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with corrections & annotations
by C.S.

PROGRAMMERS' HANDBOOK
(3rd Edition)

FOR THE

MANCHESTER ELECTRONIC COMPUTER

MARK II.

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PREFACE to the First Edition

This Handbook is written mainly for the benefit of those who will actually do programming for the Mark II machine. It is not intended to be an introduction to computing machinery, nor is it intended to explain the engineering techniques used in the machine. Although it is believed that the handbook is complete in itself, and eventually could be understood by anyone with a mathematical training, it is recommended that the reading be supplemented by some closer contact with the machine itself.

At the time of writing comparatively little programming has been done with the Mark II machine. It is hoped to issue supplements consisting of particular routines from time to time. The conventions and suggestions about programming that are given are largely based on experience gained from the Mark I machine.

Errata p 1.27
1.22

PREFACE to the Second Edition

This Handbook is written mainly for those who expect to prepare problems for the MK II machine. Although almost complete in itself it is intended to be supplemented by some closer contact with the machine and by instruction from persons already familiar with its use.

Much material has been taken over unaltered, or only slightly modified from the 1st Edition which was written by Dr. A. M. Turing. In addition some of the results of the first year's experience of programming for the MK II have been incorporated in the later chapters.

Miss C. M. Popplewell and N. E. Hoskin of the staff of this Laboratory, and A. E. Glennie of the Armament Research Establishment, Fort Halstead, Sevenoaks, Kent, were responsible for chapters 3, 7, and 6 respectively. Some of the examples of chapter 2 were originally designed as exercises for the Summer School in Programme Design held at the University Mathematical Laboratory, Cambridge, (England) in September 1951.

At all stages in the preparation of this edition Dr A. M. Turing offered valuable criticism and advice.

R. A. BROOKER.

August 1952.

PREFACE to the Third Edition

This edition has been prepared mainly because the author was dissatisfied with certain chapters of the previous edition. Having rewritten these chapters it was then decided to take the opportunity of revising the entire Handbook. Fewer topics are covered but they have been more carefully chosen and treated in greater detail. The new edition is not intended to replace the 2nd edition. instead it provides an alternative treatment of some rather fundamental topics, in particular, routine changing and the input of teleprint material.

Many of the suggestions contained in the following pages have originated from members of the staff of the Laboratory and others with experience of the MK II, in particular R. K. Livesley, A. M. Turing, A. E. Glennie, and C. Strachey.

Many of the tables in the appendix are the work of the FERUT group at Toronto University.

R. A. BROOKER.

Computing Machine Laboratory,

September 1953.

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Chapter 1

The Logical Design of the Machine and The Instruction Code.

General remarks on electronic computers

Electronic computers are intended to carry out any definite rule of thumb process which could have been done by a human operator working in a disciplined but unintelligent manner. The electronic computer should however obtain its results very much more quickly. For purposes of comparison the human computer may be imagined as supplied with various computing aids. He should have a desk machine, paper to write his results on, and more paper on which is written a detailed account of how the calculation is to be carried out. These aids have their analogues in the electronic computer. The desk machine is transformed into the arithmetical unit, and the paper becomes the information store, or more briefly the store, whether it is paper used for results or paper carrying instructions. There is also a part of the machine called the control which corresponds to the computer himself. If his possible behaviour were very accurately represented this would have to be a formidably complicated circuit. However it is only required that he should be able to obey the written instructions and these can be made so explicit that the control can be quite simple. There remain two more components of the electronic computer. These are the input and output mechanisms, by which information is to be transferred from the outside world into the store and conversely. If the analogy of the human computer is to be maintained these parts would correspond to his ears and voice, by means of which he communicates with his employer.

Scales of notation

The information stored on paper by the human computer will mostly consist of sequences of digits drawn from 0, 1, ..., 9. There may also be other symbols such as decimal points, spaces etc., and there may be occasional remarks in English, Greek letters etc. There may in fact be anything from 10 to 100 different symbols used, and there is no particular need to decide in advance how many different symbols will be concerned. With an electronic computer however such a decision has to be made and the number of symbols chosen is ruled very largely by engineering considerations.

In the vast majority of machines the number is two. The reason for this is that the "scale of 2" matches naturally with the "on-off" property of electronic switching circuits. Machines (e.g., ENIAC) have however been made with 10 different symbols. The number for the M.U. MK II machine is two, and the symbols used are 0 and 1.

It is not difficult to see that information expressed with decimal digits can be translated into information expressed with 0's and 1's by some suitable conventions, e.g., we could replace 0 by 0000, 1 by 1000, 2 by 0100, 3 by 1100, 4 by 0010, 5 by 1010, 6 by 0110, 7 by 1110, 8 by 0001, and 9 by 1001.

The scheme that is most economical in 0's and 1's however, and the one adopted for the MK II, is to use the pure binary representation of a number in which the digits, 0 or 1, are the coefficients of successive powers of 2. The most natural convention to choose is that by which the value of a 1 in the rth position from the right hand end is 2^{r-1}, so that e.g., 11001 means 25. This is the closest possible analogue of the ordinary Arabic notation for integers. The convention chosen for use with the Mark II machine is different. The value of a 1 in the rth position from the <u>left</u> hand end is 2^{r-1}, so that 25 is represented by 10011 instead of 11001. These facts may be described by saying that the machine uses the scale of two with the most significant digits at the right hand end.

Although the scale of two is appropriate for use within an electronic computer it is not so suitable for work on paper, and it is not possible to avoid paper work altogether. There are occasions when it is desirable to write down on the paper the sequence of symbols stored in some part of the machine. Suppose for instance that the sequence was

10001110111010001001100011100101010101101100100110

The copying of such sequences is slow and very liable to inaccuracy. It is very difficult to 'keep one's place'. It is therefore advisable to represent such a sequence on paper in a different form, not subject to these difficulties. The method chosen is to divide the sequence into blocks of five.

10001 11011 10100 01001 10001 11001 01010 10110 11001 00110

and then to replace each block by a single symbol, according to the table below. The above sequence then becomes Z"SLZWRFWN.

0	00000	/	11	11010	J	22	01101	P	
1	10000	E	12	00110	N	23	11101	Q	
2	01000	@	13	10110	F	24	00011	O	
3	11000	A	14	01110	C	25	10011	B	
4	00100	:	15	11110	K	26	01011	G	
5	10100	S	16	00001	T	27	11011	"	(fig. 1.1)
6	01100	I	17	10001	Z	28	00111	M	
7	11100	U	18	01001	L	29	10111	X	
8	00010	½	19	11001	W	30	01111	V	
9	10010	D	20	00101	H	31	11111	£	
10	01010	R	21	10101	Y				

These symbols are essentially the teleprinter code, except that the combinations 00000, 01000, 00100, 00010, 11011, 11111 which in true teleprint are represented by : - no effect, line feed, space, carriage return, figure shift, lettershift, respectively, have here been given the representations /, @ :, ½, ", £. These symbols have been chosen so as to enable the upper case of the typewriter to be used throughout.

The user is strongly recommended to learn the above table. In principle it is possible to do without these aids for the machine itself can do all the conversion processes required. In practice it frequently happens that some single number is required in the scale of 32, and it is found less trouble to do the conversion by hand than to use the machine. To convert a decimal fraction to the scale of 32 multiply repeatedly by 32, subtracting and recording the integral part at each stage. This can be done very quickly with a Brunsviga with transfer. The integral part obtained may then be looked up in the above table.

<u>The forms of storage used</u>

The information store in the machine consists of the <u>auxiliary</u> store and the <u>working</u> store. The capacity of the auxiliary store is

large, namely, 655,360 binary digits: in other words it corresponds to paper on which can be written 655,360 digits each of which may be either 0 or 1. But this information is not particularly readily available. It is (to maintain the analogy) as if it were written in a book. In order to find any required piece of information it is necessary to open the book at the appropriate page. The working store has a considerably smaller capacity, namely, 10240 digits, but this information is much more readily available and is to be compared rather to a number of sheets of paper exposed to the light on a table, so that any particular word or symbol becomes visible as soon as the eye focusses on it.

The magnetic store

The auxiliary store takes the form of a rotating drum whose cylindrical surface is coated with nickel. Each digit is represented by the magnetic state of one small surface area element which can be magnetised either N-S or S-N. These 655,360 areas are arranged in 256 tracks of 2560 digits each. The areas of one track lie on a circle in a plane perpendicular to the axis of the wheel. The 256 tracks are equidistant but the 2560 digit areas on one circle are not equidistant. The information in one track is further subdivided into two equal half tracks, which may be described as the left page and the right page if we continue to follow the simile of the book.

The electronic store

The information in the working store is represented by patterns of charges stored on the inner surface of the front of a cathode ray tube. There are 8 tubes each storing 1280 digits each. A tube thus contains the same amount of information as a half track of the magnetic store. A tube of information is subdivided into 64 lines of 20 digits each, the digits of one line forming a straight horizontal segment. A line of 20 digits is the basic unit of information processed by the machine. The 64 lines of a tube are to be regarded as arranged in two equal columns thus

0	32
1	33
2	34
3	35
.	.
.	.
.	.
.	.
31	63

This is in fact how they are displayed on the monitors of the machine.

The lines may be numbered consecutively through the 8 tubes. They could be numbered 0, 1, ..., 511. Instead the labelling obtained by transforming to the scale of 32, i.e., to the teleprinter code, is used. Thus the lines are known as //, E/, @/, ..., £K. In their geometrical arrangement on the tubes they are as below

Tube 0		Tube 1		Tube 7	
//	/E	/@	/A	/C	/K
E/	EE	E@	EA	EC	EK
@/	@E	@@	@A	@C	@K
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
£/	£E	£@	£A	£C	£K

Each tube is formed of two columns and the second character describing a line gives the column in which it is to be found.

Associated with each line is a line pair or long line consisting of the line together with the next line. e.g., The lines R/ and J/ form a pair and so do BK and GK. These long lines will be referred to by R/ and BK, i.e., the name of the first line of the pair. There is one exception to this rule: the line pair associated with the last line of each page consists of that line together with the first line of the same page (not the next page). Thus the line pair £E consists of £E and //.

Although the information on the magnetic wheel is arranged geometrically so differently from that in the electronic store it may be found convenient to imagine it as if it were similarly arranged. There is very little to interfere with the illusion. The only convenient method for making the content of a track visible is to copy the track on to a pair of

tubes, a process which effectively conceals the true arrangement of the digits on the track. This way of thinking permits the division of the magnetic information also into lines or line pairs. There is a good deal of ambiguity about the naming of these lines, for a page of the magnetic store can be copied on to any one of the 8 electronic tubes. No attempt is made to remove this ambiguity and it may be accepted that there are 8 alternative names for a line stored on the wheel.

Two further units of the machine must now be described, the control unit and the arithmetical unit. Roughly speaking the control unit interprets lines of the electronic store (i.e., the information held on these lines) as instructions and under their direction causes the arithmetical unit to interpret line pairs as numbers and perform certain arithmetical operations on them. These units will be described in detail.

The control unit

This unit interprets the information held in a single line of the electronic store as an instruction in accordance with a set of rules known as the instruction code of the machine. Each instruction is of the single address type, that is it refers, in general, to a line or line pair of the electronic store and causes some operation to be performed in connection with the contents of that line. This operation may involve the arithmetical unit, the input or output units, the unit controlling the transfer of information between the electronic and magnetic store, or even the control unit itself.

The control unit normally selects instructions from consecutive lines of the electronic store. The control tube records at any stage two 20 digit quantities, C and I. I is the instruction last obeyed and the first two characters of C give the address of the store line from which it was selected. The last two characters of C are irrelevant.

Certain kinds of instructions can interrupt the sequential process of selecting instructions and determine themselves which instruction is to be selected next. Such instructions are said to cause a transfer of control. Furthermore this transfer of control can be made conditional on the sign (and hence the size) of numbers in the arithmetical unit. Thus one of several courses of action can be taken according as to how the calculation is proceeding.

Representation of instructions

Each instruction takes the form s, B, f, the 20 binary digits being made up as follows.

(i) digits 0-9 give the name, s, of a line or line pair in the electronic store.

(ii) digits 10, 11, 12 are the B digits (see below) - digit 13 is spare.

(iii) digits 14-19 (called the function digits) specify the operation, f.

The meaning and use of the B digits will be described in detail later. At this stage it will be sufficient to state that they name one of eight different lines which is normally added to each instruction before it (i.e., the resulting instruction) is obeyed. This could become a nuisance but is avoided by adopting the convention that normally one of these lines (called B0 and specified by 000) is to be zero. For the present it may be assumed that digits 10, 11, 12 and 13 are all zero.

The 20 digits of a complete instruction are normally written as 4 teleprint characters. Unfortunately these do not correspond entirely with the 3 separate parts s, B, and f. The first two characters give s, but the 3rd character contains both the B digits, and one of the function digits. The remainder of the function digits are contained in the 4th character. For this reason when it is necessary to refer to the part f by itself we use the last two characters formed by taking digits 10, 11, 12, 13 to be zero. Thus, e.g., the code f = 110001 is denoted by TZ. The first character of such a pair will always be / or T according as to whether digit 14 is 0 or 1.

The six function digits provide for 64 possible operations but only 46 distinct instructions have been provided. Of these:

27 refer to the arithmetical unit;

6 are transfers of control;

6 refer to the B-lines;

2 refer to the transfer of information between the electronic and magnetic stores and to the input and output units; and

5 may be described as miscellaneous.

The Arithmetical Unit

This unit is similar in its logical design to that of a desk calculating machine, consisting of two registers for holding a 40 binary digit multiplicand and an 80 digit product. The product register is referred to as the accumulator. The content of the accumulator is denoted by A, and L and M denote respectively the 40 least and the 40 most significant digits of A. The content of the multiplicand register is denoted by D.

Representation of numbers in the store

In the arithmetical instructions the address s refers to a line pair s, s+1, 40 digits being the standard size of operands in arithmetical operations. It is possible to transfer either L or M directly to the store by a single operation. For this reason it is convenient to regard the individual operands of a product as either integers or fractions according as to whether L or M is ultimately retained. (However, by subsequent arithmetical operations it is possible to select any set of 40 digits of the product so as to work with the binary point elsewhere than at the ends of a number. Likewise by recording both halves of the product it is possible to carry out double or multiple precision operations.)

From now on S (upper case) is used to denote the content, i.e., the row of digits, held in line pair s (lower case). Alternatively for a particular s, say E:, the content is denoted by [E:]. If the row of digits S has some numerical significance, then S_+ and S_{+f} are interpreted as positive integers or fractions in accordance with the plus convention as follows.

2^0	2^1	2^2	2^3	2^4	significance of digits of S_+	2^{35}	2^{36}	2^{37}	2^{38}	2^{39}
0	0	0	0	0	- - - - - - - - - - - -	0	0	0	0	0
2^{-40}	2^{-39}	2^{-38}	2^{-37}	2^{-36}	significance of digits of S_{+f}	2^{-5}	2^{-4}	2^{-3}	2^{-2}	2^{-1}

For example:

$$////////|_{+} = 0$$
$$E/////////|_{+} = 1$$
$$AA//////|_{+} = 99$$
$$//////T|_{+} = 2^{39}$$

$$//////T|_{+f} = 1/2$$
$$//////O|_{+f} = 3/4$$
$$//////E|_{+f} = 1/32$$
$$YRYRYRYR|_{+f} = 1/3$$
$$GNIWPNIA|_{+f} = 1/10$$

The integers that can be represented in this way lie in the range $0 \leqslant S_+ \leqslant 2^{40}-1$: fractions in the range $0 \leqslant S_{+f} \leqslant 1 - 2^{-40}$

A further convention, the plus-minus convention, enables both positive and negative numbers to be represented by sacrificing one binary digit to represent the sign. This sign digit is 0 for +ve numbers and 1 for -ve numbers. This is achieved by representing negative numbers by their complements with respect to 2^{40} (or 1 in the case of fractions). In this case the most significant digit is the sign digit. If a row of digits is to be interpreted on this convention then it is accompanied by the subscript ($\pm$). For example:

$$\pounds\pounds\pounds\pounds\pounds\pounds\pounds\pounds|_{\pm} = -1$$
$$XM\pounds\pounds\pounds\pounds\pounds|_{\pm} = -99$$
$$//////T|_{\pm} = -2^{39}$$
$$//////T|_{\pm f} = -1/2$$
$$//////\pounds|_{\pm f} = -1/32$$
$$JYRYRYRY|_{\pm f} = -1/3$$
$$IWPNIWPM|_{\pm f} = -1/10$$

The ranges of numbers that can be represented on this convention are

$$-2^{39} \leqslant S_{\pm} \leqslant 2^{39} - 1$$
$$-\tfrac{1}{2} \leqslant S_{\pm f} \leqslant \tfrac{1}{2} - 2^{-40}$$

Representation of numbers in the accumulator

By considering the product of two numbers the 80 digits of the accumulator can be given the following interpretations. If both operands are interpreted as integers then A is interpreted thus

$$\underbrace{\begin{matrix} 2^0 & 2^1 & 2^2 & 2^3 & 2^4 & \cdots & 2^{35} & 2^{36} & 2^{37} & 2^{38} & 2^{39} \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \end{matrix}}_{L} \quad \underbrace{\begin{matrix} 2^{40} & 2^{41} & 2^{42} & 2^{43} & 2^{44} & \cdots & 2^{75} & 2^{76} & 2^{77} & 2^{78} & 2^{79} \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \end{matrix}}_{M}$$

If both the operands are interpreted as fractions, then A is interpreted thus

$$\overbrace{0\quad 0\quad 0\quad 0\quad 0 \ldots\ldots 0\quad 0\quad 0\quad 0\quad 0}^{L}$$

$$2^{-80}\ 2^{-79}\ 2^{-78}\ 2^{-77}\ 2^{-76}\ \ldots\ 2^{-45}\ 2^{-44}\ 2^{-43}\ 2^{-42}\ 2^{-41}$$

$$\overbrace{0\quad 0\quad 0\quad 0\quad 0 \ldots\ldots 0\quad 0\quad 0\quad 0\quad 0}^{M}$$

$$2^{-40}\ 2^{-39}\ 2^{-38}\ 2^{-37}\ 2^{-36}\ \ldots\ 2^{-5}\ 2^{-4}\ 2^{-3}\ 2^{-2}\ 2^{-1}$$

For given rows of digits as multiplier and multiplicand the digits of the product depend only on the sign conventions used for interpreting the operands. Distinct instructions are provided for setting the multiplicand on the two conventions and two further instructions relate, similarly, to the multiplier.

At this point we can examine the properties of some arithmetical instructions. As in the case of other instructions these take the form s, B, f. It will be assumed that the B digits are zero. The instructions are described by an equation or equations giving the new contents of lines altered, denoted by a prime.

<u>A first class of arithmetical instructions</u>

The members of this class are

f		Effect	
100000	T/	$A' = S_+$	
100010	T½	$A' = S_\pm$	
101100	TI	$A' = A + S_+$	$(\text{mod. } 2^{80})$
101110	TC	$A' = A + S_\pm$	
100110	TN	$A' = A - S_\pm$	
111110	TK	$A' = 2S_+$	
110110	TF	$A' = -S_\pm$	
011010	/J	$A' = A \pm 2^{40}S$ (or $M' = M + S$)	$(\text{mod. } 2^{80})$

Except in the case of the last instruction all quantities can be regarded as 80 digit integers, S_+ being extended by 40 zeros and $S_\pm$ by 40 copies of its most significant digit. Otherwise, in the case of A, the sign convention is irrelevant, all operations being taken mod. 2^{80}.

As an illustration the table below gives the changing contents of the accumulator corresponding to a consecutive set of instructions from this class. The contents of the store lines /: to JS are as shown in fig 2.1. (The significance of this particular material will be described in a subsequent chapter.)

Instructions	A: L	A: M	A_+	A_+
: S T /	////////½	////////	2^{38}	2^{38}
: S T ½	////////½	////////	2^{38}	2^{38}
I S T /	////////T	////////	2^{39}	2^{39}
I S T N	////////	E///////	2^{40}	2^{40}
I S T ½	////////T	£££££££	$2^{80}-2^{39}$	-2^{39}
E : T F	£££££££	£££££££	$2^{80}-1$	-1
E : T C	////////	////////	0	0
I S T K	////////	£££££££	$2^{80}-2^{40}$	-2^{40}
D S T I	£££££££	£££££££	$2^{80}-1$	-1
D S T N	////////	////////	0	0
: S / J	////////	////////½	2^{78}	2^{78}
: S / J	////////	////////T	2^{79}	-2^{79}
I S / J	////////	////////	0	0

There is no facility for subtracting numbers directly from M. There are a number of ways of overcoming this which will be revealed in later examples. The equation for TN should not be confused with $A' = A - S_+$, for which an instruction is, unfortunately, not provided.

<u>A second class of arithmetical instructions</u>

f		Effect
111000	TA	$S' = L,\ A' = 0$
011000	/A	$S' = M,\ M' = 0$
010100	/S	$S' = L$
010000	/E	$S' = M$
011100	/U	$S' = L,\ L' = M,\ M' = 0$
100100	T:	$A' = 0$
001100	/I	$L' = M,\ M' = L$

In these equations all quantities can be regarded simply as rows of digits. Both /S and TA can be used for removing information from L but TA also clears the accumulator and for this reason is

more frequently used.

Similarly considerations apply to /E and /A.

In the case of T: and /I the address part of the instruction is irrelevant.

The effect of /I is to interchange L and M.

/U, effecting as it does two operations at once, often proves very useful.

The multiplier: a third class of arithmetical instructions

The operation of the multiplier is fairly closely analogous to multiplication on a desk machine. The multiplicand register has already been mentioned: this closely corresponds to the lever system of, e.g., a Brunsviga. The multiplication will normally be done in two steps, the setting of D and the multiplication proper following afterwards. If an instruction is described as a multiplication it is understood to be one of the latter kind and corresponds to turning the handle of a Brunsviga. The result of the multiplication is to add to the accumulator (or possibly subtract from it) the product of the number referred to in the multiplication instruction and the number set in the multiplicand register. As with a desk machine the operation of multiplication does not alter the multiplicand setting so that if it is desired to do a further multiplication with the same multiplicand it is not necessary to set it again. A point of difference with desk machines, and one already mentioned, is the determination of the sign of the product. With desk machines one in effect multiplies |a| by b (sign a). With the computer it is possible to have either sign in the multiplicand and also in the multiplier. It is also possible to add or subtract the product according to the instructions used.

In explaining the detailed significance of the instructions it is convenient to depart from the principle that the content of a part of the machine is a row of digits rather than a number, and to say that the content of the multiplicand register, D, is an integer in the range $-2^{39} \leqslant D < 2^{40}$.

One may also put (as a definition) $D_f = 2^{-40}D$. This interpretation of D greatly simplifies the equations describing the instructions.

There are two distinct instructions for setting the multiplicand.

They are 001110 /C $D' = S_+$

011110 /K $D' = S_\pm$

There are four distinct instructions for the multiplication proper, to cover the two interpretations (plus or plus-minus) of the multiplier, and also to allow for the product to be added to or subtracted from the accumulator.

These instructions are

000010	/½	$A' = A - DS_+$
010010	/D	$A' = A - DS_\pm$
000110	/N	$A' = A + DS_+$
010110	/F	$A' = A + DS_\pm$

As in the first class of arithmetical instructions the 80 digit quantities A and DS_+ (or $DS_\pm$) are combined mod. 2^{80}, so that the sign convention of A is irrelevant. When dealing with fractional quantities it is most convenient to be able to remove products from M with a single instruction /A or /E. This assumes that a significant number of figures are retained in M. If however both operands are $O(2^{-15})$, then most of the significant figures of the product will be found in L. Alternatively such an operation can be regarded as the product of two whole numbers of $O(2^{25})$ which has overflowed into M, which is equally disastrous when working with integers. It is one of the tasks of the programmer to take account of these possibilities by the use of scale factors or double precision arithmetic.

The last sets of equations may also be written in the fractional convention as follows.

/C	$D_f' = S_{+f}$
/K	$D_f' = S_{\pm f}$
/½	$A_f' = A_f - D_f S_{+f}$
/D	$A_f' = A_f - D_f S_{\pm f}$
/N	$A_f' = A_f + D_f S_{+f}$
/F	$A_f' = A_f + D_f S_{\pm f}$

The following example, similar to that on page 1.11, provides a further illustration of the instructions concerned with multiplication.

	D	D_f	A	A_f	L	M
..T:			0	0	////////	////////
J:/C	32	2^{-35}				
F:/N			+2048	$+2^{-69}$	//@/////	////////
F:/K	64	2^{-34}				
J:/½			0	0	////////	////////
IS/C	2^{39}	½				
IS/N			$+2^{78}$	$+\frac{1}{4}$	////////	///////½
IS/K	-2^{39}	-½				
IS/N			0	0	////////	////////
IS/F			$+2^{78}$	$+\frac{1}{4}$	////////	///////½
IS/D			0	0	////////	////////
/:/C	2^{20}	2^{-20}				
DS/F			-2^{20}	-2^{-60}	////£££	£££££££
E:/F			0	0	////////	////////

Before passing to the description of further instructions two simple examples illustrating the use of some arithmetical instructions are explained.

Example 1. Three 40 digit numbers x, y, and z are given by $x = [/C]_{\pm}$, $y = [@C]_{\pm}$, $z = [:C]_{\pm}$, It is required to place x + y in IC and 2(x+y)-z in ½C. A suitable set of instructions are given below. To fix our ideas they are assumed to be located in the first few lines of column /.

/	/	/ C T ½	A' = x
	E	@ C T C	A' = x + y
	@	I C / S	copy x + y into IC
	A	I C T C	A' = 2 (x+y)
	:	: C T N	A' = 2 (x+y) -z
	S	½ C T A	transfer 2(x+y)-z to ½C

Notes

1. All operations have taken place in the lesser significant half of the accumulator.
2. $[IC]_{\pm}$ as $[IC]_{\pm}$ will only be the correct result if ~~this lies in the range~~ x+y and 2(x+y)-z both lie in the range $-2^{39} \leqslant 2(x+y)-z \leqslant 2^{39}-1$.
3. The combination 2(x+y)-z is linear, i.e., $\lambda[2(x+y)-z] = 2(\lambda x + \lambda y) - \lambda z$ so that the result will be correct if [/C], [@C], and [:C] are interpreted other than as whole numbers.

Example 2

Let $x = [/C]_{\pm f}$, $y = [@C]_{\pm f}$, and $z = [:C]_{\pm f}$. It is required to place 2(xy + yz) - xz in IC.

As in the first example the instructions are located in Column /.

/	. . T .		clears accumulator
E	@ C / K		$D'_f = y$
@	/ C / F		$A'_f = xy$
A	: C / F		$A'_f = xy + yz$
:	I C / E	}	doubles M
S	I C / J		
I	/ C / K		$D'_f = x$
U	: C / D		$A'_f = 2(xy + yz) - xz$
½	I C / A		$[IC]'_{+f} = 2(xy + yz) - xz$, M' = 0

Notes

1. Emphasis is placed on the most significant part of the accumulator since we are dealing with fractional quantities.
2. The truncated result is biased. To overcome this it is necessary to place $\frac{1}{2} \cdot 2^{-40}$ in the accumulator before starting the calculation. This can be done by altering the instruction ..T: to IST/, where IS = ///////T.
3. IC/E and /C/J can be replaced by /C/F and :C/F. These would be less economical in time (see the times of operation on page 1.28), but the result would be slightly more accurate.

A fourth class of arithmetical instructions

The instructions in this class involve the accumulator but have little numerical significance. They complete the list of instructions associated with the arithmetical unit.

The members of this class are

0 01000	/@	most significant digit	
0 01010	/R	sideways adder	
0 11001	/W	random number	
1 10010	TD	$A' = A \vee S_{\pm}$	Logical operations
1 01010	TR	$A' = A \& S_{\pm}$	
1 11010	TJ	$A' = A \not\equiv S_{\pm}$	

Most significant digit, /@

In certain kinds of numerical work it is desirable to be able to normalise numbers, i.e., to express them in the form $a.2^n$, where a lies

in the range $\frac{1}{4} \leqslant |a| < \frac{1}{2}$. This is made possible at high speed by the function /@. This function determines the position of the most significant digit of S and adds the digital position number $\mu(S)$ to M. e.g., if S = / X / R Y N / / then $\mu(S) = 28$ = M/////// is added to M. For the special case S = ////////, $\mu(S) = 63$ (definition).

Sideways adder, /R

This function determines t(S), the number of 1's in S, and adds it to M. e.g., if S = /X/RYN//, then t(S) = 11 = J/////// is added to M. The function has no relation to the interpretation of S as number.

Random number generator, /W

This function first clears the accumulator and then fills the twenty least significant digital positions with random digits. As in the case of the T: and /I instructions the address digits are irrelevant. The randomness is derived from a resistance noise generator. This facility may be useful in sampling problems..

The logical functions TD, TR, and TJ

These are operations on rows of digits and have no relation to the interpretation of these rows as numbers. The symbols V,&, and $\not\equiv$ satisfy the equations

0 V 0 = 0	0 & 0 = 0	$0 \not\equiv 0 = 0$
0 V 1 = 1	0 & 1 = 0	$0 \not\equiv 1 = 1$
1 V 0 = 1	1 & 0 = 0	$1 \not\equiv 0 = 1$
1 V 1 = 1	1 & 1 = 1	$1 \not\equiv 1 = 0$

The instructions perform these operations on the 80 digit row A and the 80 digit row obtained by extending S by 40 copies of its most significant digit. Thus, e.g., let

A = ABCDEFGHIJKLMNOP S = £/£/£/£/.

The effects of the logical instructions are then as follows

		L	M
s,TD	A' =	£B£D£F£H	IJKLMNOP
s,TR	A' =	A/C/E/G/	////////
s,TJ	A' =	MBZDVFSH	IJKLMNOP

If, however, S = /£/£/£/£ then we have

s,TD	A' =	A£C£E£G£	£££££££££
s,TR	A' =	/B/D/F/H	IJKLMNOP
s,TJ	A' =	AICPELGJ	BHTFAWUD

The main application of these operations occurs where several pieces of information are packed into one line. For this the operation & is of the most use. It can be used for breaking up a line into its various significant parts. The V operation can be used for combining the parts together. Another use is to round off a number by making the least significant digit 1. The non-equivalence operation ($\not\equiv$ can be used for changing the sign of a number in the accumulator by taking the non-equivalence with the row £££££££££ (see Coding Hints, Chapter 2).

This completes the description of the 27 arithmetical instructions. In the remaining groups of instructions the address part s refers to a single (20 digit) line and S denotes the content of that line.

The instructions for transferring control

The codes and defining equations are given below. In these equations C denotes the (20 digit) control number of which only the first two characters are significant (see page 1.6). If, as a result of one of the following operations, these characters become (say) K ❍, then the control unit will start to select instructions from lines T❍, Z❍, etc., until another transfer of control is encountered.

0 01101	/P	C' = S
0 11101	/Q	C' = C + S + 1
0 00101	/H	C' = S if $A_{\pm} \geqslant 0$, otherwise C' = C + 1
0 00111	/M	C' = C + S + 1 if $A_{\pm} \geqslant 0$; otherwise C' = C + 1

Thus to transfer control to the instruction standing in (say) line CE, the control number FE must be available in the ten least significant digits of some line s. The instruction s,/P then effects the required transfer of control. /H is similar to /P but the transfer only takes place if digit 79 of A is a 0, otherwise the next is selected in the usual way (C' = C + 1). The functions /Q and /M cause relative transfers of control. A convenient interpretation is as follows. If the integral equivalent of S is p, then s,/Q will cause control to skip p instructions. If S = ££, then s,/Q is obeyed repeatedly - a dynamic stop. If S = V£.. , then control returns to the previous instruction. And so on. The function /M is the relative form of /H.

The B tube and the associated instructions

The machine is intended to take advantage of the repetitive nature of calculations. Thus when adding up 100 numbers it is necessary to be able to tell the machine first how to do one addition and then how to repeat the operation on the remaining 99 numbers. The B tube is a device for facilitating operations of this kind.

The B tube consists of 8 lines each of 20 digits (referred to as B0, B1, B2, ..., B7) and the control unit is arranged so that the content of one of these lines - that named by the B digits of the instruction - is added to each instruction before it is obeyed. The instruction which is finally obeyed is called the actual instruction. The rule is:

presumptive instruction + content of named B line = actual instruction.

Certain instructions are exempt from this rule and in these cases the actual instruction is identical with the presumptive instruction. These, the B exceptional instructions, have the function codes

1 00011	TO
1 10011	TB
1 01011	TG
1 11011	T"
1 00111	TM
1 10111	TX
1 01111	TV
1 11111	T£

Examples of the formation of actual from presumptive instructions

Presumptive instruction	named B line	Content of named B line	Actual instruction
/C@K	B2	@///	@C@K
:CAK	B3	V£//	@C:K
:C£C	B7	V£//	@C/K
:CUK	B7	V£££	@CUK
////	B0	@C/K	@C/K

The actual instructions in all these examples are effectively identical because after the actual instruction has been formed, the digits 10, 11, 12, have no further signifance, except in cases where the actual instruction is one of those described in the next section.

