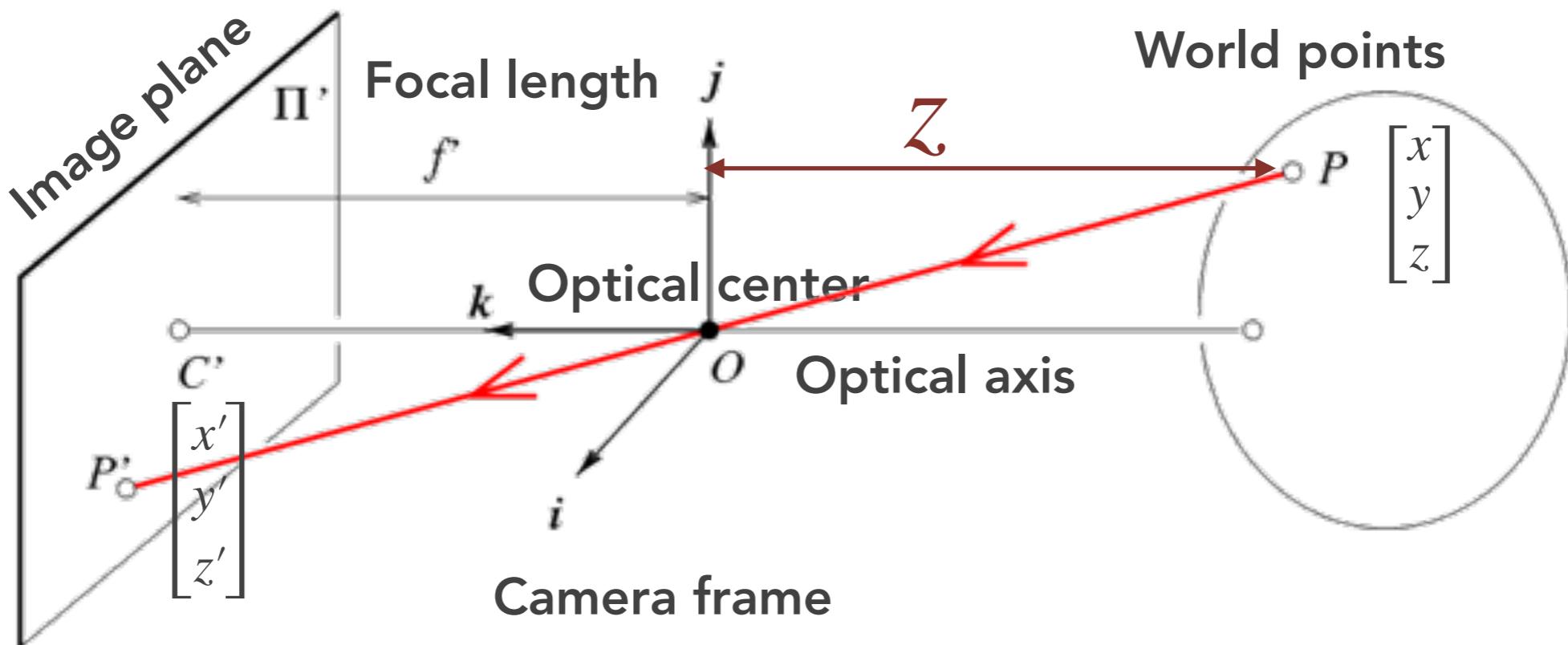
Raphael



Durer, 1525

- ❖ 公元前1世纪的壁画上已经出现了透视投影画法
- ❖ 文艺复兴期间涌现出系统性、更为成熟的透视投影画法 (约 1480-1515)

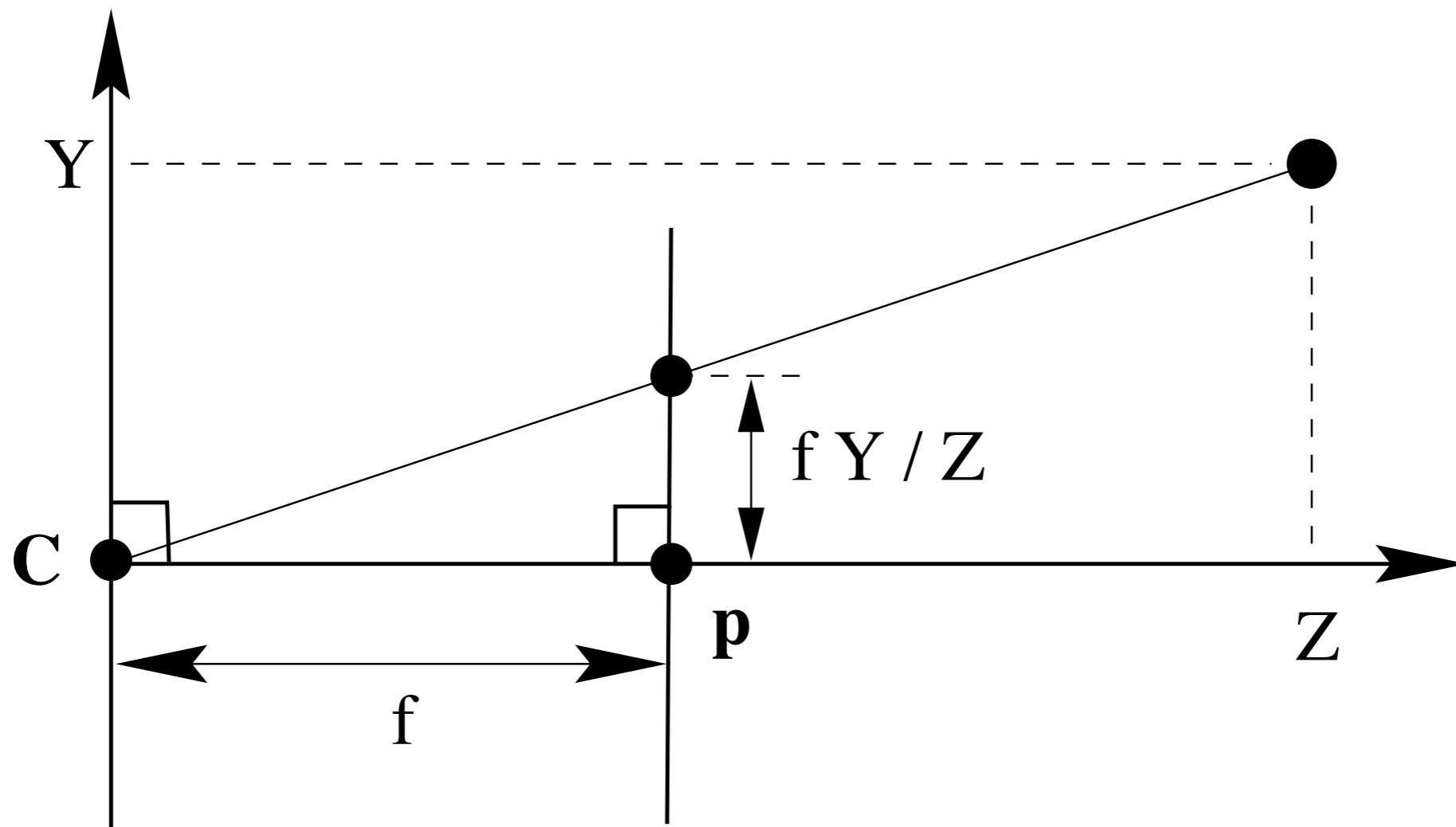
# 透视投影方程



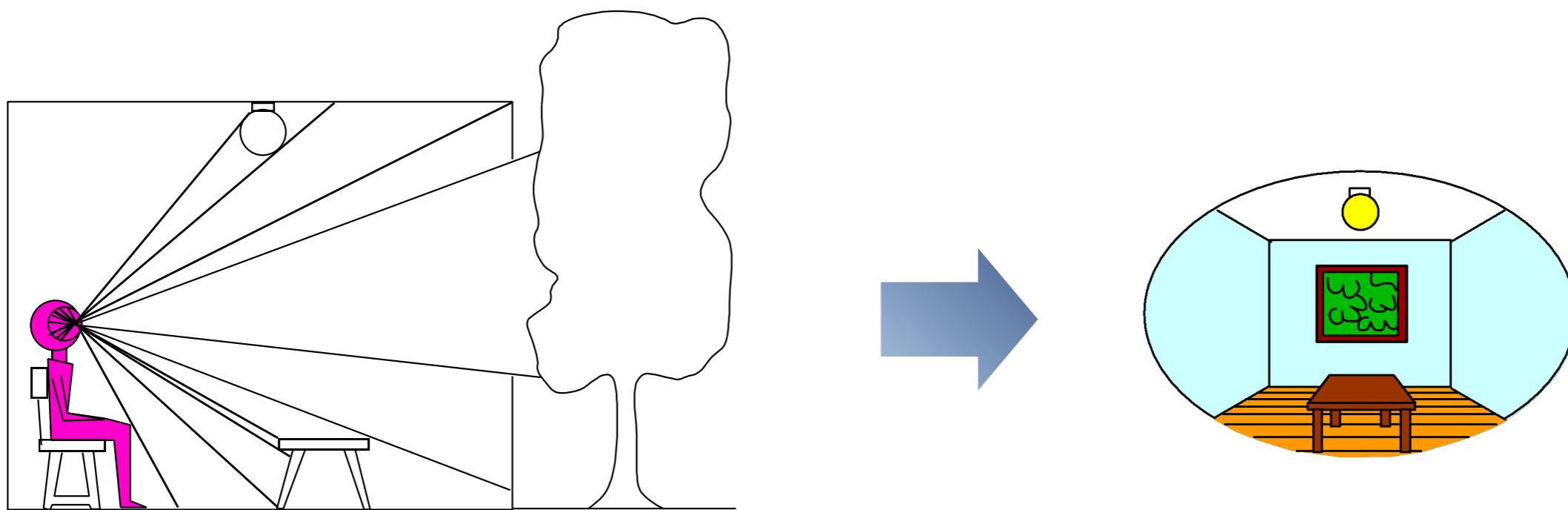
$$(x, y, z) \rightarrow \left( f \frac{x}{z}, f \frac{y}{z} \right)$$

Scen point  $\longrightarrow$  Image coordinates

Source: J. Ponce, S. Seitz



# “维度压缩机” (3D to 2D)



Point of observation

Figures © Stephen E. Palmer, 2002

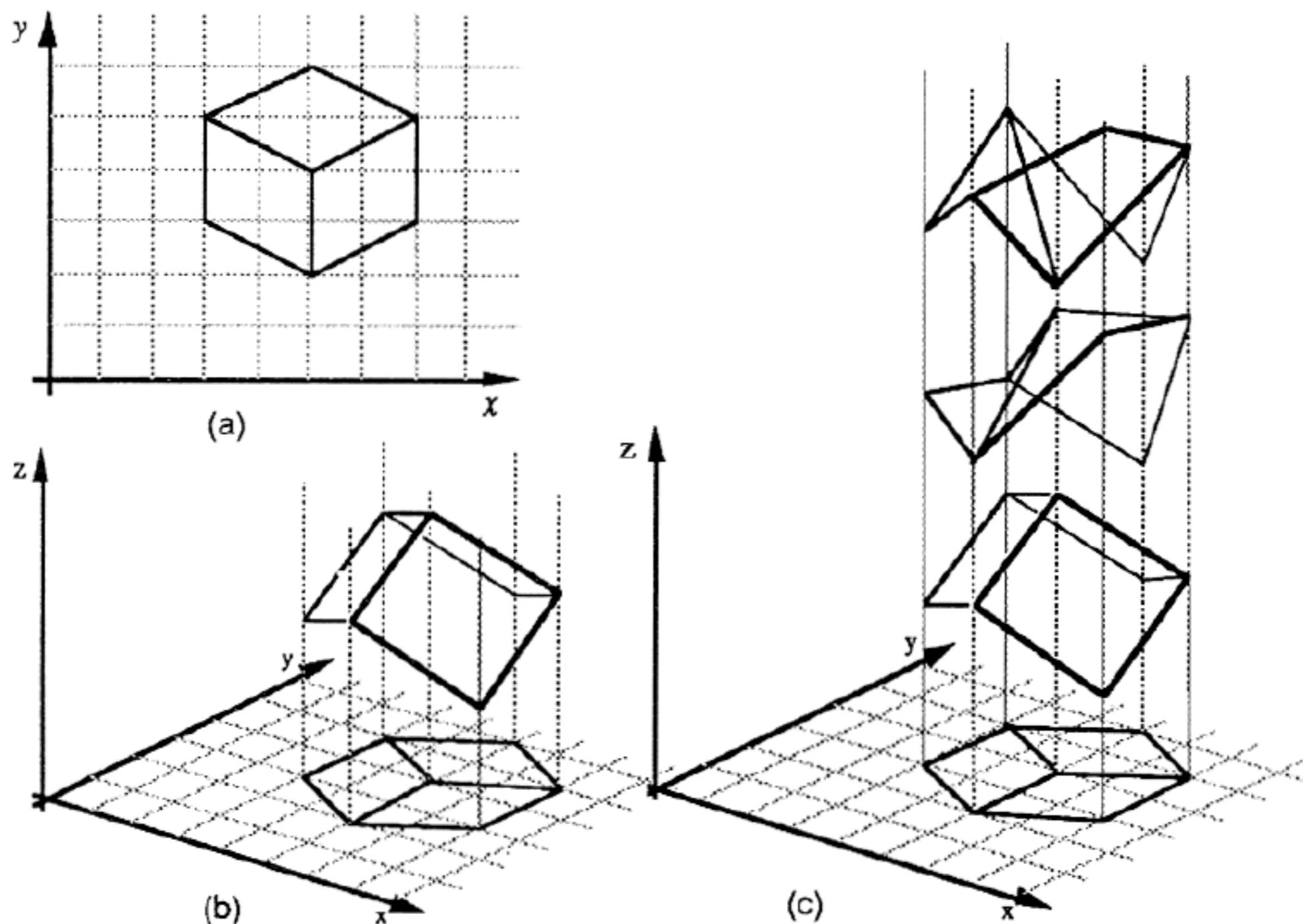


Figure 1. (a) A line drawing provides information only about the  $x$ ,  $y$  coordinates of points lying along the object contours. (b) The human visual system is usually able to reconstruct an object in three dimensions given only a single 2D projection (c) Any planar line-drawing is geometrically consistent with infinitely many 3D structures.

