CS618: Indexing and Searching Techniques in Databases
Memory-Based Index Structures

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  1. *Efficient access*: must be faster than linear scan of the entire database
  2. *Small update overhead*: if used for dynamic databases
  3. *Small size*: preferably, should fit in memory
  4. *Correctness*: answers without indexing should be the same
- Five important questions when deciding on an index structure:
  1. Type of data
     - Points or shapes or intervals
     - Discrete or continuous
     - Dimensionality and/or distance
  2. Queries on data
  3. Index data or index space where data resides
  4. Static or dynamic
  5. Memory (small) or disk-resident (large)

Classification of index structures

- Storage medium of index structure
  - Memory-based
    - Entire data fits in memory
    - No I/O cost
    - CPU efficiency
    - Generally, for low dimensions
  - Disk-based
    - Tries to reduce random (and sequential) I/O cost
    - Ignores CPU costs
    - Structure can depend on disk page size

Classification of index structures (contd.)

- Partitioning scheme for hierarchical structures
  - Space-partitioning
    - Divides *entire* space among children
    - Problem of *dead space* indexing
    - Fanout generally independent of dimensionality
    - No guarantee on space usage
  - Data-partitioning
    - Amount of data handled by each child is typically balanced
    - Eliminates *dead space* indexing
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- Type of data handled
  - Point access methods (PAM)
    - Only points are indexed
  - Spatial access methods (SAM)
    - Spatial objects (such as polygons) can be indexed

Binary search tree (BST)

- Binary tree
- Unbalanced
- If a node has key v, then every key l in left subtree and every key r in right subtree follows the properties:
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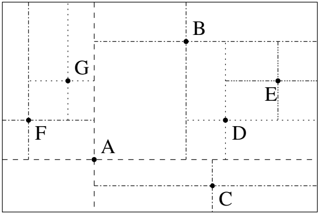
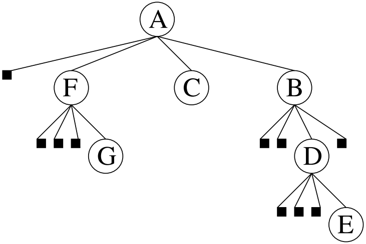
    - AVL trees
    - Red-black trees
- Not perfectly balanced but within $O(\lg n)$ bound
- Search is faster in AVL trees due to lesser height
- Insertion, etc. are slower due to more rotations

Quadtree

- Two-dimensional *unbalanced* binary search tree
- Four children corresponding to 4 quadrants
- If key in node is (x, y), then 4 partitions
  1. $(\leq x, \leq y)$
  2. $(\leq x, > y)$
  3. $(> x, \leq y)$
  4. $(> x, > y)$
- Insertion is simple but order-dependent
- These are point quadtrees





Searching in quadtrees

- Point search is simple
- Range search requires
  1. Minimum bounding rectangle computation
  2. Overlapped rectangles computation
- Average time is $O(\log n)$
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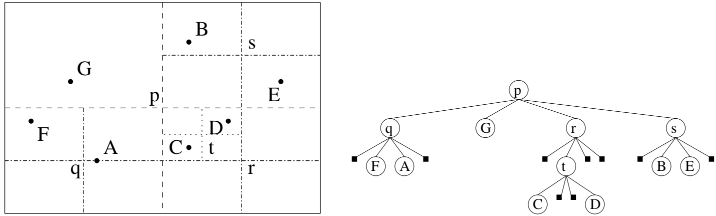
- Can be extended to d dimensions
- Fanout becomes 2^d
- Worst case searching time is $O(d.n^{(1-1/d)})$
- Called octrees in three dimensions

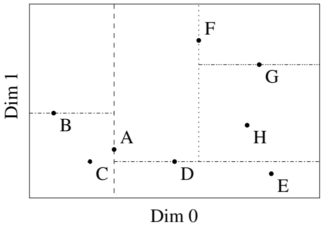
Region quadtrees

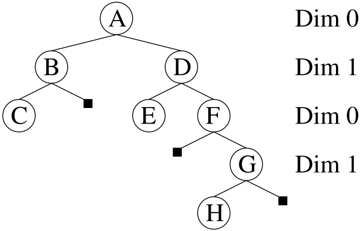
- Always divide at the geographical centre



