Technical Developments: Input and Organization of Sub-Routines for Ferut
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179[L].—T. H. Crowley, *Tables of Integrals of certain Bessel Functions.* Available at the Antenna Laboratory, Ohio State University, Columbus, Ohio.

These tables give values of the integrals

$$\int_0^u J_0(\lambda x) \sin x \, dx \quad \text{and} \quad \int_0^u J_0(\lambda x) \cos x \, dx$$

for

$$u = 0(.02)10, \quad \lambda = 0(.1)10, \quad \lambda u \leq 15.$$

Although the calculations were designed to give 4D accuracy, spot checking indicates an accuracy of 5D.

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180[L].—E. W. Pike, *Table of Parameters for the Summation Analogue of Laguerre Polynomials.* Two typewritten pages on deposit in the UMT File.

This table is of use in the design of filters for pulsed information.

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AUTOMATIC COMPUTING MACHINERY

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Technical Developments

INPUT AND ORGANIZATION OF SUB-ROUTINES FOR FERUT

1. Introduction.—Descriptions of methods for handling sub-routines on other machines have been written for the Manchester Electronic Computer,[1] EDSAC,[2] SEAC[3] and ILLIAC[4]. The purpose of this article is to discuss the input and storage of routines and numerical data and the organization of routines during the solution of a problem on Ferut.

Ferut is the serial, one address, electronic digital computer now in operation in the Computation Centre, University of Toronto, Canada. It was built by Ferranti Ltd., Manchester, England, and is similar to the computer at the University of Manchester, England. Ordinary 5-hole telegraphic punched tape is used for input with a photoelectric reader. The output mechanism is a teleprinter and punch.

Information in the machine is kept in blocks or pages. In each page there are 64 short lines of 20 binary digits each. For convenience these 20 bits are arranged in four groups of five bits. The five-bit group constitutes a digit in the scale of 32 and is represented by one teleprint character. An electronic instruction consists of one short line, two teleprint characters for the address, and two for the order. Numbers consist of two short lines or 40 bits.

The electronic storage comprises eight pages labelled $S_0, S_1, \cdots, S_7$. In addition, there is a magnetic storage of 256 tracks on a drum, each track holding two pages of information. The pages may be transferred singly or in pairs to or from a magnetic track. This is accomplished by a magnetic transfer instruction.

2. Routine Changing.—During the operation of a program, space in the electronic store is at a premium and thus the instructions necessary to carry out a problem are divided, as is customary, into a main, or master routine and a number of sub-routines. Only the set of instructions immediately in use is kept in the electronic store; the other routines are in the magnetic store. When it is desired to change from one routine to the next, it is necessary to have a group of orders which will (1) preserve, in a list called the link list, the location in magnetics of the routine being left, (2) record the line at which it is being left, (3) locate the next routine and call it down from magnetics and finally (4) enter the new routine at the appropriate line. This group of instructions is common to all routine changes and thus is kept separate from any one routine in a *Routine Changing Sequence* (R.C.S.) which is permanently down in the electronic store. (This occupies page S_2 along with powers of 2 for shifting and some other constants which are frequently required, and the page is known as PERM.)

There must be, however, in the routine being left a few instructions supplying the required information about the specific routine to be entered and a control transfer to R.C.S. This linking sequence of orders which appears in the routine being left must be kept to a minimum to conserve space, and in the case of the Ferut system it occupies four short lines. One of these lines is, of course, the magnetic transfer which calls down the next routine from the magnetic store to the electronic store.

This transfer is recorded in the link list by R.C.S. so that it can be used at a later date. The list of links grows as the control moves down a chain of sub-routines changing from the master routine (zero-th level) to the first level sub-routine, to the second level sub-routine, etc. The list is consulted by R.C.S. as the control returns back up the chain of sub-routines to the master routine.

3. Directory.—The magnetic tracks are numbered, and in writing a magnetic transfer it is necessary to specify the track number. For several reasons it may be desirable to change tracks after a problem has been coded and the tapes have been punched. You must then seek out and alter all magnetic transfers which, of course, are embedded in the program. This is not only tedious but leads to errors. To overcome this difficulty all magnetic transfers are kept separate from the routines proper in a list called the directory. In the line of the routine where the transfer should appear is placed the address in the directory of the required magnetic transfer. This line is called a false line. During input the false lines are replaced by the appropriate magnetic transfers from the directory.

