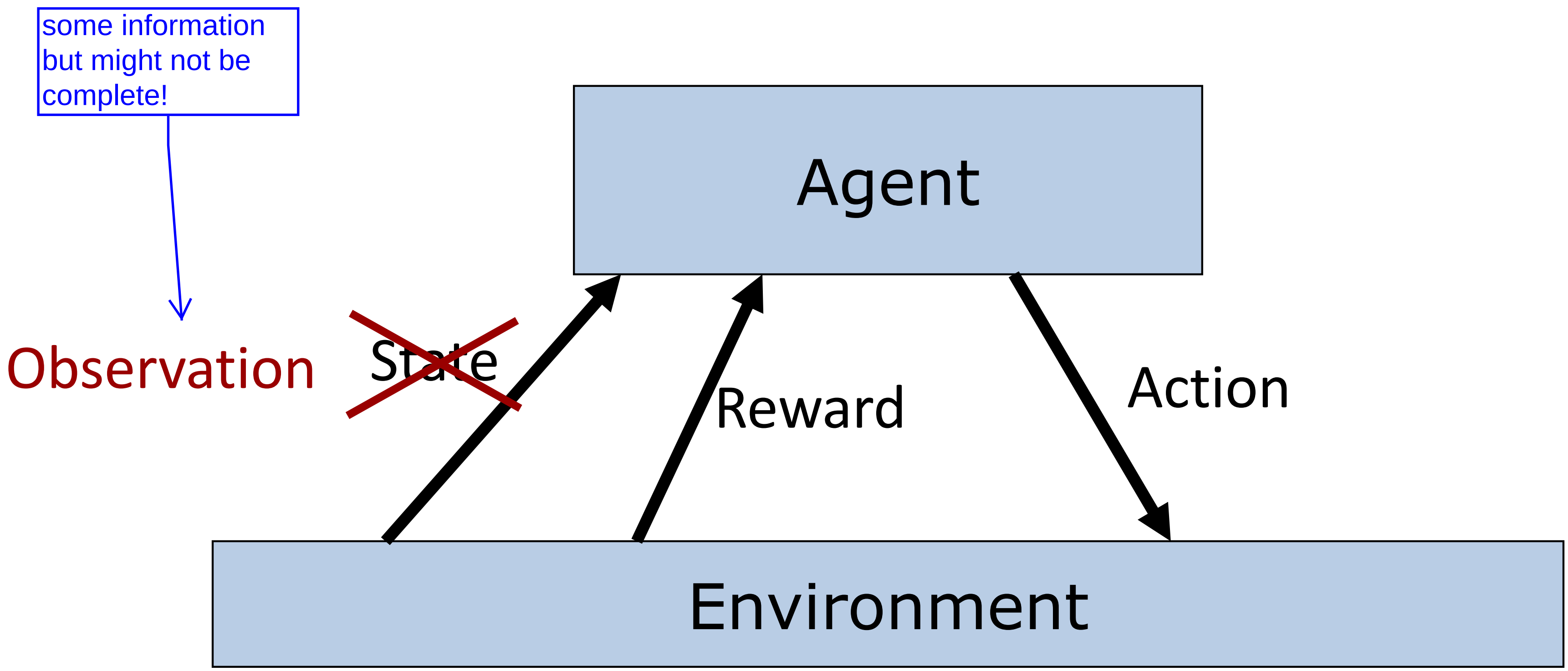
CS885 Reinforcement Learning
Lecture 11a: June 6, 2018