K-d-tree

- Multi-dimensional unbalanced binary search tree
- Splits only one dimension at a time
- Fanout is constrained to be 2
- Dimensions are ordered and splits are cycled through dimensions
- If node at level i stores object v
  - Left child: $l[i \bmod d] \leq v[i \bmod d]$
  - Right child: $r[i \bmod d] > v[i \bmod d]$
- These are point K-d-trees





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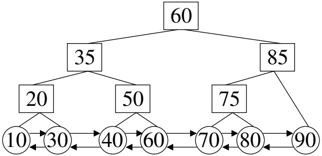
- Region K-d-trees: splits are always done at middle of range
- For uniformly distributed data, region K-d-trees are more balanced than point K-d-trees

Range tree

- Designed to support *window queries* more efficiently
- In 1-dimension, it is similar to a balanced binary search tree
- Leaves are *sorted* and *doubly linked*



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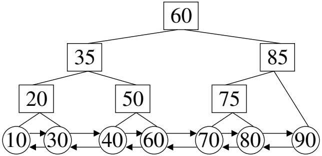

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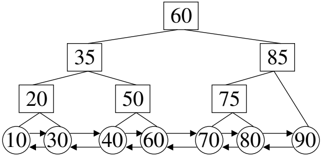

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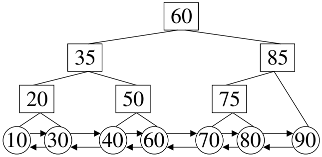

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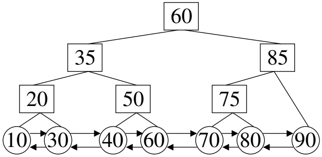

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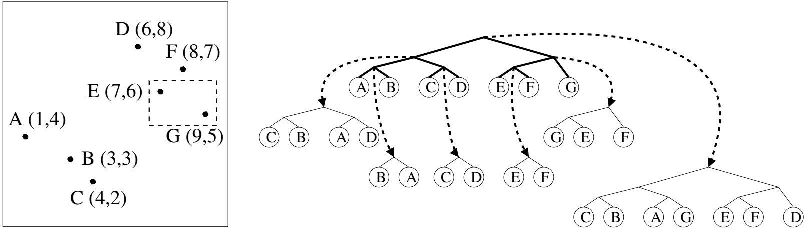
Multi-dimensional range trees

- 1-dimensional range tree of $(d-1)$-dimensional range trees
- Each node is another range tree catering to a set of points in dimensionality one less
- Order dimensions as $x_0, x_1, \ldots, x_{d-1}$
- Main range tree or "base" tree is built on dimension x_0
- For each node T_{x_0} of this base tree, a range tree R_{x_1} of dimensionality $(d-1)$ is associated
- Range tree R_{x_1} contains all points in subtree T_{x_0}, but the points are now organized according to dimension x_1



Multi-dimensional range tree searching

- Search is first through x_0
- Leaves L and R covering range in x_0 are identified
- Least common ancestor Q of L and R is determined
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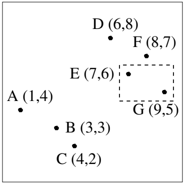

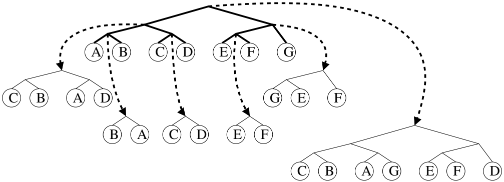

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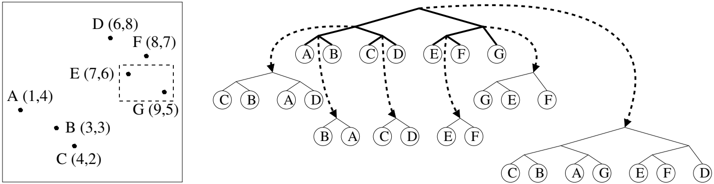

