The Seven Wonders of the World

Exercises

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Contents

1 Physics, quantities, units

For some of the following exercises you can refer to tables 1.1 and 1.2 on page 8 (reproduced from the textbook).

1.0

(Do the exercises in the main text.)

1.1

Preferably together with a friend or colleague:

If you have some large-language-model service (such as ChatGPT), ask it which physical laws are universally valid in Newtonian Mechanics and in General Relativity and in Thermodynamics and in Chemistry and in Electromagnetics.

Discuss the answer you get, based on what you have learned so far. (Note: if the answer mention a 'balance of boost momentum', that's actually correct.)

Argue with the LLM and see where the discussion goes.

1.2

Take *time* and *velocity* as primitive quantities.

1. Try to define *distance* as a derived quantity
2. Try to define *acceleration* as a derived quantity.

1.3

Which of the following quantities are *scalars*, and which are *vectors*?

- Time
- Distance
- Position
- Energy
- Velocity
- Speed
- Momentum
- Entropy
- Angular momentum
- Force
- Temperature
- Magnetic flux
- Electric charge
- Electric current
- Heat
- Power
- Volume
- Pressure

1.4

Find the correct units for the following quantities:

- *Volumic energy* or *energy density*, defined as energy divided by volume
- *Energy flux*, defined as energy divided by time.
- *Power*, defined as energy divided by time.
- *Heating*, defined as energy divided by time.
- *Magnetic flux*, which we take as a primitive quantity.
- *Electric potential difference*, defined as magnetic flux divided by time.
- *Force*, defined as momentum divided by time.
- *Momentum flux*, defined as momentum divided by time.
- *Momentum supply*, defined as momentum divided by time.
- *Pressure*, defined as force divided by area.
- *Amount of substance (or of matter)*, which we take as primitive.
- *Molar mass*, defined as mass divided by amount of substance.
- *Specific momentum*, defined as momentum divided by mass.

- *Volumic charge* or *charge density*, defined as charge divided by volume.
- *Entropy*, which we take as primitive, has dimension of energy divided by temperature.
- *Matter density*, defined as amount of substance divided by volume.
- *Matter flux*, defined as amount of substance divided by time.

1.5

With a friend or colleague:

1. Try to explain to your friend the difference between a *primitive quantity* and a *derived quantity*; then let your friend criticize unclear or incorrect points in your explanation, and comment on the good points. Then invert your roles: your friend tries to explain to you, and you criticize and comment.
2. Similarly as the previous exercise, but explaining the difference between a *scalar quantity* and a *vector quantity*.
3. If you have some large-language-model service (such as ChatGPT), ask it to explain the difference between primitive and derived quantity, and between scalar and vector quantity. Find out weak or unsure points in its answer, given what you've learned so far.

1.6

Find which of the following mathematical expressions and equalities are dimensionally incorrect, and explain why they are incorrect:

- ▹ $11\,\mathrm{J} + 4\,\mathrm{kg}$
- ▹ $\tan\left(\dfrac{a}{b}\right)$, where a has dimension length and b has dimension time
- ▹ $299\,792\,458\,\mathrm{m/s}$
- ▹ $\exp\left(\dfrac{71\,\mathrm{s}}{3\,\mathrm{s}}\right)$
- ▹ $\cos(3.14)\,\mathrm{m}$
- ▹ $m - v$, where m has dimension of mass and v of velocity

- $10\,\mathrm{N\,s} - 2\,\mathrm{kg\,m/s} = 8\,\mathrm{J\,s/m}$
- $\exp(-8\,\mathrm{J})$
- $(9\,\mathrm{m},\ 0.1\,\mathrm{rad},\ -0.5\,\mathrm{rad})$
- $8\,\mathrm{J/s} = 12\,\mathrm{N\,m} - 4\,\mathrm{N\,m}$
- $\mathrm{e}^{-8}\,\mathrm{J}$
- $\dfrac{15\,\mathrm{J}}{5\,\mathrm{kg/s^2}} = 3\,\mathrm{m}^2$
- $\sqrt{25}\,\mathrm{K} = 5$
- $(\mathrm{e}^7)^{\mathrm{s}}$
- $\tan\left(\dfrac{10\,\mathrm{m}}{5\,\mathrm{m}}\right)$
- $\sqrt{300\,\mathrm{K}}$
- $\sin(t/\mathrm{s})$, where t has dimension of time
- $\dfrac{3}{\mathrm{s}}$
- $\sin(10\,\mathrm{s})$

