tags: **ASP**

ASP HW4

Problem 1

	$\mathbf{x}(0)$	$\mathbf{v}_1(0)$	$\mathbf{v}_2(0)$	$\mathbf{v}_1(1)$	$\mathbf{v}_2(1)$	$\mathbf{v}_1(2)$	$\mathbf{v}_2(2)$	$\mathbf{v}_1(3)$	$\mathbf{v}_2(3)$	**Reas**
$\mathbf{x}(1)$	V	V								1
$\mathbf{y}(1)$	V	V			V					2
$\alpha(1)$	V	V			V					3
$\mathbf{x}(2)$	V	V		V						1
$\mathbf{y}(2)$	V	V		V			V			2
$\alpha(2)$	V	V		V	V		V			3
$\hat{\mathbf{x}}(2\|\mathbb{Y}_1)$	V	V			V					4
$\hat{\mathbf{x}}(2\|\mathbb{Y}_2)$	V	V		V	V		V			5
$\mathbf{x}(3)$	V	V		V		V				1
$\mathbf{y}(3)$	V	V		V		V			V	2
$\alpha(3)$	V	V		V	V	V	V		V	3
$\hat{\mathbf{x}}(3\|\mathbb{Y}_2)$	V	V		V	V		V			4
$\hat{\mathbf{x}}(3\|\mathbb{Y}_3)$	V	V		V	V	V	V		V	5

(1) From Process equation: $\mathbf{x}(n+1) = \mathbf{F}(n+1,n)\mathbf{x}(n) + \mathbf{v}_1(n)$, we know that $\mathbf{x}(n+1)$ is related to $\mathbf{x}(n)$ and $\mathbf{v}_1(n)$
(2) From Measurement equation: $\mathbf{y}(n) = \mathbf{C}(n)\mathbf{x}(n) + \mathbf{v}_2(n)$, we know that $\mathbf{y}(n)$ is related to $\mathbf{x}(n)$ and $\mathbf{v}_2(n)$
(3) From Innovation process: $\alpha(n) = \mathbf{y}(n) - \hat{\mathbf{y}}(n|\mathbb{Y}_{n-1})$, we know that $\alpha(n)$ is related to $\mathbf{y}(k),\ k = 1 \sim n$
(4) From the equation $\hat{\mathbf{y}}(n|\mathbb{Y}_{n-1}) = \mathbf{C}(n)\hat{\mathbf{x}}(n|\mathbb{Y}_{n-1})$, we know that $\hat{\mathbf{x}}(n|\mathbb{Y}_{n-1})$ is related to $\hat{\mathbf{y}}(n|\mathbb{Y}_{n-1})$ which is related to $\mathbf{y}(k),\ k = 1 \sim n-1$. Notice that this equation is true only if $\mathbb{E}[\mathrm{v}_2(n)\alpha^H(k)] = 0,\ k = 1 \sim n-1$ and this is proved in (3).
(5) From the equation $\hat{\mathbf{x}}(n|\mathbb{Y}_n) = \mathbf{F}(n+1,n)\hat{\mathbf{x}}(n+1|\mathbb{Y}_n)$, we know that $\hat{\mathbf{x}}(n|\mathbb{Y}_n)$ is related to $\hat{\mathbf{x}}(n+1|\mathbb{Y}_n)$ which is then related to $\mathbf{y}(k),\ k = 1 \sim n$ by (4). Notice that this equation is true only if $\mathbb{E}[\mathrm{v}_1(n)\alpha^H(k)] = 0,\ k = 1 \sim n$ and this is proved in (3).

Problem 2

a.

$\alpha(1) = x(1)$

$\alpha(2) = L_{2,1}x(1) + x(2)$

From the ortogonal property we have:

$\mathbb{E}[\alpha(2)x^H(1)] = L_{2,1}r_x(0) + r_x(1) = L_{2,1} + \frac{1}{4} = 0$

$\rightarrow L_{2,1} = -\frac{1}{4}$

b.

$$\mathbf{R}_x = \begin{bmatrix} r_x(0) & r_x(-1) \\ r_x(1) & r_x(0) \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{4} \\ \frac{1}{4} & 1 \end{bmatrix}$$

c.

$$\mathbf{R}_\alpha = \mathbb{E}[\alpha\alpha^H] = \mathbf{L}\mathbb{E}[\mathbf{x}\mathbf{x}^H]\mathbf{L}^H = \mathbf{L}\mathbf{R}_x\mathbf{L}^H = \begin{bmatrix} 1 & 0 \\ 0 & \frac{15}{16} \end{bmatrix}$$

d.

