MSP - projekt 2

Note: pri kratších úsekoch kódu je odpoveď na otázku zadania v danej sekcií ako výstup kódu.

In [1]:
```
# Imports
import pandas as pd
import numpy as np
from scipy.optimize import minimize
from scipy.special import gamma
from scipy.stats import chi2

import statsmodels.formula.api as smf
import statsmodels.api as sm
from statsmodels.stats.outliers_influence import variance_inflation_factor
import scipy.stats as stats
import matplotlib.pyplot as plt
from statsmodels.graphics.gofplots import qqplot
from copy import deepcopy
```

In [2]:
```
# Načítanie dát
file_path = 'Data_2024.xlsx'
sheet_name_ve = 'Data_věrohodnost'
sheet_name_re = 'Data_regrese'

Vdata = pd.read_excel(file_path, sheet_name=sheet_name_ve)
Rdata = pd.read_excel(file_path, sheet_name=sheet_name_re)

# Úprava stĺpcov
Vdata = Vdata.drop(columns='Unnamed: 2')
Vdata = Vdata.drop(columns='censored = 1 znamená přestal komunikovat; censored = 0 znamená změnil obor')

Vdata.columns = ['Censored', 'Time']
Rdata.columns = ['OSType', 'ActiveUsers', 'InteractingPct', 'ScrollingPct', 'Ping']


print(Vdata.head())
print(Rdata.head())
```

```
   Censored   Time
0         0  6.528
1         0  6.013
2         1  6.055
3         0  7.243
4         1  5.629
    OSType  ActiveUsers  InteractingPct  ScrollingPct  Ping
0      iOS         4113          0.8283        0.1717    47
1      iOS         7549          0.3461        0.6539    46
2  Windows         8855          0.2178        0.7822    55
3  Android         8870          0.0794        0.9206    56
4    MacOS         9559          0.7282        0.2718    76
```

Vierohodnosť

1. Weibullove rozdelenie pravdepodobnosti, logaritmická-vierohodnostná funkcia a jej parciálne derivácie

Necenzurované pozorovania

$$f_{nec}(x,k,\lambda) = \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}e^{-(\frac{x}{\lambda})^k}, L_{nec}(k,\lambda) = \prod_{i=1}^{n}\frac{k}{\lambda}(\frac{x_i}{\lambda})^{k-1}e^{-(\frac{x_i}{\lambda})^k}$$

$$l_{nec}(k,\lambda) = n\ln(k) - nk\ln(\lambda) + (k-1)\sum_{i=1}^{n}\ln(x_i) - \frac{1}{\lambda^k}\sum_{i=1}^{n}x_i^k$$

Cenzorované pozorovania

$$f_{cen}(x,k,\lambda) = 1 - F(x) = 1 - (1 - e^{-(\frac{x}{\lambda})^k}) = e^{-(\frac{x}{\lambda})^k}, L_{cen}(k,\lambda) = \prod_{i=1}^{n}e^{-(\frac{x_i}{\lambda})^k}$$

$$l_{cen}(k,\lambda) = -\frac{1}{\lambda^k}\sum_{i=1}^{n}x_i^k$$

Parciálne derivácie

$$\frac{\partial l_{nec}}{\partial\lambda} = -\frac{nk}{\lambda} + \frac{k}{\lambda^{k+1}}\sum_{i=1}^{n}x_i^k = k(-\frac{n}{\lambda} + \lambda^{-k-1}\sum_{i=1}^{n}x_i^k)$$

$$\frac{\partial l_{nec}}{\partial k} = \frac{n}{k} - n\ln(\lambda) + \sum_{i=1}^{n}\ln(x_i) - [\lambda^k\sum_{i=1}^{n}x_i^k\ln(x_i) - \ln(\lambda)\lambda^k\sum_{i=1}^{n}x_i^k]/\lambda^{2k}$$

$$\frac{\partial l_{cen}}{\partial\lambda} = k\lambda^{-k-1}\sum_{i=1}^{n}x_i^k$$

$$\frac{\partial l_{cen}}{\partial k} = -[\lambda^k \sum_{i=1}^{n} x_i^k \ln(x_i) - \ln(\lambda)\lambda^k \sum_{i=1}^{n} x_i^k]/\lambda^{2k}$$

Pre zadané dáta dokopy

$$L(k, \lambda) = L_{nec}(k, \lambda) \times L_{cen}(k, \lambda)$$

$$l(k, \lambda) = l_{nec}(k, \lambda) + l_{cen}(k, \lambda)$$

$$\frac{\partial l}{\partial \lambda} = \frac{\partial l_{nec}}{\partial \lambda} + \frac{\partial l_{cen}}{\partial \lambda}$$

$$\frac{\partial l}{\partial k} = \frac{\partial l_{nec}}{\partial k} + \frac{\partial l_{cen}}{\partial k}$$

2. Maximálne vierohodnostné odhady

In [3]:

```
# Logaritmická-vierohodnostná funkcia Weibulloveho rozdelenia
def weibull_log_likelihood(params, times, censored):
    k, lambd = params
    uncensored = 1 - censored

    f_x = (k / lambd) * (times / lambd)**(k - 1) * np.exp(-(times / lambd)**k)
    S_x = np.exp(-(times / lambd)**k)

    log_likelihood = np.sum(uncensored * np.log(f_x) + censored * np.log(S_x))
    return -log_likelihood


initial_guess_weibull = [1.0, 1.0]
result_weibull = minimize(weibull_log_likelihood, x0=initial_guess_weibull, args=(Vdata['Time'], Vdata['Censored']))

# Odhad k a lambda pre
k, lambd = result_weibull.x
print(f'Maximálne vierohodnostné odhady: tvar (k) = {k:.5f}, mierka (lambda) = {lambd:.5f}')
```

```
Maximálne vierohodnostné odhady: tvar (k) = 6.17282, mierka (lambda) = 7.42946
```

3. Test vierohodnostným pomerom s exponenciálnym rozdelením

Vypočíta sa MLE pre parameter λ, keď $k = 1$ (viď. `exponential_log_likelihood`). Rovnako ako pri Weibulovom rozdelení pravdepodobnosti.

H_0 : Exponenciálne rozdelenie je postačujúcim modelom pre dané dáta.

$$LR = 2[l_{weibull}(k, \lambda) - l_{exp}(\lambda)]$$

Doplnok kritického oboru je:

$$\overline{W}_\alpha = \langle 0; \chi^2_{1-\alpha}(1) \rangle$$

In [4]:

```
# Logaritmická-vierohodnostná funkcia Exponenciálneho rozdelenia
def exponential_log_likelihood(params, times, censored):
    lambd = params[0]
    uncensored = 1 - censored

    f_x = (1 / lambd) * np.exp(-times / lambd)
    S_x = np.exp(-times / lambd)

    log_likelihood = np.sum(uncensored * np.log(f_x) + censored * np.log(S_x))
    return -log_likelihood


initial_guess_exponential = [1.0]
result_exponential = minimize(exponential_log_likelihood, x0=initial_guess_exponential, args=(Vdata['Time'], Vdata['Censored'])
lambd_ex = result_exponential.x[0]


logL_weibull = -weibull_log_likelihood(result_weibull.x, Vdata['Time'], Vdata['Censored'])
logL_exponential = -exponential_log_likelihood(result_exponential.x, Vdata['Time'], Vdata['Censored'])

print(f'Maximálne vierohodnostný odhad lambdy pre exponenciálne rozdelenie: {lambd_ex:.5f}')


LR = 2 * (logL_weibull - logL_exponential)
critical_val = chi2.ppf(0.95, df=1) # alfa = 0.05

print(f'Likelihood Ratio (LR): {LR:.5f}')
print(f'Doplnok kritického intervalu: <0, {critical_val:.3f}>')


if 0 <= LR <= critical_val:
    print("Nezamietame H0: Exponenciálne rozdelenie je postačujúce.")
else:
    print("Zamietame H0: Exponenciálne rozdelenie nie je postačujúce.")
```

```
Maximálne vierohodnostný odhad lambdy pre exponenciálne rozdelenie: 9.05329
Likelihood Ratio (LR): 592.38982
Doplnok kritického intervalu: <0, 3.841>
Zamietame H0: Exponenciálne rozdelenie nie je postačujúce.
```

4. Bodové odhady pre strednú dobu zamestnania v obore

Stredná hodnota pre Weibullove rozdelenie pravdepodobnosti (https://en.wikipedia.org/wiki/Exponential_distribution):

$E(X) = \lambda\Gamma(1 + \frac{1}{k})$

10% percentil sa dá vypočítať z distribučnej funkcie Weibulloveho rozdelenia:

$F(x, k, \lambda) = 1 - e^{-(\frac{x}{\lambda})^k}$

Percentil x_p je hodnota, pri ktorej platí:

$F(x_p, k, \lambda) = p$

Po dosadení:

$1 - e^{-(\frac{x_p}{\lambda})^k} = p$

Po zlogaritmovaní a úprave:

$x_p = \lambda(-\ln(1 - p))^{\frac{1}{k}}$, teda

$x_{10\%} = \lambda(-\ln(1 - 0,1))^{\frac{1}{k}}$

In [5]:
```python
# Stredná hodnota
mean_time = lambd * gamma(1 + 1 / k)

# 10% percentil
x_10 = lambd * (-np.log(0.9))**(1 / k)

print(f'Stredná doba zamestnania: {mean_time:.2f} rokov')
print(f'10% percentil: {x_10:.2f} rokov')
```

```
Stredná doba zamestnania: 6.90 rokov
10% percentil: 5.16 rokov
```

Regresia

1. Nájdenie vhodného regresného modelu.
   - Úvaha nad dátami a odstránenie zjavne leneárne závislých prediktorov.
   - Štandardizácia modelu a odstránenie štatisticky nevýznamných prediktorov na základe p-hodnoty.
   - Odstránenie extrémne odľahlých hodnôt
   - Multikolinearity
   - Finálna kontrola
2. Najproblematickejšie hodnoty
3. Odozva užívateľa Windows
4. Vhodnosť výsledného modelu

1. Vhodný regresný model

Note: Priebežne v kóde sú komentáre, ktoré odkazujú všetky zmienené body.

