CS618: Indexing and Searching Techniques in Databases
Hierarchical Structures

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Mon 1200-1315, Tue 0900-1015 at CS101

B-tree

- Balanced hierarchical data structure
- Keys (and associated objects) are in secondary storage, i.e., disk
- A B-tree of order Θ has the following properties:
  1. Leaf nodes are in same level, i.e., the tree is balanced
  2. Root has at least 1 key
  3. Other internal nodes have between Θ and 2Θ keys
  4. An internal node with k keys have $k + 1$ children
  5. Child pointers in leaf nodes are null
- Branching factor is between $\Theta + 1$ and $2\Theta + 1$
- Pointer to the object corresponding to a key is stored alongside

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- Most important variety of B-tree
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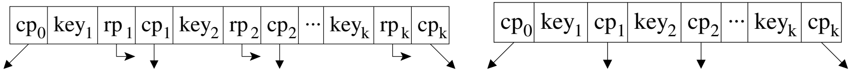

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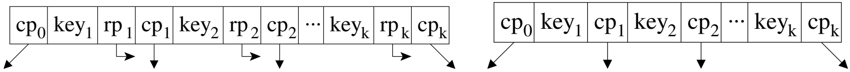

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- More keys can fit in a B+-tree
- Height may be less

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- Region splits (hyperplanes) need not alternate
- Can underflow
- Space utilization can be very low

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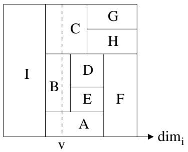

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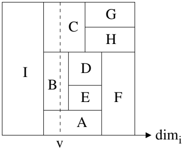

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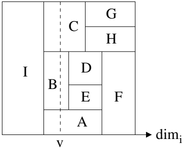

- Choice of split dimension
  - Dimension with largest range
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- Choice of split value
  - Middle of range

Framework for hierarchical object-based structures

- n objects in a d-dimensional vector space
- Object represented by a hyper-dimensional *bounding* geometric *convex* "box"
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- Root contains only one box corresponding to the entire space
- Assume that m boxes fit in a page
- Total number of leaves is $\lceil (n/m) \rceil$
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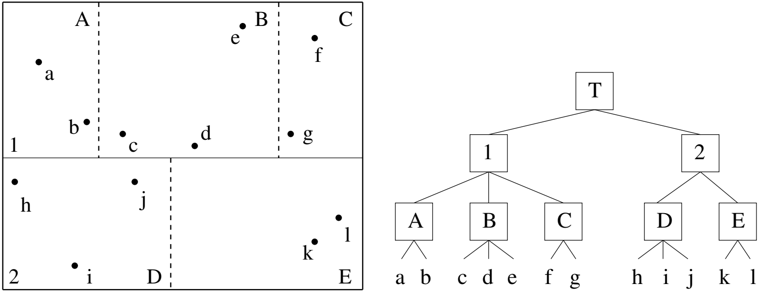
- Height is $h = \lceil \log_m n \rceil$
- Thus, any object can be located in h disk accesses

Object pyramid

- This framework is called an object pyramid
- It provides *multi-resolution representation* of database objects



Point queries

- Query for q
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  - At level i, determine if q is in object D_i
    - If no, return
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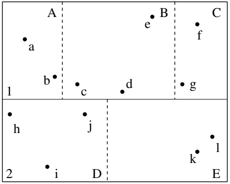
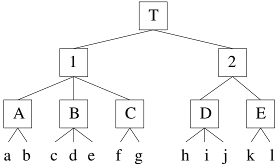
- Example: j
  - Start with T
  - Prune 1, proceed to 2 only
  - Then, only D and finally j
- May follow multiple paths when objects are non-disjoint
- May end up doing more work than sequential scan





Range queries

- Query point q and a distance radius r
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  - Can be found by region intersection
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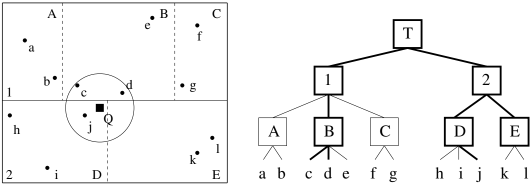
- Example
  - Both 1 and 2
  - Within 1, only B
  - Within 2, both D and E
  - Searching E is wasted but cannot be avoided
- *Window queries* are solved similarly



kNN queries

- Query point q and a positive integer k
  - Maintain max-heap of k current estimates
  - Range is dynamically maintained as k^{th} distance d_k
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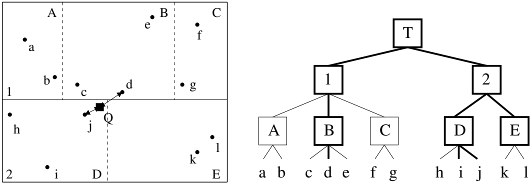
- Example
  - Assume, query first descends into 2 and then D and then j, i
  - Current kNN answer is then $\{j, i\}$ and $d_k = d(Q, i)$
  - E is traversed since minimum distance to E is less than d_k
  - No change in answer
  - Next, 1 and then B
  - Current kNN set is updated to $\{j, d\}$ with $d_k = d(Q, d)$
  - A and C are pruned since their minimum distances are greater than d_k



