Lecture 8: Policy Gradient I [1]

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CS234 Reinforcement Learning.

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- Additional reading: Sutton and Barto 2018 Chp. 13

[1] With many slides from or derived from David Silver and John Schulman and Pieter Abbeel

Last Time: We want RL Algorithms that Perform

- Optimization
- Delayed consequences
- Exploration
- Generalization
- And do it statistically and computationally efficiently

Last Time: Generalization and Efficiency

$Q(s, a, w)$

parameters

linear strong

DNN weaker

- Can use structure and additional knowledge to help constrain and speed reinforcement learning

Class Structure

- Last time: Imitation Learning
- **This time: Policy Search**
- Next time: Policy Search Cont.

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Policy-Based Reinforcement Learning

- In the last lecture we approximated the value or action-value function using parameters θ, or we often used w

$$V_\theta(s) \approx V^\pi(s)$$

$$Q_\theta(s,a) \approx Q^\pi(s,a)$$

- A policy was generated directly from the value function
  - e.g. using ϵ-greedy In q-learning we either take random action or take the action with the highest q-value

$\pi : S \to A$

- In this lecture we will directly parametrize the policy

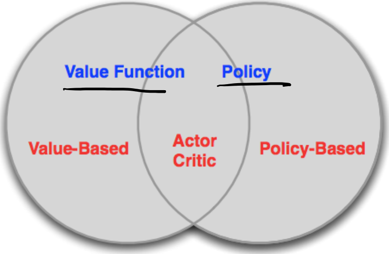
$$\pi_\theta(s,a) = \mathbb{P}[a|s;\overset{w}{\theta}]$$

- Goal is to find a policy π with the highest value function V^π
- We will focus again on model-free reinforcement learning

Value-Based and Policy-Based RL

- Value Based
  - Learnt Value Function
  - Implicit policy (e.g. ϵ-greedy)
- Policy Based
  - No Value Function
  - Learnt Policy
- Actor-Critic
  - Learnt Value Function
  - Learnt Policy

π Q / V



Advantages of Policy-Based RL

computation can matter

Advantages:

- Better convergence properties
- Effective in high-dimensional or continuous action spaces
- Can learn stochastic policies

why do we want this?
tabular MDP $\exists \pi$ which is deterministic & optimal

in tabular MDP, policies are always deterministic, so we don't need this

Disadvantages:

- Typically converge to a local rather than global optimum
- Evaluating a policy is typically inefficient and high variance

Q-learning cannot be used for partially-observable MDPs without modification. PG can!

Example: Rock-Paper-Scissors

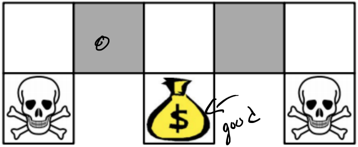
Why might we want a stochastic policy?

In this case, if you had a deterministic policy, you could be exploited by your opponent. So in this case, a random policy is the optimal policy!

- Two-player game of rock-paper-scissors
  - Scissors beats paper
  - Rock beats scissors
  - Paper beats rock
- Consider policies for iterated rock-paper-scissors
  - A deterministic policy is easily exploited
  - A uniform random policy is optimal (i.e. Nash equilibrium)

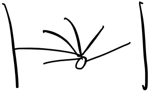
Example: Aliased Gridword (1)

This is partially observable, and you have aliasing, meaning you can't distinguish between two states based on the agent's sensors/features



good

avoid



- The agent cannot differentiate the grey states
- Consider features of the following form (for all N, E, S, W)

Features will tell the agent where they have walls

$$\phi(s, a) = \mathbb{1}(\text{wall to N}, a = \text{move E})$$

- Compare value-based RL, using an approximate value function

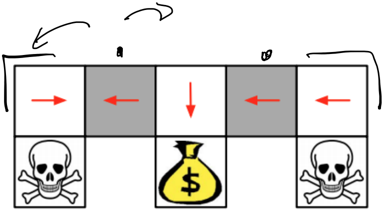
$$Q_\theta(s, a) = f(\phi(s, a); \theta)$$

- To policy-based RL, using a parametrized policy

$$\pi_\theta(s, a) = g(\phi(s, a); \theta)$$

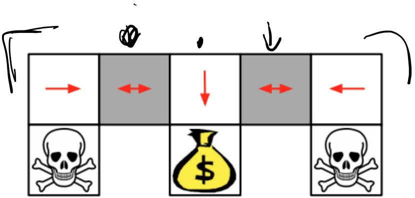
Example: Aliased Gridworld (2)

because of aliasing, the agent can't distinguish which grey state it's in, so it has to do the same thing in both states. This is not optimal!



