Project Scheduling: Networks, Duration Estimation, and Critical Path

Project Scheduling

- Identifying precedence relationships
- Sequencing activities
- Determining activity times & costs
- Estimating material & worker requirements
- Determining critical activities

Work Breakdown Structure

- 1. Project
- 2.. Major tasks in the project
- 3. Subtasks in the major tasks
- 4. Activities

Purposes of Project Scheduling

- Shows the relationship of each activity to others and to the project as a whole
- Identifies the precedence relationships among activities
- Encourages the setting of realistic time and cost estimates for each activity
- Helps make better use of people, money, and material resources by identifying critical bottlenecks

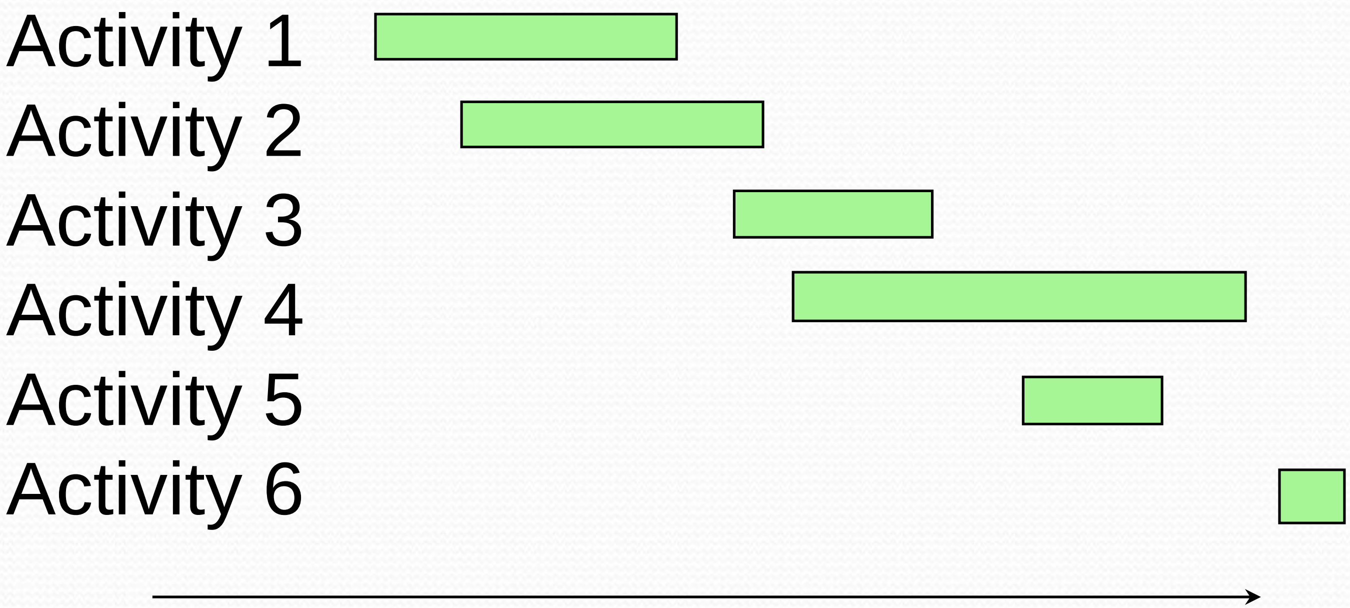
Project Scheduling Techniques

- Gantt chart
- Critical Path Method (CPM)
- Program Evaluation & Review Technique (PERT)

Project Control Charts: Gantt Chart

Vertical Axis: Always Activities or Jobs

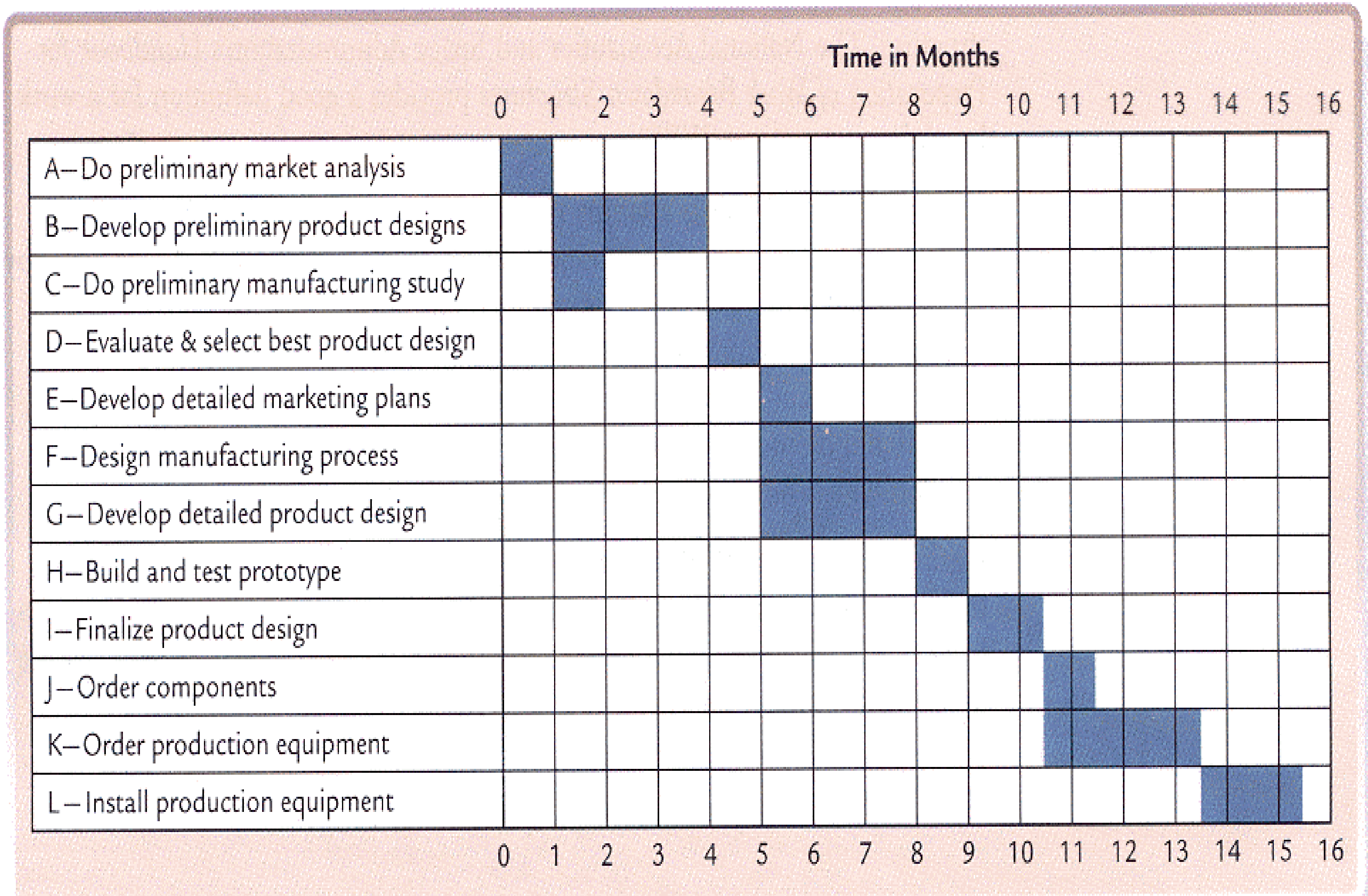
Horizontal bars used to denote time.



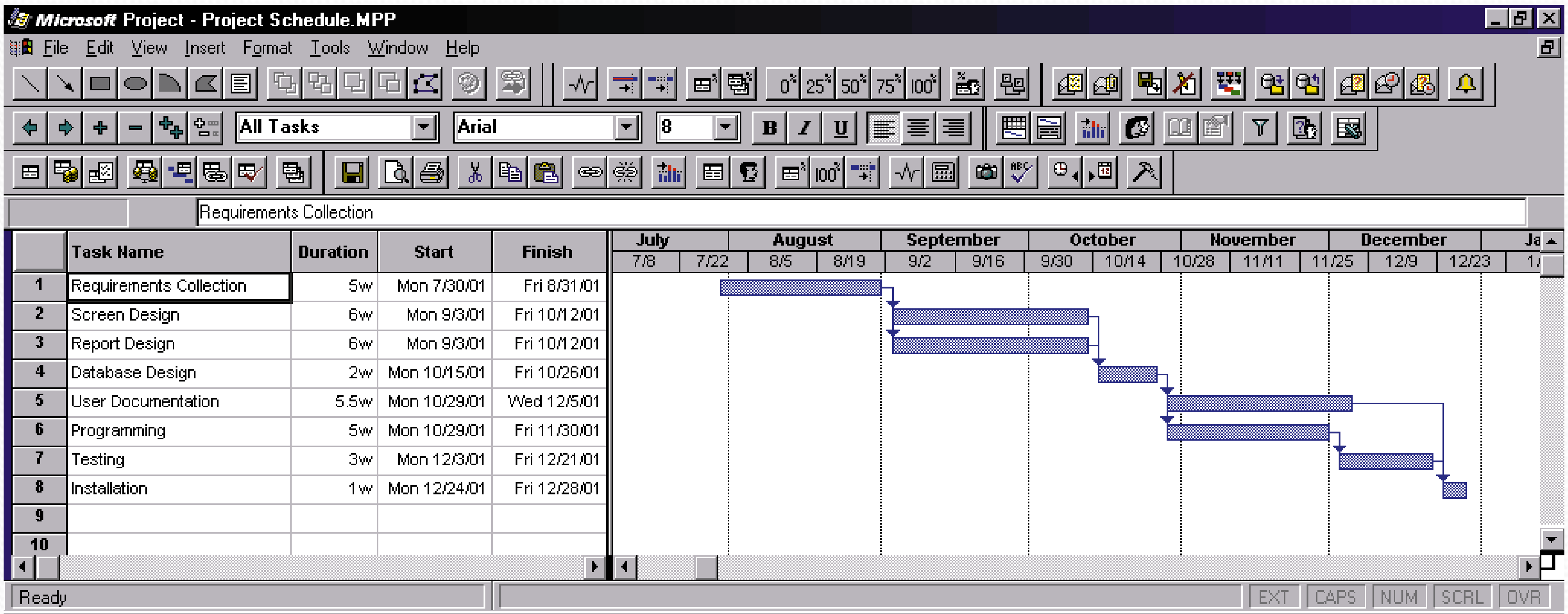
Time

Horizontal Axis: Always Time

GANTT CHART


Time in Months
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
A–Do preliminary market analysis
B–Develop preliminary product designs
C–Do preliminary manufacturing study
D–Evaluate & select best product design
E–Develop detailed marketing plans
F–Design manufacturing process
G–Develop detailed product design
H–Build and test prototype
I–Finalize product design
J–Order components
K–Order production equipment
L–Install production equipment
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Gantt Chart



Microsoft Project - Project Schedule.MPP
File Edit View Insert Format Tools Window Help
All Tasks
Arial
8
Requirements Collection
Task Name
Duration
Start
Finish
1 Requirements Collection 5w Mon 7/30/01 Fri 8/31/01
2 Screen Design 6w Mon 9/3/01 Fri 10/12/01
3 Report Design 6w Mon 9/3/01 Fri 10/12/01
4 Database Design 2w Mon 10/15/01 Fri 10/26/01
5 User Documentation 5.5w Mon 10/29/01 Wed 12/5/01
6 Programming 5w Mon 10/29/01 Fri 11/30/01
7 Testing 3w Mon 12/3/01 Fri 12/21/01
8 Installation 1w Mon 12/24/01 Fri 12/28/01
9
10
July August September October November December
7/8 7/22 8/5 8/19 9/2 9/16 9/30 10/14 10/28 11/11 11/25 12/9 12/23
Ready
EXT CAPS NUM SCRL OVR

PERT and CPM

- Network techniques
- Developed in 1950's
  - CPM by DuPont for chemical plants
  - PERT by U.S. Navy for Polaris missile
- Consider precedence relationships and interdependencies
- Each uses a different estimate of activity times

Questions Which May Be Addressed by PERT & CPM

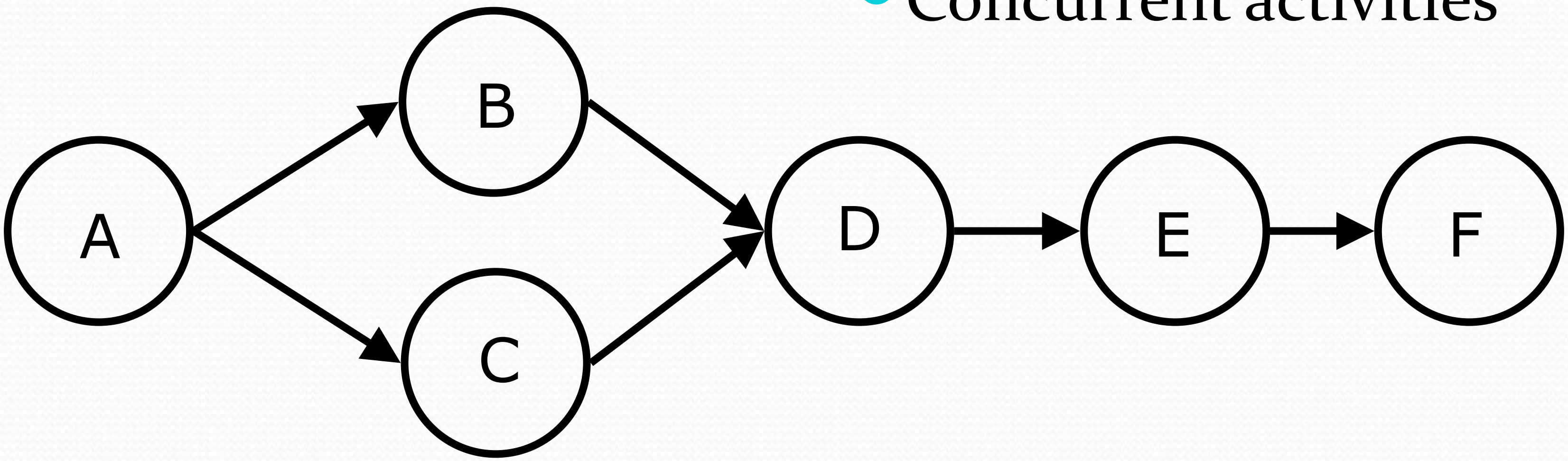
- Is the project on schedule, ahead of schedule, or behind schedule?
- Is the project over or under cost budget?
- Are there enough resources available to finish the project on time?
- If the project must be finished in less than the scheduled amount of time, what is the way to accomplish this at least cost?