0. Vytvorenie plného modelu. Pre tento účel som si vytvoril funkciu `generate_full_formula()`, ktorá vracia formulu v jazyku **R**, podľa ktorej sa vytvorí neskôr model. Čo sa týka kategoriálneho prediktoru `OSType`, tak keďže je počet jednotlivých kategórií približne rovnaký a považujem ich za celkom rovnocenné tak nebudem špecificky vyberať referenčnú kategóriu a nechám to na lexikografickom poradí.

1. **Úvaha:** Pri prvom pohľada na dáta možno pozorovať, že prediktory `ScrollingPct` a `InteractingPct` sú na sebe úplne závislé, pretože sa sčítajú do 1. Online užívatelia sú rozdelení na dve skupiny tí, ktorí interagujú s obsahom a tí čo len scrollujú. Každopádne podľa jedného z týchto prediktorov je možné určiť ten druhý. To znamená, že nám stačí jeden z nich. Ja som sa rozhodol odstrániť prediktor `ScrollingPct`, jeho druhú mocninu a interakcie druhého rádu so všetkými ostatnými prediktormi, keďže všetky tieto hodnoty (len opačné) zahŕňa prediktor `InteractingPct`, jeho druhá mocninu a interackie druhého rádu. Túto závislosť možno pekne vidieť v tabuľke korelácií `ScrollingPct` a `InteractingPct`.

2. **Odstránenie prediktorov, ktoré nepridávajú hodnotu:** Najskôr som sa rozhodol štandardizovať všetky prediktory a z nich odstrániť tie prediktory, ktoré sú štatisticky nevýznamné na základe ich p-hodnoty. Na tento účel som vytvoril funkciu `backward_elimination()`.

3. **Odstránenie odľahlých hodnôt:** Po analýze vykonanej pomocou diagnostických grafov, leverage, štandardizovaných reziduí a Cookovej vzdialenosti som našiel dva veľmi odľahlé body, ktoré mi prídu až moc podozrivé na to aby sa vyskytli pri 500 pozorovaniach len dva. Tieto body je veľmi pekne vidno v diagnostických grafoch, ale aj v tabuľke štandardizovaných reziduí alebo pri leverage.

4. Kontrola: v takto spracovanom modely sú všetky prediktory štatisticky význame, avšak podľa VIFu existuje multikolinearita medzi `ActiveUsers` a `I(ActiveUsers**2)`. `I(ActiveUsers**2)` má záporný koeficient (aj keď veľmi malý), čo mi nedáva zmysel aby ping od určitej hodnoty aktívnych užívateľov klesal, naopak by som očakával, že bude rásť. Preto som sa rozhodol odstrániť prediktor `I(ActiveUsers**2)`.

5. Finálna kontrola: Teraz sú už všetky prediktory štatisticky významné a podľa dát z VIFu a vzájomnej korelácie by som nemazal žiaden ďalší prediktor.

Čo sa týka finálneho neštandardizovaného modelu, tak niektoré prediktory pre `OSType` sa síce javia ako štatisticky nevýznamné. To však bude zrejem tým, že nehľadiac na operačný systém ping bude podobný (aspoň medzi niektorými kategóriami) v tomto prípade by som však zachoval všekty kategórie, pretože ich budeme potrebovať ak chceme porovnávať jednotlivé operačné systémy medzi sebou. Vynechať jednu kategóriu a zachovať inú mi v tomto prípade nedáva zmysel ak chceme sledovať rozdiely medzi kategóriami OS.

Rovnica finálneho modelu

$$Ping = 9,9548 + 6.7916 * Windows - 1,9029 * MacOS - 1,9057 * iOS + 0,0055 * ActiveUsers - 0,0008 * iOS * ActiveUsers - 0,0006 * Windows * ActiveUsers + 0,0020 * MacOS * ActiveUsers + 36,7221 * InteractingPct - 0,0035 * ActiveUsers * InteractingPct$$

MacOS, iOS, a Windows sú binárne premenné (1 ak ide o daný OS, inak 0) podľa kategoriálneho prediktoru `OSType`, referenčná kategória je Android.

Predpoklady lineárnej regresie a základné diagnostiky

Model dosahuje dobrú vysvetľovaciu schopnosť s hodnotami $R^2 = 0,842$ a Adjusted $R^2 = 0,839$, čo naznačuje, že približne 84% variability závislej premennej je vysvetlených modelom.

1. **Multikolinearita:** Hodnoty VIF a korelácie sú v prijateľnom rozsahu, čo znamená, že multikolinearita medzi prediktormi nie je problémom.
2. **Normalita rezíduí:** Testy Jarque-Bera $(0,104)$ a Omnibus (0.195) naznačujú, že rezíduá sa riadia normálnym rozdelením.
3. **Autokorelácia:** Hodnota Durbin-Watson $(1,913)$ je blízka 2, čo znamená absenciu autokorelácie rezíduí.

In [6]:
```
# Trieda pre prácu s formulou
class Formula:
    def __init__(self, response):
        self.response:str = response
        self.formula:set = set()

    def __str__(self):
        res = (self.response + ' ~ ')
        for i, term in enumerate(self.formula):
            res += term
            if i != len(self.formula) - 1:
                res += ' + '
            if (i+1) % 3 == 0:
                res += '\n'
        return res

    def Get(self) -> str:
        res = (self.response + ' ~ ')
        for i, term in enumerate(self.formula):
            res += term
            if i != len(self.formula) - 1:
                res += ' + '
        return res

    def Copy(self):
        return deepcopy(self)

    def Add(self, formula:str):
        self.formula.add(formula)

    def Remove(self, formula:str):
        self.formula.remove(formula)



# Funkcia pre vygenerovanie plnej formule
def generate_full_formula(data, response, predictors, category_vars) -> Formula:
    formula = Formula(response)
    dummies_names = []
    for var in predictors:
        if var in category_vars:
            dummies = pd.get_dummies(data[var], prefix=var, drop_first=True)
            dummies_names += dummies.columns.to_list()
            for dummy in dummies_names:
                formula.Add(dummy)
        else:
            formula.Add(var)
            formula.Add(f'I({var}**2)')

    all_ = dummies_names + predictors

    # Interakcie druhého rádu
    for i, var1 in enumerate(all_):
        for var2 in all_[i + 1:]:
            if (var1 in dummies_names and var2 in dummies_names) or (var1 in category_vars or var2 in category_vars):
                continue
            formula.Add(f'{var1}:{var2}')

    return formula


# ----------------------------------------------------------#
# "Plný" regresný model pre dané dáta

predictors = ['ActiveUsers', 'InteractingPct', 'ScrollingPct', 'OSType']
category_vars = ['OSType']
formula:Formula = generate_full_formula(Rdata, response="Ping", predictors=predictors, category_vars=category_vars)

# Vypíš formulu
print(formula)
```

```python
print('\n\n')

# Úprava stĺpcov pre kategoriálne dáta
dummies = dummy_ostype = pd.get_dummies(Rdata['OSType'], prefix='OSType', drop_first=True)
Rdata = pd.concat([Rdata, dummies], axis=1)

# Vytvorenie plného modelu
model = smf.ols(formula=formula.Get(), data=Rdata)
result = model.fit()
print(result.summary())
print('\n\n')


# Počty jednotlivých kategórií
# Keďže je počet jednotlivých kategórií približne rovnaký a považujem ich za celkom rovnocenné tak
# nebudem špecificky vyberať referenčnú kategóriu a nechám to na lexikografickom poradí.
print(Rdata['OSType'].value_counts())
```

```
Ping ~ I(ActiveUsers**2) + OSType_MacOS:ActiveUsers + OSType_Windows:ActiveUsers +
ActiveUsers + ActiveUsers:ScrollingPct + ScrollingPct +
I(ScrollingPct**2) + InteractingPct:ScrollingPct + OSType_Windows:InteractingPct +
InteractingPct + ActiveUsers:InteractingPct + OSType_Windows:ScrollingPct +
OSType_MacOS:ScrollingPct + OSType_MacOS + OSType_iOS:ActiveUsers +
OSType_Windows + OSType_iOS + OSType_iOS:InteractingPct +
OSType_iOS:ScrollingPct + I(InteractingPct**2) + OSType_MacOS:InteractingPct




                            OLS Regression Results
==============================================================================
Dep. Variable:                   Ping   R-squared:                       0.844
Model:                            OLS   Adj. R-squared:                  0.839
Method:                 Least Squares   F-statistic:                     187.9
Date:                Sun, 15 Dec 2024   Prob (F-statistic):          5.18e-186
Time:                        12:54:46   Log-Likelihood:                -1598.4
No. Observations:                 502   AIC:                             3227.
Df Residuals:                     487   BIC:                             3290.
Df Model:                          14
Covariance Type:            nonrobust
=====================================================================================================
                                        coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------------------------------
Intercept                            10.7060      1.150      9.311      0.000       8.447      12.965
OSType_MacOS[T.True]                  1.2156      1.270      0.957      0.339      -1.280       3.712
OSType_Windows[T.True]                5.3536      1.257      4.260      0.000       2.884       7.823
OSType_iOS[T.True]                    0.0571      1.273      0.045      0.964      -2.445       2.559
I(ActiveUsers ** 2)                 -4.17e-07    4.4e-08     -9.469      0.000   -5.03e-07    -3.3e-07
ActiveUsers                           0.0057      0.000     15.396      0.000       0.005       0.006
OSType_MacOS[T.True]:ActiveUsers      0.0014      0.000      4.536      0.000       0.001       0.002
OSType_Windows[T.True]:ActiveUsers   -0.0008      0.000     -2.505      0.013      -0.001      -0.000
OSType_iOS[T.True]:ActiveUsers       -0.0011      0.000     -3.369      0.001      -0.002      -0.000
ActiveUsers:ScrollingPct              0.0044      0.000     17.430      0.000       0.004       0.005
ScrollingPct                         -3.1171      0.888     -3.509      0.000      -4.863      -1.371
OSType_Windows[T.True]:ScrollingPct   2.4638      1.443      1.707      0.088      -0.372       5.299
OSType_MacOS[T.True]:ScrollingPct     0.7861      1.401      0.561      0.575      -1.967       3.539
OSType_iOS[T.True]:ScrollingPct      -0.1054      1.452     -0.073      0.942      -2.958       2.747
I(ScrollingPct ** 2)                 -7.9277      1.341     -5.914      0.000     -10.562      -5.294
InteractingPct:ScrollingPct           4.8106      1.317      3.654      0.000       2.224       7.398
InteractingPct                       13.8231      0.966     14.315      0.000      11.926      15.720
OSType_Windows[T.True]:InteractingPct 2.8898      1.552      1.862      0.063      -0.159       5.939
OSType_iOS[T.True]:InteractingPct     0.1625      1.525      0.107      0.915      -2.834       3.158
OSType_MacOS[T.True]:InteractingPct   0.4295      1.429      0.300      0.764      -2.379       3.238
ActiveUsers:InteractingPct            0.0013      0.000      4.853      0.000       0.001       0.002
I(InteractingPct ** 2)                9.0125      1.347      6.689      0.000       6.365      11.660
==============================================================================
Omnibus:                      228.442   Durbin-Watson:                   1.933
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3152.488
Skew:                           1.603   Prob(JB):                         0.00
Kurtosis:                      14.851   Cond. No.                     1.19e+24
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 7.57e-31. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.



OSType
MacOS      138
Windows    134
iOS        118
Android    112
Name: count, dtype: int64
```