The B instructions

The purpose of these instructions is to set, alter, and copy numbers in the B tube. Their function codes and defining equations are:

B normal	1 00001	TT	B' = S
	1 10001	TZ	S' = B
	1 01001	TL	B' = B - S
B exceptional	1 00011	TO	B' = S
	1 10011	TB	S' = B
	1 01011	TG	B' = B - S

In these equations B and B' denote the initial and final contents of the B line named by the B digits of the actual instruction (which in the case of the B exceptional instructions is identical with the presumptive instruction). Likewise S and S' refer to the initial and final contents of the short line whose address is found in the actual instruction and the equations applied are given by the function digits of the actual instruction - which may not necessarily be the same as the function digits of the presumptive instruction.

Associated with the B tube are two further conditional transfer of control instructions, s,/T (direct) and s,/O (relative) similar to /H and /M except that they discriminate on the sign of the last B line altered. (For this purpose the operation S' = B counts as an alteration.) Their codes and defining equations can be written

0 00001 /T C' = S if Q = 0; otherwise C' = C + 1

0 00011 /O C' = C + S + 1 if Q = 0; otherwise C' = C + 1.

where Q is the sign digit of the new content of the last B line to be altered. These instructions are used when testing for the last cycle of a counting operation.

The B tube and the associated instructions have three purposes. The primary purpose is the modification of presumptive instructions to give actual instructions. Secondly for the counting operations which are usually associated with the systematic modification of B additives, and lastly as a shunting station for 20 digit lines.

The B exceptional instructions

The exceptional instructions were made necessary by the fact that when one is setting a B line one does not usually wish the instruction involved to be modified by what is already in the B line in question.

However when using the B line as a shunting station, particularly when transferring to a B line from a position specified by the content of a further B line, it is necessary that the instruction involved should bring two different B lines into action, one containing information as to the source of the content of the other. These cases can be covered by the devices revealed in the following examples.

Examples illustrating some effects of the B instructions.

	Presumptive instruction	B line named by pres. instr. and its content	Actual instruction	Effect
1.	//QO	B7 content	//QO	[B7]' = [//]
2.	A:QG	B7 irrelevant	A:QG	[B7]' = [B7] - [A:]
3.	//ZT	[B1] = HUI/	HUQT	
4.	//E/	[B1] = HUPO	HUQO	
5.	////	[BO] = HUCT	HUCT	[B7]' = [HU]
6.	////	[BO] = HUQO	HUQO	
7.	V£PT	[B6] = PU//	HUQT	
8.	//QT	[B7] = @E//	@EQT	
9.	//Q/	[B7] = @E/O	@EQO	[B7]' = [@E]

It will be seen from examples 3 and 4 that the B normal instructions TT, TZ, and TL are really redundant. They achieve nothing that cannot equally well be obtained by means of other suitably chosen presumptive instructions although the device of example 7 sometimes finds application. See, for example, the routine TELEINPUT. Normally however these instructions are very seldom used. The following example illustrates a typical application of the B instructions.

Example 3

Place in /C the scalar product of the two vectors whose elements are the contents of the line pairs (i) /N, @N, ... LN, and (ii) /F,@F, ... LF, (interpreted on the ± fractional convention).

Column /

	/	L / / /	= 18
ENTER→	E	I S T /	set round-off
	@	/ / Q O	set B7 = L///
	A	/ N U K	add product to
	:	/ F U F	partial sum
B7 ≥ 0	S	A : Q G	reduce B7 by 2
	I	A : / T	test for last cycle
B7 < 0	U	/ C / A	transfer result.

cycle 10 times

IS contains ////
US " ///T
A: " @///

During the 1st cycle B7 contains L/// and hence the actual instructions corresponding to /NUK and /FUF are LNUK and LFUF. The product of the elements in LN and LF is thus taken first. On subsequent cycles B7 contains T///, C///, N///, etc. After each cycle the number in B7 is reduced by 2 and tested until eventually after the last cycle B7 becomes negative and the cycle terminates.

It will be seen that the use of the B tube for the modification of instructions fits very well with its use counting repetitions of an operation. It is not necessary to copy the partial sums out into the store as the whole process of altering the B tube and testing is done without interfering with the accumulator.

The magnetic instructions

The organisation of the magnetic store has already been described (see page 1.4). To recapitulate there are 256 tracks each consisting of a left half and a right half. The two halves of e.g., track 35 are denoted by 35L and 35 R. Each half track contains the same amount of information as an electronic tube. In both cases this amount of information is referred to as a page.

In order to make use of the information stored on the magnetic drum, there is provision for transferring the content of either a half track or a complete track to an electronic tube or pair of tubes. The process by which this is done is described as a reading transfer. If a pair of tubes is involved then it is one of the following:

tube 0 and tube 1; tube 2 and tube 3; tube 4 and tube 5; tube 6 and tube 7.

Likewise in order to store information previously held in the working store arrangements are made for transfers in the opposite direction, from an electronic tube or pair of tubes to a half track or complete track. These are called writing transfers. In addition there are magnetic checks by means of which the content of a part of the magnetic store can be compared with part of the working store. All these operations are initiated by the instruction s,B,f, where f = 000100,

with the aid of a further 20 digits of information in the single line s. This auxiliary line of information, called a magnetic instruction, names a track, a tube or pair of tubes, and the kind of operation, whether transfer or check. The effect of the original instruction is to transfer the magnetic instruction to a second control register where it is interpreted in accordance with rules given below. The instruction with the function code 000000(//) is similar to /: but interprets the row H (see page 1.27) as a magnetic instruction. This instruction like /I and T: is addressless.

When the magnetic operation has been completed the control selects the next instruction in the usual way (C' = C + 1) except in the case of a magnetic check where the check succeeds. In this case the control skips two instructions (C' = C + 3).

In the case of a reading transfer it may happen that the tube containing the original instruction is itself involved. This case is no exception, the next instruction being selected from the corresponding line in the new page of material. The 20 digits of a magnetic instruction are made up as follows.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

track characters — function characters — tube characters

Digits 0-9 name a track (digits 8, 9 are spare because there are only 256 tracks). Digits 15-19 name any column in the tube or tubes involved. (It is the function digits which determine whether one or two tubes are involved.) There is some redundancy here: if one tube is involved, then digit 15 is irrelevant; if two tubes are involved, then both digits 15 and 16 are irrelevant. Digits 10-14 specify an operation involving the named track and the tube or tube pair containing the named column. The following table defines the interpretation of this third character.

/ The content of the left half of the named track is transferred to the tube containing the named column.

E The content of the right half of the named track is transferred to the tube containing the named column.

@ The content of the left half of the named track is transferred to the even tube of the tube pair containing the named column, and the content of the right half track to the odd tube.

A The content of the left half of the named track is transferred to the odd tube of the tube pair containing the named column, and the content of the right half track to the even tube.

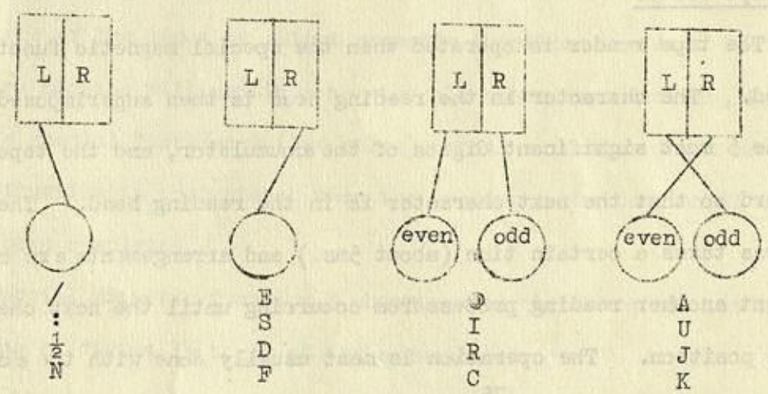
: S I U } Check operations for comparing the result of transfers /, E, @, and A. The new content of the tube or tubes involved is compared with the original material on the magnetic track.

½ D R J } Writing transfers corresponding to /, E, @, and A.

N F C K } Check operations corresponding to ½, D, R, and J.

T Z : £ } These special "magnetic" functions do not relate to the magnetic store but to the input and output units.

The possible connections between the named track and the named tube(s) are illustrated by the following diagrams.



Examples

The following examples of magnetic instructions conclude this section

//// means transfer material on half track OL to tube O

I/E½ means transfer material on half track 6R to tube 4

AE:/ means transfer material on 35L to tube 0 and that on 35R to tube 1

E:AN means transfer material on 129L to tube 7 and that on 129R to tube 6

/:½I means transfer material on tube 3 to 64L

½ED: means transfer material on tube 2 to 40R

U/R: means transfer material on tube 2 to 7L and that on tube 3 to 7R

U/J: means transfer material on tube 3 to 7L and that on tube 2 to 7R.

The Input and Output Units

Information is fed into the machine from perforated paper tape read by a photoelectric tape reader. The perforated tape is of the 5 hole teleprint type. Information can be taken out of the machine on to further teleprint tape and on to a teleprinter. This last unit is a Creed page printer modified to enable it to print 32 distinct characters on letter shift. The operation of all three units is controlled by the special magnetic instructions with 3rd character, T, Z, ..., £.

The tape reader

The tape reader is operated when the special magnetic function O is obeyed. The character in the reading head is then superimposed (' ✱ ') on the 5 most significant digits of the accumulator, and the tape moves forward so that the next character is in the reading head. The latter process takes a certain time (about 5ms.) and arrangements are made to prevent another reading process from occurring until the next character is in position. The operation is most usually done with the accumulator initially clear so that $2^{75}T$ is placed in the accumulator, where T is the integral equivalent of the character read (see fig. 1.1).

The teleprinter and punch

These units are operated when the special magnetic function T is obeyed. A 3 position switch on the console enables either or both to be used for output. The effect on the punch is to perforate the character previously set up and then to set up the

character given by the 5 most significant digits in the accumulator, i.e., the effect is the reverse of the input function.

This character is set up on the printer and the character previously set up is printed unless the printer is in the figure shift position. In this case the corresponding figure or stunt operation given by the following table is obtained.

character	/	E	⊕	A	:	S	I	U	½	D	F	Z	L	W	Y	P	M	Q	"	£
effect		0	1	2	3	4	5	6	7	8	9	.	Space	Carriage return	Line feed	Letters			+	-	=	"	£

No guarantee can be given concerning what happens when other characters are printed on figure shift. There may only be a smudge.

Arrangements are also made to do the stunt operations when the teleprinter is on letter shift. These are provided by other special magnetic functions: see table below. They also provide an alternative means of doing the stunts on figure shift. No effect is produced on the punch by these functions.

Once one of the input or output magnetic functions has been initiated, the machine will continue to obey all other instructions, including the special function B (see below), until a further input or output instruction is encountered when the machine is held up until the first has been completed.

The special magnetic function B superimposes the character set up for punching on digits 35 to 39 of the accumulator.

The following table summarises the special magnetic functions.

3rd character	effect		time and maximum rates
T	OUTPUT		
Z	SPACE	These do not	Punch: 16 chars. per. sec.
L	CAR. RET.	affect the	Printer: 6 chars. per. sec.
W	LINE FEED	punch	Tape reader: 170 chars. per. sec.
H	FIG. SHIFT		
Y	LETTER SHIFT		
P			
Q			
O	INPUT		
B	CHECK		5 beats
G			
"			
M			
X			
V			
£	DUMMY		4 beats

fig. 1.2.

Notes

1. It has been found that up to 140ms. of calculation can be arranged between two consecutive output instructions without retarding the operation of the printer. Corresponding figures for the punch and tape reader are 60 ms. and 6 ms. respectively.
2. In general an instruction calling for a carriage return should be followed by one calling for a line feed. It is recommended however that a new line should be started by 3 instructions calling for carriage return, line feed, and carriage return, in this order. This eliminates the possibility of carriage bounce and ensures an even margin throughout the length of the page.

Miscellaneous instructions

1) Dummy stops It is possible by setting certain switches to arrange that the machine stops when certain instructions are reached. The instructions in question have the function codes 001001 (/L) and 001011 (/G). Like the /I and T: instructions they are addressless, i.e., the first two characters are irrelevant. There are two on-off switches on the console labelled "/L" and "/G". If,

e.g., the "/L" switch is in the "on" position, then the machine will stop when control encounters a /L instruction; otherwise /L acts as a dummy or time wasting instruction. Similarly for the /G instruction. The instruction with function code ~~000000~~ 111111 (T£) acts as a dummy only.

Dummy stops are very useful in testing routines. The programmer is recommended to insert them at points where major operations may be considered complete (see chapter 8).

2) The hooter When an instruction with the function code ~~001111~~ 001111 (/V) is obeyed an impulse is applied to the diaphragm of a loudspeaker. By doing this repeatedly and rythmically a steady note, rich in harmonics, can be produced. This is used to enable the operator to be called to attend to the machine in some way. The simplest case is where the whole of a job is completed and it is required to clear the electronic stores and start something different. All that is then required is to repeat a cycle of instructions including a hoot, e.g.,

```
FS  NS/V
CS  FS/P
```

In this case every second instruction will put a pulse into the speaker. These pulses will occur at intervals of 8 beats i.e., 1.92ms. giving a frequency of 512 cycles (an octave above middle C). Or one could use the loop of three instructions

```
O@   /V
B@  Q@/V
G@  B@/P
```

which gives a slightly louder hoot a fifth lower in frequency. Single pulses applied to the loudspeaker are distinctly audible as something between a tap, a click, and a thump. This fact can be turned to good account. By putting hoot instructions into programmes at suitable points one is enabled to "listen in" to the progress of the routine. Some indication of what is going on is given by the clicks that are heard.

3) The hand switches One can set up a row of twenty digits on twenty switches. This row 'H' can effect the behaviour of the machine through instructions with function codes 000000 (//) or 010001 (/Z). The former of these has already been mentioned (page 1.22.). The latter

causes the number set up to be copied into the named line (S' = H). In this way small pieces of information can be put into the machine by hand. Suppose for example that we have a routine for calculating some function of a 4 character line, and suppose that the calculation takes five minutes. It would then be reasonable to put the argument in through the switches. This would be particularly so if the argument used depends partly on the judgement of the experimenter and partly on the values recently obtained, e.g., if one were trying to find a zero of the function but one was not wishing to repeat the process often enough to mechanise it fully. Again if one were playing chess against the machine this would be the natural way of registering one's own moves.

The time occupied by the individual operations

The machine is synchronous, the rhythm being divided into beats each of 240 μs. ~~The number of beats used for each operation is given in the table below.~~

The reader will observe that the instructions with positive codes, i.e., which lie in the upper half of the table, refer mainly to operations on line pairs, whilst the remainder involve only single lines. With certain exceptions the former are 5 beat codes, the latter take 4 beats. The exceptions are the multiplication codes /½, /D, /N, and /F which each take ~~8~~ 9 beats, and the random number instruction which takes 24 beats.

The times of operation of the magnetic instructions are as follows: read, read-check, write check, 35 ms; write 90 ms.

Abbreviated Instruction Code

/ /	H as mag. instr.	T /	$A' = S_+$	
/ E	S' = M	T E		
/ @	standardise	T @		
/ A	S' = M, M' = 0	T A	S' = L, A' = 0	
/ :	S' as mag. instr.	T :	A' = 0	
/ S	S' = L	T S		
/ I	L' = M, M' = L	T I	$A' = A + S_+$	
/ U	S' = L, L' = M, M' = 0	T U		
/ ½	$A' = A - D\,S_+$	T ½	$A' = S_\pm$	
/ D	$A' = A - D\,S_\pm$	T D	$A' = A \vee S_\pm$	
/ R	Sideways adder	T R	$A' = A \,\&\, S_\pm$	
/ J	M' = M + S	T J	$A' = A \not\equiv S_\pm$	
/ N	$A' = A + D\,S_+$	T N	$A' = A - S_\pm$	
/ F	$A' = A + D\,S_\pm$	T F	$A' = -S_\pm$	
/ C	$D' = S_+$	T C	$A' = A + S_\pm$	
/ K	$D' = S_\pm$	T K	$A' = 2S_\pm$	
/ T	B-cond. direct trans. control	T T	B' = S	
/ Z	S' = H	T Z	S' = B	
/ L	Dummy stop	T L	B' = B - S	
/ W	Random number generator	T W		
/ H	A-cond. direct trans. control	T H		
/ Y		T Y		
/ P	Uncond. direct trans. control	T P		
/ Q	Uncond. rel. trans. control	T Q		
/ O	B-cond. rel. trans. control	T O	B' = S	
/ B		T B	S' = B	
/ G	Dummy stop	T G	B' = B - S	
/ "		T "		
/ M	A-cond. rel. trans. control	T M		no B-addition
/ X		T X		
/ V	Hoot	T V		
/ £		T £	Dummy	

Fig 1.2

Chapter 2

Coding Examples

Introduction

The examples given in the next few pages are intended to illustrate some applications of the instruction code. They are divided into three groups which introduce the instructions in the following order.

1. The arithmetical and control transfer instructions.
2. The B-tube, the most significant digit instruction and the random number generator.
3. The magnetic instructions and the special magnetic instructions.

Conventions

There are certain conventions relating to the use of the electronic store which will be explained in detail in the next chapter. At the moment it is assumed, unless otherwise stated, that certain information is permanently available in tube 2. This consists of the powers of 2 and certain instructions, the purpose of which is explained in the next chapter. The 40 powers of 2 are so arranged that they occupy only the 41 lines /: to $\frac{1}{2}$S inclusive (see fig. 2.1). Thus e.g., the number standing in the line pair U; is 2^3 and that in I: is 2^{23}. The line pair DS is also useful.

In the solutions to the examples the sequences of instructions and any necessary working space have been arranged to occupy the first few lines of tube 0. They could of course stand anywhere in the store but this scheme matches the conventions introduced in the next chapter. The entry line of each sequence is indicated. A single dot in any character position indicates that the character is irrelevant. Unless otherwise stated quantities such as $x = [/\ C]_{\pm}$, stand in long lines.

The Powers of 2.

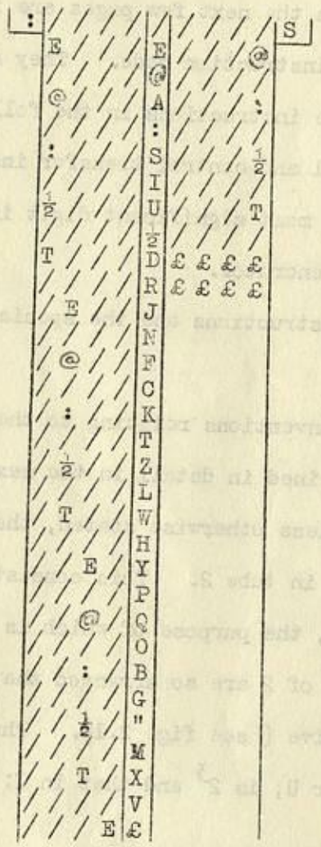


fig. 2.1.

Coding hints

A number of detached and rather trivial tricks and precepts, which are nevertheless considered worth while having in writing, have been collected together at the end of this chapter. They are intended to be studied in conjunction with the examples.

Do coding directly in teleprint code.

It is never too soon to learn the meaning of the 64 functions. The way to do so is to start coding in teleprint code straight away. Keep a list of the meanings always at hand, and refer to it as much as you wish; you will find that after a week very few references are necessary. You will not yet know all the codes, but you will know a working selection. Likewise you will eventually get to know the teleprint equivalents (fig. 1.1) but this is likely to be slower, chiefly because it is less essential to know them. Although the lines are given names which are in teleprint code, and which also correspond. to numbers, for many applications it is not necessary to know anything concerning the relation of these labellings, or even to have very much to do with the numbers at all. The names of the lines are just used as labels. Later it will be desirable to know the teleprint equivalents of the single characters by heart, but it is never necessary to know the equivalents for pairs of characters.

Examples 1

(1) Given $x = [/\ U]_{\pm f}$ and $y = [\ @\ U]_{\pm f}$, place $(x + y)$ in :U and $2x - y$ in IU.

Entry→	E	/ U T ½	x
	@	@ U T C	x + y
	A	: U T A	plants x + y
	:	/ U T K	2x
	S	@ U T N	2x - y
	I	I U T A	plants 2x - y

(2) Replace $[/\ U]_{\pm f}$ by its cube.

Entry→	E	I S T /	clears accumulator and sets round off
	@	/ U / K	x^2
	A	/ U / F	x^2
	:	/ U / A	plants x^2
	S	/ U / F	x^3
	I	/ U / A	plants x^3

The rounding off procedures are explained in Hints, 5b and 5c.

(3) Multiply $[/U]_{\pm f}$ by π^2 (assuming that $-\frac{1}{2} \leq \pi^2 [/U]_{\pm f} < \frac{1}{2}$)

Entry→	/		/
	E	I S T /	rounds off
	@	/ U / K	$x.\frac{\pi^2}{16}$
	A	/ E / N	
	:	/ U / A	
	S	. . T :	clears accumulator
	I	/ U / K	$(x.\frac{\pi^2}{16}).16 = x\pi^2$
	U	D : / N	
	½	/ U T A	transfers result.

/	T T W N	E	$[/E]_{+f} = \frac{\pi^2}{16}$
E	V H Q W		

(4) Given $z = x + iy$, where $x = [/U]_{\pm f}$ and $y = [@U]_{\pm f}$, place the real and imaginary parts of z^2 in :U and IU respectively, and $|z|^2$ in ½U.

Entry→	/		
	E	I S T /	rounds off
	@	@U / K	$-y^2$
	A	@U / D	
	:	½ U / E	$-y^2$ to ½U
	S	/ U / K	$x^2 - y^2$
	I	/ U / F	
	U	: U / A	plants $x^2 - y^2$
	½	: U T ½	
	D	½ U T N	$x^2 + y^2$
	R	½ U T N	
	J	½ U T A	plants $\|z\|^2$
	N	@U / F	
	F	@U / F	2xy.
	C	I S T I	rounds off
	K	I U / A	plants 2xy

(5) Place $|[/U]_{\pm}|$ in /U

Entry→	/	A /	
	E	/ U T ½	A' = $[/U]_{\pm}$
	@	/ / / H	tests sign
	A	/ U T F	changes sign if -ve
	:	/ U T A	transfers result

Alternatively a relative transfer of control instruction can be used. (See Hints, 10).

Entry →	/		
	E	/ U T ½	
	@	E : / M	skips one instruction if +ve
	A	/ U T F	
	:	/ U T A	

(6) Three positive numbers a, b, c are given by $[/\frac{1}{2}]_f$, $[@\frac{1}{2}]_f$, $[:\frac{1}{2}]_f$ respectively. It is known that $abc < 2^{-15}$. Form 2^{15} abc without undue loss of accuracy and place the result in /U.

Entry→	/	T :	clears acc.
	E	/ ½ / C	
	@	@ ½ / N	ms (ab) to @U
	A	/ U / U	ls (ab) to /U
	:	@U / U	
	S	: ½ / C	
	I	/ U / N	A' = 80 most significant
	U	/ U / U	figures of abc
	½	@U / N	
	D	/ U / U	ls (abc) to /U
	R	@U / U	ms (abc) to @U
	J	£ : / C	
	N	@U / N	abc x 2^{15}
	F	/ I	
	C	/ U / N	
	K	I S T I	rounds off
	T	/ U / A	ms (2^{15} abc) to /U

(7) Place the larger of the numbers $[/U]_{+f}$ and $[@U]_{+f}$ in :U.

Entry→	/	@U T /	
	E	D S T J	$[/U]_f - [@U]_f - 2^{-40}$
	@	/ U T C	
	A	A : / M	tests sign of A
	:	@U T /	
	S	E : / Q	
	I	/ U T /	
	U	: U T A	transfers result

(8) Given two numbers $x = 2^{-n} [/U]_{\pm}$, $y = 2^{-n} [@U]_{\pm}$ it is required to place the product xy in :U so that $xy = 2^{-n} [:U]_{\pm}$. (It is assumed that $-2^{39} \leq 2^{+n}xy \leq 2^{39} - 1$.) and that Transmutator is clear on entering).

Entry→	/	/ U / K	
	E	@U / F	records 80 digit product
	@	: U / U	
	A	I U / U	
	:	? / C	D' = 2^{40-n}
	S	I S T /	rounds off
	I	: U / N	
	U	: U / U	selects appropriate digits
	½	I U / F	of product
	D	: U T A	transfers result
	R		

From the 80 digit product xy are selected the 40 consecutive digits n, n+1, ..., n+39 inclusive (digit 0 being the least significant). :U is used as a working line. The quantities 2^{n-1} and 2^{40-n} are found in the powers of 2 and for this reason the address characters of the 5th instruction are not filled in.

The case n = 40 corresponds to the usual fractional convention.

In the case n = 20 the sequence can be simplified as follows.

Entry →	/	U	S	T	/
	E	/	U	/	K
	@	@	U	/	F
	A	:	U	/	U
	:	I	U	/	U
	S	S	U	T	/
	I	:	U	T	A

rounds off

selects middle 40 digits of product

(9) Given two double length numbers x, y, where

$$x = 2^{40}[\,/\,C]_{\pm} + [\,@\,C]_{+}$$

$$y = 2^{40}[\,:\,C]_{\pm} + [\,I\,C]_{+},$$

it is required to place the exact product (i.e., 160 digits) in locations /U, @U, :U and IU.

Entry →	/	[illegible]	[illegible]	T	:
	E	@	C	/	C
	@	I	C	/	N
	A	/	U	/	U
	:	@	U	/	U
	S	:	C	/	F
	I	:	U	/	U
	U	I	U	/	U
	½	/	C	/	K
	D	I	C	/	N
	R	½	U	/	U
	J	R	U	/	U
	N	@	U	T	/
	F	:	U	T	I
	C	½	U	T	I
	K	@	U	/	U
	T	I	U	T	C
	Z	R	U	T	C
	L	:	C	/	F
	W	:	U	/	U
	H	I	U	/	U

The four 80 digit products are combined as shown below (fig.2.2).

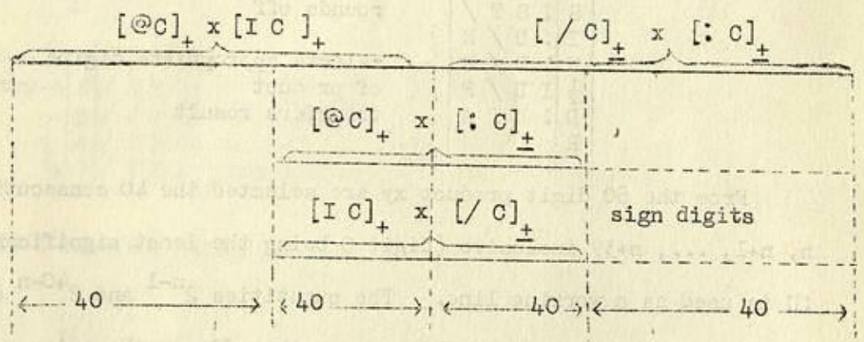


fig.2.2.

Considerable simplification is possible when only the middle 40 digits are required, as would be the case if

$$x = [\,/\,C]_{\pm} + [\,@\,C]_{f}$$

$$y = [\,:\,C]_{\pm} + [\,I\,C]_{f}$$

$$xy = [\,/\,U]_{\pm} + [\,@\,U]_{f}$$

In this case we have

	/	I	S	T	/
	E	@	C	/	C
	@	I	C	/	N
	A	/	U	/	U
	:	:	C	/	F
	S	/	C	/	K
	I	I	C	/	N
	U	@	U	/	U
	½	:	C	/	F
	D	∅	U	T	A

} see note below

Note: there is no possibility of exceeding capacity because the maximum value of $[\,@\,C]_f \times [\,I\,C]_f$ is $1 - 2.2^{-40} + 2^{-80}$.

(10) The following example illustrates a division process given by Wilkes, M.V., Wheeler, D.J., and Gill, S. in their book "Preparation of Programs for an Electronic Digital Computer", Addison-Wesley Press, Inc, Camb.Mass.

Given that $\left|[\,/\,U]_{\pm f}\right| < \frac{1}{2}\left|[\,@U]_{+f}\right|$ the effect of the following sequence is given by the equation $[\,/\,U]'_{\pm f} = [\,/\,U]_{\pm f} / [\,@\,U]_{\pm f}$; $[\,@\,U]$ is altered.

Entry →	/	/	U	T	/
	E	@	U	/	J
	@	E	:	/	M
	A	D	S	T	J
	:	A	:	/	Q
	S	@	U	/	J
	I	/	U	T	I
	U	/	U	/	S
	½	@	U	/	E
	D	S	:	/	H
	R	I	S	T	/
	J	@	U	/	K
	N	/	U	/	J
	F	/	U	/	D
	C	/	U	/	A
	K	@	U	/	D
	T	@	U	/	E
EXIT ←	Z			/	H
	L	W	/	/	P
	W	D	/		

tests sign of divisor
changes sign of divisor and dividend if divisor is negative

shifts divisor and dividend until divisor > ½.
$[\,@\,U]_{\pm f} = [\,@U]_{+f} - 1 = c_o$, $[\,/\,U]_{\pm f} = a_o$

rounds off
$D'_f = c_n$

$a_n - a_n c_n = a_{n+1}$

$-c^2_n = c_{n+1}$

tests for end of process
starts new cycle

The method uses the repetitive sequence

$$a_{n+1} = a_n(1 - c_n), \quad a_n \to \frac{a_o}{1+c_o}$$

$$c_{n+1} = -c_n^2, \quad c_n \to 0.$$

The cycle terminates when $c_{n+1} + O(2^{-40}) > 0$, where $O(2^{-40})$ is the quantity remaining in the lesser significant half of the accumulator after the previous operations.

Examples 2.

(1) Place in /U the sum modulo 2^{40} of the line pairs of tube 4.

Entry →	/	V E / /	
	E	@ / T :	clears accumulator
	@	/ / Q O	sets B7 = VE//
	A	/ ½ Q I	partial sum
	:	A : Q G	reduces B7
	S	E / / T	tests for last cycle
	I	/ U T A	

(2) Given x in the multiplicand register and $a_o, a_1, \ldots, a_n$ in locations /C, @ C, :C, etc., where B7 contains 2n, place $f(x) = a_o + a_1 x + a_2 x^2 + \ldots + a_n x^n$ in /U. (All quantities are signed fractions).

Entry →	/		
	E	I S T /	rounds off
	@	E : / Q	skips one instruction
	A	/ U / F	
	:	/ C U J	$p_{r+1} = p_r x + a_{n-(r+1)}$
	S	/ U / A	
	I	A : Q G	reduces B7
	U	A : / T	tests for last cycle
	½		

The polynomial is built up by the recurrence relation

$$p_o = a_n$$
$$p_{r+1} = p_r x + a_{n-(r+1)}$$
$$p_n = f(x)$$

For particular values of n it may be appropriate in the interests of time economy to write out the cycles explicitly. Thus, e.g., if n = 5 we have

Entry →	/	I S T /	
	E	R C / F	
	@	½ C / J	$a_5 x + a_4$
	A	/ U / A	
	:	/ U / F	
	S	I C / J	$(a_5 x + a_4) x + a_3$
	I	/ U / A	
	U	/ U / F	
	½	: C / J	$((a_5 x + a_4)x + a_3)x + a_2$
	D	/ U / A	
	R	/ U / F	
	J	@ C / J	$(((a_5 x + a_4)x + a_3)x + a_2)x + a_1$
	N	/ U / A	
	F	/ U / F	
	C	/ C / J	f(x)
	K	/ U / A	

The times involved in the two cases are: 162 beats, 100 beats.

(3) Given n numbers $[/\ \tfrac{1}{2}]_{\pm}, [@\ \tfrac{1}{2}]_{\pm}, \ldots, [/\ \tfrac{1}{2} + 2n-2]_{\pm}$, place the numerically largest in /U. B7 contains 2n-2.

Entry →	/	T :	clears @ U
	E	@ U T A	
	@	/ ½ Q ½	takes modulus of next number
	A	E : / M	
	:	/ ½ Q F	
	S	@ U T N	compares with previous largest modulus
	I	E : / M	
	U	S : / Q	
	½	@ U T C	
	D	@ U T A	transfers current largest number and its modulus
	R	/ ½ Q ½	
	J	/ U T A	
	N	A : Q G	
	F	E : / T	tests for last cycle

(4) Let $x = [/\ U]_{+f} > 0$. Replace x by $2^n x$ where $1 > 2^n x \geqslant \tfrac{1}{2}$. Place 2n in B7. Method 1 (emphasis on space economy).

