ISTANBUL TECHNICAL UNIVERSITY

Computer Engineering Department

BLG354E Signals and Systems for CE

Midterm Exam: Spring 2017 Term

Date: 10 April 2017

Time: 17:30-19:20

Rooms: EEB 5304, EEB 5305 and EEB 5307

Student Name:

Student ID#: SOLUTIONS

- **Closed Book and Closed Notes; 4 problems (105 points total); 110 minutes.**
- **YOUR FINAL RESULTS to Problems should be in the BOXES provided. OTHERWISE, your solution will NOT be graded.**
- **Be sure to justify your answers and show all your work, just writing the result does NOT get you any credit.**
- **Please make sure your full name is on all sheets. DO IT NOW!**
- **Make sure you return all the sheets of the exam before you leave the exam.**
- **Cheating will be penalized according to the university disciplinary policy.**
- **Good luck!**

Please leave the rest of this page blank for use by the graders:

Problem	No. of Points	Score	Grader
1	30		
2	25		
3	25		
4	25		
Total	105		

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Problem 1: Some of the following are True (T) / False (F) questions. Each part is 3 points.

1.1. A periodic signal can be written as a sum of (possibly infinitely many) sinusoids: T/F: T

1.2. The signal values of the impulse response of a causal LTI system must be real and positive: T/F: F

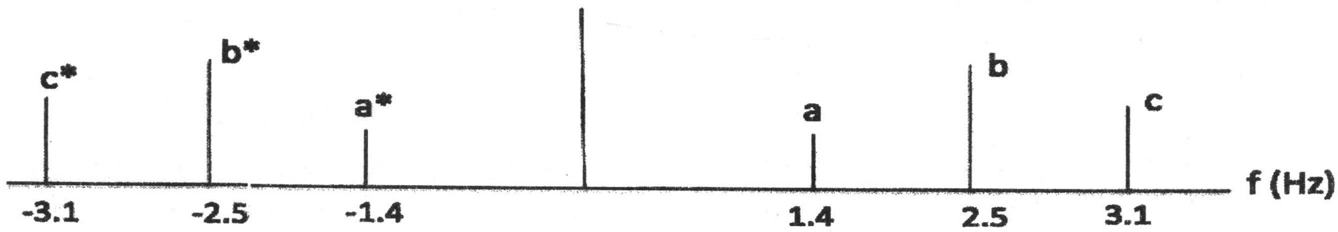
1.3. A sinusoid at frequency 1 Hz can be written as the sum of two sinusoids at frequency 0.5 Hz: T/F: F

1.4. Multiplying with $e^{j\theta}$ has an effect of rotating a phasor by θ degrees in counter-clockwise direction: T/F: T

1.5 Fourier Series coefficients of the signal, $\sin(5\pi t + \pi/4)$, is equal to zero except for only one coefficient a_1 : T/F T

1.6. What is the fundamental PERIOD of the signal $x(t)$ whose spectrum is given below? State which harmonics exist in this signal: $f_0 = 0.1$ Hz $\Rightarrow T_0 = 10$ sec $\quad k = 14, 25, 31$

Spectrum of x(t) **Complex amplitudes: a, b, c**

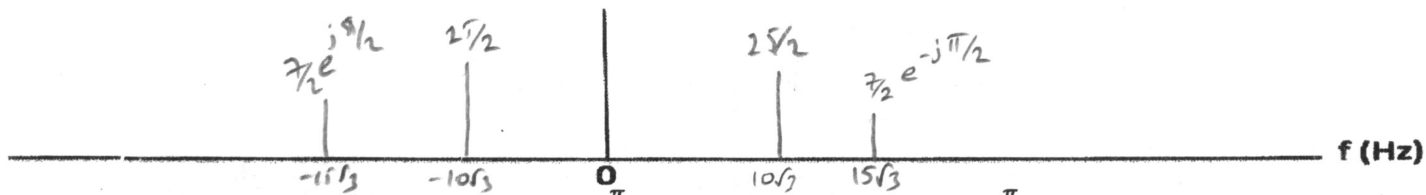


1.7. Express $u[n] - u[n-3]$ in terms of impulse sequences:

$= \delta[n] + \delta[n-1] + \delta[n-2]$

1.8. Let $x(t) = 7\sin(30\sqrt{3}\pi t) + 25\cos(5\sqrt{48}\pi t)$. Is this signal periodic? Justify your answer. If it is periodic, what is the fundamental frequency? Plot spectrum of x(t).

