Lecture 15: Batch RL

Emma Brunskill

CS234 Reinforcement Learning.

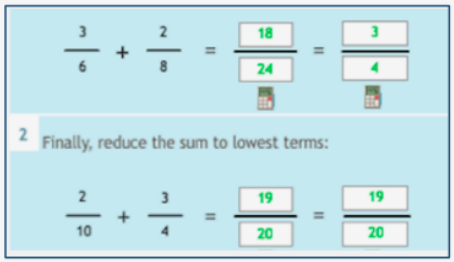
Winter 2019
Slides drawn from Philip Thomas with modifications

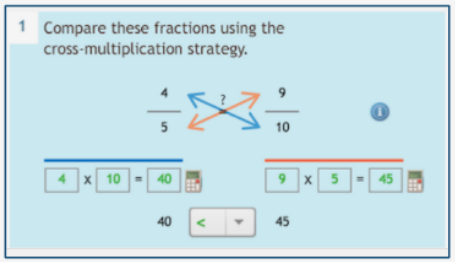
Class Structure

- Last time: Meta Reinforcement Learning
- **This time: Batch RL**
- Next time: Quiz

A Scientific Experiment

A Group





Avg Score: 95

A Scientific Experiment

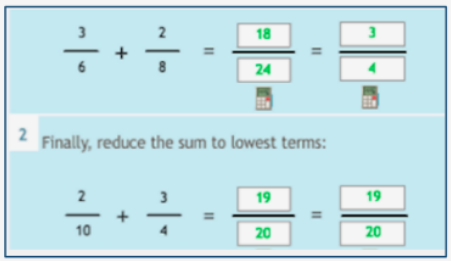
how large is NA NB

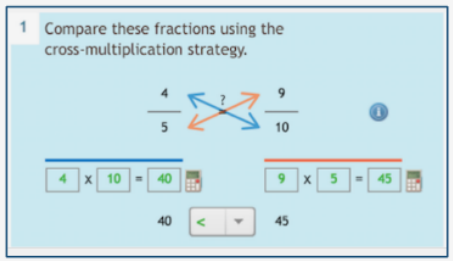
do this first

do this second

who is in the groups

A Group

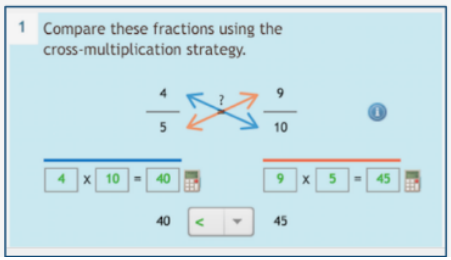


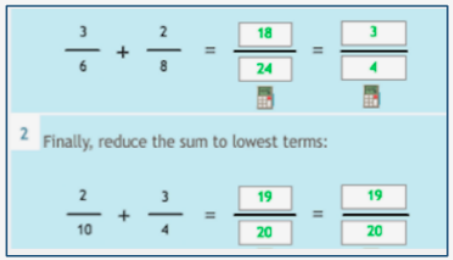


Avg Score: 95

Var

B Group





Avg Score: 92

Var

statistics

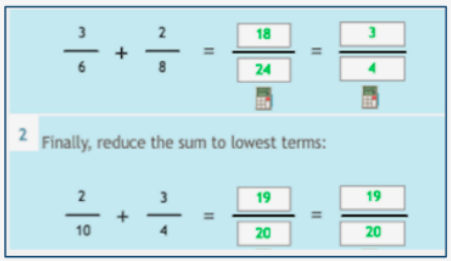
you might want to know the variance, the sample size of each, and who is in each group?

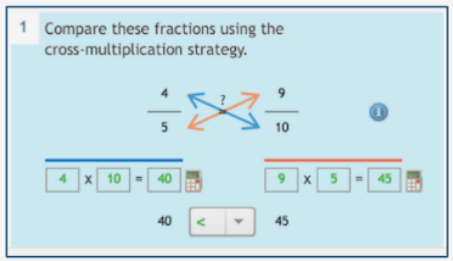
What to do for a new student?

if our goal is to maximize exam score, which sequence should we give a new student?

What Should We Do For a New Student?

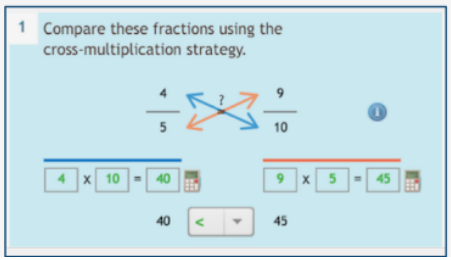
A Group

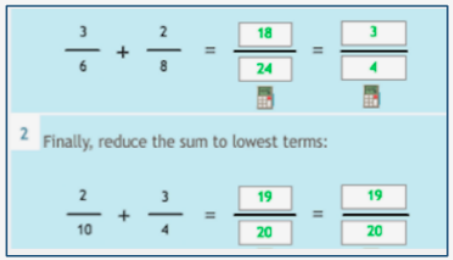




Avg Score: 95

B Group



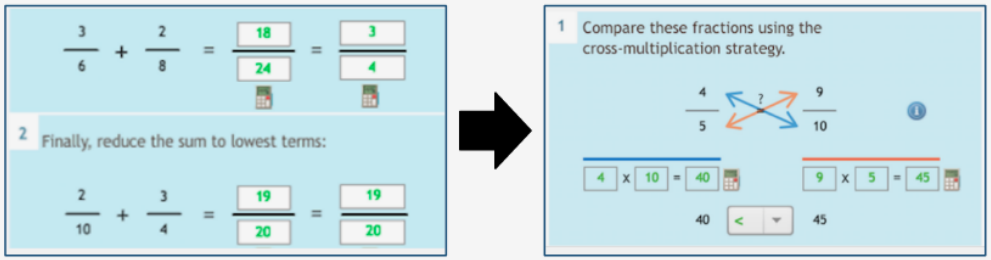


Avg Score: 92

Involves Counterfactual Reasoning

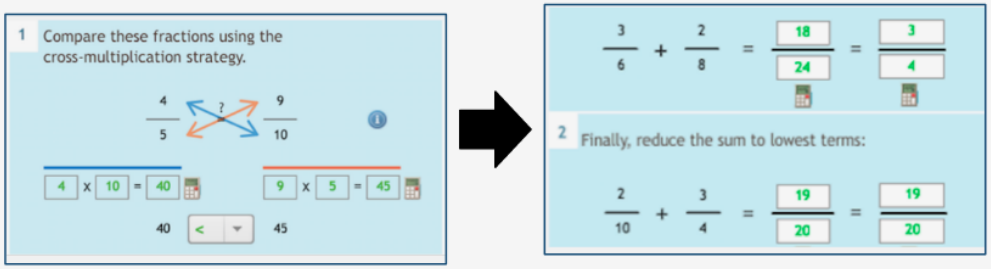
If Group A is different than Group B, we have censored data. We don't know what would have happened to group B if they had the same sequence as group A. You only get to observe the outcome for what actually happened.

A Group



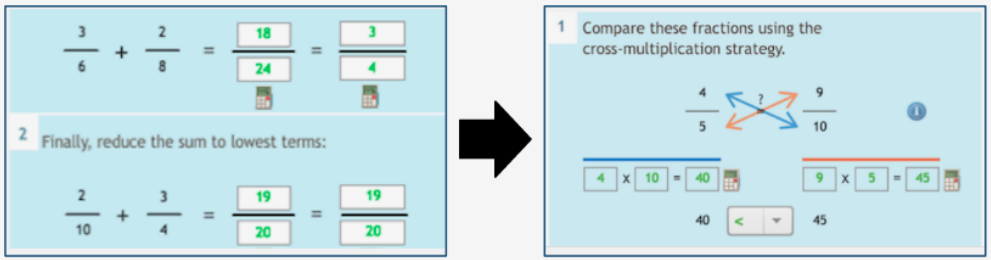
Avg Score: 95

B Group



Avg Score: 92

B Group



???

Involves Generalization

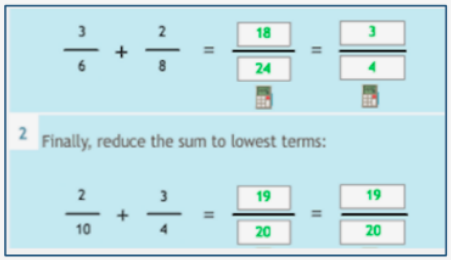
2 actions

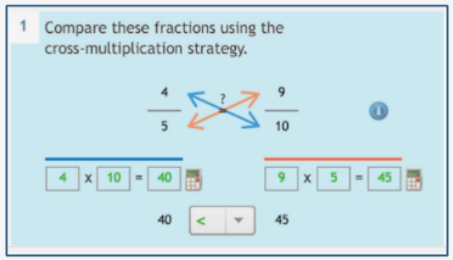
r = 0

r = 0

reward r = test score

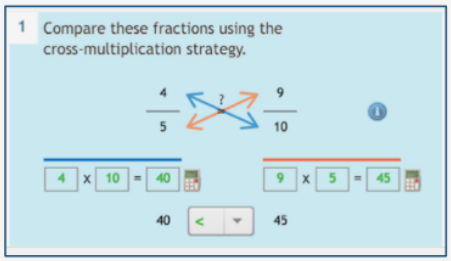
A Group

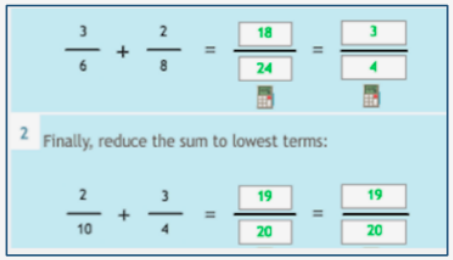




Avg Score: 95

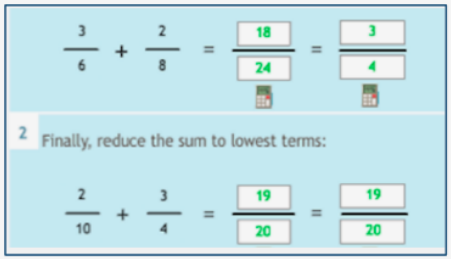
B Group

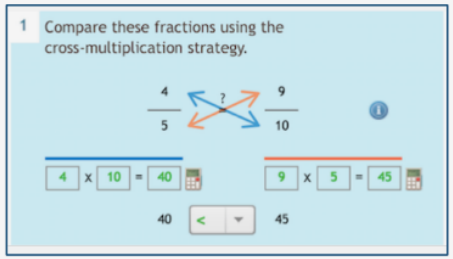




Avg Score: 92

B Group





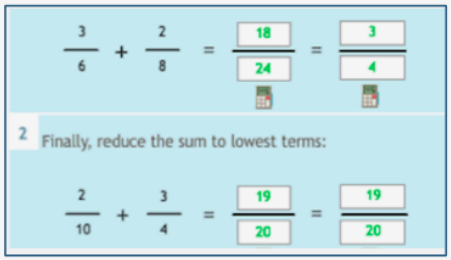
???

