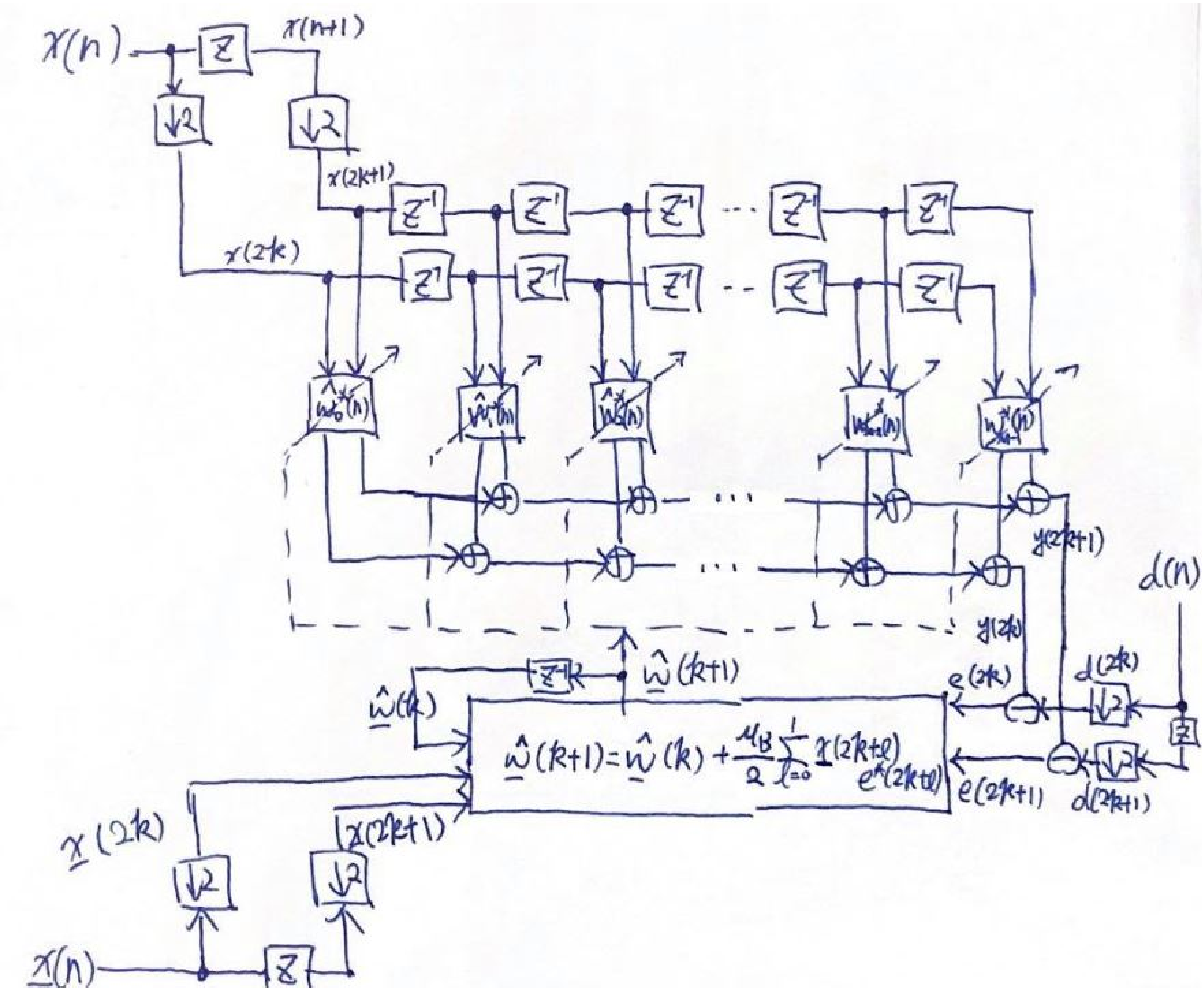
tags: **ASP**

ASP HW3

Problem 1

The block digram of $L = 2$ block-LMS is as follows:



Problem 2

By definition, $y^H = w^T A^T$ and $e^* = d - Aw$

Thus,

$< y_{LS}(n), e_{LS}(n) >$

$= y^T e^* = w^H A^H (d - Aw) = w^H (A^H d - A^H Aw) = w^H 0 = 0$

Problem 3

a.

