

# Temperature Multi-stability

From non-linear radiation terms

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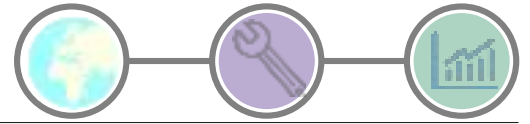


**CriticalEarth**

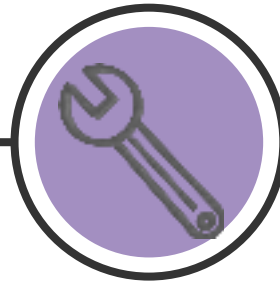


# Key Points

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The Model



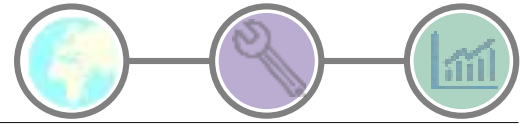
Modifications



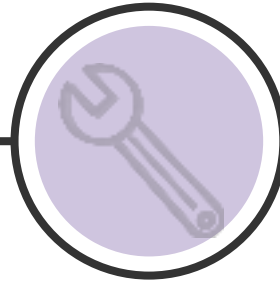
Results

# Key Points

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The Model

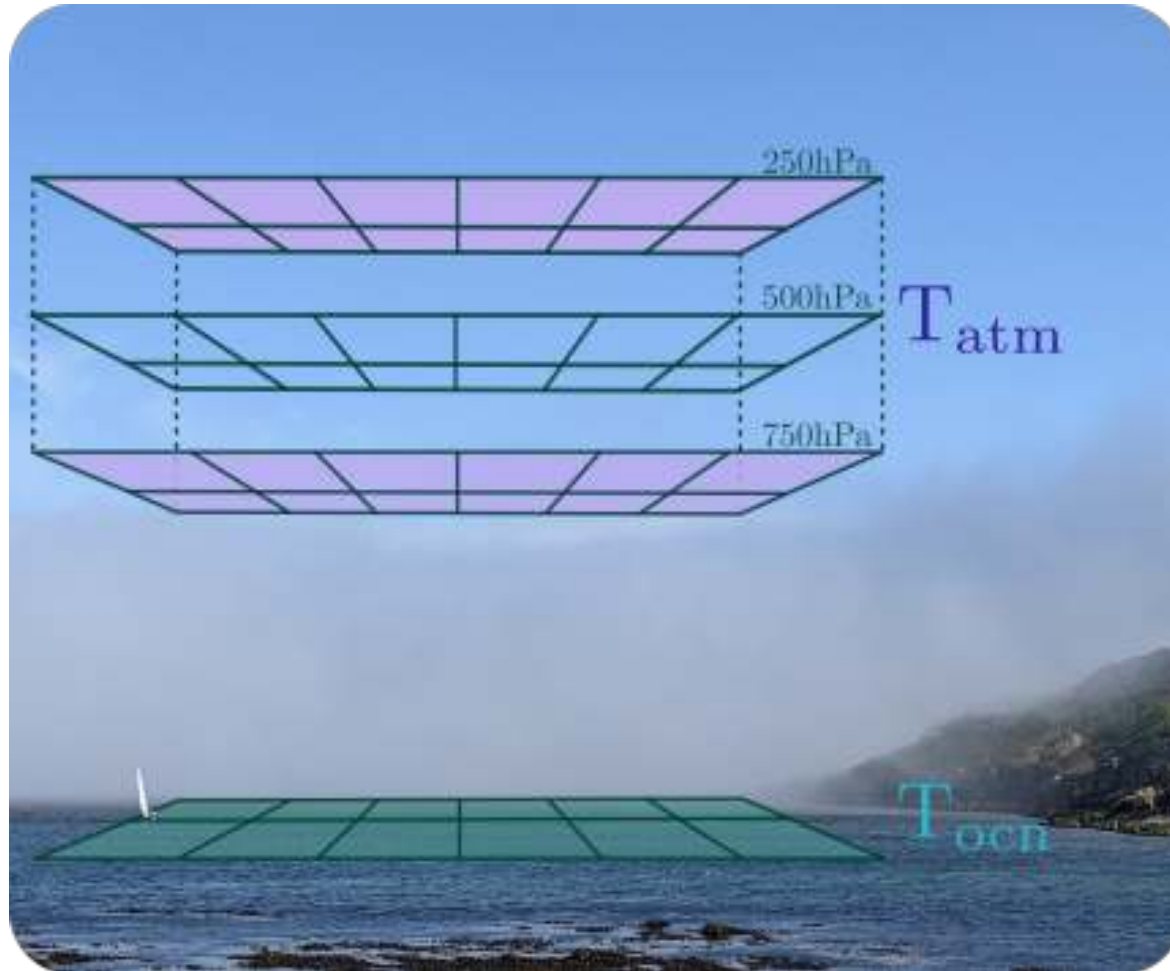


Modifications



Results

# QGS Model

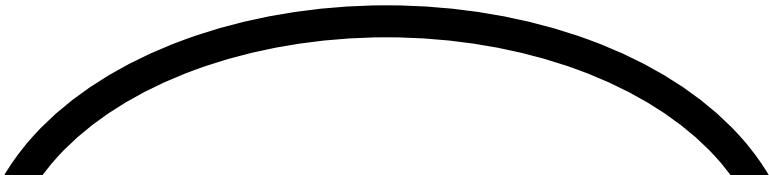


# QGS Model

## Stefan Boltzmann Law



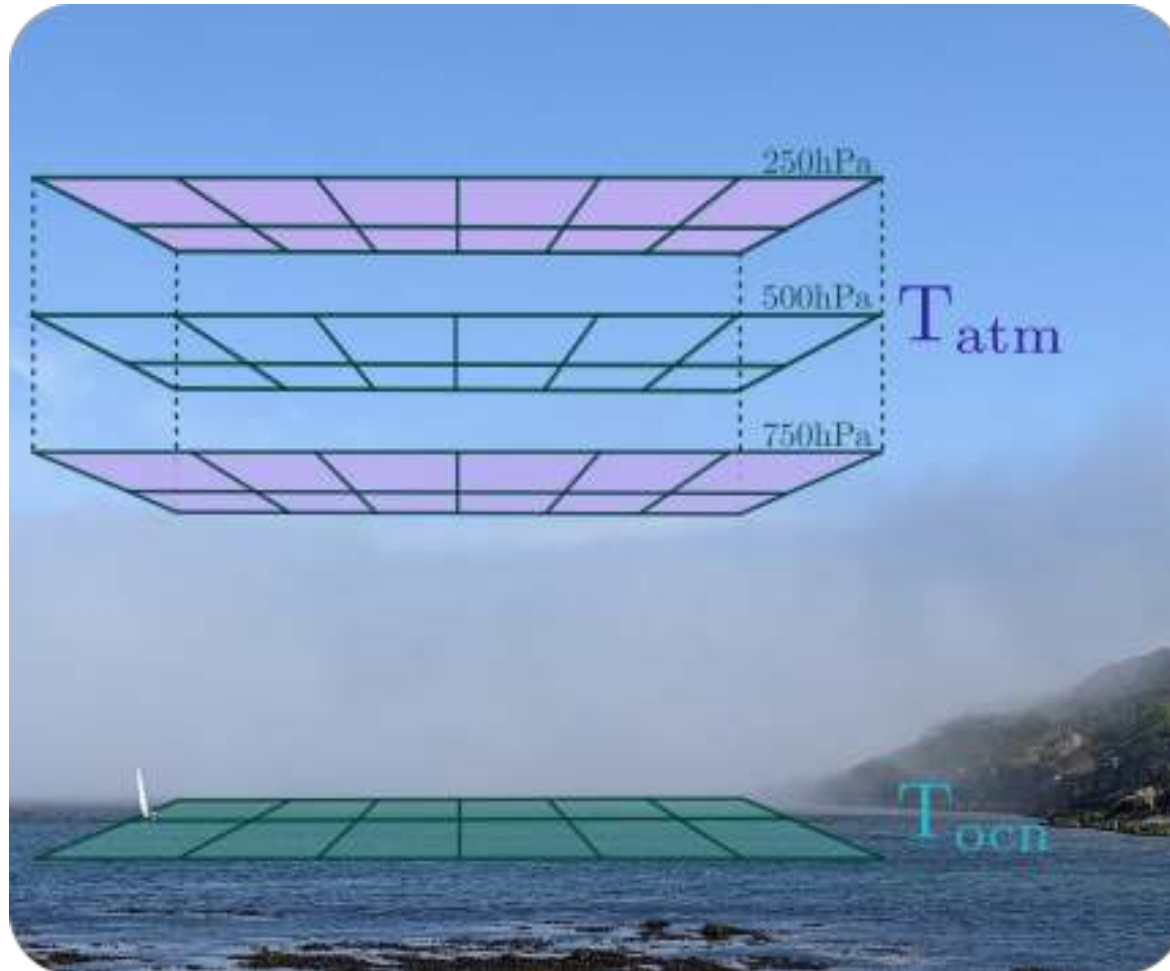
$$\sigma T^4$$



$$\epsilon \sigma T^4$$

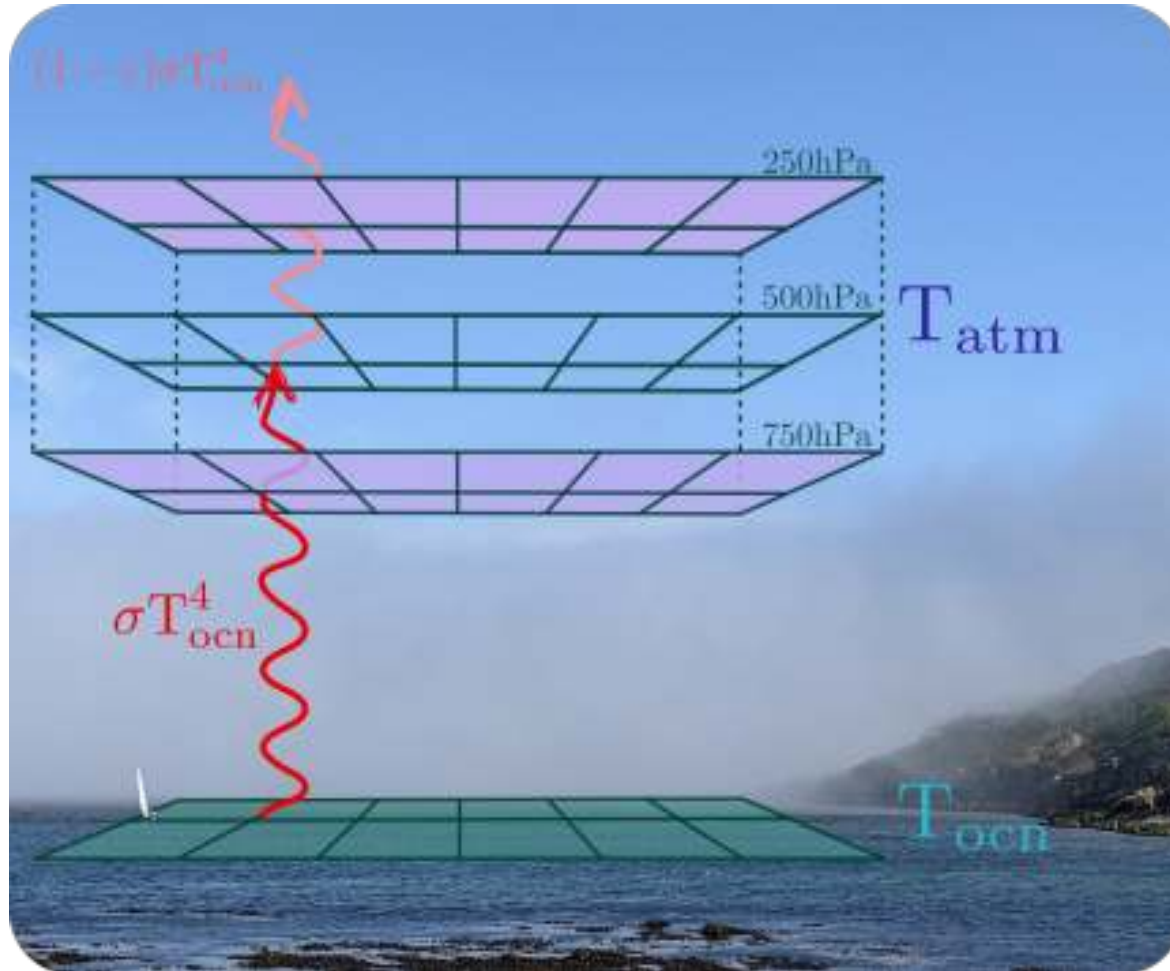
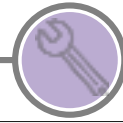