In [7]:
```python
# ----------------------------------------------------------#
# 1. Odstránenie závislosti medzi InteractingPct a ScrollingPct

X = pd.DataFrame(model.exog, columns=model.exog_names)

# VIF (kód z democvičenia)
# VIF toho moc neukázal, ale korelácia áno...
```

```python
#vif = pd.Series([variance_inflation_factor(X.values, i)
#                  for i in range(X.shape[1])],
#                  index=X.columns)

#vif_df = vif.to_frame()
#vif_df.columns = ['VIF']
#print(vif_df)
#print('\n\n\n')

# Korelácia
# Z tejto korelácie je veľmi dobre vidieť, že InteractingPct a ScrollingPct sú na sebe
# úplne závislé a stačí používať len jeden z nich.
print(X.corr().loc[['InteractingPct', 'ScrollingPct', 'InteractingPct:ScrollingPct',
                'ActiveUsers:InteractingPct', 'ActiveUsers:ScrollingPct',
                'OSType_MacOS[T.True]:InteractingPct', 'OSType_MacOS[T.True]:ScrollingPct',
                'OSType_Windows[T.True]:InteractingPct', 'OSType_Windows[T.True]:ScrollingPct',
                'OSType_iOS[T.True]:InteractingPct', 'OSType_iOS[T.True]:ScrollingPct'],
                ['InteractingPct', 'ScrollingPct']])

print('\n\n')

# Odstránenie ScrollingPct jeho druhje mocniny a interakcie druhého rádu
to_remove = ['ScrollingPct', 'I(ScrollingPct**2)', 'ActiveUsers:ScrollingPct', 'OSType_MacOS:ScrollingPct',
             'OSType_Windows:ScrollingPct', 'OSType_iOS:ScrollingPct', 'InteractingPct:ScrollingPct']
better_formula = formula.Copy()
for term_to_remove in to_remove:
    term_to_remove = term_to_remove.replace('[T.True]', '')
    term_to_remove = term_to_remove.replace(' ', '')
    better_formula.Remove(term_to_remove)

print(better_formula)
```

```
                                       InteractingPct  ScrollingPct
InteractingPct                               1.000000     -1.000000
ScrollingPct                                -1.000000      1.000000
InteractingPct:ScrollingPct                  0.022835     -0.022835
ActiveUsers:InteractingPct                   0.752979     -0.752979
ActiveUsers:ScrollingPct                    -0.711749      0.711749
OSType_MacOS[T.True]:InteractingPct          0.388633     -0.388633
OSType_MacOS[T.True]:ScrollingPct           -0.275403      0.275403
OSType_Windows[T.True]:InteractingPct        0.248636     -0.248636
OSType_Windows[T.True]:ScrollingPct         -0.262914      0.262914
OSType_iOS[T.True]:InteractingPct            0.244987     -0.244987
OSType_iOS[T.True]:ScrollingPct             -0.314208      0.314208



Ping ~ I(ActiveUsers**2) + OSType_MacOS:ActiveUsers + OSType_Windows:ActiveUsers +
ActiveUsers + OSType_Windows:InteractingPct + InteractingPct +
ActiveUsers:InteractingPct + OSType_MacOS + OSType_iOS:ActiveUsers +
OSType_Windows + OSType_iOS + OSType_iOS:InteractingPct +
I(InteractingPct**2) + OSType_MacOS:InteractingPct
```

In [8]:

```python
# ----------------------------------------------------------#
# 2. Štandardizovaný model

def standardize(data:pd.DataFrame, predictor:str):
    return(data[predictor]-((data[predictor].max()+data[predictor].min())/2))/(data[predictor].max()-data[predictor].min())*2


no_cat_predictors = ['ActiveUsers', 'InteractingPct', 'ScrollingPct']
RdataSS = Rdata.copy()
for predictor in no_cat_predictors:
    RdataSS[predictor] = standardize(RdataSS, predictor)


modelSS = smf.ols(formula=better_formula.Get(), data=RdataSS)
resultSS = modelSS.fit()
print(resultSS.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   Ping   R-squared:                       0.844
Model:                            OLS   Adj. R-squared:                  0.839
Method:                 Least Squares   F-statistic:                     187.9
Date:                Sun, 15 Dec 2024   Prob (F-statistic):          5.18e-186
Time:                        12:54:46   Log-Likelihood:                -1598.4
No. Observations:                 502   AIC:                             3227.
Df Residuals:                     487   BIC:                             3290.
Df Model:                          14
Covariance Type:            nonrobust
=====================================================================================================
                                        coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------------------------------
Intercept                            49.7322      0.700     71.082      0.000      48.358      51.107
OSType_MacOS[T.True]                  8.8844      0.784     11.337      0.000       7.345      10.424
OSType_Windows[T.True]                4.1805      0.786      5.318      0.000       2.636       5.725
OSType_iOS[T.True]                   -5.2624      0.808     -6.515      0.000      -6.849      -3.675
I(ActiveUsers ** 2)                 -10.0114      1.057     -9.469      0.000     -12.089      -7.934
ActiveUsers                          20.9152      1.160     18.036      0.000      18.637      23.194
OSType_MacOS[T.True]:ActiveUsers      6.8470      1.509      4.536      0.000       3.881       9.813
OSType_Windows[T.True]:ActiveUsers   -3.7332      1.490     -2.505      0.013      -6.661      -0.806
OSType_iOS[T.True]:ActiveUsers       -5.1859      1.539     -3.369      0.001      -8.211      -2.161
InteractingPct                        9.1270      0.982      9.293      0.000       7.197      11.057
OSType_Windows[T.True]:InteractingPct  0.2126      1.358      0.157      0.876      -2.455       2.880
OSType_iOS[T.True]:InteractingPct      0.1337      1.343      0.100      0.921      -2.505       2.773
OSType_MacOS[T.True]:InteractingPct   -0.1779      1.262     -0.141      0.888      -2.658       2.303
ActiveUsers:InteractingPct           -7.5471      0.885     -8.532      0.000      -9.285      -5.809
I(InteractingPct ** 2)               -0.9279      0.870     -1.067      0.287      -2.637       0.781
==============================================================================
Omnibus:                      228.442   Durbin-Watson:                   1.933
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3152.488
Skew:                           1.603   Prob(JB):                         0.00
Kurtosis:                      14.851   Cond. No.                         11.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

In [9]:

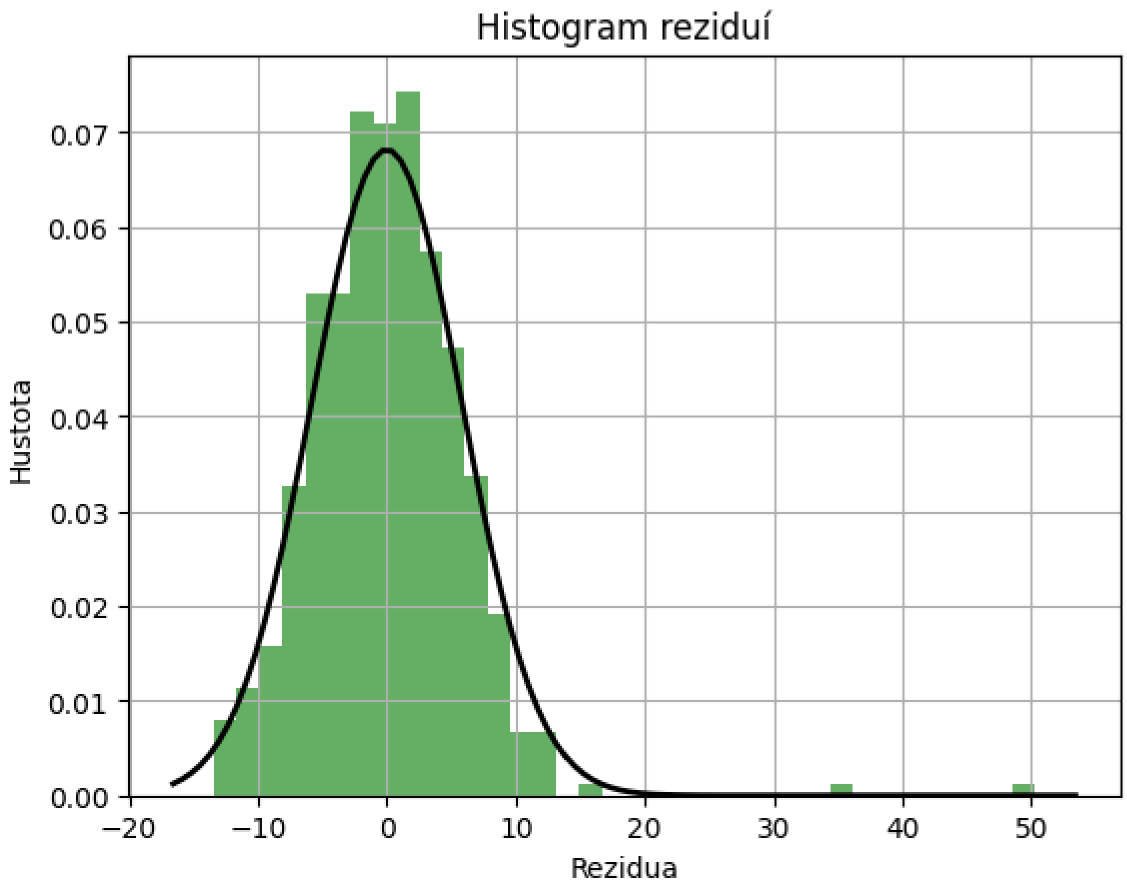
```python
# Automatizovaný algoritmus pre spätnú elimináciu
def backward_elimination(data:pd.DataFrame, initial_formula:Formula, alpha:float=0.05):
    # Začneme s plným modelom
    formula:Formula = initial_formula.Copy()
    while True:
        model = smf.ols(formula=formula.Get(), data=data)
        result = model.fit()

