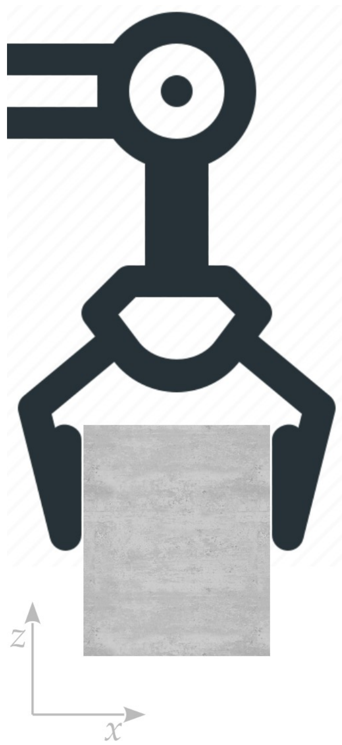
Example exam questions

Deloppgaver (a), (b), etc teller lik
Explain your reasoning, that's what gives you points. A correct numerical answer without explanation of how it is obtained does not give points. A wrong numerical answer but with a correct main explanation does give you points.

Oppgave 1



A robot arm must lift, vertically, a block of concrete having mass $m = 100\,\mathrm{kg}$ (see figure). Take the block itself as a control volume, and use coordinates (x, z) as illustrated; $z = 0\,\mathrm{m}$ corresponds to the floor. Assume that there is no atmosphere. The gravitational acceleration is $g = 9.8\,\mathrm{N/kg}$.

(a)

i. Calculate the **total** force (momentum flux) that the gripper (the robot's "hand") must exert on the block in order to lift it with a constant upward speed.

   Express this total force as a vector with x- and z-components.

   Which law do you use to calculate the total force? which constitutive relations?

ii. The left and right fingers of the gripper exert forces having these unknown components:

$$\begin{array}{cc} \text{left} & \text{right} \\ \begin{bmatrix} A \\ \mu A \end{bmatrix} & \begin{bmatrix} -B \\ \mu B \end{bmatrix} \end{array}$$

   with $A > 0$ and $B > 0$, where $\mu = 0.21$ is called sliding-friction coefficient.

   Find A and B; don't forget the physical units.

   Which principle about fluxes and volume contents are you using, in order to find these two forces from the total one?

iii. What is the **magnitude** of the force exerted by the **left** finger of the gripper?

(b)

The block is being lifted at a constant upward speed of 0.01 m/s. When the bottom of the block of concrete is at $z = 1$ m from the ground, the gripper suddenly breaks and does not exert any force anymore.

Find the magnitude of the velocity $\boldsymbol{v}$ that the block possesses right before it hits the ground:

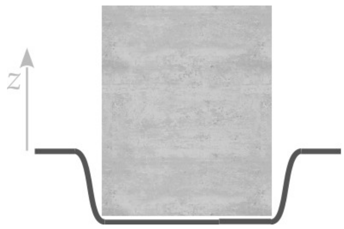
i. Find the magnitude of $\boldsymbol{v}$ using the balance of energy. Assume that the internal energy of the block is constant in time. *(Warning: remember that the block had an upward velocity while it was lifted!)*

   Explain how you use the energy balance: what is the energy content? fluxes? supply?

ii. Using *only* the balance of energy (and no intuitive reasoning), can you find not only the magnitude of $\boldsymbol{v}$, but also its x- and z-components? Explain your reasoning about why this is or is not possible. *[This question does not have a right or wrong answer; what's important is to show your reasoning.]*

iii. Use the balance of momentum to explain why the x-component of $\boldsymbol{v}$ must be zero. *(Hint: had the initial momentum any x-component? Did the total flux or supply of momentum have any x-component during the fall?)*

(c)



The floor is made of a special material that deforms (see picture) and gently stops the fall of the block.

Disregard the horizontal dimension, and consider a z-coordinate directed upward.