The Six Steps Common to PERT & CPM

- Define the project and all of its significant activities or tasks
- Develop relationships among the activities. (Decide which activities must precede and which must follow others.)
- Draw the network connecting all of the activities
- Assign time and cost estimates to each activity
- Compute the longest time path through the network. This is called the critical path
- Use the network to help plan, schedule, monitor, and control the project

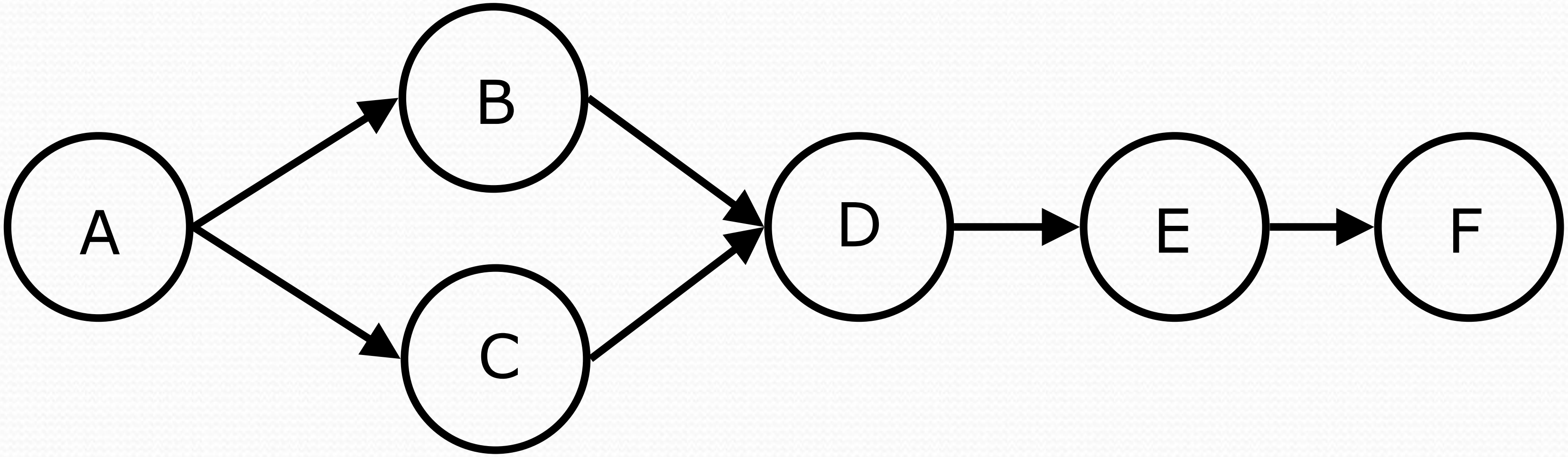
Project Scheduling Terms

- Successors
- Predecessors
- Network diagram
- Serial activities
- Concurrent activities



Project Scheduling Terms

- Node
- Path
- Critical Path



Network Diagrams

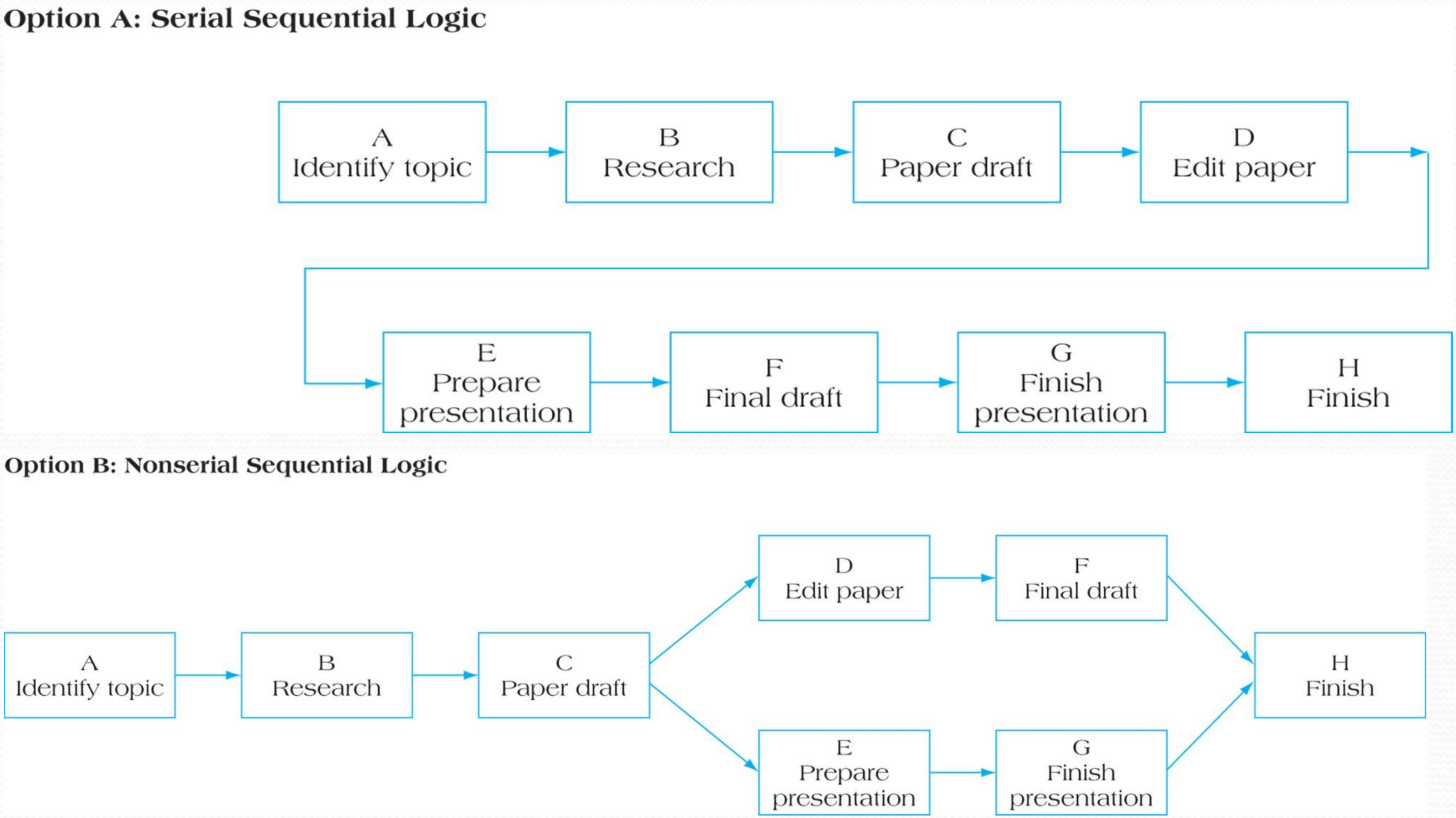
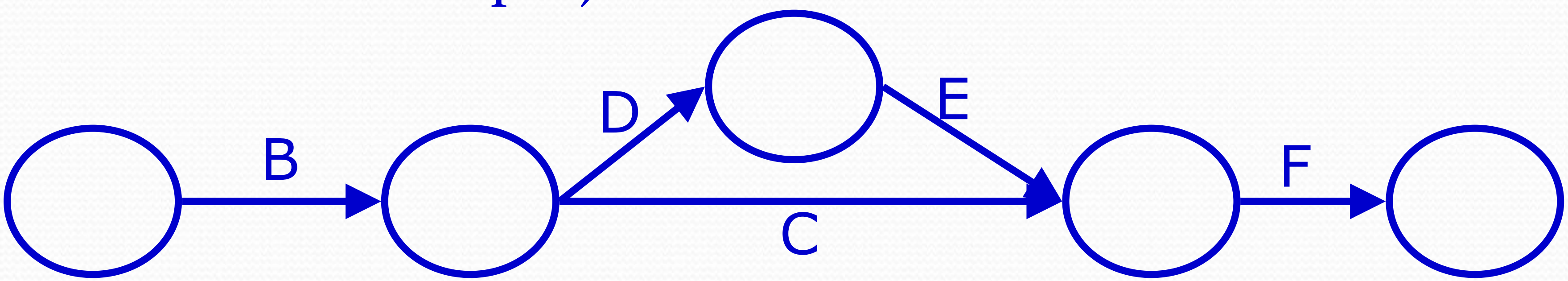


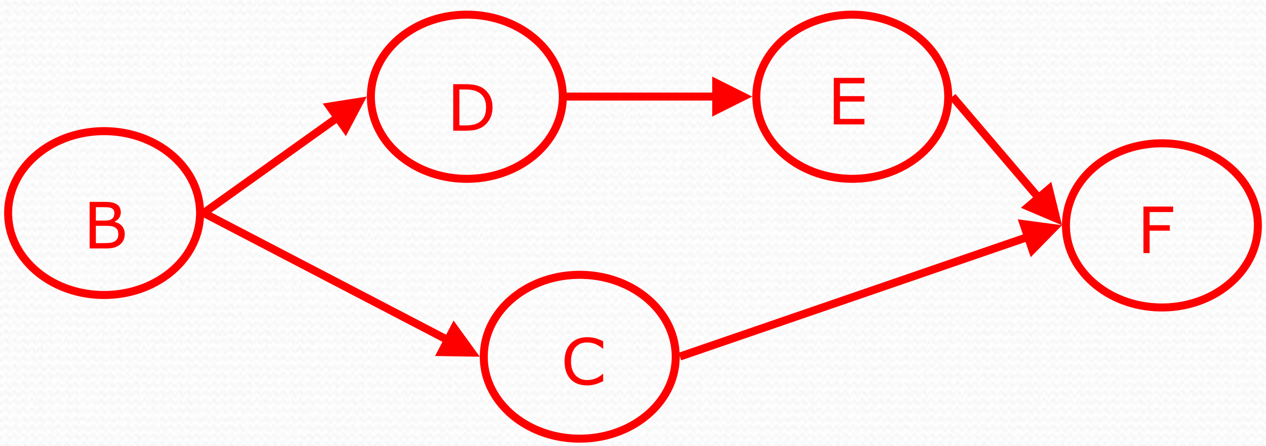
FIGURE 9.2 Alternative Activity Networks for Term Paper Assignment

AOA Versus AON

The same mini-project is shown with activities on arc…



…and activities on node.



Node Labels

Early Start	ID Number	Early Finish
Activity Float	Activity Descriptor	
Late Start	Activity Duration	Late Finish

Activity Node Labels Using MS Project 2010

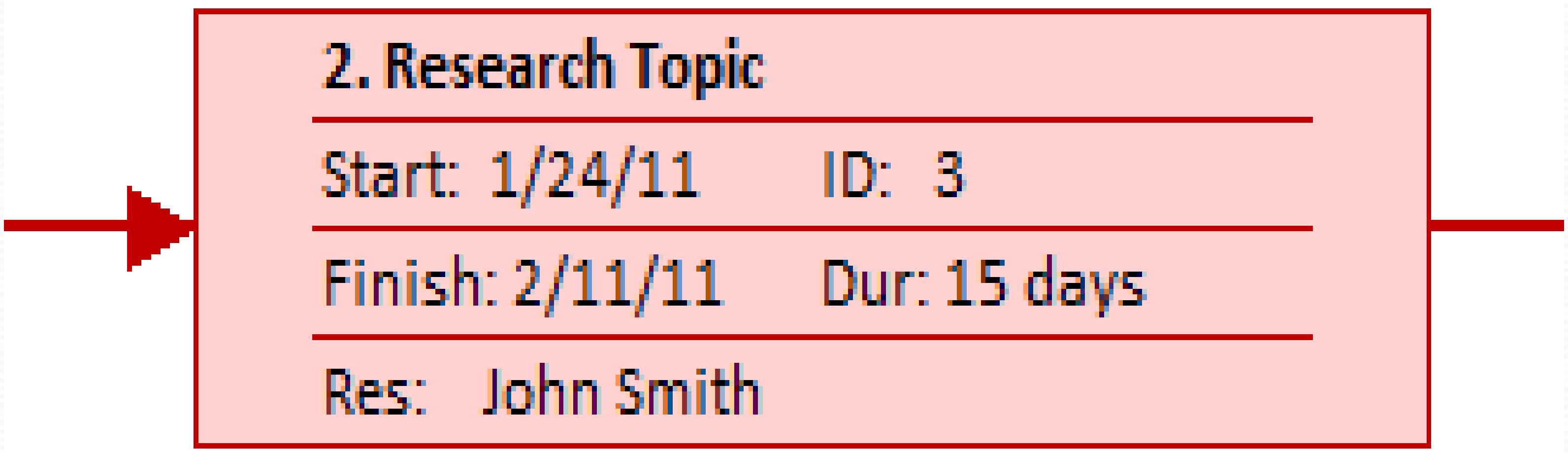


FIGURE 9.4

Serial Activities

Serial activities are those that flow from one to the next, in sequence.

FIGURE 9.5

Activities Linked in Parallel (Concurrent)

When the nature of the work allows for more than one activity to be accomplished at the same time, these activities are called ***concurrent*** and ***parallel project paths*** are constructed through the network.

