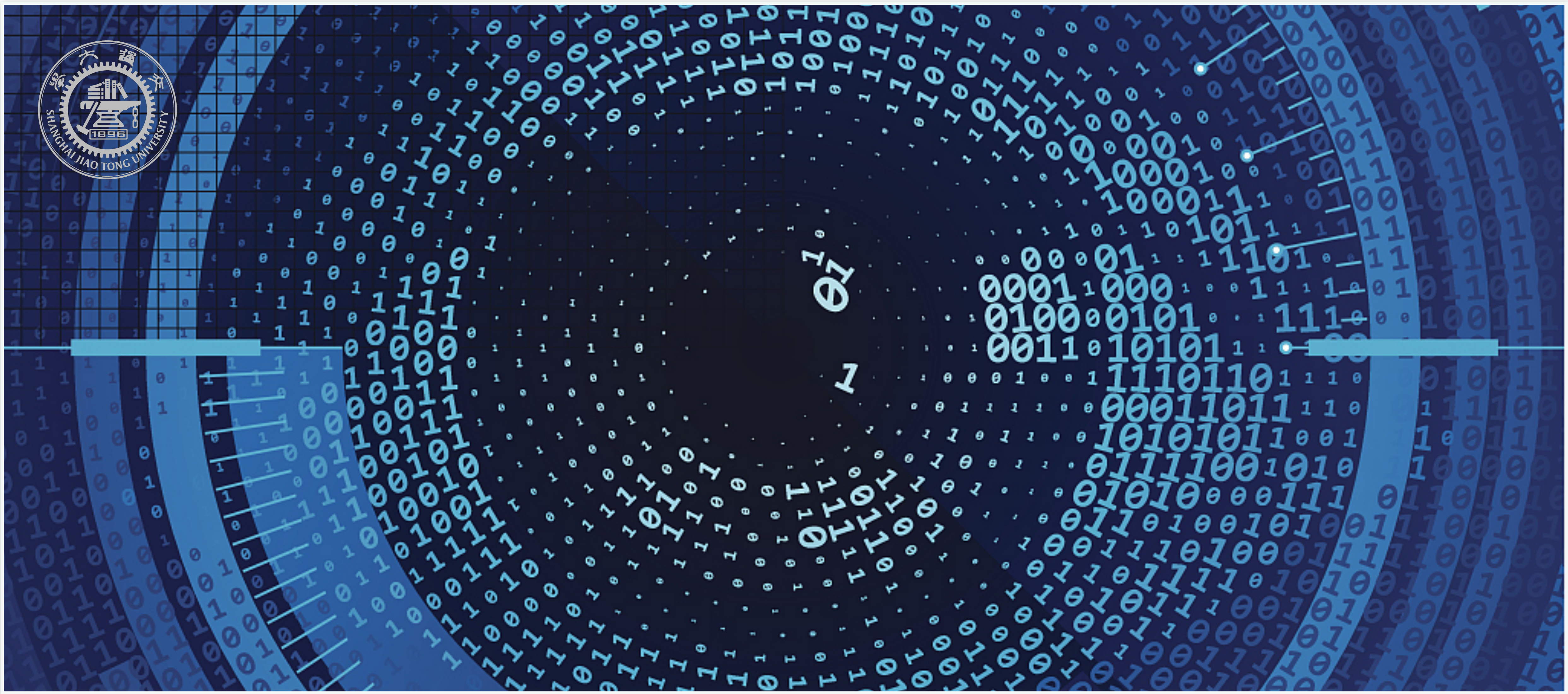

上海交通大学

计算机视觉

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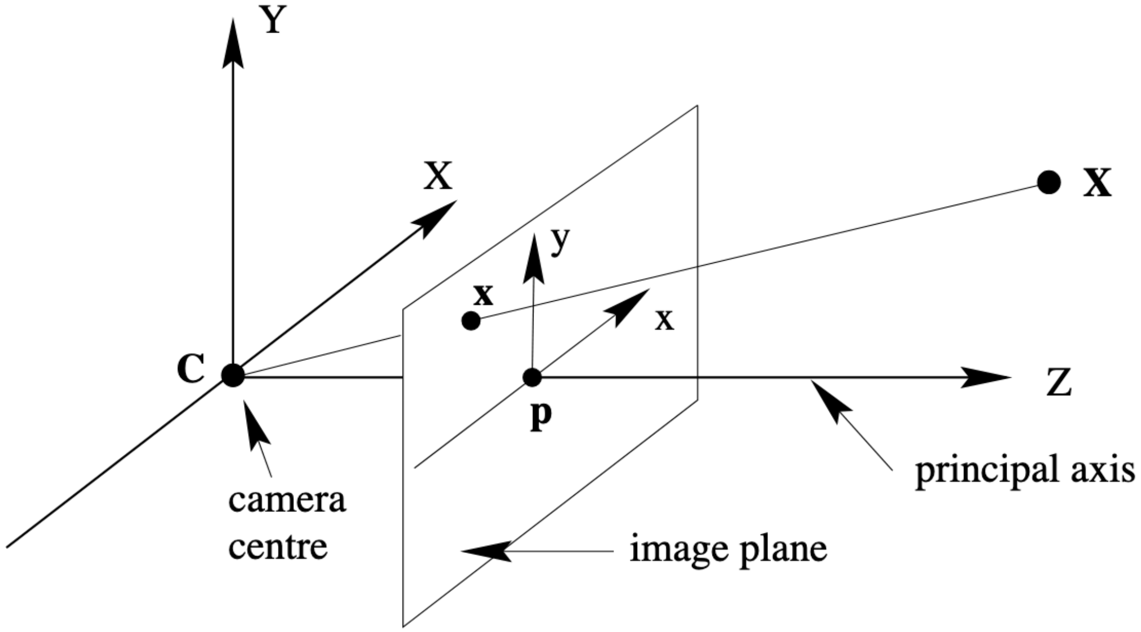
5. 摄像机参数标定