# QGS Model



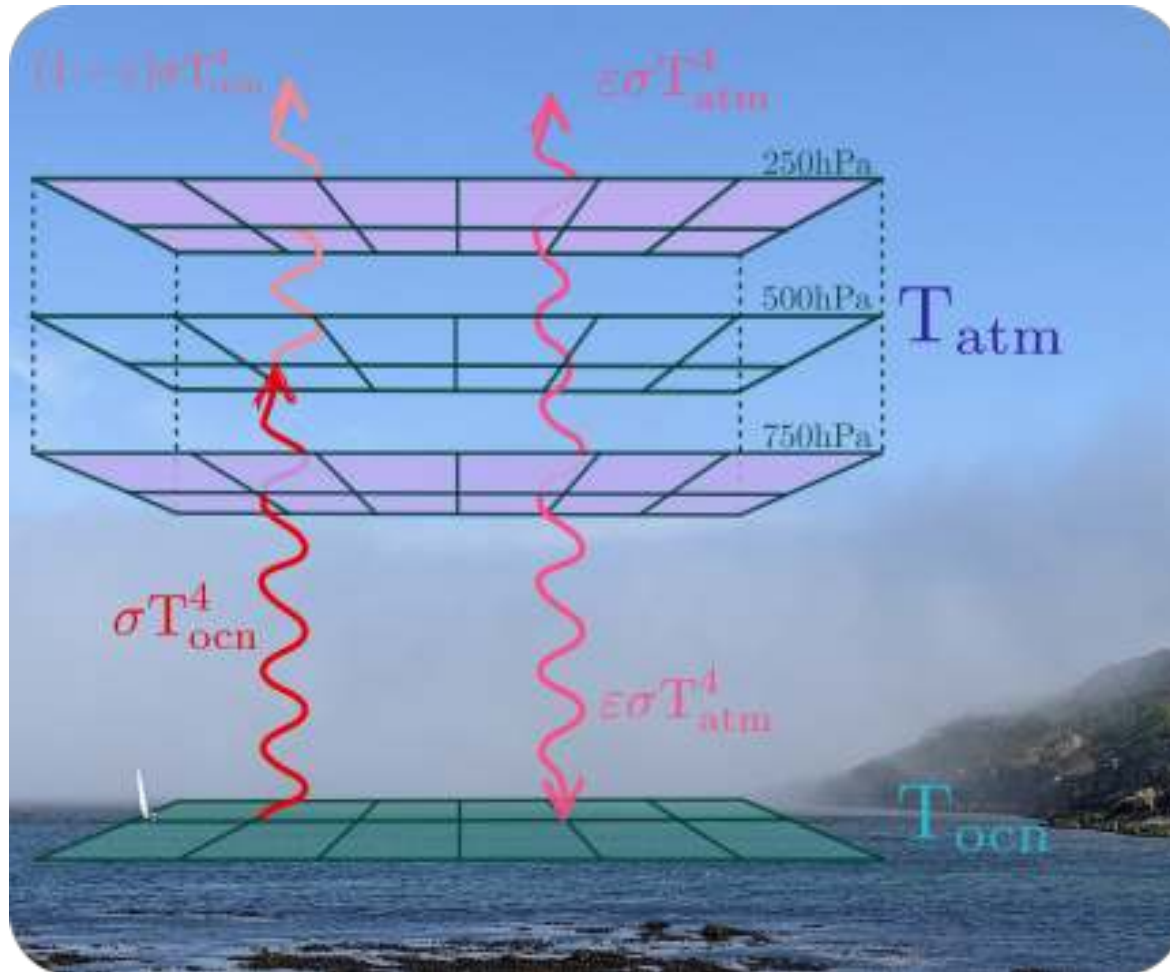
# QGS Model

## Radiation



# QGS Model

## Radiation





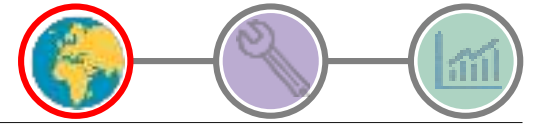
# QGS Model



## Temperature Equation

$$\gamma_o \left( \frac{\partial T_o}{\partial t} + J(\psi_o, T_o) \right) = -\lambda(T_o - T_a) - \sigma_B T_o^4 + \varepsilon \sigma_B T_a^4 + R_o$$

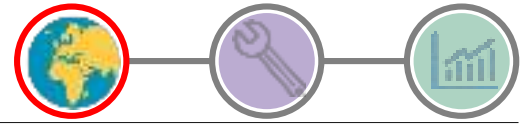
# QGS Model



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# QGS Model

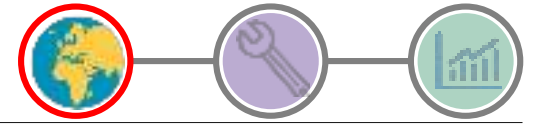


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A diagram below the equation uses physical symbols to represent each term. From left to right: a double-headed horizontal arrow for the time derivative term; a circular arrow for the flux divergence term; an upward wavy arrow from a surface for the outgoing longwave radiation term; a horizontal line with a downward wavy arrow for the incoming longwave radiation term; and a downward wavy arrow for the solar radiation term.

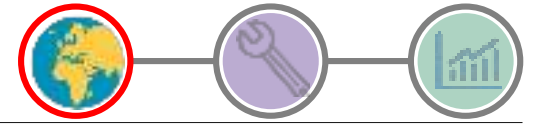
# QGS Model

## Linearisation



$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

# QGS Model



## Linearisation

$$T_o = T_{o,0} + \delta T_o(t, x, y)$$

↓

$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

# QGS Model




## Linearisation

$$T_o = T_{o,0} + \delta T_o(t, x, y)$$

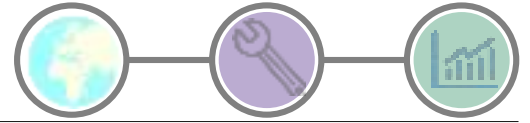
$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$

$O(\delta T_o)$

$$-4\sigma_B T_{o,0}^3 \delta T_o + 4\varepsilon \sigma_B T_{a,0}^3 \delta T_a$$


# Key Points

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The Model



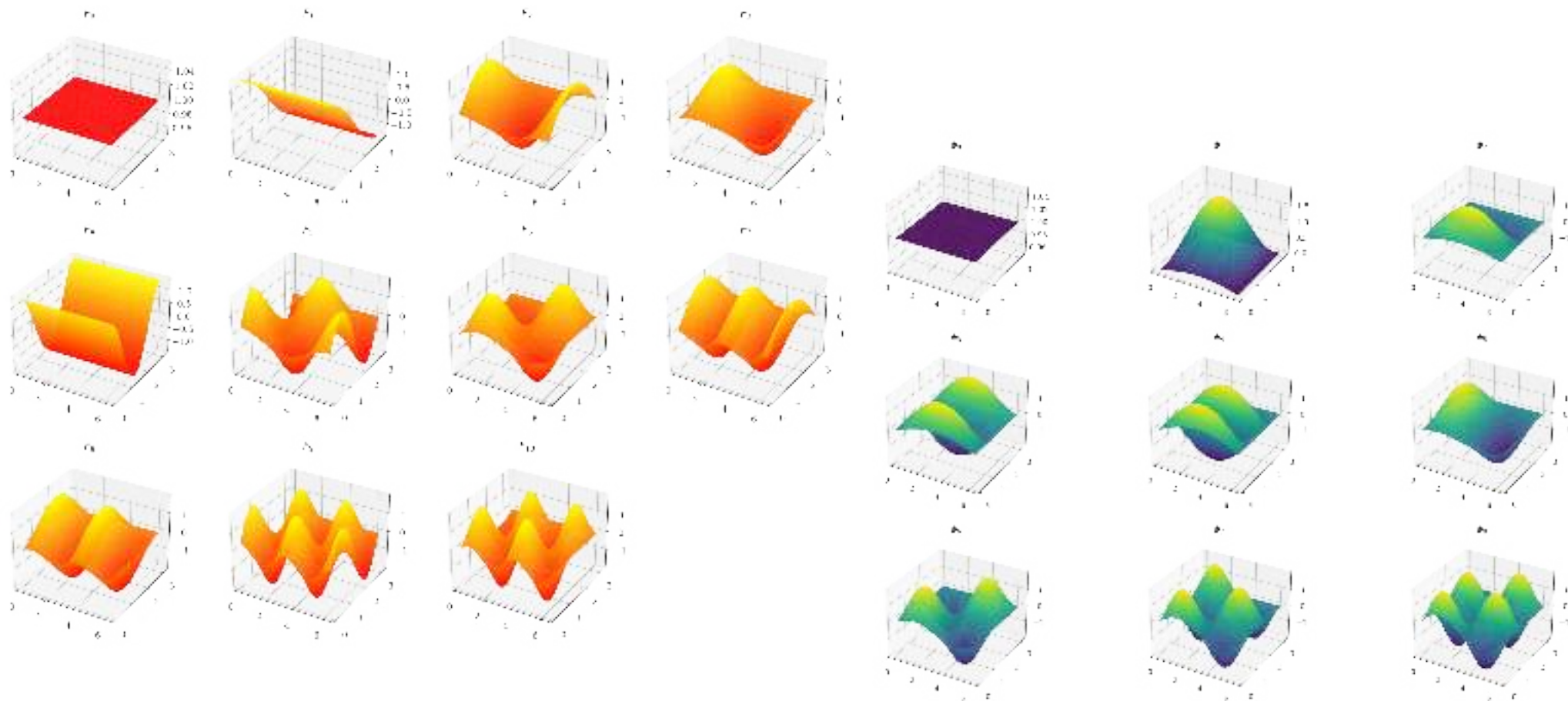
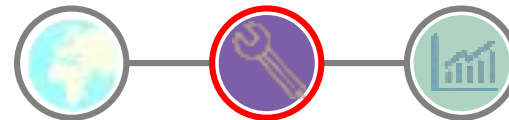
Modifications