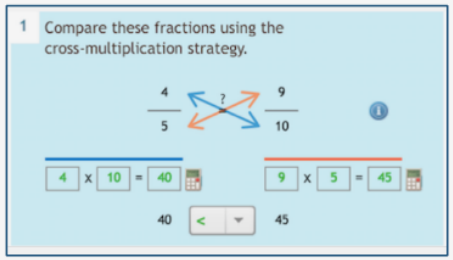
Batch Reinforcement Learning

Off Policy Offline Batch RL

generally is off-policy

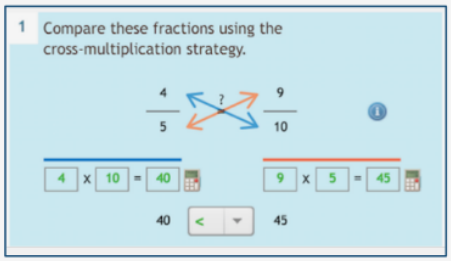
A Group

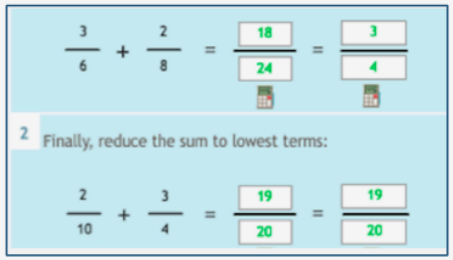




Avg Score: 95

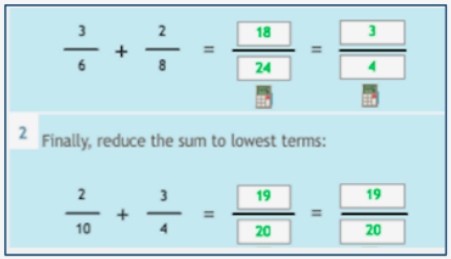
B Group

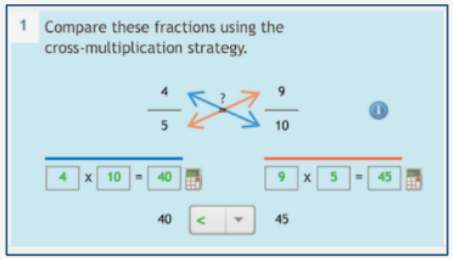




Avg Score: 92

B Group

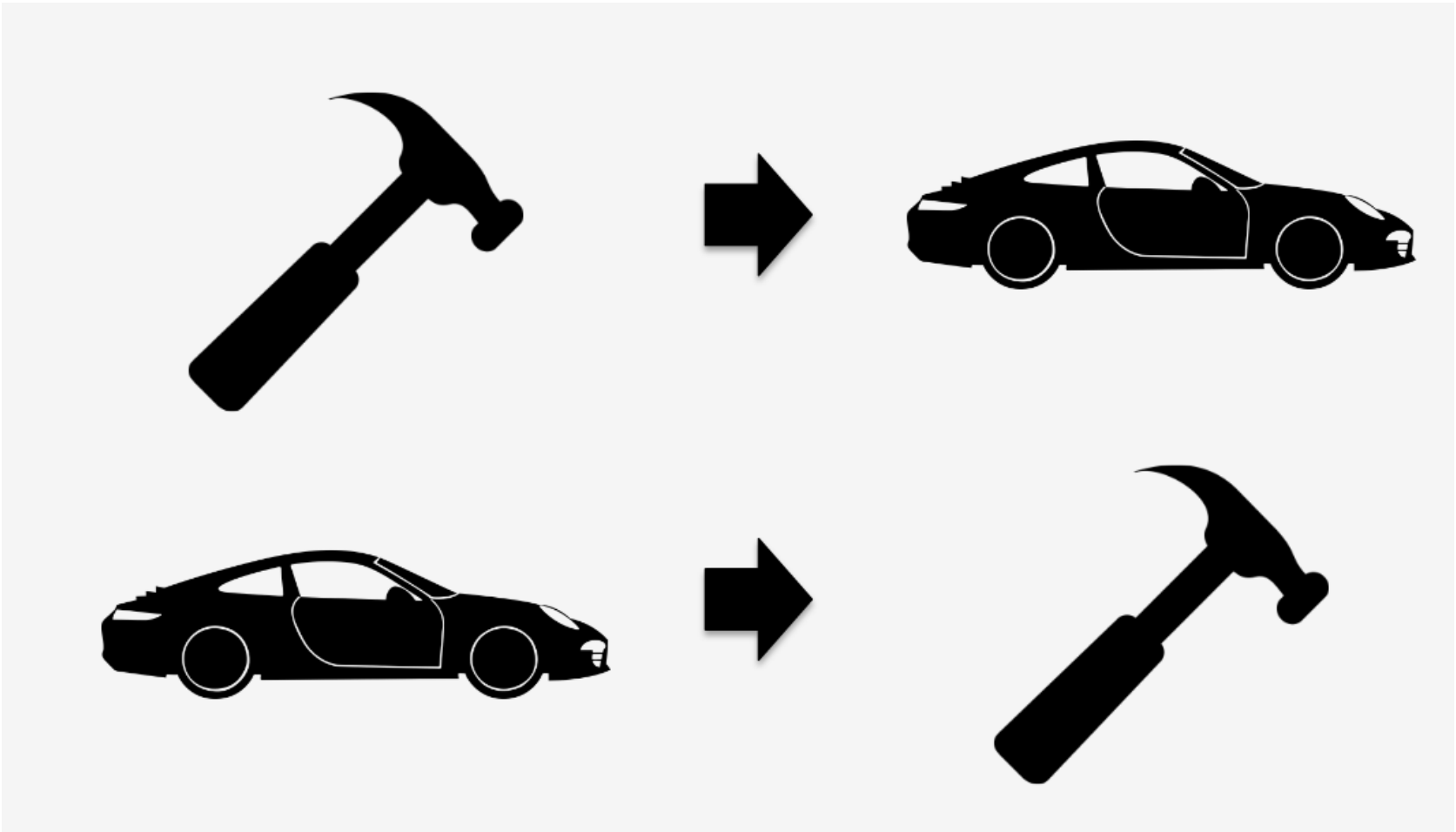




???

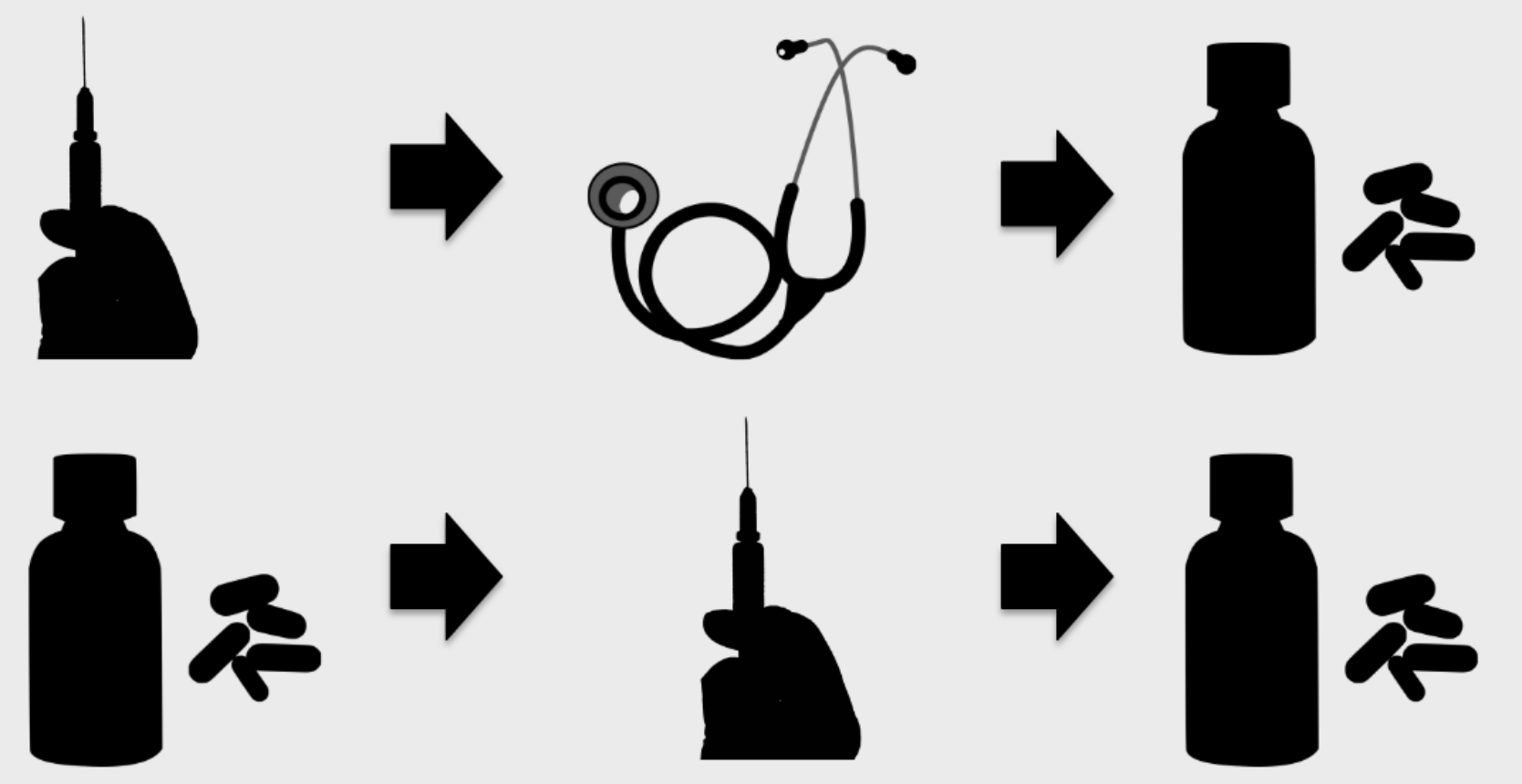
Batch RL

what sequence of maintenance should we do to maximize car longevity



Batch RL

what sequence of medicine should we give a patient to maximize health?



The Problem

- If you apply an existing method, do you have confidence that it will work?

A property of many real applications

- Deploying "bad" policies can be costly or dangerous

What property should a safe batch reinforcement learning algorithm have?

"safe" means that we're confident in the expected outcome

- Given past experience from current policy/policies, produce a new policy
  - "Guarantee that with probability at least $1-\delta$, will not change your policy to one that is worse than the current policy."
  - You get to choose δ
  - Guarantee not contingent on the tuning of any hyperparameters

Table of Contents

Notation

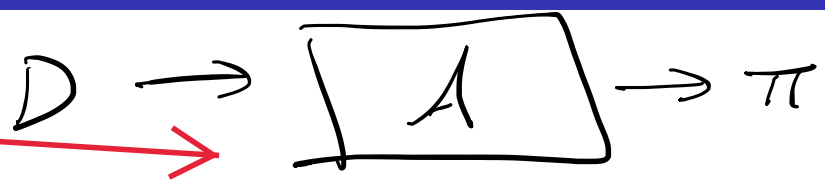
- Policy π: $\pi(a) = P(a_t = a \,|\, s_t = s)$
- Trajectory: $T = (s_1, a_1, r_1, s_2, a_2, r_2, \cdots, s_L, a_L, r_L)$
- Historical data: $D = \{T_1, T_2, \cdots, T_n\}$
- Historical data from behavior policy, π_b
- Objective:

$$V^{\pi} = \mathbb{E}[\sum_{t=1}^{L} \gamma^t R_t \,|\, \pi]$$

we're going to assume we know what the policy was when creating the data

Safe batch reinforcement learning algorithm

the algorithm will take the logged data and output a policy!



- Reinforcement learning algorithm, $\mathcal{A}$
- Historical data, D, which is a random variable
- Policy produced by the algorithm, $\mathcal{A}(D)$, which is a random variable
- a safe batch reinforcement learning algorithm, $\mathcal{A}$, satisfies:

the policy output by the algorithm is better than the policy used to generate the data with some confidence

$$\Pr(V^{\mathcal{A}(D)} \geq V^{\pi_b}) \geq 1 - \delta$$

or, in general

$$\Pr(V^{\mathcal{A}(D)} \geq V_{min}) \geq 1 - \delta$$

How do we find V(pi_b)? You could estimate it by averaging the rewards across trajectories.

How do we find V(A(D))? We'll have to do something off-policy

often, you won't just want to be better, you'll want to be significantly better

Table of Contents

Create a safe batch reinforcement learning algorithm

we need to be able to do off-policy policy evaluation. In other words, we need to be able to take the data D and estimate how good or bad an alternative policy pi(A(D)) would do. We also want to know how confident in that estimate we are. We might also want to be able to take an argmax over possible policies because we want to deploy a good policy.

$V^{A(D)}$ $V^{\pi e}$

- Off-policy policy evaluation (OPE)
  - For any evaluation policy, π_e, Convert historical data, D, into n independent and unbiased estimates of V^{π_e}
- High-confidence off-policy policy evaluation (HCOPE)
  - Use a concentration inequality to convert the n independent and unbiased estimates of V^{π_e} into a $1-\delta$ confidence lower bound on V^{π_e}

$V^{\pi e}$

- Safe policy improvement (SPI)
  - Use HCOPE method to create a safe batch reinforcement learning algorithm,

$\underset{\pi}{\text{argmax}}$ $V^{\pi e}$ w/some confidence bounds

Off-policy policy evaluation (OPE)

Historical Data, D
Proposed Policy, π_e

→ Estimate of V^{π_e}

Importance Sampling

$$IS(D) = \frac{1}{n} \sum_{i=1}^{n} \left(\prod_{t=1}^{L} \frac{\pi_e(a_t \mid s_t)}{\pi_b(a_t \mid s_t)} \right) \left(\sum_{t=1}^{L} \gamma^t R_t^i \right)$$

$$\mathbb{E}[IS(D)] = V^{\pi_e}$$

Create a safe batch reinforcement learning algorithm

- Off-policy policy evaluation (OPE)
  - For any evaluation policy, π_e, Convert historical data, D, into n independent and unbiased estimates of V^{π_e}
- High-confidence off-policy policy evaluation (HCOPE)
  - Use a concentration inequality to convert the n independent and unbiased estimates of V^{π_e} into a $1-\delta$ confidence lower bound on V^{π_e}
- Safe policy improvement (SPI)
  - Use HCOPE method to create a safe batch reinforcement learning algorithm

High-confidence off-policy policy evaluation (HCOPE)

think back to exploration

Historical Data, D
Proposed Policy, π_e
Probability, $1-\delta$

$1-\delta$ confidence lower bound on $J(\pi_e)$

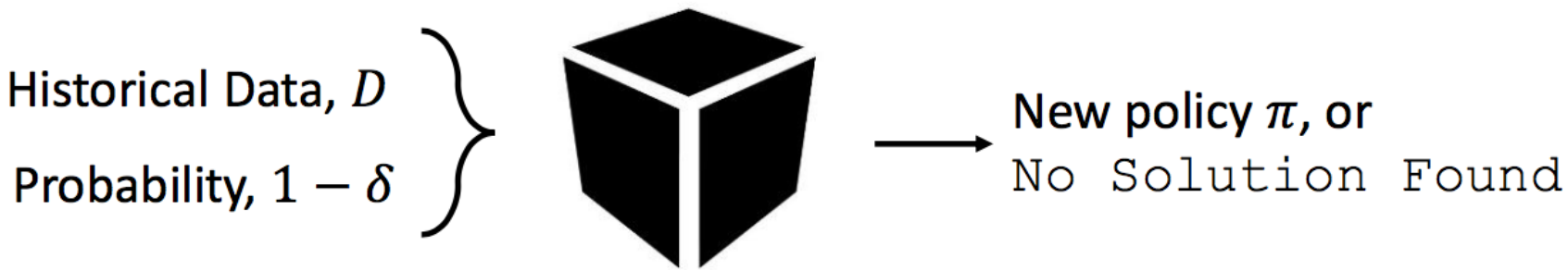
Hoeffding's inequality

- Let $X_1, \cdots, X_n$ be n independent identically distributed random variables such that $X_i \in [0, b]$
- Then with probability at least $1 - \delta$:

$$\mathbb{E}[X_i] \geq \frac{1}{n} \sum_{i=1}^{n} X_i - b \sqrt{\frac{\ln(1/\delta)}{2n}},$$