Quantity	**SI Dimension**	**Unit**
Time	time	*second* s
Length	length	*metre* m
Temperature	temperature	*kelvin* K
Matter	amount of substance	*mole* mol
Electric charge	electric charge	*coulomb* C
Magnetic flux	magnetic flux	*weber* Wb
Energy	energy, mass	*joule* J, *kilogram* kg
Momentum	force · time, mass · length/time, energy · time/length	N · s, kg · m/s, J · s/m
Angular momentum	force · length · time, mass · length^2/time, energy · time	N · m · s, kg · m^2/s, J · s
Entropy	energy/temperature	J/K

Table 1.1 Dimensions and units of the main physical quantities used in these notes. Their fluxes have the dimensions divided by time, and therefore units divided by seconds. Quantities in **boldface** are vectors, the others are scalars.

Quantity	**Volume content** [unit]	**Flux** [unit]
matter	N [mol]	J [mol/s]
electric charge	$\mathcal{Q}$ [C]	$\mathcal{I}$ [C/s or A]
magnetic flux	$\mathcal{B}$ [Wb]	$\mathcal{E}$ [Wb/s or V]
energy	E [J]	Φ [J/s or W]
momentum	$\boldsymbol{P}$ [N s]	$\boldsymbol{F}$ [N]
angular momentum	$\boldsymbol{L}$ [N m s]	$\boldsymbol{\tau}$ [N m]
entropy	S [J/K]	Π [J/(K s)]

Table 1.2 Units for volume contents and fluxes of the main seven quantities.

Example solutions

1.2

1. "Distance is the product of a time lapse and a particular velocity". See section 2.3 about *Radar distance* in our lecture notes.
2. "Acceleration is the ratio between a change in the product of a time lapse and a particular velocity, and the time taken by that change".

1.3

These quantities are scalars:

- Time
- Distance
- Energy
- Speed
- Entropy
- Temperature
- Magnetic flux
- Electric charge
- Electric current
- Heat
- Power
- Volume

These quantities are vectors:

- Position
- Velocity
- Momentum
- Angular momentum
- Force

For *pressure*, it depends on the context. In some applications it is considered a scalar, but in other applications it is considered a vector – or actually a generalized kind of vector, called *tensor*, which can be represented by a matrix.

1.4

- *Volumic energy*: $\mathrm{J/m^3}$

- *Energy flux*: J/s
- *Power*: J/s
- *Heating*: J/s
- *Magnetic flux*: Wb
- *Electric potential difference*: Wb/s
- *Force*: N
- *Momentum flux*: N
- *Momentum supply*: N
- *Pressure*: $\mathrm{N/m^2}$
- *Amount of substance*: mol
- *Molar mass*: kg/mol
- *Specific momentum*: $\mathrm{N \cdot s/kg \equiv m/s}$
- *Volumic charge*: $\mathrm{C/m^3}$
- *Entropy*: J/K
- *Matter density*: $\mathrm{mol/m^3}$
- *Matter flux*: mol/s

1.6

▹ $11\,\mathrm{J} + 4\,\mathrm{kg}$
Incorrect: cannot sum quantities of different dimension

▹ $\tan\left(\frac{a}{b}\right)$, where a dimension length and b has dimension time
Incorrect: trigonometric function must have a dimensionless argument, but a/b has dimension length/time

▹ $299\,792\,458\,\mathrm{m/s}$

▹ $\exp\left(\frac{71\,\mathrm{s}}{3\,\mathrm{s}}\right)$

▹ $\cos(3.14)\,\mathrm{m}$

▹ $m - v$, where m has dimension of mass and v of velocity
Incorrect: cannot subtract quantities of different dimension