# 齐次坐标系

- ❖ 透视投影变换是否线性变换?
  - ❖ 否， $z$ 出现在分母
- ❖ 技巧：再加一维，达成形式上的线性表示

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

齐次图像坐标

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

齐次三维点坐标

- ❖ 从齐次坐标变换回非齐次坐标

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow \left( \frac{x}{w}, \frac{y}{w} \right)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow \left( \frac{x}{w}, \frac{y}{w}, \frac{z}{w} \right)$$

# 透视投影矩阵

- ❖ 齐次坐标下，投影表示为矩阵相乘

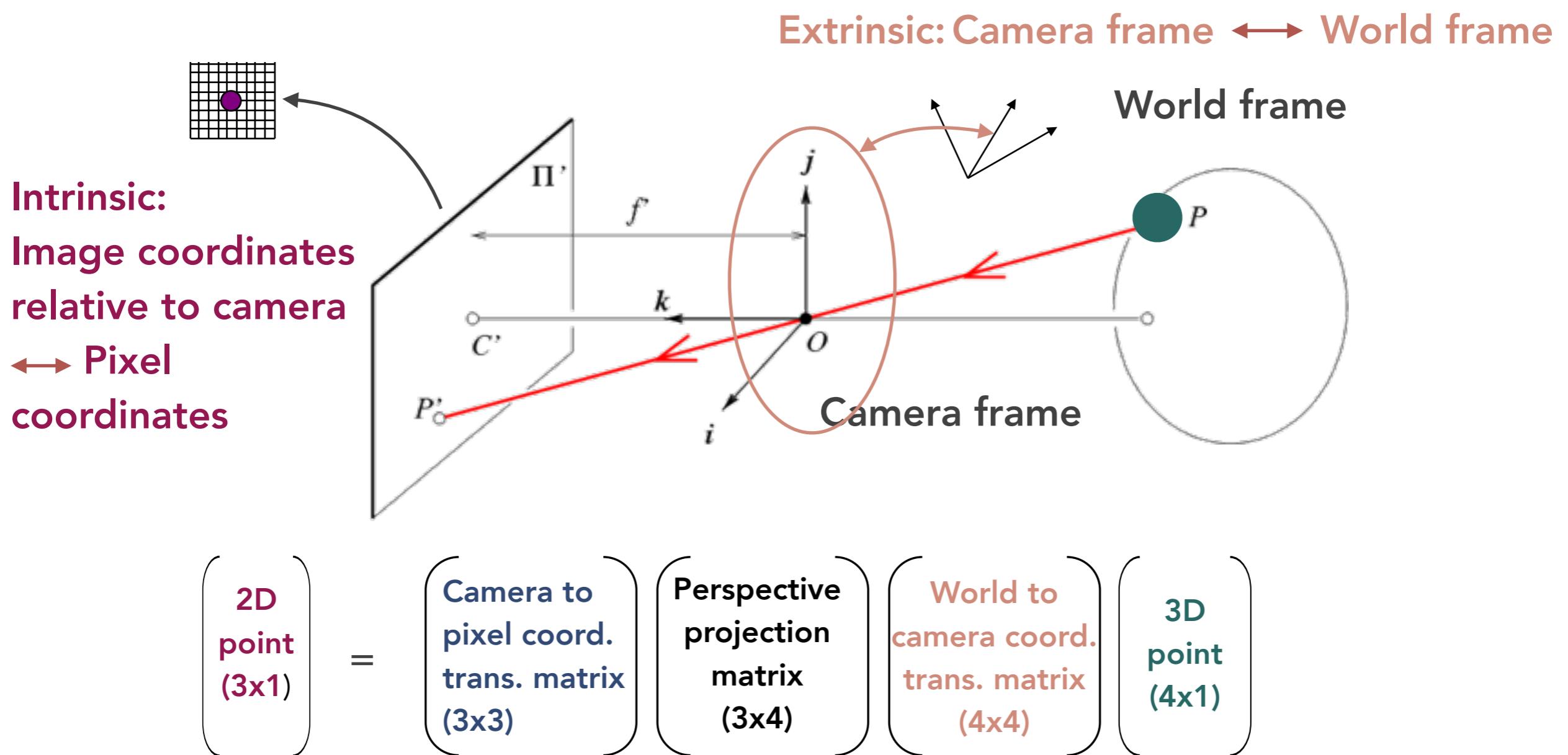
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ \frac{z}{f} \\ 1 \end{bmatrix} \Rightarrow \left( f \frac{x}{z}, f \frac{y}{z} \right)$$

- ❖ 实际上，投影矩阵可分解为不同的坐标变换矩阵

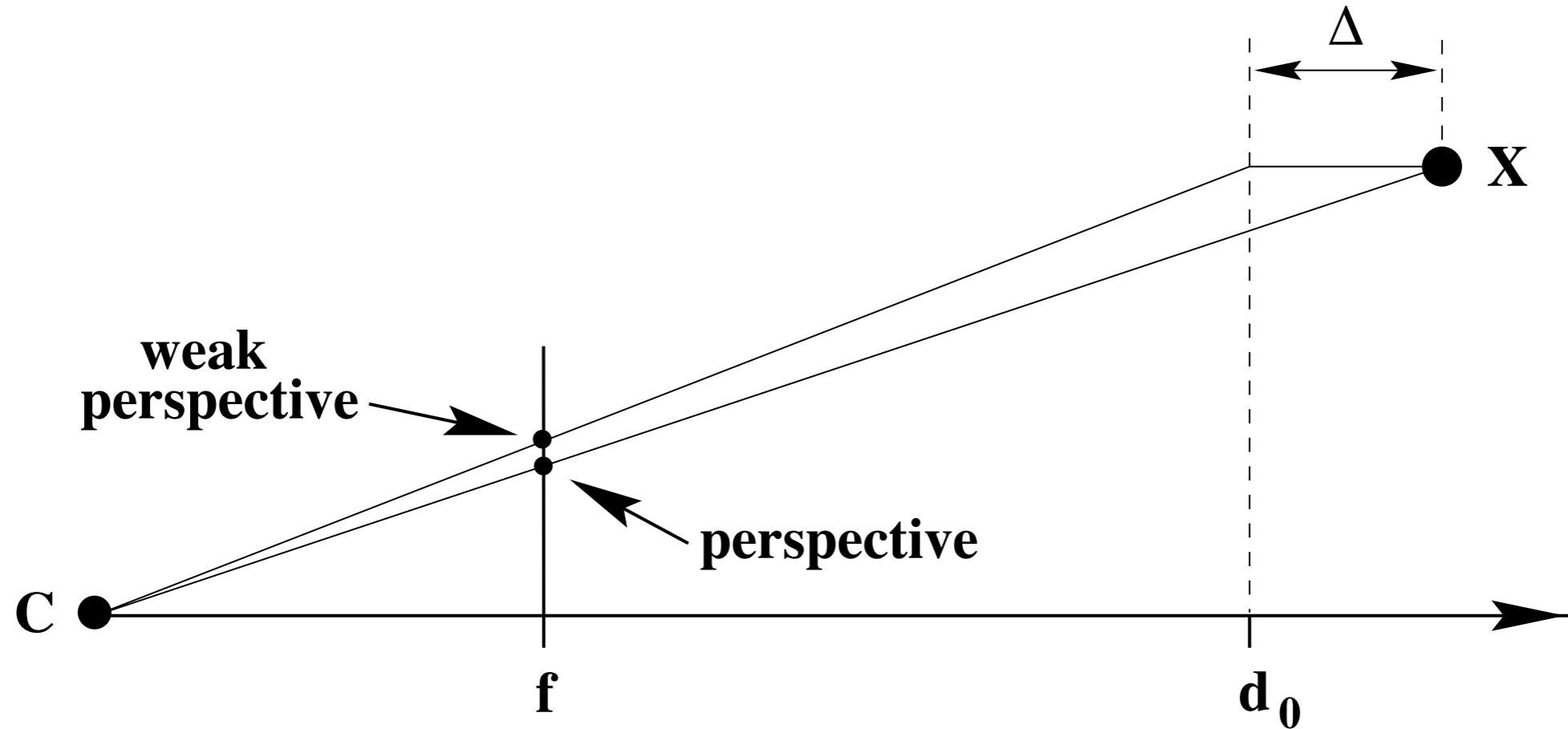
$$\begin{pmatrix} 2D \\ point \\ (3 \times 1) \end{pmatrix} = \begin{pmatrix} Camera \\ to \\ pixel \\ coord. \\ trans. \\ matrix \\ (3 \times 3) \end{pmatrix} \begin{pmatrix} Perspective \\ projection \\ matrix \\ (3 \times 4) \end{pmatrix} \begin{pmatrix} World \\ to \\ camera \\ coord. \\ trans. \\ matrix \\ (4 \times 4) \end{pmatrix} \begin{pmatrix} 3D \\ point \\ (4 \times 1) \end{pmatrix}$$