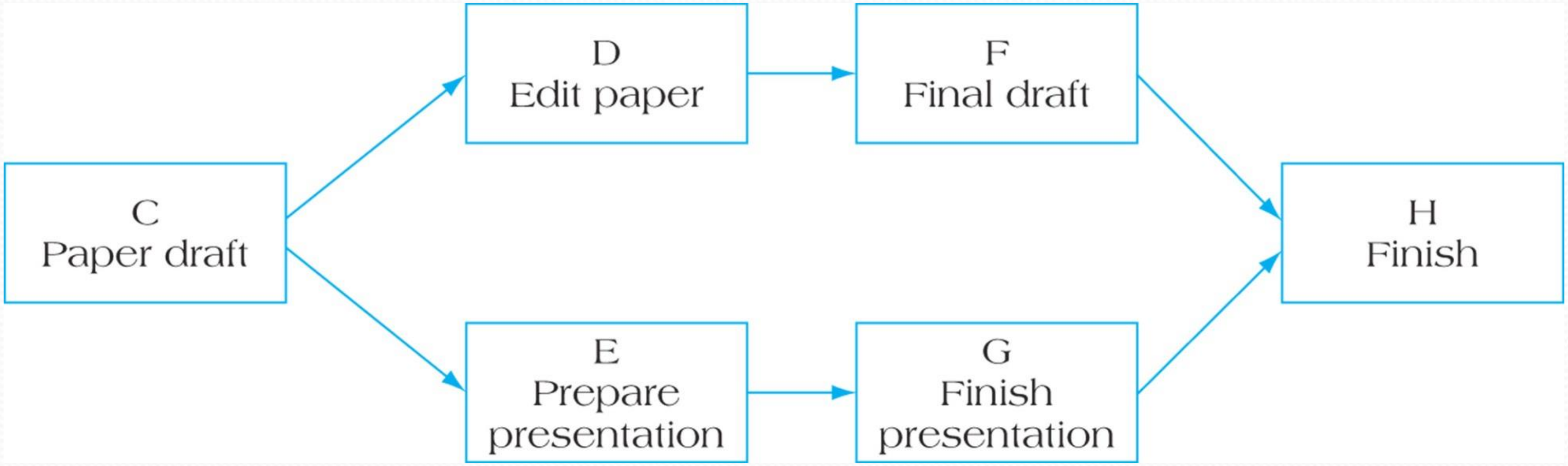


FIGURE 9.6

Merge Activity

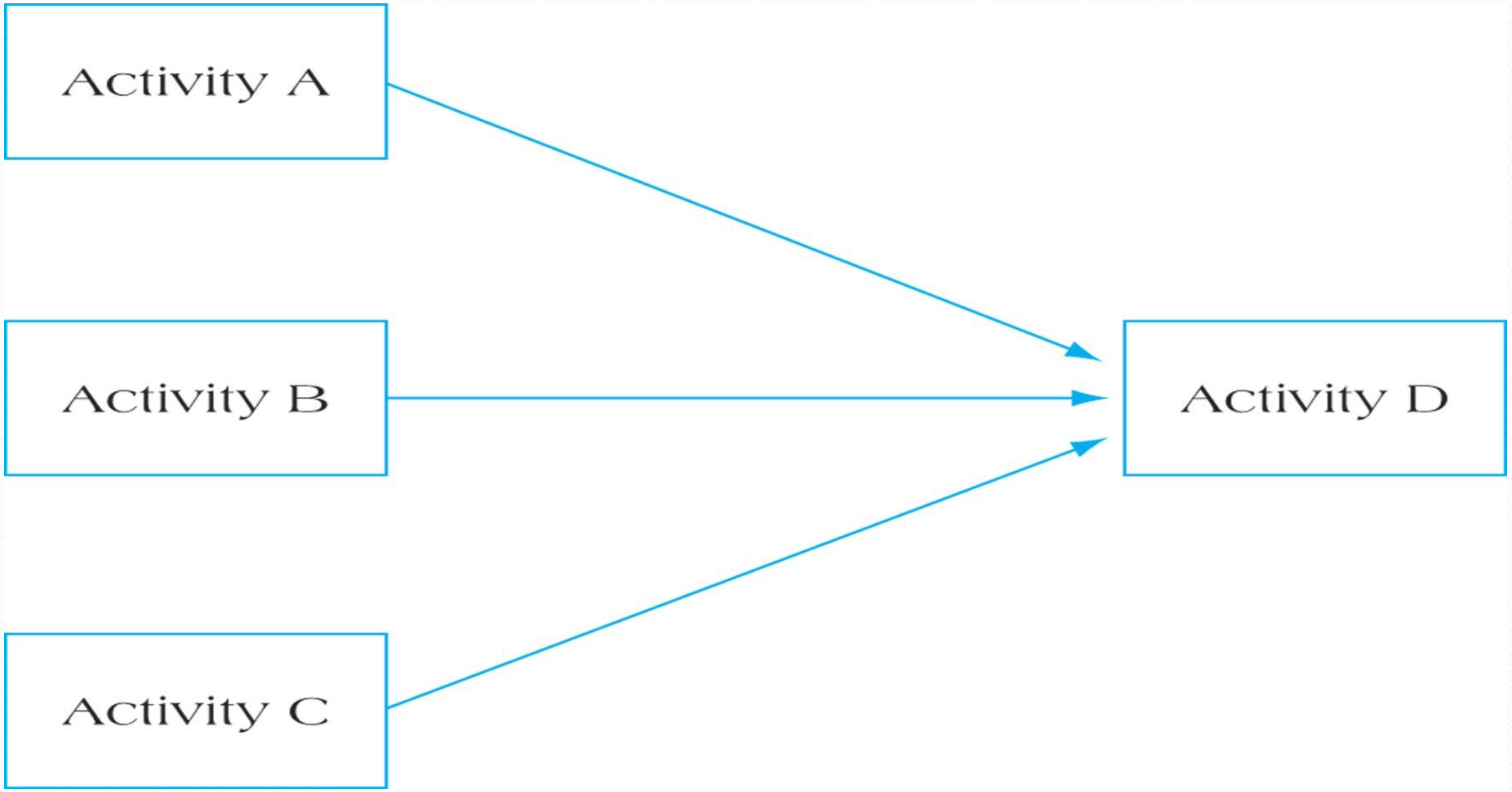


FIGURE 9.7

Burst Activity

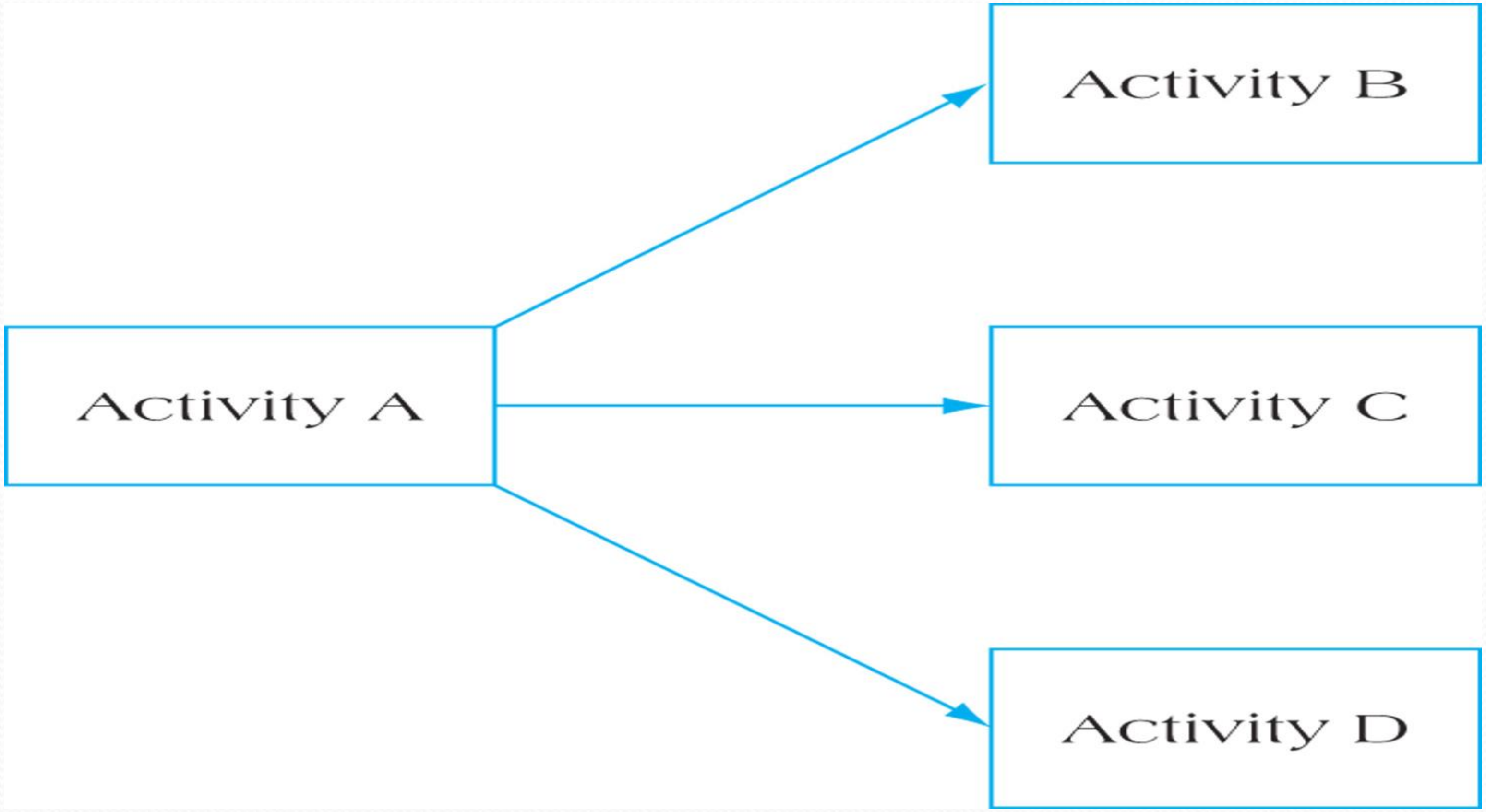


FIGURE 9.8

Complete Activity Network

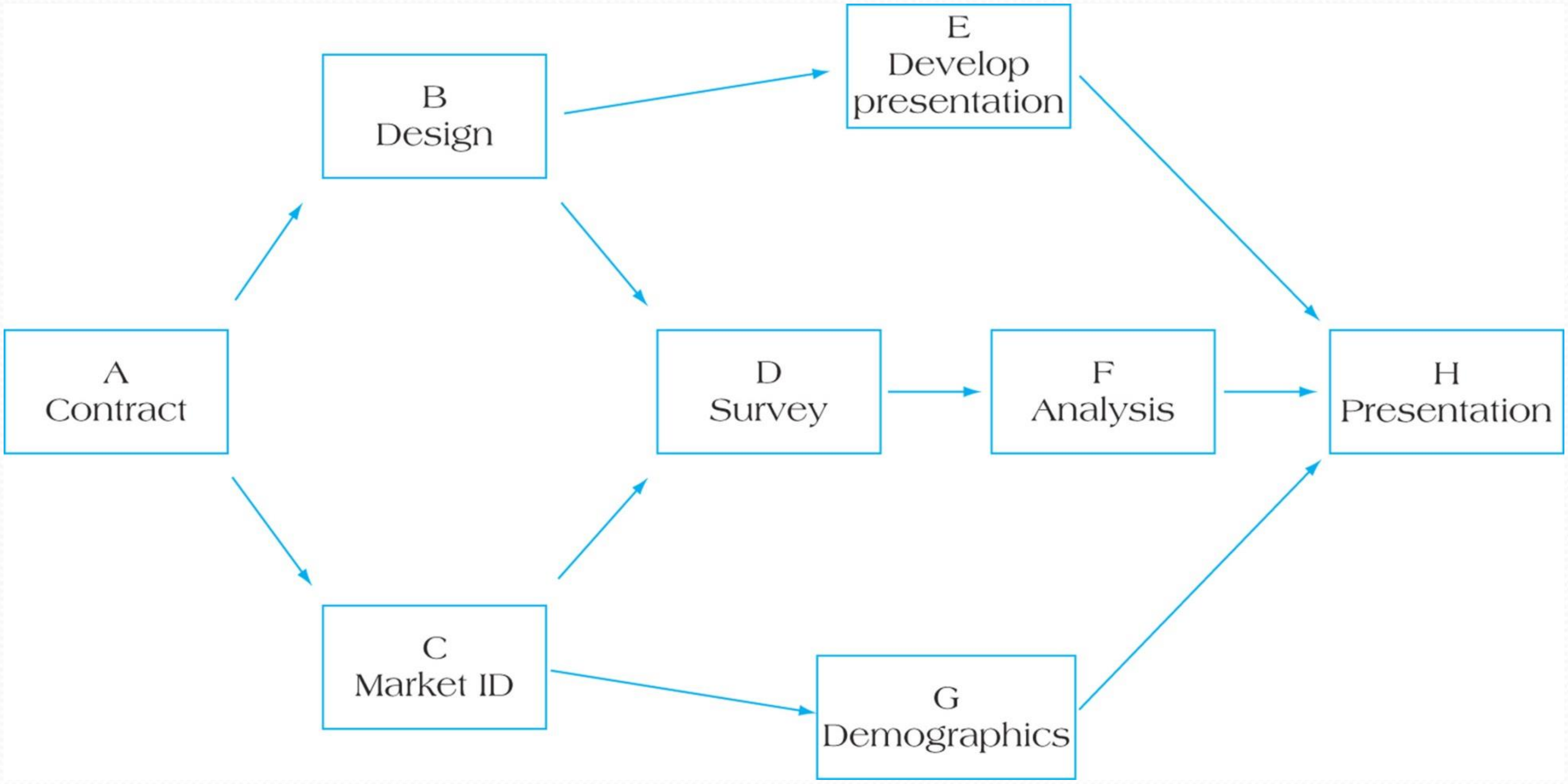
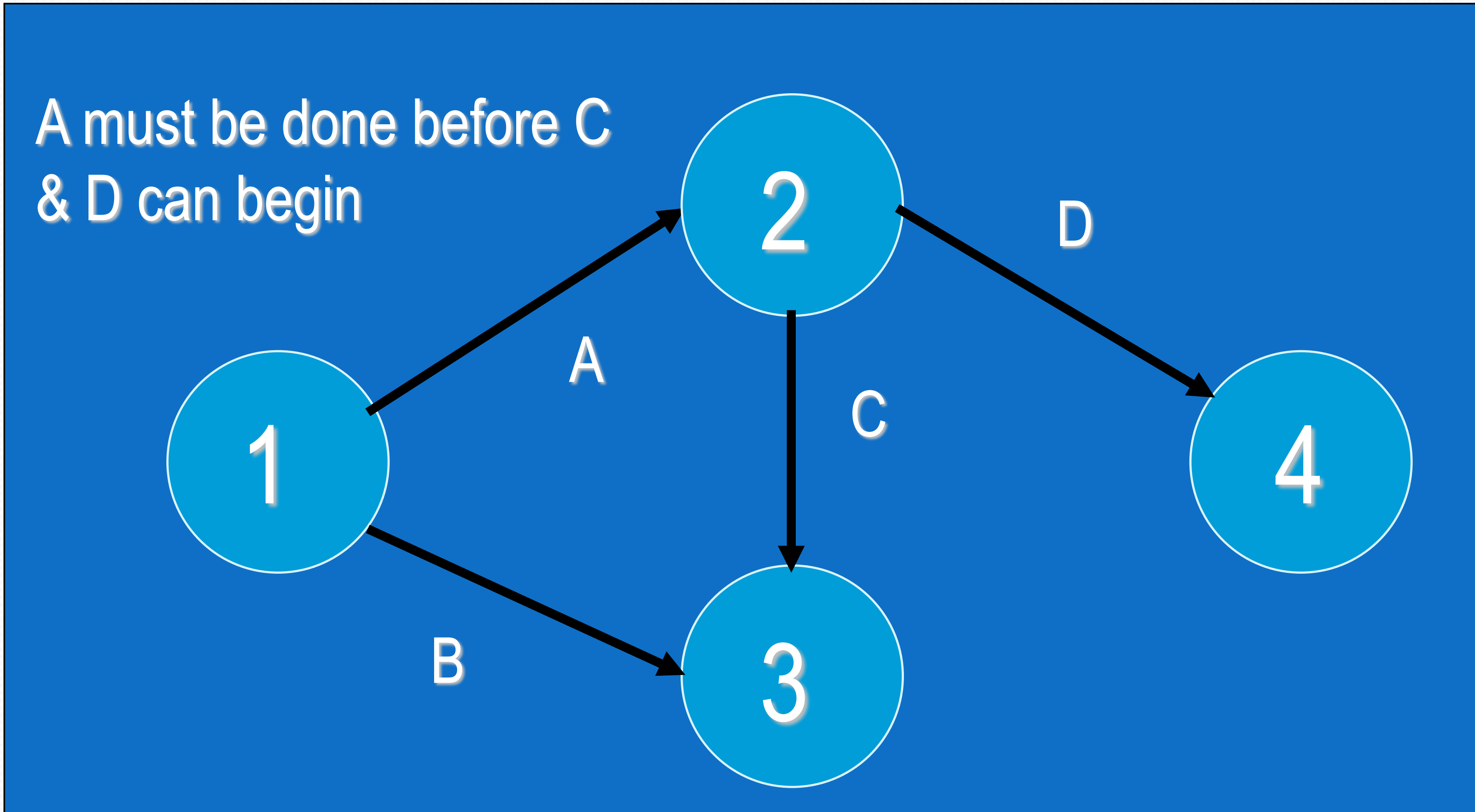
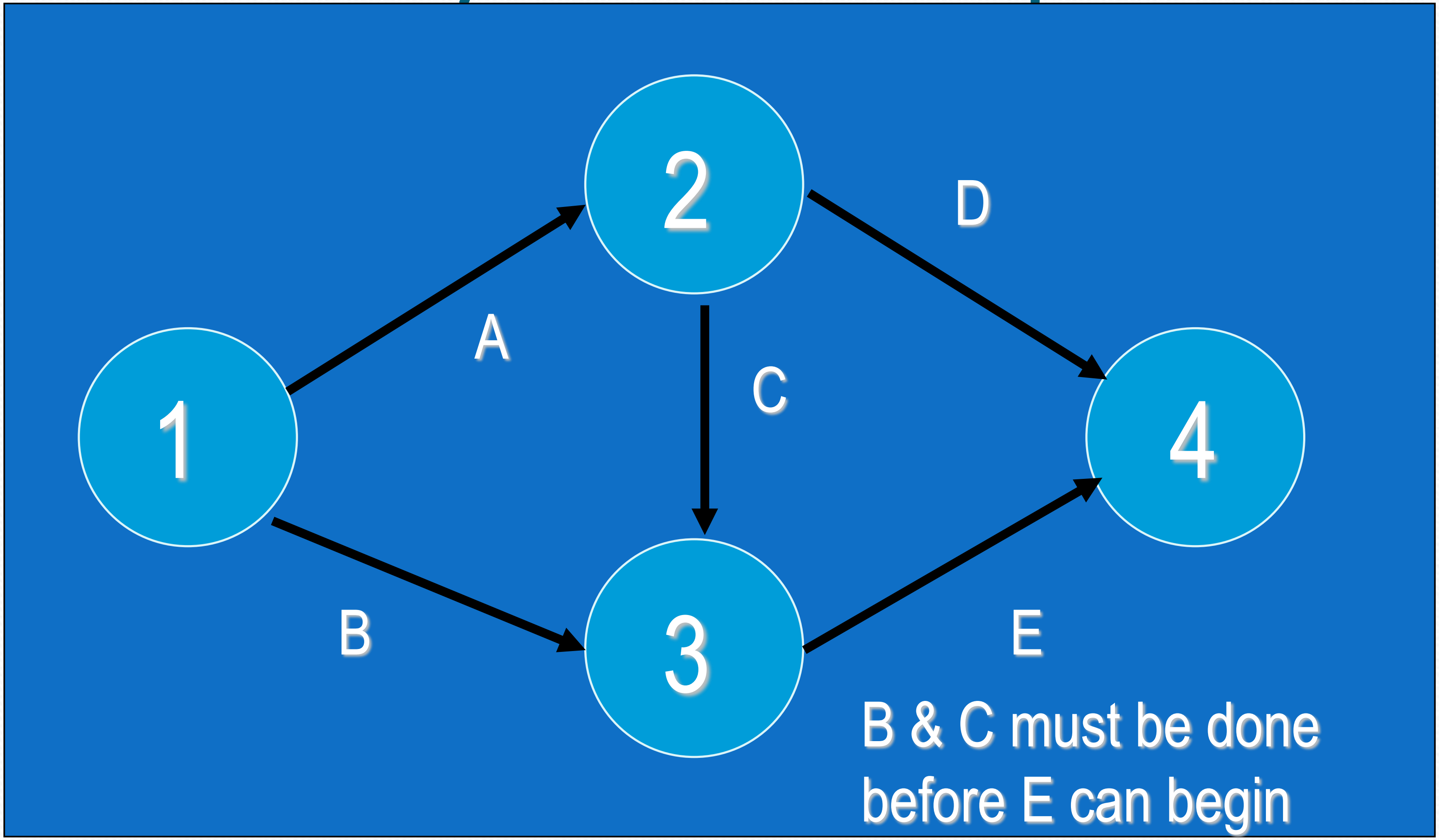


FIGURE 9.10

Activity Relationships



Activity Relationships



Types of Critical Path Methods

- CPM with a Single Time Estimate
  - Used when activity times are known with certainty.
  - Used to determine timing estimates for the project, each activity in the project, and slack time for activities.
- CPM with Three Activity Time Estimates (PERT)
  - Used when activity times are uncertain.
  - Used to obtain the same information as the Single Time Estimate model and probability information.