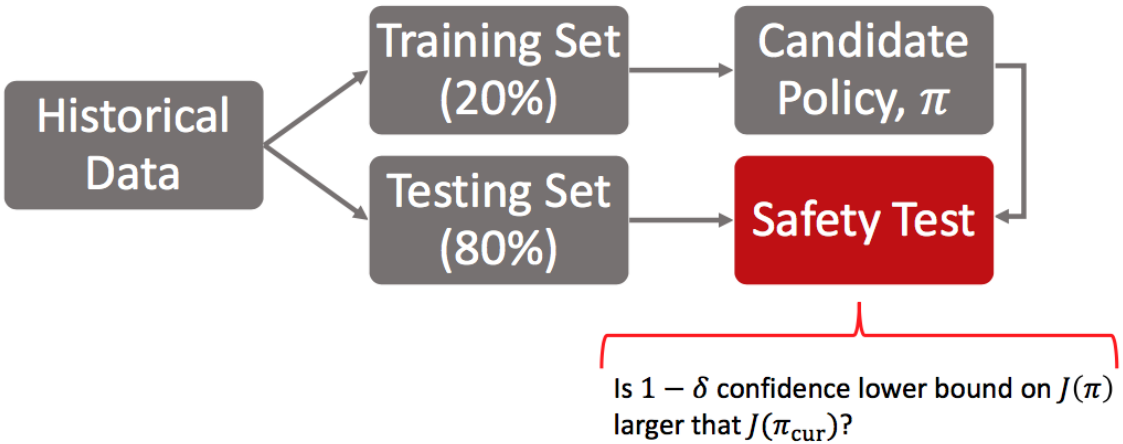
where $X_i = \frac{1}{n} \sum_{i=1}^{n} (w_i \sum_{t=1}^{L} \gamma^t R_t^i)$ in our case.

Safe policy improvement (SPI)



sometimes if you have very little data, you can't confidently output a better policy. So it's good for the algorithm to spit out "No solution found", especially in the "safe" setting.

Safe policy improvement (SPI)



Create a safe batch reinforcement learning algorithm

- Off-policy policy evaluation (OPE)
  - For any evaluation policy, π_e, Convert historical data, D, into n independent and unbiased estimates of V^{π_e}
- High-confidence off-policy policy evaluation (HCOPE)
  - Use a concentration inequality to convert the n independent and unbiased estimates of V^{π_e} into a $1-\delta$ confidence lower bound on V^{π_e}
- Safe policy improvement (SPI)
  - Use HCOPE method to create a safe batch reinforcement learning algorithm

Monte Carlo (MC) Off Policy Evaluation

Q learning off policy samples & bootstraps can be biased

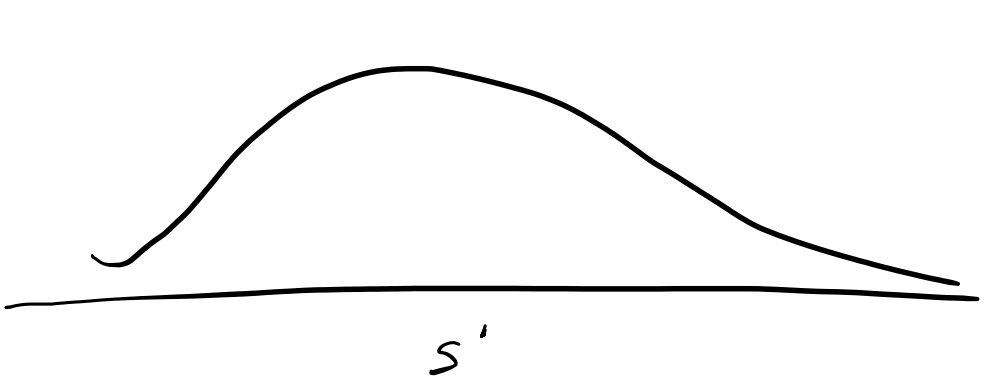
- Aim: estimate value of policy π_1, $V^{\pi_1}(s)$, given episodes generated under behavior policy π_2
- $s_1, a_1, r_1, s_2, a_2, r_2, \ldots$ where the actions are sampled from π_2
- $G_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \gamma^3 r_{t+3} + \cdots$ in MDP M under policy π
- $V^{\pi}(s) = \mathbb{E}_{\pi}[G_t | s_t = s]$
- Have data from a different policy, behavior policy π_2
- If π_2 is stochastic, can often use it to estimate the value of an alternate policy (formal conditions to follow)
- Again, no requirement that have a model nor that state is Markov

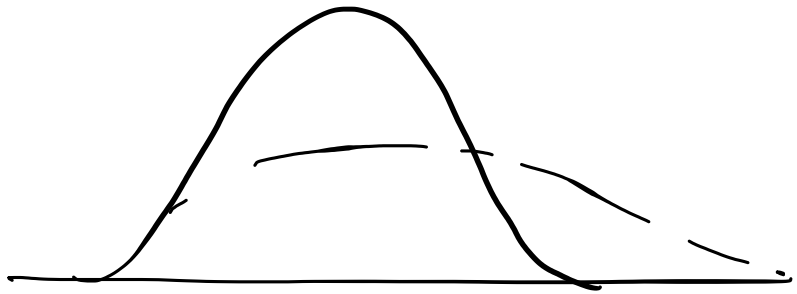
Monte Carlo (MC) Off Policy Evaluation: Distribution Mismatch

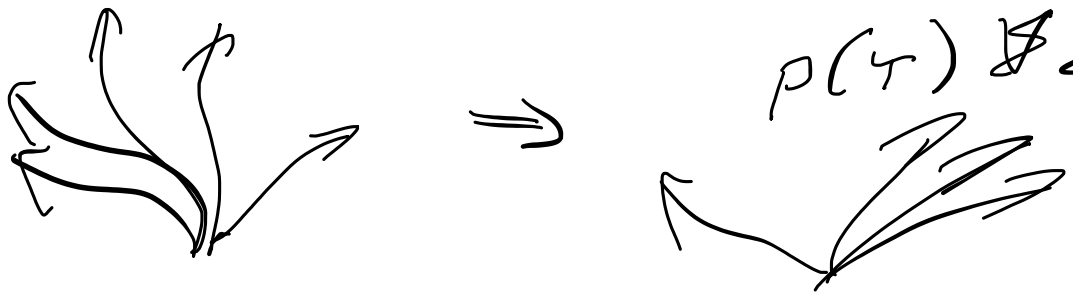
τ - $s\ a\ r\ s'\ a'\ r'\ s''\ \ldots$

- Distribution of episodes & resulting returns differs between policies

distrib of τ π_b $\neq$ distrib of τ when π_e







how do you take the trajectories in the data and change them so they look like the distribution we care about? Importance sampling.

Importance Sampling

Importance sampling is great when you have data from one distribution but you wish you had it from another. We focus on the data points that we would sample from the distribution we care about and we reweigh them so they look like they came from our desired distribution

- Goal: estimate the expected value of a function $f(x)$ under some probability distribution $p(x)$, $\mathbb{E}_{x\sim p}[f(x)]$
- Have data $x_1, x_2, \ldots, x_n$ sampled from distribution $q(s)$
- Under a few assumptions, we can use samples to obtain an unbiased estimate of $\mathbb{E}_{x\sim q}[f(x)]$

we have data from p(x) but we want data from q(x)

$$\mathbb{E}_{x\sim q}[f(x)] = \int_x q(x) f(x) \, dx$$

$$= \int_x \frac{p(x)}{p(x)} q(x) f(x) \, dx$$

$$= \int_x p(x) \left[\frac{q(x)}{p(x)} f(x)\right] dx$$

when $x_i \sim p(x)$

$$\approx 1/n \sum_i \frac{q(x_i)}{p(x_i)} f(x_i)$$

Importance Sampling (IS) for Policy Evaluation

equivalent to q(x) / p(x) on the previous slide

$$\frac{p(h_j|\pi_e)}{p(h_j|\pi_b)}$$

↑ h_1, G_1 traj, Σ rewards

↓ h_2, G_2

so we're assuming that the same trajectory was generated by two different policies pi_e and pi_b. So for this trajectory, what's the probability that you'd see this from the evaluation policy pi_e compared to the probability that you'd see it from the behavior policy pi_b. We then upweight trajectories that are more likely in the pi_e and downweight those that are less likely. That will impact our evaluation using G

could condition on full history

- Let h_j be episode j (history) of states, actions and rewards

$$h_j = (s_{j,1}, a_{j,1}, r_{j,1}, s_{j,2}, a_{j,2}, r_{j,2}, \ldots, s_{j,L_j(terminal)})$$

$$p(h_j|\pi_e) = p(s_{j,1}) \prod_{j=1}^{L_j-1} p(a_j|s_j) p(s_{j+1}|s_j, a_j) p(r|a_j, s_j)$$

policy π | trans model | reward model

$$\frac{p(h_j|\pi_e)}{p(h_j|\pi_b)} = \frac{p(s_{j,1})}{p(s_{j,1})} \prod \frac{p(a_j|s_j)^{\pi_e}}{p(a_j|s_j)^{\pi_b}} \frac{p(s_{j+1}|s_j, a_j)}{p(s_{j+1}|s_j, a_j)} \frac{p(r \ldots)}{p(r \ldots)}$$

$$= \prod \frac{\pi_e(a_j|s_j)}{\pi_b(a_j|s_j)}$$

the dynamics cancel out because they're dependent on the model, not the policy!

Importance Sampling (IS) for Policy Evaluation

- Let h_j be episode j (history) of states, actions and rewards

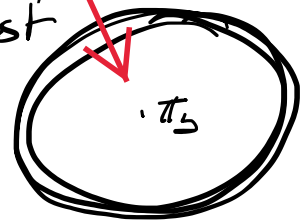
$$h_j = (s_{j,1}, a_{j,1}, r_{j,1}, s_{j,2}, a_{j,2}, r_{j,2}, \ldots, s_{j,L_j(terminal)})$$

$$\begin{aligned} p(h_j|\pi, s = s_{j,1}) =& p(a_{j,1}|s_{j,1})p(r_{j,1}|s_{j,1}, a_{j,1})p(s_{j,2}|s_{j,1}, a_{j,1}) \\ & p(a_{j,2}|s_{j,2})p(r_{j,2}|s_{j,2}, a_{j,2})p(s_{j,3}|s_{j,2}, a_{j,2}) \ldots \\ =& \prod_{t=1}^{L_j-1} p(a_{j,t}|s_{j,t})p(r_{j,t}|s_{j,t}, a_{j,t})p(s_{j,t+1}|s_{j,t}, a_{j,t}) \\ =& \prod_{t=1}^{L_j-1} \pi(a_{j,t}|s_{j,t})p(r_{j,t}|s_{j,t}, a_{j,t})p(s_{j,t+1}|s_{j,t}, a_{j,t}) \end{aligned}$$

Importance Sampling (IS) for Policy Evaluation

pretty much, you can only get a good estimate of pi_e if pi_e isn't "too different" than pi_b. If it's extremely, different, then we won't have the right data to evaluate pi_e.

- Let h_j be episode j (history) of states, actions and rewards, where the actions are sampled from π_2



$$h_j = (s_{j,1}, a_{j,1}, r_{j,1}, s_{j,2}, a_{j,2}, r_{j,2}, \ldots, s_{j,L_j(terminal)})$$

$(r_1 + \gamma r_2 + \gamma^2 r_3 \cdots)$

$$V^{\pi_1}(s) \approx \sum_{j=1}^{n} \frac{p(h_j|\pi_1, s)}{p(h_j|\pi_2, s)} G(h_j)$$

$$= \sum_{j=1}^{n} \left(\prod_{i=1}^{T} \frac{\pi_e(a_{ji}|s_{ji})}{\pi_b(a_{ji}|s_{ji})} \right) G(h_j)$$

no dynamics
" reward
" need to be Markov

greater than 0

$\pi_b(a|s) = 0$ but $\pi_e(a|s) > 0$

coverage or support

$\pi_b(a|s) = 0$ if $\pi_e(a|s) = 0$

$\pi_b(a|s) > 0$
$\forall a, s$ s.t. $\pi_e(a|s) > 0$

Importance Sampling for Policy Evaluation

1st used for RL Precup 2000?

