



# Policy Gradient Methods

Brown CSCI 1470/2470: Deep Learning

# Beyond Deep Q Networks (DQN)

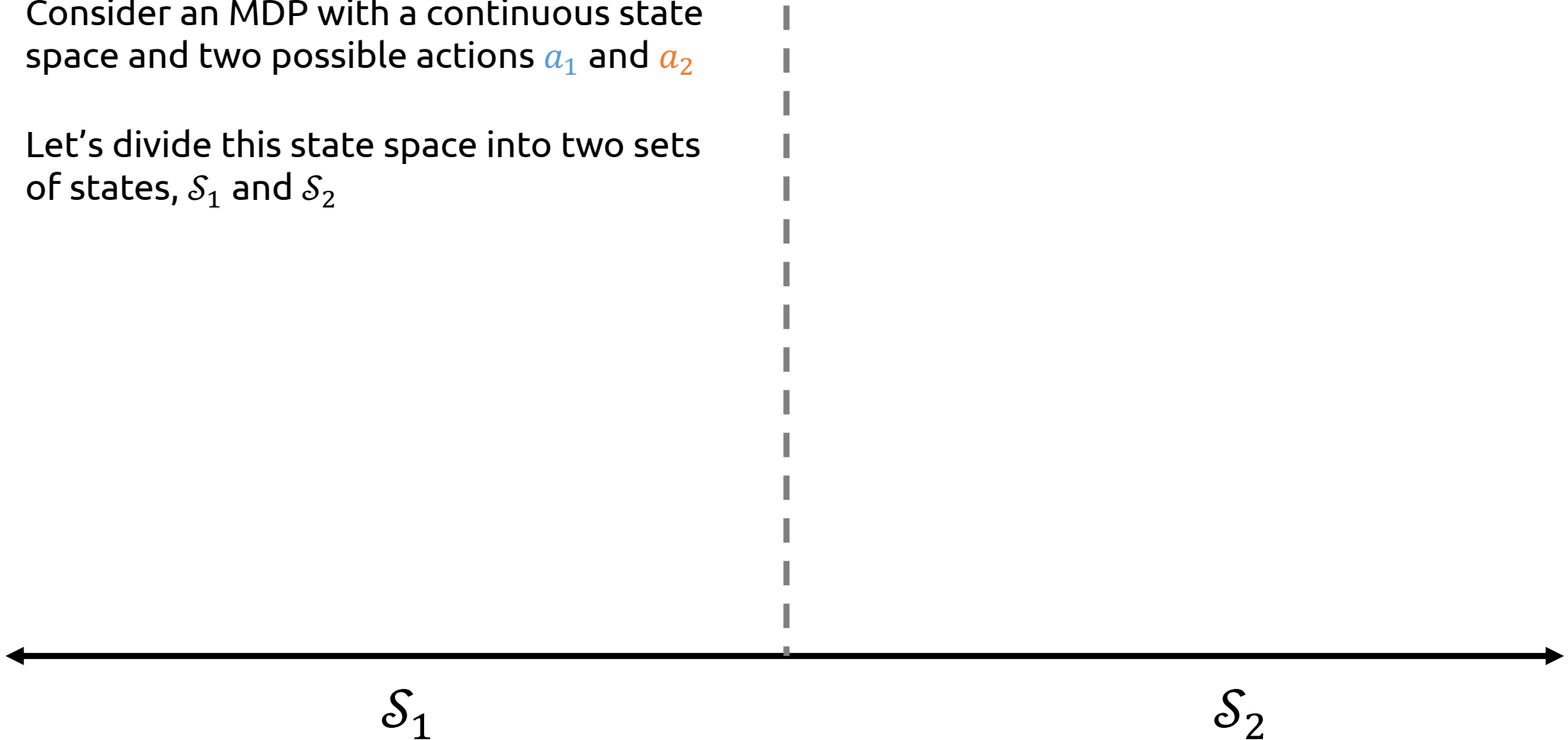
- DQN is amazing!
  - Can learn optimal play for Breakout, other Atari games given only raw pixels as input
- ***Does it have any weaknesses?***
  - DQN uses a neural net to learn an approximation of the Q function
  - Could that ever be a hard learning problem?