# 标定

- 求解相机内参数和外参数

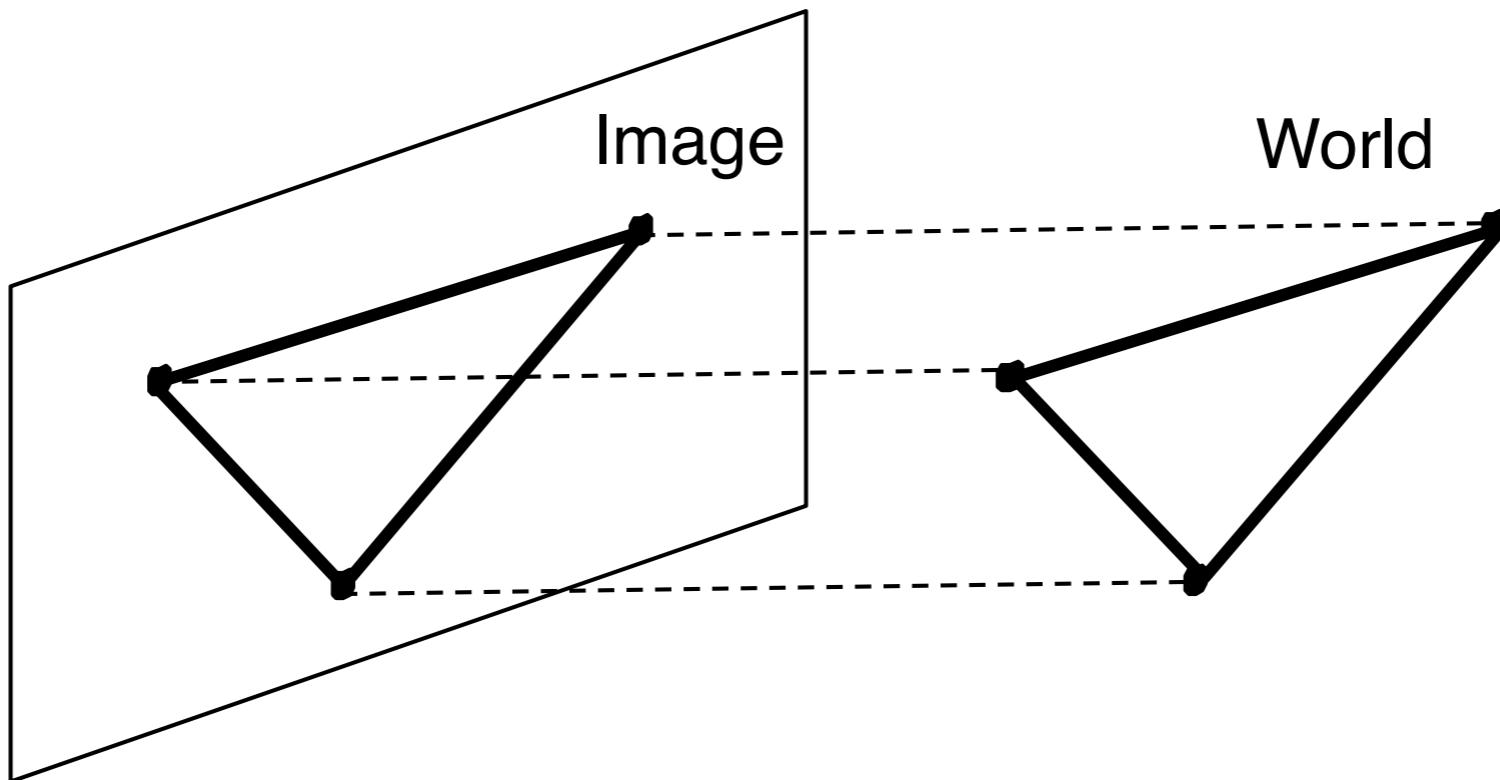


# 弱透视投影



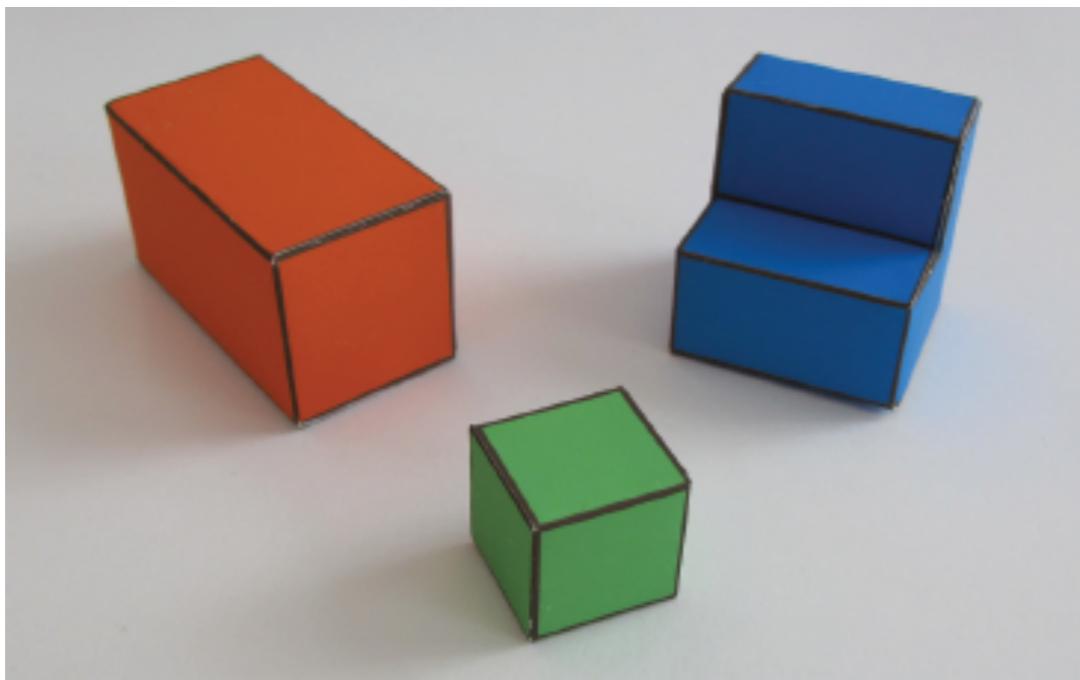
- ❖ 把一组点（深度不同）赋予同样的深度，然后按照正常的透视投影成像

# 正交投影

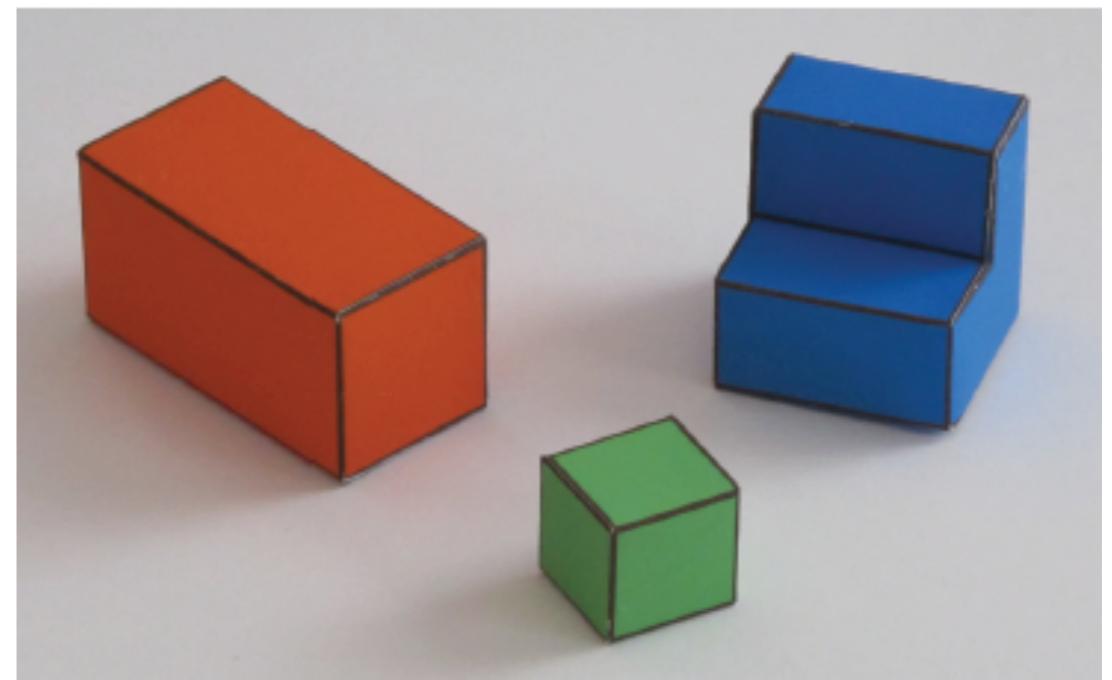


- ❖ 透视投影的特殊情形：投影中心到图像平面的距离（焦距）无限大
- ❖ 也称为“平行投影”

# 找不同



Perspective projection



Parallel (orthographic) projection

Antonio Torralba

# 三种投影类型

	3D point	2D image point
Perspective	$(x, y, z)$	$\left( \frac{fx}{z}, \frac{fy}{z} \right)$
Weak Perspective	$(x, y, z)$	$\left( \frac{fx}{z_0}, \frac{fy}{z_0} \right)$
Orthographic Projection	$(x, y, z)$	$(x, y)$

# 小孔的大小

- ❖ 小孔（光圈）的大小如何影响成像？

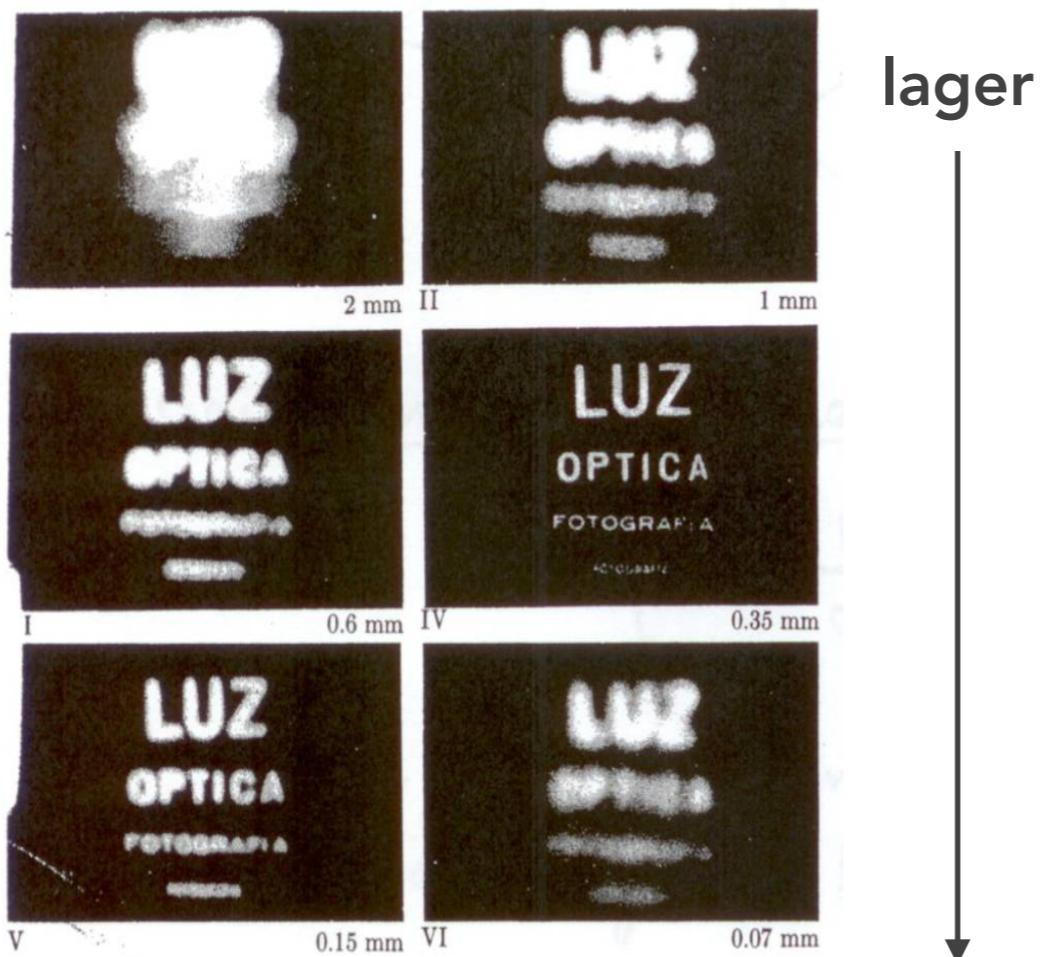
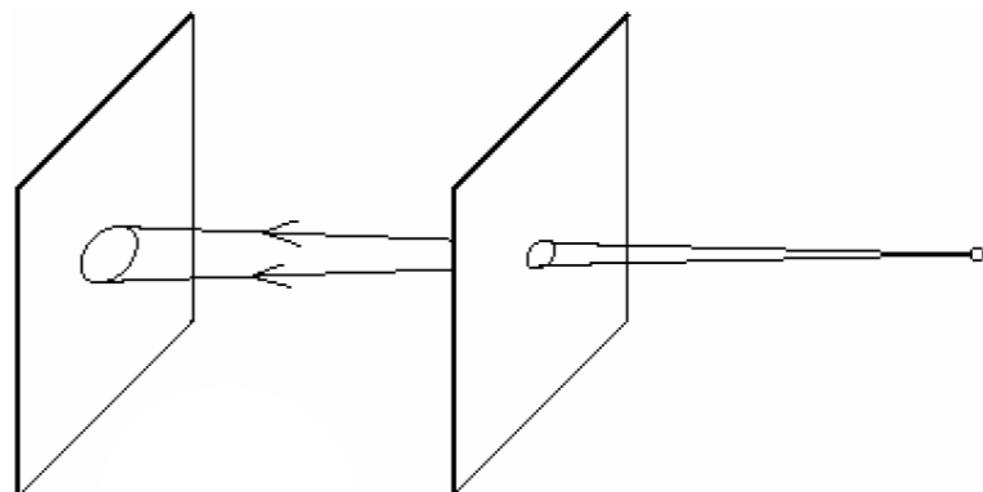
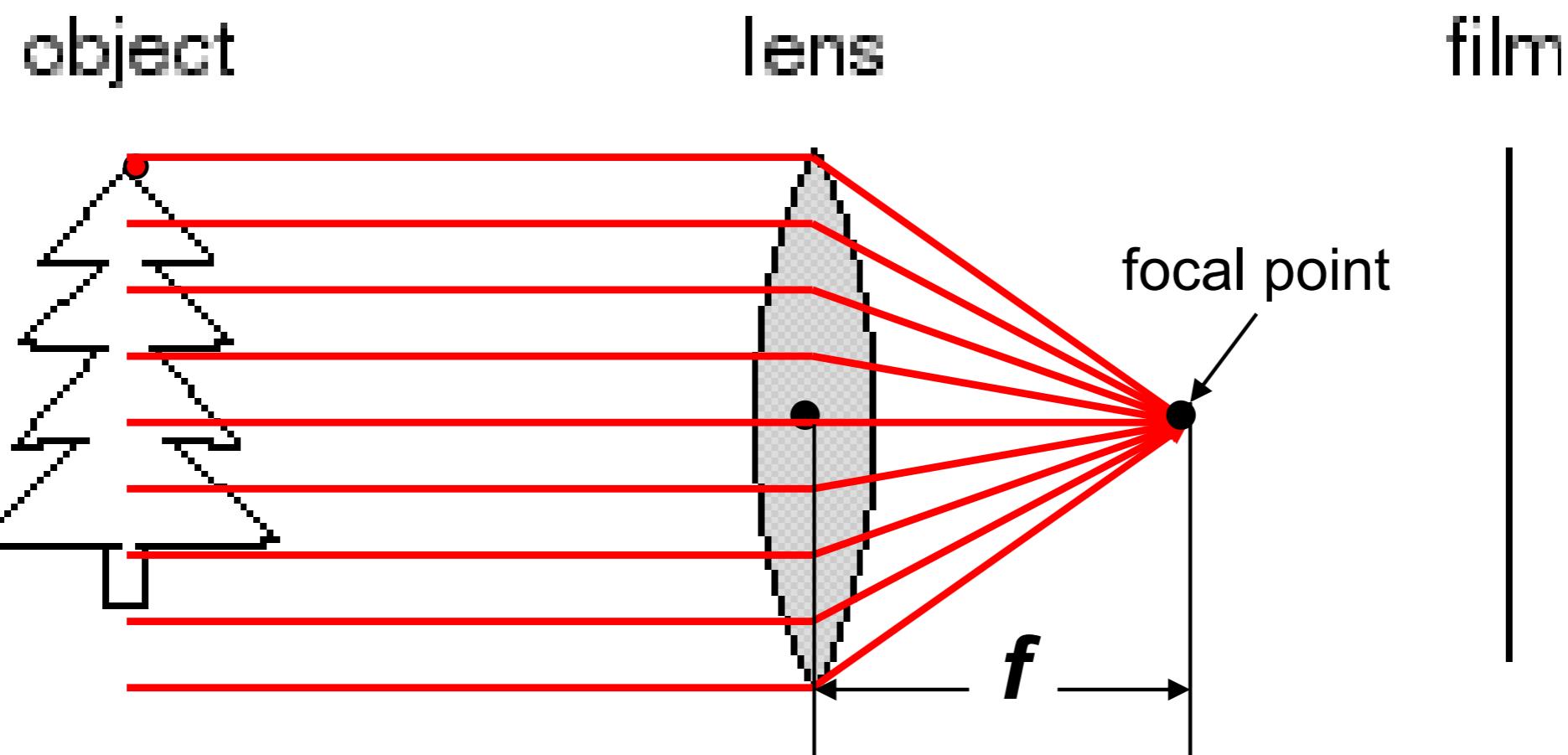


Fig. 5.96 The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joel, UNESCO.]



