WHIR 

Proximity testing for Reed–Solomon+

Gal Arnon

Giacomo Fenzi

EPFL

Alessandro Chiesa

EPFL

Eylon Yogev

Motivation

Constructing IOPs

Traditionally

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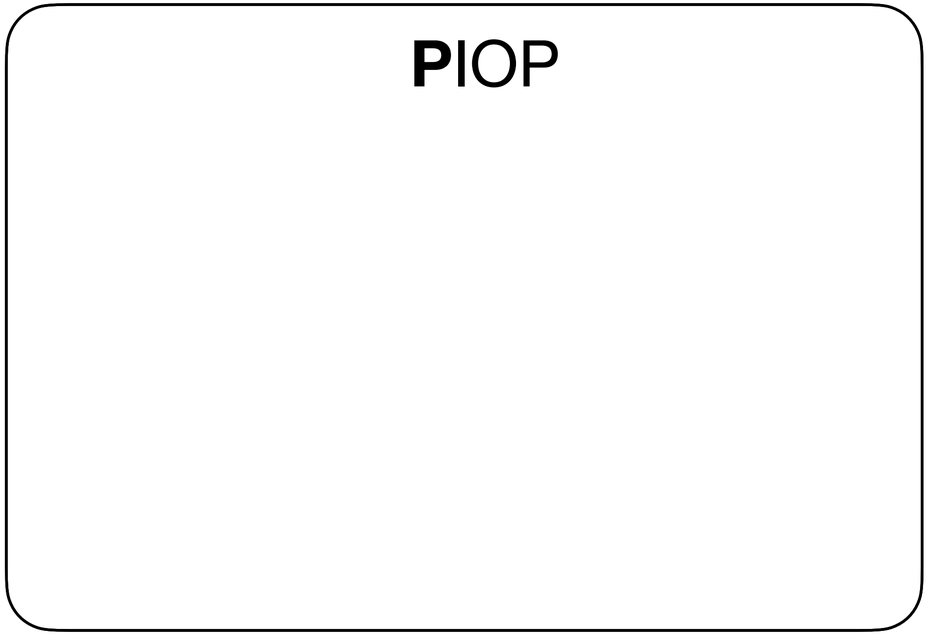

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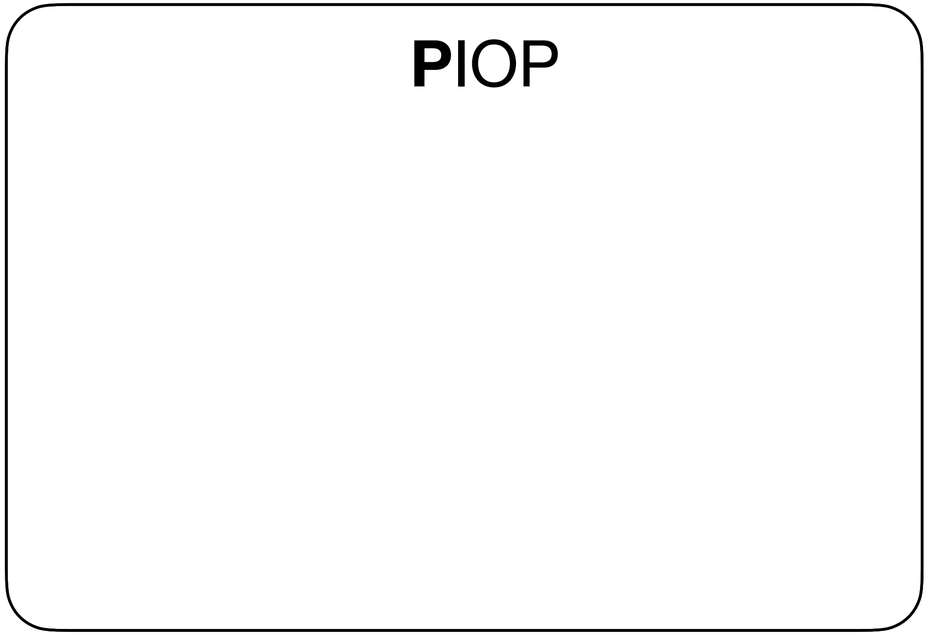

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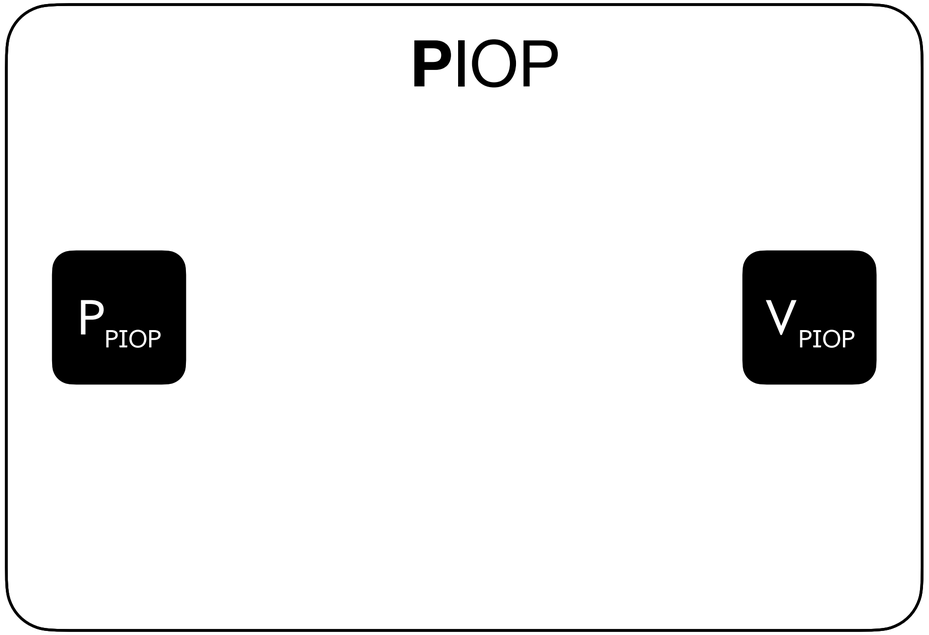

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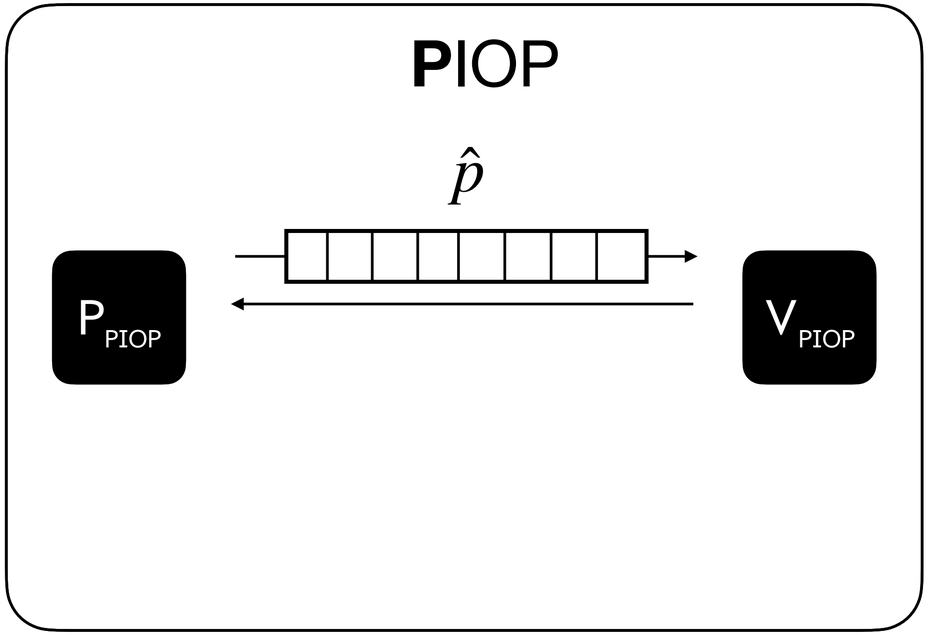

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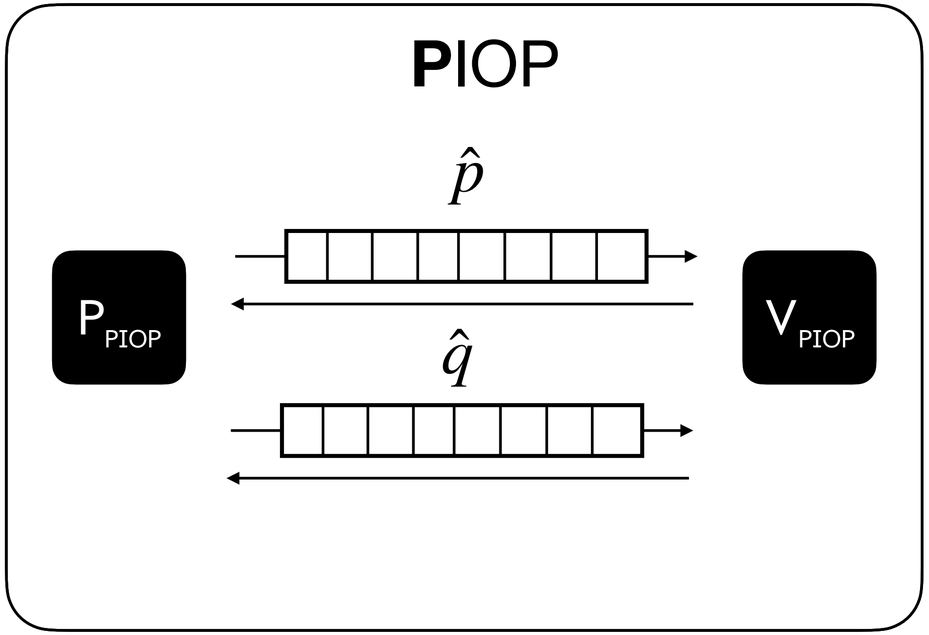

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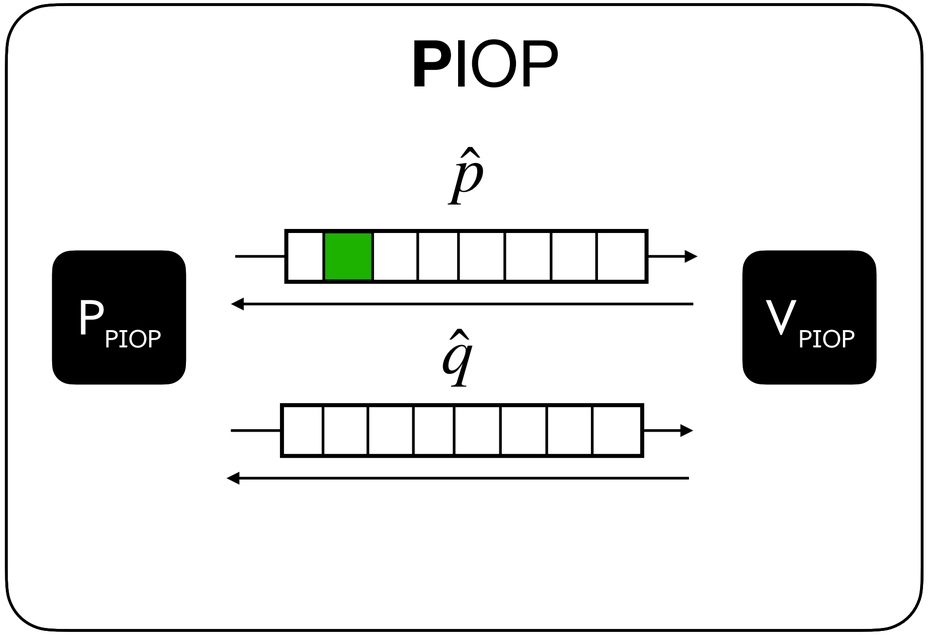

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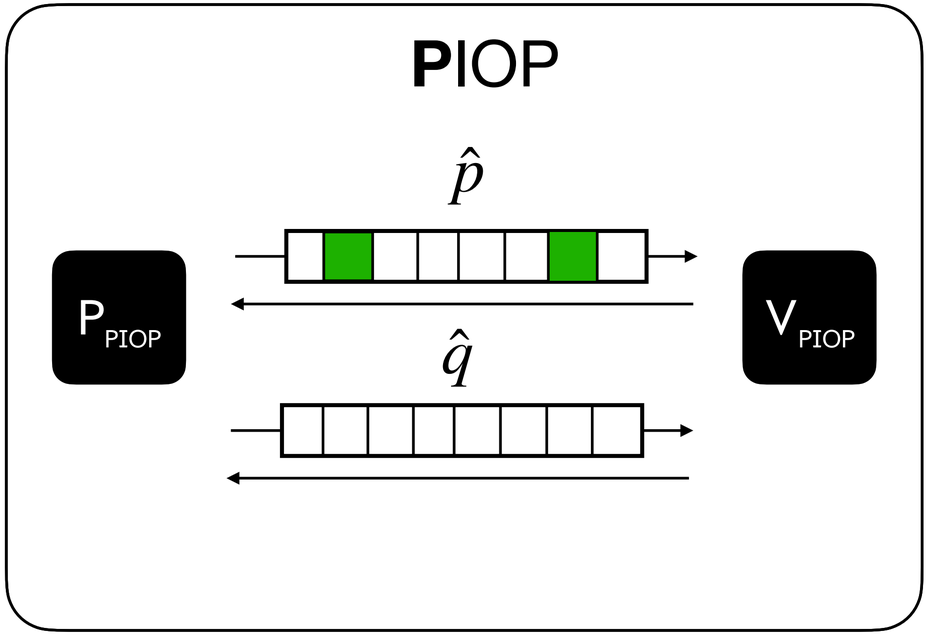