- Aim: estimate $V^{\pi_1}(s)$ given episodes generated under policy π_2
  - $s_1, a_1, r_1, s_2, a_2, r_2, \ldots$ where the actions are sampled from π_2
- Have access to $G_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \gamma^3 r_{t+3} + \cdots$ in MDP M under policy π_2
- Want $V^{\pi_1}(s) = \mathbb{E}_{\pi_1}[G_t | s_t = s]$
- IS = Monte Carlo estimate given off policy data
- Model-free method
- Does not require Markov assumption
- Under some assumptions, unbiased & consistent estimator of V^{π_1}
- Can be used when agent is interacting with environment to estimate value of policies different than agent's control policy

under a few assumptions

as $n \to \infty$, $\hat{V}^{\pi_e} \to V^{\pi_e}$

Leveraging Future Can't Influence Past Rewards

recall π gradient

$G_t = r + r' + r'' \ldots$

- Importance sampling (IS):

$$IS(D) = \frac{1}{n} \sum_{i=1}^{n} \left(\prod_{t=1}^{L} \frac{\pi_e(a_t \mid s_t)}{\pi_b(a_t \mid s_t)} \right) \left(\sum_{t=1}^{L} \gamma^t R_t^i \right)$$

- Per-decision importance sampling (PDIS)

$$PSID(D) = \sum_{t=1}^{L} \gamma^t \frac{1}{n} \sum_{i=1}^{n} \left(\prod_{\tau=1}^{t} \frac{\pi_e(a_\tau \mid s_\tau)}{\pi_b(a_\tau \mid s_\tau)} \right) R_t^i$$

only up to point of reward

Off-policy policy evaluation (revisited)

$\prod \frac{\pi_e(a|s)}{\pi_b(a|s)}$ ← might be super small

- Importance sampling (IS):

$$IS(D) = \frac{1}{n} \sum_{i=1}^{n} w_i \left(\sum_{t=1}^{L} \gamma^t R_t^i \right)$$

- Weighted importance sampling (WIS)

$$WIS(D) = \frac{1}{\sum_{i=1}^{n} w_i} \sum_{i=1}^{n} w_i \left(\sum_{t=1}^{L} \gamma^t R_t^i \right)$$

biased still consistent

lower variance

we normalize w.r.t the weights, which lowers variance. this is very common

Off-policy policy evaluation (revisited)

- Weighted importance sampling (WIS)

$$WIS(D) = \frac{1}{\sum_{i=1}^{n} w_i} \sum_{i=1}^{n} w_i \left(\sum_{t=1}^{L} \gamma^t R_t^i \right)$$

- Biased. When $n = 1, \mathbb{E}[WIS] = V(\pi_b)$
- Strongly consistent estimator of V^{π_e}
  - i.e. $\Pr(\lim_{n \to \infty} WIS(D) = V^{\pi_e}) = 1$
  - If
    - Finite horizon
    - One behavior policy, or bounded rewards

Thomas & Brunskill
ICML 2016

Control variates

- Given: X
- Estimate: $\mu = \mathbb{E}[X]$
- $\hat{\mu} = X$
- Unbiased: $\mathbb{E}[\hat{\mu}] = \mathbb{E}[X] = \mu$
- Variance: $Var(\hat{\mu}) = Var(X)$

Control variates

we might want to subtract a variable (in this case Y) if it gives us lower variance

you can subtract a variable and add on its expected value and it remains unbiased

- Given: $X, Y, \mathbb{E}[Y]$
- Estimate: $\mu = \mathbb{E}[X]$
- $\hat{\mu} = X - Y + \mathbb{E}[Y]$
- Unbiased:
  $\mathbb{E}[\hat{\mu}] = \mathbb{E}[X - Y + \mathbb{E}[Y]] = \mathbb{E}[X] - \mathbb{E}[Y] + \mathbb{E}[Y] = \mathbb{E}[X]$
- Variance:

$$Var(\hat{\mu}) = Var(X - Y + \mathbb{E}[Y]) = Var(X - Y)$$

n

Control variates

- Given: $X, Y, \mathbb{E}[Y]$
- Estimate: $\mu = \mathbb{E}[X]$
- $\hat{\mu} = X - Y + \mathbb{E}[Y]$
- Unbiased:
  $\mathbb{E}[\hat{\mu}] = \mathbb{E}[X - Y + \mathbb{E}[Y]] = \mathbb{E}[X] - \mathbb{E}[Y] + \mathbb{E}[Y] = \mathbb{E}[X] = \mu$
- Variance:

$$\begin{aligned} Var(\hat{\mu}) &= Var(X - Y + \mathbb{E}[Y]) = Var(X - Y) \\ &= Var(X) + Var(Y) - 2Cov(X, Y) \end{aligned}$$

- Lower variance if $2Cov(X, Y) > Var(Y)$ if true $Var(\hat{\mu}) < Var(X)$
- We call Y a control variate
- We saw this idea before: baseline term in policy gradient estimation

Off-policy policy evaluation (revisited)

- Idea: add a control variate to importance sampling estimators
  - X is the importance sampling estimator
  - Y is a control variate build from an approximate model of the MDP $\hat{Q}$ some estimate of state action value

Off-policy policy evaluation (revisited)

- Idea: add a control variate to importance sampling estimators
  - X is the importance sampling estimator
  - Y is a control variate build from an approximate model of the MDP

doing this is called a doubly robust estimator

2011 multi armed bandit (Dudik et.al)

- Called the doubly robust estimator (Jiang and Li, 2015)
  - Robust to (1) poor approximate model, and (2) error in estimates of π_b

control variate

    - If the model is poor,the estimates are still unbiased
    - If the sampling policy is unknown, but the model is good, MSE will still be low

if both are bad, then all bets are off

- Non-recursive and weighted forms, as well as control variate view provided by Thomas and Brunskill (ICML 2016)

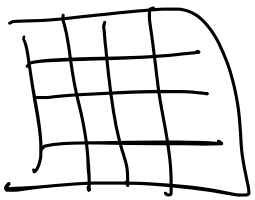
Off-policy policy evaluation (revisited)

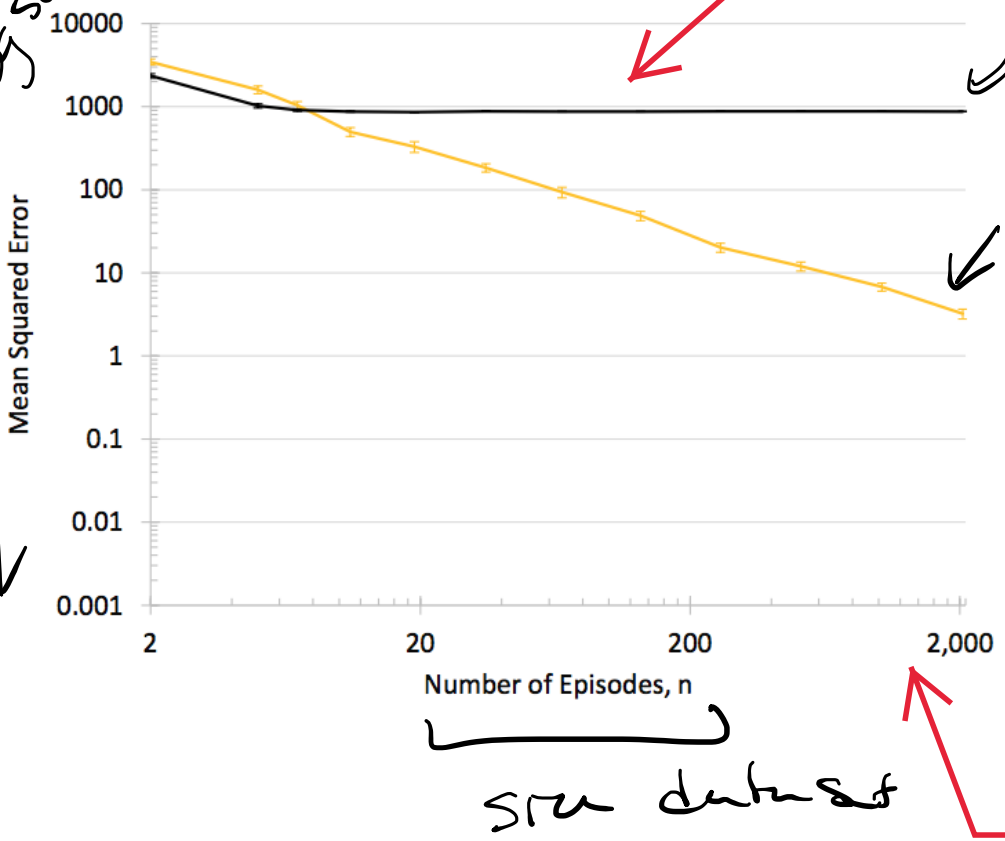
$$DR(\pi_e \mid D) = \frac{1}{n}\sum_{i=1}^{n}\sum_{t=0}^{\infty} \gamma^t w_t^i (R_t^i - \hat{q}^{\pi_e}(S_t^i, A_t^i)) + \gamma^t \rho_{t-1}^i \hat{v}^{\pi_e}(S_t^i),$$

where $w_t^i = \prod_{\tau_1}^{t} \frac{\pi_e(a_\tau \mid s_\tau)}{\pi_b(a_\tau \mid s_\tau)}$

Empirical Results (Gridworld)

using data, generate a model of the world. then use that model for simulation





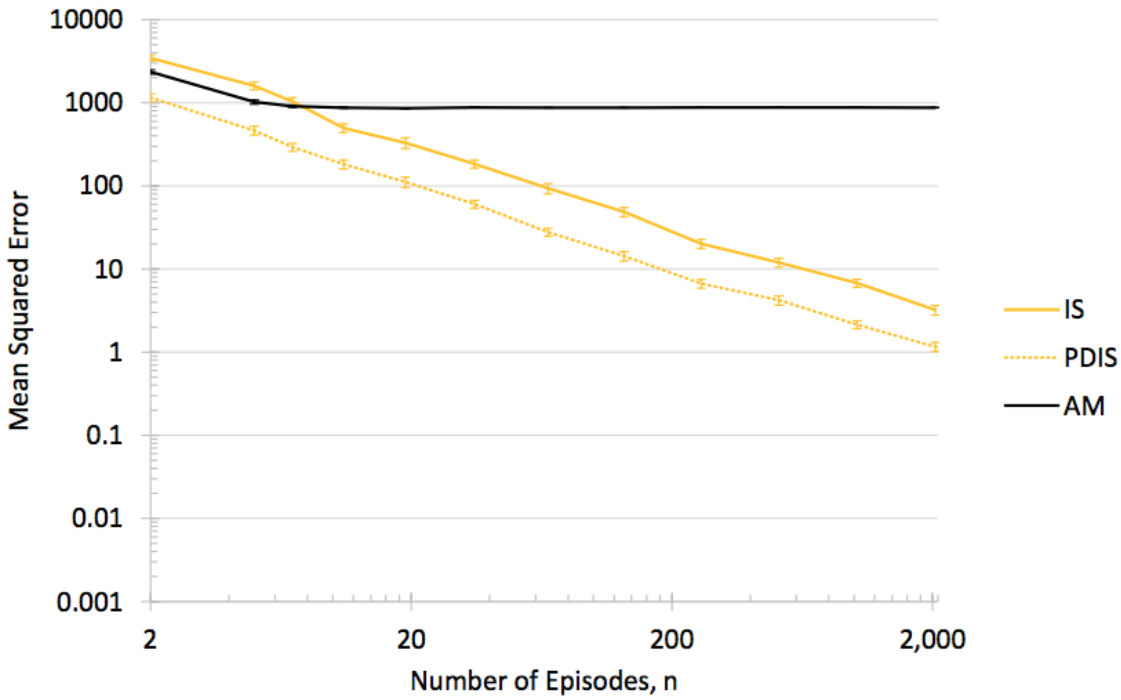
Approximate model

Direct method (Dudik, 2011)

Indirect method (Sutton and Barto, 1998)

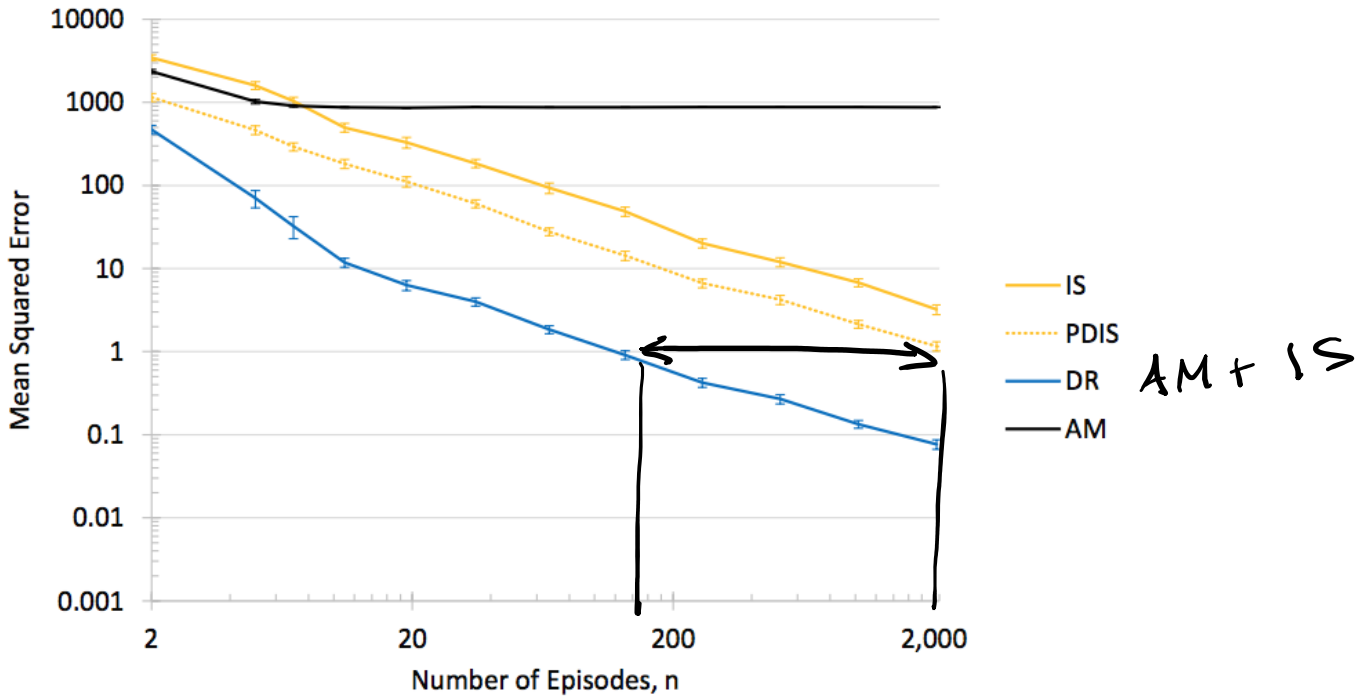
size of dataset

Empirical Results (Gridworld)