▹ $10\,\mathrm{N\,s} - 2\,\mathrm{kg\,m/s} = 8\,\mathrm{J\,s/m}$

▹ $\exp(-8\,\mathrm{J})$
Incorrect: exponential function must have a dimensionless argument, but this argument has dimension energy

- $\left(9\,\mathrm{m},\ 0.1\,\mathrm{rad},\ -0.5\,\mathrm{rad}\right)$
- $8\,\mathrm{J/s} = 12\,\mathrm{N\,m} - 4\,\mathrm{N\,m}$
  Incorrect: $\mathrm{J/s} \neq \mathrm{N\,m}$ (correct is $\mathrm{J} = \mathrm{N\,m}$)
- $\mathrm{e}^{-8}\,\mathrm{J}$
- $\dfrac{15\,\mathrm{J}}{5\,\mathrm{kg/s^2}} = 3\,\mathrm{m}^2$
- $\sqrt{25}\,\mathrm{K} = 5$
  Incorrect: both sides of an equation must have the same dimension; here the left side has dimension $\text{length}^{1/2}$, right side is dimensionless
- $\left(\mathrm{e}^7\right)^{\mathrm{s}}$
  Incorrect: cannot raise to a dimensional power
- $\tan\left(\dfrac{10\,\mathrm{m}}{5\,\mathrm{m}}\right)$
- $\sqrt{300\,\mathrm{K}}$
- $\sin(t/\mathrm{s})$, where t has dimension of time
- $\dfrac{3}{\mathrm{s}}$
- $\sin(10\,\mathrm{s})$
  Incorrect: trigonometric function must have a dimensionless argument

Time and space 2

Make sure you're familiar with the 'dot-notation' explained in § 2.8 of our text.

2.0

(Do the exercises in the main text.)

2.1

Preferably together with a friend or colleague:

The *Veritasium*[1] channel has many informative and entertaining videos on diverse scientific topics. Most of these videos are accurate and pedagogically very useful. But a couple of them contain some inaccuracies or partially faulty reasoning.

One example of partially inaccurate video is *Why no one has measured the speed of light*[2]. It contains many correct and insightful statements and explanations, but also some faulty reasoning.

Watch the video and

1. Identify and ponder about some explanations that reflect what you learned so far. (For instance, do you recognize *radar distance* between `t=3:10` and `t=3:20`?)

2. Consider the discussion between `t=4:57`[3] and `t=5:14`, and the statement "and get a response 20 minutes later". What kind of time is this statement referring to? is it proper time? if so, whose proper time? or is it coordinate time?

3. Consider the same snip and the statement "we imagine our signal takes 10 minutes to get there". Draw a spacetime diagram (similar to fig. 2.1 in our main text) illustrating this statement. In the diagram,

place the proper times on the worldline of the Earth station and on Mark's worldline; and mark the points where the signal is sent and where it is received.

How can we imagine that it takes 10 minutes to get there? Which proper time are we speaking about?

4. Consider again the snip and the statement "it's possible that our message took all 20 minutes to get there". Draw a spacetime diagram illustrating this statement. What's the difference from the previous spacetime diagram? Are the two spacetime diagrams actually different?

5. Now consider the discussion between `t=9:47`[4] and `t=10:16`, and the statement "one of the clocks will be ahead of the other". When we say *ahead*, to which kind of time are we referring? is it proper time? if so, whose proper time? Does it make sense to say that one clock is "ahead" of the other?

6. Draw one or two spacetime diagrams illustrating the discussion in the snip above. Can we make sense of the discussion using the diagrams?

7. Find parts in which the reasoning offered in the video is inconsistent. For instance, find discussions where Derek says "right now": does "right now" make sense in those discussions?

2.2

Preferably together with a friend or colleague:

A particular coordinate system (t, x, y, z) with spatial Cartesian coordinates is defined as follows:

- The time coordinate t is your proper time.
- The origin of the coordinates is your navel
- The x-axis points in front of you, the y-axis to your left, the z-axis upwards (through the top of your head).
- The unit coordinate is 1 m, measured as usual.

Answer the following questions:

1. What are your position $\boldsymbol{r}(t)$ and velocity $\boldsymbol{v}(t)$ in this coordinate system while you sleep? (Let's say that by "your position" we mean the position of your navel.)