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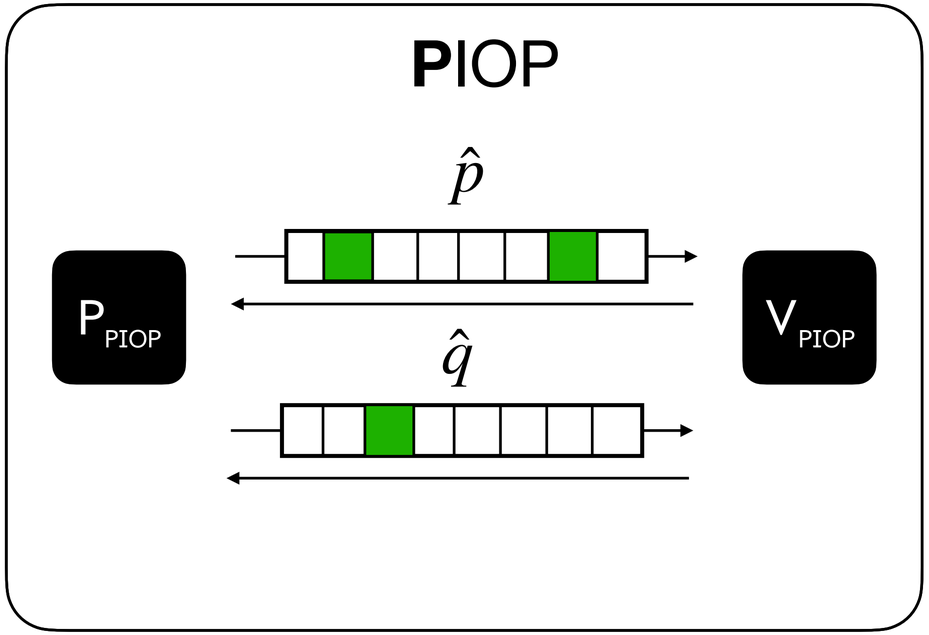

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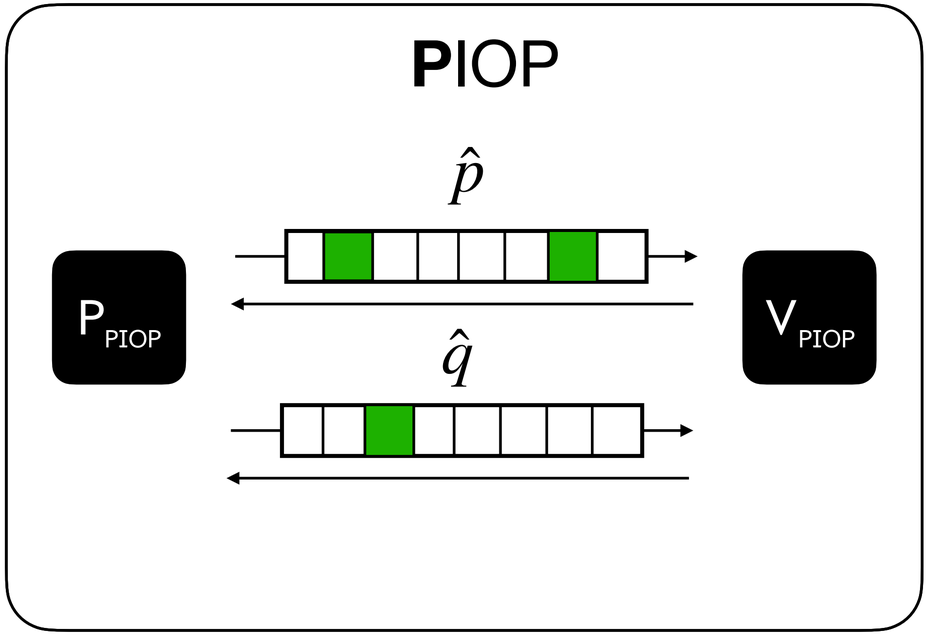

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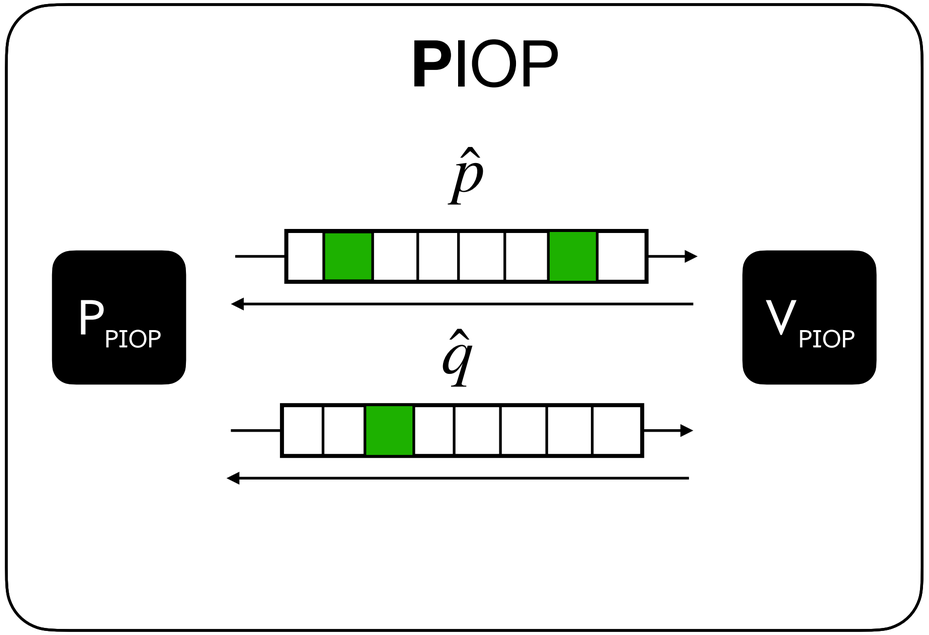

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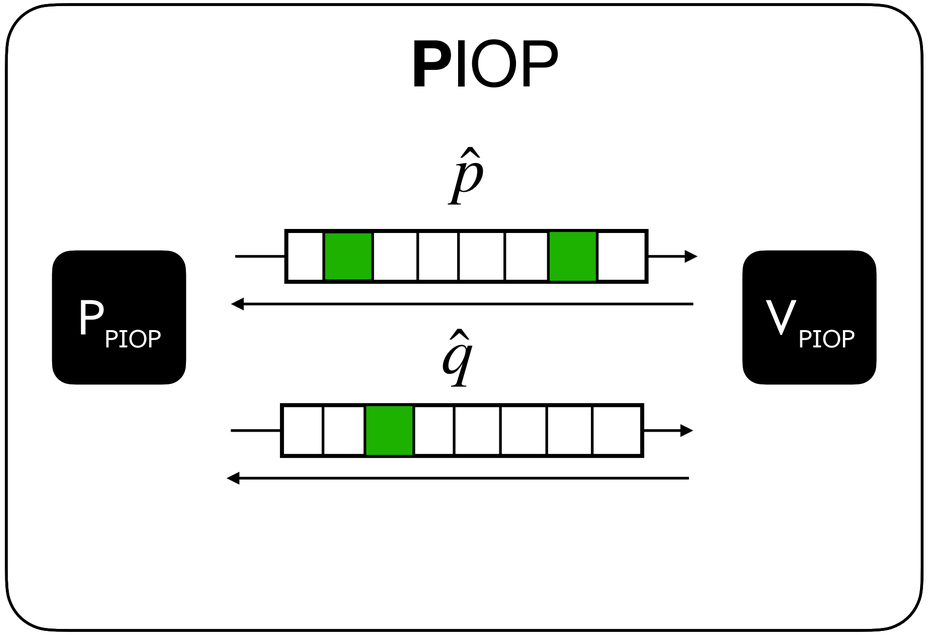

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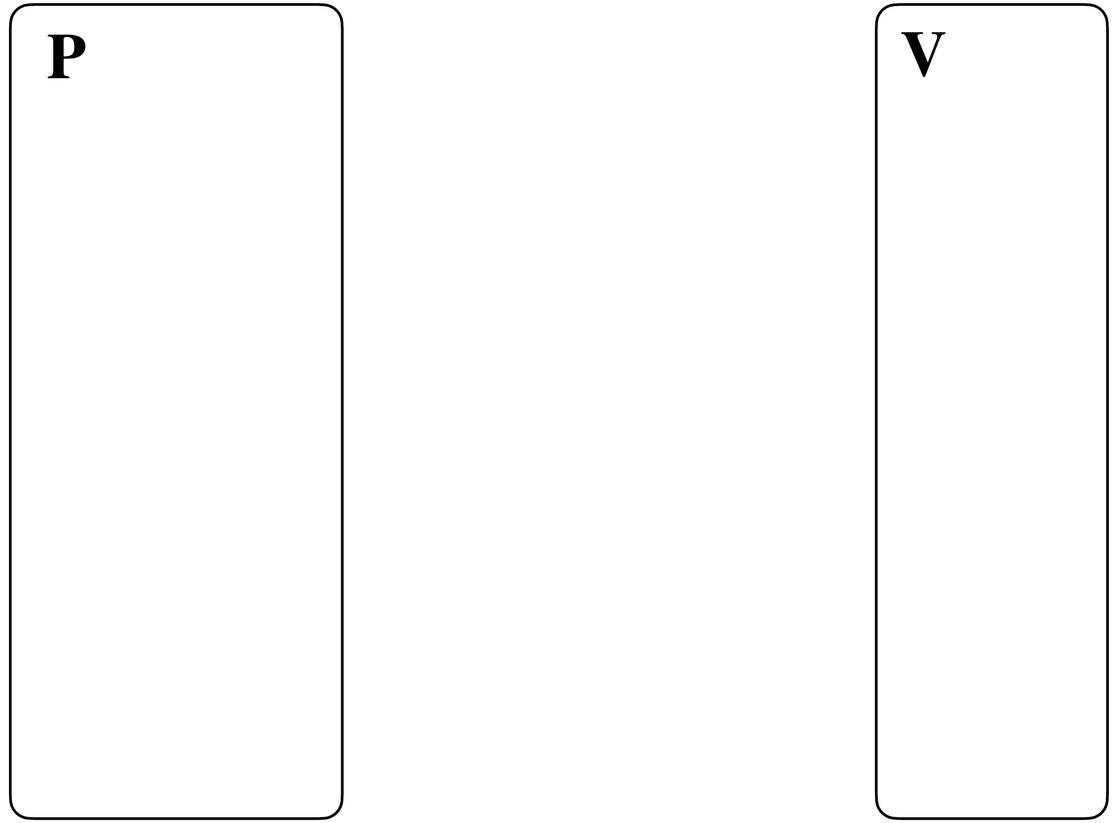

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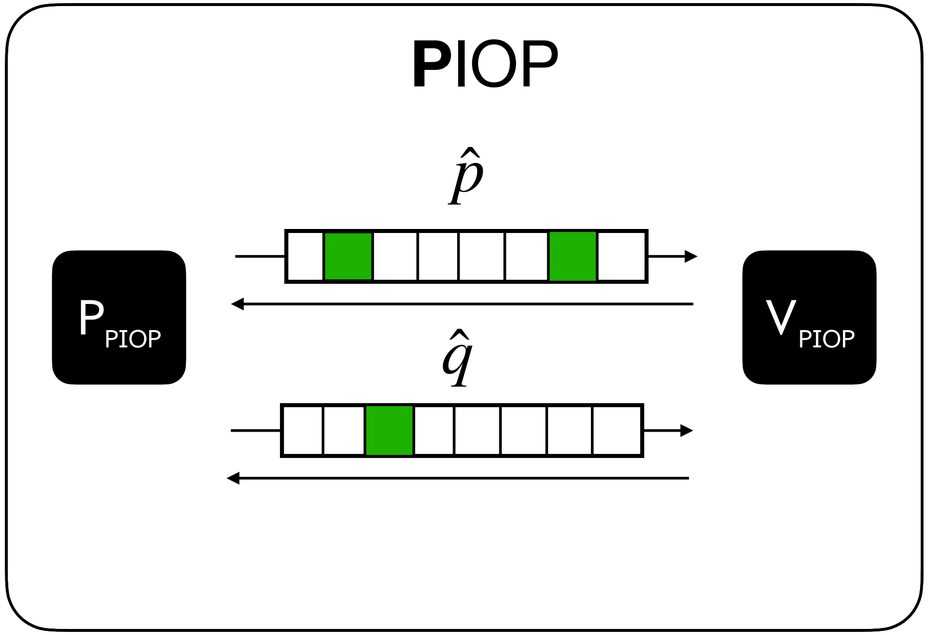

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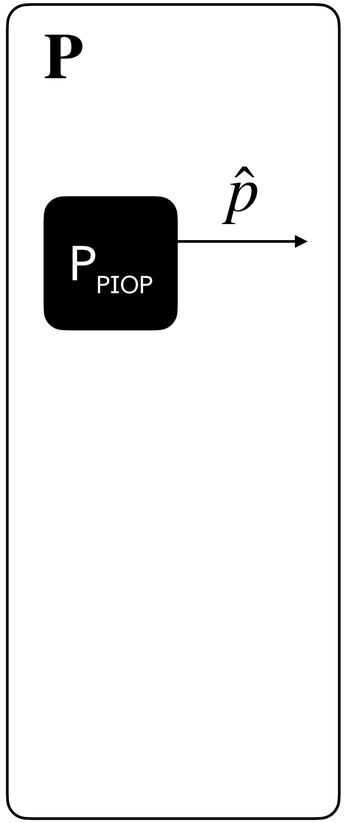

