Universal balance laws vs **constitutive relations** (reminder)

Universal balance laws (seven laws)

- Are valid for *every* phenomenon in any theory
- Relate present & *future* values of the *same* quantity
- Do *not* let quantities “talk” with one another

examples

$$N(t_1) = N(t_0) + \int_{t_0}^{t_1} J(t)\,\mathrm{d}t + \int_{t_0}^{t_1} A(t)\,\mathrm{d}t$$

$$\boldsymbol{P}(t_1) = \boldsymbol{P}(t_0) + \int_{t_0}^{t_1} \boldsymbol{F}(t)\,\mathrm{d}t + \int_{t_0}^{t_1} \boldsymbol{G}(t)\,\mathrm{d}t$$

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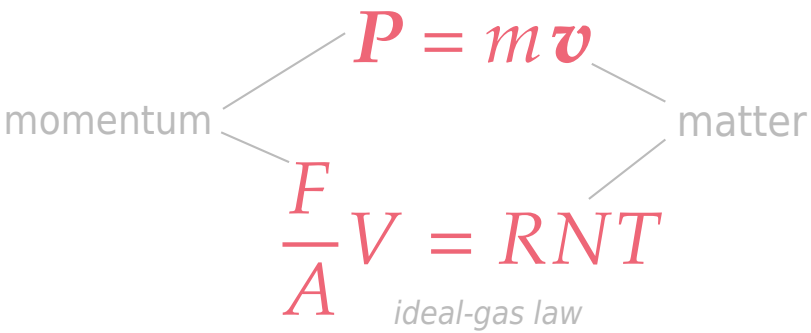
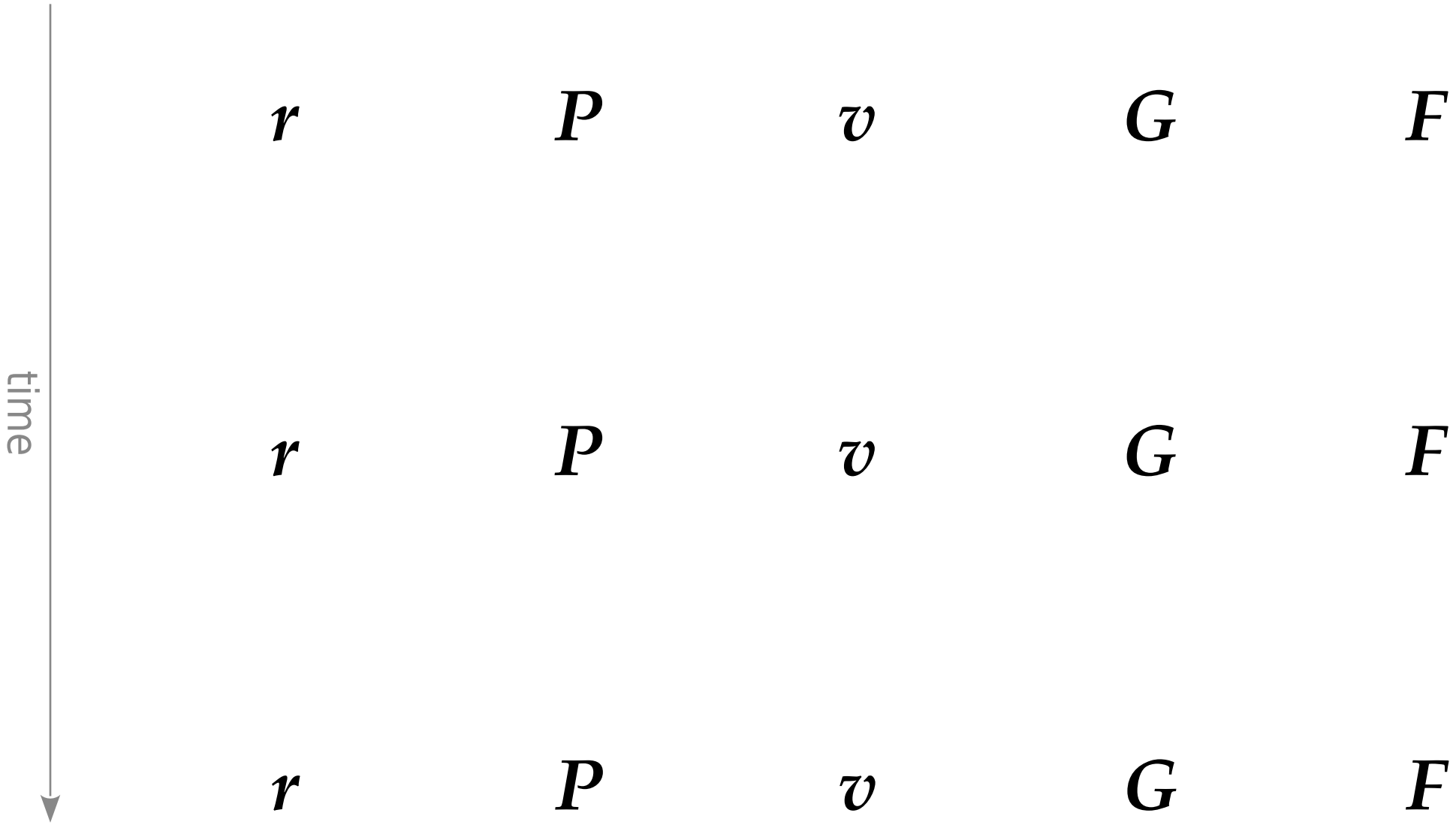
examples