Hidden Markov Models

[RusNor] Sec. 15.3 [SutBar] Sec. 17.3

Reinforcement Learning Problem



Goal: Learn to choose actions that maximize rewards

Markov Process

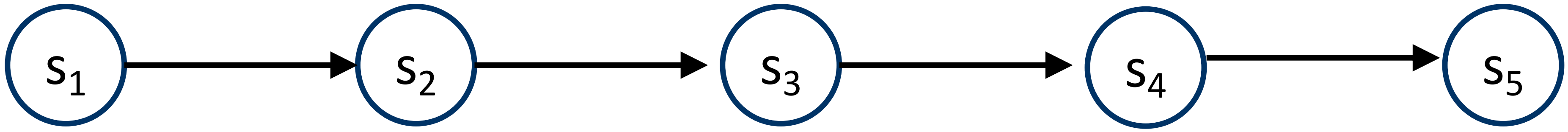
- Assumptions:
  - (first-order) Markovian:

$$\Pr(s_t|s_{t-1},\dots,s_0) = \Pr(s_t|s_{t-1})$$

  - Stationary:

$$\Pr(s_t|s_{t-1}) = \Pr(s_{t+1}|s_t)\ \forall t$$

it doesn't matter what time step we're at, the MDP is the same



Hidden Markov Model

In reality, it's very rare you get the real exact state

- Assumptions:
  - (first-order) Markovian:

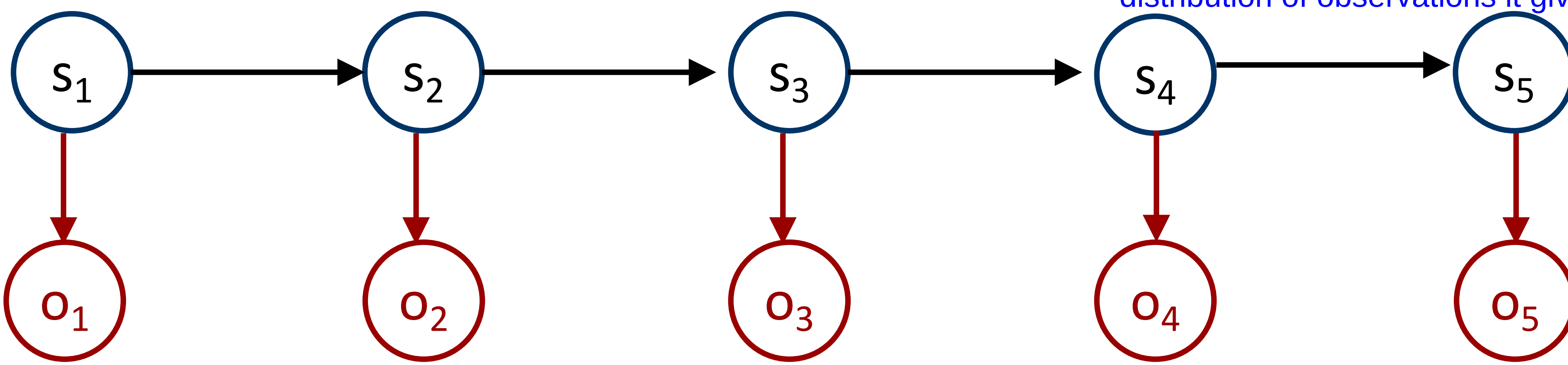
$$\Pr(s_t|s_{t-1},\ldots,s_0) = \Pr(s_t|s_{t-1})$$

  - Stationary:

$$\Pr(s_t|s_{t-1}) = \Pr(s_{t+1}|s_t)\ \forall t$$

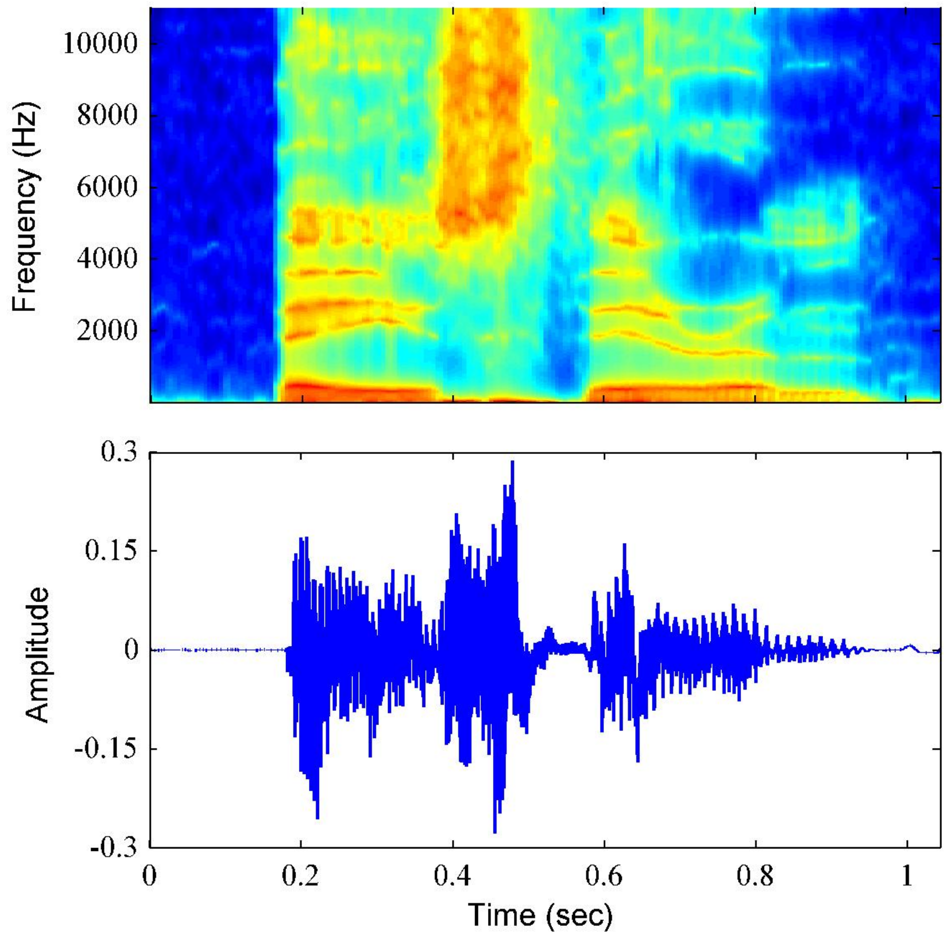
$$\Pr(o_t|s_t) = \Pr(o_{t+1}|s_{t+1})\ \forall t$$

we can model observations as the probability you receive a given observation given the actual state. So for a given state, there's a distribution of observations it gives