	/	V	£	£	£
Entry →	E	/	:	Q	O
	@	A	:	/	Q
	A	/	/	Q	G
	:	/	U	T	A
	S	/	U	T	K
	I	A	:	/	H

clears B7
skips 2 instructions
increase B7 by 2
doubles $[/U]_{\pm}$
tests sign

The time involved is 17 + 18n beats. If n > 3 then the following method (for which the time is independent of n) is faster.

Method 2 (emphasis on time economy).

	/	I	E	/	/
	E	O	V	£	£
Entry →	@			T	:
	A	/	U	/	@
	:	V	U	/	A
	S	/	/	Q	O
	I	V	U	Q	G
	U	V	U	Q	G
	½	/	:	U	C
	D	E	:	/	O
	R	D	S	U	C
	J	/	U	/	N
	N	/	U	T	A
	F	E	/	Q	G

38
-40
clears accumulator
digital position number, r.
forms 38-2r in B7
selects multiplier
multiplication
transfers result
corrects B7

The exponent n is given by 39-r, where r is the digital position number μ (/U) (see page 1.15). The multiplier 2^{39-r} is selected from PERM as follows: if $38-2r \geqslant 0$, then $[/: + 38 - 2r] = 2^{39-r}$; if $38-2r < 0$, then $[DS + 38 - 2r] = 2^{39-r}$.

Examples 3

(1) The rows of a matrix a_{ij} $(0 < i,\ j \leqslant n-1 \leqslant 31)$ are stored on n magnetic half tracks so that if the ith half track is transferred to tube 4, then $a_{ij} = [/\tfrac{1}{2} + 2j]$ $(0 \leqslant j \leqslant n-1)$. It is required to transpose the matrix so that half track i contains the elements a_{ji} $(0 \leqslant j \leqslant n-1)$. The names of the half tracks are given by a directory in tube 1: each entry is a magnetic instruction which reads the corresponding half track to tube 4. The entry for half track i stands in (/@+2i) (alternate lines are wasted). Line // contains 2n-2.

	/	(2n-2)	/	/	
Entry →	E	/	/	Q	O
	@	/	@	Q	/
	A	E	S	T	C
	:	/	E	/	S
	S	"	:	T	C
	I	E	E	T	A
	U	/	E	/	:
	½	/	E	Q	B
	D	/	E	P	O
	R	J	/	/	Q
	J	R	/	/	/
	N	/	@	I	:
	F	/	½	Q	/
	C	/	R	I	J
	K	/	R	I	U
	T	/	½	U	U
	Z	/	@	P	/
	L	"	:	T	C
	W	@	E	T	A
	H	@	E	/	:
	Y	A	:	P	G
	P	R	/	/	T
	Q	E	E	/	:
	O	A	:	Q	G
	B	E	:	/	T

sets B7 = 2n-2 = initial value of 2i
modifies ith entry of directory to refer to tube 5: plants modified transfer instruction in /E and
corresponding writing transfer in EE
reads ith row of original matrix to tube 5
sets B6 = 2j = 2i
reduces j by 1 immediately
control number
reads jth row (j < i) of original matrix to tube 4
interchanges $[/\tfrac{1}{2} + 2i]$ and $[/R + 2j]$
returns jth row to magnetic store
reduces 2j by 2
tests if $j \geqslant 0$
transfers ith column of transposed matrix to magnetic store
tests if $i \geqslant 0$

(2) It is required to clear all lines of tube 7. Which is faster: to transfer an empty page from the magnetic store or to use a cycle of instructions such as the following?

	/	V	E	/	/
Entry →	E	/	/	Q	O
	@			T	:
	A	/	C	Q	A
	:	A	:	Q	G
	S	A	:	/	T

(3) Read from the tape a positive integer n punched in decimal form and terminated by a symbol $\geqslant 10$, and place in /U.

	/				
Entry →	E			O	:
	@	E	/	/	:
	A	/	U	/	A
	:	J	:	/	C
	S	/	U	/	N
	I	/	U	/	U
	U	F	/	T	N
Exit ←	½	?		/	H
	D	F	/	T	C
	R	F	/	/	C
	J	/	U	/	N
	N	@	/	/	P
	F	R	/	/	/
	C	/	/	/	/

clears accumulator
reads decimal or other character from input tape
shifts new character to least significant 5 digits of accumulator and partial sum to /U
tests for terminating symbol
multiplies previous partial sum by 10
= 10

(4) Punch a sequence of random decimal digits on the output tape. The sequence should be interrupted by stunt characters so that when read by an auxiliary printer the resulting layout will be as follows: blocks of 5 rows, each row being partitioned into 5 groups of 5 characters, each group being separated by a space and each block by a line feed.

Entry	/ / : / C	sets multiplicand	
	E S : Y O	sets row count	
	@ S : P O	sets group count	
	A S : Q O	sets digit count	
	: A / / W	random number	
	S / E T R	in range 0 - 15	
	I : E T N		
	U : / / H	tests if ≥ 10	
	½ : E T C		
	D I E T A		
	R I E / N	prints random	
	J / I	decimal character	
	N E E / :		
	F E : Q G		
	C : / / T	tests for end of column	
	K T :		
	T @ E / J	space	
	Z E E / :		
	L E : P G		
	W A : / T	tests for end of line	
	H T :		
	Y / E / J	carriage return	
	P E E / :		
	Q T :		
	O E E / J	line feed	
	B E E / :		
	G E : Y G		
	" E : / T	tests for end of block	
	M T :		
	X E E / J	line feed	
	V E E / :		
	£ ~~Z~~ : / P		

E	/ / / K		
E	/ / O L	car. ret.	
@	/ / / W	line feed	
A	/ / / Z	space	
:	/ / / R		= 10 units
S	/ / / /		
I	working		
U	space		

Coding Hints

1. Counting procedure One of the commonest operations is a sequence of instructions to be repeated a given number of times. The counting process may be done in the B tube e.g., in B7. The most usual scheme is as follows.

↓

Set B line

Sequence ←

Reduce B line

Test

↓

The number of times that the sequence is obeyed is greater by one than the number set in the B line. It is frequently desirable to subtract something other than 1. This may be because the number of repetitions is the result of a computation, and is given with some factor applied, e.g., a power of two. Alternatively it may be desired to save a line by setting the B line with some quantity already available, e.g., to count 12 one might set GC/J into the B-tube, and subtract ///E, the former being supposed an instruction which is used in any case, the latter available in PERM.

2. Omission of counting If the operation to be repeated contains rather few instructions e.g., three, and is crucial for the speed of the whole process, then it may be best to omit the instructions concerned with counting and to repeat the instructions of the process in question the requisite number of times. Sometimes the number of repetitions may be the result of calculation, but even then the omission of counting method may still be applied, the number of repetitions being controlled through a control transfer entering the sequence of repeated operations at the appropriate point.

3. Discrimination by control transfer When two cases have to be given quite different treatment, involving different sequences of instructions, it is natural to choose the relevant sequence by a test instruction (i.e., conditional transfer /T, /H, /M, or /O). When there is a large number the best method is to manufacture a control transfer number and use a directory. A good example of this is in the input programme where the seven warning characters each have to be given a separate treatment. This is dealt with on the following lines. If α is the character in question ~~α /// is set to B6, and the instruction /AUP given.~~ $\alpha+1$ /// is set to B7 and the instruction /AUP given. The lines ~~B~~ TA to £A contain the seven control transfer numbers appropriate to the ~~thirty-two~~ sixteen possible values of $\alpha+1 \geq T$ and the sequence required is immediately entered.

4. Changing sign in the accumulator The instruction DSTJ has the effect $A'_{\pm} = -1-A_{\pm}$ which for most purposes is as good as $A'_{\pm} = -A_{\pm}$ which can only be achieved in two instructions.

5. Clearing the accumulator The beginner is liable either to leave things in the accumulator to get mixed up with the next calculation or else to put in accumulator clearing instructions which could easily have been avoided. In fact it is very seldom necessary to give a special instruction for clearing the accumulator, if the points below are born in mind.

(a) Instruction TA clears the accumulator as well as transferring L to store. If both halves of the accumulator are required to be stored one can use /U twice and the accumulator will be cleared.

(b) If an expression of form a + bc is required and the accumulator is not clear the term a should be put into the accumulator first. This applies if the final value required will be in L, and rounding off multiplications with results taken from M.

(c) Alternatively when doing multiplications with results taken from M it is not necessary to clear the whole of the accumulator in advance, but only M. The maximum error will be 1 in either case: the mean square error will be one third with clearing but only one sixth without. If the results are taken out with /A then M remains clear for another multiplication.

6. Electronic space economy measures The economising of instructions in order to reduce the space occupied in the magnetic store is seldom worth while. There are however occasions when it is worth while to economise them to save space in the electronic store. This is nearly always in order to get the instructions either into one page or into two pages (see next chapter). To do so makes the routine tidier and usually has time economy effects. Some devices for reducing the number of instructions have been mentioned, but these will mostly be learnt by experience.

7. Duplication of use of lines The chief economy measure available other than reducing the number of instructions is the use of a line for more than one purpose. One or two forms of this have already been mentioned. It is usual for instance to use the 1st 2 characters of addressless instructions as control transfer numbers. Another case was mentioned under the heading of counting. No attempt can be made to list all such devices, but there are a number associated with control transfer numbers.

These are sufficiently numerous that it should nearly always be possible to avoid using any lines specially for control transfer numbers. To make a point of doing so in cases where it is not strictly necessary is, however strongly to be discouraged, as liable to lead to a most wasteful use of programmers time. The methods already known to be available are mentioned below.

8. Sandwiching If a (short) line is sandwiched between two sequences of instructions, the instruction which uses that short line can also be used as a control transfer number for the beginning of the later sequence of instructions. A long line can only be used in this way if it ends with a pair of characters representing a harmless instruction. Lines ending with T£, Z£, .., ££ are almost the only suitable ones.

9. Positioning of dummy stops If a dummy stop or other addressless instruction be placed immediately before a junction, (i.e., an instruction to which a control transfer is made, but which is not the beginning of a straight sequence) then this addressless instruction may be used as a control transfer number in the usual way, and the control transfer instruction may also be used as control transfer number for the transfer to the junction.

10. Relative control transfers For skipping ahead the relative transfers (/Q, /M,/O) are often more convenient to use than the direct ones as the necessary relative transfer number may sometimes be found in the powers of 2. Moreover in these cases the transfer instruction remains unaltered if the routine is coded for a different set of lines. This may prove useful when preparing routines for alternative pages or when correcting mistakes of omission.

11. Inaccurate numbers If a line pair is required to be known to more than twenty but less than thirty binary digits, the first two characters of the line pair used may be changed to a control transfer number. Likewise if a 20 digit number is required to less than ten digits a control transfer number may also be concealed in it.

Chapter 3

Programming

The coding of a complete problem, a programme, is normally achieved by working up from relatively simple requirements to more complex ones. An example will be used to illustrate the ideas involved, namely, the calculation of the period of a simple pendulum swinging through a finite angle.

Let a be the length of the radius vector from the centre of the circle to the particle and $\alpha = 2\pi R$ the maximum angle which it makes with the downward vertical. It can be shown (H, Lamb, Dynamics, p.107, 2nd edition, Camb. Univ. Press, 1942) that the period of the motion is

$$4\sqrt{\frac{a}{g}}\int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{\cos^2\phi + \left(\frac{1+\cos\alpha}{2}\right)\sin^2\phi}} = 4\,\frac{a}{g}\,K.$$

The ratio which this bears to the period of an infinitely small arc is thus $2K/\pi$.

The programme is arranged to read values of R from the input tape, calculate and print the corresponding values of $2K/\pi$. The complete elliptic integral K is computed by a repetitive method based on the Gaussian form of Landen's transformation (Whittaker and Watson, Modern Analysis, p. 533, 4th edition, Camb. Univer. Press, 1950),

$$\int_0^{\frac{\pi}{2}} (a_2^2\cos^2\phi + b_2^2\sin^2\phi)^{-\frac{1}{2}}\,d\phi = \int_0^{\frac{\pi}{2}} (a_1^2\cos^2\phi + b_1^2\sin^2\phi)^{-\frac{1}{2}}\,d\phi$$

where $a_2 = \frac{1}{2}(a_1 + b_1)$, $b_2 = \sqrt{a_1 b_1}$.

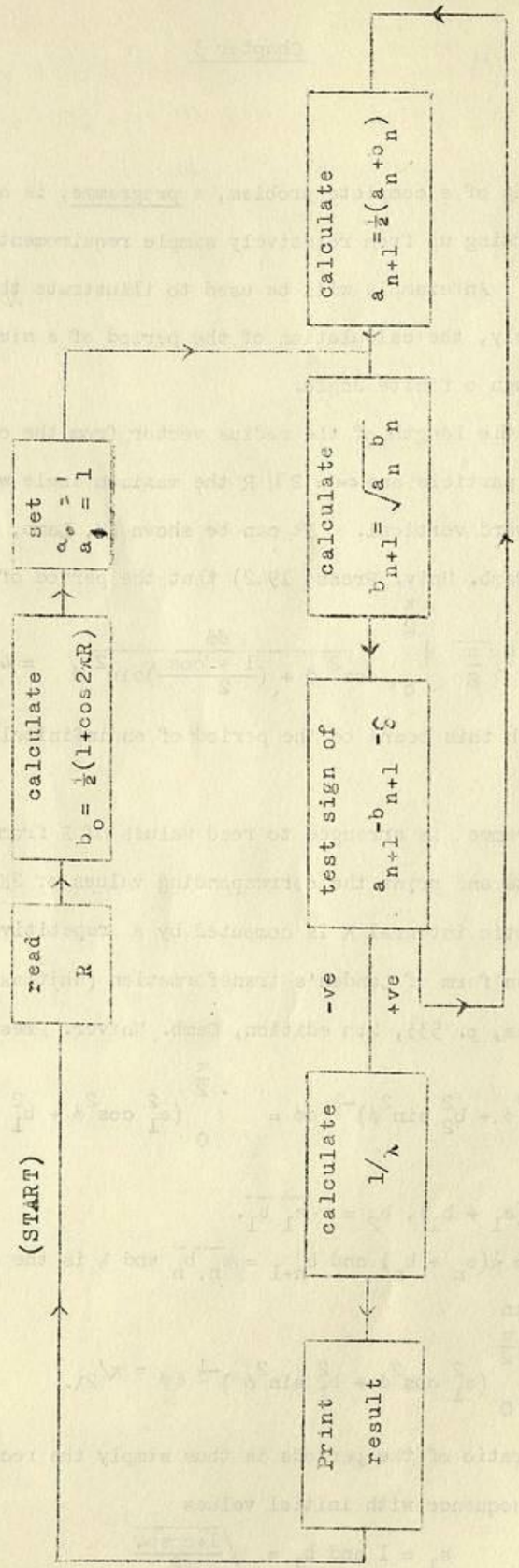
Thus if $a_{n+1} = \frac{1}{2}(a_n + b_n)$ and $b_{n+1} = \sqrt{a_n b_n}$ and λ is the common limit of a_n and b_n, then

$$\int_0^{\frac{\pi}{2}} (a_1^2\cos^2\phi + b_1^2\sin^2\phi)^{-\frac{1}{2}}\,d\phi = \pi/2\lambda.$$

The required ratio of the periods is thus simply the reciprocal of the limit of the sequence with initial values

$$a_1 = 1 \text{ and } b_1 = \sqrt{\frac{1+\cos\alpha}{2}}$$

One way of arranging the calculation is illustrated by the following flow diagram.



(The value of δ, a discriminatory constant, depends on the accuracy required.)

fig. 3.1.

Some of the steps represented are trivial needing only 2 or 3 instructions which can be written down at once in the main programme. For others one constructs a routine, a self contained sequence of instructions and constants for carrying out the process in question. A routine may extend to one or two pages of material. Arrangements are made for calling it in at the appropriate point or points in the calculation.

Using the above flow diagram as a basis one would start to make routines for

1, calculating cos $2\pi x$, given x

2, calculation of $\sqrt{x}$

3, reading a number punched on the input tape in decimal characters,

4, printing a number in decimal form.

Several questions immediately arise when designing such routines. For example: in which storage location will the cosine routine find the argument x and where will it place the result? What scale factor should be associated with the result.

What is normally required of a routine of this type is that a certain function of the content of one or more lines shall be calculated and stored in a given place, the rest of the content of the store being unaffected by the process, except certain named lines (which should be kept as few as possible) called the working space of the routine. Thus, e.g., the specification of the cosine routine might read as follows

Name: COSINE

Storage Space Occupied: Tube 1.

Effects: Replaces L by $\frac{1}{2}\cos 2\pi[L]_{\pm f}$.

Lines Altered: A, D, and B7, but no store lines outside tube 1.

Library routines

It is clear that certain processes will be common to almost all jobs, e.g., reading, printing, and division. Other processes for which it would be useful to have ready made routines are the following.

Functions: division, square root, cube root, nth root, trigonometrical, exponential, and logarithm.

Processes: linear algebra, integration.

If a library routine can be used then one is saved the task of coding an original routine and the subsequent business of eliminating errors. Moreover one can be sure that it is reasonably economical both in time of operation and storage space. If library routines are seldom used it is because, being built to meet general requirements, they are efficient for none.

The Master Routine

Consider the example already given. When the routines for the basic processes have been coded, or selected from the library, then it is necessary to arrange that they are called in in the correct sequence and that any intermediate bits of arithmetic or shunting operations that may be necessary are carried out.
This is the task of the master routine. The master routine is then a routine for calculating $1/\lambda$ and is said to use the basic routines as sub-routines. The master routine may itself be a sub-routine in a larger problem. And so on. Likewise the sub-routines of any routine may themselves have sub-routines. Eventually one arrives at a routine without sub-routines. These notions can be defined more precisely as follows. A routine A is said to use a routine B as a sub-routine if at some point the sequence of instructions of A are interrupted in order to carry out the instructions of B.

In order to return to routine A when B has completed its task it is necessary to preserve a record of the point of departure from A. This record is known as the link. If B itself uses a further sub-routine C then two links have to be preserved. And so on. At any stage in the programme the list of links forms a chain of control extending back to the master routine A. The link from B to A is said to occupy the first level; that from C to B, the second level; and so on. It is very seldom that more than two or three levels are involved.

Storage of routines

In the following conventional scheme for organising a complete programme the routines are kept on a half track or full track of the magnetic store, and transferred to the working (electronic) store when used. With each routine is associated a cue, a 40 digit line pair made up as follows.

Digits 0-9 give the control number, i.e., one less than the name of the line in which the routine starts.

Digits 10-19 are irrelevant.

Digits 20-39 give the magnetic half cue, the magnetic instruction which transfers the routine to the electronic store.

Example: S@//E@/@ is the cue of a routine which is transferred from the left half of track 65 to tube 1 and which starts with the instruction in line I@.

A page of material may contain more than one routine. In such cases the cues of these routines have the magnetic half cue in common.

When calling in a sub-routine the main routine, A, must give the cue of the new routine, B. The link, already defined, is the cue for the return from B to A.

The following outline gives the simplest form which the conventional scheme takes. Several refinements are possible in the interests of time economy but these will be described later.

1. All the routines are stored in the magnetic store on half tracks or full tracks according to their size.

2. Each routine is transferred to the electronic store when required; to tube 0 or tube 1, or tubes 0 and 1 in the case of two page routines.

3. To replace one routine by another and to manage the list of links is the purpose of a special sequence of instructions standing in lines JS to WS inclusive of tube 2. This is the routine changing sequence (R.C.S.). Each new routine is entered through this sequence and control returns to this sequence when the routine has completed its task.

The list of links (the cueue) is kept in line pairs HS, PS, etc.

4. The rest of tube 2 is occupied by the (now familiar) powers of 2 (lines /: to RS inclusive). All this material is called PERM. It is transferred from the magnetic store to tube 2 at the beginning of a programme and, as its name implies, remains there throughout the programme.

5. The rest of the working store, tubes 3,4,5,6, and 7, is available as numerical working space for the routines.

Calling in routines: the routine changing sequence

Each routine must call in the next which may be a sub-routine on the next level or a main routine on the previous level. A further relationship is possible however and is a consequence of limiting routines to two pages. (Most routines are less than one or two pages if only because they correspond to the "relatively simpler requirements"). Occasionally however, and this applies mainly to master routines, routines do extend to 3 or more pages. In such cases they are broken down into two (or possibly even more) routines, A and B, such that when A has completed its task it calls in B on the same level. In this relationship no link is involved: the new routine simply replaces the old. B is called an ad-routine of A.

Two further terms will now be defined. A closed routine is one which ultimately terminates by returning to a routine on the previous level. An open routine is one which terminates by calling in an ad-routine. The prefixes sub- and ad- describe a relationship between two routines whereas the terms open and closed describe a property of a particular routine. It is possible to use either an open or closed routine as a sub- or ad-routine.

The purpose of the routine changing sequence is to enable routines to be changed without too much preparatory bother in the routine being left. Thus to call in a sub-routine the following instructions are inserted in the main routine.

(M=0)

n-1	s, / J	M' = cue of new routine
n	n, Q O	copies itself into B7
n+1	F S / P	enters R.C.S.
n+2		← re-entry.

The line pair s contains the cue of the new routine. When the sub-routine has completed its task the main routine is re-entered at line n+2.

To call in an ad-routine the instructions are

(M=0)

n-1	s, / J	replaces cue of current routine
n	/ / E A	by cue of new routine
n+1	t, / P	enters R.C.S., where t contains the control number TS.

A closed routine is terminated simply by giving the instruction LS/P.

The significance of these instructions is revealed when the mechanism of the routine changing sequence is understood. This is:

entry for → sub-routines	J	@ / E A	adds new cue to list
	N	M £ £ £	= -4
	F	R S Q G	forms and plants control
	C	/ / Z Z	number part of the link
from closed	K	N S Z G	+4 } +2 adjusts B1.
routines →	T	A : Z G	-2 }
entry for →	Z	E / E :	transfers new routine
ad-routines	L	K S / G	dummy stop, control number
	W	/ / E P	enters new routine
	H	1st lev	CUEUE, or chain of
	Y	-el cue	cues and links
	P	2nd lev	
	Q	-el cue	

The chain of links is kept in line pairs HS, PS, OS, corresponding to the first, second, and third levels, etc. The number of levels involved is recorded in B1. Initially the (original) cue to the master routine stands in HS and B1 contains HSI/. When a sub-routine is called in the cue of the new routine is placed in PS. From n,QO, held in B7, the control line (n+1),QO is formed and copied into HS thus converting the original cue to a link. B1 is adjusted to read PSI/. Finally the magnetic instruction and the control number of the new cue are used to bring down and enter the new routine. Similarly at higher levels: whenever a sub-routine is called in [B1] is increased by 2; and subsequently decreased by 2 on exit from a closed routine.

In the case of an ad-routine the link is omitted and [B1] remains unaltered. B1 can only be used by a routine if this parameter is temporarily stored elsewhere and replaced before re-entering the R.C.S.

The /C stop was inserted to facilitate "peeping" (see Chapter 8): it enables one to make a visual examination of the routine transferred before its instructions are obeyed.

Further conventions

Design of closed sub-routines

It is now possible to answer some of the questions raised earlier about the design of sub-routines.

Lines altered by the routine should as far as possible be confined to the tube or tubes occupied by the routine itself. To achieve this use can be made of the lines occupied by instructions - after these have been used - as they will be refreshed when the routine is recalled from the magnetic store. Unless this is necessary however routines should be made self resetting so that they can be used as electronic routines(see below).

If in spite of these devices special working lines in other tubes are needed then use can be made of some of the locations j, j+2, j+4, etc., where j,I/ is the quantity recorded in B1.

All routines will make use of some information handed on by the routine it replaces. Thus function routines require an argument. Where should such parameters be located during the transition from one routine to another? The important cases are dealt with separately.

(a) A single 40 digit line is involved. This should be located in the lesser significant half of the accumulator. Thus if the quantity stands originally in VU (say) then the sub-routine can be entered with the sequence

n-2	V U T /	L' = x
n-1	s, / J	
n	n, 0 0	
n+1	F S / P	

(b) Two 40 digit lines are involved. The second line can be D. Alternate y one may wish to preserve both L and M during transition. Two cases arise.

(i)	s, T /	plants cue directly
	@ / Z A	
	set A	
n	n, 0 0	
n+1	K S / P	enters R.C.S. at line FS.

(ii) The second case arises where A is set by a previous sub-routine. (There is no difficulty in this because A is not altered when returning from a closed routine).

Fewer instructions are needed if the first sub-routine is arranged to dump the second result in (say) j+2 and the second sub-routine to recover it from this dump; or alternatively to use the dump directly as a working line. This suggests the convention that sub-routines should terminate with L as one result, M = 0, and the second result, if any, in the dump given by [B1] + 2, that is by [B1] + 4 in the main routine.

(c) If many lines are involved then these should be located entirely within a single tube, or tube pair, rather than overlap different tubes,

(d) Twenty digit parameters, e.g., names of lines, counters, can be located in B lines. (See convention below regarding the use of B lines in general.)

Electronic routines

No conventions have yet been given about the use of the tubes, 3, 4,5,6, and 7, beyond saying that they are available as working space for routines. If they are not needed in this role, then they may, as a time economy measure, be used for the (more or less) permanent storage of frequently used routines.

These electronic routines can be used in the usual way simply by inserting a dummy magnetic instruction (..£. see fig 1.2) in place of the magnetic half cue. It is necessary that such routines shall be self-resetting and designed for the tube in which they are to be stored.

In this connection it would be helpful if several versions of each library routine existed, one for each tube (other than tube 2).

The electronic routines must of course be transferred from the magnetic store initially. This can be arranged by the master routine. The question of how the master routine and PERM are initially transferred will be answered in the next chapter.

B tube conventions

These are:

(1) At the end of a routine BO = //// unless otherwise stated. Indeed any alteration of BO should be mentioned in the account of a routine.

(2) Alterations of B lines other than B7 must also be mentioned.

(3) Where a choice of B lines is available, B7 and the higher numbered B lines are to be preferred.

Cues

Cues are normally located in the routines in which they are used. Alternatively they can be assembled as an ordered list - a directory - in one of the tubes 3,4,5,6 or 7.

Replaceability conventions

Certain features of the machine are regarded as disadvantageous and it is desirable to leave open the possibility that these features might at some time be removed. This suggests the convention that no disadvantageous feature of the machine should be used in a library routine.

The features in question are mainly the function codes which are not defined in the table fig 1.3. By trial one can find out what their effects are but the maintenance staff are not responsible for the reliability of these functions. The exceptional nature of the line pairs at the end of an electronic pair is disadvantageous. Possible modification would consist in bringing these into line with the other line pairs.

The official account of a routine

The official account of a routine is designed to provide the user with all the information he needs to incorporate the routine into his programme. This information is divided in to the following sections. See, e.g., the routines of Chapter 5.

Name of routine Needed for identification purposes in verbal or written discussion. It should give some idea of the purpose of the routine.

Purpose A short description of the object of the routine.

Cue The information needed to form a cue for calling in the routine is given.

Magnetic storage: half track or full track; except for routines stored permanently on isolated tracks (see page 4.1) the word variable is also entered.

Electronic storage: the name of the tube or tubes in which the routine operates.

Control numbers: one less than the name of the line in which the routine starts. (There may be alternative entries).

Self-resetting or non-resetting: cross out which does not apply.

Link Describes how the routine is left, namely, whether it is closed or open, or whether further sub-routines are called in. Details of any ad-routines or sub-routines involved are given.

Input details See next chapter.

Effects

A complete account of the effects of the routine would amount to a set of equations giving the new content of any line altered. This might be tedious to compile but the conventions enable certain items to be omitted. No statement need be made about the new content of lines altered in the tube(s) containing the routine itself, or of B7 which is used by the R.C.S. Otherwise the information is given in a table supplemented by notes.

The Instruction Sheet

In addition to the official account the list of instructions is given. Explanatory notes are included where these are thought to have some educational value. The following conventions are observed when drawing up the list of instructions.

Lines or characters left blank indicate that they are neither relevant to, or altered by, the operation of the routine.

Lines altered are indicated thus: if the initial content of a line is irrelevant, then it is filled with the four characters USED; other lines altered are filled in with the initial content and the names of such lines are noted at the foot of the instruction sheet.

Control transfers are ruled in. Unconditional transfers of control are underlined.

Further conventions are introduced in the next chapter.

An example of a master routine

As an illustration of the conventions introduced so far the master routine for the problem given at the beginning of this chapter will be coded. The flow diagram, fig.3.1. is followed very closely. Routines for the basic processes are selected from the library routines described in Chapter 5. These routines are allocated magnetic storage as follows.

DECINPUT	20 R
DECOUTPUT	21 R
COSINE	22 R
SQUAREROOT	23 R
DIVISION	24 R

(There is no particular significance in the choice of right half tracks for the storage of these routines. Any half tracks, left or right could be used).

SQUAREROOT is used as an electronic routine being transferred to tube 1 by a separate instruction in the MASTER routine.

The question "how is the master routine started?" is postponed for the next chapter.

MASTER ROUTINE

/	Y / T :			/	V E / J	calls in DIVISION	
E	D S T /	sets $a_1 = 1$		E	E E Q O	$M'_f + L'_f = 1/x$	
@	@ I T A			@	F S / P		
A	Y E / J	calls in		A	W E / A	plants cue for	
:	: / Q O	DECINPUT: read R		:	Q E / J	DECOUTPUT: a further	
S	F S / P			S	@ / E A	alternative to those given	
I	W E / A	rejects M, if any		I	W E / J	on page 3.3.9	
U	B E / J	calls in COSINE:		U	N E P O	setting for 12 digits	
½	½ / Q O	$L'_f = \frac{1}{2} \cos 2\pi R$		½	½ E Q O	calls in DECOUTPUT	
D	F S / P			D	K S / P	to print or punch	
R	I S T I	$L'_f = \frac{1}{2}(1+\cos 2\pi R)$		R	F / / L	stop	
J	W E / A	rejects carry (if any)		J	D S / P	return to read new value of R	
W	X E / :	transfers SQUARE ROOT		N	J / / /		
F	/ / / P	routine to tube 1 jump to set $a_1 b_1$		F			
C	I S T /			C			
K	I S / C			K			
T	/ I / N	$M'_f = \frac{1}{2}(a_n + b_n)$		T			
Z	@ I / N	$= a_{n+1}$		Z			
L	@ I / C			L			
W	@ I / A	plants a_{n+1}		W	U S E D	dustbin	
H	/ I / N	$M'_f = a_n b_n$		H	U S E D		
Y	W E / U	$L'_f = a_n b_n$		Y	£ E		
P	" E / J			P	H / E @	cue to DECINPUT	
Q	O / Q O	$L'_f = \sqrt{a_n b_n} = b_{n+1}$		Q	£ E		
O	F S / P			O	Y E E @	cue to DECOUTPUT	
B	/ I T A	plants b_{n+1}		B	S @		
G	@ I T ½			G	P / E @	cue to COSINE	
"	/ I T N	$a_{n+1} - b_{n+1}$		"	£ E		
M	S : T N	$a_{n+1} - b_{n+1} - 2^{-37}$		M	£	(electronic) cue for SQUAREROOT	
X	R E / H	tests for convergence		X	Q / E @	mag-instr, for transferring SQUAREROOT	
V	/ I T /	setting for finding		V	£ E		
£	D S / C	reciprocal (see account of DIVISION).		£	O / E @	cue to DIVISION	

Strategical considerations

Conventions

The whole of what has been said about programming is an elaborate convention regarding the use of the machine and the user can ignore it altogether if he considers it desirable.

The conventions should not be regarded as absolute commands or prohibitions but rather as normal procedure any deviation from which must be noted in the descriptions of programmes and routines. Indeed they enable one to reduce considerably the lengths of official accounts since they allow a great deal to be taken for granted.

The basic plan

If the problem is a genuine numerical computation one must decide what mathematical formulae are to be used. For example if one were calculating the Bessel function $J_o(x)$ one would have, amongst others, the alternatives of using the power series in x, various other power series with other origins, interpolation from a table, various definite integrals, integration of the differential equation by small arcs, and asymptotic formulae. It may be necessary to give some consideration to a number of these taking into account the supply and demand of the economic factors involved. Thus a balance must be struck between the following incompatible desires.

To carry through the process as fast as possible on the machine.

To use as little storage space as possible.

To finish the programming as quickly as possible.

The state of the market for these economic factors will vary greatly from problem to problem. For instance there will be an enormous proportion of problems where there is no question of using the whole storage capacity of the machine, so that space is almost free. With other types of problem one can easily use ten million digits of storage and still not be satisfied. The space shortage applies mainly to working space rather than to space occupied by the routines. Since these usually have to be written down by some one this in itself has a limiting effect. Speed will usually be a factor worth considering, although there are many 'fiddling' jobs where it is almost irrelevant. For instance the calculation of tabular values for functions which are to be stored in the machine and later used for interpolation, would be in this class.