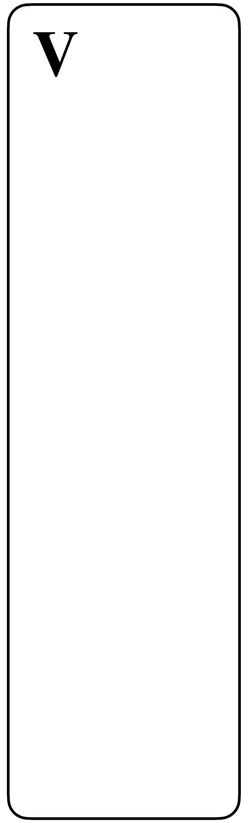

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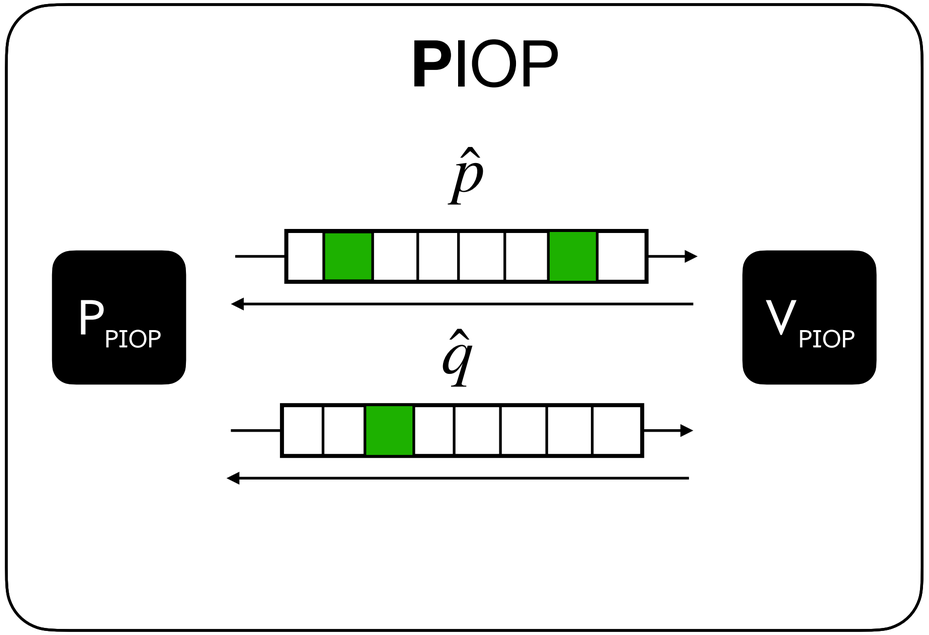

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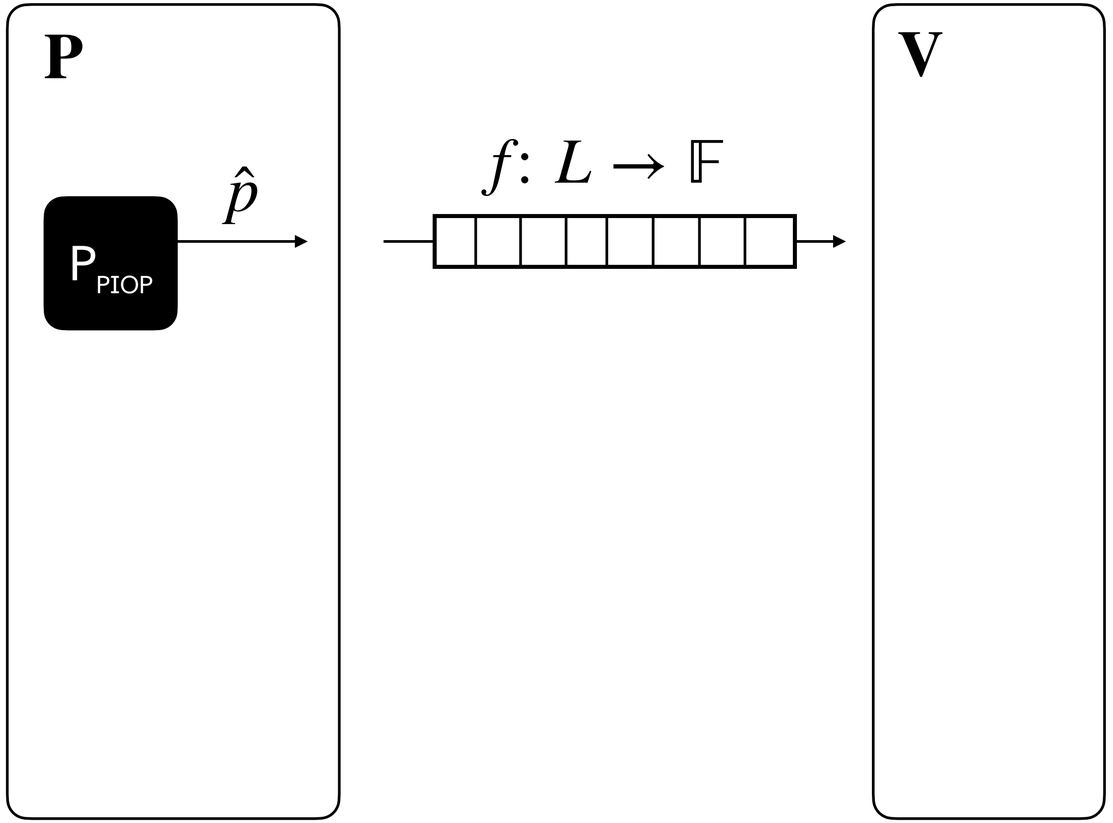

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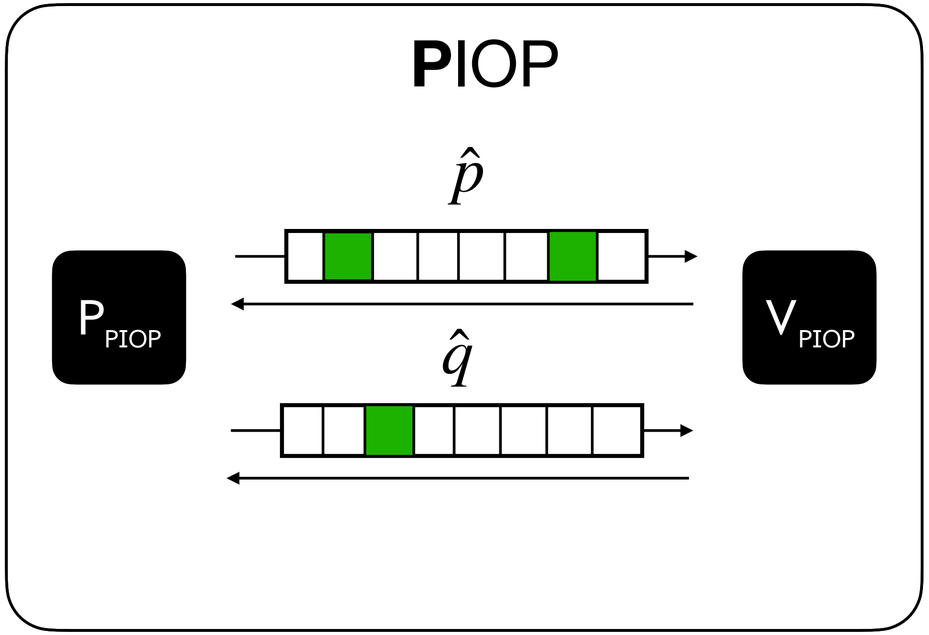

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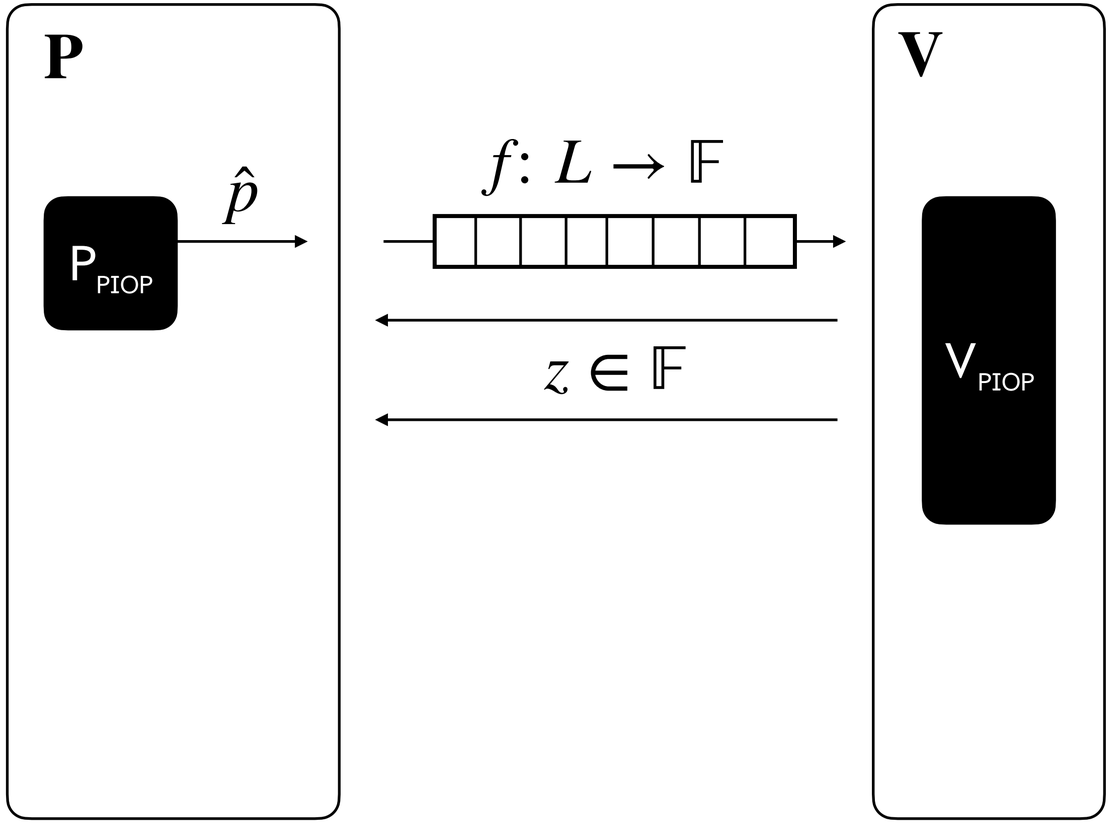

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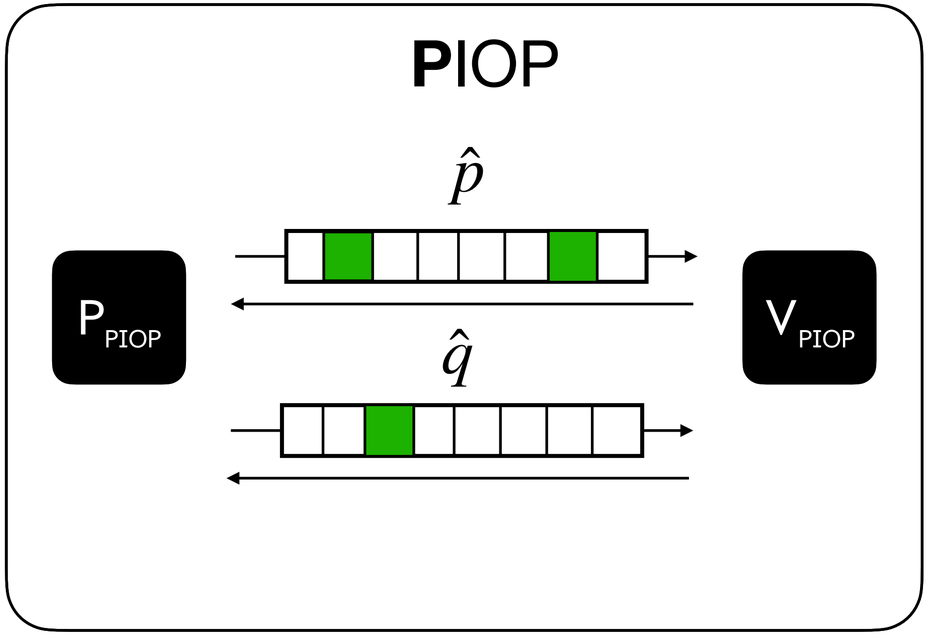

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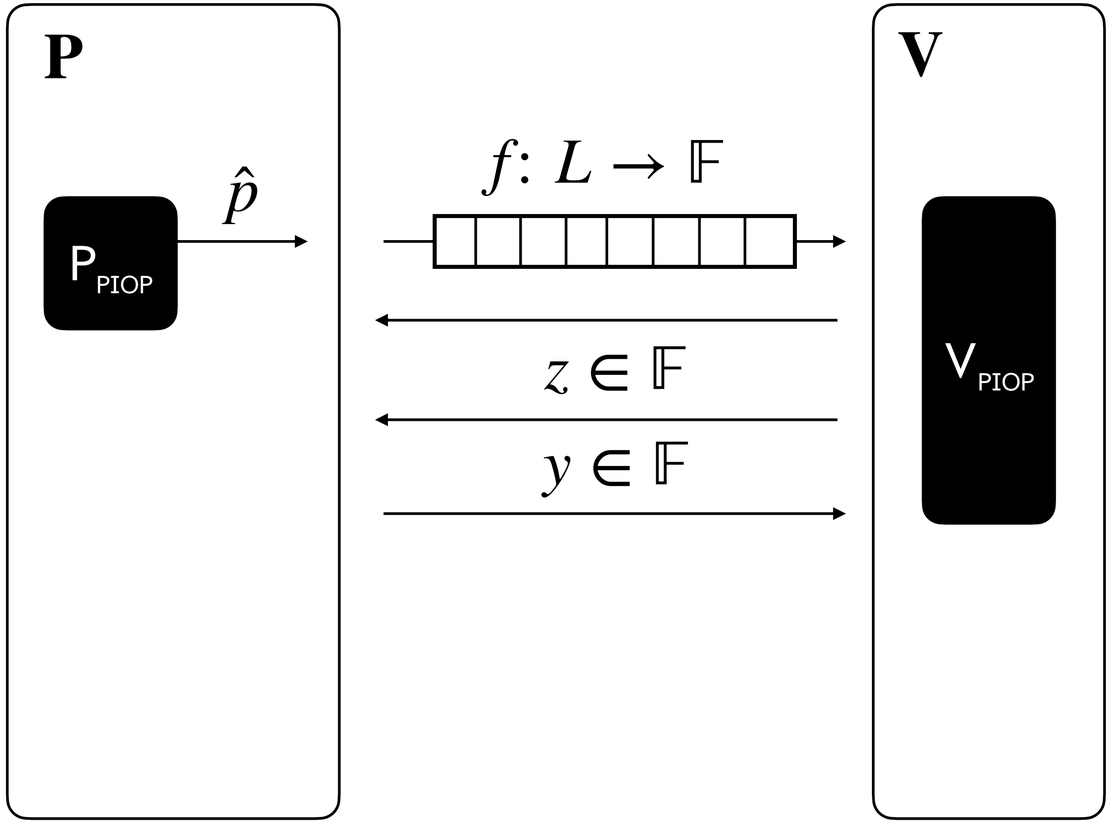

