Project-I

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1.1 Exercise1

i) After randomly initializing $x0$ and repeating the experiments. E has the eigenvalues

$$\begin{pmatrix} 1 \\ -0.5000 + 0.2887i \\ -0.5000 - 0.2887i \\ 0 \end{pmatrix}$$

two real and two complex values, the largest eigenvalue is 1. The sequence $E^1 5x0$ normalised using l2 norm, as the number of iterations increases tends to the largest eigenvector of E, the eigenvector corresponding to eigenvalue 1. eigenvector

$$\begin{pmatrix} 0.6447 \\ 0.2478 \\ 0.4154 \\ 0.5920 \end{pmatrix}$$

ii) Eigenvector

$$\begin{pmatrix} 0.2230 \\ 0.3755 \\ 0.4201 \\ 0.7955 \end{pmatrix}$$

Eigenvalues

$$\begin{pmatrix} 0 \\ 0.5614 \\ -0.2807 + 0.2640i \\ -0.2807 - 0.2640i \end{pmatrix}$$

The largest eigenvalue is 0.5614 and the sequence converges again to the largest eigenvector, up to a constant, -1 in my case.

iii) Eigenvalues of E

$$\begin{pmatrix} 1.0000 \\ -0.5000 + 0.2887i \\ -0.5000 - 0.2887i \\ -0.0000 \\ 1.0000 \\ -1.0000 \end{pmatrix}$$

There are two eigenvalues of 1. The sequence after 15 steps does not converge to the largest eigenvector (any of them)

$$\begin{pmatrix} 0.3500 \\ 0.1050 \\ 0.2099 \\ 0.3149 \\ 0.6848 \\ 0.5043 \end{pmatrix}$$

1.2 Exercise2

i) The igenvectors are -0.6446 -0.2887 + 0.5000i -0.2887 - 0.5000i 0.5345 -0.2478 -0.4330 - 0.2500i -0.4330 + 0.2500i -0.8018 -0.4154 0.5774 0.5774 0.0000 -0.5920 0.1443 - 0.2500i 0.1443 + 0.2500i 0.2673 Eigenvalues are

$$\begin{pmatrix} 1.0000 \\ -0.4250 + 0.2454i \\ -0.4250 - 0.2454i \\ -0.0000 \end{pmatrix}$$

The convergence vector is

$$\begin{pmatrix} 0.6447 \\ 0.2478 \\ 0.4154 \\ 0.5920 \end{pmatrix}$$

ii) GREEN 0.2769 -0.3641 - 0.3761i -0.3641 + 0.3761i 0.0044 0.3984 0.5959 0.5959 -0.1091 0.4548 -0.3506 + 0.2950i -0.3506 - 0.2950i -0.0657 0.7468 0.2363 + 0.3242i 0.2363 - 0.3242i 0.9918 Eigenvalues are

$$\begin{pmatrix} 0.6618 \\ -0.2427 + 0.2257i \\ -0.2427 - 0.2257i \\ -0.0264 \end{pmatrix}$$

The convergence vector is

$$\begin{pmatrix} 0.2769 \\ 0.3984 \\ 0.4548 \\ 0.7468 \end{pmatrix}$$

iii) BLUE Eigenvector of M blue

$$\begin{pmatrix} 0.5351 \\ 0.2057 \\ 0.3449 \\ 0.4914 \\ 0.3943 \\ 0.3943 \end{pmatrix}$$

The eigenvalues are

$$\begin{pmatrix} 1.0000 \\ 0.8500 \\ -0.4250 + 0.2454i \\ -0.4250 - 0.2454i \\ 0.0000 \\ -0.8500 \end{pmatrix}$$

The convergence vector is

$$\begin{pmatrix} 0.5200 \\ 0.2012 \\ 0.3358 \\ 0.4778 \\ 0.4257 \\ 0.4085 \end{pmatrix}$$

1.3 Exercise3

i) E is a stochastic column matrix for i) and iii). M is a stochastic column matrix for i) and III) again.

ii) Assume A is stochastic column matrix. A and $A^\top$ have the same eigenvalues, so if we show $A^\top$ has eigenvalue 1, implies A also has eigenvalue of 1. $det(A^\top - I) = det((a11 - 1a12..a1n)) = det(0a12a13..a1n) = 0$ When the first equality we added all the columns to the first column and the fact that the determinant of a matrix with a column of 0 equals 0. This implies AT and implicitly A have eigenvalue 1.

Assume that A, $A^\top$ have an eigenvalue greater than 1, $\lambda 0$ with eigenvector v. $A^topv = v * \lambda 0$. Then we have $A^\top * v = v$ since the sum of the elements in every row is equal to 1. Then we have $\sum aij * vi = \lambda vj$ for every j in 0..N. Let k be the index corresponding to the lasrgest entry in v. then $\sum aik * vi < \sum aijvk = vk < \lambda vk$ which is a contradiction, so 1 is the largest eigenvalue.

Acknowledgments

References