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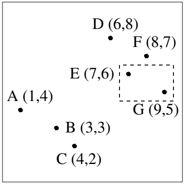

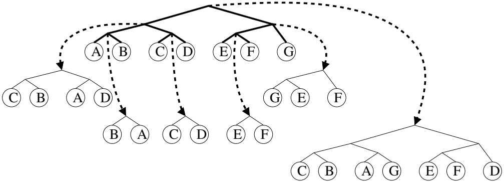

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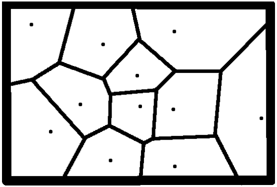
- Running time is $O(\log^d n + T)$

Voronoi diagrams

- Partitioning of a plane with n points (called sites) into *convex polygons* such that
  - Each polygon contains one and only one site
  - Every point in the polygon is closest to the site that corresponds to the polygon than any other site
- Also known as Voronoi tessellation or Thiessen tessellation
- Polygons are Voronoi regions or Voronoi cells or Thiessen polygons
- In metric spaces, structure is called Dirichlet tessellation and regions are called Dirichlet domains



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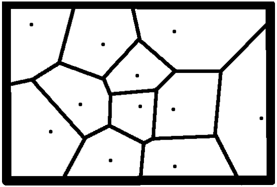
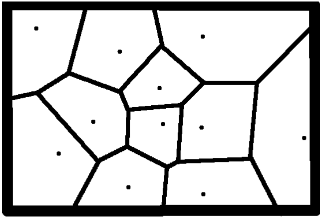
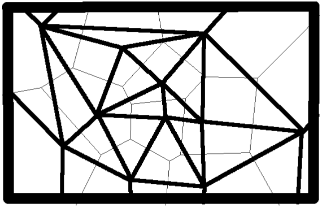


- Immediate answer to 1NN queries

Delaunay triangulation

- Dual of Voronoi diagram is Delaunay triangulation
- Triangulate such that circumcircles of every triangle is empty, i.e., it does not contain any other site
- Resulting structure resembles a graph called Delaunay graph
- Vertices of the resulting graph are sites called Voronoi vertices
- Edges are Voronoi edges





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  - Draw perpendicular bisectors between every pair of sites
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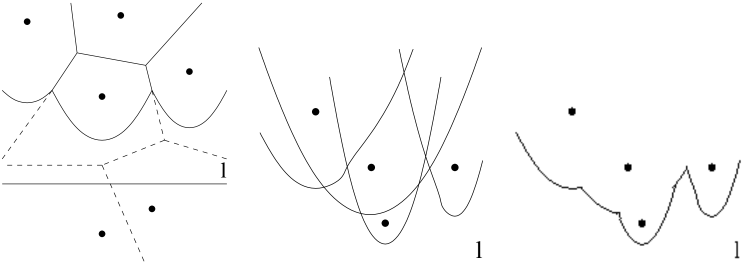
  - Draw perpendicular bisectors between every pair of sites
  - Trim the lines appropriately to form polygons
- Fortune's algorithm
  - *Plane sweep* paradigm
  - Time taken is $O(n \lg n)$
  - Optimal running time
  - Requires $O(n)$ space to store

Fortune's algorithm

- Sweep line swept from top to bottom of space
- All sites above the sweep line compute the beach line
- Beach line indicates locus of points closer to some site above the sweep line than the sweep line itself
- Beach line is actually a set of parabolic arcs
- Voronoi diagram above the beach line never changes
- So, beach line needs to be maintained correctly

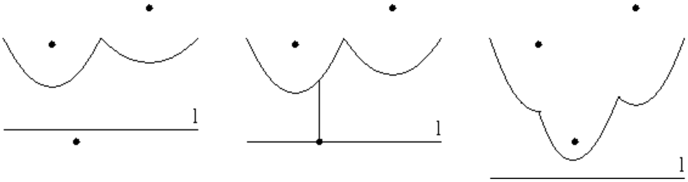


Adding a parabolic arc

- A parabolic arc gets added to the beach line

Adding a parabolic arc

- A parabolic arc gets added to the beach line only when the sweep line hits a site
- This is called a site event