Results

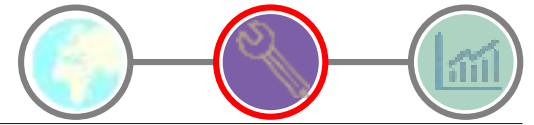
# Modifications

## New Modes






# Modifications



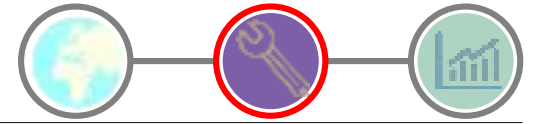
## Dynamic Equilibrium

$$T_o = T_{o,0}(t) + \delta T_o(t, x, y)$$



$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

# Modifications



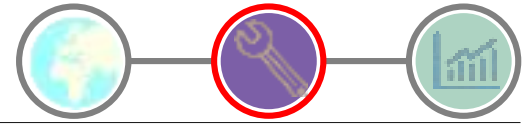
## Dynamic Equilibrium

$$T_o = T_{o,0}(t) + \delta T_o(t, x, y)$$

$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$

$O(\delta T_o)$

# Modifications



## Dynamic Equilibrium

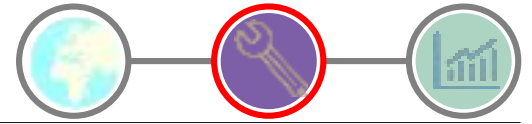
$$T_o = \boxed{T_{o,0}(t)} + \delta T_o(t, x, y)$$
$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

$O(\delta T_o)$

$$-\sigma_B T_{o,0}^4 + \varepsilon \sigma_B T_{a,0}^4 + f(\delta T_o) + g(\delta T_a)$$



# Modifications



## Non-Linear Equation

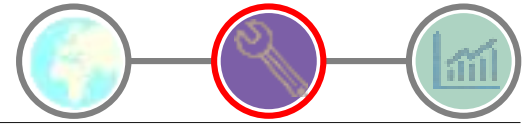
$$T_o(t, x, y)$$



$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$



# Modifications



## Non-Linear Equation

$$T_o(t, x, y)$$



$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

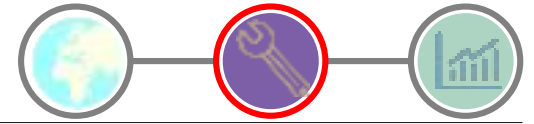
Problem:




5–6x run time



# Modifications

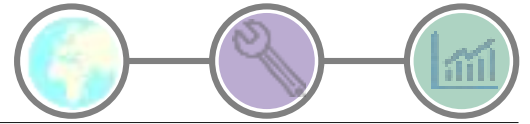
## Model Summary



	Model	$T^4$ Radiation Terms	$T_0$ Equilibrium Temperature
	Linearised	Linearised	Constant
	Dynamic Temperature	Linearised	Dynamic
	Non-Linear	Non-Linearised	Dynamic

