Lecture 13: Fast Reinforcement Learning [1]

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CS234 Reinforcement Learning

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[1] With a few slides derived from David Silver

Class Structure

- Last time: Fast Learning (Bayesian bandits to MDPs)
- **This time: Fast Learning III (MDPs)**
- Next time: Meta-learning (Guest speaker: Chelsea Finn)

Settings, Frameworks & Approaches

- Over next couple lectures will consider 2 settings, multiple frameworks, and approaches
- Settings: Bandits (single decisions), MDPs
- Frameworks: evaluation criteria for formally assessing the quality of a RL algorithm
- Approaches: Classes of algorithms for achieving particular evaluation criteria in a certain set
- Note: We will see that some approaches can achieve multiple frameworks in multiple settings

Table of Contents

Fast RL in Markov Decision Processes

- Very similar set of frameworks and approaches are relevant for fast learning in reinforcement learning
- Frameworks
  - Regret
  - Bayesian regret
  - Probably approximately correct (PAC)
- Approaches
  - Optimism under uncertainty
  - Probability matching / Thompson sampling
- Framework: Probably approximately correct

PAC RL algo:
input epsilon (how close to optimal we want to be), delta (how often we want this to occur) on all but N steps our alg will select an action where Q*(st,at) >= V*(st) - epsilon with probability at least 1-delta

where N is a polynomial function

In PAC, you're "judged" based on the states that you're in. Regret is more brutal, in that it judges you based on all states, regardless or not you're able to get to those

Model-Based Interval Estimation with Exploration Bonus (MBIE-EB)

(Strehl and Littman, J of Computer & Sciences 2008)

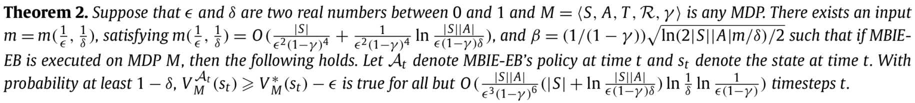
everything we're talking about is for tabular RL

```
1: Given $\epsilon$, $\delta$, $m$
2: $\beta = \frac{1}{1-\gamma}\sqrt{0.5 \ln(2|S||A|m/\delta}$
3: $n_{sas}(s,a,s') = 0 \; s \in S,\; a \in A,\; s' \in S$
4: $rc(s,a) = 0,\; n_{sa}(s,a) = 0,\; \tilde{Q}(s,a) = 1/(1-\gamma) \; \forall \; s \in S,\; a \in A$
5: $t = 0,\; s_t = s_{init}$
6: loop
7:     $a_t = \arg\max_{a \in \mathcal{A}} Q(s_t, a)$
8:     Observe reward $r_t$ and state $s_{t+1}$
9:     $n_{sa}(s_t,a_t) = n(s_t,a_t) + 1,\; n_{sas}(s_t,a_t,s_{t+1}) = n_{sas}(s_t,a_t,s_{t+1}) + 1$
10:    $rc(s_t,a_t) = \frac{rc(s_t,a_t)n_{sa}(s_t,a_t)+r_t}{(n_{sa}(s_t,a_t)+1)}$
11:    $\hat{R}(s,a) = \frac{rc(s_t,a_t)}{n(s_t,a_t)}$ and $\hat{T}(s'|s,a) = \frac{n_{sas}(s_t,a_t,s')}{n_{sa}(s_t,a_t)} \; \forall s' \in S$
12:    while not converged do
13:        $\tilde{Q}(s,a) = \hat{R}(s,a) + \gamma \sum_{s'} \hat{T}(s'|s,a) \max_{a'} \tilde{Q}(s',a') + \frac{\beta}{\sqrt{n_{sa}(s,a)}} \; \forall \; s \in S,\; a \in A$
14:    end while
15: end loop
```

Framework: PAC for MDPs

- For a given ϵ and δ, A RL algorithm $\mathcal{A}$ is PAC if on all but N steps, the action selected by algorithm $\mathcal{A}$ on time step t, a_t, is ϵ-close to the optimal action, where N is a polynomial function of $(|S|, |A|, \gamma, \epsilon, \delta)$
- Is this true for all algorithms?