2. What are your position $\boldsymbol{r}(t)$ and velocity $\boldsymbol{v}(t)$ while you run or bike or drive to school?
3. What is your acceleration $\boldsymbol{a}(t)$ in different situations?
4. Determine the z coordinate of the floor in this coordinate system, when you are standing still.
5. Determine the spatial coordinates of the tip of the index finger of your right hand, when it is extended horizontally outwards.

2.3

1. You're told that the position $\boldsymbol{r}(t)$ of an object is constant in time t. How much is the velocity $\boldsymbol{v}(t)$?
2. If the velocity $\boldsymbol{v}(t_0)$ is zero at a time t_0, must also the acceleration $\boldsymbol{a}(t_0)$ be zero at time t_0?
3. Is it possible for a coordinate velocity $v_x(t_1)$ to be positive at a time t_1, and the acceleration $a_x(t_1)$ negative at the same time? If not, explain why not. If yes, show by constructing a concrete example and explain what this situation means physically.

2.4

We have a coordinate system (t, x) with one spatial dimension only. A small object S has position $x_S(t)$ which changes with the coordinate time t. The time dependence of the position is given by

$$x_S(t) = at + b \qquad \text{with} \quad a = -3\,\mathrm{m/s}\,,\ b = 7\,\mathrm{m}\,.$$

1. Verify that the equation above is dimensionally consistent.
2. What is the spatial coordinate of S at times $t = 0\,\mathrm{s}$, $t = -10\,\mathrm{s}$, and $t = 5\,\mathrm{s}$?
3. What is the spatial coordinate of S at time $t = 10$?
4. Calculate the time dependence of the coordinate velocity of S.
5. What is the cooordinate velocity of S at time $t = 5\,\mathrm{s}$?
6. What is the *speed* of S at time $t = 5\,\mathrm{s}$?
7. Calculate the time dependence of the coordinate acceleration of S.

2.5

We have a coordinate system (t, x) with one spatial dimension only. A small object S has position $x_S(t)$ given by

$$x_S(t) = L\sin(\omega t) + b \qquad \text{with} \quad L = 2\,\mathrm{m}\,,\ \omega = \frac{\pi}{3}\,\mathrm{s}^{-1}\,,\ b = 7\,\mathrm{m}\,.$$

1. Verify that the equation above is dimensionally consistent.
2. Calculate the expressions for velocity $\dot{x}_S(t)$ and acceleration $\ddot{x}_S(t)$.
3. Find a time t_0 in which the velocity is $0\,\mathrm{m/s}$ and the acceleration is approximately $-2.2\,\mathrm{m/s^2}$.
4. Find a time t_1 in which the velocity is approximately $-2.1\,\mathrm{m/s}$ and the acceleration is $0\,\mathrm{m/s^2}$.
5. Plot $x_S(t)$ and $\dot{x}_S(t)$ as functions of time for $t \in [-4, 4]\,\mathrm{s}$.

2.6

We have a coordinate system (t, x, y, z), where the three spatial coordinates have each dimension length. A small object S has position $\boldsymbol{r}_S(t)$ given by

$$\boldsymbol{r}_S(t) = \begin{bmatrix} at + b \\ L\sin(\omega t) + b \\ 0 \end{bmatrix}$$

$$\text{with} \quad L = 2\,\mathrm{m}\,,\ \omega = \frac{\pi}{3}\,\mathrm{s}^{-1}\,,\ a = -3\,\mathrm{m/s}\,,\ b = 7\,\mathrm{m/s}\,.$$

1. Verify that the equation above is dimensionally consistent.
2. Calculate the expressions for velocity $\dot{\boldsymbol{r}}_S(t)$ and acceleration $\ddot{\boldsymbol{r}}_S(t)$.
3. Plot the three components of the velocity as functions of time for $t \in [-4, 4]\,\mathrm{s}$.