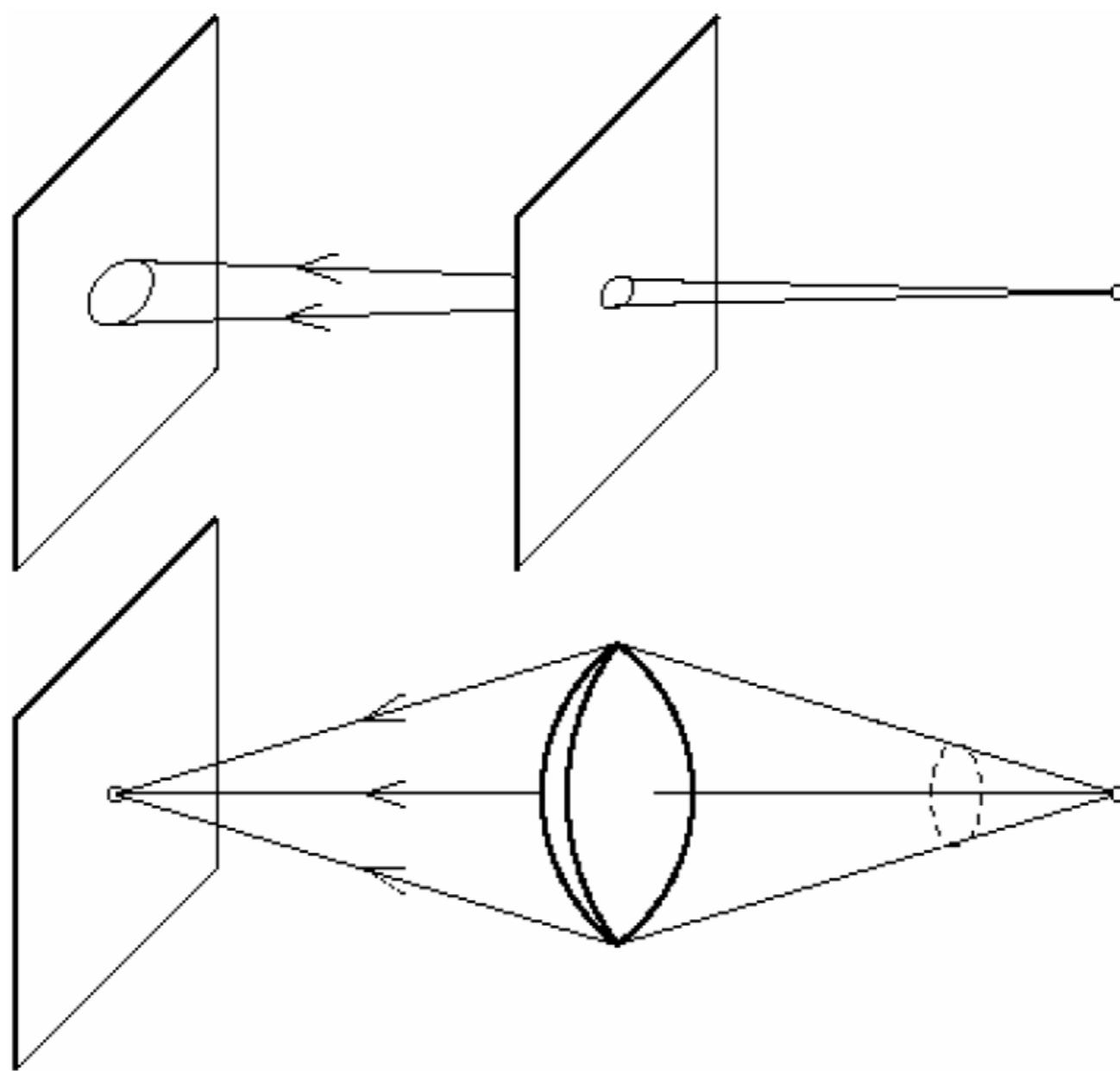
Kristen Grauman

# 镜头

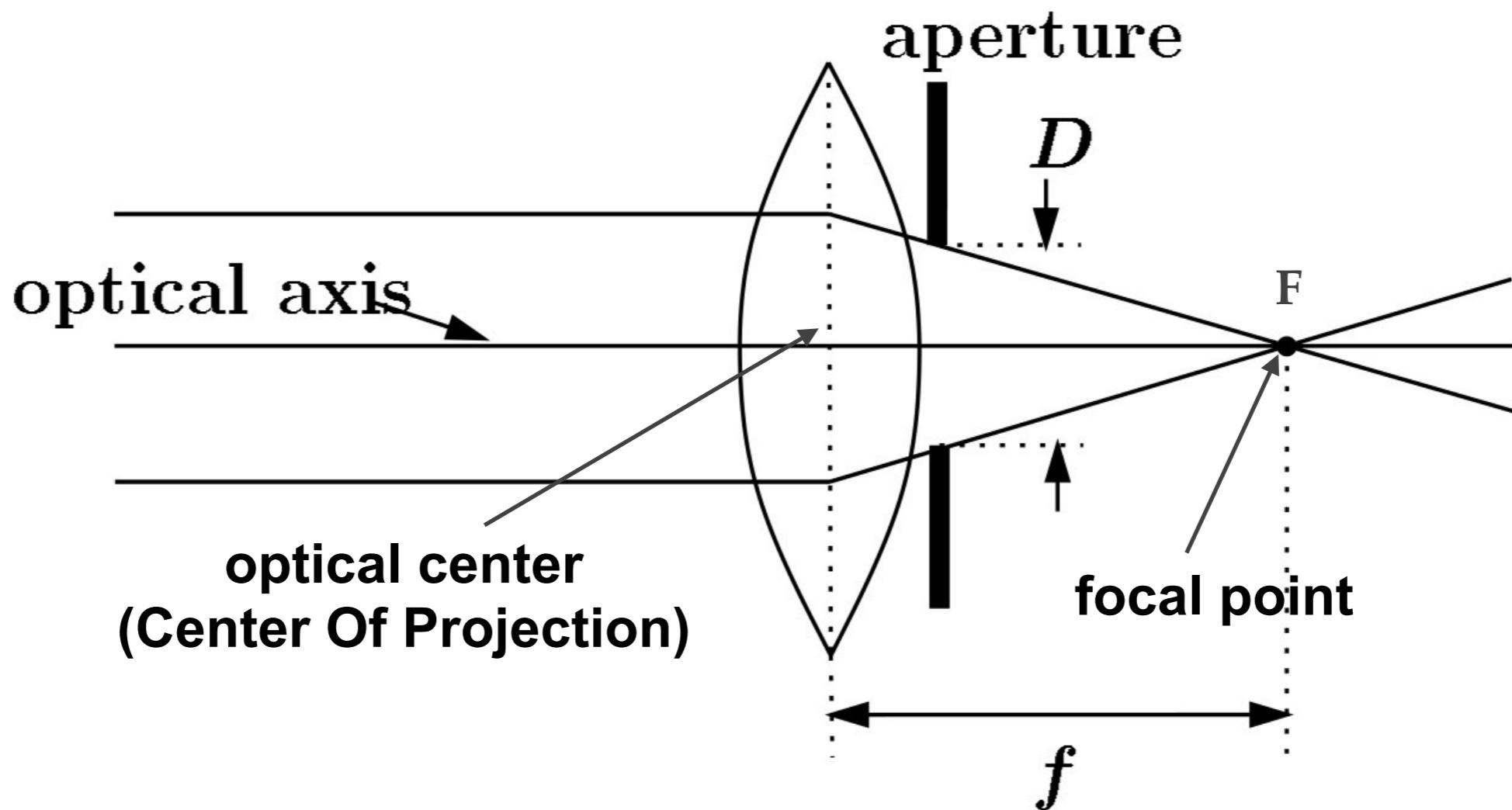


- ❖ 镜头将光线聚集到胶片上
- ❖ 经过镜头中心的光线没有任何偏曲
- ❖ 所有平行光线经过镜头后交汇于一点

# 小孔 vs. 镜头



# 带镜头的相机

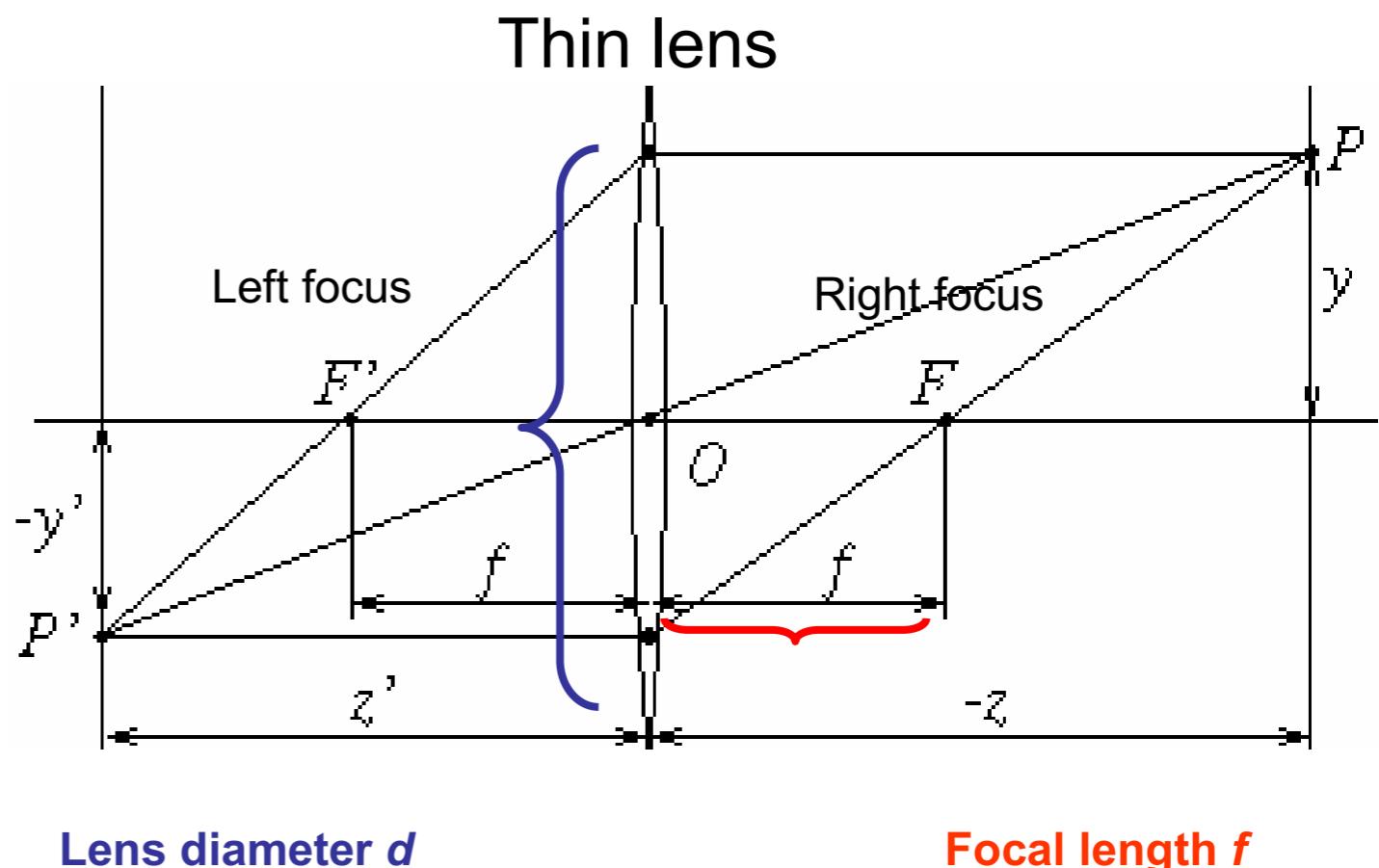


- ❖ 镜头将平行的光线聚焦到一点
- ❖ 聚集更多的光线，同时保持聚焦，使小孔模型实际可用

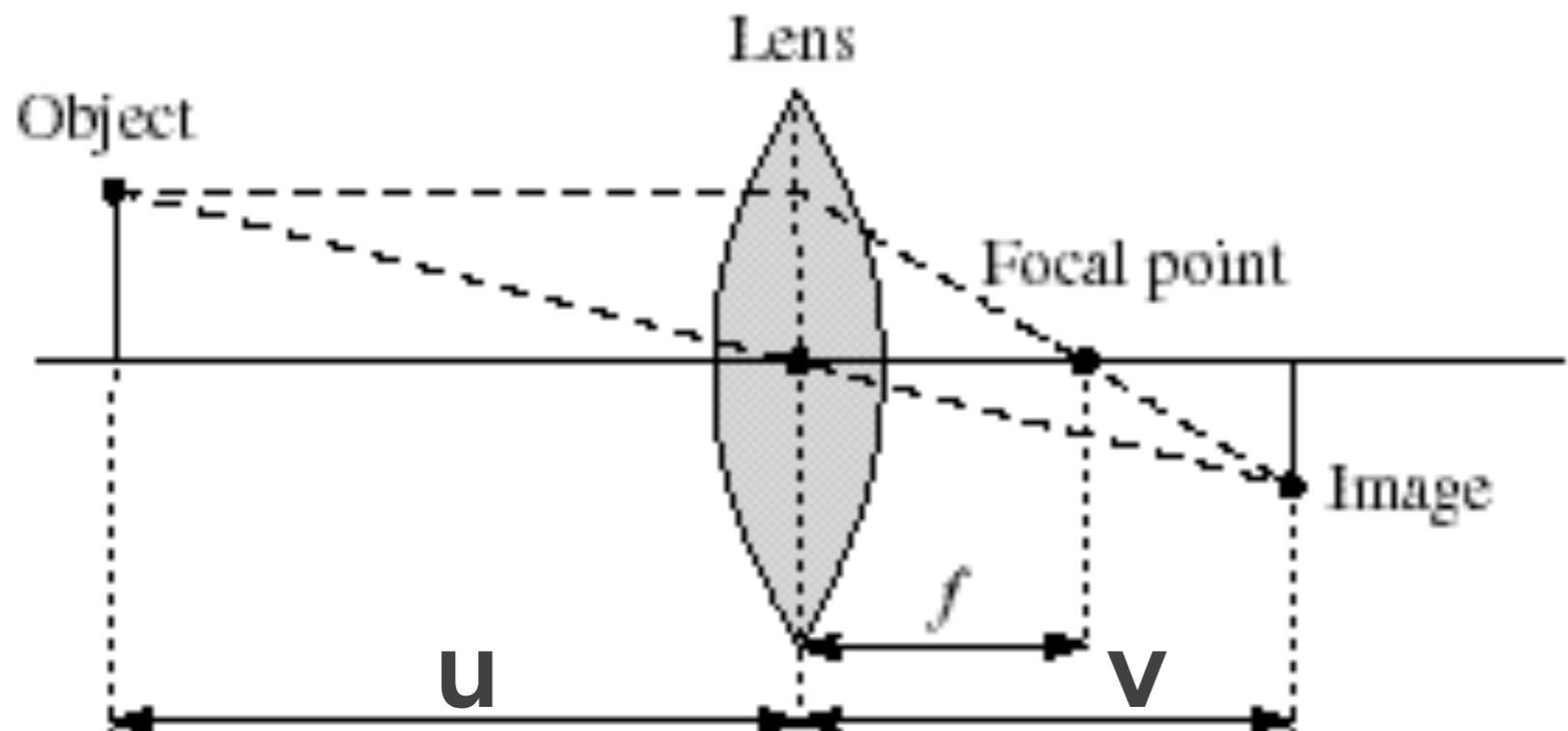
# 薄透镜

透镜的厚度与焦距的长度比较时，可以被忽略

- ❖ Rays entering parallel on one side go through focus on other, and vice versa.
- ❖ In ideal case – all rays from P imaged at P'.