From **c** we have:

$\mathbf{R}_x = \mathbf{L}^{-1}\mathbf{R}_\alpha\mathbf{L}^{-H} = (\mathbf{L}^{-1}\mathbf{R}_\alpha^{\frac{1}{2}})(\mathbf{L}^{-1}\mathbf{R}_\alpha^{\frac{1}{2}})^H$

Therefore, $\mathbf{L} = \mathbf{L}^{-1}\mathbf{R}_\alpha^{\frac{1}{2}} = \begin{bmatrix} 1 & 0 \\ \frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{\sqrt{15}}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{4} & \frac{\sqrt{15}}{4} \end{bmatrix}$

Problem 3

a.

The nummerical value of the sample covariance matrix is:

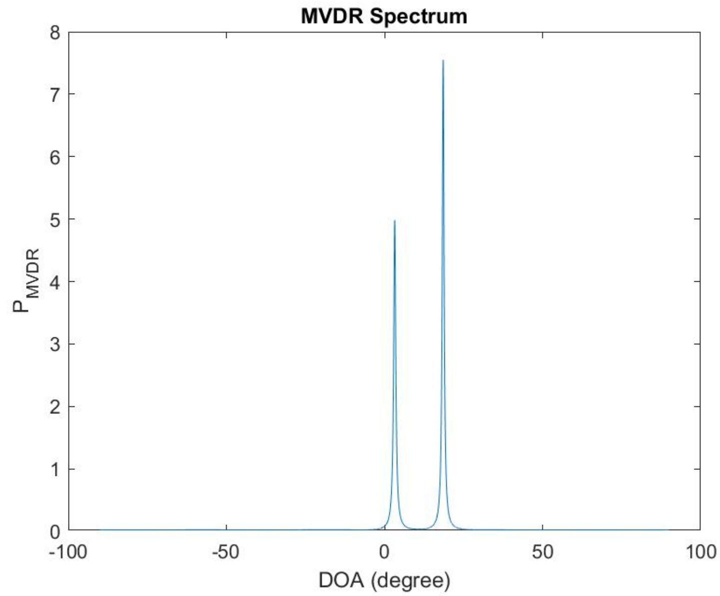
12.6627 + 0.0000i	8.9067 - 7.2179i	1.4652 - 8.5748i	-3.1576 - 3.5350i	-1.1632 + 2.4753i	5.2685 + 3.2840i	9.6392 - 2.2483i	7.1867 - 9.6166i	-0.3844 -12.2513i	-7.0559 - 8.0373i	-7.2718 - 0.6961i
8.9067 + 7.2179i	12.7772 - 0.0000i	8.9780 - 7.2449i	1.5867 - 8.6158i	-3.1957 - 3.6725i	-1.1971 + 2.3865i	5.2400 + 3.3963i	9.5660 - 2.2458i	7.2500 - 9.5898i	-0.3831 -12.4459i	-7.0464 - 8.0678i
1.4652 + 8.5748i	8.9780 + 7.2449i	12.7536 + 0.0000i	9.0266 - 7.2066i	1.5315 - 8.6385i	-3.2225 - 3.6096i	-1.2323 + 2.5292i	5.2645 + 3.3905i	9.6261 - 2.1855i	7.3474 - 9.7094i	-0.4059 -12.3869i
-3.1576 + 3.5350i	1.5867 + 8.6158i	9.0266 + 7.2066i	12.9885 + 0.0000i	9.1677 - 7.3654i	1.5464 - 8.7842i	-3.2766 - 3.7380i	-1.1959 + 2.3856i	5.2331 + 3.3270i	9.7259 - 2.3158i	7.2626 - 9.8577i
-1.1632 - 2.4753i	-3.1957 + 3.6725i	1.5315 + 8.6385i	9.1677 + 7.3654i	12.9966 + 0.0000i	9.0430 - 7.4074i	1.5031 - 8.7465i	-3.2006 - 3.6116i	-1.1629 + 2.4811i	5.3801 + 3.4329i	9.7851 - 2.4012i
5.2685 - 3.2840i	-1.1971 - 2.3865i	-3.2225 + 3.6096i	1.5464 + 8.7842i	9.0430 + 7.4074i	12.8675 + 0.0000i	9.0767 - 7.2336i	1.5766 - 8.5395i	-3.1360 - 3.5506i	-1.1314 + 2.5446i	5.4355 + 3.3633i
9.6392 + 2.2483i	5.2400 - 3.3963i	-1.2323 - 2.5292i	-3.2766 + 3.7380i	1.5031 + 8.7465i	9.0767 + 7.2336i	12.9048 - 0.0000i	8.9547 - 7.2551i	1.4697 - 8.5836i	-3.2754 - 3.5605i	-1.0836 + 2.5537i
7.1867 + 9.6166i	9.5660 + 2.2458i	5.2645 - 3.3905i	-1.1959 - 2.3856i	-3.2006 + 3.6116i	1.5766 + 8.5395i	8.9547 + 7.2551i	12.6692 - 0.0000i	8.8941 - 7.1293i	1.4967 - 8.5843i	-3.1487 - 3.5020i
-0.3844 +12.2513i	7.2500 + 9.5898i	9.6261 + 2.1855i	5.2331 - 3.3270i	-1.1629 - 2.4811i	-3.1360 + 3.5506i	1.4697 + 8.5836i	8.8941 + 7.1293i	12.6074 - 0.0000i	8.9321 - 7.2588i	1.4552 - 8.5296i
-7.0559 + 8.0373i	-0.3831 +12.4459i	7.3474 + 9.7094i	9.7259 + 2.3158i	5.3801 - 3.4329i	-1.1314 - 2.5446i	-3.2754 + 3.5605i	1.4967 + 8.5843i	8.9321 + 7.2588i	12.8972 + 0.0000i	8.9978 - 7.3153i
-7.2718 + 0.6961i	-7.0464 + 8.0678i	-0.4059 +12.3869i	7.2626 + 9.8577i	9.7851 + 2.4012i	5.4355 - 3.3633i	-1.0836 - 2.5537i	-3.1487 + 3.5020i	1.4552 + 8.5296i	8.9978 + 7.3153i	12.8479 + 0.0000i

