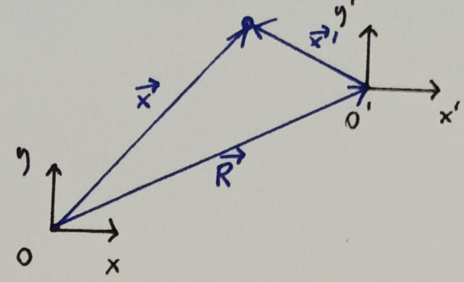

$$\vec{x}(t) = \vec{R}(t) + \vec{x}'(t) \qquad (0)$$

$$\vec{v}(t) = \dot{\vec{R}}(t) + \vec{v}'(t) \qquad (1)$$

$$\vec{a}(t) = \ddot{\vec{R}}(t) + \vec{a}'(t) \qquad (2)$$

From (2), the accelerations experienced by the system O' are given by:

$$\vec{a}'(t) = \vec{a}(t) - \ddot{\vec{R}}(t) \qquad (3)$$

Acceleration "felt" by the accelerometer in O'

Acceleration present in O (typically gravity)

Relative acceleration between systems

Examples:

1. Galilean relativity principle, or why an accelerometer cannot directly measure speeds.

If $\vec{R}(t)$ is not accelerated (for instance: $\vec{R}(t) = \vec{R}_0 + \vec{V}_0 t$), then $\ddot{\vec{R}}(t) = \vec{0}$ and, from (3), $\vec{a}'(t) = \vec{a}(t)$. So both O and O' "experience" the same physics, and thus the speed $\vec{V}_0$ is "invisible" for the accelerometer in O'.

2. Free-fallers experience no acceleration.

Gravity is already present in O, so $\vec{a}(t) = \vec{g} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$

A free-falling object accelerates with $\vec{g}$, so $\ddot{\vec{R}}(t) = \vec{g}$ too.

From (3) : $\vec{a}'(t) = \vec{0}$

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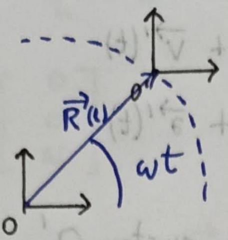
3. Centrifugal force



If O' moves in a circle around O, with constant speed, we have:

$$\vec{R}(t) = \begin{bmatrix} R_0 \cos(\omega t) \\ R_0 \sin(\omega t) \\ 0 \end{bmatrix} \Rightarrow \ddot{\vec{R}}(t) = \begin{bmatrix} -R_0 \omega^2 \cos(\omega t) \\ -R_0 \omega^2 \sin(\omega t) \\ 0 \end{bmatrix} = -\omega^2 \vec{R}(t)$$

Using (3) again:

$$\vec{a}'(t) = \vec{a}(t) + \underbrace{\omega^2 \vec{R}(t)}_{\text{Centrifugal term}} \qquad (4)$$

we see that O' "experiences" an extra acceleration.

Using the relation between linear and angular speed in a circular movement:

$$v = \omega R_0 \qquad (5)$$

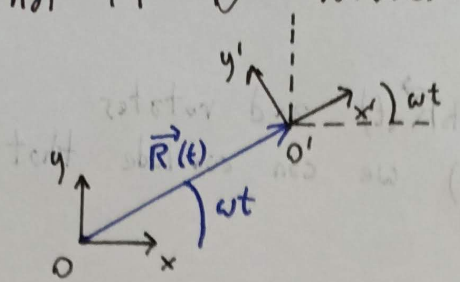
equation (4) takes the more familiar form:

$$\vec{a}'(t) = \vec{a}(t) + \frac{v^2}{R_0} \frac{\vec{R}(t)}{R_0} \equiv \vec{a}(t) + \frac{v^2}{R_0} \hat{r}(t) \qquad (6)$$

If only gravity and rotation apply:

$$\vec{a}'(t) = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \frac{v^2}{R_0} \begin{bmatrix} \cos(\omega t) \\ \sin(\omega t) \end{bmatrix} \qquad (7)$$

What if O' rotates?



All the previous equations still hold ~~as long as w~~ with the exception of (7), that requires a clarification about the coordinates used:

$$\vec{a}'(t) = \begin{bmatrix} 0 \\ -g \end{bmatrix}_O + \frac{v^2}{R_0} \underbrace{\begin{bmatrix} \cos(\omega t) \\ \sin(\omega t) \end{bmatrix}_O}_{\text{Coordinates in frame of reference O}} \quad (7.1)$$

The coordinate systems of both frames of reference are related by:

$$\begin{bmatrix} x' \\ y' \end{bmatrix}_{O'} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}_O \equiv \Omega(t) \begin{bmatrix} x \\ y \end{bmatrix}_O \quad (8) \; \cancel{(7.2)}$$

So we can express (7.1) in O' coordinates as:

$$\vec{a}'(t) = \Omega(t) \left[\begin{bmatrix} 0 \\ -g \end{bmatrix}_O + \frac{v^2}{R_0} \begin{bmatrix} \cos(\omega t) \\ \sin(\omega t) \end{bmatrix}_O \right] = \quad (7.2)$$

$$= g \begin{bmatrix} -\sin(\omega t) \\ -\cos(\omega t) \end{bmatrix}_{O'} + \frac{v^2}{R_0} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{O'}$$

Summarizing:

If our accelerometer moves according to $\vec{R}(t)$ and rotates according to $\Omega(t)$, from (3) and (8) we can conclude that it will read:

$$\underbrace{\begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix}_{O'}}_{\substack{\text{Accelerations}\\ \text{as registered by}\\ \text{the accelerometer}}} = \overset{\substack{\text{Rotation}\\ \text{matrix}}}{\Omega(t)} \left(\underbrace{\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}_{O}}_{\substack{\text{Acceleration}\\ \text{present in } O\\ \text{(typically}\\ \text{gravity)}}} - \underbrace{\begin{bmatrix} \ddot{R}_x \\ \ddot{R}_y \\ \ddot{R}_z \end{bmatrix}_{O}}_{\substack{\text{Relative}\\ \text{acceleration}}} \right) \tag{9}$$

Trick: wait a second. Aren't rotations a pain in the **s? Is there any way around it? Luckily yes!

We can exploit the fact that rotation matrices are orthogonal:

$$\Omega^t(t) = \Omega^{-1}(t) \Rightarrow \Omega^t(t)\,\Omega(t) = \mathbb{1} \tag{10}$$

and apply the norm to (9):

$$\|\vec{a}'\|^2 = \|\vec{a}_o - \ddot{\vec{R}}_o\|^2 = \|\vec{a}\|^2 + \|\ddot{\vec{R}}\|^2 - 2\,\vec{a}\cdot\vec{R} \tag{11}$$

The price we pay is that we destroy the information about orientation, but we keep the magnitudes.