- Under aliasing, an optimal deterministic policy will either
  - move W in both grey states (shown by red arrows)
  - move E in both grey states
- Either way, it can get stuck and never reach the money
- Value-based RL learns a near-deterministic policy
  - e.g. greedy or ϵ-greedy
- So it will traverse the corridor for a long time

Example: Aliased Gridworld (3)



- not Markov
- partially observable

- An optimal stochastic policy will randomly move E or W in grey states

$$\pi_\theta(\text{wall to N and S, move E}) = 0.5$$

$$\pi_\theta(\text{wall to N and S, move W}) = 0.5$$

- It will reach the goal state in a few steps with high probability
- Policy-based RL can learn the optimal stochastic policy

Policy Objective Functions

- Goal: given a policy $\pi_\theta(s, a)$ with parameters θ, find best θ
- But how do we measure the quality for a policy π_θ?
- In episodic environments we can use the start value of the policy

H steps
terminal state

$$J_1(\theta) = V^{\pi_\theta}(s_1)$$

We could just use the expected total reward of being in a distribution of starting states

There is an end. Either H steps or some terminal state

- In continuing environments we can use the average value

online

If we act forever, we will say "what's the average value of the states that we reach"

$$J_{avV}(\theta) = \sum_s \underbrace{d^{\pi_\theta}(s)} V^{\pi_\theta}(s)$$

- where $d^{\pi_\theta}(s)$ is the stationary distribution of Markov chain for π_θ.
- Or the average reward per time-step

$$J_{avR}(\theta) = \sum_s d^{\pi_\theta}(s) \sum_a \pi_\theta(s, a) R(a, s)$$

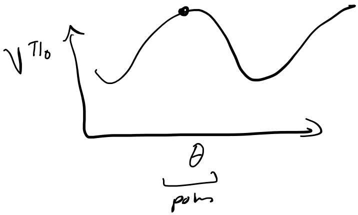
- For simplicity, today will mostly discuss the episodic case, but can easily extend to the continuing / infinite horizon case

Policy optimization

- Policy based reinforcement learning is an optimization problem
- Find policy parameters θ that maximize V^{π_θ}

Policy optimization

- Policy based reinforcement learning is an optimization problem
- Find policy parameters θ that maximize V^{π_θ}
- Can use gradient free optimization
  - Hill climbing we don't often do this for policy methods
  - Simplex / amoeba / Nelder Mead
  - Genetic algorithms
  - Cross-Entropy method (CEM)
  - Covariance Matrix Adaptation (CMA)



Human-in-the-Loop Exoskeleton Optimization (Zhang et al. Science 2017)

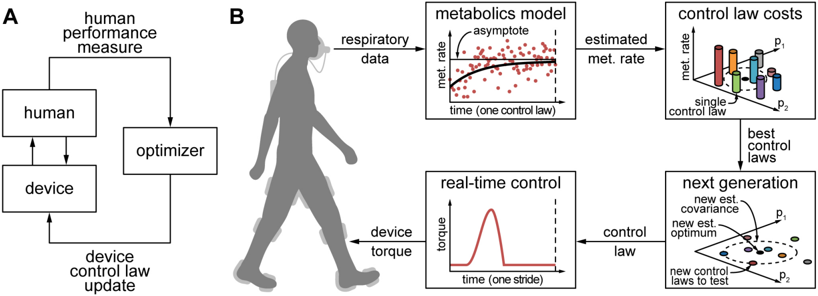


Figure: Zhang et al. Science 2017

- Optimization was done using CMA-ES, variation of covariance matrix evaluation

Gradient Free Policy Optimization

- Can often work embarrassingly well: "discovered that evolution strategies (ES), an optimization technique that's been known for decades, rivals the performance of standard reinforcement learning (RL) techniques on modern RL benchmarks (e.g. Atari/MuJoCo)" (https://blog.openai.com/evolution-strategies/)

Gradient Free Policy Optimization

- Often a great simple baseline to try
- Benefits
  - Can work with any policy parameterizations, including non-differentiable
  - Frequently very easy to parallelize
- Limitations
  - Typically not very sample efficient because it ignores temporal structure

Policy optimization

- Policy based reinforcement learning is an optimization problem
- Find policy parameters θ that maximize V^{π_θ}
- Can use gradient free optimization:
- Greater efficiency often possible using gradient
  - Gradient descent
  - Conjugate gradient
  - Quasi-newton
- We focus on gradient descent, many extensions possible
- And on methods that exploit sequential structure

