The Seven Wonders of the World

Exercises

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Physics, quantities, units 1

🧩 1.1

Take *time* and *velocity* as primitive quantities.

1. Try to define *distance* as a derived quantity
2. Try to define *acceleration* as a derived quantity.

🧩 1.2

Which of the following quantities are *scalars*, and which are *vectors*?

- Time
- Distance
- Position
- Energy
- Velocity
- Speed
- Momentum
- Entropy
- Angular momentum
- Force
- Temperature
- Magnetic flux
- Electric charge
- Electric current
- Heat
- Power
- Volume
- Pressure

🧩 1.3

Preferably together with a friend or colleague:

If you have some large-language-model service (such as ChatGPT), ask it which physical laws are universally valid in Newtonian Mechanics and in General Relativity and in Thermodynamics and in Chemistry and in Electromagnetics.

Discuss the answer you get, based on what you have learned so far. (Note: if the answer mention a 'balance of boost momentum', that's actually correct.)

Argue with the LLM and see where the discussion goes.

🧩 1.4

Find the correct units for the following quantities; refer to tables 3.1 and 4.12 in the main text.

- *Volumic energy* or *energy density*, defined as energy divided by volume
- *Energy flux*, defined as energy divided by time.
- *Power*, defined as energy divided by time.
- *Heating*, defined as energy divided by time.
- *Magnetic flux*, which we take as a primitive quantity.
- *Electric potential difference*, defined as magnetic flux divided by time.
- *Force*, defined as momentum divided by time.
- *Momentum flux*, defined as momentum divided by time.
- *Momentum supply*, defined as momentum divided by time.
- *Pressure*, defined as force divided by area.
- *Amount of substance (or of matter)*, which we take as primitive.
- *Molar mass*, defined as mass divided by amount of substance.
- *Specific momentum*, defined as momentum divided by mass.
- *Volumic charge* or *charge density*, defined as charge divided by volume.
- *Entropy*, which we take as primitive, has dimension of energy divided by temperature.
- *Matter density*, defined as amount of substance divided by volume.
- *Matter flux*, defined as amount of substance divided by time.

🧩 1.5

With a friend or colleague:

1. Try to explain to your friend the difference between a *primitive quantity* and a *derived quantity*; then let your friend criticize unclear or incorrect points in your explanation, and comment on the good points. Then invert your roles: your friend tries to explain to you, and you criticize and comment.

2. Similarly as the previous exercise, but explaining the difference between a *scalar quantity* and a *vector quantity*.

3. If you have some large-language-model service (such as ChatGPT), ask it to explain the difference between primitive and derive quantity, and between scalar and vector quantity. Find out weak or unsure points in its answer, given what you've learned so far.

🧩 1.6

Find which of the following mathematical expressions and equalities are dimensionally incorrect, and explain why they are incorrect:

▷ $11\,\mathrm{J} + 4\,\mathrm{kg}$

▷ $\tan\left(\dfrac{a}{b}\right)$, where a dimension length and b has dimension time