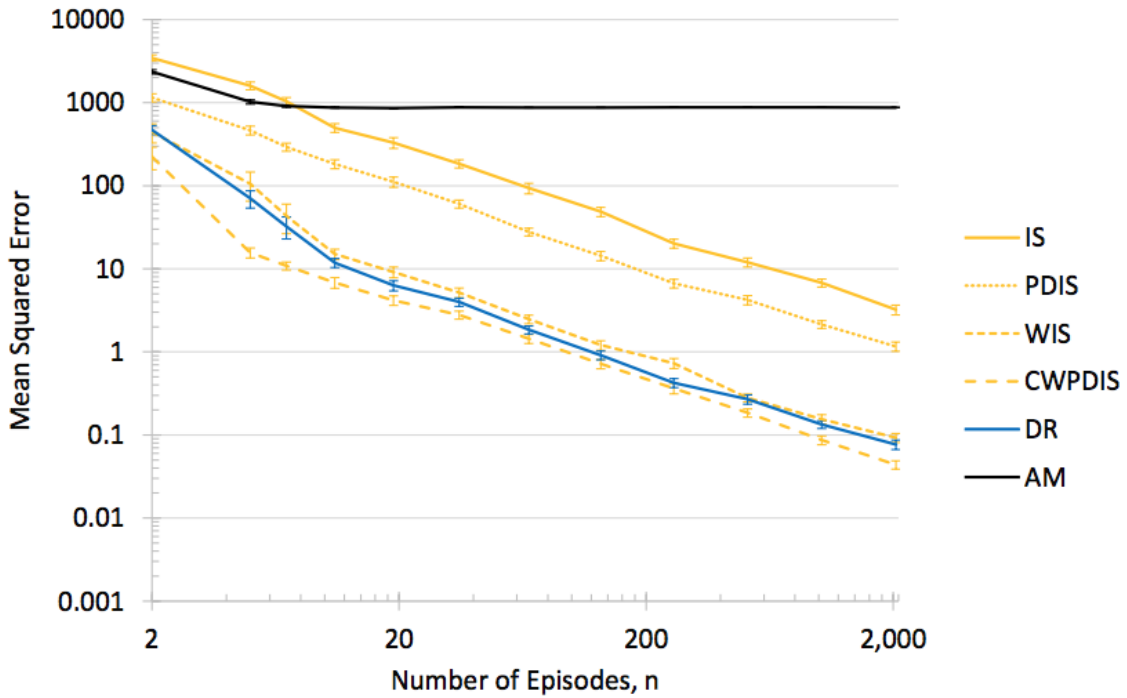


Empirical Results (Gridworld)

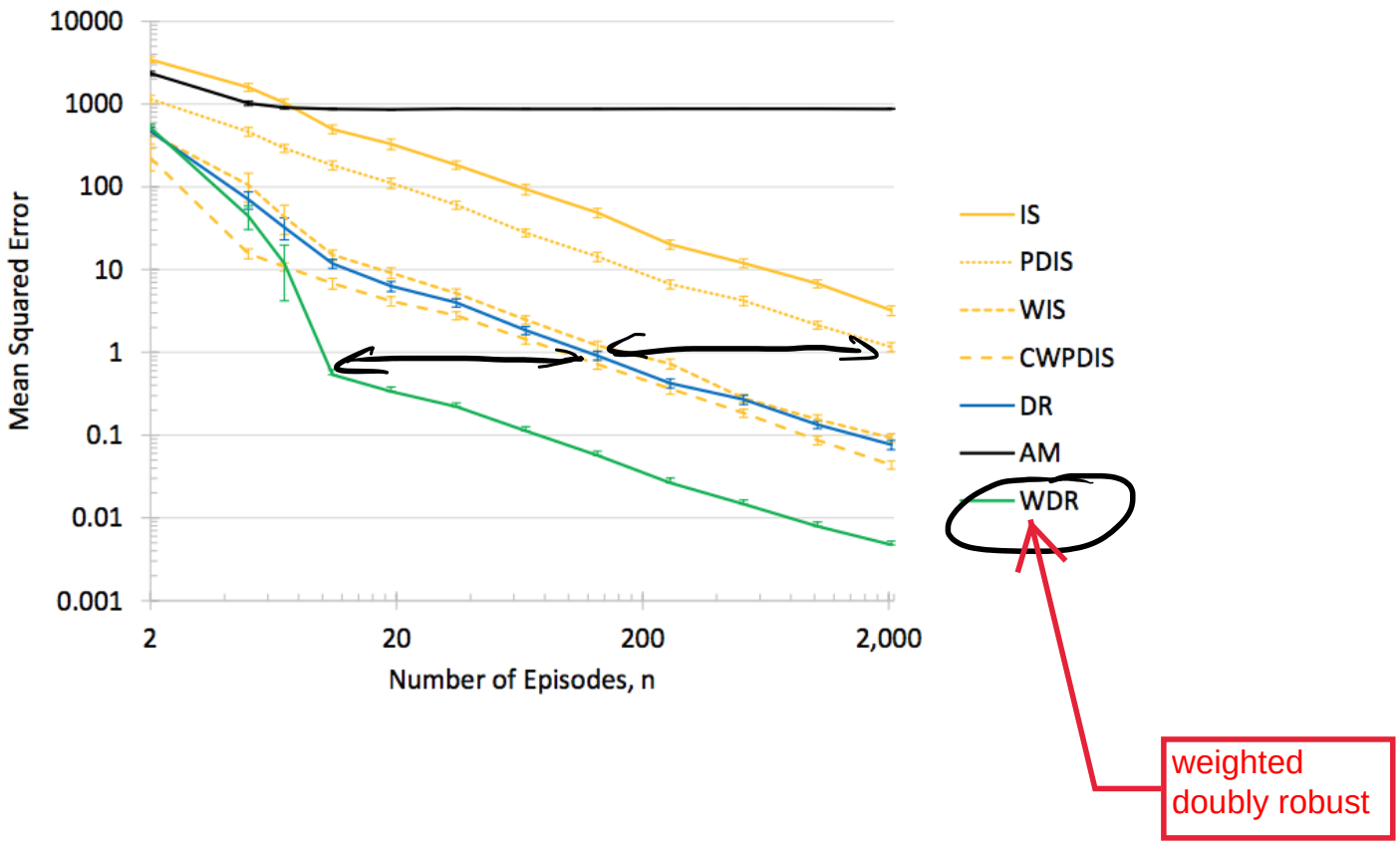
$\hat{V}^{\pi_e} - V^{\pi_e}$



Empirical Results (Gridworld)



Empirical Results (Gridworld)

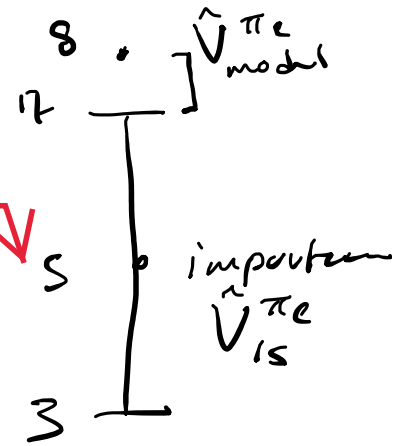


Off-policy policy evaluation (revisited): Blending

$MSE = f(\text{bias}, \text{var})$

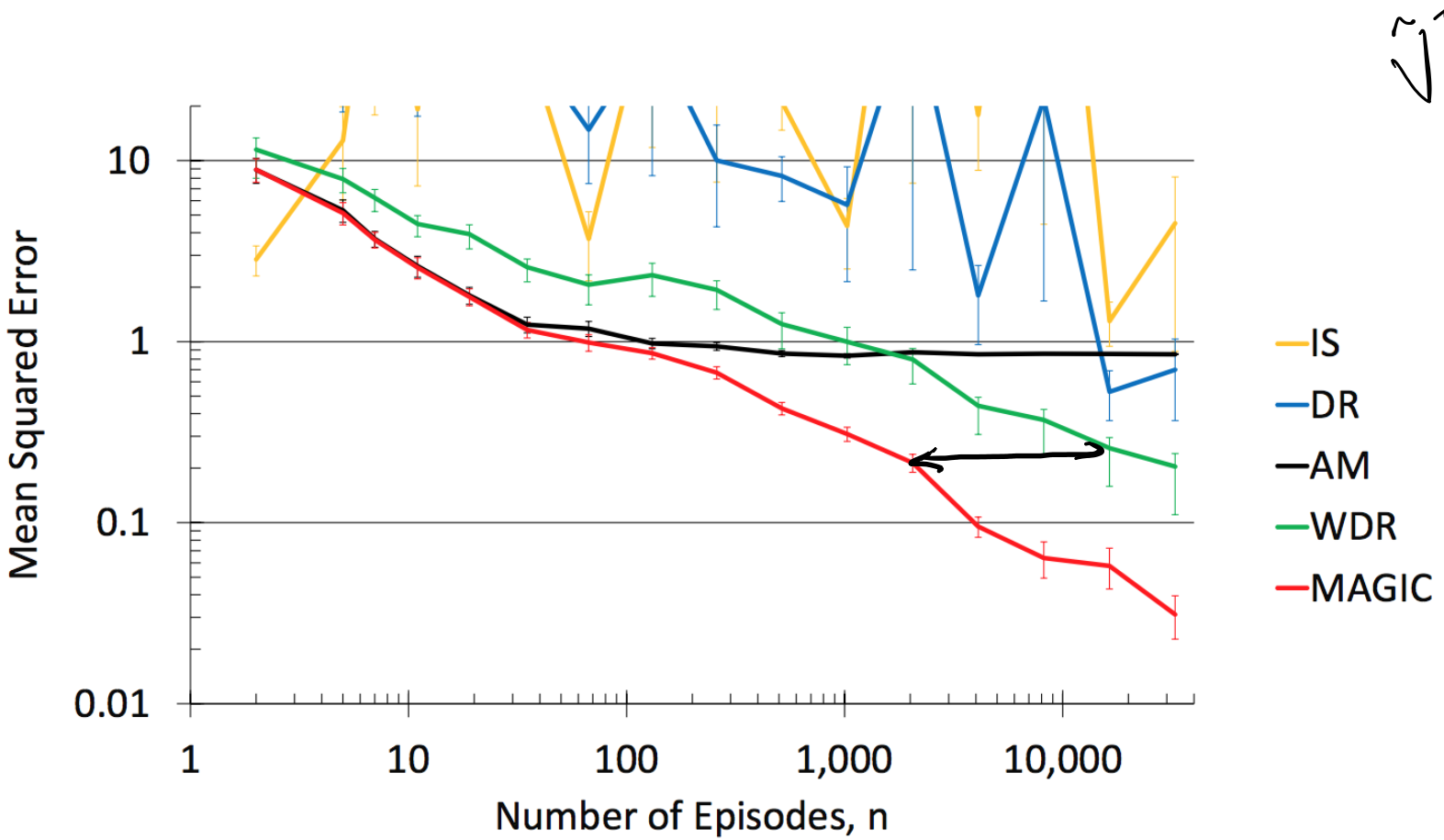
$\text{bias}: \hat{V}^{\pi_e} - V^{\pi_e}$

confidence intervals from IS



- Importance sampling is unbiased but high variance
- Model based estimate is biased but low variance
- Doubly robust is one way to combine the two
- Can also trade between importance sampling and model based estimate within a trajectory
- MAGIC estimator (Thomas and Brunskill ICML 2016) directly min MSE
- Can be particularly useful when part of the world is non-Markovian in the given model, and other parts of the world are Markov

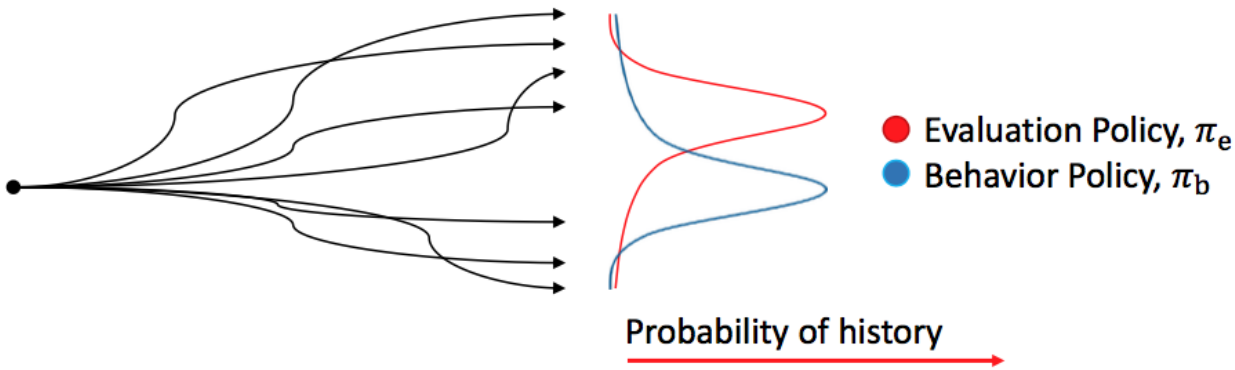
Can Need an Order of Magnitude Less Data To Get Good Estimates



