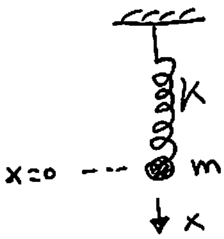
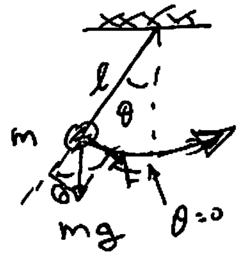
SITGES - 24-VI-2021

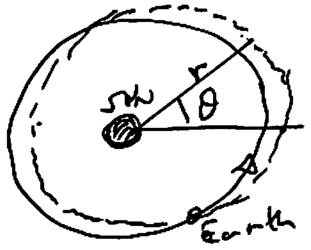
UNDERSTANDING OSCILLATIONS AROUND EQUILIBRIUM



x?

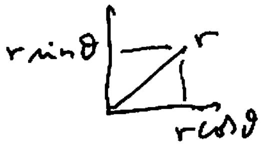


θ?

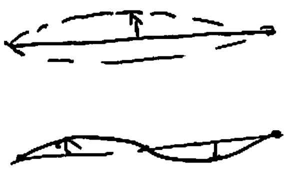


$F = -Kx \stackrel{!}{=} ma$

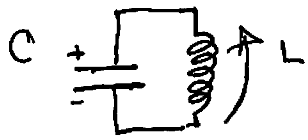
$F = -mg\sin\theta \stackrel{!}{=} ma$



$r(t)$
$\theta(t)$?



v?



→ Differential calculus → $\boxed{\text{derivatives}}$ ← $\boxed{\text{limits}}$

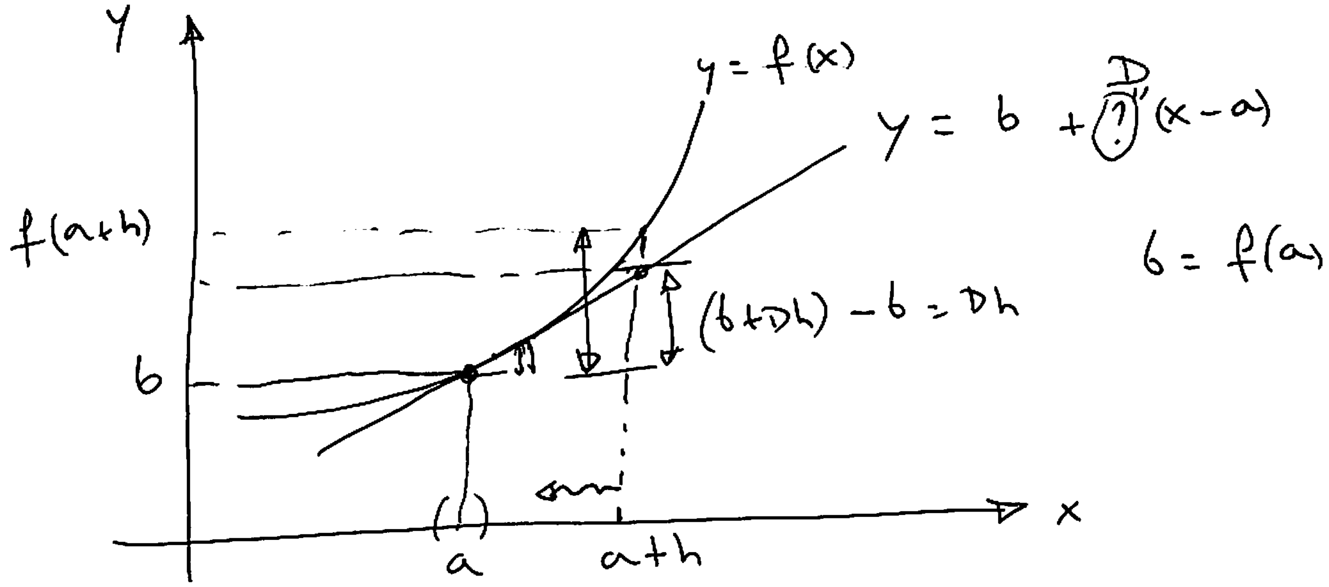
→ $\boxed{\text{differential equations}}$