$(A + UCV)(A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1})$

$= I - U(C^{-1} + VA^{-1}U)^{-1}VA^{-1} + UCVA^{-1} - UCVA^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1})$

$= I + UCVA^{-1} - U\{(C^{-1} + VA^{-1}U)^{-1} - CVA^{-1}U(C^{-1} + VA^{-1}U)^{-1}\}VA^{-1}$

$= I + UCVA^{-1} - U(I + CVA^{-1}U)(C^{-1} + VA^{-1}U)^{-1}VA^{-1}$

$= I + UCVA^{-1} - UCVA^{-1}$

$= I$

b.

Since $k(n) = \frac{\lambda^{-1}P(n-1)x(n)}{1+\lambda^{-1}x^H(n)P(n-1)x(n)}$, we have

$k(n) + \lambda^{-1}k(n)x^H(n)P(n-1)x(n) = \lambda^{-1}P(n-1)x(n)$

$\Rightarrow k(n) = (\lambda^{-1}P(n-1) - \lambda^{-1}k(n)x^H(n)P(n-1))x(n)$

$\Rightarrow k(n) = P(n)x(n)$

Problem 4

a.

$R_x = \mathbb{E}[x(t)x(t)^H] = \mathbb{E}[(As(t) + n(t))(As(t) + n(t))^H]$

$= A\mathbb{E}[s(t)s(t)^H]A^H + A\mathbb{E}[s(t)n(t)^H] + \mathbb{E}[n(t)s(t)^H]A^H + \mathbb{E}[n(t)n(t)^H]$

$= p_1 AA^H + p_n I$

Where $A = [1\ e^{j\pi sin\theta 1}\ e^{j2\pi sin\theta 1}\ \ldots\ e^{j(N-1)\pi sin\theta 1}]^T$

b.

Since $A^H A = |A|^2 = N$

And, by the matrix inversion lemma, we have

$R_x^{-1} = p_n^{-1}(I + A\frac{p1}{p_n}A^H)^{-1} = p_n^{-1}(I - A(\frac{p_n}{p1} + A^H A)^{-1}A^H)$

$= p_n^{-1}I - (\frac{p_n^2}{p1} + p_n A^H A)^{-1}AA^H = p_n^{-1}I - (\frac{p_n^2}{p1} + Np_n)^{-1}AA^H$

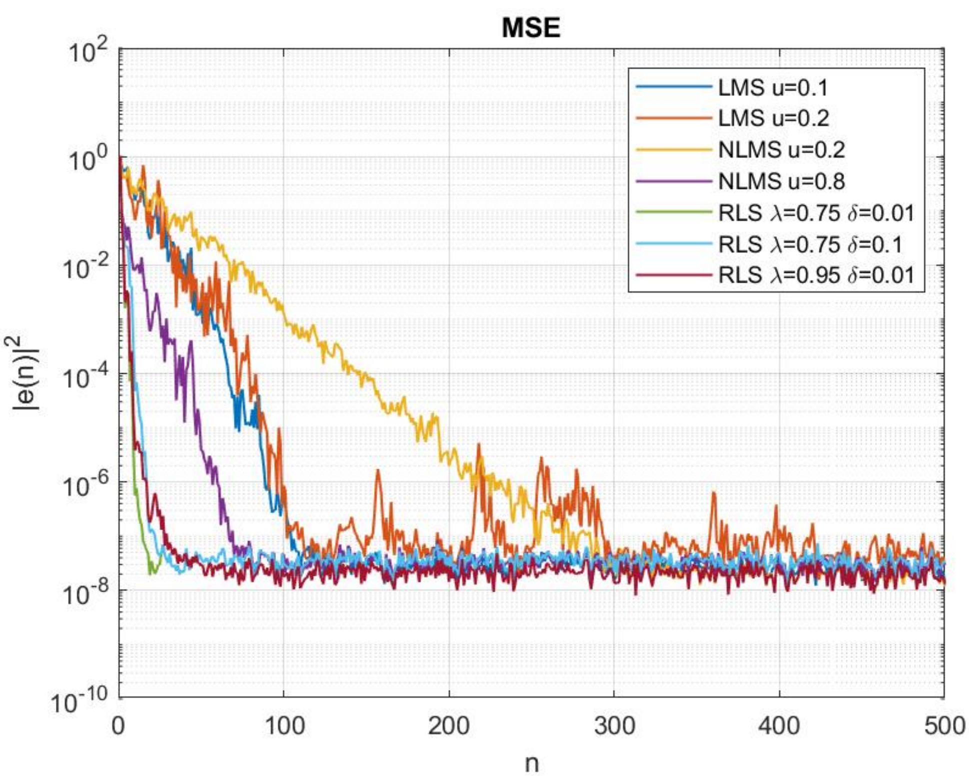
$P_{MVDR}(\theta) = \frac{1}{a^H(\theta)R^{-1}a(\theta)} \leq \frac{1}{a^H(\theta_1)R^{-1}a(\theta_1)} = P_{MVDR}(\theta_1)$
$\Leftrightarrow a^H(\theta_1)R^{-1}a(\theta_1) \leq a^H(\theta)R^{-1}a(\theta)$ ($\because R^{-1}$ is PSD and $a \neq 0$)
$\Leftrightarrow a^H(\theta_1)(p_n^{-1}I - (\frac{p_n^2}{p1} + Np_n)^{-1}AA^H)a(\theta_1) \leq a^H(\theta)(p_n^{-1}I - (\frac{p_n^2}{p1} + Np_n)^{-1}AA^H)a(\theta)$
$\Leftrightarrow a^H(\theta_1)AA^Ha(\theta_1) \geq a^H(\theta)AA^Ha(\theta)$
$\Leftrightarrow N^2 \geq |A^Ha(\theta)|^2$ ($\because \forall\theta, |a(\theta)| \leq \sqrt{N}$ $\therefore$ norm of inner product is less than N)
Q.E.D