Constitutive relations (so many!)

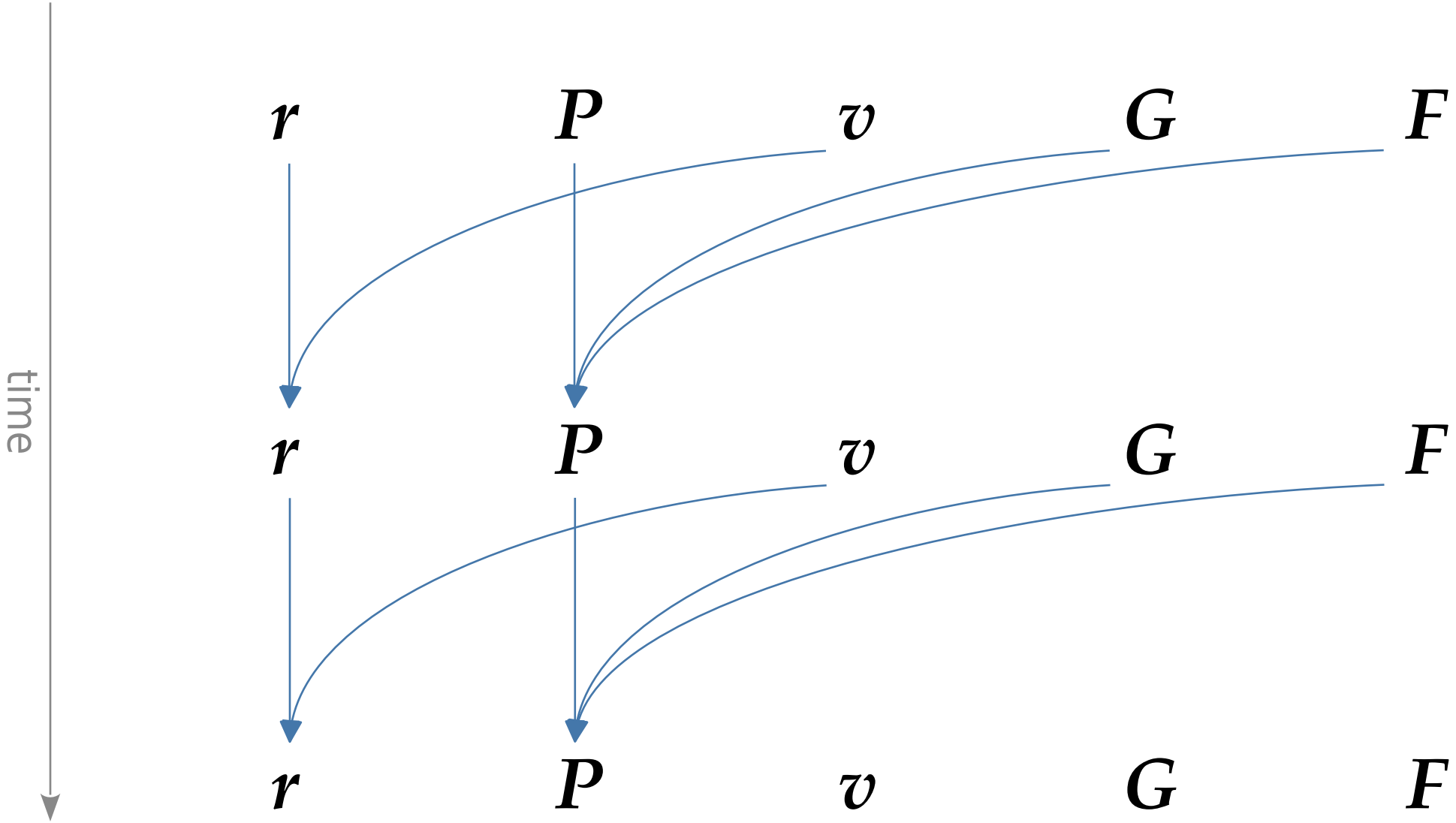
- Are only valid in *specific* phenomena or theories
- Relate *present* values of *different* quantities
- They allow quantities to “talk” with one another


r P v G F
r P v G F
r P v G F
time

Universal balance laws

$$\boldsymbol{r}(t+\Delta t) \approx \boldsymbol{r}(t) + \boldsymbol{v}(t)\,\Delta t$$

$$\boldsymbol{P}(t+\Delta t) \approx \boldsymbol{P}(t) + [\boldsymbol{F}(t) + \boldsymbol{G}(t)]\,\Delta t$$