# 薄透镜方程



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

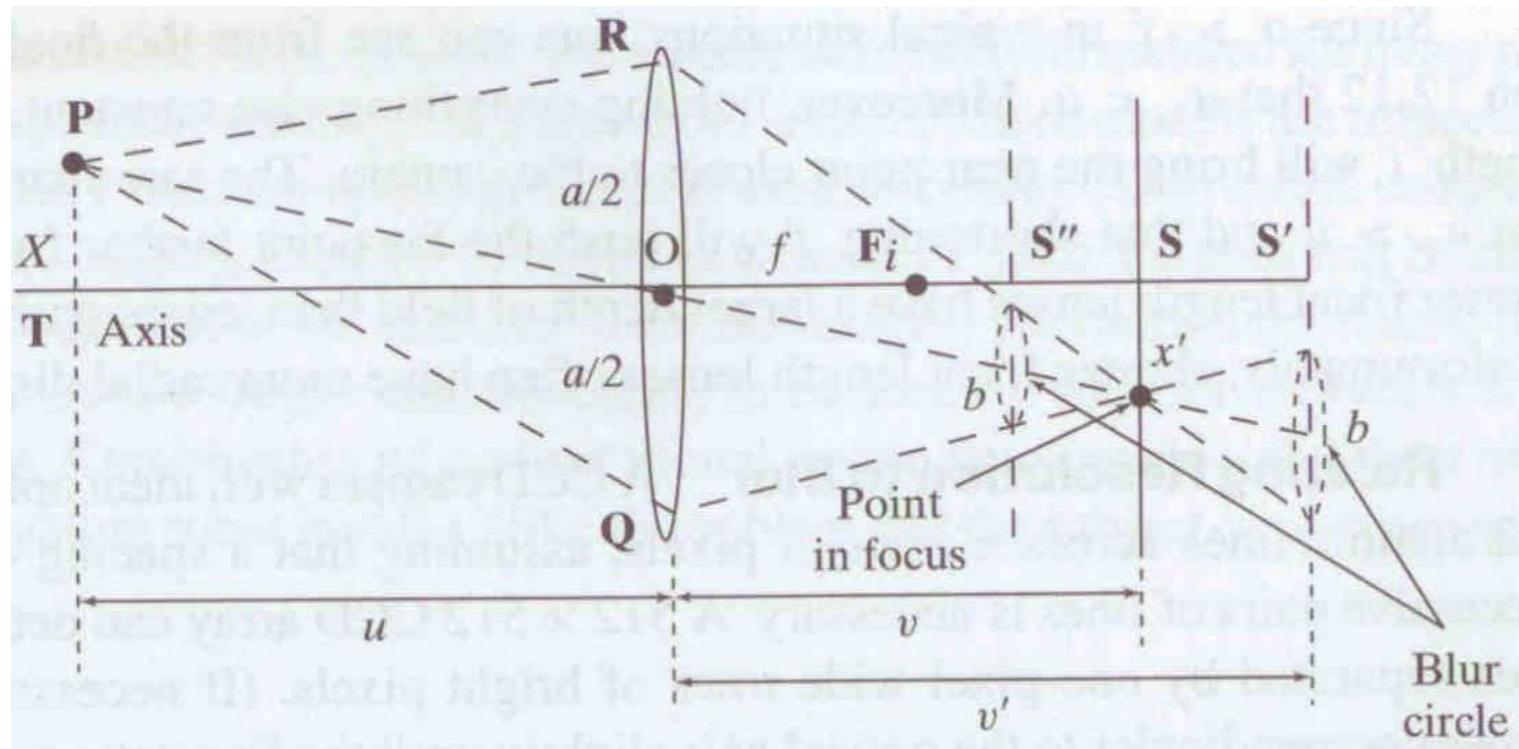
- ❖ 任何满足该方程的物点被聚焦

# 焦点与景深



DEPTH OF FIELD  
DEPTH OF FIELD

# 焦点与景深



← “circles of confusion” →

- ❖ 景深: 具有容忍范围内模糊度的不同图像平面的距离

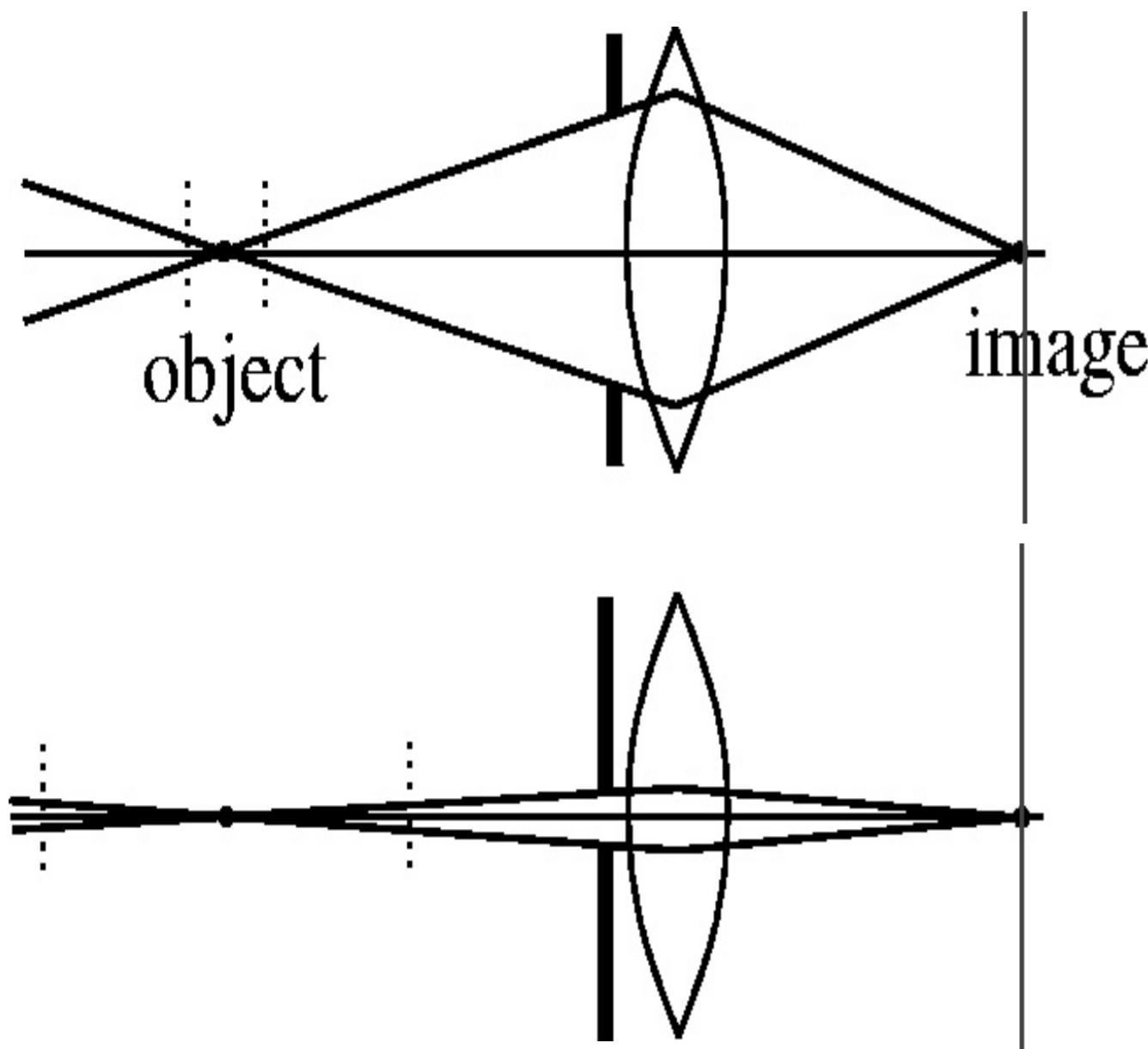
- ❖ 薄透镜: 不同深度的景物点聚焦到不同的图像平面
- ❖ 真实相机系统有更大的景深

# 通过改变光圈大小调节景深



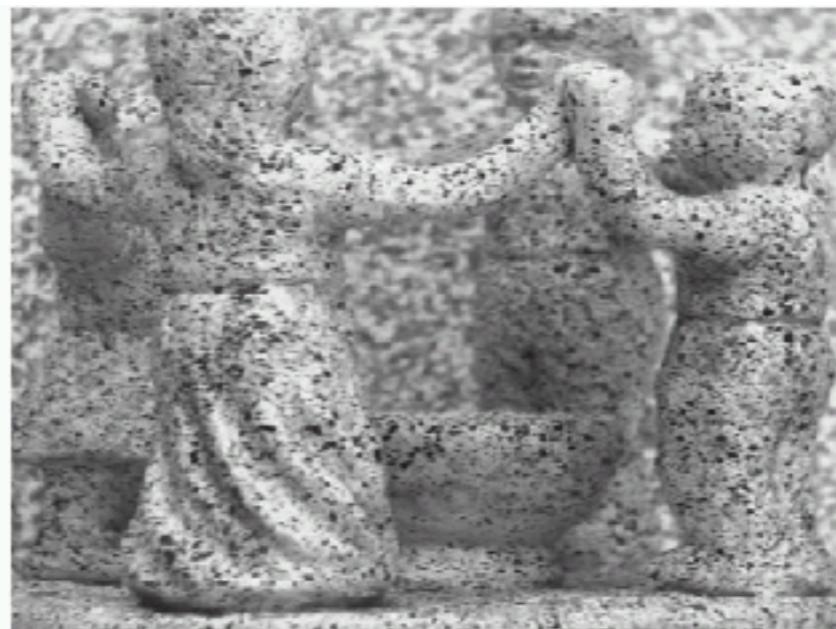
Slide by A. Efros

# 通过改变光圈大小调节景深

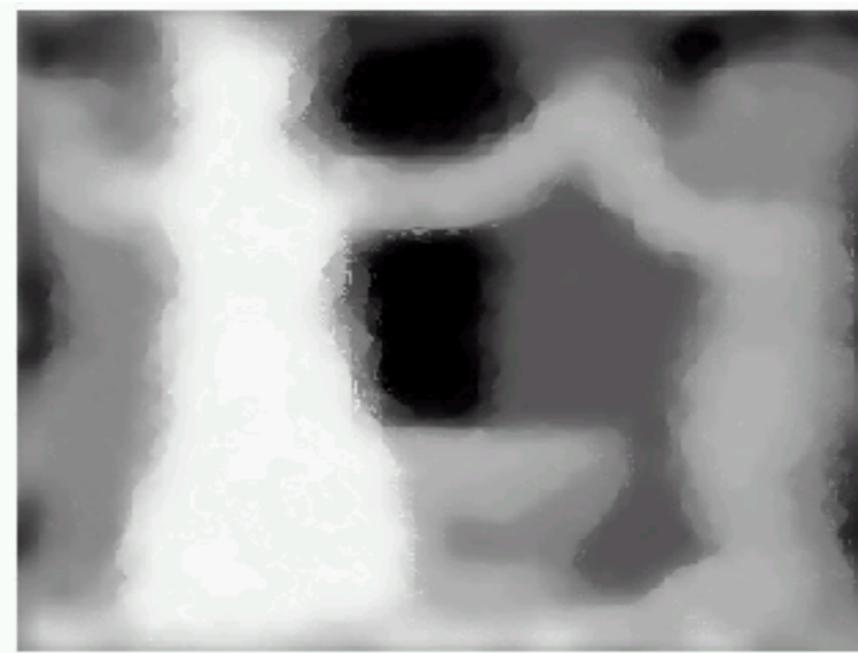
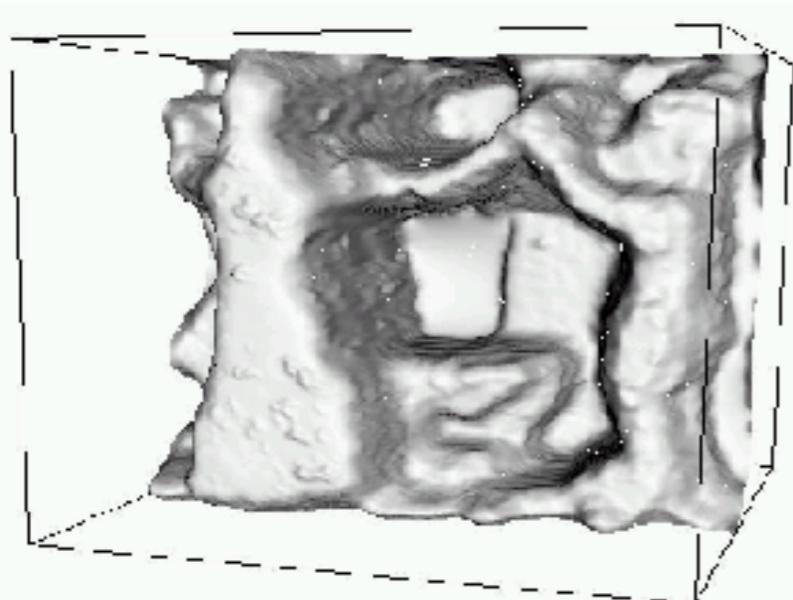


- ❖ 小光圈增大聚焦清晰的范围
- ❖ 但是小光圈降低进光量，需增加曝光时间

# 通过调焦获取深度



Images from same point  
of view, different  
camera parameters



3d shape / depth  
estimates

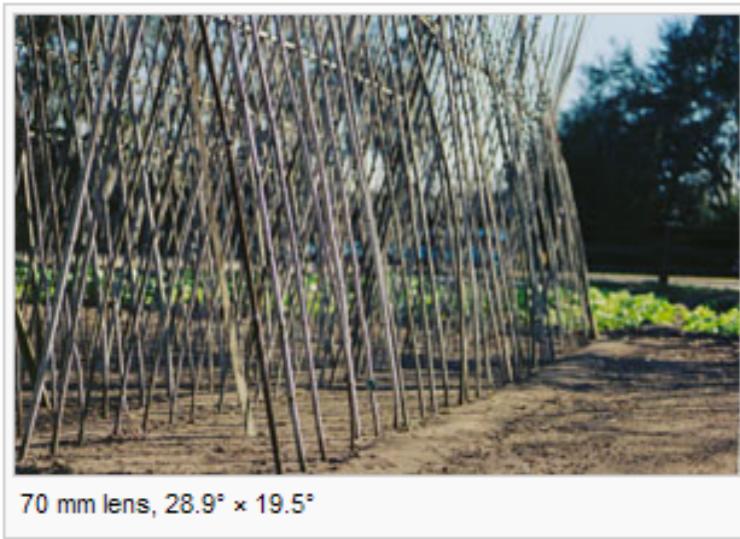
# 视场



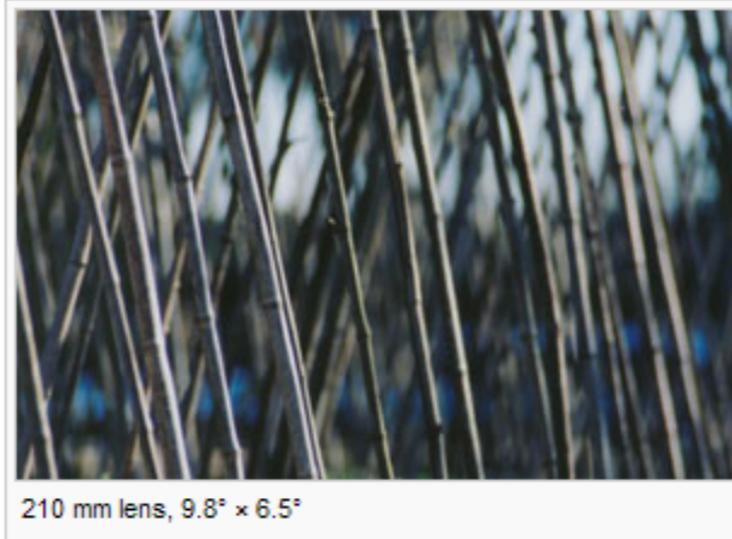
28 mm lens,  $65.5^\circ \times 46.4^\circ$