Prerequisites for Critical Path Methodology

A project must have:

well-defined jobs or tasks whose completion marks the end of the project;

independent jobs or tasks;

and tasks that follow a given sequence.

Steps in the CPM with Single Time Estimate

- 1. Activity Identification.
- 2. Activity Sequencing and Network Construction.
- 3. Determine the critical path.
  - From the critical path all of the project and activity timing information can be obtained.

Critical Path Analysis

- Provides activity information
  - Earliest (ES) & latest (LS) start
  - Earliest (EF) & latest (LF) finish
  - Slack (S): Allowable delay
- Identifies critical path
  - *Longest* path in network
  - *Shortest* time project can be completed
  - Any delay on critical path activities delays project
  - Critical path activities have 0 slack

Earliest Start and Finish Steps

- Begin at starting event and work forward
- ES = 0 for starting activities

  ES is earliest start
- EF = ES + Activity time

  EF is earliest finish
- ES = Maximum EF of all predecessors for non-starting activities

Latest Start and Finish Steps

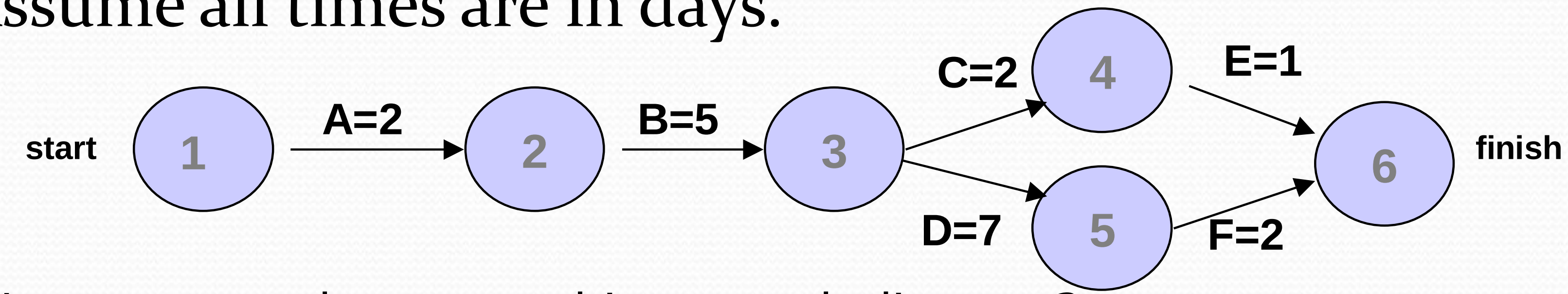
- Begin at ending event and work backward
- LF = Maximum EF for ending activities

  LF is latest finish; EF is earliest finish
- LS = LF - Activity time

  LS is latest start
- LF = Minimum LS of all successors for non-ending activities
- **Slacked Time (S): S = LS-ES = LF-EF**

Simple Example of Determining the Critical Path

- Consider the following project network diagram. Assume all times are in days.



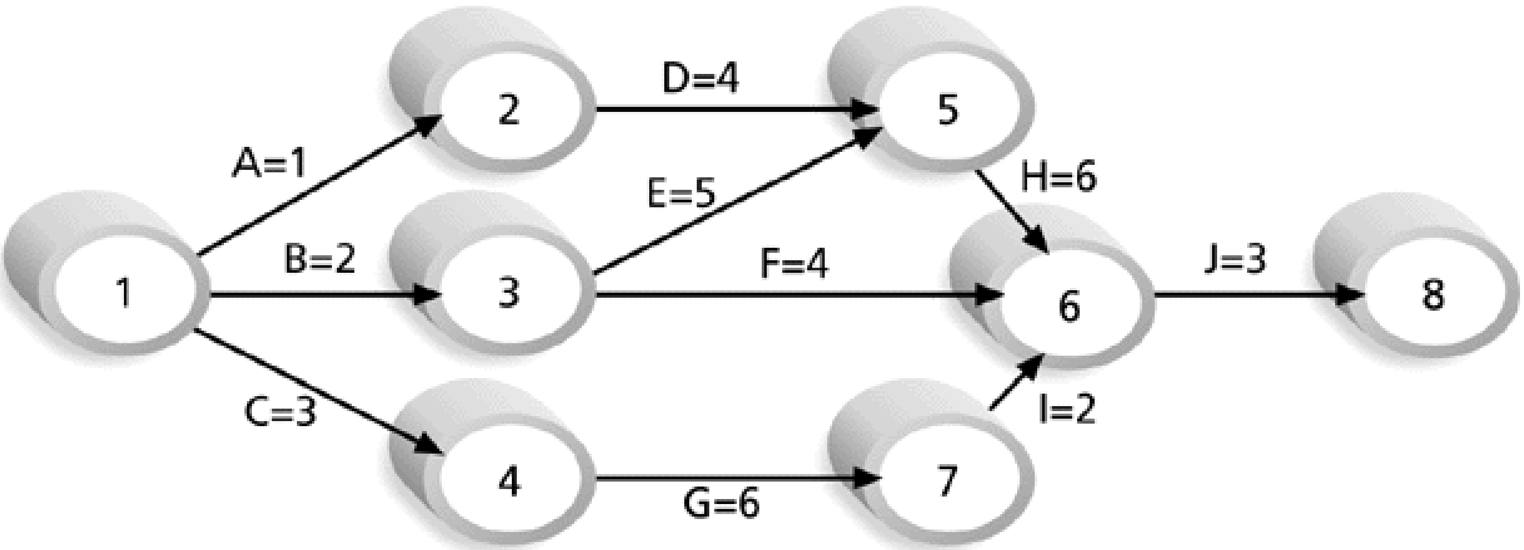
a. How many paths are on this network diagram?

b. How long is each path?

c. Which is the critical path?

d. What is the shortest amount of time needed to complete this project?

Figure 6-8. Determining the Critical Path for Project X



Note: Assume all durations are in days.

Path 1:	A-D-H-J	Length = 1+4+6+3 = 14 days
Path 2:	**B-E-H-J**	**Length = 2+5+6+3 = 16 days**
Path 3:	B-F-J	Length = 2+4+3 = 9 days
Path 4:	C-G-I-J	Length = 3+6+2+3 = 14 days

Since the critical path is the longest path through the network diagram, Path 2, B-E-H-J, is the critical path for Project X.

Example 1. CPM with Single Time Estimate

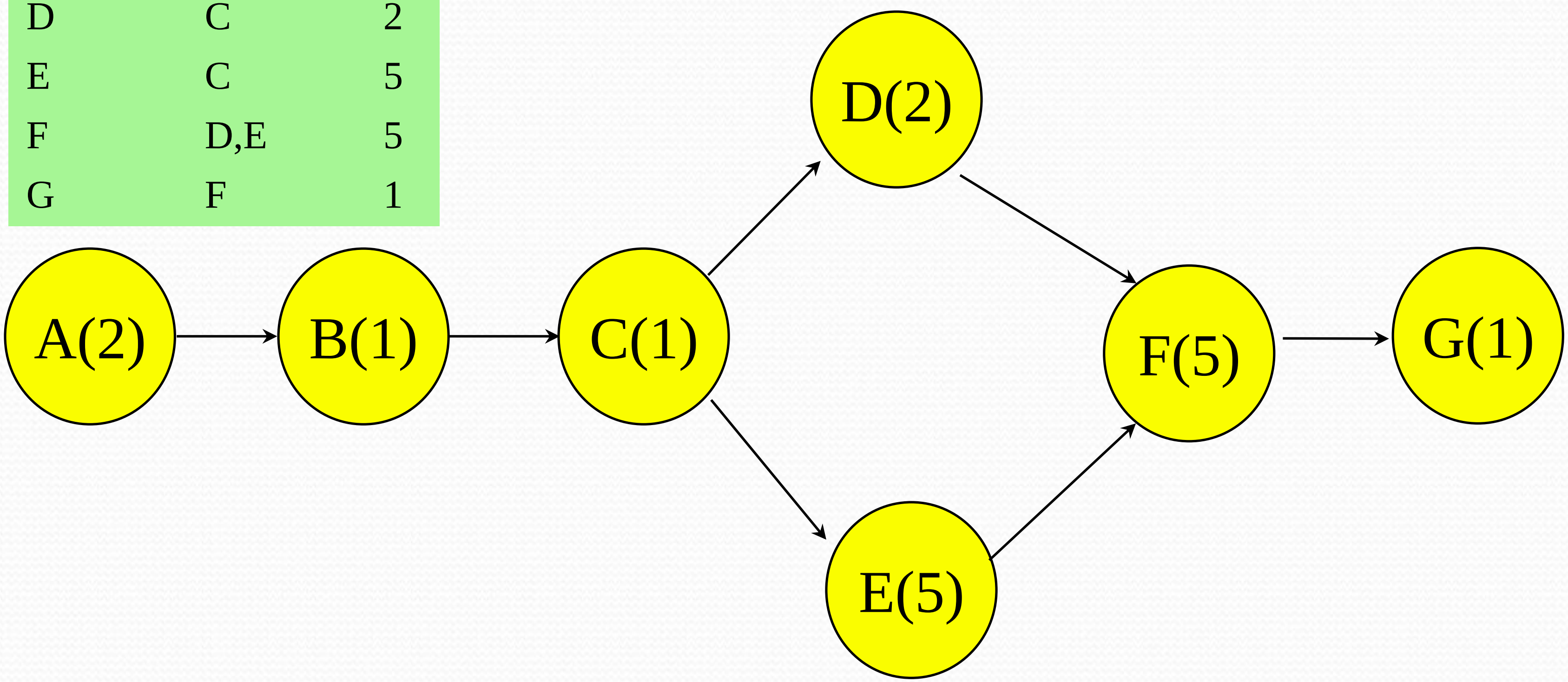
Consider the following consulting project:

Activity	Designation	Immed. Pred.	Time (Weeks)
Assess customer's needs	A	None	2
Write and submit proposal	B	A	1
Obtain approval	C	B	1
Develop service vision and goals	D	C	2
Train employees	E	C	5
Quality improvement pilot groups	F	D, E	5
Write assessment report	G	F	1

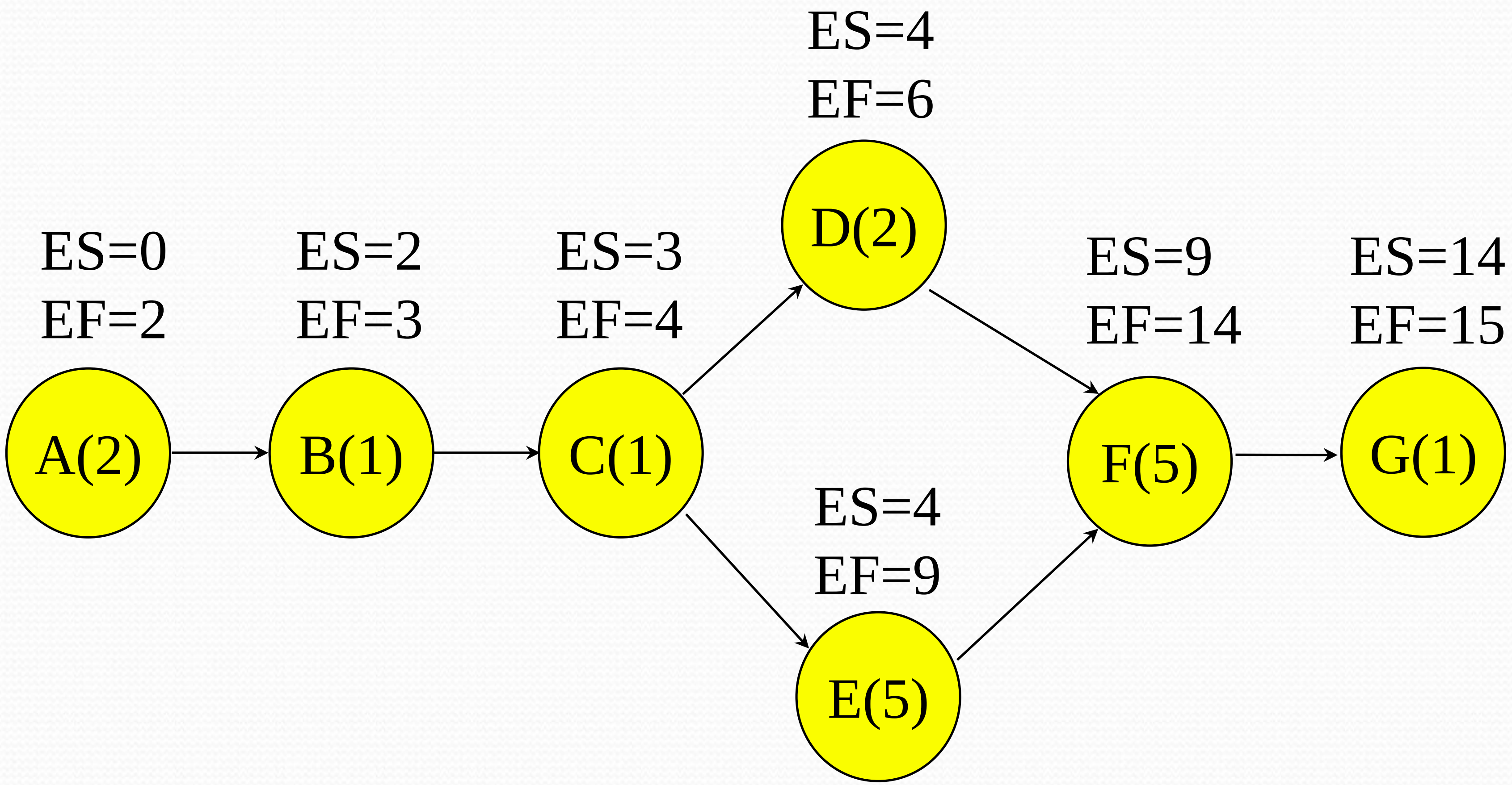
Develop a critical path diagram and determine the duration of the critical path and slack times for all activities