b.

The 11 eigen value of the sample covariance matrix is:

89.7903 + 0.0000i
48.9009 - 0.0000i
0.2957 + 0.0000i
0.2130 - 0.0000i
0.2269 - 0.0000i
0.2818 - 0.0000i
0.2348 - 0.0000i
0.2465 - 0.0000i
0.2502 + 0.0000i
0.2702 - 0.0000i
0.2620 + 0.0000i

c.

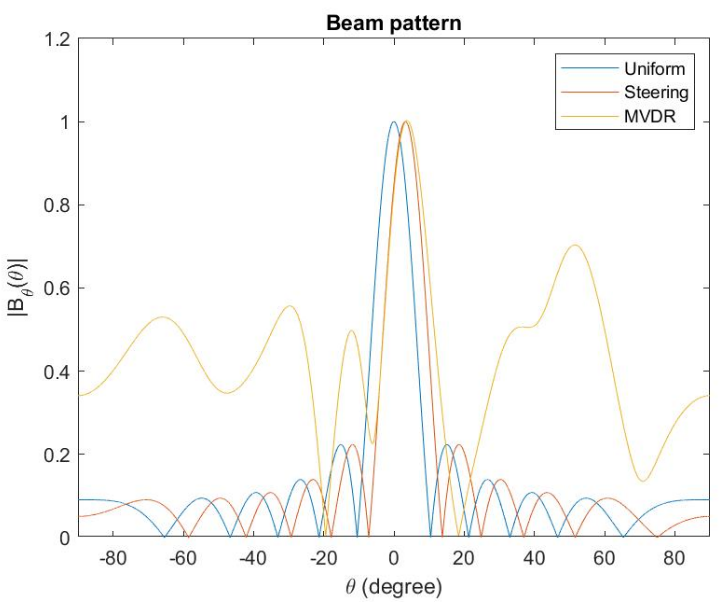


d.

By applying the 'findpeaks' function in MATLAB, the estimated DOA pf signal source is 3.25^o (Notice that the signal DOA is between 0^o and 10^o)

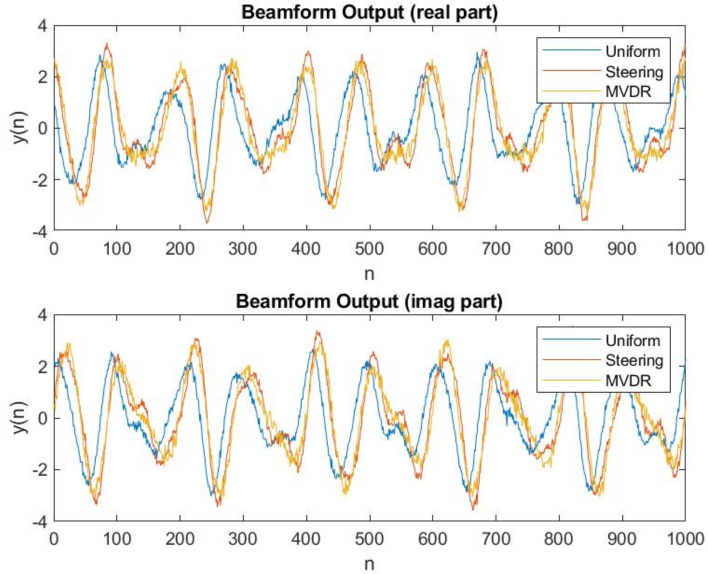
e.