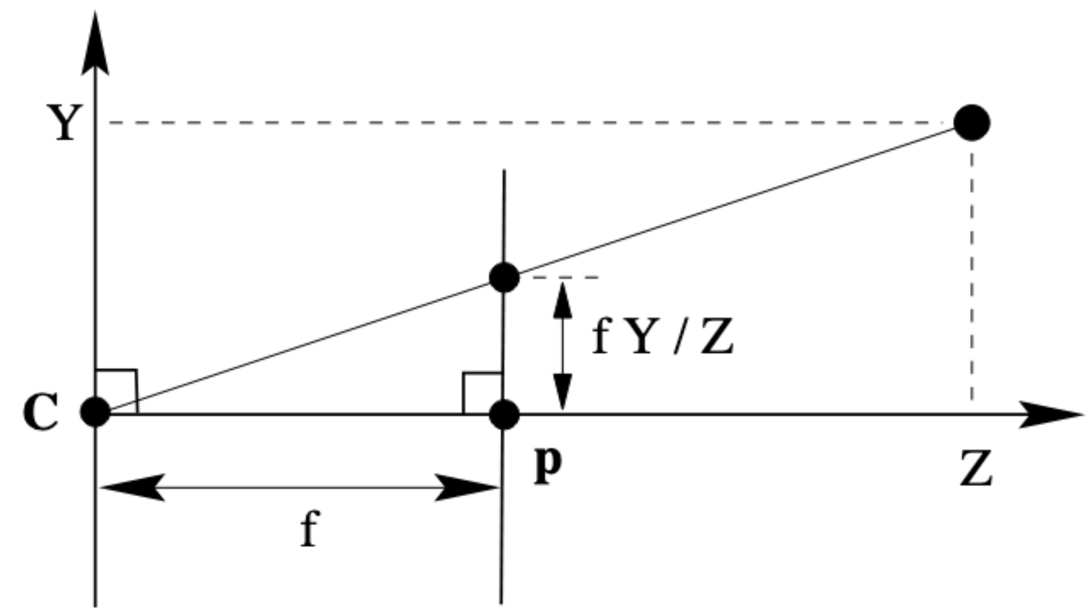
主要内容

- 摄像机模型及其内外参数
- 参数估计（摄像机标定）

摄像机模型

principal plane





$$(X, Y, Z)^{\mathsf{T}} \mapsto (fX/Z, fY/Z)^{\mathsf{T}}$$

齐次表示：

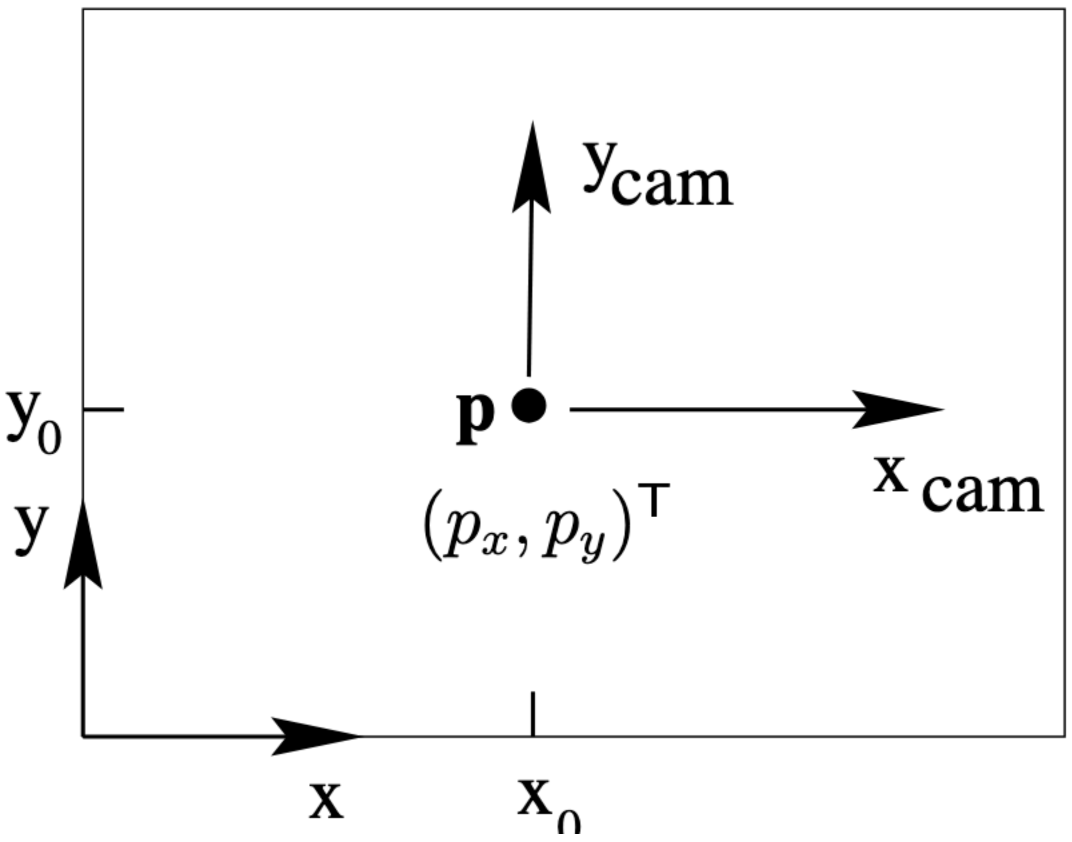
$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow$$

$$\mathbf{x} = \mathtt{P}\mathbf{X}$$

$$\mathtt{P} = \mathrm{diag}(f, f, 1)\,[\mathtt{I} \mid \mathbf{0}]$$

3×4

摄像机模型



图像平面的原点和主点不重合

$$(\mathrm{X}, \mathrm{Y}, \mathrm{Z})^\mathsf{T} \mapsto (f\mathrm{X}/\mathrm{Z} + p_x, f\mathrm{Y}/\mathrm{Z} + p_y)^\mathsf{T}$$