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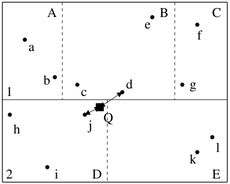
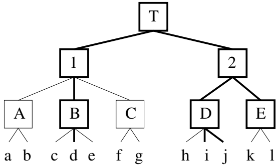
- Best-first search
  - Maintains a priority queue (min-heap) of candidates
  - Examine the next *best* candidate from the min-heap
  - May traverse the nodes in any order

Example





Extract	Insert	Priority queue	Answer	$d_{k=2}$

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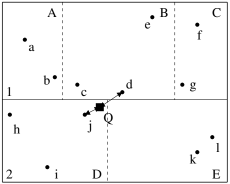

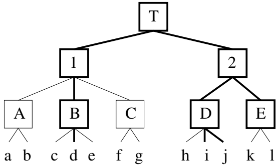

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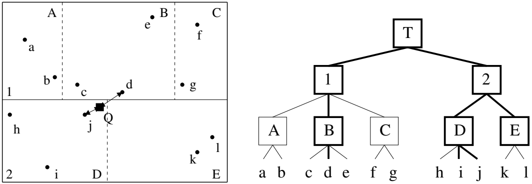

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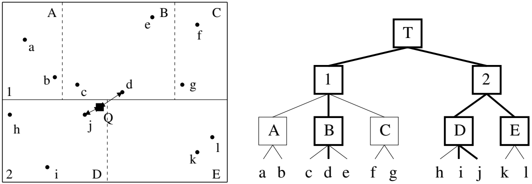

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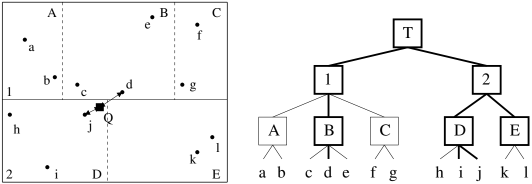

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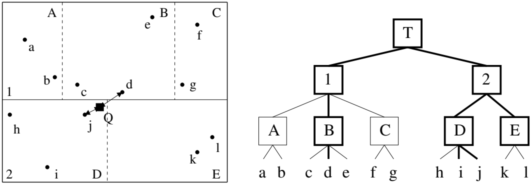

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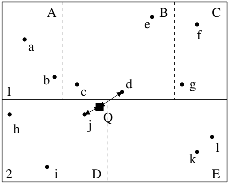

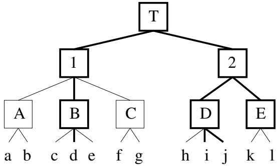

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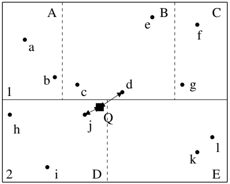

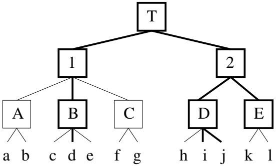

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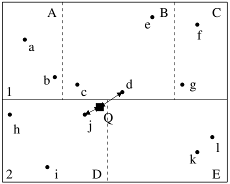

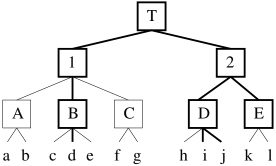

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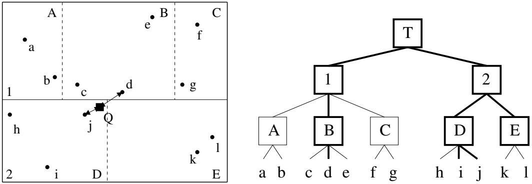

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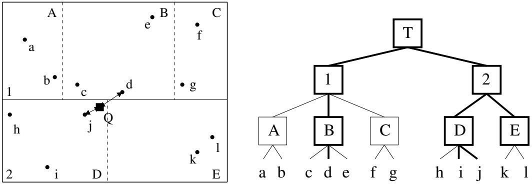

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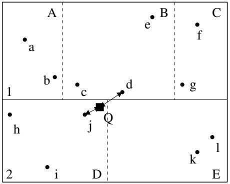
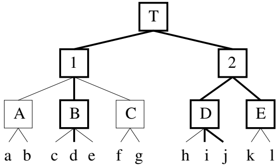
j	-	$(d,8)$, $(c,11)$, $(A,12)$, $(C,22)$, $(h,27)$, $(i,30)$, $(e,32)$, $(k,36)$, $(l,38)$	$\{j\}$	∞

Example (contd.)





