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SEPTEMBER 1958

354

PROGRAM EVALUATION RESEARCH TASK *

ERT

SUMMARY REPORT
Phase 2

SPECIAL PROJECTS OFFICE
BUREAU OF NAVAL WEAPONS
DEPARTMENT OF THE NAVY
WASHINGTON, D.C.

Dept.

OW CALLED: *
ROGRAM EVALUATION
EVIEW TECHNIQUE

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PROGRAM EVALUATION
REVIEW TECHNIQUE

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I. INTRODUCTION TO PERT

In January 1958, the Program Evaluation Research Task (PERT) was originated in the Special Projects Office (SP) of the Bureau of Ordnance. The project team was directed to the research and development of improved methods for the planning and control of the complex and far-reaching program developing the Fleet Ballistic Missile (FBM). The PERT team included members of SP, the contractor's organization, and Booz, Allen & Hamilton.

Phase I report, issued in July 1958, presented the principal theoretical basis of the work and outlined the proposed installation pattern. While some modifications to the concept have been made since that time, the basic concept remains as then discussed. The reader is referred to this earlier report for the fuller discussion of concept.

This report covers Phase II of the project, which has been essentially the development of detail procedures and mechanics, the modification of concept as experience has proved advisable, and the "prove out" application of the method in several important areas of the FBM program. This application has demonstrated considerable value. As a result, the application of the method to the entire FBM program has been authorized and is proceeding.

With full installation, the PERT method will give a continuous picture of the FBM's status - both all-encompassing and synoptic. SP will have an increased ability to appraise progress and to anticipate potential problem areas while time still remains to take some corrective action.

1. OBJECTIVES OF THE PERT METHODOLOGY. The work described in this report and its predecessor is part of the continuing development of better management systems by SP. FBM management has had well-planned reporting and evaluation procedures for some time. PERT extends the capability and builds on the firm, existent foundations of the present.

With this principle in mind, the objectives of the PERT method can be described as follows:

(1) *The Fostering of Increased Orderliness and Consistency in the Planning and Evaluating of All Areas in the FBM Project.* Order and consistency are basic requisites of a workable management system. To this end, PERT prescribes a comprehensive procedure which is applicable to all areas within the research and development program. Thus, the same procedure developed in the identical fashion is applied to all components within the FBM system. The method can accommodate activities that are principally of a production nature, as well as those more unpredictable activities inherent in research and development.

The value of this orderliness and inclusiveness lies in an increased ability to integrate all parts of the system into a meaningful whole. Evaluations can be made across the boundaries of components and subsystems and thus can achieve validity at the system level. The FBM project has uniqueness in its systems integration. Evaluation must, therefore, be aimed at the same level. Orderliness and consistency of approach enable PERT to achieve the same measure of evaluation at both the system and component levels.

However, orderliness, consistency, and inclusiveness are not virtues in themselves. They are enabling devices upon which an evaluation system can be erected.

(2) *The Providing of an Automatic Mechanism for the Identification of Potential Trouble Spots in All Areas Which Arise as a Result of a Failure in One.* The PERT system is designed to automatically identify trouble spots and infer the consequences of trouble. A complex program such as the FBM system encompasses thousands of separate activities, each tied together in terms of the ultimate system objective. Diagnosis of the status of each of these activities must be in terms of a common set of rules.

The PERT system lays out a set of rules for such identification. The sole objective of these rules is to identify those activities in the development that *may* be potentially troublesome as seen at a given point in time. By minimizing attention on the mass of activities that are of secondary importance at the moment, the managers of the system can concentrate upon those activities which most seriously jeopardize success today.

The use of the evaluation system in no way removes the necessity for wise and forceful technical decision on the part of the FBM managers. It merely helps focus their scrutiny. The role of corroborating the suggestion of trouble and the correction thereof lies entirely within the province of the cognizant technical personnel. The PERT system acts to conserve their energies for this important work.

A good evaluation system, however, should do more than show the current relative importance of activities. A good system should be capable of further aiding the managers by showing them some important consequences of decisions that they can make as they seek to prevent or offset potential trouble.

(3) *The Structuring of a Method To Give Operational Flexibility to the Program by Allowing for Experimentation in a Simulated Sense.* The consequences of a local decision on more remote events are difficult to quickly and accurately visualize. The PERT system provides a vehicle for use by the technical man to systematically observe the resultant of one or more possible decisions on the end objective.

Once the technical manager has corroborated the fact of real trouble within a group of activities, he is faced with a group of alternative decisions. Any one of these options may be capable of correcting an undesirable situation. The evaluation system will allow him to correctly appraise the time consequences of these options. Parenthetically, it should be noted that the PERT evaluation system will only aid the technical man with information relative to time. The decision is his to correctly balance the time implications of a given action with the related performance and resource implications. The PERT system does significantly reduce uncertainty in appraising the all-important time variable.

While the foregoing objectives are noteworthy, they would be of little practical consequence unless they could be practically achieved with great speed and the maximum of accuracy.

(4) *The Speedy Handling and Analysis of the Integrated Data, thus Allowing for Expeditious Correction of Recognized Trouble Areas.* The true worth of an evaluation system is measured by the speed with which it can recognize and correct for out-of-phase situations. In order to fulfill the requirement of rapid handling and analysis, the PERT system has been programmed for the Naval Ordnance Research Calculator (NORC) at the Naval Proving Ground, Dahlgren, Virginia.

Analyses and outputs can thus be available to the technical managers within a matter of hours from the time that data is submitted. Answers to some questions may be transmitted via telephone or TWX. Thus, there is no need for executive action to be delayed awaiting the mechanistic tasks of tabulating, summarizing, and analyzing of input data.

2. THE SALIENT FEATURES OF THE PERT METHOD. The major part of the theoretical foundation of this PERT system was developed early in the study. These principles are described in detail in the Phase I report dated July 1958. However, it is worthwhile to briefly review the salient features of the earlier work in order to appreciate their impact on the Phase II activity. These basic elements are described below.

(1) *Selection of Specific, Identifiable Events That Are Planned To Occur along the Way to Successful Conclusion of the Project.* A basic feature of the evaluation system is the choice of a series of detailed accomplishments that must occur in the development program if current plans are followed. These events must be both meaningful to the development plan and of a definite, recognizable nature and each must be a *point* in time.

It is known that plans will change, and hence some or all of the events will change during the course of time. However, at any point in time, a definite future plan exists. It is this plan that is typified by the set of events that are specified for accomplishment.

(2) *Linking the Planned Events So As To Graphically Portray the Interdependencies among Them.* The chosen events comprise a set of accomplishments, each of which will exist at a given point in time as the culmination of one or more activities. The actual work activities carried on in the FBM program can thus be visualized as filling the gaps between events.

The activities are the links between events. Both activities and events are portrayed in a system flow plan form in Exhibit A, following this page. The events are the numbered circles in Exhibit A and the activities are the arrow linkages.

Links between events are indicated when there is a planned dependency. Thus in Exhibit A, Event 51 will not occur until the activities starting with Events 52 and 53 are *both* complete. Event 51 does not depend on the accomplishment of Event 54, hence no link is drawn connecting these two nodes.

(3) *Estimation of the Times Necessary To Move from Event to Event Together With A Measure of Uncertainties Involved.* A second important feature of the PERT system is the gathering of elapsed time estimates for the activities in the network. This step essentially dimensions the length of the arrows for use in subsequent analysis.

The time estimates used are obtained from the technical personnel within the contractor establishments directly charged with the accomplishment of the activities. The evaluation system is dealing with the future, and the future is uncertain. PERT was designed to estimate this uncertainty. The procedure for accomplishing this is probabilistic in form. It depends upon eliciting three estimates of the time necessary to perform each activity from a technical man familiar with the activity. These three estimates (optimistic, most likely, and pessimistic) are transformed to provide the basis for the statistical inferences relative to the uncertainty in activity accomplishment.

PERT estimates the future on the basis of informed judgments. Much can be said about the validity of these judgments. However, the important fact is that these judgments constitute the most authoritative estimates that exist. Uncertainty will tend to be high and validity low when events in the remote future are estimated. Improvements will be obtained as events come closer to the present. However, it should be noted that if the system should become manned with some estimators with a low validity index, it is able, through an historical study to calibrate the level of estimators' validity. No system can do more when dealing with the vagaries of an unprecedented future.

(4) *Analysis Which Systematically Highlights Relative Criticality of Events in the Future Development.* In an earlier section, the desirability of an automatic flagging of potential trouble spots has been noted. The following paragraphs will briefly describe the procedure for such a highlighting.

The PERT analysis transforms the three time estimates for the elapsed time to complete a single activity into a probability distribution typified by its mean and variance. The procedure then moves systematically through the network of activities and events and identifies the mean and variance of the *earliest* time for each event, considering all previous events. The mean time is the earliest expected time at which the event will occur if all activities are worked at as soon as they logically (i.e., according to plan) can be. The variance is a measure of the uncertainty associated with this earliest expected time of accomplishment.

SYSTEM FLOW PLAN

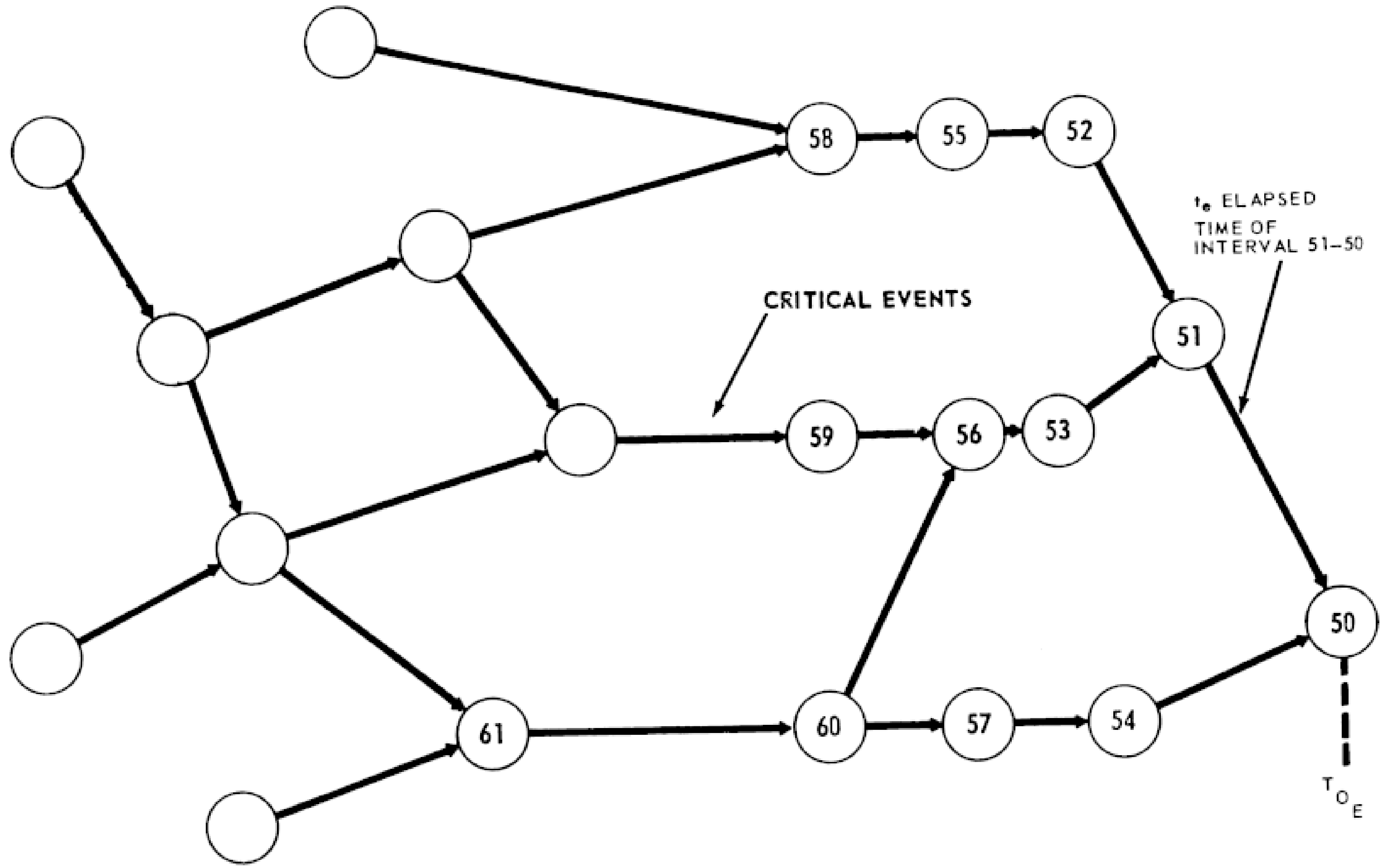

EXHIBIT A

The final event in the development system (network) is fixed at a given (desirable) point in time. The same analysis technique is then applied to the system in reverse order - taking future events first and moving back to the present.. This procedure evolves a mean and variance of the *latest* times for each event. These times are latest in the sense that they represent the latest time each event can be accomplished without jeopardizing the point in time chosen as a desirable date for accomplishing the final event of the development.

Armed with the above knowledge concerning the earliest and latest expected times for each event, a measure of "slack" in the plan is determined. Slack is defined for each event as being the difference between the expected earliest and latest allowable times. The most critical events are those which have the least slack. Any slip in the planned accomplishment of these events will result in delaying the expected completion date of the final event. The relative criticality of all events can be noted by comparing the magnitudes of their slack values. In addition, when scheduled dates exist for events, the evaluation system automatically computes the probability of meeting or beating the planned dates.

The reader should bear in mind that throughout this discussion a planned rate of resources application and a specified performance is assumed. Changes in either performance or resources application rate require reconsideration of the plan and new time estimates before PERT analysis is made.

The PERT system as conceived during Phase I is ultimately capable of accomplishing more than is indicated in the foregoing. However, the principal features of the initial installations have been herein described. More complete details will be found in the explanations shown in the Phase I report and the appendixes of this report.

Completion of Phase I in this study was followed by authorization to effect implementation of the system. Phase II has concerned itself with implementation, implementation techniques, and slight modifications of the basic model. The following section of this report will summarize the activity carried on during Phase II.

3. SUMMARY OF PHASE II ACTIVITY. The Phase II work of the PERT team has taken the basic concept as developed earlier, modified it in some aspects, and has taken steps to install the system. It is evident that the best conceived system is worthless in a practical situation without an efficient installation. Consequently, much time has been spent in carefully formulating installation plans so that all foreseeable impediments can be successfully surmounted.

Exhibit B, following this page, presents a schematic view of information flows and feedback within the operable PERT system. Phase II work has been principally devoted to the efficient achievement of the activity implied in the schematic drawing.

The most important features of the Phase II work are briefly described in the following paragraphs.

(1) *Provision for Initial Inputs to the System.* During Phase I, insights into the true nature of the FBM system were obtained through study of the propulsion component of the missile subsystem (see Exhibit C, on page 6). Phase II work involved a more careful reappraisal of the propulsion component and an extension of the application into the flight control and ballistic shell components. Input data for evaluation purposes were gathered in the field in the course of trips to the Lockheed Missile Systems Division (LMSD) at Sunnyvale, California.

These three components were integrated in terms of important common events in the systems, such as AX, A1X, BX as shown in Exhibit D, following Exhibit C. This procedure enabled the team to examine this synthesis question on a small scale. These data were submitted to a hand calculation in order to draw inferences typical of those that the system will produce. Exhibit D is representative of the results of the first hand calculations on hypothetical data. The black dots represent critical events within each component and the shaded path represents the critical path in the integrated outlook. Thus in this illustrative example, the flight control component would represent the predicted critical component in regard to achieving major event AX. The propulsion component would be critical in regard to A1X objective and flight controls again appear to be critical in objective BX.

In addition to the three missile components previously noted, field work was extended into the re-entry body and guidance components. Results of these extensions are now in the process of being evaluated and analyzed. In order to test for diverse aspects of installation, the team also gathered information on components outside of the missile subsystem. Networks were formed showing the instrumentation developments required for both the submarine and the missile test range at Patrick Air Force Base. These data were gathered in visits to the Interstate Electronics Corporation situated at Anaheim, California.

(2) *Provision for Continuous Inputs for System Analysis - the Establishment of Reporting Procedures.* The utility of the evaluation system to SP lies in the stream of outputs that it generates. Continuous inputs are therefore necessary to keep the evaluations both current and timely. The *PERT* team has recognized that it is necessary to go beyond the "one time" data-gathering expedition that validated the theoretical approach. A structure of reporting must be erected for accurate gathering and transmission of the input information to SP.

A reporting structure has been formulated and detailed by the team. This structure centers on and is controlled by Sp 12. Field implementation depends significantly on the efficient activity of the SP Field Offices - SPL, SPG, etc. Whereas the nerve center of the operation lies within SP proper, the sensory extensions lead to the field.

Official sanction for the implementing of PERT lies in an SP instruction issued in mid-October,

PERT SYSTEM IN OPERATION

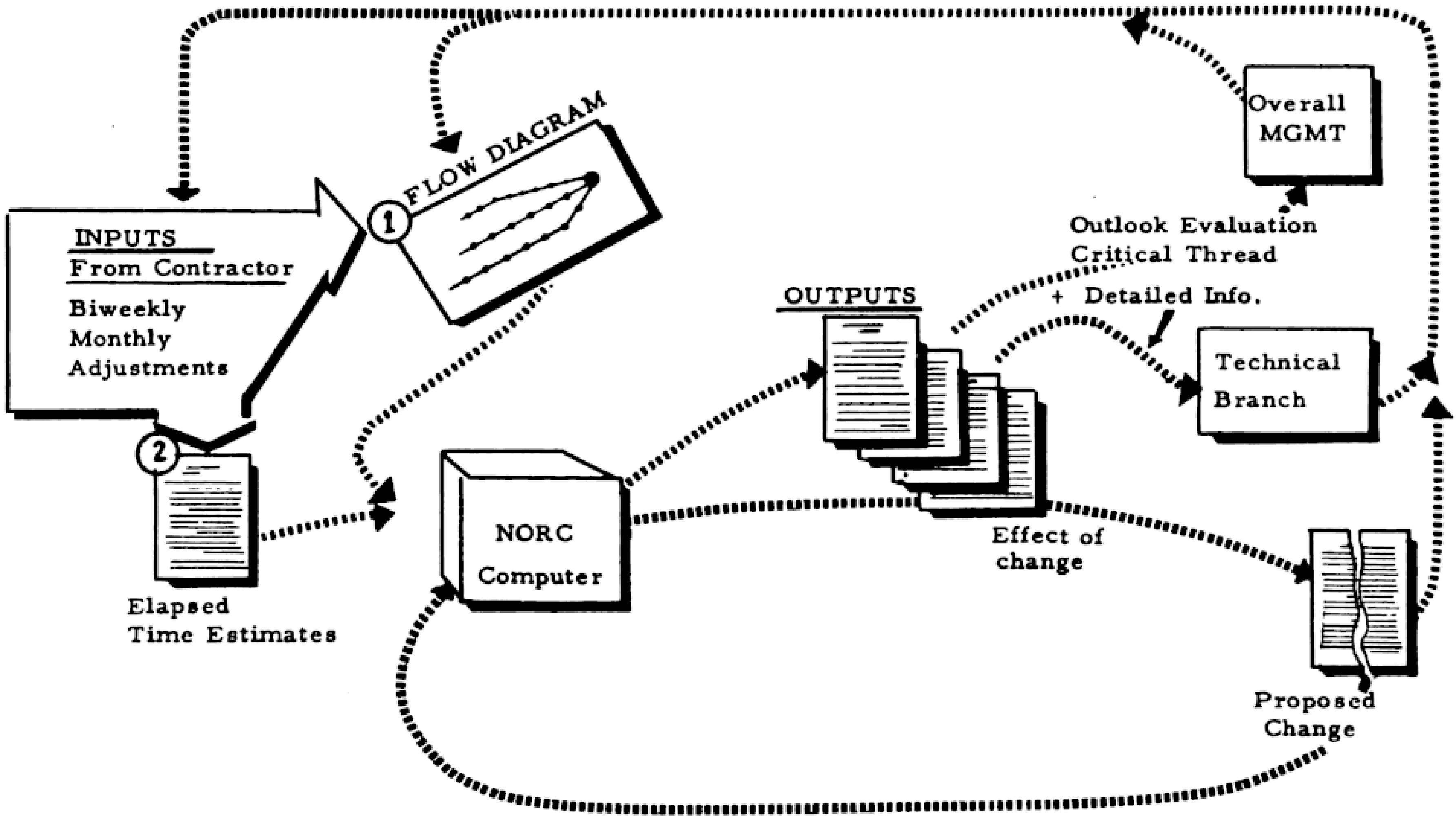

EXHIBIT B

PROPULSION FLOW CHART

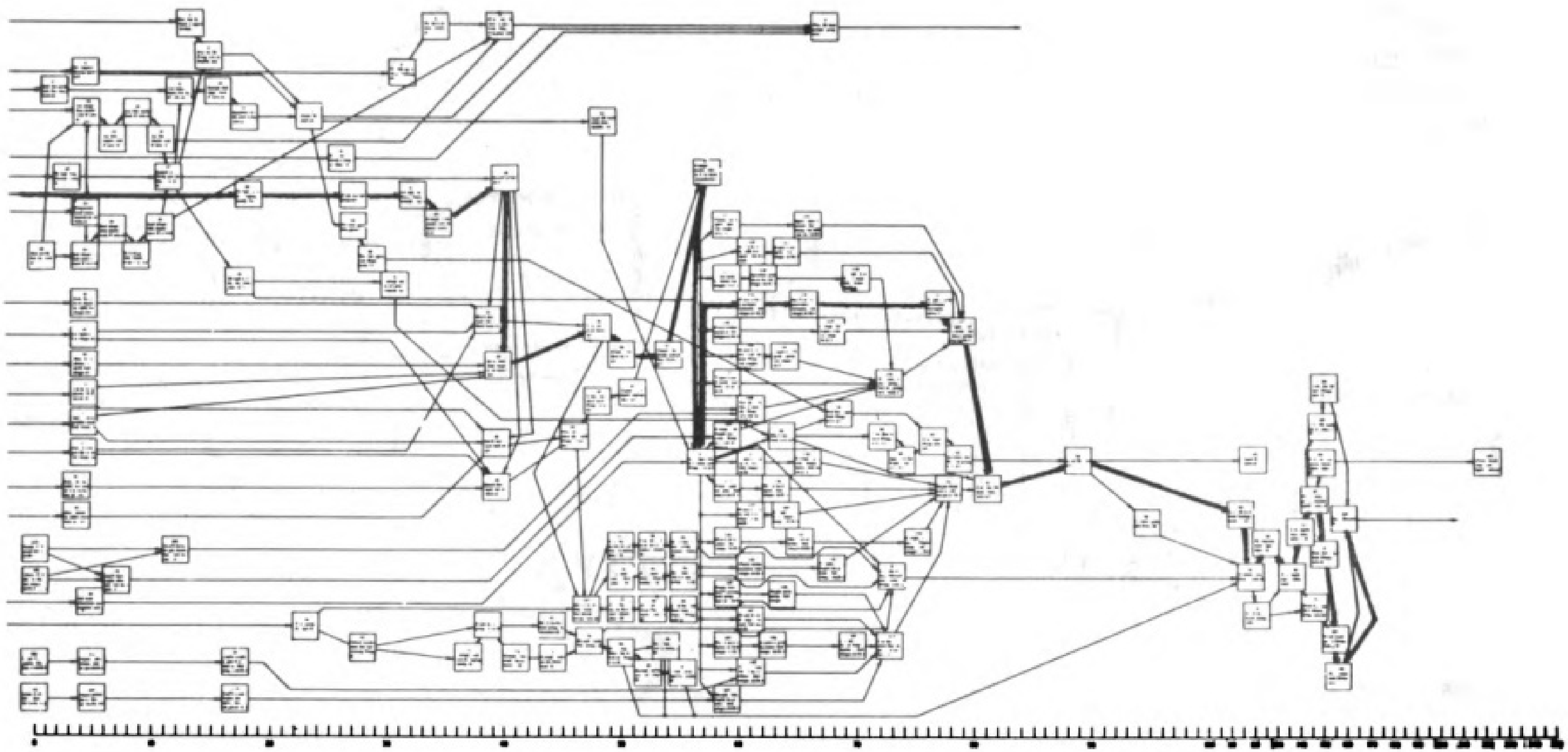

EXHIBIT C

INTEGRATED OUTLOOK

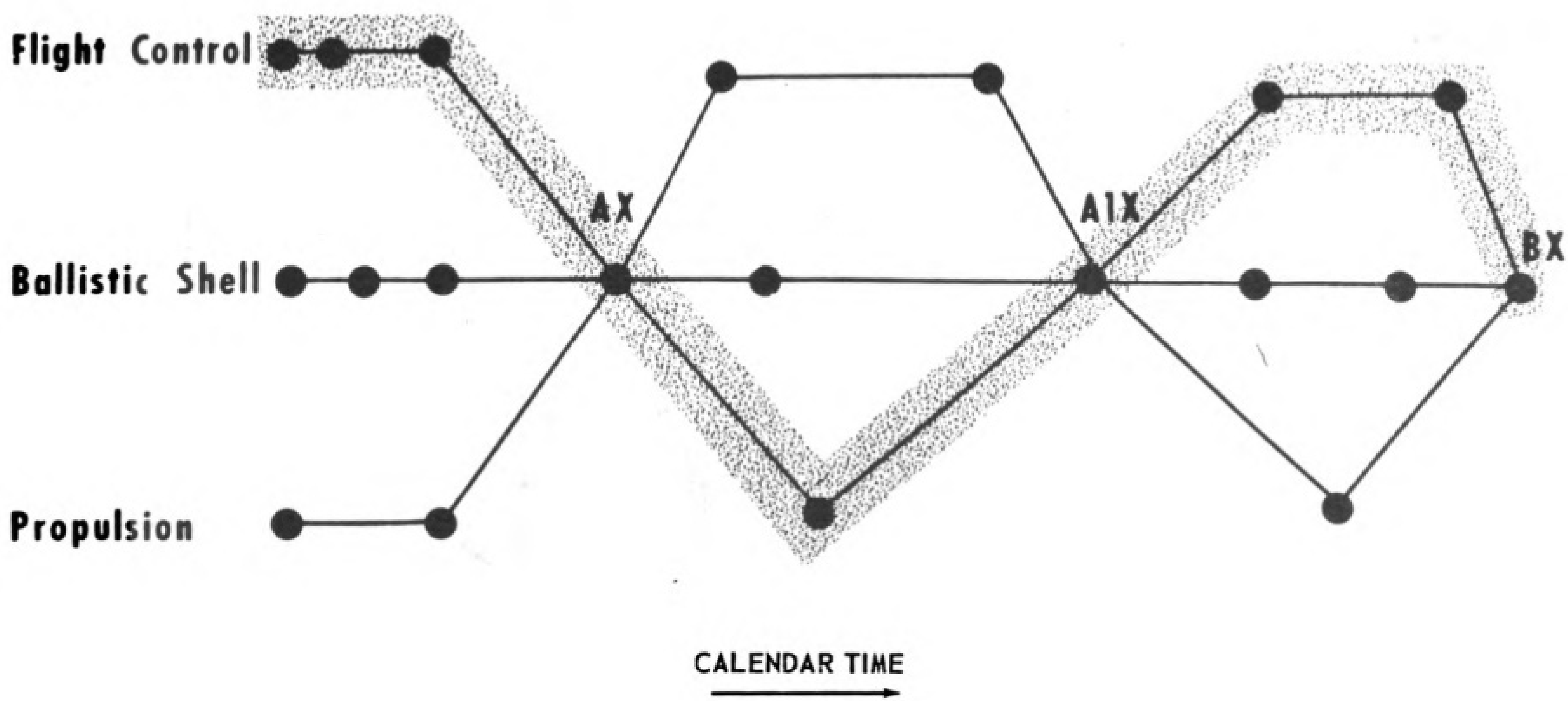

EXHIBIT D

(see Exhibit E, following this page): This instruction directs the field offices to issue technical instructions to the various contractors involved. On the basis of these "TI's," contractor activity reports flow into the field offices and thence to SP 12.

Reporting forms and reporting frequencies have been established. The layout of the report form consists of a minor revision to the currently operative Progress Reporting Form. All information contained on this current form appears on the amended one. In addition, provision is made for the three elapsed time estimates - optimistic, most likely, and pessimistic. The form layout was designed to present the incoming information in a manner that was most receptive to the subsequent computer analysis.

The frequency of reporting was established at biweekly, monthly, and quarterly intervals. This frequency will provide the system with its necessary inputs and, at the same time, will place the least additional load on the contractor.

A manual has been written to carefully describe the aims and requirements of the evaluation system. The manual is designed to facilitate installation of the PERT procedure. The distribution of the manual will include all contractors covered as well as SP personnel and field offices. The manual appears as Appendix A to this report.

(3) *Selection of a Computer Facility and Program Formulation*. The importance of speed in analysis has already been emphasized. Phase II activity included the task of selecting a computer facility as well as the programing of the computer to accommodate the PERT analysis.

A survey of Navy installations within the Washington environs resulted in an SP transfer of funds to the Naval Proving Ground at Dahlgren, Virginia, for machine time on the Naval Ordnance Research Calculator, the NORC. The NORC was most readily available and appeared to provide the greatest economies of calculation and analysis among the available computing establishments. A picture of the NORC appears in Exhibit F, on page 12.

A computer specialist on the PERT team carefully monitored the programing activity carried on at Dahlgren. A successful and efficient program has been formulated. The NORC capability was demonstrated to SP managers in early October when actual analyses were run on valid FBM data gathered during the Phase II effort.

A most important feature of the computer program as developed lies in the fact that it is completely general in nature. This feature permits FBM system changes to be made very simply and *without* requiring computer reprograming. Further, it can be used in the analysis of any R&D program, provided the inputs are in the form as developed by PERT.

(4) *Planning for SP Management*. The efficient installation of a significantly new system demands careful planning. During Phase II, the PERT team has worked to accomplish the necessary planning and thus facilitate the installation and administration of the evaluation system. This activity has been pointed in several directions beyond the establishment of the basic PERT concept.

Intensive indoctrination sessions have been held in order to apprise the Sp 12 personnel with the details of the approach. The experience gained in installation has been transferred to those who will be charged with similar work in the future.

The essential features of the computer analysis and system control were designed as part of the concept. In order to provide for computer liaison, Phase II included a test program for the choice of an acceptable individual for special training. This individual was given a short course in the NORC operation and tutored in the mathematical bases of the system.

Various techincal branches within SP were briefed on the PERT system. Emphasis was placed on the operational aspects of the system during these briefings. Procedures for updating, modifying, and experimenting with the FBM development were explained in so far as they applied to the PERT system. The significance of the critical thread analysis was also examined in detail with representatives of the Technical Division.

The future personnel and computer requirements for SP were studied and portrayed. These requirements were based on a rapid extension of the PERT system throughout the entire FBM development. The estimated personnel requirements were spelled out as were the costs occasioned by future computer usage. Chapter IV develops a detailed installation plan and personnel requirements to make a complete installation of PERT in a six-month period.

The planning described above was done in the face of incomplete information owing to the fact that the system has not been widely installed. Efficient operation of the evaluation system will require that planning be continuous. Thus, the initial activity undertaken by the PERT team must be constantly reviewed and updated in the light of experience.

(5) *Continuance of the Mathematical Appraisal of the Model*. The evaluation system has aimed at striking a balance between strict mathematical validity and operational facility. Operational considerations dictated certain features of the system in spite of the introduction of analytical biases. Phase II has continued the original mathematical work to evaluate the importance of the recognized bias. Results of this work will be issued as a supplemental report.

* * * * *

This chapter has given a synopsis of the range of activities undertaken by the PERT team during both Phases I and II. The succeeding chapters of this report will describe in greater detail the work involved in Phase II. Technical details are presented in the appendixes.

DEPARTMENT OF THE NAVY
Special Projects Office
Washington 25, D. C.

SP INST. 7720.1
Sp12-WF:aem
16 October 1958

SP INSTRUCTION 7720.1

From: Director, Special Projects
To : Distribution

Subj : Progress Reporting and Time Interval Estimating Procedures for the Program Evaluation System (PERT)

Encl : (1) Report of Progress and Time Interval Estimates

1. Introduction. The Program Evaluation System which forms the basis for this instruction is the result of the PERT project. PERT was conceived and developed to provide management of the Special Projects Office (SP) with continuous, integrated appraisals of program progress to date and the progress outlook for various FBM components, singly and in combination. The system provides a basis for re-evaluating appropriate program plans to accomplish future major objectives in the light of the FBM Program's most important commodity -- time.

2. Purpose. To set forth progress reporting requirements and procedures for PERT, and identify responsibilities for gathering information from the contractor and reporting data to Sp. Initially, this instruction is applicable to INSORD Sunnyvale and the missile subsystem contractors within their cognizance. As the PERT system is extended to other contractors, the procedures contained herein will apply.

An instruction manual developed from experience gained in early applications of the PERT methodology to the missile system will be provided to cognizant personnel.

3. Reporting Format. Enclosure (1) has been developed as the acceptable means for initiating contractor reporting of progress data and time interval estimates for PERT. In those cases where NAVEXOS Form 4153 is currently used, enclosure (1) temporarily represents an additional reporting requirement. It is a multi-purpose form and, as indicated, certain columns need not be filled out by the contractor or field office.

4. Installation of the PERT Reporting System at the Contractor Level.

a. Plan and Basic Requirements

The installation of this program evaluation system will be accomplished by use of a team approach. The Team will consist of representatives from Sp, the appropriate Sp field office, and the contractor's planning, program analysis, or other appropriate office (as the contractor may designate).

Responsibility within Sp will be divided between the designated analyst of Sp 12 and a member of the cognizant technical branch as assigned.

Basic requirements of the system are:

(1) A select list of events (definable accomplishments) on which progress will be reported;

(2) A flow chart which outlines the interconnections between events;

(3) Estimates of time required to perform the activity represented by connections between events in the flow diagram; and

(4) Periodic reports of progress and time interval estimates (Enclosure (1)).

b. Selection of Events

Events selected for this system must represent definable accomplishments for which a completion date can be established. Definitions of the events (including new or revised events) are required by Sp. For the FBM components where program management plans are available, the action milestones together with various company records (e.g., production schedules, engineering plans) will be used as sources for a tentative selection of PERT events by the team.

c. Establishing the Flow Chart

One flow chart is required for each component of the FBM Program as designated by the Team at the time of installation.

EXHIBIT E

The team, with the assistance of one or more technical individuals assigned by the head of the technical group actually engaged in the development work, will: (1) determine the final list of events, and (2) connect the events to indicate their planned relationship. This network of events so established is the flow chart.

d. Obtaining Time Interval Estimates

The next step in the installation of the Evaluation System on the contractor base involves the estimation of the time intervals for the activities represented by the connections between defined events on the flow chart.

5. Recurrent Reporting Procedures

a. Responsibility

The contractor is responsible for the preparation of the bi-weekly report. The form, Enclosure (1), with instruction for its use, is to contain the status of all events completed or scheduled for completion during the previous two (2) week period, and the status of any future events where a delay or earlier completion data is anticipated.

b. Frequency

The established reporting period is bi-weekly, with the report (Enclosure (1)) due on Tuesday of alternate weeks. Four (4) copies of each report are to be submitted as follows: two (2) copies to Sp 12, one (1) copy to cognizant Sp Technical Branch, one (1) copy to cognizant Special Projects Field Office.

The addition or revision of an event to the flow chart will require reporting of new completion dates and additional time interval estimates and/or revision of existing estimates. The contractor is to provide on the bi-weekly report all known additions or re-estimates for an activity or group of activities. This applies to future as well as short-range events. These data are to appear in the first bi-weekly report due after they become known.

c. Supplemental Reporting

Additional information may be requested on certain designated areas of the flow chart as deemed necessary by the cognizant Sp personnel and Sp field offices. Insofar as possible, Enclosure (1) will be used for reporting the data. In certain cases, the entire flow chart may be reproduced and used as the transmittal form when approved by the Sp field office.

W. F. Raborn

W. F. RABORN

Distribution:
Sp2
Sp3

EXHIBIT E

REPORT (PERT) PROGRESS & TIME INTERVAL ESTIMATES

CLASSIFICATION:

From: (Name & Location of Contractor)	To:	Contract No.	Report Period From/To:
		Flow Chart No.	

For Office use only (Do not fill in) (1)	(2)	(3)	Activity Identification: Preceding Event No. (B)	Activity Identification: Succeeding Event No. (C)	Time Int. Est.: Opt (wks) * (D)	Time Int. Est.: Most Lkly (wks) * (E)	Time Int. Est.: Pess (wks) * (F)	Completion date (G)	Do Not Fill In (4)	REMARKS (H)

Signature and Title of Responsible Official:	Date Signed:

*Columns D, E, & F: These estimates should be given for the full activity even though the activity has already started.

CLASSIFICATION:

INSTRUCTIONS FOR COMPLETION OF REPORT ON PROGRESS AND TIME INTERVAL ESTIMATES TO M

GENERAL: This report is designed to provide information on program progress and estimates of time intervals required for events included in the PERT flow chart. A separate report will be submitted for each PERT flow chart. Time interval estimates should be made by the person in the highest level of the technical organization who could perform the work himself. The best judgment of those responsible for the work is desired.

Copies will normally be submitted in writing to be received in the Sp Office by the close of each alternate Tuesday. Required timing will be specified at the installation of the Program Evaluation System for the particular component involved. Reports are required even though no events were completed or scheduled for completion during the previous two (2) week period. Verbal reports will be accepted under unusual circumstances and must be followed up with a written report. Four (4) copies should be forwarded as follows:

1. Special Projects Office, Bureau of Ordnance Navy Department, Washington 25, D. C. Attn: Sp12 (2) Sp Cognizant Technical Branch (1)
2. Cognizant Sp field office. (1)

COLUMNS 1, 2, 3, & 4: Do not fill in .

COLUMNS B & C: Enter the flow chart identification numbers for the events which form the beginning and ending points of the intervening activity currently being reported. The preceding event lies to the left or tail of the arrow on the PERT flow chart.

COLUMN D: Enter the optimistic time interval for an activity in terms of weeks (to the nearest tenth of a week) defined as: (1) An attainable time when significantly better luck than usual is expected; or (2) a minimum time with practically no hope of completing the activity in less than optimistic time.

COLUMN E: Enter the most likely or most probable time interval for an activity in terms of weeks (to the nearest tenth of a week) defined as: (1) a time longer than or equal to that entered in Column D; (2) the time that would be reasonably be expected by the person best qualified to judge; or (3) the time that would occur most often if the activity could be repeated numerous times under exactly the same conditions.

COLUMN F: Enter the pessimistic time interval for an activity in terms of weeks (to the nearest tenth of a week) defined as: (1) Time required if significantly worse luck than usual is encountered; (2) a time not influenced by strikes or by catastrophes or "acts of God," such as flood fires, etc; or (3) a time which no reasonably qualified person or persons would expect to be exceeded except in very unusual circumstances. Divided opinion among experts as to the success or failure of a project makes it necessary to include the effect of a fresh start upon the pessimistic time estimate.

COLUMN G: Report the date of completion (actual or scheduled) for all activities completed or scheduled for completion since the last bi-weekly report.

COLUMN H: Explain the reason for changes made in schedule dates and time interval estimates. Whenever an event or milestone is eliminated, added, or description is changed from the flow chart designation, such modification should be reported along with complete new information for other events and activities which may be affected by the change--i.e., in terms of completion date or time interval estimates together with the reason for change, using a supplemental sheet if necessary.

EXHIBIT E

NORC CALCULATOR

EXHIBIT F

II. THE INPUTS TO THE PERT SYSTEM

The bare concept exists, has been tested, and has proved useful. Two data inputs must be carried out to get the complete system under way. First is the initial data requirement to actually complete the installation across the entire FBM project. Second is the provision of continuing, updating data once the installation is complete.

The experience of the PERT team during Phase II furnishes guidelines for accomplishing these inputs.

1. INITIAL INSTALLATION OF THE EVALUATION SYSTEM. The steps to be taken in an initial installation in a component area are as follows:

(1) *Briefing of Contractor and Insord Representatives.* The contractors possess the most intimate knowledge of the current state of their development activities. The cognizant Insord office has the responsibility for the field monitoring of these activities. Therefore, as the first step in an initial installation, a PERT team should undertake to indoctrinate responsible representatives of both in the evaluation system requirements. These briefings should assume a general understanding of the system and should provide a basis for the designation of individuals from both contractor and Insord as members of the team for the given component installation.

Additional details should then be covered with the contractor and Insord team members. The SPINST 7720.1 (Exhibit E) and the instruction manual (Appendix A) should be carefully studied so that the detail implications of the system are understood.

(Included in Appendix A is a glossary of terms used in the PERT approach. The reader may wish to refer to this in reading the ensuing material.)

(2) *Preliminary Event Sequencing.* The SP team members (an evaluation man and a technical man) will have previously constructed a rough flow chart of the given development process. This flow chart should now be subjected to the scrutiny of the remainder of the team. Modifications will be made in the light of knowledge of the latest detailed plans.

The full team should then carefully connect the events so as to constitute the network. The connections between events are evidence of dependencies or interactions. These dependencies can be of two kinds. They can be direct technological dependencies (where one event cannot physically occur until some previous activity has been completed.) Or, the dependency may simply be a planning dependency dicated by efficient use of time. Either condition warrants the drawing of an "activity arrow."