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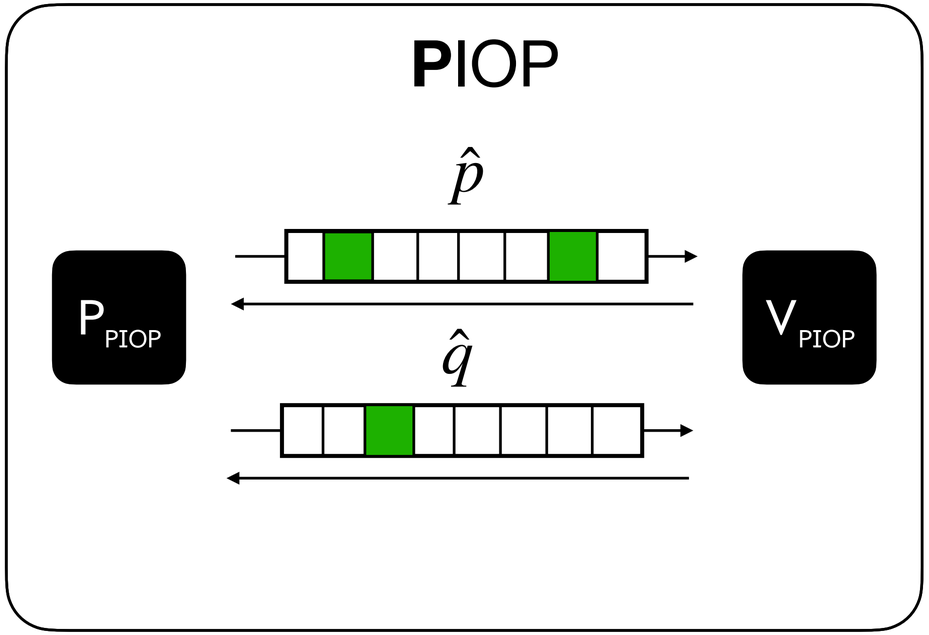

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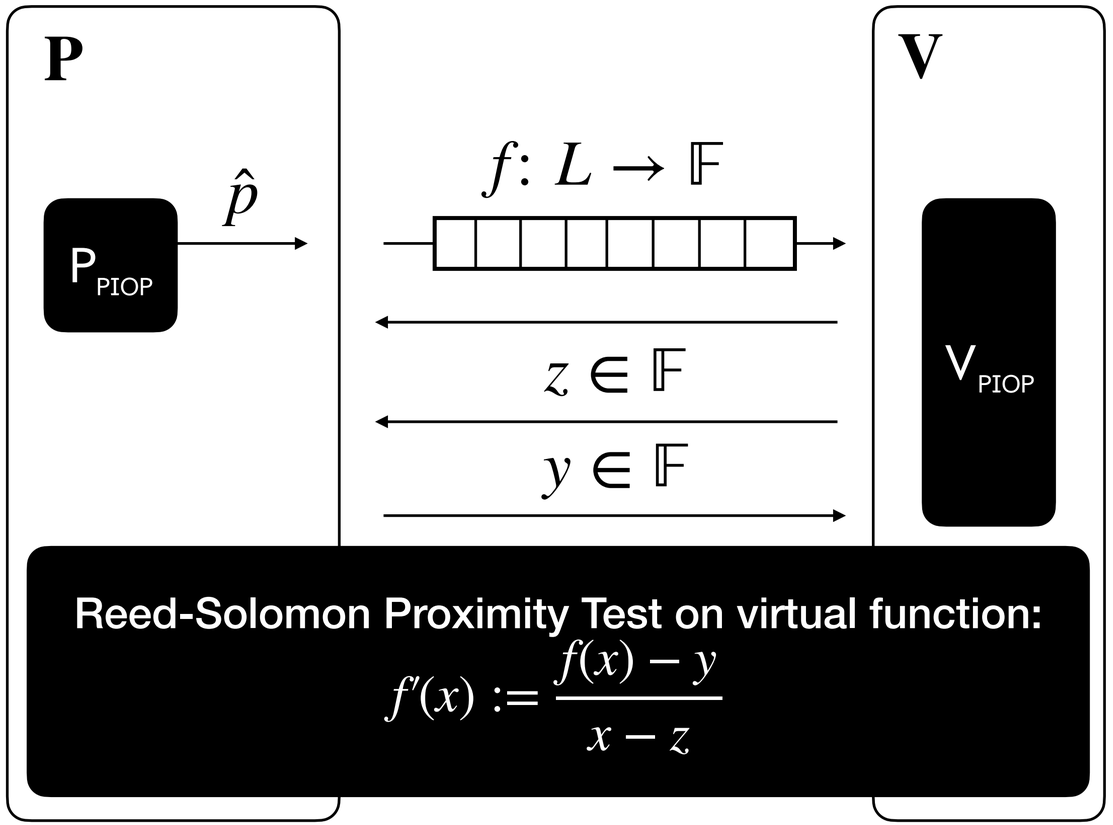

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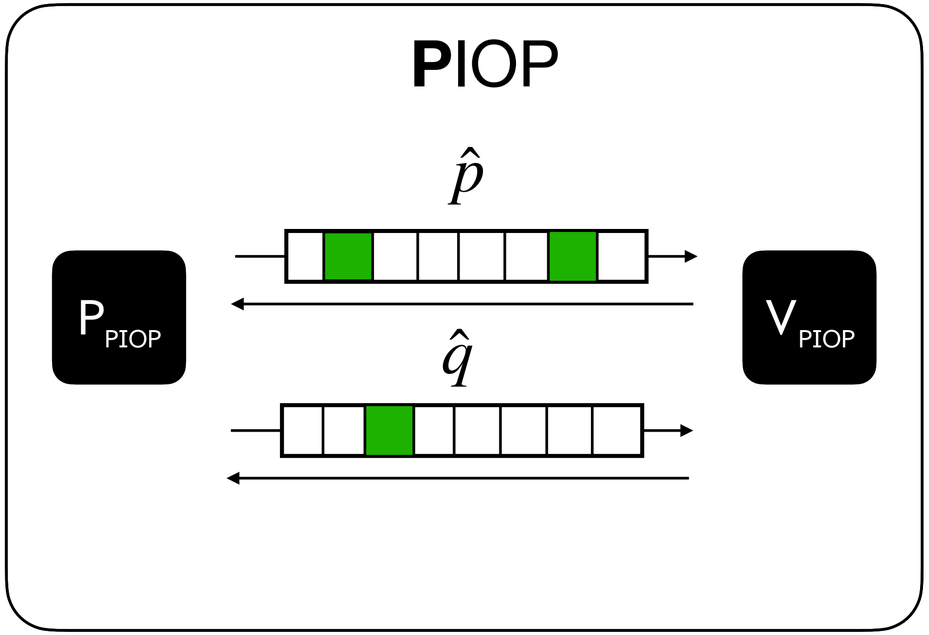

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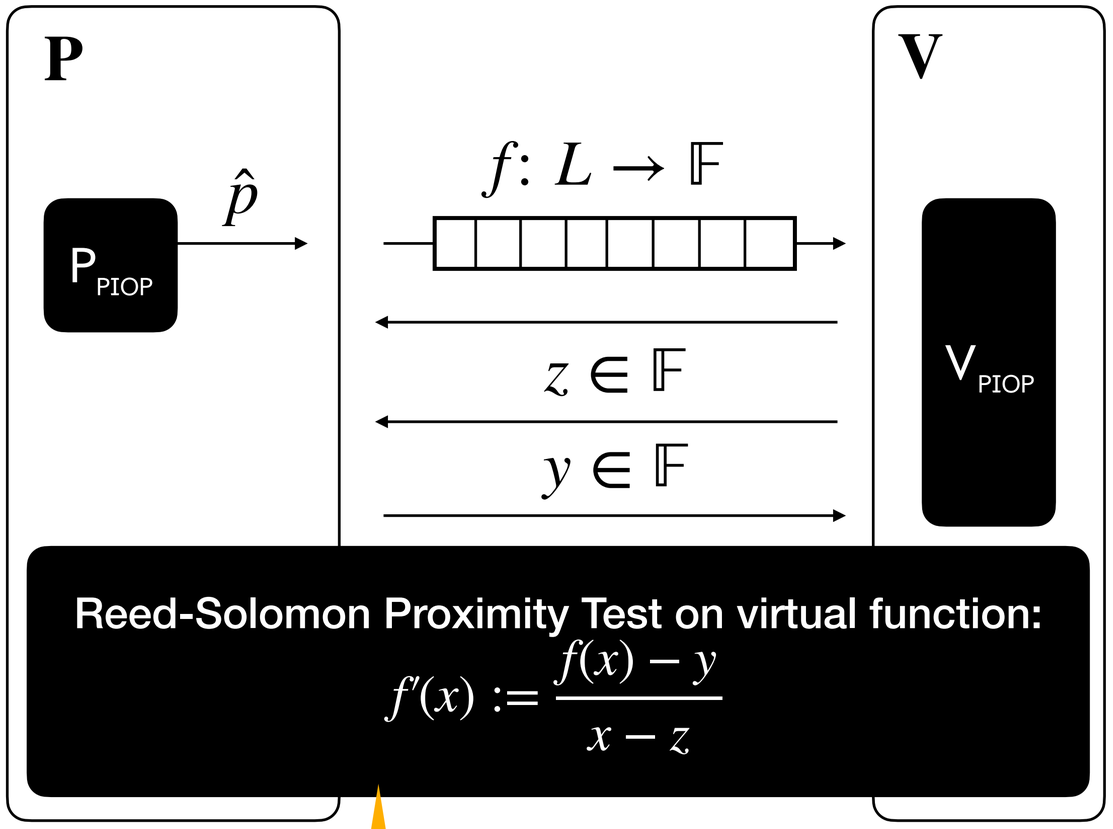

$> 80\,\%$ of argument size from proximity test!

IOP of Proximity to RS codes

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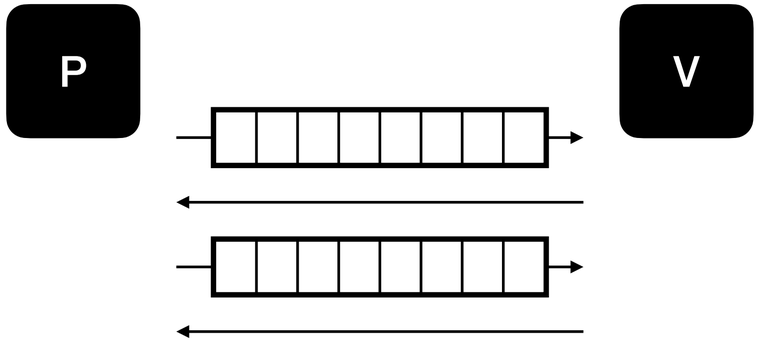

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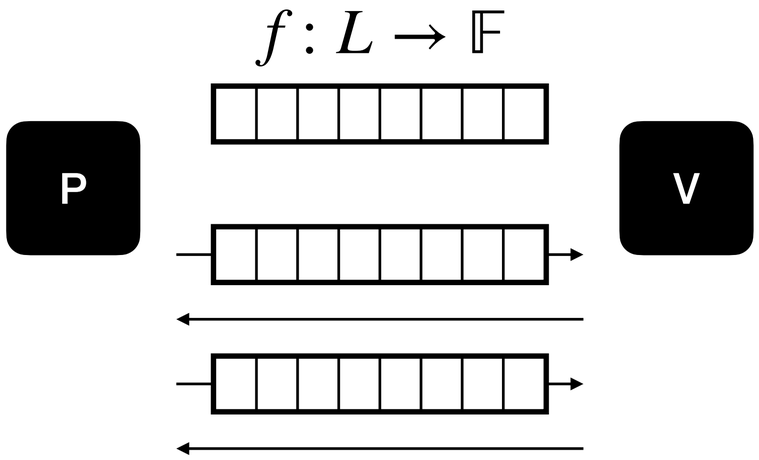

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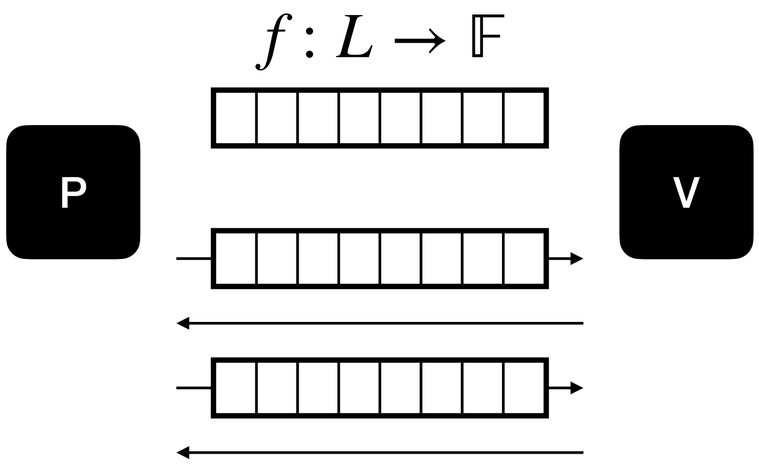

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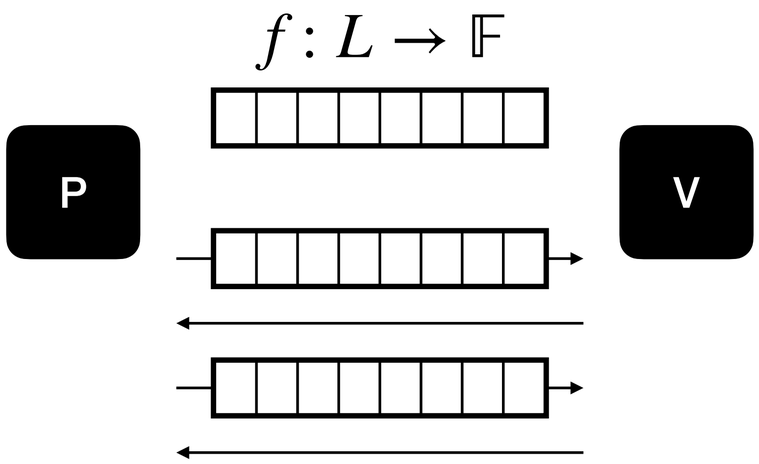

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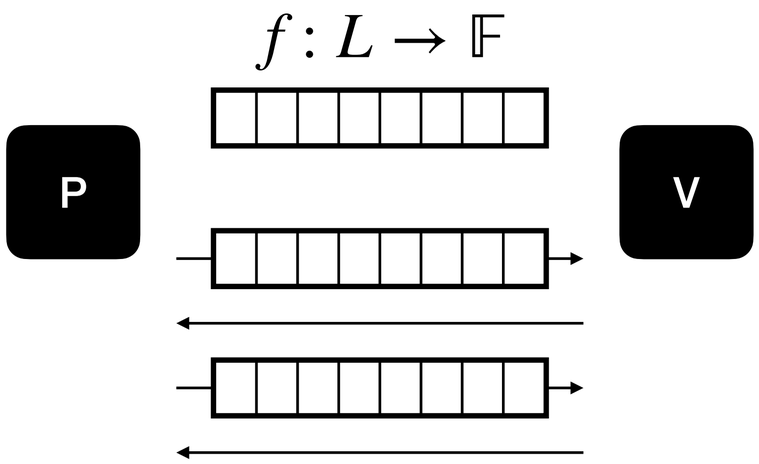

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Round by round, required by BCS transform.