        pvalues = result.pvalues

        max_pval = pvalues.drop('Intercept').max()
        if max_pval > alpha:
            term_to_remove = pvalues.idxmax()
            term_to_remove = term_to_remove.replace('[T.True]', '')
            term_to_remove = term_to_remove.replace(' ', '')
            print(f'Odstránené: {term_to_remove}, p-hodnota: {max_pval}')
            formula.Remove(term_to_remove)
        else:
            break

    return model, formula


# ----------------------------------------------------------#
# 2. Odstránenie nevýznamných prediktorov podľa p-hodnoty

# Spustenie spätnej eliminácie
best_model, best_formula = backward_elimination(RdataSS, initial_formula=better_formula)
best_result = best_model.fit()

print('\n\n')
print(best_result.summary())
```

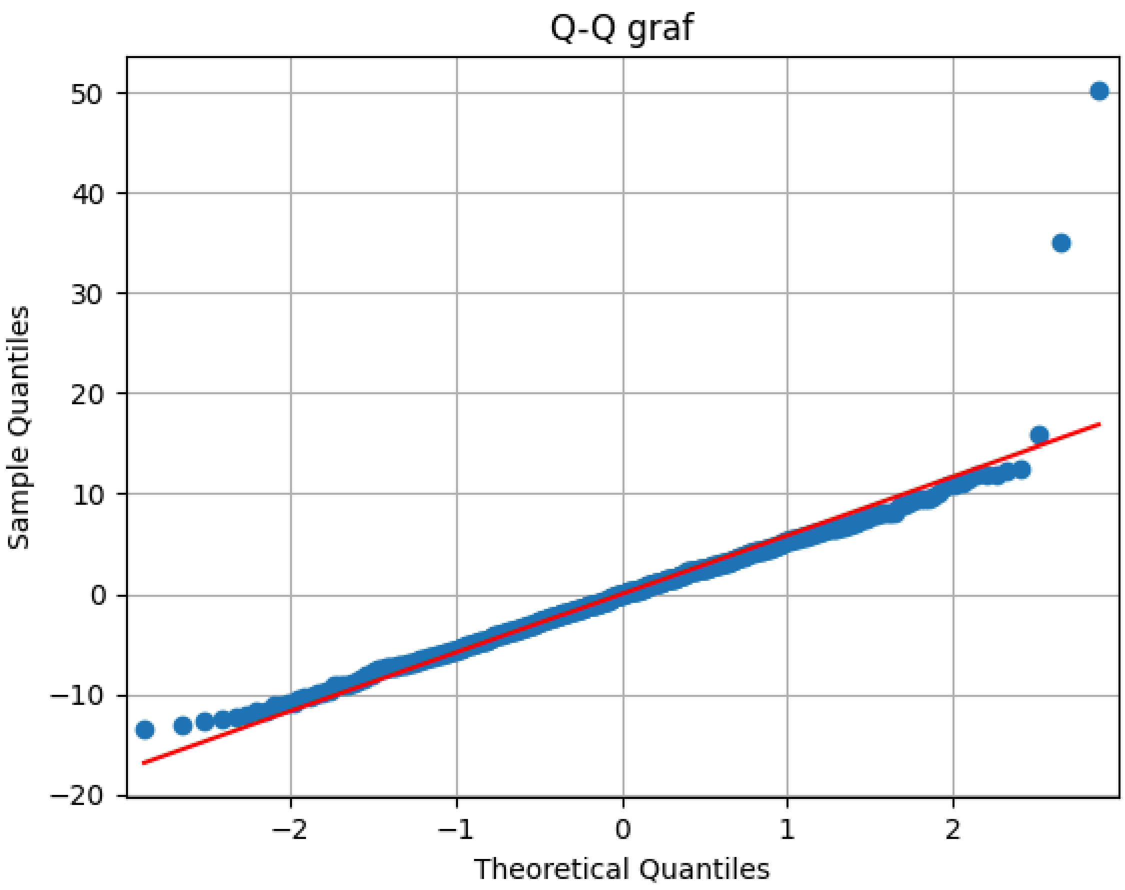
```
Odstránené: OSType_iOS:InteractingPct, p-hodnota: 0.9207665221385899
Odstránené: OSType_Windows:InteractingPct, p-hodnota: 0.9023977373454158
Odstránené: OSType_MacOS:InteractingPct, p-hodnota: 0.7602205074876843
Odstránené: I(InteractingPct**2), p-hodnota: 0.27753223057954146



                            OLS Regression Results
==============================================================================
Dep. Variable:                   Ping   R-squared:                       0.843
Model:                            OLS   Adj. R-squared:                  0.840
Method:                 Least Squares   F-statistic:                     264.4
Date:                Sun, 15 Dec 2024   Prob (F-statistic):          1.69e-190
Time:                        12:54:46   Log-Likelihood:                -1599.1
No. Observations:                 502   AIC:                             3220.
Df Residuals:                     491   BIC:                             3267.
Df Model:                          10
Covariance Type:            nonrobust
======================================================================================================
                                         coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------------------
Intercept                             49.4236      0.633     78.131      0.000      48.181      50.666
OSType_MacOS[T.True]                   8.7999      0.777     11.318      0.000       7.272      10.328
OSType_Windows[T.True]                 4.1936      0.781      5.367      0.000       2.658       5.729
OSType_iOS[T.True]                    -5.3112      0.800     -6.640      0.000      -6.883      -3.740
ActiveUsers                           20.8512      1.154     18.074      0.000      18.584      23.118
OSType_Windows[T.True]:ActiveUsers    -3.6108      1.478     -2.442      0.015      -6.515      -0.706
OSType_iOS[T.True]:ActiveUsers        -5.0588      1.530     -3.306      0.001      -8.065      -2.052
OSType_MacOS[T.True]:ActiveUsers       6.9549      1.501      4.633      0.000       4.005       9.904
I(ActiveUsers ** 2)                   -9.9907      1.054     -9.480      0.000     -12.061      -7.920
InteractingPct                         9.1649      0.453     20.253      0.000       8.276      10.054
ActiveUsers:InteractingPct            -7.5598      0.873     -8.662      0.000      -9.275      -5.845
==============================================================================
Omnibus:                      230.750   Durbin-Watson:                   1.928
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3263.977
Skew:                           1.617   Prob(JB):                         0.00
Kurtosis:                      15.066   Cond. No.                         10.6
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

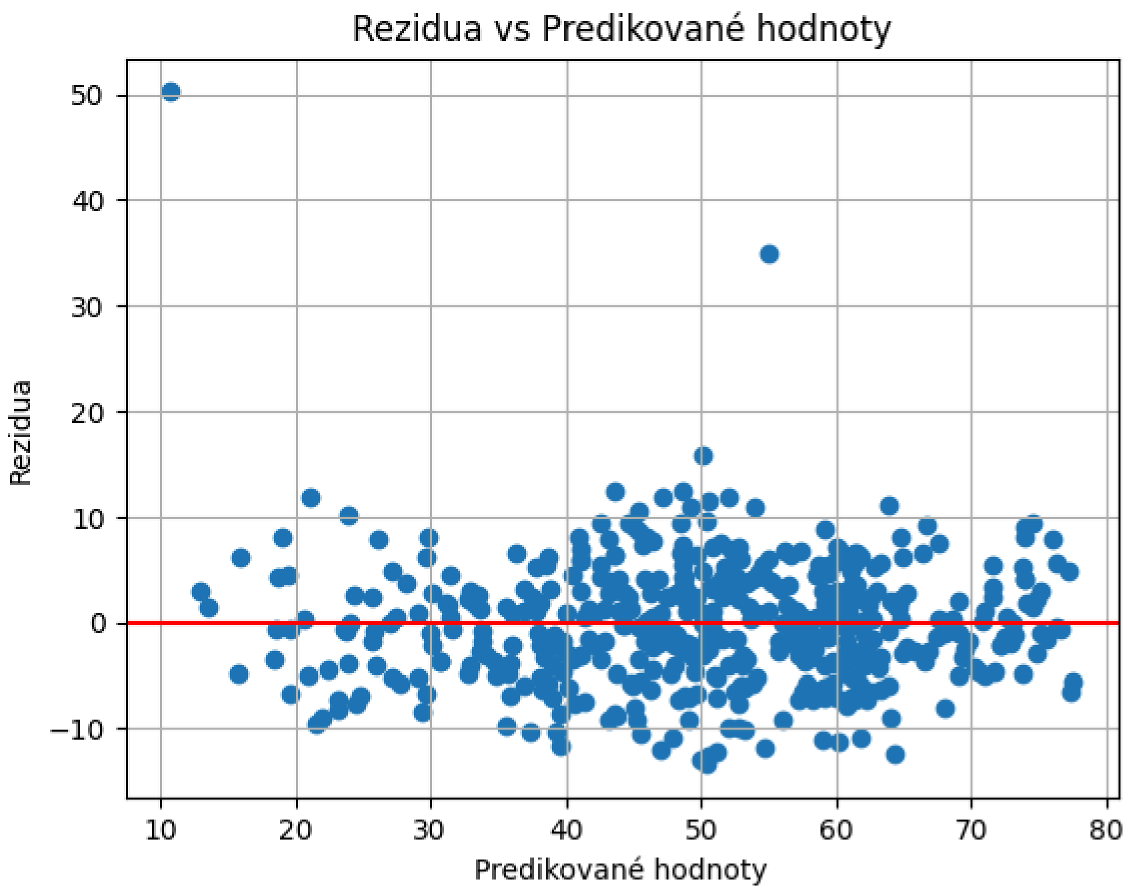
In [10]:

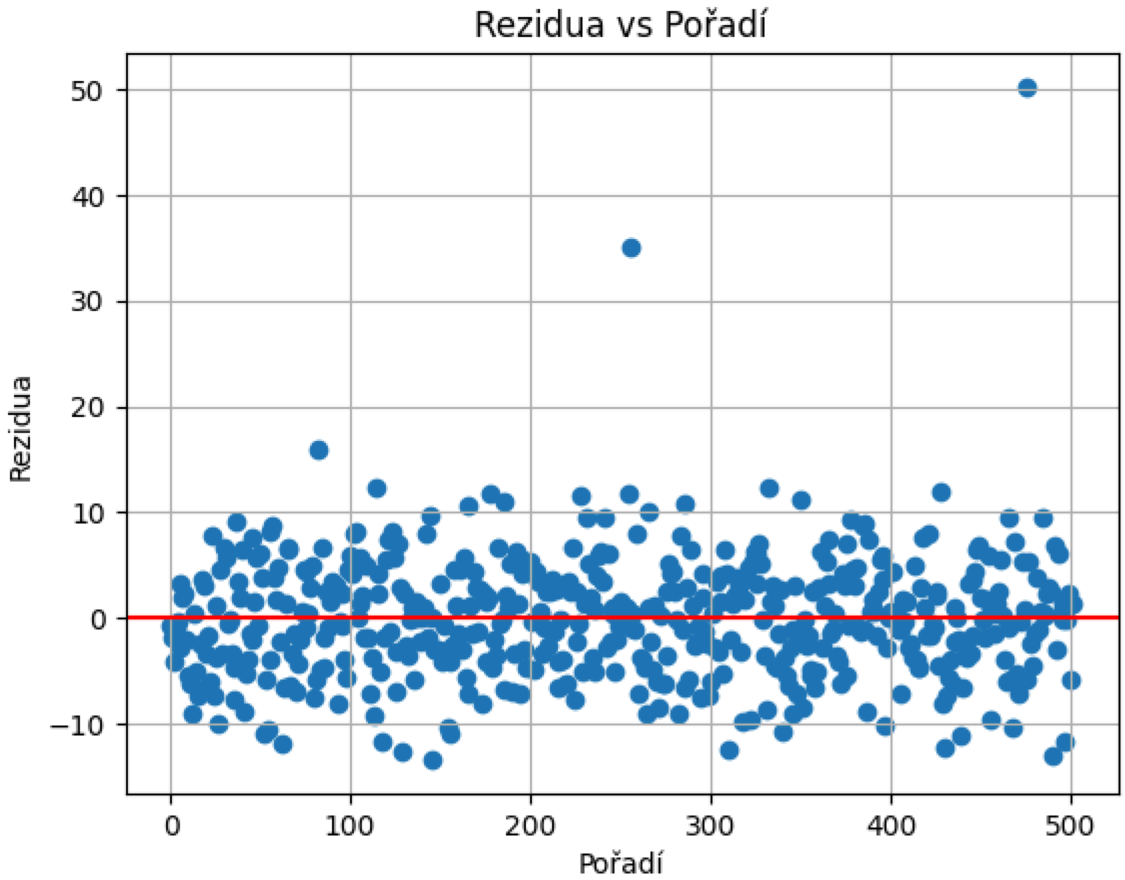
```python
# ---------------------------------------------------------- #
# 3. Odstránenie odľahlých hodnôt

##### Diagnostické grafy (kód z democvičenia) #####

plt.hist(best_result.resid, bins='auto', density=True, alpha=0.6, color='g')

# Gaussovy krivky
xmin, xmax = plt.xlim()
x = np.linspace(xmin, xmax, 100)
p = stats.norm.pdf(x, np.mean(best_result.resid), np.std(best_result.resid))
plt.plot(x, p, 'k', linewidth=2)
plt.grid(True)
plt.title("Histogram reziduí")
plt.xlabel("Rezidua")
plt.ylabel("Hustota")
plt.show()

# Q-Q graf
qqplot(best_result.resid, line='s')
plt.title('Q-Q graf')
plt.grid(True)
plt.show()

# res vs fit
plt.scatter(best_result.fittedvalues, best_result.resid)
plt.axhline(y=0, color='r', linestyle='-')
plt.grid(True)
plt.xlabel('Predikované hodnoty')
plt.ylabel('Rezidua')
plt.title('Rezidua vs Predikované hodnoty')
plt.show()

# res vs order
plt.scatter(range(len(best_result.resid)), best_result.resid)
plt.axhline(y=0, color='r', linestyle='-')
plt.grid(True)
plt.title("Rezidua vs Pořadí")
plt.xlabel("Pořadí")
plt.ylabel("Rezidua")

plt.show()


##### Leverage, štandardizované rezidua, Cookova vzdialenosť (kód z democvičenia) #####

influence = best_result.get_influence()
# Leverage
leverage = influence.hat_matrix_diag
# Cookovy D hodnoty (a p-hodnoty) ako n-tica polí [n x 2]
cooks_d = influence.cooks_distance
```

```
# Standardizované rezidua
standardized_residuals = influence.resid_studentized_internal
# Študentizované rezidua
studentized_residuals = influence.resid_studentized_external
# Výpočet p-hodnôt pre študentizované rezidua
studentized_residuals_pvalues = 2 * (1 - stats.t.cdf(np.abs(studentized_residuals), df=Rdata.shape[0]-len(best_result.params)))


outl_stats_df = pd.DataFrame({
    'Leverage': leverage,
    'Standardized Residuals': standardized_residuals,
    'Studentized Residuals': studentized_residuals,
    'Studentized Residuals p-value': studentized_residuals_pvalues,
    'Cook\'s Distance': cooks_d[0],
    'Cook\'s Distance_p-value': cooks_d[1]
}, index=Rdata.index)
# Výber "zaujímavých" hodnôt (kód z democvičenia)
outl_stats_df = outl_stats_df[(outl_stats_df['Leverage'] > 3*len(best_result.params)/Rdata.shape[0]) |
                              (np.abs(outl_stats_df['Standardized Residuals']) > 2) |
                              (outl_stats_df['Cook\'s Distance_p-value'] < 0.05)]
print(outl_stats_df)
```










```
     Leverage  Standardized Residuals  Studentized Residuals  \
62   0.012590               -2.036977              -2.043554
82   0.010646                2.699228               2.716710
114  0.012955                2.111260               2.118748
129  0.014222               -2.141213              -2.149089
145  0.023780               -2.292470              -2.302490
178  0.047086                2.054883               2.061673
254  0.011482                2.011917               2.018204
255  0.009986                5.945469               6.165493
310  0.016649               -2.111115              -2.118601
332  0.030075                2.124928               2.132592
428  0.028086                2.048785               2.055502
430  0.017414               -2.080739              -2.087844
476  0.074941                8.830417               9.618155
490  0.026903               -2.230330              -2.239431

     Studentized Residuals p-value  Cook's Distance  Cook's Distance_p-value
62                    4.153079e-02         0.004810                 1.000000
82                    6.826184e-03         0.007127                 1.000000
114                   3.461327e-02         0.005318                 1.000000
129                   3.211548e-02         0.006013                 1.000000
145                   2.172492e-02         0.011638                 1.000000
178                   3.976461e-02         0.018968                 1.000000
254                   4.411335e-02         0.004274                 1.000000
255                   1.467855e-09         0.032412                 1.000000
310                   3.462578e-02         0.006860                 1.000000
332                   3.345366e-02         0.012728                 1.000000
428                   4.035881e-02         0.011027                 1.000000
430                   3.732667e-02         0.006975                 1.000000
476                   0.000000e+00         0.574273                 0.850171
490                   2.557484e-02         0.012502                 1.000000
```

In [11]:
```
# Neštandardizovaný model, bez odľahlých hodnôt [255 a 476]
Rdata_filtered = Rdata.drop([255, 476], inplace=False)
best_model_filtered = smf.ols(formula=best_formula.Get(), data=Rdata_filtered)
best_result_filtered = best_model_filtered.fit()
print(best_result_filtered.summary())
```

```
# ---------------------------------------------------------- #
# 3. Po odstránení odľahlých hodnôt

##### Diagnostické grafy (kód z democvičenia) #####

plt.hist(best_result_filtered.resid, bins='auto', density=True, alpha=0.6, color='g')

# Gaussovy krivky
xmin, xmax = plt.xlim()
x = np.linspace(xmin, xmax, 100)
p = stats.norm.pdf(x, np.mean(best_result_filtered.resid), np.std(best_result_filtered.resid))
plt.plot(x, p, 'k', linewidth=2)
plt.grid(True)
plt.title("Histogram reziduí")
plt.xlabel("Rezidua")
plt.ylabel("Hustota")
plt.show()

# Q-Q graf
qqplot(best_result_filtered.resid, line='s')
plt.title('Q-Q graf')
plt.grid(True)
plt.show()

# res vs fit
plt.scatter(best_result_filtered.fittedvalues, best_result_filtered.resid)
plt.axhline(y=0, color='r', linestyle='-')
plt.grid(True)
plt.xlabel('Predikované hodnoty')
plt.ylabel('Rezidua')
plt.title('Rezidua vs Predikované hodnoty')
plt.show()

# res vs order
plt.scatter(range(len(best_result_filtered.resid)), best_result_filtered.resid)
plt.axhline(y=0, color='r', linestyle='-')
plt.grid(True)
plt.title("Rezidua vs Pořadí")
plt.xlabel("Pořadí")
plt.ylabel("Rezidua")

plt.show()


##### Leverage, štandardizované rezidua, Cookova vzdialenosť (kód z democvičenia) #####

influence = best_result_filtered.get_influence()
# Leverage
leverage = influence.hat_matrix_diag
# Cookovy D hodnoty (a p-hodnoty) ako n-tica polí [n x 2]
cooks_d = influence.cooks_distance
# Štandardizované rezidua
standardized_residuals = influence.resid_studentized_internal
# Študentizované rezidua
studentized_residuals = influence.resid_studentized_external
# Výpočet p-hodnôt pre študentizované rezidua
studentized_residuals_pvalues = 2 * (1 - stats.t.cdf(np.abs(studentized_residuals), df=Rdata.shape[0]-len(best_result_filtered.


outl_stats_df = pd.DataFrame({
    'Leverage': leverage,
    'Standardized Residuals': standardized_residuals,
    'Studentized Residuals': studentized_residuals,
    'Studentized Residuals p-value': studentized_residuals_pvalues,
    'Cook\'s Distance': cooks_d[0],
    'Cook\'s Distance_p-value': cooks_d[1]
}, index=Rdata_filtered.index)
# Výber "zaujímavých" hodnôt
outl_stats_df = outl_stats_df[(outl_stats_df['Leverage'] > 3*len(best_result_filtered.params)/Rdata.shape[0]) |
                              (np.abs(outl_stats_df['Standardized Residuals']) > 2) |
                              (outl_stats_df['Cook\'s Distance_p-value'] < 0.05)]
print(outl_stats_df)
```