← $\boxed{\text{series}}$

→ $\boxed{\text{Integrals}}$

→ $\boxed{\text{complex numbers}}$ i, e

DERIVATIVE



→ straight line

$$\begin{cases} x = a+h & \rightarrow \quad y = b + D\cdot(\cancel{a}+h-\cancel{a}) = b + Dh \\ x = a & \rightarrow \quad y = b \end{cases}$$

→ curve

$$\begin{cases} x = a+h & \rightarrow \quad y = f(a+h) \\ x = a & \rightarrow \quad y = f(a) \end{cases}$$

$$h \to 0 \quad \Rightarrow \quad f(a+h) - f(a) \approx Dh$$

$$D = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \equiv f'(a)$$

tangent straight line $\boxed{y = f(a) + f'(a)(x-a)}$

$$f(x) = x^2 \quad \to \quad f'(x) = 2x \equiv \frac{df(x)}{dx}$$

$$f'(a) = \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{a^2} + 2ah + h^2 - \cancel{a^2}}{h}$$

$$= \lim_{h \to 0} \left[2a + h \right] = 2a$$

$$\underbrace{\left(f(x) + g(x) \right)'}_{\parallel} = f'(x) + g'(x)$$

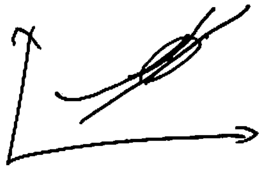
$$\lim_{h \to 0} \frac{\left[f(x+h) + g(x+h) \right] - \left[f(x) + g(x) \right]}{h} =$$

$$= \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right]$$

$$= \left(\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \right) + \left(\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \right)$$

$$= f'(x) + g'(x)$$

$$f(x) \simeq f(a) + f'(a)(x-a) \quad \text{if } x \approx a$$



$$\begin{pmatrix} x \to x+h \\ a \to x \end{pmatrix}$$

$$f(x+h) = f(x) + h\, f'(x)$$

$x \to x+h$ $\quad f(x+h) \simeq f(a) + f'(a)(x+h-a) \quad$ $x+h \approx a$

$a \to x$ $\quad f(x+h) \simeq f(x) + f'(x)(\cancel{x}+h-\cancel{x}) \quad$ $x+h \simeq x$

$$\boxed{(f(x)\,g(x))'} = \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \frac{(f(x)+f'(x)\,h)(g(x)+g'(x)\,h) - f(x)\,g(x)}{h}$$

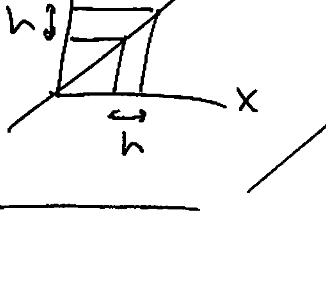
$$= \lim_{h\to 0} \frac{\cancel{f(x)\,g(x)} + f(x)\,g'(x)\,h + f'(x)\,h\,g(x) + f'(x)\,g'(x)\,h^2 - \cancel{f(x)\,g(x)}}{h}$$

$$= \lim_{h\to 0} \left(f(x)\,g'(x) + f'(x)\,g(x) + f'(x)\,g'(x)\,\underbrace{h}_{\to 0}\right)$$

$$= \boxed{f'(x)\,g(x) + f(x)\,g'(x)}$$

$$(x^2)' = (x\,x)' = (x)'\,x + x\,(x)'$$
$$= 1\cdot x + x\cdot 1$$
$$= 2x$$

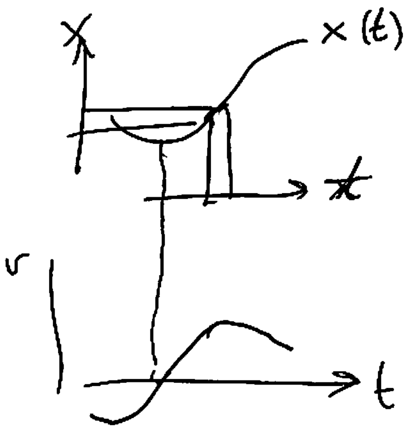
$$(x)' = \lim_{h\to 0} \frac{\cancel{(x}+h) - \cancel{x}}{h} = 1$$



$$(x^n)' = (x^{n-1}\,x)'$$
$$= (x^{n-1})'\,(x) + (x^{n-1})\,\underbrace{(x)'}_{=1}$$