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Yes! IOPP for **constrained codes**

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$\mathrm{coeff}(\hat{p}) = \mathrm{coeff}(\hat{q})$

implies that

$\hat{p}(z) = \hat{q}(z, z^2, \ldots, z^{2^{m-1}})$

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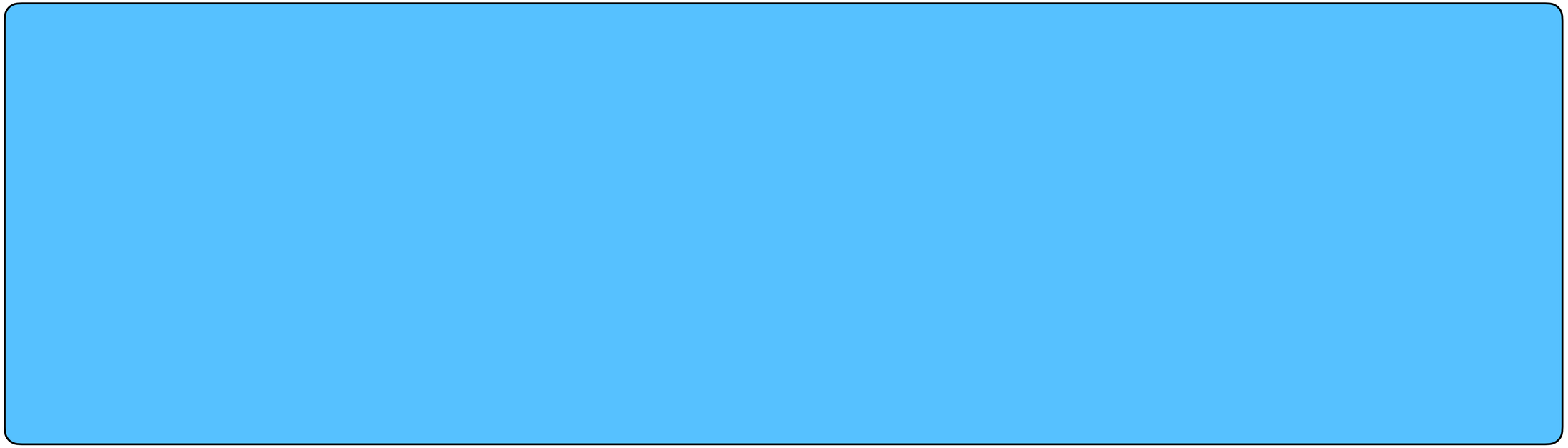
We test **proximity** to CRS!

Our results

WHIR

$k > 1$ is a folding parameter

A <u>constrained</u> Reed-Solomon proximity test



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Implementation available at WizardOfMenlo/whir

Comparison with prior work

	Queries	Verifier Time	Alphabet
BaseFold	$q_{\text{BF}} = O(\lambda \cdot m)$	$O(q_{\text{BF}})$	$\mathbb{F}^2$
FRI	$q_{\text{FRI}} = O\left(\frac{\lambda}{k} \cdot m\right)$	$O(q_{\text{FRI}} \cdot 2^k)$	$\mathbb{F}^{2^k}$
STIR	$q_{\text{STIR}} = O\left(\frac{\lambda}{k} \cdot \log m\right)$	$O(q_{\text{STIR}} \cdot 2^k + \lambda^2 \cdot 2^k)$	$\mathbb{F}^{2^k}$
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- **Further,** super-fast verification (next)

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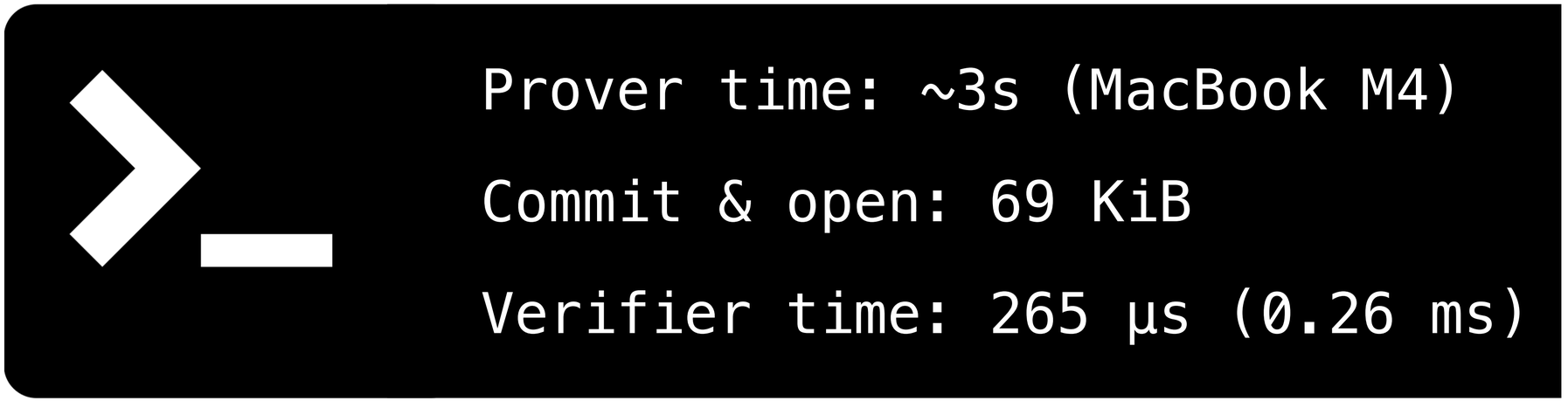

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Schemes with **trusted** setup using pairings!

Verifier time (ms)	Brakedown	Ligero	Greyhound	Hyrax	PST	KZG	WHIR-1/2	WHIR-1/16
$\lambda = 100$	3500	733	-	100	7.81	2.42	0.61	0.29
$\lambda = 128$	3680	750	130	151	9.92	3.66	1.4	0.6

Table 4: Comparison of WHIR-CB's verifier time versus other polynomial commitment schemes, on 24 variables. For the KZG degree 2^{24} is used instead.

Techniques

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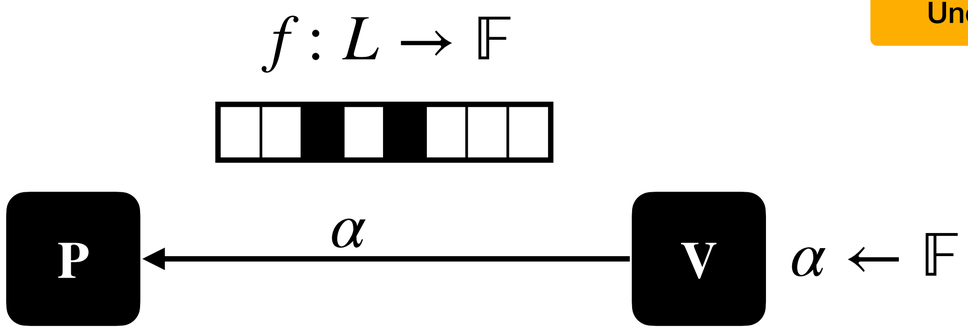

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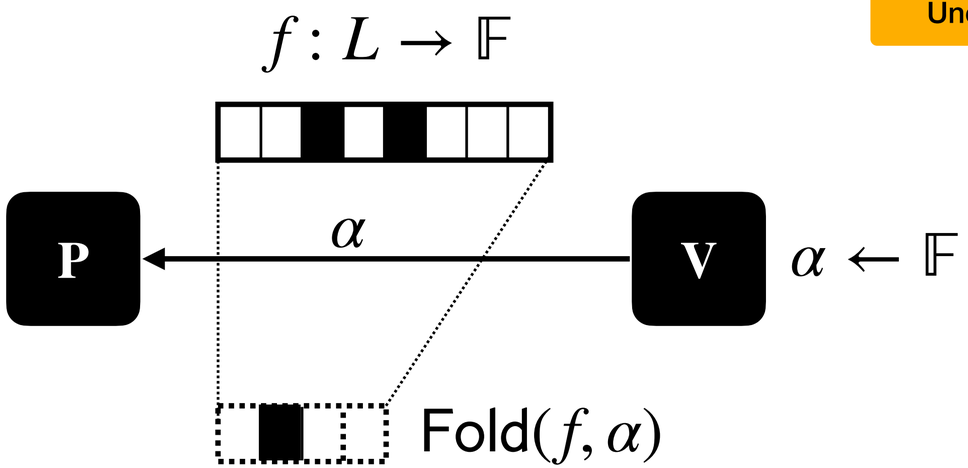

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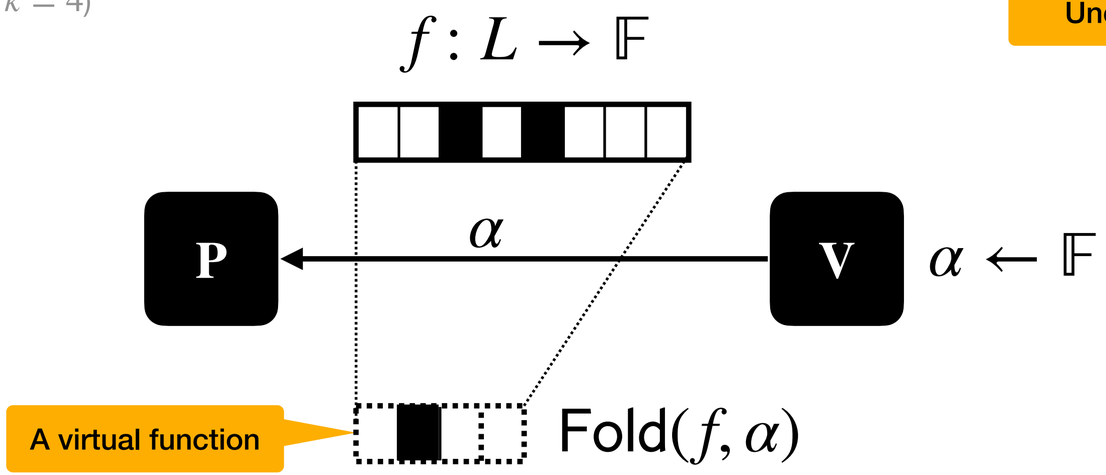

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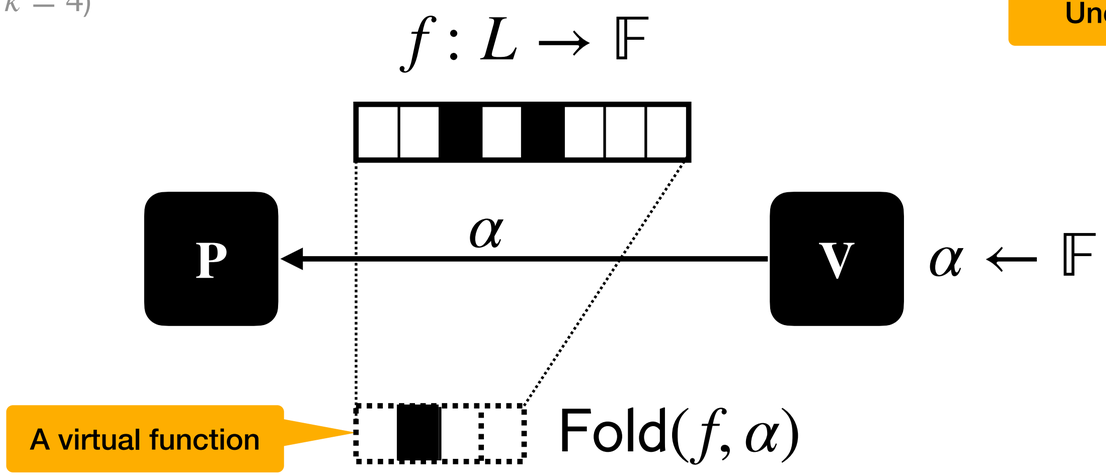