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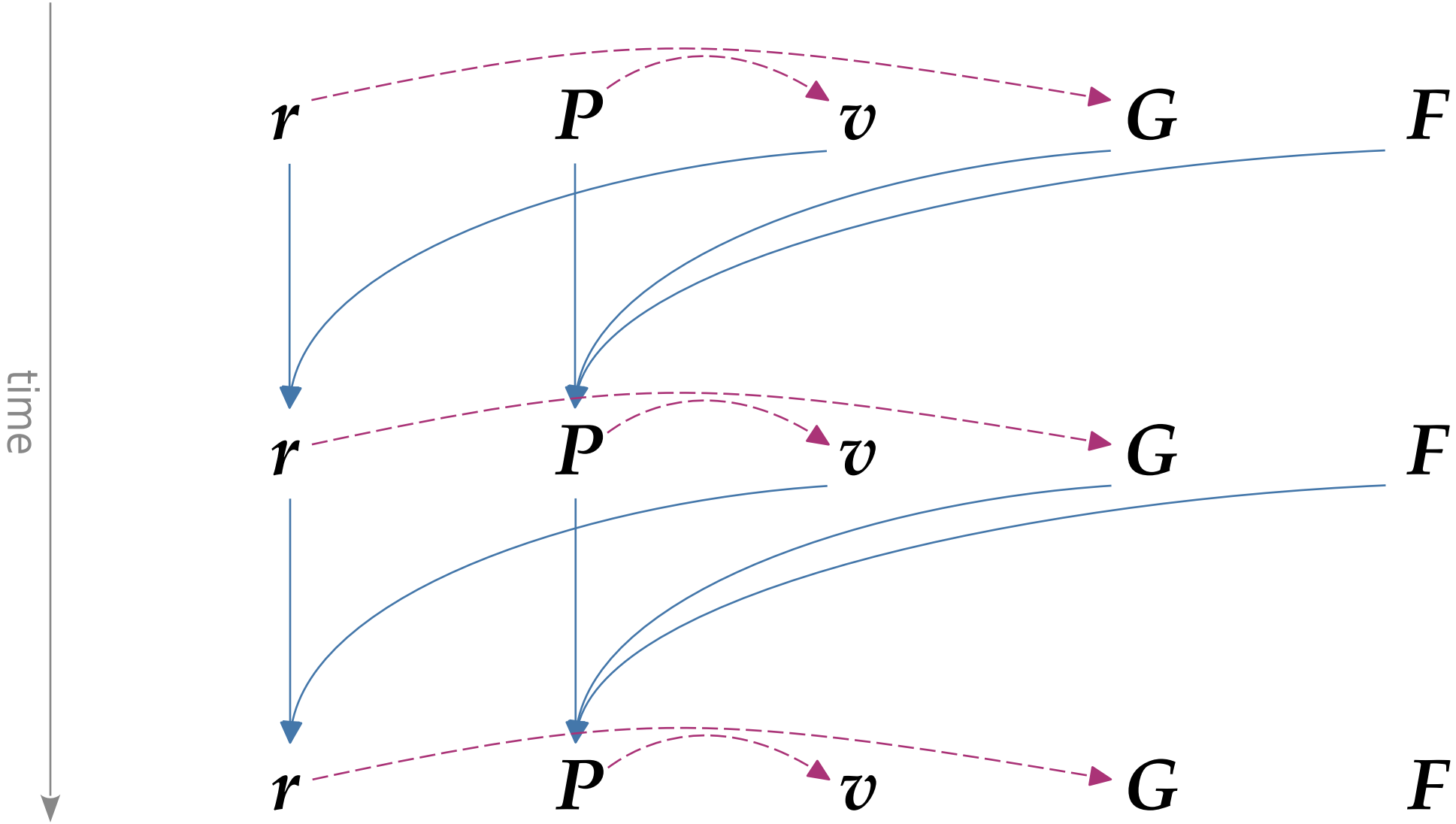
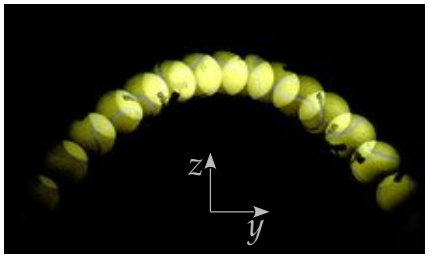
Constitutive equations