Speech Recognition

observation



| b | ey | z | th | ih | er | em |

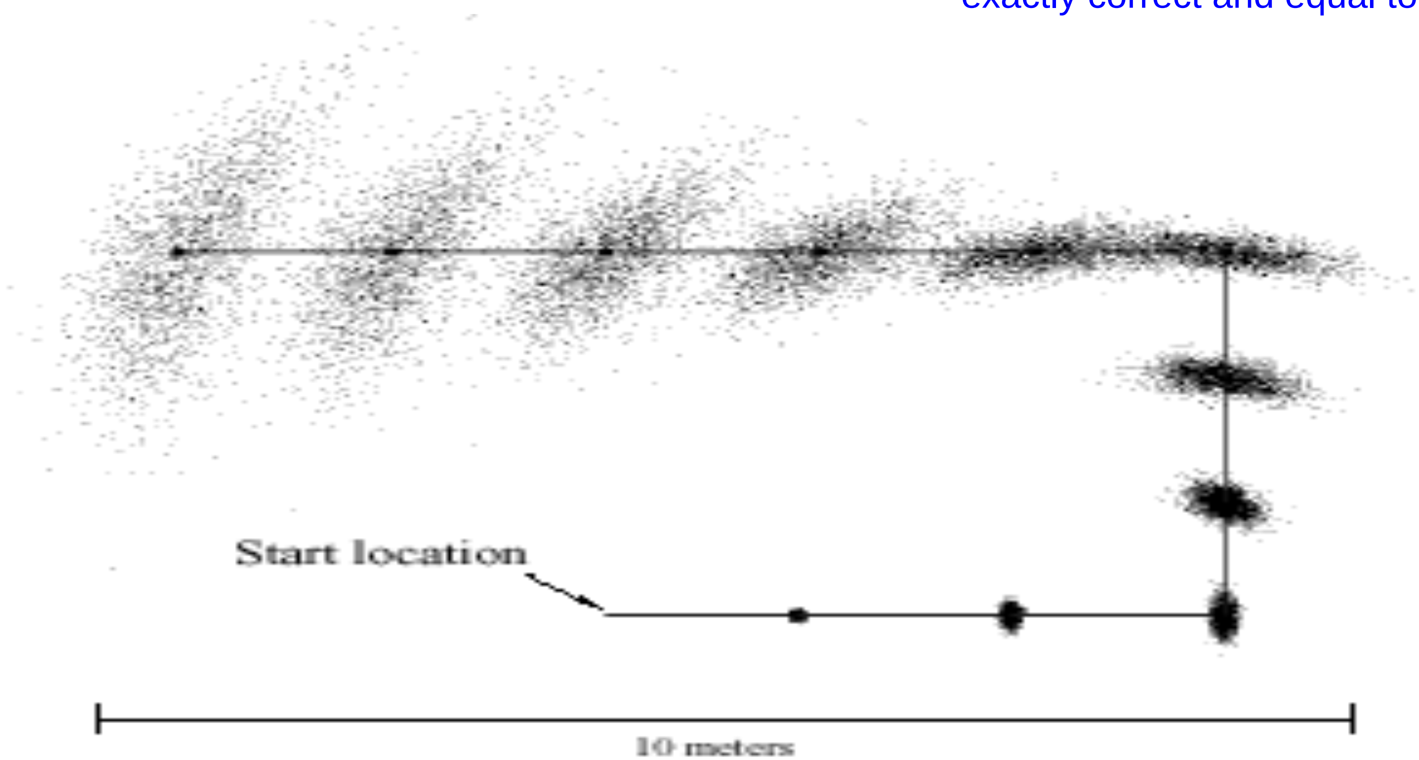
| Bayes' | Theorem |

we can think of this as the hidden state

Mobile Robot Localisation

- Example of a Markov process

even though we give it the same commands, there's stochasticity in where the robot actually goes. That has to do with the model not being exactly correct and equal to reality.



- Problem: uncertainty grows over time…

Mobile Robot Localisation

state is hidden.
we're not sure what
it is

- Hidden Markov Model:

  $\boldsymbol{s}$: coordinates of the robot on a map

  $\boldsymbol{o}$: distances to surrounding obstacles (measured by laser range finders or sonars)

  $\mathbf{Pr}(\boldsymbol{s}_t|\boldsymbol{s}_{t-1})$: movement of the robot with uncertainty

  $\mathbf{Pr}(\boldsymbol{o}_t|\boldsymbol{s}_t)$: uncertainty in the measurements provided by laser range finders and sonars these lasers are noisy as well

- **Localisation:** $\Pr(s_t|o_t, \dots, o_1)$?

what's the probability we're in
location s_t given all the
observations so far

Inference in temporal models

- Four common tasks:
  - **Monitoring:** $\Pr(s_t|o_{1..t})$
  - **Prediction:** $\Pr(s_{t+k}|o_{1..t})$
  - Hindsight: $Pr(s_k|o_{1..t})$ where $k < t$
  - Most likely explanation:
    $$argmax_{s_1,\ldots,s_t} \Pr(s_{1..t}|o_{1..t})$$

- What algorithms should we use?

we're trying to estimate the hidden state given previous observations

Monitoring

- $\Pr(s_t|o_{1..t})$: distribution over current state given observations
- Examples: robot localisation, patient monitoring
- Recursive computation:

$\Pr(s_t|o_{1..t}) \propto \Pr(o_t|s_t, o_{1..t-1})\Pr(s_t|o_{1..t-1})$ by Bayes' thm