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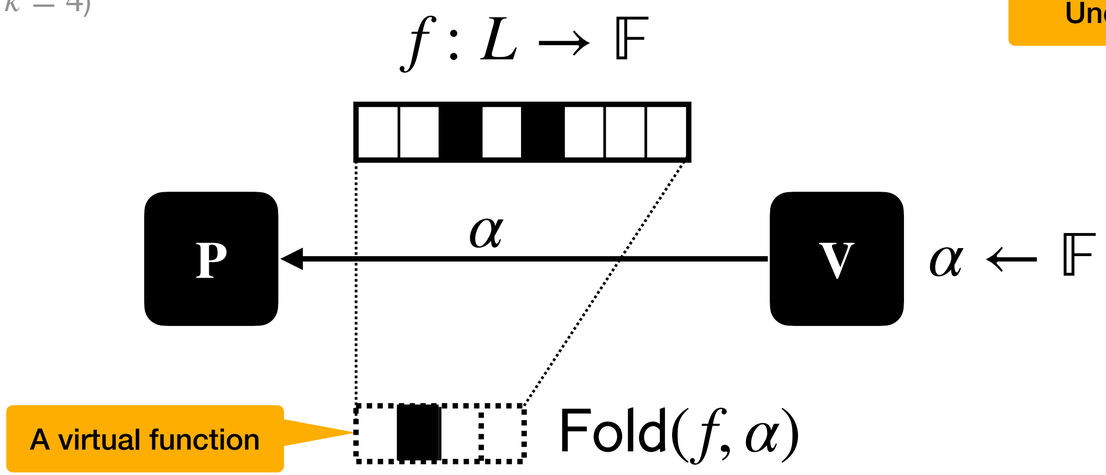

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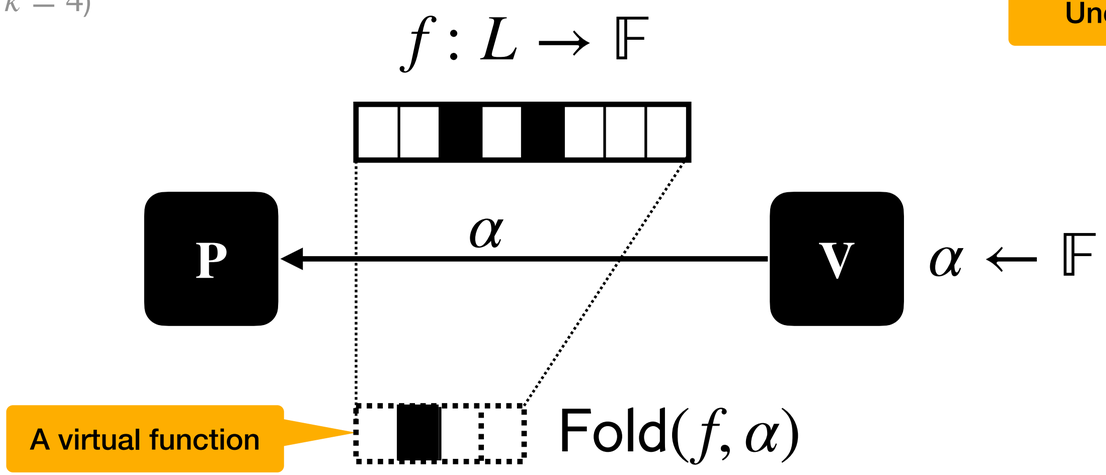

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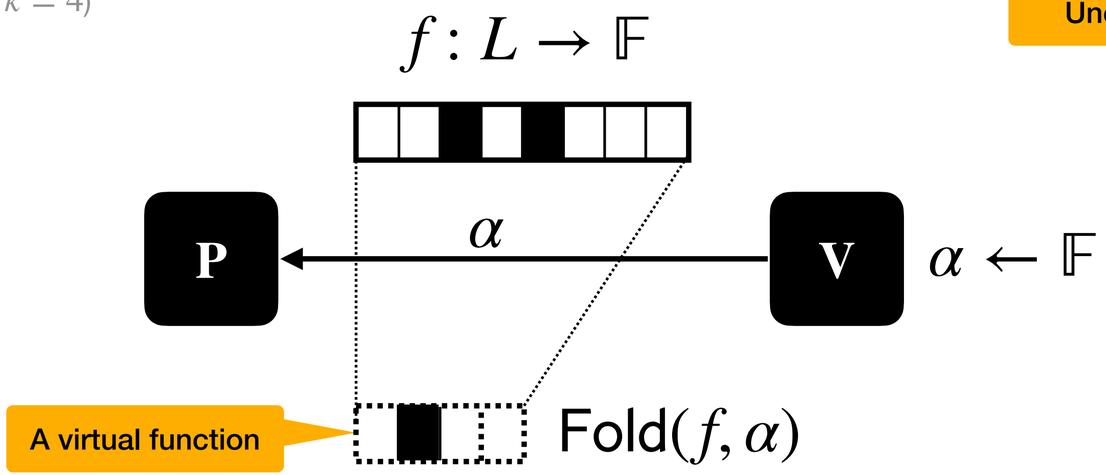

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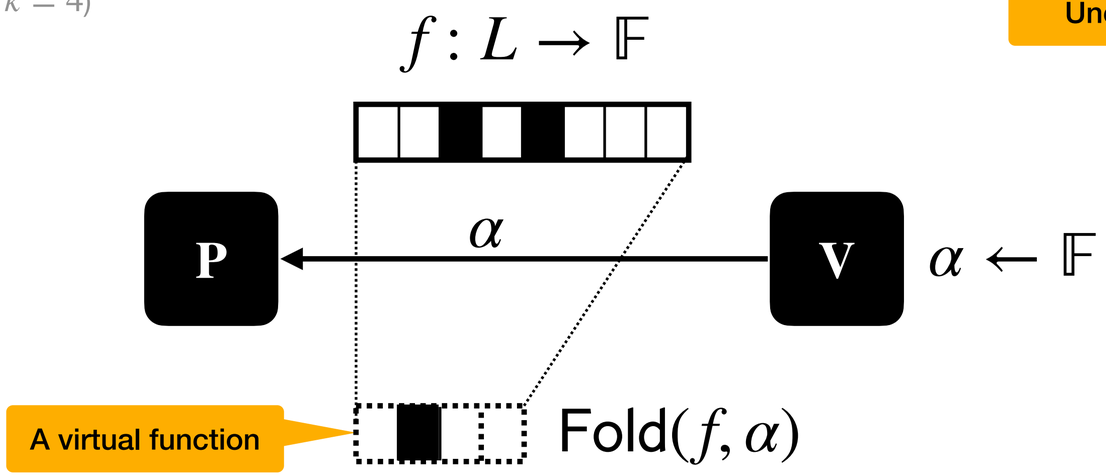

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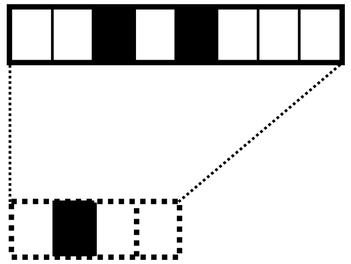
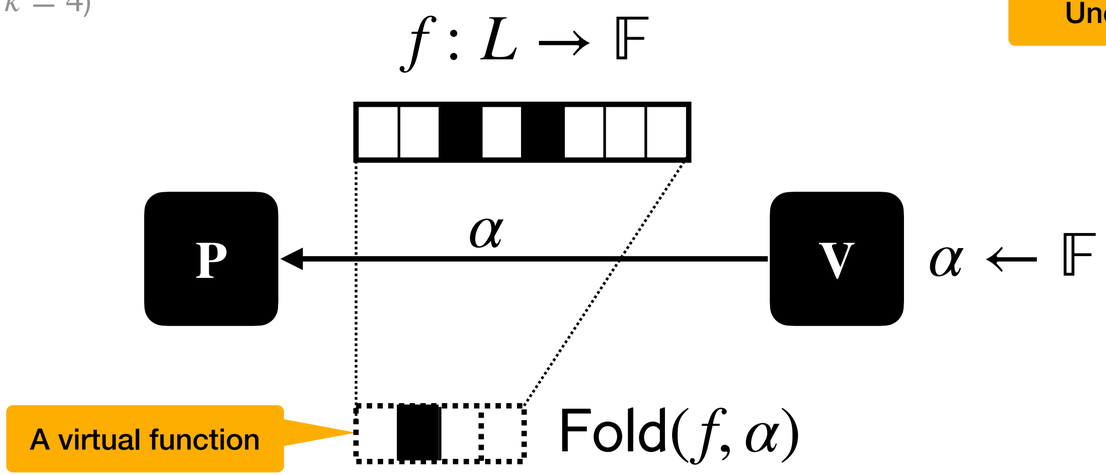


Unless w.p. $\approx \frac{\text{poly}(n, 2^m)}{|\mathbb{F}|}$, the fraction of "corrupted" entries does not decrease.

FRI & STIR Folding

Reduce $\mathrm{RS}[n, m, \rho]$ **to** $\mathrm{RS}[n/2^k, m-k, \rho]$

(Think $k = 4$)



Unchanged!

How? Inspiration from FFTs, for $k = 1$:

$$\mathrm{Fold}(f, \alpha) := f_{\text{odd}} + \alpha \cdot f_{\text{even}}$$

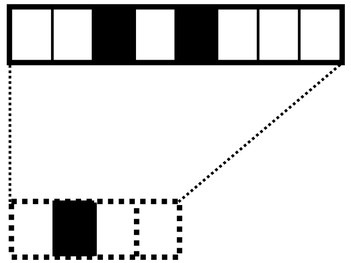
Can extend to every k that is a power of two.

Properties:

Local: compute $\mathrm{Fold}(f, \alpha)(z)$ at any point $z \in L^{2^k}$ with 2^k queries to f.

$$\delta \in \left(0, 1 - \sqrt{\rho}\right)$$

Distance preservation: if f is δ-far from $\mathrm{RS}[n, m, \rho]$, then w.h.p. $\mathrm{Fold}(f, \alpha)$ remains also δ-far from $\mathrm{RS}[n/2^k, m-k, \rho]$



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Proximity Gaps for Reed–Solomon Codes

Eli Ben-Sasson* Dan Carmon* Yuval Ishai† Swastik Kopparty ‡
Shubhangi Saraf§

July 3, 2021

Mutual correlated agreement

Test a random linear combination

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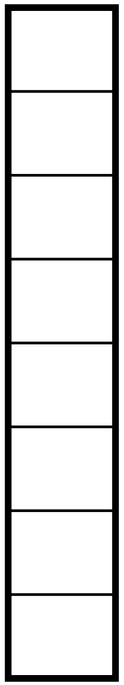

f_1

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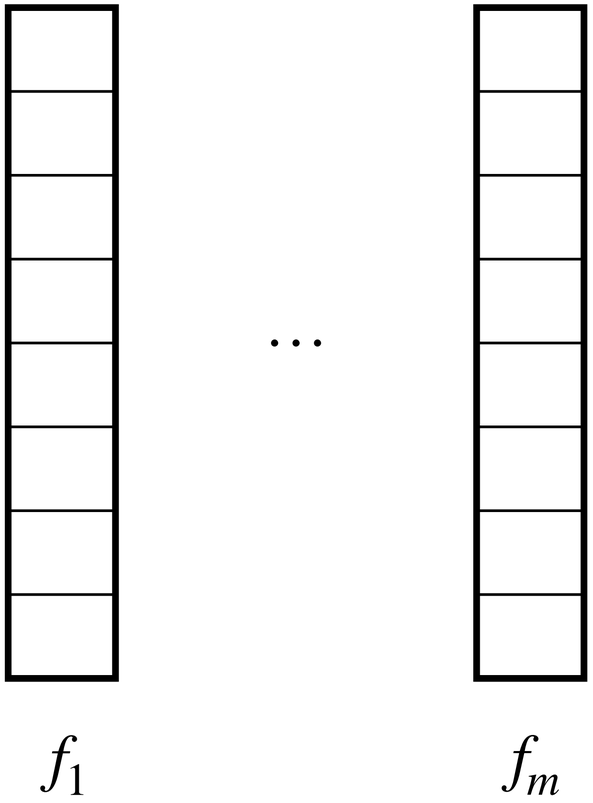

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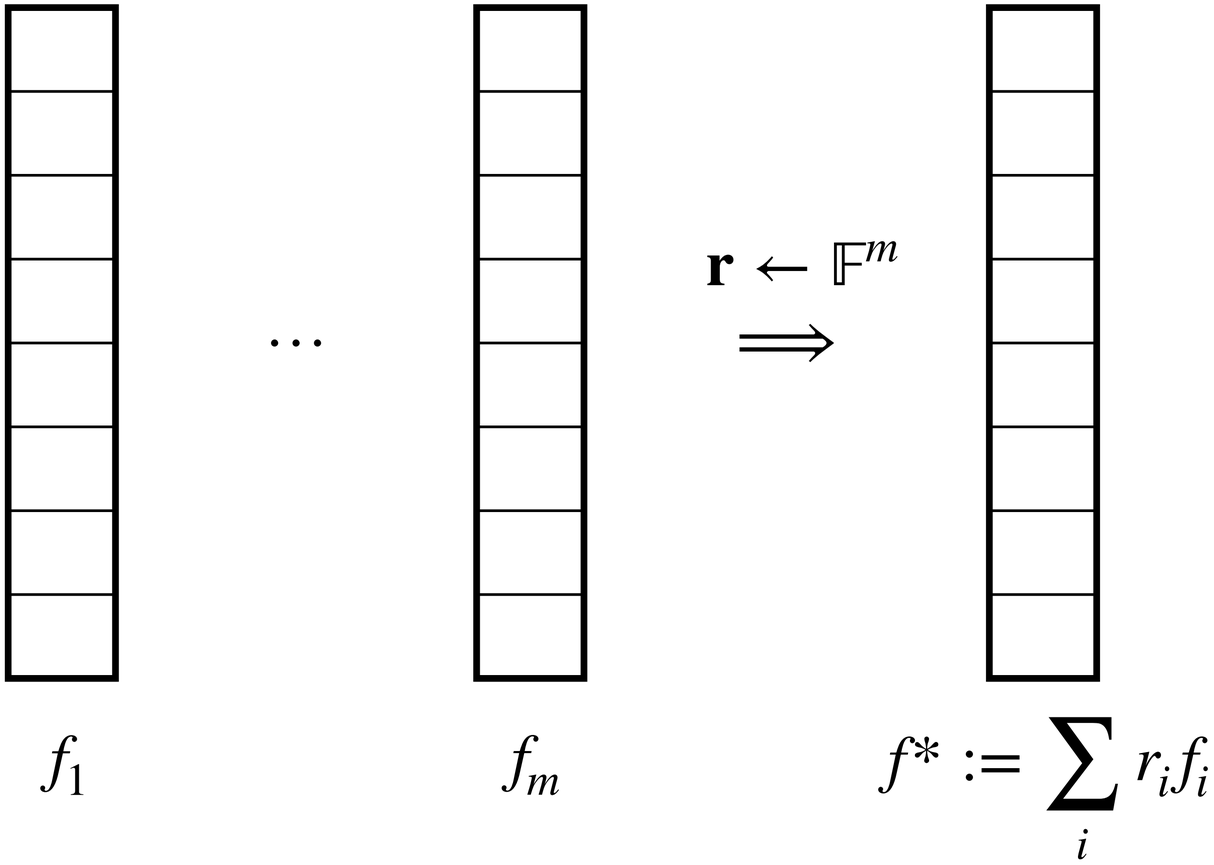