MBIE-EB is a PAC RL Algorithm



Theorem 2. *Suppose that* ϵ *and* δ *are two real numbers between* 0 *and* 1 *and* $M = \langle S, A, T, \mathcal{R}, \gamma \rangle$ *is any MDP. There exists an input* $m = m(\frac{1}{\epsilon}, \frac{1}{\delta})$*, satisfying* $m(\frac{1}{\epsilon}, \frac{1}{\delta}) = O(\frac{|S|}{\epsilon^2(1-\gamma)^4} + \frac{1}{\epsilon^2(1-\gamma)^4} \ln \frac{|S||A|}{\epsilon(1-\gamma)\delta})$*, and* $\beta = (1/(1-\gamma))\sqrt{\ln(2|S||A|m/\delta)/2}$ *such that if MBIE-EB is executed on MDP* M*, then the following holds. Let* $\mathcal{A}_t$ *denote MBIE-EB's policy at time* t *and* s_t *denote the state at time* t*. With probability at least* $1-\delta$*,* $V_M^{\mathcal{A}_t}(s_t) \geqslant V_M^*(s_t) - \epsilon$ *is true for all but* $O(\frac{|S||A|}{\epsilon^3(1-\gamma)^6}(|S| + \ln \frac{|S||A|}{\epsilon(1-\gamma)\delta}) \ln \frac{1}{\delta} \ln \frac{1}{\epsilon(1-\gamma)})$ *timesteps* t*.*

A Sufficient Set of Conditions to Make a RL Algorithm PAC

- Strehl, A. L., Li, L., Littman, M. L. (2006). Incremental model-based learners with formal learning-time guarantees. In Proceedings of the Twenty-Second Conference on Uncertainty in Artificial Intelligence (pp. 485-493)

A Sufficient Set of Conditions to Make a RL Algorithm PAC

1. Optimism $\underbrace{Q_t(s,a)}_{\text{computed value should be optimistic with respect to real Q values}} \geq Q^*(s,a) - \epsilon \quad \forall s,a,t$

2. Accuracy $V_t(s) - V^{\pi_t}_{\underbrace{\mathcal{M}_t}_{\text{will define further}}}(s) \leq \epsilon$

will define further. MDP related to true MDP & MDP defined in MBIE-EB

3) Bounded learning complexity:
- total # of updates to Q
- # times visit an "unknown" (s,a) pair

bounded by $\zeta(\epsilon, \delta)$

A RL alg that is executed on <u>any</u> MDP M will follow a $O(\epsilon)$-optimal policy on all steps but on

$O\left(\frac{\zeta(\epsilon,\delta)}{\epsilon(1-\gamma)^2} \ln\frac{1}{\delta} \ln\frac{1}{\epsilon(1-\gamma)}\right)$ timesteps w/prob $\geq 1-2\delta$

Proof MBIE-EB is optimistic

MBIE-EB uses value iteration

1st compute a Bellman backup with empirical & V^*

- for some (s,a) consider it has been experienced $n(s,a) < m$

let $X_i = r_i + \gamma V^*(s_i)$ where r_i & s_i are the ith reward & next state reached on the ith time took action a in state s

note $E[X_i] = Q^*(s,a)$

$0 \le X_i \le \frac{1}{1-\gamma} \quad \forall i = 1 \dots n(s,a)$

one can use Hoeffding * (really should do Martingale) inequalities

$\Pr\left[E[X_i] - \frac{1}{n(s,a)} \sum_{i=1}^{n(s,a)} X_i \ge \beta / \sqrt{n(s,a)}\right] \le e^{-2\beta^2(1-\gamma)^2}$

$\le \delta / 2|S||A|m$

plug in $\beta = \frac{1}{1-\gamma}\sqrt{\frac{\ln(2|S||A|m/\delta)}{2}}$

after $n(s,a) = m$ stop changing $\hat{R}(s,a)$ & $\hat{T}(s'|s,a)$

So using union bound

$\hat{R}(s,a) + \gamma \sum_{s'} \hat{T}(s'|s,a) V^*(s') - Q^*(s,a) \ge -\beta / \sqrt{n_t(s,a)}$ $\forall t \; \forall s,a$ w/prob $\ge 1 - \delta/2$

now do proof by induction

do on board

How Does MBIE-EB Fulfill these Conditions?

big idea in many analyses of optimism under uncertainty

known set

(s,a) pairs where $n(s,a)$ is large

intuitively should have good empirical estimate of these pairs

s, a else

Table of Contents

Refresher: Bayesian Bandits

we assume we have some parameteric knowledge about how the rewards are distributed

- **Bayesian bandits** exploit prior knowledge of rewards, $p[\mathcal{R}]$
- They compute posterior distribution of rewards $p[\mathcal{R} \mid h_t]$, where $h_t = (a_1, r_1, \ldots, a_{t-1}, r_{t-1})$
- Use posterior to guide exploration
    - Upper confidence bounds (Bayesian UCB)
    - Probability matching (Thompson Sampling)
- Better performance if prior knowledge is accurate

recall that probability matching refers to us selecting actions based on the probability that they're optimal

Refresher: Bernoulli Bandits

- Consider a bandit problem where the reward of an arm is a binary outcome $\{0, 1\}$ sampled from a Bernoulli with parameter θ
  - E.g. Advertisement click through rate, patient treatment succeeds/fails, ...
- The Beta distribution $Beta(\alpha, \beta)$ is conjugate for the Bernoulli distribution

alpha is the count we see a reward of +1

beta is the count we see a reward of +0

$$p(\theta|\alpha, \beta) = \theta^{\alpha-1}(1-\theta)^{\beta-1}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

where $\Gamma(x)$ is the Gamma family.