Spectrum of x(t)

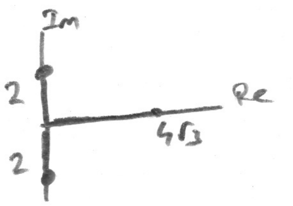


1.9. Add two sinusoids: $x_1(t) = 2\cos(10\pi t - \frac{\pi}{2})$ and $x_2(t) = 4\cos(10\pi t + \frac{\pi}{6})$ to obtain another sinusoid. Use phasor addition.

$X_1 = 2e^{-j\pi/2}$

$X_2 = 4e^{j\pi/6}$

$X_1 + X_2 = 2\sqrt{3}\, e^{j0}$



$x_1(t) + x_2(t) = 2\sqrt{3}\cos(10\pi t)$

1.10. Give examples of FIR filter coefficients to (i) smooth an input signal; (ii) detect edges in an input signal.

(i) b_k: $\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$ or $\{\frac{1}{2}, \frac{1}{2}\}$

(ii) b_k: $\{1, -1, 1\}$ or $\{1, -1\}$

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Problem 3: Suppose a continuous time music signal $x(t)$ is given below:

$$x(t) = (1 + \cos(3\pi t))\left(\sin(5\pi t + \frac{\pi}{6})\right)$$

(a) What is the fundamental angular frequency w_0 of $x(t)$? To find the existing frequencies in this signal, calculate the Fourier series coefficients, i.e., a_k of $x(t)$. State explicitly <u>each non-zero a_k to get credit</u>.

(+2) $$x(t) = \left(1 + \frac{1}{2}e^{j3\pi t} + \frac{1}{2}e^{-j3\pi t}\right)\left(\frac{1}{2j}e^{j5\pi t}e^{j\pi/6} - \frac{1}{2j}e^{-j5\pi t}e^{-j\pi/6}\right)$$

(+1) $$x(t) = \frac{1}{2}e^{-j\pi/2}e^{j\pi/6}e^{j5\pi t} + \frac{1}{2}e^{j\pi/2}e^{-j\pi/6}e^{-j5\pi t} + \frac{1}{4}e^{-j\pi/3}e^{j8\pi t} + \frac{1}{4}e^{j\pi/3}e^{-j2\pi t} + \frac{1}{4}e^{-j\pi/3}e^{j2\pi t} + \frac{1}{4}e^{j\pi/3}e^{-j8\pi t}$$

(+3) $$x(t) = \frac{1}{2}\left(e^{-j\pi/3}e^{j5\pi t} + e^{j\pi/3}e^{-j5\pi t}\right) + \frac{1}{4}e^{-j\pi/3}e^{j8\pi t} + \frac{1}{4}e^{j\pi/3}e^{-j8\pi t} + \frac{1}{4}e^{j\pi/3}e^{-j2\pi t} + \frac{1}{4}e^{-j\pi/3}e^{j2\pi t}$$

$$x(t) = \cos\left(5\pi t - \frac{\pi}{3}\right) + \frac{1}{2}\cos\left(8\pi t - \frac{\pi}{3}\right) + \frac{1}{2}\cos\left(2\pi t - \frac{\pi}{3}\right)$$

(+3) $w_0 = \gcd(2\pi, 5\pi, 8\pi)$ $\quad w_0 = \pi$ rad/s $\quad w_0 = 2\pi f_0 \Rightarrow f_0 = 0.5$ Hz