Programmers' time will usually be the main factor in special jobs, but it is relatively unimportant in fundamental (library) routines which are used in most jobs. Accuracy may compete with machine time, e.g., over such questions as the number of terms to be taken in a series, and with space over the question as to whether 20 or 40 digits of a number should be stored.

The available storage space must be allocated to various duties. This applied both to magnetic and electronic storage. It should be possible to estimate the space occupied by instructions to within (say) two tracks, for a large part will probably be previously constructed programme occupying a known number of tracks. The quantities to be held in the working space should, if possible, be arranged in packets which it is convenient to use at once, and which can be packed into a track or half track with the minimum waste of space. For instance when multiplying matrices it may be convenient to partition the matrices into four rowed or eight rowed square matrices and keep either two of the former kind in a half track or each of the latter kind in a full track. The duties of the electronic store are partly ruled by the conventions already introduced but there is still a great deal of freedom, e.g., tubes 3, 4, 5, 6, and 7 are available for working space or electronic routines.

If questions of time are at all critical the plan should include a little detailed coding of the inner loops because these are likely to consume most of the time.

The beginner will do well to ask for advice concerning plans. Bad plans lead to programmes being thrown away.

Coding the individual routines.

As with programming a whole problem a plan is needed for a routine. A convenient aid in this is a detailed flow diagram. The operations appearing as blocks may soon be replaced by actual instructions. It is not usually worth while at first to write down more than the last two characters of the presumptive instruction, i.e., the B-line and function parts.

These are quite enough to remind one of what was the purpose of the instruction. One may then write the instructions into a page deciding at the same time what are to be the addresses involved. Some of the finer points of this have been described in the last chapter.

Manoeuvring space It is seldom that one writes down a page of instructions for the first time without having forgotten one or two vital instructions. It is therefore considered desirable to aim not at pages which are chock-full, but ones which are (say) five-sixths full. The extra space is best left between sequences of consecutive instructions so that one sequence may be extended without interfering with another.

Alternative entries It is often necessary to have a number of routines differing in certain minor particulars. One convenient method is to use one assembly of instructions with different points of entry. The cues of these routines will then differ in their first ten digits. The routine DECOUTPUT provides an example of this. There is one alternative entry by which the carriage return and line feed operations are omitted, causing the number to be printed on the current line of the page.

Space economy measures

Some space economy measures were described in the last chapter. Their use may be justified for time economy reasons. By compressing the instructions into two pages means that they can be called in by a single magnetic transfer. Otherwise they must be arranged as two ad-routines involving extra magnetic transfers and consequent loss of time. Furthermore the fewer the instructions then the better chance there is of being able to use them as an electronic routine.

Chapter 4.

The INPUT Organisation

The routine TELEINPUT

This routine is stored permanently on magnetic track 16, which is isolated, i.e., so arranged that writing transfers to this track are impossible but reading transfers are unaffected. This prevents the routine being damaged by accidental writing transfers at any time. The material known as PERM is also isolated on track 17L (this involves isolating both halves of track 17).

The purpose of the TELEINPUT routine is to read into the machine information punched on tape in teleprint form. The routine is entered manually by setting the row of handswitches, H, to T/@/, pressing the key marked KEC, and then switching on the completion signals (see chapter 8). The KEC key clears all the electronic stores including the control tube and consequently when the completion signals are switched on the first instruction to be obeyed is in line E/. Since this is also empty the instruction is //// (modified by BO which is also empty), i.e., 'obey H as a magnetic transfer', which reads the TELEINPUT routine to tubes 0 and 1. Control now takes its instructions from this routine starting at line @/. The effects of the first three instructions are (i) to set B1 = HSI/ (ii) to transfer PERM to tube 2, and (iii) to transfer the content of track 15 to tubes 6 and 7. The purpose of the last operation is explained later. The first effects answers one of the questions raised in the last chapter, namely, how is PERM installed and the chain of links started? Line pair HS of PERM contains the cue to TELEINPUT. This is :///T/@/. TELEINPUT can be entered by this cue in the course of a programme, in which case the above effects are omitted, but the subsequent effects are the same as for manual entry. These are to scan the tape until one of the warning characters K,T,H,P,Q,G, or M is encountered. These characters are called warning characters and they designate the start of a meaningful sequence.

The length of the sequence and its effects depend on the warning character used. The warning characters K,T,M,G, and H are described first.

Warning character K

This permits the writing of instructions into any consecutive sequence of lines. The second and third characters specify the first line and the fourth character gives the number of lines involved. Thus, e.g., the effect of KZNAVKTAVS/E:TCC is to put VKTA into ZN, VS/E into LN, and :TCC into WN. If the fourth character is /, then the number of lines involved is 32. Thus to read a complete page of instructions into (say) tube 4 they can be punched as two meaningful sequences each of 32 instructions, namely,

K/½/ (1st column of 32 instructions)

K/D/ (2nd column of 32 instructions).

If the page is not complete but (say) the last instruction would stand in line PD if read into tube 4, then the second sequence would take the form

K/DO (2nd column of 23 instructions).

Whatever form they may take the set of meaningful sequences corresponding to a page of teleprint material is called the punching proper.

It need hardly be mentioned that extravagant effects are to be expected if this warning character is used to write over TELEINPUT or PERM itself, that is, into columns /,E,@, or A£ : or p-1 to 9 s

Warning character M (5 character sequence)

The next four characters are obeyed as a magnetic instruction if this is a transfer, otherwise there is no effect. The corresponding (write) check is also formed and obeyed immediately after the transfer. If the check fails, then a line feed is signalled to the printer and the transfer repeated until the check succeeds. The line feed is merely an acoustic means for drawing the operator's attention to the failure which may be due to a faulty track (see below) or, more commonly, to the write power being switched off (see chapter 8).

When the transfer has been successfully completed a /L stop is encountered before control returns to the input cycle. This enables the operator to examine the material transferred before more tape is read.

If the magnetic instruction is a check or a special, then there is no effect and reading of the tape continues. As in the case of warning character K applications involving tubes 0, 1, or 2 will result in novel effects.

Warning character T (3 character sequence)

The effect of Tab is to transfer control to the instruction standing in line ab + 1. with the accumulator clear. This is the principal means for quitting the TELEINPUT routine.

Warning character G (single character sequence)

The effect is to stop reading the tape if the /G stop is on; otherwise, or when surmounted, reading continues.

Warning character H (single character sequence)

The effect is to stop reading the tape and to enter a closed cycle of instructions (in tube 1) which operate the hooter (freq. 512 cycles). Uses of this warning character are covered by the remarks on page 1.27.

Example

The principal applications of the warning characters are illustrated by describing the preparation of the tape for the programme considered in the last chapter. The structure of the tape is illustrated by the following (conventional) block plan.

DECINPUT MH/DR	Console settings(See Chapter 8). H = T/@/
DECOUTPUT MY/DR	/G: on /L: off
COSINE MP/DR	write power: on other controls in the normal position.
SQUAREROOT MQ/DR	Flick K.E.C. Move C.S. to "on" position.
DIVISION MO/DR	(data tape) KHS@££//B/E/ TTS
MASTER MB/D½ H	NUMBERS

In the case of single page routines the punching proper corresponds to tube 4 or 5 according as to whether the routine ultimately operates in an even tube or an odd tube. Two page routines are punched for tubes 4 and 5.

Thus the MASTER routine is punched as follows:

K/½/Y/T:DST/.....DS/C,

K/DFVE/J.....J///,

KYDJ£E//H/E@.....O/E@.

Following each routine is the destination sequence, an M sequence which transfers the routine to the magnetic store. When the transfer is successfully completed the tape is advanced to read the next routine. Eventually the warning character H is encountered and the tape will stop and the hooter sound. The purpose of this is to make it necessary to operate KEC for the next step which is to pull the tape back to the beginning and run it in again, this time with the write current turned off.

If the material is read correctly both times, then the checks incorporated in the effects of the M sequences will still be successful. Thus the second run serves as an overall check both on the reading of the tape and the subsequent transfer to the magnetic store. When the H character is encountered for the second time the routine tape is replaced by the data tape and the KEC key is then flicked. In this way TELEINPUT is automatically re-entered (since H is unaltered) and continues to read the starting sequences. These replace the cue of TELEINPUT (in HS) by the cue of the MASTER routine and enter R.C.S. at line ZS to call in the MASTER as an ad-routine. (In view of the first instruction of the MASTER routine the content of A is irrelevant during this process.) Alternatively the MASTER routine can be called in as a sub-routine by means of the sequence

KPS@££//B/E/
TCS.

Such an entry is useful if it is necessary to return to TELEINPUT in the course of the programme in order to read in more material. In such cases part of the MASTER can be made up as a closed routine. The numbers following the starting sequence are punched in accordance with the directions given in the official account of DECINPUT (see Chapter 5).

It is customary to leave an inch or so of blank tape between the individual routines and to terminate the tape with a few £'s so as to indicate visibly which end is which. This last device is usually applied to the individual routine tapes in the library.

If the /L stop is on when the tape is read then the reading will be halted immediately after the M sequences are treated. By studying the monitors the operator can then see the material involved in the transfer. Operation of the single prepulse key will cause TELEINPUT to resume reading tape.

If the /G stop is on, then the machine will come to halt in the R.C.S. immediately after transferring the MASTER routine to tube 0.

If the single prepulse key is operated, then the programme will be initiated.

The magnetic directory

At this point it is necessary to introduce what has been a somewhat disagreeable feature of the machine in the past, namely, the unreliability of transfers involving certain tracks. The material transferred may gain or lose one or two digits during the process. The position was further complicated by the fact that the reliability depended on the material involved. Thus for a given track a certain routine could be transferred correctly but not others. Furthermore the behaviour of particular tracks varied from week to week, even from day to day. In these circumstances the successful operation of a programme requires frequent revision of the allocation of magnetic tracks. If the track references should happen to be scattered about the programme at random, then this would entail making alterations to a considerable length of tape. Clearly some means of confining these alterations to as short a length of tape as possible is essential.

The solution is to arrange for a list of recommended half tracks and tracks to be available on track 15. This magnetic directory can be set by relatively few sequences on a short length of tape. On the programme tape track numbers are replaced by references to the directory. For each routine these "false" lines are distinguished by means of sequences following the punching proper: they are examined and the reference looked up. The track number given in the directory is then substituted in the routine before this is transferred to the magnetic store. Here again the directory is referred to for the actual destination.

The means for operating in this way are provided by two further warning characters P and Q.

Warning character P (3 character sequence)

Let the content or the single line ab be pqrs. Then the effect of Pab is to replace pqrs by //rs + [pq]. Thus, e.g., if [D $\frac{1}{2}$] = VFT$\frac{1}{2}$ and [V F] = /CE/, then the effect of PD$\frac{1}{2}$ is to replace VFT$\frac{1}{2}$ by /CZ$\frac{1}{2}$. The line ab is referred to as a false line.

Warning character Q (5 character sequence)

This character is equivalent to M and P combined. The effect of Qabcd is to obey //cd + [ab] as a magnetic instruction - subject to the same provisions that apply in the case of warning character M.

The reader will recall one of the first effects of entering TELEINPUT manually, namely, to transfer the content of track 15 to tubes 6 and 7. This is to assist the purpose of the warning characters P and Q. The false lines and Q sequences are punched on the assumption that the directory entries are always standing in lines /N, EN, etc. Thus the directory can be written on track 15 at the beginning of the day, isolated, and thereafter will automatically be transferred to tubes 6, 7 whenever TELEINPUT is entered manually.

As an illustration of the directory technique the plan below shows the rearrangement for the programme already treated.

K/NIH/E/
Y/E/
P/E/
Q/E/ } directory entries
O/E/
B/E/
MK/$\frac{1}{2}$N ⟶ copies directory onto track 15.

DECINPUT Q/N$\frac{1}{2}$R	DIVISION Q:N$\frac{1}{2}$R
DECOUTPUT QEN$\frac{1}{2}$R	MASTER QSN$\frac{11}{22}$
COSINE Q@N$\frac{1}{2}$R	KHS@££//B/E/ TTS
SQUAREROOT QAN$\frac{1}{2}$R	

The MASTER routine is punched as follows.

```
K/½/Y/T:
   ....
     DS/C
K/DFVE/J
   ....
      J///
KYDJ£E//
     /N/@ ←
     £E//
     EN/@ ←
     £E//
     @N/@ ←        false lines
     £E//
     //£/
     AN/@ ←
     £E//
     :N/@ ←

PFD
POD
PGD
PXD
P£D
```

As a general rule it is advisable to use the directory technique if more than (say) half a dozen magnetic destinations are involved.

Preset parameters

The insertion of track names from a directory is one instance of the use of preset parameters, a term introduced by the EDSAC group to describe quantities associated with a routine which are not varied during the running of a programme but which may be varied in different applications of the same routine in other programmes. False lines and P sequences can also be used in these cases. For example it may be required to adjust the names of working lines and other constants of a routine in order to incorporate it into a particular programme layout. The parameters in question (20 digit lines) are placed in certain lines of tube 3 (by means of K sequences) so as to be standing in these lines when the P sequences following the routine are read from the tape. These then insert the parameters in the corresponding false lines of the routine.

Finally the routine is transferred to the magnetic store. This technique is useful when it is necessary to insert the same address digits into several different instructions. In other cases it may be simpler to follow the routine with K sequences which insert the parameters directly.

Preset lines are indicated on the instruction sheet as follows: false lines are filled in and the corresponding P sequences given at the foot of the sheet; in other cases the variable characters are replaced by stops, e.g., ..T/

Alterations

When solving differential equations it may be necessary to repeat a run with different initial conditions. If the new conditions depend on the judgement of the operator then the problem is to alter one or more constants in some particular routine which, with the rest of the programme, is already stored on the magnetic drum. This is done most readily by using TELEINPUT to read a short piece of tape bearing the correcting sequences which take the form

Mabcd

K....

Mabc'd

or, if using the directory, with the corresponding Q sequences. Here abcd is the magnetic instruction which transfers the material to be altered to (say) tube 4, and abc'd the corresponding transfer for rewriting the corrected version back to the magnetic store.

The correcting sequence may be followed by further sequences which restart the programme. If several hundred different runs are involved the successive alteration and restarting sequences may be assembled on a steering tape, arranging the programme so that at the end of each run control returns to TELEINPUT to read the next section of the tape.

Correction sequences also provide a means for correcting mistakes in routines during the development stage of the programme. Unless all the mistakes in a programme have been found it is not worth while reperforating the complete tape (see chapter 8) to correct only those mistakes found after one trial run on the machine.

Interludes

A sequence of instructions written into the electronic store can be entered directly by means of the warning character T. This facility is very useful when it is required to temporarily interrupt the action of TELEINPUT and transfer control to a separate sequence of instructions which have been read from the tape for some special purpose, e.g., to calculate a constant used in a routine, and which are then no longer required. Such a sequence should be terminated by the instruction S:/P which transfers control back to the reading cycle. This device is called an interlude.

TELEINPUT

	Column /		Column E	
	/ H S I /		/ S : / H	
Enter	E Z / / :		E E / T /	
→	@ / / Z O	sets B1	@ : A T D	forms & plants write check
	A E / / :	transfers PERM	A @ / T A	
	: C A / :	transfers DIRECTORY	: F / / :	obeys magnetic instruction
→	S X E O :	input cycle: tests for warning character, i.e., >15	S @ / / :	checks
	I S / / :		I / A Z :	signals line feed
	U V : / J		U ½ E / P	in case of failure
	½ S : / H		½ A E / L	
	D / / / A		D S : / P	
	R J : / C	& //// is formed and sent to B7, & being the character read.	P→ R / / P /	transfers false line to E/
	J / / / N		J E / T A	
	N / / / A		Q→ N F / T /	
K	F / / Q O		F I A T R	address given in false line is sent to B5
T	C / A U P	multiway switch	C @ / T A	
P	K S / / :		K @ / Y O	
Q	T V : / C		T / / Y /	
M	Z / / / A	reads next 2 characters, ab	Z E / T C	corrects false line
	L / / / N		L @ / T N	
	W S / / :		W F / T A	
	H / / / A		K H Y @ U P	2-way switch
	Y L @ U P	multiway switch	T→ Y H : / C	
Q,M→	P / / / N		P P / / / N	forms ab// and transfers this to B6
	Q S / / :	reads two further characters, cd, and forms abcd in E/	Q / / / A	
	O / / / A		O E / P O	
	B / / / N		B V : / C	
	G S / / :		G T @ U P	3-way switch
	" / / / A		K→ " S / / :	reads 4th character of K sequence and sets counter
	M T @ U P	2 way switch	M D S / J	
Q,M→	X / / T /	test if abcd represents a special or check instruction	X / / / U	
	V : A T R		V S / / :	
	£ O : T N		£ / / / A	

TELEINPUT
(cont.)

Column @

	/	/	/	/	N
	E	S	/	/	:
	@	/	/	/	A
	A	/	/	/	N
	:	S	/	/	:
	S	/	/	/	A
	I	/	/	/	N
	U	S	/	/	:
	½	/	/	/	A
P →	D	E	/	Q	O
	R	D	S	P	G
	J	£	£	P	Z
	N	V	:	T	N
	F	S	/	/	H
	C	S	:	/	P
H →	K	C	@	/	V
	T	K	@	/	P
T →	Z	E	/	/	P
	L				
	W				
	H				
	Y				
	P				
	Q				
	O				
	B				
	G				
	"				
	M				
	X				
	V				
	£				

reads and assembles the 4 characters of an instruction

transfers instr. to B7

transfers to store

adjusts counter

tests for end of sequence

returns to input cycle

hoot stop

quits TELEINPUT

Column A

control number directories

	/	G	E	W				
	E	T	@					
	@	H	E					
	A	H	E					
	:	/	/	N	/			
	S	/	/	H	/			
	I	£	£	/	/			
	U	D	E					
	½	J	E					
	D	H	E					
	R	Y	/					
	J							
	N	½	@					
	F	M	/					
	C	K	/	@	N			
	K	Y	/					
	T	C	/					
	Z	C	/					
	L	:	/					
	W	:	/					
	H	:	/					
	Y	C	@					
	P	:	/					
	Q	C	/					
	O	C	/					
	B	:	/					
G →	G	:	/	/	G			
	"	B	A	/	P			
	M	:	/					
	X	C	/					
	V	:	/					
	£	:	/					

2nd level, 3rd level, 4th level

constants

mag. instr: DIRECTORY to tubes 6,7.

(1st level) directory of control numbers

G stop

PERM

	:	/	/	/	/				
		E	/	/	/	E	/	/	/
		/	/	/	/	@	/	/	/
		@	/	/	/	A	/	/	:
		/	/	/	/	:	/	/	/
		:	/	/	/	S	/	/	½
		/	/	/	/	I	/	/	/
		½	/	/	/	U	/	/	T
		/	/	/	/	½	/	/	/
		T	/	/	/	D	£	£	£
		/	/	/	/	R	£	£	£
		/	E	/	/	J	@	/	E
		/	/	/	/	N	M	£	£
		/	@	/	/	F	R	S	Q
		/	/	/	/	C	/	/	Z
		/	:	/	/	K	N	S	Z
		/	/	/	/	T	A	:	Z
		/	½	/	/	Z	E	/	E
		/	/	/	/	L	K	S	/
		/	T	/	/	W	/	/	E
		/	/	/	/	H	:	/	/
		/	/	E	/	Y	T	/	@
		/	/	/	/	P			
		/	/	@	/	Q			
		/	/	/	/	O			
		/	/	:	/	B			
		/	/	/	/	G			
		/	/	½	/	"			
		/	/	/	/	M			
		/	/	T	/	X			
		/	/	/	/	V			
		/	/	/	E	£			

R.C.S.

link to TELEINPUT

Initial Input

It has already been explained that TELEINPUT and PERM are permanently isolated in the magnetic store. In addition a further page of material, the CONVERTER routine (see Chapter 5), is stored on the right half of track 17. It may be of interest to explain how all this material is put into the machine initially - an operation which is sometimes necessary in the event of an accident to the drum control circuits.

A simple input routine is first written into the electronic store manually with the aid of the console facilities (see Chapter 8). This is then used to read a tape on which the material to be isolated is punched as a single sequence of 256 lines. The details of this procedure are as follows.

1. The following instructions are written into tube 4 by the method described on page 8.

Column ½

	/	V	£	£	£	= -2
	E	Z	½	/	C	sets multiplicand
	@	H	½	Y	O	sets counter for 128 line pairs
	A	Z	½	P	O	clears B6
→	:	W	½	Q	O	
→	S	/	D	/	A	
	I	/	D	/	N	reads 8 characters and
	U	W	½	/	:	builds up a line pair
	½	L	½	Q	G	
	D	H	½	/	T	
	R	/	/	I	A	transfers line pair to store
	J	/	½	P	G	advances address in B6 by 2
	N	Y	½	Y	G	
	F	W	½	/	T	test for last line pair
	C	P	½	/	:	transfers TELEINPUT
	K	Q	½	/	:	" PERM and CONVERTER
	T	D	S	/	Q	dynamic stop
	Z	/	/	/	/	
	L	/	/	/	E	
	W	A	½	O	U	constants and
	H	:	½	£	A	magnetic instructions
	Y	/	/	E	/	
	P	T	/	R	/	
	Q	Z	/	R	:	

2. With the MAN-AUTO switch still set to MANUAL, control is transferred to the first instruction of INITIAL INPUT by obeying manually the instruction J½/P.

3. The tape is placed in the reader with the first character of the first line (H of HSI/) positioned over the reading holes.

4. The MAN-AUTO switch is reset to AUTO and the prepulses switched on.

INITIAL INPUT then reads the material first into tubes 0,1,2, and 3, and then transfers it to tracks 16 and 17. Finally a dynamic stop is entered.

Chapter 5

The Library Routines

In Chapter 3 the need for a library of standard routines was explained and some common processes were suggested for which it would be useful to have such routines. In this Chapter and Chapter 6, a provisional set of library routines is presented. The official account of each routine is accompanied by an explanatory list of instructions, ~~except in the case of two routines, ARCTANGENT and NATURALOG, for which a proposed specification only is given~~. All these routines have been tested on the machine.

The number of library routines has been kept as small as possible and no attempt has been made to provide alternative versions in which, e.g., emphasis might be laid on time or space economy measures. The number of "desirable" variations of any routine can be very large and individual variations are seldom used. If a standard routine is unsuitable, then a study of the instruction list will enable the user to adapt it for his own purposes.

In the writer's opinion it is far more important to build up a library of programmes to deal with complete problems. If these problems are chosen to be of a sufficiently general character, then many problems arising in physics and engineering can, when stripped of their dressings, be reformulated so as to be treated by a library programme. Except for the programme described in Chapter 7, which illustrates one general type of problem, the present Handbook stops at the point where this higher level of development should begin.

The description of a compound Tape containing these library routines is given at the end of Chapter 6

Name DIVISION

Purpose To divide giving result in the form $\frac{a}{b} = M_{\pm} + L_{f}$

Cue Magnetic storage: a single half track (variable)

Electronic storage: tube 1

Control number: £E

Self-resetting

Link Closed

Input details Conventional procedure

Effects

Name of lines altered	initial contents	final contents
D	$D_f = 1 > a \geqslant -\frac{1}{2}$ depending on whether it is set by /C or /K	irrelevant
L	$L_{\pm f} = b$	$M_{\pm} + L_{+f} = \frac{a}{b}$
M	irrelevant	

Time Approximately 90 ms.

Notes 1. Number of significant figures in result is about 2 or 3 less than the number of significant figures in a or b, whichever is the lesser.

2. As an illustration, to find the reciprocal of (say) $[/I]_{\pm f}$, the necessary instructions in the main routine are

n-3	D S / C	set a = 1 (approx.)
n-2	/ I T /	set b
n-1	s, / J	calls in DIVISION
n	n, Q O	
n+1	F S / P	

DIVISION

Column @

/	V A T A	plants divisor, b.
E	: S / N	$M' = a/4 = a_o$
@	V A T J	
A	M A / A	makes divisor +ve
:	V A / J	changing sign of dividend
S	V A T J	if necessary
I	V A / A	
U	V A / @	
½	G A / A	
D	A A Q O	
R	G A Q G	
J	G A Q G	multiplies b_o by 2^n such
N	E : / C	that $1 > 2^n b_o \geqslant \frac{1}{2}$: records
F	@ A Q G	address of 2^n in B7
C	/ : U C	
K	T :	
T	V A / N	
Z	V A T A	$[VA]_{+} = b_o$ $[VA]_{\pm} = b_o - 1 = c_o$
L	I S T /	sets round off
W	V A / K	repetitive cycle:
H	M A / J	$a_{n+1} = a_n - a_n c_n$, $a_n \to a/4 \cdot 2^n b$
Y	M A / D	$c_{n+1} = -c_n^2$, $c_n \to 0$
P	M A / A	
Q	V A / D	
O	A : / M	tests for end of process,
B	V A / A	namely, when $c_n^2 < 2^{40}$
G	" @ / P	
"	L @ T :	
M	/ : U C	multiplies result by
X	M A / F	4.2^n to correct for
V	M A / F	previous scaling
£	M A / F	

Column A

/	M A / F		
E	L S / P		closure
@	Q V £ £		= -41
A	I E / /		= 38
:			
S			
I			
U			
½			
D			
R			
J			
N			
F			
C			
K			
T			
Z			
L			
W			
H			
Y			
P			
Q			
O			
B			
G	U S E D		working lines
"	U S E D		
M	U S E D	a_n	
X	U S E D		
V	U S E D	b_n, $b_n - 1$	
£	U S E D		

The method is similar to that given in Chapter 2, except that here b_o is first multiplied by a power of 2 such that $1 > 2^n b_o \geqslant \frac{1}{2}$. In these circumstances the repetitive sequence converges rapidly, taking in the worst cases only 5 or 6 repetitions. The scaling in the quotient is corrected for.

Name SQUAREROOT

Purpose To find the square root of a number in the range $0 \leqslant x < 1$.

Cue Magnetic storage: a single half track (variable)

Electronic storage: tube 1

Control number: £E

Self resetting

Link Closed

Input details Conventional procedure

Effects

Lines altered	initial contents	final contents
L	$L_f = x$	$L_f = \sqrt{x}$
M	irrelevant	irrelevant
D	irrelevant	

Time Approximately 130 ms.

Notes Accuracy: the number of significant figures in the result is about one less than the number of significant figures in the argument. The case $x = 0$ is treated as $x = 2^{-40}$, the result being approximately 2^{-20}.

SQUAREROOT

Column @			Column A		
/	E : T D	plants x	/	M A T /	test if $b_n < 2^{-39}$:
E	V A T A		E	A : T N	if so process terminates
@	V A / @	M' = r	@	A @ / H	
A	C @ / I		A	V A T /	sets result
:	S A T R	L' = [r]	:	L S / P	closure
S	M A T A		S	V £ £ £	
I	/ : Q O		I	/ / / /	
U	M A Q G	B7' = -[r]	U	/ / / O	$= \frac{3}{4}$
½	U S U C	$D = 2^s$	½		
D	V A / N	$2^s x$	D		
R	V A T A	plants a_o	R		
J	V A / N	$2^{2s} x = c_o$	J		
N	D S T J		N		
F	E : T C	$L' = 1 - c_o = b_o$	F		
C	M A T A	plants b_o	C		
K	I S T /	sets round off	K		
T	M A / C		T		
Z	M A / N	$\rightarrow b_n^2$	Z		
L	G A / A		L		
W	V A / N	$\rightarrow a_n b_n$	W		
H	O A / A		H		
Y	G A / N	$\rightarrow b_n^3$	Y		
P	M A / A		P		
Q	M A / C	$D' = b_n^3$	Q		
O	: S / N	$\rightarrow \frac{1}{4} b_n^3$	O	U S E D	$a_n b_n$
B	I A / C		B	U S E D	
G	G A / N	$\rightarrow \frac{1}{4} b_n^3 + \frac{3}{4} b_n^2 = b_{n+1}$	G	U S E D	b_n^2
"	M A / A		"	U S E D	
M	I S / C		M	U S E D	b_n, b_n^3
X	O A / N	$\rightarrow \frac{1}{2} a_n b_n$	X	U S E D	
V	V A / J	$\rightarrow \frac{1}{2} a_n b_n + a_n = a_{n+1}$	V	U S E D	a_n
£	V A / A		£	U S E D	

working lines

The method is based on the repetitive sequences

$$a_{n+1} = a_n + \tfrac{1}{2} a_n b_n ,$$

$$b_{n+1} = b_n^2 \left(\tfrac{3}{4} + b_n/4\right),$$

Where $b_n \rightarrow o$ and $a_n \rightarrow \dfrac{a_o}{\sqrt{1 - b_o}}$

If $a_o = 2^s x$ and $1 - b_o = c_o = 2^{2s} x$, then $(a_o/\sqrt{c_o}) = \sqrt{x}$

By choosing s such that $1 > b_o \geqslant \frac{1}{4}$, rapid convergence is ensured.

Thus if r is the digital position number of L on entry, then $2s = 38 - [r]$, where $[r] = r$, if r is even and $[r] = r - 1$ if r is odd. The multiplier 2^s is selected from PERM thus

$$2^s = [E: + 2s] = [US + (2s - 38)] = [US - [r]].$$

Name COSINE

Purpose To calculate sines and cosines.

Cue Magnetic Storage: a single half track (variable)

Electronic storage: tube 1

Control numbers: (1) :@
(2) S@

Self resetting

Link Closed

Input details Conventional procedure

Effects

	lines altered	initial contents	final contents
entry (1)	L	$x = L_f$	$L_{\pm f} = \frac{1}{2} \sin 2\pi x$
entry (2)	L	$x = L_f$	$L_{\pm f} = \frac{1}{2} \cos 2\pi x$
	M	irrelevant	empty
	D	irrelevant	irrelevant

Time Approximately 70 ms.

Notes 1. Accuracy: max. error $< 16.2^{-40}$. For entry (2): if $L = 0$, then L' = M@@@@@@@K. If $L_f = \pm\frac{1}{2}$, then L' = ///////T.

Similarly for entry (1)

2. Since $L_{\pm f} = L_{+f} - 1$, $\cos 2\pi L_{\pm f} = \cos 2\pi L_{+f}$. Hence the sign convention of the argument is irrelevant.

COSINE

	Column @			Column A		
	/			/ V A / K		$[VA]'_f = (v - \frac{1}{2})^2$
	E			E V A / F		
	@			@ V A / A		
	A			A I S T /		$L_{\pm f} = -\frac{1}{2}$
	:			: S : / C		$L'_{\pm f} = 4(v - \frac{1}{2})^2 - \frac{1}{2}$
SIN→	S	: S T N	forms $x - \frac{1}{4}$	S V A / F		
COS→	I	V A T A	plants x	I V A / A		clears M
	U	V A / K		U L S / P		closure
	½	V A / F		½ T @		control number
	D	V A / E		D		
	R	V A / C	$[VA]'_f = \frac{\pi^2 x^2}{8}$	R		
	J	G A / N		J		
	N	V A / A		N E / / /		$= -\frac{1}{2}$ (suitably rounded)
	F	V A / C	$D'_f = \theta$	F / / / T		
	C	O A / J		C J Y R Y		$= \frac{1}{6} = 2^2/4!$
	K	M A / A		K R Y R S		
	T	U : Q O		T ½ D £ :		$= -2^3/6!$
	Z	M A / F	$[MA]'_f = \frac{1}{6} - \frac{\theta}{90} + \ldots$	Z X W H £		
	L	C A U J		L G / ½ A		$= +2^4/8!$
	W	M A / A		W / F / /		
	H	A : Q G		H @ N A O		$= -2^5/10!$
	Y	½ A / T		Y P £ £ £		
	P	M A / D	$M' = -\frac{\theta}{6} + \frac{\theta^2}{90} + \ldots$	P " C K :		$= +2^6/12!$
	Q	M A / A		Q / / / /		
	O	M A / F	$M' = -\frac{\theta^2}{6} + \frac{\theta^3}{90} + \ldots$	O L F V £		$= -2^7/14!$
	B	V A / J	$M' = u$	B £ £ £ £		
	G	V A / E	$M' = 2u$	G : £ I B		$= \frac{\pi^2}{8} - 1$
	"	V A / J		" M D K U		
	M	V A / C	$M' = 2u - u^2$	M U S E D		lines altered
	X	V A / ½	$= v$	X U S E D		
	V	N A / J	$M' = v - \frac{1}{2}$	V U S E D		
	£	V A / A		£ U S E D		

The method may be summarised as follows.

Let $L_f = x$; then

1. $\theta = \frac{\pi^2 x^2}{8}$
2. $u = 1 - \cos\sqrt{2\theta} = \theta - \frac{\theta^2}{6} + \frac{\theta^3}{90} - \ldots.$
3. $v = 2u - u^2 = \frac{1}{2}(1 - \cos 2\sqrt{2\theta})$
4. $\frac{1}{2}\cos 2\pi x = (2v - 1)^2 - \frac{1}{2}$

Name EXPONENT

Purpose To calculate the exponential function

Cue Magnetic storage: a single half track (variable)

Electronic storage : tube 1

Control numbers: E@

Self resetting

Link Closed

Input details Conventional procedure

Effects

lines altered	old content	new content
L	$L_{\pm f} = x$	$L'_{\pm f} = \frac{1}{2}e^x$
M	irrelevant	empty
D	irrelevant	irrelevant

Time Approximately 42 ms.