- Assume the prior over θ is a $Beta(\alpha, \beta)$ as above
- Then after observed a reward $r \in \{0, 1\}$ then updated posterior over θ is $Beta(r + \alpha, 1 - r + \beta)$

Thompson Sampling for Bandits

```
1: Initialize prior over each arm $a$, $p(\mathcal{R}_a)$
2: loop
3:     For each arm $a$ sample a reward distribution $\mathcal{R}_a$ from posterior
4:     Compute action-value function $Q(a) = \mathbb{E}[\mathcal{R}_a]$
5:     $a_t = \arg\max_{a \in \mathcal{A}} Q(a)$
6:     Observe reward $r$
7:     Update posterior $p(\mathcal{R}_a | r)$ using Bayes law
8: end loop
```

Bayesian Model-Based RL

R = very similar to bandits
T bit different
T multinomial
Dirichlet
prior
conjugate

- Maintain posterior distribution over **MDP** models
- Estimate both transition and rewards, $p[\mathcal{P}, \mathcal{R} \mid h_t]$, where $h_t = (s_1, a_1, r_1, \ldots, s_t)$ is the history
- Use posterior to guide exploration
  - Upper confidence bounds (Bayesian UCB)
  - Probability matching (Thompson sampling)

Thompson Sampling: Model-Based RL

- Thompson sampling implements probability matching

$$\begin{aligned}\pi(s, a \mid h_t) &= \mathbb{P}[Q(s,a) > Q(s,a'), \forall a' \neq a \mid h_t] \\ &= \mathbb{E}_{\mathcal{P},\mathcal{R}|h_t}\left[\mathbb{1}(a = \arg\max_{a\in\mathcal{A}} Q(s,a))\right]\end{aligned}$$

- Use Bayes law to compute posterior distribution $p[\mathcal{P}, \mathcal{R} \mid h_t]$
- **Sample** an MDP $\mathcal{P}, \mathcal{R}$ from posterior
- Solve MDP using favorite planning algorithm to get $Q^*(s, a)$
- Select optimal action for sample MDP, $a_t = \arg\max_{a\in\mathcal{A}} Q^*(s_t, a)$

Thompson Sampling for MDPs

Tabular (Finite S & A)

```
1: Initialize prior over the dynamics and reward models for each $(s, a)$, $p(\mathcal{R}_{as})$, $p(\mathcal{T}(s'|s, a))$
2: Initialize state $s_0$
3: loop
4:     Sample a MDP $\mathcal{M}$: for each $(s, a)$ pair, sample a dynamics model $\mathcal{T}(s'|s, a)$ and reward model $\mathcal{R}(s, a)$
5:     Compute $Q^*_{\mathcal{M}}$, optimal value for MDP $\mathcal{M}$
6:     $a_t = \arg\max_{a \in \mathcal{A}} Q^*_{\mathcal{M}}(s_t, a)$
7:     Observe reward $r_t$ and next state $s_{t+1}$
8:     Update posterior $p(\mathcal{R}_{a_t s_t}|r_t)$, $p(\mathcal{T}(s'|s_t, a_t)|s_{t+1})$ using Bayes rule
9:     $t = t + 1$
10: end loop
```

we have one distribution for every (s,a) pair for both the rewards and dynamics model

and now you have a MDP

now that we have the MDP, we can compute the optimal value for it.

posterior sampling

Ben Van Roy's group

Table of Contents

Generalization and Strategic Exploration

everything we've seen so far is for simple, finite states and actions. This is polynomial in the state and action space! So for large state spaces, we need to adapt it to work for other cases.

- Active area of ongoing research: combine generalization & strategic exploration
- Many approaches are grounded by principles outlined here
  - Optimism under uncertainty
  - Thompson sampling

Generalization and Optimism

- Recall MBIE-EB algorithm for finite state and action domains
- What needs to be modified for continuous / extremely large state and/or action spaces?