Off-policy policy evaluation (revisited)

support

- What if supp($\pi_e \subset$ supp(π_b))
- There is a state-action pair, (s, a), such that $\pi_e(a \mid s) = 0$, but $\pi_b(a \mid s) \neq 0$.
- If we see a history where (s, a) occurs, what weight should we give it?
- $IS(D) = \frac{1}{n} \sum_{i=1}^{n} \left(\prod_{t=1}^{L} \frac{\pi_e(a_t \mid s_t)}{\pi_b(a_t \mid s_t)} \right) \left(\sum_{t=1}^{L} \gamma^t R_t^i \right)$

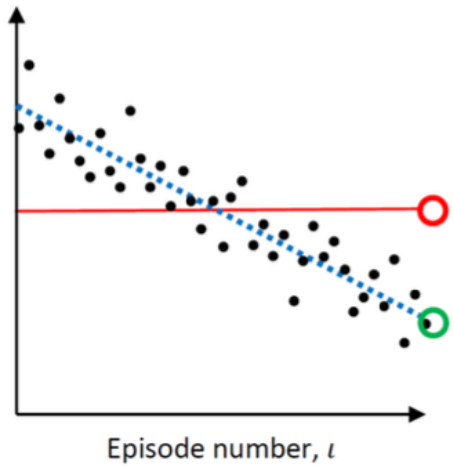


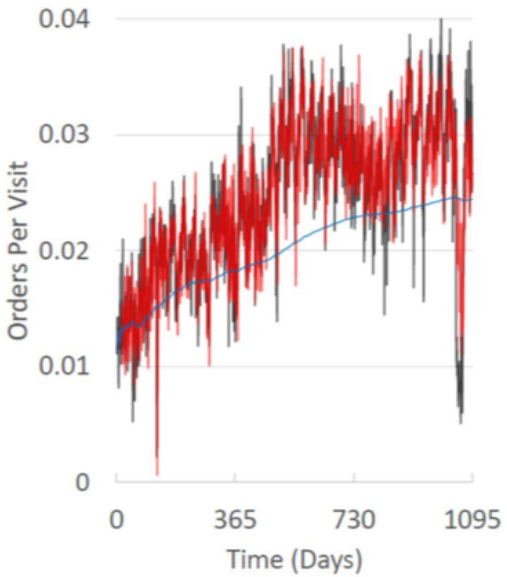
Off-policy policy evaluation (revisited)

- What if there are zero samples ($n = 0$)?
  - The importance sampling estimate is undefined
- What if no samples are in supp(π_e) (or supp(p) in general)?
  - Importance sampling says: the estimate is zero
  - Alternate approach: undefined
- Importance sampling estimator is unbiased if $n > 0$
- Alternate approach will be unbiased given that at least one sample is in the support of p
- Alternate approach detailed in Importance Sampling with Unequal Support (Thomas and Brunskill, AAAI 2017)

Off-policy policy evaluation (revisited)

- Thomas et. al. Predictive Off-Policy Policy Evaluation for Nonstationary Decision Problems, with Applications to Digital Marketing (AAAI 2017)





Create a safe batch reinforcement learning algorithm

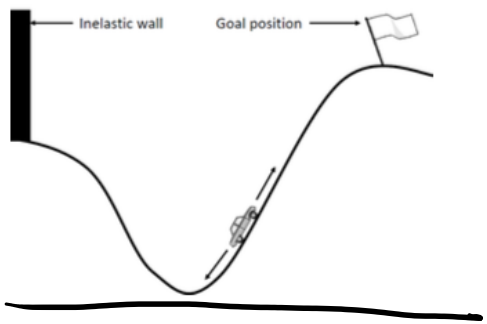
$$P(V^{A(D)} - V^{\pi_b} > 0) \geq 1 - \delta$$

- Off-policy policy evaluation (OPE)
  - For any evaluation policy, π_e, Convert historical data, D, into n independent and unbiased estimates of V^{π_e}
- High-confidence off-policy policy evaluation (HCOPE)
  - Use a concentration inequality to convert the n independent and unbiased estimates of V^{π_e} into a $1 - \delta$ confidence lower bound on V^{π_e}
- Safe policy improvement (SPI)
  - Use HCOPE method to create a safe batch reinforcement learning algorithm,

High-confidence off-policy policy evaluation (revisited)

- Consider using IS + Hoeffding's inequality for HCOPE on mountain car

exploration



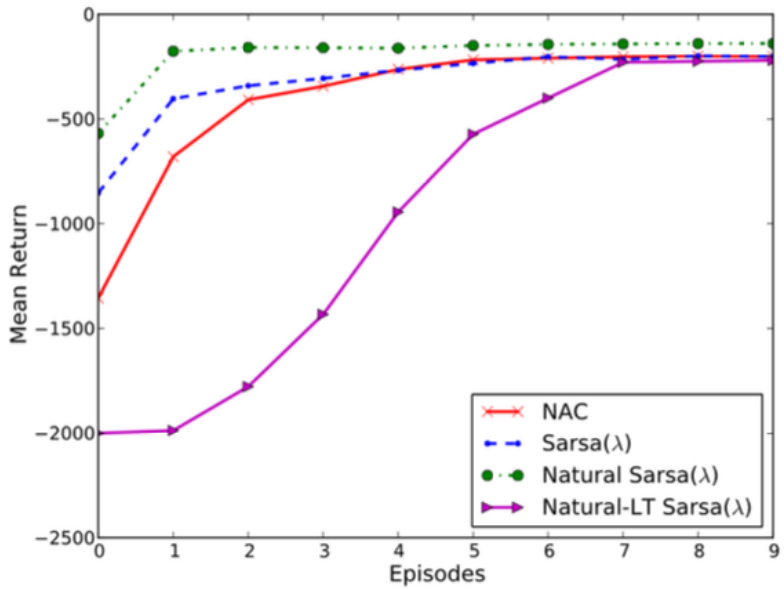


Figure 3: Mountain Car (Sarsa(λ))
Natural Temporal Difference Learning, Dabney and Thomas, 2014

Hoeffding's inequality

Hoeffding's indequality is a way to look at how different your empirical mean is from the true mean. It's a function of the amount of data we have.

empirical mean − true mean

- Let $X_1, \cdots, X_n$ be n independent identically distributed random variables such that $X_i \in [0, b]$
- Then with probability at least $1 - \delta$:

$$\mathbb{E}[X_i] \geq \frac{1}{n} \sum_{i=1}^{n} X_i - b \sqrt{\frac{\ln(1/\delta)}{2n}},$$

where $X_i = \frac{1}{n} \sum_{i=1}^{n} (w_i \sum_{t=1}^{L} \gamma^t R_t^i)$ in our case.

$b = \max\left(G \cdot \pi_{i=1}^{T} \frac{\pi_e(a_t s_t)}{\pi_b(a_t s_t)}\right)$

$(0,1)$

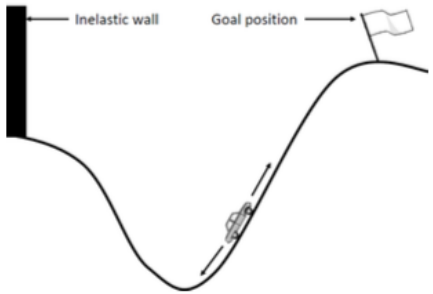
$L = 200$

$\left(\frac{1}{.1}\right)^{200} \cdot 1 \sqrt{\frac{1}{n}}$

b can get huge!

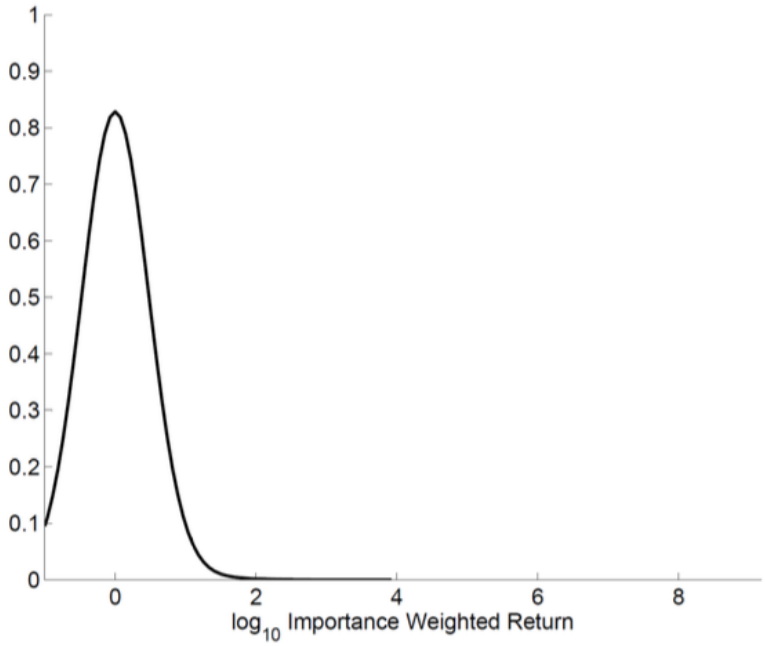
High-confidence off-policy policy evaluation (revisited)

- Using 100,000 trajectories
- Evaluation policy's true performance is $0.19 \in [0, 1]$
- We get a 95% confidence lower bound of: -5,8310,000



technically this is true, but it's not informative because we know it's between [0, 1]

What went wrong



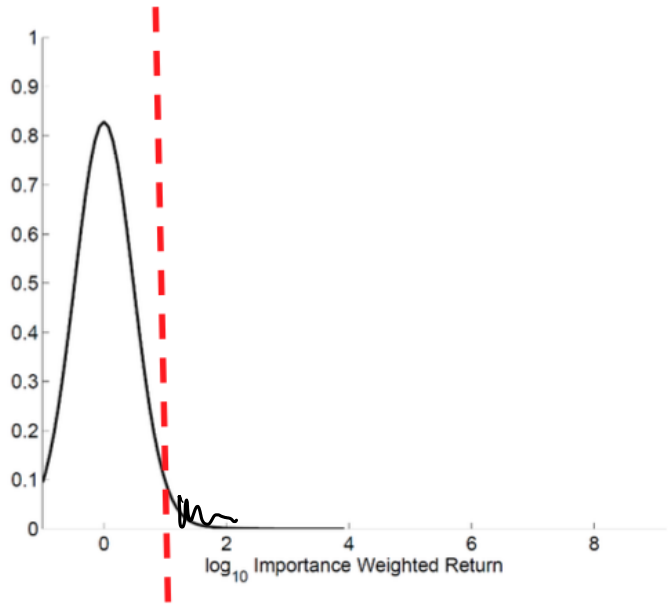
10^2

$$w_i = \prod_{t=1}^{L} \frac{\pi_e(a_t \mid s_t)}{\pi_b(a_t \mid s_t)}$$ pretty small $\left(\frac{1}{.1}\right)^L$

High-confidence off-policy policy evaluation (revisited)

- Removing the upper tail only decreases the expected value.

