

The energy-momentum tensor

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1 Peculiarities of energy and momentum

Storage and transport of energy, as well as the motion of objects and the propagation of electromagnetic fields, are constantly on the limelight in today's technologies. Our most precise theory

- (a) Why is internal energy invariant under a change of Newtonian reference frame, even non-inertial, whereas kinetic energy is not?
- (b) Consider a car accelerating on a street, say in the positive- x direction. In the frame of reference where the street is at rest, there is a flow of positive x -momentum – that is, force – from the street to the car, or equivalently a flow of negative x -momentum from the car to the street. In the frame of reference where the car is at rest, the *flow* of momentum is the same.

Not so with energy flow. In the frame of reference where the street is at rest, there is no flow of energy between street and car: the contact forces involved act on matter that has momentarily zero velocity, so their work is zero. In the reference frame where the car is at rest, there is a flow of positive energy from the car to the street: the contact forces do work on the latter.

Why do the flows of momentum and of energy behave differently under this change of frame?

- (c) In fluid mechanics, and more generally in continuum mechanics, the balance of energy can be expressed in very different forms. In one, only the time derivative of the internal energy appears, and kinetic energy is mysteriously absent; this expression of energy balance is moreover invariant with respect changes of frames, even

non-inertial. In another, kinetic energy does appear; this expression is not invariant with respect to changes of frame.

Why do we have this kind of flexibility and possible invariance with the balance of energy, but not with the balance of momentum?