The Seven Wonders of the World

Exercises

Updated 7 February 2025 07:19

Contents

1 Physics, quantities, units

For some of the following exercises you can refer to tables 1.1 and 1.2 on page 9 (reproduced from the textbook).

1.0

(Do the exercises in the main text.)

1.1

Preferably together with a friend or colleague:

If you have some large-language-model service (such as ChatGPT), ask it which physical laws are universally valid in Newtonian Mechanics and in General Relativity and in Thermodynamics and in Chemistry and in Electromagnetics.

Discuss the answer you get, based on what you have learned so far. (Note: if the answer mention a 'balance of boost momentum', that's actually correct.)

Argue with the LLM and see where the discussion goes.

1.2

Take *time* and *velocity* as primitive quantities.

1. Try to define *distance* as a derived quantity
2. Try to define *acceleration* as a derived quantity.

1.3

Which of the following quantities are *scalars*, and which are *vectors*?

- Time
- Distance
- Position
- Energy
- Velocity
- Speed
- Momentum
- Entropy
- Angular momentum
- Force
- Temperature
- Magnetic flux
- Electric charge
- Electric current
- Heat
- Power
- Volume
- Pressure

1.4

Find the correct units for the following quantities:

- *Volumic energy* or *energy density*, defined as energy divided by volume
- *Energy flux*, defined as energy divided by time.
- *Power*, defined as energy divided by time.
- *Heating*, defined as energy divided by time.
- *Magnetic flux*, which we take as a primitive quantity.
- *Electric potential difference*, defined as magnetic flux divided by time.
- *Force*, defined as momentum divided by time.
- *Momentum flux*, defined as momentum divided by time.
- *Momentum supply*, defined as momentum divided by time.
- *Pressure*, defined as force divided by area.
- *Amount of substance (or of matter)*, which we take as primitive.
- *Molar mass*, defined as mass divided by amount of substance.
- *Specific momentum*, defined as momentum divided by mass.

- *Volumic charge* or *charge density*, defined as charge divided by volume.
- *Entropy*, which we take as primitive, has dimension of energy divided by temperature.
- *Matter density*, defined as amount of substance divided by volume.
- *Matter flux*, defined as amount of substance divided by time.

1.5

With a friend or colleague:

1. Try to explain to your friend the difference between a *primitive quantity* and a *derived quantity*; then let your friend criticize unclear or incorrect points in your explanation, and comment on the good points. Then invert your roles: your friend tries to explain to you, and you criticize and comment.
2. Similarly as the previous exercise, but explaining the difference between a *scalar quantity* and a *vector quantity*.
3. If you have some large-language-model service (such as ChatGPT), ask it to explain the difference between primitive and derived quantity, and between scalar and vector quantity. Find out weak or unsure points in its answer, given what you've learned so far.

1.6

Find which of the following mathematical expressions and equalities are dimensionally incorrect, and explain why they are incorrect:

- ▹ $11\,\mathrm{J} + 4\,\mathrm{kg}$
- ▹ $\tan\left(\dfrac{a}{b}\right)$, where a has dimension length and b has dimension time
- ▹ $299\,792\,458\,\mathrm{m/s}$
- ▹ $\exp\left(\dfrac{71\,\mathrm{s}}{3\,\mathrm{s}}\right)$
- ▹ $\cos(3.14)\,\mathrm{m}$
- ▹ $m - v$, where m has dimension of mass and v of velocity

- $10\,\mathrm{N\,s} - 2\,\mathrm{kg\,m/s} = 8\,\mathrm{J\,s/m}$
- $\exp(-8\,\mathrm{J})$
- $(9\,\mathrm{m},\ 0.1\,\mathrm{rad},\ -0.5\,\mathrm{rad})$
- $8\,\mathrm{J/s} = 12\,\mathrm{N\,m} - 4\,\mathrm{N\,m}$
- $\mathrm{e}^{-8}\,\mathrm{J}$
- $\dfrac{15\,\mathrm{J}}{5\,\mathrm{kg/s^2}} = 3\,\mathrm{m}^2$
- $\sqrt{25}\,\mathrm{K} = 5$
- $(\mathrm{e}^7)^{\mathrm{s}}$
- $\tan\left(\dfrac{10\,\mathrm{m}}{5\,\mathrm{m}}\right)$
- $\sqrt{300\,\mathrm{K}}$
- $\sin(t/\mathrm{s})$, where t has dimension of time
- $\dfrac{3}{\mathrm{s}}$
- $\sin(10\,\mathrm{s})$

Quantity	**SI Dimension**	**Unit**
Time	time	*second* s
Length	length	*metre* m
Temperature	temperature	*kelvin* K
Matter	amount of substance	*mole* mol
Electric charge	electric charge	*coulomb* C
Magnetic flux	magnetic flux	*weber* Wb
Energy	energy, mass	*joule* J, *kilogram* kg
Momentum	force · time, mass · length/time, energy · time/length	N · s, kg · m/s, J · s/m
Angular momentum	force · length · time, mass · length^2/time, energy · time	N · m · s, kg · m^2/s, J · s
Entropy	energy/temperature	J/K

Table 1.1 Dimensions and units of the main physical quantities used in these notes. Their fluxes have the dimensions divided by time, and therefore units divided by seconds. Quantities in **boldface** are vectors, the others are scalars.

Quantity	**Volume content** [unit]	**Flux** [unit]
matter	N [mol]	J [mol/s]
electric charge	$\boldsymbol{Q}$ [C]	$\mathcal{I}$ [C/s or A]
magnetic flux	$\mathcal{B}$ [Wb]	$\mathcal{E}$ [Wb/s or V]
energy	E [J]	Φ [J/s or W]
momentum	$\boldsymbol{P}$ [N s]	$\boldsymbol{F}$ [N]
angular momentum	$\boldsymbol{L}$ [N m s]	$\boldsymbol{\tau}$ [N m]
entropy	S [J/K]	Π [J/(K s)]

Table 1.2 Units for volume contents and fluxes of the main seven quantities.

Example solutions

1.2

1. "Distance is the product of a time lapse and a particular velocity". See section 2.3 about *Radar distance* in our lecture notes.
2. "Acceleration is the ratio between a change in the product of a time lapse and a particular velocity, and the time taken by that change".

1.3

These quantities are scalars:

- Time
- Distance
- Energy
- Speed
- Entropy
- Temperature
- Magnetic flux
- Electric charge
- Electric current
- Heat
- Power
- Volume

These quantities are vectors:

- Position
- Velocity
- Momentum
- Angular momentum
- Force

For *pressure*, it depends on the context. In some applications it is considered a scalar, but in other applications it is considered a vector – or actually a generalized kind of vector, called *tensor*, which can be represented by a matrix.