▷ $299\,792\,458\,\mathrm{m/s}$

▷ $\exp\left(\dfrac{71\,\mathrm{s}}{3\,\mathrm{s}}\right)$

▷ $\cos(3.14)\,\mathrm{m}$

▷ $m - v$, where m has dimension of mass and v of velocity

▷ $10\,\mathrm{N\,s} - 2\,\mathrm{kg\,m/s} = 8\,\mathrm{J\,s/m}$

▷ $\exp(-8\,\mathrm{J})$

▷ $\left(9\,\mathrm{m},\ 0.1\,\mathrm{rad},\ -0.5\,\mathrm{rad}\right)$

▷ $8\,\mathrm{J/s} = 12\,\mathrm{N\,m} - 4\,\mathrm{N\,m}$

▷ $\mathrm{e}^{-8}\,\mathrm{J}$

▷ $\dfrac{15\,\mathrm{J}}{5\,\mathrm{kg/s^2}} = 3\,\mathrm{m^2}$

▷ $\sqrt{25}\,\mathrm{K} = 5$

▷ $\left(e^7\right)^{\mathrm{s}}$

▷ $\tan\left(\dfrac{10\,\mathrm{m}}{5\,\mathrm{m}}\right)$

▷ $\sqrt{300\,\mathrm{K}}$

▷ $\sin(t/\mathrm{s})$, where t has dimension of time

▷ $\dfrac{3}{\mathrm{s}}$

▷ $\sin(10\,\mathrm{s})$

Example solutions

1.1

1. "Distance is the product of a time lapse and a particular velocity". See section 2.3 about *Radar distance* in the main text.

2. "Acceleration is the ratio between a change in the product of a time lapse and a particular velocity, and the time taken by that change".

1.2

These quantities are scalars:

- Time
- Distance
- Energy
- Speed
- Entropy
- Temperature
- Magnetic flux
- Electric charge
- Electric current
- Heat
- Power
- Volume

These quantities are vectors:

- Position
- Velocity
- Momentum
- Angular momentum
- Force

For *pressure*, it depends on the context. In some applications it is considered a scalar, but in other applications it is considered a vector – or actually a generalized kind of vector, called *tensor*, which can be represented by a matrix.

1.4

- *Volumic energy*: J/m^3

- *Energy flux*: J/s
- *Power*: J/s
- *Heating*: J/s
- *Magnetic flux*: Wb
- *Electric potential difference*: Wb/s
- *Force*: N
- *Momentum flux*: N
- *Momentum supply*: N
- *Pressure*: N/m^2
- *Amount of substance*: mol
- *Molar mass*: kg/mol
- *Specific momentum*: $\text{N} \cdot \text{s}/\text{kg} \equiv \text{m}/\text{s}$
- *Volumic charge*: C/m^3
- *Entropy*: J/K
- *Matter density*: mol/m^3
- *Matter flux*: mol/s

1.6

▷ $11\,\text{J} + 4\,\text{kg}$
Incorrect: cannot sum quantities of different dimension

▷ $\tan\left(\dfrac{a}{b}\right)$, where a dimension length and b has dimension time
Incorrect: trigonometric function must have a dimensionless argument, but a/b has dimension length/time

▷ $299\,792\,458\,\text{m}/\text{s}$

▷ $\exp\left(\dfrac{71\,\text{s}}{3\,\text{s}}\right)$

▷ $\cos(3.14)\,\text{m}$

▷ $m - v$, where m has dimension of mass and v of velocity
Incorrect: cannot subtract quantities of different dimension

▷ $10\,\text{N}\,\text{s} - 2\,\text{kg}\,\text{m}/\text{s} = 8\,\text{J}\,\text{s}/\text{m}$

▷ $\exp(-8\,\text{J})$
Incorrect: exponential function must have a dimensionless argument, but this argument has dimension energy

▷ $\left(9\,\text{m},\ 0.1\,\text{rad},\ -0.5\,\text{rad}\right)$

▷ $8\,\text{J/s} = 12\,\text{N}\,\text{m} - 4\,\text{N}\,\text{m}$
Incorrect: J/s ≠ N m (correct is J = N m)

▷ $e^{-8}\,\text{J}$

▷ $\dfrac{15\,\text{J}}{5\,\text{kg/s}^2} = 3\,\text{m}^2$

▷ $\sqrt{25}\,\text{K} = 5$
Incorrect: both sides of an equation must have the same dimension; here the left side has dimension length$^{1/2}$, right side is dimensionless

▷ $\left(e^7\right)^{\text{s}}$
Incorrect: cannot raise to a dimensional power

▷ $\tan\left(\dfrac{10\,\text{m}}{5\,\text{m}}\right)$

▷ $\sqrt{300\,\text{K}}$

▷ $\sin(t/\text{s})$, where t has dimension of time

▷ $\dfrac{3}{\text{s}}$

▷ $\sin(10\,\text{s})$
Incorrect: trigonometric function must have a dimensionless argument

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