$= \Pr(o_t|s_t)\Pr(s_t|o_{1..t-1})$ by conditional independence

$= \Pr(o_t|s_t)\sum_{s_{t-1}}\Pr(s_t, s_{t-1}|o_{1..t-1})$ by marginalization

$= \Pr(o_t|s_t)\sum_{s_{t-1}}\Pr(s_t|s_{t-1}, o_{1..t-1})\Pr(s_{t-1}|o_{1..t-1})$

by chain rule

$= \Pr(o_t|s_t)\sum_{s_{t-1}}\Pr(s_t|s_{t-1})\Pr(s_{t-1}|o_{1..t-1})$ by cond. ind.

estimate in a recursive fashion

Forward Algorithm

- Compute $\Pr(s_t|o_{1..t})$ by forward computation

$\Pr(s_1|o_1) \propto \Pr(o_1|s_1)\Pr(s_1)$

start with belief about first hidden state

For $i = 2$ to t do

$$\Pr(s_i|o_{1..i}) \propto \Pr(o_i|s_i) \sum_{s_{i-1}} \Pr(s_i|s_{i-1}) \Pr(s_{i-1}|o_{1..i-1})$$

End

belief at every time step given all observations up until now

- Linear complexity in t

Prediction

- $\Pr(s_{t+k}|o_{1..t})$: distribution over future state given observations
- Examples: weather prediction, stock market prediction

- Recursive computation

  $\Pr(s_{t+k}|o_{1..t}) = \sum_{s_{t+k-1}} \Pr(s_{t+k}, s_{t+k-1}|o_{1..t})$ by marginalization

  $= \sum_{s_{t+k-1}} \Pr(s_{t+k}|s_{t+k-1}, o_{1..t}) \Pr(s_{t+k-1}|o_{1..t})$ by chain rule

  $= \sum_{s_{t+k-1}} \Pr(s_{t+k}|s_{t+k-1}) \Pr(s_{t+k-1}|o_{1..t})$ by cond. ind.

Forward Algorithm

1. Compute $\Pr(s_t|o_{1..t})$ by forward computation

   $\Pr(s_1|o_1) \propto \Pr(o_1|s_1)\Pr(s_1)$
   For $i = 1$ to t do

   $$\Pr(s_i|o_{1..i}) \propto \Pr(o_i|s_i) \sum_{y_{i-1}} \Pr(s_i|s_{i-1}) \Pr(s_{i-1}|o_{1..i-1})$$

   End

2. Compute $\Pr(s_{t+k}|o_{1..t})$ by forward computation

   For $j = 1$ to k do

   $$\Pr\left(s_{t+j}\middle|o_{1..t}\right) = \sum_{s_{t+j-1}} \Pr\left(s_{t+j}\middle|s_{t+j-1}\right) \Pr(s_{t+j-1}|o_{1..t})$$

   End

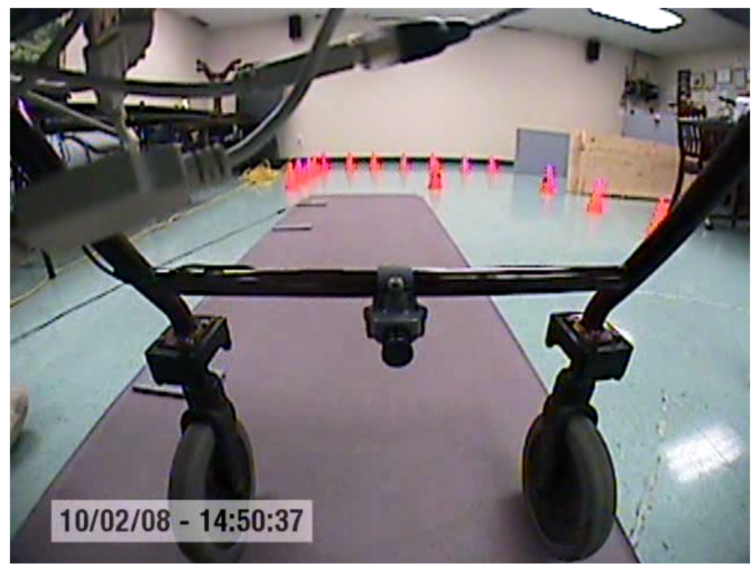
- Linear complexity in $t + k$

Case Study: Activity Recognition

- Task: infer activities performed by a user of a smart walker
  - Inputs: sensor measurements
  - Output: activity