1.4

- *Volumic energy*: $\mathrm{J/m^3}$

- *Energy flux*: J/s
- *Power*: J/s
- *Heating*: J/s
- *Magnetic flux*: Wb
- *Electric potential difference*: Wb/s
- *Force*: N
- *Momentum flux*: N
- *Momentum supply*: N
- *Pressure*: $\mathrm{N/m^2}$
- *Amount of substance*: mol
- *Molar mass*: kg/mol
- *Specific momentum*: $\mathrm{N \cdot s/kg \equiv m/s}$
- *Volumic charge*: $\mathrm{C/m^3}$
- *Entropy*: J/K
- *Matter density*: $\mathrm{mol/m^3}$
- *Matter flux*: mol/s

1.6

▹ $11\,\mathrm{J} + 4\,\mathrm{kg}$
Incorrect: cannot sum quantities of different dimension

▹ $\tan\left(\dfrac{a}{b}\right)$, where a dimension length and b has dimension time
Incorrect: trigonometric function must have a dimensionless argument, but a/b has dimension length/time

▹ $299\,792\,458\,\mathrm{m/s}$

▹ $\exp\left(\dfrac{71\,\mathrm{s}}{3\,\mathrm{s}}\right)$

▹ $\cos(3.14)\,\mathrm{m}$

▹ $m - v$, where m has dimension of mass and v of velocity
Incorrect: cannot subtract quantities of different dimension

▹ $10\,\mathrm{N\,s} - 2\,\mathrm{kg\,m/s} = 8\,\mathrm{J\,s/m}$

▹ $\exp(-8\,\mathrm{J})$
Incorrect: exponential function must have a dimensionless argument, but this argument has dimension energy

- $(9\,\mathrm{m},\ 0.1\,\mathrm{rad},\ -0.5\,\mathrm{rad})$
- $8\,\mathrm{J/s} = 12\,\mathrm{N\,m} - 4\,\mathrm{N\,m}$
  Incorrect: $\mathrm{J/s} \neq \mathrm{N\,m}$ (correct is $\mathrm{J} = \mathrm{N\,m}$)
- $\mathrm{e}^{-8}\,\mathrm{J}$
- $\dfrac{15\,\mathrm{J}}{5\,\mathrm{kg/s^2}} = 3\,\mathrm{m}^2$
- $\sqrt{25}\,\mathrm{K} = 5$
  Incorrect: both sides of an equation must have the same dimension; here the left side has dimension $\mathsf{length}^{1/2}$, right side is dimensionless
- $(\mathrm{e}^7)^{\mathrm{s}}$
  Incorrect: cannot raise to a dimensional power
- $\tan\left(\dfrac{10\,\mathrm{m}}{5\,\mathrm{m}}\right)$
- $\sqrt{300\,\mathrm{K}}$
- $\sin(t/\mathrm{s})$, where t has dimension of time
- $\dfrac{3}{\mathrm{s}}$
- $\sin(10\,\mathrm{s})$
  Incorrect: trigonometric function must have a dimensionless argument

Time and space 2

Make sure you're familiar with the 'dot-notation' explained in § 2.8 of our text.

2.0

(Do the exercises in the main text.)

2.1

Preferably together with a friend or colleague:

The *Veritasium*[1] channel has many informative and entertaining videos on diverse scientific topics. Most of these videos are accurate and pedagogically very useful. But a couple of them contain some inaccuracies or partially faulty reasoning.

One example of partially inaccurate video is *Why no one has measured the speed of light*[2]. It contains many correct and insightful statements and explanations, but also some faulty reasoning.

Watch the video and

1. Identify and ponder about some explanations that reflect what you learned so far. (For instance, do you recognize *radar distance* between `t=3:10` and `t=3:20`?)

2. Consider the discussion between `t=4:57`[3] and `t=5:14`, and the statement "and get a response 20 minutes later". What kind of time is this statement referring to? is it proper time? if so, whose proper time? or is it coordinate time?

3. Consider the same snip and the statement "we imagine our signal takes 10 minutes to get there". Draw a spacetime diagram (similar to fig. 2.1 in our main text) illustrating this statement. In the diagram,

place the proper times on the worldline of the Earth station and on Mark's worldline; and mark the points where the signal is sent and where it is received.

How can we imagine that it takes 10 minutes to get there? Which proper time are we speaking about?

4. Consider again the snip and the statement "it's possible that our message took all 20 minutes to get there". Draw a spacetime diagram illustrating this statement. What's the difference from the previous spacetime diagram? Are the two spacetime diagrams actually different?

5. Now consider the discussion between `t=9:47`[4] and `t=10:16`, and the statement "one of the clocks will be ahead of the other". When we say *ahead*, to which kind of time are we referring? is it proper time? if so, whose proper time? Does it make sense to say that one clock is "ahead" of the other?

6. Draw one or two spacetime diagrams illustrating the discussion in the snip above. Can we make sense of the discussion using the diagrams?

7. Find parts in which the reasoning offered in the video is inconsistent. For instance, find discussions where Derek says "right now": does "right now" make sense in those discussions?

2.2

Preferably together with a friend or colleague:

A particular coordinate system (t, x, y, z) with spatial Cartesian coordinates is defined as follows:

- The time coordinate t is your proper time.
- The origin of the coordinates is your navel
- The x-axis points in front of you, the y-axis to your left, the z-axis upwards (through the top of your head).
- The unit coordinate is 1 m, measured as usual.

Answer the following questions:

1. What are your position $\boldsymbol{r}(t)$ and velocity $\boldsymbol{v}(t)$ in this coordinate system while you sleep? (Let's say that by "your position" we mean the position of your navel.)

2. What are your position $\boldsymbol{r}(t)$ and velocity $\boldsymbol{v}(t)$ while you run or bike or drive to school?
3. What is your acceleration $\boldsymbol{a}(t)$ in different situations?
4. Determine the z coordinate of the floor in this coordinate system, when you are standing still.
5. Determine the spatial coordinates of the tip of the index finger of your right hand, when it is extended horizontally outwards.

2.3

1. You're told that the position $\boldsymbol{r}(t)$ of an object is constant in time t. How much is the velocity $\boldsymbol{v}(t)$?
2. If the velocity $\boldsymbol{v}(t_0)$ is zero at a time t_0, must also the acceleration $\boldsymbol{a}(t_0)$ be zero at time t_0?
3. Is it possible for a coordinate velocity $v_x(t_1)$ to be positive at a time t_1, and the acceleration $a_x(t_1)$ negative at the same time? If not, explain why not. If yes, show by constructing a concrete example and explain what this situation means physically.

2.4

We have a coordinate system (t, x) with one spatial dimension only. A small object S has position $x_S(t)$ which changes with the coordinate time t. The time dependence of the position is given by

$$x_S(t) = at + b \qquad \text{with} \quad a = -3\,\mathrm{m/s}\,,\ b = 7\,\mathrm{m}\,.$$

1. Verify that the equation above is dimensionally consistent.
2. What is the spatial coordinate of S at times $t = 0\,\mathrm{s}$, $t = -10\,\mathrm{s}$, and $t = 5\,\mathrm{s}$?
3. What is the spatial coordinate of S at time $t = 10$?
4. Calculate the time dependence of the coordinate velocity of S.
5. What is the cooordinate velocity of S at time $t = 5\,\mathrm{s}$?
6. What is the *speed* of S at time $t = 5\,\mathrm{s}$?
7. Calculate the time dependence of the coordinate acceleration of S.

2.5

We have a coordinate system (t, x) with one spatial dimension only. A small object S has position $x_S(t)$ given by

$$x_S(t) = L\sin(\omega t) + b \qquad \text{with} \quad L = 2\,\mathrm{m}\,,\ \omega = \frac{\pi}{3}\,\mathrm{s}^{-1}\,,\ b = 7\,\mathrm{m}\,.$$

1. Verify that the equation above is dimensionally consistent.
2. Calculate the expressions for velocity $\dot{x}_S(t)$ and acceleration $\ddot{x}_S(t)$.
3. Find a time t_0 in which the velocity is $0\,\mathrm{m/s}$ and the acceleration is approximately $-2.2\,\mathrm{m/s^2}$.
4. Find a time t_1 in which the velocity is approximately $-2.1\,\mathrm{m/s}$ and the acceleration is $0\,\mathrm{m/s^2}$.
5. Plot $x_S(t)$ and $\dot{x}_S(t)$ as functions of time for $t \in [-4, 4]\,\mathrm{s}$.

2.6

We have a coordinate system (t, x, y, z), where the three spatial coordinates have each dimension length. A small object S has position $\boldsymbol{r}_S(t)$ given by

$$\boldsymbol{r}_S(t) = \begin{bmatrix} at + b \\ L\sin(\omega t) + b \\ 0 \end{bmatrix}$$

$$\text{with} \quad L = 2\,\mathrm{m}\,,\ \omega = \frac{\pi}{3}\,\mathrm{s}^{-1}\,,\ a = -3\,\mathrm{m/s}\,,\ b = 7\,\mathrm{m/s}\,.$$

1. Verify that the equation above is dimensionally consistent.
2. Calculate the expressions for velocity $\dot{\boldsymbol{r}}_S(t)$ and acceleration $\ddot{\boldsymbol{r}}_S(t)$.
3. Plot the three components of the velocity as functions of time for $t \in [-4, 4]\,\mathrm{s}$.