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Policy Gradient

- Define $V(\theta) = V^{\pi_\theta}$ to make explicit the dependence of the value on the policy parameters
- Assume episodic MDPs (easy to extend to related objectives, like average reward)

Policy Gradient

- Define $V(\theta) = V^{\pi_\theta}$ to make explicit the dependence of the value on the policy parameters
- Assume episodic MDPs
- Policy gradient algorithms search for a *local* maximum in $V(\theta)$ by ascending the gradient of the policy, w.r.t parameters θ

we take the gradient wrt to the parameters that define the policy, and adjust based on some learning rate

$$\Delta\theta = \alpha \nabla_\theta V(\theta)$$

learning rate

very similar to Q/V based optimization

- Where $\nabla_\theta V(\theta)$ is the policy gradient

$$\nabla_\theta V(\theta) = \begin{pmatrix} \frac{\partial V(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial V(\theta)}{\partial \theta_n} \end{pmatrix}$$

- and α is a step-size parameter

Computing Gradients by Finite Differences

The simplest thing to do here is Finite Differences. For each of your policy parameters k, just perturb it a little bit. You'd do this for every parameter. This will give you an estimate of the gradient

- To evaluate policy gradient of $\pi_\theta(s, a)$
- For each dimension $k \in [1, n]$
  - Estimate kth partial derivative of objective function w.r.t. θ
  - By perturbing θ by small amount ϵ in kth dimension

$$\frac{\partial V(\theta)}{\partial \theta_k} \approx \frac{V(\theta + \epsilon u_k) - V(\theta)}{\epsilon}$$

where u_k is a unit vector with 1 in kth component, 0 elsewhere.

Computing Gradients by Finite Differences

- To evaluate policy gradient of $\pi_\theta(s, a)$
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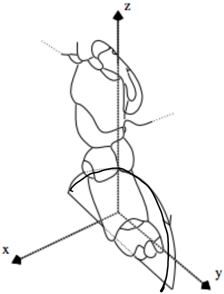
$$\frac{\partial V(\theta)}{\partial \theta_k} \approx \frac{V(\theta + \epsilon u_k) - V(\theta)}{\epsilon}$$

  where u_k is a unit vector with 1 in kth component, 0 elsewhere.

- Uses n evaluations to compute policy gradient in n dimensions
- Simple, noisy, inefficient - but sometimes effective
- Works for arbitrary policies, even if policy is not differentiable

Training AIBO to Walk by Finite Difference Policy Gradient[1]



- Goal: learn a fast AIBO walk (useful for Robocup)
- Adapt these parameters by finite difference policy gradient
- Evaluate performance of policy by field traversal time

[1]Kohl and Stone. Policy gradient reinforcement learning for fast quadrupedal locomotion. ICRA 2004. http://www.cs.utexas.edu/ ai-lab/pubs/icra04.pdf

AIBO Policy Parameterization

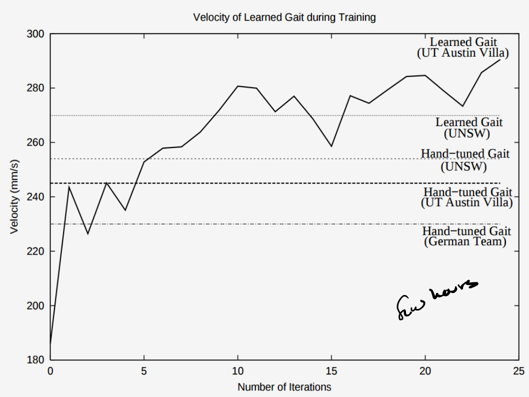
a sequence of actions that is not impacted by anything

- AIBO walk policy is open-loop policy
- No state, choosing set of action parameters that define an ellipse
- Specified by 12 continuous parameters (elliptical loci)
  - The front locus (3 parameters: height, x-pos., y-pos.)
  - The rear locus (3 parameters)
  - Locus length
  - Locus skew multiplier in the x-y plane (for turning)
  - The height of the front of the body
  - The height of the rear of the body
  - The time each foot takes to move through its locus
  - The fraction of time each foot spends on the ground
- New policies: for each parameter, randomly add (ϵ, 0, or $-\epsilon$)

AIBO Policy Experiments

- "All of the policy evaluations took place on actual robots... only human intervention required during an experiment involved replacing discharged batteries ... about once an hour."
- Ran on 3 Aibos at once
- Evaluated 15 policies per iteration.
- Each policy evaluated 3 times (to reduce noise) and averaged
- Each iteration took 7.5 minutes
- Used $\eta = 2$ (learning rate for their finite difference approach)