2.7

We have a coordinate system (t, z) with one spatial dimension only. The coordinate velocity $v_z(t)$ of a small pulse of light travelling in a particular material is given by

$$v_z(t) = c\exp(-t/\tau) \qquad \text{with} \quad c = 299\,792\,458\,\mathrm{m/s}\,,\ \tau = 0.08\,\mathrm{s}\,,$$

and the pulse is located at $z = -2\,\mathrm{m}$ at $t = 1\,\mathrm{s}$.

1. Find the expression for the position $z(t)$ of the pulse as a function of coordinate time.
2. Find the location of the pulse at time $t = 1.01\,\text{s}$.

Example solutions

2.3

1. The derivative of a constant is zero, so the velocity is $\boldsymbol{v}(t) = 0\,\mathrm{m/s}$. We must not forget the correct units!
2. No, we can have zero velocity and non-zero acceleration at a given time. See exercise 2.5 as an example.
3. No, we can have positive velocity and negative acceleration at a given time. See exercise 2.5 as an example. It means that, at that time, the movement is in the positive-x direction (positive x-velocity), and the x-velocity is decreasing – that is, it will be positive but smaller a very short time later.

2.4

1. It is, provided that t has dimension time and x has dimension length. In this case, since a has dimension length/time, then $a\,t$ has dimension length, which is added to b which also has dimension length; the left and right side have then both dimension length.
2. $x_{\mathrm{S}}(0\,\mathrm{s}) = 7\,\mathrm{m}\ , \qquad x_{\mathrm{S}}(-10\,\mathrm{s}) = 37\,\mathrm{m}\ , \qquad x_{\mathrm{S}}(5\,\mathrm{s}) = -8\,\mathrm{m}\ .$
3. The question doesn't make sense, because "$t = 10$" is dimensionless; it should have dimension length instead.
4. Denoting with $\dot{x}_{\mathrm{S}}$ the coordinate velocity of S, then $\dot{x}_{\mathrm{S}}(t) = a$, which is constant in time.
5. $\dot{x}_{\mathrm{S}}(t) = -3\,\mathrm{m/s}$ at any time.
6. The speed is $|\dot{x}_{\mathrm{S}}(t)| = 3\,\mathrm{m/s}$ at any time.
7. Denoting with $\ddot{x}_{\mathrm{S}}$ the coordinate acceleration of S, then $\ddot{x}_{\mathrm{S}}(t) = 0\,\mathrm{m/s^2}$, which is zero at all times.

2.5

1. The expression is dimensionally correct, provided t has dimension time and x has dimension length. The argument of the sine function is dimensionless, and the two terms on the right have dimension length.
2. From the rules for the derivative,

$$\dot{x}_{\mathrm{S}}(t) = \omega L \cos(\omega t)\ , \qquad \ddot{x}_{\mathrm{S}}(t) = -\omega^2 L \sin(\omega t)\ .$$

3. The time t_0 must satisfy the system of equations

$$\omega L \cos(\omega t) = 0\,\mathrm{m/s} \qquad -\omega^2 L \sin(\omega t) \approx -2.2\,\mathrm{m/s^2}\,.$$

The cosine is zero when its argument is $\pi/2$, $3\pi/2$, and so on. Let's try taking $\omega t_0 = \pi/2$, which means $t_0 = \pi/(2\omega)$. We find indeed

$$\dot{x}_\mathrm{S}(t_0) = \omega L \cos(\omega t_0) = 0\,\mathrm{m/s} \qquad \ddot{x}_\mathrm{S}(t_0) = -\omega^2 L \sin(\omega t_0) \approx -2.19\,\mathrm{m/s^2}\,.$$