2.7

We have a coordinate system (t, z) with one spatial dimension only. The coordinate velocity $v_z(t)$ of a small pulse of light travelling in a particular material is given by

$$v_z(t) = c\exp(-t/\tau) \qquad \text{with} \quad c = 299\,792\,458\,\mathrm{m/s}\,,\ \tau = 0.08\,\mathrm{s}\,,$$

and the pulse is located at $z = -2\,\mathrm{m}$ at $t = 1\,\mathrm{s}$.

1. Find the expression for the position $z(t)$ of the pulse as a function of coordinate time.
2. Find the location of the pulse at time $t = 1.01\,\mathrm{s}$.

Example solutions

2.3

1. The derivative of a constant is zero, so the velocity is $\boldsymbol{v}(t) = 0\,\mathrm{m/s}$. We must not forget the correct units!
2. No, we can have zero velocity and non-zero acceleration at a given time. See exercise 2.5 as an example.
3. No, we can have positive velocity and negative acceleration at a given time. See exercise 2.5 as an example. It means that, at that time, the movement is in the positive-x direction (positive x-velocity), and the x-velocity is decreasing – that is, it will be positive but smaller a very short time later.

2.4

1. It is, provided that t has dimension time and x has dimension length. In this case, since a has dimension length/time, then $a\,t$ has dimension length, which is added to b which also has dimension length; the left and right side have then both dimension length.
2. $x_{\mathrm{S}}(0\,\mathrm{s}) = 7\,\mathrm{m}\ , \qquad x_{\mathrm{S}}(-10\,\mathrm{s}) = 37\,\mathrm{m}\ , \qquad x_{\mathrm{S}}(5\,\mathrm{s}) = -8\,\mathrm{m}\ .$
3. The question doesn't make sense, because "$t = 10$" is dimensionless; it should have dimension length instead.
4. Denoting with $\dot{x}_{\mathrm{S}}$ the coordinate velocity of S, then $\dot{x}_{\mathrm{S}}(t) = a$, which is constant in time.
5. $\dot{x}_{\mathrm{S}}(t) = -3\,\mathrm{m/s}$ at any time.
6. The speed is $|\dot{x}_{\mathrm{S}}(t)| = 3\,\mathrm{m/s}$ at any time.
7. Denoting with $\ddot{x}_{\mathrm{S}}$ the coordinate acceleration of S, then $\ddot{x}_{\mathrm{S}}(t) = 0\,\mathrm{m/s^2}$, which is zero at all times.

2.5

1. The expression is dimensionally correct, provided t has dimension time and x has dimension length. The argument of the sine function is dimensionless, and the two terms on the right have dimension length.
2. From the rules for the derivative,

$$\dot{x}_{\mathrm{S}}(t) = \omega L \cos(\omega t)\ , \qquad \ddot{x}_{\mathrm{S}}(t) = -\omega^2 L \sin(\omega t)\ .$$

3. The time t_0 must satisfy the system of equations

$$\omega L\cos(\omega t) = 0\,\mathrm{m/s} \qquad -\omega^2 L\sin(\omega t) \approx -2.2\,\mathrm{m/s^2}\,.$$

The cosine is zero when its argument is $\pi/2$, $3\pi/2$, and so on. Let's try taking $\omega t_0 = \pi/2$, which means $t_0 = \pi/(2\omega)$. We find indeed

$$\dot{x}_S(t_0) = \omega L\cos(\omega t_0) = 0\,\mathrm{m/s} \qquad \ddot{x}_S(t_0) = -\omega^2 L\sin(\omega t_0) \approx -2.19\,\mathrm{m/s^2}\,.$$

4. The time t_1 must satisfy the system of equations

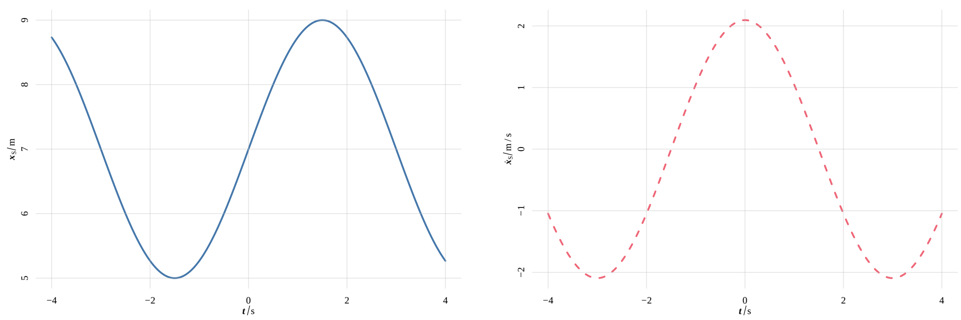
$$\omega L\cos(\omega t) = -2.1\,\mathrm{m/s} \qquad -\omega^2 L\sin(\omega t) \approx 0\,\mathrm{m/s^2}\,.$$

The sine is zero when its argument is 0, π, and so on. Let's try taking $\omega t_1 = 0$, which means $t_1 = 0\,\mathrm{s}$. We find

$$\omega L\cos(\omega t_1) \approx 2.09\,\mathrm{m/s} \qquad -\omega^2 L\sin(\omega t_1) = 0\,\mathrm{m/s^2}\,,$$

which is not what we want. Trying next $\omega t_1 = \pi$, which means $t_1 = \pi/\omega$, leads to the desired result.

5. We can plot x_S and $\dot{x}_S$ in two separate graphs:



Or we could plot them on the same graph – but only if we indicate separately the vertical axis for x_S and the one for $\dot{x}_S$ (for instance one on the left and one on the right), because these quantities have different dimensions.

2.6

1. No, the expression is not dimensionally correct, because the z component of $\boldsymbol{r}_S$ is "0", which is a dimensionless number, whereas z has dimension length. The z component should be "$0\,\mathrm{m}$".

2. See exercises 2.4 and 2.5 ☻

2.7

1. Let's call the specific position $z_0 := -2\,\mathrm{m}$ at $t_0 1\,\mathrm{s}$. The expression for $z(t)$ is found integrating $v_z(t)$ between t_0 and t, and adding the position at t_0:

$$\begin{aligned} z(t) &= z_0 + \int_{t_0}^{t} c \exp(-t/\tau)\,\mathrm{d}t \\ &= z_0 - c\tau \exp(-t/\tau)\big|_{t_0}^{t} \\ &= z_0 - c\tau\left[\exp(-t/\tau) - \exp(-t_0/\tau)\right] \\ &\qquad \text{with} \quad c = 299\,792\,458\,\mathrm{m/s}\,,\ \tau = 0.08\,\mathrm{s}\,,\ z_0 = -2\,\mathrm{m}\,,\ t_0 = 1\,\mathrm{s}\,. \end{aligned}$$

2. Substituting in the expression above, $z(1.01\,\mathrm{s}) = 8.50\,\mathrm{m}$.

URLs for chapter 2

1. https://www.youtube.com/c/veritasium/videos
2. https://www.youtube.com/watch?v=pTn6Ewhb27k
3. https://youtu.be/pTn6Ewhb27k?t=297
4. https://youtu.be/pTn6Ewhb27k?t=588

Main physical quantities 3

3.0

(Do the exercises in the main text.)

3.1

We have seen that six of the seven main physical quantities have two important properties:

- We can speak about their *content*, *flux*, and *supply*.
- They are *extensive*: for instance if a 3D region consists of two non-overlapping 3D subregions, then the content in the region is the sum of the content in the subregions. Similarly for flux and surfaces (2D regions).