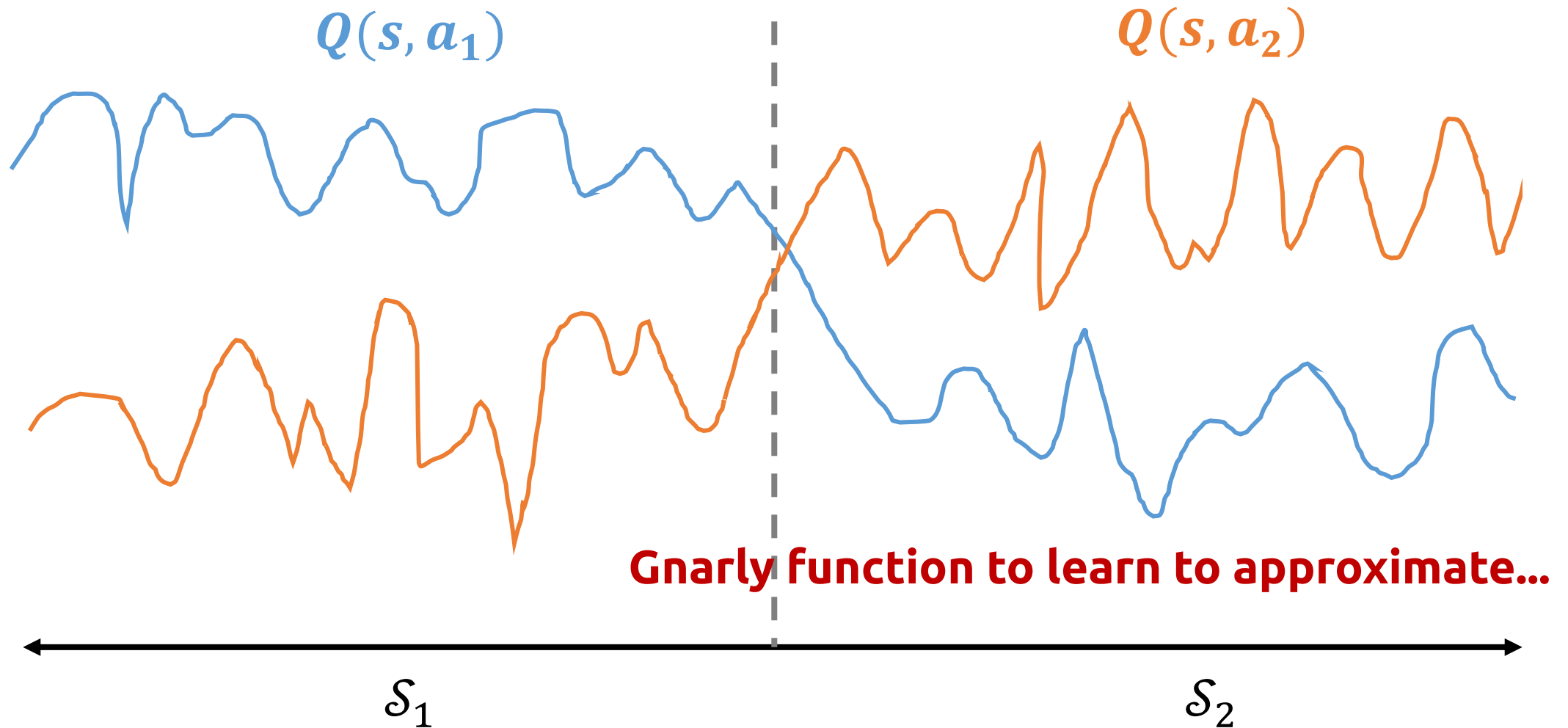
<https://www.youtube.com/watch?v=TmPfTptdgg>