$$\begin{pmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} f\mathrm{X} + \mathrm{Z}p_x \\ f\mathrm{Y} + \mathrm{Z}p_y \\ \mathrm{Z} \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \\ 1 \end{pmatrix}$$

$$\mathtt{K} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

DoFs: 3

$$\mathbf{x} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}]\mathbf{X}_{\mathrm{cam}}$$

摄像机模型

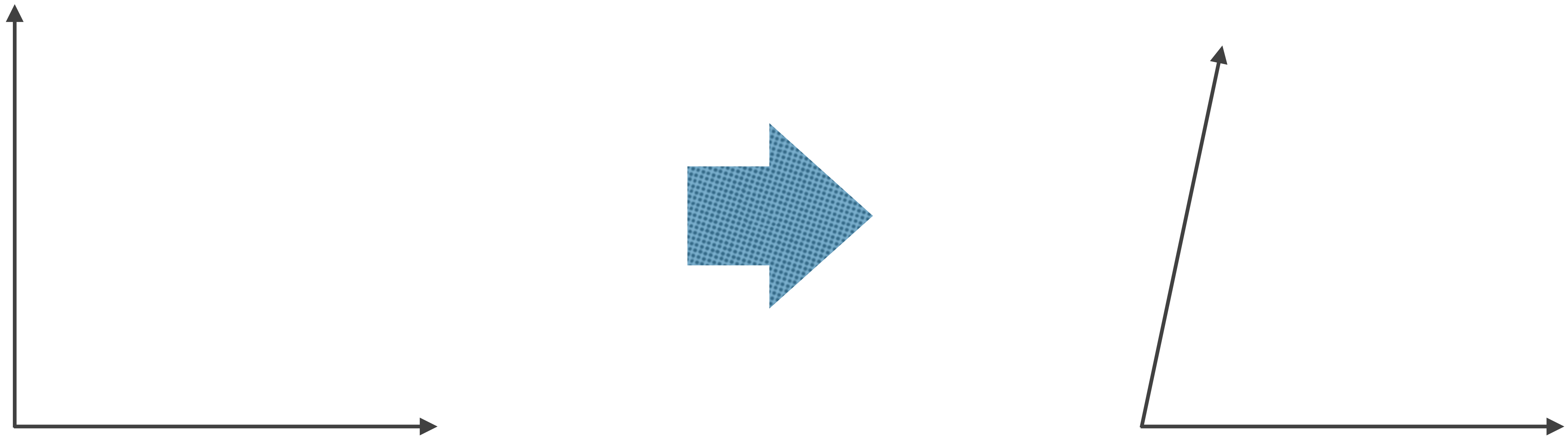
- 如果图像坐标系的x, y尺度不同
- 则在x, y轴各自的单位距离上像素数不同： m_x, m_y