Model-Based Interval Estimation with Exploration Bonus (MBIE-EB)

(Strehl and Littman, J of Computer & Sciences 2008)

can't do counts because you may never visit the same state twice

```
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14:    end while
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```

Generalization and Optimism

- Recall MBIE-EB algorithm for finite state and action domains
- What needs to be modified for continuous / extremely large state and/or action spaces?
- Estimating uncertainty
  - Counts of (s,a) and (s,a,s') tuples are not useful if we expect only to encounter any state once
- Computing a policy
  - Model-based planning will fail
- So far, model-free approaches have generally had more success than model-based approaches for extremely large domains
  - Building good transition models to predict pixels is challenging

Recall: Value Function Approximation with Control

- For Q-learning use a TD target $r + \gamma \max_a \hat{Q}(s', a'; w)$ which leverages the max of the current function approximation value

$$\Delta w = \alpha(r(s) + \gamma \max_{a'} \hat{Q}(s', a'; \underset{\text{target}}{\vec{w}}) - \hat{Q}(s, a; w))\nabla_w \hat{Q}(s, a; w)$$

Recall: Value Function Approximation with Control

- For Q-learning use a TD target $r + \gamma \max_a \hat{Q}(s', a'; w)$ which leverages the max of the current function approximation value

$$\Delta w = \alpha(r(s) + \underbrace{r_{bonus}(s,a)} + \gamma \max_{a'} \hat{Q}(s', a'; \underset{\text{target}}{\vec{w}}) - \hat{Q}(s, a; w))\nabla_w \hat{Q}(s, a; w)$$

inspired by MBIE-EB let's just put in a bonus term when we refit our weights in doing our Q-function updates

Recall: Value Function Approximation with Control

- For Q-learning use a TD target $r + \gamma \max_a \hat{Q}(s', a'; w)$ which leverages the max of the current function approximation value

$$\Delta w = \alpha(r(s) + r_{bonus}(s,a) + \gamma \max_{a'} \hat{Q}(s', a'; w) - \hat{Q}(s, a; w))\nabla_w \hat{Q}(s, a; w)$$

- $r_{bonus}(s,a)$ should reflect uncertainty about future reward from (s,a)
- Approaches for deep RL that make an estimate of visits / density of visits include: Bellemare et al. NIPS 2016; Ostrovski et al. ICML 2017; Tang et al. NIPS 2017
- Note: bonus terms are computed at time of visit. During episodic replay can become outdated.

convert to hashes; estimate density model; a few options for the bonus

Benefits of Strategic Exploration: Montezuma's revenge

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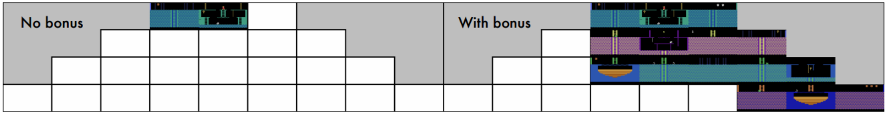

Figure 3: "Known world" of a DQN agent trained for 50 million frames with (**right**) and without (**left**) count-based exploration bonuses, in MONTEZUMA'S REVENGE.

- Bellemare et al. "Unifying Count-Based Exploration and Intrinsic Motivation"
- Enormously better than standard DQN with ϵ-greedy approach

Generalization and Strategic Exploration: Thompson Sampling

- Leveraging Bayesian perspective has also inspired some approaches
- One approach: Thompson sampling over representation & parameters (Mandel, Liu, Brunskill, Popovic IJCAI 2016)

Generalization and Strategic Exploration: Thompson Sampling

- Leveraging Bayesian perspective has also inspired some approaches
- One approach: Thompson sampling over representation & parameters (Mandel, Liu, Brunskill, Popovic IJCAI 2016)
- For scaling up to very large domains, again useful to consider model-free approaches
- Non-trivial: would like to be able to sample from a posterior over possible Q^*
- Bootstrapped DQN (Osband et al. NIPS 2016)
    - Train C DQN agents using bootstrapped samples
    - When acting, choose action with highest Q value over any of the C agents
    - Some performance gain, not as effective as reward bonus approaches

Generalization and Strategic Exploration: Thompson Sampling

- Leveraging Bayesian perspective has also inspired some approaches
- One approach: Thompson sampling over representation & parameters (Mandel, Liu, Brunskill, Popovic IJCAI 2016)
- For scaling up to very large domains, again useful to consider model-free approaches
- Non-trivial: would like to be able to sample from a posterior over possible Q^*
- Bootstrapped DQN (Osband et al. NIPS 2016)
- Efficient Exploration through Bayesian Deep Q-Networks (Azizzadenesheli, Anandkumar, NeurIPS workshop 2017)
  - Use deep neural network
  - On last layer use Bayesian linear regression
  - Be optimistic with respect to the resulting posterior
  - Very simple, empirically much better than just doing linear regression on last layer or bootstrapped DQN, not as good as reward bonuses in some cases

Table of Contents

Summary: What You Are Expected to Know

- Define the tension of exploration and exploitation in RL and why this does not arise in supervised or unsupervised learning
- Be able to define and compare different criteria for "good" performance (empirical, convergence, asymptotic, regret, PAC)
- Be able to map algorithms discussed in detail in class to the performance criteria they satisfy

Class Structure

- Last time: Fast RL
- **This time: Fast RL**
- Next time: Meta-learning