Notes (1) Accuracy: error 1.2×2^{-40}

EXPONENT

ENTRY	Column @		Column A	
	/		/ : A / U	puts result in L
	E		E L S / P	close
	@	: A T A	@	
	A	U : / C	A	
	:	: A / F	: U S E D	working
	S	: A / S	S U S E D	space
	I	: A / K	I T R Y R	$\frac{1}{3!}\ 2^{-9}$
	U	I S T I	U Y R / /	
	½	: A / A	½ J Y R Y	$\frac{1}{4!}\ 2^{-12}$
	D	/ : Q O	D R / / /	
	R	: A Q G	R L A Z ½	$\frac{1}{5!}\ 2^{-15}$
	J	: A Q G	J / / / /	
	N	N A / F	N E P S /	$\frac{1}{6!}\ 2^{-18}$
	F	R A / J	F / / / /	
	C	: A / A	C T U M O	w = -4
	K	: A / F	K H : N G	
	T	½ A / J	T £ P U Z	w = -3
	Z	: A / A	Z L V ½ Q	
	L	: A / F	L H K O A	w = -2
	W	I A / J	W K F Z H	
	H	: A / A	H O F J :	w = -1
	Y	: A / F	Y T S : L	
	P	G : / J	P / / / /	w = 0
	Q	: A / A	Q / / / T	
	O	: A / F	O P X : Z	w = 1
	B	@ S / J	B G G A C	
	G	: A / A	G L Y V M	w = 2
	"	: A / F	" " Q C N	
	M	: A / A	M I X Y T	w = 3
	X	: A / K	X Z M £ R	
	V	P A U N	V V F M Q	w = 4
	£	P A U J	£ N Z P D	

$A = 8n$, $M_{\pm} = w$
$L_{+f} = f$
$[:A]'_{+f} = f$
$D_f = [:A]_{+f}$
corrects $\bar{w}$ if $f \geqslant \frac{1}{2}$
$[:A] = w$ or $w+1$
$B7' = -2w$ or $-2(w+1)$

Builds up polynomial $1 + f.2^{-3} + \frac{f^2}{2!}2^{-6}$ etc. $= \exp\left(\frac{f}{8}\right)$ (see Example no.2.2)

forms $\exp\left(\frac{f}{8}\right) \times \frac{1}{2}\exp\left(\frac{w}{8}\right)$

Table giving $\frac{1}{2}e^{w/8}$.

The method is based on the addition law.

Let $8x = w + f$, w denoting the integral part and f the fractional part.

Two cases arise:

(i) if $\frac{1}{2} > f > 0$, then $\frac{1}{2}\exp x = \frac{1}{2}\exp\left(\frac{w}{8}\right)\exp\left(\frac{f}{8}\right)$

(ii) if $1 > f \geqslant \frac{1}{2}$, then $\frac{1}{2}\exp.x = \frac{1}{2}\exp\left(\frac{w+1}{8}\right) . \exp\left(\frac{f-1}{8}\right)$.

A table of 9 entries is used to find $\exp\frac{w}{8}$ or $\exp\frac{w+1}{8}$. The other factor is computed from the Taylor series,

$$\exp\frac{f}{8} = 1 + f.2^{-3} + \frac{f}{2!}2^{-6} + \frac{f^3}{3!}2^{-9} + \frac{f^4}{4!}2^{-12} + \frac{f^5}{5!}2^{-15} + \ldots$$

which converges rapidly, since in either case the argument lies in the range $(-1/16, +1/16)$.

Name CONVERTER

Purpose To convert the number in the accumulator interpreted in the form $M_{\pm} + L_f$ to the normalised floating binary form.

Cue Magnetic storage: a single half track (variable)

Electronic storage : tube 0

Control number : £C

Self resetting

Link Closed

Input details Conventional procedure

Effects

lines altered	initial contents	final contents
L, M	$x = M_{\pm} + L_f$	$L_{\pm f} = a$, $M_{\pm} = 2p$
D	irrelevant	

$x = a.2^p, \frac{1}{2} > |a| > 1/4$

Time Approximately 40 ms. in the case $M \neq 0$; otherwise approximately 45 ms.

Notes May be used to convert the results of the DIVISION and DECINPUT routines.

For this reason the routine is designed for tube 0.

CONVERTER

Column /			Column E		
/	E : T D	round-off	/	V E T A	
E	M E / S	copy L_f	E	V E / J	
@	V E / E	copy $M_{\pm}$	@	M E / N	
A	A : / M	tests sign of x	A	V E / U	$L_{\pm f} = a$, $M_{\pm} = 2p$
:	D S T J	changes sign if -ve	:	G E / J	
S	E : T C		S	E : T D	round off
I	O E / S	copies $\|x\|$=w+f	I	L S / P	close
U	G E / E		U	! E	
½	D S / J		½	I S / C	These instructions are obeyed if w=0 and f ≥ ½. In this case a = ½ and p = 1
D	U : / M	tests if w = 0	D	M E / N	
R	I S T I		R	M E / U	
J	C E / H	tests if f ≥ ½	J	V E / F	
N	M E T /	replaces $M_{\pm}$ by	N	V E / A	
F	V E T A	L_f and L_f by 0	F	A : / J	
C	M E T A		C	U E / P	
K	O E T /		K	: E / /	= 36
T	G E T A	replaces w by f	T	T @ / /	
Z	E : / Q		Z	/ / / /	= 80
L	T E T /	$L_{+} = 80$	L	N @ / /	
W	G E / @	$M = r$	W	/ / / /	= 76
H	P E / A		H		
Y	P E T C	forms	Y		
P	P E T C	0 + 2r - 76	P	U S E D	r
Q	L E T N	or 80 + 2r - 76	Q	U S E D	
O	G E T A		O	U S E D	f
B	K E Q O	B7 = 36	B	U S E D	
G	P E Q G	B7 = 36 - 2r	G	U S E D	w
"	P E Q G		"	U S E D	
M	/ : U C		M	U S E D	L_f
X	E : / O	These instructions multiply x by 2^{38-r}	X	U S E D	
V	D S U C		V	U S E D	$M_{\pm}$
£	V E / F		£	U S E D	

w ≠ 0

lines altered

~~Proposed Specification for a Routine to Calculate Logarithms~~

Name NATURALOG

Cue Magnetic storage : a single half track (variable)

Electronic storage : tube 1.

Control number : £E

Self resetting

Link Closed

Input details Conventional procedure

Effects

lines altered	initial contents	final contents
L	$L_{+f} = x > 0$	$M_{\pm} + L_{f} = \log_e x$
M	irrelevant	
D	irrelevant	irrelevant

Time : Should be 0 (0.1 sec.)

Accuracy: error ~~should be~~ 0 (2^{-29})

NATURALOG

Column @

/	:	A	T	A	$[:A]'_{+f} = x$
E	I	A	T	/	$[RA]'_{+f} = \frac{1}{64}\log_e\frac{3}{2}$
@	R	A	T	A	
A	:	A	/	J	
:	E	A	/	H	Test $x < \frac{1}{2}$
S			T	:	
I	:	A	/	C	$[:A]'_{+f} = x/2$
U	I	S	/	N	
½	:	A	/	A	
D	R	A	T	½	
R	A	:	/	Q	
J	R	A	T	½	
N	½	A	T	N	$[RA]'_{+f} = \frac{1}{64}\log_e\frac{3}{2} - \frac{p}{64}\log_e 2$
F	½	A	T	N	
C	R	A	T	A	
K	:	A	/	J	
T	:	A	/	J	
Z	X	@	/	H	$[:A]'_{+f} = \frac{1}{4}+2^{p-1}x$
L	:	S	/	J	
W	:	A	/	A	
H	A	A	Q	O	$[D7]' = 18$
Y	:	A	/	K	$[D]'_{\pm f} = -\frac{3}{4}\left[1-2^p\frac{2x}{3}\right]$
P	V	A	/	J	
Q	:	A	/	A	
O	:	A	/	F	
B	R	A	U	J	$[:A]'_{\pm f} = \frac{1}{64}\left[\log_e\frac{3}{2} - \log_e 2^p + \log_e 2^p\frac{2x}{3}\right]$
G	A	:	Q	G	
"	@	A	/	T	
M	:	A	/	A	
X	U	@	T	:	
V	F	:	/	C	$[D]'_{+} = 64$
£	:	A	/	F	$[M]_{\pm} + [L]_{+f} = \log_e x$

Column A

/	L	S	/	P	Closure
E	R	@	/	/	Control numbers
@	P	@	/	/	
A	L	/	/	/	18
:	U	S	E	D	
S	U	S	E	D	
I	/	P	K	:	$\frac{1}{64}\log_e\frac{3}{2}$
U	W	K	I	/	
½	£	V	Q	T	$\frac{1}{64}\log_e 2$
D	M	@	J	/	
R	U	S	E	D	
J	U	S	E	D	
N	/	M	Y	P	a_1
F	Y	R	Y	/	
C	/	H	I	C	a_2
K	M	O	Z	£	
T	/	O	F	U	a_3
Z	Z	H	N	/	
L	/	T	Y	B	a_4
W	C	J	W	£	
H	/	/	P	Q	a_5
Y	X	K	F	/	
P	/	/	½	£	a_6
Q	/	/	Z	£	
O	/	/	H	Q	a_7
B	@	Y	T	/	
G	/	/	O	D	a_8
"	V	L	N	£	
M	/	/	/	F	a_9
X	:	Q	V	/	
V	/	/	/	G	a_{10}
£	L	V	G	V	

Method:-

x is multiplied by 2^{p-1} where $\frac{1}{2} \leqslant 2^{p-1}x < 1$

$\frac{1}{64}\log_e\left(2^p \cdot \frac{2x}{3}\right)$ is computed by power series.

$$\log_e x = 64\left[\frac{1}{64}\log_e\frac{3}{2} - \frac{p}{64}\log_e 2 + \frac{1}{64}\log_e\left(2^p \cdot \frac{2x}{3}\right)\right]$$

ARCTANGENT

Column @					Column A				
/	/	A	T	A	/	U	S	E	D
E	/	A	/	J	E	U	S	E	D
@	A	:	/	M	@	U	S	E	D
A	D	S	/	J	A	U	S	E	D
:	C	@	/	H	:	U	S	E	D
S	V	A	T	/	S	U	S	E	D
I	:	A	T	A	I	E	O	F	A
U	/	A	/	K	U	N	V	S	R
½	/	A	/	F	½	O	:	Y	J
D	/	A	/	F	D	B	H	N	A
R	/	A	/	F	R	L	B	M	U
J	/	A	/	F	J	@	I	E	@
N	@	A	/	A	N	@	T	½	Y
F	@	A	/	C	F	V	Z	C	E
C	B	@	Q	O	C	"	£	/	:
K	I	A	U	J	K	£	S	:	E
T	:	A	/	½	T	Y	I	S	L
Z	:	A	/	A	Z	:	C	X	/
L	A	:	Q	G	L	C	/	T	R
W	:	@	/	T	W	D	½	O	/
H	/	A	/	K	H	P	@	S	G
Y	:	A	/	N	Y	R	½	W	/
P	:	A	/	N	P	O	L	I	/
Q	:	A	/	U	Q	@	P	F	/
O	L	S	/	P	O	A	M	J	R
B	P	/	/	/	B	T	V	U	/
G	M	@	T	/	G	"	G	B	E
"	L	S	/	P	"	O	N	A	/
M	/	/	/	/	M	N	I	/	M
X	/	/	/	O	X	Z	X	/	/
V					V	A	E	O	Q
£					£	O	A	/	/

$[/\ A]'_{\pm f} = x$

Test for case $x = -\frac{1}{2}$

$[:\ A]'_{\pm f} = \frac{a_{12}}{\pi}$ (a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})

$[@\ A]'_{\pm f} = 4x^2$

$[:\ A]'_{\pm f} = \sum_{0}^{12} \frac{a_i}{\pi}(4x^2)^i$

$= \frac{\text{arc tan } 2x}{2\pi x}$

$[L]_{\pm f} = \frac{1}{\pi}$ arc tan 2x

Closure

22.

$[L]_{\pm f} = -\frac{1}{4} = \frac{1}{\pi}\tan^{-1}(-1)$

Closure

$-\frac{1}{4}$

The arctangent is obtained by summing the Tchebychev polynomial except when $x = -\frac{1}{2}$ which is treated as a special case.

~~Proposed Specification for a Routine to Calculate Arctangents~~

Name ARCTANGENT

Cue Magnetic storage : single half track (variable)

Electronic storage : tube 1

Control number : £E

Self resetting

Link Closed

Input details Conventional procedure

Effects

lines altered	initial contents	final contents
L	$n = L_{\pm f}$	$L_{\pm f} = \frac{1}{\pi}$ arctan 2x
M	irrelevant	empty
D	irrelevant	irrelevant

Time : should be O (0.1 sec.)

Accuracy : error should be O (2^{-12})

Name DECOUTPUT

Purpose To output (i.e., to print and/or punch), with binary decimal conversion, the number in the accumulator interpreted as $M_{\pm} + L_f$.

Cue Magnetic storage: single half track (variable)

Electronic storage: tube 1

Control numbers: (1) £E, (2) U@

Non resetting

Link Closed

Input details conventional procedure

Effects

lines altered	initial content	final content
L	fractional part	empty
M	integral part	empty
D	irrelevant	
B6	n-1 (see below)	n-1 (unaltered)

The printer is first shifted to the figures position. Then the number $M_{\pm} + L_f$ is output (see note (3) below) in the following form

L W ± w. f Z,

w denoting the integral part and f the fractional part.

All unnecessary zeros are suppressed, i.e., both before and after the decimal point, which is always printed however.

The integral part may lie in the range $-2^{39} \leqslant M_{\pm} < 2^{39}$.

The fractional part is printed to n decimals (given by B6 on entry) unless it terminates beforehand. Thus if $n \geqslant 39$, then the fractional part is extended to its terminating decimal figure.

In the case of entry by (2) the operations of figure shift, carriage return and line feed are omitted.

Notes

(1) To output numbers as fractions on the + convention they should be sent to the accumulator with a T/ instruction. To obtain output on the ± convention a $T\frac{1}{2}$ instruction should be used.

(2) To output numbers as integers on the ± convention, clear the accumulator and add in with a /J instruction. Except for the space output ceases after the decimal point.

Examples L M	printed as
//////T£££££££	-.5
//////T///////	+.5
//////:S£££££££	-26.875
//////MG//////	+26.875
///////£££££££	-1.
///////////////	+.

(3) All operations connected with printing layout (i.e., space, carriage return, and line feed) are effected by sending the appropriate figure shift character (Z, L, and W) to M, and obeying special magnetic function T. This takes more space in the routine than merely obeying the special magnetic functions Z, L, and W, but has the advantage that if output is by punch, these characters are actually punched on the tape. Thus if the punched tape is subsequently read by an ordinary teleprinter (on figure shift), the layout originally planned is preserved.

This facility enables programmes in which printing takes a large proportion of the total time to be speeded up by using the punch output. The punch is more than twice as fast as the printer, and the tape produced can be printed out afterwards.

DECOUTPUT

Column @		Column A		
/	Q A / :	/	D A / H	
E	B A / A	E	N @ T F	tests for zero digit
@	N @ / J	@	Q A / M	
A	E A / :	A	A / T :	
:	V : / J	:	V : / C	
S	E A / :	S	N @ / N	outputs digit
I	@ @ / A	I	Q A T Z	(cancels suppression)
U	B A / J	U	Q @ / I	
½	/ @ P B	½	E A / :	
D	/ : Q O	D	J A T :	
R	A A / M	R	E : Q G	tests for end of digit cycle
J	D S T J	J	U A / T	
N	E : T C	N	D @ / J	outputs decimal point on 1st cycle; space on 2nd cycle
F	D S Q L	F	S A / J	
C	@ @ / A	C	E A / :	
K	Y A U J	K	A : / M	tests for end of output process
T	E A / :	T	Y @ T :	clears A
Z	: @ T A	Z	L S / P	closure
L	V A / C	L	R @ T A	
W	@ @ / N	W	Q A T B	prepares to output fractional part
H	@ @ / J	H	: @ / J	
Y	" A Q O	Y	/ @ Q O	
P	I @ / U	P	T A / P	
Q	½ @ / U	Q	I / H M	
O	M A / C	O		spare
B	I @ / N	B	U S E D	
G	I @ / U	G	U S E D	
"	½ @ / N	"	J / / /	= 11 (digit count for integral part)
M	½ @ / S	M	R / / /	= 10
X	N @ / E	X	/ / / /	
V	D S T J	V	£ P : T	= $2^{40}10^{-12}-1$
£	E : T C	£	B M S A	

/	figure shift
E	dumps M temporarily
@	carriage return
A	
:	line feed
S	
I	clears M
U	restores M
½	plants digit count
D	
R	tests sign of A
J	changes sign if -ve
N	
F	records change of sign
C	plants integral part
K	prints + or -
T	
Z	plants fractional part
L	(integral part) × $2^{40}10^{-12}$
W	
H	
Y	sets digit count for integral part
P	plants number to be converted
Q	
O	obtains next decimal digit
B	
G	
"	
M	
X	
V	tests for zero
£	remainder

Other lines altered: /@, @@ to F @ inclusive, and QA

Name SEQUENCE CONTROL for DECOUTPUT

Purpose To output a sequence of long signed numbers with automatic round off

Cue Magnetic storage: a single half track (variable)

Electronic storage: tube 0

Control number: ££

Non resetting

Link Closed. The SEQUENCE CONTROL uses DECOUTPUT as a sub-routine.

Input Details Conventional procedure: line TE involves a preset parameter, the magnetic half cue of DECOUTPUT. This is most simply arranged by following the punching proper with the sequence KTDE....

Effects

lines altered	initial contents	final contents
L	$[L]_0^{19}$ = see below $[L]_{20}^{39}$ = (2m-2)//	empty
M	irrelevant	empty
D	$2^{40} > \lambda \geqslant 0$	irrelevant
B6 B5	(n-1) /// (c-1) ///	unaltered irrelevant

$[L]_0^{19}$ = (ab):C or (ab):K according as to whether the numbers are to be treated on the + or the ± convention.

The SEQUENCE CONTROL calls in DECOUTPUT repeatedly to output a sequence of m signed numbers standing in line pairs ab + 2m-2, ab + 2m-4, etc., the last number standing in ab.

Each number is first multiplied into an empty accumulator by $\lambda + 1$, ($2^{40} > \lambda \geqslant 0$) and rounded off before calling in DECOUTPUT. The fractional part of each number is output to n decimal places, the round off quantity $\frac{1}{2}.10^{-n}$ being calculated from (n-1)///, given in B6 on entry.

The numbers are arranged in c columns (given by B5 on entry), the punching taking the form

$$LW \pm w_1 . f_1 Z, \pm w_2 . f_2 Z, \ldots\ldots\ldots\ldots, \pm w_c . f_c Z,$$
$$LW \pm w_{c+1} . f_{c+1} Z, \pm w_{c+2} . f_{c+2} Z, \ldots\ldots\ldots, \pm w_{2c} . f_{2c} Z.$$

By setting λ = £££££££££ the routine may be used to output whole numbers; $\lambda = 0$ corresponds to fractions, and λ = £££££//// enables numbers with a middle binary point (see page 2.5) to be treated.

SEQUENCE CONTROL

for

DECOUTPUT

Column /		Column E	
/	/ / H B	copies B4	/ A : / O — tests for new line
E	E / Y B	copies B5	E E / Y O — resets column count
@	/ : Y O	clears B5	@ R E Q O — alternative control number (includes c.r. and l.f.)
A	@ / T A	plants L	A @ / Z Z — plants control number part of cue for DECOUTPUT
:	@ / Q O	plants (ab) :C	: : E Q O — enters DECOUTPUT
S	B / Q B	or (ab) :K	S K S / P
I	A / H O	sets counter for sequence	I A : H G — tests for end of sequence
U	E : / N	copy λ into @/	U N E / T
½	@ / / S	$2^{40} > \lambda \geq 0$	½ / / H O — restores B4
D	D S / J	if $\lambda = 2^{40}-1$ (corresponding to integers), then the calculation of the rounding off constant is omitted.	D L S / P — closure
R	E : T C		R £ E
J	V / / H		J W E T C — rounds off (+ ve case)
N	W / T :		N P / / P
F	: / P B		F U @
C	Z E / C	calculates	C W E T A — sets rounding off
K	W E / N	$\frac{1}{2}.10^{-n}$, where n is	K N / / P — constant equal to zero
T	W E / A	the number of	T . . . @ — mag. instr. *
Z	E : P G	fractional digits to	Z G N I W — $= \frac{1}{10}$
L	N S / O	be output	L B N I A
W	: / P O	restores digit count	W / / / / — becomes $\frac{1}{2}10^{-n}$
H	D S Q O	plants electronic	H / / / T — (initially n=0)
Y	E / Z Z	link	Y
P	X / T :	clears A	P
Q	T E Q O	plants magnetic part of	Q
O	A / Z Z	cue for DECOUTPUT	O
B	U S E D	becomes (ab) :C or (ab) :K	B
G	@ / / N	multiplies [ab] by λ+1	G
"	E : / N		"
M	@ E / H	tests sign for round off purposes	M
X	W E T N	rounds off (-ve case)	X
V	F E Q O	control number*	V
£	E : Y G		£

lines altered

Other lines altered // to :/ inclusive

* control number part of cue for DECOUTPUT (omits c.r. and l.f.)

* mag. instr. for transferring DECOUTPUT to tube 1 (preset)

Name DECINPUT

Purpose To read from the tape and place in the accumulator a single number punched as an improper fraction.

Cue Magnetic storage: single half track (variable)

Electronic storage: tube 1

Control number: £E

Self resetting

Link Closed

Input details Conventional procedure

Effects

Lines altered: L, M, and D

Initial contents: irrelevant

Final contents: $M_{\pm} + L_f$ = number given by next few characters on tape. D = 4

The number is assumed to be punched as follows:

(characters < 22 in numerical value) ±w. f z,

which includes the form given by DECOUTPUT.

E.g., - 3¼ can be punched as (blank tape) -3.25Z.

Whole numbers can be punched with or without the decimal point. Thus +3 can be punched as (blank) +3.Z or, more simply, as (blank)+3Z.

DECINPUT and DECOUTPUT may be regarded as complementary routines, the one being used to reinput material produced by the other.

± punched in place of + or − cancels the entire number. Thus ±2.54 + 2.556Z will be read as: + 2.556

DECINPUT

	column @		column A	
/	£ E O :	searches tape	/ O A / J	adds new digit to
E	/ @ / :	for	E O A / A	partial sum
@	/ @ / H	character ≥ 16	@ M A / N	forms $2^3 10^{-(r+1)}$
A	@ S / J		A G A / A	$2^3 10^{-r}$
:	D S Q O	B7 -ve	: / @ / :	inputs next character
S	± @ / H	tests for - or ±	S " @ / H	tests for character < 16
I	/ : Q O	B7 +ve	I ½ A T :	clears accumulator
U	K A / J		U S : / C	D = 4
½	R @ / H	tests for +	½ O A / N	assembles complete
D	/ @ / P		D P A / J	number
R	W @	control number	R L S / T	tests if original sign char. was + or -
J	P A / A		J D S T J	changes sign of A
N	J : / C		N E : T C	if necessary
F	P A / N		F L S / P	closure
C	P A / U	digit cycle	C R / / /	= 10
K	C A T N	tests for	K / / / /	constant used in
T	Z A / H	decimal	T / / / I	testing for + char.
Z	C A T C	point, F.	Z Q @	control number
L	P A / C		L E : T N	
W	C A / N		W / @ / H	Tests for ±
H	/ @ / :	inputs next char.	H E : T C	reset accumulator
Y	½ @ / H	tests for character < 16	Y R @ / P	
P	P A T A		P U S E D	integral
Q	I A / P	omits cycle*	Q U S E D	part
O	I S T /	sets round off	O U S E D	fractional
B	O A / A	clears OA	B U S E D	part
G	V A / J	enters cycle dealing	G U S E D	$2^3 10^{-r}$
"	M @ / P	with fractional part	" U S E D	r=1, 2, etc.
M	@ A		M G N I W	$= 10^{-1}$
X	G A / C	$D_f = 2^3 10^{-r}$	X B N I A	
V	G A / A	$2^3 10^{-r} \times d_r \times 2^{-5}$, d_r	V F I W B	$= 2^3 10^{-1}$
£	G A / F	being the rth decimal digit	£ N I W B	

* omits cycle dealing with fractional part

Name SEQUENCE CONTROL for DECINPUT

Purpose To read from the tape a sequence of numbers and place them in consecutive line pairs in the store.

Cue Magnetic Storage: single half track (variable)

Electronic storage: tube 0

Control number: ££

Self resetting

Link Closed. The SEQUENCE CONTROL uses DECINPUT as a sub-routine (line pair Q/ contains the cue).

Input Details Conventional procedure. In the library version DECINPUT is given an electronic cue. This avoids the introduction of a track reference parameter and is also a time economy measure. DECINPUT must thus be standing in tube 1 when the SEQUENCE CONTROL is called in. The simplest way of arranging this is to store the two routines on the two halves of a single track and to arrange for the magnetic half cue of SEQUENCE CONTROL to transfer the full track, the SEQUENCE CONTROL to tube 0 and DECINPUT to tube 1.

Likewise the most convenient input procedure is to read the SEQUENCE CONTROL into tube 4, DECINPUT into tube 5, and then to transfer both of them to the full track in question.

Effects

lines altered	initial contents	final contents
L	$[L]_0^{19}$ = (ab)H4 $[L]_{20}^{39}$ = (2m - 2)//	empty
M	irrelevant	empty
D	$2^{40} > \lambda \geq 0$ (set by /C)	unaltered

The SEQUENCE CONTROL uses DECINPUT repeatedly to read from the tape a sequence of m signed numbers, transferring them to the consecutive line pairs ab + 2m-2, ab + 2m-4, etc., the last number read being transferred to ab. Each number, punched as described under DECINPUT, is read into the accumulator, multiplied by $(\lambda + 1)2^{-40}$, and L, the fractional part, transferred to the store.

The multiplier $(\lambda + 1)2^{-40}$ thus assumes the reverse role to that which it played in the CONTROL for DECOUTPUT, the relation being

$$(\lambda + 1)_{IN} (\lambda + 1)_{OUT} = 2^{40}.$$

Thus for fractions $\lambda_{IN} = 2^{40}-1$ and $\lambda_{OUT} = 0$; for whole numbers $\lambda_{IN} = 0$ and $\lambda_{OUT} = 2^{40}-1$.

The DECINPUT CONTROL may be used to reinput material produced by the corresponding DECOUTPUT CONTROL.

SEQUENCE CONTROL for DECINPUT

column /		column E
/	V / T A	/ U S E D
E	V / Q O	E U S E D
@	L / Q B	@ U S E D
A	X / H B	A U S E D
:	£ / H O	:
S	E : / N	S
I	V / T A	I
U	D S Q O	U
½	E / Z Z	½
D	Q / / J	D
R	R / Q O	R
J	F S / P	J
N	/ E / S	N
F	@ E / E	F
C	V / / C	C
K	/ E / N	K
T	/ E / U	T
Z	@ E / F	Z
L	U S E D	L
W	A : H G	W
H	B / / T	H
Y	X / H O	Y
P	L S / P	P
Q	£ E	Q
O	£	O
B	C S	B
G		G
"		"
M		M
X	U S E D	X
V	U S E D	V
£	U S E D	£

plants L

plants the (presumptive) transfer instruction

plants electronic link

calls in DECINPUT

multiplies $M_{\ } + Lf$ by $(\lambda + 1)2^{-40} \pm$, where $0 \leqslant \lambda \leqslant 2^{40}-1$

becomes (ab)HA, the transfer instruction

tests for end of sequence

restores B4

closure

(electronic) cue to DECINPUT

control number for re-entering R.C.S.

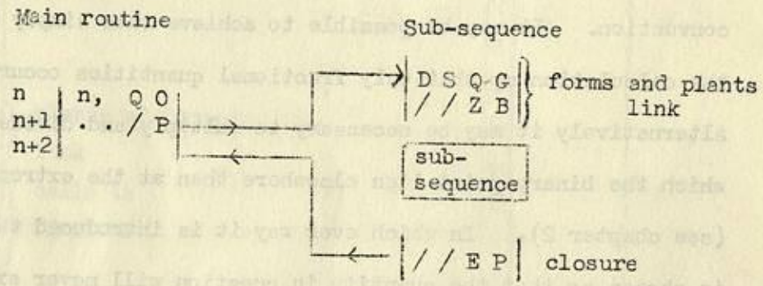
Chapter 6

Floating Operations

A necessary part of programming is to arrange the calculation so that all the quantities occurring remain within the capacity of the machine. The usual arrangement is to associate a scale factor with each quantity. Thus if a variable x takes values in the range $-10 < x < +10$, then $\frac{x}{32}$ may safely be represented on the $\pm$ fractional convention. It may be possible to achieve this simply by rearranging the calculation so that only fractional quantities occur. Alternatively it may be necessary to multiply and divide numbers in which the binary point lies elsewhere than at the extreme right hand end (see chapter 2). In which ever way it is introduced the scale factor is chosen so that the quantity in question will never exceed capacity and yet be represented at all times with an accuracy sufficient to ensure the desired precision in the results. It may not be possible to satisfy both requirements with a single fixed scale factor and it then becomes necessary either to use double length arithmetic, or to introduce an adjustable scale factor, i.e., a scale factor which is altered during the course of the calculation. It may be adjusted either continually in accordance with some criterion or it may take a preassigned set of values. For each problem there is probably a best alternative (in the sense of minimising time and space requirements) but for certain kinds of problems the determination of the optimum may not be worth the programming effort involved. For such problems a more general approach may be desirable. This will amount to associating every number occurring in the calculation with its own adjustable scale factor; the most convenient scheme being to represent numbers in the so called floating binary form $a.2^p$, where $|a|$ and p lie in given ranges. It is the purpose of this chapter to describe a scheme for carrying arithmetical and other operations on numbers expressed in this form.

The basis of the scheme is a sequence of instructions for performing the arithmetical operations of addition and multiplication, and a further sequence for normalising numbers.

These sequences are intended to be permanently available in the electronic store during those parts of the programme involving floating arithmetic. They may be regarded as electronic routines except that, for time and space economy reasons, they are not entered and quitted via the R.C.S. Instead a direct method is used which requires but two instructions in the main routine and three in the sub-sequence. The scheme may be represented as follows.



[The method is similar to that described on page 22 in "Preparation of Programmes for an Electronic Computer" by Wilkes, M.V., Wheeler, D.J., and Gill, S., Addison-Wesley Press, INC., Camb. Mass.]

In the present application a /Ꞇ stop is inserted immediately before the closure.

The following is a formal description of these electronic sequences.

NAMES (1) ADD & MULTIPLY (2) NORMALISE

PURPOSE To add, multiply, and normalise numbers expressed in the floating binary form.

CUE Magnetic storage: each sequence requires one half track

Electronic storage: NORMALISE occupies lines NI to WU inclusive of tube 3.

ADD & MULTIPLY occupies lines /½ to YD inclusive of tube 4.

The sequences must first be transferred to the electronic store before they can be entered. This may be done either in the master routine or from the input tape.

As explained above they are entered from the main routine by a couplet of the form

n	n, Q O
n+1	x, / P

,

where x refers to one of three possible control numbers within the sequences. In each case control encounters a /Ꞇ stop when the sequence has finished its task. On surmounting this stop control is returned to line n+2 of the main routine.

INPUT DETAILS Conventional procedure: NORMALISE being punched for tube 5, and ADD & MULTIPLY for tube 4.

EFFECTS It is convenient to introduce the following definitions.

Lines /I to JI inclusive are used as working space by both sequences. Of these line pairs ½I and RI are referred to as the floating binary accumulator (f.b.a); line pairs :I and II as the floating binary multiplicand register (f.b.d.)

Representation of numbers

(1) By the content of a single line pair s, s+1:

$[s]_p = a.2^p$, where $a = [s_0^{29}]_{\pm f}$, $\frac{1}{2} > |a| \geq \frac{1}{4}$,

and $2p = [s_{30}^{39}]_{\pm}$, $2^8 > p \geq -2^8$.

[s_0^{29} denotes the 29 least significant digits of line pair s,s+1; likewise s_{30}^{39} denotes the ten most significant digits]

(2) By the content of the accumulator:

$A_p = a.2^p$, where $a = L_{\pm f}$ and $2p = M_{\pm}$.