(3) *Time Estimation and General Authentication.* To this point, the contractor technical personnel (other than possible team members) have not been approached. This avoidance of contacting technical personnel has been deliberate, since the team should minimize the amount of time in which the technical people are diverted from their primary job. However, once the team has sequenced the network to the best of their own knowledge, authentication must be obtained from the most knowledgeable person - the technical man charged with the development.

After review and approval of the flow chart, the technical specialist must provide the three time estimates required for each activity link in the network. Careful explanations should be made in regard to the significance of these optimistic, most likely, and pessimistic times. Attention should be specially focused on the obtaining of best *current* estimates of *elapsed* times between pairs of events.

In some cases, the specialist may indicate the existence of optional developments in his planning. One of several options will be adopted depending upon future research. This situation can be accommodated in the evaluation. The team members should obtain time estimates for all options and note the currently preferred one - if such is the case.

Once a flow chart has been developed and has received authentication and time estimates have been rendered, the team must carefully document the identification of each event. A file should be set up to record these identifications. They should be complete and precise so that subsequent review will not be faced with a set of vague meanings.

The event identification file will be preserved both at SP, Insord, and within the contractor's establishment. The file can be very simple, as long as it presents the necessary information (see Exhibit G, following this page).

(4) *Fields of Initial Application.* The foregoing sequence of steps has been followed in the first installations of the evaluation system. Installations have been completed in several component development areas of the missile subsystem and also in certain nonairborne instrumentation development areas. The first installations have been concentrated on the missile subsystem in recognition of the current importance of this subsystem. Thus, initial flow charts have been completed for propulsion, flight control, ballistic shell, and re-entry body. Work is currently progressing toward the inclusion of the remainder of the missile.

The flow charts and time estimates for these components have been subjected to an independent reasonability review by the SP Technical Branches concerned. Such a review represents a necessary precaution for the initial incoming information from a contractor. A typical flow chart appears in Exhibit C.

2. UPDATING DATA FOLLOWING SYSTEM INSTALLATION. The dynamics of the FBM development require a continuous updating of flow chart information after it has once been obtained. The PERT system provides for this modification by prescribing periodic reports from the contractors. These reports are transmitted from the Insord office to the control center in Sp 12. The actual updating takes the following form.

(1) *Biweekly Updating.* At biweekly intervals the contractor is required to report on a standard form (see Exhibit E) the status of all events that have been completed or scheduled for completion during the past two weeks. In addition, the contractor can report on any other events when their anticipated time for accomplishment changes.

As these modifications are posted within the PERT accounting, SP will be continuously aware of the position of all phases of the development.

The biweekly updating concerns a relatively few events that are scheduled for current completion However, this information will generally provide little basis for making inferences about the course of future development. Consequently, a monthly request will arise from SP asking for more detailed information.

(2) *Monthly Updating.* The monthly request will ask for re-estimates of certain activity times in the network. Re-estimates will be required for all activities that lie along the most critical path as determined by the evaluation analysis. In addition, certain other activity times may be requested by SP in order to determine the relative validity of times previously submitted by the contractor. This latter type of request can be used as the sample control audit of SP.

All requests for the monthly updating must be transmitted by Sp 12, although they may originate in any SP office. The breaching of this requirement will only result in placing an unduly heavy load on the contractor and confusing the evaluation system. The request for the monthly updating is made on the standard reporting form (see Exhibit E). The identical form, when properly completed by the contractor, will comprise the required report.

The monthly updating provides SP with the vehicle for examining at frequent intervals those activities that are currently most critical in the achievement of the end objective. At the same time, it does not require an abundance of information about activities whose status is of relatively little concern. Both the contractor's reporting load and SP's analysis load is thus minimized without sacrificing significant information.

(3) *Quarterly Updating.* The monthly updating requests will not deal with the majority of the events in the FBM system. However, all events must be reached before the development is com-

plete. Activities differ in importance in only a relative sense. It is, therefore, necessary to periodically evaluate the totality of the network. This is accomplished in a quarterly updating report.

The quarterly updating is essentially a reconstruction of the original flow chart complete with re-estimates of activity times. The quarterly updating should generally follow the procedure set up for generating the original flow chart. It will be accomplished with SP participation along with the contractor and Insord. Some briefings that were necessary in the original installation will be found to be superfluous in this updating.

The results of the quarterly updating will provide SP with information to re-evaluate the entire development and thus obtain a fresh prognosis.

As the PERT method is applied to more and more areas of the FBM system, it will be necessary to carefully plan the times at which reports will be submitted. The report cycles should be staggered for the various subsystems and components. The biweekly reports should occur on different weeks for different components. The monthly and quarterly updatings should similarly be scheduled at different times. Such scheduling of reports should make for a constant work load with little week-to-week variation.

This duscussion has avoided many details that are necessary for a workable installation of the system. However, the main ingredients of the installation procedures have been noted. The manual (Appendix A to this report) develops the systematic step-by-step procedures that must be provided for during the installing and updating phases.

3. CONTROL AND STABILIZATION OF INPUTS. The PERT method will deal with masses of information emanating from different sources and arriving at different times. Therefore, it is of utmost importance that procedural checks be set and strictly followed.

In order to assure that all information is received, correctly handled, and transmitted, a set of logs, editings, and checks and balances has been designed. These are outlined in Appendix B, "Computer Procedures." Adherence to this design should minimize the opportunity for trouble arising within the SP system. Most importantly, however, should trouble arise, the system itself will give positive indication of such trouble. Subsequent corrective activity can then be directed toward a precise area rather than a search over all possible sources of trouble.

The set of controls must encompass transmissions to and from the computer. The details of these controls are presented in Appendix B of this report, which deals with the elements of the computer installation and analysis.

With the data-gathering structure developed, it is appropriate to discuss the output side of the PERT system.

EVENT IDENTIFICATION FILE

SUB SYSTEM 27 – MISSILE

COMPONENT 400 – BALLISTIC SHELL

EVENT	DESCRIPTION
27–400–024	Compl. Modif, and c/o by Grd. Tests Bal. Shell (A1X)
27–400–025	Arrival of Vehicle at PAFB (A1X)
27–400–026	Flt. Test Bal. Shell (A1X)
27–400–027	Comp
27–400–028	

EXHIBIT C

The output side of the evaluation system is the payoff of the system. Understanding and profitable utilization of output is a pre-requisite to the maturing of the overall procedure.

1. OUTPUT POSSIBILITIES. The PERT system is capable of presenting a large variety of information in many different forms. An initial computer output format has been designed. It is important to note that this form does not present the full range of information that can be printed if desired. However, it does exhibit those factors that will be most significant to the SP manager at this time. Subsequently, as experience and understanding mounts, the format may easily be altered to enable a more complete exploitation of the methodology.

(1) *Printed Outputs.* The form of the printed computer output appears in Exhibit H, following this page. All printed data is numeric as the NORC computer (which produces the outputs) does not work efficiently with alphabetic characters.

The first column on the output sheet shows an event code that has previously been assigned in the process of flow charting. This is a unique designator. Systems, subsystems, and components can be distinguished by their code numbers. Translation of the event codes can be easily accomplished with a listing from the previously noted "Event Identification File" (see Exhibit G.)

The second and third columns on the output report form show the values of T_E and T_L. The entries in these columns are the result of machine computation. They indicate the expected values (means) of the earliest and latest time distributions for each of the coded events. Roughly speaking, the T_E is the earliest time at which the event will occur in terms of current expectations. The value T_L is the latest time the event can occur without disturbing the date of the system objective. Printed values for T_E, T_L, and T_S are measured in weeks from the date printed at the top of the forms.

The fourth column shows the difference between the values in columns 2 and 3. This column (described as slack) will be non-negative in the present computations which set the latest time for the system objecitve at the computed earliest time it will occur. If, in determining a T_L, the system objective (end event) is set at a time before its earliest expectation, T_E, then column 4 can show negative values.

The fifth column indicates the scheduled time for an event, if such exists. It appears in the output as a coded value in the same terms as T_E and T_L. (The last column on the sheet shows the same value in decoded form, i.e., as an actual date.)

The sixth column describes the probability of meeting or beating the scheduled date in terms of current plans and anticipations.

(2) *Suppressed Values.* A record is kept in the computer of all transactions that are submitted for analysis. In addition, other pertinent factors are computed in the run and permanently kept, although not currently printed. These suppressed values can be printed with very slight modifications within the printing run.

The computing run calculates the variances of the earliest and latest time distributions for each event. These are not shown on the output, but are used in probability analyses in relation with column 6, headed P_r.

At certain times when optional plans exist, it is important to typify the resource and performance levels projected for the different plans (see Appendix A). When such levels have been indicated for the activity sequences, their values are recorded on the output cards and are available for examination, if desired. In addition, certain basic probability calculations are made in the computing run and added to the historical record. Although these calculations are not currently being used, a procedure does exist for their utilization in having the computer designate a best set of scheduled dates (see Appendix B).

(3) *Output Options.* Certain outputs have been prescribed and are currently being printed on machine runs, as indicated in Exhibit H. Other values are recorded internally within the machine memory and can be presented with some little provision for the change. There is a third possibility that should be described. It exists as an option that can be exercised with the proper setting of switches on the machine.

The totality of all events can be presented as the regular output of a machine run. This detailed information will be of interest to personnel handling the various components. Exhibit H shows a typical output of a so-called "regular" run.

For some purposes, the mass of detail shown in the "regular" run will be excessive for quick appraisal. Therefore, it is necessary to adopt some criterion for extracting from the "regular" run those particular aspects that are significant. The machine has been programed to present only the most critical events for the analyst's perusal. With the simple throwing of a switch, the output edit run will suppress the data on all events with large amounts of slack. Exhibit I, on page 17, shows how the previous data are now presented with this discriminating switch on.

The upper portion of Exhibit I merely shows those events within the ballistic shell component that have zero slack. These events are the most critical to the development of that component. Consequently they are of the most current interest. Another setting will enable the machine to print the output information on all events that have a small amount of slack. The lower portion of Exhibit I is typical of such a print-out. Here, all events that have slack within the range zero to five weeks have been laid out for special study. Thus, this optional study will give the technical manager data relative

PERT SYSTEM — BALLISTIC SHELL COMPONENT - ALL EVENTS

DATE 7|12|58 WEEK 0.0 PAGE 1

EVENT	T_E	T_L	T_L-T_E	T_S	P_r	Scheduled Date
27-400-000	0.0	0.0	0.0			
27-400-001	0.0	3.2	3.2			
27-400-002	0.0	3.2	3.2			
27-400-003	0.0	7.5	7.5			
27-400-004	4.2	15.5	11.3			
27-400-005	0.0	0.2	0.2			
27-400-006	3.0	3.0	0.0			
27-400-007	3.0	3.2	0.2			
27-400-008	3.0	3.2	0.2			
27-400-009	3.5	3.5	0.0			
27-400-010	12.0	15.5	3.5			
27-400-011	7.0	21.5	14.5			
27-400-012	7.5	7.5	0.0	9.0	.99	9-12-xx
27-400-013	15.5	15.5	0.0			
27-400-014	10.0	30.7	20.7			
27-400-015	26.0	46.7	20.7			
27-400-016	35.5	35.5	0.0			
27-400-017	21.5	21.5	0.0			
27-400-018	15.5	110.3	94.8			
27-400-019	15.5	110.3	94.8			
27-400-020	30.0	50.7	20.7			
27-400-021	51.5	51.5	0.0			
27-400-022	30.0	50.7	20.7			
27-400-023	33.0	53.7	20.7			
27-400-024	54.5	54.5	0.0			
27-400-025	55.0	55.0	0.0			
27-400-026	58.0	58.0	0.0	50.0	+0.00	6-30-xx
27-400-028	70.3	116.3	46.0			
27-400-029	96.3	96.3	0.0			
27-400-030	40.0	89.3	49.3			
27-400-031	56.0	105.3	49.3			
27-400-032	60.0	109.3	49.3			
27-400-033	60.0	109.3	49.3			
27-400-034	112.3	112.3	0.0			
27-400-035	112.3	112.3	0.0			
27-400-036	82.3	82.3	0.0			
27-400-037	63.0	112.3	49.3			
27-400-038	116.3	116.3	0.0	128.00	.99	12-31-xx

EXHIBIT H

to those activities and events that are in the second most critical range.

In summary, only the immediately pertinent information appears on the current output sheets. However, the range of output information can be expanded significantly as experience allows for its profitable utilization.

This section has described the values that appear on the output form, as well as those that can appear. The following paragraphs will describe the type of inferences that can be drawn from the output sheets.

2. INFERENCES FROM OUTPUTS. The FBM development proceeds in accordance with a plan. The PERT evaluation appraises the validity of that plan in terms of its timely accomplishment. The evaluation procedure also can give guidance to the system managers in devising modifications to the plan.

There are many ways in which the PERT outputs can be used by those charged with the successful development of the FBM. It is not possible to itemize all the ways in which the outputs can be profitably utilized. However, some of the principal types of inferences can be noted.

(1) *Determining Relative Criticality of Events.* The "regular" output of the NORC lists the slack time currently associated with events in the development. Those events with the smallest slack are the most critical events. Slippages of these events will cause the expectation of the final event to slip. Consequently, the managers should carefully monitor progress in these areas.

Contrariwise, events exhibiting large amounts of slack are in relatively good shape. The evaluation is a relative one. Surveillance over all activity is necessary, but, the relative importance of activities as shown by the slack computation enables the cognizant branch to logically allocate its supervisory activity.

(2) *Appraisal of Schedule Feasibility.* The existence of a definitive plan includes the designation of specified times for the accomplishment of events - that is, a schedule. These scheduled times are necessarily set far in advance of actual accomplishment. In some cases, it is desirable to change a scheduled date to benefit the overall program. This is possible only if the potential slippage can be seen at an early enough date.

The PERT system provides the managers with a probability measure that describes current chances of meeting future scheduled dates. A low probability of meeting a scheduled date ordinarily will generate remedial actions which will improve the observed probability. However, if thorough analysis indicates that no effective remedy is available, knowledge of the existence of the low probability can be used to alter the schedule so as to more efficiently use the resources available to the program.

(3) *Experimentation.* The existence of criticality or potential slippage is a flag that causes the system to look to available remedies for an undesirable situation. These remedies can be alternate developments, changes in the rate of resources application, or variations in the level of technical performance specified. The PERT runs can give the technical personnel insights into the varied consequences of these available options as they affect time. (Of course knowledge as to the possibilities of resources or performance trade-offs must come from the technical personnel involved or other sources external to PERT.)

Exhibit J, following this page, illustrates the use of the PERT output for this purpose. An option might exist for shortening an activity time by five weeks. See event No. 27-400-013. The effect of this shortening on an objective event is sought. The PERT experimental output indicates the results that would be obtained by this change in the large typescript. As shown, the proposed improvement has no effect in reducing the time in achieving the objective, since it remains at 112.0 weeks. Hence, it would not be wise to expend additional resources in the given area as they would be unproductive in gaining a time saving. It is possible to draw a general conclusion, or "principle," from study of this and similar cases. It becomes evident that time improvements within slack areas produce no effect on the timely accomplishment of the end objective.

Exhibit K, following Exhibit J, illustrates the result of another hypothetical change. In this case, the time improvement was made in an activity that was of a critical nature, as noted by the zero slack (event 17-400-024). The result of this reduction is

PERT SYSTEM BALLISTIC SHELL COMPONENT - CRITICAL EVENTS

DATE 7 | 12 | 58 WEEK 0.0 PAGE ____

EVENT	T_E	T_L	T_L-T_E	T_S	P_r	Scheduled Date
27-400-006	3.0	3.0	0.0			
27-400-009	3.5	3.5	0.0			
27-400-012	7.5	7.5	0.0			
27-400-013	15.5	15.5	0.0			
27-400-017	21.5	21.5	0.0			
27-400-016	35.5	35.5	0.0			
27-400-021	51.5	51.5	0.0			
27-400-024	54.5	54.5	0.0			
27-400-025	55.0	55.0	0.0			
27-400-026	58.0	58.0	0.0			
27-400-036	82.3	82.3	0.0			
27-400-029	96.3	96.0	0.0			
27-400-034	112.3	112.3	0.0			
27-400-035	112.3	112.3	0.0			
27-400-038	116.3	116.3	0.0			
27-400-001	0.0	3.2	3.2			
27-400-002	0.0	3.2	3.2			
27-400-005	0.0	0.2	0.2			
27-400-007	3.0	3.2	0.2			
27-400-008	3.0	3.2	0.2			
27-400-010	12.0	15.5	3.5			

EXHIBIT I

PERT SYSTEM

DATE 7 | 12 | 58 WEEK 0.0 PAGE ____

EVENT	T_E	T_L	$T_L - T_E$	T_S	P_r	Scheduled Date
27-400-013	15 10	27.5 27.5	12.0 17.0	Activity reduced 5 weeks in duration		
27-400-38	112.0 112.0	112.0 112.0	0.0 0.0	No effect on objective		
				PRINCIPLE:		TIME IMPROVEMENT IN SLACK AREA PRODUCES NO EFFECT ON OBJECT

EXHIBIT J

in fact beneficial. Again, a principle can be adduced. An improvement along a critical path can produce an effect on the end objective up to the amount of the original improvement.

The foregoing principles can be summarized in a slightly different fashion. Resource and performance trade-offs will tend to be productive when applied to activities that lie along the most critical path. When slack exists in a sequence of activities, nothing is to be gained by degrading performance or increasing the rate of resource application. On the contrary, when slack exists it may be possible to strive for superior performance or to extract resources for a more urgent application.

When the PERT system is used as part of such experimentation, it essentially "simulates" the future development process. The NORC speedily reveals the results of the simulation. The cognizant personnel can then make decisions with a more complete awareness of the consequences of those decisions. These consequences show not only the local effects but also the general effects on the integrated FBM development.

3. SUMMARY PRESENTATIONS. Top management must have a means for quickly grasping the essential status of the total program. Whereas the individual technical branches are concerned with parts of the whole, top management must have procedures for synthesizing the entire system. The PERT method provides procedures for this integration.

(1) *Major Event Outlook.* The NORC program is capable of extracting different levels of detail from the analysis. Previous sections have described this flexibility. Exhibit L, following this page, demonstrates a presentation that has been developed for showing the outlook for major event accomplishment.

In Exhibit L, the wealth of detail has been suppressed and only events of pre-eminent importance are shown. (The data shown in this exhibit are artificial.) In this exhibit the current outlook is visually compared with both the schedule and the last previous information. Thus, current progress can be quickly compared with desired progress. In ad-

PERT SYSTEM

DATE 7 | 12 | 58 WEEK 0.0 PAGE ____

EVENT	T_E	T_L	T_L-T_E	T_S	P_r	Scheduled Date
27-400-024	35.0	35.0	0.0			
	30.0	32.0	2.0	Activity reduced 5 weeks		
27-400-038	112	112	0.0			
	109	109	.0	Effect on end-objective 3 weeks saved		
				PRINCIPAL:		An improvement along the critical path can produce up to the amount of improvement

EXHIBIT K

MAJOR EVENT OUTLOOK

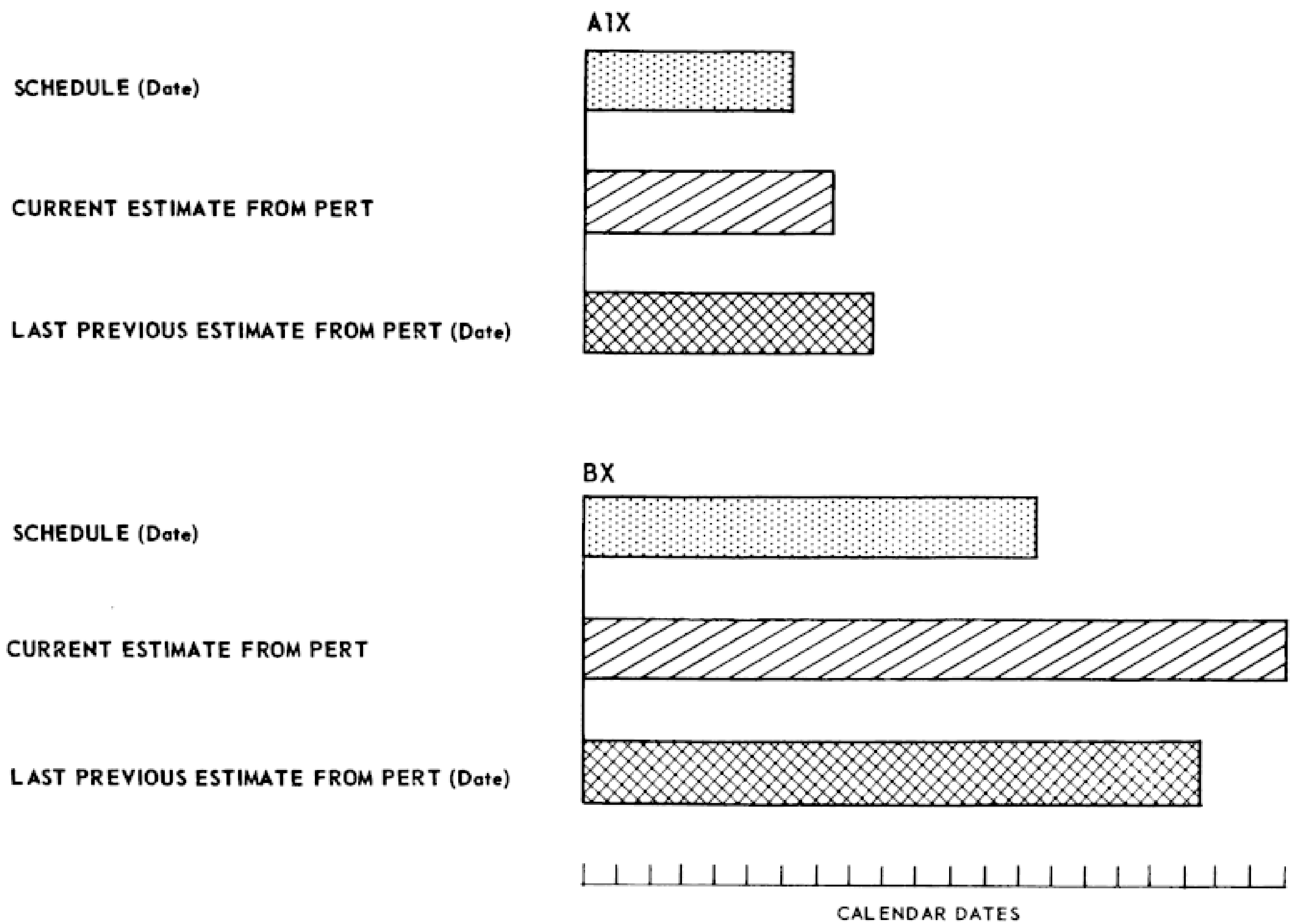

EXHIBIT L

dition, the current position is directly compared with the last estimate. Relative movement during the last report period is immediately evident.

(2) *Integrated Flow Charts.* A second mode of summary presentation is available in the development of integrated flow charts. The major subsystems are joined together in this presentation to show the entire FBM picture. Detail can be deleted from the picture and thus only those activities that are of paramount importance will be presented.

This presentation can demonstrate another major dimension of program status. Emphasis can be given to the critical path and immediately highlight to overall management those subsystems which should bear the greatest scrutiny. Exhibit D in Chapter I is illustrative of this type of presentation.

4. THE COMPUTER ANALYSIS AND OUTPUT.

(1) *Capability of the NORC.* The NORC installation at Dahlgren is the nucleus of a modern high speed computing laboratory. The NORC is currently the fastest computer operating in the world. It is from two to five times faster than any other machine. It can add or subtract 16-digit numbers at a rate of approximately 22,000 a second. Multiplications of 16-digit numbers can be carried out at the rate of 14,000 a second.

The NORC system operates with eight high speed magnetic tape storage units, each of which can transmit information within the computer at a rate of 71,400 digits per second. Input and output to the computer is via magnetic tape. The laboratory has a card-to-tape-to-card converter which will produce or accept information in a punch card medium. In addition, the Dahlgren facility has a large, well-equipped tab card installation.

(2) *Machine Runs on the NORC.* The PERT procedure has been programed on the NORC in the form of ten subprograms or machine runs. The overall program is broken down in this fashion according to the various operations that are accomplished during the cycle. Thus, ten distinct phases contribute to the outputs which have been discussed.

The various machine runs that comprise the program are described in detail in Appendix B. In summary, however, the objectives of the runs can be explained as follows:

—*Run 01 - Card to Tape Conversion.* This run takes incoming data punched in cards and translates the information into a magnetic tape form. The data can then be fed into the computer in the form of magnetized spots on plastic tape.

—*Run 02 - Data Transaction Conversion.* This operation takes the incoming data and transforms the time estimates into distribution parameters. The a, m, and b (optimistic, most likely, and pessimistic) values of the input are expressed as the mean and variance of elapsed time distributions.

—*Runs 03, 04, and 05 - Sort and Merge Operation.* The first two runs sort the incoming event information into event code order. Run 03 deals with predecessors to activities while Run 04 orders the successor events. Run 05 merges the two ordered sequences in a form amenable to subsequent operations.

—*Run 06 - Event File Updating.* The lasting historical record of previous transactions is on a magnetic tape known as the "event file." The purpose of this run is to update this "file" so as to add, delete, or change the information therein.

—*Run 07 - Sequencing.* The PERT analysis procedure requires that the event flow chart be collapsed into one straight line in such a way that all connecting links point in one direction. Run 07 accomplishes this necessary sequencing activity. The program steps for achieving this sequencing are highly novel. For this reason a detailed flow chart and program are included in Appendix B.

—*Run 08 - Slack and Probability Computations.* This run accomplishes several objectives. First, the difference between the earliest and latest times for an event is computed. Then a series of steps take place which compute probabilities of meeting schedule. These probabilities are found by reference to a table of normal probabilities recorded in the NORC's memory.

—*Run 09 - Output Sort.* This operation reorders the event information and structures it in a form that will facilitate appraisal of the output information.

—*Run 10 - Output Edit.* This run essentially selects from all the information that reposes on the tape that which will be subsequently printed. It also places this selected information in a form which will be acceptable to the tape to card conversion which follows.

Certain other runs have been designed and are now in the process of programing and checkout. These additional runs will act to simplify and coordinate the overall machine analysis. Modifications of this kind are part of a continuing program analysis that will take advantage of the experience gained in future machine runs.

(3) *Accuracy Controls in the Program.* The need for detailed controls on the flow of information outside of the computer has already been noted. The arithmetic involved in the computer operation is checked within the operation of the computer itself. In addition, good computer practice requires that the program itself include checks for accuracy. The program checks that have been designed are of the following kinds.

—*Internal Controls.* Internal controls are built into the program as a general check of consistency and reasonableness. The magnitudes of input data are checked for proper relative size. A series of additional checks assure that internal postings are made to the correct event "accounts." The efficiency of these checks should be under constant surveillance. Additional checks should be designed and incorporated into the program as they appear desirable.

—*Transaction and File Control Totals.* These controls give positive evidence that the number of transactions introduced into the machine are all processed. Trial totals are taken at various points during the runs. Machine stops occur whenever inequalities in the totals are discovered. These totals must agree with the log records kept by the SP control center.

—*Tape Label Control*. The PERT system will generate many different record tapes. Each will be applicable to a given analysis. In addition, the NORC laboratory has thousands of similar tapes descriptive of the many programs they handle. It is essential that correct tapes be used in the machine analyses. In order to uniquely identify tapes, code words are magnetically written on the tapes. These code words are checked automatically against the proper code which identifies (and accompanies) each batch of information submitted for analysis.

(4) *Costs of the NORC Operation*. SP computer expenses to date reflect two independent activities. First is the detailed program information and checkout on the current PERT system. The second is representative of work done in discovering the intrinsic validity of the current procedures. Both of these costs are one-time expenditures.

The costs incurred by SP during the initial programing phase can be divided between machine time and necessary programing labor. The total expenditure from 1 July to 31 October 1958 has been $36,095. (NORC machine time rates at $195/hour are very low for a large scale machine.)

Actual programing on the PERT methodology has been responsible for $20,645 of this expenditure. This figure represents approximately an equal division of funds between machine time and labor. The remaining funds, $15,450, have been expended in work on the validity check for the procedure.

In the future, NORC costs will tend to be directly proportional to the machine utilization at Dahlgren. This in turn will generally be a function of the rapidity with which the PERT system is installed.

The following estimates were made on the assumption that the entire FBM system would be covered in a gradual build-up over the next seven months (through June 1959). It is assumed that the number of events in the computer event file for the remainder of this fiscal year will average between 1,800 and 2,000. It is anticipated that the average number of monthly runs (both regular and experimental) will be approximately 60.

If these levels are realized, computer charges to SP will be at a $5,500/ month rate.

In addition to the charges for machine time, some program modification will be necessary. It is estimated that the necessary programing labor will come to about $5,500 in total.

Working on a basis of seven remaining months (for computing) during this fiscal year, the total expected requirement on SP is, therefore, between $45,000 and $50,000. This is an estimate that should not be exceeded in practice. Charges against a transfer of funds to the Naval Weapons Laboratory will only be made for actual labor and machine time usage. If the full FBM system is not covered by PERT by 30 June 1959, the charges will be less than the foregoing estimate.

• • • • •

The previous chapters have described the work of the PERT team during Phase II. A concept has been detailed - an installation plan has been formulated - and the first components have been covered by the system. However, the job is not complete. The following chapter describes several important areas which will need attention in order to fully realize the potential of the evaluation system.

IV. FUTURE ACTIVITY

Modification should be a continuous process as it is applied to the PERT system. The basic analysis and installation structure has been set. Improvements will be discovered and should be incorporated as their worth is established. Certain areas have already been recognized as in need of attention.

1. EXPERIMENTATION WITH OUTPUT FORMAT. The present output format is simple and easily understood. As SP works with the outputs, they undoubtedly will find variations which will lend themselves to improved efficiency. The previous chapter has indicated the extent of the available information evolved in the computer analysis. Sp 12 must work closely with the technical branches, providing them with all of the information they can meaningfully use. Only extensive experience will yield the best form for presenting the output information.

2. EXTENSION TO THE ENTIRE FBM NETWORK. The PERT method has been designed for system application. Current plans call for making the entire application by 30 June 1959. It is most important that this be done efficiently. Serious oversights can develop with less than a full implementation. A whole series of critical events may never become evident if the entire system is not incorporated in the flow network.

Extending the PERT system will involve many problems. Care must be exercised so as to prevent the substitution of a quantity of information for the necessary quality. Haphazard installations can serve to impair the worth of other better considered ones. It is recommended that contractors be carefully coached and closely watched until their compliance is satisfactory.

The SP Field Offices have an important role in the installation and data-collection process. The importance of their role must be made evident to them. Field office indoctrination should not be limited to the evaluation groups but appropriately should include the technical groups. Time spent in efficiently indoctrinating personnel in the field offices will yield a dividend in making for a smoothly working system.

Extension of the PERT procedure to the full FBM system will place a large load on the control center within Sp12. It is important that they quickly provide for accomplishing the additional work. The magnitude of this effort should not be underestimated. Exhibit M, following this page, shows the estimated personnel requirements for a full installation within the current fiscal year. This exhibit shows that at the peak of the installation, Sp 12 will need approximately 13 people. After full installation, the requirements are somewhat less. For steady state operation after installation, it is estimated that 7 to 8 total personnel will be needed in 3 different position categories as follows:

Code	*Description*
A	Direct supervision, interpretation of results, report writing, contractor, and SP Field Office indoctrination.
B	Contractor and SP Technical Liaison, data handling in and out of SP, flow diagrams, monthly and quarterly reports, interpretation.
C	NORC Liaison, computer input and output handling, checking event nomenclature file, updating.

It is important to recognize that failure to provide personnel in the number shown will result in a delay in achieving the installation objective. Time must be provided for adequate training. The development of the foregoing personnel estimates is treated in Appendix D "PERT Installation Personnel Requirements."

It is imperative that Sp 12 gear up their operation so as to effectively accommodate information as it arrives. Present activity emphasizes field installation. When these installations are complete, data will flow in relatively large volume. Future success of the system can depend to a considerable measure on the wisdom and efficiency with which initial inputs are handled.

3. REFINEMENTS IN ANALYSIS.

(1) *Improved Computer Programing.* Previous chapters of this report have highlighted the need for making the computer program even more flexible than it currently is. Activity is now underway to accomplish certain improvements. These are described in Appendix B. This work must be closely monitored by SP so as to ensure the most effective processing.

(2) *Calibration of Estimators.* Inferences from the PERT system depend upon estimates of the future. Individuals within the different contractors' establishments will exhibit varying abilities to make unbiased estimates. As experience mounts, it should be possible to make statistical analyses that will allow for calibrating the validity of a contractor's or individual's estimating ability. The basis for such analysis will lie in the historical transaction file within the NORC.

(3) *Sensitivity Analysis of the Present Method.* The PERT networks frequently show a situation where there are two or more sequences of activities that are necessary in moving from one event to another. The present PERT calculation takes that sequence with the longest expected time as the only constraint on the accomplishment of the second event. This procedure will underestimate the time necessary to move between the two events. Occasionally, a sequence with a relatively short time expectation will slip significantly. The resultant slippage can make the given sequence the major constraint of the movement. The present procedure does not accommodate this possibility.

Mathematical analysis of this type of situation has been made. Current work is proceeding toward estimating the extent to which the bias introduced by the present method underestimates predicted times. A separate report will be issued in which the results of the present work will be given. The mathematics involved in obtaining an unbiased analysis are complex. If the bias in the present computation is small, the inaccuracies should be overlooked. If the bias is large, further work should be directed toward correcting the situation. Appendix C of this report describes the original mathematical work leading to the accurate solution of the network problem.

(4) *Relation of PERT to Other Sp 12 Controls.* Broadly speaking, the PERT computer program can be fed information in sufficient fineness of detail to encompass all line-of-balance and PMP activity. However, a serious question is immediately apparent. Is this amount of detail necessary to effectively perform the job of SP management? Corollary to this is the question "How many detailed production-type activities are needed in order to make the PERT outlooks meaningful? Are not production activities more definable and less subject to variance in their achievement times?

These questions are important in structuring the networks and in decisions as to the number of detailed events to be included. To date, the philosophy of the PERT approach has been to move to a position halfway between the coverage detail of PMP and the line-of-balance charts. In individual branches, it will be found desirable to go into more or less detail as the situation requires or as is desired.

One way of approaching the resolution to these questions (which we recommend to be answered only as greater experience with the system is acquired) is to consider the NORC as a possible repository of all information relating to performance as scheduled.

With this thought in mind, if it seems desirable to include more details such as the line-of-balance

TOTAL REQUIREMENT
(INSTALLATION AND OPERATION)

In Man-Wks/Mo.*

Figures in parentheses ( ) indicate total equivalent number of personnel required

Month	Total Personnel	Position Category A		B		C	
1	(8.1)	7.98	(1.8)	18.87	(4.4)	8.25	(1.9)
2	(9.1)	8.46	(1.95)	20.94	(4.8)	9.75	(2.3)
3	(10.0)	8.94	(2.06)	23.01	(5.3)	11.25	(2.6)
4	(11.0)	9.42	(2.18)	25.08	(5.8)	12.75	(3.0)
5	(11.9)	9.90	(2.29)	27.15	(6.3)	14.25	(3.3)
6 (Target)	(12.9)	10.38	(2.40)	29.22	(6.8)	15.75	(3.7)
7 Operating rate	(7.5)	3.36	(.77)	14.49	(3.3)	10.50	(2.4)

*The above does not provide for training of installation staff.

EXHIBIT M

information in regard to any component, it is a very simple matter as far as the NORC is concerned. One has merely to define and code the events, indicate their predecessors and successors, provide the three time estimates (they may all be equal), and process as an input to the NORC analysis. Then, if reports are required periodically, the routine reporting procedure described can be used to provide inputs to the NORC that will in turn provide printed output sheets showing status of these more detailed events.

As a display tool, we believe that line-of-balance has great utility at both the contractor and SP levels. We recommend that the question posed here - whether to use this detail in PERT - be answered by experimenting with such use and evaluating the experience gained.

* * * * *

The foregoing sections discuss some of the more important work that remains to be done before PERT is a smoothly functioning, accurate evaluation system. The concept of PERT is sound. Initial installations have been successful. Top management has provided strong backing to the procedure. The work has matured to a point beyond the "project" status, as is indicated by a significant change in designation. PERT, as a code word, now signifies "Program Evaluation and Review Technique." Continued good work on the part of SP will result in a system that will provide far-reaching insights into the FBM development. Hopefully, the system will augment the Navy's currently operable policies and aid in the successful and timely production of the Fleet Ballistic Missile.

APPENDIX A

INSTRUCTION MANUAL
and
SYSTEMS AND PROCEDURES
for
THE PROGRAM EVALUATION SYSTEM (PERT)

TABLE OF CONTENTS

INDEX OF EXHIBITS IN APPENDIX A

I. INTRODUCTION

The Program Evaluation System (PERT) is a method for evaluating progress to improve the planning of a major research and development program. It is a tool for management control. Initially, the system has been developed and applied to the Fleet Ballistic Missile (FBM) Program.

The FBM program is very similar in form to any other type of plan since it is composed of a series of activities or tasks scheduled in anticipated sequence culminating in the attainment of some final major objective. Product performance and resource allocation operate as constraints on the time of accomplishment.

The problem of evaluating a research and development program is acknowledged as difficult by experienced research people because of the nature of the activities - intellectual, pioneering, and unpredictable.

How can the Program Evaluation System (PERT) improve on a day-to-day comparison of progress with planned schedule dates? Basically, the system can provide at a minimum (1) orderliness and consistency in planning and evaluating, (2) automatic identification of possible future trouble spots as a result of failure in one area, (3) speed in integrating progress evaluation, and (4) throughout these previous points, accurate portrayal of the dynamic research situation.

II. PURPOSE

1. GENERAL DESCRIPTION. The purpose of this Program Evaluation System (PERT) is to provide a means of handling the large and complex problem of rapid evaluation and reprograming in the FBM program. The Systems and Procedures Manual provides a description of the methods used to obtain and process the required information necessary for the effective operation of the system. This manual is intended as a guide for all personnel directly concerned with operation of the system.

Sections of the manual contain information to cover the answers to the general questions: Who? What? When? Where? How? Specifically, operation of the Program Evaluation System must develop along the following lines:

(1) Select specific, identifiable events* which must occur along the planned progress toward some goal.

(2) Sequence these events and establish the relationships which exist among them.

(3) Estimate the time and a measure of the variability of this time for the activity joining each pair of events.

(4) Apply electronic data-processing equipment (computer) to process and integrate these data.

(5) Establish information flow channels to assemble progress and change data.

2. IMPORTANCE OF PROPER PROCEDURES. Efficient operation of any evaluation system is dependent upon timely receipt of accurate information. The procedures outlined herein are designed to obtain the necessary information rapidly and with minimum effort. In order to decrease the amount of work involved and minimize the problems of installation and operation, the procedures have been tailored to fit in with existing milestone progress reporting procedures wherever practicable. The effective operation of the Program Evaluation System is highly dependent on up-to-date computer inputs and rapid dissemination of the outputs to management. The capability of the NORC computer installation is utilized for processing the information. The computer can process some 10,000 activity times in less than half an hour.

*Exhibit I, on the following page, is a glossary of terms that apply to the Program Evaluation System.

III. INSTALLING THE SYSTEM

1. PRELIMINARY MEETINGS. There can be no substitute for adequate preparation of all levels of management within the contractor organization prior to beginning the actual installation. Probably the best approach to use is similar to the one frequently employed by SP management for originating milestone progress reporting procedures. Briefly, this involves briefing certain contractor personnel in PERT methodology and outlining the procedures to be used for gathering and transmitting essential data. This can be accomplished, in most cases, at the time of a visit to the contractor's plant by a well-trained team from SP with representatives from and arrangement by the SP Field Office.

The importance of the acceptance and approval of management within the contractor organization should not be underestimated. "Getting off on the right foot," goes a long way toward making an effective installation of the system.

2. ORIGINATING THE FLOW CHART - THE VISIT OF THE EVALUATION TEAM.

(1) *Selecting the Events.* After the Evaluation Team has been properly set up and generally briefed and trained, a meeting with the contractor-assigned members of the team should be planned and held to select the events to be included on the flow chart. It is possible that SP milestones used in existing progress reporting after examination and necessary revision can become the events that are needed to form the basis for the flow chart.

It is important to analyze each event description considered for use on the flow chart by asking appropriate questions in the following categories:

Category 1 - Does the event represent a definite discernible beginning or ending point of some activity or group of activities? (Such words as firm, finalize, freeze, ship, etc. are to be avoided. In their place use such phrases as, "design release

GLOSSARY
(PERT)
FBM PROGRAM EVALUATION SYSTEM

ACTIVITY

An event is separated from other events by activities. An activity is a time-consuming element in the development process. It is represented on a flow chart by an arrow. An activity cannot be started until its preceding event has been accomplished. A succeeding event to an activity cannot be accomplished until the activity is complete.

ACTIVITY TIMES

Estimates of the elapsed time necessary to complete an activity in a specified manner are activity times. They are represented by estimates indicating:

- Optimistic times
- Most likely times
- Pessimistic times

CRITICAL PATH

A critical path is that particular sequence of activities in a flow chart that comprise the most rigorous time constraint in the accomplishment of the end event.

EVALUATION TEAM

The Evaluation Team (Team) is descriptive of a group of designated individuals charged with furthering the SP evaluation effort as it bears on a given sector of the FBM development. Generally, a Team will be comprised of individuals representing the SP Technical and the SP Evaluation divisions, SP Field Office, and the Contractor.

EVENT

A meaningful specified accomplishment in the FBM development program. An event should be recognizable as a particular instant in time.

FLOW CHART OR PLAN

The sequenced diagrammatic representation of events and activities.