Problem 5

$H(z) = \frac{1+\frac{1}{10}z^{-1}}{(1-\frac{1}{2}x^{-1})(1+\frac{1}{3}z^{-1})} = \frac{\frac{18}{25}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{7}{25}}{1+\frac{1}{3}z^{-1}}$
$\Rightarrow h(n) = \{\frac{18}{25}(\frac{1}{2})^n + \frac{7}{25}(-\frac{1}{3})^n\}u(n)$

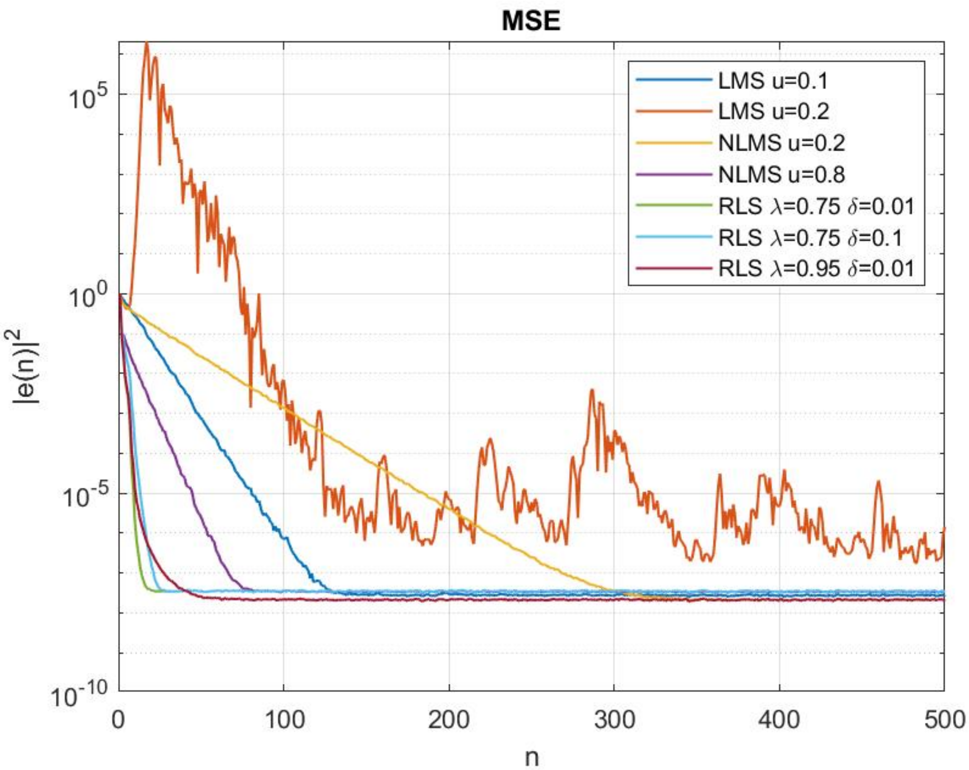
a.