(3) By the floating binary accumulator:

$f.b.a = a.2^p$, where $a = [\frac{1}{2}I]_{\pm f}$ and $2p = [RI]_{\pm}$.

(4) By the floating binary multiplicand:

$f.b.d = a.2^p$, where $a = [\cdot I]_{\pm f}$ and $2p = [II]_{\pm}$

If in (2), (3), & (4), |a| and p lie in the ranges given in (1), then the number in question is said to be normalised or in normal form.

If the accumulator is clear, then a number represented in the form (1) can be converted to the form (2) simply by (signed) multiplication by 2^{10}. It is intended that 2^{10} be retained as a multiplicand (D) for this purpose. The reverse operation is only slightly more com-

plicated: instructions EU to ;U inclusive of NORMALISE illustrate this operation.

The detailed effects of the two sequences will now be described. For both sequences the list of lines altered should include B7 and the link line j, given by the first two characters of B1.

ADD and MULTIPLY entry $\binom{n,\ Q\ O}{Y\ D\ /\ P}$

line pairs altered	initial contents	final content
/I ⊗I ½I RI L M D	Irrelevant $f.b.a = a.2^p$ $A_F = b.2^q$ irrelevant	Irrelevant $f.b.a. = a.2^p + b.2^q$ $L = 0$ $M = 0$ 2^{10}

The sum is formed as follows.

If $q \geqslant p$, then $c\ 2^r = (a.2^{p-q} + b)\ 2^q$

If $q < p$, then $c.2^r = (a + b2^{q-p})\ 2^p$

The quantities $\frac{c}{2}$ and r are actually formed first: if then $\left|\frac{c}{2}\right| \geqslant \frac{1}{4}$ the result is left in the form $\frac{c}{2}.2^{r+1}$; if $\frac{c}{2} < \frac{1}{4}$, then the result is placed in the form $c.2^r$. Finally c is rounded by making the last digit 1.

Entry $\binom{n,\ Q\ O}{D\ \frac{1}{2}\ /\ P}$

Line pairs altered	initial content	final content
/I ⊗I ½I RI L M D	irrelevant $f.b.a = a.2^p$ $A_F = b.2^q$ irrelevant	irrelevant $f.b.a = a.2^p + (b.2^q) \times (c.2^r)$ $L = 0$ $M = 0$ 2^{10}

where $f.b.d = c.2^r$

The product $(b.2^q) \times (c.2^r)$ is first formed $(bc.2^{q+r})$ and then added to f.b.a as described above. If the sequence is entered by $\binom{n,\ £\ O}{D\ \frac{1}{2}\ /\ P}$, then f.b.a is replaced by the product, i.e.,

$$(f.b.a)' = A_F \times (f.b.d).$$

NORMALISE

Entry $\binom{n,\ Q\ O}{C\ U\ /\ P}$

line pairs altered	initial content	final content
⊗I	irrelevant	irrelevant
½I RI	$a.2^p$	$a'2^{p'}$ = normal form of $a.2^p$
L	irrelevant	$L_F = a.2^p$ (see note below)
M D	" "	clear 2^{10}

Before "repacking" the two parts of the (normalised) f.b.a. the magnitude of the exponent is tested and corrected as follows. If $p > 255$, then a dynamic stop in line UU is encountered; if $p < -256$, then L_F is replaced by a number approximately equal to 2^{-130}. (N.B., L_F denoted the 40 digit line L interpreted in the manner (1) above).

Representations by a single line pair divided in ratios other than 30 : 10 may be used by making the appropriate alterations to lines EU, IU, NU, TU, and ID. It is left as an exercise to the reader to determine the necessary changes for any given case.

Before using the sequences for the first time it is necessary to set $D = 2^{10}$ and to take account of the initial setting of f.b.a. The punching proper for the sequence NORMALISE includes the settings of the line pairs /I to JI. These are empty except for the f.b.a for which [½I] = ///////½, [RI] = //£££££ (p = -512). Thus for the purposes of comparison of exponents f.b.a. may be regarded as "clear".

INPUT and OUTPUT operations on numbers expressed in the floating binary form.

For input purposes the simplest scheme is to use DECINPUT to read each number into A in the form $M_{\pm} + L_F$, followed by the CONVERTER routine to transform it to the form (2) given above.

To output numbers with "reasonable" exponents the reverse operation can be used, namely to convert each number to the form $M_{\pm} + L_F$ and then to use DECOUTPUT. The conversion is very simply achieved. E.g., to convert the f.b.a

	T :	Clears A
	½ I / K	number part as multiplicand
	R I Q O	
	/ T	tests sign of exponent
(numbers > $\frac{1}{4}$) E : U N	½ S U N	
/ P	@ I / A	@I used as a working line — numbers < $\frac{1}{4}$
	@ I T ½	
		calls in DECOUTPUT

If it is desired to output correctly numbers with exponents < -20, then a more elaborate set of instructions is needed such as, e.g., is found in COSINE/F.

The Calculation of functions expressed in the floating binary form

Routines are available for calculating the common functions for floating binary arguments. These form a very useful supplement to NORMALISE and ADD & MULTIPLY. They are all designed to replace x = f.b.a. by its corresponding function value, f(x). The following is a formal description of these routines:

Function routines

Names: RECIPROCAL/F, $f(x) = 1/x$

SQUAREROOT/F, $f(x) = \sqrt{x}$

COSINE/F, $f(x) = \cos 2\pi x$

EXPONENTIAL/F, $f(x) = \exp x$

NATURALOG/F $f(x) = \log_e x$

ARCTANGENT/F $f(x) = \frac{\arctan x}{\pi}$

Purposes: To replace f.b.a (not necessarily normalised) by the corresponding function vlaue in normal form.

Cue: Magnetic storage: variable half track } plus the storage requirements of auxiliary fixed point sub-routine (see below).

Electronic storage: tube 0

Control numbers (1) ££ (2) E/ (see note (1))

Self resetting.

Link: Closed. Each routine (except RECIPROCAL/F) makes use of the corresponding fixed point routine (as given in Chapter 5) as a sub-routine. Each routine also makes use of the sequence NORMALISE; and ARCTANGENT/F makes use of ADD & MULTIPLY. These sequences must therefore be available in tubes 3 or 4 when the routine is called.

Input Details: In the library versions of these routines the corresponding fixed point sub-routine is given an electronic cue. This avoids the introduction of a track reference parameter and is also a time economy measure. The sub-routine must thus be standing in tube 1 when the main routine proper is called in. The simplest way of arranging this is to store the main routine and the sub-routine on the left and right halves of the same track and to arrange for the magnetic half cue of the main routine to transfer the full track, the main routine to tube 0 and the sub-routine to tube 1.

Likewise the most convenient input procedure is to read the main routine into tube 4, the sub-routine into tube 5, and then to transfer both of them to the magnetic store by means of a single destination sequence referring to the full track in question.

If subsequently, it is decided to allocate a permanent place in the magnetic store to any of these routines, then both parts will be stored on a single track as described above.

Effect

lines altered	initial content	final content	
/I	irrelevant	irrelevant	(see note 2)
@I	"	"	
½I	f.b.a = x	f.b.a = f(x)	(see note 1)
RI			
L	irrelevant	L_F = f(x)	
M	"	empty	
D	"	2^{10}	
plus any further lines which may be altered by the corresponding fixed point sub-routine.			

Notes. (1) Each routine begins and terminates by normalising the f.b.a. If the argument is known to be in normal form then the first operation can be omitted by entering the routine with control number (2); otherwise control number (1) is used.

(2) Applies only to the routine ARCTANGENT/F which uses ADD & MULTIPLY.

Method, Accuracy, Time: These details are given below under the individual routine headings. In all cases the time given is approximate and does not include the time for the initial normalising operation nor the time spent in the R.C.S.

RECIPROCAL/F

Method $1/(a.2^p) = [(\frac{1}{4})/_{2a}]\ 2^{3-p}$, the ratio being calculated by the usual repetitive sequence.

Accuracy The number of significant digits in the result is almost the same as that in the normalised argument, except where this is the maximum (40), in which case about two significant digits are lost.

Time Approx. 130 m.s.

SQUAREROOT/F

Method $\sqrt{a.2^p} = \sqrt{a}.2^{p/2} = \frac{\sqrt{a}}{2}\ 2^{(\frac{p}{2}+1)}$ if p is even,

$= \frac{\sqrt{a}}{\sqrt{2}}\ 2^{(\frac{p}{2}+\frac{1}{2})}$ if p is odd.

The square root is calculated by SQUAREROOT.

Accuracy Remarks similar to those given in the case of RECIPROCAL/F apply here also.

Time Approx. 70 m.s. + that required by SQUAREROOT.

NATURALOG/F

Method $\text{Log}_e\ (a.2^p) = p \log_e 2 + \log_e a$

$$= \left(\frac{p \log_e 2 + \log_e a}{2^9}\right) \times 2^9$$

This result is then normalised.

NATURALOG is used to calculate $\log_e a$

Accuracy The number of significant digits in the result is approximately equal to (30 + the number of digits in p)

Time 65 m.s. + that required by NATURALOG

COSINE/F

Method The relations $\cos 2\pi(M_{\pm} + L_f) = \cos 2\pi L_f = \cos 2\pi L_{\pm f}$ are used. The argument is first converted to the form $M_{\pm} + L_f$, the integral part is rejected, and COSINE used to find $\frac{1}{2} \cos 2\pi L_{\pm f}$.

Accuracy The number of accurate digits in the result before it is finally normalised is approximately 40-p. The number of significant digits depends on the magnitude of the result.

Time 70 m.s. + that required by COSINE.

EXPONENTIAL/F

Method There are two cases:

(1) $p < 0$, $\exp\ (a.2^p) = \frac{(\exp\ (a.2^{-|p|})}{4}.\ 2^2$

(2) $p \geqslant 0$, $\exp\ (a.2^p) = \left(\left[\frac{\exp a}{4}\right].2^2\right)^{2p}$.

In both cases EXPONENTIAL is used to find the function: in the former case the argument is first denormalised; in the latter case the result is squared p times and put directly into normal form after each squaring.

Accuracy In case (1) p significant digits are lost.

In case (2) the sum of the significant digits in the exponent and the numerical part is roughly constant.

Time In case (1): 100 ms + that required by EXPONENTIAL

In case (2): (70 + 18p) ms + that required by EXPONENTIAL.

ARCTANGENT/F

Method There are two cases:

(1) if $p < 2$, then $\frac{1}{\pi} \arctan\ (a.2^p) = \frac{1}{\pi}\ [\arctan\ 2(a \times 2^{-|p-1|})].2^0$

(2) if $p \geqslant 2$, then $\frac{1}{\pi} \arctan\ (a.2^p) = \frac{1}{\pi}[\frac{1}{2} - \arctan 2(\frac{1}{2a.2^p})].2^0$

In both cases ARCTANGENT is used to calculate the (principal) value of the function. In the second case the reciprocal of the argument is first found and then denormalised as in case (1).

Accuracy $|p|$ significant digits are lost.

Time In case (1): 50 ms + that required by ARCTANGENT

In case (2): 150 ms + that required by ARCTANGENT

NORMALISE

Column I

	/	U S E D	
	E	U S E D	
	@	U S E D	
	A	U S E D	
	:	U S E D	number
	S	U S E D	part f.b.d.
	I	U S E D	exponent
	U	U S E D	
	½	/ / / /	number
	D	/ / / ½	part f.b.a.
	R	/ / £ £	exponent
	J	£ £ £ £	
Enter →	N	½ I T ½	
	F	E : / M	plants modulus of
	C	D S T J	number part of f.b.a.
	K	E : T D	in @I
	T	@ I T A	
	Z	@ I / @	most significant digit of [@ I]
	L	@ I / A	
	W	D S Q G	plants link
	H	/ / Z Z	
	Y	Z U Q O	
	P	@ I Q G	B7' = 36 - 2r
	Q	@ I Q G	
	O	/ : U C	selects multiplicand
	B	E : / O	from PERM
	G	D S U C	
	"	½ I / F	normalises number
	M	½ I T A	part
	X	@ I T K	corrects exponent
	V	R I T C	2p' = 2p - 2(38 - r)
	£	L U T N	

Other line pairs altered: ½I, RI.

NORMALISE

Column U

	/	R I T A	plants corrected exponent
	E	H : / C	
	@	½ I / N	packs up number part
	A	@ I / U	and exponent as L_F
	:	R I / F	
	S	I I Q O	tests magnitude of
	I	W : O G	exponent : if p ≤ -256,
	U	D S / O	then stops; if p < 256
	½	T U Q G	then L_F is replaced by
	D	E : / O	"zero"
	R	O I T /	
	J	@ I / A	clears M
	N	Y : / C	restores multiplicand, 2^{10}
	F	E : T D	rounds off
	C	J I / ↯	dummy stop
	K	/ / E P	returns to main sequence
	T	/ / £ £	-1024
	Z	: E / /	36 constants
	L	N @ / /	76
	W	/ / / /	
	H		
	Y		
	P		
	Q		
	O		
	B		
	G		
	"		
	M		
	X		
	V		
	£		

ADD & MULTIPLY

Column $\frac{1}{2}$

	/	£	£	V	O	
(X)→	E	I	I	/	J	$M' = 2(q + r)$
	@	/	I	/	U	dumps b
	A	@	I	/	U	dumps 2(q+r)
	:	/	I	/	K	forms rounded
	S	:	I	/	F	product bc
	I	I	S	T	I	
	U	/	I	/	U	$L'_{\pm f} = bc$
	$\frac{1}{2}$	@	I	/	J	$M'_{\pm} = 2(q + r)$
(+)→	D	/	$\frac{1}{2}$	Q	G	plants link & tests whether
	R	/	/	Z	Z	entry instruction was
	J	H	$\frac{1}{2}$	/	T	n,QO or n,£O → to WD
	N	/	I	/	U	dump number part (b)
	F	@	I	/	U	dump exponent (2q)
	C	@	I	Q	O	forms 2(q - p)
	K	R	I	Q	G	
	T	I	S	/	C	$D' = \frac{1}{2}$
	Z	T	D	/	T	tests: jump if $q \geqslant p$ → to RD
	L	$\frac{1}{2}$	I	/	F	$M'_f = a/2$
	W	/	I	/	K	$D' = b$
	H	L	D	Q	G	forms in B7 the address
	Y	S	:	/	O	of $2^{-\lvert p-q \rvert-1}$
	P	Z	D	Q	G	
	Q	A	:	/	O	
	O	A	:	/	Q	case: $\lvert p-q \rvert \geqslant 40$
	B	/	/	/	O	
	G	/	:	U	N	forms sum
	"	$\frac{1}{2}$	I	/	A	plants sum
	M	$\frac{1}{2}$	I	T	K	double
	X	D	I	Q	O	tests if $\lvert sum \rvert \geqslant \frac{1}{4}$
	V	B	$\frac{1}{2}$	Q	G	if so, then twice the sum is used
	£	S	:	/	O	

ADD & MULTIPLY

Column D

from $£\frac{1}{2}$	/	R	I	T	$\frac{1}{2}$	
	E	A	:	T	C	corrects exponent $2p' = 2p + 2$
	@	R	I	T	A	
	A	$\frac{1}{2}$	I	T	/	replace original sum
	:	E	:	T	D	rounds off
	S	$\frac{1}{2}$	I	T	A	
	I	Y	:	/	C	restores multiplicand
	U	:	D	/	Y	
	$\frac{1}{2}$	/	/	E	P	
	D	W	$\frac{1}{2}$			control number
	R	@	I	T	/	
from $J\frac{1}{2}$	J	R	I	Q	O	forms
	N	@	I	Q	G	2(p - q)
	F	R	I	T	A	replaces p by q
	C	/	I	/	F	$M'_f = {}^{b}/_{2}$
	K	$\frac{1}{2}$	I	/	K	$D' = a$
	T	D	D	/	P	
from $T\frac{1}{2}$	Z	Q	V	£	£	constants
	L	G	V	£	£	
	W	R	I	/	E	these instructions are obeyed in the case
	H	U	D	/	P	where the product is to replace the f.b.a.
	Y	$\frac{1}{2}$	$\frac{1}{2}$			control number for + entry
	P					
	Q					
	O					
	B					
	G					spare
	"					lines
	M					
	X					
	V					
	£					

these instructions are obeyed in the case $q \geqslant p$: they reverse the roles of addend & augend

RECIPROCAL/F

Column /

/	/ / Q O	normalise argument
E	C U / P	
@	B / T K	exponent of result
A	R I T N	$2p' = 2(3 - p)$
:	R I T A	
S	: S T /	
I	½ I / J	
U	B / / M	tests sign of divisor
½	D S T J	if divisor -ve changes
D	E : T C	sign of divisor and
R	E : / J	dividend
J	½ I / S	plants $a_o = \pm\frac{1}{4}$
N	@ I / E	doubles divisor: $M_{+f} = 2a$; $M_{\pm f} = 2a-1 = c_o$
F	@ I / J	
C	@ I / A	plants c_n
K	@ I / K	$a_{n+1} = a_n(1-c_n)$
T	½ I / D	
Z	½ I / J	
L	½ I / A	plants a_{n+1}
W	@ I / D	$c_{n+1} = -c_n^2$
H	E : / M	tests for end of process
Y	" / / P	
P	P / Q O	normalises result
Q	C U / P	
O	L S / P	closure
B	A / / /	
G	/ / / /	
"	F /	control number
M		
X		
V		
£		

SQUAREROOT/F

Column /

/	/ / Q O	normalises argument, $x = a.2^p$
E	C U / P	
@	½ I T /	sets argument for SQUAREROOT
A	D S Q O	plants electronic link
:	E / Z Z	
S	" / / J	calls in SQUAREROOT,
I	I / Q O	$L'_f = \sqrt{a}$
U	F S / P	
½	½ I T A	
D	: S / C	if p is even, then $A'_{39} = 0$, $[RI]'_{\pm} = \frac{p}{2}$
R	R I / F	if p is odd, then $A'_{39} = 1$, $[RI]'_{\pm} = \frac{p-1}{2}$
J	R I / A	
N	/ I	$A'_{79} = A_{39}$
F	I S / C	$D' = \frac{1}{2}$ if p is even
C	E : / M	$= \frac{1}{\sqrt{2}}$ if p is odd
K	B / / C	
T	I ST /	forms number
Z	½ I / N	part of result
L	½ I / A	
W	R I T K	plants exponent
H	A : T C	of result
Y	R I T A	
P	P / Q O	normalises result
Q	C U / P	
O	L S / P	closure
B	G £ N I	$\frac{1}{\sqrt{2}}$
G	K O H P	
"	£ E	(electronic) cue to
M	£	SQUAREROOT
X		
V		
£		

COSINE/F

Column /		Column E	
/	/ / Q O		
E	C U / P		
@	½ I / K		
A	N / T :		
:	R I Q O		
S	X / / T		
I	" / Q G		
U	: E / O		
½	G / Q G		
D	E : / O		
R	E : / O		
J	/ : U N		
N	@ I / U		
F	D S Q O		
C	E / Z Z		
K	O / / J		
T	T / Q O		
Z	F S / P		
L	½ I T A		
W	A : T /		
H	R I T A		
Y	Y / Q O		
P	C U / P		
C	L S / P		
O	S @		
B	£		
G	Q V £ £		
"	O V £ £		
M	T @ / /		
X	M / Q G		
V	D S / O		
£	" / Q G		

/	E : / O	
E	G / Q G	
@	/ : U N	
A	@ I / A	rejects integral part
:	A / / P	jumps to enter COSINE

normalises argument, $x = a.2^p$

$D' = a$

clears accumulator

tests sign of exponent: jump if $\geqslant 0$.

case $p < 0$: multiplies a by $2^{-|p|}$ if $p \geqslant -40$; otherwise by zero.

product is displaced to L

plants electronic link

calls in COSINE, $L'_f = \frac{1}{2}\cos 2\pi L_f$

plants number part of result

plants exponent of result

normalises result

closure

(electronic) cue to COSINE

-41, -40, 80 constants

these instructions are obeyed in the case $p \geqslant 0$: multiplies a by 2^p if $p \leqslant 39$; otherwise dynamic stop is entered.

EXPONENTIAL/F

Column /		Column E	
/	/ / Q O		
E	C U / P		
@	D E T :		
A	R I Q O		
:	P E / T		
S	½ I / K		
I	R I T A		
U	L E Q G		
½	Z E / O		
D	W E Q G		
R	E : / O		
J	E : / O		
N	/ : U N		
F	@ I / U		
C	D S Q O		
K	E / Z Z		
T	K E / J		
Z	Z / Q O		
L	F S / P		
W	½ I T A		
H	½ I / C		
Y	I S / N		
P	I S T I		
Q	½ I / A		
O	R I Q O		
B	S : T /		
G	R I T A		
"	@ / / P		
M	@ E		
X	R I T K		
V	A : T C		
£	R I T A		

/	½ I / K	squares number
E	½ I / F	part
@	I S T I	rounds off
A	½ I / A	
:	R I T ½	corrects
S	A : T N	exponent
I	R I T A	
U	½ I / J	doubles number part
½	½ I / J	
D	M / / H	
R	A : Q G	tests for last
J	D Σ / T	squaring
N	N Σ Q C	normalises result
F	C U / P	
C	L S / P	closure
K	E @	(electronic) cue
T	£	to EXPONENT
Z	A /	control number
L	O V £ £	constants
W	Q V £ £	
H	F /	control number
Y	½ I T /	case $p \geqslant 0$
P	H E / P	
Q		
O		
B		
G		
"		
M		
X		
V		
£		

normalising cycle

normalises argument

$x = a.2^p$

$A' = 0$

tests sign of exponent: jump if $p \geqslant 0$

in the case $p < 0$ the argument is "denormalised" (these instructions are similar to those in lines I/ to N/ inclusive of COSINE/F)

plants electronic link

calls in EXPONENT, $L'_{\pm f} = \frac{1}{2}\exp(L_{\pm f})$

number part of f.b.a. becomes $1/4 \exp a$ if $p \geqslant 0$, $1/4 \exp(a2^{-|p|})$ if $p < 0$

B7 gives number of subsequent squarings

plants exponent of f.b.a.

$2p' = 4p + 2$ squares f.b.a.

NATURALOG/F

~~(untested)~~

Column /		
/	/ / Q O	normalises argument, $x = a.2^p$
E	C U / P	
@	½ I T /	sets argument for NATURALOG
A	D S Q O	plants electronic link
:	E / Z Z	
S	G / / J	calls in NATURALOG,
I	I / Q O	$M'_{\pm} + L'_{f} = \log_e a$
U	F S / P	
½	R I / K	$M'_{\pm} + L'_{f} = \log_e a + p \log_e 2 = \log_e a.2^p$
D	O / / N	
R	½ I / U	scaling operations
J	R I / U	
N	P : / C	$[\frac{1}{2}I]'_{\pm f} = 2^{-9} \log_e(a.2^p)$
F	½ I / N	
C	½ I / U	
K	R I / F	
T	½ I T A	
Z	P / T /	plants exponent of result
L	R I T A	
W	W / Q O	normalises result
H	C U / P	
Y	L S / P	closure
P	L / / /	exponent for result = 18 (= 2 × 9)
Q	/ / / /	
O	~~Y Y Y S~~ ~~R J X R~~	
B	~~T M C T~~ ~~E E S >~~	$\pm\log_e 2$
G	£ E	(electronic) cue to NATURALOG
"	£	
M		
X		
V		
£		

ARCTANGENT/F

~~(untested)~~

Column /		
/	/ / Q O	normalises argument,
E	C U / P	$x = a.2^p$
@	A : Q O	sets switch for the
A	H U Q B	case $x < 1$
:	R I T ½	2(p-2)
S	S : T N	
I	O E / H	test: if p≥2 finds (1/x)
U	A : T C	2p←2p-2 (halves f.b.a)
½	R I T A	
D	D / Q O	"denormalises" ($^x/2$)
R	Y D / P	(see note below)
J	½ I T /	sets argument for ARCTAN.
W	D S Q O	plants electronic
F	E / Z Z	link
C	V E / J	calls in ARCTANGENT,
K	K / Q O	$L'_{\pm f} = \frac{1}{\pi} \arctan (2L_{\pm f})$
T	F S / P	
Z	H U / Q	
L	D S T J	forms $\frac{1}{2} - \frac{1}{\pi} \arctan (^1/x)$
W	I S T I	$= \frac{1}{\pi} \arctan (^1/x)$
H	½ I T A	plant $\frac{1}{\pi} \arctan (^1/x)$
Y	R I T A	with zero exponent
P	P / Q O	normalises result
Q	C U / P	
O	L S / P	closure
B	R ~~/~~	
G	A : T /	tests magnitude of exponent of normalised reciprocal: if p<1 jump
"	R I T N	
M	B E / H	
X	M E T /	if p=2 replace
V	½ I T A	f.b.a by $\frac{1}{2}.2^0$
£	R I T A	

Column E		
/	B / / P	jump to enter ARCTANGENT clears the switch
E	H U T B	
@	B E T K	These instructions are identical with those of lines @ ~~/~~ to "/ of routine RECIPROCAL/F The effect is to replace f.b.a (already in normal form) by its normalised reciprocal.
A	R I T N	
:	R I T A	
S	: S T /	
I	½ I / J	
U	B E / M	
½	D S T J	
D	E : T C	
R	E : / J	
J	½ I / S	
N	@ I / E	
F	@ I / J	
C	@ I / A	
K	@ I / K	
T	½ I / D	
Z	½ I / J	
L	½ I / A	
W	@ I / D	
H	E : / M	
Y	" E / P	
P	P E Q O	
Q	C U / P	
O	/ E / P	
B	A / / /	
G	/ / / /	
"	F E	
M	£ £ £ £	$= \frac{1}{2}$ (approx)
X	£ £ £ K	
V	£ E	(electronic) cue to ARCTANGENT
£	£	

Note. For space economy reasons the usual method for denormalising the argument (as used, e.g., in COSINE/F) is not used here. Instead the effect is obtained by adding 0.2^0 to the f.b.a. with the aid of ADD & MULTIPLY.

The Library Tape

Normally the following material is isolated in the magnetic store.

TRACK	MATERIAL
16	TELEINPUT
17	(L) PERM (R) CONVERTER

In these circumstances the library proper can be put into the magnetic store by using TELEINPUT to read the LIBRARY COMPOUND TAPE. This is a single length of tape bearing the punching proper of every routine in the library. Each routine is accompanied by a destination sequence which transfers it to the magnetic store in accordance with the following table.

18	(L)	SEQ.CONTROL	(R)	DECINPUT
19	(L)	SEQ.CONTROL	(R)	DECOUTPUT
20	(L)	ADD & MULTIPLY	(R)	NORMALISE
21	(L)	RECIPROCAL/F	(R)	DIVISION
22	(L)	SQUAREROOT/F	(R)	SQUAREROOT
23	(L)	COSINE/F	(R)	COSINE
24	(L)	EXPONENT/F	(R)	EXPONENT
25	(L)	NATURALOG/F ~~(Untested)~~	(R)	NATURALOG ~~(not yet made)~~ ~~proposed extension to compound tape.~~
26	(L)	ARCTANGENT/F ~~(Untested)~~	(R)	ARCTANGENT ~~(not yet made)~~

This allocation of magnetic storage is a suggestion only. The user is at liberty to make his own compound tape. The details of the tape are given below. To reduce the length of the tape only the relevant lines in each routine are punched. The last K sequence is followed by a £ symbol. This is not a warning character but indicates where the punching proper ends in the case where the last few characters of the sequence are /'s as these cannot be distinguished from blank tape.

To prevent superposition of successive routines the contents of an empty track are transferred to tubes 4 and 5 before each routine is read from the tape. In this way each routine is read onto a clean "backcloth". (Alternative backcloths are suggested on page 8.7)

M K// R ½ (clear track 15)

SEC.CONTROL

DECINPUT

M L / R ½

M K / @ ½ (clear tubes 4 & 5)

SEC.CONTROL

DECOUTPUT

M W / R ½

M K / @ ½

ADD & MULTIPLY

NORMALISE

M H / R ½

M K / @ ½

RECIPROCAL/F

DIVISION

M Y / R ½

M K / @ ½

SQUAREROOT/F

SQUAREROOT

M P / R ½

M K / @ ½

COSINE/F

COSINE

M Q / R ½

M K / @ ½

EXPONENTIAL/F

EXPONENTIAL

M O / R ½

M K / @ ½

NATURALOG/F

NATURALOG

M B / R ½

M K / @ ½

ARCTANGENT/F

ARCTANGENT

M G / R ½

H £££££ (hoot stop)

Operating details

With the tape in the reader

1. /L off.
2. Set H = T / @ /.
3. Operate K.E.C.
4. Switch on prepulses.

Note

The SEQUENCE CONTROL for DECOUTPUT contains the necessary transfer for calling in DECOUTPUT.

Chapter 7

Introduction

In this chapter an example of a reasonably complex problem is prepared for treatment by the machine. All stages of the preparation are described, namely, the programming, coding and input procedure. Compared with the example given in chapter 3, this problem makes heavier demands on running time and storage space and for this reason may serve to illustrate some of the strategical principles discussed in chapter 3.

The Problem

The problem is the numerical reduction of a linear symmetric matrix operator to the Lanczos "codiagonal" form by the method of minimised iterations (C.Lanczos, An Iteration Method for the Solution of the Eigenvalue Problem of Linear Differential and Integral Operators, J.Research N.B.S., 45, 255 (1950) RP 2133).

Only those details of the method which are necessary to follow the coding are reproduced here. For further information the reader is referred to the above paper.

The method of minimised iterations leads to the construction of a set of orthogonal vectors.

$b_o, b_1, \ldots\ldots\ldots, b_n$, starting with an arbitrary trial vector b_o.

The successive members of this set are generated by means of a recurrence relation,

$$b_{i+1} = Ab_i - \alpha_i b_i - \beta_{i-1} b_{i-1}, \qquad i \geqslant 1$$

where $\alpha_i = (b_i.Ab_i)/(b_i.b_i)$, $\beta_{i-1} = (b_i.b_i)/(b_{i-1}.b_{i-1})$

In the case $i = 0$ the relation used is

$$b_1 = Ab_o - \alpha_o b_o, \text{ where } \alpha_o = (b_o.Ab_o)/(b_o.b_o).$$

Here A denotes the matrix operator. Note that it is not necessary to know A explicitly but only to have rules for forming Ab, where b is an arbitrary vector. In cases where A is large and contains relatively few non zero elements arranged in a definite pattern it may be more convenient to introduce A in this way instead of storing the n^2 elements of A and then forming Ab by a formal matrix multiplication.

It is necessary to carry a fixed number of significant figures in the calculation : where the vectors are concerned this means that the lengths of the vectors must be represented to this number of significant figures. An essential part of the method is the compensation for any loss of significant figures that may occur through cancellation in forming the sum

$$Ab_i - \alpha_i b_i - \beta_{i-1} b_{i-1}.$$

This correction is made by reorthogonalising b_{i+1} with respect to all the previous vectors. Thus as each b_{i+1} is formed it is multiplied with each of the previous b_j and corrected by the addition of the vector quantity

$$-\left(\frac{b_{i+1} \cdot b_j}{b_j \cdot b_j}\right) b_j.$$

As a result b_{i+1} is then strictly orthogonal to every b_j, $j \neq i$. Hence b_n must come out as identically zero to the number of significant figures carried.

If the b_i vectors are introduced as an auxiliary reference system the original matrix A is transformed into the following "codiagonal" form

$$H A H^{-1} = \begin{bmatrix} \alpha_0 & 1 & & & \\ \beta_0 & \alpha_1 & 1 & & \\ & \beta_1 & \alpha_2 & 1 & \\ & & & \ddots & \\ & & & \beta_{n-2} & \alpha_{n-1} \end{bmatrix},$$

where $H' = [\, b_0, b_1, \text{-----} b_{n-1}]$ i.e., the rows of H are the vectors b_0, ---b_{n-1}.

When the α's, β's, and the b_i are available the solution of the principal axis problem is comparatively straightforward. For further details see the reference already mentioned and also, Lanczos and Others, Separation of Close Eigenvalues of a Real Symmetric Matrix, J.Research N.B.S., 47, 291 (1951) RP 2256.

The Programming

The conventions introduced in chapter 3 will be adhered to. With this in mind the first step is to consider the storage requirements.

Clearly unless n is trivially small the main sequence of b_i vectors must be stored in the magnetic store. Similar considerations apply to the rows or columns of the matrix operator. In order to carry out conveniently the operation of vector addition, which the basic recurrence relation involves, it is necessary to be able to retain two vectors in the electronic store at some stage. This suggests the following layout of the electronic store

tubes	0, 1	routines
tube	2	FERM
tube	3	Cues, special working space
tubes	4, 5	a vector
tubes	6, 7	a second vector

It will be seen later that the most convenient method of carrying out the operation Ab requires space for 3 vectors in the electronic store (b, Ab, and a matrix row). A means for arranging this is suggested.

