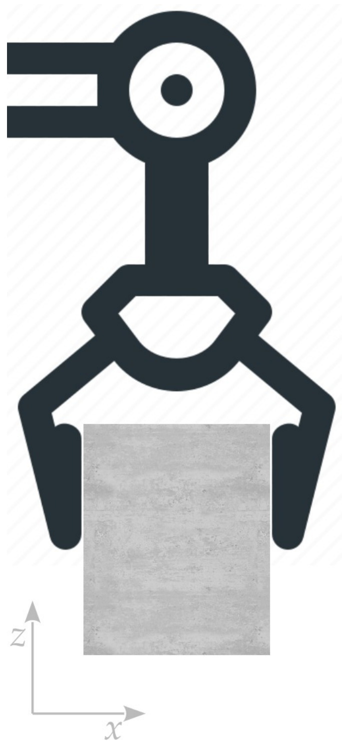
Example exam questions

1



A robot arm must lift, vertically, a block of concrete having mass $m = 100\,\mathrm{kg}$, as illustrated in the figure. Take the block itself as a control volume, and adopt coordinates (x, z) as illustrated; $z = 0\ \mathrm{m}$ corresponds to the floor. Assume that there's no atmosphere. The gravitational acceleration is $g = 9.8\,\mathrm{N/kg}$.

(a)

i. Calculate the **total** force (momentum flux) that the gripper must exert on the block in order to lift it with a constant (upward) speed.

   Express this total force as a vector with x- and z-components.

   Explain which law you use to calculate the total force.

ii. The left and right fingers of the gripper exert forces of this form:

$$\underset{\text{left}}{\begin{bmatrix} A \\ \mu\, A \end{bmatrix}} \qquad \underset{\text{right}}{\begin{bmatrix} -B \\ \mu\, B \end{bmatrix}}$$

   with $A > 0$ and $B > 0$, where $\mu = 0.21$ is called sliding-friction coefficient.

   Find A and B; don't forget the physical units. Which principle about fluxes and volume contents are you using, in order to find these two forces from the total one?

iii. What is the magnitude of the force exerted by the left finger of the gripper?

(b)

The block is being lifted at a constant upward speed of $0.01\ \mathrm{m/s}$. But when the block of concrete is at $z = 1\ \mathrm{m}$ from the ground (distance measured from the bottom of the block to the ground), suddenly the gripper breaks, and doesn't exert any force anymore.

We want to find the magnitude of the velocity $\boldsymbol{v}$ that the block possesses right before it hits the ground.

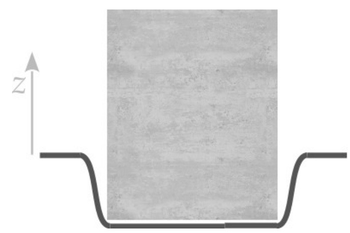
i. Find the magnitude of $\boldsymbol{v}$ using the balance of energy. Assume that the internal energy of the block is constant. *(Warning: remember that the block had an upward velocity while it was lifted!)*

   Explain how you use the energy balance: what is the energy content? fluxes? supply?

ii. Using *only* the balance of energy (and no intuitive reasoning), can you find not only the magnitude of $\boldsymbol{v}$, but also its x- and z-components? Explain your reasoning about why this is or is not possible.

iii. Use the balance of momentum to explain why the x-component of $\boldsymbol{v}$ must be zero. *(Hint: had the initial momentum any x-component? Did the total flux or supply of momentum have any x-component during the fall?)*

(c)



Luckily the floor is made of a special deformable material that dampens and stops the fall of the block, so that it does not get too damaged.

Disregard the horizontal dimension, and consider a z-coordinate directed upward.