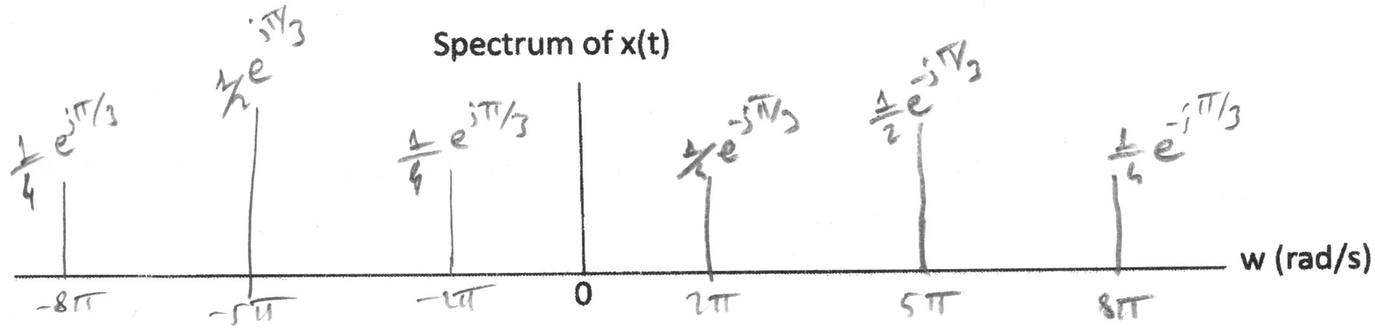
(+6) We have $a_2 = \frac{1}{4}e^{-j\pi/3}$ $\quad a_5 = \frac{1}{2}e^{-j\pi/3}$ $\quad a_8 = \frac{1}{4}e^{-j\pi/3}$

$a_{-2} = \frac{1}{4}e^{j\pi/3}$ $\quad a_{-5} = \frac{1}{2}e^{j\pi/3}$ $\quad a_{-8} = \frac{1}{4}e^{j\pi/3}$

2nd, 5th and 8th harmonics

(b) Plot the spectrum of the signal $x(t)$ versus w (rad/sec) on the provided axis below.

(6)



(c) Why is the spectrum of this music signal two-sided? What kind of signals have one-sided spectrum?

(4)

Two sided because this is a real signal (+2) recall Euler expansion on cosines & sines. Only complex signals have one-sided spectrum (+2), which end-up complex

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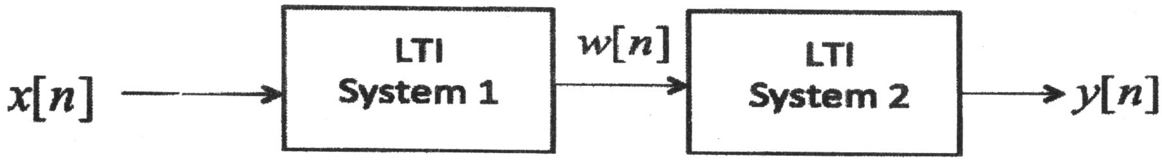
Problem 2: Consider the cascade connection of two LTI systems with impulse responses $h_1[n]$ and $h_2[n]$.



The second LTI system is a first-order difference system, hence its input-output relation is given by: $y[n] = w[n] - w[n-1]$. The step response of the overall cascade system is: $y[n] = u[n] - u[n-1]$.

Recall that the step response is obtained when the input to the system is set to the unit step sequence.

(a) Is the second system an FIR filter? Why? (Only 1 sentence). What is its impulse response $h_2[n]$? What are the filter coefficients b_k?

+1 Yes $b_k = \{1, -1\} \rightarrow$ FIR: finite number of filter coefficients +2

$h_2[n] = \delta[n] - \delta[n-1]$ or $h_2(n) = \{1, -1\}$

(b) Find the impulse response $h[n]$ of the overall cascade system.

+1 $u[n] \longrightarrow \boxed{S} \longrightarrow u[n] - u[n-1] = s_1(n)$

In LTI system

+2 $u[n-1] \longrightarrow \boxed{S} \longrightarrow u(n-1) - u(n-2) = s_2(n)$

+2 $\delta[n] = u[n] - u[n-1]$

$h[n] = s_1[n] - s_2[n]$

$h[n] = u(n) - 2u[n-1] + u[n-2]$ +3

(c) Find the impulse response of the first system $h_1[n]$.