1. Can you think of some physical quantity that has the property of extensivity, but for which it doesn't make sense to speak of "content" or of "flux" or of "supply"?

2. Vice versa, can you think of some physical quantity for which we can speak of content, flux, supply, but which doesn't have the property of extensivity?

3.2

Do a little research, and find out whether there are any physics disciplines in which *mass* is usually measured in units of *energy*.

3.3

Do a little research, and find out whether there are any physics disciplines in which *mass* is usually measured in units of *energy*.

3.4

With a friend or colleague or a large language model:

Take turns to explain to each other what is the difference between *matter* and *mass*. Try to find weak points in each other's explanations.

3.5

1. Imagine that someone tells you this:

   > An important difference between *matter* and *electric charge* is that electric charge can be both positive and negative, whereas matter can only be positive.

   How would you reply? Can you give counterexamples to this statement?

2. The same person tells you:

   > In nature we observe both positive and negative electric charge equally easily. But we mostly observe 'positive' matter, and very rarely 'negative' matter (antimatter).

   Do some research and find out whether this statement is true.

3.6

Someone tells you:

> *Mass* and *energy* are two different things. I can experimentally prove it to you: Take a battery for example, and weigh it to measure its mass. Now use the battery for some device. As you use it, the battery loses energy; in fact eventually it can't power the device anymore. But if you weigh it again, you'll find that it has the same mass as before. Therefore mass and energy must be two different things.

How would you reply to this person?

3.7

Let's say you use a coordinate system (x, y, z). In a given 3D region of space you measure a net amount (content) of momentum $\boldsymbol{P} = [2, -3, 0]\,\mathrm{N\,s}$. Which of the following statements are true? which false? Explain why.

1. There must be some non-zero electric charge in the 3D region.
2. The net amount of momentum has zero z-component.
3. There must be some matter (or antimatter) in the 3D region.
4. The 3D region must be enough small.
5. Some kind of motion, with respect to your coordinate system, must be occurring in the 3D region.
6. Any scientist measuring the net momentum in the 3D region would agree that its value is $[2, -3, 0]\,\mathrm{N\,s}$.
7. Whatever it is that contributes to the net momentum, it must be uniformly spread out through the whole 3D region.

Example solutions

3.1

[Before reading this answer, keep in mind that the word volume *has two different meanings: sometimes we use it in the sense of "3D region of space"; sometimes in the sense of the* size *of a 3D region of space (measured in cubic metres for instance).]*

1. One example is the *volume* of a 3D region. It is extensive, because the volume of two non-overlapping 3D regions together is the sum of their volumes. But it doesn't make much sense to speak of the "flux" of volume through a surface. Similarly for *area*.
2. The writer of this solution doesn't know any example of such a physical quantity. It is possible to define mathematical objects that behave this strange way, but no physical quantity seems to be represented by such objects.

3.3

One example is particle physics[1], where the rest mass of subatomic particles is measured in *electronvolts*, denoted 'eV', which is a unit for energy equal to $1.602\,176\,634 \times 10^{-19}$ J. Other examples are special relativity and general relativity.

3.7

1. *False.* There can be momentum in a region even if the net electric charge is zero.
2. *True.* z is the third coordinate, and the third momentum component is $0\,\mathrm{N\,s}$.
3. *False.* Electromagnetic fields have momentum, so in the region there could be only an electromagnetic field, such as a beam of light, but no matter.
4. *False.* We can speak of the net amount of momentum in arbitrarily large regions.
5. *True.* Momentum is associated with the motion of matter or of electromagnetic fields.

6. *False.* The amount of momentum depends on the coordinate system we choose; so scientists that use coordinates different from yours will measure a different net momentum – it could even be completely zero.
7. *False.* The matter or electromagnetic field that possess the momentum might be concentrated in one or several small regions within the 3D region, for example.

URLs for chapter 3

1. https://cms.cern/content/glossary#E

Volume contents, fluxes, supplies 4

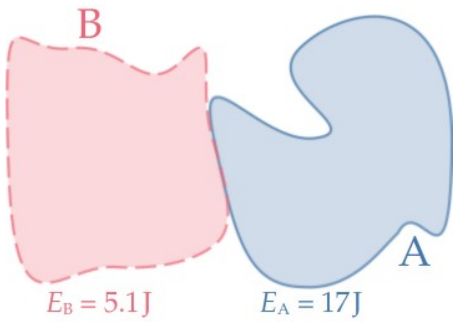
4.0

(Do the exercises in the main text.)

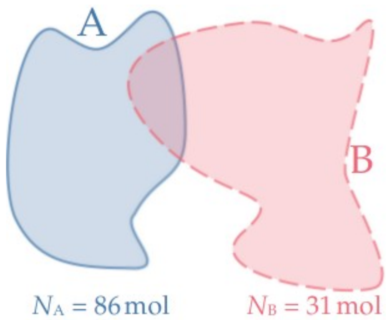
4.1

In the following figures, region A is indicated in blue and a solid contour, and region B is indicated in red and a dashed contour (they are 2D simplified representations of 3D regions). For each figure determine, if possible, the net volume content of the corresponding quantities in the region comprising A and B. If not possible, explain why.

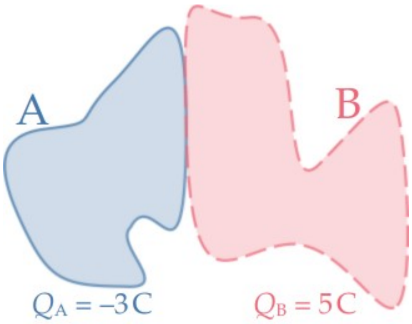
1. Energy-mass:



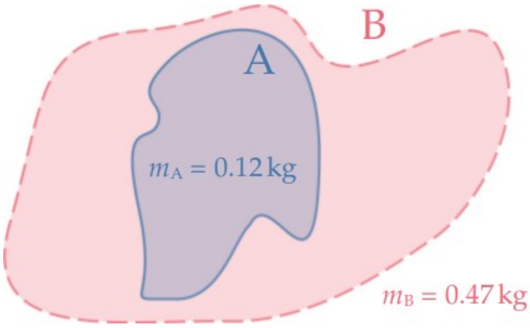
2. Matter:



3. Electric charge:



4. Energy-mass:

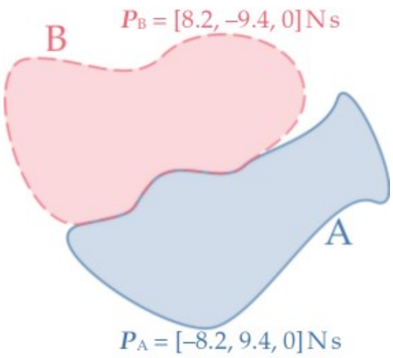


4.2

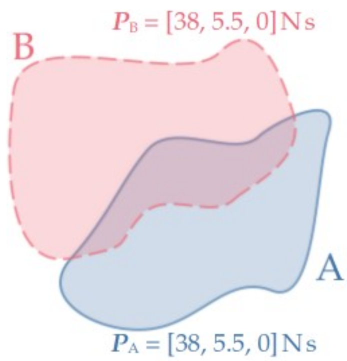
In the following figures, region A is indicated in blue and a solid contour, and region B is indicated in red and a dashed contour (they are 2D simplified representations of 3D regions). Use a coordinate system (x, y, z) where x is horizontal to the right, y is vertical upwards on the page, and z comes out of the page towards you. For each figure:

- draw the vectors representing the momentum contents of A and B;
- determine, if possible, the net volume content of momentum in the region comprising A and B; if not possible, explain why.