# Q Functions can be complex...

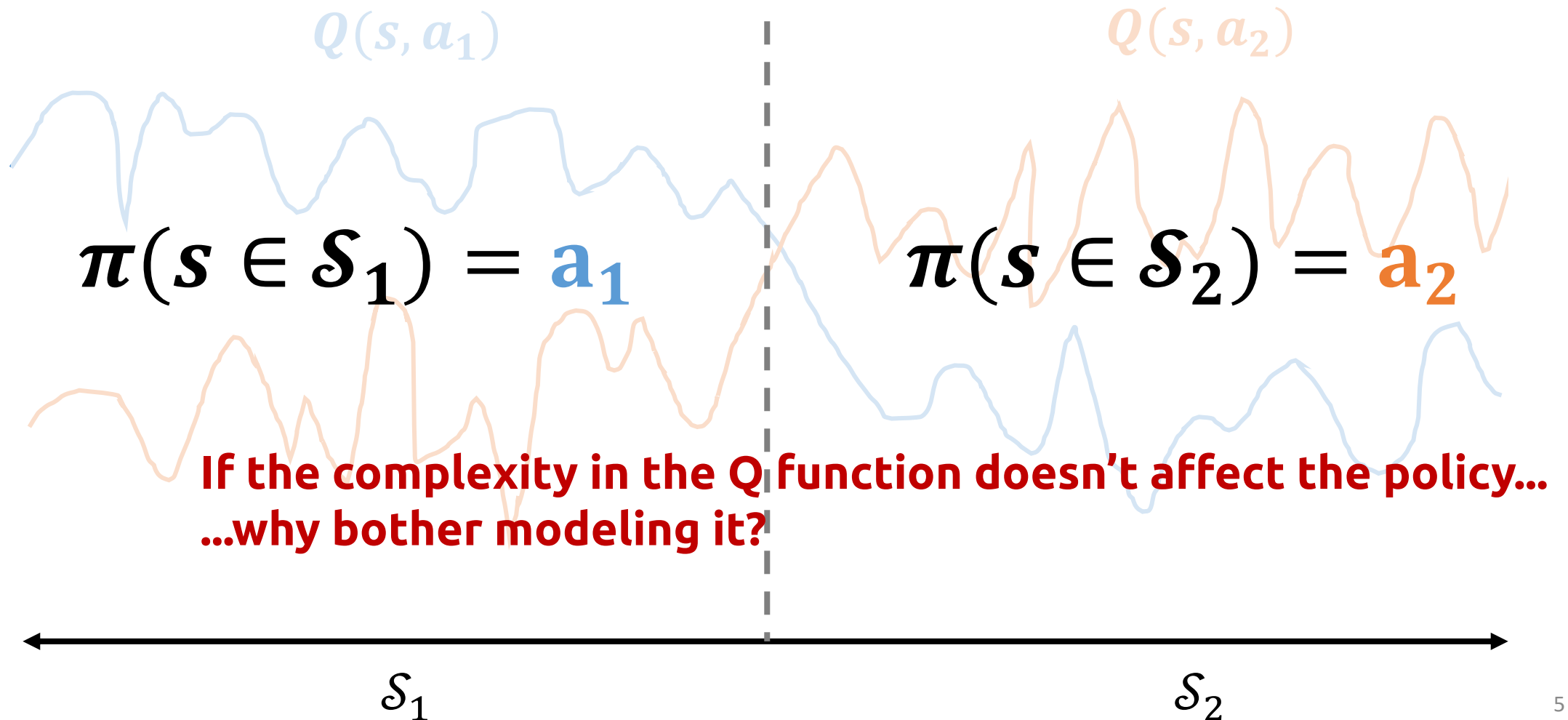
- Consider an MDP with a continuous state space and two possible actions  $a_1$  and  $a_2$
- Let's divide this state space into two sets of states,  $\mathcal{S}_1$  and  $\mathcal{S}_2$



