

3 - Constant Product AMM Math | DeFi

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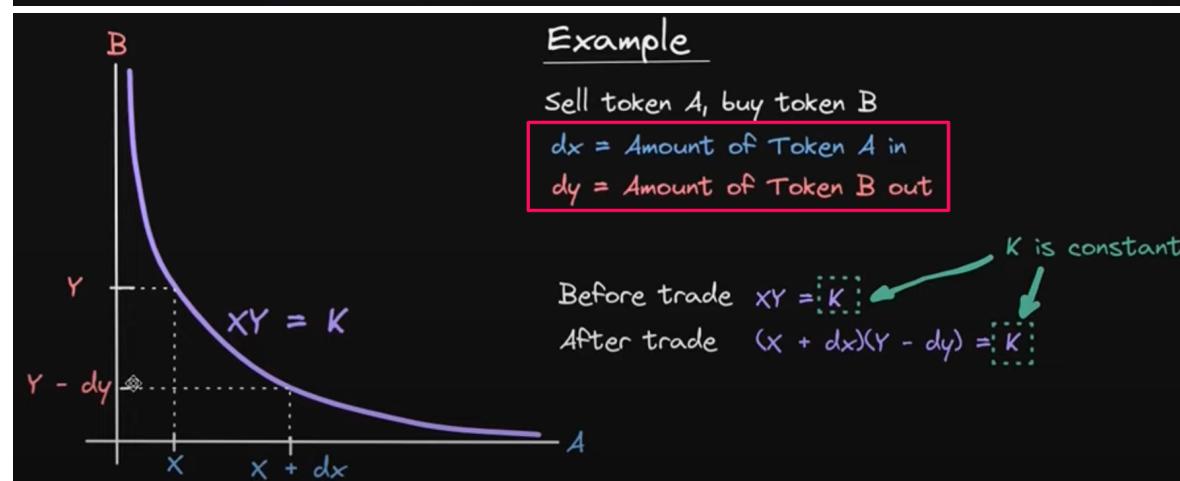
What is a Constant Product AMM (CPAMM)

- A CPAMM is an Auto Market Maker that determines the price of the tokens based on the equation: $X \cdot Y = K$
- When doing trades in this type of AMM, the number of each token can vary, but the constant K must remain the same

Price of tokens are determined by the equation

$$XY = K$$

X = Amount of Token A in the AMM
 Y = Amount of Token B in the AMM



Swapping Tokens

- Before and after all trades, the constant K must remain the same

Before swap

$$XY = K$$

After swap

$$(X + dx)(Y - dy) = K$$

- The total number of tokens to return in a swap is determined by the next equation (Which is the result of applying some math to the swap's function)

Answer **tokens to return**

$$dy = \frac{Ydx}{X + dx}$$

Swap's formula



To move canvas, hold mouse wheel or spacebar while dragging

Swap - How many token to return in a trade?