$$
\mathtt{K} = \begin{bmatrix} \alpha_x & & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}
$$

DoFs: 4

$$\alpha_x = f m_x \qquad \alpha_y = f m_y$$

摄像机模型

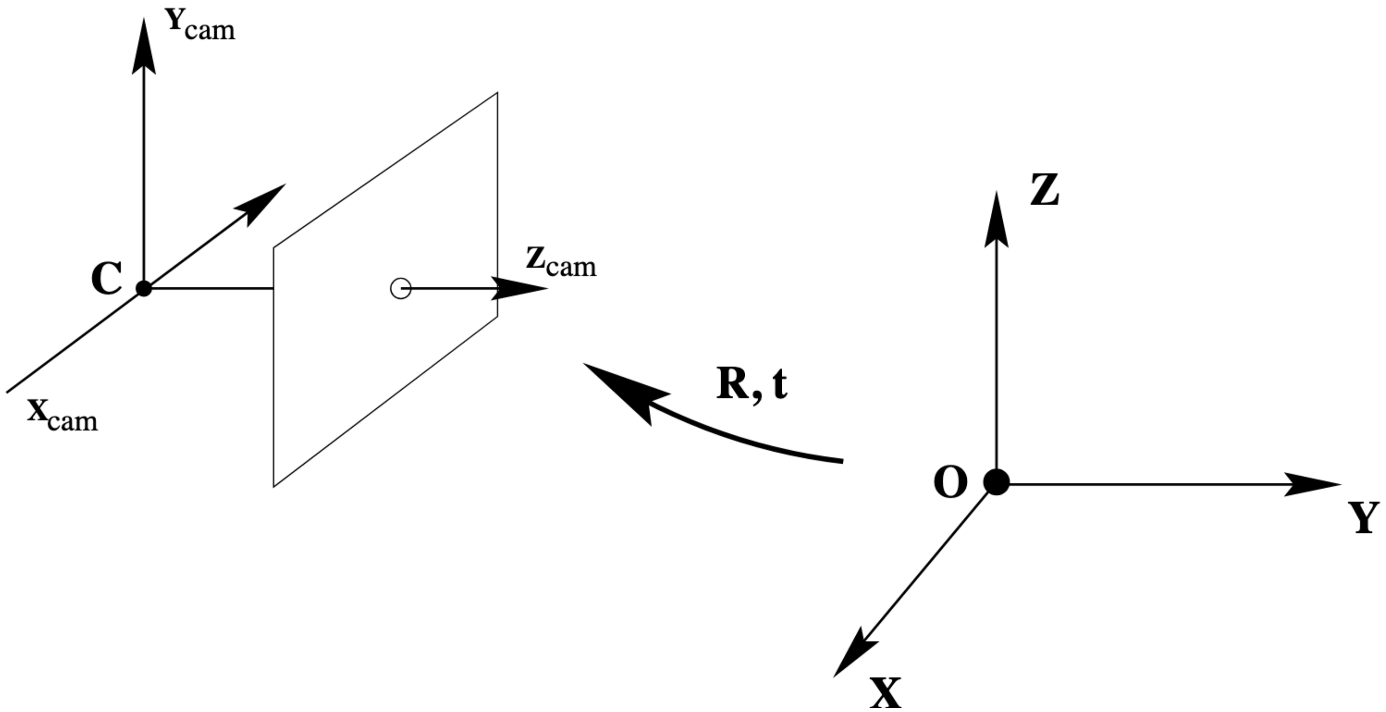


s - skew（偏斜）参数

$$\mathtt{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

DoFs: 5

摄像机模型



$$\left\{\begin{aligned} \mathbf{X}_{\mathrm{cam}} &= \begin{bmatrix} \mathrm{R} & -\mathrm{R}\widetilde{\mathbf{C}} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \\ 1 \end{pmatrix} = \begin{bmatrix} \mathrm{R} & -\mathrm{R}\widetilde{\mathbf{C}} \\ 0 & 1 \end{bmatrix} \mathbf{X} \\ \mathbf{x} &= \mathrm{K}[\mathrm{I} \mid \mathbf{0}]\mathbf{X}_{\mathrm{cam}} \end{aligned}\right.$$

