1

SIGNALS

INTRODUCTION, REPRESENTATION OF DISCRETE-TIME SIGNAL – GRAPHICAL REPRESENTATION, FUNCTIONAL REPRESENTATION, TABULAR REPRESENTATION, SEQUENCE REPRESENTATION, ELEMENTARY SIGNALS – UNIT STEP FUNCTION, UNIT RAMP FUNCTION, UNIT PARABOLIC FUNCTION, UNIT IMPULSE FUNCTION

SHORT QUESTIONS

Q.1. Define signal.

Ans. A signal can be defined as a function of one or more independent variables which conveys information. A signal is shown in fig.1.1. It can be a function of time, temperature, pressure, position, distance etc. Nowadays, some signals are available in the form of speech, music, image and video signals.

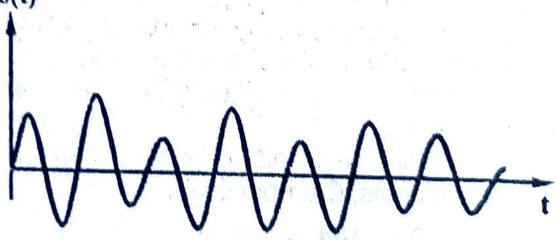


Fig. 1.1

The signals can be divided into two categories i.e., one dimensional signals or multidimensional signals.

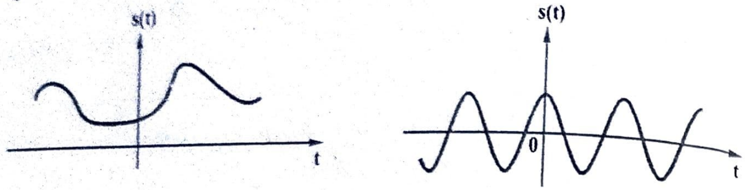
One Dimensional Signals – The signal is known as one dimensional when the function depends on a single variable. Speech signal is an example of one dimensional signal.

Multi-dimensional Signals – The signal is known as multidimensional when the function depends on two or more variables. The image signal is an example of multidimensional signal. Image signal is a two dimensional signal with horizontal co-ordinate and vertical co-ordinate.

Q.2. What are the classification of signal ?

Ans. The signal are of two types –

(i) Continuous Time Signal – It is defined as the mathematical continuous function. This function can be defined continuously in the time domain. A signal that varies continuously with time is known as continuous time signal. It is represented by s(t) where s represents the shape of the signal and t represents that the variable is time. A continuous time signal is shown in fig. 1.2.



(a) (b)

Fig. 1.2 Continuous Time Signal

Therefore, we can say that a signal of continuous amplitude and time is called a continuous signal. A continuous signal is also called an analog signal. The example of a continuous time signal are temperature, sound, sine wave and cosine wave etc.

(ii) Discrete Time Signals – Discrete time signals are defined for discrete values of an independent variable (time). Discrete time signal is not defined at instants between two successive samples. It is incorrect to think that s(n) is equal to zero if n is not an integer. Simply the signal s(n) is not defined for noninteger values of n. A sequence of number s in which the n^{th} number in the sequence is denoted as s(n) and it is in formal manner written by the expression as –

$$s = s(n) \quad \text{for} -\infty < n < \infty$$

The discrete time signals i.e., sequences are graphically shown in fig. 1.3.

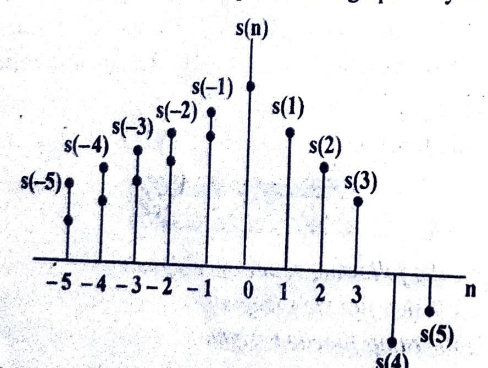


Fig. 1.3 Graphical Representation of a Discrete Time Signal

Q.3. How discrete time signals are represented ?

Ans. Discrete time signals can be represented in following ways –

(i) Graphical Representation – Discrete time signals can be represented by a graph when the signal is defined for every integer value of n for $-\infty < n < \infty$ as shown in fig. 1.3.

(ii) Functional Representation – Discrete time signals can be represented functionally as given below –

$$s(n) = \begin{cases} 2, & \text{for } n = 1,\ 3 \\ 4, & \text{for } n = 2 \\ 0, & \text{elsewhere} \end{cases}$$

(iii) Tabular Representation – Discrete time signals can also be represented by a table as –

n	......	−3	−2	−1	0	1	2	3	4	5	......
s(n)		0	0	0	1	2	1	0	0	0	

(iv) Sequence Representation – An infinite duration $(-\infty \le n \le \infty)$ signal with the time as origin (n = 0) and indicated by the symbol ↑.

$$s(n) = \ldots\ldots\ 0,\ 0,\ 0,\ \underset{\uparrow}{1},\ 3,\ 1,\ 0,\ 0$$

Q.4. What are the elementary signals ?

Ans. There are some elementary signals which play an important role in the study of signals and systems. These elementary signals are used to model a large number of physical signals that take place in nature. These elementary signals are also known as standard signals. The elementary signals are as follows –

(i) Unit step function (ii) Unit impulse function
(iii) Unit ramp function (iv) Unit parabolic function
(v) Real exponential signal (vi) Complex exponential signal
(vii) Sinusoidal function (viii) Rectangular pulse function
(ix) Triangular pulse function (x) Signum function
(xi) Sinc function (xii) Gaussian function.

LONG QUESTIONS

Q.5. Define the following elementary signals –
(i) Unit step function (signal)
(ii) Unit ramp function (signal).

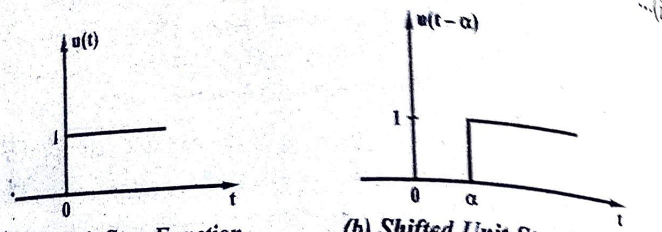
Ans. (i) Unit Step Function – The unit step function exists only for positive value of time (t > 0) and is zero for negative value of time (t < 0). The function is said to be unit step function when a step function has unity magnitude.

(a) Continuous-time Unit Step Function – The continuous time unit step function u(t) is shown in fig. 1.4 (a). Mathematically, it is expressed as –

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} \quad ...(i)$$

The shifted unit step function u(t – α) [see fig. 1.4 (b)] can be expressed as follows –

$$u(t-\alpha) = \begin{cases} 0, & t < \alpha \\ 1, & t \geq \alpha \end{cases} \quad ...(ii)$$



(a) Unit Step Function for Continuous Time *(b) Shifted Unit Step Function for Continuous Time*

Fig. 1.4

(b) Discrete-time Unit Step Function – The discrete time unit step function is denoted by u(n). Mathematically, it is expressed as –

$$u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases} \quad ...(iii)$$

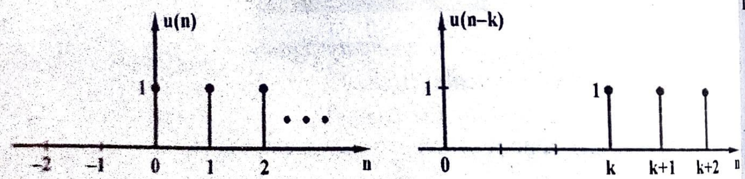
In the form of sequence, it can be written as

$$u(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1, ...\}$$

The shifted unit step sequence u(n – k) can be expressed as

$$u(n-k) = \begin{cases} 0, & n < k \\ 1, & n \geq k \end{cases} \quad ...(iv)$$

The unit step function and shifted unit step function for discrete time are shown in figs. 1.5 (a) and (b).



(a) Unit Step Function for Discrete Time *(b) Shifted Unit Step Function for Discrete Time*

Fig. 1.5

(ii) Unit Ramp Function –

(a) Continuous Time Unit Ramp Function – It is denoted by $u_r(t)$ which starts at t = 0 and rises linearly with time. Mathematically, it is expressed as –

$$u_r(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$
$$= t\,u(t)$$

The unit ramp function is calculated by integrating the unit step function. This implies that, unit step function u(t) is calculated by differentiating unit ramp function $u_r(t)$.

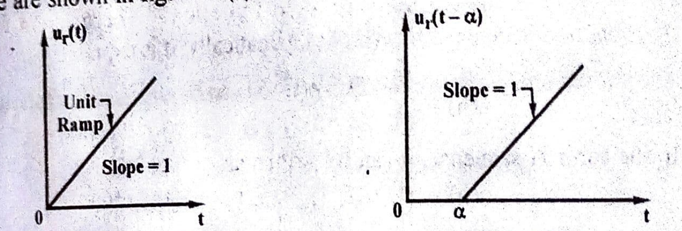
$$\therefore \quad u_r(t) = \int u(t)\,dt = \int dt = t, \quad t \geq 0$$

and
$$u(t) = \frac{d}{dt} u_r(t)$$

The shifted unit ramp function $u_r(t - \alpha)$ can be expressed as follows –

$$u_r(t-\alpha) = \begin{cases} 0, & t < \alpha \\ t-\alpha, & t \geq \alpha \end{cases}$$
$$= (t - \alpha)\,u(t - \alpha)$$

The unit ramp function and shifted unit ramp function for continuous time are shown in figs. 1.6 (a) and (b).



(a) Unit Ramp Function for Continuous Time *(b) Shifted Unit Ramp Function for Continuous Time*

Fig. 1.6

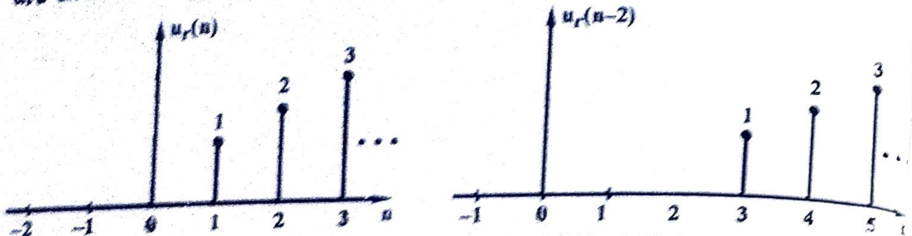
(b) Discrete-time Unit Ramp Function – Discrete-time unit ramp function is denoted by $u_r(n)$. Its value increases linearly with sample number n. Mathematically, it is expressed as

$$u_r(n) = \begin{cases} 0, & n < 0 \\ n, & n \geq 0 \end{cases}$$
$$= n\,u(n)$$

The shifted unit ramp function $u_r(n - 2)$ is expressed as

$$u_r(n-2) = \begin{cases} 0, & n < 2 \\ n-2, & n \geq 2 \end{cases}$$
$$= (n - 2)\,u(n - 2)$$

The unit ramp function and shifted unit ramp function for discrete time are shown in figs. 1.7 (a) and (b).



(a) Unit Ramp Function for Discrete Time

(b) Shifted Unit Ramp Function for Discrete Time

Fig. 1.7

Q.6. Explain the unit parabolic function in detail.

Ans. (i) Continuous-time Unit Parabolic Function – The continuous time unit parabolic function is denoted by p(t) which starts at t = 0. The continuous-time unit parabolic function is also known as unit acceleration signal. Mathematically, it is expressed as follows –

$$p(t) = \begin{cases} 0, & t < 0 \\ t^2/2, & t \geq 0 \end{cases} \quad ...(i)$$

$$= \frac{t^2}{2} u(t)$$

The shifted unit parabolic function $p(t-\alpha)$ can be expressed as follows –

$$p(t-\alpha) = \begin{cases} 0, & t < \alpha \\ \dfrac{(t-\alpha)^2}{2}, & t \geq \alpha \end{cases}$$

$$= \frac{(t-\alpha)^2}{2} u(t-\alpha)$$

Figs. 1.8 (a) and (b) show the unit parabolic function and shifted unit parabolic function for continuous time.

It can be determined by single integrating the unit ramp function or double intergrating the unit step function as follows –

$$p(t) = \int u_r(t)\, dt \quad ...(ii)$$

and

$$p(t) = \iint u(t)\, dt \quad ...(iii)$$

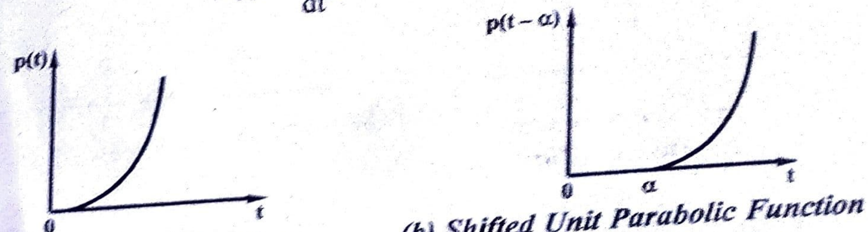
$$= \int t\, dt = \frac{t^2}{2}, t \geq 0$$

The unit ramp function is equal to the differentiating of unit parabolic function.

$$u_r(t) = \frac{d}{dt} p(t) \quad ...(iv)$$

The unit step function is equal to the double differentiating of parabolic function.

$$u(t) = \frac{d^2}{dt^2} p(t) \quad ...(v)$$



(a) Unit Parabolic Function for Continuous Time

(b) Shifted Unit Parabolic Function for Continuous Time

Fig. 1.8

(ii) Discrete-time Unit Parabolic Function – The discrete-time unit parabolic function is denoted by p(n). Mathematically, it is expressed as

$$p(n) = \begin{cases} 0, & n < 0 \\ n^2/2, & n \geq 0 \end{cases}$$

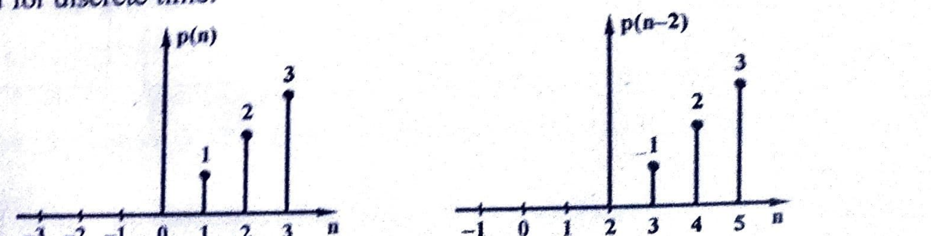
$$= \frac{n^2}{2} u(n)$$

The shifted unit parabolic function $p(n-2)$ is expressed as

$$p(n-2) = \begin{cases} 0, & n < 2 \\ \dfrac{(n-2)^2}{2}, & n \geq 2 \end{cases}$$

$$= \frac{(n-2)^2}{2} u(n-2)$$

Figs. 1.9 (a) and (b) show the unit parabolic signal and shifted parabolic signal for discrete time.



(a) Unit Parabolic Signal for Discrete-time

(b) Shifted Parabolic Signal for Discrete-time

Fig. 1.9

Q.7. Define the continuous-time unit impulse (sample) function and its properties.

Ans. The continuous-time unit impulse function is also known as Dirac delta function. It is denoted by $\delta(t)$. The Dirac delta function $\delta(t)$ is an extremely important function used for the analysis of signals and systems. Mathematically it can be expressed as follows –

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ 1, & t = 0 \end{cases} \qquad \text{...(i)}$$

The area under the unit impulse is given as

$$= \int_{-\infty}^{\infty} \delta(t)\, dt = 1 \qquad \text{...(ii)}$$

That is, u(t) is the running integral of the unit impulse function. From this it is obtained that –

$$\delta(t) = \frac{du(t)}{dt} \qquad \text{....(iii)}$$

There is clearly some formal difficulty with this as a definition of the unit impulse function since u(t) is discontinuous at $t = 0$ and consequently is formally not differentiable. Equation (iii) can be interpreted by considering u(t) as the limit of a continuous function. Therefore, let us define $u_\Delta(t)$ as shown in fig. 1.10 so that u(t) equals the limit of $u_\Delta(t)$ as $\Delta \to 0$, and let us define $\delta_\Delta(t)$ as –

$$\delta_\Delta(t) = \frac{du_\Delta(t)}{dt} \qquad \text{....(iv)}$$

It is observed that $\delta_\Delta(t)$ has unity area for any value of Δ and is zero outside the interval $0 \le t \le \Delta$. As $\Delta \to 0, \delta_\Delta(t)$ becomes narrower and higher to maintaining its unit area. Its limiting form,

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t) \qquad \text{....(v)}$$

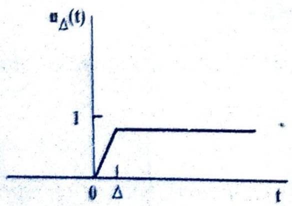


Fig. 1.10 Continuous Approximation to the Unit Step

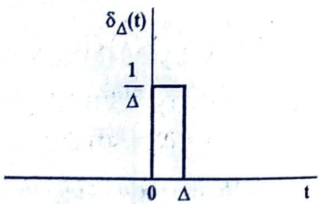


Fig. 1.11 Derivative of $u_\Delta(t)$

This will be depicted as indicated in fig. 1.12. In general, a scaled impulse $k\delta(t)$ will have an area k and therefore,

$$\int_{-\infty}^{t} k\,\delta(\tau)\,\delta\tau = ku(t)$$

A scaled impulse is indicated in fig. 1.13. Although the "value" at $t = 0$ is infinite, the height of the arrow used to indicate the scaled impulse will be selected to be representative of its area.

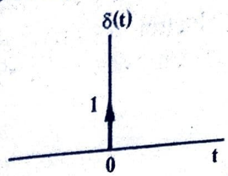


Fig. 1.12 Unit Impulse

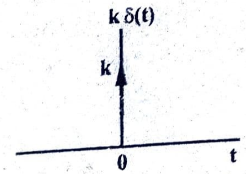


Fig. 1.13 Scaled Impulse

The shifted unit impulse function $\delta(t-\alpha)$ can be expressed as follows –

$$\delta(t-\alpha) = \begin{cases} 0, & t \neq \alpha \\ 1, & t = \alpha \end{cases}$$

The graphical representation of shifted unit impulse function $\delta(t-\alpha)$ is shown in fig. 1.14.

The properties of continuous time unit impulse function are as given below –

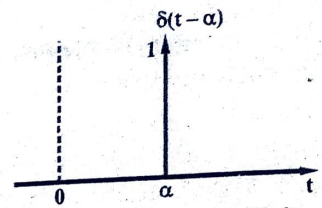


Fig. 1.14 Shifted Unit Impulse Function

(i) $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

(ii) Continuous-time unit impulse function is an even function of time t i.e.,

$$\delta(t) = \delta(-t)$$

(iii) $\int_{-\infty}^{\infty} s(t)\,\delta(t)\, dt = s(0)$

(iv) $\int_{-\infty}^{\infty} s(t)\,\delta(t-t_0)\, dt = s(t_0)$

(v) $\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$

(vi) $\int_{-\infty}^{\infty} s(\lambda)\,\delta(t-\lambda)\, d\lambda = s(t)$

(vii) $s(t_0) = s(t)\,\delta(t-t_0) = s(t_0)\,\delta(t-t_0)$

(ix) $s(0) = s(t)\,\delta(t) = s(0)\,\delta(t)$

Q.8. Define the unit impulse function for discrete time and its properties.

Ans. In discrete-time the unit impulse (or unit sample) is defined as –

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases} \qquad \text{....(i)}$$

which is shown in fig. 1.15. It should be noted that unlike its continuous-time counterpart, there are no analytical difficulties in defining $\delta[n]$.

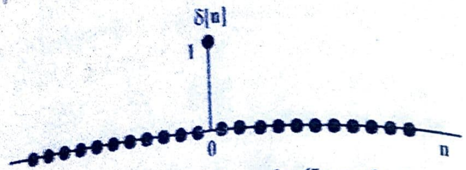


Fig. 1.15 Unit Sample (Impulse)

Discrete-time unit sample has many properties which closely parallel the characteristics of the continuous-time unit impulse. For example, since $\delta[n]$ is nonzero (and equal to 1) only for $n = 0$, it is immediately seen that –

$$x[n]\ \delta[n] = x[0]\ \delta[n] \quad \text{...(ii)}$$

Similar to continuous time impulse which is formally the first derivative of the continuous-time unit step, the discrete-time unit impulse is the first difference of the discrete-time step.

$$\delta[n] = u[n] - u[n-1] \quad \text{...(iii)}$$

In the same way, while the continuous-time unit step is the running integral of $\delta(t)$, the discrete-time unit step is the running sum of the unit sample. That is given as –

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \quad \text{...(iv)}$$

which is shown in fig. 1.16. As the only nonzero value of the unit sample is at the point at which its argument is zero, it is seen from the fig. that the running sum is 0 for $n < 0$ and 1 for $n \geq 0$. The discrete-time unit step can also be written in terms of the unit sample as –

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k] \quad \text{...(v)}$$

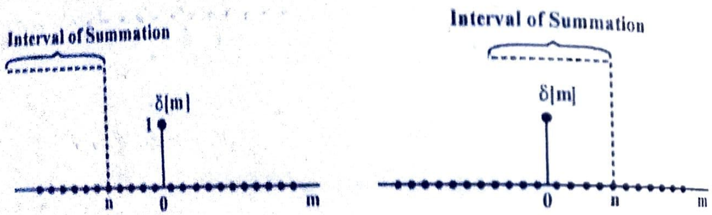


Fig. 1.16 Running Sum of Equation (a) $n < 0$ (b) $n > 0$.