Extract	Insert	Priority queue	Answer	$d_{k=2}$
B	c, d, e	$(j, 6)$, $(d, 8)$, $(c, 11)$, $(A, 12)$, $(C, 22)$, $(h, 27)$, $(i, 30)$, $(e, 32)$, $(k, 36)$, $(l, 38)$	Φ	∞
j	-	$(d, 8)$, $(c, 11)$, $(A, 12)$, $(C, 22)$, $(h, 27)$, $(i, 30)$, $(e, 32)$, $(k, 36)$, $(l, 38)$	$\{j\}$	∞
d	-	$(c, 11)$, $(A, 12)$, $(C, 22)$, $(h, 27)$, $(i, 30)$, $(e, 32)$, $(k, 36)$, $(l, 38)$	$\{j, d\}$	8

Incremental nearest neighbors

- Obtain 10 nearest neighbors
- Not satisfied with the results
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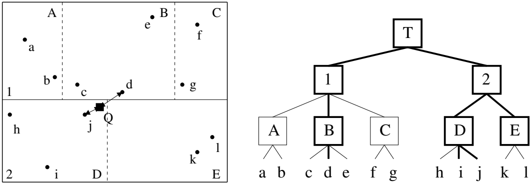
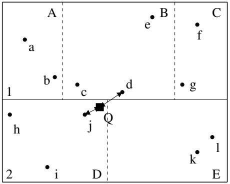
- Incremental nearest neighbors are best solved by *best-first search*

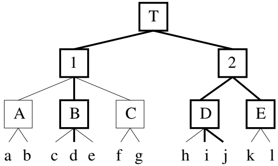
Example



Extract	Insert	Priority queue	Answer	$d_{k=4}$

Example





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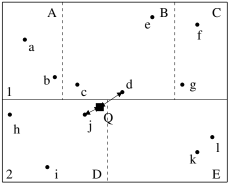

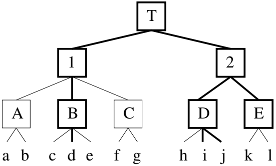

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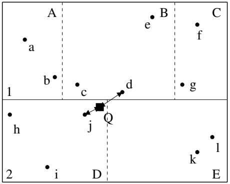

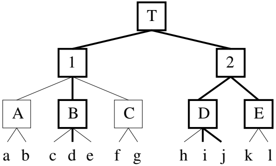

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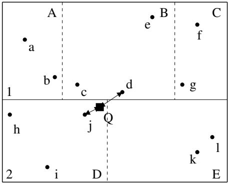

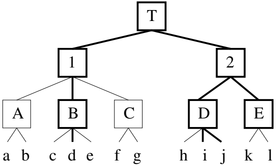

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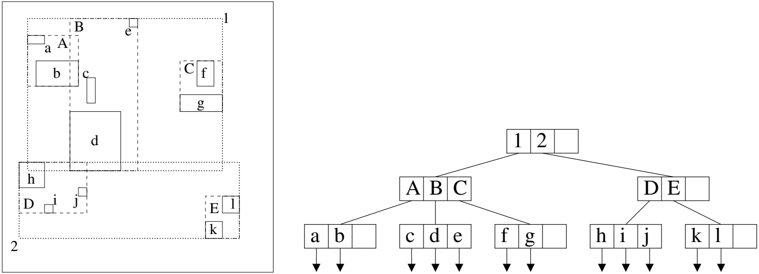
R-tree

- Multi-dimensional B+-tree
- Can store hyper-dimensional points, lines, shapes, etc.
- Objects bounded by a hyper-dimensional minimum bounding rectangle (MBR)
- In d dimensions, hyper-rectangle is specified by $2d$ parameters
- 2 in each dimension to indicate the minimum and maximum values
- Data are contained in leaves only which are all at the same level
- Does *not* index *dead space*
- Children *may* overlap