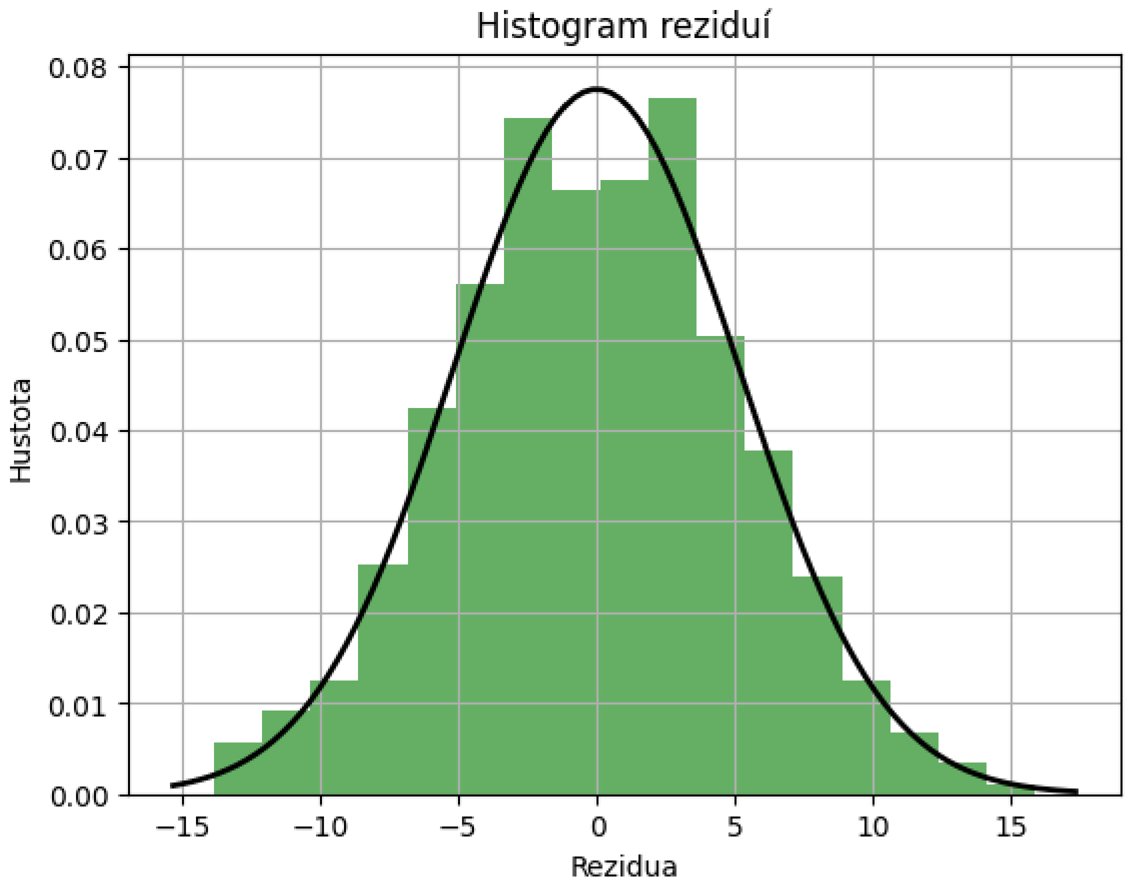
OLS Regression Results

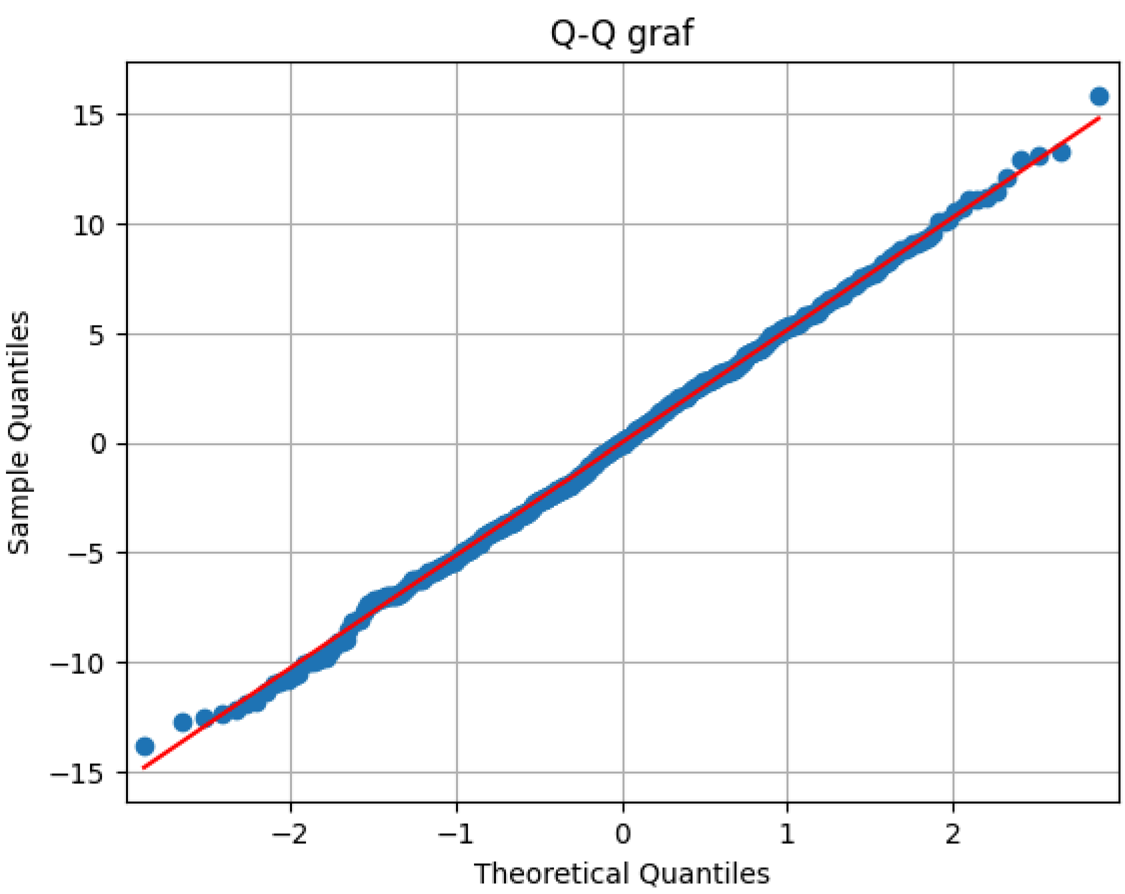
Dep. Variable:	Ping	R-squared:	0.877
Model:	OLS	Adj. R-squared:	0.875
Method:	Least Squares	F-statistic:	349.9
Date:	Sun, 15 Dec 2024	Prob (F-statistic):	1.28e-215
Time:	12:54:46	Log-Likelihood:	-1528.7
No. Observations:	500	AIC:	3079.
Df Residuals:	489	BIC:	3126.
Df Model:	10		
Covariance Type:	nonrobust		

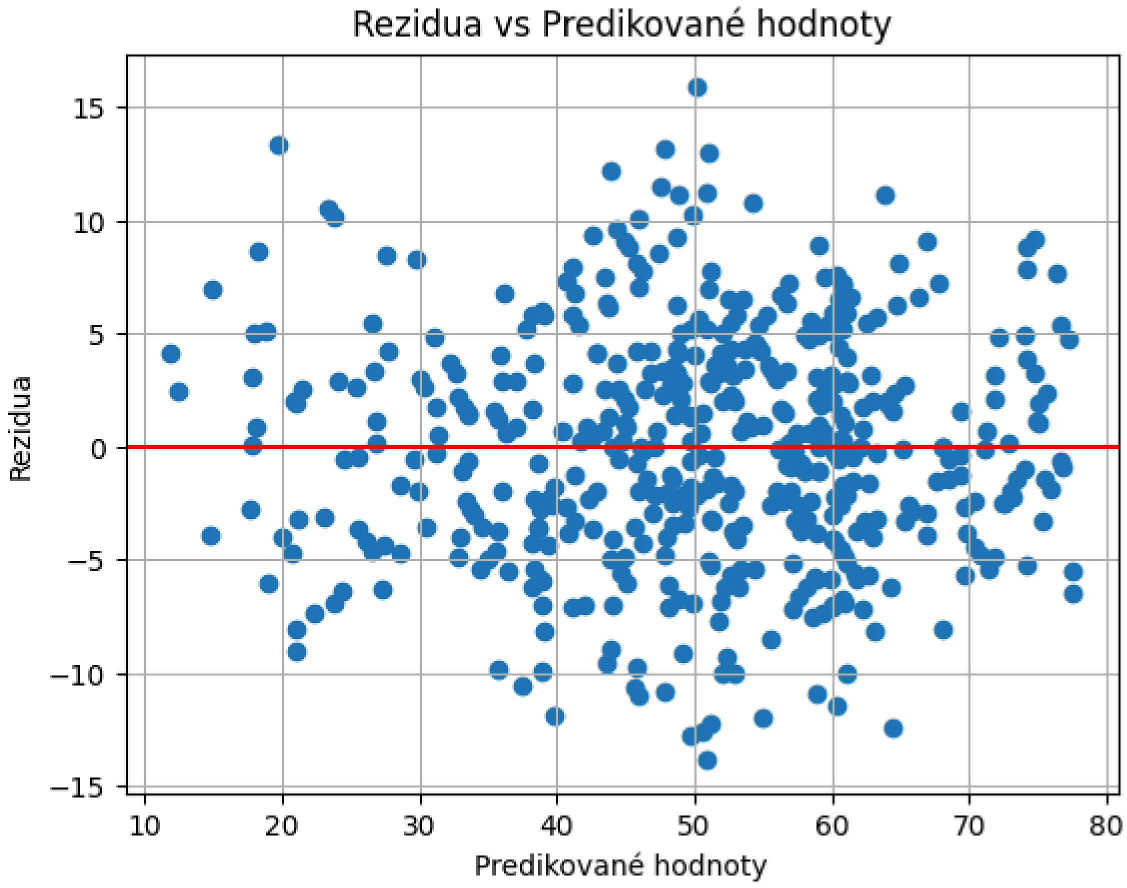
	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-1.8398	1.832	-1.004	0.316	-5.439	1.759
OSType_MacOS[T.True]	-0.5427	1.677	-0.324	0.746	-3.838	2.753
OSType_Windows[T.True]	7.7823	1.646	4.729	0.000	4.549	11.016
OSType_iOS[T.True]	0.1798	1.667	0.108	0.914	-3.096	3.455
ActiveUsers	0.0107	0.001	21.064	0.000	0.010	0.012
OSType_Windows[T.True]:ActiveUsers	-0.0008	0.000	-2.827	0.005	-0.001	-0.000
OSType_iOS[T.True]:ActiveUsers	-0.0011	0.000	-3.915	0.000	-0.002	-0.001
OSType_MacOS[T.True]:ActiveUsers	0.0017	0.000	6.415	0.000	0.001	0.002
I(ActiveUsers ** 2)	-4.63e-07	3.91e-08	-11.832	0.000	-5.4e-07	-3.86e-07
InteractingPct	35.9786	1.879	19.146	0.000	32.286	39.671
ActiveUsers:InteractingPct	-0.0034	0.000	-10.752	0.000	-0.004	-0.003