Backward view

Forward view



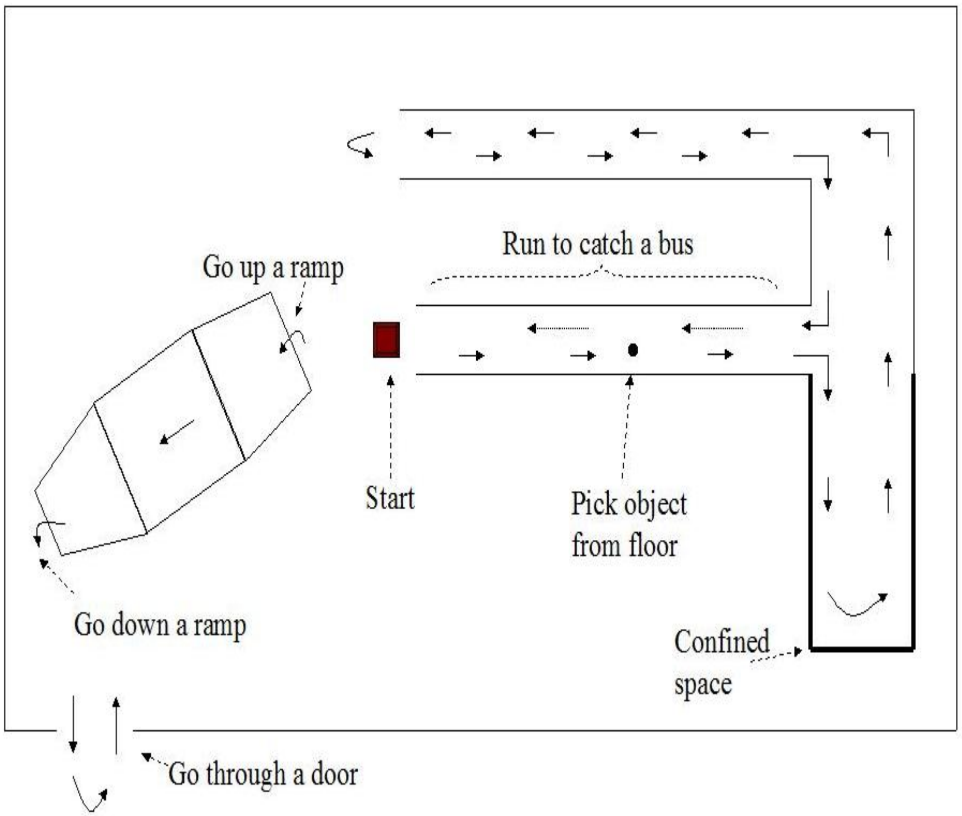
Inputs: Raw Sensor Data

- 8 channels:
  - Forward acceleration
  - Lateral acceleration
  - Vertical acceleration
  - Load on left rear wheel
  - Load on right rear wheel
  - Load on left front wheel
  - Load on right front wheel
  - Wheel rotation counts (speed)

- Data recorded at 50 Hz and digitized (16 bits)

Data Collection

- 8 walker users at Winston Park (84-97 years old)
- 12 older adults (80-89 years old) in the Kitchener-Waterloo area who do not use walkers