so we underestimate the goodness of our policy, which is okay for safe RL



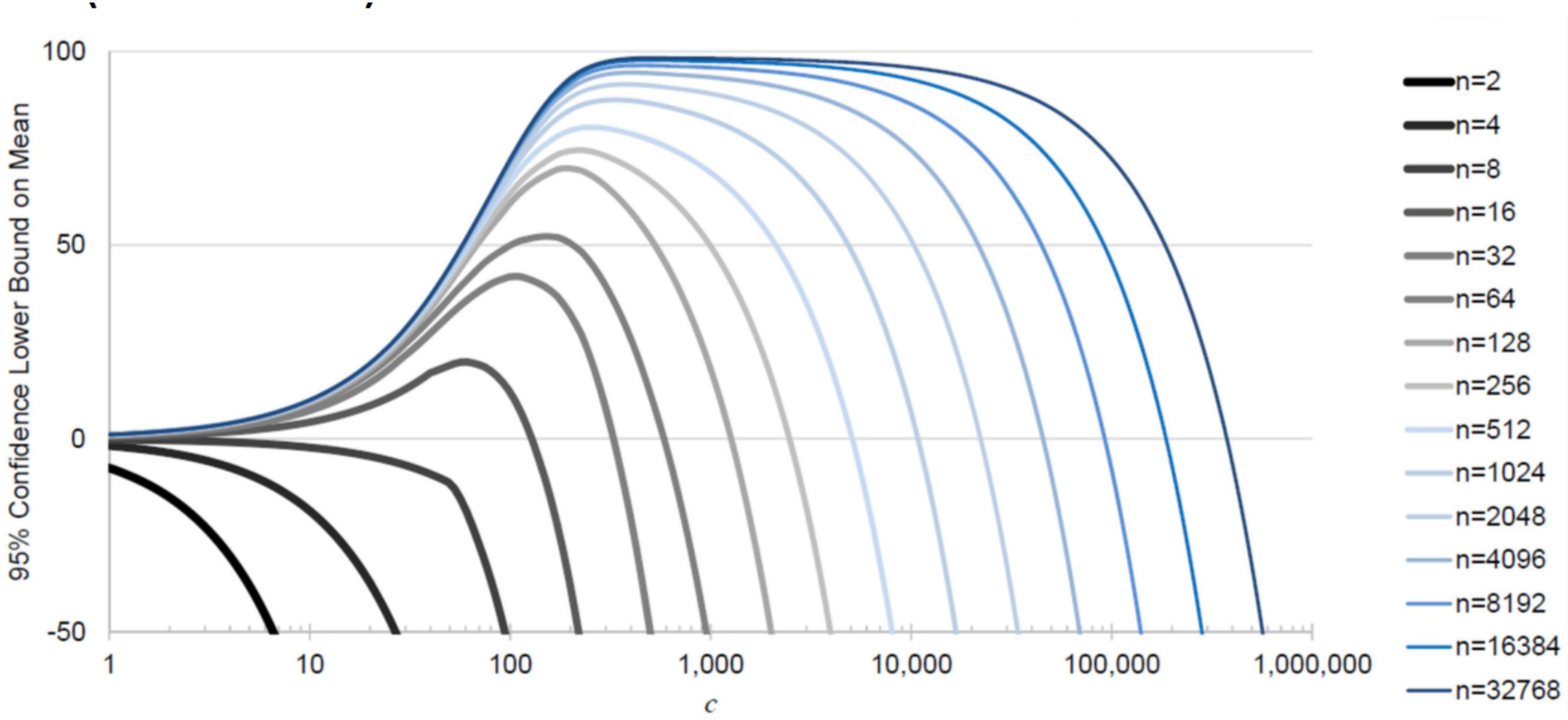
High-confidence off-policy policy evaluation (revisited)

- Thomas et. al, High confidence off-policy evaluation, AAAI 2015

Theorem 1. *Let $X_1, \ldots, X_n$ be n independent real-valued random variables such that for each $i \in \{1, \ldots, n\}$, we have $\mathbb{P}[0 \leq X_i] = 1$, $\mathbb{E}[X_i] \leq \mu$, and some threshold value $c_i > 0$. Let $\delta > 0$ and $Y_i := \min\{X_i, c_i\}$. Then with probability at least $1 - \delta$, we have*

$$\mu \geq \underbrace{\left(\sum_{i=1}^{n} \frac{1}{c_i}\right)^{-1} \sum_{i=1}^{n} \frac{Y_i}{c_i}}_{\text{empirical mean}} - \underbrace{\left(\sum_{i=1}^{n} \frac{1}{c_i}\right)^{-1} \frac{7n \ln(2/\delta)}{3(n-1)}}_{\text{term that goes to zero as } 1/n \text{ as } n \to \infty} - \underbrace{\left(\sum_{i=1}^{n} \frac{1}{c_i}\right)^{-1} \sqrt{\frac{\ln(2/\delta)}{n-1} \sum_{i,j=1}^{n} \left(\frac{Y_i}{c_i} - \frac{Y_j}{c_j}\right)^2}}_{\text{term that goes to zero as } 1/\sqrt{n} \text{ as } n \to \infty}. \quad (3)$$

High-confidence off-policy policy evaluation (revisited)



High-confidence off-policy policy evaluation (revisited)

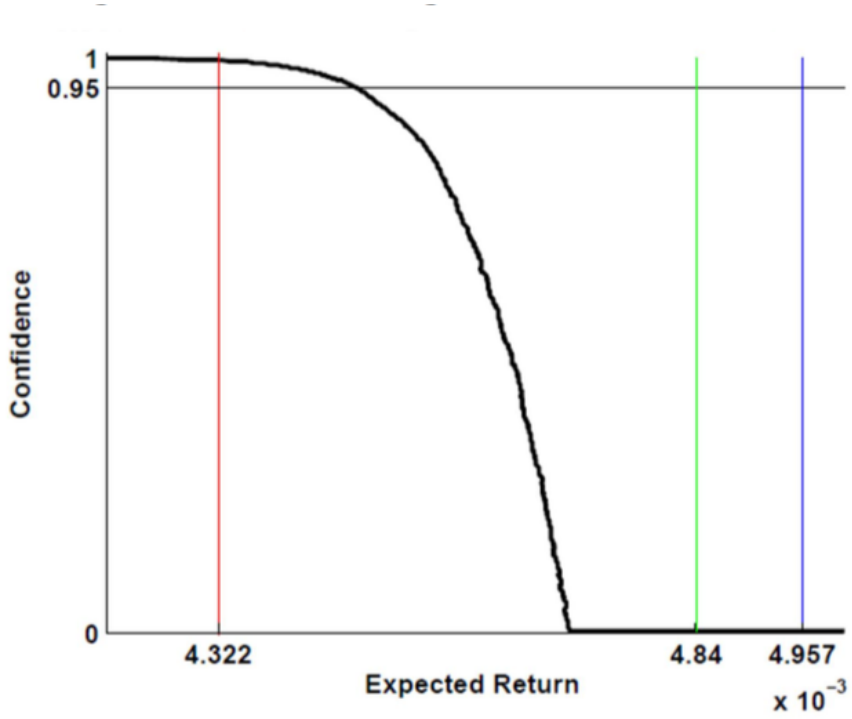
- Use 20% of the data to optimize c (cutoff)
- Use 80% to compute lower bound with optimized c
- Mountain car results:

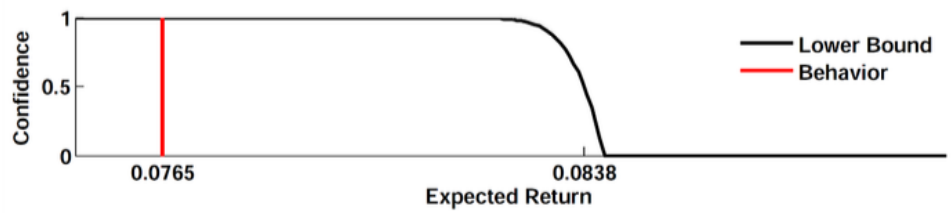
	CUT	Chernoff-Hoeffding	Maurer	Anderson	Bubeck et al.
95% Confidence lower bound on the mean	0.145	−5,831,000	−129,703	0.055	−.046

this is our lower bound on V(pi_e)

High-confidence off-policy policy evaluation (revisited)

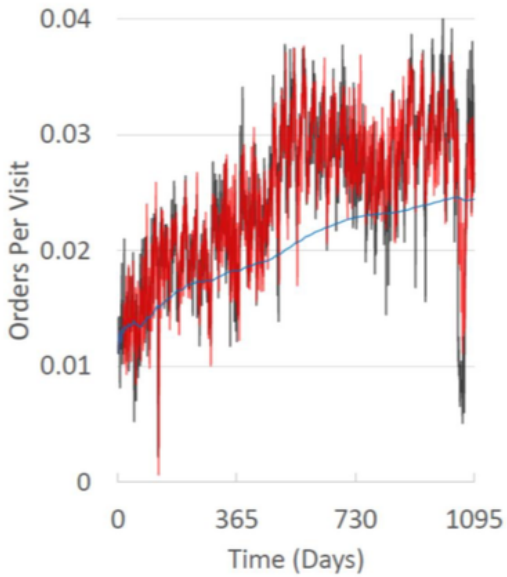
Digital marketing:





High-confidence off-policy policy evaluation (revisited)

Cognitive dissonance:



$$\mathbb{E}[X_i] \geq \frac{1}{n}\sum_{i=1}^{n} X_i - b\sqrt{\frac{\ln(1/\delta)}{2n}}$$

High-confidence off-policy policy evaluation (revisited)

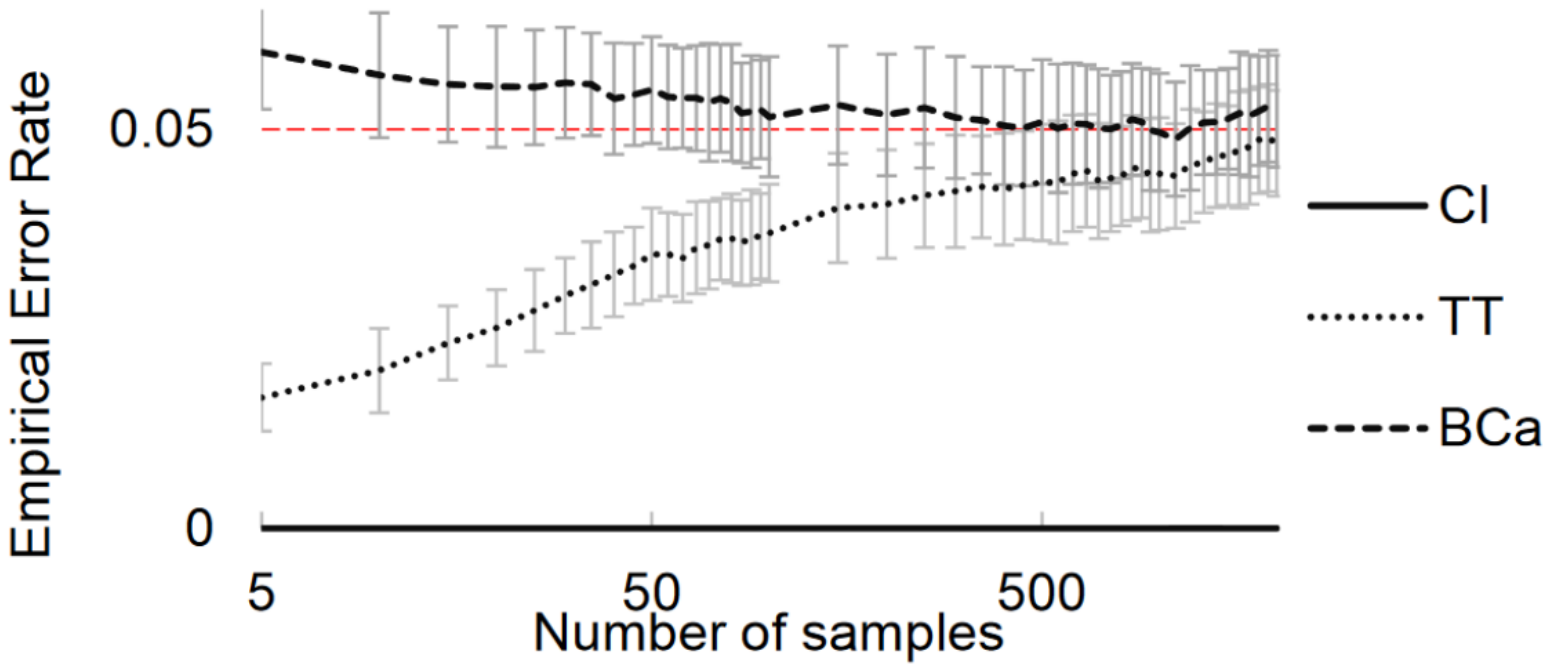
- Student's t-test
  - Assumes that $IS(D)$ is normally distributed
  - By the central limit theorem, it (is as $n \rightarrow \infty$)

$$\Pr\left(\mathbb{E}[\frac{1}{n}\sum_{i=1}^{n} X_i] \geq \frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2}}{\sqrt{n}} t_{1-\delta,n-1}$$

$$\geq 1 - \delta$$

  - Efron's Bootstrap methods (e.g., BCa)
    - Also, without importance sampling: Hanna, Stone, and Niekum, AAMAS 2017

High-confidence off-policy policy evaluation (revisited)



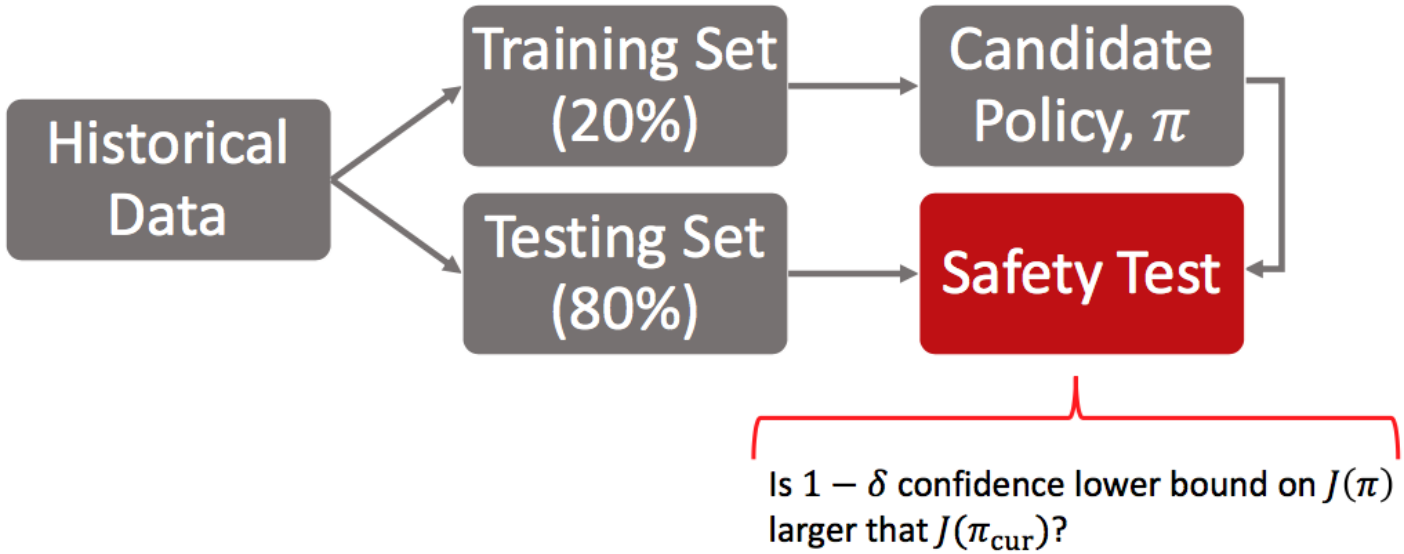
P. S. Thomas. Safe reinforcement learning (PhD Thesis, 2015)