Representation of vectors

The elements of each vector are associated with a common scale factor and a common exponent. This material is arranged as follows. Let the vector be given by

$$b = u.2^p = \begin{pmatrix} {}_0u \\ {}_1u \\ \vdots \\ {}_{n-1}u \end{pmatrix} 2^p$$

The notation $u = n\{b\}$, $p = e\{b\}$ may also be used.

Then, if the pages in question are tubes 6, 7, we have

$$[\,/\,N]_{\pm} = 2p \quad [:N+2i]_{\pm f} = {}_iu.$$

The allowance of two pages for this material enables cases for which $n \leqslant 62$ to be treated. If $n \leqslant 30$ only one page of material is involved. In such cases half tracks only are necessary for storing the vectors in the magnetic store. However no advantage is taken of the space thus made available in the electronic store.

The ${}_iu$ are scaled so that the length ρ, given by $\rho^2 = {}_0u^2 + {}_1u^2 + \ldots + {}_{n-1}u^2$, is less than $\frac{1}{2}$. A vector e.g., that standing in S6,7 is said to be <u>normalised</u> or in <u>normal form</u> if (i) ρ lies in the range $\frac{1}{4} \leqslant \rho < \frac{1}{2}$ and (ii) $[@N]_{\pm f} = \rho^2$ $(\frac{1}{16} \leqslant \rho^2 < \frac{1}{4})$.

Note: the dot product b.b is given by $\rho^2 2^{2p}$.

<u>Arithmetical operations on vectors</u>

Let $a = u.2^p$ denote the vector standing in tubes 6,7.

Let $b = v.2^q$ denote the vector standing in tubes 4,5.

<u>Dot product</u>

The dot product is given by

$$a.b = u.v\ 2^{p+q} = \left(\sum_{i=0}^{n-1} {}_iu\ {}_iv\right) 2^{p+q}$$

The lengths of u and v are each less than $\frac{1}{2}$ so that the sum of products can be formed directly without exceeding capacity.

<u>Multiplication by a scalar, λ</u>

$$n\{\lambda a\} = n\{\lambda\} \times n\{a\} \qquad e\{\lambda a\} = e\{\lambda\} + e\{a\}$$

<u>Addition of two vectors</u> and

<u>Normalisation of a vector</u>

These operations are sufficiently complex to require the use of sub-routines. VECTOR ADD and VECTOR NORMALISE have been designed for these purposes. The methods used are explained in the official accounts and instruction lists which are given in the following pages.

<u>Magnetic storage of the vectors, the VECTOR DIRECTORY</u>

Each vector may occupy a complete track+ of the magnetic store so that a maximum of 62 tracks may be required for this purpose. The magnetic addresses of these vectors are given by a VECTOR DIRECTORY, a page-pair of material normally stored on a magnetic track and transferred to tubes 4, 5 when required. With reference to these tubes this directory material may be described as follows.

Single line $(/\frac{1}{2} + 2i)$ contains the magnetic instruction which transfers b_i to tubes 4,5.

Single line $(/\frac{1}{2} + 2i + 1)$ contains the magnetic instruction which transfers b_{i+1} from tubes 6,7 to the magnetic store.

The punching proper for the VECTOR DIRECTORY is given below. This corresponds to using track (30 + i) for storing the vector b_i.

+ It has already been mentioned that in the cases $n \leqslant 30$ half tracks only are involved. In view of statements made elsewhere (see page 4.6) about the unreliability of the magnetic wheel it may be necessary to take advantage of this fact.

Punching proper for VECTOR DIRECTORY (2 pages)

½	K / ½ /			D
	V / @ ½	/		
	V / R N	E		
	£ / @ ½	@		
	£ / R N	A		
	/ E @ ½	:		
	/ E R N	S		
		I		
	etc.	U		
		½		
		D		
		R		
		J		
		N		

<u>Representation of α's and β's</u>

These are computed in floating binary form using one line pair each for the numerical part and the exponent. In this connection a special division routine is needed, DIVISION (SPECIAL), similar to DIVISION, but giving the result in floating binary form. (For details see page 7.23).

The programme developed below is arranged to print the values of α_i and β_{i-1}, but in addition stores them in the magnetic store for possible future use. Four complete tracks are used for this purpose, one each for the n possible values of $n\{\alpha_i\}$, $e\{\alpha_i\}$, $n\{\beta_{i-1}\}$, and $e\{\beta_{i-1}\}$, $(0 \leqslant i \leqslant n-1)$.

Each set is stored so that if the track in question were transferred to tubes 4, 5 then

$$[\,/\,\tfrac{1}{2} + 2\,i]_{\pm f} = n\{\alpha_i\}, \quad n\{\beta_{i-1}\}$$

$$[\,/\,\tfrac{1}{2} + 2\,i]_{\pm} = e\{\alpha_i\}, \quad e\{\beta_{i-1}\}.$$

As each new value of α_i and β_{i-1} is computed it is added to the corresponding list in the magnetic store.

The Matrix Operator A

This is introduced by means of a conventional closed sub-routine OPERATOR which is arranged as follows: given a (normalised) vector b standing in tubes 4,5 (dimension given in line £I or preset) the routine places the vector Ab (not necessarily normalised) in tubes 6,7; alterations of lines outside these tubes being in accordance with the following rules.

1. Lines in tubes 2,3,4 and 5 must not be altered.
2. B6 (and of course B0 & B1) must not be altered.
3. The accumulator must be empty on exit.

Rules 1 and 2 mean that the content of these lines must be the same on exit as on entry. It is permissible however to temporarily transfer (say) the contents of tubes 2,3 to a magnetic track (other than those given below) in order to provide more working space in the electronic store. For example the successive rows of the matrix A may be transferred to these tubes. However any complexities of this kind should be buried out of sight and the routine arranged so that it is entered and left in the conventional manner.

For the purpose of this example it is assumed that the user will draw up this routine and arrange to input such numerical material as may be required in connection with it. For the general case of a symmetric matrix involving n(n+1)/2 distinct elements a library routine should be made available.

Summary giving magnetic storage of material involved

ROUTINES	Magnetic track
OPERATOR*	20
VECTOR ADD	21L } (see official account
VECTOR NORMALISE	21R } of these routines)
DIVISION (SPECIAL)	22R
DECOUTPUT	23R
MASTER PRINT	23L
MASTER	24

Other teleprint material	Magnetic track
TUBE 3 (this material also serves as a reading-in directory)	102L
VECTOR DIRECTORY	25

Numerical data	Magnetic track
List of n $\{\alpha_i\}$	26
List of e $\{\alpha_i\}$	27
List of n $\{\beta_i\}$	28
List of e $\{\beta_i\}$	29
The b_i vectors	30 onwards

*This table omits any details of numerical data associated with the matrix OPERATOR routine.

The MASTER routine (2 pages)

It is the MASTER routine which generates the vector sequence from the basic recurrence relation, calling in as sub-routines, OPERATOR, DIVISION (SPECIAL), VECTOR ADD, and VECTOR NORMALISE where necessary. Permanent material and working lines on tube 3 are used. Details of the MASTER routine are provided by the following notes which refer to numbered sections of the punching proper given on page 7.11.

1. Sets i (initially i = 0)
2. Transfers b_i to tubes 4,5
3. Forms the vector Ab_i in tubes 6,7 (sub-routine)
4. Normalises Ab_i
5. Takes the dot product $b_i.Ab_i$

$$\left[I\ \ U\right]_{\pm f} = n\left\{b_i.\ Ab_i\right\} = n\left\{b_i\right\} \times n\left\{Ab_i\right\}$$

$$\left[\tfrac{1}{2}\ \ U\right]_{\pm} = 2e\left\{b_i.\ Ab_i\right\} = 2(e\left\{Ab_i\right\} + e\left\{b_i\right\})$$

6. Transfers the dot product $b_i.\ b_i$

$$\left[@\ \ U\right]_{\pm f} = n\left\{b_i.\ b_i\right\}$$

$$\left[:\ \ U\right]_{\pm} = 2e\ \left\{b_i.b_i\right\} = 4e\ \left\{b_i\right\}$$

7.
8. } Two cycles
9.

First cycle

7. Forms $\alpha_i = (b_i.Ab_i)/(b_i.b_i)$.

The sub-routine DIVISION (SPECIAL) provides

$$M_{\pm} = 2e\left\{\frac{n\{b_i.Ab_i\}}{n\{b_i.b_i\}}\right\} \quad L_{\pm f} = n\left\{\frac{n\{b_i.Ab_i\}}{n\{b_i.b_i\}}\right\}$$

$$[I\ U]'_{\pm f} = n\{\alpha_i\} = n\left\{\frac{n\{b_i.Ab_i\}}{n\{b_i.b_i\}}\right\}$$

$$[\tfrac{1}{2}\ U]'_{\pm} = 2e\{\alpha_i\} = 2e\left\{\frac{n\{b_i.Ab_i\}}{n\{b_i.b_i\}}\right\} + 2e\{b_i.Ab_i\} - 4e\{b_i\}$$

8. Multiplies the vector b_i by $-\alpha_i$

$$n\{-\alpha_i b_i\} = -n\{\alpha_i\} \times n\{b_i\}$$
$$e\{-\alpha_i b_i\} = e\{\alpha_i\} + e\{b_i\}$$

9. The vector $-\alpha_i b_i$ is added to Ab_i. The sum is normalised and placed in tubes 6,7. (Sub-routine)

The instructions standing in the following lines are used in connection with the two cycles.

LE, WE } arranges to omit 2nd cycle in case i = 0

ZE, G/ } transfers b_{i-1} to tubes 4,5 at beginning of 2nd cycle

O/, HE, YE } counts the two cycles

Second cycle

7. Forms $\beta_{i-1} = (b_i.b_i)/(b_{i-1}.b_{i-1})$

$$[@U]'_{\pm i} = n\{\beta_{i-1}\} = n\left\{\frac{n\{b_i.b_i\}}{n\{b_{i-1}.b_{i-1}\}}\right\}$$

$$[:\ U]'_{\pm} = 2e\{\beta_{i-1}\} = 2e\left\{\frac{n\{b_i.b_i\}}{n\{b_{i-1}.b_{i-1}\}}\right\} + 2e\{b_i.b_i\} - 4e\{b_{i-1}\}$$

8. Multiplies the vector b_{i-1} by $-\beta_{i-1}$.

$$n\{-\beta_{i-1}\ b_{i-1}\} = -n\{\beta_{i-1}\}.n\{b_i\}$$

$$e\{-\beta_{i-1}\ b_{i-1}\} = e\{\beta_{i-1}\} + e\{b_{i-1}\}$$

9. The vector $-\beta_{i-1}\ b_{i-1}$ is added to the vector standing in tubes 6,7., namely, $Ab_i - \alpha_i b_i$. The result, b_{i+1}, is normalised and placed in tubes 6,7.

10,11,12,13,14,15. Cycle i+1 times

10. Transfers b_j to tubes 4,5. (Initially j = i)

11. Takes the dot product $b_j.\ b_{i+1}$

$$[R\ U]'_{\pm f} = n\{b_j.\ b_{i+1}\} = n\{b_j\}\ .\ n\{b_{i+1}\}$$

12. Forms $(b_j.b_{i+1})/(b_j.b_{i+1})$

$$[R\ U]'_{\pm f} = n\left\{\frac{n\{b_j.b_{i+1}\}}{n\{b_j.b_j\}}\right\} \quad L_{\pm} = e\left\{\frac{n\{b_j.b_{i+1}\}}{n\{b_j.b_j.\}}\right\}$$

13. Multiplies the vector b_j by $-(b_j.b_{i+1})/(b_j.b_j)$

$$n\left\{-\left(\frac{b_j.b_{i+1}}{b_j.b_j}\right)b_j\right\} = -n\left\{\frac{n\{b_j.b_{i+1}\}}{n\{b_j.b_j\}}\right\}.n\{b_j\}$$

$$e\left\{-\left(\frac{b_j.b_{i+1}}{b_j.b_j}\right)b_j\right\} = e\left\{\frac{n\{b_j.b_{i+1}\}}{n\{b_j.\ b_j\}}\right\} + e\{b_{i+1}\}$$

14. Corrects b_{i+1} by the addition of $-\left(\frac{b_j.b_{i+1}}{b_j.\ b_j}\right) b_j$.

The corrected b_{i+1} is normalised and left in tubes 6,7.

15. Reduces j by 1 and tests: if $j \geqslant 0$ returns to 10.

16. Transfers reorthogonalised b_{i+1} to magnetic store

17. Adds n $\{\alpha_i\}$ to list in magnetic store

18. Adds e $\{\alpha_i\}$ " " " " "

19. Adds n $\{\beta_{i-1}\}$ " " " " "

20. Adds e $\{\beta_{i-1}\}$ " " " " "

21. Tests i: if i = n stop

22. Advance i by 1

23. Calls in MASTER PRINT (see below) to print α_i and β_{i-1}, eventually returning to 2.

Punching proper for MASTER ROUTINE

| | | |

TUBE 3 MATERIAL

The role of tube 3 is as follows. Column 1 contains mainly cues and magnetic instructions needed by the MASTER routine. The first few lines of column U are used for working space, the rest of the lines being spare. Details are given below. The magnetic entries correspond to the table on page 7.6.

/	: /			
E	T / @ /	cue to TELEINPUT	/ / / /	(2i)//
@	£ £		E / / / /	
A	H / @ /	cue to OPERATOR	@	
:	£ E		A	
S	Y / E @	cue to VECTOR NORMALISE	:	
I	£ £		S	
U	Y / @ /	cue to VECTOR ADD	I	working space
½	£ E		U	
D	P / E @	cue to DIVISION(SPECIAL)	½	
R	U @		D	
J	Q / E @	cue to DECOUTPUT	R	
N	£ £		J	
F	Q / / /	cue to MASTER PRINT		
C	£ E			
K	Q / E @	cue to DECOUTPUT		
T	£ £			
Z	O / @ /	cue to MASTER		
L				
W				
H		spare lines		
Y				
P	B / @ ½	transfers VECTOR DIRECTORY to tubes 4,5		Magnetic instructions.
Q	G / @ ½	" " list of $n\{\alpha_i\}$ " " " "		
O	" / @ ½	" " " " $e\{\alpha_i\}$ " " " "		
B	M / @ ½	" " " " $n\{\beta_i\}$ " " " "		
G	X / @ ½	" " " " $e\{\beta_i\}$ " " " "		
"	G / R ½	" " " " $n\{\alpha_i\}$ " magnetic store		
M	" / R ½	" " " " $e\{\alpha_i\}$ " " "		
X	M / R ½	" " " " $n\{\beta_i\}$ " " "		
V	X / R ½	" " " " $e\{\beta_i\}$ " " "		
£	. . . .			

Preset parameter:- (2n-2)//, where n is the number of elements in each vector.

Other lines altered: /U, EU.
* (alternative entry by omitting c.r.,l.f)

Punching proper for Tube 3 MATERIAL

		K / F @	
K / N L	: / / /	/ / / /	
	: A @ /	E / / / /	
	£ £ / /	@	
	H / @ /	A	
	£ E / /	:	
	Y / E @	S	
	£ £ / /	I	
	Y / @ /	U	
	£ E / /	½	
	P / E @	D	
	U @ / /	R	
	Q / E @	J	
	£ £ / /	N	
	Q / / /	F	
	£ E / /	C	
	Q / E @	K	
	£ £ / /	T	
	O / @ /	Z	
		L	
		W	
		H	
		Y	
K P N D	B / @ ½	P	
	G / @ ½	Q	
	" / @ ½	O	
	M / @ ½	B	
	X / @ ½	G	
	G / R ½	"	
	" / R ½	M	
	M / R ½	X	
	X / R ½	V	
		£	

The MASTER PRINT routine (1 page)

This routine is called in as an ad-routine by the MASTER routine at the end of each step. It arranges to print on a new line the values of α_i, β_{i-1}, and 2e $\{b_{i+1}\}$, calling in DECOUTPUT for these purposes. The significance of the last quantity lies in the fact that the difference e $\{b_{i+1}\}$ - e $\{b_i\}$ measures the cancellation that occurs in the formation of b_{i+1}. On the last step, i = n-1, all figures should cancel. If the matrix is defective this may happen for values of $i < n-1$, in which case these values give the dimensions of invariant subspaces.

The details of the MASTER PRINT routine are given below.

Column /

/	S : Y O	sets B5
E	@ U S K	$D_f = n\{\alpha_i\}$ on 1st cycle
@	: U Y /	$= n\{\beta_i\}$ on 2nd cycle
A	R U T A	B7= $2e\{\alpha_i\}$ on 1st cycle
:	R U Q O	$= 2e\{\beta_i\}$ on 2nd cycle
S	R I Y /	plants cue to DECOUTPUT
I	@ / Z A	
U	£ / / T	tests sign of exponent
½	I S T /	sets round-off
D	/ E Q G	
R	S : / O	case of -ve exponent:
J	E E Q G	selects correct
N	A : / O	multiplier from PERM
F	A : / Q	
C	L / / /	= digit counter: corresponds
K	/ : U N	to 9 decimals
T	R U / A	prepares to output α_i,
Z	R U T ½	β_{i-1} as fractions
L	C / P O	sets digit count
W	W / Q O	
H	K S / P	calls in DECOUTPUT
Y	S : Y G	counts two cycles
P	/ : / T	
Q	/ N / J	prepare to output 2e $\{b_{i+1}\}$
O	O / Q O	calls in DECOUTPUT (the appropriate cue is
B	K S / P	already standing in list of links)
G	T I / J	
"	/ / E A	calls in MASTER routine
M	@ E / P	
X	Z /	
V	E : U N	case of + ve exponent
£	X / / P	

Column E

/	O V £ £	discriminatory
E	Q V £ £	constants used
@	T S	in selecting
		multiplier
		from PERM

Cycle twice:
on 1st cycle α_i is printed on a new line, i.e., preceeded by carriage return and line feed; on 2nd cycle β_{i-1} is printed on the same line.

Printing of α_i and β_{i-1}. The method described in chapter 6 has been extended here to deal with the case of negative exponents lying in the ranges $-40 \leq e < -20$ and $e < -40$. The limitation on the range of positive exponents, i.e., $0 < e \leq 19$ remains as this range is considered sufficiently large to cover all likely cases.

Punching proper for MASTER PRINT

K	/	½	/		K	/	D A
S	:	Y	O	/	O	V	£ £
@	U	S	K	E	Q	V	£ £
:	U	Y	/	@	T	S	/ /
R	U	T	A	A			
R	U	Q	O	:			
R	I	Y	/	S			
@	/	Z	A	I			
£	/	/	£	U			
I	S	T	/	½			
/	E	Q	G	D			
S	:	/	O	R			
E	E	Q	G	J			
A	:	/	O	N			
E	:	/	Q	F			
L	/	/	/	C			
/	:	U	N	K			
R	U	/	A	T			
R	U	T	½	Z			
C	/	P	O	L			
W	/	Q	O	W			
K	S	/	P	H			
S	:	Y	G	Y			
/	:	/	T	P			
/	N	/	J	Q			
O	/	Q	O	O			
K	S	/	P	B			
T	I	/	J	G			
/	/	E	A	"			
@	E	/	P	M			
Z	/	/	/	X			
E	:	U	N	V			
X	/	/	P	£			

Official Accounts of Sub-routines Used

Name VECTOR NORMALISE

Purpose To normalise vectors represented in floating binary form

Cue Magnetic storage: half track, variable, but see account of VECTOR ADD

Electronic storage: tube 1

Control number: £E

Self resetting

Link Closed

Input details Conventional procedure, but see also corresponsing section of account of VECTOR ADD. Lines /@ and B@ involve a common parameter. These are most conveniently preset by following the punching proper with the overwriting sequences

K/RE(s)QO
KBRE(s)QO, where s is the name of a line containing the quantity (2n-2)//, n being the number of elements in the vector.

Effects Let $u.2^p = \begin{pmatrix} {}_0u \\ {}_1u \\ {}_2u \\ \vdots \\ {}_{n-1}u \end{pmatrix} .2^p$ denote the vector standing in tubes 6,7, being represented as follows

$$[\ /\ N\ ]_{\pm} = 2p$$

$$[:N + 2i]_{\pm f} = {}_iu$$

Let ρ denote the length of u, that is,

$$\rho^2 = {}_0u^2 + {}_1u^2 + \dots\dots\dots + {}_{n-1}u^2.$$

In normal form ρ lies in the range $\frac{1}{4} \leqslant \rho < \frac{1}{2}$, i.e., $\frac{1}{16} \leqslant \rho^2 < \frac{1}{4}$.

It is assumed that $\rho < \frac{1}{2}$ initially. The process of normalising is as follows. Each element of the vector is multiplied by 2^m, where m is such that $\frac{1}{16} \leqslant 2^{2m}\rho^2 < \frac{1}{4}$, i.e., such that $\frac{1}{4} \leqslant 2^m\rho < \frac{1}{2}$.

lines altered	initial content	final content
A	empty	empty
D	irrelevant	irrelevant
relevant lines of tubes 6,7	$u.2^p$	$(u.2^m).2^{p-m}$
@N	irrelevant	$[@N]_{\pm f} = 2^{2m}\rho^2$

Column @		Column A	
/	. . Q O	/ M A / N	multiplies ρ^2 by 2^m
E	: N U F	E M A / U	
@	: N U K	@ V A / N	
A	A : Q G	A V A / U	
:	N S / O	: M A / N	multiplies $\rho^2 2^m$ by a further 2^m:
S	E : T D	S M A / U	
I	V A / A	I V A / N	
U	G A / S	U E : T D	$[@N]'_{\pm f} = m.s.\rho^2.2^{2m}$
½	M A T A	½ @ N T A	
D	V A T F	D / N T /	
R	£ @ / M	R G A T N	corrects exponent:
J	V A T /	J C A T N	
N	G A T A	N / N T A	2p' = 2p-2m
F	C A / J	F L S / P	closure
C	G A /@	C ½ E / /	= 40
K	G A / U	K / / / /	
T	D S T J	T Q V £ £	=-41
Z	C A T @	Z V £ £ £	
L	A : T N	L £ £ £ £	
W	Z A T R	W	
H	G A T A	H	
Y	G A Q O	Y	
P	E : / O	P	
Q	T A Q G	Q	
O	/ : U C	O	
B	. . Q O	B	
G	I S T /	G U S E D	
"	: N U F	" U S E D	
M	: N U A	M 1.s $\{\rho^2\}$	working space
X	A : Q G	X	
V	N S / O	V m.s $\{\rho^2\}$	
£	A / T :	£	

$A = \rho^2$

preventive round-off

most sig. $\{\rho^2\}$ to VA

least sig. $\{\rho^2\}$ to GA

and to MA

skip if most sig. $\{\rho^2\}=0$

replace [GA] by most sig. $\{\rho^2\}$

M' = 40

M'=r, where r is digital position number of the 80 digit quantity ρ^2

$[GA]_{\pm} = 37-r$ (r odd)
$= 36-r$ (r even) $\}(2m-40)$

$D = 2^m$ is selected from PERM

sets counter corresponding to number of elements.

multiplies each element of vector by 2^m

clears A

If r is the digital position number of the exact (80 digit) product ρ^2, then m is given by

$$2m = 76 - r \qquad (r \text{ even}),$$

$$2m = 77 - r \qquad (r \text{ odd})$$

Name VECTOR ADD

Purpose To add vectors represented in floating binary form

Cue Magnetic storage: 1 half track (but see next section)

Electronic storage: tube 0

Control number: ££

Self resetting

Link Open. When VECTOR ADD has completed its task control is transferred to the first instruction of tube 1 which is intended to contain VECTOR NORMALISE. In this case the two routines act as a single closed sub-routine. If they are both stored on the same magnetic track, then they can be transferred to the electronic store by means of a single magnetic instruction, and hence called in with a single cue.

Input details Likewise both routines may be input into the electronic store (VECTOR ADD into tube 4 and VECTOR NORMALISE into tube 5) and then transferred to the magnetic store with a single destination sequence. Line K/ of VECTOR ADD involves a parameter. This is most conveniently preset by following the routine on the tape by the overwriting sequence KK½E(s)QO, where s is the name of a line containing the quantity (2n-2)//, n being the number of elements in the vector.

Effects Let the vector standing in tubes 4,5 be denoted by $v.2^q$ and the vector standing in tubes 6,7 by $u.2^p$. This material is represented in the following way.

$$v = \begin{bmatrix} {}_0v \\ {}_1v \\ \cdot \\ \cdot \\ \cdot \\ {}_{n-1}v \end{bmatrix} \qquad u = \begin{bmatrix} {}_0u \\ {}_1u \\ \cdot \\ \cdot \\ \cdot \\ {}_{n-1}u \end{bmatrix}$$

$$[/\tfrac{1}{2}]_{\pm} = 2q$$

$$[/N]_{\pm} = 2p$$

$$[\tfrac{1}{2} + 2i]_{\pm f} = {}_iv$$

$$[N + 2i]_{\pm f} = {}_iu$$

lines altered	initial content	final content
A	irrelevant	empty
D	"	irrelevant
relevant line-pairs in tubes 6,7	$u.2^p$	$w.2^r = u.2^p + v.2^q$

The vector sum is formed as follows, two cases arising:

1. $p > q$, $\quad w.2^r = \frac{w}{2}\,2^{r+1} = \left(\frac{u}{2} + \frac{v}{2}\,2^{(q-p)}\right)\,2^{p+1}$

2. $p \leqslant q$, $\quad w.2^r = \frac{w}{2}\,2^{r+1} = \left(\frac{u}{2}\,2^{(p-q)} + \frac{v}{2}\right)\,2^{q+1}$

The result is left in the form $\left(\frac{w}{2}\right)\,2^{r+1}$ so that it is necessary to normalise this in some way in order to prevent a steady loss of figures "off the back". In this connection see under LINK above.

VECTOR ADD

Column /		Column E		
/ : E T /		/ / N T A		replaces p by q
E / ½ Q O		E I E T /		
@ / N Q G	forms q-p	@ ½ E / P		
A I / / T	Jump if $q \geqslant p$	A K /		control number
: F / T A		: M E / S	$p > q$	One of these pairs of instructions is planted in line pair F/.
S I S T /	$L_{+f} = 2^{-1}$ (multiplier of larger vector)	S V E / A		
I M / Q G		I V E / S	$p \leqslant q$	
U S : / O	selects multiplier for smaller vector	U K E / A		
½ J / Q G		½ A /		control number
D A : / O	$M_{+f} = 2^{-\lvert p-q \rvert -1}$	D		
R A : / Q		R		
J Q V £ £	= -41	J		
N / : U J		N		
F M E / S	In case $q \geqslant p$ these instrs. become VE/S ME/A. They plant the two multipliers, 2^{-1} and $2^{-\lvert p-q \rvert -1}$	F		
C V E / A		C		
K . . Q O		K		
T M E / C		T		
Z : N U F	combines vectors with appropriate multipliers	Z		
L V E / C		L		
W : ½ U F	if $p > q$, then $\frac{1}{2}u + \frac{1}{2}v.2^{-(p-q)}$	W		
H : N U A		H		
Y A : Q G	if $q > p$, then $\frac{1}{2}u.2^{-(q-p)} + \frac{1}{2}v$	Y		
P A E / T		P		
Q / N T /	corrects exponent of result	Q		
O A : T C		O		
B / N T A		B		
G " / / P	to NORMALISE	G		
" £ E	control number	"		
M G V £ £	= -38	M U S E D		
X / N Q O	forms p-q — These instructions are obeyed in the case $q \geqslant p$	X U S E D		
V / ½ Q G		V U S E D		
£ / ½ T /		£ U S E D		

Punching proper for VECTOR ADD and VECTOR NORMALISE

	K	/	½	/	K	/	D D
/	:	E	T	/	/	N	T A E
	/	½	Q	O	E	I	E T /
	/	N	Q	G	@	½	E / P
	I	/	/	T	A	K	/ / /
	F	/	T	A	:	M	E / S
	I	S	T	/	S	V	E / A
	M	/	Q	G	I	V	E / S
	S	:	/	O	U	M	E / A
	J	/	Q	G	½	A	/ / /
	A	:	/	O	D		
	A	:	/	Q	R		
	Q	V	£	£	J		
	/	:	U	J	N		
	M	E	/	S	F		
	V	E	/	A	C		
	/	/	Q	O	K		
	M	E	/	C	T		
	:	N	U	F	Z		
	V	E	/	C	L		
	:	½	U	F	W		
	:	N	U	A	H		
	A	:	Q	G	Y		
	A	E	/	T	P		
	/	N	T	/	Q		
	A	:	T	C	O		
	/	N	T	A	B		
	"	/	/	P	G		
	£	E	/	/	"		
	G	V	£	£	M		
	/	N	Q	O	X		
	/	½	Q	G	V		
	/	½	T	/	£		

$KK\frac{1}{2}E(s)QO$

[preset parametars punched by user]

	K	/	R	/	K	/	J W
@	/	/	Q	O	/	M	A / N A
	:	N	U	F	E	M	A / U
	:	N	U	K	@	V	A / N
	A	:	Q	G	A	V	A / U
	N	S	/	O	:	M	A / N
	E	:	T	D	S	M	A / U
	V	A	/	A	I	V	A / N
	G	A	/	S	U	E	: T D
	M	A	T	A	½	@	N T A
	V	A	T	F	D	/	N T /
	£	@	/	M	R	G	A T N
	V	A	T	/	J	C	A T N
	G	A	T	A	N	/	N T A
	C	A	/	J	F	L	S / P
	G	A	/	@	C	½	E / /
	G	A	/	U	K	/	/ / /
	D	S	T	J	T	Q	V £ £
	C	A	T	C	Z	V	£ £ £
	A	:	T	N	L	£	£ £ £
	Z	A	T	R	W		
	G	A	T	A	H		
	G	A	Q	O	Y		
	E	:	/	O	P		
	T	A	Q	G	Q		
	/	:	U	C	O		
	/	/	Q	O	B		
	I	S	T	/	G		
	:	N	U	F	"		
	:	N	U	A	M		
	A	:	Q	G	X		
	N	S	/	O	V		
	A	/	T	:	£		

K/RE(s)QO

KBRE(s)QO

Name DIVISION (SPECIAL)

Purpose To divide giving result in floating binary form (normalised)

Cue Magnetic storage: 1 half track, variable

Electronic storage: tube 1

Control number: £E

Self resetting

Link Closed routine

Input details Conventional procedure

Effects

lines altered	initial content	final content
L	$L_{\pm f} = b$	$L_{\pm f} = n\left\{\frac{a}{b}\right\}$ $\frac{1}{4} \leqslant \left\lvert n\left\{\frac{a}{b}\right\}\right\rvert < \frac{1}{2}$
M	irrelevant	$M_{\pm} = 2e\left\{\frac{a}{b}\right\}$
D	$D = a$ $-\frac{1}{2} \leqslant a < \frac{1}{2}$	irrelevant

DIVISION (SPECIAL)

Column @

/	V A T A
E	I S / N
@	V A T J
A	M A / A
:	V A / J
S	V A T J
I	V A / A
U	V A / @
½	G A / A
D	Z A Q O
R	G A Q G
J	G A Q G
N	E : / O
F	W A Q G
C	/ : U C
K	L @ T :
T	V A / N
Z	V A T A
L	I S T /
W	V A / K
H	M A / J
Y	M A / D
P	M A / A
Q	V A / D
O	A : / M
B	V A / A
G	K @ / P
"	M A T ½
M	E : / M
X	M A T F
V	E : T D
£	V A T A

These instructions are similar to those in the corresponding lines of DIVISION. The effects may be described by the equations

$$[MA]'_{\pm f} = \frac{a}{2^{40-r}\,b}$$

$$[GA]'_{\pm} = r.$$

Note that here the dividend is initially multiplied by ½ instead of ¼ as in the DIVISION routine, where the dividend may lie in the range $-\frac{1}{2} \leqslant a < 1$

plants modulus of quotient in VA

Column A

/	V A / @
E	V A / A
@	V A T /
A	G A T N
:	A : T C
S	G A T A
I	L A Q O
U	V A Q G
½	V A Q G
D	E : / O
R	W A Q G
J	/ : U C
N	M A / F
F	V A / A
C	G A / J
K	G A / J
T	L S / P
Z	I E / /
L	: E / /
W	Q V £ £
H	
Y	
P	
Q	
O	
B	
G	U S E D
"	U S E D
M	U S E D
X	U S E D
V	U S E D
£	U S E D

$[VA]'_{+}$ = digital position number, r', of quotient

forms 2-r+r'

forms 36-2r'

multiplies $(a/2^{40-r}b)$ by $2^{38-r'}$, i.e., normalises the quotient

$$L'_{\pm f} = n\left\{a/b\right\} = \frac{a\ 2^{38-r'}}{b\ 2^{40-r}}$$

$$M'_{\pm} = 2e\left\{\frac{a}{b}\right\} = 2-r+r'$$

closure
= 38
= 36
= -41

The punching proper for DIVISION (SPECIAL)

	K / R /		K / J H		
@	V A T A	/	V A / @	A	
	I S / N	E	V A / A		
	V A T J	@	V A T /		
	M A / A	A	G A T N		
	V A / J	:	A : T C		
	V A T J	S	G A T A		
	V A / A	I	L A Q O		
	V A / @	U	V A Q G		
	G A / A	½	V A Q G		
	Z A Q O	D	E : / O		
	G A Q G	R	W A Q G		
	G A Q G	J	/ : U C		
	E : / O	N	M A / F		
	W A Q G	F	V A / A		
	/ : U C	C	G A / J		
	L @ T :	K	G A / J		
	V A / N	T	L S / P		
	V A T A	Z	I E / /		
	I S T /	L	: E / /		
	V A / K	W	Q V £ £		
	M A / J	H			
	M A / D	Y			
	M A / A	P			
	V A / D	Q			
	A : / M	O			
	V A / A	B			
	K @ / P	G			
	M A T ½	"			
	E : / M	M			
	M A T F	X			
	E : T D	V			
	V A T A	£			

Input of material

The following layout (which is similar to that on p.4.7.) illustrates the make up of the input tape.