$$\boldsymbol{v}(t) = \boldsymbol{P}(t)/m$$

$$\boldsymbol{G} = [0\ ,\ 0\ ,\ -m\,g]$$



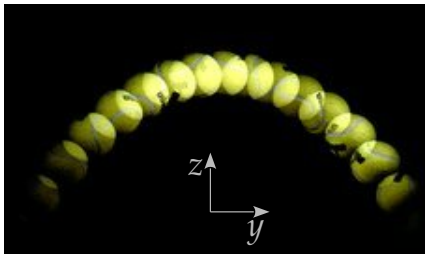
Simulation: flight of tennis ball in 2D



Which physical laws we use:

Script:

Simulation: flight of tennis ball in 2D



Which physical laws we use:

Universal balances

$$y(t+\Delta t) \approx y(t) + v_y(t)\,\Delta t$$

$$z(t+\Delta t) \approx z(t) + v_z(t)\,\Delta t$$

$$P_y(t+\Delta t) \approx P_y(t) + [F_y(t) + G_y(t)]\,\Delta t$$

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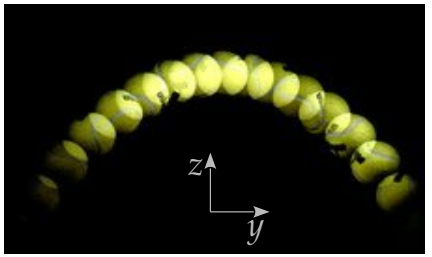

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Script:

$$y \leftarrow y + v_y\,\Delta t$$

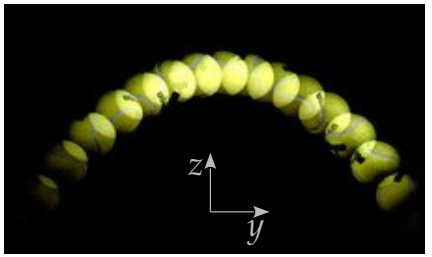
$$z \leftarrow z + v_z\,\Delta t$$

$$P_y \leftarrow P_y + (F_y + G_y)\,\Delta t$$

$$P_z \leftarrow P_z + (F_z + G_z)\,\Delta t$$

step forward in time

Simulation: flight of tennis ball in 2D



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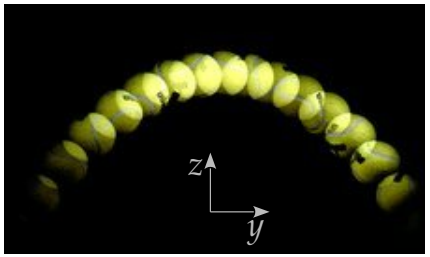

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Constitutive relations

$$v_y(t) = P_y(t)/m$$

$$v_z(t) = P_z(t)/m$$

$$G_y(t) = 0$$

$$G_z(t) = -m\,g$$

Script:

step forward in time

$$y \leftarrow y + v_y\,\Delta t$$

$$z \leftarrow z + v_z\,\Delta t$$

$$P_y \leftarrow P_y + (F_y + G_y)\,\Delta t$$

$$P_z \leftarrow P_z + (F_z + G_z)\,\Delta t$$

$$t \leftarrow t + \Delta t$$

same-time

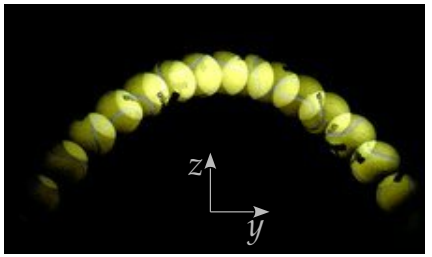
$$v_y \leftarrow P_y/m$$

$$v_z \leftarrow P_z/m$$

$$G_y \leftarrow 0$$

$$G_z \leftarrow -m\,g$$

Simulation: flight of tennis ball in 2D



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$$v_y(t) = P_y(t)/m$$

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$$G_z(t) = -m\,g$$

Boundary conditions

$$F_y(t) = 0$$

$$F_z(t) = 0$$

Script:

$F_y \leftarrow 0$

$F_z \leftarrow 0$

step forward in time

$y \leftarrow y + v_y\,\Delta t$

$z \leftarrow z + v_z\,\Delta t$

$P_y \leftarrow P_y + (F_y + G_y)\,\Delta t$

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$v_y \leftarrow P_y/m$

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