The properties of discrete-time unit impulse function are as follows –

(i) $\delta(n) = u(n) - u(n-1)$ (ii) $\delta(n-k) = \begin{cases} 0, & n \neq k \\ 1, & n = k \end{cases}$

(iii) $s(n_0) = \sum_{n=-\infty}^{\infty} s(n)\ \delta(n - n_0)$ (iv) $\sum_{k=-\infty}^{\infty} s(k)\ \delta(n-k) = s(n)$

SINUSOIDAL SIGNAL, REAL EXPONENTIAL SIGNAL, COMPLEX EXPONENTIAL SIGNAL, RECTANGULAR PULSE FUNCTION, TRIANGULAR PULSE FUNCTION, SIGNUM FUNCTION, SINC FUNCTION, GAUSSIAN FUNCTION

SHORT QUESTIONS

Q.9. What is meant by sinusoidal signal ? Explain.

Ans. **Continuous Time Sinusoidal Signal** – A sinusoidal signal is an example of a periodic signal. The sinusoidal signals contain sine and cosine signals. A continuous-time sinusoidal signal is expressed as

$$s(t) = A \sin \omega t = A \sin (2\pi ft) \quad \text{...(i)}$$

where A is the amplitude and ω is angular frequency (in radian).

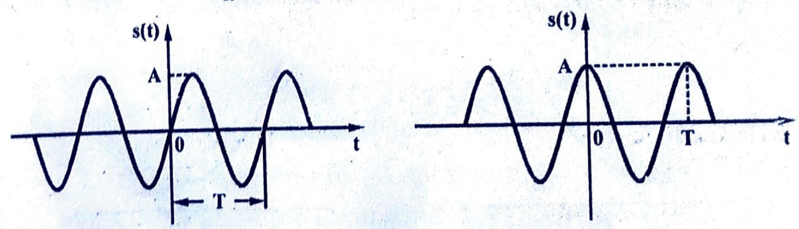
A cosine signal is expressed as –

$$s(t) = A \cos \omega t = A \cos (2\pi ft) \quad \text{...(ii)}$$

The waveforms of sine signal and cosine signal are shown in fig. 1.17 (a) and (b).

The time period T of a sinusoidal signal can be defined as follows –

$$T = \frac{2\pi}{\omega}$$



(a) Sine Signal ***(b) Cosine Signal***

Fig. 1.17 Sinusoidal Signal

Discrete-time Sinusoidal Signal – The discrete time sinusoidal signal is denoted by s(n) and is expressed as –

$$s(n) = A \sin \omega n = A \sin (2\pi fn)$$

where A is the amplitude, ω is angular frequency in radians and n is an integer.

The period of the discrete time sinusoidal signal is given by

$$N = \frac{2\pi}{\omega} m$$

The waveform of discrete-time sinusoidal signal is shown in fig. 1.18.

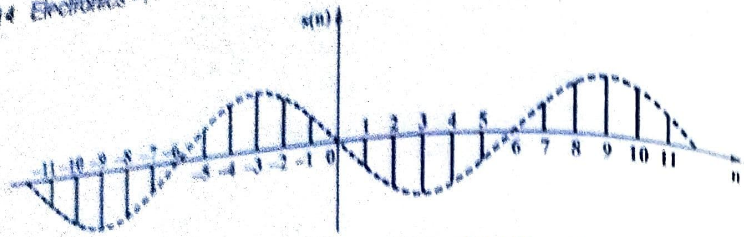


Fig. 1.18 Discrete Time Sinusoidal Signal

Similarly, a discrete-time cosine signal is defined as –

$$s(n) = A \cos \omega n = A \cos(2\pi fn)$$

Q.10. Write short note on real exponential signal.

Ans. **Continuous-time Real Exponential Signal** – A continuous-time real exponential signal is of the following form –

$$s(t) = Ce^{\alpha t} \quad ...(i)$$

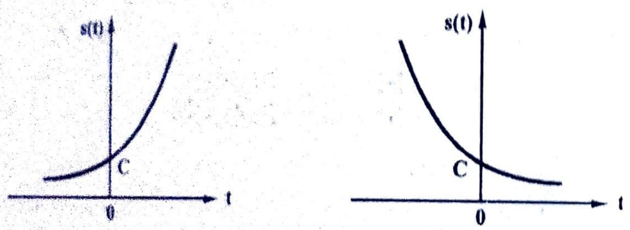
where both C and α are real.

The parameter α can be either positive or negative. There are two types of exponential signals.

(i) The signal s(t) is a rising exponential signal when α is positive (i.e. $\alpha > 0$).

(ii) The signal s(t) is a decaying exponential signal when α is negative (i.e. $\alpha < 0$).

The rising and decaying exponential signals are shown in figs. 1.19 (a) and (b).



(a) Rising Exponential Signal *(b) Decaying Exponential Signal*

Fig. 1.19

Mathematically, they are represented as under –

Rising exponential signal is

$$s(t) = e^{\alpha t} \quad ...(ii)$$

Decaying exponential signal is

$$s(t) = e^{-\alpha t} \quad ...(iii)$$

We have considered C = 1 for both the equation stated above.

Discrete-time Real Exponential Signal – A discrete-time real exponential signal is denoted by s(n) and can be expressed as

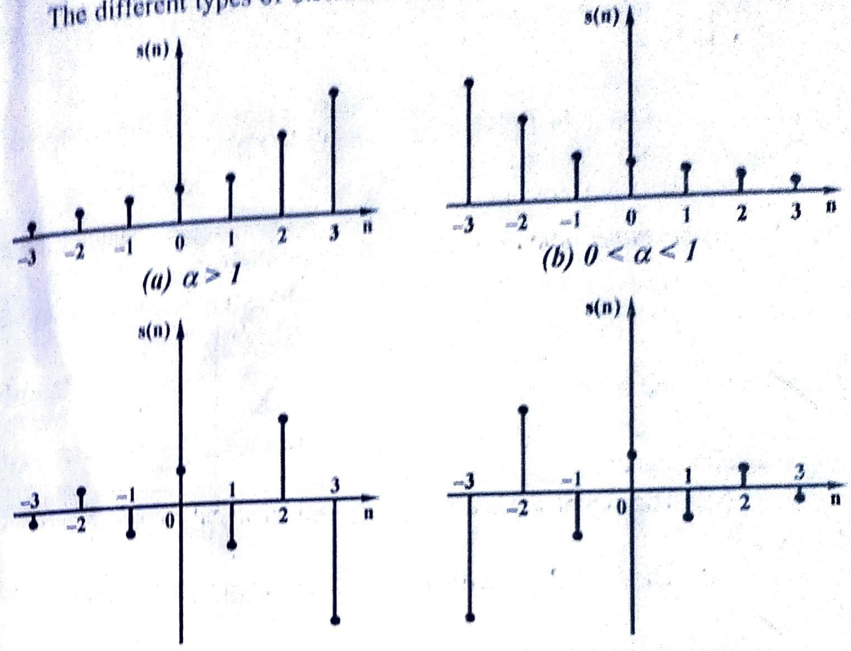
$$s(n) = \alpha^n \text{ for all } n$$

Depending on the values of α, we have four different cases as follows –

(i) Case 1 – $\alpha > 1$ (ii) Case 2 – $0 < \alpha < 1$

(iii) Case 3 – $\alpha < -1$ (iv) Case 4 – $-1 < \alpha < 0$.

The different types of discrete-time signals are shown in fig. 1.20.



(a) $\alpha > 1$ *(b) $0 < \alpha < 1$*

(c) $\alpha < -1$ *(d) $-1 < \alpha < 0$*

Fig. 1.20 Discrete Time Real Exponential Signal

LONG QUESTIONS

Q.11. Explain complex exponential signal.

Ans. **Continuous-time Complex Exponential Signal** – The general complex exponential signal is expressed as

$$s(t) = C\,e^{\alpha t} \quad ...(i)$$

where, C = amplitude

α = complex variable

The complex variable is given by

$$\alpha = r + j\omega_0$$

From equation (i), we get

$$s(t) = C\,e^{(r+j\omega_0)t}$$

$$= C\,e^{rt}.e^{j\omega_0 t}$$

$$= C\,e^{rt}(\cos \omega_0 t + j \sin \omega_0 t) \quad ...(ii)$$

For r = 0, we get

$$s(t) = C\,e^0(\cos\omega_0 t + j\sin\omega_0 t) \qquad [\because e^0 = 1]$$
$$= C(\cos\omega_0 t + j\sin\omega_0 t) \qquad ...(iii)$$

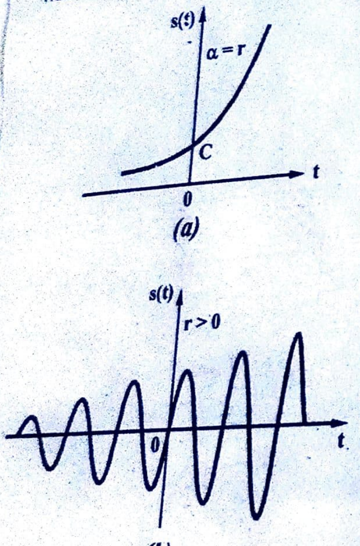
Therefore, the real and imaginary parts of the complex exponential are **sinusoidal.**

For r > 0,

If r is positive (r > 0), then e^{rt} in equation (ii) will be a rising exponential signal. A rising exponential signal is shown in fig. 1.21.

For r < 0

If r < 0, then e^{rt} in equation (ii) will be a decaying exponential signal which is shown in fig. 1.22.



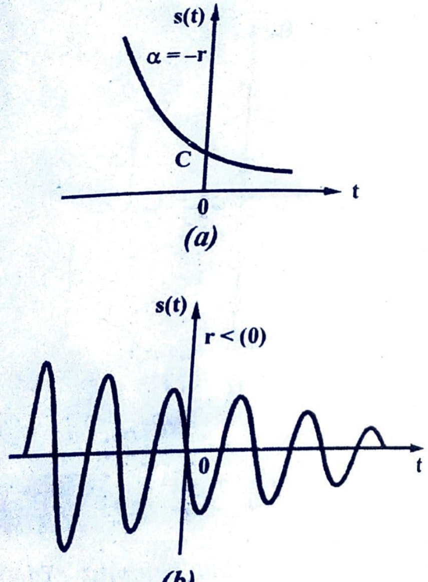


Fig. 1.21 Rising Exponential Signal *Fig. 1.22 Decaying Exponential Signal*

Discrete time complex exponential signal – The discrete time complex exponential signal is denoted by s(n). It is expressed as

$$s(n) = \alpha^n e^{j(\omega_0 n+\theta)}$$
$$= \alpha^n[\cos(\omega_0 n+\theta) + j\sin(\omega_0 n+\theta)] \qquad ...(iv)$$

The real and imaginary parts of complex exponential signal are sinusoidal for the value of $|\alpha| = 1$.

The value of amplitude of the sinusoidal signal exponentially decays for the value of $|\alpha| < 1$ as shown in fig. 1.23 (a).

The value of amplitude of the sinusoidal signal exponentially rises for the value of $|\alpha| > 1$ as shown in fig. 1.23 (b).

Ans.

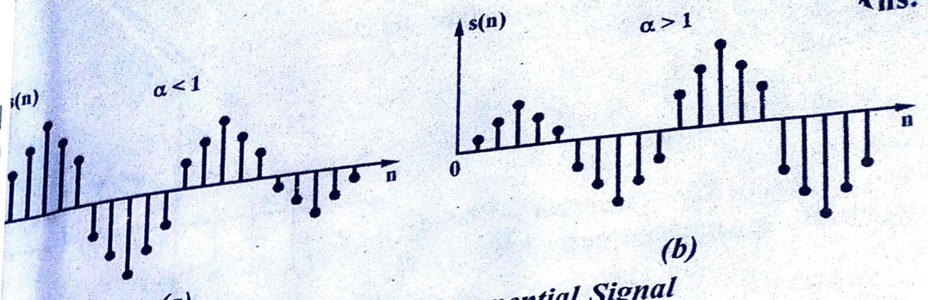


Fig. 1.23 Complex Exponential Signal

Q.12. Define the following elementary signals –
(i) Rectangular pulse function (ii) Triangular pulse function
(iii) Signum function (iv) Sinc function
(v) Gaussian function.

Ans. (i) Rectangular Pulse Function – A rectangular pulse function rect (t/T) of unit amplitude and duration T is shown in fig. 1.24. Mathematically, it can be expressed as –

$$\text{rect}(t/T) = \begin{cases} 1, & -\dfrac{T}{2} \le t \le \dfrac{T}{2} \\ 0, & \text{Otherwise} \end{cases}$$

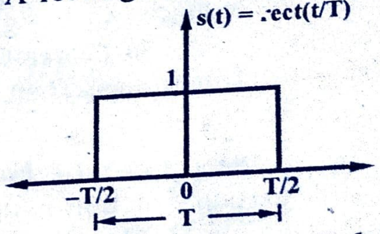


Fig. 1.24 Rectangular Pulse Function

(ii) Triangular Pulse Function – The triangular pulse function is denoted by Δ(t/T). Mathematically, it can be expressed as –

$$\Delta(t/T) = \begin{cases} 1-(2|t|/T), & |t| < (T/2) \\ 0, & |t| > (T/2) \end{cases}$$

The unit triangular pulse function is shown in fig. 1.25. It is an even function of t.

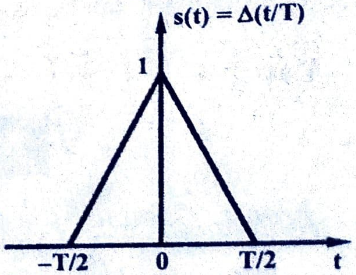


Fig. 1.25 Triangular Pulse Function

(iii) Signum Function – The signum function is denoted by sgn (t) and it is shown in fig. 1.26. Mathematically, it can be expressed as –

$$\text{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$

The signum function is defined in terms of unit step as follows –

$$\text{sgn}(t) = 2u(t) - 1$$

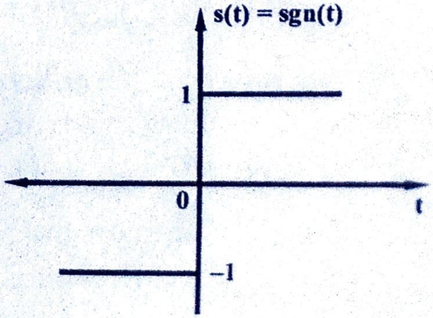


Fig. 1.26 Signum Function

Similarly, a discrete-time signum function sgn(n) shown in fig. 1.27 expressed as –

$$\text{sgn}(n) = \begin{cases} 1, & n > 0 \\ 0, & n = 0 \\ -1, & n < 0 \end{cases}$$

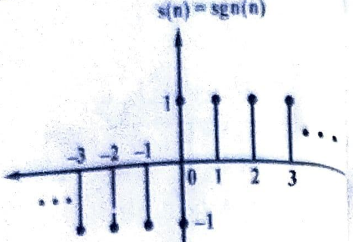


Fig. 1.27 Discrete Time Signum Function

(iv) Sinc Function – Mathematically, it can be expressed as –

$$\text{sinc}(t) = \frac{\sin(\pi t)}{(\pi t)} \quad \text{for } t = 0$$

where t is the independent variable.

$\text{sinc}(t) = 1$ at $t = 0$

and $\text{sinc}(t) = 0$ at $t = \pm 1, \pm 2, \pm 3$

The sinc function has the shape of a sine wave as shown in fig. 1.28. Its amplitude goes on decreasing as the value of |t| increases. Therefore, sinc $\to 0$ when $t \to \infty$.

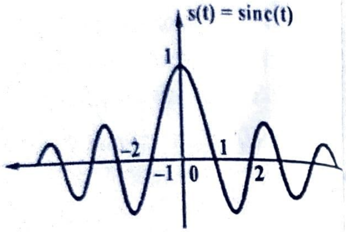


Fig. 1.28 Sinc Function

(v) Gaussian Function – The Gaussian function is denoted by $g_a(t)$ (see fig. 1.29). This function is extremely useful in probability theory. Mathematically, it can be expressed as –

$$g_a(t) = e^{-at^2}, \quad -\infty < t < \infty$$

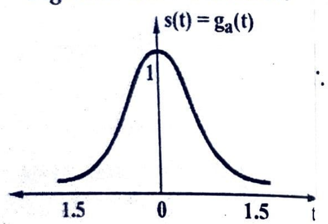


Fig. 1.29 Gaussian Function

NUMERICAL PROBLEMS

Prob.1. Compute the following integrals –

(i) $s(t) = \int_0^{\infty} t^3 \delta(t-5)\, dt$ (ii) $s(t) = \int_{-\infty}^{\infty} e^{-j2\omega t} \delta(t)\, dt$

(iii) $s(t) = \int_{-\infty}^{\infty} \delta(t+1) e^{-t} dt$ (iv) $s(t) = \int_0^{3} \delta(t) \sin 3\pi t\, dt$

(v) $s(t) = \int_{-\infty}^{\infty} (t-3)^2 \delta(t-3)\, dt$

(vi) $s(t) = \int_{-\infty}^{\infty} [\delta(t-1)\cos 3t + \delta(t-1)\sin 3t]\, dt$

Sol. (i) We know that

$$\delta(t-5) = \begin{cases} 0, & t \neq 5 \\ 1, & t = 5 \end{cases}$$

$\therefore$ $s(t) = \int_0^{\infty} t^3 \delta(t-5)\, dt = [t^3]_{t=5} = 5^3 = \mathbf{125}$ **Ans.**

(ii) We know that,

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

$\therefore$ $s(t) = \int_{-\infty}^{\infty} \delta(t) e^{-j2\omega t} dt = [e^{-j2\omega t}]_{t=0} = e^0 = \mathbf{1}$ **Ans.**

(iii) We know that

$$\delta(t+1) = \begin{cases} 0, & t \neq -1 \\ 1, & t = -1 \end{cases}$$

$\therefore$ $s(t) = \int_{-\infty}^{\infty} \delta(t+1) e^{-t} dt = [e^{-t}]_{t=-1} = e^1 = \mathbf{e}$ **Ans.**

(iv) We know that

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

$\therefore$ $s(t) = \int_0^{3} \delta(t) \sin 3\pi t\, dt = [\sin 3\pi t]_{t=0} = \sin 0 = \mathbf{0}$ **Ans.**

(v) We know that

$$\delta(t-3) = \begin{cases} 0, & t \neq 3 \\ 1, & t = 3 \end{cases}$$

$\therefore$ $\int_{-\infty}^{\infty} (t-3)^2 \delta(t-3)\, dt = \left[(t-3)^2\right]_{t=3} = (3-3)^2 = \mathbf{0}$ **Ans.**

(vi) $s(t) = \int_{-\infty}^{\infty} [\delta(t-1)\cos 3t + \delta(t-1)\sin 3t]\, dt$

We know that

$$\delta(t-1) = \begin{cases} 0, & t \neq 1 \\ 1, & t = 1 \end{cases}$$

$\therefore$ $\int_{-\infty}^{\infty} [\delta(t-1)\cos 3t + \delta(t-1)\sin 3t]\, dt = [\cos 3t]_{t=1} + [\sin 3t]_{t=1}$

$= \mathbf{\cos 3 + \sin 3}$ **Ans.**

Prob.2. Calculate the following summations –

(i) $s(n) = \sum_{n=-\infty}^{\infty} \delta(n-3)\, e^{n^3}$ (ii) $s(n) = \sum_{n=-\infty}^{\infty} e^{4n} \delta(n-4)$

(iii) $s(n) = \sum_{n=-\infty}^{\infty} \delta(n-3)\cos 4n$ (iv) $s(n) = \sum_{n=0}^{\infty} \delta(n+2)\, 2^n$

(v) $s(n) = \sum_{n=-\infty}^{\infty} n^2 \delta(n+3)$

Sol. (i) $s(n) = \sum_{n=-\infty}^{\infty} \delta(n-3)\, e^{n^3}$

We know that

$$\delta(n-3) = \begin{cases} 0, & n \neq 3 \\ 1, & n = 3 \end{cases}$$

$$\therefore \quad s(n) = \sum_{n=-\infty}^{\infty} \delta(n-3) e^{n^3} = [e^{n^3}]_{n=3} = e^{3^3} = e^{27}$$ A

(ii) $s(n) = \sum_{n=-\infty}^{\infty} e^{4n}\, \delta(n-4)$

We know that

$$\delta(n-4) = \begin{cases} 0, & n \neq 4 \\ 1, & n = 4 \end{cases}$$

$$\therefore \quad s(n) = \sum_{n=-\infty}^{\infty} e^{4n}\, \delta(n-4) = [e^{4n}]_{n=4} = e^{4\times 4} = e^{16}$$ Ans.

(iii) $s(n) = \sum_{n=-\infty}^{\infty} \delta(n-3)\cos 4n$

We know that

$$\delta(n-3) = \begin{cases} 0, & n \neq 3 \\ 1, & n = 3 \end{cases}$$

$$\therefore \quad s(n) = \sum_{n=-\infty}^{\infty} \delta(n-3)\cos 4n = [\cos 4n]_{n=3} = \cos 4 \times 3 = \cos 12$$ A

(iv) $s(n) = \sum_{n=0}^{\infty} \delta(n+2)\, 2^n$

We know that

$$\delta(n+2) = \begin{cases} 0, & n \neq -2 \\ 1, & n = -2 \end{cases}$$

$$\therefore \quad s(n) = \sum_{n=0}^{\infty} \delta(n+2)\, 2^n = 0$$ Ans.

(v) $s(n) = \sum_{n=-\infty}^{\infty} n^2 \delta(n+3)$

We kow that,

$$\delta(n+3) = \begin{cases} 0, & n \neq -3 \\ 1, & n = -3 \end{cases}$$

$$\therefore \quad s(n) = \sum_{n=-\infty}^{\infty} n^2 \delta(n+3) = [n^2]_{n=-3} = (-3)^2 = 9$$ Ans.