PRECEDING EVENT

See ACTIVITY

SLACK PATHS

Slack paths are sequences of activities that do not lie on the critical paths. Slack may exist in varying amounts.

SUCCEEDING EVENT

See ACTIVITY

EXHIBIT I

complete," "production prototype design complete," "approve for captive test," "arrival of 1st flight test vehicle at AFMTC," etc.)

Category 2 - Is the description detailed and complete? Does it tell who does it? Where and what is done? (Be sure model descriptions are complete. Use specific FTV, CTV, and other available code numbers to name specific components, subsystems, and systems. Use test and model numbers.)

Category 3 - Does the technical man who is to do the estimating understand the events as beginning and ending points of some clearly defined activity? Is he able to fix in his own mind what has to take place *at the completion* of the preceding event before the immediately succeeding event is reached? (This category is used when interrogation of the estimator takes place in obtaining time interval estimates.)

(2) *Laying Out the Chart.* After the events have been properly selected, the next step is to place the descriptions on a flow chart. This can be accomplished on a "first cut" basis by drawing a series of circles (or boxes) about 1-1/2 inches across on a large sheet of paper. The event descriptions are handwritten in the circles. (They may be abbreviated as long as a complete list of events and their description is maintained.) Events are placed vertically according to category (subcomponent on a component chart, subsystem on a system chart, etc.) and along the horizontal axis roughly according to schedule dates. It is best not to place a permanent horizontal scale on the chart. If this is done, the estimator may use the divisions to judge the planned amount of time scheduled to elapse between events and, thereby, introduce a bias into his estimates.

At the conclusion of this step in the process, the chart is nothing more than a series of circles with written descriptions in them. (For the most part, a sheet of paper about 24 by 36 inches should provide enough space for about 200 events with a 2-year scale along the horizontal axis.)

(3) *Connecting Events.* To aid in connecting events, the head of the technical group directly engaged in doing the development work under consideration should be requested to provide qualified personnel. They should be persons of a high level in the technical organization and yet should be capable of doing the development work themselves. In other words, they should be aware of the overall planning as it applies to the area covered by the flow chart.

Technical personnel are to indicate the connections or arrows to be drawn between events according to present plans and expectations. The criterion to use is, for example:

> "According to present plans it is expected that events numbered 10 and 22 are related and that event number 10 will have to be reached before event number 22 can be accomplished. Events number 10 and 22 are so related and interdependent that unless 10 is completed the activity between them cannot be started. Furthermore, it will be impossible to reach the point in time represented by 22 until the activity represented by the arrow between them is completed. More specifically: an inspection of an assembly cannot be started until it is assembled, the assembly cannot begin until the components are fabricated, etc."

10 22

It is advantageous to start at the left of the chart where some events may have been completed already or scheduled for completion momentarily. Some events are considered as having no preceding event. These events start as of the time "0" or "now."

Arrows leading into these events originate at the "0" point. The connecting process is continued across the chart to the extreme right until each event is properly related to all the other events either directly or indirectly. Events 1, 3, and 5 of Exhibit II, following this page, are events of this type.

The statement is sometimes made by the technical man, "But this event does not have to be accomplished before we can start the indicated subsequent activity."

The question to ask in this case is, "Does the relationship as shown by the arrow, represent current planning within your group?"

(4) *Estimating Elapsed Times.* The same technical man, if available, who indicated the connections to be made between events should be consulted to provide estimates of elapsed times and associated variabilities for the activities represented by the arrows between pairs of events. It is important that the technical man be fully aware of the actions within his group which must take place between the events. Also, he must be able to provide three time estimates corresponding to three different sets of conditions as indicated below:

1. *Optimistic Time* - The first estimate is an "optimistic" one, in that it gives the best or shortest time. There is little hope of completing the activity in less than the optimistic time.

2. *Most Likely Time* - The "most likely" time estimate is that which would be given if only one time estimate was requested. It is the time that would occur most often if the activity was repeated under *exactly* the same conditions many times. If many knowledgeable people were asked for the most likely time, the value given most often would form the most likely time estimate.

3. *Pessimistic Time* - If significantly worse luck than usual occurs, the "pessimistic time" estimate indicates the longest time that the activity would take.

Time estimates should be entered on the flow charts along the arrows to which they apply. A good

ORIGINAL FLOW CHART
(DUMMY)

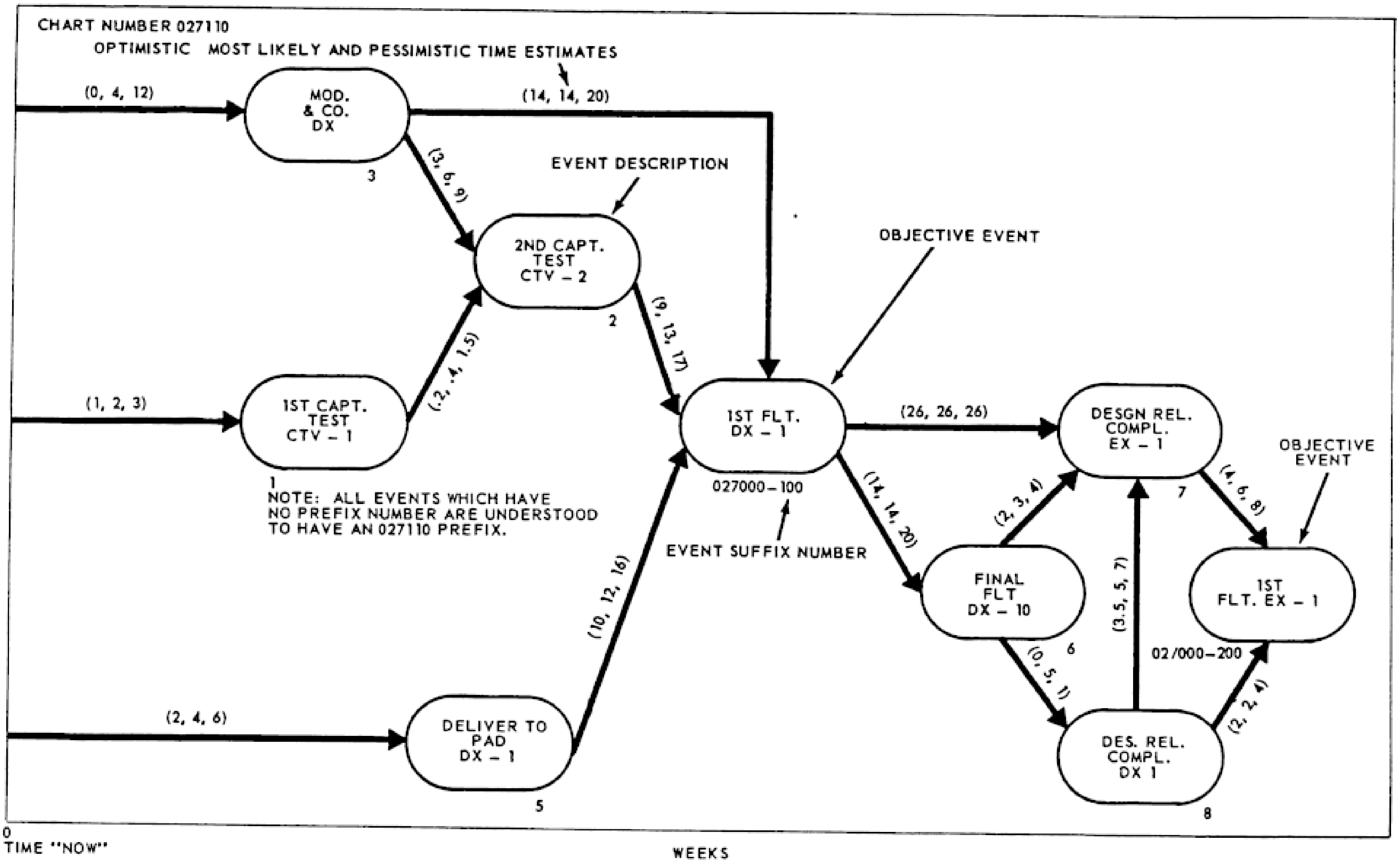

EXHIBIT II–1

RE-DRAWN FLOW CHART
(DUMMY)

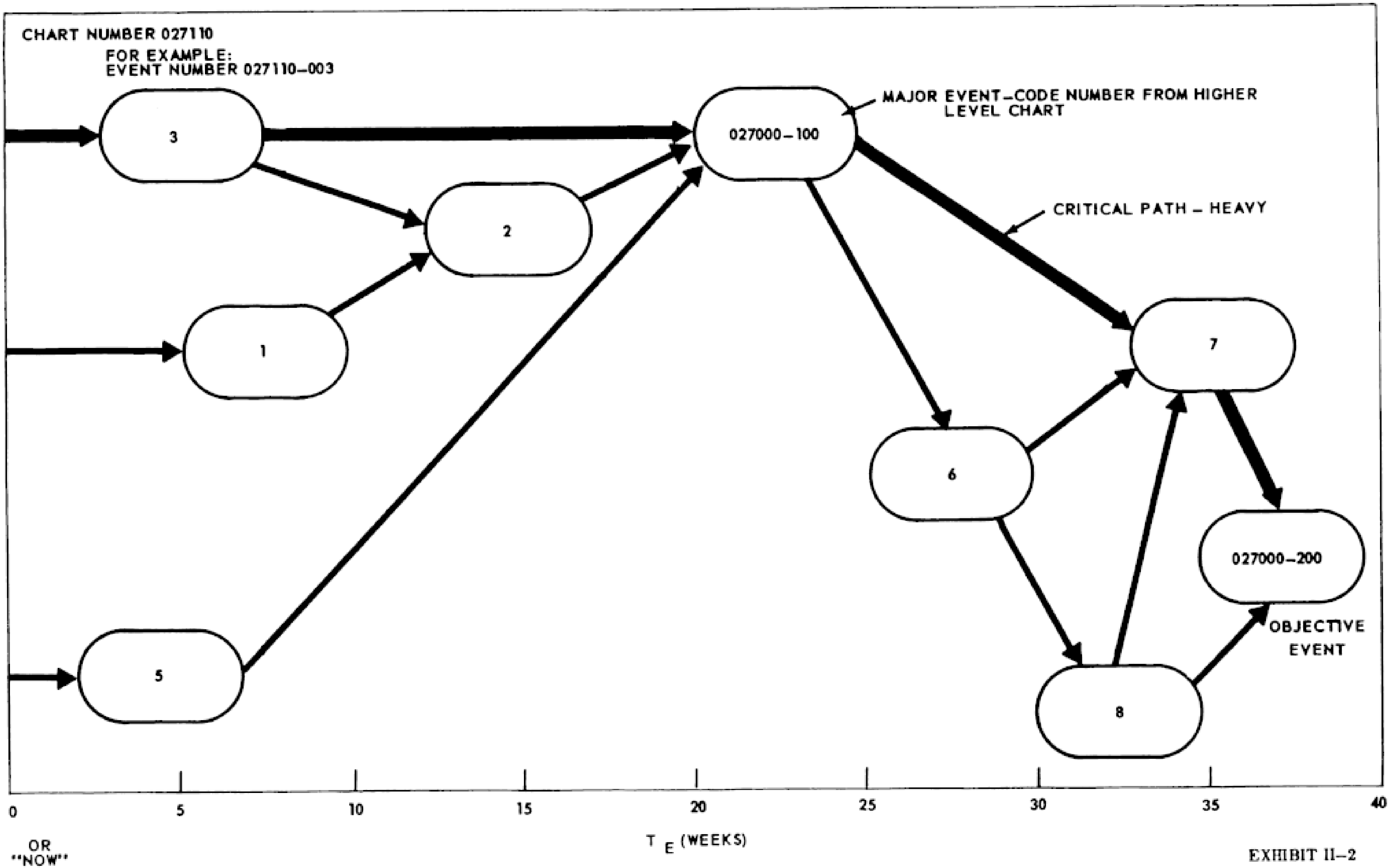

EXHIBIT II–2

form to use is illustrated in the original flow chart of Exhibit II. When all three time estimates (optimistic, most likely and pessimistic) are equal, the activity should be analyzed carefully for possible errors in estimation.

(5) *Identifying Events and Assigning Code Numbers.* In order to adequately identify events in a form amenable to computer analysis and to transmit information about events in an unclassified manner, it is necessary to assign code numbers to the various events.

The code number consists of two parts: a six-digit prefix and a three-digit suffix, separated by a hyphen. The prefix identifies the highest level chart number upon which the event appears. For example, most events on a component chart numbered 027110 would have this particular six-digit number as a prefix. However, certain important events which appear on a subsystem or system flow chart would be numbered according to the topmost chart upon which they appeared. Typically, the flow chart numbering would appear as one of the following examples:

Chart Number	-	*General Type*
020000	-	System.
027000	-	Subsystem (may contain 020000 numbers)
027100	-	Component (may contain 020000 and 027000 numbers).
027110	-	Subcomponent (may contain 020000, 027000, and 027100 numbers).
027111	-	Use of the last right hand column indicates a special chart.

Note: It is possible to have a short-range system chart and number it 020001. Events on it, however, would be numbered 02000 *if* they appeared on the 02001 chart.

An index should be kept in SP probably by computer liaison, of all high level chart numbers of a general nature that are likely to appear on lower level charts, i.e., events that concern a combination of two or more systems, subsystems, etc. A flight test, for example, would probably concern ballistic shell, flight control, guidance, and propulsion, etc. When constructing a new flow chart or network, all events of a general nature should be checked with a copy of the index. When event descriptions are the same, numbering of the highest level chart must take precedence. Notice the numbering of events in Exhibit II.

The suffix consists of three digits that are assigned sequentially as the chart is drawn. Descriptions of events are noted on the flow chart according to the form and order illustrated by the following dummy.

(Begin)	Key word
(Flight Test from Flat Pad)	Complete description
(AFMTC)	Location, if necessary
(Propulsion s/s)) (CX-1))	Identification
(16)	Prefix or sequence number

After all the events have been placed on the chart, suffix numbers are assigned: 001 to the first event picked (not in any prescribed order), 002 to the second, 003 to the third, etc. It is best to keep an index of these events. The above example would appear as follows:

INDEX

Code Number	*Other Identification Number* (If Any)*	*Description*	*Responsibility*	*Schedule Date*
027100-001		Begin Flight Test from Flat Pad, AFMTC Propulsion s/s CX-1	LMSD	(---)

*Any other number, such as chart and line number of a PMP which should be retained.

(6) *Completing the Chart.* The flow chart is to be used as the basis for conveying the initial data to Sp 12. In its original "work sheet" form, it must be classified because of the event description. If the chart is redrawn with no time scale or dates shown and the identification code numbers are placed in the event boxes instead of the descriptions, the chart can be transmitted unclassified. However, an event index list is required in addition to the flow chart to explain the meaning of each code number.

3. TRANSMITTING INFORMATION.

(1) *Using Flow Charts To Convey Information.* The chart should be redrawn so that it is readily reproducible. If placed on tracing paper with India ink lines, it can be reproduced by the Ozalid process. Original charts and reproducible masters should be kept by the contractor member of the Program Evaluation Team and copies submitted through the SP Field Office as per the data flow chart in Exhibit III, following this page.

(2) *Translating the Flow Chart to a Standard Progress Form.* Information as presented on the flow chart cannot be fed directly into the computer. At some point in transmission, the flow chart has to be converted into numerical data. Each activity time and its corresponding boundary events are translated into entries on the Report of Progress

DATA FLOW DIAGRAM - PERT
ORIGINATING AND REVISING FLOW CHARTS

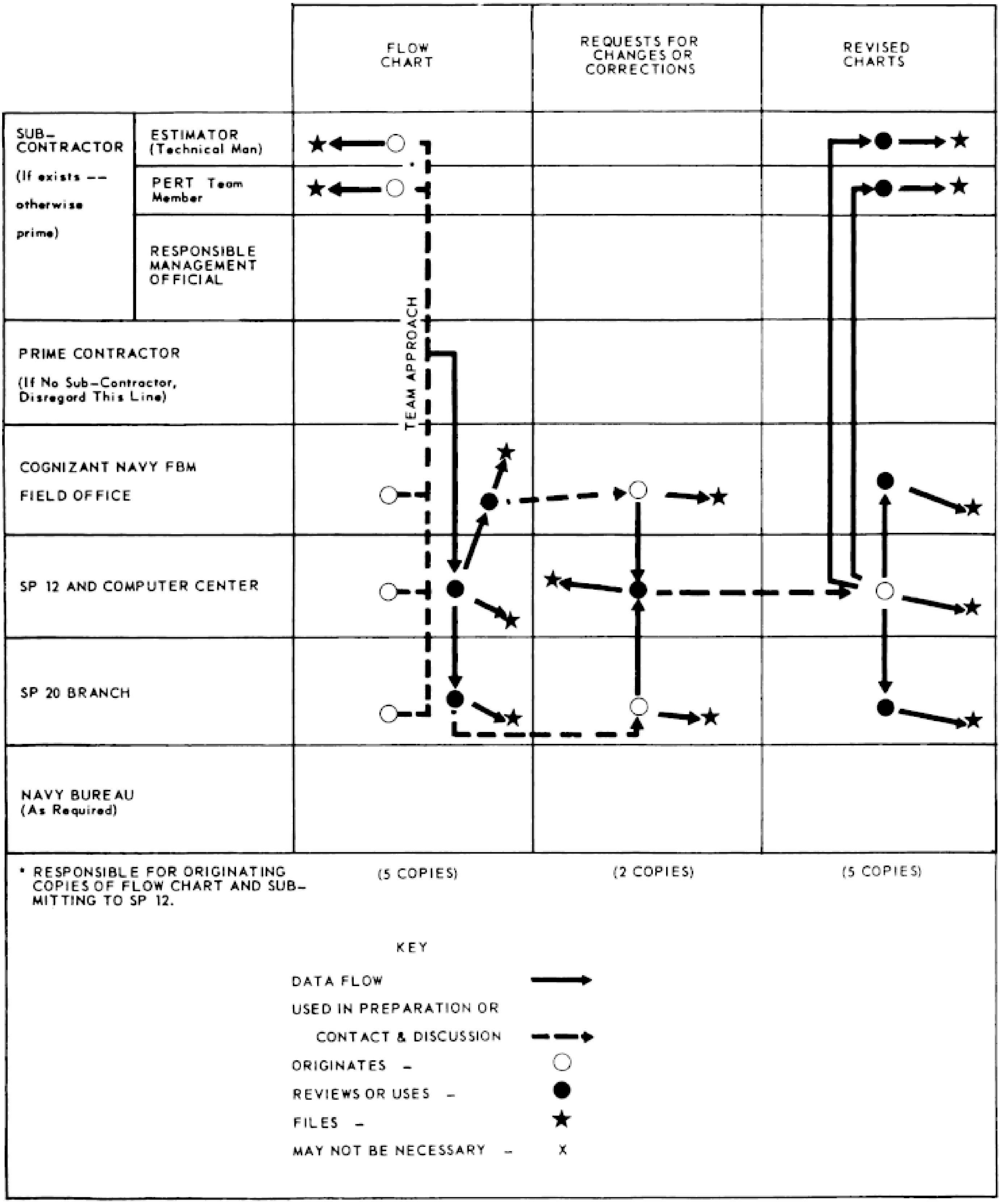

EXHIBIT III

(PERT)
REPORT OF PROGRESS & TIME INTERVAL ESTIMATES

DUMMY

CLASSIFICATION
(WHEN FILLED IN)

FROM: (NAME & LOCATION OF CONTRACTOR)	TO:	CONTRACT NO.	REPORT PERIOD FROM/TO
SP 12	COMPUTER CENTER	---	---

FOR OFFICE USE ONLY (DO NOT FILL IN)			EVENT IDENTIFICATION NO.		TIME INT. EST.			COMPLE-TION DATE	DO NOT FILL IN	REMARKS
			PRECEDING	SUCCEEDING	OPT. (WKS)	MOST LKLY (WKS)	PESS (WKS)			
(1)	(2)	(3)	(B)	(C)	(D)	(E)	(F)	(G)	(4)	(H)
4	200	50	027100–296	027100–340	2	6	12	(NOT NEEDED)	(NOT USED)	Effect of Proposed Changes on Entire Program
4	200	50	027100–297	027100–339	6	7	8			
4	200	50	027100–298	027100–323	12	26	32			
				(ETC.)						

SIGNATURE AND TITLE OF RESPONSIBLE OFFICIAL

DATE SIGNED

CLASSIFICATION
(WHEN FILLED IN)

EXHIBIT IV

and Time Interval Estimates Form illustrated in Exhibit IV, following Exhibit III. The conversion should take place in the data flow process as soon as possible after the origination of the chart. A logical place for this routine clerical operation would be in the SP Field Office or in the contractor's program plans or program analysis group. The decision as to who does this will be made by the Sp 12 representative at the time of installation. Required copies of the completed forms are to be forwarded to SP for distribution and analysis as indicated in Exhibit III.

(3) *Coding the Standard Progress Form.* Certain sections of the Report of Progress and Time Interval Estimates Form are reserved for the coding section of the Computer Center or other authorized groups. Entries in column (1) must be made as follows:

Reason for Line Entry on Form	*Code Number*
New or original estimate of an activity (adding a connecting link)	1
Re-estimate of existing activity	2
Completed activity	3
Static change - correction by computer staff	4
Deletion of an activity	5

Columns (2) and (3) are specifically set aside for SP, FBM managers. Special computer runs can be made when requested. In order to code computer runs, entries in these two columns will be supplied by the individuals within SP Technical Branch or the SP Field Office that originates the request for a special run. These entries will reflect any hypothetical or proposed changes in resources (column 2) and/or performance (column 3). Entries should be in the form of 3 digits representing per cent of performance or resource allocation. Dropping performance to 50% of original plans would be indicated by an 050 entry; similarly resource allocation of 500% (5 times original plans) would be shown by a 500 entry.

Column (4) is provided for the computer staff. Entries here translate a calendar date from column (G) into a code number understandable by the computer. Tables will be provided the computer clerical staff so that they can make the conversions.

*Leave this section out of manuals intended for use solely by contractor personnel.

4. HANDLING OF ORIGINAL DATA.

(1) *Revising the Original Flow Chart Drawing.* Computer analysis of the original data will determine for each event an earliest time or a time when the event is expected to occur. After this is done, revised flow charts will be drawn by Sp 12 Graphics using earliest times for determining the positions of various events along the horizontal axis. Copies of these revised original charts will be transmitted to the cognizant SP Technical Branch and to the estimator making the estimates. Any necessary changes or corrections must be submitted to Sp 12 Computer Center on the chart or on the proper form Exhibit V, following this page.

If there are corrections or changes to be made, Sp 12 Graphics will revise the chart and notify the original team of the changes (see Exhibit III).

IV. MAINTAINING THE DATA FLOW

1. GENERATING THE BIWEEKLY REPORT. The form used with its attendant instructions is described in Exhibit VI, on page 39. Transmission of the form is outlined in Exhibit VII, on page 40.

2. REQUESTING ADDITIONAL INFORMATION ON CRITICAL AREAS. As a result of computer analysis, specific paths on the flow chart will show up as critical. At the end of each monthly period, the contractor will be required to submit re-estimates of all time intervals along the critical path. The requests will be handled as set forth in the data flow diagrams of Exhibit VIII, on page 41, and will require either completion of a form similar to that illustrated in Exhibit V or a revision of a flow chart as provided by Sp 12. Requests from Sp 12 may originate on the form mentioned, in which case, it will be necessary for contractor personnel to finish the entry for each partially completed line and return the report to the Computer Center. Outputs will be distributed as illustrated in the data flow chart, Exhibit VIII.

3. FULFILLING "SPECIAL" REQUESTS FOR SUPPLEMENTARY DATA. A "special" request for supplementary data may originate at any time as a result of endeavors by SP Technical Management to develop new or revised plans and schedules. The method to follow in originating and transmitting these requests is outlined in the data flow diagram of Exhibit VIII.

The process begins with the SP Technical Branch and the SP Field Office writing up the request on a standard form (Exhibit V) and submitting this to Sp 12. If more information is required on the form from the contractor, the partially completed form is transmitted as shown in Exhibit VIII. Outputs generated as a result of such hypothetical or proposed changes are returned to the requesting body.

4. REVISING EXISTING FLOW CHARTS QUARTERLY. Essentially the same procedure is followed in revising flow charts quarterly as is used

(PERT)
REPORT OF PROGRESS & TIME INTERVAL ESTIMATES

DUMMY

CLASSIFICATION (WHEN FILLED IN)

FROM: (NAME & LOCATION OF CONTRACTOR)	TO:	CONTRACT NO	REPORT PERIOD FROM/TO
LMSD, SUNNYVALE, CALIFORNIA	SP 12	(INSERT CONTRACT NO.)	9/16 TO 9/30

FOR OFFICE USE ONLY (DO NOT FILL IN)			EVENT IDENTIFICATION NO.		TIME INT. EST.			COMPLE-TION DATE	DO NOT FILL IN	REMARKS
			PRECEDING	SUCCEEDING	OPT. (WKS)	MOST LKLY (WKS)	PESS (WKS)			
(1)	(2)	(3)	(B)	(C)	(D)	(E)	(F)	(G)	(4)	(H)
3			"0"*	027110–001				9/22	12.6**	Completed event
2			027110–003	027110–005	6	8	9			Re-estimate because of change in plans
5			027110–002	027110–004						
5			027110–004	027110–005						Event # 027110–004 deleted
1			027110–002	027110–196	3	9	12			
1			027110–196	027110–005	4.5	6.5	8			Event # 027110–196 added – schedule date: 2/59

SIGNATURE AND TITLE OF RESPONSIBLE OFFICIAL

DATE SIGNED

* No predecessor

** Filled in by Sp 12 or Computer Personnel

CLASSIFICATION (WHEN FILLED IN)

EXHIBIT V

(PERT)
REPORT OF PROGRESS & TIME INTERVAL ESTIMATES

DUMMY

CLASSIFICATION (WHEN FILLED IN)

FROM: (NAME & LOCATION OF CONTRACTOR)	TO:	CONTRACT NO.	REPORT PERIOD FROM/TO
LMSD – SUNNYVALE CALIFORNIA	SP 12	(INSERT CONTRACT NO.)	- - -

FOR OFFICE USE ONLY (DO NOT FILL IN)			EVENT IDENTIFICATION NO.		TIME INT. EST.			COMPLE-TION DATE	DO NOT FILL IN	REMARKS
			PRECEDING	SUCCEEDING	OPT. (WKS)	MOST LKLY (WKS)	PESS. (WKS)			
(1)	(2)	(3)	(B)	(C)	(D)	(E)	(F)	(G)	(4)	(H)
1*			027110–001	027110–002	** .1	.5	1	9/58 ***		
1			NOW OR "0"	027110–003	10	12	16	12/58		
1			027110–002	027110–004	.2	.8	1.2	10/58		
1			027110–004	027110–005	12	16	22	2/59		
1			027110–003	027110–005	3	4	5	2/59		
1			027110–005	02700–100	.2	.2	1	1/59		(Notice use of 02700 numbers indicating general high level, type of event)
1			02700–100	027110–006	3	3	3	3/15/59		
1			027110–009	027110–010	5	5	10	5/59		

SIGNATURE AND TITLE OF RESPONSIBLE OFFICIAL

DATE SIGNED

* Filled in by other than Contractor Personnel

** Column (D), (E), (F) and (G) filled out by Contractor

*** A date given as a month and a year will be considered as equal to the last day of that month.

CLASSIFICATION (WHEN FILLED IN)

DATA FLOW DIAGRAM - PERT
TRANSMITTING BIWEEKLY REPORTS

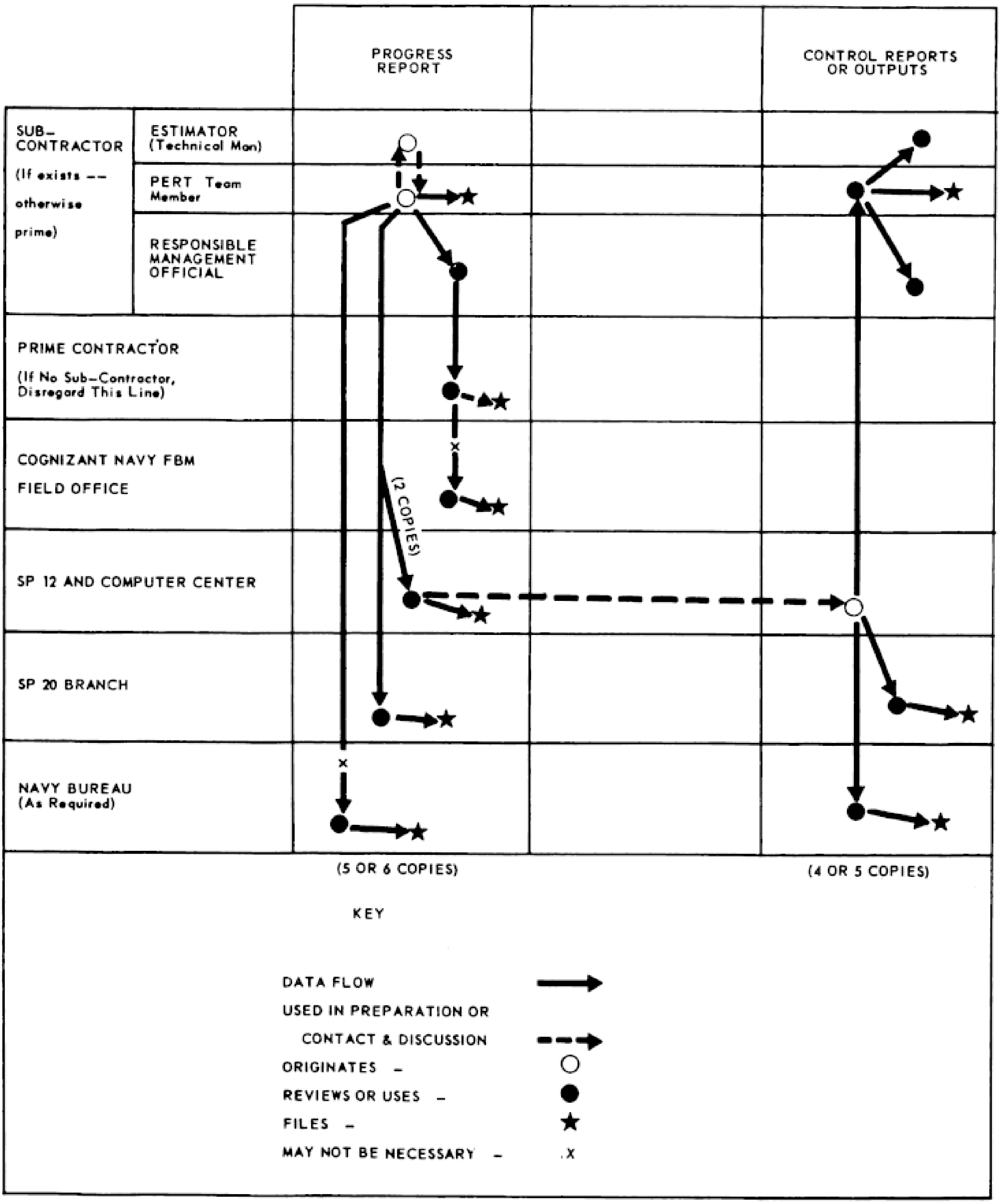

EXHIBIT VII

DATA FLOW DIAGRAM - PERT
REQUESTING ADDITIONAL INFORMATION

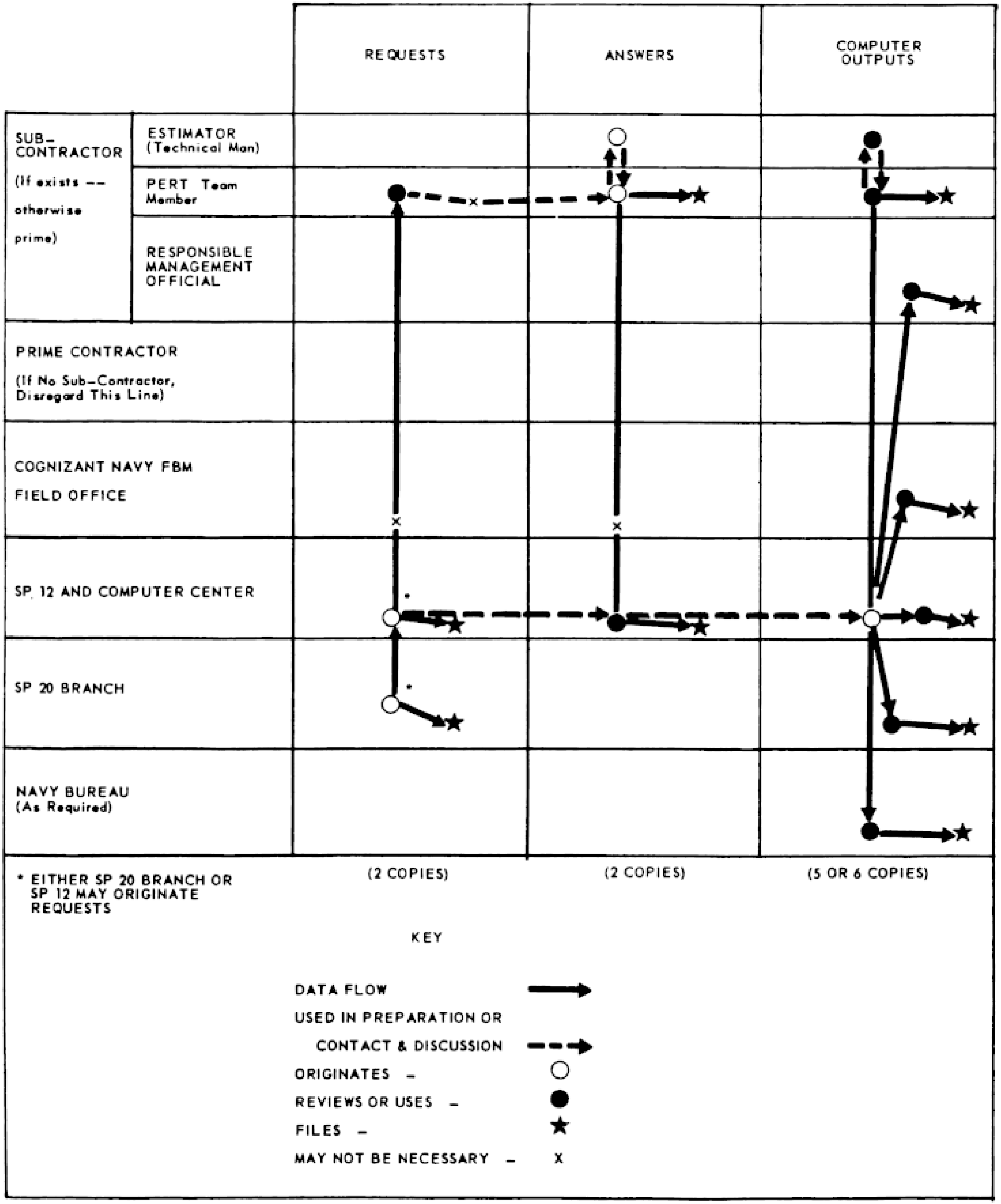

EXHIBIT VIII

in setting them up originally. A team of representatives as described in Section III critically examines all events and times in the existing flow chart. Revisions and re-estimates are made wherever necessary. All changes are communicated as outlined in the flow chart of Exhibit III and are incorporated as soon as available to Sp 12 in subsequent computer runs.

V. UTILIZING OUTPUTS FOR MANAGEMENT DECISION MAKING

1. INTERPRETING AND ANALYZING OUTPUTS. The output sheets as illustrated in Exhibit IX, following this page, provide valuable information as to progress to date and anticipated future progress. The T_L - T_E column indicates the amount of slack in a particular event. The presence of a large amount of positive slack in an event is an indication of a place where resources might be available for possible "trade-offs." A slack value approaching zero would indicate that the event in question is likely to be a potential trouble spot. Critical paths are determined by highlighting (drawing in red or otherwise emphasizing) those lines which pass through events with zero slack. Slippage of an event along the zero slack path will cause a corresponding slip in some major end objective.

Monthly requests for additional data are an attempt to focus attention on relatively critical paths and to audit a few specified events. Re-estimates are requested for all those times that lie along the critical paths. This serves two purposes: (1) emphasis is placed on "tight" areas, and (2) estimates in these areas are marked for critical analysis with an effort to produce more accurate data where it is needed most.

Auditing seeks to improve the accuracy of the flow chart by requesting estimates on a small group of selected activities which differ from month to month. It is expected that all events on the chart will be covered at one time or another.

The T_S column is merely a conversion of the schedule date for an event to the coded date as utilized by the computer. The P_r column is the result obtained by comparing on a probability basis the expected or earliest time (T_E) for the occurrence of an event with the schedule time (T_S). The number in the P_r column indicates the probability of reaching the event on or ahead of schedule. Obviously, if the P_r value is low (below .05), the probability of meeting or beating the schedule is quite remote. Values of about 0.5 are good, while values close to 1.0 are excellent.

2. DEVELOPING NEW OR REVISED PLANS. As a result of examining initial outputs on a particular component or subsystem, the Technical Branch of SP may decide to develop and to test hypothetical plans. Those events that are marked as probably being reached ahead of schedule indicate possible areas where resource trade-offs might be arranged. Events along the critical (zero slack) path indicate possible areas for performance degradation or increased resource application.

If the schedule for a major event is in jeopardy, it may be possible to replan in a fashion that will improve the outlook of meeting schedule. This replanning could take the form of altering the planned sequence of events, if that is possible. Such an alteration could postpone certain activities to a later point in time. E.g., it might be possible in certain situations to forego some testing before a flight. Thus the replanning would not force the flight to await all of the preliminary tests that were originally planned.

In addition to the changes already mentioned, it is possible to set up the computer so that it will develop a new schedule based on some arbitrary set of criteria. This is described in detail in the Phase I report of the PERT project - available from Sp 12.

Changes like those above can be indicated on a form of the type presented in Exhibit IV, and the computer will make the analysis within a few hours after the origination of the request. The speed with which the computer operates allows many hypothetical situations to be posed and tested in a short period of time. The difficult part of the process is the translation of the physical situation envisioned in the mind of the planner into the objective terms which form the language of the computer. There can be nothing hazy or indefinite about the information or instructions that are fed into the computer if accurate and reliable results are to be obtained. A computer liaison man will be available in Sp 12 for consultation on problems of computer interpretation.

PERT SYSTEM Biweekly Report Output

DATE 9|16|58 WEEK 11.8 PAGE ____

EVENT	T_E	T_L	$T_L - T_E$	T_S	P_r	Scheduled Date
027110-001	1.1	1.1	0	1.5	.65	9/26/58
027110-004	4.3	5.6	1.3	6.0	1.00-	10/58
027110-005	29.3	29.3	0	25.0	.00+	2/59
027110-196	30.7	32.9	2.2	25.0	.00+	2/59

EXHIBIT IX

APPENDIX B

COMPUTER PROCEDURES

TABLE OF CONTENTS

INDEX OF EXHIBITS IN APPENDIX B

I. INTRODUCTION

The advantages that will accrue from the PERT system depend on ability to speedily analyze a large mass of numerical data. The analysis itself involves many arithmetic operations. Human capacity is unable to accurately cope with both the volume of data and complexity of analysis that is necessary for timely evaluation. Recognition of this fact (which is conditioned by the immensity of the FBM program) inevitably led to the necessity of a computer analysis.

A computer is a mechanism for following man-made rules with great speed and with great accuracy. In the PERT application, the computer exists as a facilitating tool for technical managers. It is an adjunct that quickly summarizes for the manager a group of facts that bear on the FBM program. These facts will not make decisions for the manager. They will, however, expand the base of knowledge which is necessary for wise decision. The computer processing of the PERT procedure will provide important help in getting the necessary job done on time.

The PERT system analysis is being run on the Naval Ordnance Research Calculator (NORC) at the Naval Proving Ground, Dahlgren, Virginia. The planning for this application has been done by a Booz, Allen & Hamilton staff specialist who, in addition, has monitored the machine coding and checkout. The actual machine coding has been carried out by programers from the Naval Proving Ground (NPG) installation.

During a large part of the installation period, the Special Projects Office was without an available computer liaison man. Consequently, it is particularly necessary to provide an appropriate continuity of effort and transfer of knowledge and experience. This appendix will provide such continuity and transfer by a detailed description of all phases of the PERT computer installation.

The remainder of this appendix will trace the history of the installation, note the problems encountered and their solutions, and, in addition, will highlight certain important considerations for continuing computer application to PERT.

II. CHOICE OF THE COMPUTER FACILITY

1. GENERAL CONSIDERATIONS. The anticipated size of the combined PERT networks for the complete FBM system immediately directed attention to the need for a high speed electronic computer. Preliminary analyses of the magnitude of the job indicated that complete coverage of the FBM system would involve 5,000 to 10,000 events. Thus, a high speed computing system is needed to maintain accurate, detailed records of the events and to produce timely reports.

From the preliminary analysis, it was clear that the data-processing logic for PERT could be handled by computer of the IBM 650 class. The need for rapid reporting and possible expansion to more complex calculation led to the consideration of large computing systems. A further consideration of importance was the availability of such systems within the Naval establishment.

Consequently, a survey of nearby Navy establishments was undertaken in order to choose that facility which would best do the job from the standpoints of time and cost.

The equipment involved in the basic choice were the Univac 1103-A at APL, Johns Hopkins University, and the IBM NORC at the Naval Proving Ground. The comparative costs and operating capabilities of both systems are shown on Exhibit I, following this page. This analysis and some subjective consideration of the availability of personnel lead to the selection of the NORC installation.