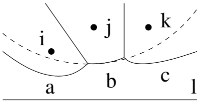


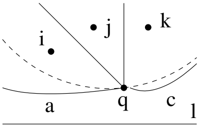
Deleting a parabolic arc

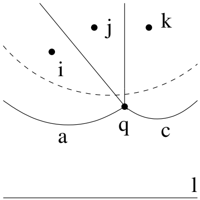
- A parabolic arc gets deleted from the beach line

Deleting a parabolic arc

- A parabolic arc gets deleted from the beach line only when its contribution shrinks to a point and it disappears
- This is called a circle event
- When arc b reduces to a point, the point q is equidistant from all the three sites i, j, and k
- A circumcircle can be defined that passes through all i, j, and k
- Circumcircle also touches the sweep line, i.e., it is tangent to it
- As soon as sweep line moves downward a little, the arc b disappears







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- Voronoi diagram of order-k
  - Partition according to k closest sites
  - Useful for kNN searches
  - Order-$(n-1)$ is called farthest point Voronoi diagram
  - In two dimensions, requires $O(k(n-k))$ space and $O(n \lg n + k^2(n-k))$ time

Tries

- Comes from the word retrieval
- Mostly used for strings
- Structure of a *basic* trie
  - Root represents null string
  - Each edge defines the next character
  - Each node stores a string or a prefix of a string
  - Strings with same prefix share the path
- Advantages over binary search trees
  - Search time is $O(m)$ where m is the length of the query
  - Generally, $m \ll \lg n$
- Also called a prefix tree

Example

- Trie for strings "steam", "string", "strong", "tea", and "team"

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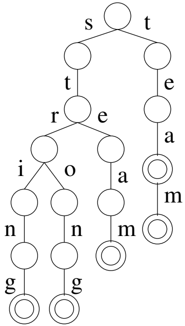

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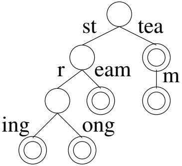

Suffix trees

- For efficient *substring* searching
- It encodes all the suffixes of a string
- Every edge represents the next character(s) from a suffix
- Every leaf represents a suffix of the string
- A compressed trie of all the suffixes
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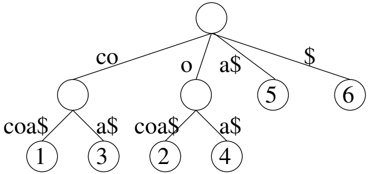

Suffix trees

- For efficient *substring* searching
- It encodes all the suffixes of a string
- Every edge represents the next character(s) from a suffix
- Every leaf represents a suffix of the string
- A compressed trie of all the suffixes
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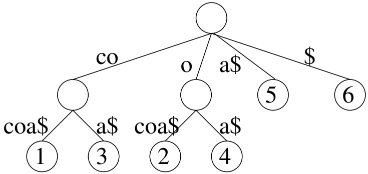

- A special end-of-string symbol ($) is assumed
- This ensures that no suffix is a prefix of another suffix

Generalized suffix trees

- A generalized suffix tree is built from multiple strings

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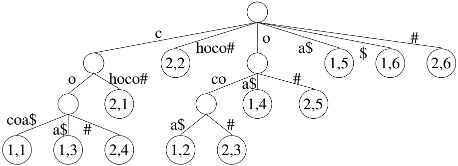

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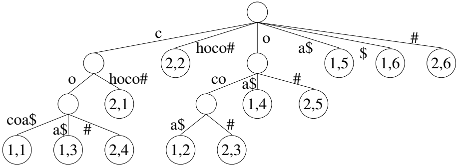

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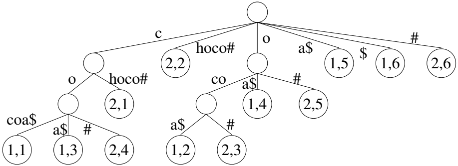