BASIC OPERATIONS ON SIGNALS – TIME SHIFTING, TIME REVERSAL, AMPLITUDE SCALING, TIME SCALING, SIGNAL ADDITION, SIGNAL MULTIPLICATION

SHORT QUESTIONS

Q.13. Write down the basic operation on signals. Explain any one.

Ans. There are following basic operations on signals as given below –

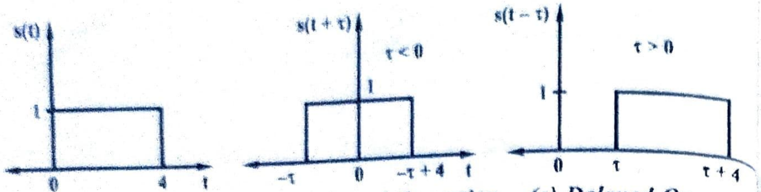
(i) Time shifting
(ii) Time folding or reversal
(iii) Time scaling
(iv) Amplitude scaling
(v) Signal addition
(vi) Signal multiplication.

Time Shifting –

For Continuous Time Signal – The time shifting of a continuous time signal s(t) is given by

$$y(t) = s(t - \tau) \qquad ...(i)$$

For $\tau < 0$, the shift is to the left and then the shifting advances the signal, and for $\tau > 0$, the shift is to the right and then the shifting delays the signal. A signal s(t), its advanced operation and delayed operation are shown in fig. 1.30 (a), (b) and (c).



(a) Signal Operation *(b) Advanced Operation* *(c) Delayed Operati*

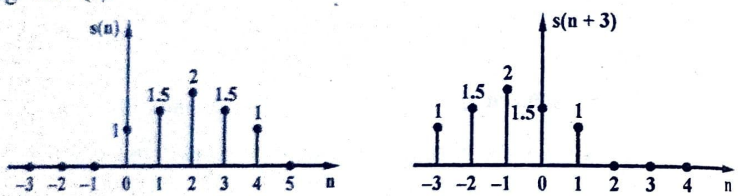
Fig. 1.30

For Discrete-time Signal – The time shifting of a discrete time sig
s(n) is given by

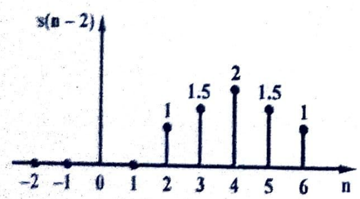
$$y(n) = s(n-k) \quad ...($$

A signal y(n) is obtained by time shifting the signal s(n) by k units. equation (ii), if k is negative ($k < 0$) the shift is to the left and then shifti advances the signal, and if k is positive ($k > 0$) the shift is to the right and th the shifting delays the signal.

Fig. 1.31 (a) shows the arbitrary signal s(n). A signal s(n + 3) is obtain by shifting s(n) to the left by 3 units which is shown in fig. 1.31 (b). A sign s(n – 2) is obtained by shifting s(n) to the right by 2 units which is shown fig. 1.31 (c).



(a) Sequence s(n) *(b) s(n + 3)*



(c) s(n – 2)

Fig. 1.31

Q.14. What is meant by amplitude scaling ?

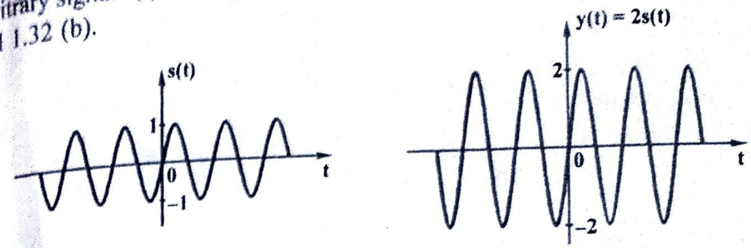
Ans. **For Continuous Time Signal** – The amplitude of the signal changed with amplitude scaling. The amplitude scaling of a continuous-tin signal s(t) is given by

$$y(t) = Bs(t), -\infty < t < \infty$$

where B is real constant quantity.

The amplitude of y(t) at any instant is equal to the B times the amplitude of s(t) at that instant, but the shape of y(t) is identical as the shape of s(t). For the value of $B < 1$, it is attenuation and for the value of $B > 1$, it is amplification.

Here the amplitude is rescaled and is called the amplitude scaling. An arbitrary signal s(t) and a scaled signal y(t) = 2s(t) are shown in figs. 1.32 (a) and 1.32 (b).



(a) s(t) = sin ωt *(b) y(t) = 2s(t)*

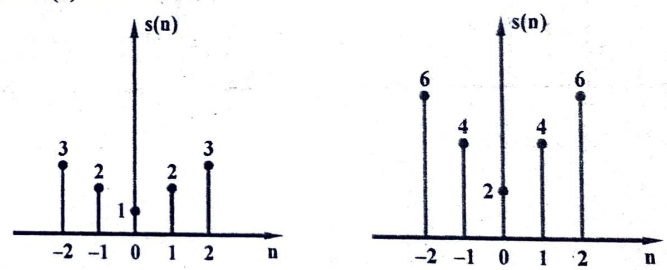
Fig. 1.32

For Discrete-Time Signal – The amplitude scaling of a discrete-time signal is given by

$$y(n) = bs(n)$$

where b is real constant quantity.

For the value of $b > 1$, it is amplification and for the value of $b < 1$, it is attenuation. An arbitrary signal s(n) and a scaled signal y(n) = 2s(n) are shown in figs. 1.33 (a) and 1.33 (b).



(a) Signal s(n) *(b) y(n) = 2s(n)*

Fig. 1.33

Q.15. Discuss signal multiplication in brief.

Ans. **For Continuous Time Signal** – Let $s_1(t)$ and $s_2(t)$ be the two continuous time signal. Then, their multiplication is given as

$$y(t) = s_1(t).s_2(t)$$

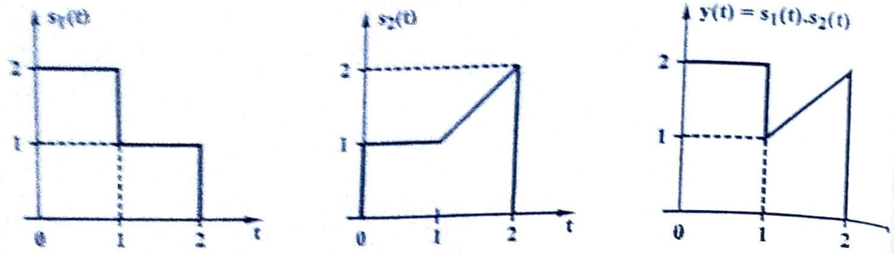
Figs. 1.34 (a) and (b) show the two continuous time signals $s_1(t)$ and $s_2(t)$. The signals $s_1(t)$ and $s_2(t)$ are multiplied as shown below to obtain $s_1(t).s_2(t)$ shown in fig. 1.34 (c).

For $0 \le t \le 1$, $\quad s_1(t) = 2$ and $s_2(t) = 1$

$$y(t) = s_1(t).s_2(t) = 2 \times 1 = 2$$

For $1 \le t \le 2$, $s_1(t) = 1$ and $s_2(t) = 1 + (t - 1)$

$$y(t) = s_1(t).s_2(t) = 1 \times [1 + (t-1)]$$
$$= 1 + (t - 1)$$



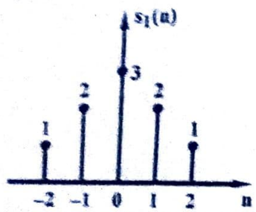
(a) (b) (c)

Fig. 1.34 Multiplication of Continuous Time Signal

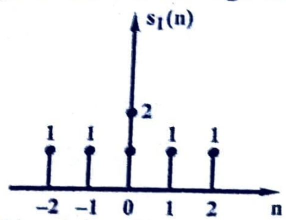
For Discrete-time Signal – Let us consider $s_1(n)$ and $s_2(n)$ be the tw discrete time signals [fig. 1.35 (a) and (b)]. Then, their multiplication is given a

$$y(n) = s_1(n).s_2(n)$$

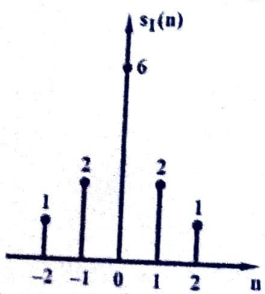
Multiplication of two discrete-time signals is shown in fig. 1.35.



(a) $s_1(n) = \{1, 2, 3, 2, 1\}$



(b) $s_2(n) = \{1, 1, 2, 1, 1\}$



(c) $y(n) = s_1(n).s_2(n) = \{1 \times 1, 2 \times 1, 3 \times 2, 2 \times 1, 1 \times 1\} = \{1, 2, 6, 2, 1$

Fig. 1.35 Multiplication of Discrete time Signal

LONG QUESTIONS

Q.16. Write short on time reversal.

***Ans.* For Continuous-time Signal** – The time reversal is also known time folding. The time reversal (or folding) of a continuous-time signal $s(t)$ obtained by folding the signal about $t = 0$. The time folding operation is used

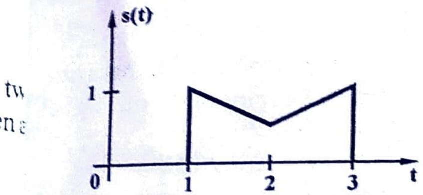
convolution and denoted by $s(-t)$. The time folding of a signal is obtained by replacing t with $-t$, i.e.

$$y(t) = s(-t)$$

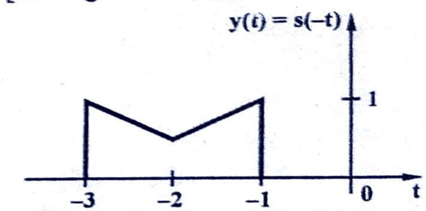
Here, $y(t)$ is time folded or reflected version of $s(t)$.

An arbitrary signal $s(t)$ and its reflection $s(-t)$ are shown in figs. 1.36 (a) and 1.36 (b).

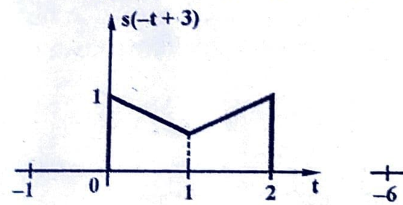
The signal $s(-t + 3)$ can be obtained by delaying (shifting to the right) the time reversed signal $s(-t)$ right by 3 units of time [see fig. 1.36 (c)]. The signal $s(-t - 2)$ can be obtained by advancing (shifting to the left) the time reversed signal $s(-t)$ by 2 units of time [see fig. 1.36 (d)].



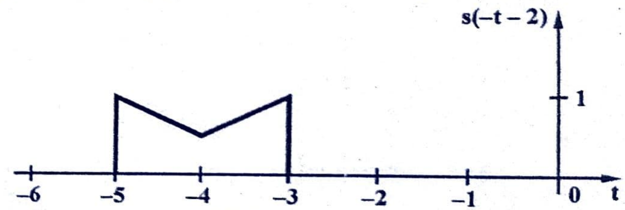
(a) Arbitrary Signal s(t)



(b) Time Reversal Signal s(–t)



(c) Time Reversed and Delayed Signal



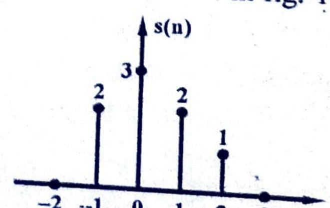
(d) Time Reversed and Advanced Signal

Fig. 1.36

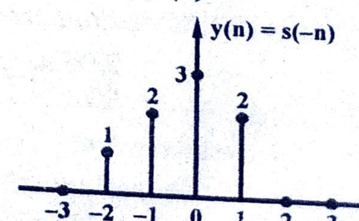
For Discrete-time Signal – The time reversal operation of a discrete time signal $s(n)$ is obtained by the folding sequence about $n = 0$. The time reversal is obtained by replacing n with $-n$ i.e.

$$y(n) = s(-n)$$

An arbitrary signal $s(n)$ and its time reversed $s(-n)$ are shown in figs. 1.37 (a) and fig. 1.37 (b). The delayed and advanced operations of reversed signal $x(-n)$ are shown in fig. 1.38 (a) and fig. 1.38 (b).

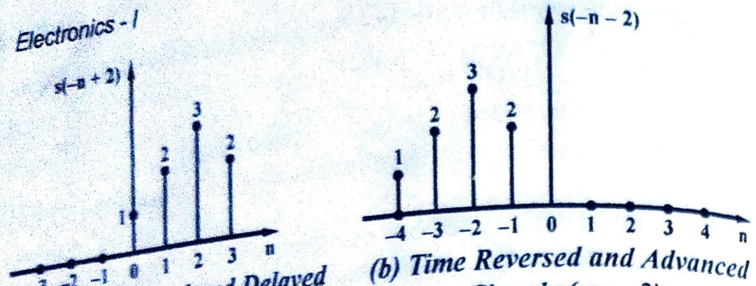


(a) Signal s(n)



(b) Time Reversed Signal s(–n)

Fig. 1.37



(a) Time Reversed and Delayed Signal $s(-n+2)$

(b) Time Reversed and Advanced Signal $s(-n-2)$

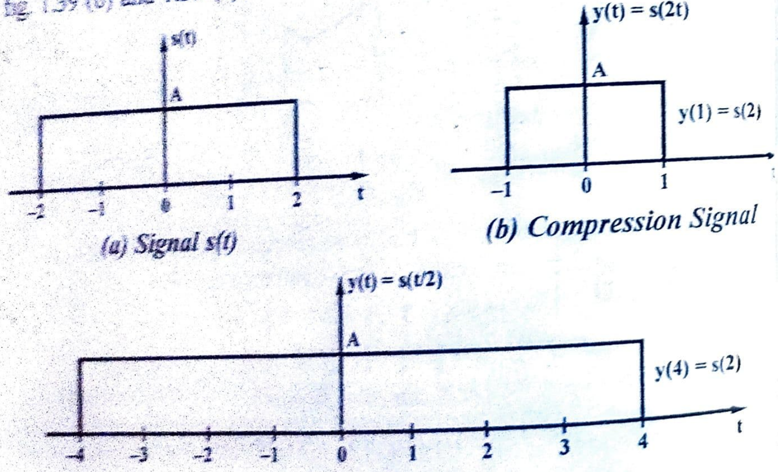
Fig. 1.38

Q.17. Discuss time scaling operation.

***Ans.* For Continuous-time Signal** – Time scaling can be tim compression or time expansion. The time scaling of a continuous-time signa $s(t)$ is obtained by replacing t by αt. It is defined as

$$y(t) = s(\alpha t)$$

For the value of $\alpha > 1$, it is time compression and for the value of $\alpha <$ it is time expansion by a factor α. Let us assume a signal s(t) is shown in fi 1.39 (a). The time compression signal and time expansion signal are shown fig. 1.39 (b) and 1.39 (c).



(a) Signal s(t)

(b) Compression Signal

(c) Expansion Signal

Fig. 1.39 Time Scaling on Continuous Time Signal

For Discrete-time Signal – The time scaling of a discrete time sign s(n) is obtained by replacing n by αn. It is defined as

$$y(n) = s(\alpha n)$$

For the value of $\alpha > 1$, it is time compression and for the value of $\alpha <$ it is time expansion.

Let us consider s(n) be a sequence as shown in fig. 1.40 (a).

$$y(n) = s(2n), \text{ for } \alpha = 2$$

Then, we have

$$y(0) = s(0) = 0$$
$$y(1) = s(2) = 1$$
$$y(2) = s(4) = 2$$

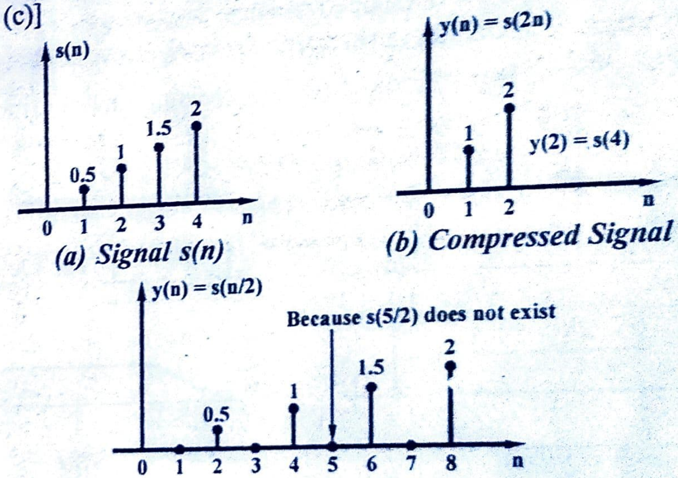
Hence, the signal is compressed [See fig. 1.40 (b)]. Alternate samples of s(n) are skipped. This is known as sub-sampling.

$$y(n) = s(n/2), \quad \text{for } \alpha = 1/2$$

Then, we have

$$y(0) = s(0)$$
$$y(2) = s(2/2) = s(1)$$
$$y(4) = s(4/2) = s(2)$$
$$y(6) = s(6/2) = s(3)$$

Here, it should be noted that $y(1) = s(1/2)$, which is not available. Thus, y(1), y(3) and y(5) can be considered zero. Hence, the signal is expanded [see fig. 1.40 (c)]



(a) Signal s(n)

(b) Compressed Signal

(c) Expanded Signal

Fig. 1.40 Time Scaling on Discrete-time Signal

Q.18. Explain signal addition and signal subtraction.

***Ans.* For Continuous-time Signal** – Let us consider $s_1(t)$ and $s_2(t)$ be the two continuous-time signals. Then, the addition of $s_1(t)$ and $s_2(t)$ can be expressed as

$$y(t) = s_1(t) + s_2(t)$$

Similarly, the subtraction of $s_1(t)$ and $s_2(t)$ is expressed as –

$$y(t) = s_1(t) - s_2(t)$$

The two signals $s_1(t)$ and $s_2(t)$ are shown in fig. 1.41 (a) and 1.41 (b).

The addition of two signals $s_1(t)$ and $s_2(t)$ is obtained by assuming diffe values of time intervals as given below.

For $0 \le t \le 1$, $s_1(t) = 1$ and $s_2(t)$ is increasing linearly from 0 to Therefore,

$$y(t) = s_1(t) + s_2(t) = 1 + 0.5 = 1.5$$

For $1 \le t \le 3$, $s_1(t) = 0.5$ and $s_2(t) = 0.5$.

Therefore, $\quad y(t) = s_1(t) + s_2(t) = 0.5 + 0.5 = 1$

For $3 \le t \le 4$, $s_1(t) = 1$ and $s_2(t)$ is decreasing linearly from 0.5 Therefore,

$$y(t) = s_1(t) + s_2(t) = 1 + 0 = 1$$

The addition of $s_1(t)$ and $s_2(t)$ is shown in fig. 1.41 (c).

The subtraction of two signals $s_1(t)$ and $s_2(t)$ is obtained by assum different values of time intervals as given below –

For $0 \le t \le 1$, $s_1(t) = 1$, and $s_2(t)$ is increasing linearly from 0 to Therefore,

$$y(t) = s_1(t) - s_2(t) = 1 - 0.5 = 0.5$$

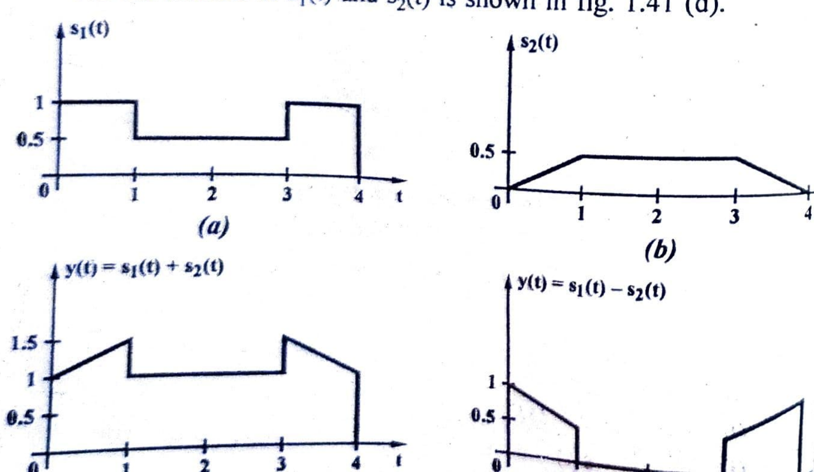
For $1 \le t \le 3$, $s_1(t) = 0.5$ and $s_2(t) = 0.5$. Therefore,

$$y(t) = s_1(t) - s_2(t) = 0.5 - 0.5 = 0$$

For $3 \le t \le 4$, $s_1(t) = 1$ and $s_2(t)$ is decreasing linearly from 0.5 to Therefore,

$$y(t) = s_1(t) - s_2(t) = 1 - 0 = 1$$

The subtraction of $s_1(t)$ and $s_2(t)$ is shown in fig. 1.41 (d).