Create a safe batch reinforcement learning algorithm

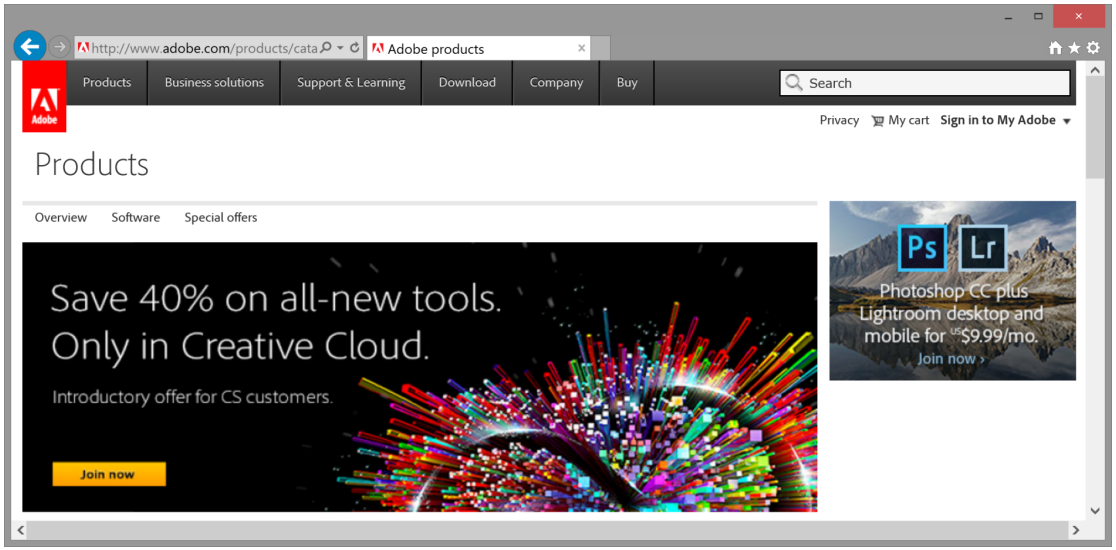
- Off-policy policy evaluation (OPE)
  - For any evaluation policy, π_e, Convert historical data, D, into n independent and unbiased estimates of V^{π_e}
- High-confidence off-policy policy evaluation (HCOPE)
  - Use a concentration inequality to convert the n independent and unbiased estimates of V^{π_e} into a $1-\delta$ confidence lower bound on V^{π_e}
- Safe policy improvement (SPI)
  - Use HCOPE method to create a safe batch reinforcement learning algorithm

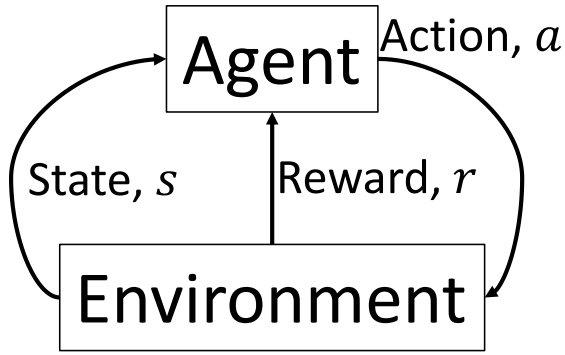
Safe policy improvement (revisited)

Thomas et. al, ICML 2015

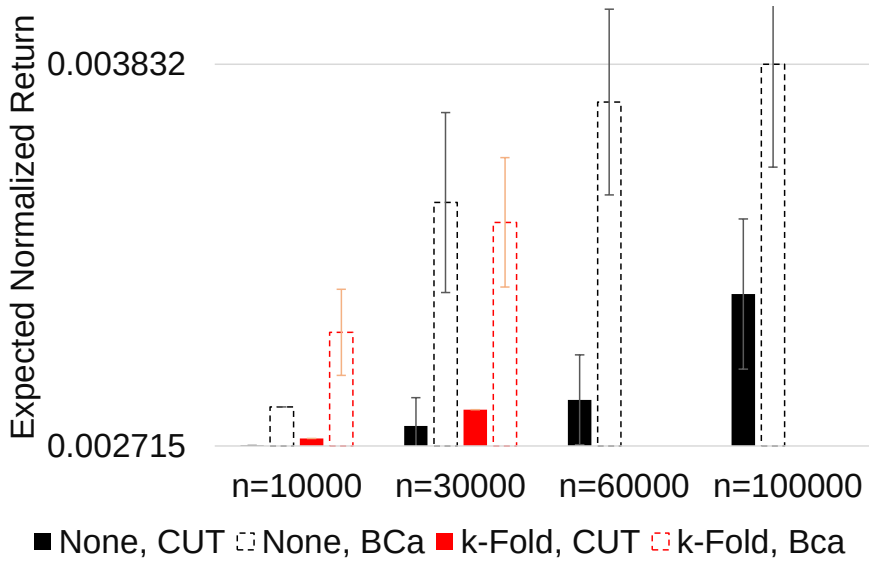


Empirical Results: Digital Marketing

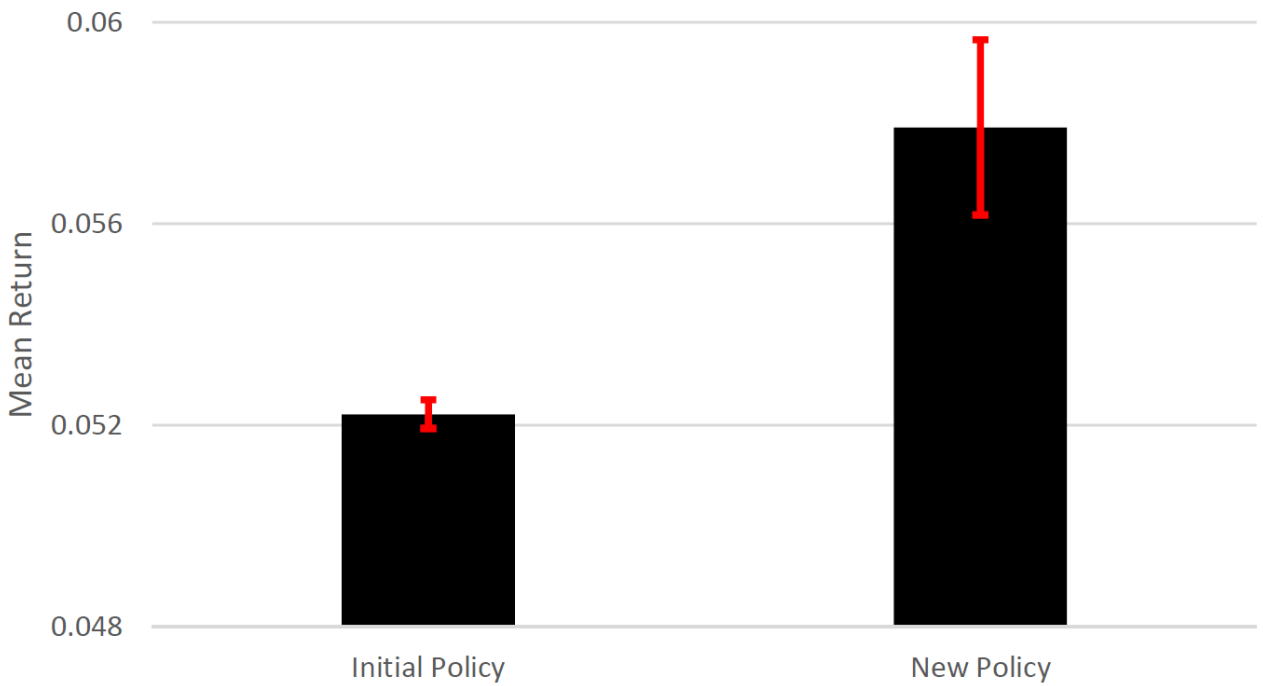




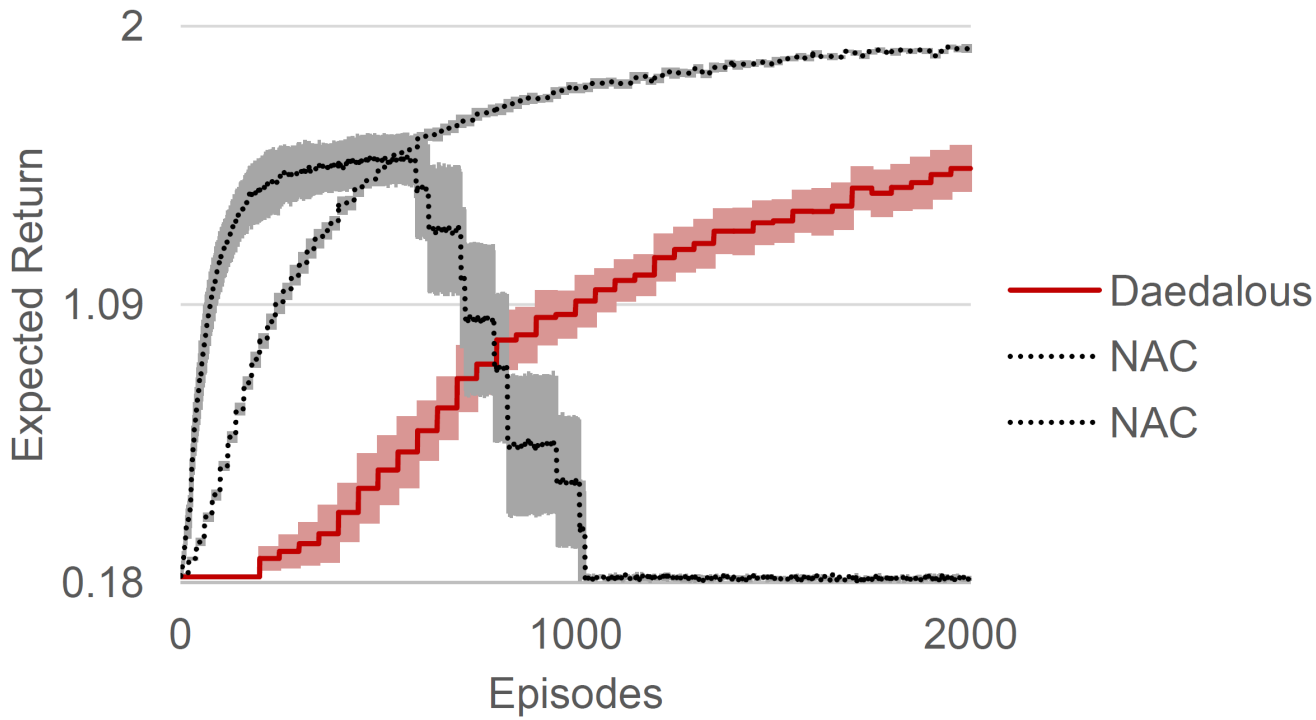
Empirical Results: Digital Marketing



Empirical Results: Digital Marketing



Empirical Results: Digital Marketing



Other Relevant Work

- How to deal with long horizons? (Guo, Thomas, Brunskill NIPS 2017)
- How to deal with importance sampling being "unfair"? (Doroudi, Thomas and Brunskill, best paper UAI 2017)
- What to do when the behavior policy is not known? (Liu, Gottesman, Raghu, Komorowski, Faisal, Doshi-Velez, Brunskill NeurIPS 2018)
- What to do when the behavior policy is deterministic?
- What to do when care about safe exploration?
- What to do when care about performance on a single trajectory
- For last two, see great work by Marco Pavone's group, Pieter Abbeel's group, Shie Mannor's group and Claire Tomlin's group, amongst others

Off Policy Policy Evaluation and Selection

- Very important topic: healthcare, education, marketing, ...
- Insights are relevant to on policy learning
- Big focus of my lab
- A number of others on campus also working in this area (e.g. Stefan Wager, Susan Athey...)
- Very interesting area at the intersection of causality and control

What You Should Know: Off Policy Policy Evaluation and Selection

$D \rightarrow$ good V^{π_e}

- Be able to define and apply importance sampling for off policy policy evaluation
- Define some limitations of IS (variance)
- List a couple alternatives (weighted IS, doubly robust)
- Define why we might want safe reinforcement learning
- Define the scope of the guarantees implied by safe policy improvement as defined in this lecture

Class Structure

- Last time: Meta Reinforcement Learning
- **This time: Batch RL**
- Next time: Quiz

Off-policy policy evaluation (revisited)

- Weighted per-decision importance sampling
  - Also called consistent weighted per-decision importance sampling
  - A fun exercise!