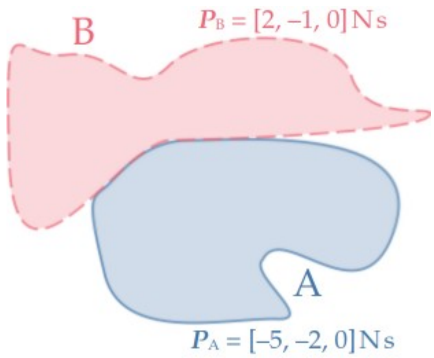
1.



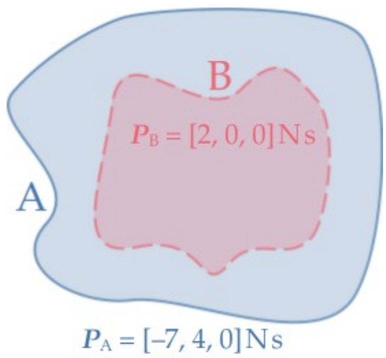
2.


$P_B = [38, 5.5, 0]$ Ns
B
A
$P_A = [38, 5.5, 0]$ Ns

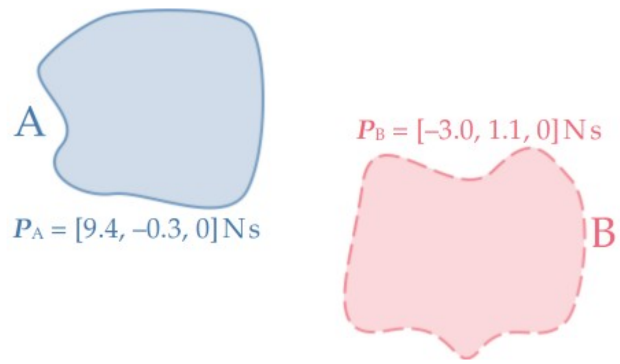
3.


B
$P_B = [2, -1, 0]$ Ns
A
$P_A = [-5, -2, 0]$ Ns

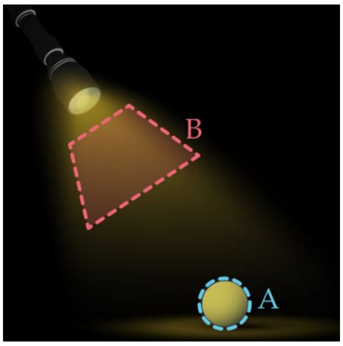
4.


B
$P_B = [2, 0, 0]$ Ns
A
$P_A = [-7, 4, 0]$ Ns

5.


A
$P_A = [9.4, -0.3, 0]$ Ns
$P_B = [-3.0, 1.1, 0]$ Ns
B

4.3

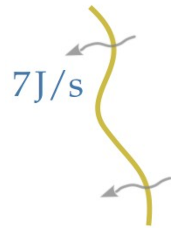


In the side figure, consider volume A, which contains the ball, and volume B, occupied by the light beam. Imagine there's no air (although in reality if there were no air we wouldn't be able to see the light beam).

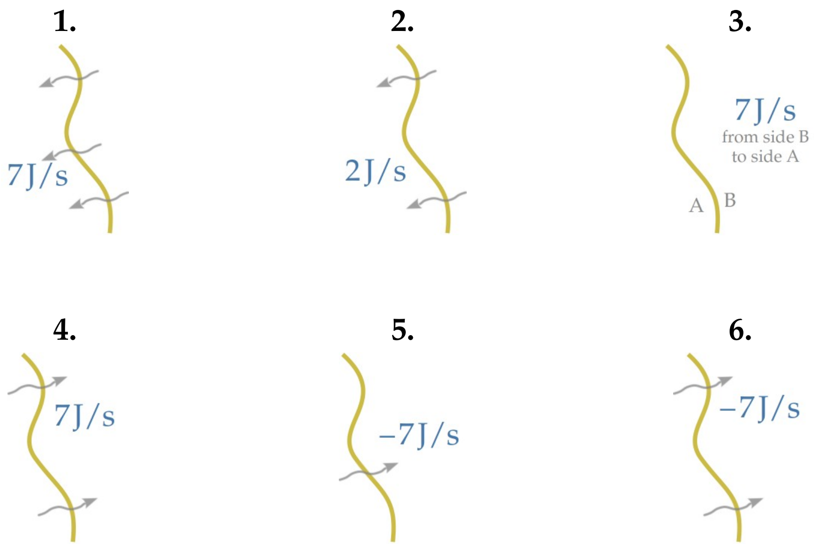
In a given coordinate system, the momentum content of volume A is $\boldsymbol{P}_{\mathrm{A}} = [1, 0, 0]\,\mathrm{N\,s}$, and that of volume B is $\boldsymbol{P}_{\mathrm{B}} = [2, -2, 1] \times 10^{-16}\,\mathrm{N\,s}$.

Does it really make sense to speak of the momentum in volume B (remember there's no air there)? If it does make sense, how much is the net momentum content of volumes A and B considered together?

4.4



Take a look at the energy flux through a surface represented in the side figure. Which of the representations below is completely equivalent to the one on the side? Which does represent a different flux instead?



4.5

A cuboid region is delimited by a closed control surface that can be divided into six parts; call them 'up', 'down', 'front', 'back', 'left', 'right'. They are all have an inward crossing direction, except for 'back' which has an outward crossing direction.

Here are the fluxes of various quantities through the six surfaces in the given crossing directions. Calculate the total net **in**fluxes.

1. $J_\mathrm{u} = 23.9\,\mathrm{mol/s}$ $J_\mathrm{d} = -8.1\,\mathrm{mol/s}$ $J_\mathrm{f} = 0.9\,\mathrm{mol/s}$
   $J_\mathrm{b} = -30.5\,\mathrm{mol/s}$ $J_\mathrm{l} = 2.3\,\mathrm{mol/s}$ $J_\mathrm{r} = 37.6\,\mathrm{mol/s}$

2. $\Phi_\mathrm{u} = -24.6\,\mathrm{J/s}$ $\Phi_\mathrm{d} = 2.4\,\mathrm{J/s}$ $\Phi_\mathrm{f} = 1.3\,\mathrm{J/s}$
   $\Phi_\mathrm{b} = 10.8\,\mathrm{J/s}$ $\Phi_\mathrm{l} = 15.4\,\mathrm{J/s}$ $\Phi_\mathrm{r} = -2.1\,\mathrm{J/s}$

3. $\mathcal{I}_\mathrm{u} = -9.7\,\mathrm{C/s}$ $\mathcal{I}_\mathrm{d} = 27.4\,\mathrm{C/s}$ $\mathcal{I}_\mathrm{f} = -6.3\,\mathrm{C/s}$
   $\mathcal{I}_\mathrm{b} = 16.4\,\mathrm{C/s}$ $\mathcal{I}_\mathrm{l} = -25.1\,\mathrm{C/s}$ $\mathcal{I}_\mathrm{r} = -20.0\,\mathrm{C/s}$

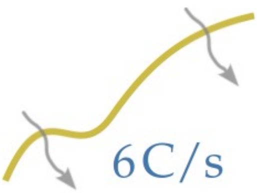
4. $\Pi_\mathrm{u} = 31.7\,\mathrm{J/(K\,s)}$ $\Pi_\mathrm{d} = 17.9\,\mathrm{J/(K\,s)}$ $\Pi_\mathrm{f} = 7.2\,\mathrm{J/(K\,s)}$
   $\Pi_\mathrm{b} = -20.4\,\mathrm{J/(K\,s)}$ $\Pi_\mathrm{l} = -16.5\,\mathrm{J/(K\,s)}$ $\Pi_\mathrm{r} = -4.8\,\mathrm{J/(K\,s)}$

5. $\boldsymbol{F}_\mathrm{u} = [25.2, -42.7, 4.1]\,\mathrm{N}$ $\boldsymbol{F}_\mathrm{d} = [-46.3, -5.9, -33.3]\,\mathrm{N}$
   $\boldsymbol{F}_\mathrm{f} = [10.2, -16.2, -36.7]\,\mathrm{N}$ $\boldsymbol{F}_\mathrm{b} = [-9.8, 19.5, -60.0]\,\mathrm{N}$
   $\boldsymbol{F}_\mathrm{l} = [29.5, 18.3, 58.4]\,\mathrm{N}$ $\boldsymbol{F}_\mathrm{r} = [16.0, 1.2, -3.2]\,\mathrm{N}$

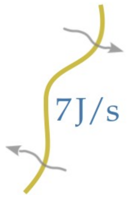
4.6

Which of the following graphical representations of fluxes do make sense? Which don't? Explain why.