(a) (b)

(c) Addition of Continuous Time Signal

(d) Subtracton of Continuous Time Signal

Fig. 1.41

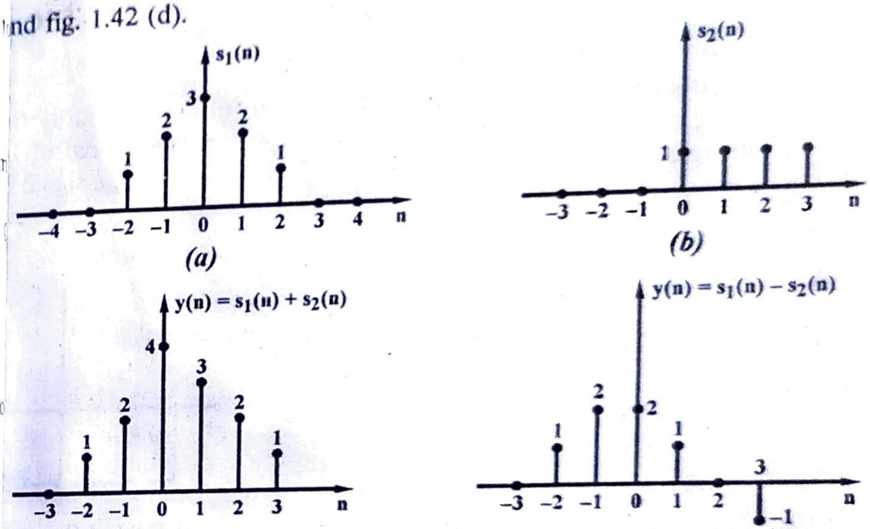
For Discrete-time Signal – Let us consider $s_1(n)$ and $s_2(n)$ be the two iscrete-time signals. Then the addition of $s_1(n)$ and $s_2(n)$ is expressed as

$$y(n) = s_1(n) + s_2(n)$$

Similarly, the subtraction of $s_1(n)$ and $s_2(n)$ is expressed as

$$y(n) = s_1(n) - s_2(n)$$

The two sequences $s_1(n)$ and $s_2(n)$ are shown in fig. 1.42 (a) and 1.42 (b). The addition and subtraction of two sequences are shown in fig. 1.42 (c) nd fig. 1.42 (d).



(a) (b)

c) $y(n) = \{1 + 0, 2 + 0, 3 + 1, 2 + 1, 1 + 1, 0 + 1\} = \{1, 2, \underset{\uparrow}{4}, 3, 2, 1\}$

(d) $y(n) = \{1 - 0, 2 - 0, 3 - 1, 2 - 1, 1 - 1, 0 - 1\} = \{1, 2, \underset{\uparrow}{2}, 1, 0, -1\}$

Fig. 1.42 Addition and Subtraction of Discrete-time Signal

NUMERICAL PROBLEMS

Prob.3. Draw the waveforms represented by the following unit step unctions –

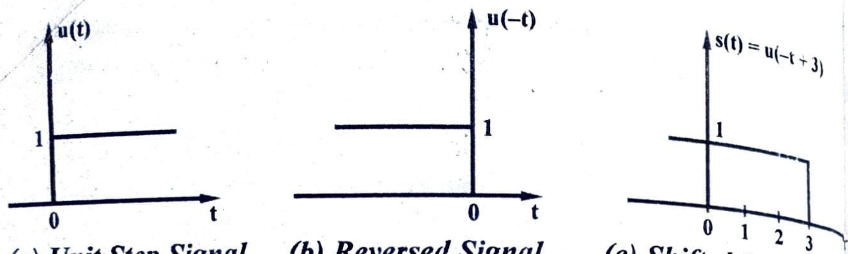
(i) $s(t) = u(-t + 3)$

(ii) $s(t) = -3u(t + 3)$

(iii) $s(t) = 2u(t - 1)1.$

Sol. (i) Given $\quad s(t) = u(-t + 3)$

The signal $s(t) = u(-t + 3)$ shown in fig. 1.43 (c) is obtained by first lrawing the unit step signal [see fig. 1.43 (a)], then time reversing the signal ı(t) about t = 0 to obtain u(–t) [see fig. 1.43 (b)], and then the time reversing ignal u(–t) is shifting to the right by 3 units.

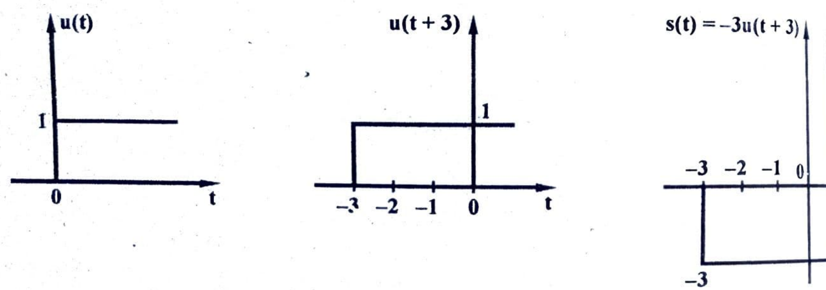


(a) Unit Step Signal *(b) Reversed Signal u(–t)* *(c) Shifted Reversed Si s(t) = u(–t + 3)*

Fig. 1.43

(ii) Given s(t) = –3u(t + 3)

The signal s(t) = –3u(t + 3) shown in fig. 1.44 (c) is obtained by drawing the unit step signal u(t) [see fig. 1.44(a)], then the signal u(shifted to the left by 3 units of time [see fig. 1.44 (b)] and then the signa + 3) is multiplied by –3.

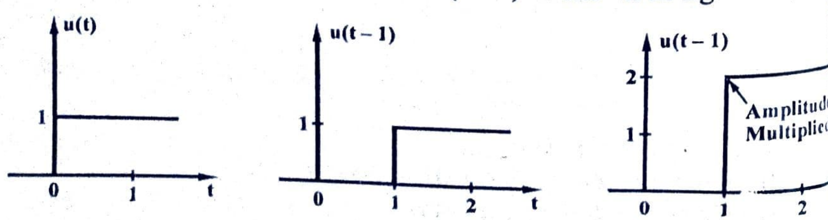


(a) Unit Step Signal *(b) Delayed Signal u(t + 3)* *(c) Scaled Sign s(t) = –3u(t +*

Fig. 1.44

(iii) Given s(t) = 2u(t – 1)

The signal s(t) = 2u(t – 1) is obtained by the first drawing the unit function u(t) [see fig. 1.45 (a)], then the signal u(t) is shifted to the righ 1 unit of time to obtain u(t – 1) as shown in fig. 1.45 (b) and then the si u(t – 1) is multiplied by 2 to obtain 2u(t – 1) as shown in fig. 1.45 (c).



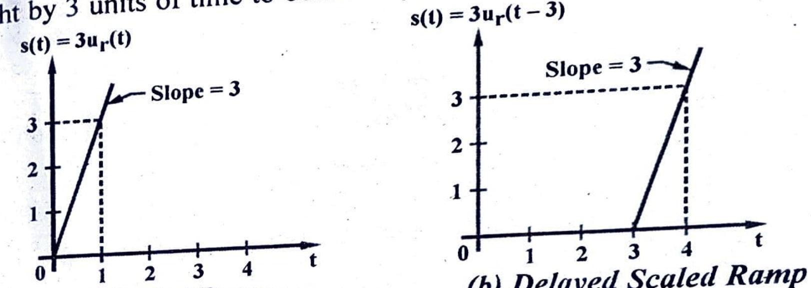
(a) Unit Step Signal u(t) *(b) Shifted Signal u(t – 1)* *(c) Scaled Signal 2u(*

Fig. 1.45

Prob.4. Draw the waveform represented by the following ramp signal –
(i) $s(t) = 3u_r(t-3)$ (ii) $s(t) = -4u_r(t)$ (iii) $s(t) = u_r(-t+2)$.

Sol. (i) Given $s(t) = 3u_r(t-3)$

The signal $s(t) = 3u_r(t-3)$ is obtained by first drawing the ramp signal $u_r(t)$ with slope of 3[see fig. 1.46 (a)] and then the signal $3u_r(t)$ is shifted to right by 3 units of time to obtain $3u_r(t-3)$ as shown in fig. 1.46 (b).



(a) Scaled Ramp *(b) Delayed Scaled Ramp*

Fig. 1.46

(ii) $s(t) = -4u_r(t)$

Fig. 1.47 shows the signal s(t) is a ramp signal h a slope of –4.

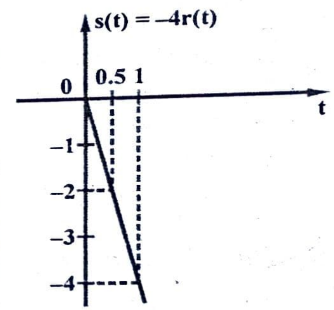
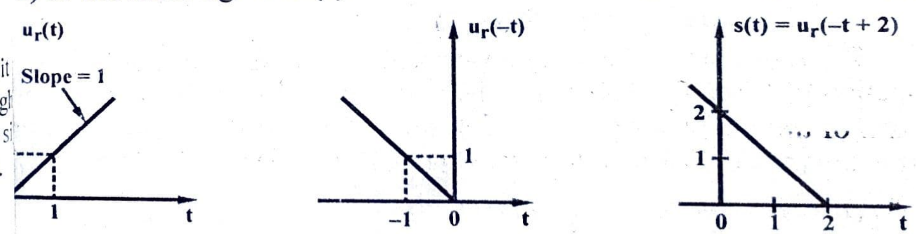


Fig. 1.47 $s(t) = -4u_r(t)$

(iii) $s(t) = u_r(-t+2)$

The signal $s(t) = 2u_r(-t+2)$ is obtained by first wing the ramp signal $u_r(t)$ [see fig. 1.48 (a)], then time reversing the signal $u_r(t)$ about t = 0 to obtain –t) as shown in fig. 1.48 (b) and then the signal –t) is shifted to the right by 2 units to obtain $u_r(-$ 2) as shown in fig. 1.48 (c).



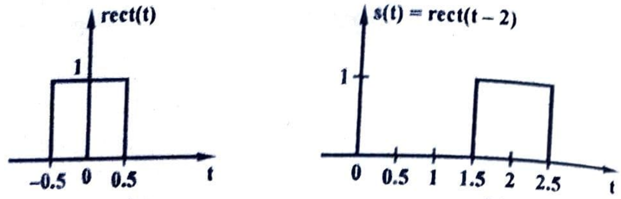
) Ramp Signal (b) Reversed Ramp Signal (c) Shifted Reversed Signal

Fig. 1.48

Prob.5. Sketch the following signals –
(i) s(t) = rect(t – 2) (ii) s(t) = rect(2t + 3).

Sol. (i) Given s(t) = rect (t – 1)

The signal s(t) = rect (t – 2) is obtained by first drawing rectangular ction rect(t) as shown in fig. 1.49 (a). Then the signal rect(t) is shifted to ri by 2 units of time to obtain rect (t – 2) as shown in fig. 1.49 (b).



(a) (b)

Fig. 1.49

(ii) Given $s(t) = \text{rect}(2t + 3)$

Let us consider rect(t) is a rectangular pulse of amplitude 1 and dur
$-1 \leq t \leq 1$.

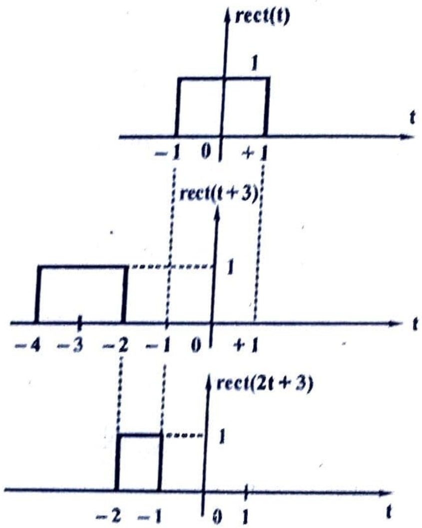


Fig. 1.50

(i) First, we shift rect(t) to left by 3, to get rect(t + 3). Fig.
(a) shows this plot of rect(t + 3).

(ii) Then, we compress rect(t + 3) by 2 to get rect(2t + 3).
1.50 (b) shows the plot of rect(2t + 3).

Prob.6. Draw the following signals –

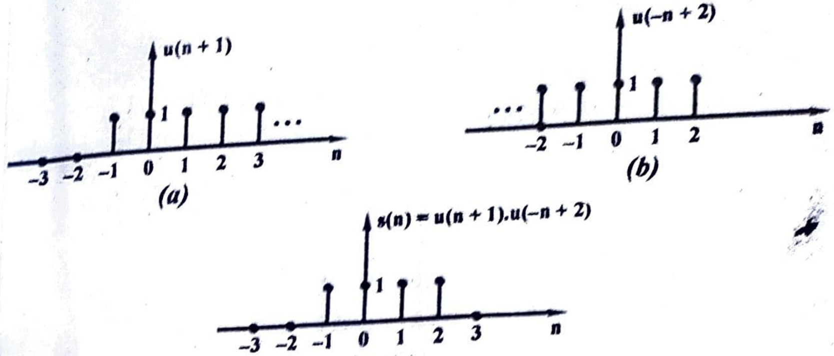
(i) $s(n) = u(n + 1)\, u(-n + 2)$

(ii) $s(n) = u(n + 3) - u(n - 2)$.

Sol. (i) Given $s(n) = u(n + 1)\, u(-n + 2)$

The signal $s(n) = u(n + 1)\, u(-n + 2)$ is obtained by first plotting the si
$u(n + 1)$ [see fig. 1.51 (a)], then drawing $u(-n + 2)$ as shown in fig. 1.51
Then multiplying these sequences element by element to obtain s(n) as sh
in fig. 1.51 (c).

$s(n) = 0$ for $n < -1$ and $n > 2$; $s(n) = 1$ for $-1 < n < 2$

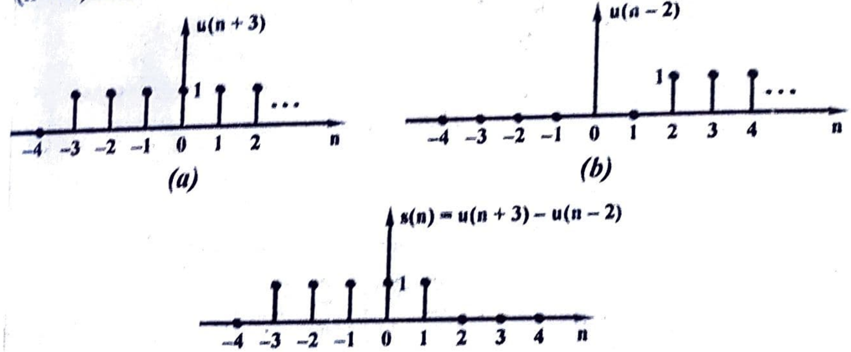


(a) (b) (c)

Fig. 1.51

(ii) Given $s(n) = u(n + 3) - u(n - 2)$

The signal $s(n) = u(n + 3) - u(n - 2)$ is obtained by first drawing $u(n + 3)$
s shown in fig. 1.52 (a), then drawing $u(n - 2)$ as shown in fig. 1.52 (b) and
hen subtracting each element of $u(n - 2)$ from the corresponding element of
$(n + 3)$ to obtain s(t) as shown in fig. 1.52 (a).



(a) (b) (c)

Fig. 1.52

Prob.7. A discrete time signal s[n] is shown in fig. 1.53. Sketch and abel carefully each of the following signals –

(i) $s[2n + 1]$ (ii) $s[n]\, u[2 - n]\, s[n]$.

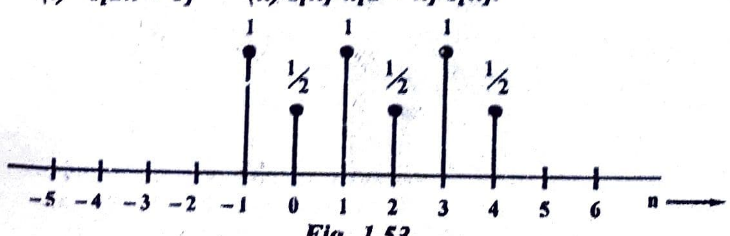


Fig. 1.53

Sol. (i) The given system is

$$y[n] = s[2n + 1]$$

(a) First we shift s[n] to left by +1 to get s[n + 1]

(b) Then, we compress s[n] by 2 to get s[2n + 1]

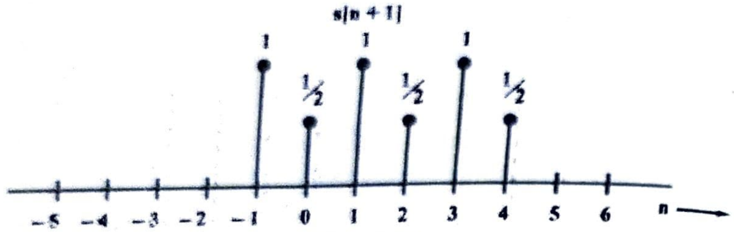


Fig. 1.54

Put $n = 0$ $\quad y[0] = s[1] = 1/2$

$n = 1$ $\quad y[1] = s[3] = 1/2$

$n = 2$ $\quad y[2] = s[5] = 0$

$n = 3$ $\quad y[3] = s[7] = 0$

Similarly

$n = -1$ $\quad y[-1] = s[-1] = 1/2$

$y[-2] = s[-3] = 0$

$y[-3] = s[-5] = 0$

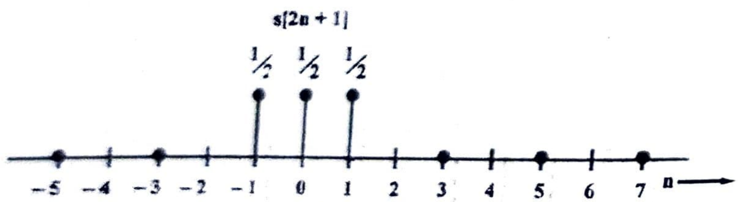


Fig. 1.55

(ii) The given system is

$$y[n] = s[n]\, u[2 - n]\, s[n]$$

(a) First, we draw the unit step function the condition of step function is

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

(b) Second, shift the u[n] to right by 2 to get u[n – 2]

(c) Third, the delayed unit step function is then folded in ti

Hence, we get

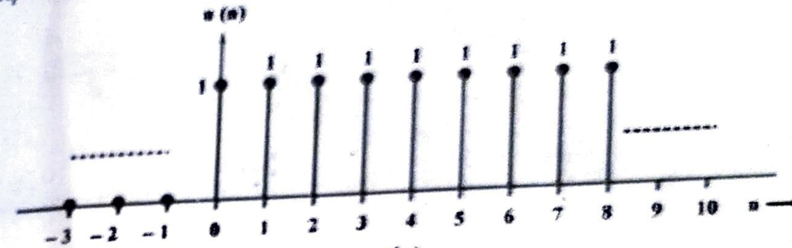
$$f_1(n) = u[-(n-2)]$$

$$f_1(n) = u(-n+2)$$

$$f_1(n) = u[2-n]$$

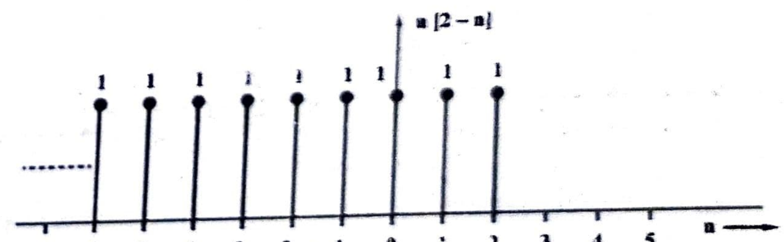
(d) Multiply the given discrete time signal with $f_1[n]$ from step 3 to get s[n] u[2 – n]

(e) Again, multiply the waveform found in step-4 to the given input discrete time signal s[n] to get the final waveform as s[n] u[2 – n] s[n].

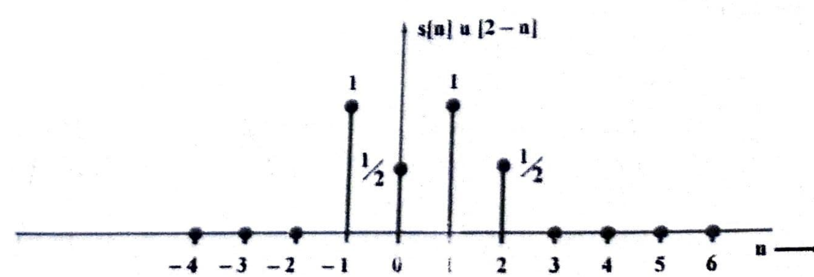


(a)

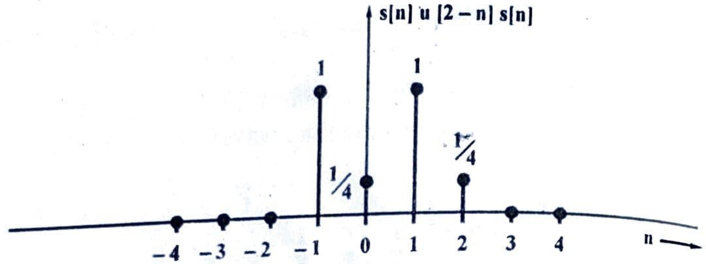
(b)



(c)



(d)



(e)

Fig. 1.56

Prob.8. Prove the following relationship between functions –

(i) $\delta(n) = u(n) - u(n-1)$

(ii) $u(n) = \sum_{k=-\infty}^{n} \delta(k)$

(iii) $u(n) = \sum_{k=0}^{\infty} \delta(n-k)$.

Sol. (i) Given $\delta(n) = u(n) - u(n-1)$

We know that

$$u(n) = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}, \quad u(n-1) = \begin{cases} 0, & n<1 \\ 1, & n \geq 1 \end{cases}$$

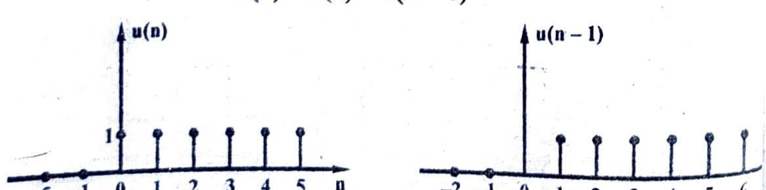
Therefore, we get

$$u(n) - u(n-1) = \begin{cases} 0, & n \geq 1 \\ 1, & n = 0 \\ 0, & n<0 \end{cases} = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

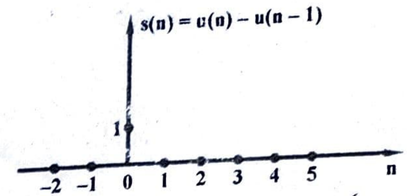
$$u(n) - u(n-1) = \delta(n)$$

The signals u(n) and u(n – 1) are shown in fig. 1.57 (a) and fig. 1.57 The subtraction $\delta(n) = u(n) - u(n-1)$ is shown in fig. 1.57 (c). All the va become zero except n = 0. Hence, the signal of fig. 1.57 (c) is nothing $\delta(n)$. This means that

$$\delta(n) = u(n) - u(n-1)$$



(a) Unit Step Signal u(t) *(b) Shifted Unit Step Function by 1*



(c) Subtraction s(n) = u(n) – u(n – 1)

Fig. 1.57 u(n) – u(n – 1) gives Unit Sample Function

(ii) $u(n) = \sum_{k=-\infty}^{n} \delta(k)$

$$\sum_{k=-\infty}^{n} \delta(k) = \begin{cases} 1, & k \geq 0 \\ 0, & k<0 \end{cases}$$

The right hand side of the above equation is unit sample u(n). Therefore, the equation is proved.