The floor exerts a vertical force (momentum flux) F on the block. This force has constitutive equation

$$F = -k\,z - \mu\,v$$

where $z \leq 0$ is the position of the block's bottom and v its velocity (velocity is positive if it is upward). The constant k is an elastic constant, and μ is a dampening coefficient.

i. Find the units of k and those of μ.

ii. Assume that $k = 1500$ and $\mu = 1000$, measured in the units you found. How much is the force F (including its sign) the moment the block hits the floor? *(Hint: you already calculated the velocity at impact in a previous question, and the floor correspond to the zero of the z coordinate.)*

iii. Between the moment t_0 when the block hits the floor, and the moment t_1 when it stops, the integrated flux of momentum given to the block by the floor is

$$\int_{t_0}^{t_1} F(t)\,\mathrm{d}t = +3392.42\,\mathrm{N\,s}$$

   Use the balance of momentum in integral form to find how much is the integrated supply of momentum G between t_0 and t_1. *(Hint: How* *much is the final momentum, when the block has stopped? and how much was the initial momentum?)*

iv. The integrated supply of momentum that you just found is, by integration, also equal to

$$\int_{t_0}^{t_1} G(t)\,\mathrm{d}t = -m\,g\,(t_1 - t_0)$$

Find the time $(t_1 - t_0)$ that it took for the floor to completely stop the fall of the block of concrete.

2

We want to find how much the floor was deformed by stopping the block of concrete, or in other words how deep is the "hole" that the block made on the floor. This depth corresponds to the final z position of the block's bottom, once it has stopped.

We decide to find this numerically, by numerically integrating the motion of the block of concrete as it is slowed down by the deformable floor. A simple script to implement this is shown in the next page. The script continues as long as the velocity is higher than 0.001 m/s (that is, 1 mm/s), then outputs the final depth `z` of the block and the time `t` elapsed.

(a)

i. Explain which balance law is implemented in the script.

ii. Locate the line, in the while-loop, where the balance law is implemented. Explain the meaning of the various terms on this line.

iii. Which constitutive relations are used in the while-loop?

(b)

i. Add comments to each of the lines 21–25, shortly explaining what each does.

ii. Explain why the lines are in the order given. Could any of those lines be moved to another position in the while-loop?

Script for numerical simulation of floor deformation

```
%%% Numerical time integration of dampened fall
%% vertical z coordinate, positive upward
%% SI base units used throughout
m = 100; % mass
z0 = 0; % initial position
v0 = -4.4272; % initial velocity
k = 1500; % elastic constant
h = 1000; % viscosity coefficient
g = 9.8; % gravitational acceleration
t0 = 0; % initial time
dt = 0.01; % time step
%% Numerical time integration
%% Initialize
t = t0;
z = z0;
v = v0;
P = m*v0; % initial momentum
G = -m*g; % constant momentum supply
%% stop when |v| < 1 mm/s
while (abs(v) > 0.001)
  t = t + dt;
  F = -k*z - h*v;
  P = P + (F + G)*dt;
  z = z + v*dt;
  v = P/m;
end
disp(z)
disp(t)
```

(c)

Assume that the energy of the floor consists only of internal energy U. Also assume that this energy increases only from the mechanical power $F \cdot v$ coming from the force exerted by the block on the floor (no heat flux).

Take the initial energy of the floor to be 0 J. Add two lines to the script:

- one that initializes the value of the internal energy
- one that updates the internal energy, using the balance of energy.

3

(a)

i. Explain what is the difference between a universal law and a constitutive relation. Give at least two examples of constitutive relations.

ii. Why do we consider Newton's formula $\boldsymbol{P} = M\boldsymbol{v}$ (relating momentum, mass, velocity) to be a constitutive relation, rather than a universal law?

(b)

What is the difference between a conservation law and a balance law? Which quantities do you know that are conserved, and which balanced but not conserved?

(c)

You and a friend of yours synchronize your clocks. Your friend goes to live for a while in an orbiting space station. After one year of your time, your friend comes back and you two meet again. You notice that your friend has aged three years instead of one.

How large is the mass M of the hypothetical planet in which you two live? Use the approximate formula

$$\Delta t_{\text{friend}} = \left[1 + \frac{G}{c^2}\frac{M}{R}\right]\Delta t_{\text{you}}$$

where Δt_{friend} is your friend's proper time while in orbit, Δt_{you} is your proper time, $G = 6.7\times10^{-11}\,\mathrm{m^3/(kg\cdot s^2)}$, $c = 3\times10^8\,\mathrm{m/s}$, and assume your planet has radius $R = 7\times10^7$ m.