# Key Points

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The Model



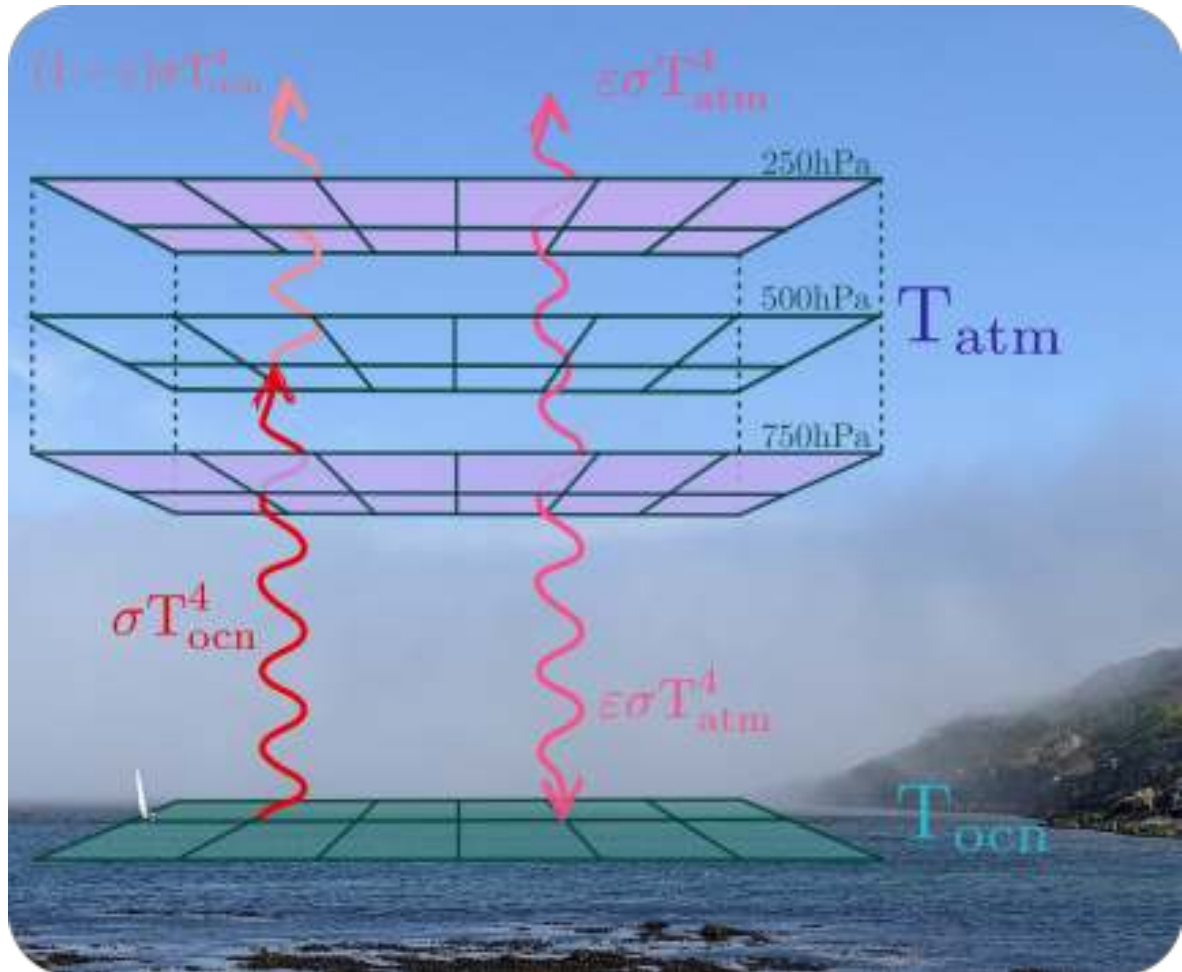
Modifications



Results

# Model Parameters

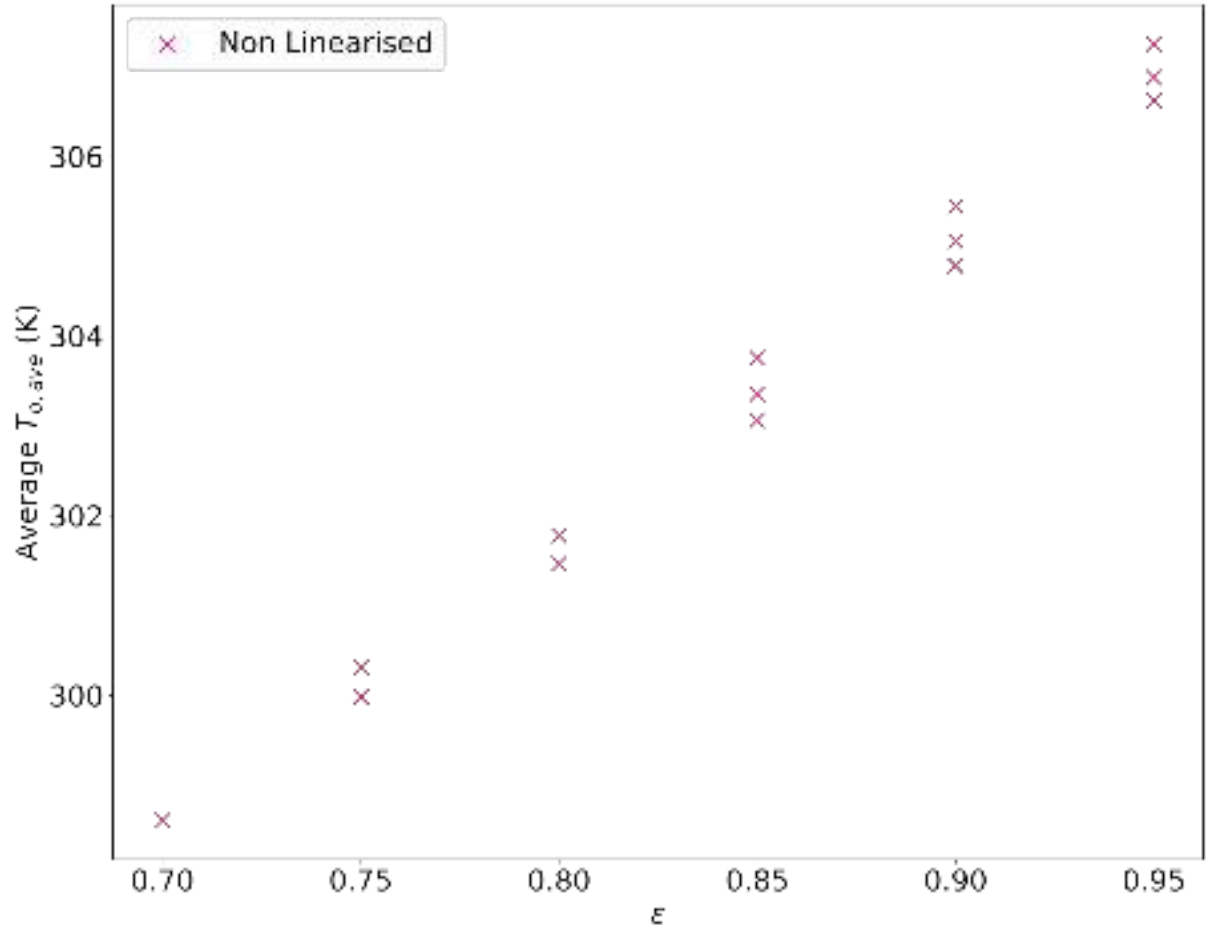
## Emissivity $\epsilon$





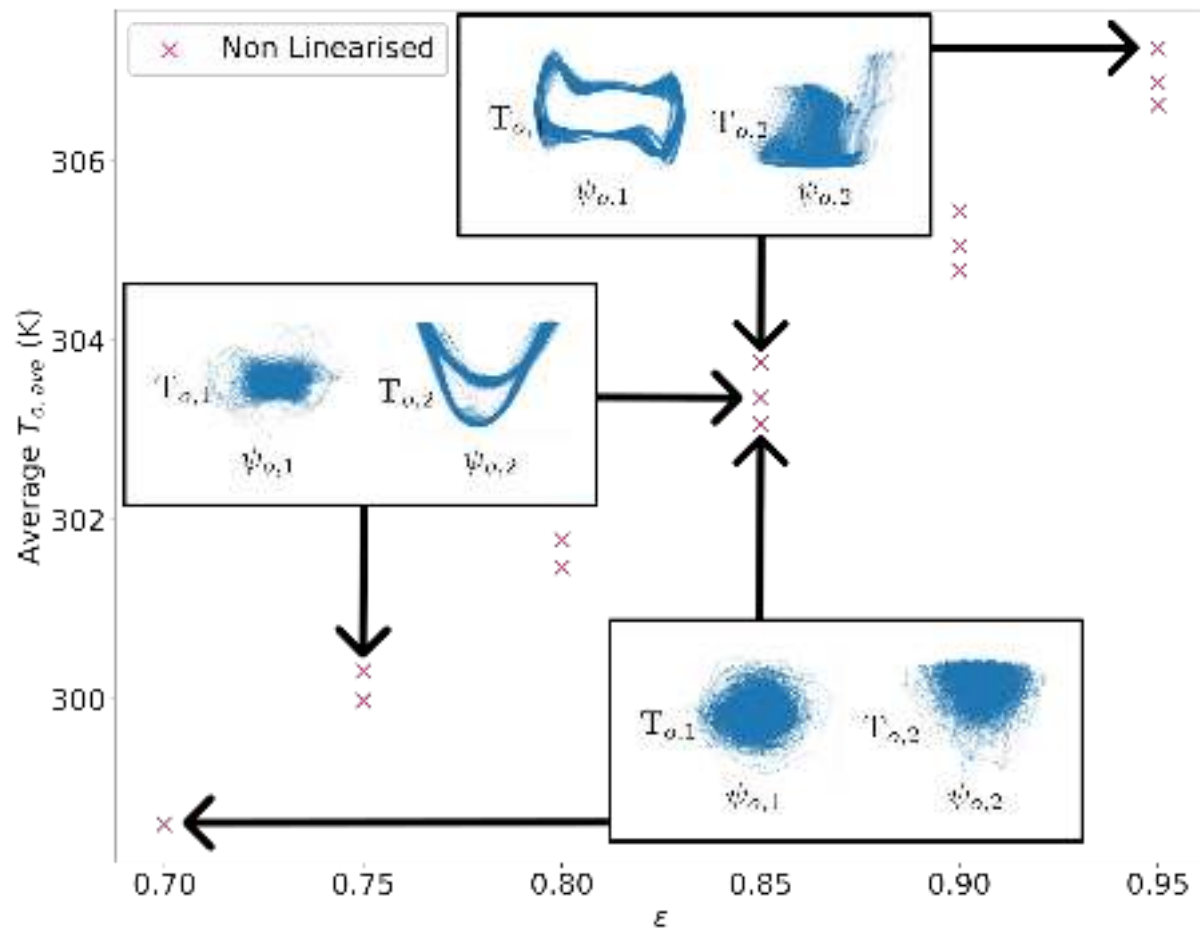
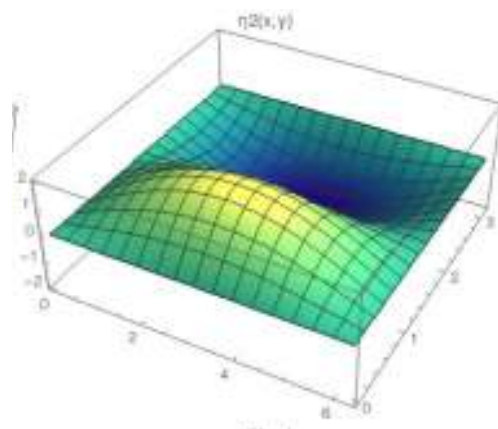
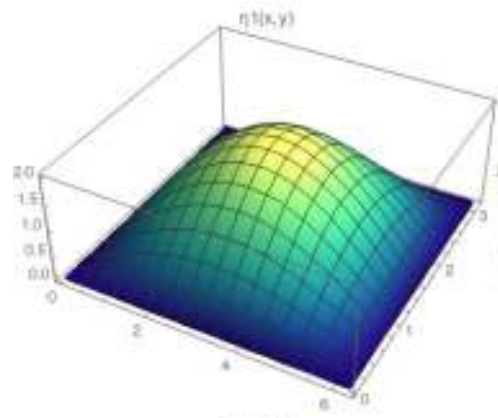
# Model Parameters