TUBE 3 MATERIAL		
MK/½N		
VECTOR DIRECTORY	STARTING TAPE	
QPN½/	MK//I K£IE(2n-2)// KHS££//0/2/	enters the constant which gives the vector size.
VECTOR ADD	TTS	calls in MASTER ROUTINE
VECTOR NORMALISE		
KK½E£IQO K/RE£IQO KHRE£IQO QUN11/22		preset parameters: see official accounts of these routines
DIVISION (SPECIAL)		
QDN11/22		
MASTER PRINT		
DECOUTPUT		
QFNR½		
MASTER		
QZN11/22		
INITIAL VECTOR (b_0)		(see note below)
MV/R½ H		(Hoot stop)

It is necessary for the user to prepare a further tape to input the OPERATOR routine and any associated numerical material.

The initial vector b_0 accompanies the routines and other material on the input tape and is ultimately transferred directly to track 30 (in accordance with the first entry of the VECTOR DIRECTORY).
If the initial vector takes a simple form, e.g., $(1, 0, 0, \ldots 0) \times 2^0$, then the corresponding teleprint material is easily written down and the box

INITIAL VECTOR

represents the punching proper in the usual way. If it is desired to choose the elements in any other way, e.g., randomly, then this box may represent an interlude (see chapter 4), the effect of which is to compute and place the vector in tubes 4,5, afterwards returning control to the input cycle of TELEINPUT.

Chapter 8

This chapter is concerned with some practical details of the operation of the machine. They fall under four headings.

1. The Console
2. The Tape Preparation Equipment
3. Eliminating mistakes in programme
4. Reliability

1. The Console

This is a control desk by means of which the behaviour of the machine can be controlled manually. In practice this is best studied in connection with the machine itself. For this reason only a brief description giving its essential logical features will be given here.

A switch is a 2-position manual control which is stable in either position.

A key is one which must be kept pressed if it is to remain in the "active" position.

The various keys and switches are classified as follows.

1.1. Dummy stop switches and the hand switches

These are described in Chapter I.

1.2. MAN-AUTO switch

Manual instruction switches

When the MAN-AUTO switch is in the AUTO position the instructions from the electronic store are selected and obeyed in the manner determined by the switches in group 3 (below). When this switch is in the MAN position a different arrangement applies. In this case the actual instruction is the combination set up on the set of 20 manual instruction switches. A number of restrictions are associated with this facility and its use by programmers is not recommended except in one important case, namely, for the alteration of lines (see below). It is useful, however, to the maintenance engineer.

1.3. Switches for the control of completion signals

With the MAN-AUTO switch in the AUTO position the instructions can be selected and obeyed from the electronic store in the following ways.

(a) Full speed ahead, i.e., in the manner already explained in detail in Chapter I.

(b) 50 per second.

(c) Singly under the operation of the single prepulse key.

(d) Printing and/or punching a record of the 4th character.

This last mode of operation is described below in the section on fault diagnosis.

1.4. Clearing keys

By means of these the operator can clear the entire electronic store or its individual units, that is the accumulator, the B-tube, the control lines, the multiplicand register, and all 8 storage tubes (individual storage tubes cannot be cleared).

1.5. Manual writing keys

These provide means for inserting or removing digits from any line of the electronic store.

1.6. Input and Output control switches

These enable the operator to select the printer and/or the punch as the o tput unit and to inhibit the operation of the output unit and/or the tape reader.

1.7. Writing suppression switches

The effect of undesired magnetic writing transfers can be disastrous. In many problems no such transfers need be made. During the operation of such programmes it is usual to suppress all writing transfers by switching off the write power. It is also possible to suppress the write power on certain blocks of 16 tracks, namely the groups // to K/, T/ to /E, etc. It is not (easily) possible to suppress writing on individual tracks.

The monitors

Monitor tubes and neons are provided by means of which the behaviour of the machine can be observed. The contents of the control tube, the B-tube, the accumulator, and the multiplier registers are displayed on 4 different 3" cathode ray tubes. The content of any two stores can be displayed on two 6" tubes.

Two neon lamps indicate the sign digits of the multiplicand register and the last B line altered (on which /T discriminates), the neon being lit if the corresponding digit is a 1.

Numerous special points arise in connection with the precise b haviour of certain keys and switches, and with the interpretation of the monitors in special circumstances. No attempt will be made to list them here as they are not regarded as important.

Apart from the control of completion signals and the selection of output unit there are two important uses of the keys and switches. These are to carry out magnetic transfers and to alter lines in the electronic store.

Manual magnetic transfers

For these purposes the completion signals must be switched off and the MAN-AUTO switch in the AUTO position.

Method 1 (reading transfers only)

Set up the magnetic instruction on H.

Operate (i.e., momentarily press) K.E.C. (clear everything key).

Operate the single prepulse key.

The last operation causes the transfer. The explanation can be found on page 4.1.

Method 2.

Set up the magnetic instruction on H.

Press K.C.C. (clear control key) and keep pressed.

Operate the single prepulse key.

Finally release K.C.C.

Roughly speaking what happens is that the control unit is forced to obey //// as an instruction, and hence the row H as a magnetic instruction.

Alteration of lines

Here also the completion signals must be switched off but the MAN-AUTO switch is now moved to the MAN position.

Set up the new content of the line to be altered on H.

Set up s,/Z on the manual instruction switches (s being the name of the line to be altered).

Operate the single prepulse key.

This is probably the principal application of the /Z instruction!

The two applications often go hand in hand. Thus to alter a line of a page of the magnetic store, the page in question is first transferred to (say) tube 0 using either method 1 or 2 , the appropriate line of tube 0 is then altered as described above and finally the new version is transferred to the magnetic store (using method 2). After a little practice one can carry through the complete operation in a few seconds.

2. The Tape Preparation Equipment

This equipment is concerned with the physical preparation of programme tapes for the machine.

There are two keyboard perforators. These are instruments, similar to typewriters, for punching combinations of holes in paper tape. They have 32 keys on which are engraved the characters /, E,...£, and which when depressed, punch the corresponding combination of holes on the tape.

At the same time the corresponding teleprint signal is transmitted along a teleprint line. There are also keys engraved with symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ·, (SP), (CR), (LF), (FIGS), (LETTERS), +, =, -. whose effects are respectively identical with /, E, @, A, :, S, I, U, ½, D, F, Z, L, W, H, Y, P, Q, M.

There is also a reperforator which accepts signals from a teleprint line and punches the corresponding combinations of holes. Associated with each keyboard is a tape reader which reads tape and provides an electrical output of teleprint signals. By coupling the reperforator to a keyboard and reader one may copy tape with interpositions of new material and possibly with omissions. One may also use the reader and reperforator to copy several short tapes on to one length of tape.

There is also a mechanism for printing the contents of tapes. In the letter shift position this gives the standard characters; on figure shift the equivalent symbol or operation will apply, namely

0 1 2 3 4 5 6 7 8 9 · SP CR LF + = " - £

/ E @ A : S I U ½ D F Z L W P Q " M £

The changes from figure to letter shift and back are manually controlled. The printer stops automatically at the end of a line and the carriage return and line feed can be operated by the appropriate keys on the keyboard. This applies to both the letter and figure shift position but in the latter case these operations also take place whenever L and W occur on the tape.

3. Fault Diagnosis in Programmes

This section is concerned with the problem of finding mistakes in programmes. The incidence of such mistakes and the difficulty of locating them may prove serious obstacles to the efficient use of the machine. Moreover experience with machines has shown that the occurence of mistakes is not a temporary evil due to lack of experience but is likely to remain as long as programmes are drawn up by human beings. Examples of the sort of mistakes to bear in mind are the omission of B digits from B modified instructions, the use of wrong control numbers (this may occur as a result of correcting other mistakes by interpositions of instructions), or arithmetical mistakes such as forgetting to clear the accumulator or using T/ instead of T½. Whilst a lot of such mistakes can be avoided by reasonable care and attention, there frequently remain mistakes which could only have been detected in the early stages by prolonged and laborious study.

One method for locating programme mistakes is known as "peeping". This is a derogatory term for the following procedure. Seated at the console and with the aid of the dummy stops and single prepulse key the mathematician works through his programme, or sections of it, instruction by instruction observing from the monitors the behaviour of certain storage locations. This is not a very efficient procedure because the machine can obey several hundreds of instructions in the time taken to reach for a switch or to note the new content of a line.

However the facilities provided on the console do enable one to correct errors on the spot when they have been found, so that, in the hands of an experienced operator, this procedure may involve less machine time than the alternative one of vacating the machine every time a mistake is found, adding a correction to the programme tape, and then rerunning the corrected version later on.

Some of the basic techniques of fault diagnosis will now be described. A comprehensive account of these and other more sophisticated techniques has been given by S.Gill (The Diagnosis of Mistakes in Programmes on the EDSAC, Proc.Roy.Soc., 206, 538-554,1951).

The basic idea is to observe as much evidence as possible from the behaviour of the machine whilst the programme is running.

At the input stage the actual progress of the tape through the reader provides evidence of any mistakes that may have occurred in preparing the tape. Thus if the tape unaccountably stops, then the place should be noted. If incorrect positioning of the write power switches and the dummy stops is not the cause, then an examination of the punching in the neighbourhood of the stopping place should quickly reveal the source of trouble.

When the programme proper has been read into the machine and initiated, then the D switch technique may be used. This is a switch on the console which enables the operator to change from the normal mode of operation to an instruction printing (or punching) mode. In this the 4th character (i.e., the 5 most significant function digits) of each actual instruction is output immediately after it has been obeyed. Symbols that should normally be output by the programme under test become interspersed with the main output. If the punch only is used then the procedure is quite fast and does not have to be manually interrupted in order to provide carriage returns and line feeds, as is necessary if the printer is used. The aggregate of output symbols thus forms a complete record of the course of the programme which can be torn off, taken away, and printed and studied at leisure. Such a record is extremely useful when testing any routine for the first time.

The change from the normal mode to the printing mode and back is most conveniently made at the /G stop in the R.C.S.

In other techniques the original programme is modified so as to provide further evidence of its progress when running. A simple example of this procedure is the alteration of a programme to cause the printing of intermediate results which are not ultimately required, the additional values serving as checks.

Another anticipatory measure is to insert an output instruction at the head of each routine so as to give a characteristic symbol when called in. The appearance of such symbols will thus be an indication of the state of the programme.

Another device is to input all routines onto a "backcloth" of (say) /L instructions. This may be done by transferring to tubes 4 and 5 a page of such instructions immediately before each routine is read from the tape. Thus if control should subsequently depart from the instructions (as would happen in the case of a wrong control number) then knowing the point at which it encounters a /L stop may reveal the source of trouble. The backcloth of /L instructions can be generated by the following interlude which may be included at the head of the programme tape. Thus

K / U R	/ / / L	
	/ / / L	
	M A / /	
	S U / /	Interlude for filling tubes
	@ U Q O	4 & 5 with ///L instructions
	/ U T /	
	/ ½ U S	
	A : Q G	
	A U / T	
	S : / P	returns control to TELEINPUT
T A U		enters interlude
M £ U R ½		transfers the backcloth to (for example) tract 255

If now each routine is preceeded by MEU@½, then those (irrelevant) lines not altered by the punching proper remain as ///L when the routine is transferred to the magnetic store and subsequently used.

Alternatively the dummy stops can be included in the punching proper.

"Post Mortem" Tapes

The "post mortem" technique is used when it is required to know the numerical contents of certain parts of the magnetic store. If SEQUENCE CONTROL + DECOUTPUT is already in the magnetic store, then by reading a short steering tape the material in question can be transferred to the electronic store and the relevant lines printed or punched.

Thus e.g., let it be required to know the contents, interpreted as fractions, of the last two line pairs of half track 50R. The appropriate steering tape is as follows.

M L E E ½	Transfers material to tube 4	
K P S @	(cue to SEQUENCE CONTROL)	
K / U J C S / /		interlude for calling in SEQUENCE CONTROL as a sub-routine of TELEINPUT (see page 4.5)
N / / /		
M D : K	ab = MD	
@ / / /	(m = 2)	
E U P O	(n = 13)	
/ : Y O	(c = 1)	
D U / C	(λ = o)	
@ U T /	sets L	
/ U / P	enters R.C.S	
/ / / /		
/ / / /		
T A U		enters Interlude

When the output routine has completed its task control returns to TELEINPUT via the link which is normally sitting in line pair HS.

4. Measures Against Machine Breakdown

Warning

It should be clearly understood that the machine is liable to carry out certain operations incorrectly. This malfunctioning can be either chronic or transient. Everything is being done on the engineering side to eliminate these undesirable features, but in the meantime programmers are obliged to minimise the effects of errors by incorporating checks of some kind, preferably overall mathematical checks, in the programme.

Nature of faults

In the case of many units, e.g., the power supplies, the effects of malfunctioning are usually obvious at once. When such a fault is detected then the maintenance engineers should be informed immedietely.

Transient faults in the arithmetical units are liable to pass unnoticed.

A further kind of fault consists of the appearance of spurious digits in one or more of the electronic stores. This is known as clodding. Faults of this kind in tubes containing instructions may easily cause the programme to breakdown.

Other possible faults of a transient nature are incorrect magnetic transfers. These have already been described in Chapter 4. If the transfer involves numerical information, then it may pass unnoticed. If a routine is transferred incorrectly, then the programme will most likely break down. Magnetic write checks may be used to check the results of writing transfers. The difficulty with the use of the read check for reading transfers is that the signal at its most vulnerable level passes through exactly the same mechanism in both operations : the effectiveness is thus very questionable.

As already explained in chapter 4 users are advised to arrange their programmes so that they are not tied to any fixed tracks. Instead they should think in terms of entries in a directory, a list of recommended tracks which can be varied either by himself or by the machine in accordance with the state of the wheel.

Fault detection and precautions

A suitable technique for minimising the effects of machine breakdown is the following.

The calculation is broken down into steps and the programme is so arranged that a decision as to whether each step has or has not been carried out correctly can be taken by an operator on the basis of information given by the machine at the end of each step; and that, if necessary, the calculation can be restarted at the beginning of each step by (say) simply operating K.E.C.

If the programme is made as far as possible self checking then the information given to the operator can be kept to a minimum. In the case where overall mathematical checks are available for each step, or where each step really consists of a smaller step repeated once, and followed by a comparison of the results, then the required information need consist of only one binary digit - YES or NO, according to whether the check has succeeded or not. This information is most simply conveyed to the operator by arranging the machine to scream if the check succeeds, not otherwise. For this to be effective the steps should all be approximately equal in time. The duration of such steps depends on how the machine is behaving, but the programmer must be prepared to arrange for steps as small as 1 minute. If the machine is functioning well, however, then fault-free runs of up to ½ hr can be relied on.

APPENDIX 1.

Abbreviated Instruction Code.

/ /	H as mag. instr.	T /	$A' = S_+$
/ E	S' = M	T E	
/ @	standardise	T @	
/ A	S' = M, M' = 0.	T A	S' = L, A' = 0
/ :	S as mag. instr.	T :	A' = 0
/ S	S' = L	T S	
/ I	L' = M, M' = L	T I	$A' = A + S_+$
/ U	S' = L, L' = M, M' = 0	T U	
/ ½	$A' = A - D\,S_+$	T ½	$A' = S_{\pm}$
/ D	$A' = A - D\,S_{\pm}$	T D	$A' = A \vee S_{\pm}$
/ R	Sideways adder	T R	$A' = A \,\&\, S_{\pm}$
/ J	M' = M + S	T J	$A' = A \neq S_{\pm}$
/ N	$A' = A + D\,S_+$	T N	$A' = A - S_{\pm}$
/ F	$A' = A + D\,S_{\pm}$	T F	$A' = -S_{\pm}$
/ C	$D' = S_+$	T C	$A' = A + S_{\pm}$
/ K	$D' = S_{\pm}$	T K	$A' = 2S_{\pm}$
/ T	B-cond.direct trans.control	T T	B' = S
/ Z	S' = H	T Z	S' = B
/ L	stop	T L	B' = B - S
/ W	Random number generator	T W	B' = B - S
/ H	A-cond.direct trans.control	T H	
/ Y		T Y	
/ P	Uncond.direct trans.control	T P	
/ Q	Uncond.rel.trans.control	T Q	
/ O	B-cond.rel.trans.control	T O	B' = S
/ B		T B	S' = B
/ G	Stop	T G	B' = B - S
/ "		T "	B' = B - S
/ M	A-cond. rel trans. control	T M	
/ X		T X	
/ V	Hoot	T V	
/ £		T £	Dummy

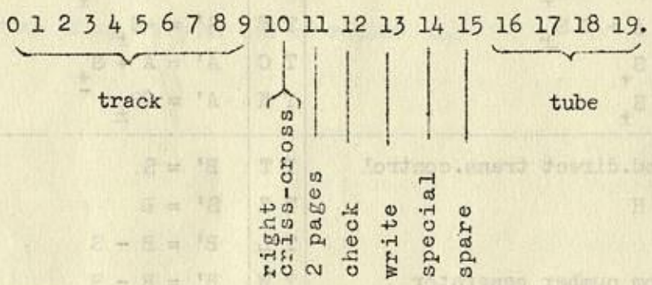
no B-addition

The reader will observe that the instructions with positive codes, i.e., which lie in the upper half of the table, refer mainly to the accumulator, whilst the remainder are transfers of control or refer to the B-tube. With certain exceptions the former are 5 beat codes, the latter take 4 beats.

The exceptions are the multiplication codes /½, /D, /N, and /F which each take 9 beats, the random number instruction which takes 24 beats, and the magnetic instructions which take times as follows: read, read check, write check 35 ms; write 90 ms.

Magnetic Instructions.

The purpose of the 20 digits of a magnetic instruction can be represented as follows



A special magnetic instruction (input-output) is interpreted according to its 3rd character as follows

T	Print and Punch
Z	Space
L	Carriage return
W	Line feed
H	Figures
Y	Letters
O	Input
B	Check output
£	Official Dummy.

The Teleprinter Code

0	00000	/	0	11	11010	J		22	01101	P	+
1	10000	E	1	12	00110	N		23	11101	Q	
2	01000	@	2	13	10110	F		24	00011	O	
3	11000	A	3	14	01110	C		25	10011	B	
4	00100	:	4	15	11110	K		26	01011	G	
5	10100	S	5	16	00001	T		27	11011	"	
6	01100	I	6	17	10001	Z		28	00111	M	-
7	11100	U	7	18	01001	L		29	10111	X	
8	00010	½	8	19	11001	W		30	01111	V	
9	10010	D	9	20	00101	H		31	11111	£	
10	01010	R		21	10101	Y					

Where the figure shift equivalent is not shown there may only be a smudge.

Aids to Calculation in the Scales of 2 and 32.

Fig.1 Multiplication by powers of 2.

The product is given one character at a time. Each character is determined by two adjacent characters of the multiplicand. The less significant is shown on the left of the table, the more significant above it. Thus, e.g., to multiply the array IRELAND by 8, the pattern is first bordered by a couple of /'s, thus /IRELAND/, and the characters in the result are then given as follows

D and / give @
N " D " J,
A " N " /, and so on,

the final result being TZRTM/J@.

Fig.2 Teleprint Equivalents of Some Powers of 10.

Fig.3 Integral Equivalents of Pairs of Teleprint characters.

E.G., that for YR = 1010101010 is given at the intersection of Y on the horizontal scale with R on the vertical scale, which is 341. It may be used to facilitate the binary conversion of a decimal fraction. The fraction is multiplied by 1024 and the pair of teleprint characters corresponding to the integral part looked up in the table. The process is repeated with the fractional part.

Fig.4 Inverse Powers of Two in Decimal Form.

Fig.5 Powers of Two in Decimal Form.

Fig.6 Some Useful Constants and their Teleprint Equivalents.

Fig.7 Decimal Round Off Constants, Factorials and their Reciprocals.

(X 2)

	/ E @ A : S I U ½ D R J N F C K
	T Z L W H Y P Q O B G " M X V £
/ E @ A : S I U ½ D R J N F C K	/ @ : I ½ R N C T L H P O G M V
T Z L W H Y P Q O B G " M X V £	E A S U D J F K Z W Y Q B " X £

(X 4)

		/ E @ A : S I U
		½ D R J N F C K
		T Z L W H Y P Q
		O B G " M X V £
	/ E @ A : S I U	/ : ½ N T H O M
	½ D R J N F C K	E S D F Z Y B X
	T Z L W H Y P Q	@ I R C L P G V
	O B G " M X V £	A U J K W Q " £

(X 16)

		/ E @ A
		: S I U
		½ D R J
		N F C K
		T Z L W
		H Y P Q
		O B G "
		M X V £
/ E @ A : S I U ½ D R J N F C K T Z L W H Y P Q O B G " M X V £	/ T Z L W H Y P Q O B G " M X V £	

(X 8)

/ E @ A : S I U ½ D R J N F C K T Z L W H Y P Q O B G " M X V £	/ ½ T O E D Z B @ R L G A J W " : N H M S F Y X I C P V U K Q £

Fig.1. Multiplication by powers of 2.

10	R	$2^{40} \times 10^{-1}$	G N I W B N I A
10^2	: A	$2^{45} \times 10^{-2}$	O Y U R O Y U R
10^3	½ £	$2^{45} \times 10^{-3}$	B J G F L O / E
10^4	T O D	$2^{50} \times 10^{-4}$	M O G S C " ½ A
10^5	/ Y E A	$2^{55} \times 10^{-5}$	C M ½ N F Z K R
10^6	/ L T V	$2^{55} \times 10^{-6}$	" @ H Q Q Z E E
10^7	/ H S Z D	$2^{60} \times 10^{-7}$	X Y I S £ J J A
10^8	/ ½ O J £ @	$2^{65} \times 10^{-8}$	@ I @ Q A W Q R
10^9	/ T L Y B X	$2^{65} \times 10^{-9}$	W / X J T J @ E
10^{10}	/ / B Q / R D	$2^{70} \times 10^{-10}$	O C P £ F V F A
$2^{-10} \times 10^{11}$	G F U : X @	$2^{75} \times 10^{-11}$	I K E £ £ G £ R
$2^{-10} \times 10^{11}$	: R R R A X	$2^{75} \times 10^{-12}$	Q : T B M S A E
$2^{-10} \times 10^{13}$	½ S U U E A D	$2^{80} \times 10^{-13}$	D @ G : M L T A
$2^{-10} \times 10^{14}$	T H U ½ N V G @	$2^{85} \times 10^{-14}$	S H I M I D ½ J
$2^{-10} \times 10^{15}$	/ F N L G K F M	$2^{85} \times 10^{-15}$	H " W K X / : E
$2^{-15} \times 10^{16}$	@ M Q D V I M ½	$2^{90} \times 10^{-16}$	B V S S J D W A
$2^{-20} \times 10^{17}$	B C E K S G O @	$2^{95} \times 10^{-17}$	" ½ I R X X T J
$2^{-20} \times 10^{18}$	P W C P P S O "	$2^{95} \times 10^{-18}$	D U G X L M : E
$2^{-25} \times 10^{19}$	S L / A B Z Y ½	$2^{100} \times 10^{-19}$	Q X W L M E P A
$2^{-30} \times 10^{20}$	P S V G Z Q P @	$2^{105} \times 10^{-20}$	D L N ½ ½ B B J

The figures given have an accuracy of ± 1.

Fig.2 Teleprint Equivalents of Some Powers of 10.

	/	E	@	A	:	S	I	U	½	D	R	J	N	F	C	K	T	Z	L	W
/	0										10									
E	32								40										50	
@	64						70										80			
A	96				100										110					
:	128		130										140							
S	160										170									
I	192								200										210	
U	224						230										240			
½	256				260										270					
D	288		290										300							
R	320										330									
J	352								360										370	
N	384						390										400			
F	416				420										430					
C	448		450										460							
K	480										490									
T	512								520										530	
Z	544						550										560			
L	576				580										590					
W	608		610										620							
H	640										650									
Y	672								680										690	
P	704						710										720			
Q	736				740										750					
O	768		770										780							
B	800										810									
G	832								840										850	
"	864						870										880			
M	896				900										910					
X	928		930										940							
V	960										970									
£	992								1000										1010	

Fig.3 Integral Equivalents of Pairs of Teleprint characters.

	H	Y	P	Q	O	B	G	"	M	X	V	£
/	20										30	31
E	52								60			63
@	84						90					95
A	116				120							127
:	148		150									159
S	180										190	191
I	212								220			223
U	244						250					255
½	276				280							287
D	308		310									319
R	340										350	351
J	372								380			383
N	404						410					415
F	436				440							447
C	468		470									479
K	500										510	511
T	532								540			543
Z	564						570					575
L	596				600							607
W	628		630									639
H	660										670	671
Y	692								700			703
P	724						730					735
Q	756				760							767
O	788		790									799
B	820										830	831
G	852								860			863
"	884						890					895
M	916				920							927
X	948		950									959
V	980										990	991
£	1012								1020			1023

/	.015625	.00000	/
E	.046875	.03125	E
@	.078125	.0625	@
A	.109375	.09375	A
:	.140625	.125	:
S	.171875	.15625	S
I	.203125	.1875	I
U	.234375	.21875	U
½	.265625	.25	½
D	.296875	.28125	D
R	.328125	.3125	R
J	.359375	.34375	J
N	.390625	.375	N
F	.421875	.40625	F
C	.453125	.4375	C
K	.484375	.46875	K
T	.515625	.50	T
Z	.546875	.53125	Z
L	.578125	.5625	L
W	.609375	.59375	W
H	.640625	.625	H
Y	.671875	.65625	Y
P	.703125	.6875	P
Q	.734375	.71875	Q
O	.765625	.75	O
B	.796875	.78125	B
G	.828125	.8125	G
"	.859375	.84375	"
M	.890625	.875	M
X	.921875	.90625	X
V	.953125	.9375	V
£	.984375	.96875	£

Fig.3 Integral Equivalents in Pairs of Teleprint characters.

POWERS OF TWO IN DECIMAL FORM.

2^n	n
1	0
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
1 024	10
2 048	11
4 096	12
8 192	13
16 384	14
32 768	15
65 536	16
131 072	17
262 144	18
524 288	19
1 048 576	20
2 097 152	21
4 194 304	22
8 388 608	23
16 777 216	24
33 554 432	25
67 108 864	26
134 217 728	27
268 435 456	28
536 870 912	29
1 073 741 824	30
2 147 483 648	31
4 294 967 296	32
8 589 934 592	33
17 179 869 184	34
34 359 738 368	35
68 719 476 736	36
137 438 953 472	37
274 877 906 944	38
549 755 813 888	39
1 099 511 627 776	40
1 125 899 906 842 624	50

Fig.4 Powers of Two in Decimal Form.

INVERSE POWERS OF TWO IN DECIMAL FORM

n	$\frac{1}{2^n}$
0	1.0
1	0.5
2	.25
3	.125
4	.062 5
5	.031 25
6	.015 625
7	.007 812 5
8	.003 906 25
9	.001 953 125
10	.000 976 562 5
11	.000 488 281 25
12	.000 244 140 625
13	.000 122 070 312 ...
14	.000 061 035 156 ...
15	.000 030 517 578 ...
16	.000 015 258 789 ...
17	.000 007 629 394 ...
18	.000 003 814 697 ...
19	.000 001 907 348 ...

n	$\frac{1}{2^n}$ X 10^6
20	0.953 674 ...
21	.476 837 ...
22	.238 419 ...
23	.119 209 ...
24	.059 604 6..
25	.029 802 3..
26	.014 901 2..
27	.007 450 6..
28	.003 725 290..
29	.001 862 645..
30	0.000 931 323...
31	.000 465 661...
32	.000 232 831....
33	.000 116 415....
34	.000 058 207 66 ...
35	.000 029 103 83..
36	.000 014 551 92..
37	.000 007 275 96..
38	.000 003 637 98 ..
39	.000 001 818 989 ..
40	.000 000 909 495 ..
50	.000 000 008 881 79..

Fig.5 Inverse Powers of Two in Decimal Form.

Number	Decimal	Teleprint	
π	3.141 592 653 590	- - - -	- - - -
$\pi/4$	0.785 398 163 397	E Z ½ Y	X U : B
$\pi/20$	0.157 079 632 679	B D " X	S " / S
$\pi/60$	0.052 359 877 560	V F V D	Q W Y E
$\pi/600$	0.005 235 987 756	F E W R	L J S /
$1/\pi$	0.318 309 886 184	U B F A	N V S R
$1/2\pi$	0.159 154 943 092	W M P E	I £ @ S
$\sqrt{\pi}/2$	0.886 226 925 453	½ F A D	M K J M
$1/\sqrt{\pi}$	0.564 189 583 548	Y T / Y	J Q E J
e	2.718 281 828 459	- - - -	- - - -
$2^{-3}e$	0.339 785 228 557	Z @ J H	@ V " R
$2^{-2}\sqrt{e}$	0.412 180 317 675	O A C N	R @ I F
γ	0.577 215 664 902	V " O K	I @ K L
ln2	0.693 147 180 560	L V X K	E B S P
$2^{-3}\ln 10$	0.287 823 136 624	Y P " F	N Q I D
$2^{-3}\log_2 10$	0.415 241 011 861	U K D O	W I D F
lg e	0.434 294 481 903	M : Y O	V P M F
lg 2	0.301 029 995 664	V W T G	: ½ H D
$2^{-2}\log_2 e$	- - - -	C S S "	Z R Z J
$\sqrt{\frac{1}{2}}$	0.707 106 781 186	" £ N I	K @ H P
$1/\sqrt{3}$	- - - -	@ : J H	W I K L
$\sqrt{3}/4$	0.433 012 701 892	E J ½ Q	V N " F
1/3	0.333 333 333 333	Y R Y R	Y R Y R
1/5	0.200 000 000 000	W B N I	W B N I
1/7	0.142 857 142 857	L : D L	: D L :
1/9	0.111 111 111 111	Z A U C	M O Z A
1/11	0.090 909 090 909	X @ X @	X @ X @
1/13	0.076 923 076 923	Z X : P	W O C @
1/15	0.066 666 666 667	Z ½ : @	Z ½ : @
1/17	0.058 823 529 412	K O A V	T U M E
1/19	0.052 631 578 947	T L Q I	H M Y E
2/3	0.666 666 666 667	J Y R Y	R Y R Y
3/4	0.750 000 000 000	/ / / /	/ / / O

ln x = 2.302 585 092 994 . lg x

lg x = 0.434 294 481 903 . ln x

(Maximum error is ± 1 in 10^{12}.)

Fig.6 Some Useful Constants and their Teleprint Equivalents.

n	5×10^{-n}	
1	/ / / /	/ / / T
2	F I W B	N I W E
3	" A S M	G A S /
4	I X I D	N T / /
5	X @ Q F	H E / /
6	P P O U	S / / /
7	M " O T	/ / / /
8	/ P Y E	/ / / /
9	G J S /	/ / / /
10	I Z / /	/ / / /
11	Q E / /	/ / / /
12	S / / /	/ / / /
13	/ / / /	/ / / /

n	n!	1/n!
0	1	1.0
1	1	1.0
2	2	0.5
3	6	.166 666 666 667 ..
4	24	.041 666 666 667 ..
5	120	.008 333 333 333 ..
6	720	.001 388 888 889 ..
7	5 040	.000 198 412 698 ..
8	40 320	.000 024 801 587 ..
9	362 880	.000 002 755 732 ..
10	3 628 800	.000 000 275 573 ..
11	39 916 800	.000 000 025 052 ..
12	479 001 600	.000 000 002 088 ..
13	6 227 020 800	.000 000 000 161 ..
14	87 178 291 200	.000 000 000 011 ..
15	1 307 674 368 000	
16	20 922 789 888 000	
17	355 687 428 096 000	

Fig.7 Decimal Round-Off Constants, Factorials and their Reciprocals.