Structure of an R-tree node

- Design is based on disk page size
- *Underflow* and *overflow* parameters α and β
- α guarantees space utilization (is 2 for root)
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- Disk page capacity is C bytes
- Key consumes γ bytes
- Pointer to key (or object) requires η bytes

$$C \geq 2.d.\beta.\gamma + \beta.\eta$$
$$\Rightarrow \beta_{\text{R-tree}} = \left\lfloor \frac{C}{2.d.\gamma + \eta} \right\rfloor$$

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- Choose child requiring *least volume enlargement*
  - If tie, then child with *least volume*, and then *lesser number of children*
- Ensures least dead space indexing
- If no overflow at leaf, insert
- Otherwise, split
- If overflows continue to root, height gets incremented

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  3. Linear

Quadratic split

- Choose a pair of entries (called *seeds*) that is the *most wasteful*
- Waste for two entries a and b is $w = vol(U) - vol(a) - vol(b)$ where U is the *covering* hyper-rectangle of a and b
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- When one group has $\beta - \alpha + 1$ entries, put the rest into the other group to avoid underflow

Linear split

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Linear split

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- Rest of the procedure is same
- For each dimension, choose entries with *highest minimum* and *lowest maximum* values along that dimension
- Normalize the difference by dividing by the total range along the dimension
- Choose the entries with the *largest difference* as the seeds
- Requires *linear* time

Discussion

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- R-trees are quite suitable for relational databases

R*-tree

- Three engineering optimizations over R-trees
  1. Insertion
  2. Splitting
  3. Forced re-insertion during overflow

Insertion

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- If child pointers are internal nodes
  - Choose subtree with least volume increase (same as R-tree)
- If child pointers are leaf nodes (i.e., level is one up from leaves)
  - Choose subtree with *least overlap enlargement*
- For child entries $E_1, \ldots, E_k$, overlap of E_i is $\sum_{j=1, j \neq i}^{k} volume(E_i \cap E_j)$
- Overlap is sum of intersected volumes
- Quadratic amount of computations

Splitting

- Considers 3 parameters:
  1. Sum of volumes
  2. Sum of margins, i.e., lengths of sides of hyper-rectangles
  3. Overlap, i.e., common space (in other words, the intersection of the volumes)

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  - All possible $(\beta - 2\alpha + 2)$ distributions are considered
  - For each distribution, *goodness value* is computed using the three parameters
- Split axis is best chosen using sum of margins
- Groups are best partitioned using overlap (and then sum of volumes to break ties)

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- Experimentally, close reinsert better with $p = 30\,\%$
- Done only once at each level to avoid infinite loop
- In other words, if all entries are inserted in the same node again, splitting is applied

R+-tree

- Mix of R-tree's data partitioning with K-d-B-tree's space-partitioning
- Reduces dead space problem of K-d-B trees by using index rectangles
- Reduces node overlap by making siblings *disjoint*
- Data object may be split into disjoint hyper-rectangles
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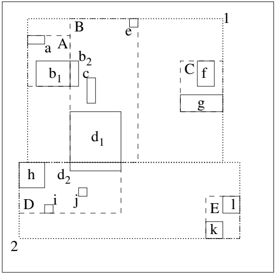
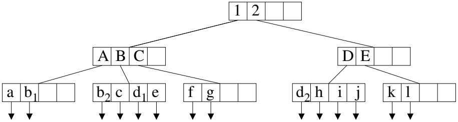
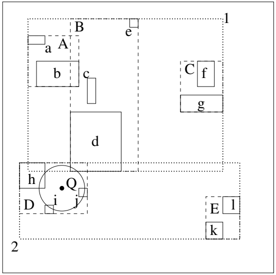
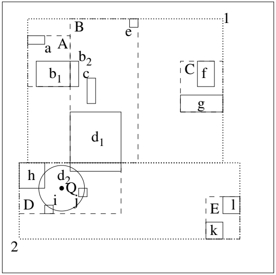
- Data object b broken into b_1 and b_2
- As a result, nodes A and B do not overlap





Searching





R-tree R+-tree

- Consider query Q with range
- In R-tree, both nodes 1 and 2 are searched
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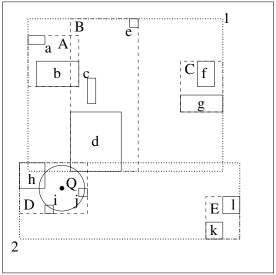

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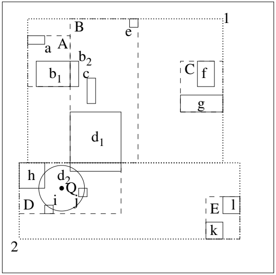