Training AIBO to Walk by Finite Difference Policy Gradient Results



- Authors discuss that performance is likely impacted by: initial starting policy parameters, ϵ (how much policies are perturbed), η (how much to change policy), as well as policy parameterization

AIBO Walk Policies

- link

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Computing the gradient analytically

only converge local optima

- We now compute the policy gradient *analytically* most commonly used now
- Assume policy π_θ is differentiable whenever it is non-zero
- and we know the gradient $\nabla_\theta \pi_\theta(s, a)$ We just assume that we can compute our policy gradient somehow

computable

Likelihood Ratio Policies

terminate

- Denote a state-action trajectory as $\tau = (s_0, a_0, r_0, ..., s_{T-1}, a_{T-1}, r_{T-1}, s_T)$
- Use $R(\tau) = \sum_{t=0}^{T} R(s_t, a_t)$ to be the sum of rewards for a trajectory τ

Likelihood Ratio Policies

- Denote a state-action trajectory as
  $\tau = (s_0, a_0, r_0, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T)$
- Use $R(\tau) = \sum_{t=0}^{T} R(s_t, a_t)$ to be the sum of rewards for a trajectory τ
- Policy value is

Probability that we observe a particular trajectory

Reward for that trajectory

Policy value: expected sum of rewards we get for following this policy

$$V(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{T} R(s_t, a_t); \pi_\theta\right] = \sum_{\tau} P(\tau; \theta) R(\tau)$$

- where $P(\tau; \theta)$ is used to denote the probability over trajectories when executing policy $\pi(\theta)$
- In this new notation, our goal is to find the policy parameters θ:

so now our goal is to find policy params theta that maximize this!

The change we made is that now theta only appears in terms of the distribution of trajectories under this policy

$$\arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau; \theta) R(\tau)$$

Likelihood Ratio Policy Gradient

- Goal is to find the policy parameters θ:

$$\arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau;\theta)R(\tau)$$

- Take the gradient with respect to θ:

$$\nabla_\theta V(\theta) \quad = \quad \nabla_\theta \sum_{\tau} P(\tau;\theta)R(\tau)$$

$$= \sum_{\tau} R(\tau) \nabla_\theta P(\tau;\theta)$$

we can bring the gradient inside the sum

$$= \sum_{\tau} R(\tau) \frac{P(\tau;\theta)}{P(\tau;\theta)} \nabla_\theta P(\tau;\theta)$$

multiply by the same thing

$$\nabla_\theta \log P(\tau;\theta) = \frac{1}{P(\tau;\theta)} \nabla_\theta P(\tau;\theta)$$

$$= \sum_{\tau} R(\tau) P(\tau;\theta) \nabla_\theta \log P(\tau;\theta)$$

now we can reexpress this

This log trick is helpful.
It's helpful because it will be useful when we want to do all this when we don't know the dynamics P(tau) or reward model. We want to be able to evaluate the gradient of the policy without knowing the dynamics model. This trick will help us get there.

Likelihood Ratio Policy Gradient

- Goal is to find the policy parameters θ:

$$\arg\max_\theta V(\theta) = \arg\max_\theta \sum_\tau P(\tau;\theta)R(\tau)$$

- Take the gradient with respect to θ:

$$\begin{aligned}
\nabla_\theta V(\theta) &= \nabla_\theta \sum_\tau P(\tau;\theta)R(\tau) \\
&= \sum_\tau \nabla_\theta P(\tau;\theta)R(\tau) \\
&= \sum_\tau \frac{P(\tau;\theta)}{P(\tau;\theta)} \nabla_\theta P(\tau;\theta)R(\tau) \\
&= \sum_\tau P(\tau;\theta)R(\tau) \underbrace{\frac{\nabla_\theta P(\tau;\theta)}{P(\tau;\theta)}}_{\text{likelihood ratio}} \\
&= \sum_\tau P(\tau;\theta)R(\tau)\nabla_\theta \log P(\tau;\theta)
\end{aligned}$$

We have a sum over all traj, we don't have access to all, but we can sample them to approximate.

Likelihood Ratio Policy Gradient

- Goal is to find the policy parameters θ:

$$\arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau;\theta)R(\tau)$$

Issue: We don't know P, so we can't analytically solve this!

