BLG354E-Spring2018-Midterm_last

12 Mayıs 2018 Cumartesi 13:55

NAME: ________________________________ **STUDENT ID:** _______________

Problem 1: Each part is 3 points.

1.1. For which of two signals with given Fourier Series (FS) coefficients, the FS approximation of the signal converges faster to the original signal as we add more FS coefficients? (Explain your reasoning briefly):

i) $a_k = \begin{cases} \frac{1}{j\pi k}, k = \mp1, \mp3, \dots \\ 0, k = \mp2, \mp4, \dots \\ \frac{1}{2}, k = 0 \end{cases}$ ii) $a_k = \begin{cases} \frac{1}{j\pi^2 k^2}, k = \mp1, \mp3, \dots \\ 0, k = \mp2, \mp4, \dots \\ \frac{1}{2}, k = 0 \end{cases}$ Since difference will be less for second option as we add coefficients, it will be converging faster (with less coefficients)

1.2. Rate of oscillation of discrete time signal increases as ω increases up to ..2π..

1.3. The reason of Gibbs phenomenon is discontinuity of the signal

1.4. What are the frequency values f_1 and f_2 of the following beat signal: $\sin(2\pi 210t) + sin(2\pi 200t) = 2\cos(2\pi f_1 t) sin(2\pi f_2 t)$?

$f_1 + f_2 = 210 \quad f_1 - f_2 = 200 \qquad 2f_1 = 410 \quad f_1 = 205 \qquad f_2 = 5$

1.5. Calculate Fourier Transform of $\delta(t)$ using Fourier Transform formula: $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$.

$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t} dt = e^{-j\omega 0} = 1$

1.6. $y(t) = \frac{dx}{dt} + x(t)$, determine system properties: Linear or nonlinear; Time invariant or time varying; Causal or non-causal)

System is not linear and not causal because of the component $\frac{dx}{dt}$

It is time varying.

1.7. Express $u[n+2] + u[n+1] - u[n] - u[n-3]$ in terms of impulse sequences:

$\delta[n+2] + 2\delta[n+1] + 2\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$

1.8. Calculate Discrete Fourier Transform coefficients of $x[0] = 8, x[1] = 4, x[2] = 8, x[3] = 4$ (Recall $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$).

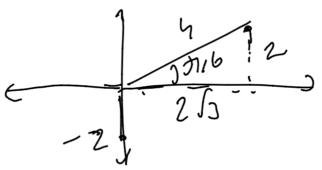
$X_1 = 8 \cdot e^{\frac{-j2\pi \cdot 0 \cdot 1}{4}} + 4 \cdot e^{-j\frac{\pi}{2}1} + 8 \cdot e^{-j\pi} + 4e^{-2j\pi} = 8 + 4j - 8 + 4 = -4j + 4$

$X_2 = 8 \cdot e^{\frac{-j2\pi \cdot 0 \cdot 2}{4}} + 4e^{-j\pi} + 8 \cdot e^{-2j\pi} + 4e^{-4j\pi} = 8 - 4 + 8 + 4 = 16$

$X_3 = 8 + 4e^{-2j\pi} + 8e^{-4j\pi} + 4e^{-8j\pi} = 8 + 4 + 8 + 4 = 24$

$X_4 = 8 + 4e^{-4j\pi} + 8e^{-8j\pi} + 4e^{-16j\pi} = 8 + 4 + 8 + 4 = 24$

1.9. Add two sinusoids: $x_1(t) = 2\sin(5\pi t - \frac{\pi}{2})$ and $x_2(t) = 4\cos(5\pi t + \frac{\pi}{6})$ to obtain another sinusoid. Use phasor addition.



$x_1(t) + x_2(t) = 2\sqrt{3}\cos(5\pi t)$

1.10. Fourier Transform of $x(t) = \sin(\omega_0 t)$ is equal to $\frac{\pi}{j}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$

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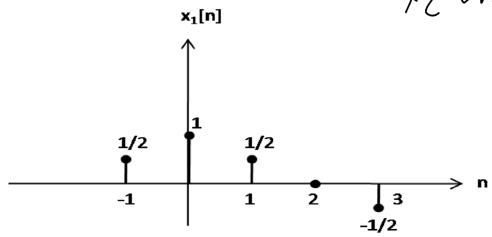
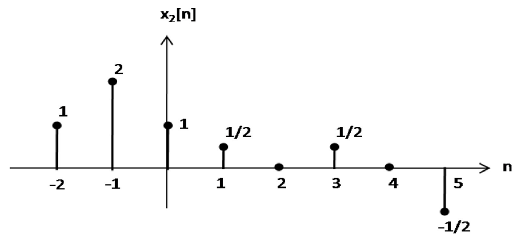
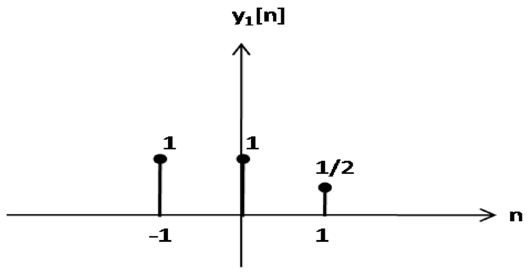
Problem 2: Problem parts (a) and (b) are not related.