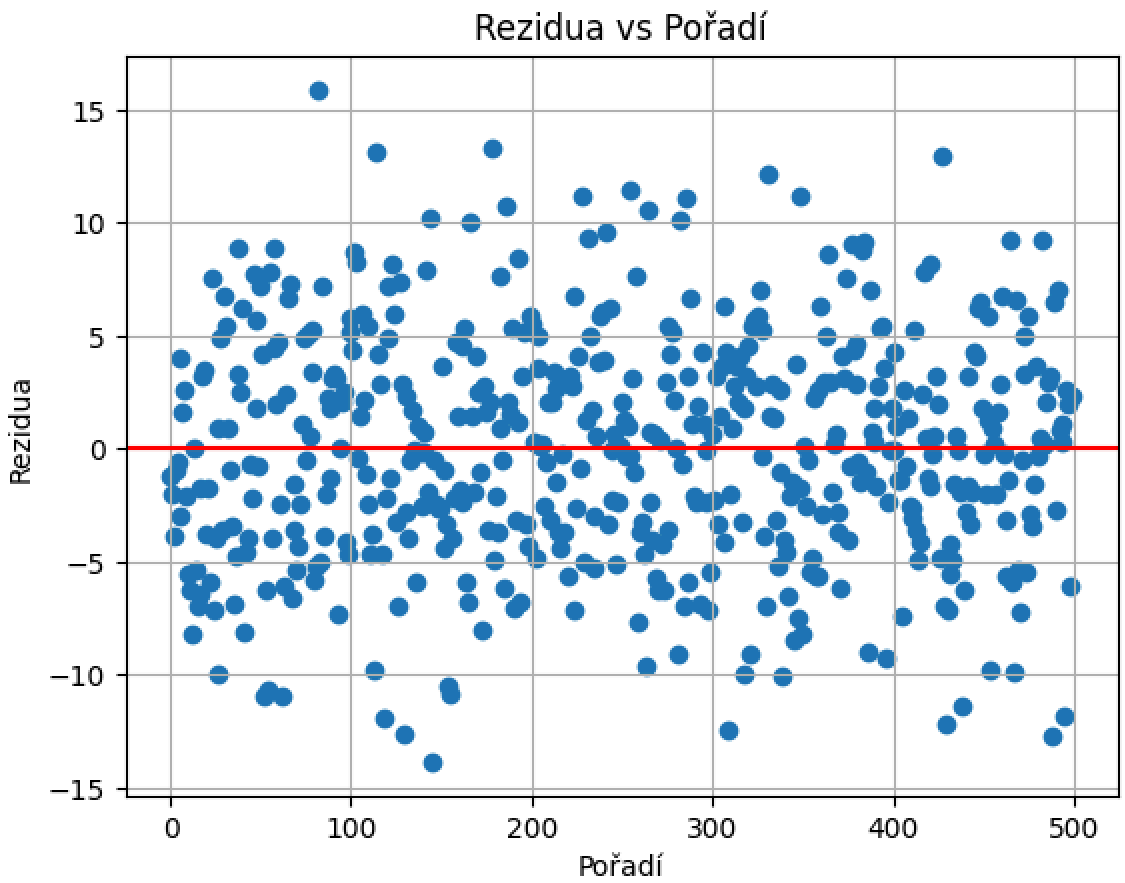
Omnibus:	0.661	Durbin-Watson:	1.990
Prob(Omnibus):	0.719	Jarque-Bera (JB):	0.750
Skew:	0.014	Prob(JB):	0.687
Kurtosis:	2.812	Cond. No.	5.66e+08

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 5.66e+08. This might indicate that there are
strong multicollinearity or other numerical problems.






Rezidua vs Predikované hodnoty
Rezidua
Predikované hodnoty
15
10
5
0
−5
−10
−15
10
20
30
40
50
60
70
80


Rezidua vs Pořadí
Rezidua
Pořadí
15
10
5
0
−5
−10
−15
0
100
200
300
400
500

```
     Leverage  Standardized Residuals  Studentized Residuals  \
52   0.018193               -2.120187              -2.127821   
54   0.026599               -2.074386              -2.081442   
62   0.012941               -2.120597              -2.128236   
82   0.010775                3.066469               3.093217   
114  0.013115                2.538616               2.552897   
118  0.011757               -2.308893              -2.319207   
129  0.014368               -2.433998              -2.446373   
144  0.039306                2.000763               2.006948   
145  0.023860               -2.693010              -2.710429   
154  0.016115               -2.036268              -2.042864   
155  0.013696               -2.095390              -2.102707   
178  0.047766                2.619997               2.635883   
186  0.014604                2.084850               2.092036   
228  0.011648                2.162591               2.170785   
254  0.011564                2.216516               2.225456   
265  0.022286                2.051585               2.058364   
286  0.034934                2.176925               2.185313   
310  0.016784               -2.402386              -2.414218   
332  0.030087                2.373472               2.384820   
350  0.009601                2.154338               2.162420   
428  0.028485                2.526195               2.540241   
430  0.017415               -2.367111              -2.378354   
439  0.017725               -2.206767              -2.215569   
490  0.026953               -2.480917              -2.494125   
497  0.012697               -2.289823              -2.299843   

     Studentized Residuals p-value  Cook's Distance  Cook's Distance_p-value  
52                        0.033849         0.007573                      1.0  
54                        0.037911         0.010690                      1.0  
62                        0.033815         0.005360                      1.0  
82                        0.002093         0.009312                      1.0  
114                       0.010984         0.007786                      1.0  
118                       0.020793         0.005766                      1.0  
129                       0.014780         0.007851                      1.0  
144                       0.045303         0.014889                      1.0  
145                       0.006955         0.016116                      1.0  
154                       0.041599         0.006174                      1.0  
155                       0.036000         0.005543                      1.0  
178                       0.008657         0.031303                      1.0  
186                       0.036948         0.005856                      1.0  
228                       0.030426         0.005010                      1.0  
254                       0.026504         0.005225                      1.0  
265                       0.040082         0.008722                      1.0  
286                       0.029338         0.015595                      1.0  
310                       0.016134         0.008957                      1.0  
332                       0.017466         0.015886                      1.0  
350                       0.031068         0.004090                      1.0  
428                       0.011385         0.017010                      1.0  
430                       0.017772         0.009028                      1.0  
439                       0.027179         0.007989                      1.0  
490                       0.012955         0.015499                      1.0  
497                       0.021876         0.006130                      1.0  
```

In [12]:

```python
# 4. Kontrola multikolinearity

X = pd.DataFrame(best_model_filtered.exog, columns=best_model_filtered.exog_names)

# VIF (z democvičenia)
vif = pd.Series([variance_inflation_factor(X.values, i)
                 for i in range(X.shape[1])],
                index=X.columns)

vif_df = vif.to_frame()
vif_df.columns = ['VIF']
print(vif_df)
print('\n\n\n')

# Korelácia
print(X.corr()[['ActiveUsers', 'I(ActiveUsers ** 2)']])
```

```
                                            VIF
Intercept                             61.935626
OSType_MacOS[T.True]                  10.328997
OSType_Windows[T.True]                 9.758714
OSType_iOS[T.True]                     9.248966
ActiveUsers                           30.613252
OSType_Windows[T.True]:ActiveUsers    10.038729
OSType_iOS[T.True]:ActiveUsers         8.997457
OSType_MacOS[T.True]:ActiveUsers      10.528764
I(ActiveUsers ** 2)                   22.999789
InteractingPct                         5.712444
ActiveUsers:InteractingPct             8.720958




                                    ActiveUsers  I(ActiveUsers ** 2)
Intercept                                   NaN                  NaN
OSType_MacOS[T.True]                   0.006710            -0.000737
OSType_Windows[T.True]                 0.000596             0.008306
OSType_iOS[T.True]                    -0.065845            -0.057381
ActiveUsers                            1.000000             0.977659
OSType_Windows[T.True]:ActiveUsers     0.257546             0.261636
OSType_iOS[T.True]:ActiveUsers         0.188768             0.183669
OSType_MacOS[T.True]:ActiveUsers       0.242860             0.228658
I(ActiveUsers ** 2)                    0.977659             1.000000
InteractingPct                         0.036544             0.030781
ActiveUsers:InteractingPct             0.590081             0.575407
```