+2 $h_1[n] * h_2[n] = h[n] = u[n] - 2u[n-1] + u[n-2]$

+2 $h_1[n] * (\delta[n] - \delta[n-1]) = u[n] - 2u[n-1] + u[n-2]$

+3 $h_1[n] - h_1[n-1] = h_1[n] = u[n] - u[n-1]$

d) Find the output of the overall system when the input to the system is: $x[n] = \{2, 1\}$ starting at $n = 0$.

$x[n] = +2\delta[n] + \delta[n-1] \Rightarrow y[n] = 2h[n] + h[n-1]$ +2

$y[n] = x[n] * h[n] = 2u[n] - 4u[n-1] + 2u[n-2] + u[n-3]$ +2

$y(n) = 2u[n] - 3u[n-1] + u[n-3]$ +1

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Problem 4. For the following systems in parts (a) and (b), determine separately whether or not the system is (i) linear; (ii) time-invariant; (iii) causal; (iv) stable. Justify your answer to get any credits. If you determine that the given system is LTI, use the corresponding tests of Stability and Causality for LTI systems to get credit.

(a) $y[n] = \left(\frac{1}{3}\right)^n \cdot x[n+1]$.

(i) Linear: Y or N (why?) Y

4

$x_1[n] \to \boxed{S} \to y_1[n] = (\frac{1}{3})^n x_1[n+1]$

$x_2[n] \to \boxed{S} \to y_2[n] = (\frac{1}{3})^n x_2[n+1]$

$\oplus \to a_1(\frac{1}{3})^n x_1[n+1] + a_2(\frac{1}{3})^n x_2[n+1] = w[n]$

$a_1x_1[n] + a_2x_2[n] \to \boxed{S} \to (\frac{1}{3})^n \cdot (a_1x_1[n+1] + a_2x_2[n+1]) = y[n] \Rightarrow y[n] = w[n]$

(ii) Time-Invariant: N

3

$x[n] \to \boxed{\text{Delay}} \to x[n-n_0] \to \boxed{S} \to (\frac{1}{3})^n x[n-n_0]$

$x[n] \to \boxed{S} \to (\frac{1}{3})^n x[n] \to \boxed{\text{Delay}} \to (\frac{1}{3})^{n-n_0} x[n-n_0]$

(iii) Stable: N

3

Let $|x[n]| \leq M$ for $M \leq \infty$

$|y[n]| = |(\frac{1}{3})^n x[n+1]| \leq |(\frac{1}{3})^n| |x[n+1]| \leq M|(\frac{1}{3})^n| < \infty$

$n \to \infty$
$= \infty$

(iv) Causal: N

3

let $n=1$ $y[1] = \frac{1}{3}x[2] \to$ No

(b) $y[n] = (x[n] - x[n-2])$

(i) Linear: Y or N (why?) Y

3

$x_1[n] \to \boxed{S} \to y_1[n] = (x_1[n] - x_1[n-2])$

$x_2[n] \to \boxed{S} \to y_2[n] = (x_2[n] - x_2[n-2])$

$\oplus \to y[n] = a_1(x_1[n] - x_1[n-2]) + a_2(x_2[n] - x_2[n-2])$

$a_1x_1[n] + a_2x_2[n] \to \boxed{S} \to a_1x_1[n] + a_2x_2[n] - a_1x[n-2] - a_2x[n-2]$ =

(ii) Time-Invariant: Y

3

$x[n] \to \boxed{\text{Delay}} \to x[n-n_0] \to \boxed{S} \to x[n-n_0] - x[n-n_0-2]$

$x[n] \to \boxed{S} \to x[n] - x[n-2] \to \boxed{\text{Delay}} \to x[n-n_0] - x[n-2-n_0]$ =

(iii) Stable:

3

LTI system $h[n] = \delta[n] - \delta[n-2]$

$\sum |h[n]| = 2 < \infty \Rightarrow$ stable

(iv) Causal:

3

$h[n] = 0$ for $n < 0$ Causal