50 mm lens,  $39.6^\circ \times 27.0^\circ$



70 mm lens,  $28.9^\circ \times 19.5^\circ$



210 mm lens,  $9.8^\circ \times 6.5^\circ$

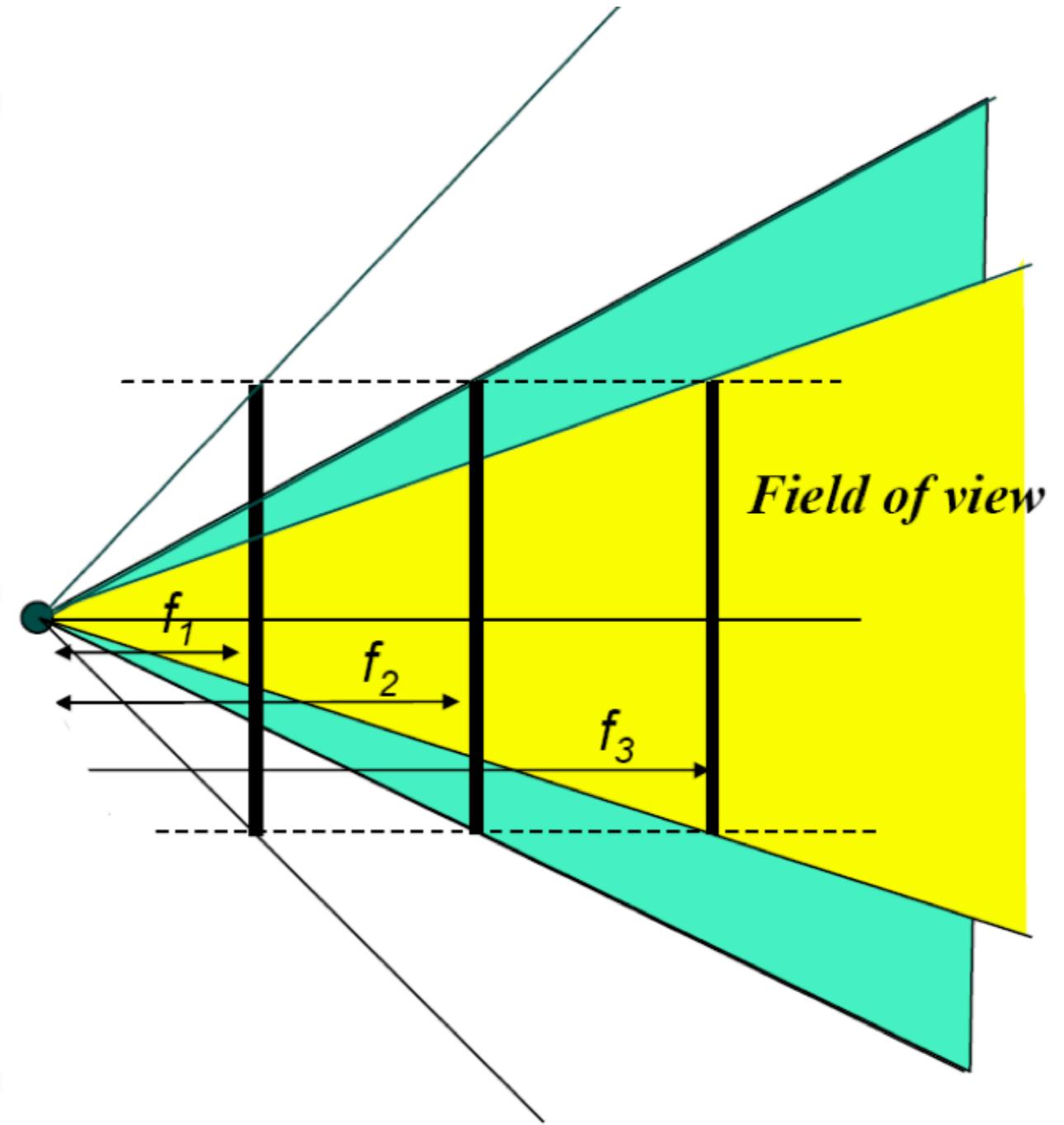
Angular measure  
of portion of 3d  
space seen by  
the camera

Images from [http://en.wikipedia.org/wiki/Angle\\_of\\_view](http://en.wikipedia.org/wiki/Angle_of_view)

Kristen Grauman

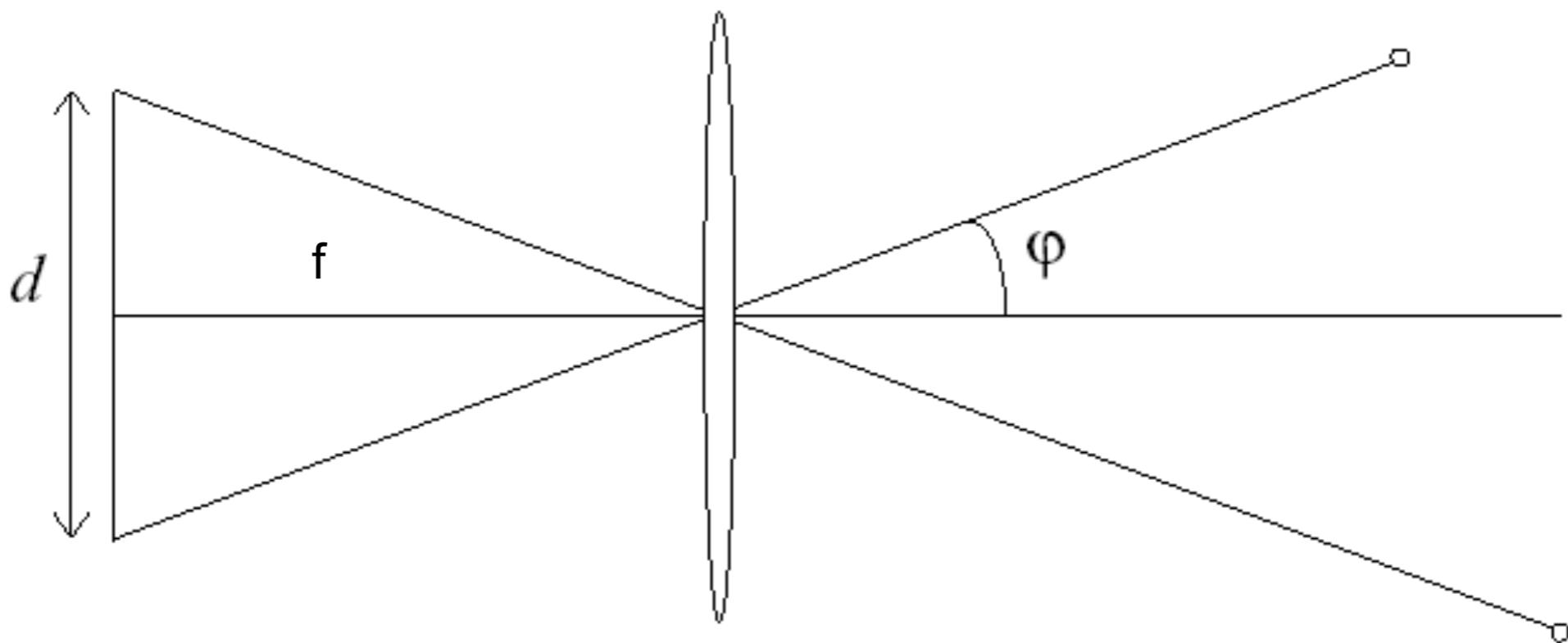
# 视场取决于焦距

- ❖ 焦距减小，视场角增大
- ❖ 焦距增大，视场角减小



from R. Duraiswami

# 视场取决于焦距

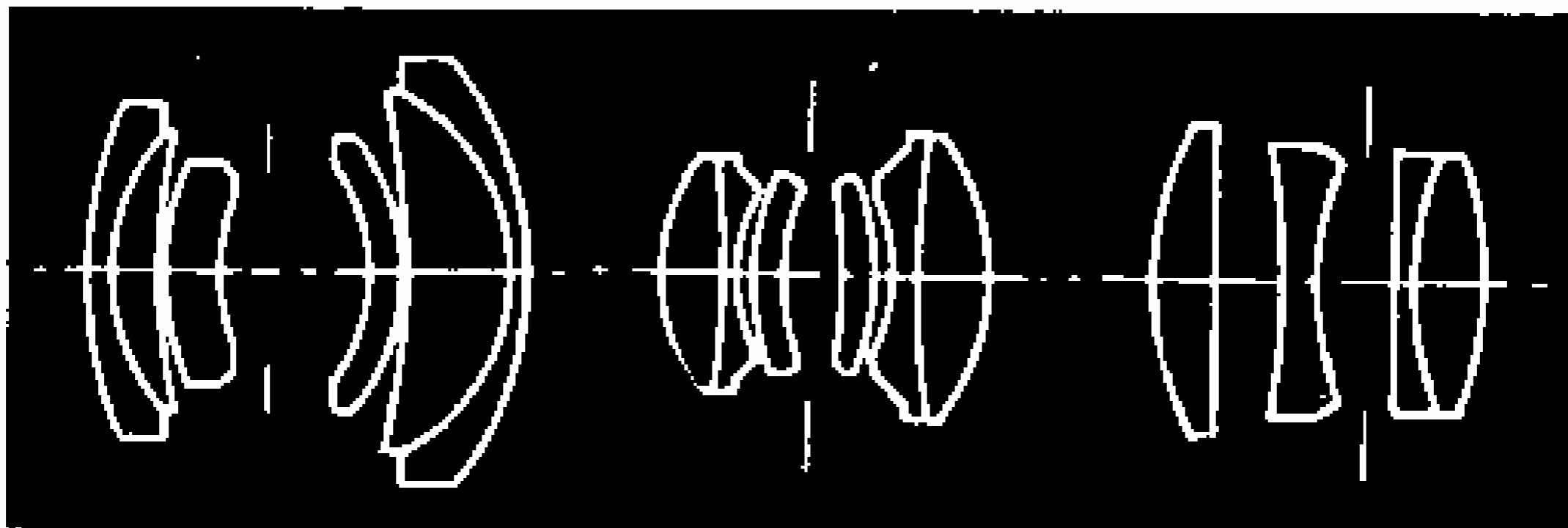


FOV depends on focal length and size of the camera retina

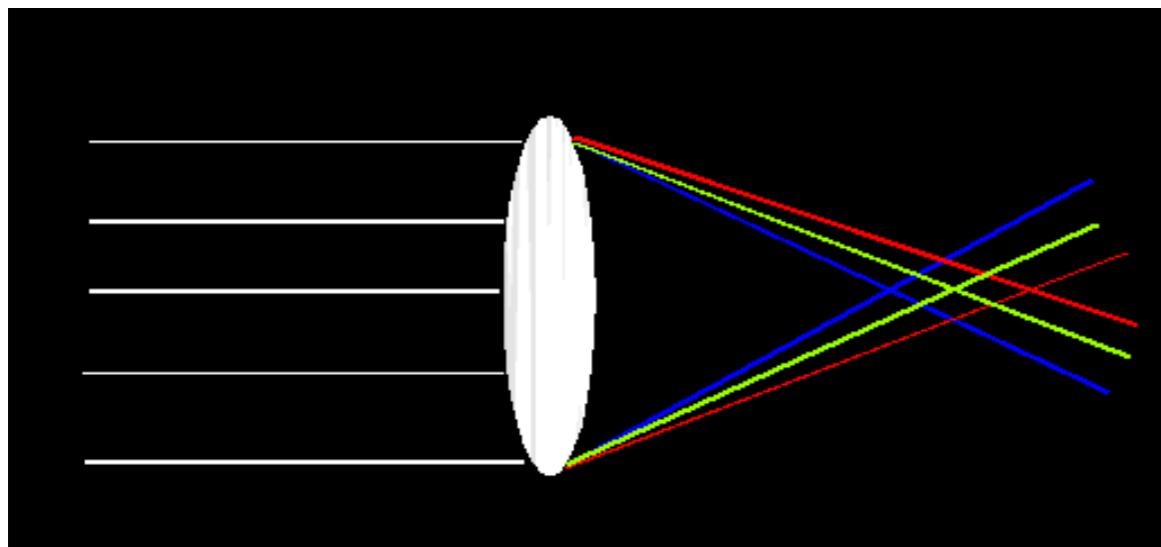
$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Smaller FOV = larger Focal Length