Example 1: First draw the network

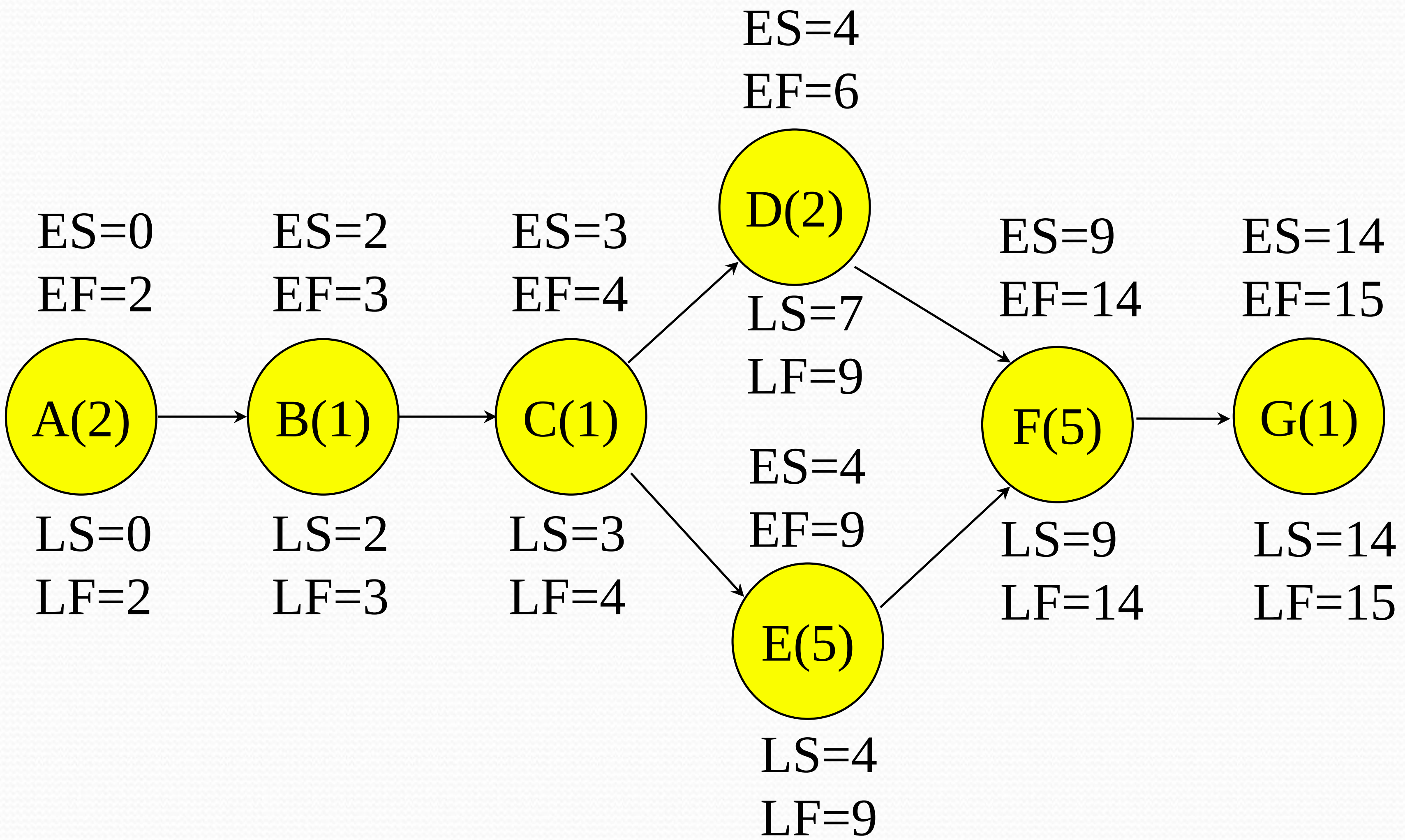
Act.	Imed. Pred.	Time
A	None	2
B	A	1
C	B	1
D	C	2
E	C	5
F	D,E	5
G	F	1



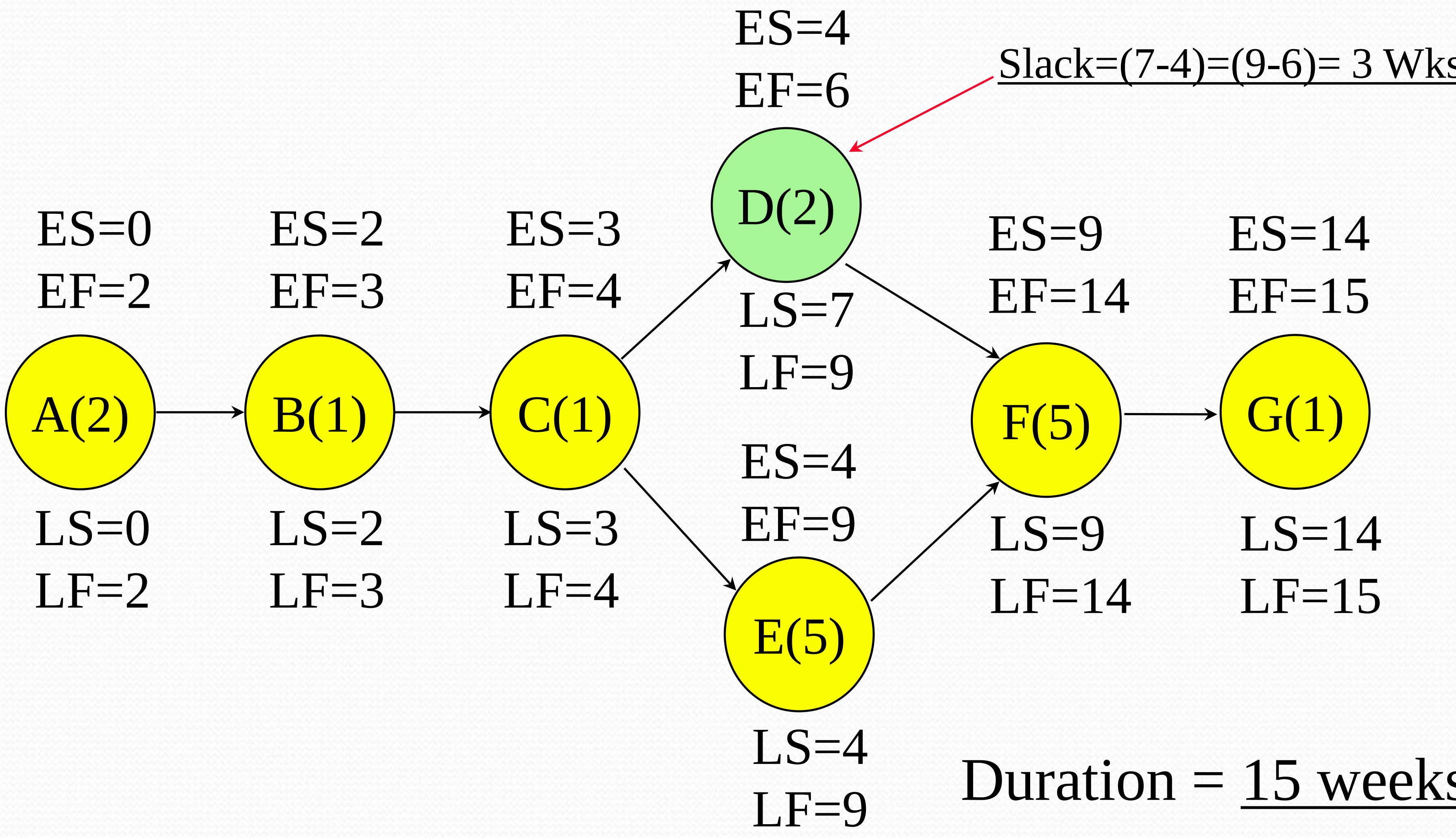
Example 1: Determine early starts and early finish times



Example 1: Determine late starts and late finish times



Example 1: Critical Path & Slack



Duration = 15 weeks

CPM Example

FORWARD PASS

activity	duration	predecessor
A requirements analysis	3 weeks	-
B programming	7 weeks	A
C get hardware	1 week	A
D train users	3 weeks	B, C

Early Start Schedule

schedule A	start	0	finish	0+3	=3
schedule B		3		3+7	=10
& C		3		3+1	=4
schedule D		10		10+3	=13

CPM Example

backward pass

schedule D	finish	13	late start=	13-3	= 10
schedule B		10		10-7	= 3
& C		10		10-1	= 9
schedule A		3		3-3	= 0

Late Start Schedule

slack	A	LF=	3	EF=	3	3-3	= 0
	B	LF=	10	EF=	10	10-10	= 0
	C	LF=	10	EF=	4	10-4	= 6
	D	LF=	13	EF=	13	13-13	= 0

critical path: A-B-D

CPM

- can have more than one critical path

activity	duration	predecessor
A requirements analysis	3 weeks	-
B programming	7 weeks	A
C get hardware	7 weeks	A
D train users	3 weeks	B, C

- critical paths A-B-D

A-C-D

both with duration of 13 weeks

PERT

Uncertain activity completion times

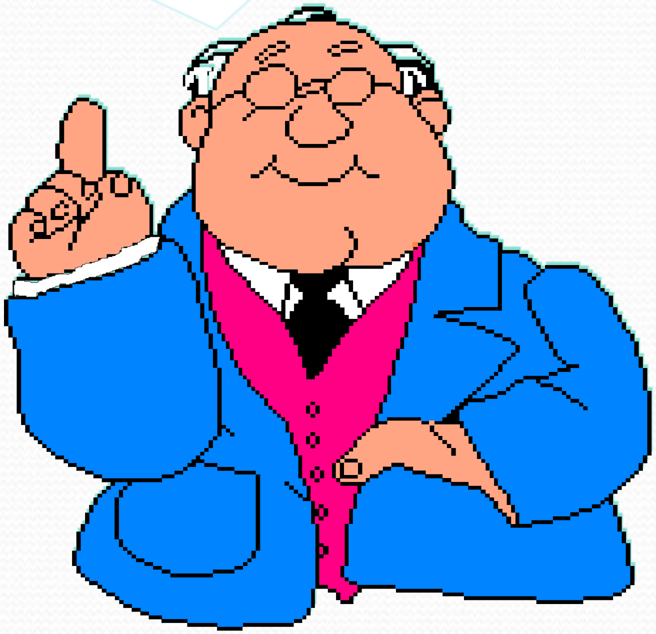
Uncertain activity completion times

- One important extension to the basic network analysis technique relates to uncertain activity completion times. In this extension to the basic method we give, for each activity, not a single completion time but three times:
- **optimistic time (a)** - the completion time if all goes well
- **most likely (modal) time (m)** - the completion time we would expect under normal conditions
- **pessimistic time (b**) - the completion time if things go badly.

Project Times

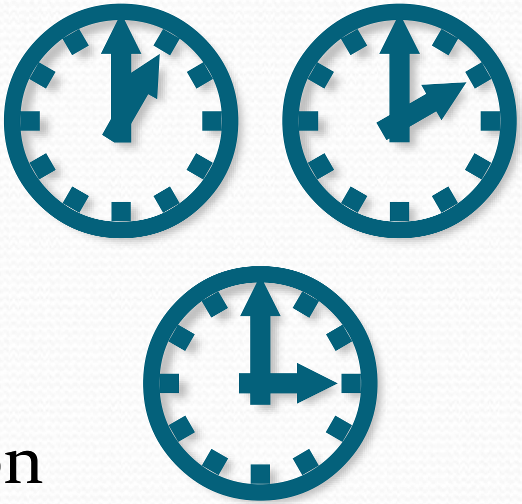
- Expected project time (T)
  - Sum of critical path activity times, t
- Project variance (V)
  - Sum of critical path activity variances, v

Used to obtain probability of project completion!



PERT Activity Times

- 3 time estimates
  - Optimistic times (a)
  - Most-likely time (m)
  - Pessimistic time (b)
- Follow normal or beta distribution
- Expected time: $t = (a + 4m + b)/6$
- Variance of times: $v = (b - a)^2/6$



PERT ACTIVITY TIMES

- These three times are combined into a single number, the expected activity completion time, given by

Expected time=(a + 4m + b)/6.

Essentially we can find answers to questions like:

- What is the probability that the project will take longer than…?
- What is the probability that the project will be finished by…?