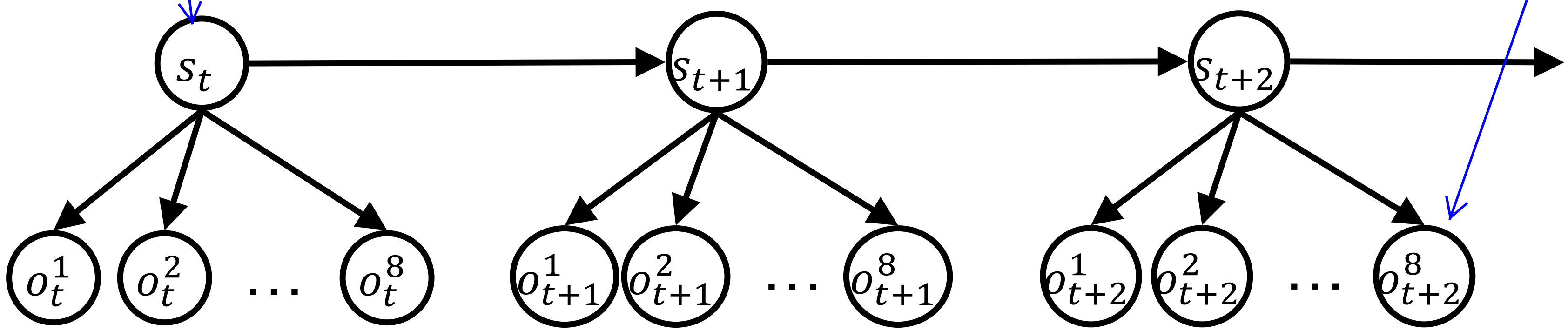
Output: Activities

- Not Touching Walker (NTW)
- Standing (ST)
- Walking Forward (WF)
- Turning Left (TL)
- Turning Right (TR)
- Walking Backwards (WB)
- Sitting on the Walker (SW)
- Reaching Tasks (RT)
- Up Ramp/Curb (UR/UC)
- Down Ramp/Curb (DR/DC)

the activity

Hidden Markov Model (HMM)