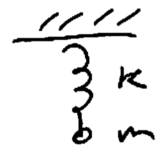
$$(x^n)' = x\,(x^{n-1})' + x^{n-1}$$
$$\ldots = n\,x^{n-1}$$

$$v = \frac{dx}{dt} \equiv \dot{x}$$



$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$\equiv \frac{d^2x}{dt^2} \equiv \ddot{x}$$



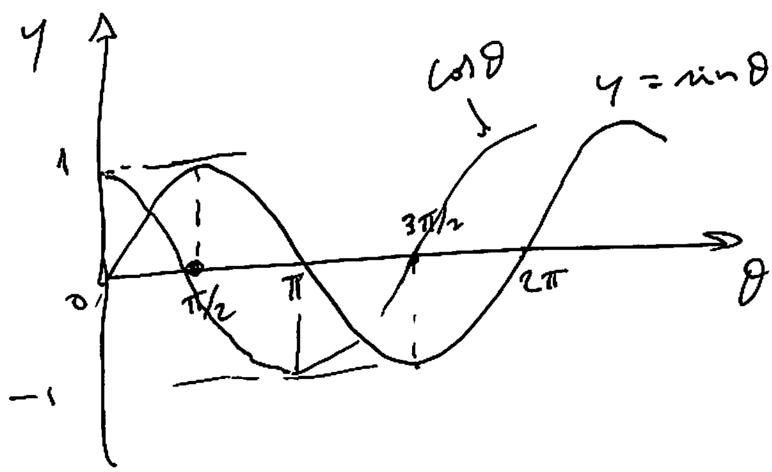
$$ma = -kx$$

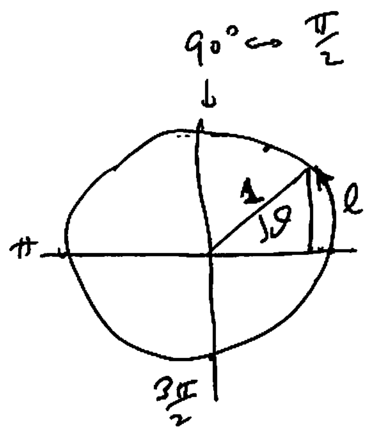
$$m\ddot{x} = -kx$$

$$\boxed{x = A\cos\left(\sqrt{\frac{k}{m}}\,t\right) + B\sin\left(\sqrt{\frac{k}{m}}\,t\right)}$$

$$(\cos(\theta))' = -\sin(\theta)$$

$$(\sin(\theta))' = \cos(\theta)$$



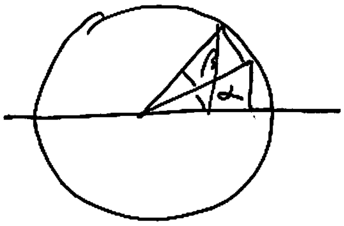


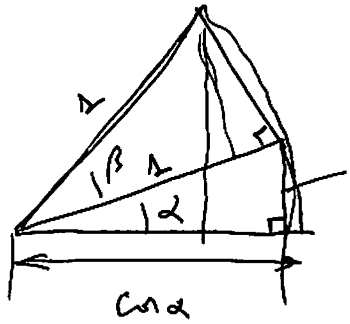
$$2\pi r = L_{circle}$$

$$\frac{l}{r} = \theta$$

(1)

$$\cos(\alpha+\beta) = (\cos\alpha)\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$





(2) Derivative

(3) Limits → (3') $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

$\approx 2.71828\ldots$

"e" number