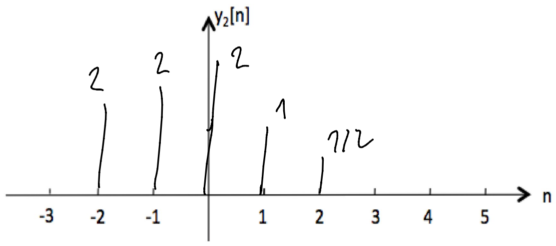
2. (a) Consider a linear time-invariant system whose response to $x_1[n]$ is the signal $y_1[n]$. Sketch carefully the response of the system $y_2[n]$ to the input $x_2[n]$ in the given axis below. Justify your reasoning clearly.

$x_2[n] = 2x_1[n+1] + x_1[n-1]$



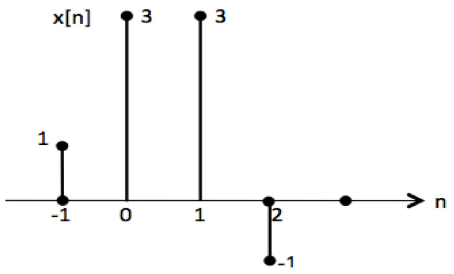


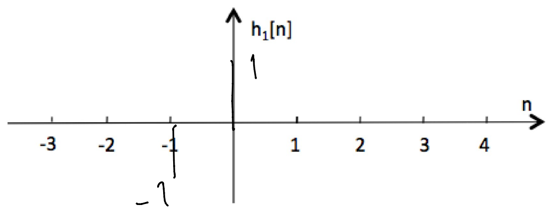


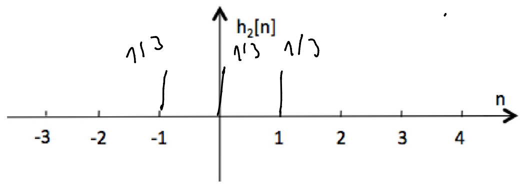


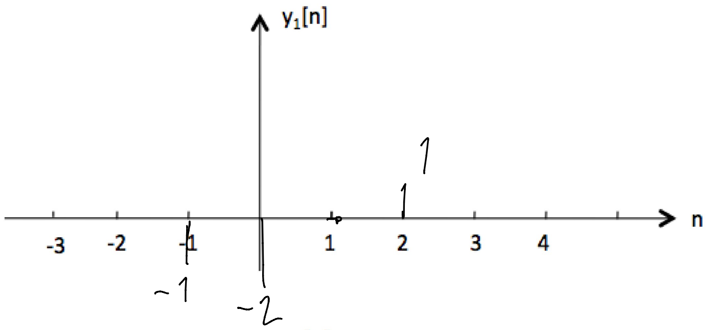
(b) A discrete time signal $x[n]$, whose plot is given below, is provided as an input to two systems that you will define:

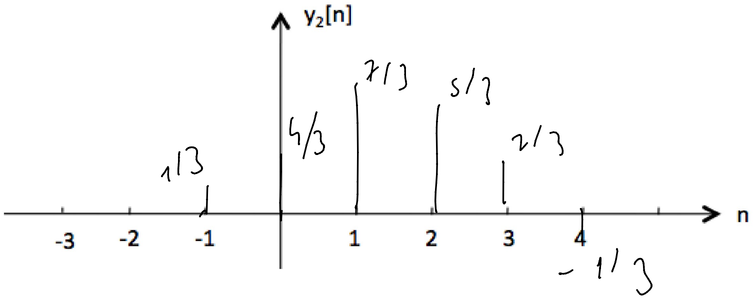
S1: a 2-pt filter that detects abrupt changes in the signal; S2: a 3-pt filter that smooths the signal. Plot the impulse responses $h_1[n]$ and $h_2[n]$ corresponding to S1 and S2, respectively, in the given axes below. Also, plot the output of the system S1, i.e. $y_1[n]$, and output of S2, $y_2[n]$, in response to input x[n] in the given axes below. Justify your solution.