1.

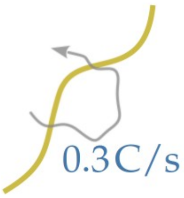


2.



3.

4.



5.

6.

7.

8.

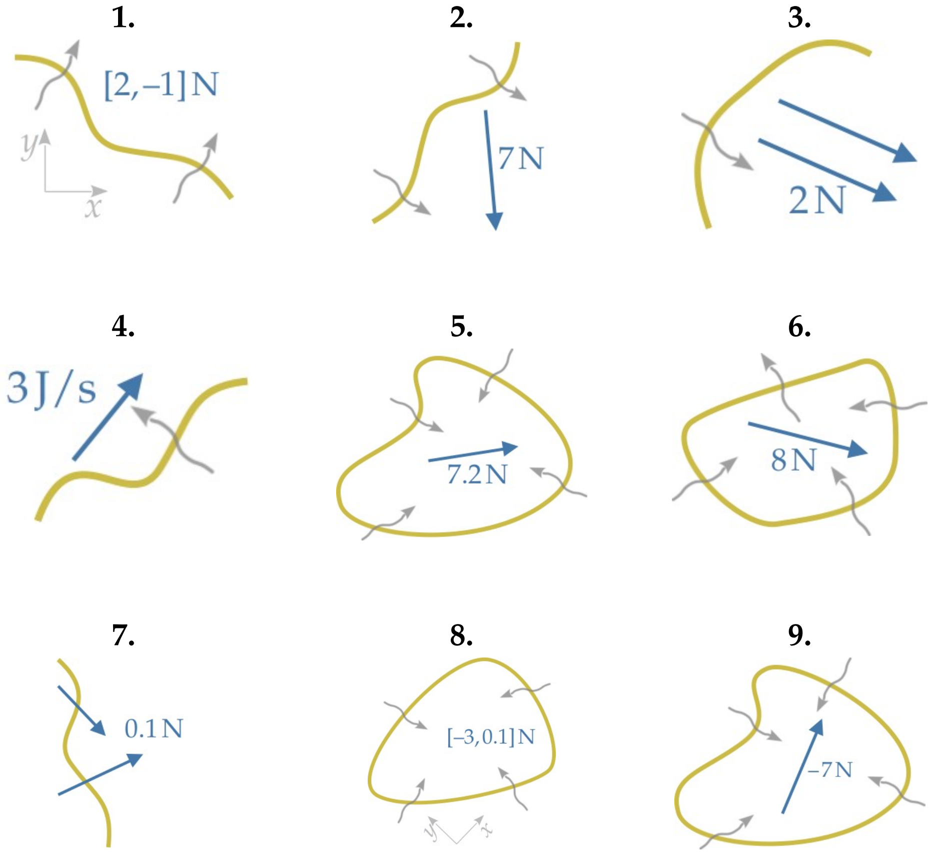
9.

4.7

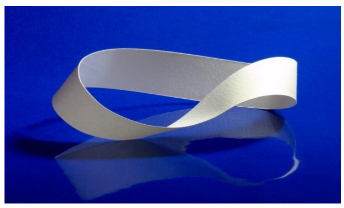
1. Suppose that through a control surface, in a given crossing direction, there is a non-zero flux of momentum. Momentum is a quantity related to movement. Does this mean that the control surface must be in motion?
2. If through a control surface there's a non-zero flux of some quantity, does it mean that the fluxes of all other quantities must be zero?
3. Can there be a flux of temperature through a control surface?

4.8

Which of the following graphical representations of fluxes do make sense? Which don't? Explain why.



4.9



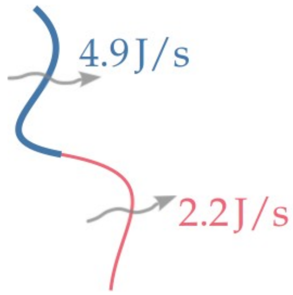
The Möbius band[1] is a surface that, in a certain sense, has only *one* side. It is quite easy to make with a strip of paper and some tape.

Do you manage to choose a definite crossing direction on this surface? Do you think it could be chosen as a control surface?

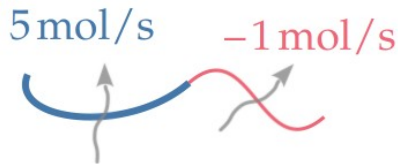
4.10

For each figure, calculate and draw, or describe precisely, the net total flux through the composite control surface. (The different parts are differentiated by different colours and thicknesses.) Don't forget to specify the crossing direction when you speak about a flux.

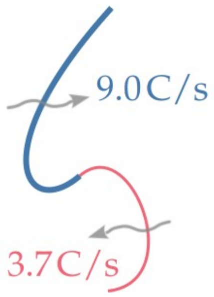
1.



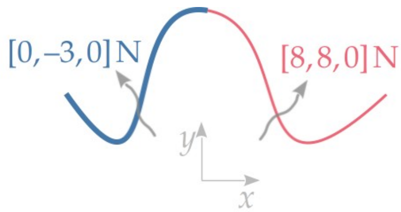
2.



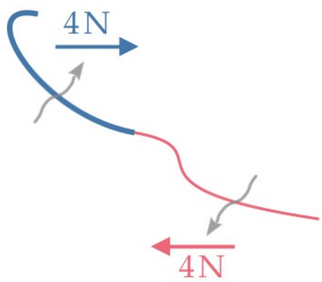
3.



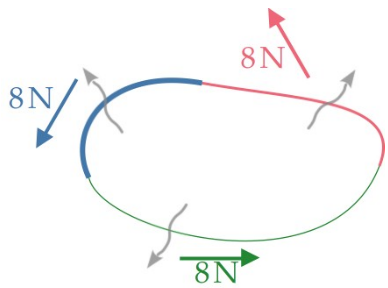
4.



5.



6.



4.11

1. Through a particular closed control surface there's a continuous influx of electric charge $\mathcal{I}(t)$ with the following time dependence:

$$\mathcal{I}(t) = A\sin(\omega t) \qquad \text{with} \quad A = 30\,\mathrm{C/s}\,, \quad \omega = 376.991\,184\,\mathrm{s}^{-1}\,.$$

   How much is the time-integrated influx of charge between times $t_0 = 10\,\mathrm{s}$ and $t_1 = 40\,\mathrm{s}$?

2. Imagine a closed control surface around the planet Mercury. The energy $\Phi(t)$ influx through this control surface is approximately given by

$$\Phi(t) = W + u\ \cos(\sigma t)$$
$$\text{with} \quad W = 2 \times 10^{17}\,\mathrm{J/s}\,, \quad u = 8 \times 10^{16}\,\mathrm{J/s}\,, \quad \sigma = 8 \times 10^{-7}\,\mathrm{s}^{-1}\,.$$

Mercury's year lasts around 88 Earth-days. How much is the time-integrated influx of energy over a year on Mercury?

3. A football has a mass-energy $m = 0.4\,\mathrm{kg}$. The football has a continuous supply of momentum $\boldsymbol{G}(t)$ from the Earth's gravitational field. This supply is constant in time and given by

$$\boldsymbol{G}(t) = m\boldsymbol{g} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

where $\boldsymbol{g} = 9.8\,\mathrm{N/kg}$, and we are using a coordinate system (t, x, y, z) where z is vertical with respect to the ground and points upwards.