4. The time t_1 must satisfy the system of equations

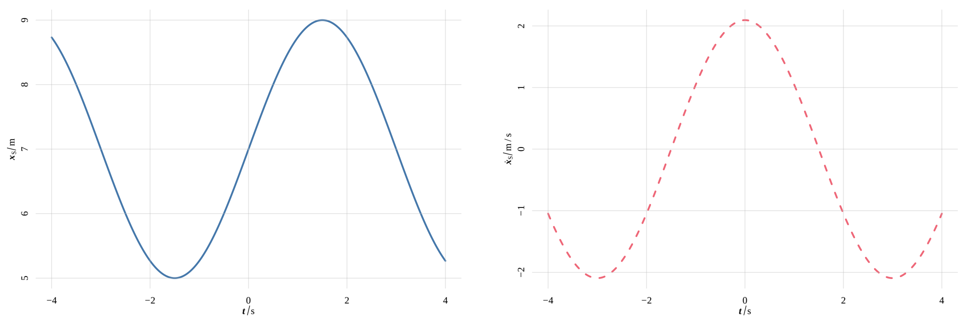
$$\omega L \cos(\omega t) = -2.1\,\mathrm{m/s} \qquad -\omega^2 L \sin(\omega t) \approx 0\,\mathrm{m/s^2}\,.$$

The sine is zero when its argument is 0, π, and so on. Let's try taking $\omega t_1 = 0$, which means $t_1 = 0\,\mathrm{s}$. We find

$$\omega L \cos(\omega t_1) \approx 2.09\,\mathrm{m/s} \qquad -\omega^2 L \sin(\omega t_1) = 0\,\mathrm{m/s^2}\,,$$

which is not what we want. Trying next $\omega t_1 = \pi$, which means $t_1 = \pi/\omega$, leads to the desired result.

5. We can plot x_S and $\dot{x}_\mathrm{S}$ in two separate graphs:



Or we could plot them on the same graph – but only if we indicate separately the vertical axis for x_S and the one for $\dot{x}_\mathrm{S}$ (for instance one on the left and one on the right), because these quantities have different dimensions.

2.6

1. No, the expression is not dimensionally correct, because the z component of $\boldsymbol{r}_\mathrm{S}$ is "0", which is a dimensionless number, whereas z has dimension length. The z component should be "$0\,\mathrm{m}$".

2. See exercises 2.4 and 2.5 ☻

2.7

1. Let's call the specific position $z_0 \coloneqq -2\,\mathrm{m}$ at $t_0 1\,\mathrm{s}$. The expression for $z(t)$ is found integrating $v_z(t)$ between t_0 and t, and adding the position at t_0:

$$
\begin{aligned}
z(t) &= z_0 + \int_{t_0}^{t} c \exp(-t/\tau)\,\mathrm{d}t \\
&= z_0 - c\tau \exp(-t/\tau)\big|_{t_0}^{t} \\
&= z_0 - c\tau \big[\exp(-t/\tau) - \exp(-t_0/\tau)\big] \\
&\qquad \text{with} \quad c = 299\,792\,458\,\mathrm{m/s}\,,\ \tau = 0.08\,\mathrm{s}\,,\ z_0 = -2\,\mathrm{m}\,,\ t_0 = 1\,\mathrm{s}\,.
\end{aligned}
$$

2. Substituting in the expression above, $z(1.01\,\mathrm{s}) = 8.50\,\mathrm{m}$.

URLs for chapter 2

1. https://www.youtube.com/c/veritasium/videos
2. https://www.youtube.com/watch?v=pTn6Ewhb27k
3. https://youtu.be/pTn6Ewhb27k?t=297
4. https://youtu.be/pTn6Ewhb27k?t=588

Volume contents, fluxes, supplies 3

3.0

(Do the exercises in the main text.)

Example solutions

Physical laws 4

4.0

(Do the exercises in the main text.)

Example solutions

The Seven Wonders of the world 5

5.0

(Do the exercises in the main text.)

Example solutions

Conservation & balances of matter 6

6.0

(Do the exercises in the main text.)

Example solutions

Conservation of electric charge 7

7.0

(Do the exercises in the main text.)

Example solutions

Conservation of magnetic flux 8

8.0

(Do the exercises in the main text.)

Example solutions

9 Balance of momentum

9.0

(Do the exercises in the main text.)

Example solutions

10 Balance of energy

10.0

(Do the exercises in the main text.)

Example solutions

Balance of angular momentum 11

11.0

(Do the exercises in the main text.)

Example solutions

12 Remarks on momentum and energy

12.0

(Do the exercises in the main text.)

Example solutions

Balance of entropy 13

13.0

(Do the exercises in the main text.)

Example solutions

Constitutive relations 14

14.0

(Do the exercises in the main text.)

Example solutions