- Take the gradient with respect to θ:

$$\nabla_\theta V(\theta) \quad = \quad \sum_{\tau} \underbrace{P(\tau;\theta)}R(\tau)\nabla_\theta \log P(\tau;\theta)$$

- Approximate with empirical estimate for m sample paths under policy π_θ:

we can estimate the probability of a given trajectory by taking samples

Great, we got rid of one P. But we still have logP, which we don't know

$$\nabla_\theta V(\theta) \quad \approx \quad \hat{g} = \underbrace{(1/m)\sum_{i=1}^{m}} R(\tau^{(i)})\nabla_\theta \log P(\tau^{(i)};\theta)$$

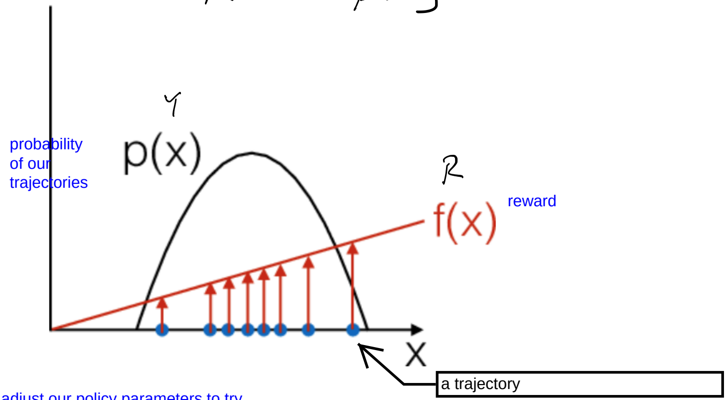
the gradient is then the reward we get for a given trajectory times the gradlog probability of that trajectory

- Consider generic form of $R(\tau^{(i)})\nabla_\theta \log P(\tau^{(i)};\theta)$:
  $\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$
- $f(x)$ measures how good the sample x is.
- Moving in the direction $\hat{g}_i$ pushes up the logprob of the sample, in proportion to how good it is
- *Valid even if $f(x)$ is discontinuous, and unknown, or sample space (containing x) is a discrete set*

Score Function Gradient Estimator: Intuition

$$\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$$

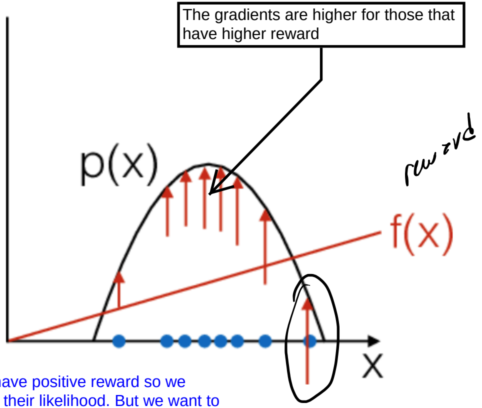
R policy



We want to adjust our policy parameters to try to increase our probability of trajectories that reusult in high reward

Score Function Gradient Estimator: Intuition

$$\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$$



All these trajectories have positive reward so we want to increase all of their likelihood. But we want to increase the trajectories that lead to higher reward even more!

Decomposing the Trajectories Into States and Actions

- Approximate with empirical estimate for m sample paths under policy π_θ:

so we have to be able to approximate this term. So we have to be able to determine the probability of a trajectory under a policy with certain params

unknown

probability of a certain starting state. we don't know this

$$\nabla_\theta V(\theta) \approx \hat{g} = (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \nabla_\theta \log P(\tau^{(i)})$$

$$\nabla_\theta \log P(\tau^{(i)}; \theta) = \nabla_\theta \log \Big(\underbrace{\mu(s_0)}_{\text{initial state}} \prod_{j=0}^{T-1} p(s_{j+1}^i \mid s_j^i, a_j^i)\, \pi_\theta(a_j^i \mid s_j^i) \Big)$$

$$= \nabla_\theta \log \mu(s_0) + \sum_{j=0}^{T-1} \nabla_\theta \log p(s_{j+1}^i \mid s_j^i, a_j^i) + \sum_{j=0}^{T-1} \nabla_\theta \log \pi_\theta(a_j^i \mid s_j^i)$$

1 $\to 0$ 2 $\to 0$ 3

the derivative of this wrt to theta is zero because we don't have theta

the product of the probability of observing the next state given our current state and the action that was taken, times the probability of taking that action given our current policy

So, doing the log transformation was helpful because it allowed us to take the product of the probability of the action we took and the state transitions and decompose it into sums! Then we can apply the derivative separately, and some terms disappear! So now we don't need to know the transition model!

we can rewrite: log(AB) = logA + logB

the derivative of this wrt to theta is 0 because this doesn't depend on theta!