(iii) Given $u(n) = \sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 1, & k \geq 0 \\ 0, & k<0 \end{cases}$

The right hand side of the above equation is unit sample signal u(n). Therefore, the equation is proved.

Prob.9. A continuous-time signal s(t) is shown in fig. 1.58. Sketch the following –

(i) $s(t-2)$ *and* $s(t+2)$

(ii) $s(2t+2)$ *and* $s\left(\frac{1}{2}t - 2\right)$

(iii) $s\left(\frac{5}{3}t\right)$ *and* $s\left(\frac{3}{5}t\right)$.

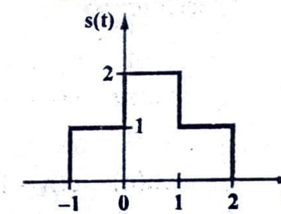
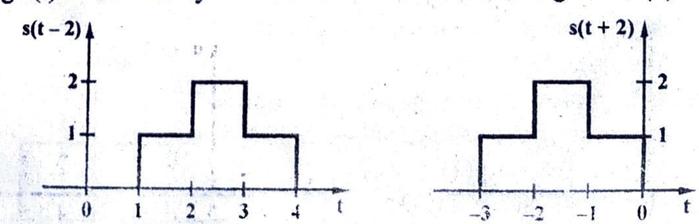


Fig. 1.58

Sol. (i) The signal s(t – 2) is obtained by shifting s(t) to the right by 2 units of time as shown in fig. 1.59 (a) and the signal s(t + 2) is obtained by shifting s(t) to the left by 2 units of time as shown in fig. 1.59 (b).



(a) (b)

Fig. 1.59

(ii) $s(2t+2)$ and $s\left(\frac{1}{2}t-2\right)$

The signal $s(t+2)$ is compressed by a factor of 2 as shown in fig. 1.60 (a). The signal $s(t-2)$ is expanded by a factor (1/2) as shown in fig. 1.60 (b).

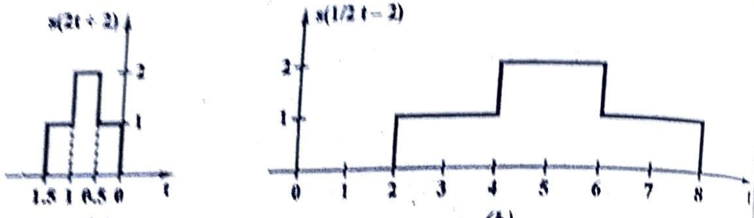
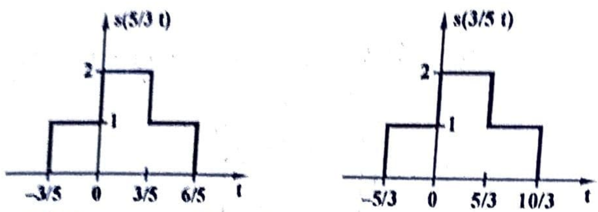


Fig. 1.60

(iii) The signal $s(5/3\ t)$ is obtained by compressing the signal $s(t)$ (3/5) times as shown in fig. 1.61 (a) and the signal $s(3/5\ t)$ is obtained expanding the signal $s(t)$ by (5/3) times as shown in fig. 1.61 (b). The zero point remains as it is because $0 \times a = 0$ itself.



(a) Compressed Signal (b) Expanded Signal

Fig. 1.61

Prob.10. *For the signal defined as –*

$$s(t) = \begin{cases} 1, & -1 \le t \le 1 \\ 0, & \text{Otherwise} \end{cases}$$

Draw the following signal is –

(i) $y(t) = s(2t)$

(ii) $y(t) = s(-2+t)$.

Sol. The given signal is shown in fig. 1.62.

(i) Given $y(t) = s(2t)$

Here, $s(2t)$ represents time scaled operation

Now, substituting the several values of "t",

At $t = 1$, we get $y(1) = s(2)$

It is value of $s(t)$ at $t = 2 = 0$

At $t = -1$, we get $y(-1) = s(-2)$

It is value of $s(t)$ at $t = -2 = 0$

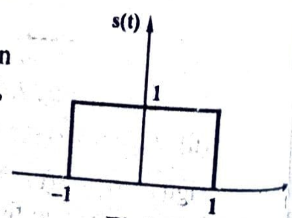


Fig. 1.62

At $t = 0.5$, we get $y(0.5) = s(1)$

It is value of $s(t)$ at $t = 1 = 1$

At $t = -0.5$, we get $y(-0.5) = x(-1)$

Similarly, other values can be determined.

This is known as compression shown in fig. 1.63.

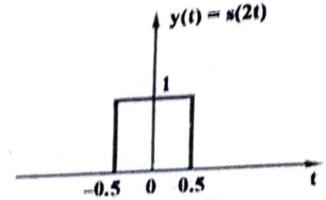


Fig. 1.63

(ii) Given $y(t) = s(-2+t) = s(t-2)$

It is represented that $s(t)$ is shifted by 2 as shown in fig. 1.64.

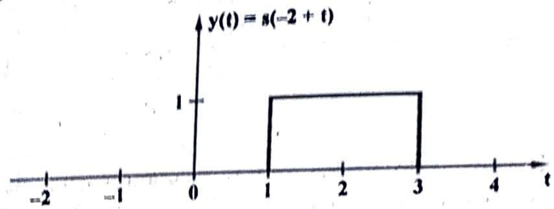


Fig. 1.64

Prob.11. *A signal s(t) is shown in fig. 1.65. Draw and label the following signals –*

(i) $y(t) = s(-t)$ (ii) $y(t) = s(t-3)$ (iii) $y(t) = s(2t)$.

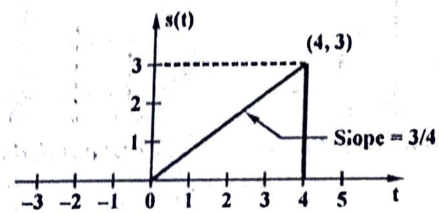


Fig. 1.65

Sol. (i) Given $y(t) = s(-t)$ –

It is reversing operation as shown in fig. 1.66.

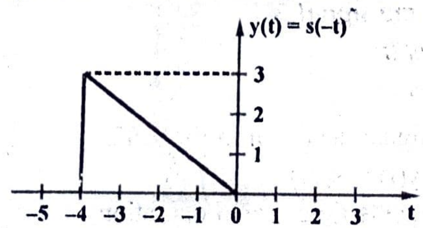


Fig. 1.66 Reversed Signal of s(t)

(ii) Given $y(t) = s(t-3)$

The signal $y(t) = s(t-3)$ is obtained by shifting $s(t)$ towards right by 3 units of time as shown in fig. 1.67.

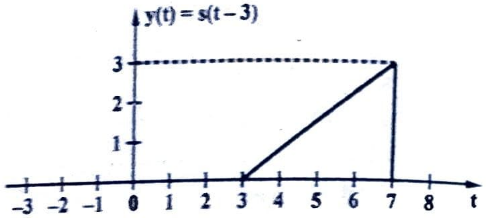


Fig. 1.67

(iii) Given y(t) = s(2t).

Here, slope of line is 3/4. A signal represents time scaling operatio shown in fig. 1.68.

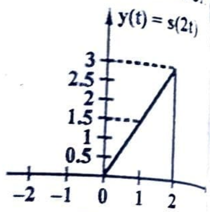


Fig. 1.68

For t = 0, $y(0) = s(0) = 0$

For t = 1, $y(1) = s(2) = 2 \times \frac{3}{4} = \frac{3}{2} = 1.5$

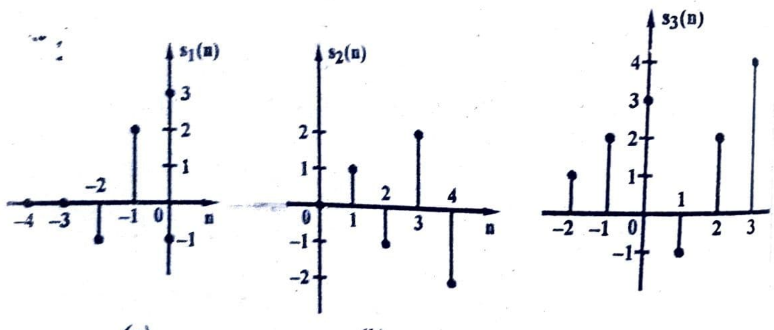
For t = 2, $y(2) = s(4) = 4 \times \frac{3}{4} = 3$

Prob.12. Represent the following signals graphically –

(i) $s_1(n) = \{0, 0, -1, 2, \underset{\uparrow}{3}\}$ *(ii)* $s_2(n) = \{\underset{\uparrow}{0}, 1, -1, 2, -2\}$

(iii) $s_3(n) = \{1, 2, \underset{\uparrow}{3}, -1, 2, 4\}$.

Sol. These signals have been shown in fig. 1.69 (a), (b) and respectively.



(a) (b) (c)

Fig. 1.69

Prob.13. A discrete time signal s(n) is shown in fig. 1.70. Draw the following signals –

(i) $y(n) = s(n - 3)$ *(ii)* $y(n) = s(3 - n)$

(iii) $y(n) = s(2n)$ *(iv)* $y(n) = s(n).s(3 - n)$.

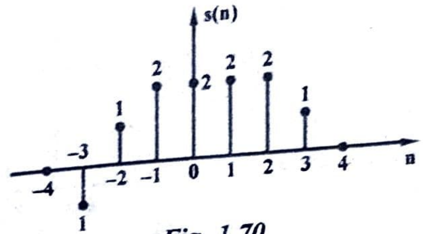


Fig. 1.70

Sol. The given signal is written in sequential form as –

$$s(n) = \{-1, 1, 2, \underset{\uparrow}{2}, 2, 2, 1\}$$

(i) Given y(n) = s(n – 3)

The signal y(n) = s(n – 3) is obtained by shifting s(t) towards right by 3 positions as shown in fig. 1.71.

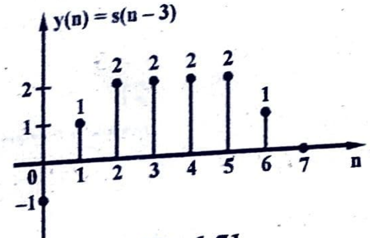


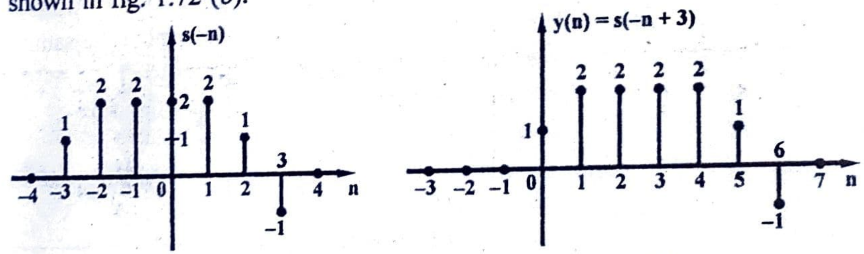
Fig. 1.71

Therefore,

$$y(n) = s(n - 3) = \{\underset{\uparrow}{-1}, 1, 2, 2, 2, 2, 1\}$$

(ii) Given y(n) = s(3 – n) = s(–n + 3)

Hence, s(–n) represents reversing operation. Therefore, s(–n + 3) represents shift of reversed signal, s(–n) by 3 positions to obtain s(–n + 3) as shown in fig. 1.72 (b).



(a) Reversed Signal of s(–n) (b)

Fig. 1.72

Here, $s(-n) = \{1, 2, 2, \underset{\uparrow}{2}, 2, 1, -1\}$

and $s(-n+3) = \{\underset{\uparrow}{1}, 2, 2, 2, 2, 1, -1\}$

(iii) Given $y(n) = s(2n)$

The signal $s(2n)$ is obtained by substituting different values of n. T range of n in $s(n)$ is −3 to 3.

At $n = -2$, $y(-2) = s(-4) = 0$

At $n = -1$, $y(-1) = s(-2) = 1$

At $n = 0$, $y(0) = s(0) = 2$

At $n = 1$, $y(1) = s(2) = 2$

At $n = 2$, $y(2) = s(4) = 0$

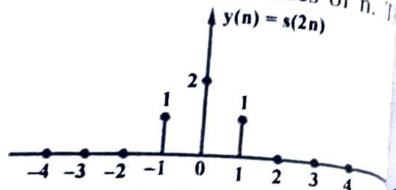


Fig. 1.73

The signal $y(n) = s(2n)$ is shown in fig. 1.73.

(iv) Given $y(n) = s(n).s(3-n)$

$$s(n) = \{-1, 1, 2, \underset{\uparrow}{2}, 2, 2, 1\}$$

and $$s(3-n) = \{\underset{\uparrow}{1}, 2, 2, 2, 2, 1, -1\}$$

$$y(n) = s(n).s(3-n)$$
$$= \{2 \times 1, 2 \times 2, 2 \times 2, 1 \times 2\}$$
$$= \{\underset{\uparrow}{2}, 4, 4, 2\}$$

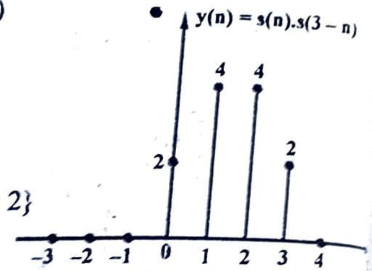


Fig. 1.74

The signal $y(n) = s(n).s(3-n)$ is shown in fig. 1.74.

Prob.14. A discrete-time signal x(k) is defined as –

$$x(k) = k(0.2)^{|k|} \quad -4 \le k \le +4$$

(i) Determine its values and sketch the signal x(k)

(ii) Determine $y(k) = x(k-4) + x(k+8)$

(iii) Obtain $y(k-4).1(k)$

(iv) Determine $y(k+4) - y(k-2)$.

where, $1(k) = 1 \ \forall\, k \ge 0$.

Sol. (i) Given, $x(k) = k(0.2)^{|k|}$, $-4 \le k \le +4$ we determine the sam values of $x(k)$ by substituting the value of k.

$x(-4) = -4(0.2)^{|-4|} = -0.0064$

$x(-3) = -3(0.2)^{|-3|} = -0.024$

$x(-2) = -2(0.2)^{|-2|} = -0.08$

$x(-1) = -1(0.2)^{|-1|} = -0.2$

$x(0) = 0(0.2)^{|0|} = 0$

$x(1) = 1(0.2)^{|1|} = 0.2$

$x(2) = 2(0.2)^{|2|} = 0.08$

$x(3) = 3(0.2)^{|3|} = 0.024$

$x(4) = 4(0.2)^{|4|} = 0.0064$

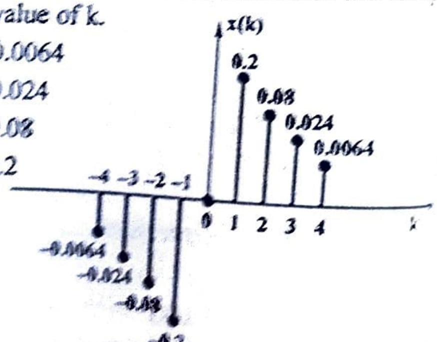


Fig. 1.75 Representation of x(k)

Thus,

$$x(k) = (-0.0064, -0.024, -0.08, -0.2, \underset{\uparrow}{0}, 0.2, 0.08, 0.024, 0.0064)$$

Graphical representation of x(k)

(ii) Given, $y(k) = x(k-4) + x(k+8)$

y(k) can also be written as,

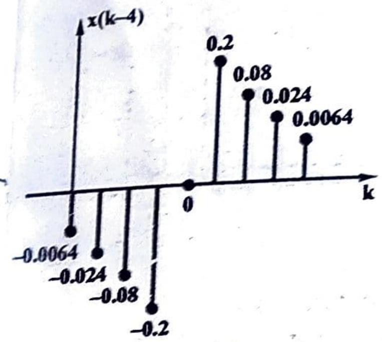
$$y(k) = x_1(k) + x_2(k)$$

where $x_1(k) = x(k-4)$ and $x_2(k) = x(k+8)$

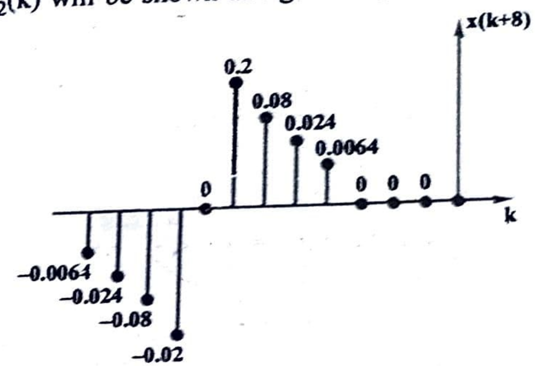
Now, $x_1(k) = x(k-4)$ is the delayed sequence by 4 unit, can be illustrated in fig. 1.76 (a).

and $x_2(k) = x(k+8)$ can be represented in fig. 1.76 (b).

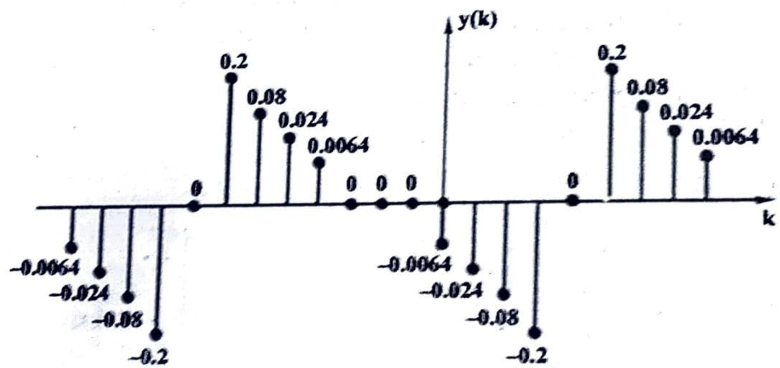
Therefore, $y(k) = x_1(k) + x_2(k)$ will be shown in fig. 1.76 (c).



(a)



(b)



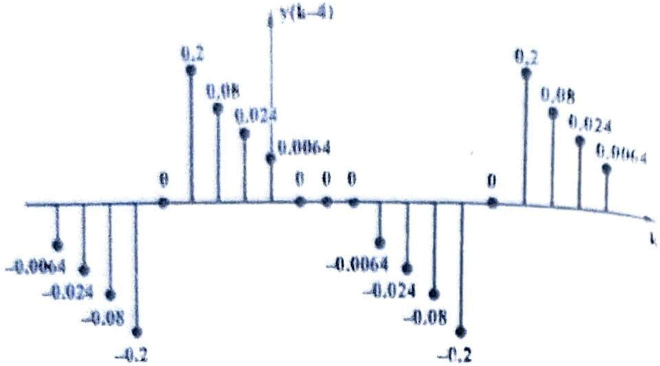
(c)

Fig. 1.76 Representation of $y(k) = x(k-4) + x(k+8)$

(iii) Given $y(k-4)1(k)$ and

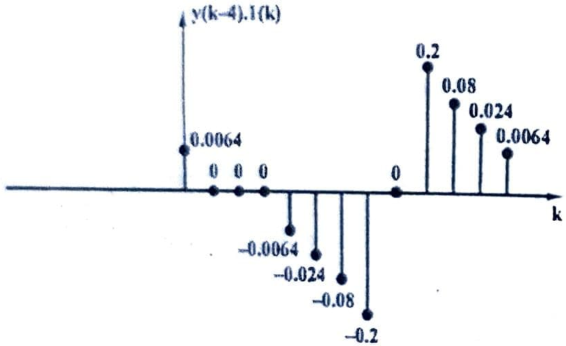
$$1(k) = 1 \text{ for all } k \ge 0$$

$y(k-4)$ is the delayed sequence of $y(k)$ by 4 unit can be represented as



(a)

Now y(k − 4).1k is represented as

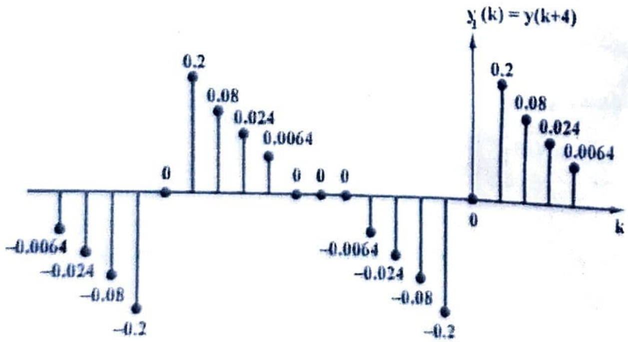


(b)

Fig. 1.77 Representation of y(k − 4) 1 (k)

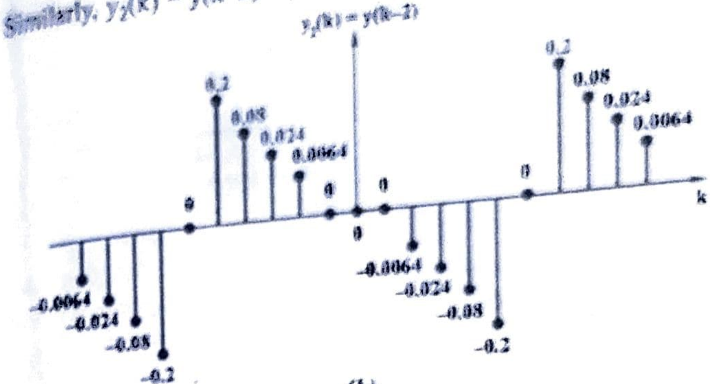
(iv) $y(k) = y(k+4) - y(k-2) = y_1(k) - y_2(k)$

$y_1(k) = y(k+4)$ can be represented as



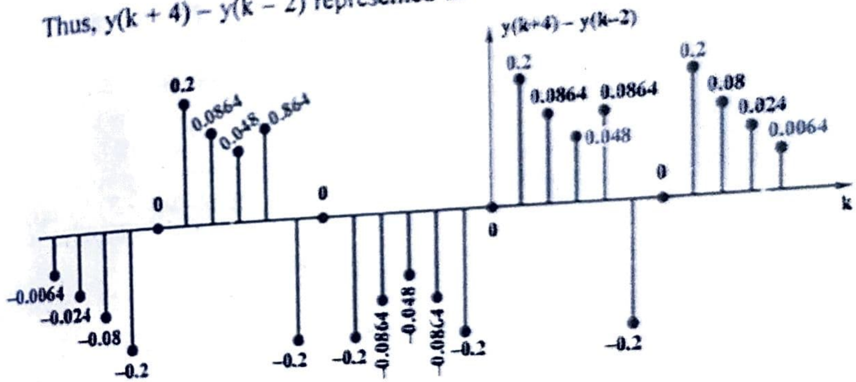
(a)

Similarly, $y_2(k) = y(k-2)$ represented as



(b)

Thus, $y(k+4) - y(k-2)$ represented as



(c)

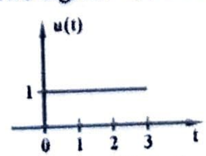
Fig. 1.78 Determination of y(k) = y(k + 4) − y (k − 2)

Prob.15. Sketch the signal $s(t) = u(t) + u_r(t-1) - 2u(t-3)$.