$$\mathbf{x} = \mathrm{KR}[\mathrm{I} \mid -\widetilde{\mathbf{C}}]\mathbf{X}$$

$$\mathrm{P} = \mathrm{KR}[\mathrm{I} \mid -\widetilde{\mathbf{C}}]$$

$$\mathbf{x} = \mathrm{P}\mathbf{X}$$

K - 相机内参数矩阵

$R, \tilde{C}$ - 相机外参数

DoFs: 11

摄像机模型

$$\mathtt{P} = \mathtt{KR}[\mathtt{I} \mid -\widetilde{\mathbf{C}}]$$

3 × 4 齐次矩阵

$$\mathtt{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

$$\mathtt{M} = \mathtt{KR}$$

3 × 3 非奇异矩阵

R - 旋转矩阵

通过RQ分解可以得到K和R

<u>有限射影相机的相机矩阵等价于3 × 4齐次矩阵，且该矩阵的左3 × 3矩阵为非奇异矩阵</u>

$$\mathtt{P} = [\mathtt{M} \mid \mathbf{p}_4]$$

P：3 × 4 矩阵，$rank(P) = 3$

P存在1维右零空间（存在秩3列线性相关）

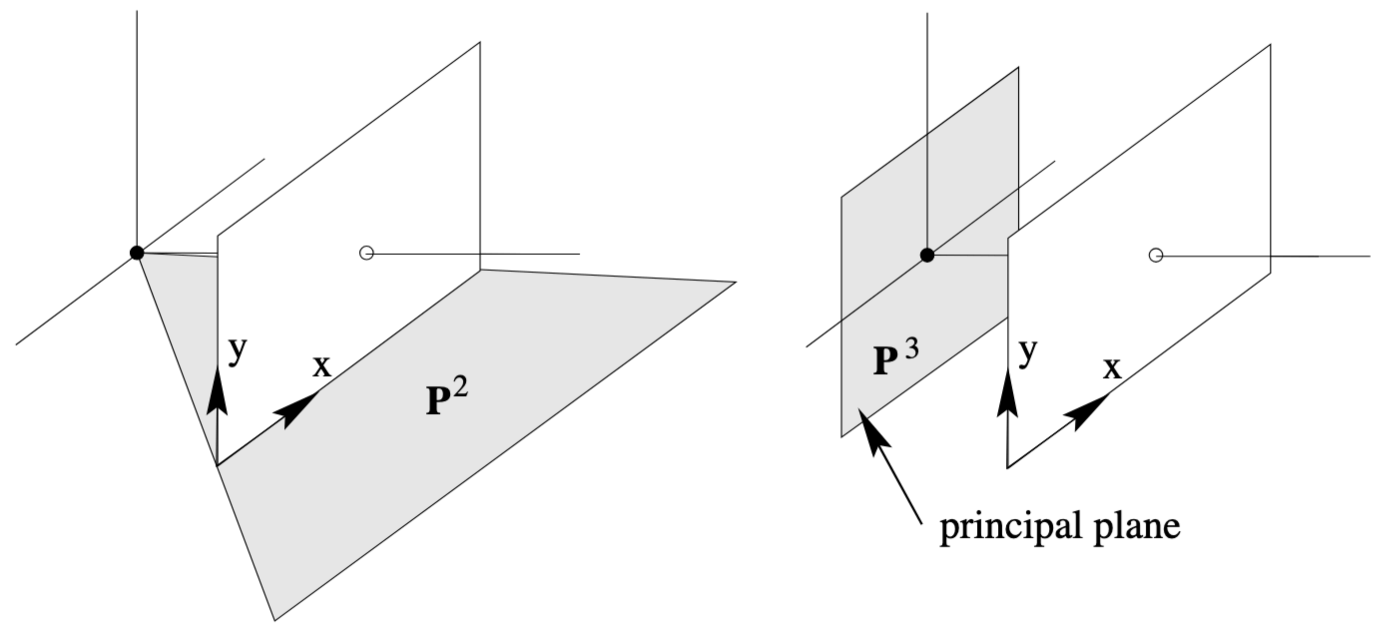
$$\mathtt{P}\mathbf{C} = \mathbf{0}$$

通过SVD分解可得C

C-相机中心

摄像机模型

$$
\begin{array}{cc}
 & \begin{array}{cccc} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4 \end{array} \\
\mathtt{P} = & \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{1\mathsf{T}} \\ \mathbf{P}^{2\mathsf{T}} \\ \mathbf{P}^{3\mathsf{T}} \end{bmatrix}
\end{array}
$$