# Q Functions can be complex...

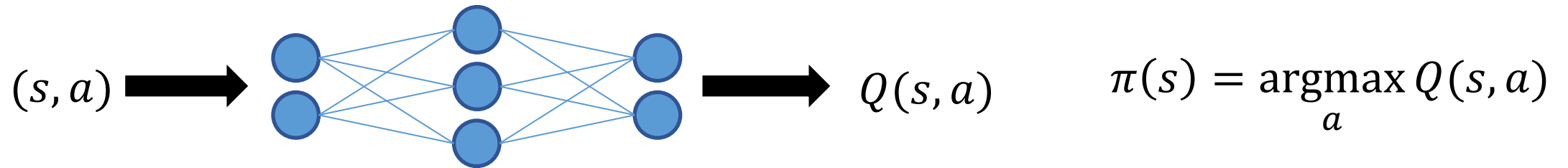


...but policies can still be simple

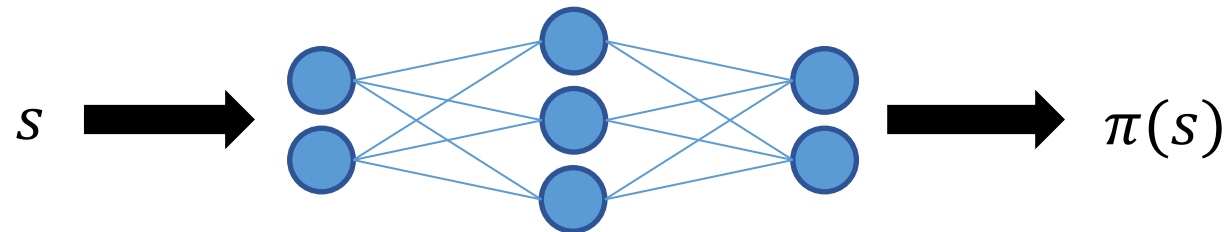


# An Idea:

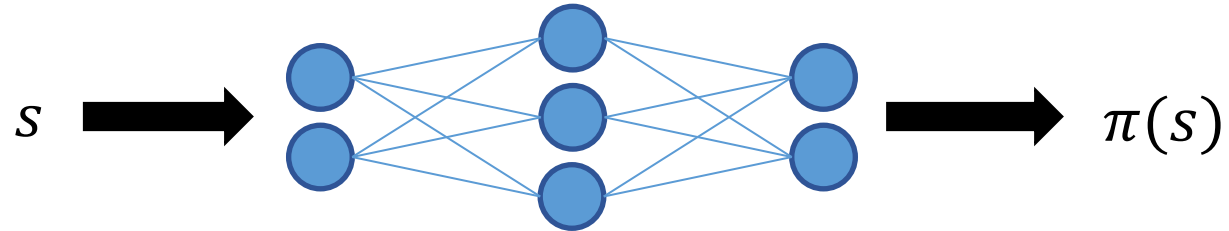
- Instead of learning a Q Network, and then extracting the policy from it:



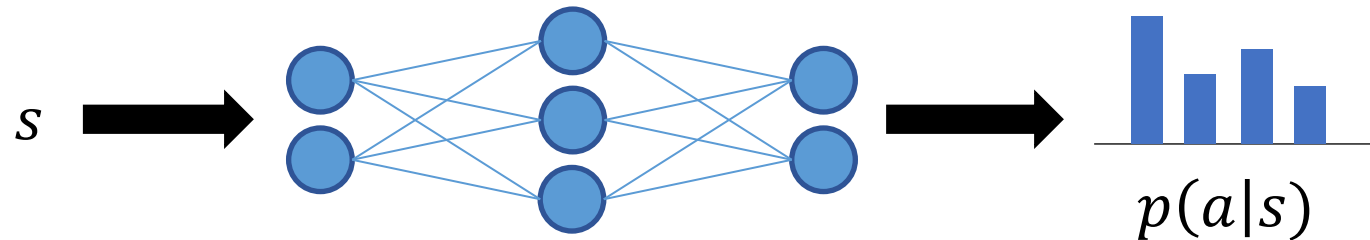
- ...why don't we just directly learn a ***Policy Network***?
  - i.e. have a neural net that takes in a state and outputs an action



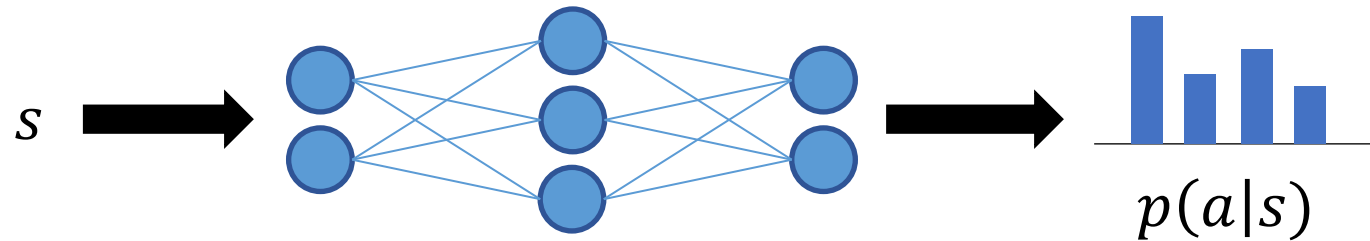
# Policy Networks



- Q:  $\pi(s)$  is a discrete action...how to make the network output that?
- A: Treat it like a classification problem—have the network output a ***probability distribution*** over actions



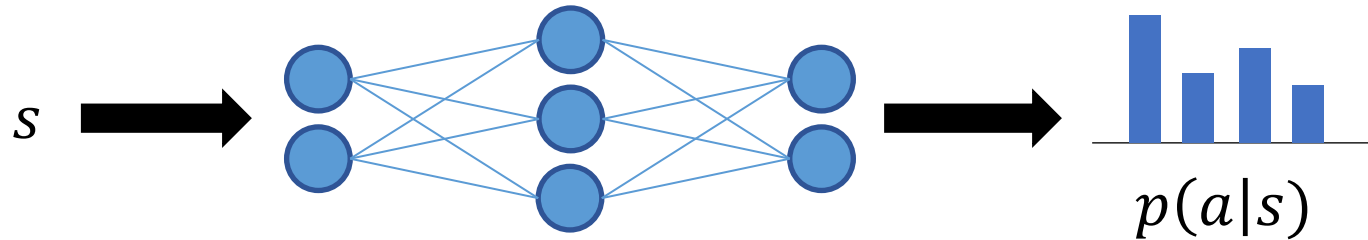
# Using a Policy Network



- Q: How to get a discrete action out of this distribution?
- A: Two possibilities:
  1.  $\pi(s) = \underset{a}{\operatorname{argmax}} p(a|s) \rightarrow$  Deterministic policy (just like Q learning)
  2.  $\pi(s) = \operatorname{sample}(p(a|s)) \rightarrow$  **Stochastic** policy
    - Don't always take the same action in the same situation
    - Arguably, more "naturalistic" behavior