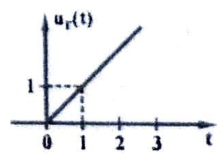
Sol. The given signal is

$$s(t) = u(t) + u_r(t-1) - 2u(t-3)$$

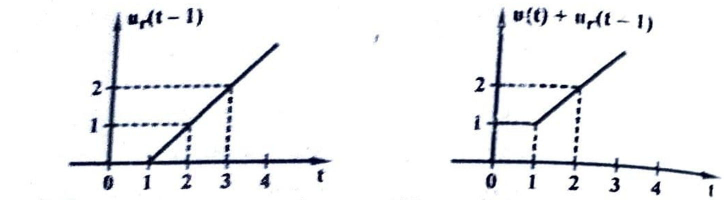
This signal $u(t) + u_r(t-1)$ is obtained by first drawing unit step function u(t) [see fig. 1.79 (a)] and ramp function $u_r(t)$ [see fig. 1.79 (b)]. Then unit ramp signal u_r is shifted by 1 to the right as shown in fig. 1.79 (c). The addition of two signals u(t) and $u_r(t-1)$ is shown in fig. 1.79 (d).



(a) Unit Step Function u(t)



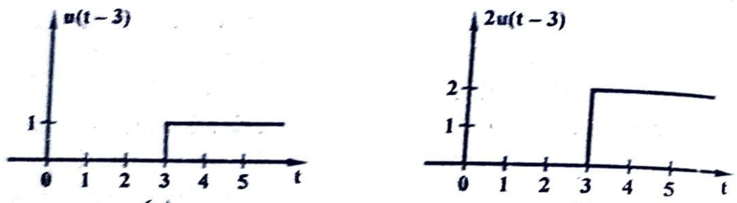
(b) Ramp Function $u_r(t)$



(c) *Shifted Ramp Function* (d) *Addition of u(t) and r(t – 1)*

Fig. 1.79

Now, the term $2u(t-3)$ is assumed. It is unit step function whi shifted by 3 positions as shown in fig. 1.80 (a) and its magnitude is multip by 2 as shown in fig. 1.80 (b).



(a) (b)

Fig. 1.80

The signal s(t) is obtained by subtracting the signal $2u(t-3)$ by u(t $u_r(t-1)$ as shown in fig. 1.81.

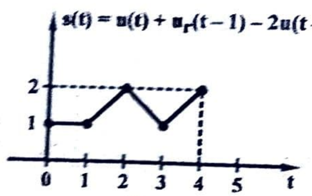
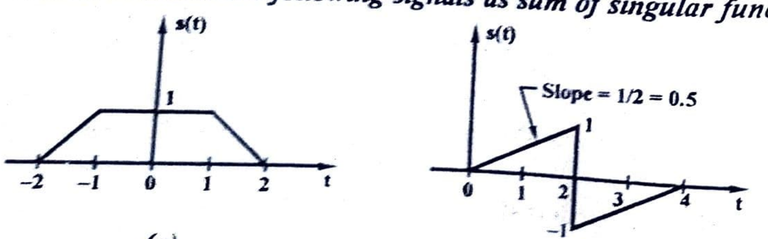


Fig. 1.81

Prob.16. Evaluate the following signals as sum of singular function



(a) (b)

Fig. 1.82 Waveforms for Prob.16

***Sol.* (a)** The signal is starting at $t = -2$ with a slope 1 and extends up t $t = -1$. Hence,

From $-2 \le t \le -1$, $s(t) = u_r(t+2)$

For $t = -1$, the slope is changing from 1 to 0 and for $t = 1$, we have to add a ramp with a slope of –1. This 0 slope is maintained up to $t = 1$. Hence,

From $-2 \le t \le 1$,

$$s(t) = u_r(t+2) - u_r(t+1)$$

For $t = 1$, the slope is changing from 0 to –1 and for $t = 1$, we have to add a ramp with a slope of –1. This –1 slope is maintained up to $t = 2$. Hence,

From $-2 \le t \le 2$,

$$s(t) = u_r(t+2) - u_r(t+1) - u_r(t-1)$$

For $t = 2$, the slope is changing from –1 to 0 and for $t = 2$, we have to add a ramp with a slope of 1. This 0 slope is maintained upto $t = \infty$. Hence,

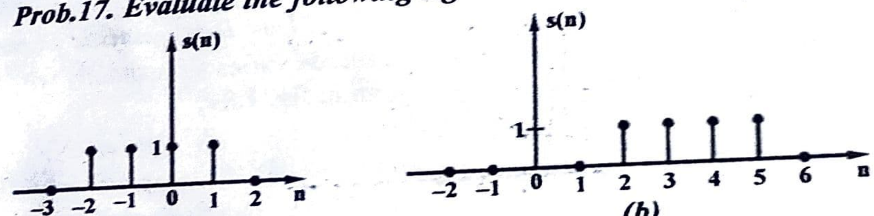
From $-2 \le t \le \infty$

$$\mathbf{s(t) = u_r(t+2) - u_r(t+1) - u_r(t-1) + u_r(t-2)} \quad \textbf{Ans.}$$

(b) The signal is starting from $t = 0$ with a slope (1/2). It is ramp function (1/2) $u_r(t)$. The value of (1/2) $u_r(t) = 1$ for $t = 2$. For $t = 2$, the amplitude is varying from 1 to –1 and then increasing linearly with same slope (1/2). Therefore for $t = 2$, we have to add a step of –2 amplitude [i.e. $-2u(t-2)$]. The signal is terminated for $t = 4$. Hence for $t = 4$, we have to add a ramp with a slope –(1/2) [i.e. –(1/2) $u_r(t-4)$].

$$\mathbf{s(t) = (1/2)\, u_r(t) - 2u(t-2) - (1/2)\, u_r(t-4)} \quad \textbf{Ans.}$$

Prob.17. Evaluate the following signals –



(a) (b)

Fig. 1.83 Waveforms of Prob.17

***Sol.* (a)** The given signal is shown in fig. 1.83 (a). This signal is given by

$$s(n) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1)$$

$$s(n) = \begin{cases} 0, & n \le -3 \\ 1, & -2 \le n \le 1 \\ 0, & n \ge 2 \end{cases} = \mathbf{u(n+2) - u(n-2)} \quad \textbf{Ans.}$$

(b) The signal is shown in fig. 1.83 (b). This signal is given by

$$s(n) = \delta(n-2) + \delta(n-3) + \delta(n-4) + \delta(n-5)$$

$$s(n) = \begin{cases} 0, & n \le 1 \\ 1, & 2 \le n \le 5 \\ 0, & n \ge 6 \end{cases} = \mathbf{u(n-2) - u(n-6)} \quad \textbf{Ans.}$$

CLASSIFICATION OF SIGNALS – DETERMINISTIC AND RANDOM SIGNALS, PERIODIC AND NON-PERIODIC SIGNALS, ENERGY AND POWER SIGNALS, CAUSAL AND NON-CAUSAL SIGNALS, EVEN AND ODD SIGNALS

SHORT QUESTIONS

Q.19. Write short note on deterministic and non-deterministic (rando signals.

Ans. Deterministic signals are completely specified in time. T deterministic signal pattern is regular and is characterized mathematically. Al the nature and amplitude of deterministic signal at any time is predicted.

Following are the examples of deterministic signal –

(i) $s(t) = at$

This is a ramp signal. For this signal the amplitude rises linearly with time and slope is a.

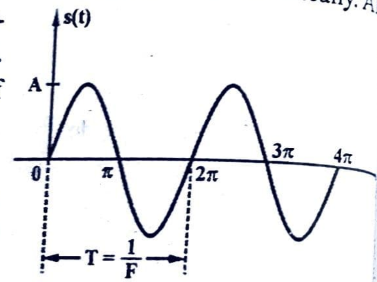


Fig. 1.84 Deterministic Signal

(ii) $s(t) = A \sin \omega t$

This is a sinusoidal signal whose amplitude varies sinusoidally with tin and its maximum amplitude is A, as shown in fig. 1.84.

(iii) $s(n) = \begin{cases} 2, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$

This is a discrete time signal. For this signal, the amplitude is 2 for t sampling instants $n \geq 0$ and the amplitude is zero for all other samples.

Thus, we observe that the amplitude at any time instant is predicted i advance for all the above signal. Hence, all the above signal are deterministic signals.

Whereas, random signal is one whose occurrence is always random in-nature. The pattern of random signal is quite irregular. Non-deterministic signals are known as random signals.

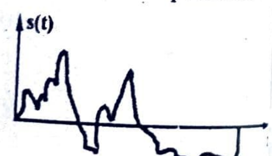


Fig. 1.85 Random Signa

A typical example of non-deterministic signals is thermal noise generated in a electric circuit. The random signal is shown in fig. 1.85.

Q.20. Write down the comparison of energy signal and power signal.

Ans. Comparison of energy and power signals are given below –

S.No.	Energy Signal	Power Signal
(i)	Energy signal are time limited.	Power signals can exist over infinite time.
(ii)	Non-periodic signals are energy signals.	Practical periodic signal are power signals.
(iii)	The energy signal is $E = \int_{-\infty}^{\infty} \lvert s(t)\rvert^2\, dt$	The average power is $P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \lvert s(t)\rvert^2\, dt$
(iv)	Total energy is non zero and finite.	The average power is non-zero and finite.

Q.21. Discuss causal and non-causal signals.

Ans. A continuous-time signal s(t) is called causal when $s(t) = 0$ for the value of t is negative ($t < 0$), otherwise the signal is non-causal. A signal s(t) is called anti-causal when $s(t) = 0$ for the value of t is positive ($t > 0$).

For $t < 0$, a causal signal does not exist and for $t > 0$, an anti-causal signal does not exist. A signal which exists in negative as well as positive time is neither causal nor anti-causal signal. This is non-causal signal. An unit step signal u(t) is a causal signal and reversed unit step signal u(–t) is anti-causal signal.

A discrete-time signal s(n) is called causal when $s(n) = 0$ for the value of n is negative (i.e. $n < 0$), otherwise the signal is non-causal. A signal s(n) is called anti-causal when $s(n) = 0$ for the value of n is positive (i.e. $n > 0$).

LONG QUESTIONS

Q.22. Classify the signal for continuous-time.

Ans. The signals for continuous-time are given belows –

(i) Deterministic and Random (non-deterministics) Signals – Refer the ans. of Q.19.

(ii) Periodic and Non-periodic Signals – A ***periodic*** signal is one which repeats itself after every time interval T_0 which is called the time period of the signal. Mathematically, a function $g_p(t)$ is said to be periodic when it satisfies the condition

$$g_p(t) = g_p(t + T_0)$$

Signals which do not satisfy the above condition are called ***non-periodic*** or ***aperiodic*** signals.

(iii) Energy and Power Signals – Signals can also be classified as those having finite energy or finite average power. The total energy and the

average power normalized to unit resistance of any arbitrary signal x(t) ca
defined as,

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt \quad \text{Joules}$$

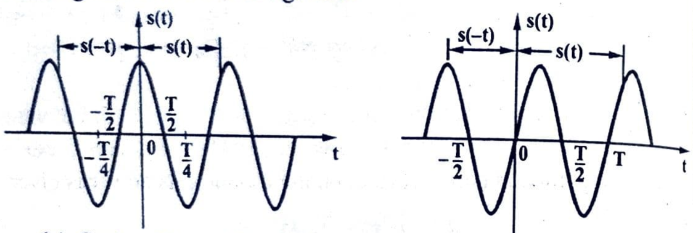
and

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \quad \text{watts}$$

The ***energy signal*** is one which has finite energy and zero average po
i.e., x(t) is an energy signal if $0 < E < \infty$, and $P = 0$. The ***power signal*** is
which has finite average power and infinite energy, i.e., $0 < P < \infty$, and E
If the signal does not satisfy any of these two conditions, then it is neithe
energy nor a power signal.

(iv) Causal and Non-causal Signals – Refer the ans. of Q.21.

(v) Even and Odd Signals – A continuous-time signal s(t) is said
be symmetrical or even if it satisfies the symmetry condition, i.e., s(t) = s(
But on the other hand a signal s(t) is said to be antisymmetrical or odd i
satisfies the antisymmetry condition, i.e., s(t) = – s(–t). The examples e
and odd signals are shown in fig. 1.86.



(a) Cosine Wave s(t) = s(–t)
Even Signal

(b) Sine Wave s(t) = –s(–t)
Odd Signal

Fig. 1.86 Even and Odd Signal

Any continuous-time signal is expressed as the summation of even p
and odd part i.e.

$$s(t) = s_e(t) + s_o(t) \quad ...($$

Putting t = –t in equation (i), we get

$$s(-t) = s_e(-t) + s_o(-t) \quad ...($$

Expression for $s_e(t)$ –

For the even signal, we have

$$s_e(t) = s_e(-t) \quad ...(i$$

and for the odd signal, we have

$$s_o(t) = -s_o(-t) \quad ...(i$$

Substituting equations (iii) and (iv) in equation (ii), we get

$$s(-t) = s_e(t) - s_o(t) \quad ...($$

Adding equation (i) and (v), we get

$$2s_e(t) = s(t) + s(-t)$$

$$s_e(t) = \frac{1}{2}[s(t) + s(-t)] \quad ...(vi)$$

The above equation gives even components of s(t).

Expression for $s_o(t)$ –

Now, subtracting equation (v) from equation (i), we get

$$s(t) - s(-t) = 2s_o(t)$$

or

$$s_o(t) = \frac{1}{2}[s(t) - s(-t)] \quad ...(vii)$$

The above equation gives odd components of s(t).

Q.23. Explain the various types of signal for discrete-time.

Ans. There are following types of discrete-time signal as given below.

(i) Deterministic and Random Signals – Refer the ans. of Q.19.

(ii) Periodic and Non-periodic Signals – A signal s(n) is periodic with period N(N > 0) if and only if

$$s(n + N) = s(n) \quad \text{for all } n \quad ...(i)$$

The smallest value of N for which equation (i) holds is called the period. If there is no value of N that satisfies equation (i), the signal is called nonperiodic or aperiodic. The energy of a periodic signal s(n) over a single period say over the interval $0 \le n \le N-1$ is finite if s(n) takes on finite values over the period. On the other hand the average power of the periodic signal is finite and it is equal to the average power over a single period. Thus if s(n) is a periodic signal with fundamental period N takes on finite values, its power is given by

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |s(n)|^2$$

Consequently, periodic signals are power signals.

(iii) Energy and Power Signals – The energy E of a signal s(n) is defined as –

$$E = \sum_{n=-\infty}^{\infty} |s(n)|^2$$

The energy of a signal can be finite or infinite. If E is finite then s(n) is called an energy signal, many signals that possess infinite energy, have a finite average power. It is defined as –

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |s(n)|^2$$

Signal energy of s(n) over the finite interval $-N \le n \le N$ as –

$$E_N = \sum_{n=-N}^{N} |s(n)|^2$$

(iii) Given $y(n) = s(2n)$

The signal $s(2n)$ is obtained by substituting different values of n. T
range of n in $s(n)$ is -3 to 3.

At $n = -2$, $y(-2) = s(-4) = 0$

At $n = -1$, $y(-1) = s(-2) = 1$

At $n = 0$, $y(0) = s(0) = 2$

At $n = 1$, $y(1) = s(2) = 2$

At $n = 2$, $y(2) = s(4) = 0$

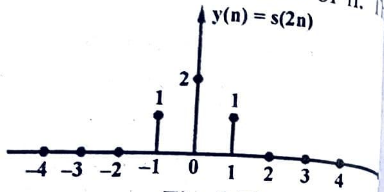


Fig. 1.73

The signal $y(n) = s(2n)$ is shown in fig. 1.73.

(iv) Given $y(n) = s(n).s(3 - n)$

$$s(n) = \{-1, 1, 2, \underset{\uparrow}{2}, 2, 2, 1\}$$

and $$s(3 - n) = \{\underset{\uparrow}{1}, 2, 2, 2, 2, 1, -1\}$$

$$y(n) = s(n).s(3 - n)$$
$$= \{2 \times 1, 2 \times 2, 2 \times 2, 1 \times 2\}$$
$$= \{\underset{\uparrow}{2}, 4, 4, 2\}$$

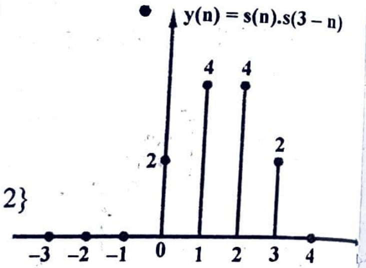


Fig. 1.74

The signal $y(n) = s(n).s(3 - n)$ is shown in fig. 1.74.

Prob.14. A discrete-time signal x(k) is defined as –

$$x(k) = k(0.2)^{|k|} \quad -4 \le k \le +4$$

(i) Determine its values and sketch the signal x(k)

(ii) Determine $y(k) = x(k-4) + x(k+8)$

(iii) Obtain $y(k-4).1(k)$

(iv) Determine $y(k+4) - y(k-2)$.

where, $1(k) = 1 \; \forall \; k \ge 0$.

Sol. (i) Given, $x(k) = k(0.2)^{|k|}$, $-4 \le k \le +4$ we determine the samp
values of x(k) by substituting the value of k.

$$x(-4) = -4(0.2)^{|-4|} = -0.0064$$
$$x(-3) = -3(0.2)^{|-3|} = -0.024$$
$$x(-2) = -2(0.2)^{|-2|} = -0.08$$
$$x(-1) = -1(0.2)^{|-1|} = -0.2$$
$$x(0) = 0(0.2)^{|0|} = 0$$
$$x(1) = 1(0.2)^{|1|} = 0.2$$
$$x(2) = 2(0.2)^{|2|} = 0.08$$
$$x(3) = 3(0.2)^{|3|} = 0.024$$
$$x(4) = 4(0.2)^{|4|} = 0.0064$$

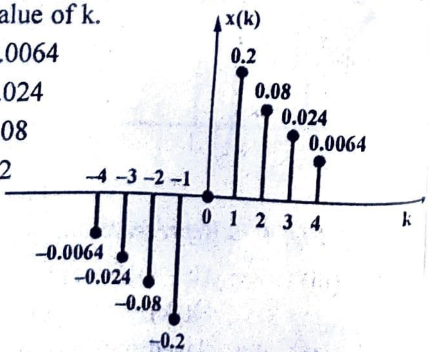


Fig. 1.75 Representation of x(k)

Thus,

$$x(k) = (-0.0064, -0.024, -0.08, -0.2, \underset{\uparrow}{0}, 0.2, 0.08, 0.024, 0.0064)$$

Graphical representation of x(k)

(ii) Given, $y(k) = x(k-4) + x(k+8)$

y(k) can also be written as,

$$y(k) = x_1(k) + x_2(k)$$

where $x_1(k) = x(k-4)$ and $x_2(k) = x(k+8)$

Now, $x_1(k) = x(k-4)$ is the delayed sequence by 4 unit, can be illustrated in fig. 1.76 (a).

and $x_2(k) = x(k+8)$ can be represented in fig. 1.76 (b).

Therefore, $y(k) = x_1(k) + x_2(k)$ will be shown in fig. 1.76 (c).