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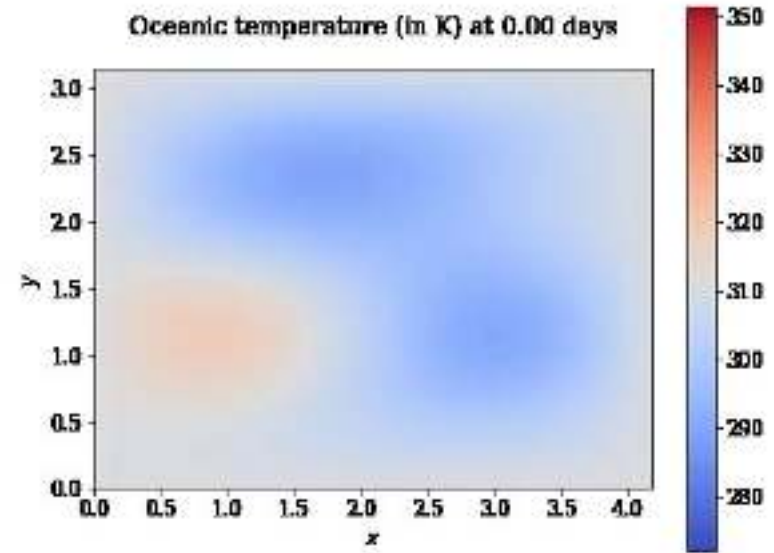
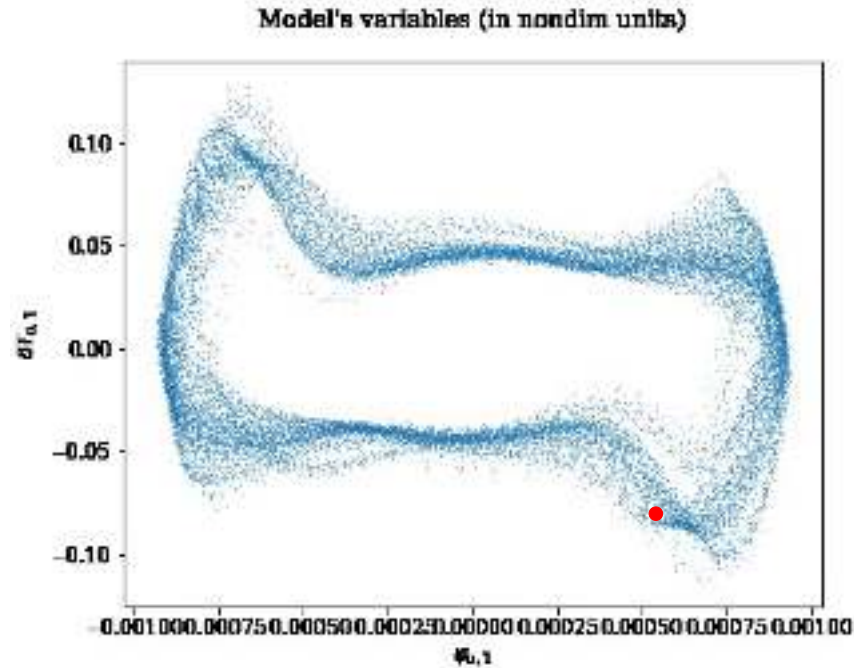
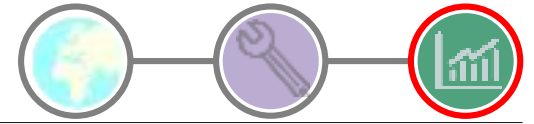
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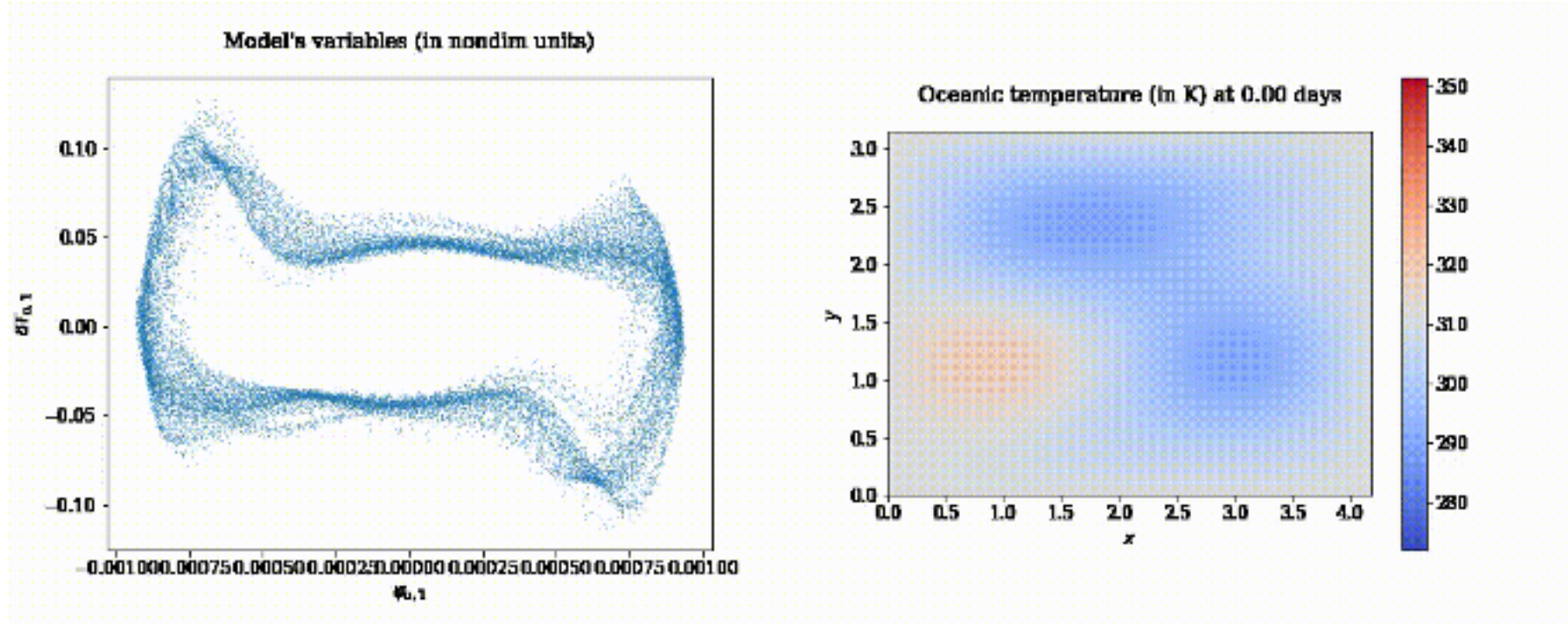
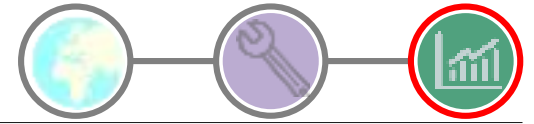