Empirical learning curves for realization $R = 10$



b.

Empirical learning curves for realization $R = 10$



Problem 6

We can have the following observation from the above experiments.

a.

First, we calculate J_{min} by the optimal weight

Since

$\mathbb{E}[x(n)v^*(n-k)] = \mathbb{E}[\sum_t h(t)v(n-t)v^*(n-k)] = \sum_t h(t)\mathbb{E}[v(n-t)v^*(n-k)] = h(k)$

We have

$r_x(k) = \mathbb{E}[x(n)x^*(n-k)] = \mathbb{E}[\sum_t h(t)v(n-t)x^*(n-k)] = \sum_t h(t)\mathbb{E}[v(n-t)x^*(n-k)]$

$= \sum_t h(t)\mathbb{E}[x(n)v^*(n+k-t)]^* = \sum_t h(t)h^*(t-k)$

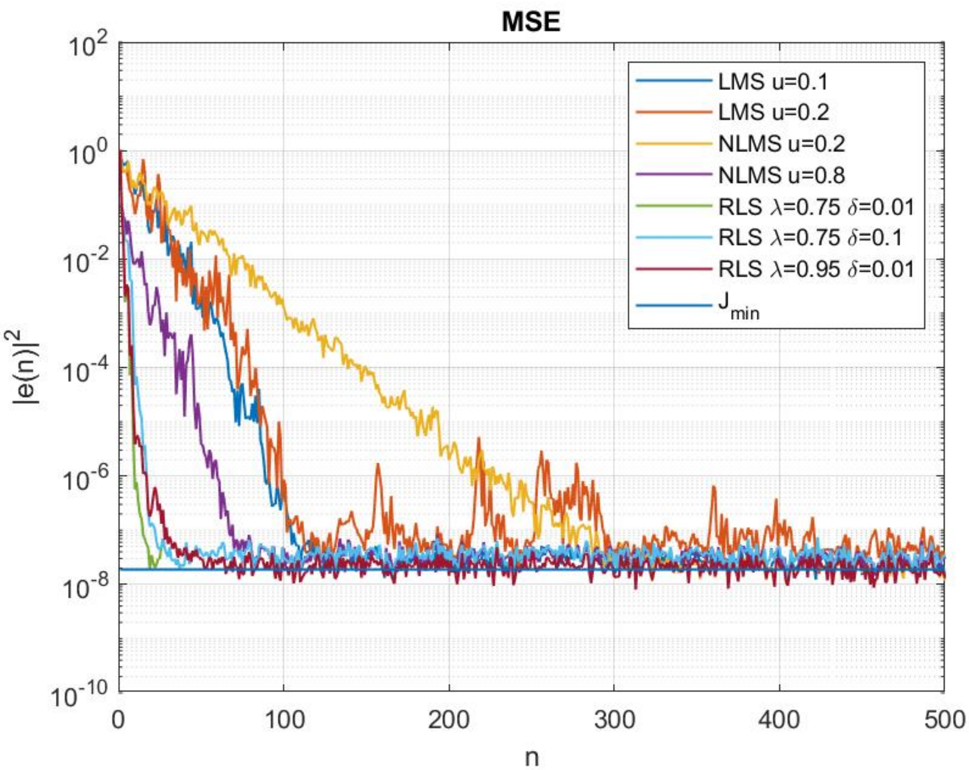
$$p = \mathbb{E}[\mathbf{x}(n)d^*(n)] = \mathbb{E}[\mathbf{x}(n)v^*(n)] = \begin{bmatrix} h(0) \\ h(-1) \\ \vdots \\ h(-4) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