# 真实镜头组



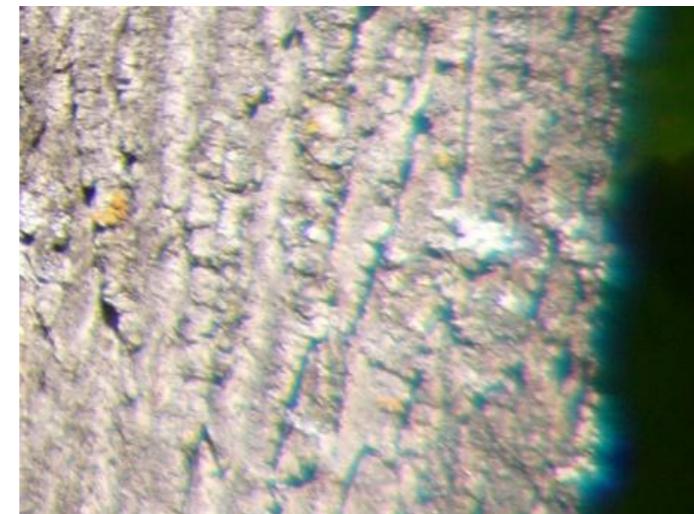
# 镜头缺陷：色差



镜头对不同波长的光线有不同的折射率，导致彩色边纹

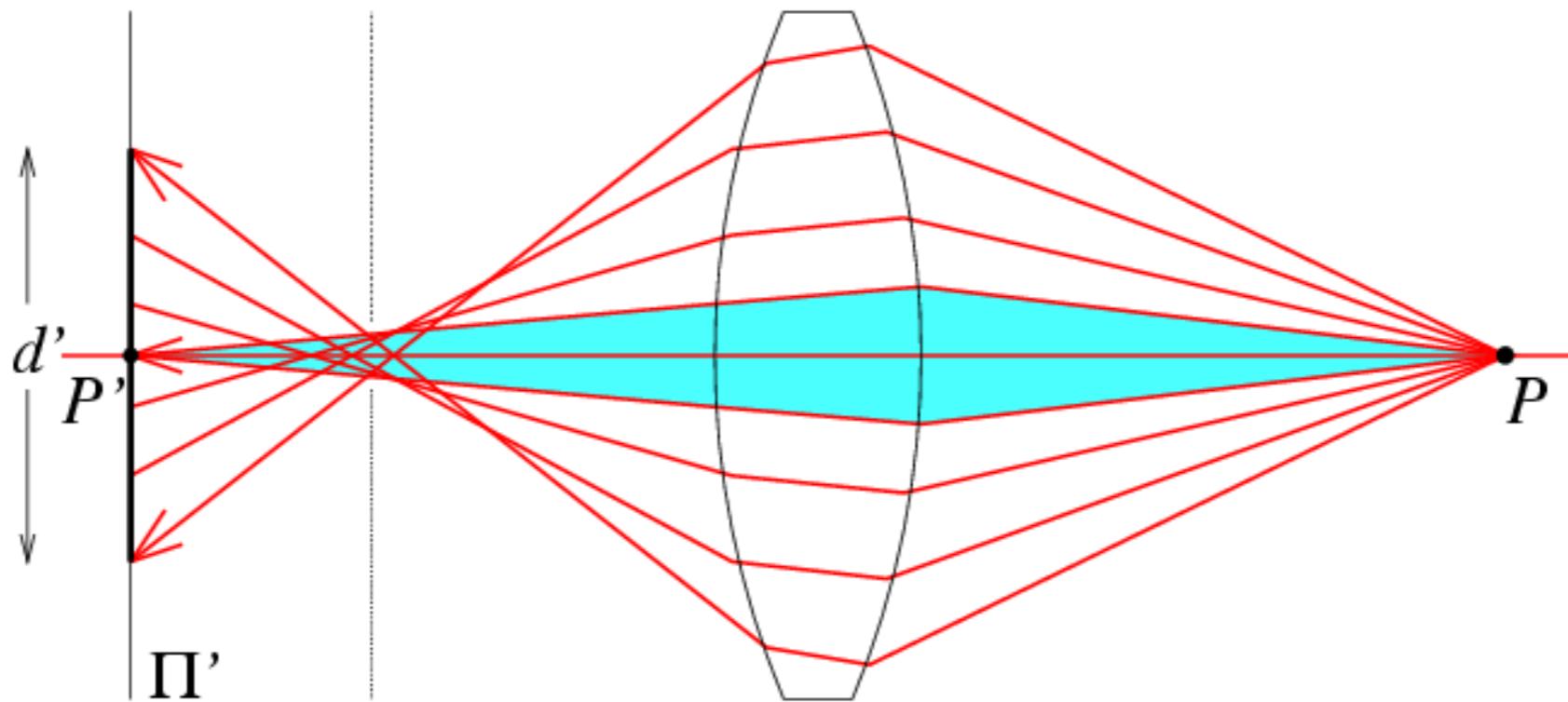


Near Lens Center



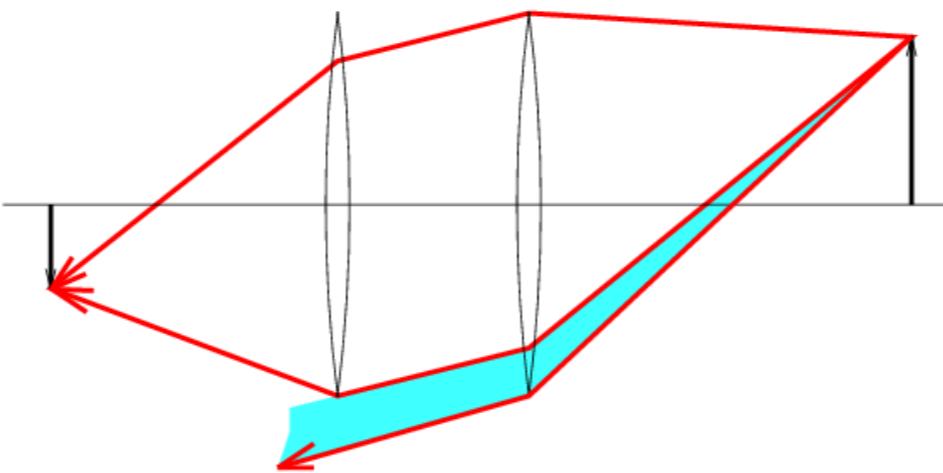
Near Lens Outer Edge

# 镜头缺陷: 球面像差



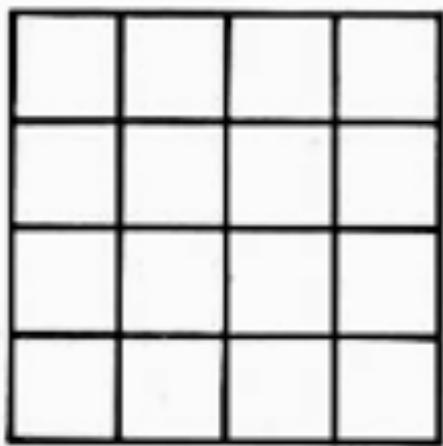
- ❖ 球状镜头不能完美聚焦
- ❖ 远离光轴的光线聚焦到更近的地方

# 镜头缺陷: 晕影

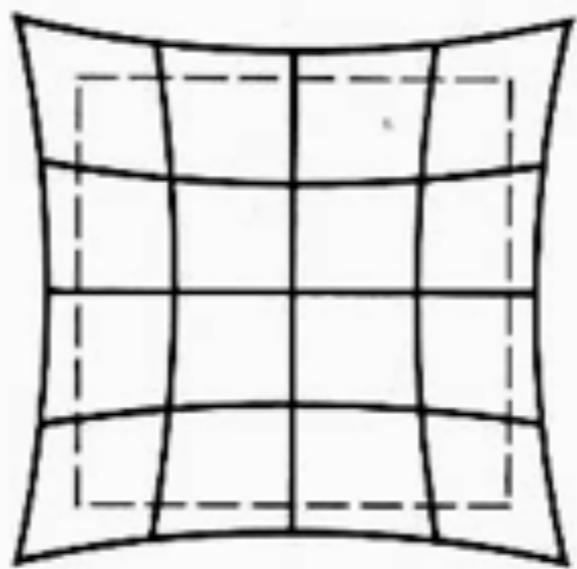


slides from Rob Fergus

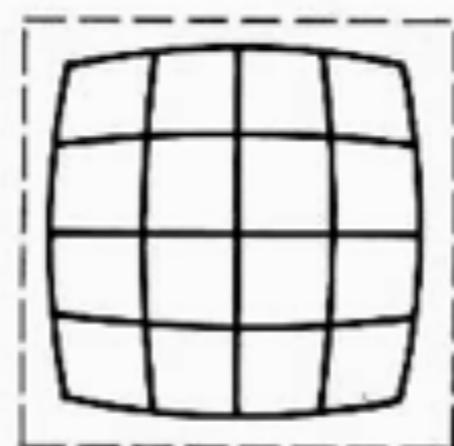
# 径向畸变



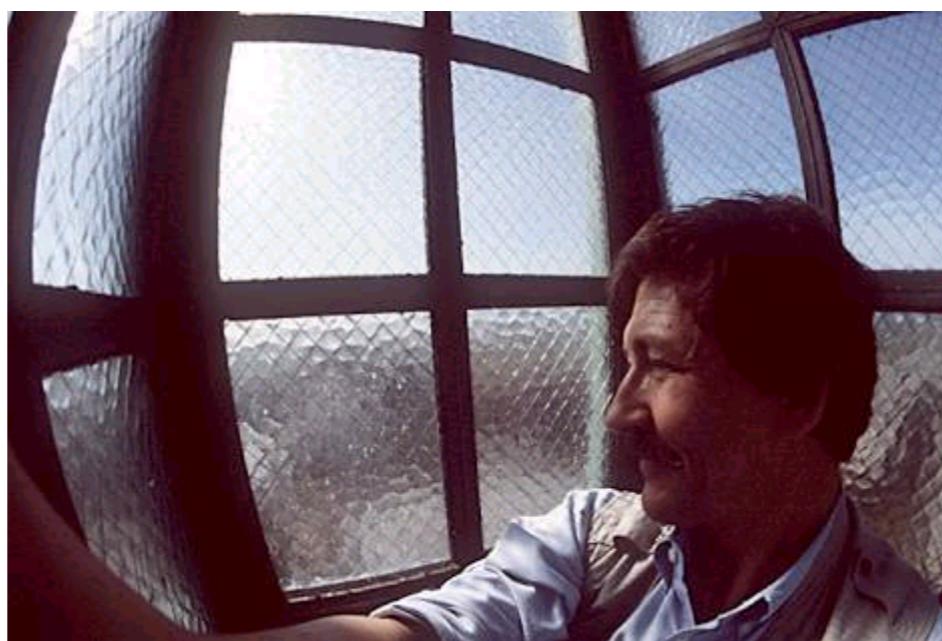
无畸变



枕形畸变

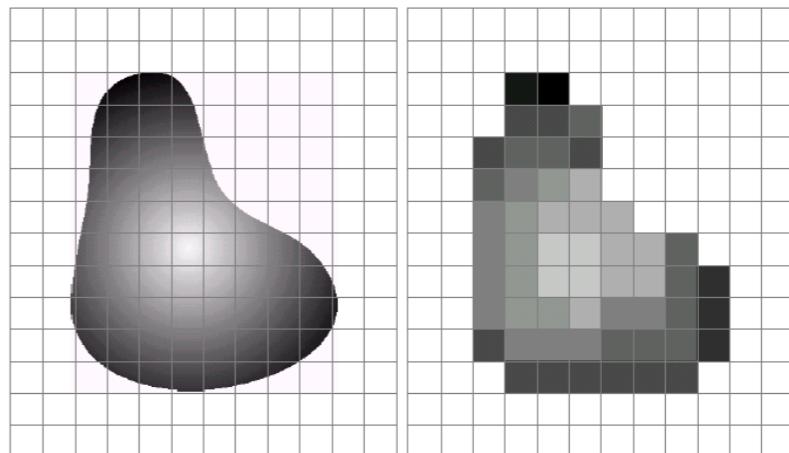


桶形畸变



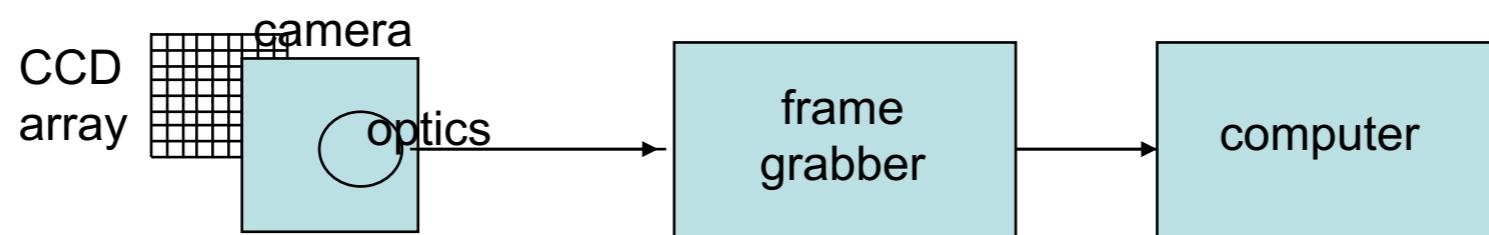
slides from Rob Fergus

# 数码相机



a b

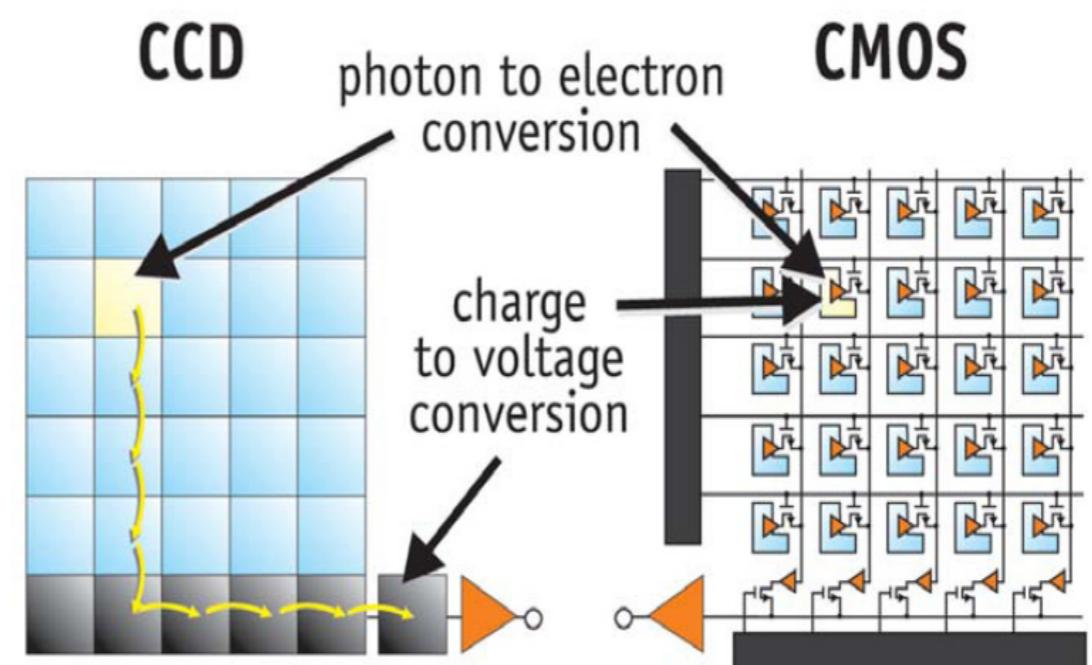
**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.



- ❖ 胶片 -> 芯片
- ❖ 电荷耦合器件阵列
- ❖ CCD：光敏二极管-将光信号转变为电信号

# CCD vs. CMOS

- ❖ **CCD:** transports the charge across the chip and reads it at one corner of the array. An **analog-to-digital converter (ADC)** then turns each pixel's value into a digital value by measuring the amount of charge at each photosite and converting that measurement to binary form
- ❖ **CMOS:** uses several transistors at each pixel to amplify and move the charge using more traditional wires. The CMOS signal is digital, so it needs no ADC.