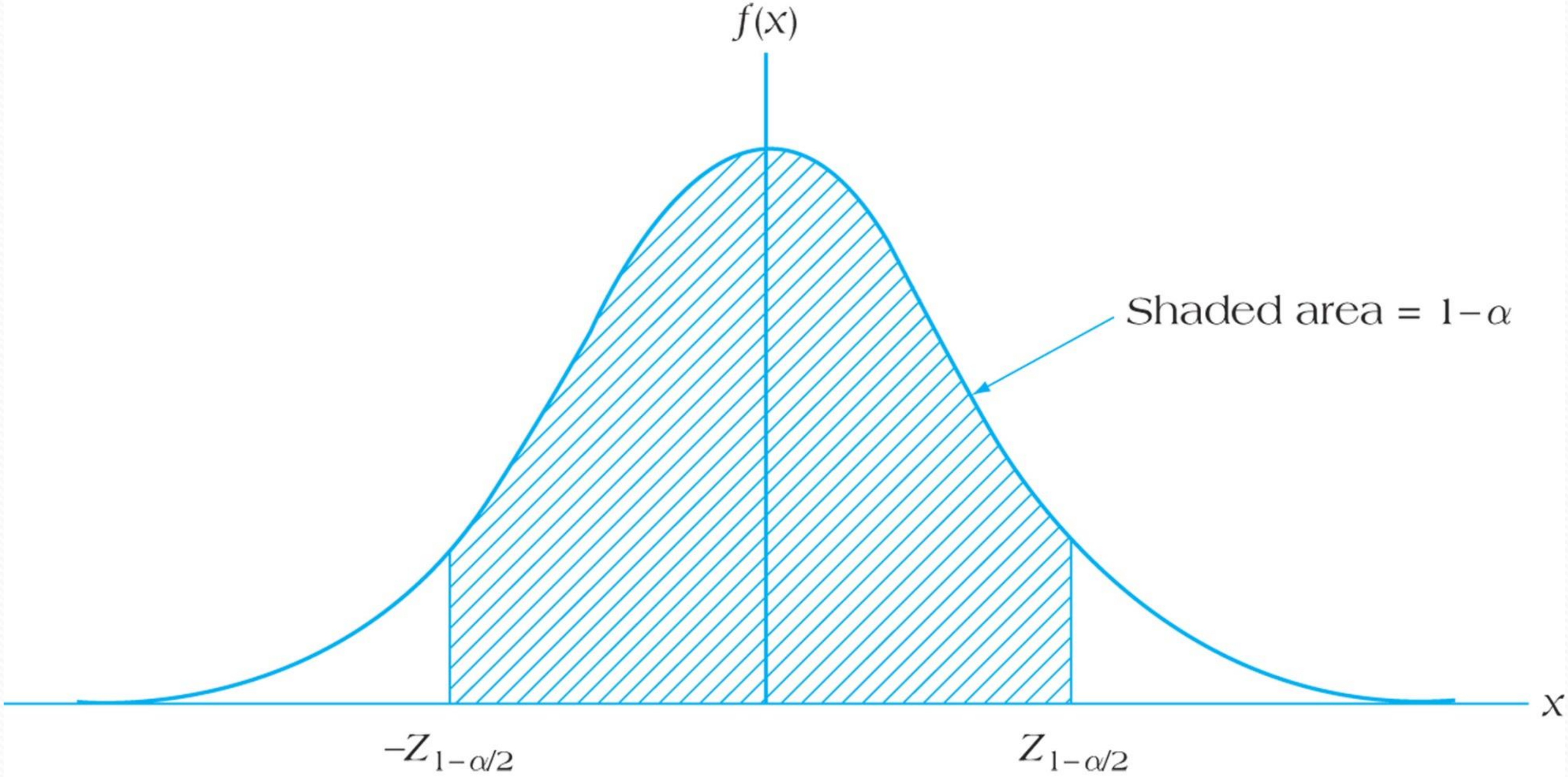


FIGURE 9.14 Symmetrical (Normal) Distribution for Activity Duration Estimation

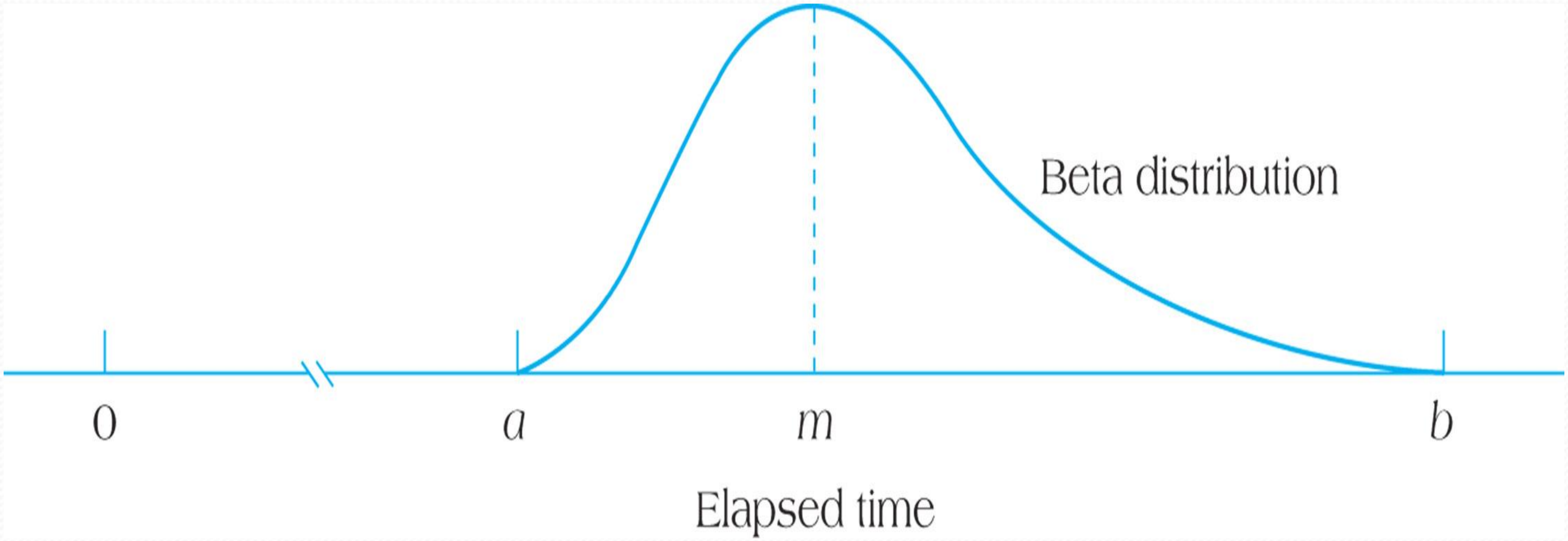


FIGURE 9.15 Asymmetrical (Beta) Distribution for Activity Duration Estimation

Example: CPM with Three Activity Time Estimates

Task	Immediate Predecesors	Optimistic	Most Likely	Pessimistic
A	None	3	6	15
B	None	2	4	14
C	A	6	12	30
D	A	2	5	8
E	C	5	11	17
F	D	3	6	15
G	B	3	9	27
H	E,F	1	4	7
I	G,H	4	19	28

Example 2. Expected Time Calculations

Task	Immediate Predecesors	Expected Time
A	None	7
B	None	5.333
C	A	14
D	A	5
E	C	11
F	D	7
G	B	11
H	E,F	4
I	G,H	18

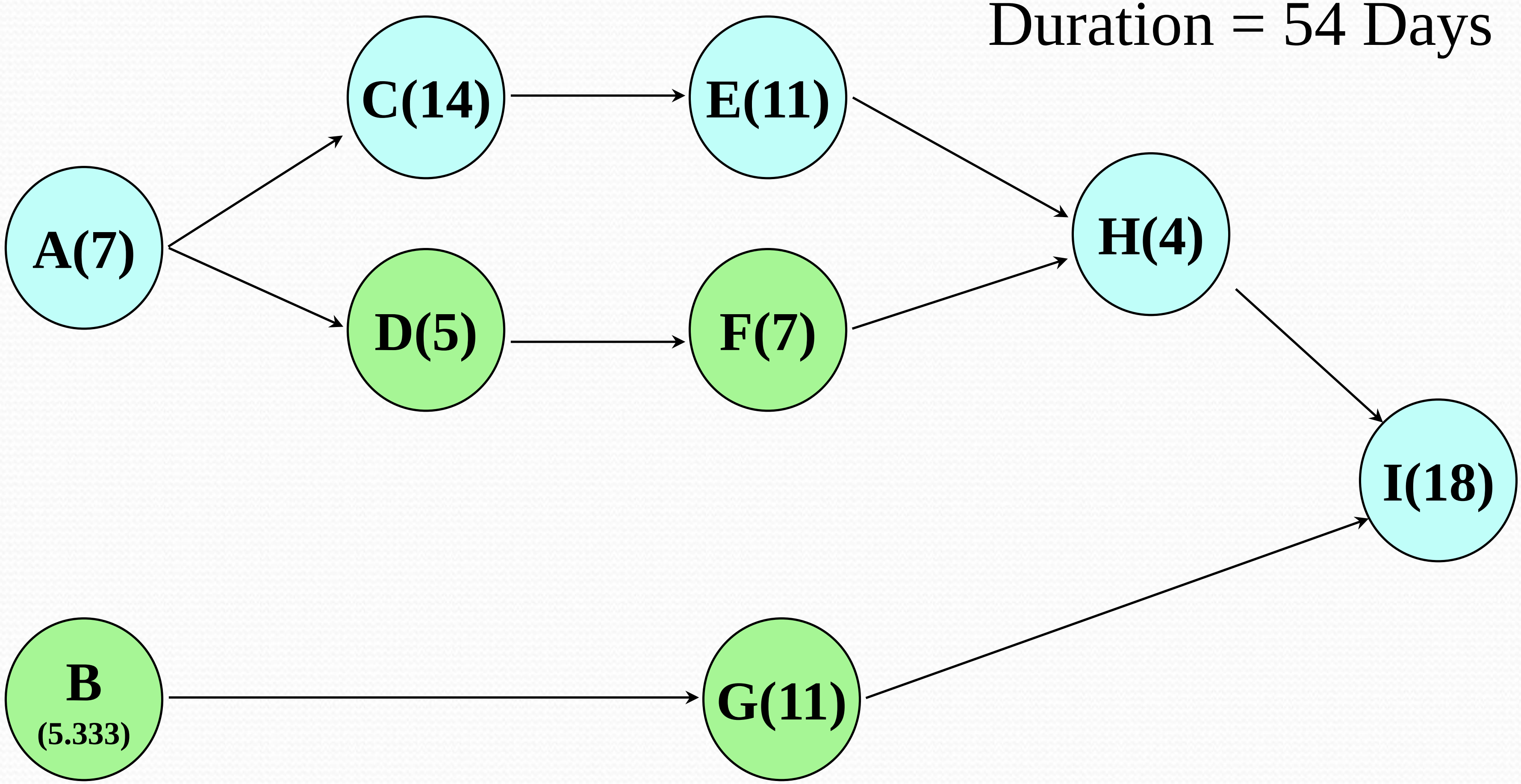
$$ET(A) = \frac{3+4(6)+15}{6}$$

$$ET(A)=42/6=7$$

$$\text{Expected Time} = \frac{\text{Opt. Time} + 4(\text{Most Likely Time}) + \text{Pess. Time}}{6}$$

Example 2. Network

Duration = 54 Days



$$\text{Activity variance, } \sigma^2 = \left(\frac{\text{Pessim. - Optim.}}{6}\right)^2$$

Task	Optimistic	Most Likely	Pessimistic	Variance
A	3	6	15	4
B	2	4	14	
C	6	12	30	16
D	2	5	8	
E	5	11	17	4
F	3	6	15	
G	3	9	27	
H	1	4	7	1
I	4	19	28	16

(Sum the variance *along the critical path*.) $\sum \sigma^2 = 41$

Project Delta Information

Activity	Description	Predecessors	Optimistic	Likely	Pessimistic
A	Contact signing	None	3	4	11
B	Questionnaire design	A	2	5	8
C	Target Market ID	A	3	6	9
D	Survey sample	B, C	8	12	20
E	Develop presentation	B	3	5	12
F	Analyze results	D	2	4	7
G	Demographic analysis	C	6	9	14
H	Presentation to client	E, F, G	1	2	4

Project Delta Information

Project Delta

Activity	**Description**	**Predecessors**	**Estimated Duration**
A	Contract signing	None	5
B	Questionnaire design	A	5
C	Target market ID	A	6
D	Survey sample	B, C	13
E	Develop presentation	B	6
F	Analyze results	D	4
G	Demographic analysis	C	9