The beam pattern of different beamforming weights is as follows:



f.

The beamform output is as follows:



Problem 4

a.

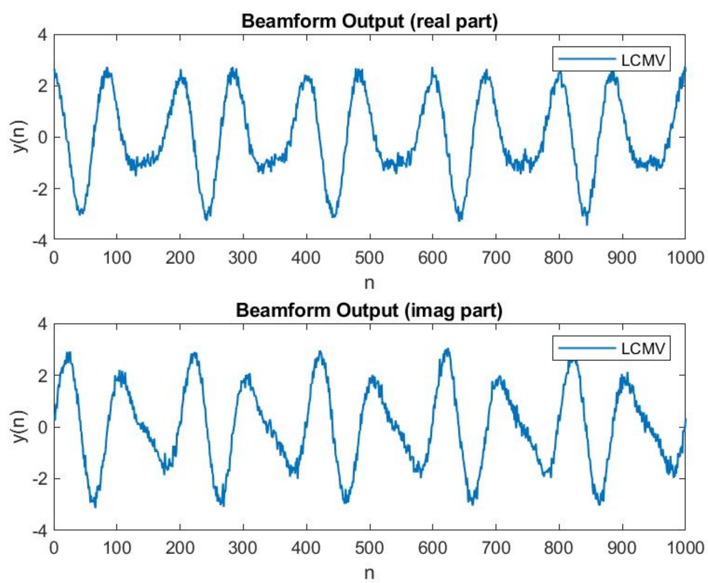
(1) Utilizing sample matrix inversion to estimate the correlation matrix $\mathbf{R}$

(2) Utilizing ESPIRIT algorithm for more accurate DOA estimation

(3) Utilizing LCMV algorithm so that the signal DOA is at peak and the interference DOA is at null.

b.

The output of our Beamformer is as follows:

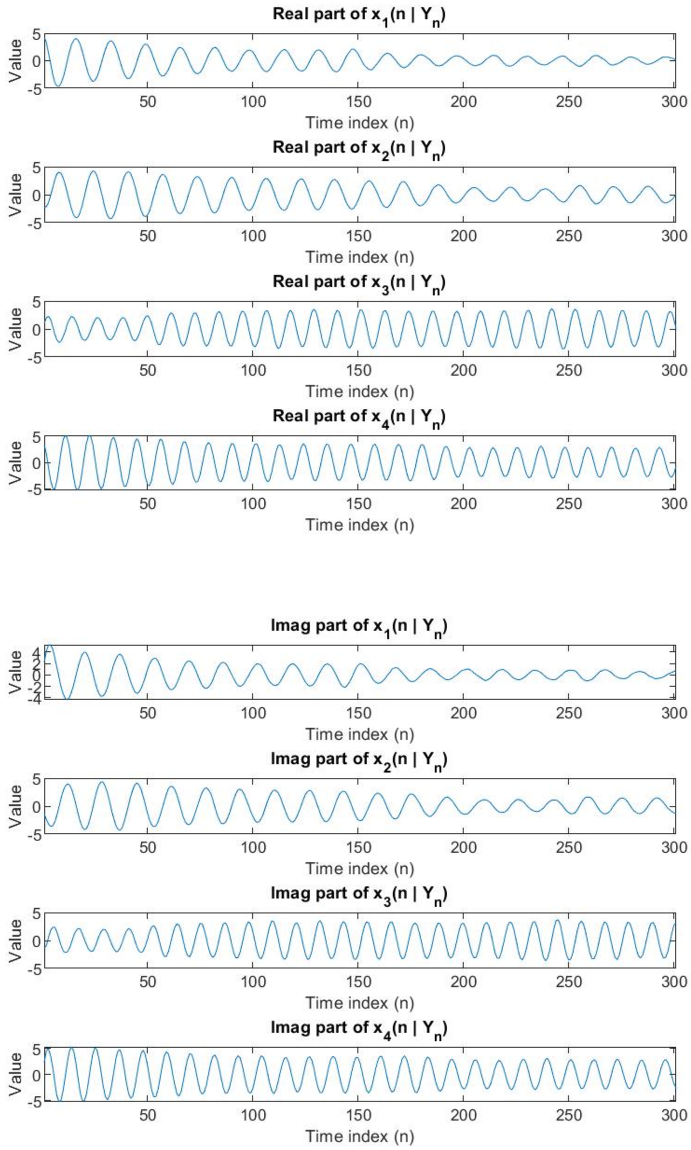


Problem 5

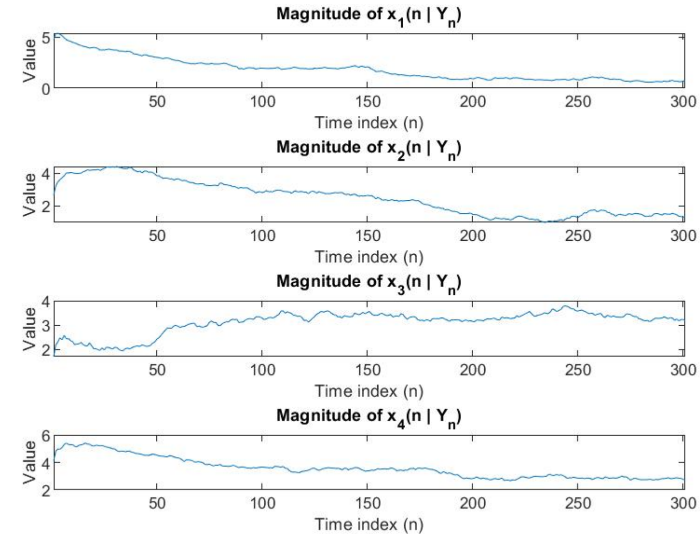
a.

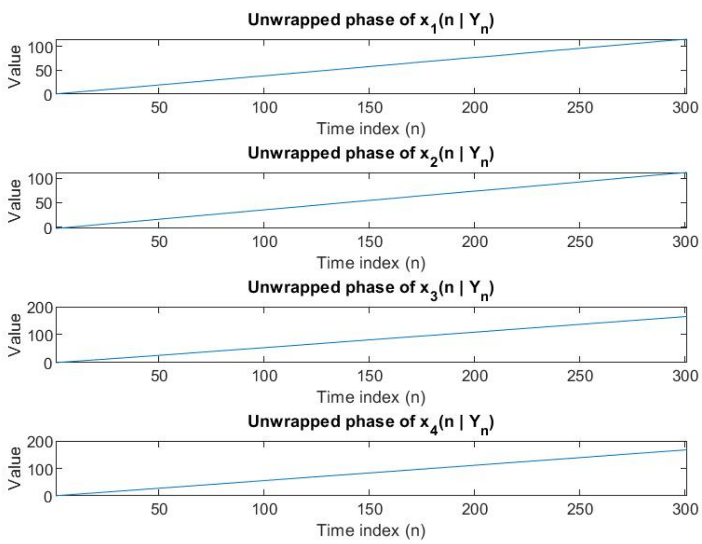
$M = 4,\ N = 9,\ L = 301$

b.



Real part of $x_1(n | Y_n)$
Real part of $x_2(n | Y_n)$
Real part of $x_3(n | Y_n)$
Real part of $x_4(n | Y_n)$
Imag part of $x_1(n | Y_n)$
Imag part of $x_2(n | Y_n)$
Imag part of $x_3(n | Y_n)$
Imag part of $x_4(n | Y_n)$
Value
Time index (n)
5
0
-5
4
2
0
-2
-4
50
100
150
200
250
300





c.

I design a ϵ-NLMS estimator, where the input signal is the timing index and the desire signal is the unwrapped phase. Then, the weight of the filter is then the slope of the unwrapped phase.

The pseudo code is as follows:

```
w = zeros(M, 1);
u_tilde = 0.2;
eps = 0.001;
dis = 1;

for k = 1 : M
    phase_x = unwrap(angle(x_tilde_o(k,:)));
    res_phase = phase_x(2:L) - phase_x(1);

    for x = 1 : L-1
        y = w(k) * x;
        e = res_phase(x) - y;
        w(k) = w(k) + u_tilde / (x^2 + eps) * x * e;
    end
end

w_pi = w / pi;

for i = 1 : M
    fprintf(strcat("The slope of unwrapped phase of x", num2str(i), "(n) is: " ...
            ,num2str(round(w_pi(i), 3)), " pi \n"));
end
```

And the results is as follows:

```
The slope of unwrapped phase of x1(n) is: 0.121 pi
The slope of unwrapped phase of x2(n) is: 0.121 pi
The slope of unwrapped phase of x3(n) is: 0.176 pi
The slope of unwrapped phase of x4(n) is: 0.177 pi
```