By Wiener-Hopf equation, the optimzal weight is

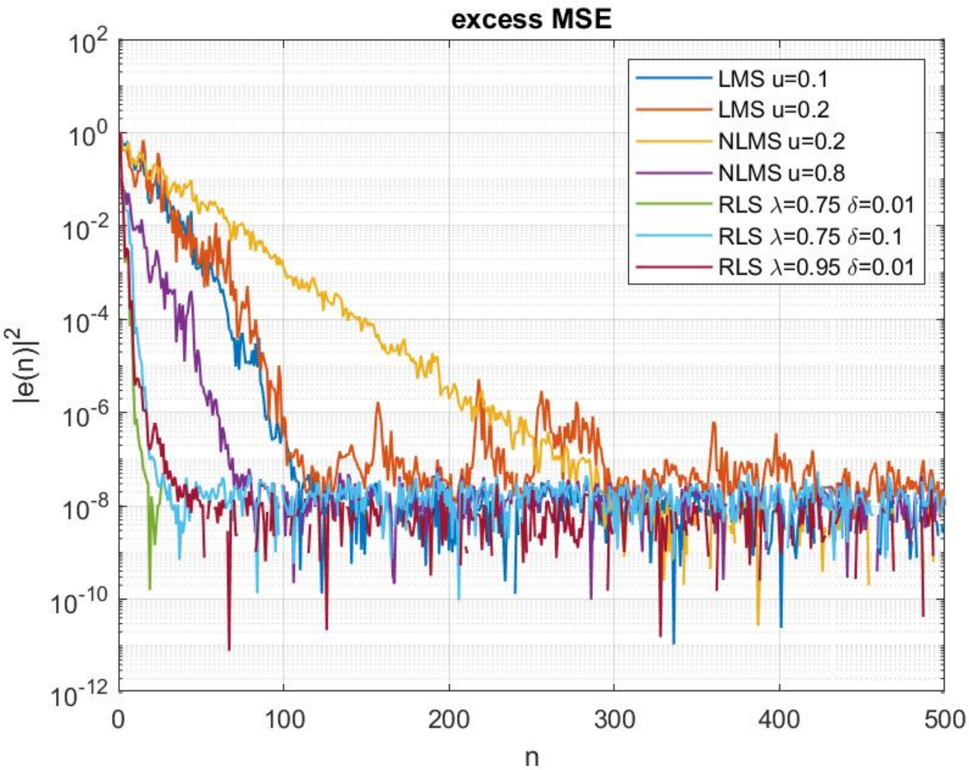
$$w_{opt} = R^{-1}p = \begin{bmatrix} 1 \\ -0.2667 \\ -0.14 \\ 0.014 \\ -0.0014 \end{bmatrix}$$

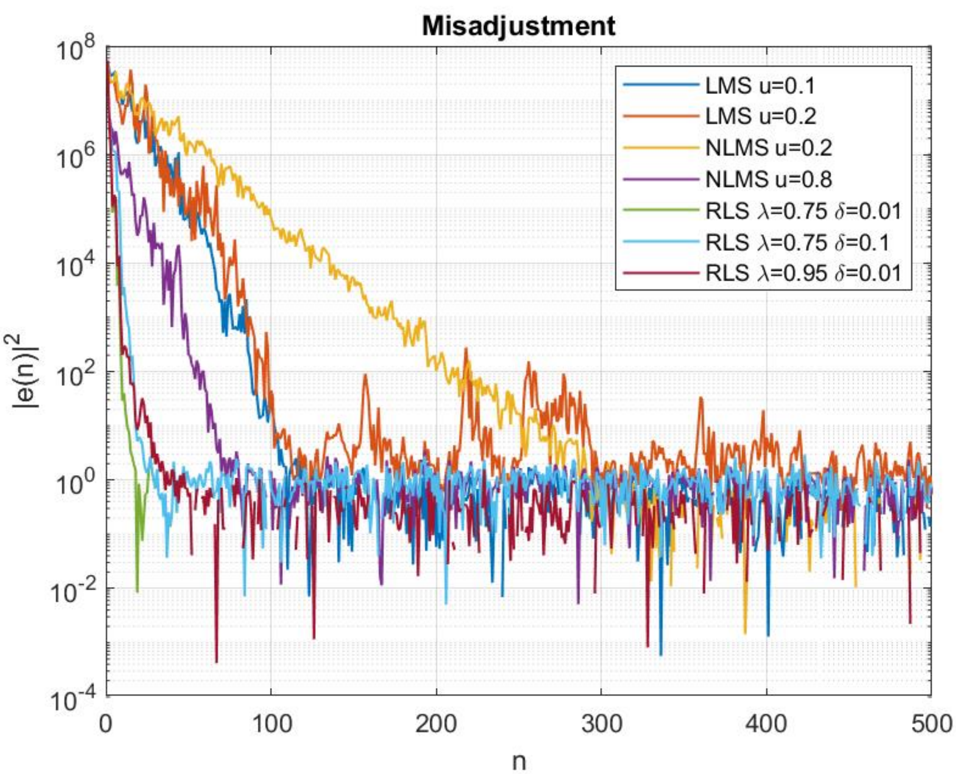
$$J_{min} = \sigma_d^2 - p^H w = 1.8835 \times 10^{-8}$$