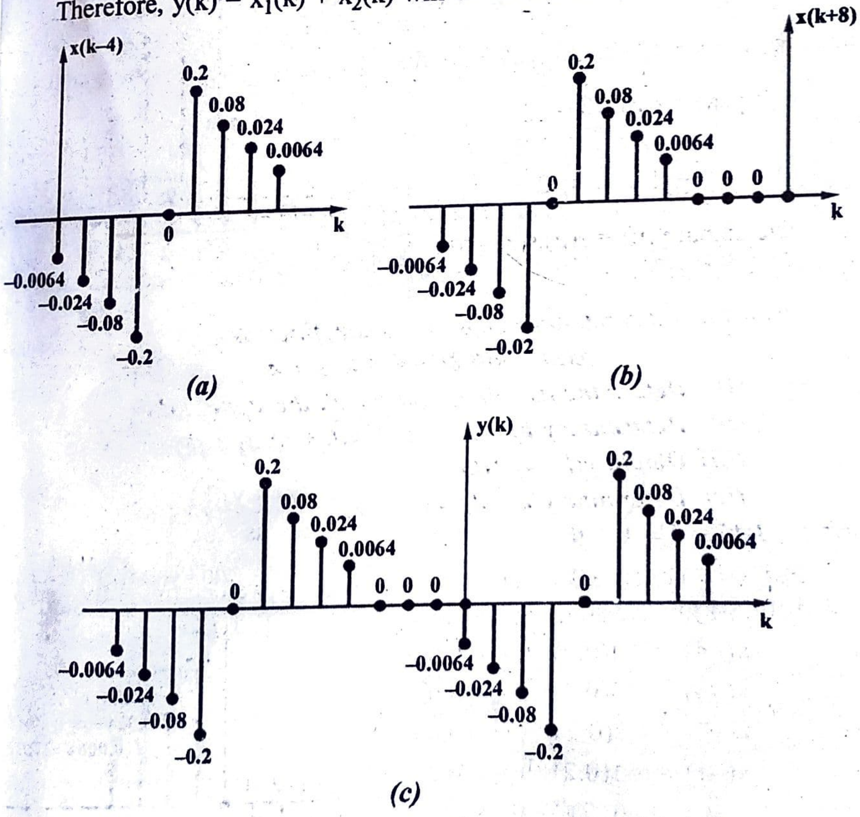
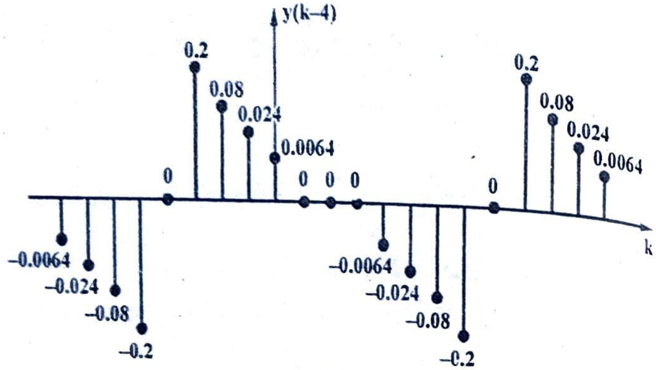


Fig. 1.76 Representation of $y(k) = x(k-4) + x(k+8)$

(iii) Given $y(k-4)1(k)$ and

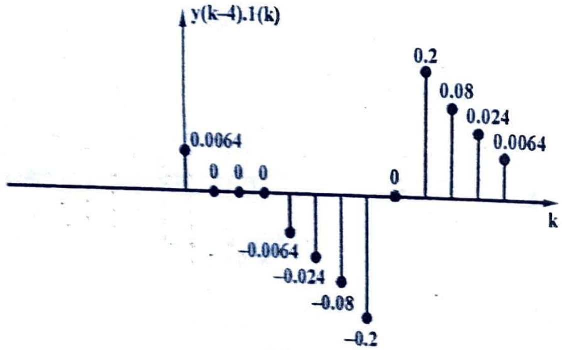
$$1(k) = 1 \text{ for all } k \ge 0$$

$y(k-4)$ is the delayed sequence of y(k) by 4 unit can be represented as



(a)

Now y(k – 4).1k is represented as

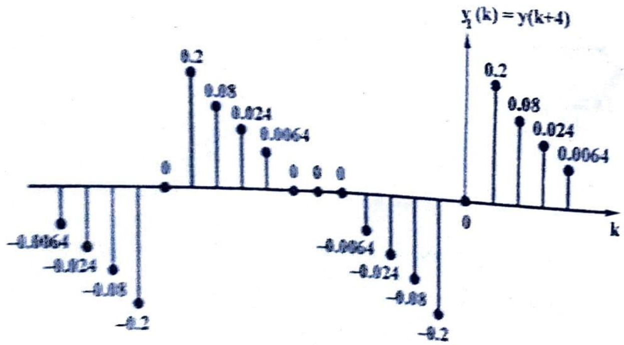


(b)

Fig. 1.77 Representation of y(k – 4) 1 (k)

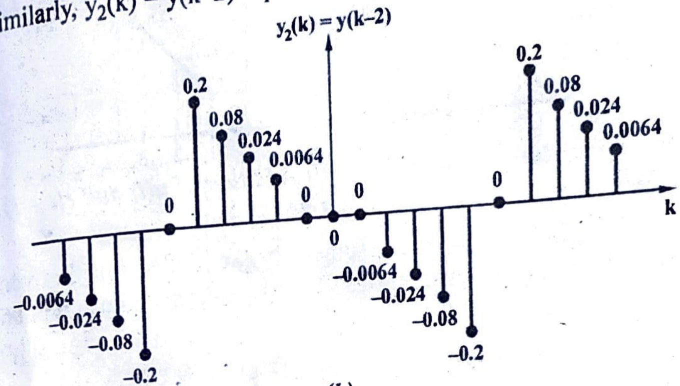
(iv) $y(k) = y(k + 4) - y(k - 2) = y_1(k) - y_2(k)$

$y_1(k) = y(k + 4)$ can be represented as



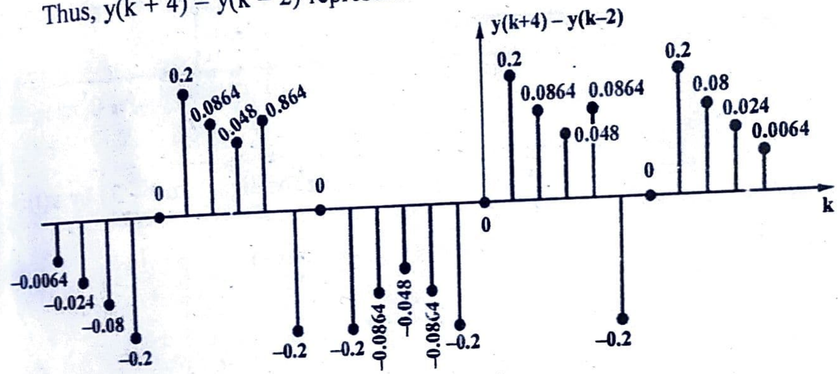
(a)

Similarly, $y_2(k) = y(k-2)$ represented as



(b)

Thus, y(k + 4) – y(k – 2) represented as



(c)

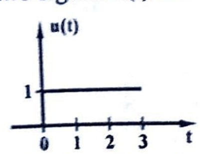
Fig. 1.78 Determination of y(k) = y(k + 4) – y (k – 2)

Prob.15. Sketch the signal $s(t) = u(t) + u_r(t - 1) - 2u(t - 3)$.

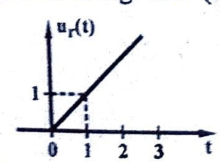
Sol. The given signal is

$$s(t) = u(t) + u_r(t - 1) - 2u(t - 3)$$

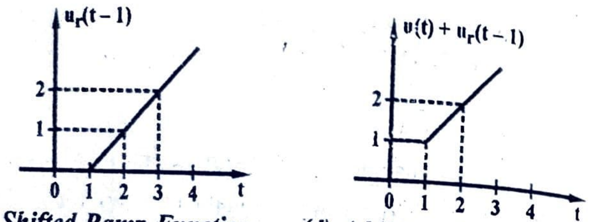
This signal $u(t) + u_r(t - 1)$ is obtained by first drawing unit step function u(t) [see fig. 1.79 (a)] and ramp function $u_r(t)$ [see fig. 1.79 (b)]. Then unit ramp signal u_r is shifted by 1 to the right as shown in fig. 1.79 (c). The addition of two signals u(t) and $u_r(t - 1)$ is shown in fig. 1.79 (d).



(a) Unit Step Function u(t)



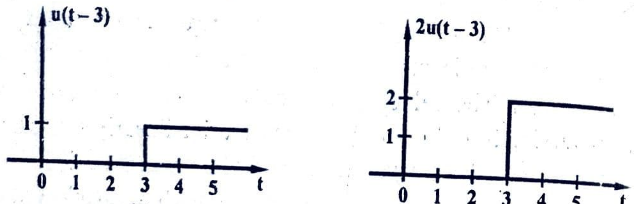
(b) Ramp Function $u_r(t)$



(c) Shifted Ramp Function *(d) Addition of u(t) and r(t – 1)*

Fig. 1.79

Now, the term $2u(t-3)$ is assumed. It is unit step function which shifted by 3 positions as shown in fig. 1.80 (a) and its magnitude is multiplied by 2 as shown in fig. 1.80 (b).



(a) *(b)*

Fig. 1.80

The signal s(t) is obtained by subtracting the signal $2u(t-3)$ by $u(t)$ $u_r(t-1)$ as shown in fig. 1.81.

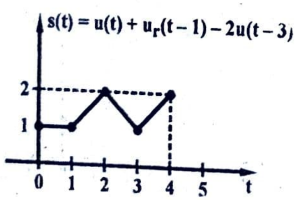
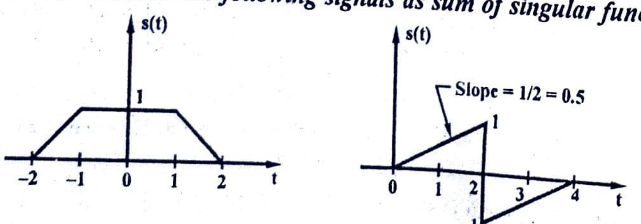


Fig. 1.81

Prob.16. Evaluate the following signals as sum of singular functions



(a) *(b)*

Fig. 1.82 Waveforms for Prob.16

Sol. **(a)** The signal is starting at $t = -2$ with a slope 1 and extends up to $t = -1$. Hence,

From $-2 \le t \le -1$, $s(t) = u_r(t+2)$

For $t = -1$, the slope is changing from 1 to 0 and for $t = 1$, we have to add a ramp with a slope of –1. This 0 slope is maintained up to $t = 1$. Hence,

From $-2 \le t \le 1$,

$$s(t) = u_r(t+2) - u_r(t+1)$$

For $t = 1$, the slope is changing from 0 to –1 and for $t = 1$, we have to add a ramp with a slope of –1. This –1 slope is maintained up to $t = 2$. Hence,

From $-2 \le t \le 2$,

$$s(t) = u_r(t+2) - u_r(t+1) - u_r(t-1)$$

For $t = 2$, the slope is changing from –1 to 0 and for $t = 2$, we have to add a ramp with a slope of 1. This 0 slope is maintained upto $t = \infty$. Hence,

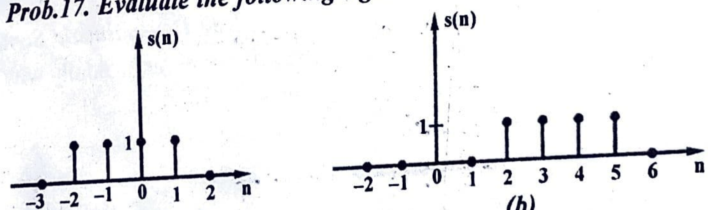
From $-2 \le t \le \infty$

$$\mathbf{s(t) = u_r(t+2) - u_r(t+1) - u_r(t-1) + u_r(t-2)} \quad \textbf{Ans.}$$

(b) The signal is starting from $t = 0$ with a slope (1/2). It is ramp function $(1/2)\, u_r(t)$. The value of $(1/2)\, u_r(t) = 1$ for $t = 2$. For $t = 2$, the amplitude is varying from 1 to –1 and then increasing linearly with same slope (1/2). Therefore for $t = 2$, we have to add a step of –2 amplitude [i.e. $-2u(t-2)$]. The signal is terminated for $t = 4$. Hence for $t = 4$, we have to add a ramp with a slope –(1/2) [i.e. $-(1/2)\, u_r(t-4)$].

$$\mathbf{s(t) = (1/2)\, u_r(t) - 2u(t-2) - (1/2)\, u_r(t-4)} \quad \textbf{Ans.}$$

Prob.17. Evaluate the following signals –



(a) *(b)*

Fig. 1.83 Waveforms of Prob.17

Sol. **(a)** The given signal is shown in fig. 1.83 (a). This signal is given by

$$s(n) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1)$$

$$s(n) = \begin{cases} 0, & n \le -3 \\ 1, & -2 \le n \le 1 \\ 0, & n \ge 2 \end{cases} = \mathbf{u(n+2) - u(n-2)} \quad \textbf{Ans.}$$

(b) The signal is shown in fig. 1.83 (b). This signal is given by

$$s(n) = \delta(n-2) + \delta(n-3) + \delta(n-4) + \delta(n-5)$$

$$s(n) = \begin{cases} 0, & n \le 1 \\ 1, & 2 \le n \le 5 \\ 0, & n \ge 6 \end{cases} = \mathbf{u(n-2) - u(n-6)} \quad \textbf{Ans.}$$

CLASSIFICATION OF SIGNALS – DETERMINISTIC AND RANDOM SIGNALS, PERIODIC AND NON-PERIODIC SIGNALS, ENERGY AND POWER SIGNALS, CAUSAL AND NON-CAUSAL SIGNALS, EVEN AND ODD SIGNALS

SHORT QUESTIONS

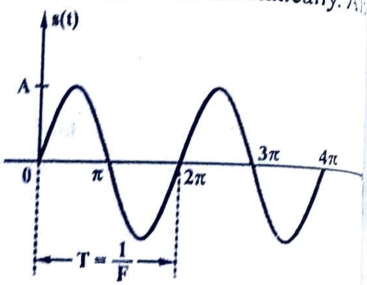
Q.19. Write short note on deterministic and non-deterministic (rando[m]) signals.

Ans. Deterministic signals are completely specified in time. T[he] deterministic signal pattern is regular and is characterized mathematically. Al[so] the nature and amplitude of deterministic signal at any time is predicted.

Following are the examples of deterministic signal –

(i) $s(t) = at$

This is a ramp signal. For this signal the amplitude rises linearly with time and slope is a.



(ii) $s(t) = A \sin \omega t$

Fig. 1.84 Deterministic Signal

This is a sinusoidal signal whose amplitude varies sinusoidally with ti[me] and its maximum amplitude is A, as shown in fig. 1.84.

(iii) $s(n) = \begin{cases} 2, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$

This is a discrete time signal. For this signal, the amplitude is 2 for t[he] sampling instants $n \geq 0$ and the amplitude is zero for all other samples.

Thus, we observe that the amplitude at any time instant is predicted [in] advance for all the above signal. Hence, all the above signal are deterministic signals.

Whereas, random signal is one whose occurrence is always random in-nature. The pattern of random signal is quite irregular. Non-deterministic signals are known as random signals.

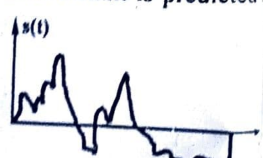


Fig. 1.85 Random Sign[al]

A typical example of non-deterministic signals is thermal noise generated in a electric circuit. The random signal is shown in fig. 1.85.

Q.20. Write down the comparison of energy signal and power signal.

Ans. Comparison of energy and power signals are given below –

S.No.	Energy Signal	Power Signal
(i)	Energy signal are time limited.	Power signals can exist over infinite time.
(ii)	Non-periodic signals are energy signals.	Practical periodic signal are power signals.
(iii)	The energy signal is $E = \int_{-\infty}^{\infty} \lvert s(t)\rvert^2 \, dt$	The average power is $P = \lim\limits_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \lvert s(t)\rvert^2 \, dt$
(iv)	Total energy is non zero and finite.	The average power is non-zero and finite.

Q.21. Discuss causal and non-causal signals.

Ans. A continuous-time signal s(t) is called causal when $s(t) = 0$ for the value of t is negative $(t < 0)$, otherwise the signal is non-causal. A signal s(t) is called anti-causal when $s(t) = 0$ for the value of t is positive $(t > 0)$.

For $t < 0$, a causal signal does not exist and for $t > 0$, an anti-causal signal does not exist. A signal which exists in negative as well as positive time is neither causal nor anti-causal signal. This is non-causal signal. An unit step signal u(t) is a causal signal and reversed unit step signal $u(-t)$ is anti-causal signal.

A discrete-time signal s(n) is called causal when $s(n) = 0$ for the value of n is negative (i.e. $n < 0$), otherwise the signal is non-causal. A signal s(n) is called anti-causal when $s(n) = 0$ for the value of n is positive (i.e. $n > 0$).

LONG QUESTIONS

Q.22. Classify the signal for continuous-time.

Ans. The signals for continuous-time are given belows –

(i) Deterministic and Random (non-deterministics) Signals – Refer the ans. of Q.19.

(ii) Periodic and Non-periodic Signals – A ***periodic*** signal is one which repeats itself after every time interval T_0 which is called the time period of the signal. Mathematically, a function $g_p(t)$ is said to be periodic when it satisfies the condition

$$g_p(t) = g_p(t + T_0)$$

Signals which do not satisfy the above condition are called ***non-periodic*** or ***aperiodic*** signals.

(iii) Energy and Power Signals – Signals can also be classified as those having finite energy or finite average power. The total energy and the

average power normalized to unit resistance of any arbitrary signal x(t) ca
defined as,

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\, dt \quad \text{Joules}$$

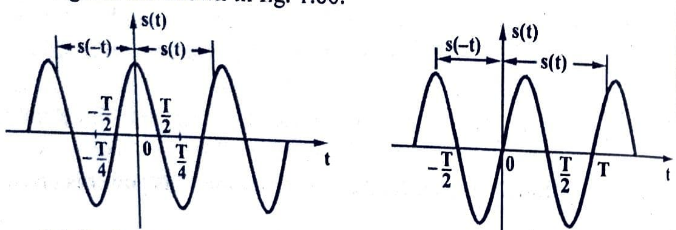
and

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt \quad \text{watts}$$

The ***energy signal*** is one which has finite energy and zero average po
i.e., x(t) is an energy signal if $0 < E < \infty$, and $P = 0$. The ***power signal*** is
which has finite average power and infinite energy, i.e., $0 < P < \infty$, and E =
If the signal does not satisfy any of these two conditions, then it is neither
energy nor a power signal.

(iv) Causal and Non-causal Signals – Refer the ans. of Q.21.

(v) Even and Odd Signals – A continuous-time signal s(t) is said
be symmetrical or even if it satisfies the symmetry condition, i.e., s(t) = s(–
But on the other hand a signal s(t) is said to be antisymmetrical or odd if
satisfies the antisymmetry condition, i.e., s(t) = – s(–t). The examples of e
and odd signals are shown in fig. 1.86.



(a) Cosine Wave s(t) = s(–t) Even Signal *(b) Sine Wave s(t) = –s(–t) Odd Signal*

Fig. 1.86 Even and Odd Signal

Any continuous-time signal is expressed as the summation of even pa
and odd part i.e.

$$s(t) = s_e(t) + s_o(t) \quad ...($$

Putting t = –t in equation (i), we get

$$s(-t) = s_e(-t) + s_o(-t) \quad ...($$

Expression for $s_e(t)$ –

For the even signal, we have

$$s_e(t) = s_e(-t)$$

and for the odd signal, we have ...(ii

$$s_o(t) = -s_o(-t) \quad ...(iv$$

Substituting equations (iii) and (iv) in equation (ii), we get

$$s(-t) = s_e(t) - s_o(t) \quad ...($$

Adding equation (i) and (v), we get

$$2s_e(t) = s(t) + s(-t)$$

or

$$s_e(t) = \frac{1}{2}[s(t) + s(-t)] \quad ...(vi)$$

The above equation gives even components of s(t).

Expression for $s_o(t)$ –

Now, subtracting equation (v) from equation (i), we get

$$s(t) - s(-t) = 2s_o(t)$$

or

$$s_o(t) = \frac{1}{2}[s(t) - s(-t)] \quad ...(vii)$$

The above equation gives odd components of s(t).

Q.23. Explain the various types of signal for discrete-time.

Ans. There are following types of discrete-time signal as given below.

(i) Deterministic and Random Signals – Refer the ans. of Q.19.

(ii) Periodic and Non-periodic Signals – A signal s(n) is periodic with period N(N > 0) if and only if

$$s(n + N) = s(n) \quad \text{for all } n \quad ...(i)$$

The smallest value of N for which equation (i) holds is called the period. If there is no value of N that satisfies equation (i), the signal is called nonperiodic or aperiodic. The energy of a periodic signal s(n) over a single period say over the interval $0 \le n \le N - 1$ is finite if s(n) takes on finite values over the period. On the other hand the average power of the periodic signal is finite and it is equal to the average power over a single period. Thus if s(n) is a periodic signal with fundamental period N takes on finite values, its power is given by

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |s(n)|^2$$

Consequently, periodic signals are power signals.

(iii) Energy and Power Signals – The energy E of a signal s(n) is defined as –

$$E = \sum_{n=-\infty}^{\infty} |s(n)|^2$$

The energy of a signal can be finite or infinite. If E is finite then s(n) is called an energy signal, many signals that possess infinite energy, have a finite average power. It is defined as –

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |s(n)|^2$$

Signal energy of s(n) over the finite interval $-N \le n \le N$ as –

$$E_N = \sum_{n=-N}^{N} |s(n)|^2$$

Then we can express the signal energy E as

$$E = \lim_{N \to \infty} E_N$$

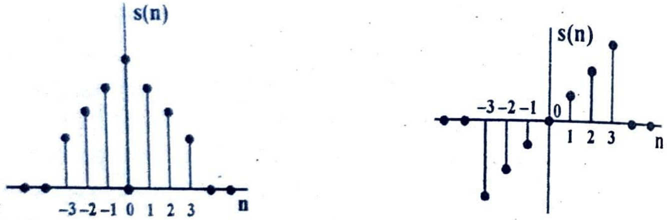
and average power of the signal s(n) as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} E_N$$

Clearly, if E is finite, P = 0 and if E is infinite the average power P ma[y be] either finite or infinite. If P is finite (non-zero) the signal is called a power sig[nal].