2. DESCRIPTION OF NORC. NORC is a high speed, stored-program computer. The computer proper consists of a 2,000-word cathode ray tube high speed memory, an electronic control section, an arithmetic section, eight high speed magnetic tape memory units, two line-at-a-time printers, and a console allowing manual direction of the computer. Associated, but separate, equipment includes a card-to-tape-to-card converter and a complete selection of card-processing equipment, 407 printers, 514 reproducers, 077 collators, 082 sorters, and standard key-punch machines.

Three modifications to the NORC system have been authorized and will be installed during the next two years. The first modification (now available) is a cathode ray tube high speed photographic printer. (Secondly, the memory will be expanded to 20,000 words (magnetic cores) during the next year. Lastly, the Universal Data Transcriber (UDT) is being designed and built by the NPG staff. This is a unit containing a small scale computer designed specifically to handle the problems of converting one input media to another. Generally, the UDT will convert information in any media and code to any other media and code. Included in its capability is the transfer from one tape code to another, card-to-tape, tape-to-card, paper tape to magnetic tape, magnetic tape to paper tape, tape-to-printer, card-to-printer, etc.

NORC carries out its calculations on decimal numbers arranged in 16-digit groups called words, under control of a three address stored program. An instruction word is arranged in five fields. Two 2-digit fields (P and Q) specify the form and operation of an instruction, and three 4-digit fields (R, S, and T) specify the high speed memory locations required for execution of an instruction. A data word consists of 16 decimal digits which when used in floating print operation consists of a 2-digit coefficient, a 1-digit sign, and a 13-digit mantissa. In this manner, NORC can calculate using up to 13 significant figures in the range 10^{-30} to 10^{+30}.

The speed of the NORC is an important consideration as shown earlier (see Exhibit I) and can be summarized as below:

Salient Speed Characteristics

Memory Access Time	8 microseconds
Instruction Time	(Average executive times for arithmetic operations including access time)
Add-Subtract	46 microseconds
Multiplication	72 microseconds
Division	272 microseconds

Tape Time

Tape packing	510 digits/inch
Tape transport time	140 inches/second
Tape information rate	71,400 digits/second

The off-line card-to-tape-to-card converter (CTC) has a basic limiting speed of either reading or punching cards at 100 cards/minute.

Essentially NORC speed can be summarized by stating that NORC is at least two times, and in many cases four to five times, as fast as any other machine now in use, no matter how the problem is considered.

NORC is operated on three shifts daily, five days a week. The primary shift is devoted primarily to testing of programs, while the second and third shifts are utilized for production runs. Maintenance is scheduled and is conducted mainly during the second and third shifts. Utilization of the equipment is in excess of 90%, when good time is expressed as a percentage of scheduled good time. All maintenance and operating functions are performed by the Navy.

Having described both the facility that was chosen as well as the considerations that led to its choice, it is appropriate to associate the computer with the problem at hand. The computer processing of the PERT information can be considered as two classes of activity. On one hand, a structure must be erected in the computer so as to accommodate data and provide the framework within which analysis can take place. Secondly, the analysis itself must be accomplished. The following two chapters will describe the work involved in establishing an efficient program in each of the foregoing classes.

III. PROVIDING A STRUCTURE FOR ANALYSIS

In order to provide a framework within which machine analysis can take place, two jobs must be performed. Initially, identifiable and meaningful data must be introduced into the machine. In addition, however, these data must be carefully placed within the machine so as to expedite further analysis and subsequent outputs.

The general flow of the data in the PERT system is symbolically portrayed in Exhibit II, on page 52. This flow chart shows the handling of the stream of incoming information from the time it is received by Sp 12 to its reappearance as a NORC output.

1. INPUTS TO THE COMPUTER

(1) *Key-Punch Procedure.* Upon arrival at the Computer Center, a batch of transactions will be assigned to the key-punch group. Here, input information will be key-punched in cards while maintaining batch identity. After key-punching, the cards will be key-verified. The key-punch time can be estimated at *100* cards per hour, and key-verifying can be estimated at *100* cards per hour. The following are estimates for the weekly key-punching and key-verifying times for given sizes of event files.

Number of Events	Cards/Week	Key-Punch and Verify Time (Hours)
500	100	2.0
1,000	150	3.0
2,500	375	7.5
5,000	600	12.0
10,000	1,000	20.0

(2) *Card Layout.* Functionally, the card is laid out to meet the NORC operating requirements. As such, the NORC input card is constructed with four 16-digit words beginning at column 12 and ending in column 75 (see Exhibit III, on page 54). Within these four 16-digit words, items of information can be laid out with little or no restriction on the fields. However, to conserve on NORC operating time and coding, it is advantageous to keep the fields within each word and, when possible, to keep each input transaction within one card. In developing the system for mechanizing the evaluation system, it was possible to satisfy both conditions. It is important to note that all NORC input must be either significant digits or zeros. Blanks or the absence of data are indicated by zeros.

(3) *Card-to-Tape Conversion (Run 01).* After the cards have been punched and verified, they enter into what has been designated as computer Run 01. The card-to-tape conversion is not a programed run in the sense that a stored program is necessary to direct the operation. The CTC is an auxiliary unit in the NORC Computer Center which functions either to produce punched cards from magnetic tape or to produce standard NORC magnetic tapes from properly prepared cards. The CTC unit contains its own 100-word cathode ray tube memory, a separate magnetic tape unit and a modified IBM 519 reproducing punch, which acts as a card punch and card reader. The unit operates at a rate of 100 cards per minute either punching or reading cards.

The 100-word memory is the heart of the CTC operation, as it acts as a buffer or speed exchanger between the high speed, high density magnetic tape

COMPARISON OF COMPUTER FACILITIES AVAILABLE FOR PERT

	NORC	Available 1103-A Type
Cost		
Machine Time	$195/hr.	$220/hr.
Efficiency	3 - 5	1
Relative Cost (Cost/unit of work)	$40-$65/hr.	$220/hr.
Programming Cost	Labor and Station O. H.	Fixed Cost(1)
Flexibility		
Staff	Large	Small
Experience	10 Years	1 - 2 Years
Peripheral Equipment	Extensive	Limited(2)
Interchangeability	None	Extensive
Navy Control	Direct (BuOrd)	Indirect

(1) Approximately equal to NORC Labor and Overhead.

(2) No off-line card-to-tape, or tape-to-card equipment available. Serious cost limitation.

EXHIBIT I

PERT DATA PROCESSING FLOW CHART
(PART 1)

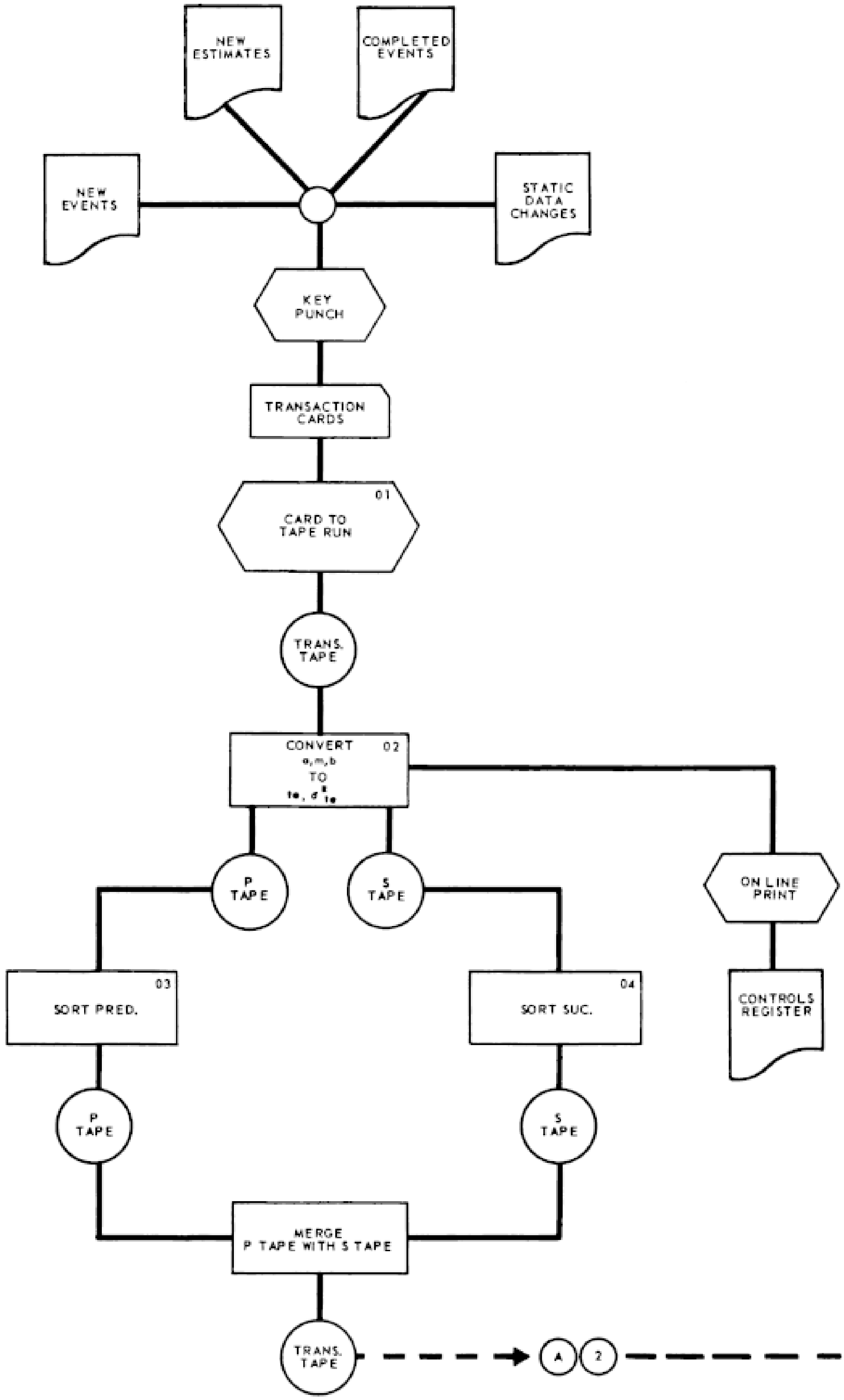

EXHIBIT II

PERT DATA PROCESSING FLOW CHART
(PART 2)

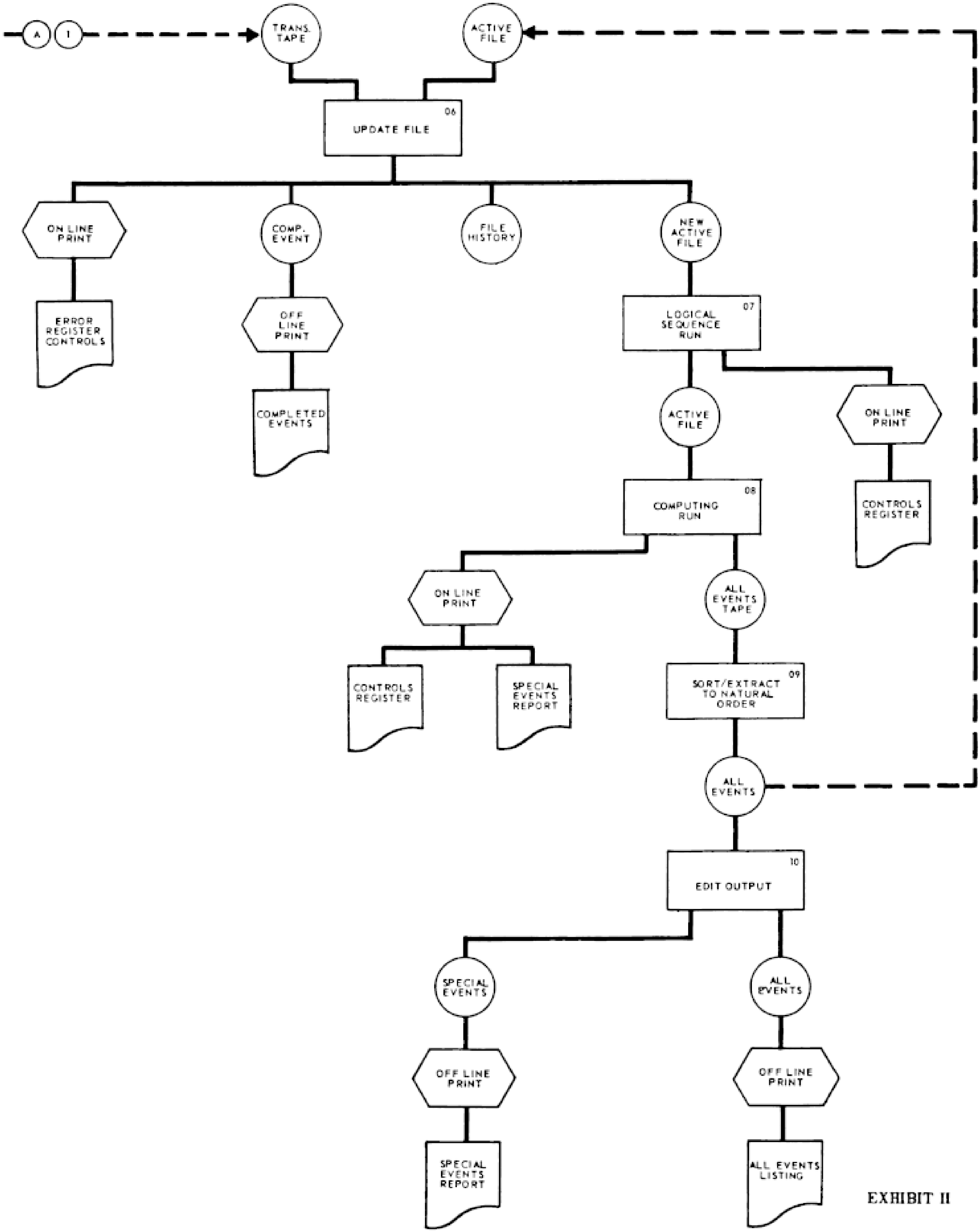

EXHIBIT II

INPUT CARD LAYOUT

ZERO	BATCH NUMBER	KEYPUNCH DATE	CARD CODE	RESOURCE RATE	PERFORMANCE RATE	ZERO	PREDECESSOR CODE	ZEROS	SUCCESSOR CODE	REPORT CODE	OPTIMISTIC ESTIMATE (a)	MOST LIKELY ESTIMATE (m)	PESSIMISTIC ESTIMATE (b)	COMPLETION OR SCHEDULE WEEK	SCHEDULE DATE MO. DAY YR.	ZEROS	ZERO	PERT.
1	2 5	6 7 8 9 10 11	12	13 14	15 16	17	18 26	27 33	34 42	43	44 47	48 51	52 55	56 59	60 65	66 75	76	77 80
			I					II			III				IV			

EXHIBIT III

and the relatively slow speed, low density punched cards. In the card-to-tape operation the unit reads 24 PERT system input cards into the memory. This group is preceded by a 1-word card, indicating the beginning of the group or block and is followed by a 1-word card indicating the end of the block. When the memory is loaded with the 100-word information, it is read out of memory into magnetic tape. Thus the 24 IBM cards are compressed into 3.33 inches of tape. In the card-to-tape processing the CTC unit provides a check digit for each word. This is later used as a part of the internal automatic checking operation which is part of the basic NORC design. The large difference in speed between the card and tape functions allows the cards to be read continuously while the tape transfer takes place between the reading and movement cycles of the 26th and 27th cards, and then again between the 52nd and 53rd card, etc.

As a result of this operation the PERT input tape is created, and a detailed section of it is shown in Exhibit IV, on page 56. The tape is organized in 100 word blocks or sub-blocks.

2. POSITIONING THE INPUTS WITHIN THE COMPUTER. The magnetic tape output of the card-to-tape conversion is the vehicle for actually introducing information into the computer. However, before the input information is used in an analysis run, it is necessary to precisely locate it and change it to a form most amenable to the subsequent analysis. In order to accomplish this, several computer runs are made. The result of these runs is the establishment of a basic event file, the final repository of the basic inputs (see Exhibit II).

(1) *Transaction Data Conversion (Run 02).* The input to Run 02, the transaction data conversion run, is the tape from the CTC, Run 01. The input is arranged in 100 word blocks, 24 transactions to a block. Each transaction is arranged in four word messages, see Exhibit IV. Transactions are not ordered in any sense for this run.

The primary purpose for this run is to convert the three time estimates—a, m, and b—into values of t_e and $\sigma^2_{t_e}$ and to set up two output tapes for updating predecessors and successors.

In converting a, m and b to t_e and $\sigma^2_{t_e}$, the program uses formulae of the following form.

$$t_e = K_1 m + K_2 (a + b)$$

$$\sigma^2_{t_e} = \left(\frac{b - a}{K_3}\right)^2$$

The three parameters K_1, K_2, and K_3 are inserted in the program at the time the run is set up and can thus be changed at will if needed. At the present time the values used are as follows:

$K_1 = 0.6666666666667$
$K_2 = 0.1666666666667$
$K_3 = 6.0$

The run 02 also tests the relationship:

$a \leq m \leq b$

and rejects all transactions where this inequality is not met. These are printed out so they may be corrected. Also the test $\sigma^2_{t_e} \geq 100.0$ is performed and, although the transaction is not rejected, a print-out is made for further outside verification of this large value of $\sigma^2_{t_e}$. The values for t_e and $\sigma^2_{t_e}$ are computed and stored in the floating point mode using a full NORC word. In addition to the printed outputs noted above, the program prints out the input and output labels for the tapes themselves.

The transaction data conversion program prepares two output tapes, one to be used in setting up (or updating) the activity times for the predecessor event and the other to be used in setting up (or updating) the successor event. A code identifies which of the two events is being handled (successor or predecessor). A six is used in the case of the predecessor event updating and a seven for the successor. The successor event identifies the file event and carries all the static data for the event file.

(2) *Sort and Merge Transaction Tape (Runs 03, 04, 05).* Runs 03, 04, and 05 are shown on the flow chart (Exhibit II) as separate logical functions and are separate runs at the present time. However, they will be combined into a single run in the near future and can be considered as a single function.

The purpose of these runs is to sort the converted transaction tapes in event order sequence and then merge them. The predecessor updating transactions are sorted into event order on Run 03. Then the successor updating transactions are sorted into event order on Run 04: Following this, the two sorted tapes are used as the input tapes for the merge run, Run 05. The two tapes are merged, maintaining event order with the predecessor events going on to the output tape first in the case of identical event codes. The resulting output tape (in event order) is arranged in 196 word blocks with transaction messages seven words in length. Each transaction with a predecessor event code has sufficient information to update the successor *activity* of the event indicated by the predecessor code. Each transaction with a successor event code has the information necessary to update the predecessor *activity* of the successor event. In addition, such information as schedule date, T_S, completion date (in week code form), resource, and performance rate codes are associated with the successor event transaction.

The output of the three runs is a tape with ordered lists of successor and predecessor events together with associated information. An additional run is now necessary before the event file contains the input data. This run (updating) is programed to accommodate either an original file set up or the altering of an existent file of events.

(3) *Event File Updating (Run 06).* The file updating run is designed to add, delete, or change information in the event files. Two input tapes are required, first the even file tape and, second, the transaction tape. Both of these are in event order. This program will handle five and only five transactions.

INPUT TAPE LAYOUT

WORD #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	CARD CODE	RES. RATE		PREF. RATE		—	PREDECESSOR CODE									—	√
	X	X	X	X	X	O	X	X	X —	X	X	X —	X	X	X	O	X
2	—	—	—	—	—	—	SUCCESSOR CODE									RPT. CODE	√
	O	O	O	O	O	O	X	X	X —	X	X	X —	X	X	X	X	X
3	OPTIMISTIC EST. (a)				MOST LIKELY EST. (m)				PESSIMISTIC EST. (b)				COMPLETION OR SCHEDULE DATE				√
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
4	SCHEDULE DATE (CAL.)						—	—	—	—	—	—	—	—	—	—	√
	X	X —	X	X —	X	X	O	O	O	O	O	O	O	O	O	O	X

EXHIBIT IV

—*Transaction Code 1.* Code 1 indicates that the transaction message contains a new *activity* which is not now in the file. An option switch described later also allows this type of transaction to set up a new file from blank tape. Transactions with Code 1 add a new activity to the file event message in either the predecessor or successor group. In addition, transaction Code 1 will set up new information in the performance and resource rate fields, as well as schedule information.

—*Transaction Code 2.* Transaction Code 2 indicates that the transaction message contains data to be used to add a re-estimate of the activity time for the indicated predecessor or successor activity of the event specified. Only transactions with Codes 1 and 2 contain t_e and $\sigma^2_{t_e}$ data.

—*Transaction Code 3.* Transaction Code 3 indicates that the transaction message contains information that an activity has been completed. The transaction removes the activity from the file and inserts a completion date in the file when the last predecessor is completed.

—*Transaction Code 4.* Transaction Code 4 indicates that the transaction message contains data to be used in making changes in the static portion of an event record. The static data is contained in the first seven words of the file. A static data change will affect only schedule dates, and report codes.

—*Transaction Code 5.* Transaction Code 5 indicates that the transaction message contains data to be used in deleting an activity from the predecessor event (a successor activity) and from the successor event (a predecessor activity). "Delete activity" transactions are similar to completion transactions in overall function except that they do not adjust the completion date.

The program operates in the following manner: The first block on the file tape and the first block on the transaction tape are read into memory. The first transaction event code and the event code for the first event in the file are compared through a subtraction. If the transaction code is larger, the next event in the file is selected and the process repeated. If the transaction is equal to file event code, the program branches into the updating subroutine. The file event message at this point is extracted from the read-in area and is placed in a working memory position. The transaction code and the predecessor-successor update code are tested. A new activity code (Code 1) will cause a new successor or predecessor activity code and the associated activity time estimates to be added to the event file. A re-estimate code (Code 2) will cause a new set of activity time estimates to replace the existing estimates in the file event. . . successor or predecessor activities. A completion code (Code 3) will cause a predecessor-successor activity to be removed from the file event record and the successor or predecessor index to be adjusted. In addition, the predecessor index will be tested after modification. If it is zero, the completion date will be inserted in the completion date field. Following this, the entire event file record will be read out into the completed event tape. A static change code (Code 4) will cause the file event static data to be replaced with the static data indicated in the transaction message. Note the only static data change permissible is a change in schedule date, report code of T_S. A deleted activity code (Code 5) will cause the indicated predecessor or successor activity to be removed from the event file record. No test is made of the predecessor index and no write-out of deleted events is made.

The cases for the transaction code's being greater than or equal to the file event code have been described. If the transaction code is less than the file event code, the transaction code is tested. If the transaction code is a new transaction code (Code 1) and the updating code (predecessor-successor) indicates a successor event to be updated, a new event is constructed in working storage and then read out into the output tape block. If any other code combination is identified, then the transaction is rejected and the transaction data printed on the error control register. When a transaction is rejected, the next transaction is extracted