$$\mathbf{p}_i, i = 1,2,3$$

3个列向量分别对应于世界坐标系X,Y,Z轴对应的消影点

$$\mathbf{D} = (1, 0, 0, 0)^{\mathsf{T}}$$

$$\mathbf{p}_1 = \mathtt{P}\mathbf{D}$$

摄像机模型

$$\mathtt{P} = [\mathtt{M} \mid -\mathtt{M}\widetilde{\mathbf{C}}] = \mathtt{K}[\mathtt{R} \mid -\mathtt{R}\widetilde{\mathbf{C}}]$$

一旦求得P，通过RQ分解可以得到K和R

$$\mathtt{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

相机矩阵P的计算

平面单应的求解：

$$\mathbf{x}' = \mathtt{H}_{\mathrm{P}}\mathbf{x} = \begin{bmatrix} \mathtt{A} & \mathbf{t} \\ \mathbf{v}^{\mathsf{T}} & v \end{bmatrix} \mathbf{x}$$

$$\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$$

相机矩阵的求解：

$$\mathbf{x} = \mathtt{P}\mathbf{X}$$

$$\mathtt{P} = [\mathtt{M} \mid -\mathtt{M}\widetilde{\mathbf{C}}] = \mathtt{K}[\mathtt{R} \mid -\mathtt{R}\widetilde{\mathbf{C}}]$$

$$\mathbf{x}_i \leftrightarrow \mathbf{X}_i$$

相机矩阵P的计算

$$\mathbf{x}_i = (x_i, y_i, \omega_i)^T$$

$$\mathbf{x}_i \times P\mathbf{X}_i = 0$$

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i^\top & y_i\mathbf{X}_i^\top \\ w_i\mathbf{X}_i^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i^\top \\ -y_i\mathbf{X}_i^\top & x_i\mathbf{X}_i^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}.$$

$$\underbrace{\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i^\top & y_i\mathbf{X}_i^\top \\ w_i\mathbf{X}_i^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i^\top \end{bmatrix}}_{2 \times 12} \underbrace{\begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix}}_{12 \times 1} = \mathbf{0}$$

相机矩阵P的计算：最小配置解

- $\mathbf{x}_i = P\mathbf{X}_i$, $\mathbf{x}_i \leftrightarrow \mathbf{X}_i$
- $A\mathbf{p} = 0$
  - A 为$2n \times 12$ 矩阵
- $\mathbf{p}$有12个元素，11个自由度，因此，**最小配置解**需要5.5个点对应，这时候**A**为11×12矩阵
- 通过SVD分解可以求得$\mathbf{p}$

相机矩阵P的计算：DLT

- $\mathbf{x}_i = P\mathbf{X}_i$, $\mathbf{x}_i \leftrightarrow \mathbf{X}_i$
- 超定解：当有更多的点对应时（$n \geq 6$）
  - 优化问题：$\min \|A\mathbf{p}\|$ 服从约束：$\|\mathbf{p}\| = 1$
- 通过SVD分解求解

相机矩阵P的计算：黄金标准算法

Objective

Given $n \geq 6$ world to image point correspondences $\{\mathbf{X}_i \leftrightarrow \mathbf{x}_i\}$, determine the Maximum Likelihood estimate of the camera projection matrix $\mathtt{P}$, i.e. the $\mathtt{P}$ which minimizes $\sum_i d(\mathbf{x}_i, \mathtt{P}\mathbf{X}_i)^2$.

Algorithm

(i) **Linear solution.** Compute an initial estimate of $\mathtt{P}$ using a linear method such as algorithm 4.2(*p*109):

  (a) **Normalization:** Use a similarity transformation $\mathtt{T}$ to normalize the image points, and a second similarity transformation $\mathtt{U}$ to normalize the space points. Suppose the normalized image points are $\tilde{\mathbf{x}}_i = \mathtt{T}\mathbf{x}_i$, and the normalized space points are $\tilde{\mathbf{X}}_i = \mathtt{U}\mathbf{X}_i$.