If the magnetic transfer required to call down a certain routine is stored in line ab of the directory, ab is said to be the *Directory Number* of the routine. This directory number is what is written in a false line. When a routine has been tested and is considered useful, it may be assigned a

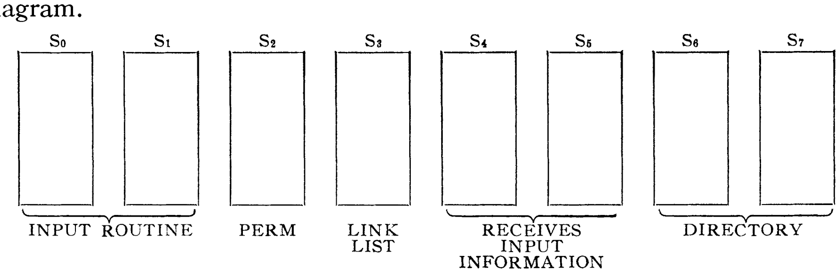
Library Number which will be its permanent directory number. Sixteen lines at the beginning of the directory are kept open for routines of a particular problem. These directory numbers are temporary.

4. Input and Output.—To perform input or output operations it is necessary to combine the simple input or output orders of the machine with other orders in an input or output routine. The function of the Ferut *Input Routine* is to read information from tape, perform certain alterations on routines or numerical data and store the routines or data in assigned locations in the machine.

The input routine reads one character at a time from the tape testing it until it finds a *Warning Character*. There are eleven warning characters. Each of the warning chs. causes control to be taken to a certain section of the input routine where a prescribed group of instructions is obeyed.

An attempt has been made to choose, for warning chs., teleprint characters which are suggestive of the operations which the warning chs. perform. For example—the warning ch. J followed by n/ causes the n chs. following the / to be printed out by the machine. (The ch. J suggests the word JOT.) After the Jn/ the machine reads in n chs. before it begins again to examine the tape for the next warning ch. Since / (zero) is not a warning ch., blank tape is examined rapidly and passed by. Thus between meaningful sequences on the tape (such as the Jn/ followed by n chs.) it does no harm to have spaces, and they are a great assistance when reading the tape by eye.

The use of the electronic pages during input is best described by a diagram.



After input only page S_2 and one half of S_3 are used. Routines normally operate from S_0 and S_1.

The list of warning characters is as follows:

STOP—Stops the tape—control is returned to input to search for the next warning ch. when a prepulse is given manually.

Jn/—Prints out the n chs. on the tape following Jn/—used for titling routines (J suggests JOT).

E ab—Transfers control to line (ab + 1) (E suggests ENTER).

: abcd—Causes abcd to be obeyed as a magnetic instruction (: suggests /: the "obey as a magnetic instruction" order of Ferut).

K ab n—Reads in from the tape n short lines starting the entries in line ab of the store. A maximum of 32 short lines can be read in at a time. (K suggests absolutely nothing but is used for this purpose by all the other input systems.)

$\left.\begin{matrix}\text{U ab—Writes Up}\\ \text{D ab—Reads Down}\end{matrix}\right\}$ the routine whose directory number is ab $\begin{Bmatrix}\text{from}\\ \text{to}\end{Bmatrix}$ page S_4 (or pages S_4 and S_5 if it is a 2 page routine) $\begin{Bmatrix}\text{to}\\ \text{from}\end{Bmatrix}$ the magnetic track assigned in the directory. Check of the magnetic transfer is performed, and a warning signal indicates failure.

$\left.\begin{array}{r}\text{@ abcd}\\ \text{R abcd}\end{array}\right\}$—Calls in as an $\left\{\begin{array}{c}\text{ad-routine}\\ \text{sub-routine}\end{array}\right\}$ the routine whose directory number is ab and enters it at line (cd + 1). (@ sounds like ad—and R is suggestive of Routine.)

F ab—Replaces the false line ab of a routine by the contents of a line of the directory. (Explanation: In order to incorporate a magnetic transfer within a routine in line ab the directory number of the appropriate magnetic transfer is placed in the routine itself in line ab. This is a false line. The warning ch. F replaces this by the appropriate true line, i.e., the magnetic transfer as it appears in the directory. For example: One line of the linking sequence is such a false line and any routine containing a linking sequence must be followed by the proper F sequence.)

C/(2n)—Adds together the 32 long lines of page n and compares the sum modulo 2^{40} with the known *Check Sum* of this page. If the two differ, a signal occurs. The known check sum must, of course, be planted in a certain line of the store.