UPDATING FLOW CHART
RUN #06

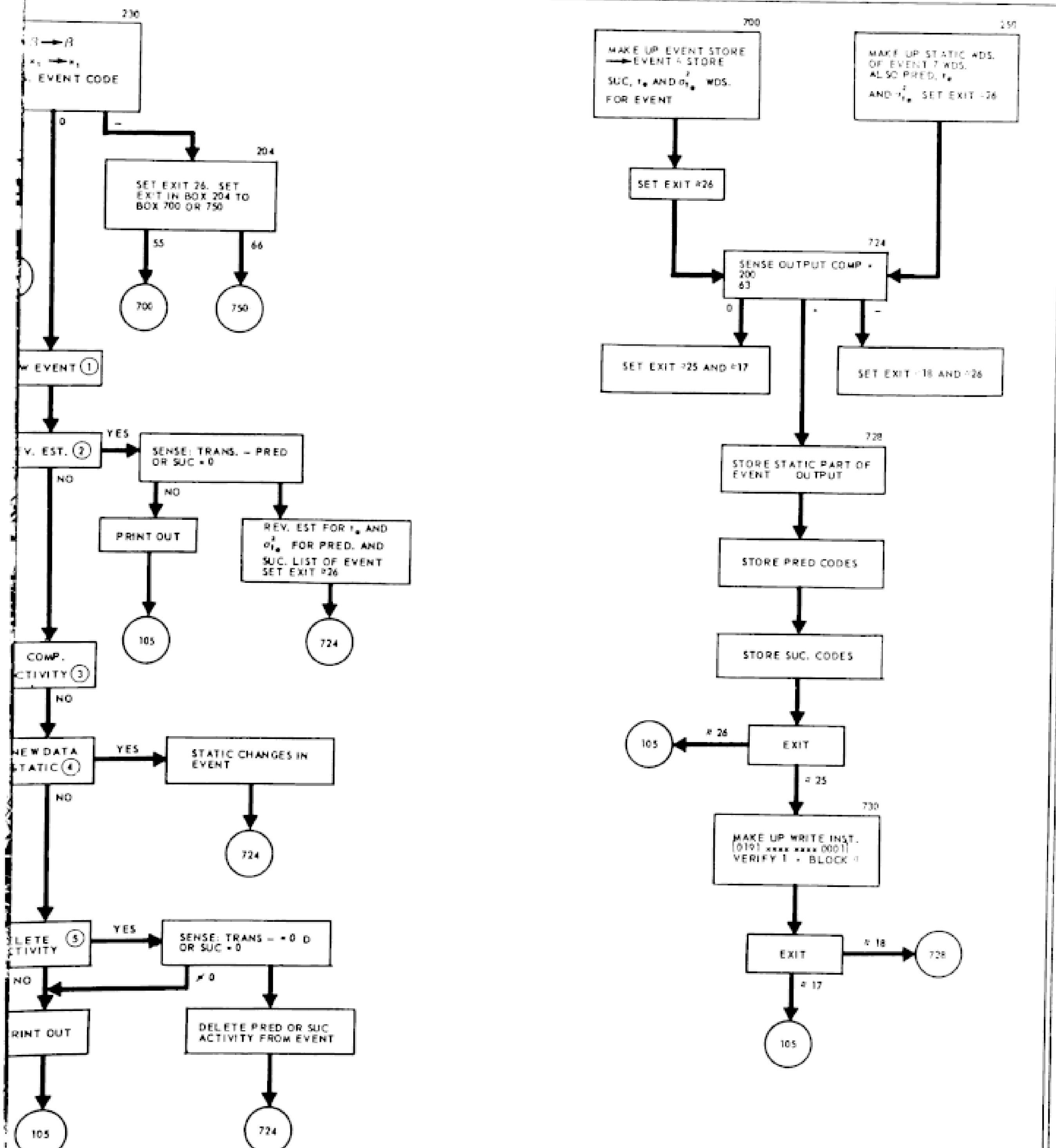

EXHIBIT V

UPDATING ROUTINE

page 1

```
-02 91 0051 0056 0000   09 96 0008 1727 0001   00 68 0000 0000 0006   00 60 0000 0000 0009
 00 00 0000 0000 0000   00 61 0000 0000 0001   00 00 0000 0000 3951   02 91 0051 0056 0000-


-02 91 0051 1770 0001   00 00 0000 0000 0000   00 54 0000 0000 0000   02 94 1031 1031 0000
 08 68 0011 0000 1687   00 61 0000 0000 0088   00 40 1553 1578 1578   00 70 1031 0000 0016
 00 61 0000 0000 0000   00 60 1553 0008 0000   00 40 1578 1583 1578   01 91 1578 1578 0000
 08 60 0019 0000 1704   00 74 0000 0000 0027   03 94 1032 1032 0000   08 68 0022 0000 1687
 00 40 1579 1552 1579   00 60 0000 0000 0000   00 70 1579 1032 0027   00 61 0000 0000 0000
 00 80 1032 0001 0000   00 80 1578 0002 0000   00 80 1588 0003 0000   00 80 0000 0004 0000
 00 80 0000 0005 0000   00 80 0000 0006 0000   00 83 0000 0007 0000   02 94 0629 0000 0001
 08 68 0035 0000 1687   00 67 0000 0000 0599   00 60 0000 1554 0000   04 42 0000 1585 1555
 08 42 1554 1586 0000   00 41 1555 0000 1556   00 40 1587 0000 1556   00 40 0034 1587 0034
 00 51 0000 0000 0000   00 60 0000 0000 0000   00 74 0000 0000 0057   03 94 0830 0000 0001
 08 68 0047 0000 1687   00 67 0000 0000 0607   00 60 0000 1558 0000   04 42 0000 1585 1555
 08 42 1558 1586 0000   00 41 1555 0000 1559   00 40 1587 0000 1559   00 52 0000 0000 0000
 00 40 0046 1587 0046   00 60 0000 0000 0000   00 41 1556 1557 0000   00 63 0063 0060 0000
 00 61 0000 0000 0000   00 60 1669 0065 0000   00 60 1588 1557 0000   00 60 1589 0044 0034
 00 60 4629 1033 0000   00 40 0063 1590 0063   04 58 0001 0007 0063   96 60 1591 0000 0000
 00 40 0065 0000 0065   00 60 1592 0063 0000   00 40 1557 1591 1557   00 42 1036 1593 1560
 00 70 1593 0000 0074   00 70 1560 1594 0081   00 61 0000 0000 0000   00 60 1607 0146 0000
 00 60 1614 0254 0000   00 60 1614 0291 0000   00 60 0341 0329 0000   00 60 1680 0422 0000
 00 60 1666 0414 0000   00 60 1668 0342 0521   00 60 1608 0146 0000   00 60 1633 0254 0000
 00 60 1635 0291 0000   00 60 1614 0329 0000   00 60 1656 0422 0000   00 60 1614 0414 0000
 00 60 1614 0342 0551   00 80 1553 0007 0000   00 80 1031 0001 0000   00 80 1588 0003 0000
 00 80 0000 0004 0000   00 80 0000 0005 0000   00 80 1552 0006 0000   00 81 1588 0002 0019
 00 60 0000 0000 0000   00 41 1559 1561 0000   00 63 0102 0099 0000   00 61 0000 0000 0000
 00 60 1588 1561 0000   00 60 1595 0056 0000   00 60 1601 0124 0046   00 60 1588 1563 0000
 00 60 1588 1564 0000   00 60 6836 1562 0000   02 42 1562 1582 1563   00 42 1562 1596 1564
 00 40 1563 1597 0000   50 24 1598 0000 0000   00 34 1599 0000 1563   00 40 1564 1597 0000
 50 24 1598 0000 0000   00 34 1599 0000 1564   00 60 1591 1565 0000   00 72 1563 1588 0116
 00 60 1670 0130 0118   00 72 1564 1588 0118   00 60 1671 0138 0000   00 40 1563 1564 0000
 00 40 1565 0000 1565   00 59 0000 0000 0000   00 42 0000 1585 1575   00 60 6830 1041 0000
 00 40 0122 1590 0122   06 58 0001 0007 0122   00 60 1600 0122 0000   00 42 0124 1585 0132
 96 60 1563 0000 0000   00 40 0132 0000 0000   00 40 1602 0000 0132   00 60 6830 1048 0000
 00 40 0130 1590 0130   00 00 0000 0000 0000   00 60 1603 0130 0000   00 42 0132 1585 0140
 96 60 1564 0000 0000   00 40 0140 0000 0000   00 40 1604 0000 0140   00 60 6830 1148 0000
 00 40 0138 1590 0138   00 00 0000 0000 0000   00 60 1605 0138 0000   96 60 1565 0000 0000
 00 40 0124 0000 0124   00 40 1561 1565 1561   15 60 1033 1569 0000   00 42 1033 1606 1568
 00 42 1042 1606 1567   00 70 1568 1567 0150   00 60 1637 0224 0163   00 41 1568 1567 0000
 00 63 0152 0235 0411   00 70 1587 1569 0161   00 80 1033 0001 0000   00 80 1034 0002 0000
 00 80 1035 0003 0000   00 80 1036 0004 0000   00 80 1037 0005 0000   00 80 1038 0006 0000
 00 83 1039 0007 0000   00 60 1638 0095 0057   00 60 1637 0224 0163   00 60 0000 0000 0000
 00 60 1591 1565 0000   00 40 1563 1564 0000   00 40 1565 0000 1565   00 40 1570 1565 0000
 00 41 1609 0000 0000   00 63 0172 0169 0171   00 60 0224 0234 0000   00 60 1610 0224 0172
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UPDATING ROUTINE

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00 60 1611 0234 0225   00 54 0000 0000 0000   97 60 1041 0000 0000   03 60 0000 1041 0000
02 42 1047 1582 0000   88 34 1623 0000 0000   87 60 0000 0000 0000   00 40 1041 0000 1041
00 42 1047 1596 0000   88 34 1623 0000 0000   87 60 0000 0000 0000   00 40 1041 0000 1041
87 60 1591 0000 0000   00 40 1041 0000 1041   13 60 0000 1565 0000   00 60 1041 9348 0000
00 40 0186 1612 0186   00 60 0000 0000 0000   08 58 0001 0007 0186   00 60 1613 0186 0000
00 60 1614 0172 0000   00 72 1563 1588 0194   00 60 0000 0000 0203   00 42 0189 1585 0202
96 60 1563 0000 0000   00 40 1616 0000 0000   00 40 0202 0000 0202   00 60 1048 9348 0000
00 40 0198 1612 0198   00 70 9348 1593 0588   00 40 1577 1587 1577   00 00 0000 0000 0000
00 60 1617 0198 0000   00 60 1588 1577 0000   00 72 1564 1588 0207   00 60 0000 0000 0217
00 59 0000 0000 0000   96 42 0000 1582 0216   96 60 1564 0000 0000   00 40 1618 0000 0000
00 40 0216 0000 0216   00 60 1148 9348 0000   00 40 0212 1612 0212   00 70 9348 1593 0594
00 40 1577 1587 1577   00 00 0000 0000 0000   00 60 1619 0212 0000   00 60 1588 1577 0000
96 60 1565 0000 0000   00 40 0189 0000 0189   00 60 1588 1563 0000   00 60 1588 1564 0000
00 40 1570 1565 1570   00 00 0000 0000 0000   96 60 1570 0000 0000   00 41 0000 1590 1570
00 40 1620 0000 0228   00 00 0000 0000 0000   08 60 0229 0000 1704   00 40 1620 1587 1620
00 60 1621 0189 0000   00 60 1615 0172 0000   00 60 1588 1570 0000   00 00 0000 0000 0000
00 70 1569 1587 0422   00 70 1569 1622 0254   00 70 1569 1623 0342   00 70 1569 1624 0329
00 70 1569 1625 0291   00 80 1578 0001 0000   00 80 1584 0002 0000   94 80 1553 0003 0000
00 80 1588 0004 0000   00 80 0000 0005 0000   00 80 0000 0006 0000   00 81 0000 0007 0153
00 80 1578 0001 0000   00 80 1568 0002 0000   00 80 1043 0003 0000   00 80 1044 0004 0000
00 80 1045 0005 0000   00 80 1046 0006 0000   00 81 1036 0007 0377   00 00 0000 0000 0000
00 42 1036 1606 1568   00 42 1148 1606 0000   00 70 1568 0000 0263   00 40 0256 1626 0256
00 40 1571 1623 1571   00 41 1564 0000 0000   00 63 0254 0000 0000   00 60 1683 0410 0404
00 42 1036 1593 0000   00 41 1036 0000 1036   04 42 0256 1586 0000   00 40 1590 0000 0000
00 40 1628 0000 0268   00 00 0000 0000 0000   00 40 0268 1629 0270   00 00 0000 0000 0000
00 60 1630 0256 0000   00 60 1588 1571 0000   00 60 1638 0095 0057   00 42 1033 1606 1568
00 42 1048 1606 0000   00 70 1568 0000 0282   00 40 0275 1626 0275   00 40 1623 1571 1571
00 41 1563 0000 0000   00 63 0275 0000 0000   00 60 1684 0410 0404   04 42 0275 1586 0000
00 40 1597 0000 0000   00 40 1631 0000 0285   00 00 0000 0000 0000   00 40 0285 1629 0287
00 00 0000 0000 0000   00 60 1632 0275 0000   00 60 1588 1571 0000   00 60 1638 0095 0057
00 00 0000 0000 0000   00 42 1036 1606 1568   00 42 1148 1606 0000   00 70 1568 0000 0300
00 40 0293 1626 0293   00 40 1571 1623 1571   00 41 1564 0000 0000   00 63 0293 0000 0000
00 60 1685 0410 0404   04 42 0293 1586 0000   00 40 1634 0000 0302   00 00 0000 0000 0000
00 40 0302 1590 0304   00 00 0000 0000 0000   00 40 0304 1590 0306   00 00 0000 0000 0000
00 41 1047 1587 1047   00 60 1630 0293 0000   00 60 1588 1571 0290   00 42 1033 1606 1568
00 42 1048 1606 0000   00 70 1568 0000 0318   00 40 0311 1626 0311   00 40 1571 1623 1571
00 41 1563 0000 0000   00 63 0311 0000 0000   00 60 1686 0410 0404   98 60 1587 0000 0000
00 41 1047 0000 1047   04 42 0311 1586 0000   00 40 1636 0000 0322   00 00 0000 0000 0000
00 40 0322 1590 0324   00 00 0000 0000 0000   00 40 0324 1590 0326   00 00 0000 0000 0000
00 60 1632 0311 0000   00 60 1588 1571 0290   00 00 0000 0000 0000   01 60 1042 0000 0000
99 60 0000 1042 0000   00 42 1036 1587 0000   00 40 1042 0000 1042   96 60 1047 0000 0000
04 60 0000 1047 0000   88 42 1039 1582 0000   00 40 1047 0000 1047   00 42 1041 1584 1041
10 42 1039 1580 0000   00 40 1041 0000 1041   00 60 0000 0000 0057   00 00 0000 0000 0000
00 42 1033 1606 1568   00 42 1048 1606 0000   00 70 1568 0000 0351   00 40 0344 1626 0344
00 40 1571 1623 1571   00 41 1563 0000 0000   00 63 0344 0000 0000   00 60 1632 0344 0403
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EXHIBIT VI

UPDATING ROUTINE

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04 42 0344 1586 0000	00 40 1634 0000 0353	00 00 0000 0000 0000	00 40 0353 1590 0355
00 00 0000 0000 0000	00 40 0355 1590 0357	00 00 0000 0000 0000	98 60 1587 0000 0000
00 41 1047 0000 1047	00 60 1632 0344 0365	06 60 1041 0000 0000	94 60 0000 1041 0000
10 42 1039 1580 0000	00 40 1041 0000 1041	96 60 1586 0000 0000	00 42 1047 0000 1576
92 60 1047 0000 0000	08 60 0000 1047 0000	92 42 1039 1582 0000	00 60 0000 0000 0000
00 40 1047 0000 1047	00 40 1576 0000 1047	00 41 1563 1623 0000	00 63 0376 0247 0000
00 61 0000 0000 0000	00 60 1588 1571 0290	90 80 1041 0003 0000	00 80 1047 0002 0000
00 80 1578 0001 0000	00 80 1588 0004 0000	00 80 0000 0005 0000	00 80 0000 0006 0000
00 83 0000 0007 0376	00 42 1036 1606 1568	00 42 1148 1606 0000	00 70 1568 0000 0392
00 40 0385 1626 0385	00 40 1571 1623 1571	00 41 1564 0000 0000	00 63 0385 0391 0391
00 60 1630 0385 0402	04 42 0385 1586 0000	00 40 1634 0000 0394	00 00 0000 0000 0000
00 40 0394 1590 0396	00 00 0000 0000 0000	00 40 0396 1590 0398	00 00 0000 0000 0000
00 60 1630 0385 0000	00 41 1047 1587 1047	00 60 1588 1571 0290	00 60 1681 0410 0404
00 60 1682 0410 0000	00 80 1578 0001 0000	00 80 1567 0002 0000	00 80 1568 0003 0000
00 80 1588 0004 0000	00 80 1569 0005 0000	00 80 1571 0006 0000	00 00 0000 0000 0000
00 72 1587 1569 0153	96 60 1565 0000 0000	00 41 0124 0000 0124	00 00 0000 0000 0000
00 41 1650 1623 0095	00 60 1664 0587 0000	00 60 1588 1563 0553	00 60 1667 0095 0000
00 60 1664 0547 0000	00 60 1563 1564 0526	00 60 1678 0453 0439	00 60 0000 0000 0000
00 42 1036 1606 1568	00 42 1148 1606 1562	00 41 1568 0000 0000	00 63 0428 0000 0439
00 61 0000 0000 0000	00 40 0424 1626 0424	00 40 1623 1571 1571	00 60 1148 1248 0000
00 40 0430 1629 0432	00 00 0000 0000 0000	00 40 0432 1629 0434	00 00 0000 0000 0000
00 40 0434 1629 0430	00 41 1564 1571 0000	00 63 0424 0454 0000	00 61 0000 0000 0000
00 60 0000 0000 0000	00 42 1036 1593 0000	00 41 1036 0000 1036	00 42 0430 1585 0000
00 40 1639 0000 0444	00 00 0000 0000 0000	00 40 0444 1629 0446	00 00 0000 0000 0000
00 40 0446 1629 0448	00 00 0000 0000 0000	04 60 1626 0000 0000	00 40 0430 0000 0430
00 60 1587 1574 0000	00 40 1047 1587 1047	00 60 1640 0439 0428	00 60 1614 0439 0000
00 70 1587 1574 0457	00 60 1655 0453 0440	00 40 1564 1623 1564	00 60 1641 0430 0000
00 60 1588 1571 0000	00 42 0424 1582 0000	00 40 1630 0000 0424	00 60 1248 1148 0000
00 40 0462 1629 0462	00 40 1571 1587 1571	00 41 1564 0000 0000	00 63 0462 0468 0000
00 61 0000 0000 0000	00 60 1642 0462 0000	00 60 1588 1571 0000	00 60 1588 1574 0290
00 72 1563 1588 0473	00 60 1679 0504 0489	00 42 1033 1606 1568	00 42 1048 1606 1562
00 41 1568 0000 0000	00 63 0478 0000 0489	00 61 0000 0000 0000	00 40 0474 1626 0474
00 40 1623 1571 1571	00 60 1048 1248 0000	00 40 0480 1629 0482	00 00 0000 0000 0000
00 40 0482 1629 0484	00 00 0000 0000 0000	00 40 0484 1629 0480	00 41 1563 1571 0000
00 63 0474 0505 0000	00 61 0000 0000 0000	00 60 0000 0000 0000	04 42 0480 1585 0000
00 40 1658 0000 0492	00 00 0000 0000 0000	96 42 0492 1582 0000	00 40 1657 0000 0000
00 40 1590 0000 0496	00 00 0000 0000 0000	00 40 0496 1629 0498	00 00 0000 0000 0000
04 60 1626 0000 0000	00 40 0480 0000 0480	00 60 1587 1574 0000	98 60 1587 0000 0000
00 40 1047 0000 1047	00 60 1659 0489 0478	00 60 1614 0489 0000	00 70 1587 1574 0508
00 60 1661 0504 0490	00 40 1563 1623 1563	00 60 1588 1571 0000	00 60 1662 0480 0000
00 42 0474 1582 0000	00 40 1632 0000 0474	00 60 1248 1048 0000	00 40 0513 1629 0513
00 40 1571 1587 1571	00 41 1563 0000 0000	00 63 0513 0519 0000	00 61 0000 0000 0000
00 60 1673 0513 0000	00 60 1588 1571 0470	00 74 1033 1042 0523	00 60 0000 0000 0095
00 42 0521 1585 0000	00 41 0521 0000 0521	00 40 0521 1623 0521	00 42 1033 1606 1568
00 42 1042 1606 1567	15 60 1033 1569 0000	00 72 1587 0000 0153	00 72 1568 1567 0545

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00 60 1568 1042 0000	00 42 1036 1593 0000	00 41 1036 0000 1036	00 60 1036 1148 0000
00 40 0534 1629 0536	00 00 0000 0000 0000	00 40 0536 1629 0538	00 00 0000 0000 0000
00 42 0538 1585 0000	00 40 1590 0000 0000	00 40 1639 0000 0534	00 40 1564 1623 1564
00 40 1047 1587 1047	00 60 0000 0000 0057	00 60 1646 0573 0000	00 60 1649 0534 0000
00 60 1645 0224 0163	00 70 1570 1588 0619	00 40 1653 1587 0234	00 60 0000 0000 0225
00 74 0000 0000 0553	00 60 0000 0000 0095	00 42 1036 1606 1568	00 42 1042 1606 1567
15 60 1033 1569 0000	00 72 1587 0000 0153	00 72 1568 1567 0585	00 42 1036 1593 0000
00 41 1036 0000 1042	10 42 1039 1580 1041	00 60 1588 1043 0000	00 60 0000 1044 0000
00 60 0000 1045 0000	00 60 0000 1046 0000	96 60 1047 0000 0000	04 60 0000 1047 0000
88 42 1039 1582 0000	00 40 1047 0000 1047	07 42 1033 1643 0000	00 40 1047 0000 1047
07 42 1033 1644 0000	00 40 1047 0000 1047	00 42 1033 1606 1048	96 42 0573 1582 0000
00 40 1590 0000 0000	00 40 1631 0000 0577	00 00 0000 0000 0000	00 40 0577 1629 0579
00 00 0000 0000 0000	00 40 0573 1623 0573	98 60 1587 0000 0000	00 40 1047 0000 1047
00 40 1563 1623 1563	00 60 0000 0000 0057	00 60 1649 0534 0000	00 60 1646 0573 0000
00 60 1650 0224 0163	00 40 0198 1674 0198	00 41 0202 1675 0202	00 41 1563 1623 1563
00 41 0000 1577 0000	00 63 0198 0203 0000	00 61 0000 0000 0000	00 40 0212 1674 0212
00 41 0216 1675 0216	00 41 1564 1623 1564	00 41 0000 1577 0000	00 63 0212 0217 0593
00 74 1676 0550 0548	00 70 1572 1623 0616	00 60 1622 1572 0000	00 60 1637 0057 0000
00 60 1614 1638 0000	00 60 0161 0151 0000	00 72 1570 1588 0163	00 60 0000 0000 0096
03 70 1572 1622 0616	00 60 1623 1572 0000	00 60 1614 0552 0000	00 60 1672 0522 0000
00 60 1677 0234 0000	00 60 1033 1042 0000	00 60 1588 1563 0000	00 60 1588 1564 0000
00 60 1588 1047 0225	00 70 1570 1588 0618	00 60 0000 0000 0627	03 98 0000 0000 0000
01 91 0000 0000 0000	02 98 0000 0000 0000	01 98 0000 0000 0000	09 98 0000 0000 0000
00 60 0518 0001 0001	00 40 1663 1575 0625	00 00 0000 0000 0000	00 60 0000 0000 0102
00 74 1676 0550 0549	00 60 0000 0000 0549	00 00 0000 0000 0000	00 00 0000 0000 0000
00 00 0000 0000 0000	00 00 0000 0000 0000	00 00 0000 0000 0000	59 05 0000 0000 0000
39 09 0000 0000 0000	11 11 1100 0000 0000	00 00 0011 1111 0000	00 00 0000 0000 1111
00 01 0000 0000 0000	11 11 1111 1100 0000	00 00 0000 1111 0000	00 00 1111 0000 0000
00 00 0000 0000 0001	00 00 0000 0000 0000	00 60 0000 0000 0043	00 00 0000 0001 0000
00 00 0000 0000 0007	00 60 4629 1033 0000	55 00 0000 0000 0000	66 00 0000 0000 0000
00 60 0000 0000 0102	00 00 0000 0000 0011	12 00 0000 0000 0000	00 03 0000 0000 0000
00 01 0000 0000 0000	00 60 6830 1041 0000	06 58 0001 0007 0122	06 58 0001 0000 0130
00 60 6830 1048 0000	06 58 0001 0000 0138	00 60 6830 1148 0000	00 00 0011 1111 1110
00 42 1033 1606 1568	00 42 1036 1606 1568	00 00 0000 0000 0200	00 60 0000 0000 0225
00 60 0000 0000 0172	00 00 0001 0000 0000	00 60 1043 9348 0000	00 60 0000 0000 0000
00 54 0000 0000 0000	08 58 0001 0000 0198	00 60 1048 9348 0000	08 58 0001 0000 0212
00 60 1148 9348 0000	01 91 1348 1348 0001	08 58 0001 0007 0186	00 00 0000 0000 0002
00 00 0000 0000 0003	00 00 0000 0000 0004	00 00 0000 0000 0005	00 80 0003 0000 0000
00 42 1248 1606 0000	00 60 1037 0000 0000	00 00 0001 0001 0000	00 42 1148 1606 0000
00 60 1034 0000 0000	00 42 1048 1606 0000	00 60 0000 0000 0274	00 80 1593 0000 0000
00 60 0000 0000 0310	00 60 1568 1573 0000	00 60 0000 0000 0096	00 60 0000 0000 0145
00 60 1036 0000 0000	00 60 0000 0000 0628	00 60 1148 1248 0000	00 60 1248 1148 0000
01 10 0000 0000 0000	00 01 1000 0000 0000	00 60 1588 1047 0531	00 42 1033 1606 1048
00 60 0000 0000 0203	00 60 0000 0000 0217	00 60 1036 1148 0080	00 60 1588 1047 0558
00 60 1845 0224 0000	00 60 1650 0224 0548	00 60 0000 0000 0618	00 60 1640 0439 0428

EXHIBIT VI

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                                    UPDATING  ROUTINE                                      page 5

00 60 1654 0453 0457    00 60 0000 0000 0471    00 60 1034 0000 0000    00 42 1033 1606 0000
00 60 0000 0000 0478    00 60 1659 0489 0478    00 60 1660 0504 0508    00 60 1048 1248 0000
00 52 0000 0000 0000    00 60 1665 0274 0548    00 60 0000 0000 0624    00 60 0000 0000 0418
00 60 0000 0000 0526    00 60 0000 0000 0384    04 58 0001 0007 0063    00 60 0000 0000 0133
00 60 0000 0000 0141    00 60 0000 0000 0526    00 60 1748 1048 0000    00 00 0002 0000 0000
00 00 0000 0003 0000    00 60 1614 0274 0163    00 60 0000 0000 0070    00 60 1654 0453 0000
00 60 1660 0504 0000    00 70 1564 1548 0421    00 81 1564 0007 0401    00 81 1563 0007 0376
00 81 1564 0007 0271    00 81 1563 0007 0288    00 81 1564 0007 0308    00 81 1563 0007 0327
00 40 1702 0000 1694    92 60 0000 0000 0000    00 40 1703 0000 1690    00 00 0000 0000 0000
00 00 0000 0000 0007    00 00 0000 0000 0000    00 00 0000 0000 0000    00 00 0000 0000 0000
00 61 0000 0000 1696    12 60 1692 1691 0000    88 42 1702 0000 0000    00 40 1701 0000 1690
00 60 1700 1693 1690    00 68 1695 1693 1691    88 40 1691 1702 1691    00 59 9999 9132 0001
00 59 9998 1692 1696    00 40 1723 0000 1716    91 60 0000 0000 0000    01 40 1724 0000 1708
00 40 1727 1718 0000    00 00 0000 0000 0000    00 68 0000 0000 0000    12 42 1721 1705 0000
88 40 1726 0000 1712    00 00 0000 0000 0000    00 41 1712 1725 1714    00 00 0000 0000 0000
00 68 0000 0000 1718    00 00 0000 0000 0000    00 61 1721 0000 1727    00 00 0000 0000 0000
00 40 1721 1726 1720    00 00 0000 0000 0000    00 00 0000 0000 0000    00 60 1717 1718 1709
00 59 9999 9140 0001    05 99 9981 7210 0000    00 01 0000 0000 0000    00 02 0000 0030 0097
00 60 1712 1718 1718    02 91 0051 1770 0001-

* 944 input-output locations inserted here.
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EXHIBIT VI

and the entire last block of the event file is retested.

In addition to the regular updating operations just discussed, the program performs several other housekeeping functions. First, if transactions remain after the event file tapes are exhausted, each remaining transaction is tested for a new activity code (Code 1) and a successor updating code. If the test indicates a new activity and a successor updating, a new event is constructed and added to the event file. Any other combination of transaction and updating codes will reject the transaction in the usual manner.

As the run is being made, the program counts the number of transactions processed into the event file After the run is completed, the program prints the number of transactions successfully processed and the input and output tape labels at the end of the reject register.

The output tapes from the file updating run (Run 06) are first a new (updated) event file tape, and second, a completed events tape which lists the complete file for each event that has had all its predecessor activities completed.

The detailed program flow chart and absolute coding for Run 06 are to be found in Exhibits V and VI, on pages 57 through 61.

The outputs of Run 06 are of much importance to the analysis. The completed events file represents a detailed history of experience which will be available for separate analysis work. The event file tape represents the current picture of future activity It is the basis of the subsequent evaluation analysis. The composition of the event file tape can now be discussed in detail.

(4) *Event File Tape.* The information that is kept in the event file is by nature variable in length although it is made up of fixed length items. The length of the file message or record is determined by the number of predecessors and successors an event has. In order for an event to exist in the file, there must be at least one successor or predecessor event related to it. The minimum size of a record is thus fixed. Theoretically, there is no upper limit to the size of an event record. However, a practical limitation has been imposed by limiting a tape block length to 200 words, thus restricting the maximum event record length to one full block or 200 words.

The static or fixed data in the file record require 7 words. In addition, a single predecessor or successor reference requires 3 words, so that the minimum record length is 10 words. The maximum length of the record is 200 words, which limits the event complexity to no more than a total of 64 predecessors and successors. This does not appear to be a serious restriction at this time.

The NORC is a fixed word machine. Thus, in order to use the variable length record technique, it was necessary to add three control words. The first controls the length of the record itself. The remaining two control the number of predecessor and successor references within the record.

A NORC word consists of 16 decimal digits available for variable data usage and a 17th digit used as a word check digit. The check digit is used to verify tape and high speed memory transfers and is not available for actual use as data storage. Although NORC is designed to handle only decimal digits, it is possible to generate and handle alphabetic information with the NORC itself. However, two digits are required to represent each alphabetic character and certain programing restrictions are required. Fortunately, alphabetic information is not required in the PERT system.

All data items in the PERT system are fixed in length. The economy that might be achieved in file space by using variable length data items is far outweighed by the large increase in programing complexity that would be incurred. There are very few data items for which the ratio of maximum length to average length is very large.

NORC will operate arithmetically either in fixed point or floating point rotation depending upon the actual programing codes selected. When operating in the floating point mode, a full word is required

DATA ITEM REFERENCE
FILE LAYOUT CROSS-REFERENCE

	Data Item	Number of Digits	Word Number	Word Position
1.	Message length	3	1	1 – 3
2.	Probability $P_r(T_L \geq T_S \geq T_E)$	2	1	5 – 6
3.	Sign $T_L - T_E$	1	8	2
4.	$T_L - T_E$	4	1	7 – 10
5.	Schedule date	6	1	11 – 16
6.	Predecessor counter	2	2	1 – 2
7.	Successor counter	2	2	3 – 4
8.	Probability $P_r(T_E \geq T_S)$	2	2	5 – 6
9.	Event code	9	2	7 – 15
10.	Report code	1	2	16
11.	T_E	16	3	1 – 16
12.	$\sigma^2{}_{T_E}$	16	4	1 – 16
13.	T_L	16	5	1 – 16
14.	$\sigma^2{}_{T_L}$	16	6	1 – 16
15.	Sign T_S	1	1	4
16.	T_S	4	7	1 – 4
17.	Completion week	4	7	5 – 8
18.	Resource code	2	7	9 – 10
19.	Specification code	2	7	11 – 12
20.	Predecessor index	2	7	13 – 14
21.	Successor index	2	7	15 – 16
22.	Predecessor code	9	8	7 – 15
23.	t_e (Predecessor)	16	9	1 – 16
24.	$\sigma^2_{t_e}$ (Predecessor)	16	10	1 – 16
25.	t_e (Successor)	16	11	1 – 16
26.	$\sigma^2_{t_e}$ (Successor)	16	12	1 – 6

EXHIBIT VII

EVENT FILE TAPE LAYOUT

WORD #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	MESSAGE LENGTH			$\pm T_S$	P_r		$T_L - T_E$				SCHEDULE DATE						✓
	X	X	X	X	.X	X	X	X	X	− X	X	X −	X	X −	X	X	X
2	PRED. COUNTER		SUC. COUNTER		P_r		EVENT CODE									REPT CODE	✓
	X	X	X	X	.X	X	X	X	X	— X	X	X —	X	X	X	X	X
3					$\sigma_{T_E}^2$												✓
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
4					σ_{T_E}												✓
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
5					T_L												✓
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
6					$\sigma_{T_L}^2$												✓
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
7	T_S				COMPLETION DATE CODE				RESOURCE CODE		SPEC. CODE		PRED. IN.		SUC. IN.		✓
	X	X	X	.X	X	X	X	.X	X	.X	X	.X	X	X	X	X	X
8	—	±	—	—	—	—	PREDECESSOR CODE									—	✓
	O	X	O	O	O	O	X	X	X	X	X	X	X	X	X	O	X
9						t_e											✓
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
10						$\sigma_{t_e}^2$											✓
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
11						t_e											✓
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
12						$\sigma_{t_e}^2$											✓
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
13						t_e											✓
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
14						$\sigma_{t_e}^2$											✓
	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

EXHIBIT VIII

for each data item. The first two digits of the word specify the location of the decimal point. The third digit indicates the algebraic sign of the number and the remaining 13 digits, the significant digits of the number. In order to preserve as much precision as possible throughout the PERT calculations, it is necessary to perform some of the arithmetic operations in the floating point mode. Data items referring to T_E, T_L, $\sigma_{T_E}^2$, $\sigma_{T_L}^2$, t_e, and $\sigma_{t_e}^2$ are all carried as full NORC words in the floating point mode. The remaining numeric data items are carried in a compact fixed point notation.

Two restrictions were made on the actual layout of the event file. First, it should be as compact as possible, consistent with good coding techniques, and second, no data item should be spread over two words. The event file tape is summarized in Exhibit VII, on opposite page, and shown in layout form in Exhibit VIII above.

The basic event file tape has now been generated. The next step in the cycle is the analysis run.

1. THE COMPUTING RUN

(1) *Sequencing (Run 07).* The method of elapsed time computation as outlined in the Phase I PERT project report outlines in detail the requirements for the sequence of events needed to perform the calculations. It requires that calculations for the events proceed in accordance with an ordered sequence. This ordered sequence essentially collapses the PERT network into a single line. The line of events is constituted so that all arrows point in the same direction. Thus, it is necessary to arrange the events within the machine in such a particular sequence. The computing sequence run accomplishes this by mechanizing the method outlined in the Phase I report. In order to mechanize this method, a few changes in details were necessary. Essentially, however, the method used is the one described in the Phase I report.

The only input required is a file of events, arranged in any order. (Most frequently they will be arranged in event code order, since the input to Run 07 will generally be the output of Run 06. This is not a necessary condition, however.)

The first operation of the program is to scan the input tape (Tape 1) for events having no predecessors (the initial events in the sequence). These events are extracted from the input tape and put on the tape to be used for events for which the above predecessor condition has been satisfied (Tape 2). Both tapes are then rewound. The first event on Tape 2 is extracted, as well as its successor code. The input tape (Tape 1) is searched for the event which matches the successor of the first event on Tape 2. The matching operation is performed through a subtraction. The event from the satisfied predecessor tape, Tape 2, is subtracted from the event on the tape being searched. Thus, as long as the result of the subtraction remains positive, the search continues. A zero result indicates the required event has been located. A negative result indicates that either the network was constructed illogically and is in error or that the program must branch and search elsewhere for the required event.

On the first search on the input tape (Tape 1), the event must be found or the negative result from the searching test indicates an error. During the search for the first event on the input tape, the input tape, Tape 1, is transferred directly to one of the intermediate work tapes (Tape 5 or 6). When the event is found on Tape 1, its predecessor is compared to the search criterion. If they are equal, the predecessor counter is increased by one and compared to the predecessor index. If the latter are equal, the event is transferred to the conjugate or continuation of Tape 2, Tape 4. If they are unequal, the event is transferred to the intermediate or work tape (Tape 5 or Tape 6). The successor counter of the event from Tape 2, the search criterion, is increased by one and compared to the successor index. If these two are equal, the event is removed from Tape 2 and transferred to the final sequenced output tape, Tape 3. If the comparison is unequal, the next successor of the first event on Tape 2 becomes the search criterion; if the comparison is equal the first successor of the next event in Tape 2 becomes the search criterion.

The search on the input tape (Tape 1) continues with transfers to the intermediate work tape (Tape 5 or 6). When searching Tape 1, if the search comparison becomes negative (Tape 2 minus Tape 1) then the remainder of the block is transferred to the intermediate work tape (Tape 5 or 6). The work tape just written on is rewound, and the search begins on that tape. As either Tape 5 or Tape 6 is searched, the tape being searched is written on the other work tape. For example, if Tape 5 is being searched, the information is also transferred to Tape 6. As an item is found on a work tape and all the predecessors are satisfied (i.e., the predecessor counter equal to the predecessor index), they are transferred to Tape 4 or Tape 2. Tapes 2 and 4 are used alternatively as the storage for events whose predecessors are satisfied. If Tape 2 is being read, then Tape 4 is used for storage. If Tape 4 is being read, then Tape 2 is being used for storage.

If the search criterion (Tape 2—Tape 5 or 6) gives a negative result with one of the work tapes, the search will stop. The remainder of the file on the tape being searched is then transferred to the other work tape. The search will then begin on the other work tape after it has been rewound. If the end of a work tape is reached and there has been a previous negative search test on the other work tape, the search will then begin on the input tape (Tape 1). Note that Tape 1 is never rewound. The search on the input tape always progresses from the last stopping point.

Generally, events flow selectively from the input tape (Tape 1), to the work tapes (Tapes 5 and 6). From the work tapes, the events flow to the tape which contains all events for which the predecessors have been satisfied (Tapes 2 and 4). When all the successors of the events on Tapes 2 and 4 have been satisfied, the events flow to the output tape (Tape 3) and all the sequencing criteria necessary for computation are satisfied. Tape 3 is the input for the succeeding run (Run 08).

A standard stop for the routine is reached when both Tapes 2 and 4 are empty and the input tape (Tape 1) is exhausted.

In relating the program to the method described in the Phase I report, it can be noted that *Tape 1* and Tapes 5 and 6 are equivalent to Pile 1. *Tape 2* and Tape 4 are equivalent to Pile 2; *Tape 3* is equivalent to Pile 3. Exhibit IX, on the following page, gives a simplified flow diagram of the program. A detailed program flow chart and absolute coding for Run 07 is found in Exhibits X and XI, following.

In addition to listing the beginning and ending tape labels, the computing sequence run (Run 07)

INFORMATION FLOW DIAGRAM
COMPUTATION SEQUENCE RUN (RUN 07)

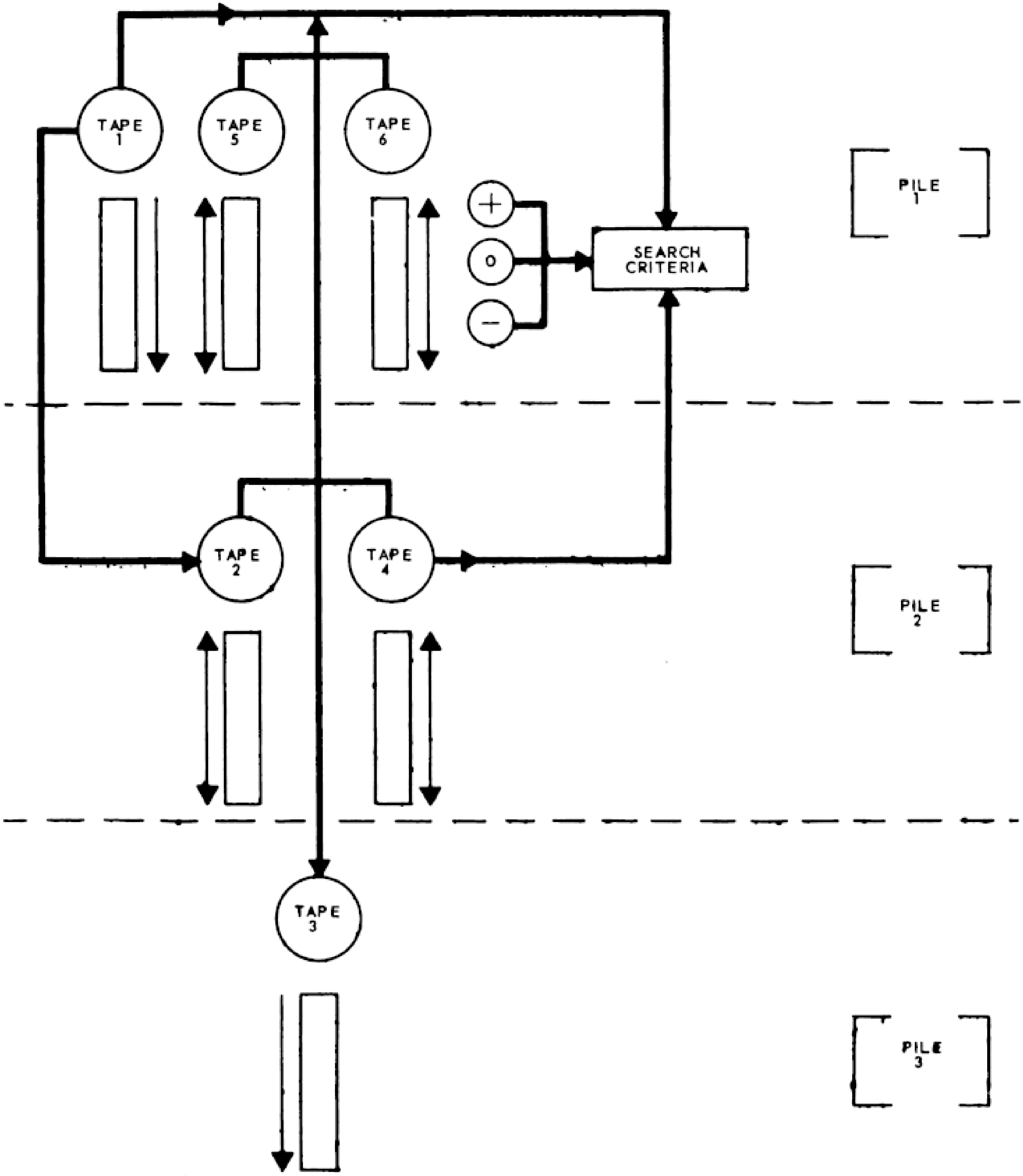

EXHIBIT IX

produces a total of the number of events on the final output tape.

(2) *Slack and Probability Computations (Run 08).* Once a sequenced event file has been prepared, the computing of the outputs can follow immediately. The logic of the computing run is completely dependent upon sets of sequenced events as input.

During the forward analysis of the sequenced event file, T_E and $\sigma^2_{T_E}$ are calculated and stored in the file. After the T_E and $\sigma^2_{T_E}$ have been calculated for the last event in the file, the process is reversed. The sequenced net of events is analyzed in reverse by reading the tape in reverse. Now T_L and $\sigma^2_{T_L}$ are calculated and stored on the event file. As the T_L and $\sigma^2_{T_L}$ calculations are completed for each event, the program branches and calculates the slack $T_L - T_E$. The value obtained for the slack $T_L - T_E$ is tested and if the result lies between 0 and 5, it is flagged by putting a 9 code in the report code field. This will allow the event to be selected automatically for special output reporting.

In order to operate, the program needs only one input tape, a file of sequenced events, and the output of the sequencing run (Run 07). The program then tests for the presence of a scheduled time T_S. If a T_S exists in the file for the event, the probability calculation for $P_r(T_S \leq T_{E'})$ is made. In addition, the calculations $P_r(T_{L'} \geq T_S)$ and $P_r(T_{E'} \leq T_S \leq T_{L'})$. ($T_{E'}$ and $T_{L'}$ are random variables.)

The first block on the input tape is read into memory and the first event is extracted for computation. The first event on the tape by definition will have no predecessors, this also could be the case for one or more of the events following the first event in the file. The program goes through a dummy calculation cycle and develops a zero T_E and $\sigma^2_{T_E}$. The program then constructs a working tape for later use in the calculation. Three words are read on to this working tape, the event code, the T_E value, and the $\sigma^2_{T_E}$ value. The nine digit event code is read into the first word of the working tape message. $T_{E'}$, in full NORC word floating point form, is read into the second word of the working tape message and $\sigma^2_{T_E}$, again in full NORC word floating point form, is read into the third word of this working tape message.

The working tape is a method used to decrease the size of the file needed for computational look-up. As such, it increases the efficiency of the program and reduces the overall running time substantially. The full event file contains as a minimum 10 words for each event. The more complex events require 20 to 30 words of tape and memory storage. For computation purposes, the working tape using 3 words per event is equivalent to the full event file.

After computing the T_E and $\sigma^2_{T_E}$ for the first and following "no predecessor" events, the program examines the next event, the first event with predecessors. The program selects the first predecessor and goes back to this predecessor event and tests for the presence of completion date. If there is a completion date, the program picks up the report week constant and computes the difference. This difference is then applied to the predecessor t_e as a correction for time "now." The proportional change in t_e is applied to $\sigma^2_{t_e}$ to adjust the variance. With these adjustments made, the computation of T_E and $\sigma^2_{T_E}$ is extracted. The sums are computed and the results stored. The process is repeated for each of the predecessors of the event being considered. The stored results for the nominated T_E's and $\sigma^2_{T_E}$'s are compared. The larger T_E is selected and it, along with its $\sigma^2_{T_L}$, are stored in the main event file. The temporary working tape message is constructed and read out. The next event on the event file is extracted and the computation cycle is repeated.

The adjustment subroutine, completed or beginning events, is only repeated when reference is made to beginning or completed events. If the event is a beginning event and no completion date is carried, the time adjustment is made from time zero.

The time adjustment is so programed as to prevent the adjustment on a t_e value to less than one week.

The forward computation cycle continues until the input file is exhausted. The last event on the file is assumed to be the objective event and *all event files* and networks *must* be adjusted to reflect this condition. After computing T_E and $\sigma^2_{T_E}$ for the last or objective event, the program tests two option switches. If the first switch is "on," the program will select the T_S for this objective event and set it up as the T_L for the last event. If the first switch is "off," the program will test the second switch. If this switch is "on," the program will pick up a manually inserted value and set it up as the T_L for the last event. If both switches are off, the program will select the T_E for the last event and set it up as the T_L for the last event.

The working tape used in the forward calculation has no further value to the operation. However, a new working tape is set up using the event code T_L and $\sigma^2_{T_L}$ in the same manner as before. Before the program goes into the next event in the reverse

PROGRAM FLOW CHART
SEQUENCING RUN #07

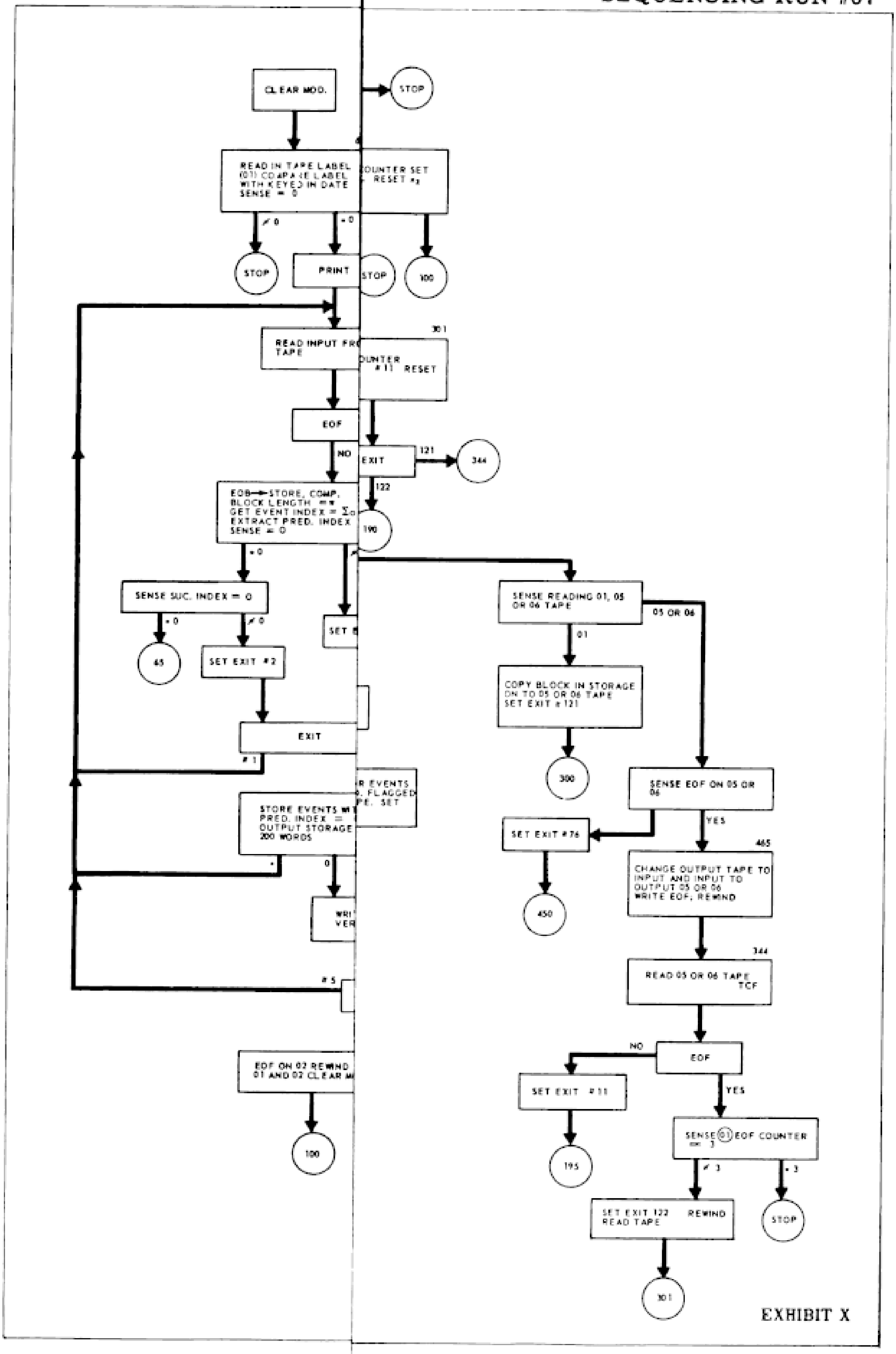

EXHIBIT X

SEQUENCING ROUTINE page 1

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-02 91 0051 0056 0000   09 96 0008 1997 0001   00 68 0000 0000 0006   00 60 0000 0000 0008
 00 00 0000 0000 0000   00 61 0000 0000 0001   00 00 0000 0000 3953   02 91 0051 0056 0000-


-02 91 0051 2040 0001   03 94 0650 0650 0000   08 68 0009 0000 1931   00 61 0000 0000 0615
 02 91 0000 0000 0000   08 60 0012 0000 1948   00 67 0000 0000 0015   00 61 0000 0000 0000
 00 57 0000 0000 0000   00 40 1729 1687 1687   00 70 0650 0000 0019   00 61 0000 0000 0000
 00 40 1748 1729 1729   00 40 1686 0000 1686   01 91 1686 1686 0000   08 60 0022 0000 1948
 03 94 0684 0000 1000   08 68 0024 0000 1931   00 67 0000 0000 0265   00 60 0000 1689 0000
 04 42 1689 1730 1690   08 42 1689 1731 0000   00 41 1690 0000 1691   00 40 1732 0000 1691
 00 51 0000 0000 0000   00 40 0023 1732 0023   00 41 1691 1692 0000   00 63 0039 0136 0000
 00 61 0000 0000 0000   00 60 1733 1692 0000   00 60 1743 0052 0000   00 60 0000 0000 0029
 13 60 4684 1696 0000   02 42 4690 1734 1694   00 42 4690 1735 1695   00 70 1694 1733 0044
 00 60 1770 0079 0045   00 60 1744 0079 0000   96 34 1694 1747 1694   96 34 1695 1747 1695
 00 42 1694 1746 1694   00 42 1695 1746 1695   00 40 1692 1696 1692   00 60 4684 0885 0000
 00 40 0050 1737 0050   04 58 0001 0007 0050   00 60 1738 0050 0000   00 72 1694 1733 0056
 00 60 0000 0000 0064   96 60 1694 0062 0000   00 42 0057 1730 0000   00 40 0062 0000 0062
 00 40 1739 0000 0067   00 60 4684 0892 0000   00 40 0060 1737 0060   00 00 0000 0000 0000
 00 60 1740 0060 0000   00 72 1695 1733 0066   00 60 0000 0000 0077   00 59 0000 0000 0000
 04 42 0000 1731 0073   96 60 1695 0000 0000   00 40 0073 0000 0073   00 40 1741 0000 0073
 00 60 4684 0988 0000   00 40 0071 1737 0071   00 00 0000 0000 0000   00 60 1742 0071 0000
 00 60 1733 0887 0000   00 60 1733 0888 0000   96 60 1696 0000 0000   00 40 0052 0000 0052
 00 00 0000 0000 0000   08 42 0891 1731 0000   00 40 1745 0000 1697   00 42 1686 1746 0000
 00 40 1745 0000 1698   50 22 0000 1697 1699   00 60 0989 1700 0000   00 70 1700 1733 0097
 50 22 1700 1699 1701   50 22 0000 1748 0000   00 63 0092 0092 0000   00 60 1748 1701 0092
 00 00 0000 0000 0000   50 26 1701 1700 1702   00 60 1701 0989 0000   00 60 0990 1700 0000
 50 24 1702 0000 1701   00 60 0000 0990 0000   00 40 0085 1747 0085   00 40 0093 1749 0093
 00 40 0094 1747 0094   00 40 0096 1749 0096   04 60 1749 0000 0000   00 40 1703 0000 1703
 00 41 1695 0000 0000   00 63 0085 0106 0000   00 61 0000 0000 0000   00 60 1733 1703 0000
 00 60 1750 0085 0000   00 60 1751 0093 0000   00 60 1752 0094 0000   00 60 1753 0096 0000
 00 60 1733 1700 0112   00 54 0000 0000 0000   00 60 1764 6652 0000   00 40 1704 1732 1704
 00 60 8885 6653 0000   00 50 0000 0001 0000   08 58 0001 0007 0115   00 54 0000 0000 0000
 96 60 1694 0000 0000   00 40 1756 0000 0124   00 70 1694 1733 0125   00 60 8892 6653 0000
 00 50 0000 0001 0000   00 00 0000 0000 0000   00 54 0000 0000 0000   96 60 1695 0000 0000
 00 40 1758 0000 0132   00 72 1695 1733 0130   00 60 0000 0000 0133   00 60 8988 6653 0000
 00 50 0000 0001 0000   00 00 0000 0000 0000   00 50 0000 0001 0000   00 40 1696 1704 1704
 00 60 0000 0000 0146   96 60 1704 0000 0000   00 41 0000 1737 0000   00 40 1760 0000 0139
 00 00 0000 0000 0000   08 60 0140 0000 1948   00 40 1760 1732 1760   00 60 1733 1704 0000
 00 52 0000 0000 0000   00 60 0000 0000 0036   00 60 0000 0000 0000   00 40 1705 1749 1705
 00 41 1763 1705 0000   00 63 0156 0155 0152   00 60 1733 1705 0000   00 40 1768 1767 0168
 00 60 1766 0157 0153   00 61 0000 0000 0000   00 60 1836 0164 0000   00 60 1767 0166 0033
 00 60 1769 0170 0157   00 60 1765 0170 0000   00 42 0886 1764 1084   04 60 1749 0000 0000
 00 40 0157 0000 0157   11 34 1699 1748 1699   89 42 0000 1746 1726   98 42 1699 1825 0000
 00 40 1726 0000 0000   00 40 1084 0000 1084   00 40 0164 0566 0164   00 60 0887 1085 0000
 00 40 0166 1749 0166   00 60 0888 1086 0000   00 40 0168 1749 0168   00 00 0000 0000 0000
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SEQUENCING ROUTINE page 2

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 04 91 1084 1683 0001   08 60 0172 0000 1948   00 40 0171 1732 0171   00 00 0000 0000 0000
 96 60 1694 0000 0000   00 40 1772 0000 0181   00 54 0000 0000 0000   00 42 1084 1764 1706
 00 42 8892 1764 1707   00 70 1706 1707 0187   00 00 0000 0000 0000   00 40 0178 1747 0178
 00 40 1700 1749 1700   00 41 1763 0000 0000   00 63 0175 0241 0000   00 61 0000 0000 0000
 04 42 0178 1731 0000   00 40 1737 0000 1699   00 59 0000 0000 0000   92 42 0000 1746 0000
 00 40 1773 0000 0000   00 40 1699 0000 0218   00 42 0178 1731 1726   00 40 1837 0000 0195
 00 00 0000 0000 0000   00 70 1728 1733 0218   00 40 1745 0000 1728   00 40 1726 1841 0199
 00 00 0000 0000 0000   00 40 1728 0000 1728   00 42 0218 1731 0091   00 40 1842 0000 0203
 00 00 0000 0000 0000   00 40 0091 1843 0205   00 00 0000 0000 0000   50 22 0000 1748 0000
 00 63 0209 0209 0000   00 60 1748 1701 0000   04 60 0091 0091 0000   00 40 1844 0091 0211
 00 00 0000 0000 0000   00 40 1845 0091 0213   00 00 0000 0000 0000   00 40 1846 0091 0216
 00 40 1847 0091 0217   00 00 0000 0000 0000   00 00 0000 0000 0000   00 00 0000 0000 0000
 00 40 0218 1768 0000   00 40 1732 0000 0221   00 00 0000 0000 0000   50 22 1701 1708 0000
 00 63 0000 0226 0228   00 60 1701 1708 0000   00 60 1702 1709 0228   50 22 1702 1709 0000
 00 63 0224 0000 0000   00 40 1749 1703 1703   04 60 1703 0000 0000   00 41 1694 0000 0000
 00 63 0182 0233 0000   00 61 0000 0000 0000   00 60 1733 1703 0000   00 60 1774 0178 0000
 00 60 1733 1700 0000   00 60 1708 0887 0000   00 60 1709 0888 0000   00 60 1733 1708 0000
 00 60 0000 1709 0000   00 60 0000 0000 0112   00 74 0000 0000 0000   00 42 0171 1746 1710
 00 40 0254 1710 0260   00 41 1710 1732 0000   00 40 0249 0000 0252   00 60 1775 0174 0000
 00 60 1794 0241 0171   04 91 0000 0000 0000   04 95 0000 0000 0000   00 67 0000 0000 0252
 00 61 0000 0000 0000   00 00 0000 0000 0000   00 67 0000 0000 0251   04 94 1084 0000 0000
 08 68 0255 0000 1931   00 67 0000 0000 0251   00 41 0252 1732 0252   00 60 1795 0240 0000
 00 60 1796 0241 0175   00 00 0000 0000 0000   08 68 0261 0000 1931   00 67 0000 0000 0251
 00 60 1744 0241 0000   00 60 1797 0240 0112   00 40 1777 1777 0000   00 40 1767 0000 1767
 00 40 1767 1768 0168   00 60 1807 1765 0000   00 60 1808 0258 0000   00 60 1767 0166 0000
 00 60 1757 0154 0000   00 60 1733 1705 0000   00 60 1848 0160 0000   00 60 1744 0153 0000
 00 60 1809 0259 0000   00 60 1810 0264 0000   00 60 1778 0157 0000   00 42 0139 1746 0000
 00 40 0287 0000 0287   02 91 0000 0000 0000   02 95 0000 0000 0000   00 67 0000 0000 0284
 00 61 0000 0000 0000   03 98 0000 0000 0000   04 98 0000 0000 0000   00 57 0000 0000 0000
 02 95 0884 0000 0000   08 68 0288 0000 1931   00 67 0000 0000 0622   00 60 0000 1689 0000
 04 42 0000 1730 1690   00 41 0287 1732 0287   08 42 1689 1731 0000   00 41 1690 0000 1691
 00 40 1691 1792 1691   00 60 0000 0000 0000   00 41 1691 1692 0000   00 63 0303 0444 0000
 00 61 0000 0000 0000   00 60 1733 1692 0000   00 60 1781 0306 0000   00 60 1771 0304 0287
 00 54 0000 0000 0000   96 60 1691 0000 0000   00 40 1776 0000 0308   00 70 0884 1764 0310
 00 41 0306 1777 0306   00 00 0000 0000 0000   00 61 0000 0000 0000   00 42 0306 1731 0000
 00 40 0000 1777 1711   00 44 0306 1777 0306   00 60 1761 0304 0000   00 60 0000 0000 0000
 00 40 1711 1779 0000   00 40 1780 0000 0317   00 00 0000 0000 0000   02 42 1693 1734 1694
 00 42 1693 1735 1695   96 34 1694 1747 0000   00 42 0000 1746 1694   96 34 1695 1747 0000
 00 42 0000 1746 1695   00 60 1736 1696 0000   00 40 1694 1695 0000   00 40 1696 0000 1696
 00 40 1692 0000 1692   00 40 1732 1692 1692   96 40 1696 1732 0000   00 41 0308 0000 0308
 00 51 0000 0000 0000   00 40 1782 1711 0333   00 60 4000 0885 0000   00 40 0333 1737 0333
 04 58 0001 0007 0333   00 42 0333 1731 0000   00 40 1783 0000 0343   96 60 1694 0345 0000
 00 42 0335 1730 0000   00 40 0345 0000 0000   00 40 1784 0000 0345   00 70 1694 1733 0346
 00 00 0000 0000 0000   00 40 0343 1737 0343   00 00 0000 0000 0000   00 42 0343 1731 0000
 00 40 1786 0000 0353   96 60 1695 0355 0000   00 42 0345 1730 0000   00 40 0355 0000 0000
```