  (b) **DLT:** Form the $2n \times 12$ matrix $\mathtt{A}$ by stacking the equations (7.2) generated by each correspondence $\tilde{\mathbf{X}}_i \leftrightarrow \tilde{\mathbf{x}}_i$. Write $\mathbf{p}$ for the vector containing the entries of the matrix $\tilde{\mathtt{P}}$. A solution of $\mathtt{A}\mathbf{p} = \mathbf{0}$, subject to $\|\mathbf{p}\| = 1$, is obtained from the unit singular vector of $\mathtt{A}$ corresponding to the smallest singular value.

(ii) **Minimize geometric error.** Using the linear estimate as a starting point minimize the geometric error (7.4):

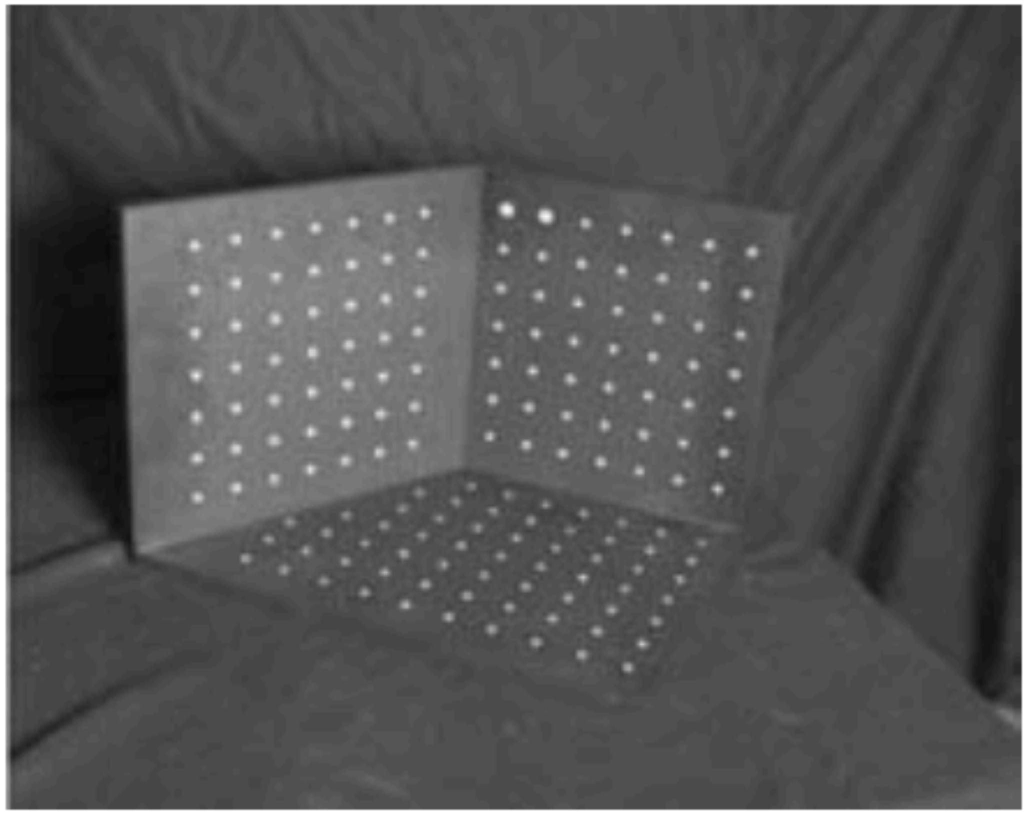
$$\sum_i d(\tilde{\mathbf{x}}_i, \tilde{\mathtt{P}}\tilde{\mathbf{X}}_i)^2$$

over $\tilde{\mathtt{P}}$, using an iterative algorithm such as Levenberg–Marquardt.