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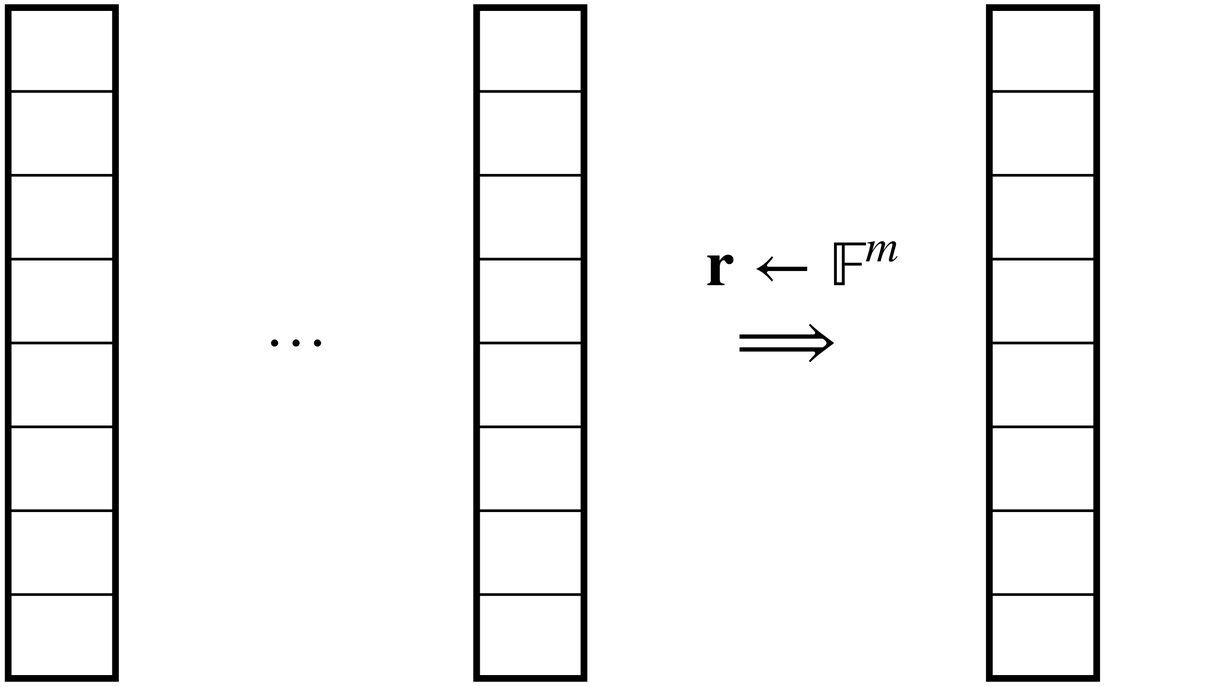

f_1 f_m $f^* := \sum_i r_i f_i$

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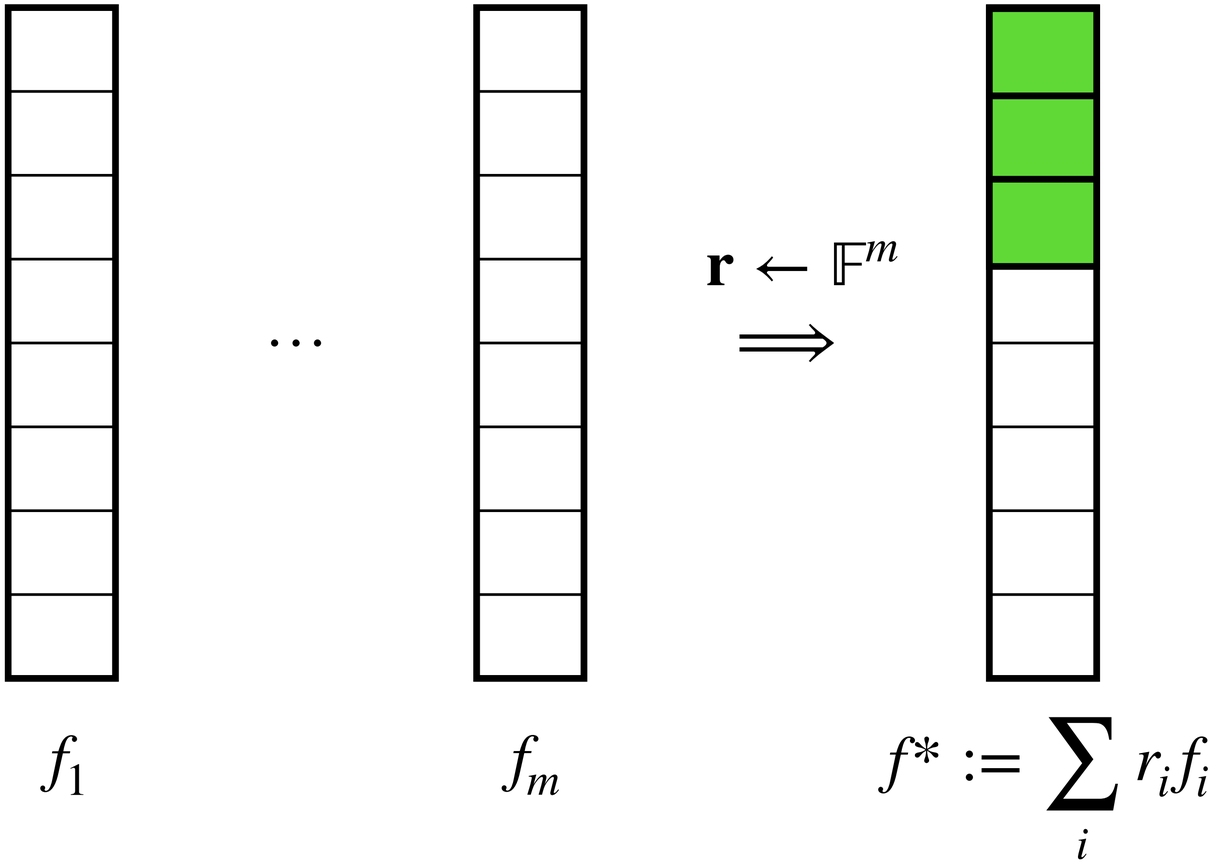

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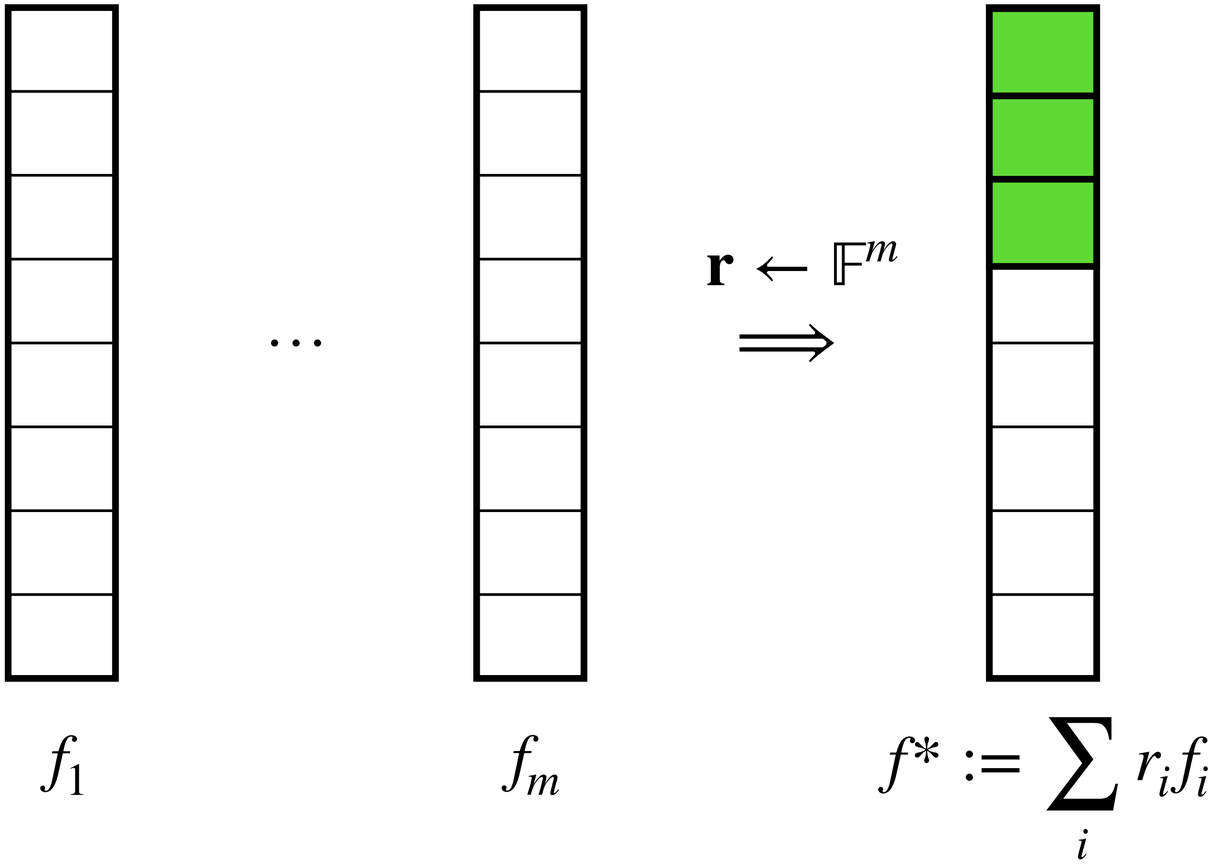

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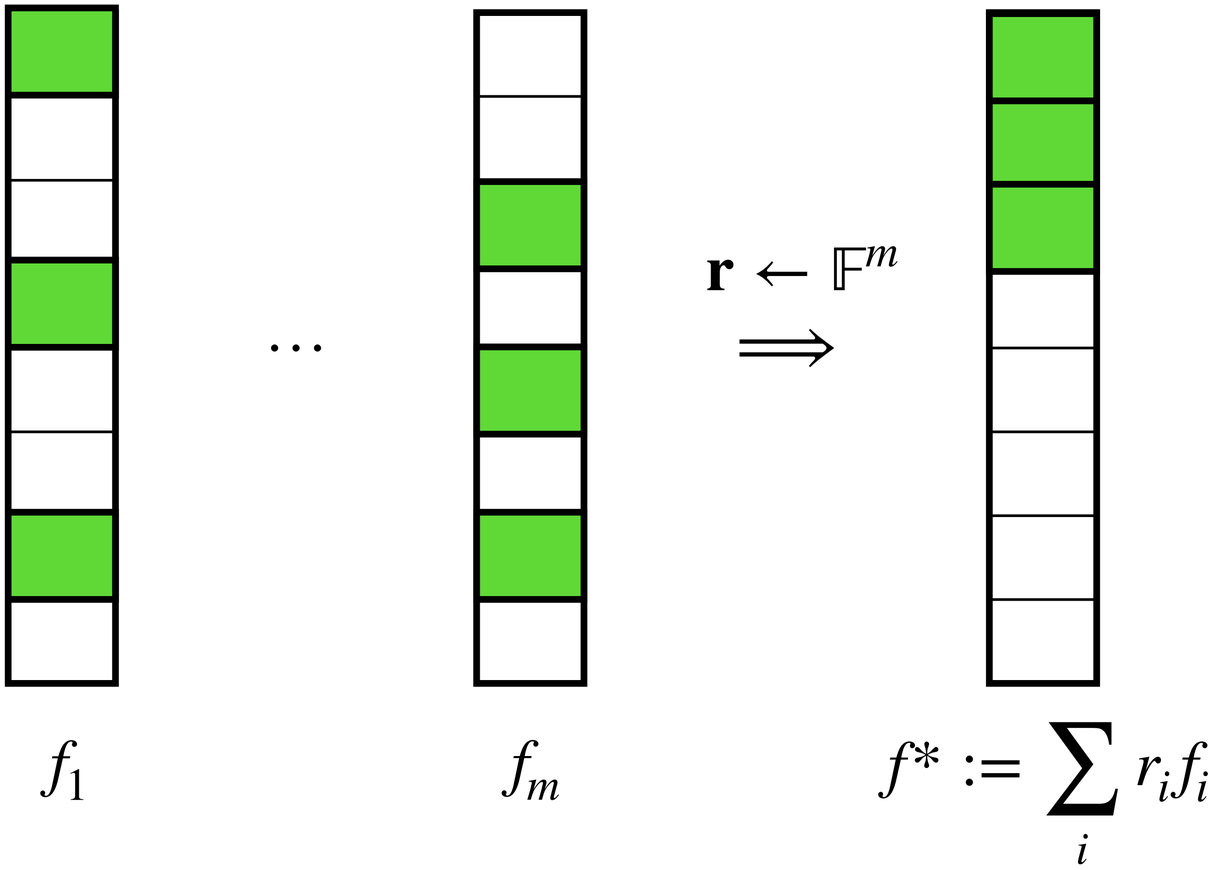