R+-tree

- Consider query Q with range
- In R-tree, both nodes 1 and 2 are searched
- In R+-tree, only node 2 is searched
- Trade-off between overlapped searches and increased height
- Not much better than R*-trees
- Maintenance is harder in R+-trees

Hilbert R-tree

- A mix of B+-tree and R-tree
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Hilbert R-tree

- A mix of B+-tree and R-tree
- An ordering on data hyper-rectangles that guides how splits are handled
- A Hilbert space-filling curve grid is imposed on the space
- For a data object, the *Hilbert key* is that of its center
- For an internal node, the *Hilbert key* is the *largest* Hilbert key among its children

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- Only when all such s co-operating sibling nodes overflow, the entries in them are re-assigned to $s+1$ new nodes
- Increasing s increases space utilization but simultaneously increases the insertion cost
- Overall, $s=2$, i.e., a 2-to-3 split, is best empirically

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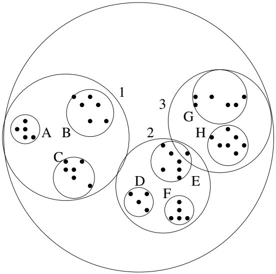
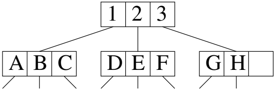
- This ordering is similar to that in a B+-tree
- Performs better than R*-tree when data is skewed
- Performance deteriorates rapidly for higher dimensions as the property of nearby objects in space enjoying proximity in the Hilbert key ordering gets less respected

SS-tree

- Uses minimum bounding spheres (MBS) instead of MBRs
- Motivated by range and kNN queries which are hyper-spheres
- Center of hyper-sphere is centroid of hyper-spheres representing children nodes
- Radius is the tightest one that covers all children
- Maintains total number of points in the subtree
- Higher fanout due to smaller storage requirements: $d + 1$ parameters instead of $2d$ per child





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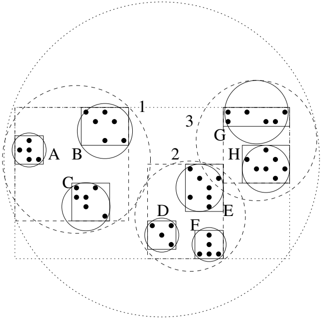
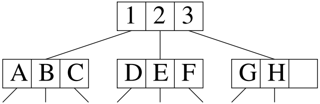
- Ideally, an index node should combine short diameter regions of SS-tree with small volume regions of R*-tree

SR-tree

- Uses *both* bounding spheres and bounding rectangles
- Index node is *intersection* of these two
  - Tighter
  - Index is implicitly stored
  - Explicitly, both the geometries are stored
- Also maintains number of data objects per child entry





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- Together, faster search than SS-trees and R*-trees
- Mostly, hyper-rectangles are more useful especially at higher dimensions

Fanout example

- What is the maximum fanout of an internal node for an R*-tree, an SS-tree and an SR-tree indexing 5-dimensional values of 8 bytes each with child pointers of size 4 bytes for a page size of 4 KB? Assume that integers require 4 bytes of storage

Tree	R*-tree	SS-tree	SR-tree
Child pointer	4	4	4
Bounding rectangle	$2 \times 5 \times 8 = 80$	-	$2 \times 5 \times 8 = 80$
Bounding sphere	-	$5 \times 8 + 8 = 48$	$5 \times 8 + 8 = 48$
Number of objects	-	4	4
Total size	84	56	136
Fanout	$\lfloor 4096/84 \rfloor = 48$	$\lfloor 4096/56 \rfloor = 73$	$\lfloor 4096/136 \rfloor = 30$

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- Map convex polyhedron from d-dimensional *attribute space* to hyper-rectangle at m-dimensional *orientation space*
- Polyhedra intersect if and only if corresponding hyper-rectangles intersect

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- Bottom-up methods decide on the layout of leaves first
- n objects
- Disk page has capacity for c objects
- Least number of data pages is $p = \lceil n/c \rceil$
- Pack into p leaves
- For next level, pack these p leaves into $\lceil p/c \rceil$ pages, and so on

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- *Clustering* algorithms are good examples as well

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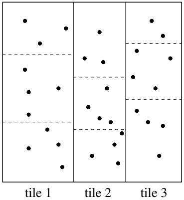

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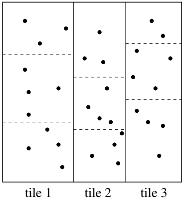

- In d dimensions, $s' = \lceil p^{\frac{1}{d}} \rceil$ partitions are created
- Each slice contains $n/s' = c.p^{\frac{d-1}{d}}$ objects in $(d-1)$ dimensions