A wide variety of routines have been devised for handling the input and output of numerical data depending on the position of decimal or binary point, method of conversion, number of significant figures, layout, etc. These are entered as sub-routines from the input routine by an R warning ch. or from the master routine of the problem by a linking sequence. When numerical data are put in, the pages of information can be given directory numbers and thus stored and brought down in a manner similar to that used in handling sub-routines.

The following checks are a part of this system of input. Each Up or Down transfer is checked; thus the contents of magnetic and electronic storage are known to be identical. The transfer from tape to electronic storage is checked by having a known check sum of a routine or of numerical data read in and compared with the check sum of the contents of the electronic page by means of the C warning ch. Alternately, if the check sum is not known, the tape may be read in twice, once with current on which writes from electronic to magnetic storage, the second time with this write current off. The Up transfer check then effectively compares what was read in the first time from tape with what was read in the second time.

Most problem tapes consist of sequences of characters which (i) print the title, (ii) read in and store the directory, (iii) read in, alter and store routines and (iv) start the problem. Titling (see i) is useful for identifying tapes. Process (ii) consists of reading the magnetic transfers for the routines being used into the appropriate lines of the directory. The directory is stored by means of the : warning ch. All routines are read (see iii) by K-sequences into S_4 (or S_4 and S_5 in the case of two-page routines). They are checked if desired by the C ch., altered if necessary, i.e., false lines are replaced by F-sequences, and then stored by the appropriate Up order. The problem is started (see iv) by entering the master routine as an ad-routine by means of an @ sequence on the tape.

Routines may be altered during input by bringing them down with a D-sequence, altering the desired lines by K-sequences and then sending them back up to magnetics by a U-sequence.

Library tapes may be used without alteration in a problem tape. They consist of a titling sequence, K-sequences reading the routine into S_4 or S_4 and S_5 and then the Up order using the library number of the routine. Library tapes have a check sum taken by means of the Down order followed by the C warning ch. After this false lines are replaced by F-sequences, and the routine is stored again by the U-sequence.

5. Example.—Consider the organization of the routines for the two-dimensional Fourier synthesis

$$\rho(y, z) = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} F_{kl} \cos 2\pi(ky + lz)$$

where the coefficients F_{kl} are tabulated. The program for this problem will consist of four routines and the data. Three of the routines to be used are library routines, namely, Decimal Input, Decimal Output, and Cosine. The fourth is the Master routine. The latter must call in Cosine and Decimal Output as sub-routines and thus has a false line in each of the linking sequences calling in these two routines. These false lines have addresses a_1b_1 and a_2b_2. Transfers calling down the data require false lines in a_3b_3, a_4b_4, etc.

The coefficients F_{kl} are taken in and converted from decimal to binary form by the Decimal Input routine before they are stored in the magnetics. This is accomplished during input by entering Decimal Input as a sub-routine from the tape by means of the R warning ch. Details for the routines and data are given in Table I.

The magnetic transfer calling down the Master will be (t_1) $\delta_2 S_0$, where (t_1) represents the two teleprint character track address, S_0 indicates the electronic storage from which the routine operates, and δ_2 denotes that it is a two-page routine. This magnetic transfer must be recorded in line /N of the directory.

TABLE I

Description	Directory Number	Magnetic Storage	Electronic Storage
Master	/N	t_1	S_0, S_1
Decimal Input	SF	t_2	S_0, S_1
Decimal Output	ON	t_3	S_0, S_1
Cosine	XN	t_4 (left half)	S_0
Data 1 (two pages)	EN	t_5	S_4, S_5
Data 2 (two pages)	@N	t_6	S_6, S_7
Data 3 (two pages)	AN	t_7	S_4, S_5
etc.			

The tape is organized in the following way:

Punching	*Description*
T T T T T	T is a letter which is not a warning ch. but indicates the beginning of the tape to the operator.
spaces	. . . .
JZ / FOURIER: SYNTHESIS	Title (Z is the teleprint character for 17.)
spaces	. . . .
	The K-warning ch. reads into the directory the magnetic transfer for bringing down:
K / N E (t_1) δ_2 S_0	the Master
K S F E (t_2) δ_2 S_0	Decimal Input
K O N E (t_3) δ_2 S_0	Decimal Output
K X N E (t_4) δ_1 S_0	Cosine