How much is the time-integrated influx of momentum in the football between times $t_0 = 0\,\mathrm{s}$ and $t_1 = 3600\,\mathrm{s}$?

Example solutions

4.1

1. Net energy-mass content is $E_{\mathrm{A}} + E_{\mathrm{B}} = 22.1\,\mathrm{J}$.
2. Unknown: A and B are overlapping, so we cannot simply add their contents. The result depends on the net matter content in the overlap region, which is unknown.
3. Net electric-charge content is $Q_{\mathrm{A}} + Q_{\mathrm{B}} = 2\,\mathrm{C}$.
4. A and B are overlapping, so we cannot simply add their mass-energy contents. But in this case A is a subregion of B, so the full region occupied by both is B itself. The net mass-energy content in this region is therefore $m_{\mathrm{B}} = 0.47\,\mathrm{kg}$. We can also deduce that the net mass-energy content in the region between A and B is $m_{\mathrm{B}} - m_{\mathrm{A}} = 0.35\,\mathrm{kg}$.

4.2

1. Net momentum content is $\boldsymbol{P}_{\mathrm{A}} + \boldsymbol{P}_{\mathrm{B}} = [0, 0, 0]\,\mathrm{N\,s}$.
2. Unknown: A and B are overlapping, so we cannot simply add their momentum contents. The result depends on the net momentum content in the overlap region, which is unknown.
3. Net momentum content is $\boldsymbol{P}_{\mathrm{A}} + \boldsymbol{P}_{\mathrm{B}} = [-3, -3, 0]\,\mathrm{N\,s}$.
4. A and B are overlapping, so we cannot simply add their momentum contents. But in this case B is a subregion of A, so the full region occupied by both is A itself. The net momentum content in this region is therefore $\boldsymbol{P}_{\mathrm{A}} = [-7, 4, 0]\,\mathrm{N\,s}$. We can also deduce that the net momentum content in the region between B and A is $\boldsymbol{P}_{\mathrm{A}} - \boldsymbol{P}_{\mathrm{B}} = [-9, 4, 0]\,\mathrm{N\,s}$.
5. Net momentum content is $\boldsymbol{P}_{\mathrm{A}} + \boldsymbol{P}_{\mathrm{B}} = [6.4, 0.8, 0]\,\mathrm{N\,s}$. The region consisting of A and B together is a spatially disconnected region, but we can speak of volume content even for such kind of regions.

4.3

Yes, it does make sense. First of all, we can *always* speak of the momentum content in *any* 3D region; it isn't important whether there's matter or electromagnetic fields or not in that region. If it's completely empty, its momentum content is simply zero. Second, in volume B in the figure there's an electromagnetic field, which does have momentum just like matter.

The net momentum content in the volumes A and B considered together is $\boldsymbol{P}_\mathrm{A} + \boldsymbol{P}_\mathrm{B} \approx [1, -2 \times 10^{-16}, 1 \times 10^{-16}]\,\mathrm{N\,s}$.

4.4

1.	2.	3.
Same 👍 Same crossing direction, same flux magnitude. Wiggly arrows only indicate the crossing direction, it doesn't matter how many there are.	**Different** 👎 This flux has a different magnitude.	**Same** 👍 Same crossing direction, same flux magnitude. Crossing direction is indicated verbally.
4.	**5.**	**6.**
Different 👎 This flux has same magnitude but opposite direction.	**Same** 👍 Same flux, by the principle of symmetry of flux.	**Same** 👍 Same flux, by the principle of symmetry of flux.

4.5

Because of extensivity:

1. $J_\mathrm{tot} = J_\mathrm{u} + J_\mathrm{d} + J_\mathrm{f} - J_\mathrm{b} + J_\mathrm{l} + J_\mathrm{r} = 87.1\,\mathrm{mol/s}$
2. $\Phi_\mathrm{tot} = \Phi_\mathrm{u} + \Phi_\mathrm{d} + \Phi_\mathrm{f} - \Phi_\mathrm{b} + \Phi_\mathrm{l} + \Phi_\mathrm{r} = -18.4\,\mathrm{J/s}$
3. $\mathcal{I}_\mathrm{tot} = \mathcal{I}_\mathrm{u} + \mathcal{I}_\mathrm{d} + \mathcal{I}_\mathrm{f} - \mathcal{I}_\mathrm{b} + \mathcal{I}_\mathrm{l} + \mathcal{I}_\mathrm{r} = -50.1\,\mathrm{C/s}$
4. $\Pi_\mathrm{tot} = \Pi_\mathrm{u} + \Pi_\mathrm{d} + \Pi_\mathrm{f} - \Pi_\mathrm{b} + \Pi_\mathrm{l} + \Pi_\mathrm{r} = -50.1\,\mathrm{C/s}$
5. $\boldsymbol{F}_\mathrm{tot} = \boldsymbol{F}_\mathrm{u} + \boldsymbol{F}_\mathrm{d} + \boldsymbol{F}_\mathrm{f} - \boldsymbol{F}_\mathrm{b} + \boldsymbol{F}_\mathrm{l} + \boldsymbol{F}_\mathrm{r} = [44.4, -64.8, 49.3]\,\mathrm{C/s}$

4.6

1. **Makes sense**	**2.** **No** Crossing direction is unclear.	**3.** **No** 'J' is not the unit of a flux.
4. **No** Crossing direction is unclear.	**5.** **No** Crossing direction is missing.	**6.** **Makes sense** The arrows for the crossing direction are very tilted, but crossing direction is still clear.
7. **No** 'N' is the unit of a **vector** flux, but there's no vector or vector components here.	**8.** **Makes sense** Alternative way of indicating the crossing direction, but perfectly clear.	**9.** **Makes sense**

4.7

1. No, the control surface may also be completely still (in that coordinate system). For several reasons:
   - *Volume content* of momentum means that something (matter or electromagnetic field) inside the volume must be moving.
   - A control surface may be completely imaginary, so it doesn't need to be something that is affected by any physical influence.

2. No: *in principle* the fluxes of the seven quantities are independent and can all be non-zero. In fact, in many physical situations they are connected by particular physical laws.

3. No: we can speak of the temperature at a point at a given time, but we cannot speak of volume content or flux or supply of temperature. Temperature is not an extensive quantity.

4.8

1.
Makes sense 👍
This vector flux is expressed in components, and the coordinates are shown: we can reconstruct the vector if needed.

2.
Makes sense 👍

3.
No 👎
There should be only one vector indicating the flux. Unclear if this flux should have magnitude 4 N.

4.
No 👎
'J/s' is the unit of a **scalar** flux, but there's a vector here. Either the unit is wrong or the vector is there by mistake.