As soon as the block is in contact with the floor, the floor starts exerting a vertical force (momentum flux) F on the block. This force has constitutive equation

$$F(t) = -k\,z(t) - \mu\,v(t)$$

where $z(t) \leq 0$ is the position of the block's bottom and $v(t)$ its velocity, at time t. The velocity is positive if it points upward. The constant k is an elastic constant, and μ is a dampening constant.

i. Find the units of k and those of μ.

ii. Assume that $k = 1500$ and $\mu = 1000$, measured in the units you found. How much is the force $F(t_0)$ (including its sign) at the very moment

t_0 the block hits the floor? *(Hint: you already calculated the velocity at impact in a previous question, and the floor correspond to the zero of the z coordinate.)*

iii. Between the moment t_0 when the block hits the floor, and the moment t_1 when it stops, the integrated flux of momentum given to the block by the floor is

$$\int_{t_0}^{t_1} F(t)\,\mathrm{d}t = +3392.42\,\mathrm{N\,s}$$

Use the balance of momentum in integral form to find the integrated supply of momentum G between t_0 and t_1. *(Hint: How much is the final momentum, when the block has stopped? how much was the initial momentum? Note that you do not need to solve any integrals.)*

iv. The integrated supply of momentum that you just found is, by analytical integration, also equal to

$$\int_{t_0}^{t_1} G(t)\,\mathrm{d}t = -m\,g\,(t_1 - t_0)$$

From this expression and your previous result, find the time $(t_1 - t_0)$ that it took for the floor to completely stop the fall of the block of concrete.

Oppgave 2

We want to find how deep is the "hole" that the block made on the floor (see previous picture). This depth corresponds to the final z position of the block's bottom, once it has stopped.

We can find this by numerically integrating the motion of the block in contact with the floor. The script to do this is shown on the next page. The script is designed to continue as long as the velocity is higher than 0.001 m/s, then it outputs the final depth `z` of the block and the time `t` elapsed.

(a)

i. Explain which balance law is implemented in the script.

ii. Locate the line, in the while-loop, where the balance law is implemented. Explain the meaning of the various terms on this specific line.

Script for numerical simulation of floor deformation

```
1  %%% Numerical time integration of dampened fall
2  %% vertical z coordinate, positive upward
3  %% SI base units used throughout
4  m = 100; % mass
5  z0 = 0; % initial position
6  v0 = -4.4272; % initial velocity
7  k = 1500; % elastic constant
8  h = 1000; % viscosity coefficient
9  g = 9.8; % gravitational acceleration
10 t0 = 0; % initial time
11 dt = 0.01; % time step
12 %% Numerical time integration
13 %% Initialize
14 t = t0;
15 z = z0;
16 v = v0;
17 P = m*v0; % initial momentum
18 G = -m*g; % constant momentum supply
19 %% stop when |v| < 1 mm/s
20 while (abs(v) > 0.001)
21   t = t + dt;
22   F = -k*z - h*v;
23   P = P + (F + G)*dt;
24   z = z + v*dt;
25   v = P/m;
26 end
27 disp(z)
28 disp(t)
```

(b)

i. Which constitutive relations are used in the while-loop?

ii. Add comments to each of the lines 21–25, shortly explaining what each does.

(c)

Assume that the total energy E of the floor consists only of internal energy U (no kinetic or potential energies). Also assume that this energy increases only from the mechanical power $F \cdot v$ coming from the force exerted by the block on the floor; that is the energy flux is adiabatic.

Take the initial energy of the floor to be $U(t_0) = 0\,\mathrm{J}$. Add two lines to the script, in appropriate places:

- one that initializes the value of the internal energy

- one that updates the internal energy, using the balance of energy.

that is, tell between which lines of the script above you would insert your lines, and what the code of your new lines would be.

Oppgave 3

(a)

i. Explain what is the difference between a universal law and a constitutive relation. Give at least two examples of constitutive relations.

ii. Why do we consider Newton's formula $\boldsymbol{P} = \boldsymbol{M v}$ (relating momentum, mass, velocity) to be a constitutive relation, rather than a universal law? *[This question does not have a right or wrong answer; simply tell what you know.]*

(b)

What is the difference between a *conservation* law and a *balance* law? Which quantities do you know that are conserved, and which that are balanced but not conserved?

(c)

You and a friend of yours synchronize your clocks. Your friend goes to live for a while in an orbiting space station. After one year of your time, your friend comes back and you two meet again. You notice that your friend has aged three years instead of one.

How large is the mass M of the hypothetical planet in which you two live? Use the approximate formula

$$\Delta t_{\text{friend}} = \left[1 + \frac{G}{c^2}\frac{M}{R}\right]\Delta t_{\text{you}}$$

where Δt_{friend} is your friend's proper time while in orbit, Δt_{you} is your proper time, $G = 6.7 \times 10^{-11}\,\text{m}^3/(\text{kg}\cdot\text{s}^2)$, $c = 3 \times 10^8\,\text{m/s}$, and assume your planet has radius $R = 7 \times 10^7$ m.