Swap from token A to token B

dx = Amount of Token A in

dy = Amount of Token B out

Before swap

$$XY = K$$

After swap

$$(X + dx)(Y - dy) = K$$

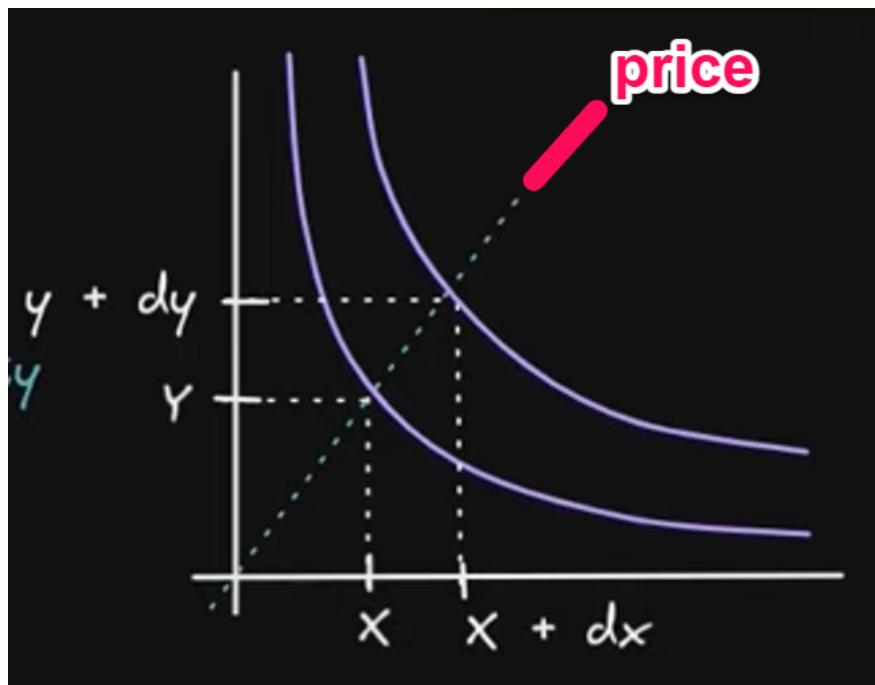
Answer

$$dy = \frac{Ydx}{X + dx}$$

tokens to return

Adding Liquidity

- When adding liquidity, the price must not change
- The price is calculated by dividing the total number of tokens X by the total number of tokens Y
- When adding liquidity, the same price ratio must remain intact
 - That means that depending on the number of tokens X & Y in the AMM, it'll be the number of required tokens X & Y to be added as liquidity (May vary)
 - The ratio of the number of tokens to add must be equal to the ratio of tokens before adding liquidity



$$dy = \frac{Ydx}{X}$$

$$\frac{Xdy}{Y} = dx$$

$$\frac{dx}{dy} = \frac{X}{Y}$$

Formula to determine the number of shares to mint

To move canvas, hold mouse wheel or spacebar while dragging

Add liquidity - How many shares to mint?

Answer

$$s = \frac{dx}{X} T = \frac{dy}{Y} T$$

Minting Shares

- The increase in liquidity is proportional to the increase in shares

- If the liquidity is increased in 10%, the total tokens must increase in 10% too
- shares to mint = change in liquidity = \sqrt{xy}
- The below formula determines the number of shares to mint when adding liquidity

Formula to determine the total shares to mint

How many shares to mint?

Increase in liquidity is proportional to increase in shares

L_0 = Total liquidity before

L_1 = Total liquidity after

T = Total shares before

s = shares to mint

$$\frac{L_1}{L_0} = \frac{T + s}{T} \quad \xrightarrow{\text{Find } s} \quad \frac{L_1}{L_0} T = T + s$$

$$\frac{L_1}{L_0} T - T = s$$

$$\frac{L_1 - L_0}{L_0} T = s$$

Simplifying the original formula to determine the number of tokens to mint using Algebra

$$\frac{L_1 - L_0}{L_0} T = s \quad + \quad \frac{L_1 - L_0}{L_0} = \frac{dx}{x} = \frac{dy}{y} \quad \rightarrow \quad s = \frac{dx}{x} T = \frac{dy}{y} T$$

How to measure liquidity in a Constant Product AMM

- The increase in liquidity is proportional to the increase in shares
 - The increase in liquidity should be linear - To reduce the difference in total liquidity and the number of tokens inside the AMM

How to measure total liquidity from X and Y?

$$f(X, Y) = \text{total liquidity}$$

$$f(X, Y) = \sqrt{XY}$$

increasing liquidity - visual representation

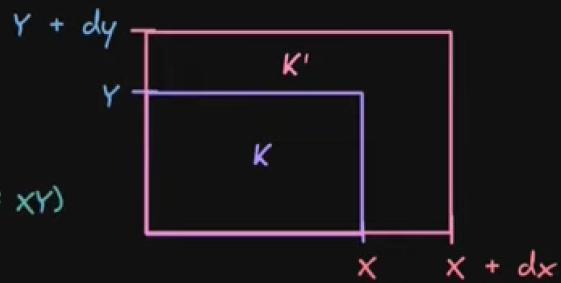


Motivation

$$XY = K$$

$$(X + dx)(Y + dy) = K'$$

Adding liquidity increases K (area of XY)



Removing Liquidity

- The number of tokens to withdraw must be proportional to the number of shares that the provider is burning

Formula to determine the number of tokens to return when withdrawing liquidity

Answer

$$dx = X \frac{s}{T}$$

↳

$$dy = Y \frac{s}{T}$$

Math behind the formula to determine the number of tokens to return when withdrawing liquidity

Remove liquidity - How many tokens to withdraw?

Withdraw tokens proportional to shares

$$a = \text{Amount out} = f(dx, dy) = \sqrt{dxdy}$$

$$L = \text{Total liquidity} = f(X, Y) = \sqrt{XY}$$

s = Amount of shares to burn

T = Total shares

$$\frac{a}{L} = \frac{s}{T}$$

Find a

$$a = L \frac{s}{T}$$

$$\sqrt{dxdy} = \sqrt{XY} \frac{s}{T}$$

Resources

🔗 <https://www.youtube.com/watch?v=QNPY...>