5.
Makes sense 👍
This is a total influx.

6.
No 👎
The wiggly arrows that indicate the crossing direction are not mutually consistent.

7.
No 👎
Unclear if one of the two arrows indicates a crossing direction.

8.
Makes sense 👍
Total influx again. The flux is expressed in components, and the coordinates are shown.

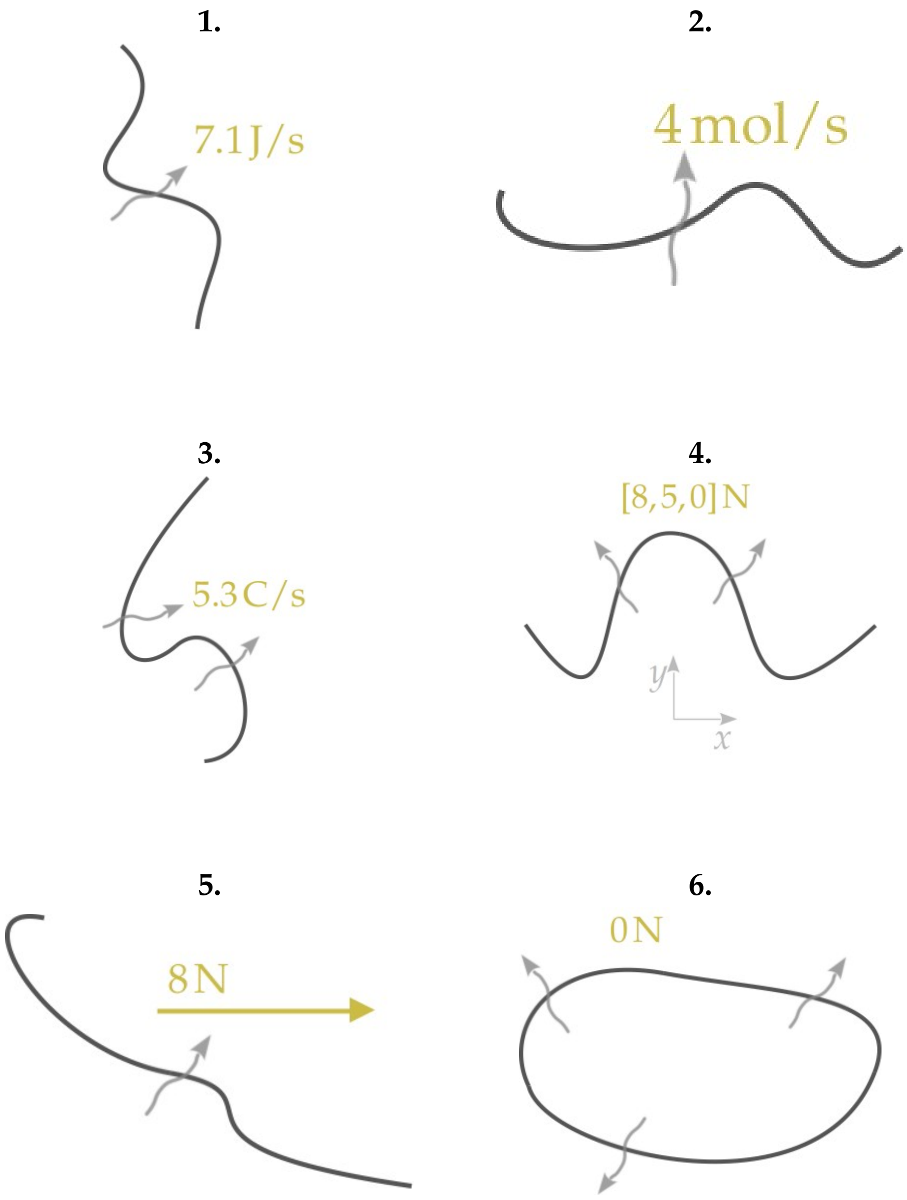
9.
No 👎
The magnitude of a vector cannot be negative.

4.9

You have noticed that if you try to choose consistently a crossing direction on the whole band, you end up where you started but with an *opposite* crossing direction. For this reason the Möbius band cannot be used as a control surface: it cannot be given an overall crossing direction. The lack of crossing direction is related to another peculiarity: it is impossible to extend this surface in such a way as to enclose a three-dimensional region of space.

If we clip a part of the Möbius band, that part can be used as a control surface. Or if we clip the band so that it gets an extra border, then it becomes the same as a twisted rectangle, and so it can be used as a control surface.

4.10



4.11

1. The time-integrated flux of charge is found with a time integral. Note that the result has a dimension of electric charge: this is the net amount of charge that flowed through the control surface, in the given crossing direction, between the two times:

$$\begin{aligned}\int_{t_0}^{t_1} \mathcal{I}(t)\,\mathrm{d}t = \int_{t_0}^{t_1} A\sin(\omega t)\,\mathrm{d}t &= -\frac{A}{\omega}\cos(\omega t)\Big|_{t_0}^{t_1} \\ &\approx -796\,\mathrm{C}\cdot(1-1) = 0\,\mathrm{C}\,.\end{aligned}$$

So even if the flux of charge most of the time is non-zero, reaching a maximum of $\pm 30\,\mathrm{C/s}$, the *net* amount of charge crossing the surface in the given direction during the 30 s is zero. This is because the flux is sometimes positive, sometimes negative.

2. 88 Earth-days are approximately equal to $88\cdot 24\cdot 60\cdot 60\,\mathrm{s}\approx 7\,600\,000\,\mathrm{s}$. So we time-integrate between $t_0 = 0\,\mathrm{s}$ and $t_1 = 7\,600\,000\,\mathrm{s}$:

$$\begin{aligned}\int_{t_0}^{t_1} \Phi(t)\,\mathrm{d}t = \int_{t_0}^{t_1} \bigl[W + u\,\cos(\sigma t)\bigr]\,\mathrm{d}t &= Wt\Big|_{t_0}^{t_1} + \frac{u}{\sigma}\sin(\sigma t)\Big|_{t_0}^{t_1} \\ &\approx 1.5\times 10^{24}\,\mathrm{J} + 1\times 10^{23}\,\mathrm{J}\cdot(-0.2-0) \\ &\approx 1.5\times 10^{24}\,\mathrm{J}\,.\end{aligned}$$

3. Recall that time-integrating a vector simply means time-integrating each component. In this case the supply is constant, so integration is easy:

$$\begin{aligned}\int_{t_0}^{t_1} \boldsymbol{G}(t)\,\mathrm{d}t &= \int_{t_0}^{t_1} m\boldsymbol{g}\begin{bmatrix}0\\0\\-1\end{bmatrix}\mathrm{d}t \\ &= m\boldsymbol{g}\begin{bmatrix}\int_{t_0}^{t_1} 0\,\mathrm{d}t\\ \int_{t_0}^{t_1} 0\,\mathrm{d}t\\ -\int_{t_0}^{t_1} 1\,\mathrm{d}t\end{bmatrix} = m\boldsymbol{g}\begin{bmatrix}0\,\mathrm{s}\\0\,\mathrm{s}\\-t\end{bmatrix}\Bigg|_{t_0}^{t_1} = m\boldsymbol{g}\left(\begin{bmatrix}0\,\mathrm{s}\\0\,\mathrm{s}\\-t_1\end{bmatrix} - \begin{bmatrix}0\,\mathrm{s}\\0\,\mathrm{s}\\-t_0\end{bmatrix}\right) \\ &\approx 3.9\,\mathrm{N}\cdot\begin{bmatrix}0\\0\\-3600\end{bmatrix}\mathrm{s} \approx \begin{bmatrix}0\\0\\-1.4\times 10^{4}\end{bmatrix}\mathrm{N\,s}\,.\end{aligned}$$

This net amount of momentum is vertical, pointing downward. But note that we don't know whether the football contains this much momentum after 3600 s, because we don't know how much the flux of momentum into the football was. It's possible that the flux of momentum completely balanced this supply of momentum.

URLs for chapter 4

1. https://mathworld.wolfram.com/MoebiusStrip.html

Physical laws 5

5.0

(Do the exercises in the main text.)

Example solutions

The Seven Wonders of the world 6

6.0

(Do the exercises in the main text.)

Example solutions

Conservation & balances of matter 7

7.0

(Do the exercises in the main text.)

Example solutions

Conservation of electric charge 8

8.0

(Do the exercises in the main text.)

Example solutions

Conservation of magnetic flux 9

9.0

(Do the exercises in the main text.)

Example solutions

Balance of momentum 10

10.0

(Do the exercises in the main text.)

Example solutions

Balance of energy 11

11.0

(Do the exercises in the main text.)

Example solutions

Balance of angular momentum 12

12.0

(Do the exercises in the main text.)

Example solutions

Remarks on momentum and energy 13

13.0

(Do the exercises in the main text.)

Example solutions

Balance of entropy 14

14.0

(Do the exercises in the main text.)

Example solutions

Constitutive relations 15

15.0

(Do the exercises in the main text.)

Example solutions