Punching	*Description*
K E N E (t_5) δ_2 S_4	Data 1
K@ N E (t_6) δ_2 S_6	Data 2
K A N E (t_7) δ_2 S_4	Data 3
etc.	(E is teleprint for 1.)
: I / R N	I / R N is the magnetic transfer instruction which stores the directory on track.
S T O P	
Library tape for Decimal Input Library tape for Decimal Output Library tape for Cosine	these routines all have check sums
Master routine punched with the K-sequences which read it into S_4 and S_5.	
Fa_1b_1 Fa_2b_2 Fa_3b_3 Fa_4b_4, etc.	Replaces false lines in Master by the magnetic transfers from the directory.
U / N	Stores Master on track t_1.
S T O P	
R S F c d Data 1	Enters Decimal Input as a sub-routine at line cd + 1 whereupon Data 1 punched in the appropriate form are read in and converted.
U E N	Stores Data 1 on track t_5.
R S F c d Data 2	Reads in and converts Data 2.
U @ N	Stores Data 2.
R S F c d Data 3	Reads in and converts Data 3.
U A N	Stores Data 3.
etc.	
S T O P	
@ / N c d	Enters the Master as an ad-routine at line cd + 1.
$ $ $ $	A non-warning ch. indicating the end of the tape.

6. Summary.—The Ferut System has the following features:

(1) No reference to fixed magnetic tracks is made except in a special list called the directory.

(2) Minimum time and mental effort are consumed in changing from one routine to another during the operation of a problem.

(3) The details of the input are easy to remember.

(4) Alterations may be made to routines or pages of data with a minimum of effort.

(5) Certain automatic checks are built into the system.

(6) Library tapes may be used without alteration in a problem tape.

This system of input and routine organization was adopted after studying the various systems in use at the University of Manchester.[1] The details were devised by the author in cooperation with C. STRACHEY, National Research Development Corporation, London, England, and H. GELLMAN, Chalk River, Canada, with the advice and criticism of C. C. GOTLIEB and the staff of the Computation Centre, University of Toronto.

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[1] *The Programmers Handbook for the Manchester Electronic Computer (Mark II)*. mimeographed, Univ. of Manchester, England, Aug. 1952, 9 chapters.

[2] M. V. WILKES, D. J. WHEELER & S. GILL, *The Preparation of Programs for an Electronic Digital Computer*. Addison-Wesley Press, Inc., Cambridge, Mass., 1951.

D. R. HARTREE, *Numerical Analysis*. Chapter XII, Oxford University Press, 1952.

[3] Anon., "The incorporation of subroutines into a complete problem on the NBS Eastern Automatic Computer," *MTAC*, v. 4, 1950, p. 164–168.

[4] D. J. WHEELER, *Program Organization for the University of Illinois Digital Computer*. Internal Report No. 33, University of Illinois, May 16, 1952.

DISCUSSIONS

Programs for Computing the Hypergeometric Series

The hypergeometric series

$$F(a, b; c; z) = 1 + \frac{ab}{c}\frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{z^2}{2!} + \cdots$$

appears in the solution of various problems in applied mathematics and it is therefore desirable to have efficient methods for evaluating this series. To meet these needs a program of investigating available methods for computing the hypergeometric series is being conducted. The present note discusses two computations on SEAC.

A code for real values of parameters and argument has been checked out with a, b, c equal to various combinations of 1, 2, 3 and $-.9 \leqq x \leqq .9$ where many of the answers are known functions. The results were accurate to nine and more often ten significant figures. For each set of parameters it took approximately twenty-five minutes to tabulate the series for $x = -.9(.1).9$.

A code for complex values of parameters and argument has been checked with one of the real cases above giving the same accuracy, and with $a = 1 + i.2$, $b = -2$, $c = 2 + i$, $z = .5 - i.8$ where the hypergeometric series is a polynomial. The agreement in this case was exact.

A previous hand computation of $F(a, b; c; z)$ for

$$a = -11.753 + i12.204, \quad b = 12.753 - i12.204, \quad c = 2, \quad z = .5$$

gave the result

$$F(a, b; c; z) = 832{,}109 - i827{,}535$$

where the result was accurate to four figures. Using these same parameters and argument and carrying all computations to at least five decimal places and computing until the truncation error was less than 10^{-2} (54 terms) SEAC gave

$$F(a, b; c; z) = 832{,}103.8 - i827{,}536.8.$$

Repeating the computations to at least seven decimal places and computing first until the truncation error was less than 10^{-3} (57 terms) and then until the truncation error was less than 10^{-5} (65 terms) SEAC gave the identical results

$$F(a, b; c; z) = 832{,}108.0985 - i827{,}540.7310.$$

As a check on this result $F(a - 1, b; c; z)$ and $F(a, b + 1; c + 1; z)$ were computed and the following recursion formula yielded the results