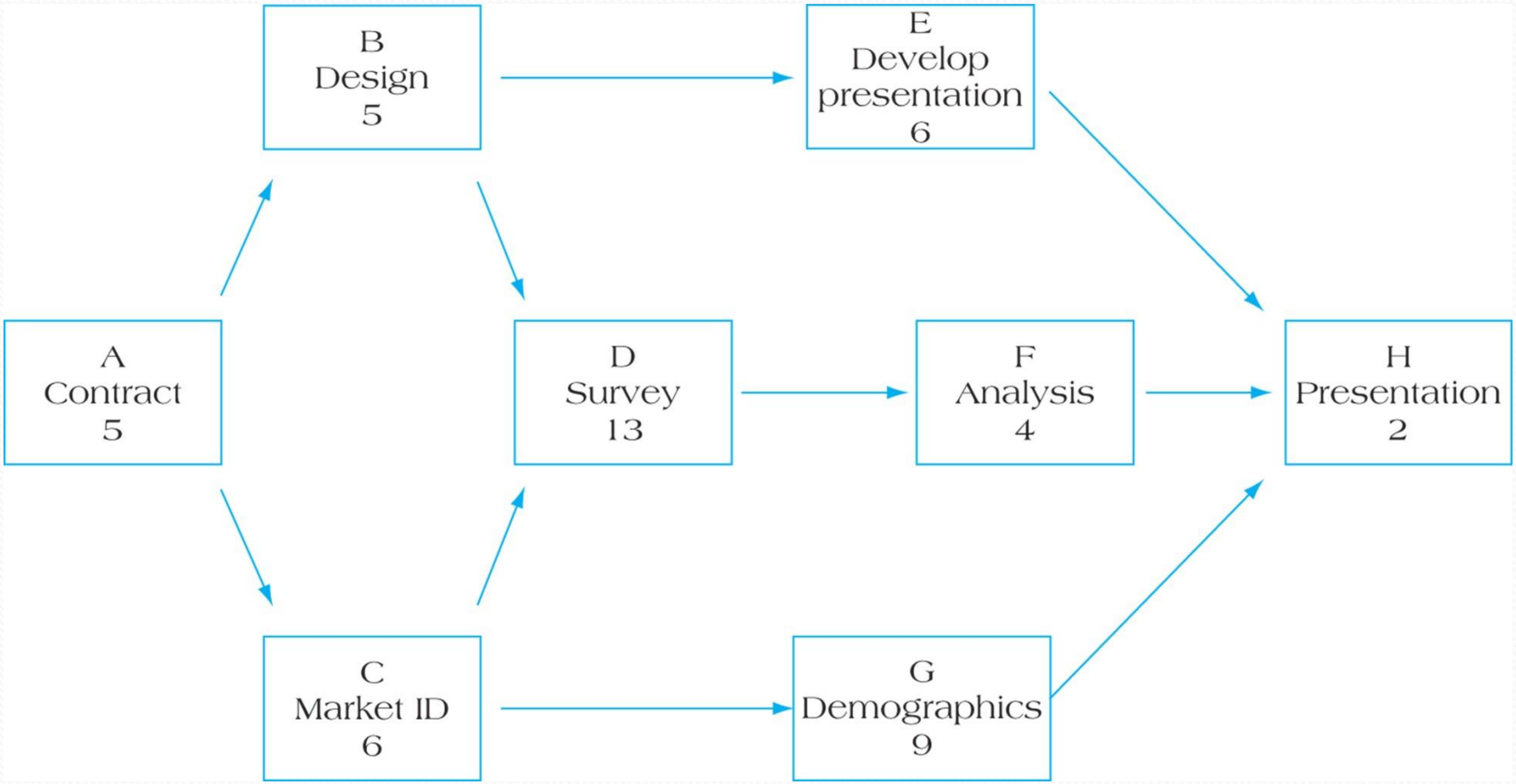


FIGURE 9.16 Partial Project Activity Network with Task Durations

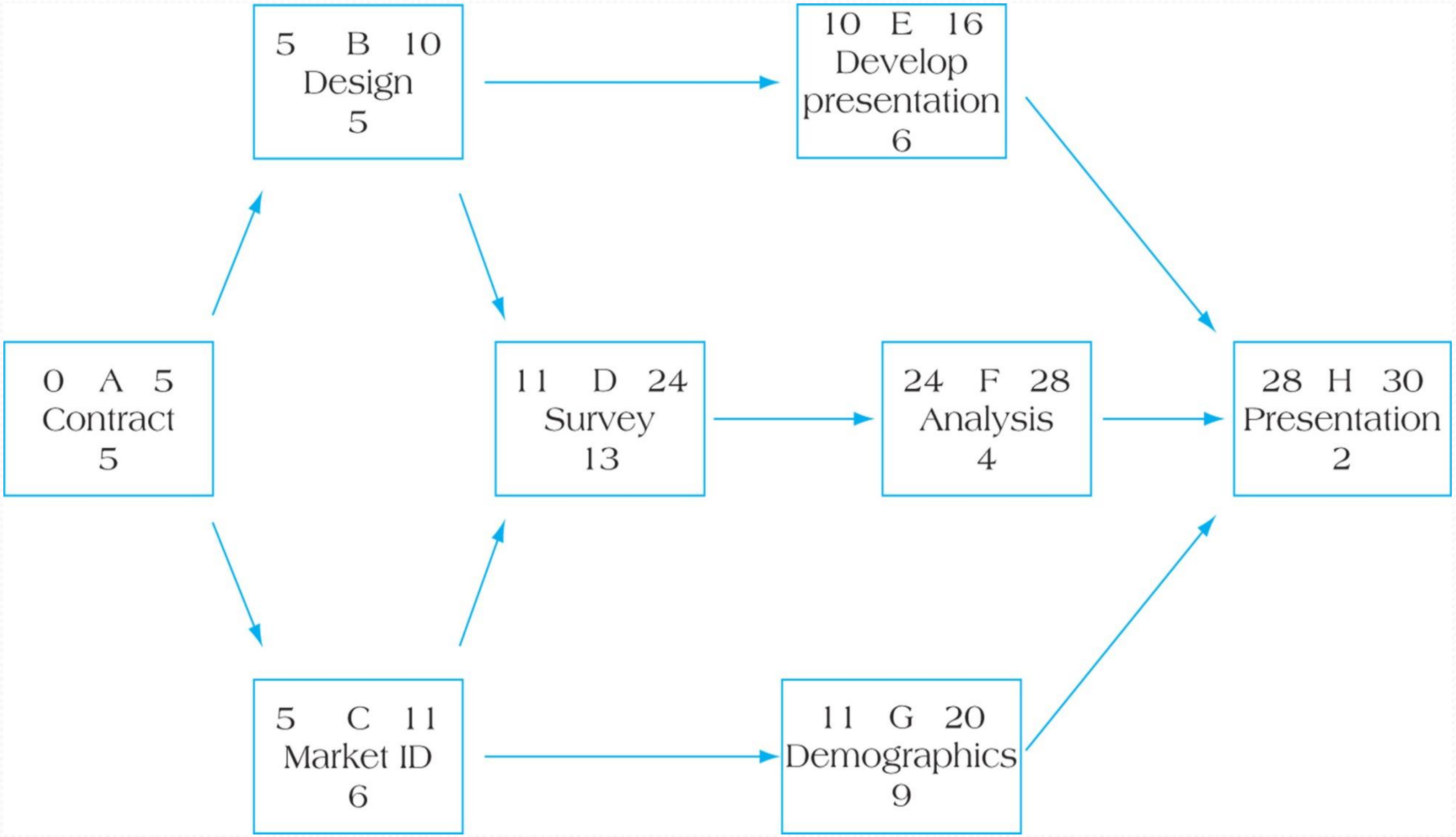


FIGURE 9.18 Activity Network with Forward Pass

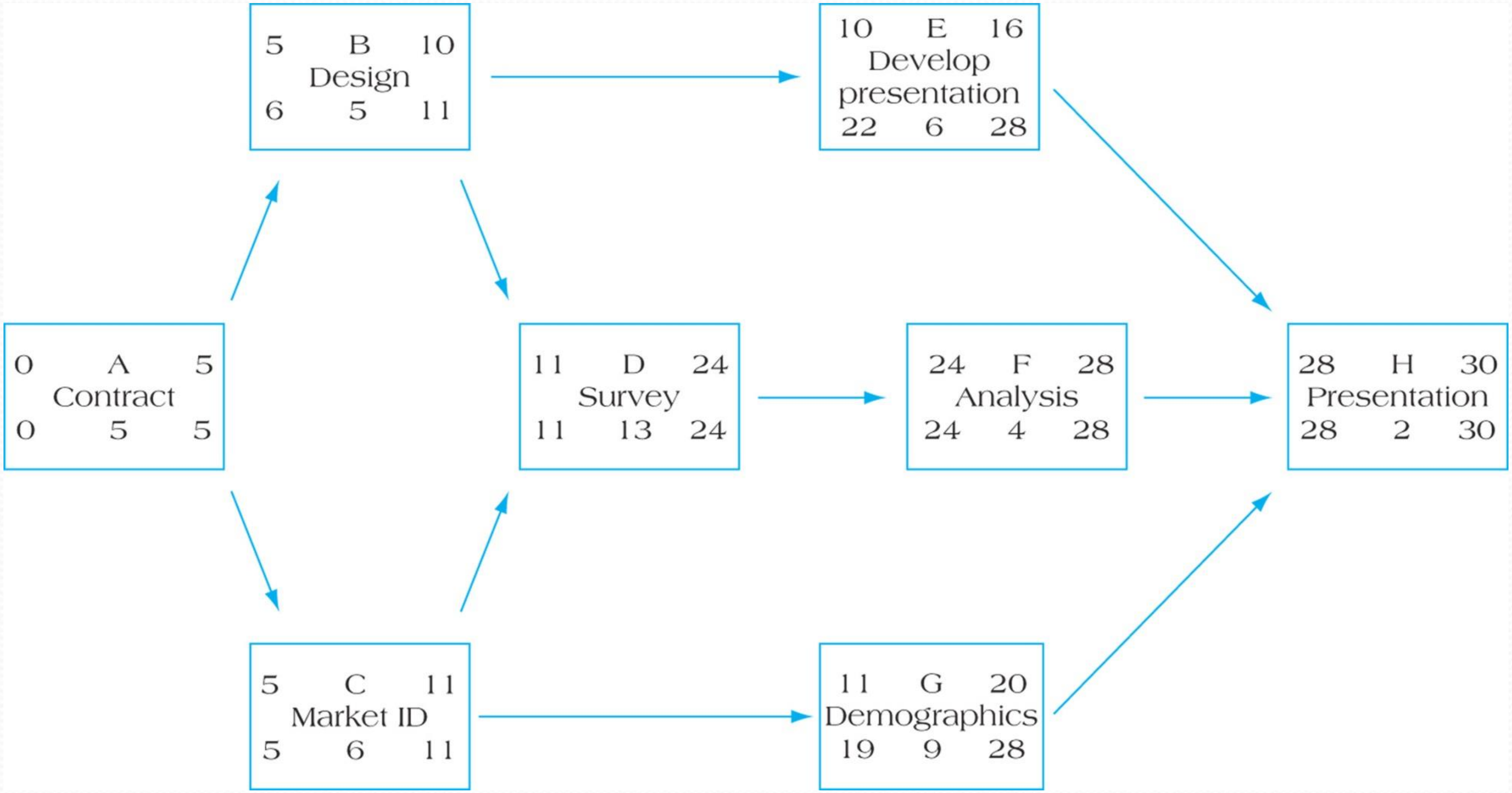


FIGURE 9.19 Activity Network with Backward Pass

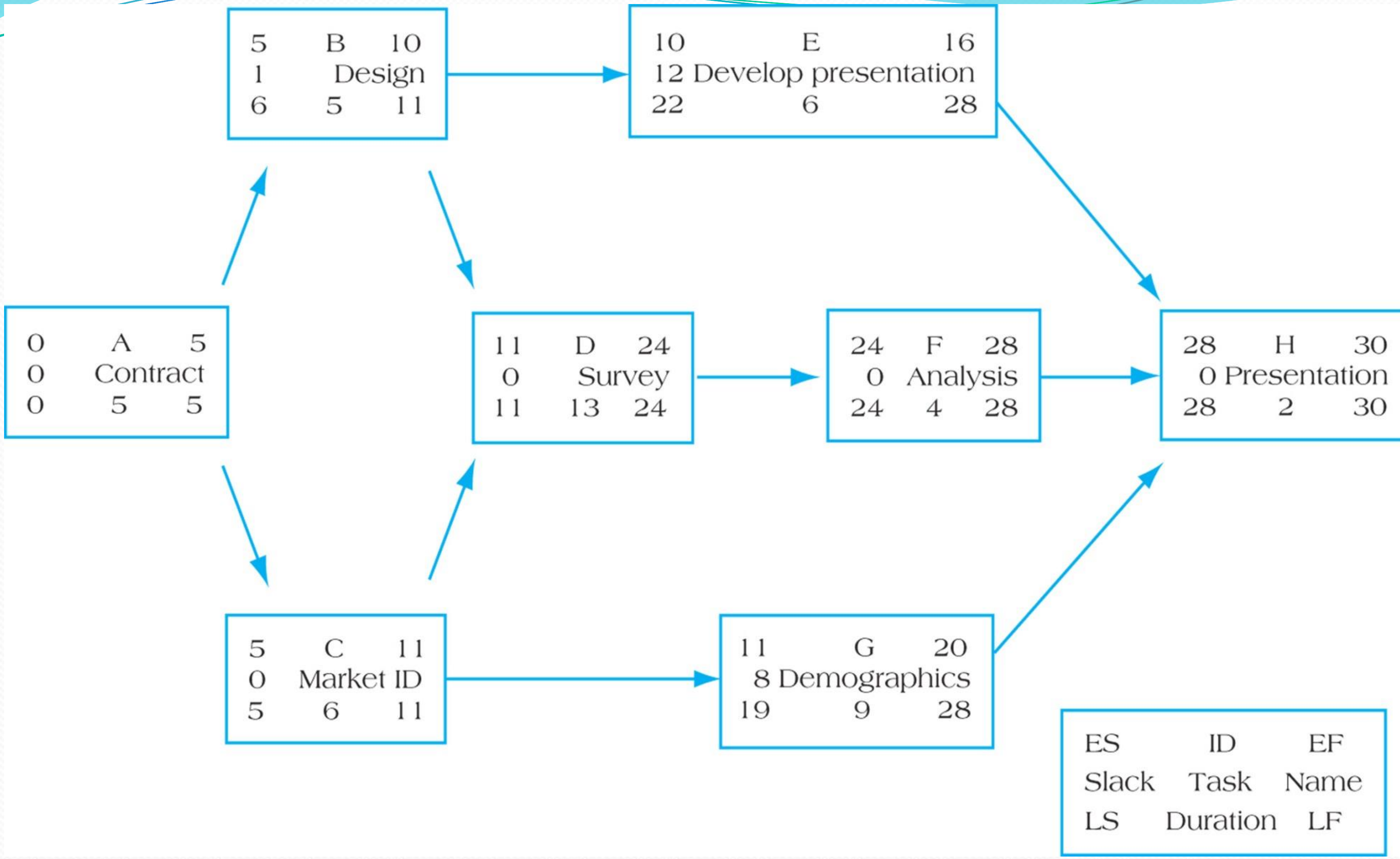


FIGURE 9.20 Project Network with Activity Slack and Critical Path
Note: Critical path is indicated with bold arrows.

Reducing the Critical Path

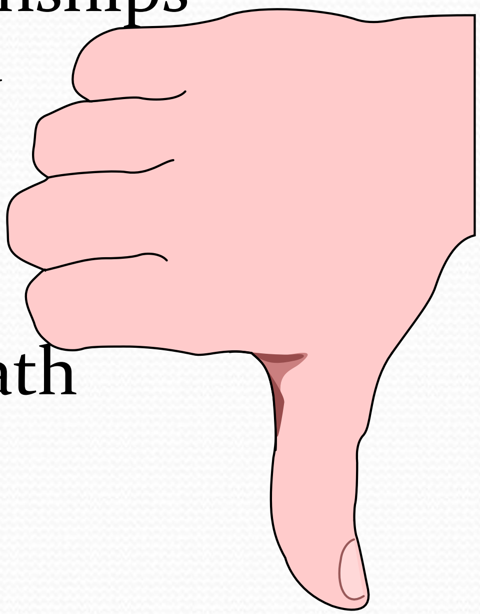
- Eliminate tasks on the CP
- Convert serial paths to parallel when possible
- Overlap sequential tasks
- Shorten the duration on critical path tasks
- Shorten
  - early tasks
  - longest tasks
  - easiest tasks
  - tasks that cost the least to speed up

Advantages of PERT/CPM

- Networks generated provide valuable project documentation and graphically point out who is responsible for various project activities
- Applicable to a wide variety of projects and industries
- Useful in monitoring not only schedules, but costs as well

Limitations of PERT/CPM

- Assumes clearly defined, independent, & stable activities
- Specified precedence relationships
- Activity times (PERT) follow beta distribution
- Subjective time estimates
- Over-emphasis on critical path



Probability Analysis

Probabilistic Network Analysis

Determine probability that project is completed within specified time

$$Z = \frac{x - \mu}{\sigma}$$

where

$\mu = t_p$ = ***project mean time***

σ = ***project standard deviation***

x = ***proposed project time***

Z = ***number of standard deviations x is from mean***

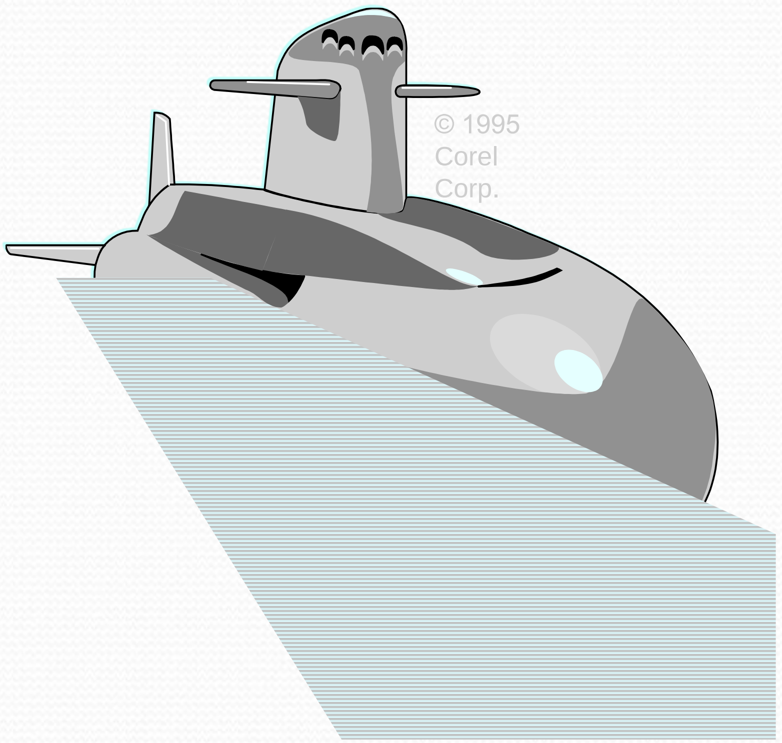
Calculation

- **D= desired completion date**
- **Te= expected completion date**
- **Sum of variance along the critical path = project variance**

$$Z = \frac{D - T_E}{\sqrt{\sum \sigma_{cp}^{2}}}$$

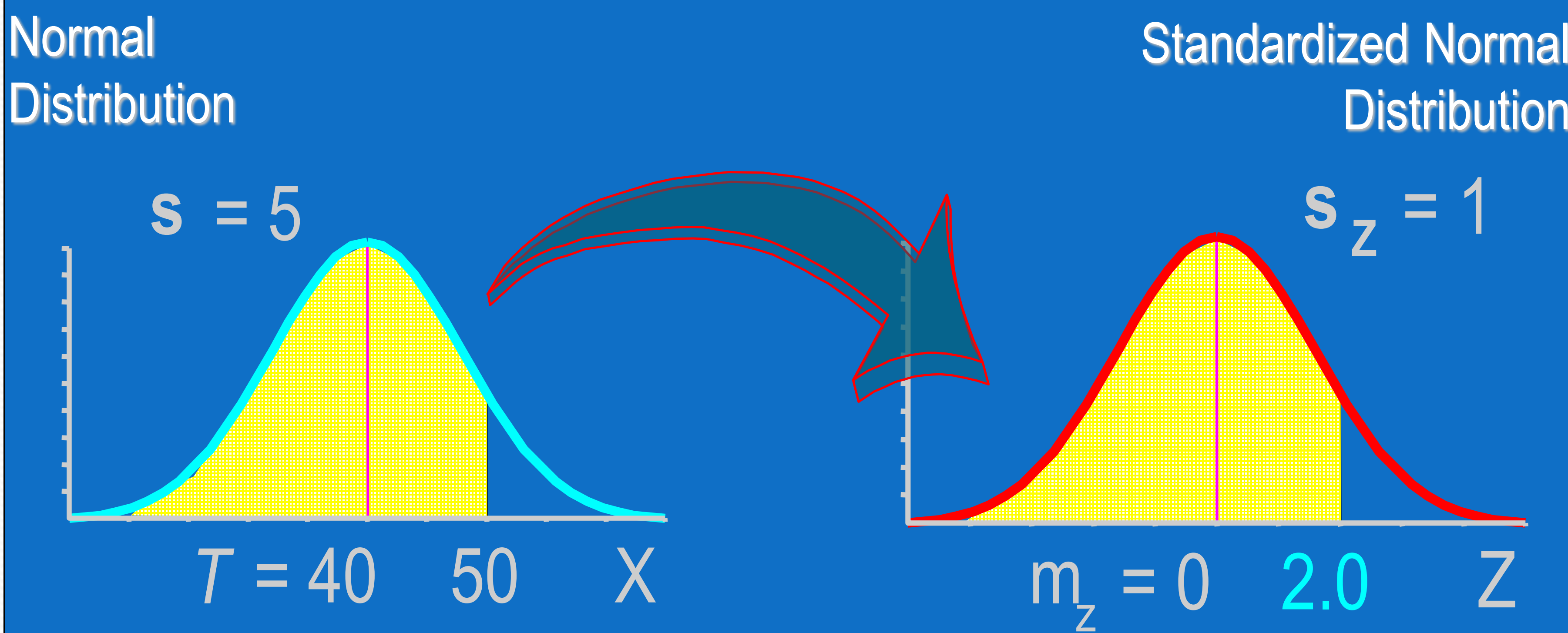
PERT Probability Example

You're a project planner for General Dynamics. A submarine project has an expected completion time of 40 weeks, with a standard deviation of 5 weeks. What is the probability of finishing the sub in 50 weeks or less?