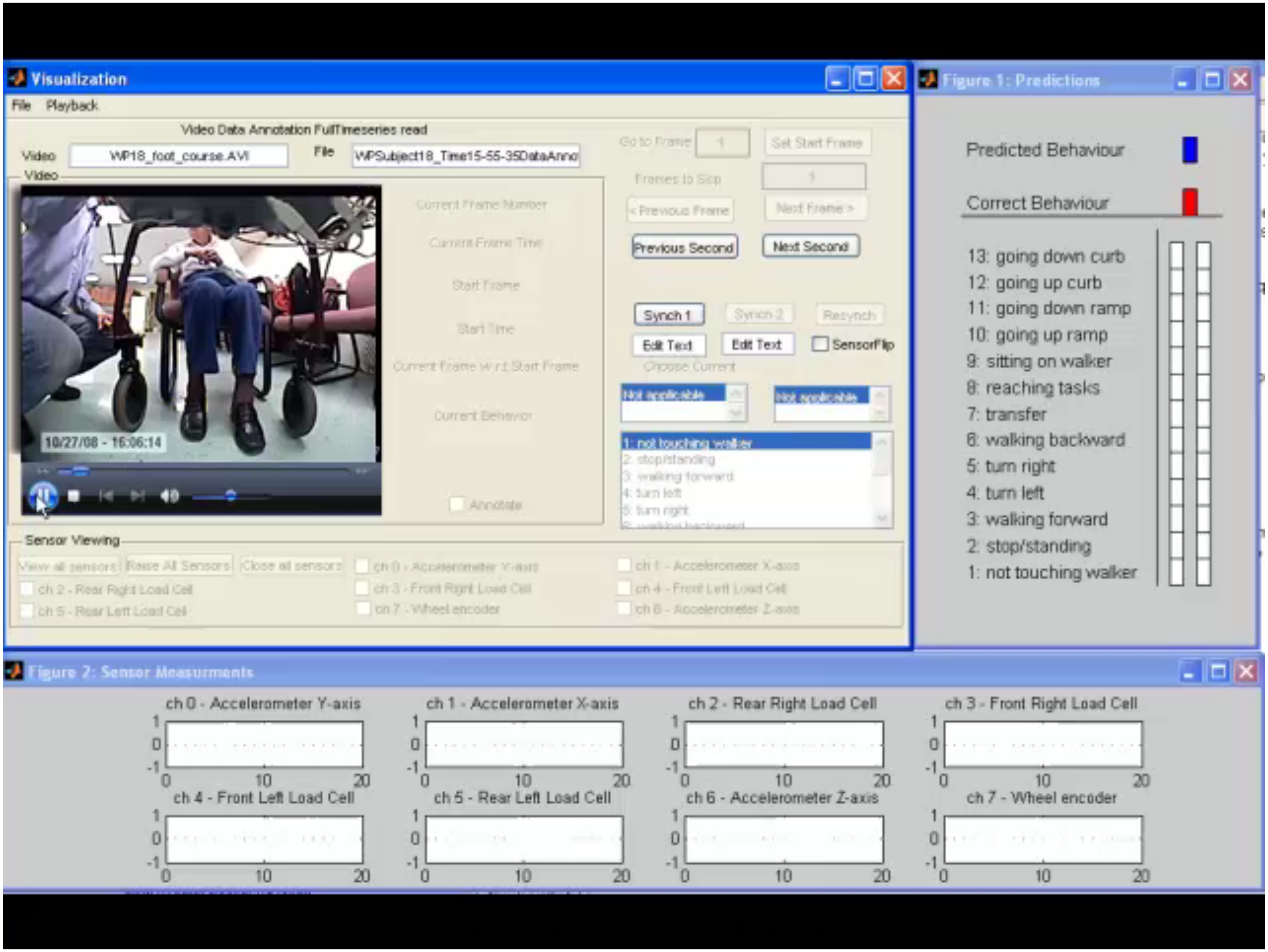
8 sensors represent the observations



- Parameters
  - Initial state distribution: $\psi_{class} = \Pr(s_1 = class)$
  - Transition probabilities: $\theta_{class'|class} = \Pr(s_{t+1} = class'|s_t = class)$
  - Observation probabilities: $\phi^i_{val|class} = \Pr(o^i_t = val|o_t = class)$

    or $N(val|\mu^i_{class}, \sigma^i_{class}) = \Pr(o^i_t = val|o_t = class)$
- Maximum likelihood:
  - Supervised: $\psi^*, \theta^*, \phi^* = argmax_{\psi,\theta,\phi} \Pr(s_{1:T}, o_{1:T}|\psi, \theta, \phi)$
  - Unsupervised: $\psi^*, \theta^*, \phi^* = argmax_{\psi,\theta,\phi} \Pr(o_{1:T}|\psi, \theta, \phi)$

Demo


Visualization
File Playback
Video Data Annotation FullTimeseries read
Video WP18_foot_course.AVI
File WPSubject18_Time15-55-35DataAnno
10/27/08 - 16:06:14
Current Frame Number
Current Frame Time
Start Frame
Start Time
Current Frame w.r.t Start Frame
Current Behavior
Annotate
Go to Frame
Set Start Frame
Frames to Skip
< Previous Frame
Next Frame >
Previous Second
Next Second
Synch 1
Synch 2
Resynch
Edit Text
Edit Text
SensorFlip
Choose Current
Not applicable
Not applicable
1: not touching walker
2: stop/standing
3: walking forward
4: turn left
5: turn right
Sensor Viewing
View all sensors
Raise All Sensors
Close all sensors
ch 0 - Accelerometer Y-axis
ch 1 - Accelerometer X-axis
ch 2 - Rear Right Load Cell
ch 3 - Front Right Load Cell
ch 4 - Front Left Load Cell
ch 5 - Rear Left Load Cell
ch 7 - Wheel encoder
ch 6 - Accelerometer Z-axis
Figure 1: Predictions
Predicted Behaviour
Correct Behaviour
13: going down curb
12: going up curb
11: going down ramp
10: going up ramp
9: sitting on walker
8: reaching tasks
7: transfer
6: walking backward
5: turn right
4: turn left
3: walking forward
2: stop/standing
1: not touching walker
Figure 2: Sensor Measurments
ch 0 - Accelerometer Y-axis
ch 1 - Accelerometer X-axis
ch 2 - Rear Right Load Cell
ch 3 - Front Right Load Cell
ch 4 - Front Left Load Cell
ch 5 - Rear Left Load Cell
ch 6 - Accelerometer Z-axis
ch 7 - Wheel encoder