By plotting J_{min} in the figure in problem 5, we can see that all algorithm might satisfy the property $lim_{n\to\infty}\hat{J}(n) = J_{min}$



Notice that $\hat{J}(n) - J_{min}$ is define as the excess MSE and $\frac{\hat{J}(n) - J_{min}}{J_{min}}$ is define as misadjustment, and the result of realization $R = 10$ is plot in the figure below.





b.

LMS

The “sufficient” condition for LMS to converge is:

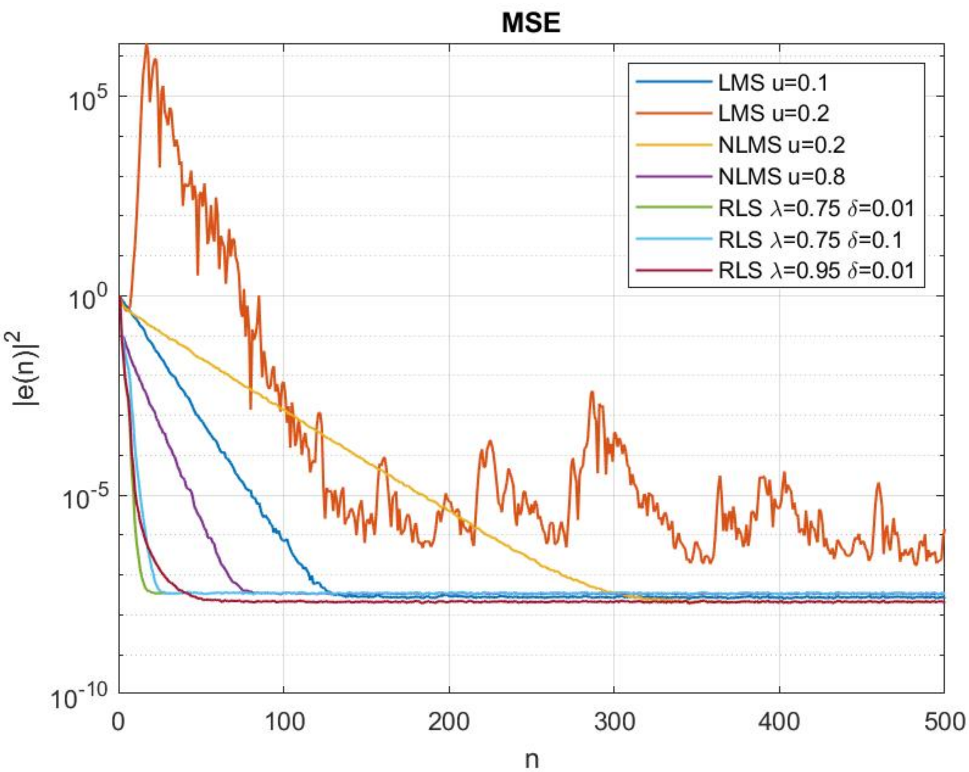
$0 < \mu < \min_{n=1\sim N} \frac{1}{|x(n)|^2}$

For $R = 10$, we find the upper bound of μ be 0.1179.
For $R = 1000$, we find the upper bound of μ be 0.0643.
Though both μ is greater than the upper bound, we find that only the case $\mu = 0.2$ diverges.

NLMS

For the NLMS case, since the step size is normalized by the squared of the value, it will not diverges. Though NLMS has the potential to converge faster than LMS, it is not guaranteed that NLMS converges faster than LMS.

RLS



Smaller δ is much more accurate for estimating matrix P thus faster convergence; however, it might have numerical issues.
As for the parameter λ, it represents the memory of the previous iteration. Thus, larger λ results in slower conevrgence.