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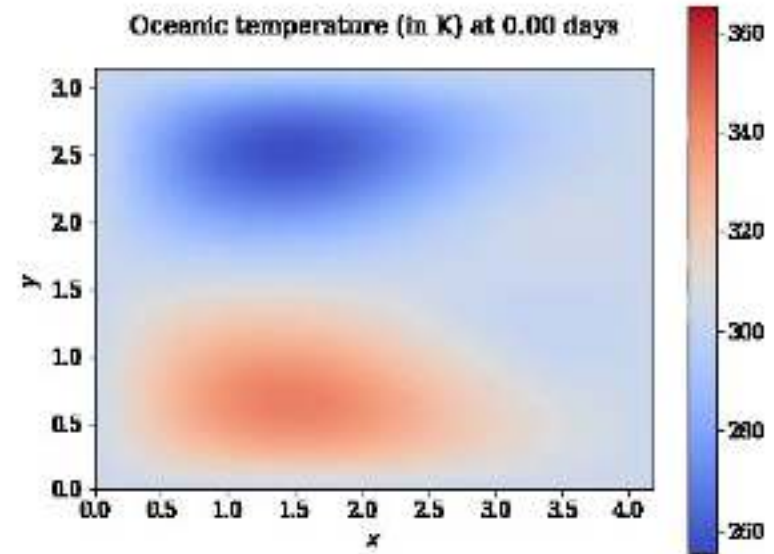
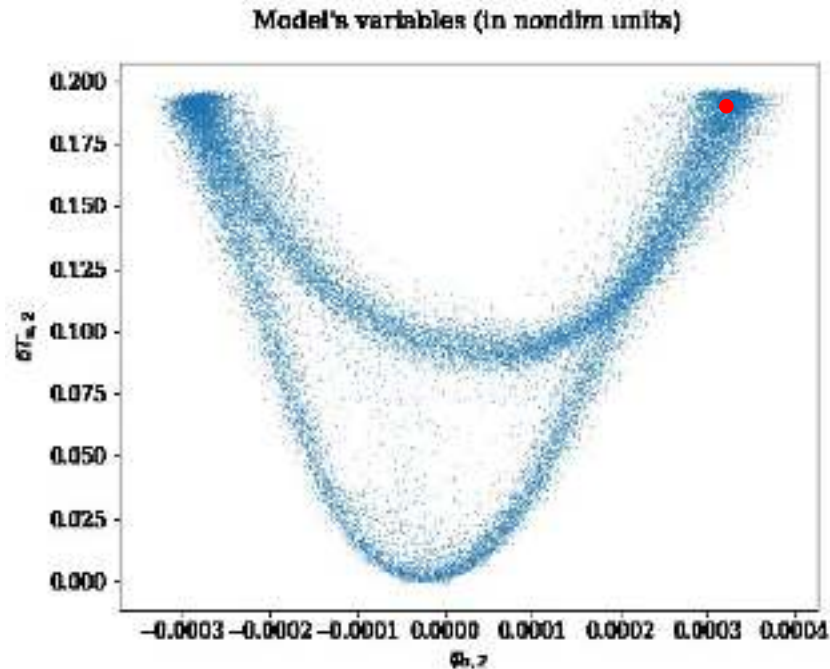
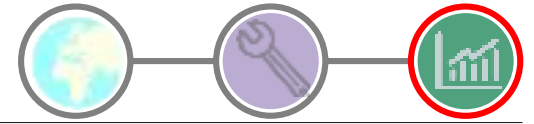
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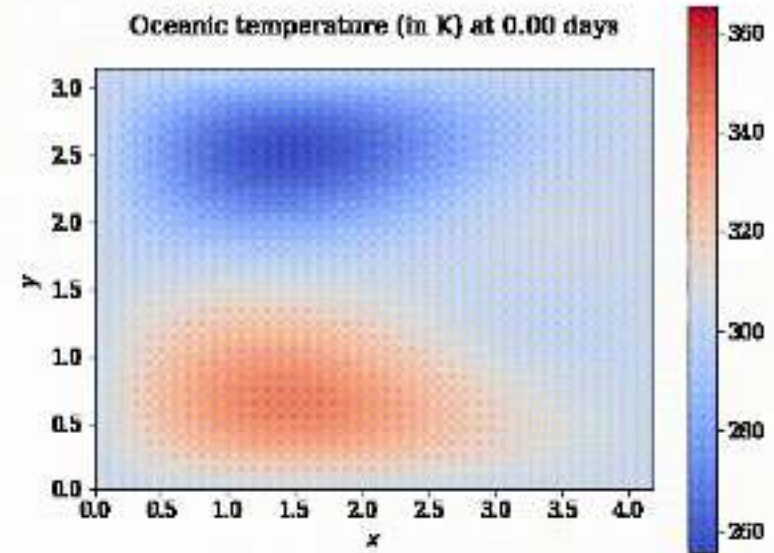
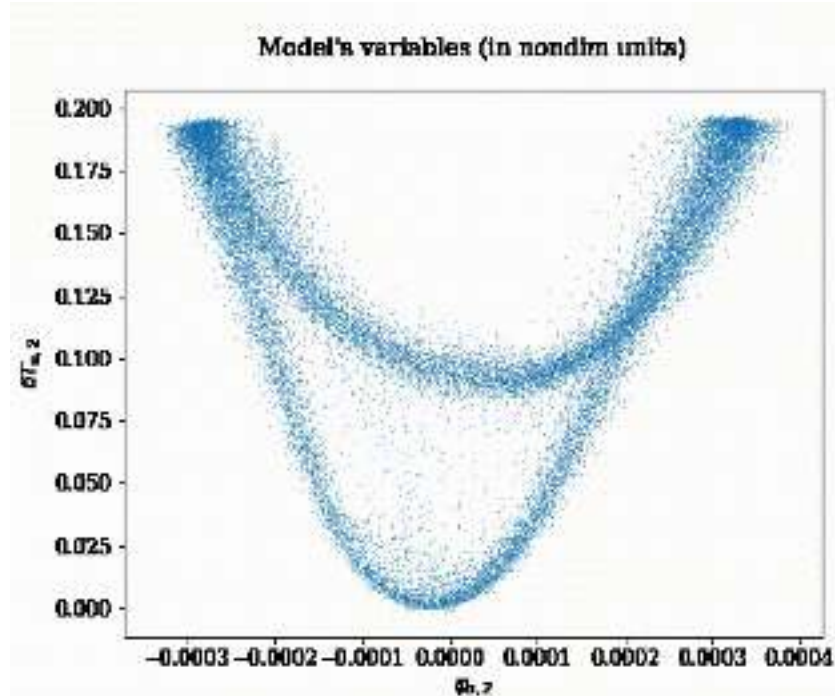
# Model Parameters