Decomposing the Trajectories Into States and Actions

- Approximate with empirical estimate for m sample paths under policy π_θ:

$$\nabla_\theta V(\theta) \approx \hat{g} = (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \nabla_\theta \log P(\tau^{(i)})$$

$$\begin{aligned}
\nabla_\theta \log P(\tau^{(i)}; \theta) &= \nabla_\theta \log \left[\underbrace{\mu(s_0)}_{\text{Initial state distrib.}} \prod_{t=0}^{T-1} \underbrace{\pi_\theta(a_t|s_t)}_{\text{policy}} \underbrace{P(s_{t+1}|s_t, a_t)}_{\text{dynamics model}} \right] \\
&= \nabla_\theta \left[\log \mu(s_0) + \sum_{t=0}^{T-1} \log \pi_\theta(a_t|s_t) + \log P(s_{t+1}|s_t, a_t) \right] \\
&= \sum_{t=0}^{T-1} \underbrace{\nabla_\theta \log \pi_\theta(a_t|s_t)}_{\text{no dynamics model required!}}
\end{aligned}$$

have to evaluate it

We still have to be able to evaluate the derivative of our log policy.

You might have to estimate that derivative itself using finite approximation or brute force, or computation.

But if you choose a particular form of a parameterized policy, this will be analytic

Score Function

we need to be able to evaluate this

- Define score function as $\underbrace{\nabla_\theta \log \pi_\theta(s, a)}$

Likelihood Ratio / Score Function Policy Gradient

- Putting this together
- Goal is to find the policy parameters θ:

$$\arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau;\theta) R(\tau)$$

- Approximate with empirical estimate for m sample paths under policy π_θ using score function:

$$\begin{aligned} \nabla_\theta V(\theta) &\approx \hat{g} = (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \nabla_\theta \log P(\tau^{(i)};\theta) \\ &= (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)}) \end{aligned}$$

we sum rewards for entire trajectory

- Do not need to know dynamics model

Policy Gradient Theorem

Before we took the sum of all rewards for a trajectory/episode. But we can actually put different values here too. We could instead use the average reward per timestep, or the average value. Therefore we can extend this to the continuous case, where there aren't finite episodes

- The policy gradient theorem generalizes the likelihood ratio approach

Theorem

For any differentiable policy $\pi_\theta(s, a)$,
for any of the policy objective function $J = J_1$, (episodic reward), J_{avR} (average reward per time step), or $\frac{1}{1-\gamma} J_{avV}$ (average value),
the policy gradient is

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s, a) Q^{\pi_\theta}(s, a)]$$

so we can generalize this to any objective function!

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Likelihood Ratio / Score Function Policy Gradient

We take our policy, we run it M times. For each of those m times, we get a sequence of states, actions, and rewards. And then we average. This is an unbiased estimate of the policy gradient, but it's very noisy!

This type of sampling is known as a Monte Carlo estimate!

$$\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$

- Unbiased but very noisy
- Fixes that can make it practical
  - Temporal structure
  - Baseline
- Next time will discuss some additional tricks

Alternatives to Monte Carlo...

- bootstrapping (temporal difference methods)
--- This will help reduce variance
--- We replace R with somethign else

Policy Gradient: Use Temporal Structure

- Previously:

sum of rewards you get

sum of your derivative wrt to policy params

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau \left[\left(\sum_{t=0}^{T-1} r_t \right) \left(\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \right) \right]$$

- We can repeat the same argument to derive the gradient estimator for a single reward term $r_{t'}$.

derivative of the expected reward for t'

only summing up to t' so we don't sum over all the future timesteps too. So it's a shortened trajectory looking at values only up until t'

$$\nabla_\theta \mathbb{E}[r_{t'}] = \mathbb{E}\left[r_{t'} \sum_{t=0}^{t'} \nabla_\theta \log \pi_\theta(a_t|s_t) \right]$$

- Summing this formula over t, we obtain

$$\nabla_\theta V(\theta) = \nabla_\theta \mathbb{E}[R] = \mathbb{E}\left[\sum_{t'=0}^{T-1} r_{t'} \sum_{t=0}^{t'} \nabla_\theta \log \pi_\theta(a_t|s_t) \right]$$

now we can reorder terms

$$= \mathbb{E}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t, s_t) \sum_{t'=t}^{T-1} r_{t'} \right]$$

this is just the return starting at time step t

Policy Gradient: Use Temporal Structure

- Recall for a particular trajectory $\tau^{(i)}$, $\sum_{t'=t}^{T-1} r_{t'}^{(i)}$ is the return $G_t^{(i)}$

$$\nabla_\theta \mathbb{E}[R] \approx (1/m) \sum_{i=1}^{m} \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t, s_t) G_t^{(i)}$$

this has a slightly lower variance.