# Training Policy Networks

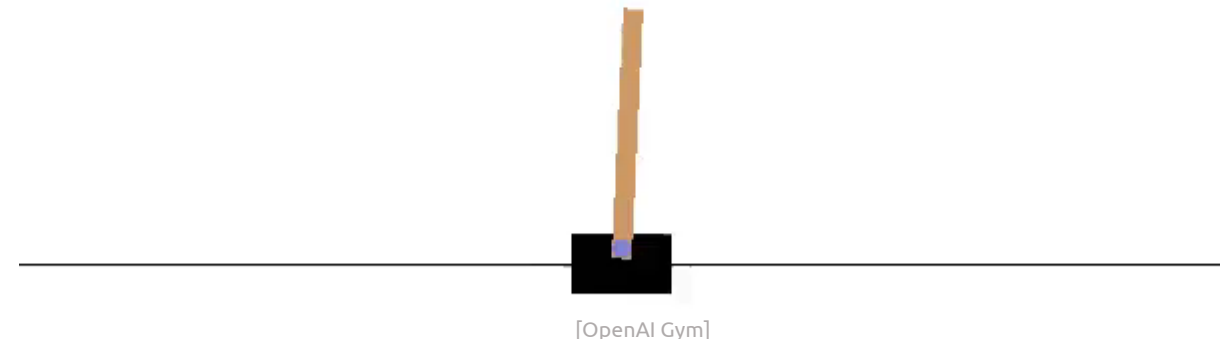


- How do we train a network like this?
- We can't just “adapt Q learning” somehow—this is a fundamentally different beast
- The study of how to learn policy networks lies at the core of all modern deep reinforcement learning research
- Family of learning algorithms known as ***Policy Gradient*** methods
- Let's make this concrete via a specific example...

# The “Cart Pole” Environment

# Cart Pole

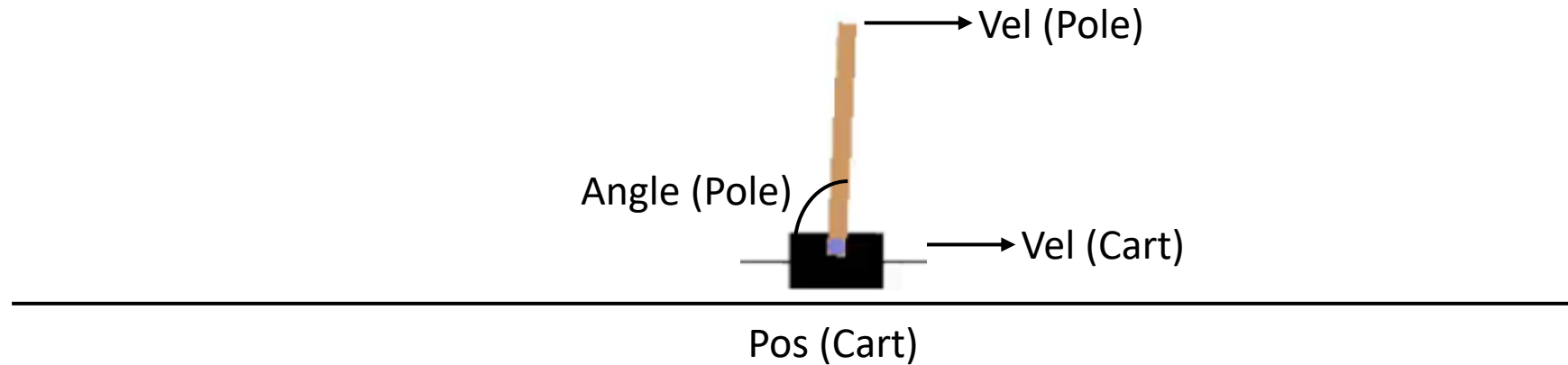
- Attempt to keep a pole vertically balanced on a moving cart
- Continuous-state MDP
  - Not solvable with tabular Q-learning
- Still a “toy problem”
  - This is a an instance of a dynamic equilibrium problem in classical robotics / control theory.
  - There exist [closed-form solutions](#) to the problem.
  - But it's also a fun test-case for RL 😊



*Note: the ‘jumps’ in the video are from the agent failing and the simulation restarting again*