Converting to Standardized Variable

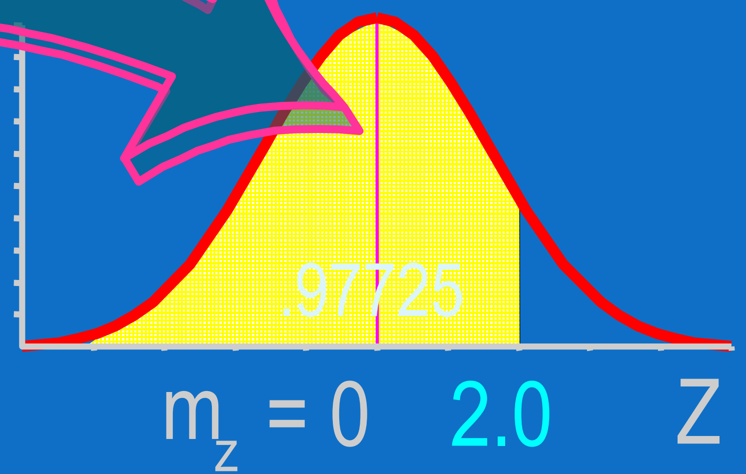
$$Z = \frac{D - T}{s} = \frac{50 - 40}{5} = 2.0$$



Obtaining the Probability

Standardized Normal Probability Table (Portion)

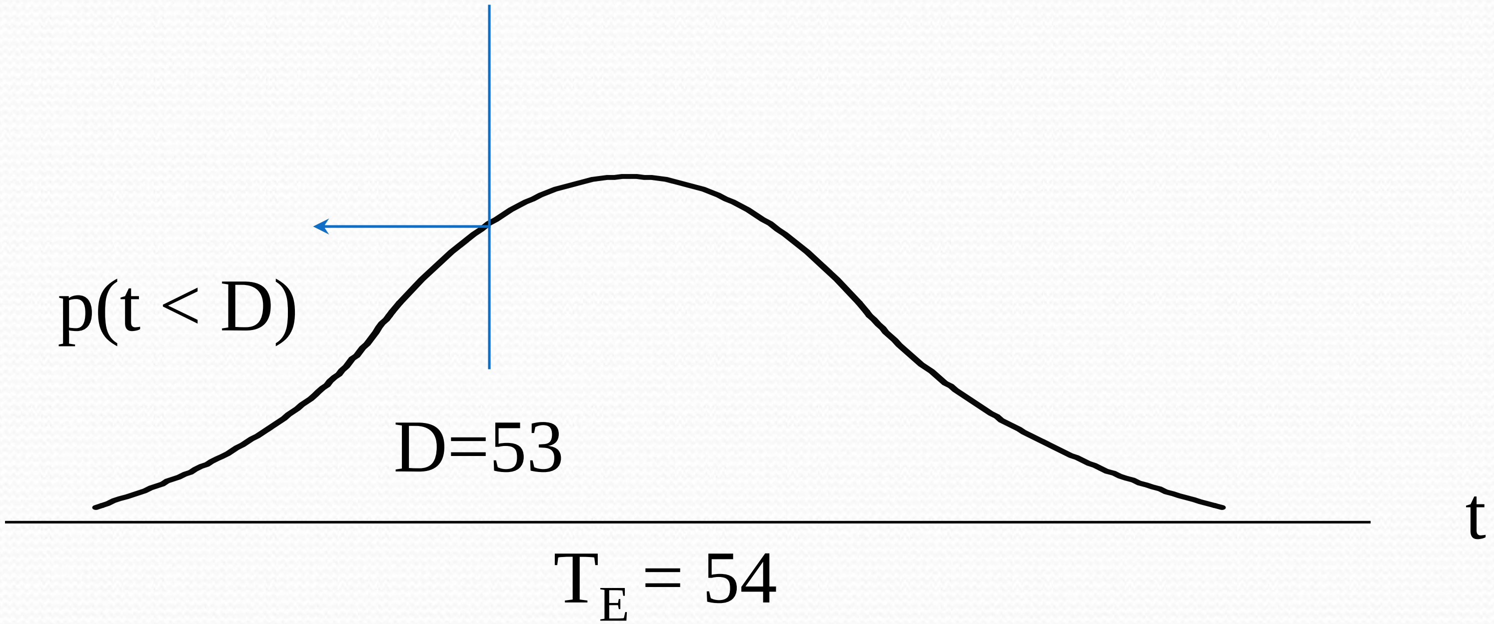
Z	.00	.01	.02
0.0	.50000	.50399	.50798
:	:	:	:
2.0	.97725	.97784	.97831
2.1	.98214	.98257	.98300



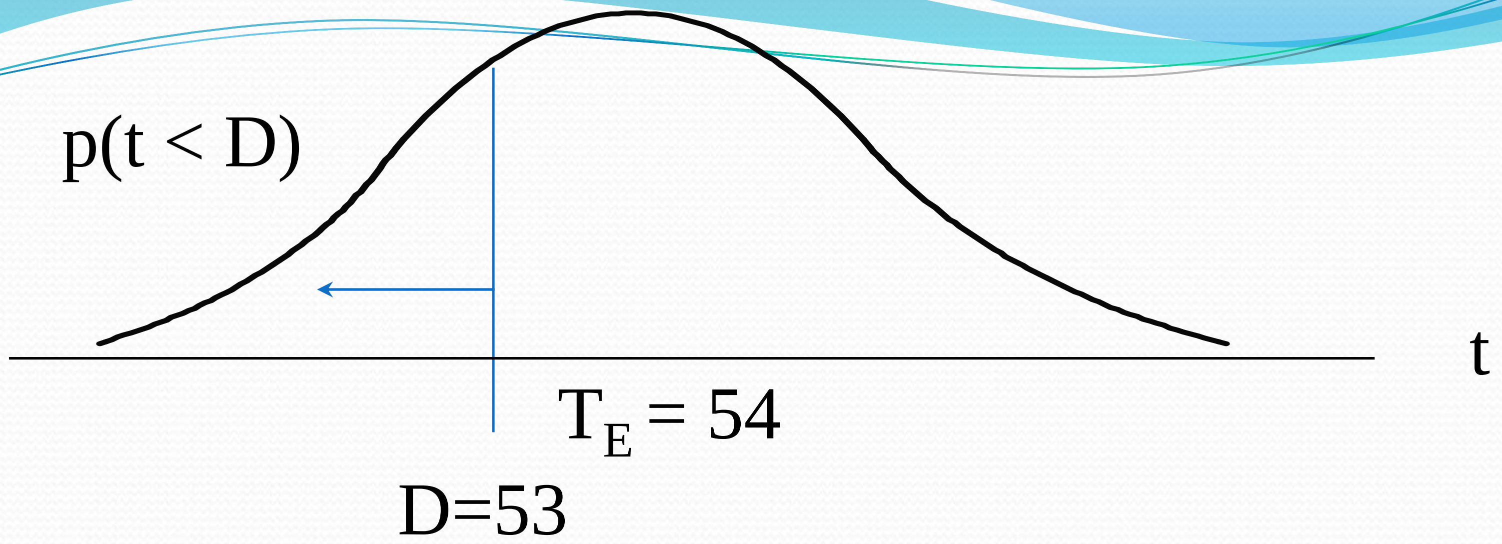
Probabilities in body

Example Probability Exercise (example 2 previous slights pp: 50)

What is the probability of finishing a project in
less than 53 days if expected completion time is 54 weeks?



$$Z = \frac{D - T_E}{\sqrt{\sum \sigma_{cp}^{2}}}$$

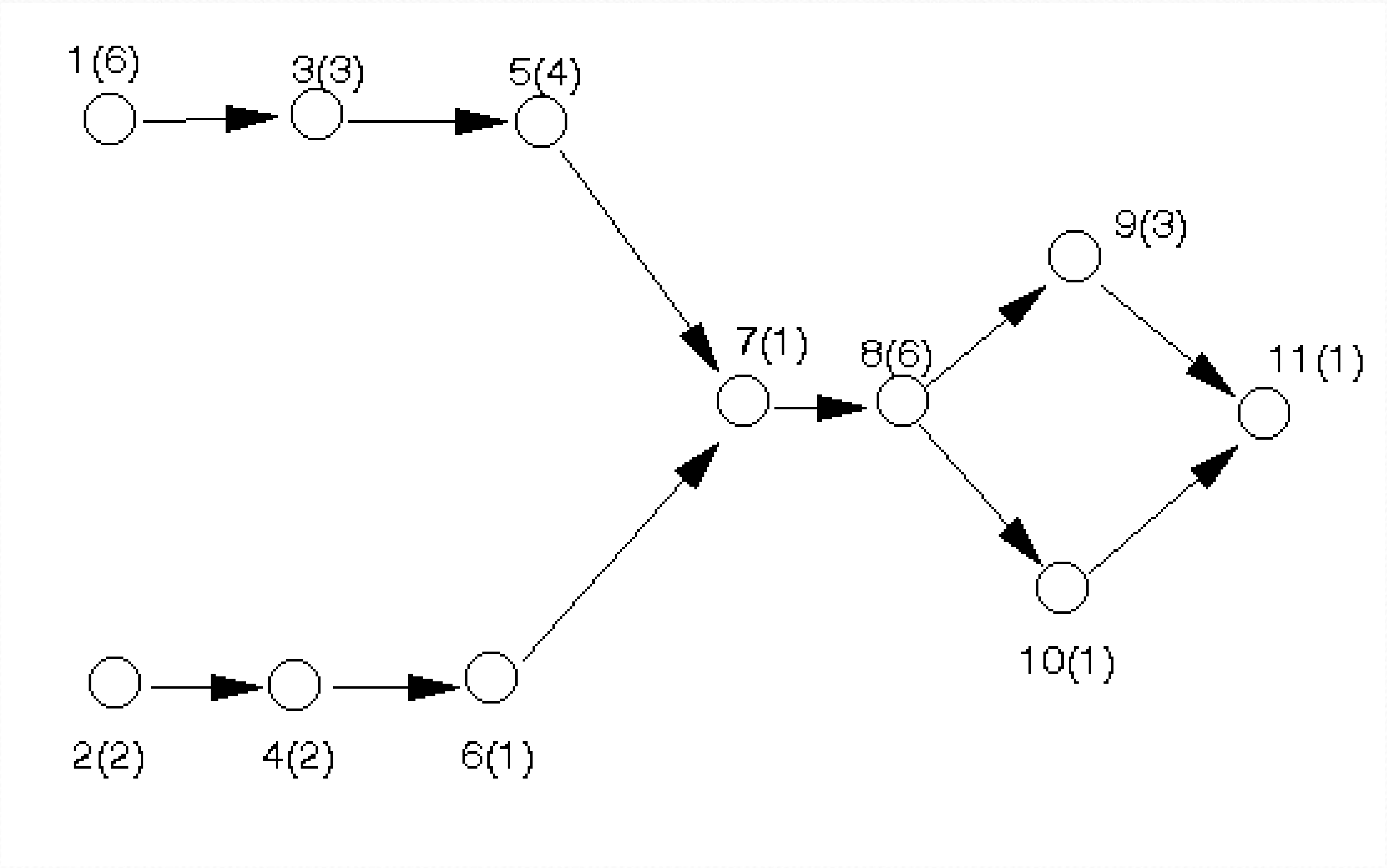


$$Z = \frac{D - T_E}{\sqrt{\sum \sigma_{cp}^{2}}} = \frac{53 - 54}{\sqrt{41}} = -.156$$

$p(Z < -0.156) = 0.436$, or 43.6 %

There is a 43.6% probability that this project will be completed in less than 53 weeks.

Example



we have an optimistic time equal to one-half the most likely time and a pessimistic time equal to twice the most likely time.

Activity Number	Activity Name	Immediate Predecessor (list number/name, separated by ',')	Optimistic time (a)	Most likely time (m)	Pessimistic time (b)
1	1		3	6	12
2	2		1	2	4
3	3	1	1.5	3	6
4	4	2	1	2	4
5	5	3	2	4	8
6	6	4	0.5	1	2
7	7	5,6	0.5	1	2
8	8	7	3	6	12
9	9	8	1.5	3	6
10	10	8	0.5	1	2
11	11	9,10	0.5	1	2

This standard deviation is calculated

Activity Name	On Critical Path	Activity Mean Time	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Slack (LS-ES)	Activity Time Distribution	Standard Deviation
1	Yes	6.5	0	6.5	0	6.5	0	3-Time estimate	1.5
2	no	2.1667	0	2.1667	8.6667	10.8333	8.6667	3-Time estimate	0.5
3	Yes	3.25	6.5	9.75	6.5	9.75	0	3-Time estimate	0.75
4	no	2.1667	2.1667	4.3333	10.8333	13	8.6667	3-Time estimate	0.5
5	Yes	4.3333	9.75	14.0833	9.75	14.0833	0	3-Time estimate	1
6	no	1.0833	4.3333	5.4167	13	14.0833	8.6667	3-Time estimate	0.25
7	Yes	1.0833	14.0833	15.1667	14.0833	15.1667	0	3-Time estimate	0.25
8	Yes	6.5	15.1667	21.6667	15.1667	21.6667	0	3-Time estimate	1.5
9	Yes	3.25	21.6667	24.9167	21.6667	24.9167	0	3-Time estimate	0.75
10	no	1.0833	21.6667	22.75	23.8333	24.9167	2.1667	3-Time estimate	0.25
11	Yes	1.0833	24.9167	26	24.9167	26	0	3-Time estimate	0.25
Project	Completion	Time	=	26.00	weeks				
Number of	Critical	Path(s)	=	1					

What is the probability of completing the project in less than 25 days?

Activity Name	On Critical Path	Activity Mean Time	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Slack (LS-ES)	Activity Time Distribution	Standard Deviation
1	Yes	6.5	0	6.5	0	6.5	0	3-Time estimate	1.5
2	no	2.1667	0	2.1667	8.6667	10.8333	8.6667	3-Time estimate	0.5
3	Yes	3.25	6.5	9.75	6.5	9.75	0	3-Time estimate	0.75
4	no	2.1667	2.1667	4.3333	10.8333	13	8.6667	3-Time estimate	0.5
5	Yes	4.3333	9.75	14.0833	9.75	14.0833	0	3-Time estimate	1
6	no	1.0833	4.3333	5.4167	13	14.0833	8.6667	3-Time estimate	0.25
7	Yes	1.0833	14.0833	15.1667	14.0833	15.1667	0	3-Time estimate	0.25
8	Yes	6.5	15.1667	21.6667	15.1667	21.6667	0	3-Time estimate	1.5
9	Yes	3.25	21.6667	24.9167	21.6667	24.9167	0	3-Time estimate	0.75
10	no	1.0833	21.6667	22.75	23.8333	24.9167	2.1667	3-Time estimate	0.25
11	Yes	1.0833	24.9167	26	24.9167	26	0	3-Time estimate	0.25
Project	Completion	Time	=	26.00	weeks				
Number of	Critical	Path(s)	=	1					

Sum of variances in critical path:

$1,5^2+0,75^2+1^2+0,25^2+1,5^2+0,75^2+0,25^2=6,75$

$z=(25-26)/(6,75)^{0,5}=-0,3849$ $P(z<-0,3849)=0,3501$