Recall before: $\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$

We had to separately sum up all of our rewards, and then multiply that by the full sum of the derivative of the logs, now we only sum up the rewards from the current timestep onward. Before we summed rewards over the entire trajectory.

Monte-Carlo Policy Gradient (REINFORCE)

sample from your current policy to get an entire trajectory

- Leverages likelihood ratio / score function and temporal structure

$$\Delta\theta_t = \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) G_t$$

well first we have to decide how we are going to parameterize our policy

REINFORCE:

Initialize policy parameters θ arbitrarily

for each episode $\{s_1, a_1, r_2, \cdots, s_{T-1}, a_{T-1}, r_T\} \sim \pi_\theta$ **do**

  for $t = 1$ to $T - 1$ **do**

    $\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) G_t$

  endfor

endfor

return θ

for every timestep in that trajectory you update your policy params

sum of rewards for being in s_t, a_t onwards

one critical question: how do we compute the gradient the policy wrt the params?

What type of policys do people typically consider that have nice differentiable forms?
1) softmax, 2) NN, 3) Gaussians

Differentiable Policy Classes

- Many choices of differentiable policy classes including:
  - Softmax
  - Gaussian
  - Neural networks

Softmax Policy

features of our state and action space

weights of our policy params

Softmax: Have a linear combination of features and take an exponential weight of them

Reasonable for DISCRETE action space

This is what we do for Multinomial classification

- Weight actions using linear combination of features $\phi(s,a)^T\theta$
- Probability of action is proportional to exponentiated weight

To get a probability of taking an action given a state, we take an exponential of our weighted features

$$\pi_\theta(s,a) = e^{\phi(s,a)^T\theta} / (\sum_a e^{\phi(s,a)^T\theta})$$

sum over all actions

- The score function is

$$\nabla_\theta \log \pi_\theta(s,a) = \phi(s,a) - \mathbb{E}_{\pi_\theta}[\phi(s,\cdot)]$$

the value of our features we observed!

e.g. angles in a robot

"average" feature values you would observe over all the possible actions you might take given our policy and state

Gaussian Policy

common in controls & robotics

- In continuous action spaces, a Gaussian policy is natural
- Mean is a linear combination of state features $\mu(s) = \phi(s)^T\theta$
- Variance may be fixed σ^2, or can also parametrised
- Policy is Gaussian $a \sim \mathcal{N}(\mu(s), \sigma^2)$
- The score function is

'a' is drawn from a Gaussian distribution using this mean of our features. So we compare our current state to the mean, and then select an action

$$\nabla_\theta \log \pi_\theta(s, a) = \frac{(a - \mu(s))\phi(s)}{\sigma^2}$$

Likelihood Ratio / Score Function Policy Gradient

$$\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$

- Unbiased but very noisy
- Fixes that can make it practical
  - Temporal structure
  - **Baseline**
- Next time will discuss some additional tricks

Policy Gradient: Introduce Baseline

- Reduce variance by introducing a *baseline* $b(s)$

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t, \theta) \left(\sum_{t'=t}^{T-1} r_{t'} - b(s_t) \right) \right]$$

- For any choice of $b(s)$, gradient estimator is unbiased.
- Near optimal choice is expected return, $b(s_t) \approx \mathbb{E}[r_t + r_{t+1} + \cdots + r_{T-1}]$
- Interpretation: increase logprob of action a_t proportionally to how much returns $\sum_{t'=t}^{T-1} r_{t'}$ are better than expected

Baseline $b(s)$ Does Not Introduce Bias–Derivation

$$
\mathbb{E}_\tau[\nabla_\theta \log \pi(a_t|s_t, \theta) b(s_t)]
$$
$$
= \mathbb{E}_{s_{0:t}, a_{0:(t-1)}}\left[\mathbb{E}_{s_{(t+1):T}, a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t, \theta) b(s_t)]\right]
$$