(iv) Causal and Non-causal Signals – Refer the ans. of Q.21.

(v) Even and Odd Signals – A real valued signal s(n) is cal[led] symmetric (even) if s(–n) = s(n). But on the other hand a signal s(n) is cal[led] antisymmetric (odd) if s(–n) = –s(n). We note that if signal s(n) is odd th[e] signal s(0) = 0. The even signals and odd signals symmetry example are sho[wn] in fig. 1.87.



(a) Graphical Representation of Even Signals

(b) Graphical Representation of Odd Signals

Fig. 1.87 Example of Representation of Even and Odd Signals

Any arbitrary signal can be expressed as the sum of two sign[al] components, one of which is even and the other is odd. The even sign[al] component is formed by adding s(n) to s(–n) and dividing by 2. It is obtain[ed] by the expression as –

$$s_e(n) = \frac{1}{2}[s(n) + s(-n)] \quad ...$$

In a similar manner we form an odd signal component $s_0(n)$ accordi[ng] to the relation as –

$$s_0(n) = \frac{1}{2}[s(n) - s(-n)] \quad ...($$

Now if we add two signal components defined by equations (i) and (then we obtain the signal s(n) in the form as –

$$s(n) = s_e(n) + s_0(n) \quad ...(i$$

NUMERICAL PROBLEMS

***Prob.18.** Determine the following signals are periodic or not. If t[he] signal is periodic, find the fundamental period.*

(i) $s_1(t) = 8 \sin (400\,\pi t)$ *(ii)* $s_2(t) = 5 + t^2$

(iii) $s_3(t) = \sin 20\,\pi t$ *(iv)* $s_4(t) = \sin\sqrt{2}\pi t$

(v) $s_5(t) = \sin 10\,\pi t$ *(vi)* $s_6(t) = s_3(t) + s_5(t)$.

Sol. (i) Given $s_1(t) = 8 \sin(400\,\pi t)$

The standard equation is

$$s_1(t) = A \sin \omega t$$

Therefore,

$$\omega = 400\,\pi$$
$$2\pi f = 400\,\pi$$
$$f = 200$$

The fundamental period is,

$$T = \frac{1}{f} = \frac{1}{200} \text{ sec.}$$

Hence, the above signal is **periodic** because it is ratio of two integer and fundamental period is 1/200 second. ...(i)

(ii) Given $s_2(t) = 5 + t^2$

Put t = t + T in equation (i), we get

$$s_2(t+T) = 5 + (t+T)^2 \quad ...(ii)$$
$$= 5 + t^2 + 2tT + T^2$$

For any value of T, equations (i) and (ii) cannot be made equal. Hence, the given signal is **non-periodic.**

(iii) Given $s_3(t) = \sin(20\,\pi t)$

Therefore

$$\omega = 20\,\pi$$
$$2\pi f = 20\,\pi$$
$$f = 10$$

The fundamental period is,

$$T = \frac{1}{f} = \frac{1}{10} = 0.1 \text{ second}$$

Hence, the given signal is **periodic** and fundamental period is 0.1 second.

(iv) Given $s_4(t) = \sin(\sqrt{2}\pi t)$

Therefore,

$$\omega = \sqrt{2}\pi$$
$$2\pi f = \sqrt{2}\pi$$
$$f = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

The fundamental period is,

$$T = \frac{1}{f} = \frac{1}{1/\sqrt{2}} = \sqrt{2} = 1.414$$

Hence, the signal is **periodic** and fundamental period is 1.414 second.

(v) Given, $s_5(t) = \sin(10\,\pi t)$

$$\omega = 10\,\pi$$
$$2\pi f = 10\,\pi$$
$$f = 5$$

The fundamental period is,

$$T = \frac{1}{f} = \frac{1}{5} = 0.2 \text{ sec.}$$

Hence, the signal is **periodic** and fundamental period is 0.2 second

(vi) Given $s_6(t) = s_3(t) + s_5(t)$

The fundamental period of $s_3(t) = T_3 = 0.1$ second and the fundam period of $s_5(t) = T_5 = 0.2$ second. The ratio of fundamental period is gi

$$\frac{T_3}{T_5} = \frac{0.1}{0.2} = \frac{1}{2}, \text{ can be expressed as a ratio of inte}$$

Hence, the signal $s_6(t)$ is **periodic**.

Prob.19. Examine whether the signal –

$$s(n) = \cos\left(\frac{n}{10}\right)\cos\left(\frac{n\pi}{10}\right) \text{ is a periodic or not.}$$

Sol. Given $\quad s(n) = \cos\left(\frac{n}{10}\right)\cos\left(\frac{n\pi}{10}\right)$

Assume, $\quad s_1(n) = \cos\left(\frac{n}{10}\right),\ s_2(n) = \cos\left(\frac{n\pi}{10}\right)$

The standard signal is,

Now, we get $\quad s(n) = \cos\omega n = \cos(2\pi\ fn)$

$$2\pi f_1 = \frac{1}{10} \text{ or } f_1 = \frac{1}{20\pi}$$

and

$$2\pi f_2 = \frac{\pi}{10} \text{ or } f_2 = \frac{1}{20}$$

Hence, the signal s(n) is **non-periodic** because f_1 is not a rational num

Prob.20. Determine whether the following discrete-time signals periodic or not. If the signal is periodic, find the fundamental period.

(i) $s(n) = \cos(5\pi n)$ (ii) $s(n) = \sin(5n)$

(iii) $s(n) = \sin(\pi + 0.4\ n)$ (iv) $s(n) = e^{(j\pi/4)n}$

Sol. (i) Given $s(n) = \cos(5\pi n)$

The standard equation is given by

$$s(n) = \cos(\omega n)$$
$$= \cos(2\pi\ fn)$$
$$2\pi f = 5\pi$$

$$f = \frac{5}{2} = \frac{K}{N}$$

Hence, this signal is **periodic** and the fundamental period, N = 2.

(ii) Given, $s(n) = \sin(5n)$

Therefore. $\quad 2\pi f = 5$

$$f = \frac{5}{2\pi} = \frac{K}{N}$$

which is not the ratio of two integers. Hence, this signal is **non-periodic.**

(iii) Given, $s(n) = \sin(\pi + 0.4\ n)$

Comparing above equation with,

$$s(n) = \sin(\theta + 2\pi fn) = \sin(\theta + \omega n)$$
$$\theta = \pi(\text{phase shift}) \text{ and } 2\pi fn = 0.4\ n$$

We have

$$f = \frac{0.2}{\pi} = \frac{2}{10\pi} \text{ which is not rational.}$$

Therefore, the given signal is **non-periodic.**

(iv) Given, $s(n) = e^{(j\pi/4)n}$

$$= \cos\frac{\pi}{4}n + j\sin\frac{\pi}{4}n \qquad ...(i)$$

After simplifying,

$$s(n) = \cos 2\pi fn + j\sin 2\pi fn \qquad ...(ii)$$

Comparing the equations (i), and (ii), we get

$$2\pi fn = \frac{\pi}{4}n$$
$$f = \frac{1}{8} = \frac{K}{N}$$

which is rational. Therefore, this signal is **periodic** and fundamental period is N = 8.

Prob.21. Check whether the following signals are energy or power signals and determine the value.

$$s(t) = e^{-\alpha t}\,u(t)$$

Sol. Given, $\quad s(t) = e^{-\alpha t}\,u(t)$

Energy of this signal will be

$$E = \int_{-\infty}^{\infty} |s(t)|^2\,dt = \int_{-\infty}^{\infty} |e^{-\alpha t}u(t)|^2\,dt$$

Because u(t) exist for $0 \le t \le \infty$, therefore

$$= \int_0^{\infty} e^{-2\alpha t}\,dt = \frac{1}{-2\alpha}\left[e^{-2\alpha t}\right]_0^{\infty}$$

$$= -\frac{1}{2\alpha}\left[e^{-\infty} - e^{0}\right] = -\frac{1}{2\alpha} \times -1 = \frac{1}{2\alpha} \qquad \textbf{Ans.}$$

The given signal is an **energy signal** because the total energy is fin... non-zero.

We know that, the power of the signal is zero when the energy ... signal is finite. Hence,

$$P = 0$$

Prob.22. Prove the following –

(i) The power of the energy signal is zero over infinite time.

(ii) The energy of the power signal is infinite over infinite ti...

Sol. (i) Let us consider s(t) be an energy signal. The power is express...

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |s(t)|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\left[\lim_{T\to\infty}\int_{-T/2}^{T/2} |s(t)|^2\, dt\right]$$

$$= \lim_{T\to\infty} \frac{1}{T}\left[\int_{-\infty}^{\infty} |s(t)|^2\, dt\right]$$

Limits are changed from $-\infty$ to ∞ as $T \to \infty$, we get

$$P = \lim_{T\to\infty} \frac{1}{T} \times E = 0 \qquad \left[\therefore E = \int_{-\infty}^{\infty} |s(t)|^2 ...\right.$$

Hence, the power of energy signal is zero over infinite time.

(ii) The energy of the signal is expressed as

$$E = \int_{-\infty}^{\infty} |s(t)|^2\, dt$$

We can the change the limits of integration as $-\frac{T}{2}$ to $\frac{T}{2}$ and take $\lim_{T\to ...}$

Hence, we get

$$E = \lim_{T\to\infty}\int_{-T/2}^{T/2} |s(t)|^2\, dt = \lim_{T\to\infty}\left[T.\frac{1}{T}\int_{-T/2}^{T/2} |s(t)|^2 ...\right.$$

$$= \lim_{T\to\infty} T\left[\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} |s(t)|^2\, dt\right]$$

$$= \lim_{T\to\infty} T \times P = \infty$$

Hence, energy of the power signal is ∞ over infinite time.

Prob.23. Find whether the following signals are energy signal, po... signals or neither of them –

***(i)** $x(n) = (-0.3)^n u(n)$ **(ii)** $x(n) = 2u(n)$ **(iii)** $y(t) = t^n u(n)$*

Also determine energy/power for these signals.

Sol. (i) Given,

$$x(n) = (-0.3)^n u(n)$$

The signal is not periodic therefore strictly we can say that the given signal is energy signal.

Thus, we have $\quad E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} \left[(-0.3)^n\right]^2 u(n) = \sum_{n=0}^{\infty} (0.09)^n$

But, we know that

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \text{ for } |a| < 1,$$

Then, the above expression will be

$$E = \frac{1}{1-\frac{1}{0.09}} = -\mathbf{0.098}$$

Ans.

(ii) Given, $x(n) = 2\, u(n)$

This signal is periodic and of infinite duration. Hence, it may be a power signal.

Thus,

$$P = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=0}^{N} (2.1)^2 \qquad \because u(n) = 1$$

$$= 4 \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=0}^{N} (1)^2$$

Here, $\sum_{n=0}^{N} 1^2 = 1 + 1 + 1 + 1$ .... for n = 0 to N

Therefore,

$$P = 4 \lim_{N\to\infty} \frac{1}{2N+1}.(N+1)$$

$$P = 4 \lim_{N\to\infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}} = 4 \times \frac{1}{2} = \mathbf{2}$$

Ans.

(iiii) Given, $y(t) = t^n u(n)$

Since, the given signal is non-periodic signal and the energy of signal must be infinite therefore it is neither energy signal nor power signal. **Ans.**

Prob.24. Find the following signals are causal or non-causal.

***(i)** $s(t) = \sin 2t\, u(t)$* *(ii) $s(t) = e^{-3t} u(-t+2)$*

(iii) $s(t) = 2u(-t)$ *(iv) $s(n) = u(-n)$*

(v) $s(n) = u(n+4) - u(n-2)$

(vi) $s(n) = \left(\frac{1}{4}\right)^n u(n+2) - \left(\frac{1}{2}\right)^n u(n-4).$

Sol. (i) Given $s(t) = \sin 2t\, u(t)$

For t is negative (i.e. $t < 0$), the given signal is **causal** because s(t) =

(ii) Given $s(t) = e^{-3t}\, u(-t+2)$

For t is negative (i.e. $t < 0$), the given signal is **non-causal** because s(t) =

(iii) Given $s(t) = 2u(-t)$

The signal s(t) exists only for t is negative (i.e. $t < 0$). Therefore, it **anti-causal.** It is also known as **non-causal.**

(iv) $s(n) = u(-n)$

For $n < 0$, the signal s(n) exists. Therefore, the signal is anti-causal. It also known as non-causal.

(v) $s(n) = u(n+4) - u(n-2)$

The signal s(n) exists from $n = -4$ to 1 because $s(n) \neq 0$ for n is negat (n < 0). Therefore, the given signal is **non-causal.**

(vi) $s(n) = \left(\frac{1}{4}\right)^n u(n+2) - \left(\frac{1}{2}\right)^n u(n-4)$

The signal s(n) exists for n is negative (i.e., $n < 0$). Therefore, the sig is **non-causal.**

Prob.25. The signal is shown in fig. 1.88. Sketch even and odd part the signal.

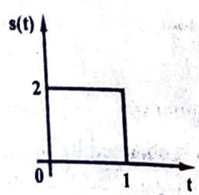


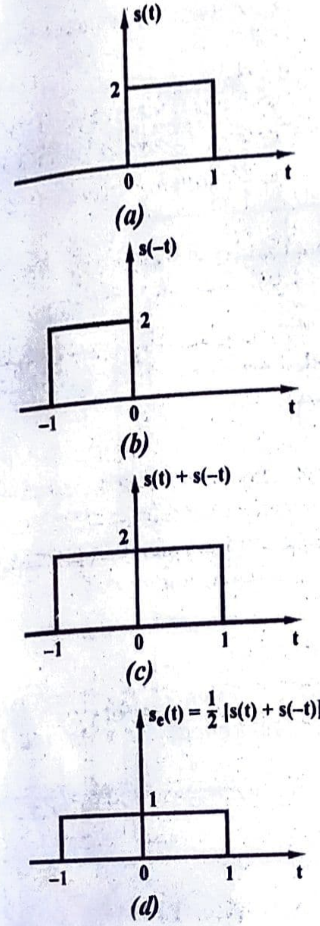
Fig. 1.88

Sol. The even and odd parts are given below –

$$s_e(t) = \frac{1}{2}[s(t) + s(-t)]$$

and
$$s_o(t) = \frac{1}{2}[s(t) - s(-t)]$$

An even part of the signal can be obtained by the first drawing the sign s(t), then draw the reversed of the signal s(–t) and then the signal s(t) an s(–t) are added. Finally, we divide the addition by 2 to obtain $s_e(t)$ as shown fig. 1.89.



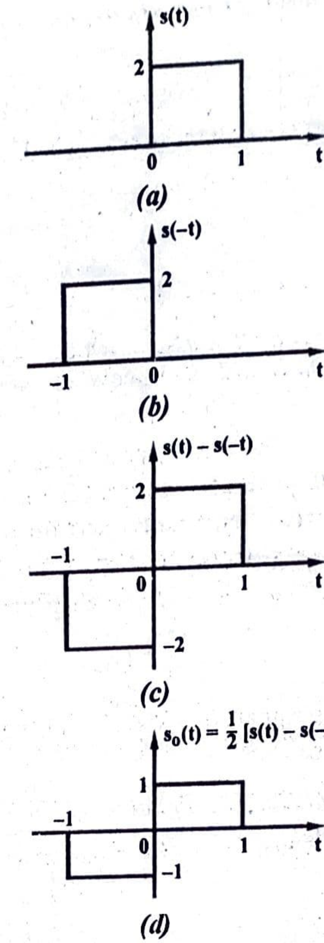


Fig. 1.89 Even Part of the Signal s(t) *Fig. 1.90 Odd Part of the Signal s(t)*

An odd part of the signal is obtained by first drawing signal s(t), then we draw the reversed of the signal s(–t) and then the signal s(–t) from s(t) are subtracted. Finally, we divide the subtraction by 2 to obtain $s_o(t)$ as shown in fig. 1.90.

Prob.26. Fig. 1.91 shows the even and odd parts of a signal. Draw the signal s(t).

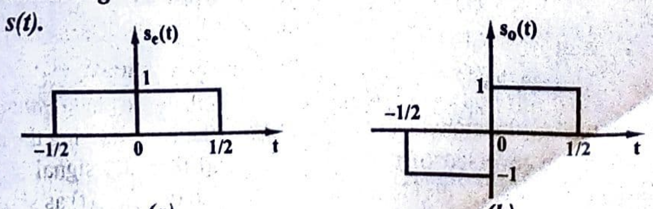


Fig. 1.91

Sol. The signal s(t) is,

$$s(t) = s_e(t) + s_o(t)$$

Fig. 1.92 shows the addition of $s_e(t)$ and $s_o(t)$.

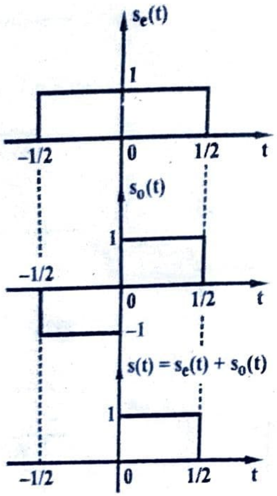


Fig. 1.92

Prob.27. A discrete-time signal s(n) is shown in fig. 1.93. Dr the even and odd parts of s(n). Write the sequences for even and parts.

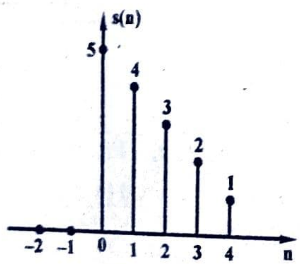


Fig. 1.93

Sol. Even and odd parts of the signal s(n) are given by

$$s_e(n) = \frac{1}{2}[s(n) + s(-n)]$$

$$s_o(n) = \frac{1}{2}[s(n) - s(-n)]$$

An even part of the signal is shown in fig. 1.94 and odd part of the sig is shown in fig. 1.95.

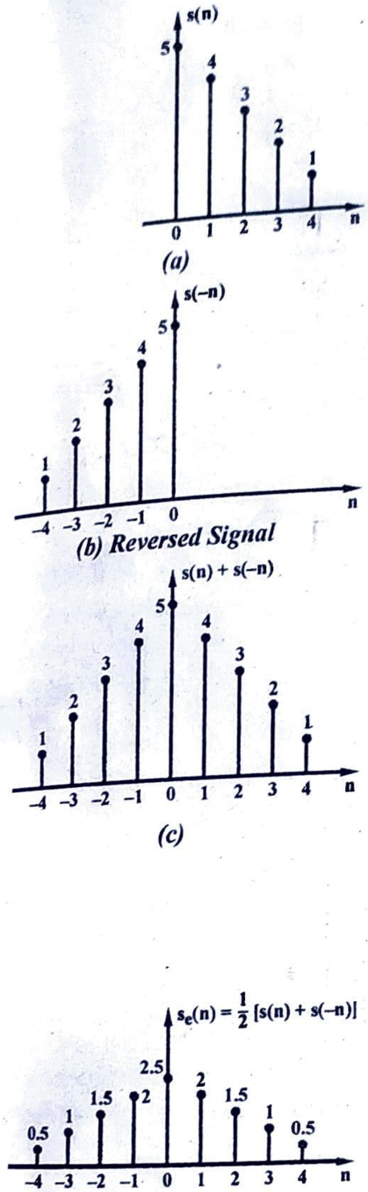


Fig. 1.94

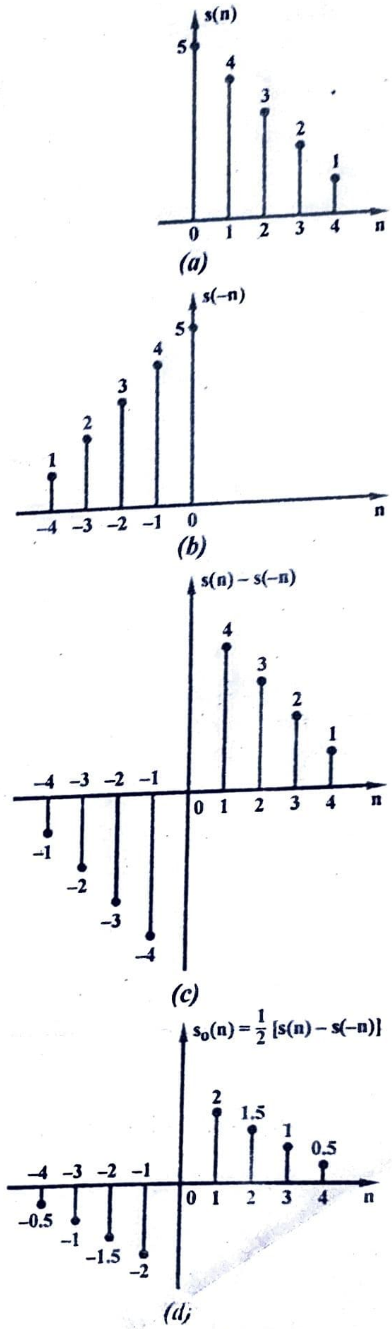


Fig. 1.95

For even and odd parts, the sequences are given by

$$s_e(n) = \{0.5,\ 1,\ 1.5,\ 2,\ \underset{\uparrow}{2.5},\ 2,\ 1.5,\ 1,\ 0.5\}$$

and $$s_o(n) = \{-0.5, -1, -1.5, -2, \underset{\uparrow}{0}, 2, 1.5, 1, 0.5\}$$