Hi Michael,

PDF is easier cause email doesn't have math maarkup.

Here.

Generic concept of a regulator. Suppose there is a sum $\sum_n a_n$ of some infinite sequence a_n and suppose it is not absolutely convergent, for whatever reason. Maybe the a_n never get small, maybe they alternate sign, maybe they alternate sign and get large. There are three "basic" regulators: the zeta regulator

$$\zeta(s) = \sum_n a_n^{-s}$$

which usually converges if you pick real part of s large enough. This is usually "easy" to extended to the entire complex plane, and you will get a pole located at each divergence of the sum. Note there may be more than one pole! This is not uncommon!

Note that this is the same thing as the "zeta generating function", although GF's are used to get the series, whereas for regulators, the question is "what is the value at $\zeta(1)$?" or at least "describe $\lim_{s\to 1} \zeta(s)$"

The exponential regulator is

$$e(t) = \sum_n a_n e^{-t|a_n|}$$

and the question is "what is $\lim_{t\to 0} e(t)$?" and note that when the limit exists one has

$$\lim_{t\to 0} e(t) = \lim_{s\to 1} \zeta(s)$$

and more generally, you can get $e(t)$ from $\zeta(s)$ and vice-versa with a Laplace transform. (and its usually easy to do. For example, if $a_n \sim n^3$ then $\zeta(s)$ has a simple pole at $s = 3$ and $e(t) \sim 1/t^3 + \mathscr{O}\left(1/t^2\right)$ as $t \to 0$.

Both of the above suck for numerical work. What works great for numerical work is

$$g(t) = \sum_n a_n e^{-t^2|a_n|^2}$$

but this is much harder to relate to the other two, analytically. Numerically, it becomes easy to take the limit $t \to 0$.

There is a small ocean of work on this stuff from the late 19th through early 20th cent and I guess up to modern times. I've got a book "Buck & Boas" or os it "Boas & Creighton" I don't recall, dealing with series of "exponential type" and of "psi type" See

- https://en.wikipedia.org/wiki/Exponential_type
- https://en.wikipedia.org/wiki/Nachbin%27s_theorem
- https://en.wikipedia.org/wiki/Paley%E2%80%93Wiener_theorem

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