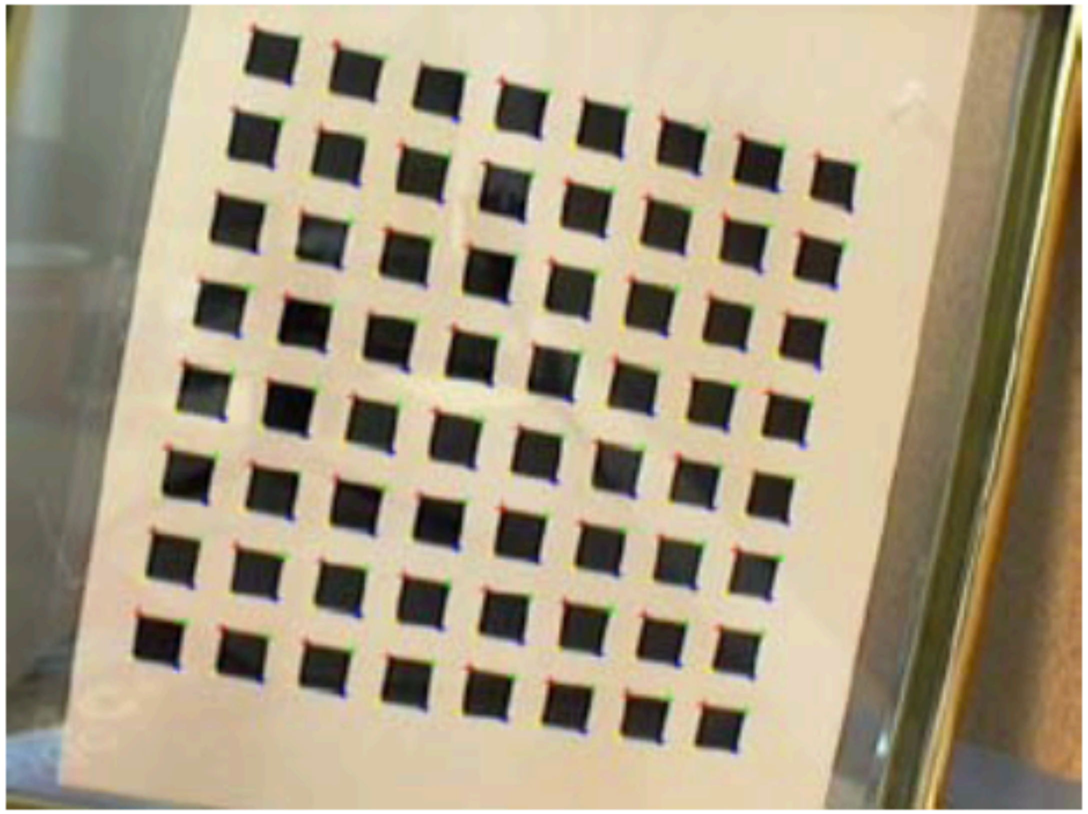
(iii) **Denormalization.** The camera matrix for the original (unnormalized) coordinates is obtained from $\tilde{\mathtt{P}}$ as

$$\mathtt{P} = \mathtt{T}^{-1}\tilde{\mathtt{P}}\mathtt{U}.$$

标定物

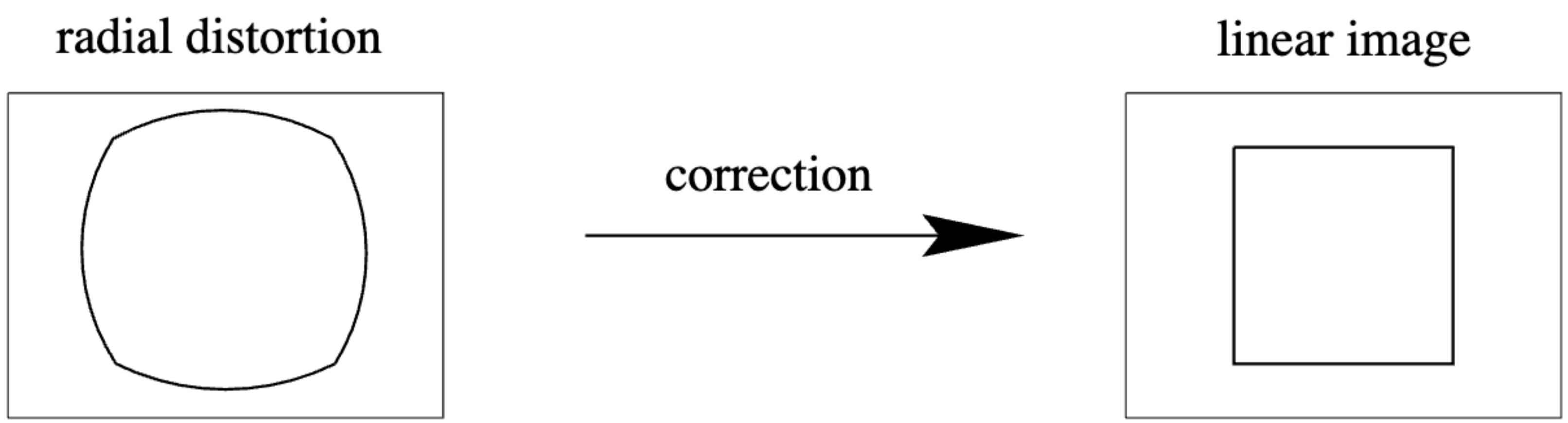


3D



2D

畸变矫正



a

b