# Cart Pole MDP Formulation

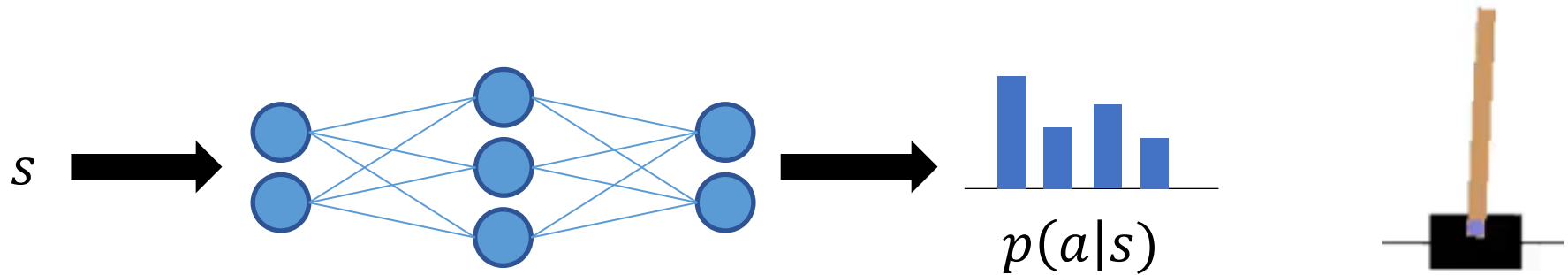
- State: cart position, cart velocity, pole angle, pole tip velocity



- Actions: push cart to **left** or **right**
- Transition function: (deterministic) simulation of Newtonian physics
- Reward function: 1 for every step taken
  - i.e. rewards keeping the pole balanced for as many steps as possible

# Training a Policy Network for Cart Pole

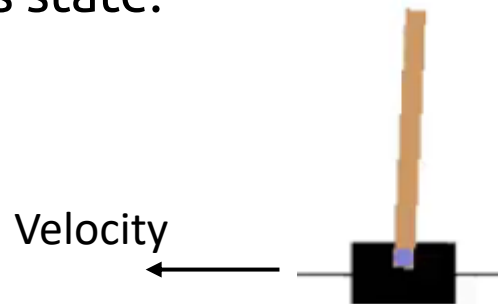
- Would be easy to do with supervised learning (i.e. if we had a ground-truth expert demonstration to follow)



- Just use cross-entropy loss on the ground-truth “correct” action at every time step
- But we don’t have supervision in RL...so what do we do instead?

# Training a Policy Network for Cart Pole

- Naïve loss function: Play an episode of the simulation, record the states/actions taken  $(\mathbf{s}, \mathbf{a}) = (s_1 \dots s_T, a_1 \dots a_T)$ , maximize the reward received at each timestep
  - i.e.  $L(s_t, a_t) = r(s_t, a_t, s_{t+1})$
- ***Why is this not a good loss function?***
  - Just because an action keeps the pole up for one more timestep, doesn't mean it will lead to keeping the pole up for the long term
  - E.g. consider this state:



- Moving the cart the left will not make the pole tip over immediately (so you'll get a reward of 1), but it will hasten the pole's eventual tipping

# Training a Policy Network for Cart Pole

- Better loss function: maximize the expected future return that you'll get from taking an action (not the immediate reward)
  - i.e.  $L(s_t, a_t) = \mathbb{E}[G_t | a_t]$
- What's another name for the expected future return?
  - The Q function!  $L(s_t, a_t) = \mathbb{E}[G_t | a_t] = Q(s_t, a_t)$
- We don't know Q, though (we're trying to **avoid** estimating it)
- But, we can play an entire simulation episode to completion, and then see what future reward we got ***in that single episode***.
  - i.e. if the episode lasts  $T$  steps, then  $L(s_t, a_t) = \sum_{i=t}^T \gamma^{i-1} r(s_i, a_i, s_{i+1})$

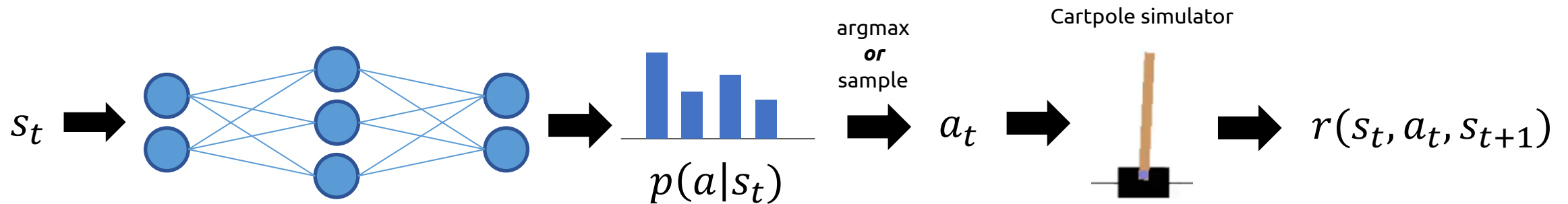
# Training a Policy Network for Cart Pole