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- Every leaf under this subtree ("cocoa$", "coa$", "co#") returns an answer

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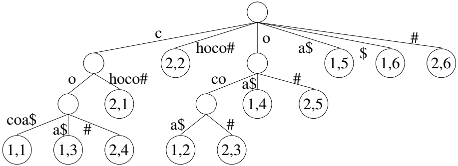

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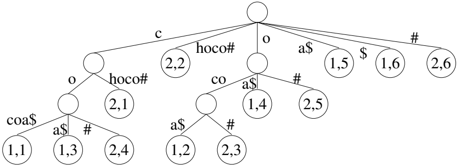

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- Suppose, “co” is searched
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- Every leaf under this subtree (“cocoa$”, “coa$”, “co#”) returns an answer
- Searching for “cha” will end without reaching any node
- Naïve construction requires $O(m^2)$ time for a m-length string
- Can be brought down to $O(m)$
- For multiple strings of length $m_1, m_2, \ldots$, can be done in $O(m_1 + m_2 + \ldots)$ time

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- Considers substrings from the beginning
- *Online* in nature
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  1. Path for $S[j \dots i]$ ends in a leaf: $S[i+1]$ is added to the path
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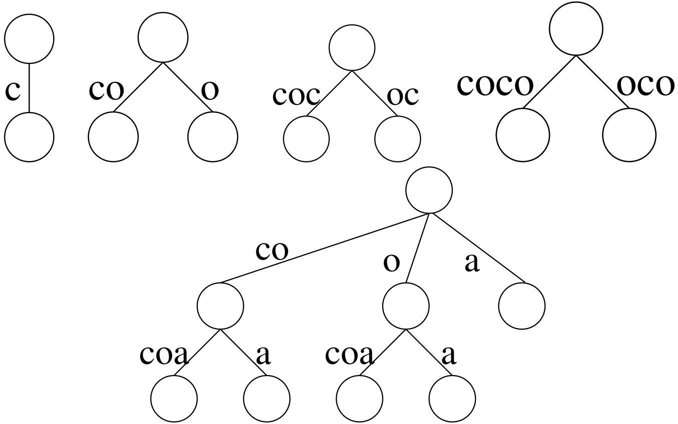
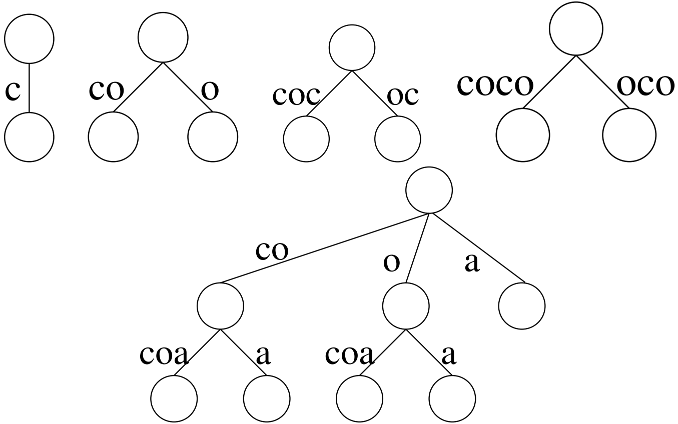
  3. Path for $S[j \ldots i]$ ends in an internal node and there is a path from that node starting with $S[i + 1]$: Nothing is done

Example



- Adding 'o' to "c":

Example



- Adding ‘o’ to “c”: first kind
- Adding ‘o’ to “”:

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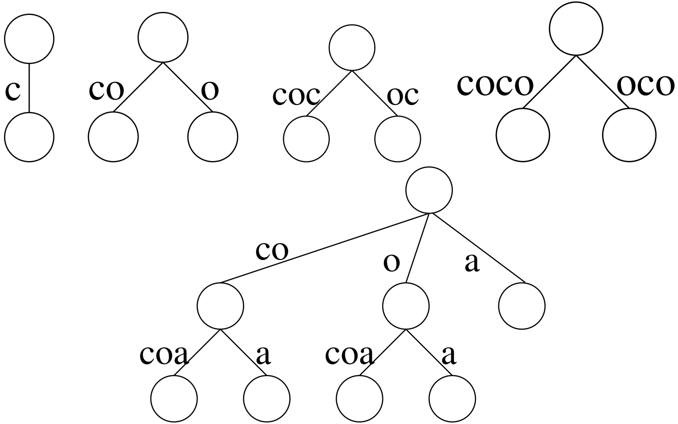

- Adding ‘o’ to “c”: first kind
- Adding ‘o’ to “”: second kind
- Adding second ‘c’:

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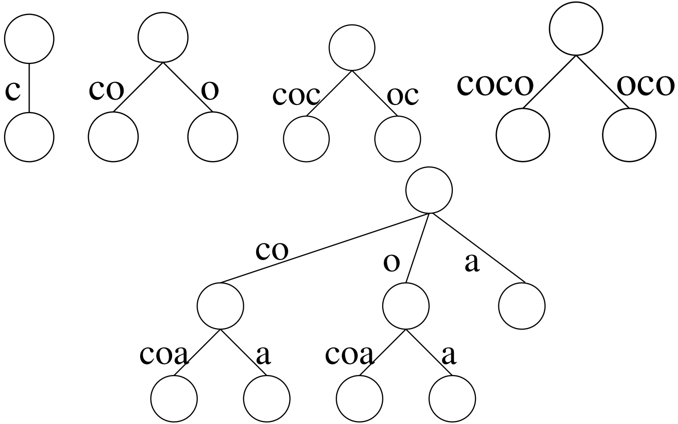

- Adding 'o' to "c": first kind
- Adding 'o' to "": second kind
- Adding second 'c': third kind

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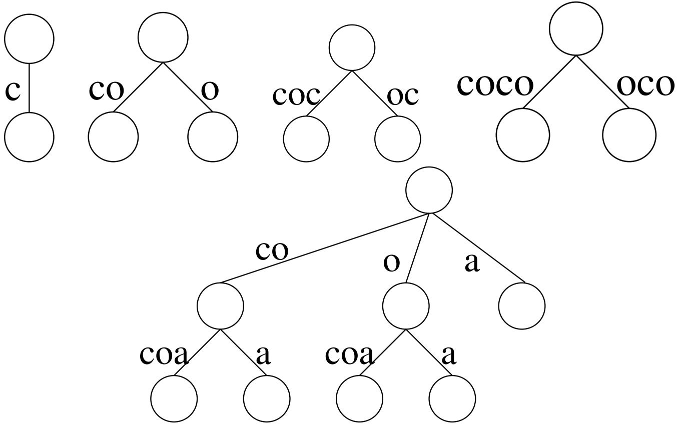

- Adding ‘o’ to “c”: first kind
- Adding ‘o’ to “”: second kind
- Adding second ‘c’: third kind
- Naïve implementation is *not* $O(m)$
- Improved by implementation tricks

Bitmap index

- Attribute domain consists of a small number of distinct values
- Each distinct value has a bitmap or a bit vector
- Length of bit vector is size of database
- Suppose number of objects is n
- Number of distinct values is m $(v_1, v_2, \ldots, v_m)$
- If value of attribute for i^{th} object is v_j,
  - i^{th} bit of j^{th} bit vector is 1
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- If value of attribute for i^{th} object is v_j,
    - i^{th} bit of j^{th} bit vector is 1
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- Can be stored as a regular two-dimensional bit array

Example

Object	Gender	Grade
O_1	male	C
O_2	female	A
O_3	female	C
O_4	male	D
O_5	male	A

Example

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- Two sets of bit vectors, corresponding to each type of attribute
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- O/S allows efficient bitmap operations when packed in word sizes
- Example: Find male students who got C
  - bitmap(male) (bitwise)-AND bitmap(C)
  - 10011 AND 10100 = 10000
  - Object O_1
- Null values require a special bitmap for null in relational databases