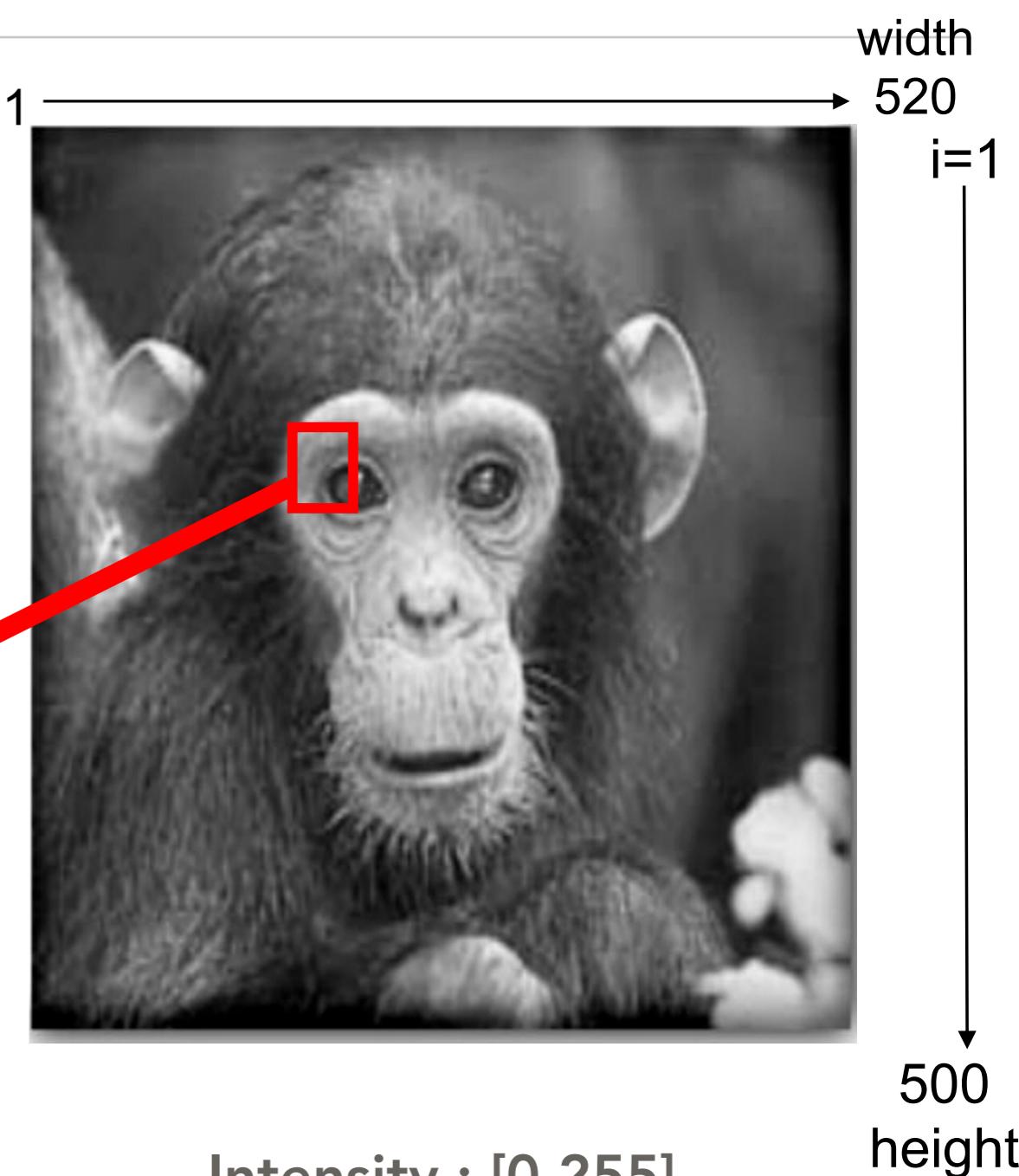
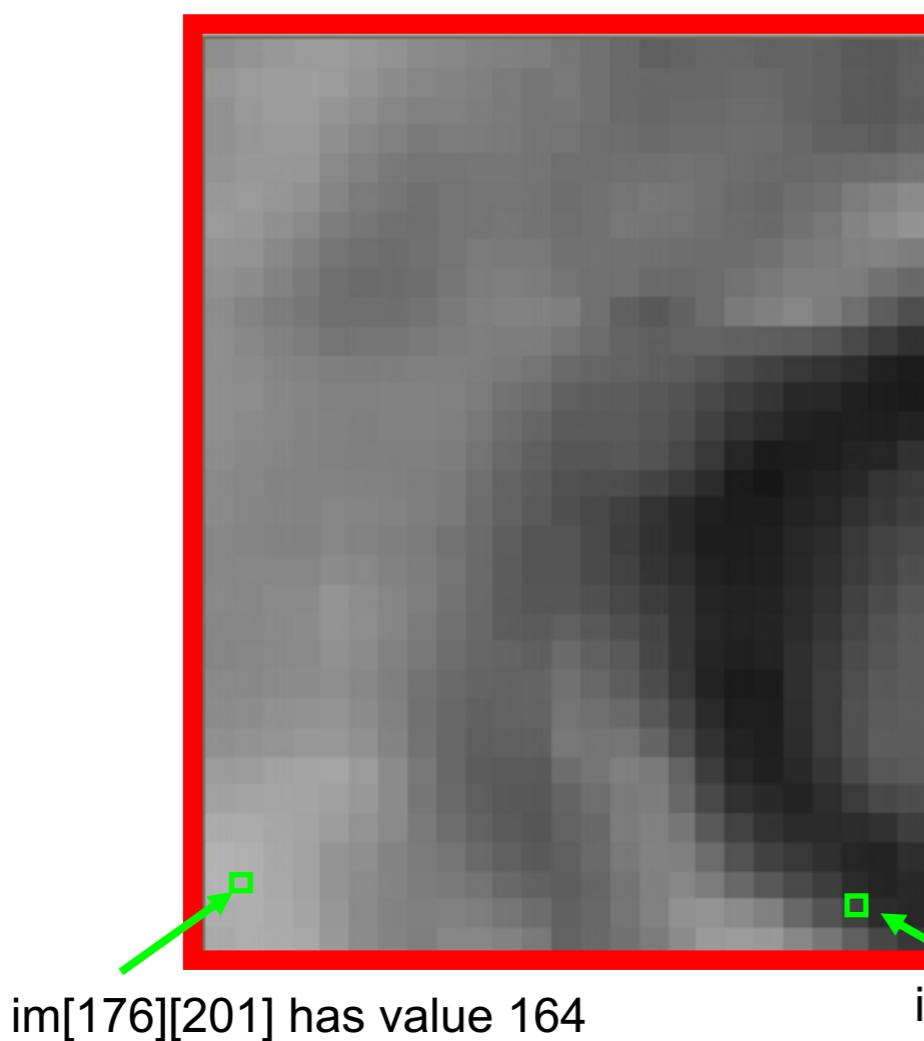
*CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.*

[http://www.dalsa.com/shared/content/pdfs/CCD\\_vs\\_CMOS\\_Litwiller\\_2005.pdf](http://www.dalsa.com/shared/content/pdfs/CCD_vs_CMOS_Litwiller_2005.pdf)

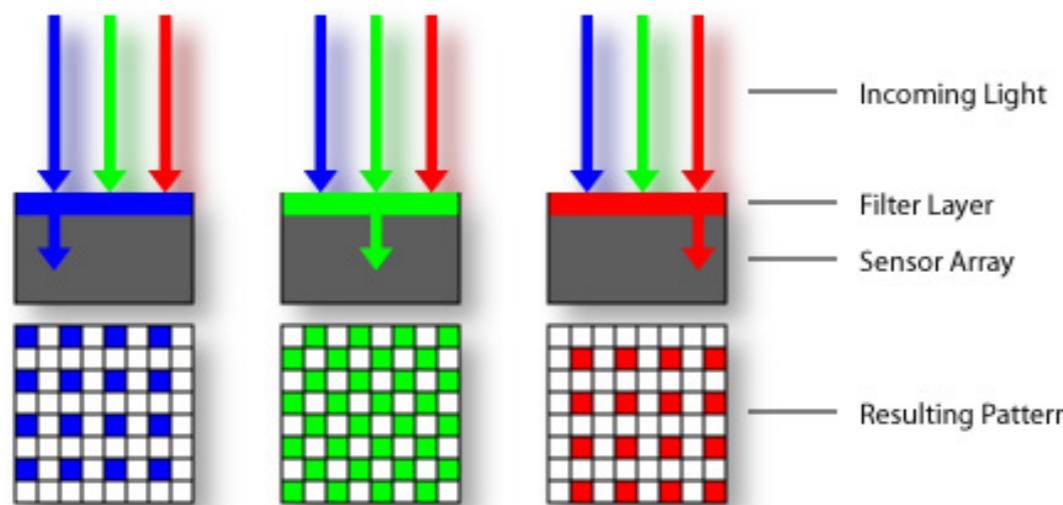
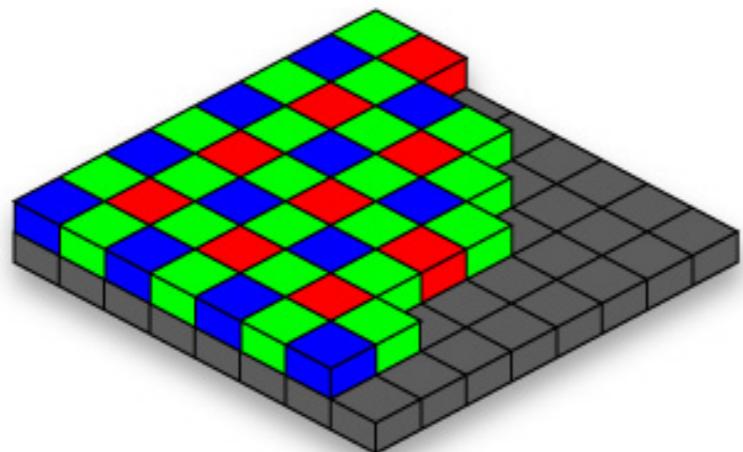
<http://electronics.howstuffworks.com/digital-camera.htm>

# 数字图像

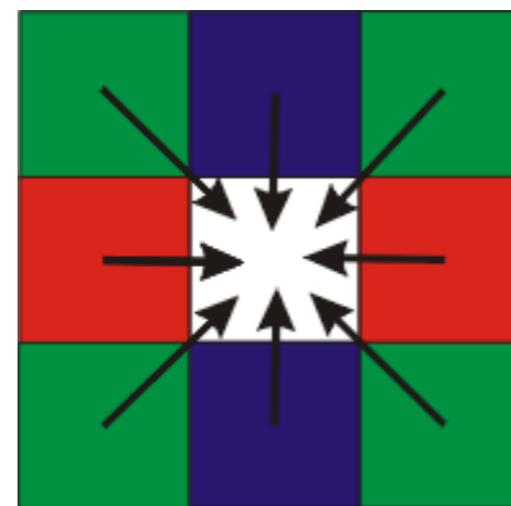
图像可以看成由CCD 阵列形成的矩阵.



# 颜色感知：彩色滤光阵列

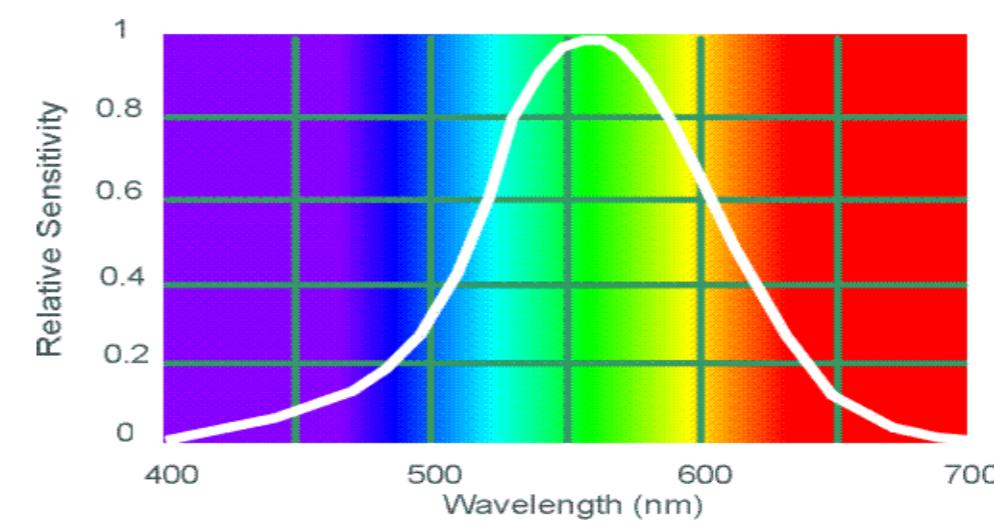


Bayer grid



Estimate missing components from neighboring values (demosaicing)

Why more green?



Human Luminance Sensitivity Function

# 颜色计算

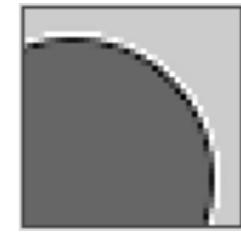
?	red	?	red	?	red
?	?	?	?	?	?
?	red	?	red	?	red
?	?	?	?	?	?
?	?	?	red	?	red
?	?	?	?	?	?

green	?	green	?	green	?
?	green	?	green	?	green
green	?	green	?	green	?
?	green	?	green	?	green
green	?	green	?	green	?
?	green	?	green	?	green

?	?	?	?	?	?
blue	?	?	?	?	?
?	?	?	?	?	?
?	?	?	?	?	?
blue	?	?	?	?	?
?	?	?	?	?	?

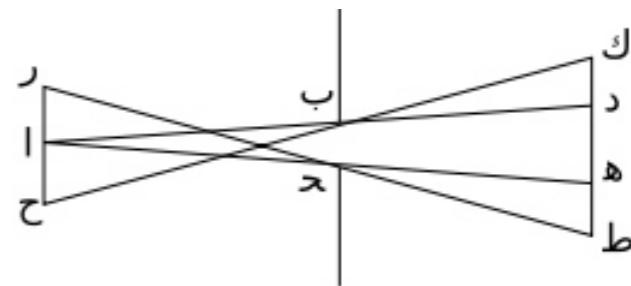
# 数字图像的失真

- ❖ 噪声
  - ❖ 光线暗的时候容易引入噪声
  - ❖ 光线敏感度增加 (ISO) 会引入更多噪声
  - ❖ 坏点
- ❖ 相机内部的图像处理
  - ❖ 过度锐化生成光晕
  - ❖ 压缩带来失真、块效应
  - ❖ 散焦效应
    - ❖ 电荷溢出到相邻像素
- ❖ 颜色失真
  - ❖ 紫晕 (微镜头)
  - ❖ 白平衡

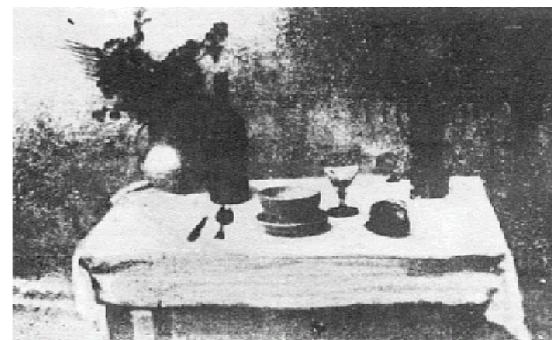


# 相机历史回顾

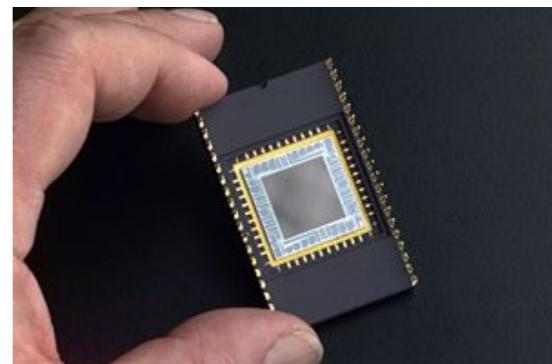
- ❖ **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- ❖ **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- ❖ **Camera obscura:** Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- ❖ **First photo:** Joseph Nicéphore Niépce (1822)
- ❖ **Daguerreotypes** (1839)
- ❖ **Photographic film** (Eastman, 1889)
- ❖ **Cinema** (Lumière Brothers, 1895)
- ❖ **Color Photography** (Lumière Brothers, 1908)
- ❖ **Television** (Baird, Farnsworth, Zworykin, 1920s)
- ❖ **First consumer camera with CCD:** Sony Mavica (1981)
- ❖ **First fully digital camera:** Kodak DCS100 (1990)



Alhacen's notes



Niépce, "La Table Servie," 1822



CCD chip

Slide credit: L. Lazebnik

# 总结

- ❖ 影响图像生成的主要因素：几何、光度测量、光学
- ❖ 透视投影方程描述了三维点如何被映射为图像点
- ❖ 齐次坐标使透视投影可以被描述为线性系统
- ❖ 镜头的使用使小孔成像变为实际照相机
- ❖ 多个参数会影响到成像 (焦距, 光圈大小, 镜头直径,...)