- Let's call this the discounted future reward function:
  - $D(s_t, a_t) = \sum_{i=t}^T \gamma^{i-t} r(s_i, a_i, s_{i+1})$
- This gives us a good idea for our ideal loss function: across every step of our simulated training episode  $(s, a)$ , maximize the discounted future reward:
  - $L(s, a) = \sum_{t=1}^T D(s_t, a_t)$
- Brilliant! Let's simulate some episodes, throw them at our favorite SGD optimizer, and call it a day :)



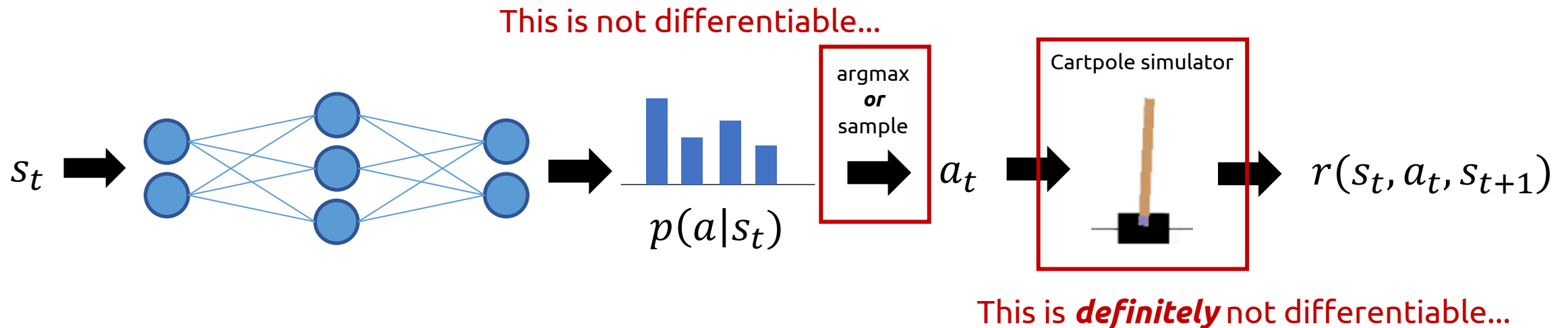
# Not so fast...

- Let's take a look at the computation graph for a single term of the discounted future reward function  $D(s_t, a_t) = \sum_{i=t}^T \gamma^{i-t} r(s_i, a_i, s_{i+1})$



# Not so fast...

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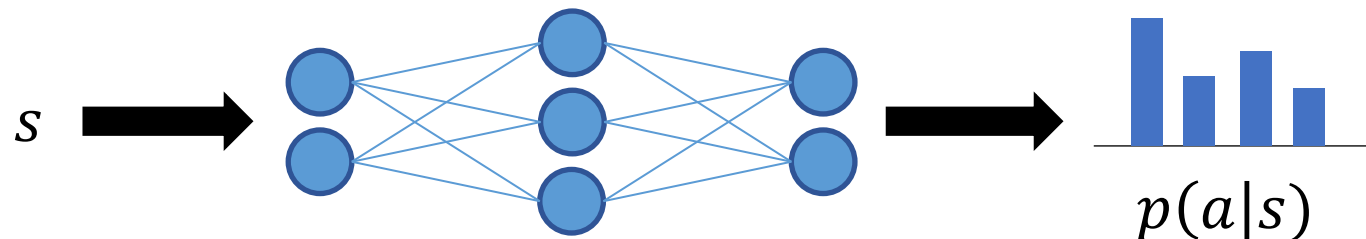
- Uh oh...it looks like we can't use SGD because we don't have an end-to-end differentiable function!

# The Policy Gradient Theorem to the Rescue

- Fortunately, it turns out that we can get the behavior we want by running SGD with the following gradient:

$$-\sum_{t=1}^T \underbrace{\nabla \log p(a_t | s_t)}_{\text{policy network}} D(s_t, a_t)$$

We only need the gradient of this part, which is our (fully differentiable) policy network!