EXHIBIT XI

SEQUENCING ROUTINE page 3

00 40 1787 0000 0355	00 70 1695 1733 0356	00 00 0000 0000 0000	00 40 0353 1732 0353
00 00 0000 0000 0000	00 72 1695 1733 0453	00 75 0000 0000 0360	00 76 0000 0000 0364
00 60 0000 0000 0367	00 60 0000 0000 0000	12 60 0891 0000 0000	00 40 1745 0000 1712
00 60 1788 0360 0368	00 60 0000 0000 0000	00 60 1688 1712 0000	00 60 1788 0364 0368
00 60 0887 1712 0000	00 60 0000 0000 0000	00 60 1712 0889 0000	00 60 1733 0890 0000
00 60 1789 0367 0373	00 60 1717 0889 0000	02 60 1731 0000 0000	00 42 0885 0000 0000
00 41 0885 0000 0885	50 22 0889 0887 1697	50 23 0000 1733 0000	50 22 1790 0000 0000
00 63 0384 0000 0000	50 22 1697 1733 0000	00 63 0000 0000 0632	50 22 1685 1697 0000
00 63 0000 0000 0632	01 60 0886 0000 0000	99 60 0000 0886 0000	00 40 0886 1791 0886
00 60 0000 0000 0389	00 00 0000 0000 0000	00 42 1697 1825 0000	00 72 1825 0000 0393
50 30 1697 1833 1697	00 60 1748 1726 0395	00 60 1733 1726 0000	50 30 1697 1790 1697
00 34 1792 1697 1697	94 60 0000 0000 0000	00 40 0885 0000 0885	12 60 0891 0000 0000
00 72 1733 0000 0497	00 60 0000 0000 0401	00 54 0000 0000 0000	92 41 1695 1732 0000
00 40 1793 0000 0408	00 72 1695 1733 0406	00 60 0000 0000 0411	96 60 1695 0000 0000
00 40 1798 0000 0410	00 00 0000 0000 0000	00 41 0408 1768 0408	00 00 0000 0000 0000
00 54 0000 0000 0000	92 41 1694 1732 0000	00 40 1799 0000 0420	00 42 0408 1730 0000
00 40 0420 0000 0420	00 72 1694 1733 0418	00 60 0000 0000 0423	96 60 1694 0000 0000
00 40 1800 0000 0422	00 00 0000 0000 0000	00 41 0420 1768 0420	00 00 0000 0000 0000
00 54 0000 0000 0000	00 42 0420 1730 0000	00 40 1801 0000 0426	00 00 0000 0000 0000
00 41 0426 1768 0426	08 58 0001 0007 0426	00 40 1727 1732 1727	00 42 0426 1730 0000
96 40 1759 0000 0439	00 40 1834 0000 0438	08 42 0000 1731 0000	00 40 0438 0000 0438
00 40 0439 1835 0439	08 42 0000 1731 0000	00 40 0439 0000 0439	00 00 0000 0000 0000
00 00 0000 0000 0000	00 40 1696 1704 1704	96 60 1696 0000 0000	00 41 1793 0000 1793
00 60 0000 0000 0146	92 41 1704 1732 0000	00 41 1802 0000 0446	00 00 0000 0000 0000
08 60 0447 0000 1948	00 40 1802 1732 1802	96 60 1704 0000 0000	00 40 1793 0000 1793
00 60 1733 1704 0000	00 60 0000 0000 0300	96 60 1695 0000 0000	00 40 1803 0000 0450
00 52 0000 0000 0000	00 42 1084 1764 1706	00 42 0698 1764 1707	00 70 1706 0000 0465
00 00 0000 0000 0000	00 40 0456 1747 0456	00 40 1700 1749 1700	00 41 1763 0000 0000
00 63 0453 0241 0000	00 61 0000 0000 0000	00 42 0456 1731 0000	00 40 1777 0000 1699
00 59 0000 0000 0000	00 42 0000 1730 1711	00 40 1804 0000 0000	00 40 1699 0000 0471
00 00 0000 0000 0000	00 40 1711 1805 0475	00 40 1699 1777 0000	00 40 0475 0000 0475
00 00 0000 0000 0000	00 60 1744 0476 0479	50 22 1701 1708 0000	00 63 0483 0481 0479
00 60 1701 1708 0000	00 60 1702 1709 0483	50 22 1702 1709 0000	00 63 0479 0000 0000
00 40 1749 1703 1703	04 60 1703 0000 0000	00 41 1695 0000 0000	00 63 0460 0488 0000
00 61 0000 0000 0000	00 60 1733 1703 0000	00 60 1774 0456 0000	00 60 1733 1700 0000
00 60 1708 0889 0000	00 60 1709 0890 0000	00 60 1733 1708 0000	00 60 0000 1709 0000
00 60 1806 0476 0000	00 60 0000 0000 0373	12 60 0891 1713 0000	99 42 0895 1748 0000
00 40 1713 0000 1713	00 40 1745 0000 1713	50 22 0000 1684 1713	96 60 0891 0000 0000
04 60 0000 0891 0000	00 42 0885 1748 0000	00 41 0885 0000 0885	00 42 1713 1825 0000
00 72 0000 1825 0511	00 40 0885 1748 0385	50 30 1713 1833 1687	00 60 0000 0000 0512
50 30 1713 1790 1687	00 34 1792 1687 1687	88 60 1687 0000 0000	00 40 0891 0000 0891
00 60 1744 0561 0000	00 72 0890 1733 0518	00 60 1748 1714 0520	00 60 0519 0000 1972
17 14 0520 0000 0890	50 22 1713 0889 0000	50 26 0000 1714 1715	50 28 1812 1715 0000
00 63 0526 0000 0000	00 42 1715 1825 0000	00 40 1813 0000 1715	00 42 1715 1825 0000
00 72 0000 1825 0530	50 20 1826 1715 1715	00 60 1820 0560 0532	50 20 1814 1715 1715

SEQUENCING ROUTINE page 4

00 60 1828 0560 0000	00 34 1815 1715 0000	00 40 1818 0000 1716	50 26 0000 1790 0000
00 34 1816 0000 1717	96 40 1817 0000 1718	96 40 0000 1737 1719	00 40 0000 1777 1720
00 40 1819 1717 0000	50 24 0000 1790 1721	50 22 1716 0000 0000	50 26 0000 1790 1722
00 40 1718 1820 0000	00 40 1719 0000 0549	00 40 1718 1824 0551	00 40 1719 1821 0555
00 40 1720 1820 0556	00 40 1718 1823 0557	00 00 0000 0000 0000	50 24 1722 0000 0000
00 00 0000 0000 0000	50 22 1722 1748 0000	50 24 1722 0000 0000	50 24 0000 1830 1723
00 00 0000 0000 0000	00 00 0000 0000 0000	00 00 0000 0000 0000	50 24 1723 0000 0000
50 20 0388 0000 1724	00 00 0000 0000 0000	00 00 0000 0000 0000	00 60 1831 0561 0000
00 40 0559 1732 0559	00 72 0888 1733 0567	00 60 1748 1714 0569	00 00 0003 0000 0003
00 60 0568 0000 1972	17 14 0569 0000 0888	50 22 1713 0887 0000	50 26 0000 1714 1715
50 28 1812 1715 0000	00 63 0575 0000 0000	00 42 1715 1825 0000	00 40 1813 0000 1715
00 42 1715 1825 0000	00 72 1825 0000 0580	00 40 1829 1732 0560	50 20 1826 1715 1715
00 60 0000 0000 0582	00 40 1828 1732 0560	50 20 1814 1715 1715	00 34 1715 1815 1716
00 40 1818 0000 1716	00 60 0000 0000 0534	00 41 0559 1732 0559	50 24 1774 1725 1715
50 28 1748 1715 0000	00 63 0590 0000 0000	00 60 1785 1715 0000	00 42 1715 1825 0000
00 70 1825 0000 0594	50 20 1715 1814 1715	00 60 0000 0000 0595	50 20 1715 1826 1715
00 34 1815 0000 1715	00 42 0885 0598 0885	00 60 0000 0000 0599	11 11 0011 1111 1111
99 42 1715 1735 0000	00 40 0885 0000 0885	00 42 1725 1825 0000	00 70 1825 0000 0605
50 20 1814 1725 1725	00 60 0000 0000 0606	50 20 1725 1826 1725	50 28 1748 1725 0000
00 63 0609 0000 0000	00 60 1785 1725 0000	00 34 1815 1725 1725	06 60 0886 0000 0000
06 60 0000 0886 0000	90 42 1725 1735 0000	00 40 0886 0000 0886	00 60 0000 0000 0400
00 80 0650 0001 0000	00 80 1687 0002 0000	00 80 1686 0003 0000	00 80 1688 0004 0000
00 80 1684 0005 0000	00 80 1685 0006 0000	00 83 1733 0007 0011	00 70 1704 1733 0630
00 60 1832 0447 0444	01 91 0000 0000 0000	01 98 0000 0000 0000	02 91 0000 0000 0000
04 91 0000 0000 0000	09 98 0000 0000 0000	00 60 0018 0001 0001	00 60 1838 0615 0000
00 60 1839 0621 0615	00 42 0886 1732 0000	00 72 1791 0000 0387	01 60 0886 0000 0000
99 60 0000 0886 0387	00 59 0000 0000 0000	04 42 0000 1731 0008	00 42 0052 1730 0000
00 41 0000 0008 0000	00 63 0050 0000 0000	00 61 0000 0000 0000	00 59 0000 0000 0000
04 42 0000 1731 0008	00 42 0062 1730 0000	00 41 0000 0008 0000	00 63 0060 0641 0641
00 59 0000 0000 0000	04 42 0000 1731 0008	00 00 0000 0000 0000	00 00 0000 0000 0000

1076 input-output locations inserted here.

00 00 0000 0000 0000	00 00 0000 0000 0000	39 07 0000 0000 0000	00 00 0000 1111 0000
00 00 1111 0000 0000	00 00 0000 0000 0001	00 00 0000 0000 0000	00 00 0000 0000 1100
00 00 0000 0000 0011	00 00 0000 0000 0007	00 00 0000 0001 0000	00 60 4684 0885 0000
04 58 0001 0000 0060	00 60 4684 0892 0000	04 58 0001 0000 0071	00 60 4684 0988 0000
04 58 0001 0007 0050	00 60 0000 0000 0000	11 00 0000 0000 0000	00 00 0000 0000 1111
00 00 0003 0000 0000	00 01 0000 0000 0000	00 00 0000 0003 0000	00 60 0989 1700 0000
00 60 1701 0989 0000	00 60 0990 1700 0000	00 60 0000 0990 0000	00 00 0000 0000 0200
00 60 0000 0000 0149	08 58 0001 0000 0122	00 60 1767 0166 0297	08 58 0001 0000 0130
00 00 0000 0008 0000	02 91 0652 0652 0001	00 60 0000 0000 0306	00 60 1744 0144 0000
00 00 0000 0600 0000	00 00 0011 1111 1110	00 60 0000 0000 0033	00 42 0886 1764 1084
00 60 0887 1085 0000	00 00 0001 0001 0000	00 60 1755 0174 0000	00 60 0000 0000 0175
96 60 1691 0000 0000	08 58 0003 0000 0179	50 20 0893 0000 1701	00 42 1084 1764 1706
00 60 0000 0000 0241	08 58 0001 0000 0306	00 00 0001 0000 0000	00 42 0886 1764 1084
00 00 0006 0000 0000	00 60 0000 1693 0000	00 70 0886 1764 0310	00 60 4000 0885 0000

EXHIBIT XI

SEQUENCING ROUTINE page 5

```
00 60 0000 0892 0000   04 58 0001 0000 0342   99 09 9000 0000 0000   00 60 0000 0988 0000
04 58 0001 0000 0352   00 60 0000 0000 0368   00 60 0000 0000 0372   98 05 0000 0000 0000
00 00 0000 0000 0009   89 01 0000 0000 0000   00 60 0988 0884 0000   00 60 0000 0000 0248
00 60 0000 0000 0260   00 60 0000 0000 0252   00 60 0000 0000 0112   08 58 0001 0000 0408
CC 60 0892 0000 0000   08 58 0001 0000 0420   00 60 0891 0000 0000   01 91 0884 0884 0001
06 58 0003 0000 0457   50 22 0000 0989 1701   50 20 0000 0990 1702   00 60 1744 0476 0479
00 60 0000 0000 0297   00 60 1795 0496 0000   00 60 1796 0241 0453   00 60 1811 0496 0398
00 60 0000 0000 0373   00 04 0000 0000 0000   00 03 9900 0000 0000   97 05 0000 0000 0000
90 01 0000 0000 0000   88 01 0000 0000 0000   00 00 0000 0000 1849   10 00 0000 0000 0000
12 00 0000 0000 0000   50 22 0000 0000 0000   50 24 0000 1822 0000   00 02 0000 0000 0000
50 20 0000 0000 0000   50 20 0000 0000 0388   00 10 0000 0000 0000   97 15 0000 0000 0000
99 15 0000 0000 0000   50 20 1830 0000 1724   50 20 1827 0000 1724   99 05 0000 0000 0000
00 60 0000 0000 0585   00 60 0000 0000 0630   98 15 0000 0000 0000   00 42 0000 1764 0000
00 40 0000 1726 0000   00 40 1084 0000 1084   11 42 0000 1840 1728   00 80 1686 0001 0000
00 83 1727 0007 0624   01 11 1000 0000 0000   02 42 0000 1818 0000   00 70 1733 0000 0218
50 22 0000 1728 1701   50 26 1701 0000 1702   50 24 1702 0001 1702   00 60 1701 0000 0000
00 60 1702 0001 0000   00 60 0000 0000 0166   00 00 0000 0000 0000   99 00 1994 0000 0000
99 00 3983 0000 0000   99 00 5962 0000 0000   99 00 7926 0000 0000   99 00 9871 0000 0000
99 01 1791 0000 0000   99 01 3683 0000 0000   99 01 5542 0000 0000   99 01 7364 0000 0000
99 01 9146 0000 0000   99 02 0884 0000 0000   99 02 2575 0000 0000   99 02 4215 0000 0000
99 02 5804 0000 0000   99 03 0114 0000 0000   99 02 8814 0000 0000   99 03 0234 0000 0000
99 03 1594 0000 0000   99 03 2894 0000 0000   99 03 4134 0000 0000   99 03 5314 0000 0000
99 03 6433 0000 0000   99 03 7493 0000 0000   99 03 8493 0000 0000   99 03 9435 0200 0000
99 04 0320 0000 0000   99 04 1149 0000 0000   99 04 1924 0000 0000   99 04 2647 0000 0000
99 04 3319 0000 0000   99 04 3943 0000 0000   99 04 4520 0000 0000   99 04 5053 0000 0000
99 04 5543 0000 0000   99 04 5994 0000 0000   99 04 6407 0000 0000   99 04 6784 0000 0000
99 04 7128 0000 0000   99 04 7441 0000 0000   99 04 7725 0000 0000   99 04 7982 0000 0000
99 04 8214 0000 0000   99 04 8422 0000 0000   99 04 8610 0000 0000   99 04 8778 0000 0000
99 04 8928 0000 0000   99 04 9061 0000 0000   99 04 9180 0000 0000   99 04 9286 0000 0000
99 04 9379 0000 0000   99 04 9461 0000 0000   99 04 9534 0000 0000   99 04 9598 0000 0000
99 04 9653 0000 0000   99 04 9702 0000 0000   99 04 9744 0000 0000   99 04 9781 0000 0000
99 04 9813 0000 0000   99 04 9841 0000 0000   99 04 9865 0000 0000   99 04 9886 0000 0000
99 04 9903 0000 0000   99 04 9918 0000 0000   99 04 9391 0000 0000   99 04 9942 0000 0000
99 04 9952 0000 0000   99 04 9960 0000 0000   99 04 9966 0000 0000   99 04 9972 0000 0000
99 04 9977 0000 0000   99 04 9981 0000 0000   99 04 9984 0000 0000   99 04 9987 0000 0000
99 04 9989 0000 0000   99 04 9991 0000 0000   99 04 9993 0000 0000   99 04 9994 0000 0000
99 04 9995 0000 0000   99 04 9996 0000 0000   99 05 0000 0000 0000   99 05 0000 0000 0000
00 40 1946 0000 1938   92 60 0000 0000 0000   00 40 1947 0000 1934   00 00 0000 0000 0000
00 00 0000 0000 0000   00 00 0000 0000 0000   00 00 0000 0000 0000   00 00 0000 0000 0000
00 61 0000 0000 1940   12 60 1936 1935 0000   88 42 1946 0000 0000   00 40 1945 0000 1934
00 60 1944 1937 1934   00 68 1939 1937 1935   88 40 1935 1946 1935   00 59 9999 9132 0001
00 59 9998 1936 1940   00 40 1967 0000 1960   91 60 0000 0000 0000   01 40 1968 0000 1952
00 60 1971 1942 0000   00 00 0000 0000 0000   00 68 0000 0000 0000   12 42 1965 1949 0000
88 40 1970 0000 1956   00 00 0000 0000 0000   00 41 1956 1969 1958   00 00 0000 0000 0000
00 68 0000 0000 1962   00 00 0000 0000 0000   00 61 1965 0000 1971   00 00 0000 0000 0000
```

SEQUENCING ROUTINE page 6

```
00 40 1965 1970 1964   00 00 0000 0000 0000   00 00 0000 0000 0000   00 60 1961 1962 1955
00 59 9999 9140 0001   05 99 9981 9650 0000   00 01 0000 0000 0000   00 02 0000 0030 0097
00 60 1956 1962 1962   00 60 0000 1997 0000   08 60 0000 0000 0000   00 40 1993 0000 1991
92 60 1997 0000 0000   00 40 1994 0000 1978   16 60 0000 1997 0000   00 00 0000 0000 0000
00 63 0000 1991 1992   00 40 1996 0000 0000   03 60 0000 0000 0000   11 34 1995 0000 0000
86 60 0000 0000 0000   00 40 1996 0000 0000   00 40 1997 0000 1997   50 36 1978 0000 0000
50 32 0000 1997 0000   50 34 1995 0000 0000   50 20 1997 0000 0000   00 72 0000 1997 1985
00 00 0000 0000 0000   00 61 0000 0000 1991   00 60 1997 0000 0000   50 20 0000 0000 1978
99 05 0000 0000 0000   67 01 0000 0000 0000   00 00 0000 0000 0000   02 91 0051 2040 0001=
```

EXHIBIT XI

sequence, it first calculates the value for $T_L - T_E$, the slack. This is stored in the event file but is not needed on the working tape. After calculation, the value of $T_L - T_E$ is tested by subtracting 5.0 from it. If the result is zero or negative, a 9 code is generated and stored in the record code position. The existence of a value for T_S is then tested if present, the values

$$\frac{T_S - T_E}{\sigma_{T_E}} \text{ and } \frac{T_L - T_S}{\sigma_{T_L}}$$

are calculated. T_S represents a measure of the elapsed time from time "now" to the scheduled date. As time "now" is advanced, the T_S or elapsed time must be reduced. This is adjusted by manually storing the time differential from the last reporting date, or the difference from time zero for the first run after setting up the system. This adjustment is applied to every T_S in the file. The adjusted value is stored in the event file after each computation of T_L.

These values are looked up in a previously stored table of the normal probability function. The resulting probability expression for

$$\frac{T_S - T_E}{\sigma_{T_E}}$$

is stored in the event file. The product of the two values resulting from the table look-up is also stored in the event file.

The T_L calculations follow in a similar manner for each event in the event file. The reverse calculation is based on the successor events rather than the predecessor events, since these are the predecessor events in the reverse sense. The slack ($T_L - T_E$) and the probability calculations are carried out as indicated above. Adjustments in the final T_E and $\sigma^2_{T_E}$ values are made for the activities leading up to time now as in the earlier T_E calculations.

A detailed program flow chart and the absolute coding for Run 08 are found in Exhibits XII and XIII, following this page. The generation of the output tape from Run 08 represents the end of the computer calculations. It is now necessary to translate the findings to a usable form.

2. OUTPUT PROVISIONS

(1) *Output Sort (Run 09).* Run 09, the output sort run, has two functions. First, it re-sorts the event file tape into exact event order and prepares an input tape for use in the file updating run (Run 06). Generally, when the system is set up and operating for large networks, the utility of the outputs will be enhanced if they are presented in an order which allows easy reference to selected events. The input to Run 09 is the same computed output event tape in ascending order of the event code. The program itself is almost identical to Runs 03 and 04. Further discussion of sorting programs is found there.

(2) *Output Edit (Run 10).* The output edit run is a necessary function for two reasons. First, the event file contains a great deal more information than is useful on a report. Therefore, some of the information in the file should be screened out before the report is printed. Secondly, as noted earlier, in order to maintain a reasonable amount of precision in the computations, it is necessary to use a full NORC word and floating point arithmetic operations. In reporting values, however, it is only reasonable and consistent to report T_E and T_L to one decimal place. The inherent accuracy of the data is no better than this level, and it is more readily understood in this format. The output editing routine accomplishes both these results.

The input to the output edit run is a computed event file tape. The resulting output is an edited output tape containing all events and a second edited output tape containing all events having a report code. A report code can be inserted into the file through the updating routine or it can be generated internally as a result of the critical slack calculation. The format of both tapes is identical, using a four word event message. The output tapes are then ready to be converted to cards on the CTC operation. It should be noted that these cards can be sorted and manipulated using the IBM 083 card sorter. Thus, the printed reports can be prepared in any order.

The program operations are straightforward. A block is read into memory from the input tape. The program extracts the first event, and sets up the event code in word 1 of the output tape storage. The event code is also placed in word 1. The program then selects the value for T_E, converts it to fixed point form, rounds off the number to one decimal place, develops the coded sign of the value, and places the entire result in the last five digits of word 1. In a similar manner, it constructs the correct form for values of T_L, $T_L - T_E$, and T_S placing them in word 2. The probability value and the schedule date are placed in word 3. Zeros (NORC blanks) are placed in word 4, and the entire result is placed on the output tape. The program selects the next event, and the cycle is repeated until all events have been edited and placed on the output tape.

PROGRAM FLOW CHART
COMPUTATION RUN #08

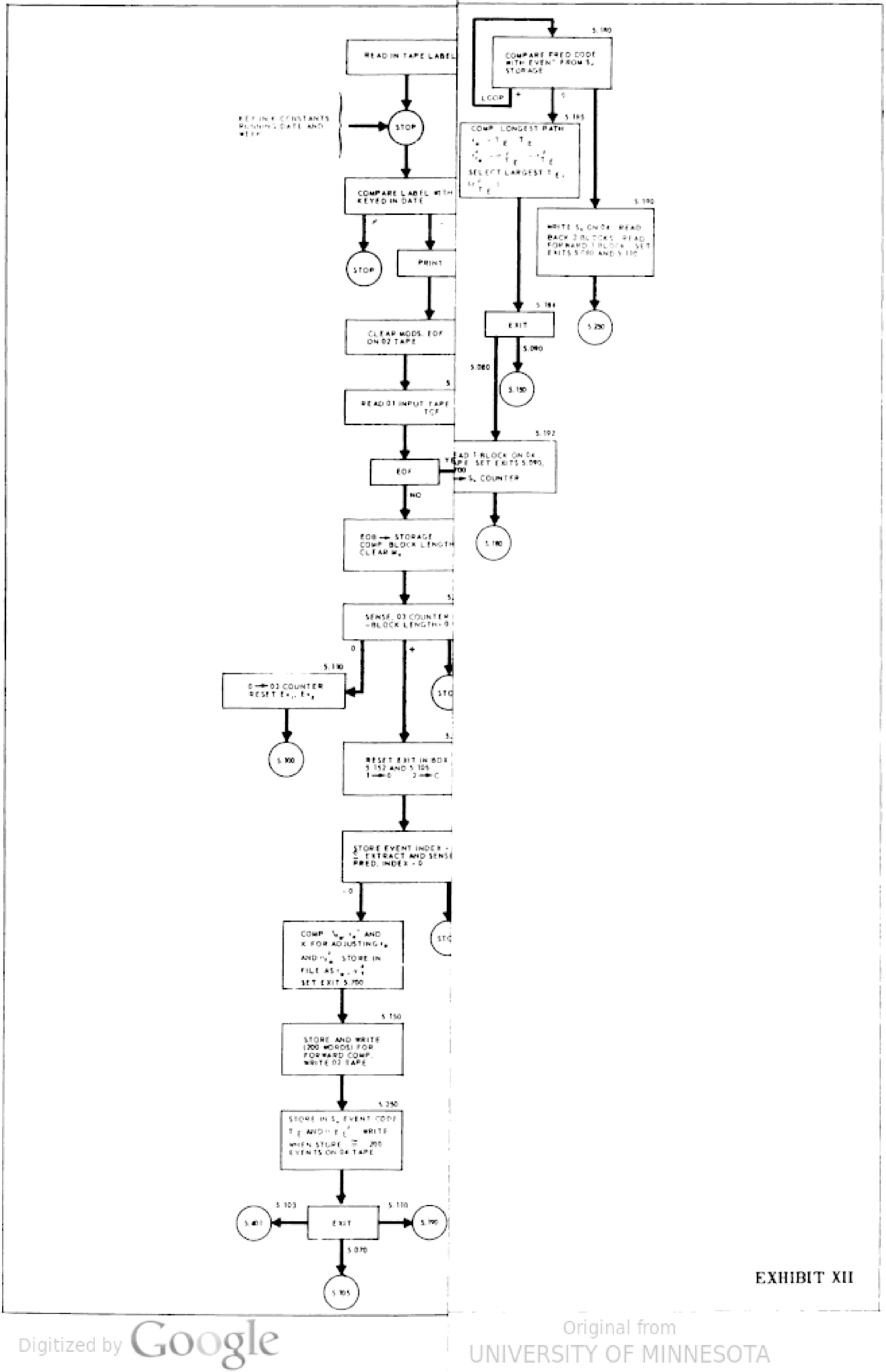


EXHIBIT XII

COMPUTATION ROUTINE page 1

```
-02 91 0051 0056 0000    09 96 0008 1753 0001    00 68 0000 0000 0006    00 60 0000 0000 0008
 00 00 0000 0000 0000    00 61 0000 0000 0001    00 00 0000 0000 3952    02 91 0051 0056 0000-

-02 91 0051 1796 0001    01 94 1602 1602 0000    08 68 0009 0000 1713    00 61 0000 0000 0088
 00 40 1631 1697 1631    00 70 1602 0000 0014    00 61 0000 0000 0000    00 40 1710 1697 0000
 00 40 1629 0000 1629    03 91 1629 1629 0000    08 60 0017 0000 1730    00 57 0000 0000 0000
 01 94 0397 0000 0001    08 68 0020 0000 1713    00 67 0000 0000 0086    00 60 0000 1603 0000
 04 42 0000 1637 1604    08 42 1603 1638 0000    00 41 1604 0000 1605    00 40 1639 0000 1605
 00 52 0000 0000 0000    00 41 1605 1606 0000    00 63 0035 0031 0000    00 61 0000 0000 0000
 00 60 1658 0050 0000    00 40 0019 1639 0019    00 60 1640 1606 0000    00 60 1645 0036 0019
 13 60 6397 1607 0000    00 60 0403 1608 0000    02 42 1608 1642 1609    00 42 1608 1643 1610
 00 72 1609 1640 0044    00 60 1646 0054 0000    00 70 1610 1640 0044    00 60 0000 0000 0045
 00 60 0000 0000 0198    00 60 1647 0054 0000    96 60 1607 0000 0000    00 40 0050 0000 0050
 00 54 0000 0000 0000    00 60 6397 8598 0000    00 50 0000 0000 0001    06 58 0001 0000 0048
 00 40 1607 1606 1606    92 60 1607 0000 0000    00 40 0036 0000 0036    00 60 0000 0000 0028
 00 40 1611 1607 0000    00 41 1650 0000 0000    00 63 0000 0060 0059    00 50 1647 0069 0062
 00 60 0058 0079 0070    00 60 1647 0079 0000    00 60 1646 0069 0062    00 51 0000 0000 0000
 96 60 1607 0000 0000    00 40 1649 0000 0068    00 60 4598 0798 0000    00 40 0065 1648 0065
 00 40 1611 1639 1611    00 00 0000 0000 0000    00 00 0000 0000 0000    96 60 1611 0000 0000
 00 41 0000 1648 0000    00 40 1652 0000 0073    00 00 0000 0000 0000    08 60 0074 0000 1730
 00 60 0000 0000 0000    00 60 1651 0065 0000    00 40 1652 1639 1652    00 60 1640 1611 0000
 00 00 0000 0000 0000    01 98 0000 0000 0000    02 91 0000 0000 0000    02 98 0000 0000 0000
 00 57 0000 0000 0000    00 40 1681 1652 1652    00 60 1640 1606 0095    00 70 1611 1640 0080
 00 60 1646 0079 0070    00 80 1602 0001 0000    00 80 1631 0002 0000    00 80 1629 0003 0000
 00 80 1640 0004 0000    00 80 0000 0005 0000    00 80 0000 0006 0000    00 83 0000 0007 0011
 02 94 0397 0000 0000    08 68 0096 0000 1713    00 67 0000 0000 0311    00 60 0000 1603 0000
 04 42 0000 1637 1604    08 42 1603 1638 0000    00 41 1604 0000 1605    00 40 1639 0000 1605
 00 51 0000 0000 0000    00 60 0000 0000 0000    01 94 0999 0000 0001    08 68 0106 0000 1713
 00 67 1653 1634 0348    00 60 0000 1613 0000    00 40 0105 1639 0105    00 60 1653 1671 0000
 04 42 1613 1637 1604    08 42 1613 1638 0000    00 41 1604 0000 1614    00 40 1639 0000 1614
 00 52 0000 0000 0000    00 60 1646 0140 0000    00 60 0000 0000 0000    00 41 1605 1606 0000
 00 63 0124 0121 0000    00 61 0000 0000 0000    00 60 1640 1606 0000    00 60 1645 0174 0000
 00 60 1656 0104 0095    00 60 0403 1608 0000    13 60 4397 1607 0000    02 42 1608 1642 1609
 00 42 1608 1643 1610    00 72 1640 1609 0131    00 72 1610 1640 0131    00 60 1699 0140 0133
 00 72 1610 1640 0133    00 60 1693 0140 0133    96 34 1609 1644 1609    96 34 1610 1644 1610
 00 42 1609 1711 1609    00 42 1610 1711 1610    92 60 1607 0000 0000    00 40 0124 0000 0124
 00 40 1607 1606 1606    00 60 0000 0000 0000    00 41 1614 1615 0000    00 63 0149 0144 0000
 00 61 0000 0000 0000    00 60 1640 1615 0000    00 60 1659 0149 0000    00 60 1657 0117 0000
 00 60 1660 0167 0000    00 60 0000 0000 0105    00 60 1005 1616 0000    13 60 6999 1619 0000
 02 42 1616 1642 1617    00 42 1616 1643 1618    00 72 1617 1640 0156    00 60 0155 0169 0156
 00 60 1646 0169 0141    96 34 1617 1644 1617    96 34 1618 1644 1618    92 60 1619 0000 0000
 00 40 0149 0000 0149    00 42 1617 1711 1617    00 42 1618 1711 1618    00 40 1619 1615 1615
 96 60 1619 0000 0000    00 40 0167 0000 0167    00 60 6999 0598 0000    00 40 0165 1548 0165
 06 98 0001 0000 0165    00 60 1661 0165 0000    00 60 0000 0000 0000    92 60 1609 0000 0000
```

COMPUTATION ROUTINE page 2

```
00 40 1672 0000 0000    00 40 1694 0000 0174    00 42 0599 1655 1620    00 00 0000 0000 0000
00 41 0000 1620 0000    00 63 0000 0178 0207    00 60 0197 0196 0179    00 60 1640 1635 0227
00 40 1611 1619 0000    00 41 1650 0000 0000    00 63 0185 0182 0183    00 60 0196 0206 0184
00 60 1700 0206 0198    00 60 0043 0196 0000    00 54 0000 0000 0000    96 60 1619 0000 0000
00 40 1662 0000 0194    00 42 0599 1655 0000    00 72 1636 0000 0191    00 61 0000 0000 0000
00 60 8598 1200 0000    00 40 0191 1648 0191    00 40 1639 1611 1611    00 00 0000 0000 0000
00 42 0599 1655 1636    00 00 0000 0000 0000    00 60 0000 0000 0141    96 60 1611 0000 0000
00 41 0000 1648 0000    00 40 1664 0000 0201    00 00 0000 0000 0000    08 60 0202 0000 1730
00 40 1664 1639 1664    00 60 1640 1611 0000    00 60 1663 0191 0000    00 00 0000 0000 0000
00 72 1653 1621 0373    00 60 1704 0148 0000    00 60 0177 0169 0177    00 42 0201 1653 0000
00 72 1666 0000 0218    00 40 1664 1653 1664    00 60 1701 0338 0000    00 40 1683 1666 0223
00 40 1684 1666 0224    00 40 1684 1666 0343    00 60 0000 0000 0223    00 41 1664 1653 1664
00 40 1653 1701 0338    00 40 1683 1688 0223    00 40 1684 1688 0224    00 40 1684 1688 0343
00 00 0000 0000 0000    00 00 0000 0000 0000    00 60 1689 0148 0000    00 60 1640 1636 0144
00 42 0605 1655 1620    00 42 4398 1655 0000    00 70 1620 0000 0234    00 40 1644 0227 0227
00 40 1707 1624 1624    00 72 0000 1617 0227    00 61 0000 0000 0000    00 60 1640 1624 0000
00 60 1669 0227 0000    00 40 0599 1653 0599    14 60 0000 1632 0000    02 42 0604 1542 0000
00 41 0000 1632 0000    00 63 0000 0282 0251    00 60 0177 0250 0000    00 60 1667 0272 0000
02 60 1653 0000 0000    00 40 4398 0000 4398    12 42 4398 1671 1632    00 42 4403 1643 0000
00 41 0000 1632 0000    00 63 0000 0253 0252    00 40 1672 1644 1672    00 00 0000 0000 0000
00 61 0000 0000 0000    00 61 0000 0000 0000    92 60 1698 1672 0000    88 42 4403 1643 0000
00 41 4398 0000 4398    00 40 1625 1607 0000    00 41 1650 0000 0000    00 63 0000 0260 0261
00 60 1646 0271 0262    00 60 1678 0271 0262    00 60 0259 0281 0273    00 54 0000 0000 0000
00 40 1612 1639 1612    96 60 1607 0000 0000    00 40 1674 0000 0270    00 60 4397 1401 0000
00 40 0266 1648 0266    00 40 1625 1639 1625    00 50 0001 0000 0000    00 00 0000 0000 0000
00 00 0000 0000 0000    00 00 0000 0000 0000    96 60 1625 0000 0000    00 41 0000 1648 0000
00 40 1677 0000 0276    00 00 0000 0000 0000    08 60 0277 0000 1730    00 40 1677 1639 1677
00 60 1640 1625 0000    00 60 1673 0266 0000    00 00 0000 0000 0000    00 60 0197 0250 0000
00 60 1699 0272 0000    88 42 0604 1642 0000    00 41 0599 0000 0599    00 40 1619 1626 0000
00 41 1650 0000 0000    00 63 0000 0290 0292    00 60 1646 0300 0293    00 60 0301 0310 0000
00 60 1682 0300 0293    00 60 0289 0310 0302    00 54 0000 0000 0000    96 60 1619 0000 0000
00 40 1679 0000 0299    00 60 8598 0798 0000    00 40 0296 1648 0296    00 40 1626 1639 1626
00 00 0000 0000 0000    00 00 0000 0000 0000    00 60 0000 0000 0243    96 60 1626 0000 0000
00 41 0000 1648 0000    00 40 1652 0000 0305    00 00 0000 0000 0000    08 60 0306 0000 1730
00 40 1652 1639 1652    00 60 1640 1626 0000    00 60 1680 0296 0000    00 00 0000 0000 0000
00 70 1626 1640 0313    00 60 1686 0310 0302    00 40 1630 1653 1630    00 72 1681 0000 0316
00 60 0000 0000 0395    00 42 0095 1653 0000    00 72 1681 0000 0325    00 40 0095 1681 0095
00 41 1652 1681 1652    00 40 1685 1683 0332    00 40 1681 1684 0333    00 40 1685 1684 0334
00 40 1681 1683 0335    00 60 0333 0336 0332    00 41 0095 1681 0095    00 40 1652 1681 1652
00 40 1683 1681 0332    00 40 1684 1685 0333    00 40 1684 1681 0334    00 40 1683 1685 0335
00 60 0333 0336 0332    00 00 0000 0000 0000    00 00 0000 0000 0000    00 00 0000 0000 0000
00 00 0000 0000 0000    00 00 0000 0000 0000    00 60 0000 0000 0121    00 00 0000 0000 0000
08 68 0339 0000 1713    00 67 1688 1628 0343    00 60 0000 1613 0000    00 60 1666 1621 0111
00 00 0000 0000 0000    00 60 1685 1634 0000    00 60 1691 0148 0144    00 00 0000 0000 0000
00 00 0000 0000 0000    00 60 1708 0105 0381    03 91 0000 0000 0000    01 98 0000 0000 0000
```

EXHIBIT XIII

```
                          COMPUTATION  ROUTINE                          page 3

02 98 0000 0000 0000   03 98 0000 0000 0000   04 98 0000 0000 0000   05 98 0000 0000 0000
06 98 0000 0000 0000   09 98 0000 0000 0000   00 60 0013 0001 0001   00 00 0000 0000 0000
00 70 1611 1640 0210   00 60 1712 0206 0198   00 72 1628 1688 0365   00 60 1640 1628 0000
00 60 1646 0169 0000   00 60 1689 0148 0359   00 60 1706 0345 0338   00 80 1629 0001 0000
00 80 1640 0002 0000   00 80 0000 0003 0000   00 80 0000 0004 0000   00 80 0000 0005 0000
00 80 0000 0006 0000   00 83 1612 0007 0349   00 60 1668 0148 0000   00 60 0177 0169 0177
00 59 0000 0000 0000   04 42 0000 1637 0380   00 41 1650 0000 0000   00 63 0165 0000 0000
00 61 0000 0000 0000   00 00 0000 0000 0000   00 72 1634 1640 0383   00 61 0000 0000 0000
00 40 1635 1639 1635   00 41 1707 0000 0000   00 63 0362 0362 0000   00 60 0174 0387 0000
00 00 0000 0000 0000   00 80 0000 0001 0000   00 80 1629 0007 0000   00 80 1612 0007 0000
00 80 1620 0003 0000   00 80 1640 0004 0000   00 80 0000 0005 0000   00 81 0000 0006 0382
00 70 1625 1640 0366   00 60 1709 0281 0273   00 00 0000 0000 0000   00 00 0000 0000 0000
1236 input-output locations inserted here.
00 00 0000 0000 0000   00 00 0000 0000 0000   00 00 0000 1111 0000   00 00 1111 0000 0000
00 00 0000 0000 0001   00 00 0000 0000 0000   00 00 0000 0000 0004   00 00 0000 0000 1100
00 00 0000 0000 0011   00 00 0003 0000 0000   00 60 0403 1608 0000   00 60 0000 0000 0000
00 60 0000 0000 0028   00 00 0000 0001 0000   04 58 0001 0000 0065   00 00 0000 0000 0200
00 60 4598 0798 0000   02 91 0798 0798 1000   01 00 0000 0000 0000   00 00 1000 0000 0000
00 00 0011 1111 1110   00 60 1640 1630 0118   00 60 1646 0169 0141   06 58 0001 0000 0048
00 60 1005 1616 0000   06 58 0001 0000 0165   00 60 6999 0598 0000   08 58 0001 0000 0191
00 60 8598 1200 0000   05 91 1200 1200 0500   00 60 0000 0000 0179   05 00 0000 0000 0000
00 60 1699 0196 0179   00 60 0000 0000 0361   00 42 0605 1655 1670   00 00 0001 0000 0000
00 11 0000 0000 0000   00 00 0007 0000 0000   00 60 4397 1401 0000   08 58 0001 0000 0266
00 60 1657 0196 0185   00 60 1696 0196 0338   03 91 1401 1401 1000   00 60 0272 0281 0273
08 58 0001 0000 0296   00 60 8598 0798 0000   02 00 0000 0000 0000   00 60 0000 0000 0302
00 91 0000 0000 0000   00 98 0000 0000 0000   04 00 0000 0000 0000   00 60 0000 0000 0313
00 60 0000 0000 0170   06 00 0000 0000 0000   00 60 0000 0000 0338   00 95 0000 0000 0000
00 60 1640 1628 0105   00 40 0167 0000 0167   00 60 1699 0272 0256   00 42 4397 1655 0000
00 50 0000 0000 0000   00 60 0000 0000 0346   39 08 0000 0000 0000   00 00 0000 0300 0007
00 60 1646 0140 0118   00 60 0000 0000 0185   05 94 0999 0000 0000   00 60 1703 0206 0198
00 60 0000 0000 0197   00 60 0000 0000 0359   00 60 1691 0148 0144   00 60 1705 0345 0361
00 00 0000 0000 0003   00 60 0000 0000 0383   00 60 0000 0000 0366   00 01 0000 0000 0000
00 00 0000 0000 1111   00 60 0000 0000 0210   00 40 1728 0000 1720   92 60 0000 0000 0000
00 40 1729 0000 1716   00 00 0000 0000 0000   00 00 0000 0000 0000   00 00 0000 0000 0000
00 00 0000 0000 0000   00 00 0000 0000 0000   00 61 0000 0000 1722   12 60 1718 1717 0000
88 42 1728 0000 0000   00 40 1727 0000 1716   00 60 1726 1719 1716   00 68 1721 1719 1717
88 40 1717 1728 1717   00 59 9999 9132 0001   00 59 9998 1718 1722   00 40 1749 0000 1742
91 60 0000 0000 0000   01 40 1750 0000 1734   00 60 1753 1744 0000   00 00 0000 0000 0000
00 68 0000 0000 0000   12 42 1747 1731 0000   88 40 1752 0000 1738   00 00 0000 0000 0000
00 41 1738 1751 1740   00 00 0000 0000 0000   00 68 0000 0000 1744   00 00 0000 0000 0000
00 61 1747 0000 1753   00 00 0000 0000 0000   00 40 1747 1752 1746   00 00 0000 0000 0000
00 00 0000 0000 0000   00 60 1743 1744 1735   00 59 9999 9140 0001   05 99 9981 7470 0000
00 01 0000 0000 0000   00 02 0000 0030 0097   00 60 1738 1744 1744   02 91 0051 1796 0001-
```

EXHIBIT XIII

(3) *Output Tape and Output Card.* The output tape format is shown on Exhibit XIV, following this page. Word 1 contains the event code in positions 2 through 10. The report code is in position 11. The sign of T_E is in position 12 and T_E occupies positions 13 through 16. T_E is expressed in weeks and tenths of weeks. Word 2 contains the sign of T_L in position 1 and T_L in positions 2 through 5. The sign of $T_L - T_E$ is in position 6, and $T_L - T_E$ is in positions 7 through 10. Position 11 is zero (blank). Position 12 contains the signs of T_S, while T_S is in positions 13 through 16. Word 3 contains the probability value in positions 3 and 4, and the schedule date is in positions 5 through 10. The remainder of word 3 and all of word 4 are filled with zeros. These zero positions could be used for additional output information at some time in the future.