Baseline $b(s)$ Does Not Introduce Bias–Derivation

$$
\begin{aligned}
&\mathbb{E}_\tau[\nabla_\theta \log \pi(a_t|s_t,\theta) b(s_t)] \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[\mathbb{E}_{s_{(t+1):T},a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t,\theta) b(s_t)]\right] \text{ (break up expectation)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\mathbb{E}_{s_{(t+1):T},a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t,\theta)]\right] \text{ (pull baseline term out)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}[b(s_t)\mathbb{E}_{a_t}[\nabla_\theta \log \pi(a_t|s_t,\theta)]] \text{ (remove irrelevant variables)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\sum_a \pi_\theta(a_t|s_t)\frac{\nabla_\theta \pi(a_t|s_t,\theta)}{\pi_\theta(a_t|s_t)}\right] \text{ (likelihood ratio)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\sum_a \nabla_\theta \pi(a_t|s_t,\theta)\right] \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\nabla_\theta \sum_a \pi(a_t|s_t,\theta)\right] \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}[b(s_t)\nabla_\theta 1] \\
&= \mathbb{E}_{s_0:t,a_0:(t-1)}[b(s_t)\cdot 0] = 0
\end{aligned}
$$

"Vanilla" Policy Gradient Algorithm

Initialize policy parameter θ, baseline b
for iteration$=1, 2, \cdots$ **do**
Collect a set of trajectories by executing the current policy
At each timestep in each trajectory, compute
the *return* $R_t = \sum_{t'=t}^{T-1} r_{t'}$, and
the *advantage estimate* $\hat{A}_t = R_t - b(s_t)$.
Re-fit the baseline, by minimizing $||b(s_t) - R_t||^2$,
summed over all trajectories and timesteps.
Update the policy, using a policy gradient estimate $\hat{g}$,
which is a sum of terms $\nabla_\theta \log \pi(a_t|s_t;\theta)\hat{A}_t$.
(Plug $\hat{g}$ into SGD or ADAM)
endfor

Practical Implementation with Autodiff

- Usual formula $\sum_t \nabla_\theta \log \pi(a_t|s_t;\theta)\hat{A}_t$ is inifficient–want to batch data
- Define "surrogate" function using data from current batch

$$L(\theta) = \sum_t \log \pi(a_t|s_t;\theta)\hat{A}_t$$

- Then policy gradient estimator $\hat{g} = \nabla_\theta L(\theta)$
- Can also include value function fit error

$$L(\theta) = \sum_t \left(\log \pi(z_t|s_t;\theta)\hat{A}_t - ||V(s_t) - \hat{R}_t||^2\right)$$

Value Functions

- Recall Q-function / state-action-value function:

$$Q^{\pi,\gamma}(s,a) = \mathbb{E}_\pi \left[r_0 + \gamma r_1 + \gamma^2 r_2 \cdots | s_0 = s, a_0 = a \right]$$

- State-value function can serve as a great baseline

$$\begin{aligned} V^{\pi,\gamma}(s) &= \mathbb{E}_\pi \left[r_0 + \gamma r_1 + \gamma^2 r_2 \cdots | s_0 = s \right] \\ &= \mathbb{E}_{a \sim \pi}[Q^{\pi,\gamma}(s,a)] \end{aligned}$$

- Advantage function: Combining Q with baseline V

$$A^{\pi,\gamma}(s,a) = Q^{\pi,\gamma}(s,a) - V^{\pi,\gamma}(s)$$

N-step estimators

- Can also consider blending between TD and MC estimators for the target to substitute for the true state-action value function.

$$\hat{R}_t^{(1)} = r_t + \gamma V(s_{t+1})$$
$$\hat{R}_t^{(2)} = r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2}) \qquad \cdots$$
$$\hat{R}_t^{(\text{inf})} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+1} + \cdots$$

- If subtract baselines from the above, get advantage estimators

$$\hat{A}_t^{(1)} = r_t + \gamma V(s_{t+1}) - V(s_t)$$
$$\hat{A}_t^{(2)} = r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2}) - V(s_t)$$
$$\hat{A}_t^{(\text{inf})} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+1} + \cdots - V(s_t)$$

- $\hat{A}_t^{(a)}$ has low variance & high bias. $\hat{A}_t^{(\infty)}$ high variance but low bias. (Why? Like which model-free policy estimation techniques?)
- Using intermediate k (say, 20) can give an intermediate amount of bias and variance.

Application: Robot Locomotion

Learning to Walk in 20 Minutes

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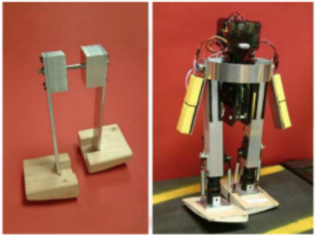

Class Structure

- Last time: Imitation Learning
- **This time: Policy Search**
- Next time: Policy Search Cont.