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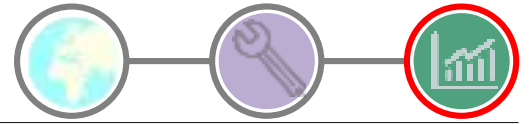


# Model Parameters

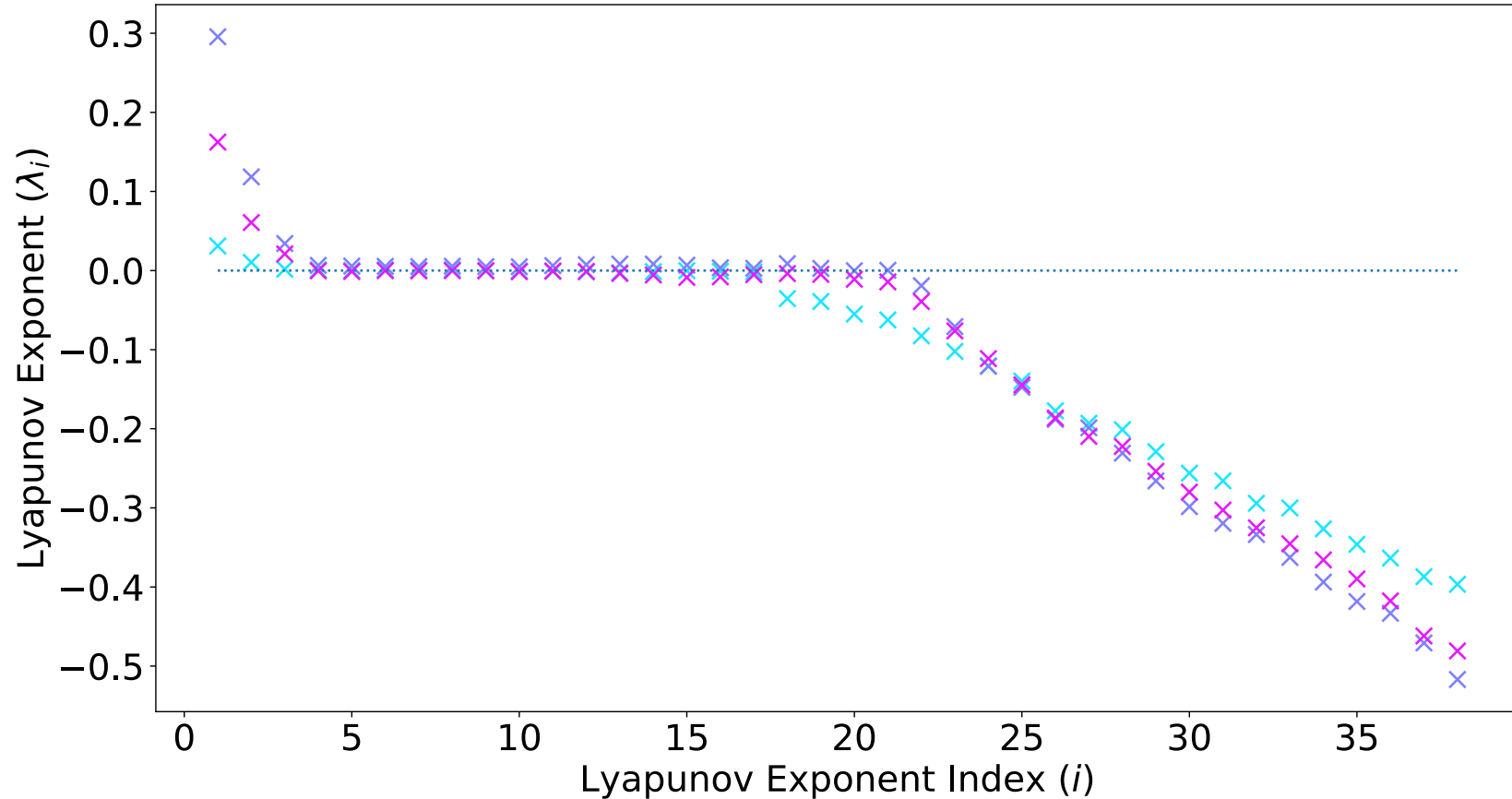
## Emissivity $\varepsilon$



# Model Outputs



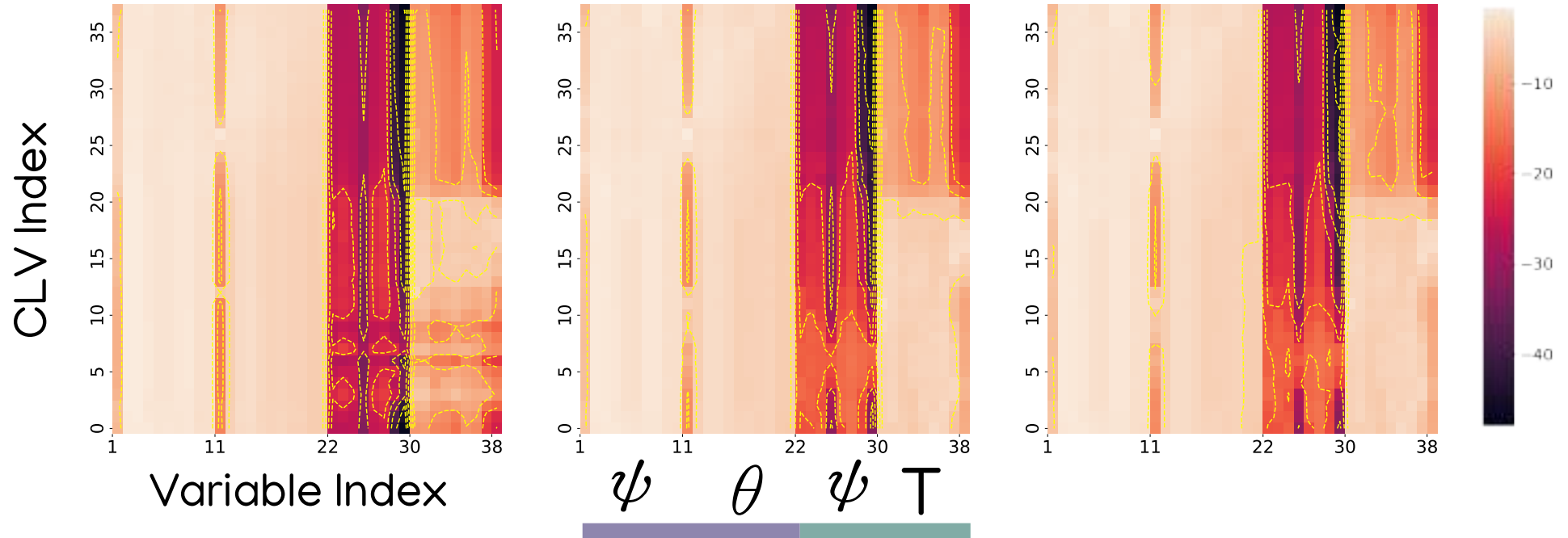
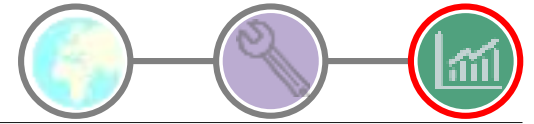
## Lyapunov Exponents





# Model Outputs

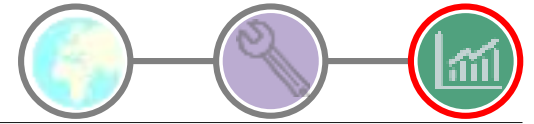
## Lyapunov Exponents





# Conclusion

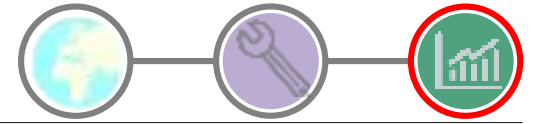
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Non linear radiation terms produce temperature multi-stabilities

# Conclusion

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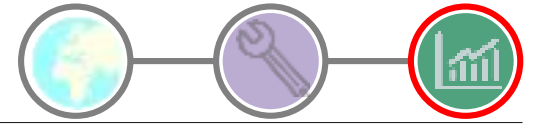


Non linear radiation terms produce temperature multi-stabilities

Multi-stabilities produce distinct behaviour

# Conclusion

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Non linear radiation terms produce temperature multi-stabilities

Multi-stabilities produce distinct behaviour

Multi-stabilities in majority of cases produced by dynamic equilibria

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# Thank you

oisin.hamilton@meteo.be



De Cruz et al. (2016)  
The Modular Arbitrary-Order  
Ocean-Atmosphere Model:  
MAOOAM v1.0