The output card is shown on Exhibit XV, on page 78, and follows exactly the same format of the output tape.

(4) *Output Form.* The output form contains only the information needed for currently planned evaluation. The computer system has been designed to allow for the generation of additional factors that may be needed for future analyses. Additional information is carried in the main event file or could be generated from this file by relatively minor modifications in the computer programs.

The output printer has been programed to recognize a change in the first three digits of the event code. A change here designates a change in a major system and thus in the responsible SP Technical Branch. Upon recognizing this change, the printer will cause the paper to advance, and it will start printing the next system on a new page. The printer has been further programed to effect a triple space on a page when there is a change in the second group of three digits. The latter designates a change in subsystem within a major system. The capacity of this coding system has been examined and is considered adequate for the complete FBM system.

The output form has been pictured in the main body of this report. The information processing procedure places constraints on the magnitudes of the numbers that can be printed on the output form.

EDITED OUTPUT TAPE LAYOUT

WORD #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	—	EVENT CODE									RPT. CODE	±	T_E				✓
	O	X	X	X	X	X	X	X	X	X	X	X	X	X	X	.X	X
2	±	T_L				±	$T_L - T_E$				—	±	T_S				✓
	X	X	X	X	.X	X	X	X	X	.X	O	X	X	X	X	.X	X
3	—	—	P_r		SCHEDULE DATE						—	—	—	—	—	—	✓
	O	O	.X	X	X	X	X	X	X	X	O	O	O	O	O	O	X
4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	✓
	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	X

EXHIBIT XIV

These constraints are summarized below.

Designation	Number of Digits	Range
Col. (2) – T_E	4	1.0 – 999.9+
Col. (3) – T_L	4	999.9+ – 999.9-
Col. (4) – Slack	4	999.9+ – 999.9-
Col. (5) – T_S	4	999.9+ – 999.9-
Col. (6) – P_r	2	.00 – .99
Col. (7) – Schedule Date	6	Month-Day-Year
Col. (8) –	24	Available for any additional data

The foregoing sections have described the logic of the computer program. The computer program has been carefully designed to automatically discover inconsistencies and errors. The following section focuses upon the detailed checks that are the mark of a mature system.

3. ACCURACY CONTROL. The general need for accuracy control techniques and procedures in a high speed data-processing system cannot be overemphasized. The very nature of any such system is to remove from visual and manual observation data which in the normal course of handling would be sight verified many times by many people. It is not enough to establish exacting rules for processing the data in the computer program. In addition, it is further necessary to establish and maintain standards of accuracy and control for the data flowing into and out of the machine operations.

For the SP PERT system, the problem is aggravated by the fact that NPG is acting as a service function for Sp 12. As such, PERT will occupy less than 5% of their total time. In this position, SP cannot be assured of a continuing close follow-up by NPG personnel over the life of the project. It is necessary, therefore, that sufficient controls be developed and administered by SP so that they can be assured of the accuracy and fidelity of the operations at NPG.

(1) *Accuracy Control Techniques.* Accuracy control for the NORC-PERT system is developed and maintained in three ways.

—*Internal Control.* In addition to the machine checks of the NORC itself, several additional checks have been added to the programs. These checks consist mainly of tests of reasonableness and consistency as applied to the actual file identification and computative data. For example, before updating any record, the event codes of the file and the transaction are verified. In a similar way, before events are qualified for repositioning in the sequencing operation, the event and its predecessor are verified with the corresponding event and its successor.

A reasonableness check is applied to the input data before updating to determine that $a \leq m \leq b$ and further that

$$\sigma^2_{T_E} \leq 99.9.$$

Other similar checks can be applied. Moreover, the entire system should be examined periodically to consider the efficiency of current checks and the possible addition of new internal controls.

—*Transaction and File Control Totals.* The total transactions in each batch are counted before they are transmitted to NPG. During the file maintenance run, the total transactions processed are counted. The figure is balanced with the total transactions key-punched. Finally, this count is transmitted to SP with the output sheets, and should be balanced with the total transmitted to NPG.

OUTPUT CARD LAYOUT

ZEROS	ZERO	EVENT CODE	REPORT CODE	$\pm$ T_E	$\pm$ T_L	$\pm T_L - T_E$	ZERO	$\pm$ T_S	ZERO	P_Y	SCHEDULE DATE	ZEROS	ZEROS
1 – 12	13	14 – 22	23	24 – 28	29 – 33	34 – 38	39	40 – 44	45 – 46	47 – 48	49 – 54	55 – 60	61 – 80
	I	I	I	I	I	II	II	II	III	III	III	III	IV

EXHIBIT XV

A manual running control of the total events in the file is maintained at Sp 12. A beginning total is established and adjusted by the additions and deletions of events. Both sequencing and computing runs produce a total of events in the file for control purposes. These should be balanced to each other so as to test the consistency of operation at NPG. The computing run at NPG also generates the total number of predecessors and successors. These totals should be equal and individually give the total activities current in the file.

—*Tape Label Control.* Each tape used in the management data-processing activity at NPG has a tape label which uniquely identifies it. The tape label consists of four items in a NORC word.

Problem Number	Run Number	Date	Sequence Number	Week Number
XX	XX	XX-XX-XX	XX	XXX.X

The first two digits represent the problem number—the PERT problem number is 39—and will remain unchanged during its life. In the PERT system there are ten runs, as described earlier. The date is a six-digit calendar date. The sequence number is used to identify the multiple runs during a day. The week number is the week count which is identical to the date and used in the updating and computing runs to establish week "now." Normally, the regular runs will be numbered 0000, and special runs assigned numbers in sequence. Each run will produce control data showing the beginning and ending tape label for the run. A simple log of the tapes and their labels for each run will protect the validity of the output data and isolate special runs made for experimental purposes. It is proposed that NPG hold transaction tapes for three consecutive updating runs before destroying the data. Completely analyzed files (after computing) will be consolidated and held for six months. A flow chart of the NORC control system is shown in Exhibit XVI, following this page.

(2) *Accuracy Control Procedures*

—Input data will be transmitted to NPG in batches. Batches will be registered by dispatch clerk at Sp 12 and the batch number assigned. The number of lines (transactions) will be counted before they are given to the dispatch clerk. The dispatch clerk will then count, verify, and initial each batch. Batches will be combined at SP or NPG only with permission of the Sp 12 liaison officer.

—Output data will be returned to the dispatch clerk, where it will be logged in and the actual control data entered in the dispatch book. The form for the NORC Data Dispatch and Control Sheet is shown on Exhibit XVII, on page 81. When it has been verified that the batch has been correctly processed, the dispatch clerk will issue instructions to NPG to destroy the input cards.

—A running control of the number of events in the file will be maintained by the dispatch clerk. The beginning number of events will be entered on the control sheet and additions, deletions, and completed event totals will be entered by date and batch number. The output control data will be examined and the event total determined by the NORC will be entered in the running control. *Output data will not be released until running controls and batch controls are balanced.*

—NPG will use the last good file for each updating and computing run. A log or register of the tape labels will be maintained at SP and NPG. The layout of the SP tape register is shown on Exhibit XVIII, on page 82. The corrected tape label for each run must be verified by reference to the output control data before *any output data is released by SP.* If special runs have been made in the period between two regular runs, the liaison officer must issue specific instructions to NPG indicating the correct file tapes to be used.

* * * * *

The last two chapters have been concerned with the "hardware" aspects of the computer installation and the programing steps taken to ensure efficient machine runs. However, a going data-processing system must provide for more than a well-conceived program. In addition, SP must underwrite the necessary costs of the system and provide the personnel for running the system. The following chapter will deal with these "nonhardware" elements that comprise the computer environment.

V. COMPUTER REQUISITES

1. COSTS OF OPERATION

(1) *Program Formulation Costs.* Exhibit XIX, on page 83, shows a breakdown of the costs incurred at the Computation Laboratory, Naval Weapons Laboratory, Dahlgren, Virginia.

For the PERT computation system, approximately $20,645 of the total of $36,095 was spent on programing, coding, and program checkout. Ten different programs or computer runs were prepared. The total cost is apportioned over 3,600 program steps. This is a cost of $5.74 per step. Approximately 221 man-days were used in completing the programing effort. This is a rate of 16.3 steps per day.

(2) *Future Cost Requirements.* On the basis of an assortment of approximately 60 runs per month, the running costs at NORC are estimated to be $5,500/month. The estimated machine time for these runs is 25 to 28 hours per month. Preliminary estimates of running time rates are shown in Exhibit XX, on page 84. These estimates are

DATA FLOW CHART
SP-12 NORC CONTROL SYSTEM

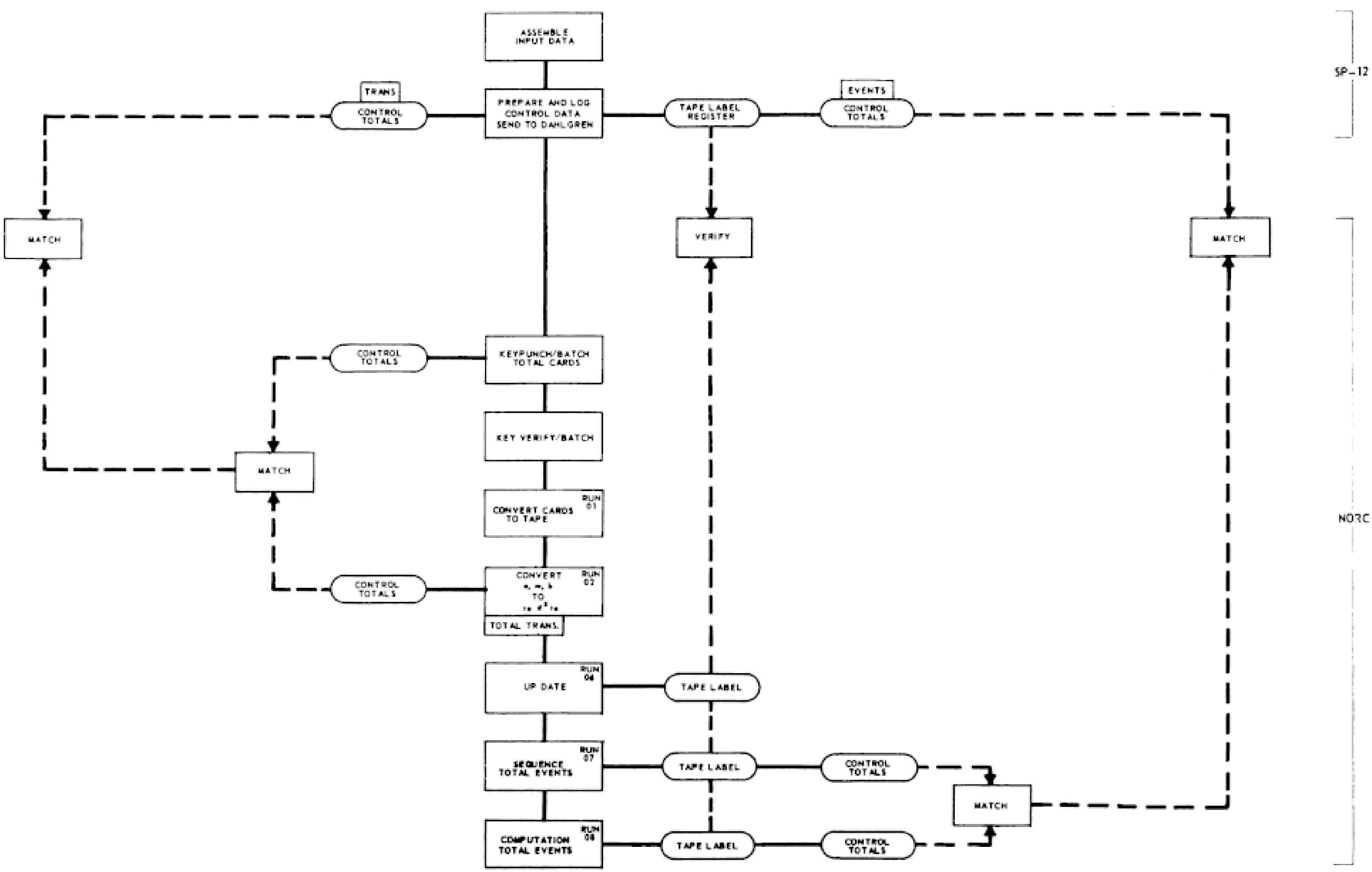


EXHIBIT XVI

SP-12 NORC DATA DISPATCH AND CONTROL SHEET

DATE	BATCH*	TOTAL TRANS		CHANGES IN EVENTS				TOTAL EVENTS		TOTAL ACTIVITIES		REMARKS
		SP-12	NORC	DELETE	COMPLETE (NORC)	NEW	NET	SP-12	NORC	PRED.	SUC.	

EXHIBIT XVII

SP-12 NORC TAPE REGISTER

DATE	TAPE LABEL			REMARKS
	06	07	08	

EXHIBIT XVIII

based on very early figures from checkout operations and are not completely accurate. Over the first three months of operations, the operating time should be monitored closely and more accurate computations prepared.

On the basis of the continuing needs of the PERT program, additional programing and follow-up effort at Dahlgren will be required. Two man-months will be needed during November and December. During the remainder of the project, support should continue at the rate of one man-week per month. These personnel costs are in addition to the machine costs.

A small quantity of the preprinted PERT report form was supplied to Dahlgren. Continuing effort should require on the average of 2,000 sets of this form each month beginning January 1.

2. SP (COMPUTER) PERSONNEL

(1) *Personnel Responsibilities.* The staff needed at SP to efficiently perform the computer liaison function should consist of two people, once a full load of events is being processed in the system. There is a need for a computer liaison officer and a general clerical helper. The computer liaison officer should be a professional Sp 12 staff man who is fully versed in the PERT methodology. He must have a detailed knowledge of the individual computer programs. The computer liaison officer should have sufficient detailed knowledge to prepare specifications for new programs or revisions to existing programs. In addition, he should be able to verify that such new or revised programs meet the required or modified specifications. These responsibilities fit the flow and control of data to and from the computer group at NORC as previously described.

A complete PERT system will require a control and dispatch clerk to perform the routine functions of maintaining log books and directing the flow of data.

The dispatch control clerk will be responsible for gathering all input sheets together, scanning them for accuracy, and originating the batch counts. It will be his duty to prepare the transmittal envelope and to log the data as transmitted to Dahlgren. The control data and the output sheets will be returned to the control dispatch clerk, and it will be his duty to log in the output and prove the controls before releasing the reports to the designated people.

(2) *Training of Computer Personnel.* Five members of the Sp 12 professional staff were given the "IBM Aptitude Test for EDPM Programers" (55-234TB). The scores are listed below.

68)	
63)	A
62)	
56)	B
49)	C

The staff member previously selected for the position of computer liaison scored the highest grade. However, there is no reason to expect that the first four people could not be successful in computer work. The person selected began with the PERT team August 10 and was briefed extensively on the PERT system and also on the computer work as it was under way at Dahlgren. Several tutoring sessions were conducted on the computer system and programs penetrating well into the logic of each computer run. The computer liaison trainee attended a NORC computer programing school at Dahlgren and participated as a regular computer programer trainee

COST NPG - JULY 1 - OCTOBER 31

NORC Time	Labor and O.H.	Total
$21,740	$14,355	$36,095

Approximate Distribution/COST CENTER

NORC TIME		
Production (Tables)	$ 8,000	$
Debugging (Tables and PERT)	13,740	
		21,740
LABOR AND O.H.		
Tables	$ 4,015	
PERT	10,340	
		14,355
		$36,095

Approximate Distribution/PROBLEM

TABLES		
NORC Production	$ 8,000	
Labor and O.H.	4,015	
NORC Checkout (1/4)	3,435	
		$15,450
PERT		
NORC Checkout (3/4)	$10,305	
Labor and O.H.	10,340	
		20,645
		$36,095

EXHIBIT XIX

PRELIMINARY ESTIMATES OF RUNNING TIME

Run	Disc	Factor	Standard	Time (Min.)	Setup
01	CTC	Cards	100 Cards (Trans)	1.0	2.0
02	A, M, B Conversion	Trans	100 Trans	1.3	1.0 (1)
03	Pred Sort	Trans	100 Trans	.5	1.0 (1)
04	Suc Sort	Trans	100 Trans	.5	1.0 (1)
05	Merge	Trans	100 Trans	.5	1.0 (1)
06	Update File	File	1,000 Events	2.0	1.0 (2)
07	Sequency	File	1,000 Events	9.0	1.0 (2)
08	Computing	File	1,000 Events	4.0	1.0 (2)
09	Sorting Output	File	1,000 Events	2.0	1.0 (3)
10	Edit Output	File	1,000 Events	1.0	1.0 (3)
-	Tape to Card	File	1,000 Cards	10.0	2.0
-	407 Printing	File	1,000 Cards	10.0	2.0

(1) Setup time for four runs could be reduced to a total of 10 minutes if programs combined.

(2) Setup time for three runs could be reduced to a total of 1.0 minutes if programs combined.

(3) Setup time for two runs could be reduced to a total of 1.0 minutes if programs combined.

EXHIBIT XX

at Dahlgren. As a part of his general training, he has participated in several of the programing progress conferences held at Dahlgren. It is desirable that Sp 12 select and train a second member of its staff in the computer operations so as to allow sufficient backup in the operations over the next two to three years.

VI. PRODUCTION OF TABLES ON NORC

The analysis computation that has been programed on NORC accepts the longest time path to an event as the governing restraint on its accomplishment. Use of this procedure introduces a bias into the computation. Theoretical development has produced a technique for an unbiased approach. The NORC has been used in generating tables that will uncover the extent of the bias in the present computations. This chapter will discuss the role of NORC in the production of the tables. (The tables themselves will appear in a separate report.)

The derivation of the formulae for the expected value, variance, and correlation coefficient of the greatest of a finite set of random variables are developed and presented in Appendix C.

The mathematical tables were developed for two separate situations.

1. TABLE A, THE GREATER OF TWO RANDOM VARIABLES. As shown in the derivation, the functions used to relate the two variables under consideration are normalized to fix the mean of one of the variables at zero and its standard deviation at unity. The expected value and variance of the greater of two random variables then have the parameters:

- μ — The mean of unnormalized random variable.
- σ — The standard deviation of the unnormalized random variable.
- ρ — The correlation coefficient between the random variables under consideration.

The range of the parameters for the tables was selected to be as follows:

μ — −5.0 to +5.0 (intervals of 0.1)
σ — 0 to 1.0 (intervals of 0.2)
ρ — −1.0 to +1.0 (intervals of 0.5)

The parameters were selected so as to examine the function in sufficient detail to identify its critical regions. The potential usage of the tables would not be a tabular form, but in the form of regression equations fitted to the tables. The detail of computations is sufficient for general usage of the tables and for the development of regression equations.

Table A not only presents the expected value and standard deviation but also the measures of skewness and kurtosis which are calculated functions of the third and fourth moments about the mean. These extra moments were calculated in order to examine the normality of the functions involved. (This is important to later usage.) It can be reported that, except for a few unusual cases, the functions being considered closely approximate normality as indicated by the coefficients of kurtosis and skewness.

2. TABLE B*, THE TABLE FOR THE CORRELATION COEFFICIENT BETWEEN A THIRD RANDOM VARIABLE AND THE GREATER OF TWO RANDOM VARIABLES. Table B* was developed in a manner similar to Table A, and the derivation of the formulae is covered in Appendix C. In this case, the functions are normalized so as to make means and variances of two of the three variables zero and unity respectively. Thus a nine parameter problem is effectively reduced to a five parameter problem with:

- μ — The mean of remaining unnormalized variable.
- σ — The standard deviation of the remaining unnormalized variable.
- ρ — The correlation coefficient between the first pair of variables.
- R — The correlation coefficient between the second pair of variables.
- S — The correlation coefficient between the third pair of variables.

The range of parameters used for Table B* is the same as those used for Table A. In both cases there is a series of formulae for a group of special cases. The tabular computation for these special cases consists of an evaluation of a known experimental function. The more general case for both Table A and Table B* requires numerical integration of nontrivial functions.

3. ACCURACY OF TABLES. The general accuracy consideration for the tables was set at four decimal places. This decision involved a balance of both the cost of computation and the potential use of the computed tables. The accuracy dictates the precision of numerical integration. The production runs of the numerical integration used Simpson's method. Before the production runs were made, however, separate test integrations were made using the Runge-Kulta method, a longer and slower method which generally yields more accuracy. For the test cases, both methods match to 0.00005.

The integration was carried out, starting at an interval for x, the variable of the integration, of 0.5. The next step was to divide the interval in half and to carry out the integration again. This process was continued until the results of the integration changed less than .00005. At that point the last integration was accepted and used in later calculations for the final determination of the tabular value.

The functions used changed markedly for various values of the parameters. It was, therefore, necessary to compute the range of integration before the computing. Several formulae were tested and finally a simple set of rules were devised. The range of integration was selected to be the greatest of either

$$\mu + 5.\sigma \text{ or } + 7.0$$

and the least of either

$$\mu - 5.\sigma \text{ or } - 7.0$$

In addition to checking the tables with machine and hand calculations at NORC, additional hand calculations were performed at SP verifying 18 points in Table A. For Table B*, 9 points were verified by hand calculation. All points were verified to 4 decimal places.

VII. PROGRAM MODIFICATIONS

Any design program which is to meet a scheduled date for production must have its specifications frozen and made inviolable at some point in its development. The specifications for the PERT system programs were frozen early in August. Since that time only minor coding changes have been made.

However, a number of important changes have been analyzed for their impact on the improvement of the overall operating efficiency of the program and the analysis. These changes are discussed in succeeding sections of this chapter. They are listed in the order of their relative importance.

1. INDIVIDUAL COMPONENT ANALYSES. At the time that the design specifications were frozen, only integrated analysis runs were contemplated. It has since become advisable to alter the original plan so as to provide for both component and system analyses. The programs presented in the earlier chapters can evolve component analysis. However, in so doing they require tight controls and introduce a strong possibility of human error.

In order to minimize the possibility of errors, it is advisable to alter the original program so as to accommodate the component analysis without using "ad hoc" procedures. The necessary changes have been studied and subsequently detailed as follows:

(1) *General Specifications*

—Analyze each component system within a subsystem independently and compute all outputs based on component system only. For example, analyze ballistic shell, propulsion, etc. each by itself. The primary concern at this point is to show "zero" slack paths for each component system.
—Analyze each subsystem independently and compute all outputs based on the subsystem only. For example, analyze the missile subsystem and the navigation subsystem and others showing the relationship only within each subsystem. Here, the primary concern is to show the "zero" slack path for each subsystem.
—Analyze the overall FBM system and compute all outputs based on the entire FBM system, as proposed in earlier reports.

The proposed method of analysis requires that each component system be maintained on a separate tape reel. It will be necessary to separate all transactions into batches by component system. This will call for small batches in the establishing and updating and maintaining each component file. Component files will range from 30 to 500 events, with transaction batches probably varying from size 2 to 50. If these estimates are correct, the PERT system at Dahlgren will consist of small tape files with weekly or biweekly updating runs on each tape of only 2 to 50 transactions. Techniques can be developed to organize these small files so that updating will be accomplished without 150 to 300 separate machine setups. Control systems, to assure accurate operations, must be maintained for each of the separate component system files.

The actual increase in computer time will be a factor of 1.7 times that involved in the integrated computer run. The following program changes should be made if component runs are regularly made.

(2) *Data Flow.* Exhibit XXI, following this page, shows the modified computer system flow chart expanded to show only a representative section of the entire component subsystems. The operations for *each* component system are processed as indicated in Exhibit II.

The sequenced component systems tapes (resulting from Run 07) are merged and combined on Run 11. Each combined subsystem is then reprocessed beginning with computation run (Run 08). At this point it may be desirable to suppress certain outputs. This can be done by controlling the parameters of the sort and edit operations (Run 09 and 10).

The input tapes to the FBM system merge-combine run must be sequenced subsystem tapes. After the subsystem tapes are merged and combined onto one FBM system tape, the outputs are computed on Run 08. The actual number and type of outputs can be chosen, depending upon the selection of the parameters of Runs 09 and 10.

(3) *Accuracy Control.* Separate control systems, i.e., separate control books, must be maintained for each component system. Both control registers must be maintained for each component system. The tape label register must be modified to carry the component identification number (the first three digits of the event code) in the tape label. In the same sense, transaction batches must be set up separately by component system, and in no case can batches of transactions be combined in the updating operation.

The analysis and correction of errors in the system will be much faster and easier, as the reconstruction of any one piece will require the analysis of a smaller group of transactions and events. Nevertheless, the maintenance of the controls for a larger group of files will require more effort.

MODIFIED SYSTEM FLOW CHART

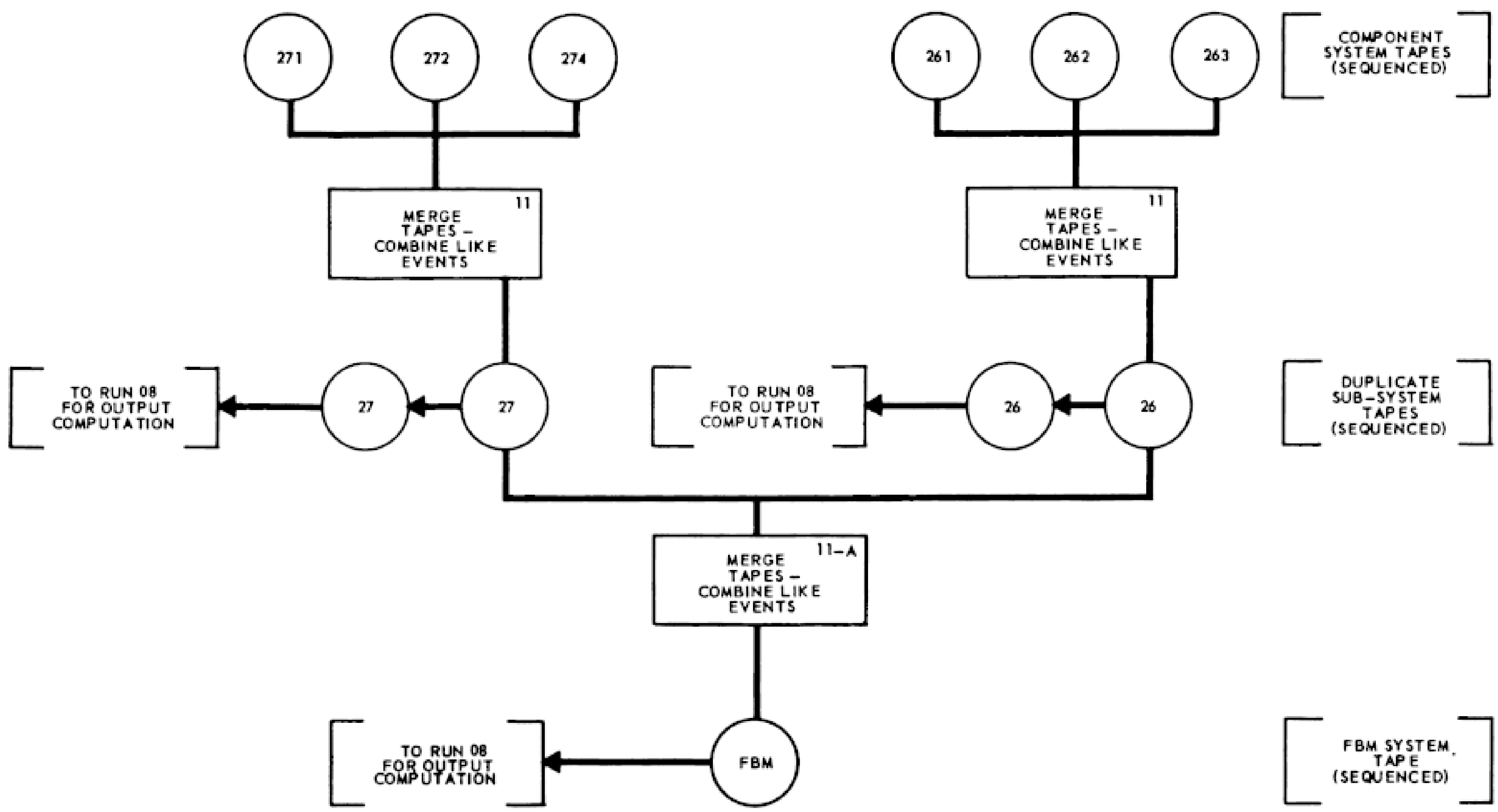


EXHIBIT XXI

(4) *Modification of Present Programs.* All programs must be modified to verify and print a tape label with a component system code as follows:

Problem Number	Run Number	Date	Sequence Number	Component System Code Number
XX	XX	XX-XX-XX	XX	XXXX

As shown, the component system code replaces the week identification.

This change will necessitate a slight change in the computation program. The week number (the identification of week "now") must be made a part of the setup operation. The program must be modified to pick up this item from the new location.

(5) *New Runs To Be Programed*

—*Run 11—Component System Merge-Combine.* This program will merge unlike events from a group of component system tapes and combine identical events (key events) from all tapes. As events are combined, all the combined predecessors and successors will be added in their correct positions within the event message on the combined file tape. The resulting output tape will be in sequence and thus ready for the output computing run (Run 08). The run itself will not produce the duplicate tapes needed to continue both the output computing and the subsystem merge-combine run. Thus, it will be necessary to duplicate the output tape from Run 11 in order to have input tapes for Run 11-A, (see Exhibit XXI).

(6) *Input Restrictions.* Event codes for key events, common events among components or subsystems, must have at least one digit in the event code which is unique to these key events such as 0200000 - xxx.

Input tapes must be sequenced, i.e., output tapes from either Run 07 or 08.

The program can accept no more than six component systems in any one pass, so if more than six component systems are to be merged the following procedures must be used. First, select six component systems and merge-combine them. Take the output of this run, mount it with remaining component systems and merge-combine them. Then duplicate the final output. This restriction is a function of the eight tape units of the NORC computer, one of which must be reserved for the program tape and a second for the output tape. *Note:* Under certain operating and setup conditions, it is possible to use the program tape unit for either output or input.

The difference between Run 11 and Run 11-A (see Exhibit XXI), is a minor one. The programs are identical except that the setup operation for the component system merge-combine run includes control data and instructions. These check the input tapes to verify that they are for the same subsystem. This is not necessary for the subsystem merge-combine runs, Run 11-A.

(7) *Mode of Operation (New Program) (Run 11).* Six component systems are mounted on Tape units 1 through 6. The output tape is mounted on Tape 7 and the program on Tape 8. Tape 1 is transferred to the output tape (Tape 7) until a key event is sensed. Tape 2 is then transferred to the output tape until a key event is sensed. The process is continued until a key event is sensed on each of the six input tapes. The key event on Tape 1 is compared with the key event on Tape 2. If they are equal, the predecessors and successors of the key event on Tape 2 are added to the key event on Tape 1. The process is repeated until all the predecessors and successors have been added to the key event of Tape 1.

At this point, the key event of Tape 1 is read out to the output tape. If the key events on any of the Tapes 2 through 6 are not equal to the key event on Tape 1, they are flagged. The program now continues the process of reading the unflagged input tapes to the output tapes until the next key event on each tape is sensed. At this point, the program again tests the key event on Tape 1 with those on Tapes 2 through 6. Before the test is made, all flags on key events are removed. They are only reestablished if indicated by succeeding comparison operations. Identical events are combined by adding their successors and predecessors to the tape. The program continues through the merge-combine cycles until all input tapes are appropriately transferred

2. INPUT CONVERSION (RUN 02). The earlier section that describes the input form and procedure requires that the date of completion and the scheduled date (T_S) be coded as a number of elapsed weeks. It has been suggested that the accuracy of the input would be improved if this coding were done internally. To make this change, Run 02 must be modified to perform an arithmetic conversion on the normal calendar date to develop the elapsed time in weeks and tenths of weeks. The procedure would be as follows: Pick up the date to be converted from the input message. Extract the month portion from the calendar date and subtract 7 (July) from it. Multiply the resulting number by 4.33 to develop the first factor in the conversion and add it to a counter (Counter A). Extract the day portion from the calendar date and subtract 12 (for July 12) from it. Multiply the resulting number by 1/7 and add it to the Counter A. Finally extract the year portion of the calendar date under consideration and subtract 58 (for July 12, 1958) from it. Multiply the resulting number by 52 and add it to Counter A. The week code read out of Counter A will be equal to the elapsed time from 12/7/58 to the date under consideration. This can be added as a subroutine to the input conversion run (Run 02).

3. SHORTEST PATH COMPUTATION. It has been recognized that in some uses it may be desirable to consider the shortest rather than the longest path. However, there had been little or no field experience to indicate the extent to which this is necessary. The necessity arises when several concurrent development activities are going on and the plan is to accept the one which meets specifications in the shortest elapsed time. The problem is

confounded by the time restraints of a decision date. I.E., a decision will be made at a certain point in time regardless of whether any development is finished.

This situation can be included in the present system by making the following changes. First, the event record must be changed to allow the use of a special indicator or flag. This flag will be attached at the confluence of the several parallel developments wherein a lesser of the restraints type of decision is to be made. Once flagged, the calculation proceeds as usual, except that instead of using the logic which selects the greatest of several paths, the program branches and will select the least of the several paths from preceding events. After selecting the least of the several restraints leading to the event, the program should now extract the adjusted T_S value. This is compared with the selected least path and the greater of the two, T_S or T_E, is chosen. At this point, the program will continue in the normal manner until it reaches another event flagged with the special calculation code.

4. A PROCEDURE FOR RESCHEDULING. The procedure for rescheduling events described in the Phase I PERT report is quite capable of being mechanized and can be done as follows. First, the probability calculation routine in the computation run (Run 08) should be added to the edit run (Run 10). Then the following rescheduling subroutine should be added to the edit routine. Both of these routines should be added on an option switch. With these routines, any computed, sorted, or sequenced tape can be used in the rescheduling procedure. This will allow experimentation at any time with any set of data and will immediately print the revised experimental schedule without disturbing the existing file data. If it is desired to add the revised scheduled dates to the file, the output cards can be reproduced into input format on the IBM 519 card reproducer. The revisions in schedule can then be processed as regular transactions using the Transaction Code 5, static change.

The program would first compare T_L, T_E, and T_S. If T_S is less than T_L, a new T_S equal to T_L is inserted on the output message in Word 4. The probability calculation would also be inserted in Word 4. If T_S is greater than T_L, and T_L is not equal to T_E, a new T_S is generated equal to T_L plus $1/2\,(T_L - T_E)$. At this point the probability calculations are carried out and the result stored in memory. Then an iteration loop would calculate several values greater than T_S and less than T_S. After completing the iteration loop, the T_S yielding the largest probability value would be written out on the output message in Word 4, along with the new probability calculation. The program would then proceed to the next event.

5. EDIT FOR MISCELLANEOUS OUTPUT. Several of the runs produce monitoring or control outputs on the on-line printer. At the present time this output is being prepared using the standard NORC five or six words per line unedited form. Output in this form is readily understood by only those familiar with the NORC system. If interest grows at SP in these intermediate outputs (such as control totals and completed events) they should be edited in their respective routines.

6. MORE EFFICIENT SORTING ROUTINES. The present sorting runs, Runs 03, 04, and 09, use strings-of-two sorting methods. There are several improvements in this method generally classified under the term "progressive" sorting. Such sorting takes advantage of the order which already might exist in the data. Strings-of-two sorting ignores this advantage. If the future the sorting load builds up to a point where it absorbs more than 15% of the operating time, improvements in the sorting runs would be in order. Exploration of "progressive" sorting and strings-of-four sorting techniques would be suggested.

7. GENERAL NOTES. Program revision is a necessary part of the continuing data-processing operation on any computer. There must be a continuing review of methods and techniques. Over a period of time it is possible to reduce the overall operating time by as much as 50% by review and adjustment. The costs of such a program can be recovered by increased operating efficiency.

APPENDIX C

SUPPORTING MATHEMATICS

THE GREATEST OF A FINITE SET OF RANDOM VARIABLES

I. INTRODUCTION

Given random variables $x_1, \ldots, x_n$ together with their joint probability distribution, we would like to construct the distribution of max $(x_1, \ldots, x_n)$, the greatest of the n variables. However, we restrict the analysis to an approximate solution of the following special case.

We assume that a subset of $x_1, \ldots, x_n$ have a normal joint probability distribution. The complement of this subset consists of variables each of which is constant with probability one.

Let x and y be any pair of $x_1, \ldots, x_n$. We shall compute the first four moments of max (x, y). If t is a third variable, we shall compute the coefficient of correlation between t and max (x, y).

These results permit the following approximate solution of the original problem. The expected value and variance of max (x_1, x_2) are computed. Assuming that the distribution of max (x_1, x_2) can be adequately approximated by the normal distribution, we approximate the coefficient of correlation between x_3 and max (x_1, x_2). Still using a normal approximation, we approximate the expected value and variance of max $[x_3, \max(x_1, x_2)]$ which is max (x_1, x_2, x_3). After a number of steps we reach max $(x_1, x_2, \ldots, x_n)$.

The third and fourth moments of max (x, y) are not used in the above computation. These moments give some insight into the accuracy of the normal approximation.