Just likely computing gradients through a classification network

# Policy Gradient: Why It Works

$$-\sum_{t=1}^T \nabla \log p(a_t|s_t) D(s_t, a_t)$$

- It's possible to rigorously prove that this gradient does the right thing...
- ...but instead, we're going to focus on the ***intuition*** behind what it does

# Policy Gradient: Why It Works

$$-\sum_{t=1}^T \nabla \log p(a_t | s_t) D(s_t, a_t)$$

- **This part** says “maximize the probability of taking this action”
- If the sequence of actions  $\mathbf{a} = a_1 \dots a_T$  from our episode were given by a ground truth demonstration, then this would be all we need.
- But they’re not. So, some of these actions that we took in our episode might not be so good, so we shouldn’t just blindly maximize them.

# Policy Gradient: Why It Works

$$-\sum_{t=1}^T \nabla \log p(a_t|s_t) D(s_t, a_t)$$

- **This part** says “weight how much we maximize the probability of this action by how good that action was in the long term”
  - If it led to positive reward in the long term, we try to maximize the probability
  - If it led to zero reward in the long term, we leave the probability unchanged
  - If it led to **negative** reward in the long term, we try to **minimize** the probability

# Policy Gradient: Why It Works

$$-\sum_{t=1}^T \nabla \log p(a_t|s_t) D(s_t, a_t)$$

- There are, in fact, many different approaches that fall under the umbrella of “policy gradient methods” and which look something like this
- This particular one is the simplest, and is known as **REINFORCE**
  - No, it’s not an acronym for anything. The authors of the original paper just thought that shouting their algorithm name in all-caps would be a good idea...

# REINFORCE: Pseudo Code

Initialize model weights  $\theta$

Repeat until done (converge, time limit expired, etc.):

Run N episodes of environment simulation, each for  $T$  timesteps

For each episode

For  $t = 1$  to  $t = T$

$\theta \leftarrow \theta + \text{OptimizerStep}(\nabla \log p(a_t|s_t) D(s_t, a_t))$

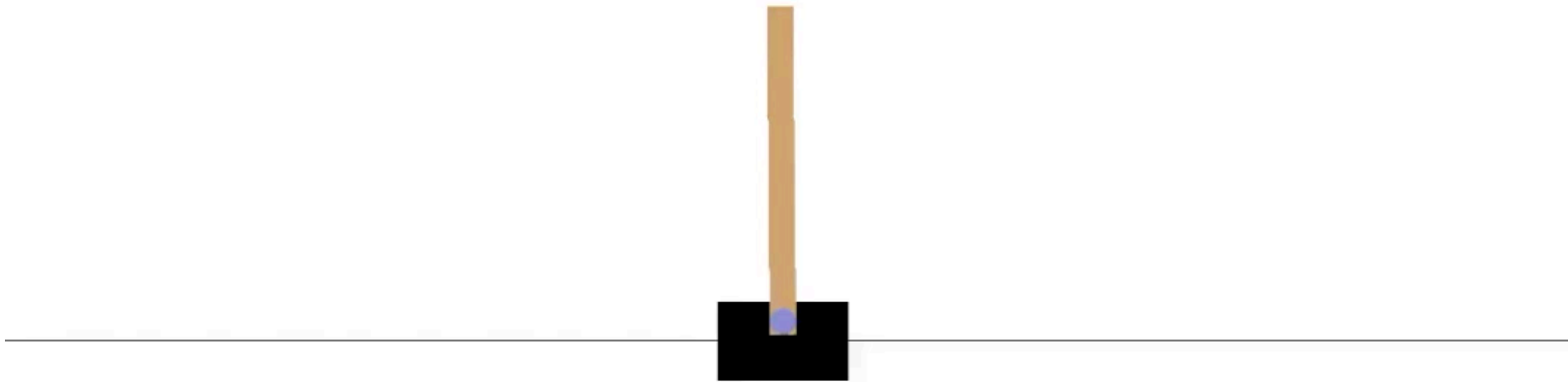
Return  $\theta$

**Your favorite optimizer (SGD, Adam, ...)**



# REINFORCE in action on Cart Pole

Episode= 1  
Episode reward= 10.0  
Average of last 500 rewards= 10.0  
Average of last 100 rewards= 10.0



# Reinforce vs DQN

## Pros

- Policy often easier to learn than Q function
- Automates explore vs. exploit tradeoff
  - Policy network starts off random and gradually becomes better as it is trained for more and more episodes
- Can learn stochastic policies
  - More naturalistic behavior
- In practice, can converge faster than DQN

## Cons

- Finds local optima more often than DQN...
- Unstable training
- Gradient updates only at end of each game (DQN updates after every step)

**We'll see how to fix these two issues in the next lecture...**

# Acknowledgments

- The following people contributed to these slides:
  - Josh Roy (UTA Fall '19)