In [13]:

```
# Problém druhej mocniny prediktoru ActiveUsers

# Ostránenie I(ActiveUsers**2)
the_best_formula:Formula = best_formula.Copy()
the_best_formula.Remove('I(ActiveUsers**2)')

best_model_filtered = smf.ols(formula=the_best_formula.Get(), data=Rdata_filtered)
best_result_filtered = best_model_filtered.fit()
print(best_result_filtered.summary())

print('\n\n')

# ----------------------------------------------------------- #
# Finálna kontrola

X = pd.DataFrame(best_model_filtered.exog, columns=best_model_filtered.exog_names)

# VIF (z democvičenia)
vif = pd.Series([variance_inflation_factor(X.values, i)
                 for i in range(X.shape[1])],
                 index=X.columns)

vif_df = vif.to_frame()
vif_df.columns = ['VIF']
print(vif_df)
print('\n\n')

# Korelácia
print(X.corr())

# Finálna formula
print('\n\n')
print(the_best_formula)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   Ping   R-squared:                       0.842
Model:                            OLS   Adj. R-squared:                  0.839
Method:                 Least Squares   F-statistic:                     290.8
Date:                Sun, 15 Dec 2024   Prob (F-statistic):          3.35e-190
Time:                        12:54:47   Log-Likelihood:                -1591.6
No. Observations:                 500   AIC:                             3203.
Df Residuals:                     490   BIC:                             3245.
Df Model:                           9
Covariance Type:            nonrobust
==================================================================================================
                                     coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------------
Intercept                          9.9548      1.741      5.717      0.000       6.534      13.376
OSType_MacOS[T.True]              -1.9029      1.896     -1.004      0.316      -5.628       1.822
OSType_Windows[T.True]             6.7916      1.862      3.647      0.000       3.133      10.450
OSType_iOS[T.True]                -1.9057      1.878     -1.015      0.311      -5.596       1.785
ActiveUsers                        0.0055      0.000     19.205      0.000       0.005       0.006
OSType_Windows[T.True]:ActiveUsers    -0.0006      0.000     -2.137      0.033      -0.001   -5.19e-05
OSType_iOS[T.True]:ActiveUsers    -0.0008      0.000     -2.454      0.014      -0.001      -0.000
OSType_MacOS[T.True]:ActiveUsers     0.0020      0.000      6.428      0.000       0.001       0.003
InteractingPct                    36.7221      2.128     17.257      0.000      32.541      40.903
ActiveUsers:InteractingPct        -0.0035      0.000     -9.767      0.000      -0.004      -0.003
==============================================================================
Omnibus:                        4.530   Durbin-Watson:                   1.913
Prob(Omnibus):                  0.104   Jarque-Bera (JB):                3.273
Skew:                           0.022   Prob(JB):                        0.195
Kurtosis:                       2.606   Cond. No.                     8.65e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 8.65e+04. This might indicate that there are
strong multicollinearity or other numerical problems.



                                          VIF
Intercept                           43.594257
OSType_MacOS[T.True]                10.280475
OSType_Windows[T.True]               9.733446
OSType_iOS[T.True]                   9.145571
ActiveUsers                          7.525168
OSType_Windows[T.True]:ActiveUsers  10.026870
OSType_iOS[T.True]:ActiveUsers       8.912828
OSType_MacOS[T.True]:ActiveUsers    10.474528
InteractingPct                       5.706057
ActiveUsers:InteractingPct           8.714959



                                    Intercept  OSType_MacOS[T.True]  \
Intercept                                 NaN                   NaN
OSType_MacOS[T.True]                      NaN              1.000000
OSType_Windows[T.True]                    NaN             -0.369828
OSType_iOS[T.True]                        NaN             -0.341441
ActiveUsers                               NaN              0.006710
OSType_Windows[T.True]:ActiveUsers        NaN             -0.323462
OSType_iOS[T.True]:ActiveUsers            NaN             -0.296499
OSType_MacOS[T.True]:ActiveUsers          NaN              0.885295
InteractingPct                            NaN              0.089866
ActiveUsers:InteractingPct                NaN              0.056752

                                    OSType_Windows[T.True]  \
Intercept                                              NaN
OSType_MacOS[T.True]                             -0.369828
OSType_Windows[T.True]                            1.000000
OSType_iOS[T.True]                               -0.334582
ActiveUsers                                       0.000596
OSType_Windows[T.True]:ActiveUsers                0.874629
OSType_iOS[T.True]:ActiveUsers                   -0.290542
OSType_MacOS[T.True]:ActiveUsers                 -0.327407
InteractingPct                                   -0.018190
ActiveUsers:InteractingPct                        0.001144

                                    OSType_iOS[T.True]  ActiveUsers  \
Intercept                                          NaN          NaN
OSType_MacOS[T.True]                         -0.341441     0.006710
OSType_Windows[T.True]                       -0.334582     0.000596
OSType_iOS[T.True]                            1.000000    -0.065845
ActiveUsers                                  -0.065845     1.000000
OSType_Windows[T.True]:ActiveUsers           -0.292635     0.257546
OSType_iOS[T.True]:ActiveUsers                0.868375     0.188768
OSType_MacOS[T.True]:ActiveUsers             -0.302277     0.242860
InteractingPct                               -0.063760     0.036544
ActiveUsers:InteractingPct                   -0.086494     0.590081

                                    OSType_Windows[T.True]:ActiveUsers  \
Intercept                                                          NaN
OSType_MacOS[T.True]                                         -0.323462
OSType_Windows[T.True]                                        0.874629
OSType_iOS[T.True]                                           -0.292635
```

```
ActiveUsers                                            0.257546
OSType_Windows[T.True]:ActiveUsers                     1.000000
OSType_iOS[T.True]:ActiveUsers                        -0.254117
OSType_MacOS[T.True]:ActiveUsers                      -0.286360
InteractingPct                                         0.009830
ActiveUsers:InteractingPct                             0.163177

                                    OSType_iOS[T.True]:ActiveUsers  \
Intercept                                                      NaN
OSType_MacOS[T.True]                                     -0.296499
OSType_Windows[T.True]                                   -0.290542
OSType_iOS[T.True]                                        0.868375
ActiveUsers                                               0.188768
OSType_Windows[T.True]:ActiveUsers                       -0.254117
OSType_iOS[T.True]:ActiveUsers                            1.000000
OSType_MacOS[T.True]:ActiveUsers                         -0.262490
InteractingPct                                           -0.051566
ActiveUsers:InteractingPct                                0.052500

                                    OSType_MacOS[T.True]:ActiveUsers  \
Intercept                                                        NaN
OSType_MacOS[T.True]                                        0.885295
OSType_Windows[T.True]                                     -0.327407
OSType_iOS[T.True]                                         -0.302277
ActiveUsers                                                 0.242860
OSType_Windows[T.True]:ActiveUsers                         -0.286360
OSType_iOS[T.True]:ActiveUsers                             -0.262490
OSType_MacOS[T.True]:ActiveUsers                            1.000000
InteractingPct                                              0.071554
ActiveUsers:InteractingPct                                  0.192962

                                    InteractingPct  ActiveUsers:InteractingPct
Intercept                                      NaN                         NaN
OSType_MacOS[T.True]                      0.089866                    0.056752
OSType_Windows[T.True]                   -0.018190                    0.001144
OSType_iOS[T.True]                       -0.063760                   -0.086494
ActiveUsers                               0.036544                    0.590081
OSType_Windows[T.True]:ActiveUsers        0.009830                    0.163177
OSType_iOS[T.True]:ActiveUsers           -0.051566                    0.052500
OSType_MacOS[T.True]:ActiveUsers          0.071554                    0.192962
InteractingPct                            1.000000                    0.752473
ActiveUsers:InteractingPct                0.752473                    1.000000


Ping ~ OSType_Windows:ActiveUsers + ActiveUsers + OSType_MacOS +
OSType_iOS:ActiveUsers + OSType_Windows + OSType_iOS +
InteractingPct + ActiveUsers:InteractingPct + OSType_MacOS:ActiveUsers
```

2. Najproblematickejšie hodnoty

In [14]:
```
Rdata_predicted = Rdata_filtered.copy()
grid_predictions = best_result_filtered.get_prediction(Rdata_filtered)
Rdata_predicted['PredictedPing'] = grid_predictions.summary_frame()['mean']

# Výber maxima
max_index = Rdata_predicted['PredictedPing'].idxmax()
max_row = Rdata_predicted.iloc[max_index]
print(max_row)
```

```
OSType                 MacOS
ActiveUsers             9953
InteractingPct        0.6729
ScrollingPct          0.3271
Ping                      76
OSType_MacOS            True
OSType_Windows         False
OSType_iOS             False
PredictedPing      83.441842
Name: 227, dtype: object
```

3. Odozva užívateľa Windows

In [15]:
```
au_mean = Rdata_filtered['ActiveUsers'].mean()
inter_mean = Rdata_filtered['InteractingPct'].mean()
scroll_mean = 1 - inter_mean
ping_mean = Rdata_filtered['Ping'].mean()

pred_data = pd.DataFrame({'OSType_Windows': [1],
                          'OSType_MacOS': [0],
                          'OSType_iOS': [0],
                          'OSType_Windows:ActiveUsers': [1 * au_mean],
                          'OSType_MacOS:ActiveUsers': [0 * au_mean],
                          'OSType_iOS:ActiveUsers': [0 * au_mean],
                          'ActiveUsers:InteractingPct': [au_mean * inter_mean],
                          'ActiveUsers': [au_mean],
                          'InteractingPct': [inter_mean],
                          'ScrollingPct': [scroll_mean],
                          'Ping': [ping_mean]})
```

```python
prediction = best_result_filtered.get_prediction(pred_data)
pred_summary = prediction.summary_frame()

point_estimate = pred_summary['mean'].iloc[0]
confidence_interval = prediction.conf_int()[0]
prediction_interval = (pred_summary['obs_ci_lower'].iloc[0], pred_summary['obs_ci_upper'].iloc[0])


print(f"Bodový odhad odozvy: {point_estimate} ms")
print(f"Konfidenčný interval: [{confidence_interval[0]} ; {confidence_interval[1]}]")
print(f"Predikčný interval: [{prediction_interval[0]} ; {confidence_interval[1]}]")
```

```
Bodový odhad odozvy: 51.85528705390375 ms
Konfidenčný interval: [50.84899581144395 ; 52.861578296363554]
Predikčný interval: [40.225513989290576 ; 52.861578296363554]
```

4. Vhodnosť výsledného modelu

Model má celkom celkom dobré $R^2 = 0,842$ a Adj. $R^2 = 0.839$ naznačuje, že model nie je preťažený zbytočnými prediktormi. Pokiaľ model štandardizujeme, tak všetky prediktory sú štatisticky významné a podľa hodnôt VIF a korelácie tu nie je ani multikolinearita. Rozhodol som sa zachovať všetky kategórie, pretože sú približne rovnako zastúpené a nevidím dôvod nejakú z modelu odstrániť. Durbin-Watsonova štatistika = 1,913 naznačuje, že reziduá nie sú výrazne autokorelované. Podľa testov Jarque-Bera $(0,104)$ a Omnibus $(0,195)$ sa reziduá riadia normálnym rozdelením. Omnibus je v tomto prípade smerodatnejší, keďže máme len 500 pozorovaní.

Podľa koeficientov prediktor najviac ovplyvňuje hodnotu ping pomer aktívnych interagujúcich užívateľov, pričom vzťah je lineárny.