II. NORMALIZATION

We set aside the trivial case in which the variances of both x_1 and x_2 are zero. We apply a linear transformation $ax_1 + b$ simultaneously to x_1 and x_2 to obtain the transformed variables x and y with $E(x) = 0$, $V(x) = 1$, $E(y) = \mu$, $V(y) = \sigma^2$, $0 \leq \sigma^2 \leq 1$, and $\rho(x, y) = r$, where E, V, and ρ denote expected value, variance and linear correlation coefficient, respectively.

As we consider $\rho\,[x_3, \max(x, y)]$, we note that ρ is undefined if $V(x_3) = 0$. This special case is set aside. Then a linear transformation reduces x_3 to t such that $E(t) = 0$, $V(t) = 1$, $\rho(t, x) = R$, and $\rho(t, y) = S$. Since our interest at this point is in $\rho\,[x_3, \max(x_1, x_2)]$, the linear transformation applied to x_3 is different from the transformation applied simultaneously to x_1 and x_2.

III. TABULATION OF RESULTS

Tables have been constructed from the following formulas. They are in the possession of SP. A separate report will be issued in which the use of these results will be demonstrated. Although the following formulas are expressed in moment notation, their subsequent utilization should be in the form of expected values, standard deviations, coefficients of skewness and coefficients of excess (kurtosis).

<u>LIST OF RESULTS:</u>

Let ν_i, $i = 1, \ldots, 4$ be the moments about zero of max (x, y).

<u>CASE I.</u> If $0 < \sigma \leq 1$, $-1 < r < 1$, then

$$\nu_i = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^i \left\{ \exp\left(-\frac{x^2}{2}\right) F\left(\frac{\frac{x-\mu}{\sigma} - rx}{\sqrt{1-r^2}}\right) + \frac{1}{\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] F\left(\frac{x - r\,\frac{x-\mu}{\sigma}}{\sqrt{1-r^2}}\right) \right\} dx,$$

where

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{t^2}{2}\right) dt.$$

CASE II. If $0 < \sigma \leq 1$, $r = \pm 1$, but not simultaneously $\sigma = r = 1$, then

$$\nu_1 = \mu \quad F\left(\frac{\mu}{1 \mp \sigma}\right) + \frac{1 \mp \sigma}{\sqrt{2\pi}} \quad \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right],$$

$$\nu_2 = 1 + (\mu^2 + \sigma^2 - 1) \quad F\left(\frac{\mu}{1 \mp \sigma}\right) + \frac{\mu\,(1 \mp \sigma)}{\sqrt{2\pi}} \quad \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right],$$

$$\nu_3 = (\mu^3 + 3\mu\sigma^2) \quad F\left(\frac{\mu}{1 \mp \sigma}\right) + \frac{\mu^2\,(1 \mp \sigma) + 2\,(1 \mp \sigma)^3}{\sqrt{2\pi}} \quad \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right],$$

$$\nu_4 = 3 + (\mu^4 + 6\mu^2\sigma^2 + 3\sigma^4 - 3) \quad F\left(\frac{\mu}{1 \mp \sigma}\right) + \frac{1}{\sqrt{2\pi}}\,[\mu^3\,(1 \mp \sigma) + \mu\,(3 \pm 3\sigma + 3\sigma^2 \mp 5\sigma^2)]$$

$$\exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right].$$

CASE III. If $\sigma = r = 1$, $\mu \leq 0$, then $\nu_1 = 0$, $\nu_2 = 1$, $\nu_3 = 0$, $\nu_4 = 3$.

CASE IV. If $\sigma = r = 1$, $\mu > 0$, then

$$\nu_1 = \mu, \ \nu_2 = 1 + \mu^2, \ \nu_3 = 3\mu + \mu^3, \ \nu_4 = 3 + 6\mu^2 + \mu^4.$$

CASE V. If $\sigma = 0$, then

$$\nu_1 = \mu \quad F(\mu) + \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\mu^2}{2}\right),$$

$$\nu_2 = 1 + (\mu^2 - 1) \quad F(\mu) + \frac{1}{\sqrt{2\pi}}\,\mu \quad \exp\left(-\frac{\mu^2}{2}\right),$$

$$\nu_3 = \mu^3 \quad F(\mu) + \frac{1}{\sqrt{2\pi}}\,(\mu^2 + 2) \quad \exp\left(-\frac{\mu^2}{2}\right),$$

$$\nu_4 = 3 + (\mu^4 - 3) \quad F(\mu) + \frac{1}{\sqrt{2\pi}}\,(\mu^3 + 3\mu) \quad \exp\left(-\frac{\mu^2}{2}\right).$$

CASE VI. If $0 < \sigma \leq 1$, $r \neq \pm 1$, then with the notation $R = \rho\,(t, x)$ and $S = \rho\,(t, y)$ we have

$$E\,[t \max (x, y)] = \frac{R}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 \exp\left(-\frac{x^2}{2}\right) F\left(\frac{\frac{x-\mu}{\sigma} - rx}{\sqrt{1-r^2}}\right) dx$$

$$+ \frac{S}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{x\,(x-\mu)}{\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] F\left(\frac{x - r\,\frac{x-\mu}{\sigma}}{\sqrt{1-r^2}}\right) dx$$

$$+ \frac{\mu\,(1 - r\sigma)\,(rS - R + r\sigma R - \sigma S)}{\sqrt{2\pi}\,(1 - 2r\sigma + \sigma^2)^{3/2}} \exp\left[-\frac{\mu^2}{2\,(1 - 2r\sigma + \sigma^2)}\right].$$

CASE VII. If $0 \leq \sigma \leq 1$, $r = \pm 1$, but not simultaneously $\sigma = r = 1$, then

$$E\,[t \max (x, y)] = R\left[1 - (1 \mp \sigma)\; F\left(\frac{\mu}{1 \mp \sigma}\right)\right].$$

CASE VIII. If $\sigma = r = 1$, then $E\,[t \max (x, y)] = R$.

CASE IX. If $\sigma = 0$, then $E\,[t \max (x, y)] = R\,[1 - F\,(\mu)]$.

INTEGRALS REQUIRED IN THE DERIVATIONS.

Well-known properties of the normal distribution give:

$$\int_{-\infty}^{\infty} \exp\left[-\frac{(x-a)^2}{2b^2}\right] dx = b\sqrt{2\pi}\;,$$

$$\int_{-\infty}^{\infty} x \exp\left[-\frac{(x-a)^2}{2b^2}\right] dx = \int_{-\infty}^{\infty} [(x-a) + a] \exp\left[-\frac{(x-a)^2}{2b^2}\right] dx = 0 + ab\sqrt{2\pi} = ab\sqrt{2\pi}\;,$$

$$\int_{-\infty}^{c} \exp\left[-\frac{(x-a)^2}{2b^2}\right] dx = b\sqrt{2\pi}\; F\left(\frac{c-a}{b}\right).$$

$$\int_{-\infty}^{c} x \exp\left[-\frac{(x-a)^2}{2b^2}\right] dx = \int_{-\infty}^{c} [(x-a) + a] \exp\left[-\frac{(x-a)^2}{2b^2}\right] dx = -b^2 \exp\left[-\frac{(c-a)^2}{2b^2}\right]$$

$$+ ab\sqrt{2\pi}\; F\left(\frac{c-a}{b}\right).$$

If we use the notation

$$I_i = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} x^i \exp\left(-\frac{x^2}{2}\right) dx,$$

$$J_i = \frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} x^i \exp\left(-\frac{x^2}{2}\right) dx,$$

we compute using integration by parts

$$I_1 = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{a^2}{2}\right),$$

$$I_2 = \frac{1}{\sqrt{2\pi}}\, a \exp\left(-\frac{a^2}{2}\right) + F(a),$$

$$I_3 = -\frac{1}{\sqrt{2\pi}} (a^2 + 2) \exp\left(-\frac{a^2}{2}\right),$$

$$I_4 = -\frac{1}{\sqrt{2\pi}} (a^2 + 3a) \exp\left(-\frac{a^2}{2}\right) + 3\, F(a),$$

$$J_1 = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{a^2}{2}\right),$$

$$J_2 = \frac{1}{\sqrt{2\pi}}\, a \exp\left(-\frac{a^2}{2}\right) + 1 - F(a),$$

$$J_3 = \frac{1}{\sqrt{2\pi}} (a^2 + 2) \exp\left(-\frac{a^2}{2}\right),$$

$$J_4 = \frac{1}{\sqrt{2\pi}} (a^3 + 3a) \exp\left(-\frac{a^2}{2}\right) + 3 - 3\, F(a).$$

DERIVATIONS.

CASE I. The probability density of x and y is:

$$f(x, y) = \frac{1}{2\pi\sigma\sqrt{1 - r^2}} \exp\left\{-\frac{1}{2(1 - r^2)}\left[x^2 - 2rx\frac{y - \mu}{\sigma} + \left(\frac{y - \mu}{\sigma}\right)^2\right]\right\}$$

which can be reduced to the following two forms

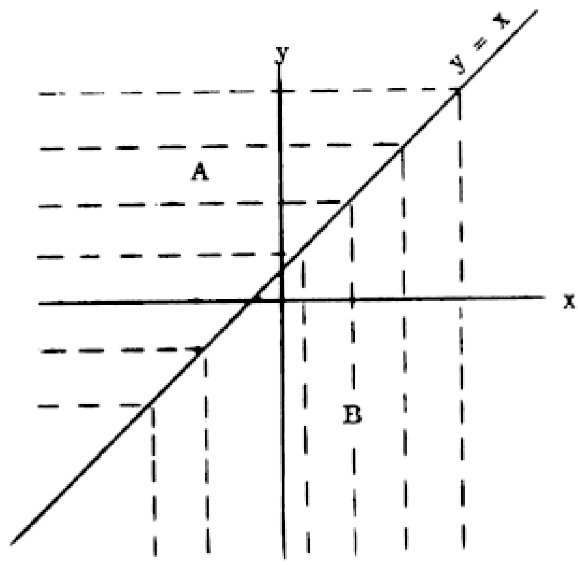
$$f\ (x,\ y) = \frac{1}{2\pi\sigma\sqrt{1-r^2}} \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right] \exp\left[-\frac{1}{2\ (1-r^2)}\left(x - r\,\frac{y-\mu}{\sigma}\right)^2\right],$$

$$f\ (x,\ y) = \frac{1}{2\pi\sigma\sqrt{1-r^2}} \exp\left(-\frac{x^2}{2}\right) \exp\left[-\frac{1}{2\ (1-r^2)}\left(\frac{y-\mu}{\sigma} - rx\right)^2\right].$$

We have

$$\nu_i = \iint_{-\infty}^{\infty} [\max\ (x,\ y)]^i\ f\ (x,\ y)\ dx\ dy.$$

We shall use notation whose meaning is indicated in the figure.



$$\nu_i = \iint_A + \iint_B$$

We note that max $(x,\ y) = y$ in region A and max $(x,\ y) = x$ in region B.

$$\iint_A = \frac{1}{2\pi\sigma\sqrt{1-r^2}} \int_{-\infty}^{\infty} y^i \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right] dy \int_{-\infty}^{y} \exp\left[-\frac{1}{2\ (1-r^2)}\left(x - r\,\frac{y-\mu}{\sigma}\right)^2\right] dx$$

$$= \frac{1}{2\pi\sigma\sqrt{1-r^2}} \int_{-\infty}^{\infty} y^i \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right] \sqrt{2\pi}\ \sqrt{1-r^2}\ F\left(\frac{y - r\,\dfrac{y-\mu}{\sigma}}{\sqrt{1-r^2}}\right) dy$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} y^i \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right]\ F\left(\frac{y - r\,\dfrac{y-\mu}{\sigma}}{\sqrt{1-r^2}}\right) dy.$$

$$\iint_B = \frac{1}{2\pi\sigma\sqrt{1-r^2}} \int_{-\infty}^{\infty} x^i \exp\left(-\frac{x^2}{2}\right) dx \int_{-\infty}^{x} \exp\left[-\frac{1}{2\ (1-r^2)}\left(\frac{y-\mu}{\sigma} - rx\right)^2\right] dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^i \exp\left(-\frac{x^2}{2}\right) F\left(\frac{\dfrac{x-\mu}{\sigma} - rx}{\sqrt{1-r^2}}\right) dx.$$

The result for Case I given above is obtained directly.

CASE II. Since $r = \pm 1$, $\dfrac{y-\mu}{\sigma} = \pm x$, $y = \mu \pm \sigma x$.

$$x > y => x > \mu \pm \sigma x,\ \ x(1 \mp \sigma) > \mu\ \ x > \frac{\mu}{1 \mp \sigma}.$$

The last relation has meaning because we do not have $\sigma = r = 1$.

$$\operatorname{Max}(x, y) = \begin{cases} y = \mu \pm \sigma x, \text{ if } x < \dfrac{\mu}{1 \mp \sigma}, \\ x, \text{ if } x \geq \dfrac{\mu}{1 \mp \sigma}. \end{cases}$$

Hence

$$\nu_i = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\mu}{1 \mp \sigma}} (\mu \pm \sigma x)^i \exp\left(-\frac{x^2}{2}\right) dx + \frac{1}{\sqrt{2\pi}} \int_{\frac{\mu}{1 \mp \sigma}}^{\infty} x^i \exp\left(-\frac{x^2}{2}\right) dx.$$

Using integrals listed above, we write

$$\nu_1 = \mu\ F\left(\frac{\mu}{1 \mp \sigma}\right) \pm \sigma \left\{\frac{-1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]\right\} + \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]$$

$$= \mu\ F\left(\frac{\mu}{1 \mp \sigma}\right) + \frac{1 \mp \sigma}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]$$

$$\nu_2 = \mu^2\ F\left(\frac{\mu}{1 \mp \sigma}\right) \pm 2\mu\sigma \left\{\frac{-1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]\right\}$$

$$+ \sigma^2 \left\{\frac{-1}{\sqrt{2\pi}} \frac{\mu}{1 \mp \sigma} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right] + F\left(\frac{\mu}{1 \mp \sigma}\right)\right\}$$

$$+ \frac{1}{\sqrt{2\pi}} \frac{\mu}{1 \mp \sigma} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right] + 1 - F\left(\frac{\mu}{1 \mp \sigma}\right)$$

$$= 1 + (\mu^2 + \sigma^2 - 1)\ F\left(\frac{\mu}{1 \mp \sigma}\right) + \frac{\mu(1 \mp \sigma)}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]$$

$$\nu_3 = \mu^3 \; F\left(\frac{\mu}{1 \mp \sigma}\right) \pm 3\,\mu^2\,\sigma \left\{\frac{-1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]\right\}$$

$$+ 3\,\mu\,\sigma^2 \left\{\frac{-1}{\sqrt{2\pi}} \;\frac{\mu}{1 \mp \sigma} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right] + F\left(\frac{\mu}{1 \mp \sigma}\right)\right\}$$

$$\pm \sigma^3 \left\{\frac{-1}{\sqrt{2\pi}}\left[\frac{\mu^2}{(1 \mp \sigma)^2} + 2\right] \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]\right\}$$

$$+ \frac{1}{\sqrt{2\pi}}\left[\frac{\mu^2}{(1 \mp \sigma)^2} + 2\right] \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]$$

$$= (\mu^3 + 3\,\mu\,\sigma^2)\; F\left(\frac{\mu}{1 \mp \sigma}\right) + \frac{\mu^2\,(1 \mp \sigma) + 2\,(1 \mp \sigma^3)}{\sqrt{2\pi}} \exp\left[\frac{-1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]$$

$$\nu_4 = \mu^4 \; F\left(\frac{\mu}{1 \mp \sigma}\right) \pm 4\,\mu^3\,\sigma \left\{\frac{-1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]\right\}$$

$$+ 6\,\mu^2\sigma^2 \left\{\frac{-1}{\sqrt{2\pi}} \;\frac{\mu}{1 \mp \sigma} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right] + F\left(\frac{\mu}{1 \mp \sigma}\right)\right\}$$

$$\pm 4\,\mu\,\sigma^3 \left\{\frac{-1}{\sqrt{2\pi}}\left[\frac{\mu^2}{(1 \mp \sigma)^2} + 2\right] \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]\right\}$$

$$+ \sigma^4 \left\{\frac{-1}{\sqrt{2\pi}}\left[\frac{\mu^3}{(1 \mp \sigma)^3} + \frac{3\,\mu}{1 \mp \sigma}\right] \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right] + 3\; F\left(\frac{\mu}{1 \mp \sigma}\right)\right\}$$

$$+ \frac{1}{\sqrt{2\pi}}\left[\frac{\mu^3}{(1 \mp \sigma)^3} + \frac{3\,\mu}{1 \mp \sigma}\right] \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right] + 3 - 3\; F\left(\frac{\mu}{1 \mp \sigma}\right)$$

$$= 3 + (\mu^4 + 6\,\mu^2\sigma^2 + 3\,\sigma^4 - 3)\; F\left(\frac{\mu}{1 \mp \sigma}\right)$$

$$+ \frac{\mu^3\,(1 \mp \sigma) + \mu\,(3 \pm 3\sigma + 3\sigma^2 \mp 5\sigma^3)}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right]$$

CASE III. If $\sigma = r = 1$ and $\mu \leq 0$, then clearly $\max(x, y) = x$. Since x is a standard normal variable, $\nu_1 = 0$, $\nu_2 = 1$, $\nu_3 = 0$, $\nu_4 = 3$. These moments as well as the moments given in Case IV are well known parameters of the normal distribution.

CASE IV. If $\sigma = r = 1$, and $\mu > 0$, then clearly max (x, y) = y and $\nu_1 = \mu$, $\nu_2 = \mu^2 + 1$, $\nu_3 = \mu^3 + 3\mu$, $\nu_4 = \mu^4 + 6\mu^2 + 3$.

CASE V. If $\sigma = 0$, then $\max(x, y) = \begin{cases} \mu, \text{ if } x \leq \mu \\ x, \text{ if } x > \mu. \end{cases}$

$$\nu_i = \mu^i \ F(\mu) + \frac{1}{\sqrt{2\pi}} \int_{\mu}^{\infty} x^i \exp\left(-\frac{x^2}{2}\right) dx.$$

$$\nu_1 = \mu \ F(\mu) + \frac{1}{\sqrt{2\pi}} \text{esp} \left(-\frac{\mu^2}{2}\right).$$

$$\nu_2 = 1 + (\mu^2 - 1) \ F(\mu) + \frac{1}{\sqrt{2\pi}} \mu \exp\left(-\frac{\mu^2}{2}\right).$$

$$\nu_3 = \mu^3 \ F(\mu) + \frac{1}{\sqrt{2\pi}} (\mu^2 + 2) \exp\left(-\frac{\mu^2}{2}\right).$$

$$\nu_4 = 3 + (\mu^4 - 3) \ F(\mu) + \frac{1}{\sqrt{2\pi}} (\mu^3 + 3\mu) \exp\left(-\frac{\mu^2}{2}\right).$$

CASE VI. The covariance matrix of t, x, and y is

$$\Delta = \begin{pmatrix} 1 & R & S \\ R & 1 & r \\ S & r & 1 \end{pmatrix}.$$

The determinate $|\Delta|$ of Δ is $1 + 2\,rRS - r^2 - R^2 - S^2$. We know that r, R, and S must satisfy $|\Delta| \geq 0$. We assume that the joint distribution of t, x, and y is normal. As indicated in the section on Normalization, we assume that $E(t) = E(x) = 0$, $V(t) = V(x) = 1$, $E(y) = \mu$, and $V(y) = \sigma^2$. Hence, if $|\Delta| > 0$, the joint probability density of t, x, and y is

$$f(t, x, y) = \frac{1}{(2\pi)^{3/2} \sigma \sqrt{|\Delta|}} \exp\left(-\frac{\phi}{2}\right)$$

where

$$\phi = \frac{1}{|\Delta|} \left[(1 - r^2)\, t^2 + (1 - S^2)\, x^2 + (1 - R^2) \left(\frac{y - \mu^2}{\sigma}\right) + 2\,(rS - R)\, tx \right.$$
$$\left. + 2\,(rR - S)\, t \frac{y - \mu}{\sigma} + 2\,(RS - r)\, x \frac{y - \mu}{\sigma} \right].$$

The limiting case with $|\Delta| = 0$ is discussed below, and we now assume that $|\Delta| > 0$.

Since we shall integrate first with respect to t, we reduce ϕ to

$$\phi = \frac{1 - r^2}{|\Delta|}\left(t + \frac{rS - R}{1 - r^2}\,x + \frac{rR - S}{1 - r^2}\,\frac{y - \mu}{\sigma}\right)^2 + \frac{1}{1 - r^2}\left[x^2 - 2\,r\,x\,\frac{y - \mu}{\sigma} + \left(\frac{y - \mu}{\sigma}\right)^2\right].$$

Using this expression for ϕ we write

$$E\,[t \max(x, y)] = \frac{1}{(2\pi)^{3/2}\,\sigma\sqrt{|\Delta|}}\iiint_{-\infty}^{\infty} t \max(x, y) \exp\left(-\frac{\phi}{2}\right) dt\,dx\,dy$$

$$= \frac{1}{(2\pi)^{3/2}\,\sigma\sqrt{|\Delta|}}\iint_{-\infty}^{\infty} \max(x, y)\,\exp\left\{-\frac{1}{2\,(1 - r^2)}\left[x^2 - 2rx\,\frac{y - \mu}{\sigma}\right.\right.$$

$$\left.\left. + \left(\frac{(y - \mu)}{\sigma}\right)^2\right]\right\} dx\,dy \int_{-\infty}^{\infty} t\,\exp\left[-\frac{1 - r^2}{2\,|\Delta|}\left(t + \frac{rS - R}{1 - r^2}\,x + \frac{rR - S}{1 - r^2}\,\frac{y - \mu}{\sigma}\right)^2\right] dt.$$

As indicated in the section above concerning Integrals, this reduces to

$$\frac{1}{(2\pi)^{3/2}\,\sigma\sqrt{|\Delta|}}\iint_{-\infty}^{\infty} \max(x, y)\,\exp\left\{-\frac{1}{2\,(1 - r^2)}\left[x^2 - 2r\,x\,\frac{y - \mu}{\sigma} + \left(\frac{y - \mu}{\sigma}\right)^2\right]\right\} \times$$

$$\left[-\frac{rS - R}{1 - r^2}\,x - \frac{rR - S}{1 - r^2}\,\frac{y - \mu}{\sigma}\right]\frac{\sqrt{|\Delta|}}{\sqrt{1 - r^2}}\sqrt{2\pi}\;dx\,dy.$$

We shall simplify and proceed as in Case I.

$$E\,[t \max(x, y)] = \iint_{-\infty}^{\infty} \frac{\max(x, y)}{2\pi\sigma\sqrt{1 - r^2}}\left[\frac{R - rS}{1 - r^2}\,x + \frac{S - rR}{1 - r^2}\,\frac{y - \mu}{\sigma}\right] \times$$

$$\exp\left\{-\frac{1}{2\,(1 - r^2)}\left[x^2 - 2rx\,\frac{y - \mu}{\sigma} + \left(\frac{y - \mu}{\sigma}\right)^2\right]\right\} dx\,dy = \iint_A + \iint_B$$

where the meaning of A and B is indicated in the figure above.

Repeating some manipulations used in Case I, we write,

$$\iint\limits_{A} = \frac{1}{2\pi\sigma\sqrt{1-r^2}} \int_{-\infty}^{\infty} y \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right] dy \int_{-\infty}^{\infty} \left(\frac{R-rS}{1-r^2}\,x + \frac{S-rR}{1-r^2}\,\frac{y-\mu}{\sigma}\right) x$$

$$\exp\left[-\frac{1}{2(1-r^2)}\left(x - r\,\frac{y-\mu}{\sigma}\right)^2\right] dx$$

$$= \frac{1}{2\pi\sigma\sqrt{1-r^2}} \int_{-\infty}^{\infty} y \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right] dy \int_{-\infty}^{y} \left[\frac{R-rS}{1-r^2}\left(x - r\,\frac{y-\mu}{\sigma}\right)\right.$$

$$\left. + \frac{R-rS}{1-r^2}\,r\,\frac{y-\mu}{\sigma} + \frac{S-rR}{1-r^2}\,\frac{y-\mu}{\sigma}\right] \exp\left[-\frac{1}{2(1-r^2)}\left(x - r\,\frac{y-\mu}{\sigma}\right)^2\right] dx$$

$$= \frac{1}{2\pi\sigma\sqrt{1-r^2}} \int_{-\infty}^{\infty} y \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right] \left\{\frac{R-rS}{1-r^2}\,[-(1-r^2)] \exp\left[-\frac{\left(y - r\,\frac{y-\mu}{\sigma}\right)^2}{2(1-r^2)}\right]\right.$$

$$\left. + \frac{rR - r^2S + S - rR}{1-r^2}\,\frac{y-\mu}{\sigma}\,\sqrt{1-r^2}\,\sqrt{2\pi}\,F\left(\frac{y - r\,\frac{y-\mu}{\sigma}}{1-r^2}\right)\right\} dy$$

$$= \frac{rS-R}{2\pi\sigma\sqrt{1-r^2}} \int_{-\infty}^{\infty} y \exp\left\{-\frac{1}{2(1-r^2)}\left[y^2 - 2ry\,\frac{y-\mu}{\sigma} + \left(\frac{y-\mu}{\sigma}\right)^2\right]\right\} dy$$

$$+ \frac{S}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{y\,(y-\mu)}{\sigma} \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right] F\left(\frac{y - r\,\frac{y-\mu}{\sigma}}{\sqrt{1-r^2}}\right) dy.$$

$$\iint\limits_{B} = \frac{1}{2\pi\sigma\sqrt{1-r^2}} \int_{-\infty}^{\infty} x \exp\left(-\frac{x^2}{2}\right) dx \int_{-\infty}^{x} \left[\frac{S-rR}{1-r^2}\left(\frac{y-\mu}{\sigma} - r\,x\right)\right.$$

$$\left. + \frac{S-rR}{1-r^2}\,r\,x + \frac{R-rS}{1-r^2}\,x\right] \exp\left[-\frac{1}{2(1-r^2)}\left(\frac{y-\mu}{\sigma} - rx\right)^2\right] dy$$

$$= \frac{1}{2\pi\sigma\sqrt{1-r^2}} \int_{-\infty}^{\infty} x \exp\left(-\frac{x^2}{2}\right) \left\{ \frac{S - rR}{1 - r^2} [-(1-r^2)\sigma] \exp\left[-\frac{\left(\frac{x-\mu}{\sigma} - rx\right)^2}{2(1-r^2)}\right] \right.$$

$$\left. + \frac{rS - r^2 R + R - rS}{1 - r^2} x \sigma \sqrt{1-r^2} \sqrt{2\pi} F\left(\frac{\frac{x-\mu}{\sigma} - rx}{\sqrt{1-r^2}}\right) \right\} dx$$

$$= \frac{rR - S}{2\pi\sqrt{1-r^2}} \int_{-\infty}^{\infty} x \exp\left\{-\frac{1}{2(1-r^2)}\left[x^2 - 2r x \frac{x-\mu}{\sigma} + \left(\frac{x-\mu}{\sigma}\right)^2\right]\right\} dx$$

$$+ \frac{R}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 \exp\left(-\frac{x^2}{2}\right) F\left(\frac{\frac{x-\mu}{\sigma} - r x}{\sqrt{1-r^2}}\right) dx.$$

In the expressions for $\iint\limits_A$ and $\iint\limits_B$ we have

$$\int_{-\infty}^{\infty} x \exp\left\{-\frac{1}{2(1-r^2)}\left[x^2 - 2r x \frac{x-\mu}{\sigma} + \left(\frac{x-\mu}{\sigma}\right)^2\right]\right\}$$

which will be evaluated next. The integral reduces to

$$\int_{-\infty}^{\infty} x \exp\left\{-\frac{1 - 2r\sigma + \sigma^2}{2\sigma^2(1-r^2)}\left[x + \frac{\mu(r\sigma - 1)}{1 - 2r\sigma + \sigma^2}\right]^2\right\} \exp\left[-\frac{\mu^2}{2(1 - 2r\sigma + \sigma^2)}\right] dx$$

$$= \exp\left[-\frac{\mu^2}{2(1 - 2r\sigma + \sigma^2)}\right] \frac{\mu(1 - r\sigma)}{1 - 2r\sigma + \sigma^2} \frac{\sigma\sqrt{1-r^2}}{\sqrt{1 - 2r\sigma + \sigma^2}} \sqrt{2\pi}$$

$$= \frac{\sqrt{2\pi}\,\mu\sigma(1 - r\sigma)\sqrt{1-r^2}}{(1 - 2r\sigma + \sigma^2)^{3/2}} \exp\left[-\frac{\mu^2}{2(1 - 2r\sigma + \sigma^2)}\right].$$

We substitute this last expression in $\iint\limits_A + \int\limits_B$ and get

$$E[t \max(x, y)] = \left(\frac{rS - R}{2\pi\sigma\sqrt{1-r^2}} + \frac{rR - S}{2\pi\sqrt{1-r^2}}\right) X$$

$$\frac{\sqrt{2\pi}\ \mu\sigma(1-r\sigma)\sqrt{1-r^2}}{(1-2r\sigma+\sigma^2)^{3/2}}\exp\left[-\frac{\mu^2}{2(1-2r\sigma+\sigma^2)}\right]$$

$$+\frac{S}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{y(y-\mu)}{\sigma}\exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right]F\left(\frac{y-r\dfrac{y-\mu}{\sigma}}{\sqrt{1-r^2}}\right)dy$$

$$+\frac{R}{\sqrt{2\pi}}\int_{-\infty}^{\infty}x^2\exp\left(-\frac{x^2}{2}\right)F\left(\frac{\dfrac{x-\mu}{\sigma}-rx}{\sqrt{1-r^2}}\right)dx$$

which reduces immediately to the result stated above for Class VI.

The formula for E [t max (x, y)] is defined and continuous along $|\Delta| = 0$ for fixed r. Hence, limiting values of E [t max (x, y)] are correctly computed by the above formula. The continuity of E [t max (x, y)] along $|\Delta| = 0$ has not been completely investigated by the author. However, the following special cases are considered.

First suppose that $R = \pm 1$. Then $1 + 2rRS - r^2 - R^2 - S^2 = -(r \mp S)^2$. Since $|\Delta| \geq 0$, we must have $S = \pm r$ and $|\Delta| = 0$. In this case $t = \pm x$ and (since $r \neq \pm 1$ in Case VI) the probability density of x and y is

$$\frac{1}{2\pi\sigma\sqrt{1-r^2}}\exp\left\{-\frac{1}{2(1-r^2)}\left[x^2-2rx\frac{y-\mu}{\sigma}+\left(\frac{y-\mu}{\sigma}\right)^2\right]\right\}.$$

$$E\,[t\max(x,y)] = \frac{1}{2\pi\sigma\sqrt{1-r^2}}\iint_{-\infty}^{\infty}\pm x\max(x,y)\exp\left\{-\frac{1}{2(1-r^2)}\left[x^2-2rx\frac{y-\mu}{\sigma}+\left(\frac{y-\mu}{\sigma}\right)^2\right]\right\}dx\,dy.$$

In the general discussion above in which $|\Delta| > 0$, one finds the relation

$$E\,[t\max(x,y)] = \iint_{-\infty}^{\infty}\frac{\max(x,y)}{2\pi\sigma\sqrt{1-r^2}}\left[\frac{R-rS}{1-r^2}x+\frac{S-rR}{1-r^2}\frac{y-\mu}{\sigma}\right]\times$$

$$\exp\left\{-\frac{1}{2(1-r^2)}\left[x^2-2rx\frac{y-\mu}{\sigma}+\left(\frac{y-\mu}{\sigma}\right)^2\right]\right\}dx\,dy.$$

If in this expression we replace R by ± 1 and S by $\pm r$ (as established above), we obtain the immediately preceding expression for E [t max (x, y)]. Hence, the result when $R = \pm 1$ is obtained from the general result with $R = \pm 1$, $S = \pm r$.

In the same way we can show that the general formula can be used if $S = \pm 1$. In this case $R = \pm r$.

CASE VII. First assume that $R = + \pm 1$. Since $r = \pm 1$, $\frac{y - \mu}{\sigma} = \pm x$, $y = \mu \pm \sigma x$, and $x > y$ if and only if $x > \mu \pm \sigma x$ or $x > \frac{\mu}{1 \mp \sigma}$. Hence,

$$\max(x, y) = \begin{cases} \mu \pm \sigma x, \text{ if } x < \dfrac{\mu}{1 \mp \sigma} \\ x, \text{ if } x \geq \dfrac{\mu}{1 \mp \sigma}. \end{cases}$$

Since the joint probability density of t and x is

$$\frac{1}{2\pi\sqrt{1 - R^2}} \exp\left[-\frac{1}{2(1 - R^2)}(t^2 - 2Rtx + x^2)\right]$$

$$= \frac{1}{2\pi\sqrt{1 - R^2}} \exp\left[-\frac{1}{2(1 - R^2)}(t - Rx)^2\right] \exp\left(-\frac{x^2}{2}\right)$$

$$E[t \max(x, y)] = \frac{1}{2\pi\sqrt{1 - R^2}} \int_{-\infty}^{\infty} \max(x, y) \exp\left(-\frac{x^2}{2}\right) dx \int_{-\infty}^{\infty} t \exp\left[-\frac{(t - Rx)^2}{2(1 - R^2)}\right] dt$$

$$= \frac{1}{2\pi\sqrt{1 - R^2}} \int_{-\infty}^{\infty} \max(x, y) \exp\left(-\frac{x^2}{2}\right) Rx \sqrt{1 - R^2} \sqrt{2\pi} \, dx$$

$$= \frac{R}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\mu}{1 \mp \sigma}} (\mu \pm \sigma x)\, x \exp\left(-\frac{x^2}{2}\right) dx + \frac{R}{\sqrt{2\pi}} \int_{\frac{\mu}{1 \mp \sigma}}^{\infty} x^2 \exp\left(-\frac{x^2}{2}\right) dx.$$

sing the table of integrals derived above, we reduce this last expression to

$$= R\mu \frac{-1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right] \pm R\sigma \left\{\frac{-1}{\sqrt{2\pi}} \frac{\mu}{1 \mp \sigma} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right] + F\left(\frac{\mu}{1 \mp \sigma}\right)\right\}$$

$$+ R\left\{\frac{1}{\sqrt{2\pi}} \frac{\mu}{1 \mp \sigma} \exp\left[-\frac{1}{2}\left(\frac{\mu}{1 \mp \sigma}\right)^2\right] + 1 - F\left(\frac{\mu}{1 \mp \sigma}\right)\right\}$$

$$= R\left[1 - (1 \mp \sigma) F\left(\frac{\mu}{1 \mp \sigma}\right)\right].$$

If $R = \pm 1$, then $t = \pm x = Rx$, and

$$E[t \max(x, y)] = \frac{1}{2\pi} \int_{-\infty}^{\frac{\mu}{1 \mp \sigma}} Rx(\mu \pm \sigma x) \exp\left(-\frac{x^2}{2}\right) dx + \frac{1}{\sqrt{2\pi}} \int_{\frac{\mu}{1 \mp \sigma}}^{\infty} Rx^2 \exp\left(-\frac{x^2}{2}\right) dx.$$

Since this identical expression arose above when $R \neq \pm 1$, the results derived above still hold when $R = \pm 1$.

(The case $S = \pm 1$ is included in the case $R = \pm 1$ because $S = \pm 1$ and $r = \pm 1$ imply $R = \pm 1$; indeed $R = rS$.)

The analysis of Case VII is complete.

CASE VIII. If $\sigma = r = 1$, we consider four cases, namely

(a) $\mu \leq 0$, $R \neq \pm 1$ (c) $\mu > 0$, $R \neq \pm 1$

(b) $\mu \leq 0$, $R = \pm 1$ (d) $\mu > 0$, $R = \pm 1$.

In case (a) we have $\max(x, y) = x$.

$$E[t \max(x, y)] = \frac{1}{2\pi\sqrt{1 - R^2}} \iint_{-\infty}^{\infty} t\, x \exp\left[-\frac{1}{2(1 - R^2)}(t^2 - 2Rtx + x^2)\right] dt\, dx$$

$$= \frac{1}{2\pi\sqrt{1 - R^2}} \int_{-\infty}^{\infty} x \exp\left(-\frac{x^2}{2}\right) dx \int_{-\infty}^{\infty} t \exp\left[-\frac{1}{2(1 - R^2)}(t - Rx)^2\right] dt$$

$$= \frac{1}{2\pi\sqrt{1 - R^2}} \int_{-\infty}^{\infty} x \exp\left(-\frac{x^2}{2}\right) Rx \sqrt{1 - R^2} \sqrt{2\pi}\, dx$$

$$= \frac{R}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 \exp\left(-\frac{x^2}{2}\right) dx$$

$$= R$$

because the coefficient of R is the variance of a normal distribution.

Case (b). We have $t = Rx$, $\max(x, y) = x$, and

$$E[t \max(x, y)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Rx^2 \exp\left(-\frac{x^2}{2}\right) dx = R.$$

Case (c). We have max (x, y) = y, and

$$E[t \max(x, y)] = \frac{1}{2\pi\sigma\sqrt{1 - S^2}} \iint_{-\infty}^{\infty} t\, y \exp\left\{ -\frac{1}{2(1 - S^2)} \left[t^2 - 2\, St \frac{y - \mu}{\sigma} + \left(\frac{y - \mu}{\sigma} \right)^2 \right] \right\} dt\, dy$$

$$= \frac{1}{2\pi\sigma\sqrt{1 - S^2}} \int_{-\infty}^{\infty} y \exp\left[-\frac{1}{2} \left(\frac{y - \mu}{\sigma} \right)^2 \right] dy \int_{-\infty}^{\infty} t \exp\left[-\frac{1}{2(1 - S^2)} \left(t - 2S \frac{y - \mu}{\sigma} \right)^2 \right] dt$$

$$= \frac{1}{2\pi\sigma\sqrt{1 - S^2}} \int_{-\infty}^{\infty} y \exp\left[-\frac{1}{2} \left(\frac{y - \mu}{\sigma} \right)^2 \right] 2S \frac{y - \mu}{\sigma} \sqrt{1 - S^2} \sqrt{2\pi}\, dy$$

$$= \frac{S}{\sigma^2 \sqrt{2\pi}} \int_{-\infty}^{\infty} y (y - \mu) \exp\left[-\frac{1}{2} \left(\frac{y - \mu}{\sigma} \right)^2 \right] dy$$

$$= \frac{S}{\sigma^2 \sqrt{2\pi}} \int_{-\infty}^{\infty} (y - \mu)^2 \exp\left[-\frac{1}{2} \left(\frac{y - \mu}{\sigma} \right)^2 \right] dy + \frac{S}{\sigma^2 \sqrt{2\pi}} \int_{-\infty}^{\infty} \mu (y - \mu) \exp\left[-\frac{1}{2} \left(\frac{y - \mu}{\sigma} \right)^2 \right] dy$$

$$= \frac{S}{\sigma^2} (\sigma^2) + 0$$

$$= R$$

because $S = R$ when $r = 1$.

Case (d). We have $t = \pm \frac{y - \mu}{\sigma} = R \frac{y - \mu}{\sigma}$, max (x, y) = y, and

$$E[t \max(x, y)] = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} R \frac{y - \mu}{\sigma}\, y \exp\left[-\frac{1}{2} \left(\frac{y - \mu}{\sigma} \right)^2 \right] dy = R$$

as above.

The analysis of Case VIII is complete.

CASE IX. When $\sigma = 0$, we have

$$\max(x, y) = \begin{cases} \mu, \text{ if } x < \mu, \\ x, \text{ if } x \geq \mu. \end{cases}$$

If $R \neq +1$, $E[t, \max(x, y)] =$

$$\frac{1}{2\pi\sqrt{1-R^2}} \iint_{-\infty}^{\infty} t \max(x, y) \exp\left[-\frac{1}{2(1-R^2)}(t^2 - 2Rtx + x^2)\right] dt\, dx$$

$$= \frac{1}{2\pi\sqrt{1-R^2}} \int_{-\infty}^{\infty} \max(x, y) \exp\left(-\frac{x^2}{2}\right) dx \int_{-\infty}^{\infty} t \exp\left[-\frac{1}{2(1-R^2)}(t - Rx)^2\right] dt$$

$$= \frac{1}{2\pi\sqrt{1-R^2}} \int_{-\infty}^{\infty} \max(x, y) \exp\left(-\frac{x^2}{2}\right) Rx\sqrt{1-R^2}\sqrt{2\pi}\, dx$$

$$= \frac{R}{\sqrt{2\pi}} \int_{-\infty}^{\mu} \mu x \exp\left(-\frac{x^2}{2}\right) dx + \frac{R}{\sqrt{2\pi}} \int_{\mu}^{\infty} x^2 \exp\left(-\frac{x^2}{2}\right) dx$$

$$= R\mu\left[\frac{-1}{\sqrt{2\pi}} \exp\left(-\frac{\mu^2}{2}\right)\right] + R\left[\frac{1}{\sqrt{2\pi}} \mu \exp\left(-\frac{\mu^2}{2}\right) + 1 - F(\mu)\right]$$

$$= R[1 - F(\mu)]$$

If $R = \pm 1$, then $t = \pm x = Rx$, and

$$E[t \max(x, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Rx \max(x, y) \exp\left(-\frac{x^2}{2}\right) dx$$

$$= \frac{R}{\sqrt{2\pi}} \int_{-\infty}^{\mu} \mu x \exp\left(-\frac{x^2}{2}\right) dx + \frac{R}{\sqrt{2\pi}} \int_{\mu}^{\infty} x^2 \exp\left(-\frac{x^2}{2}\right) dx$$

$$= R$$

as above.

The analysis of Case IX is complete.