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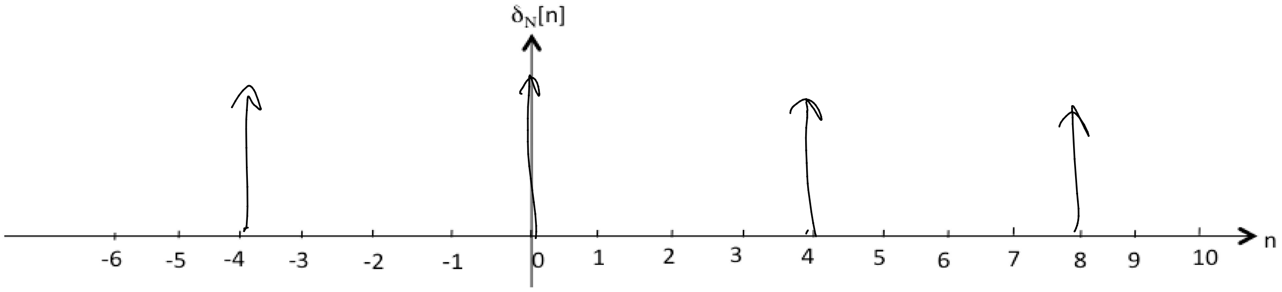
Problem 3. Consider the periodic impulse train: $\delta_N[n] = \sum_{m=-\infty}^{\infty} \delta[n - mN]$.

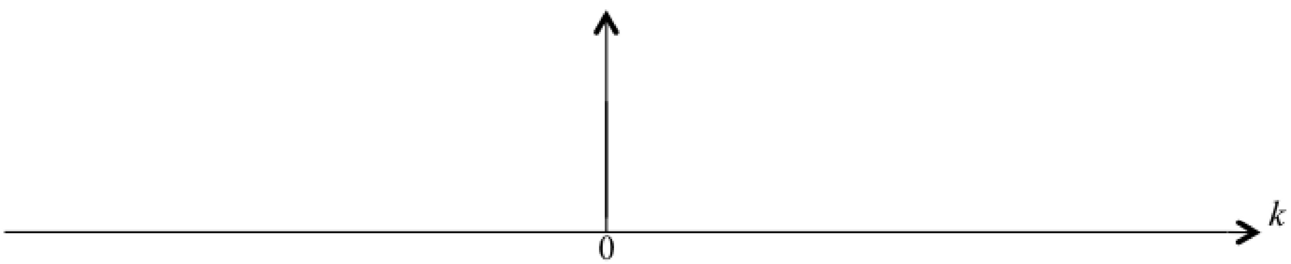
3.1. Plot $\delta_N[n]$ for N=4.



3.2. Calculate the Discrete Time Fourier Series coefficients c_k of $\delta_N[n]$, for all k integers: $c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$

3.3. Using the coefficients calculated in 3.2, write down the Fourier series representation of $\delta_N[n]$.

3.4. Plot the frequency spectrum of $\delta_N[n]$. State which frequency k=1 corresponds to?



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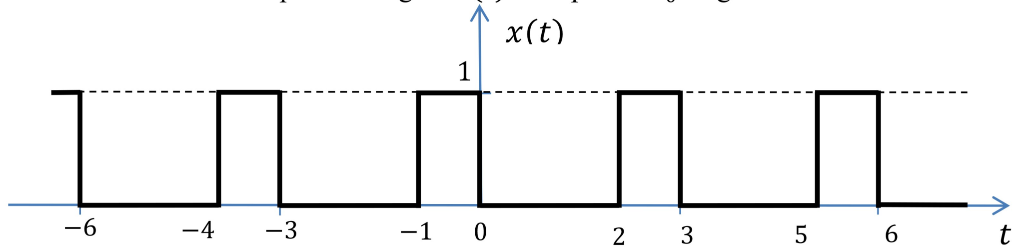
Problem 4: A continuous time periodic signal $x(t)$ with period T_0 is given below:



a) Derive a general formula for the Fourier series coefficients: a_0 and a_k (k: integers, $k \neq 0$) for $x(t)$. $a_k = \frac{1}{T_o} \int_{To} x(t) e^{-j2\pi f_0 k t} dt$

$a_0 = \frac{1}{3} \int x(t) e^{-j2\pi \frac{1}{3} \cdot 0 \cdot t} dt = \frac{1}{3} \int x(t) dt = \frac{1}{3} \int u(t-2) - u(t-3) dt = \frac{1}{3}.$

$a_k = \frac{1}{3} \int x(t) e^{-j2\pi \frac{1}{3} kt} dt = \frac{1}{3} \int_2^3 e^{-j2\pi kt} dt = \frac{1}{3} \frac{e^{-j2\pi kt}}{j2\pi k} \Big|_2^3$

$a_k = \frac{1}{3} \left(\frac{e^{-j6\pi k} - e^{-j4\pi k}}{j\pi k} \right)$

b) Compute the Fourier series coefficients a_k for $-3 \leq k \leq 3$ in polar form and plot the spectrum for those harmonics.

(Use $\theta \triangleq \text{atan}\left(\frac{\sqrt{3}}{3}\right)$ if needed.)