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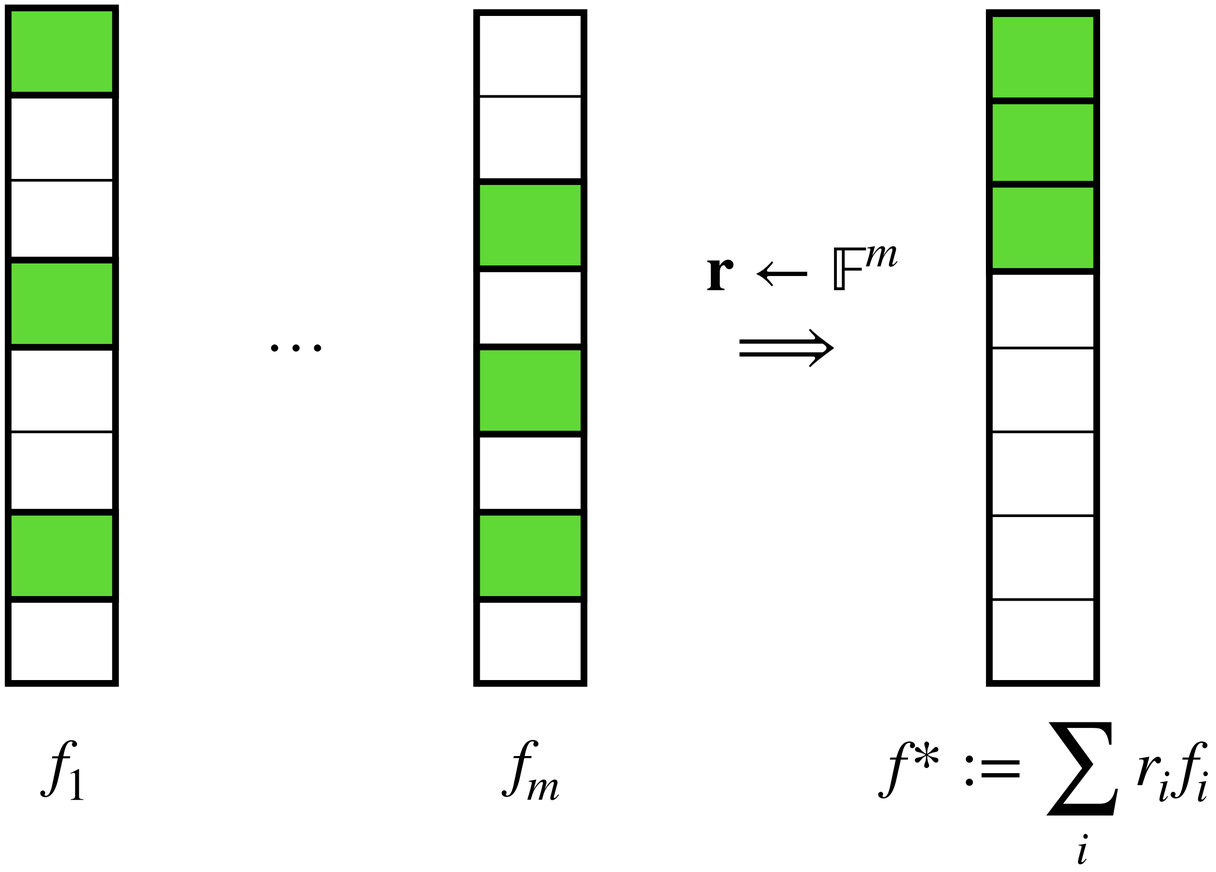

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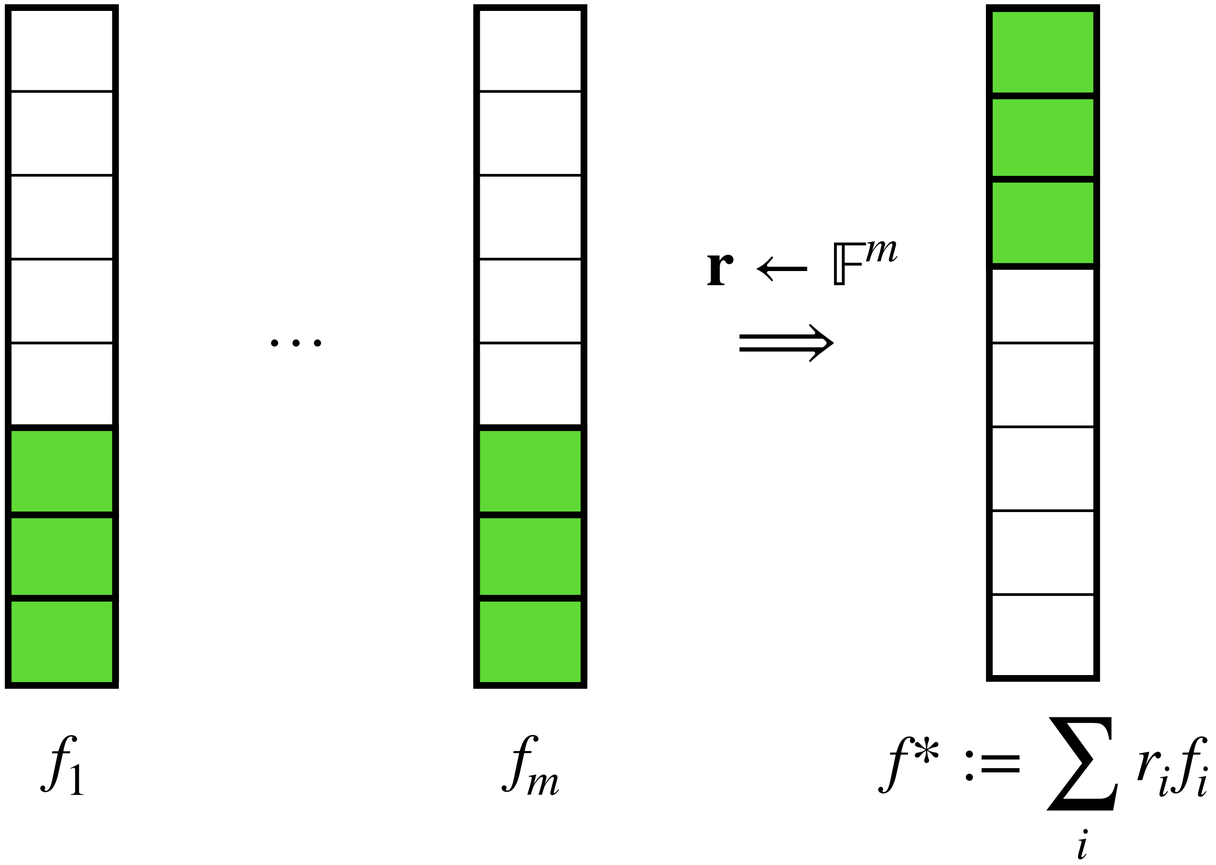

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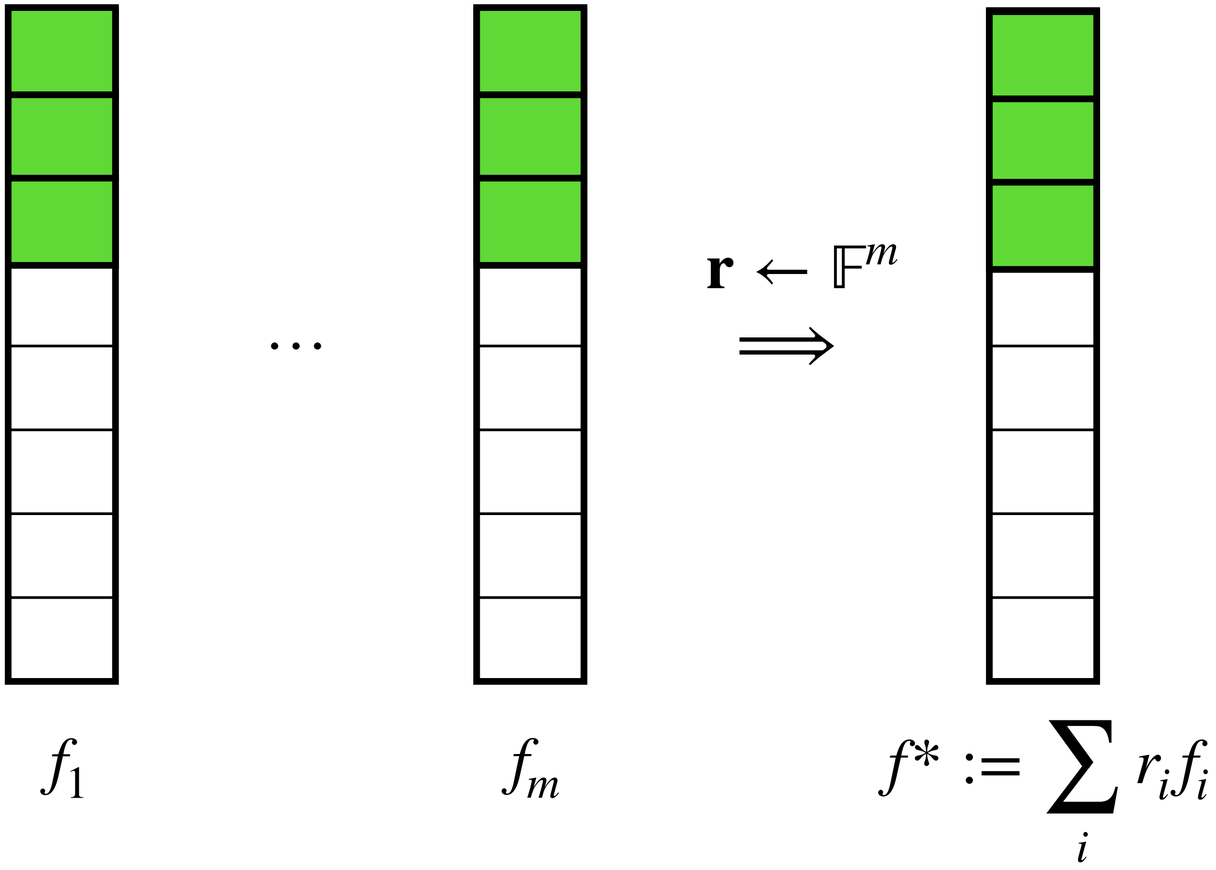

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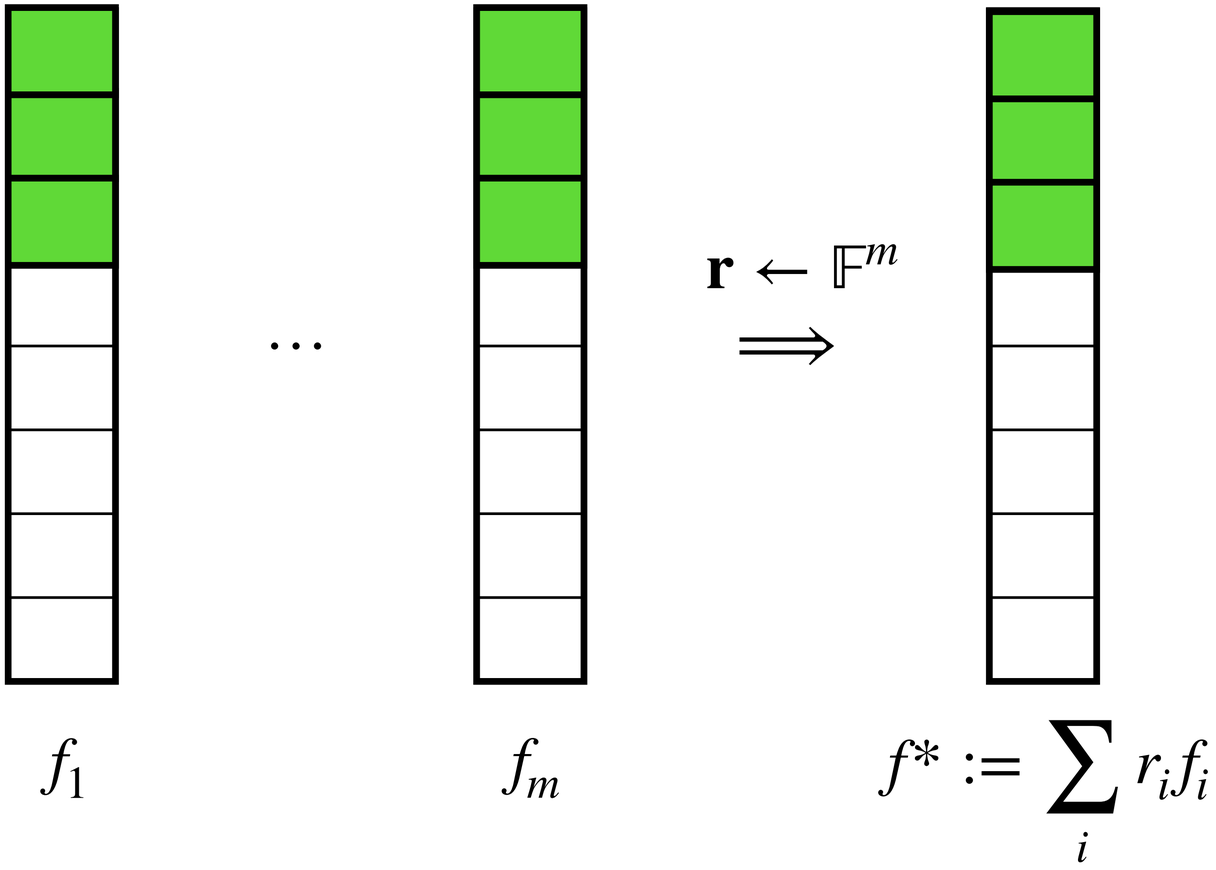

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Mutual correlated agreement: the stripe in which $f_1, \ldots, f_m$ agree with $\mathscr{C}$ is the same on which f^* does:

"*No new correlated domains appear*"

Folding and lists commute

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

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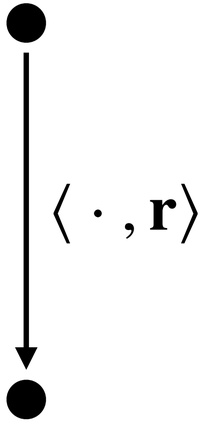

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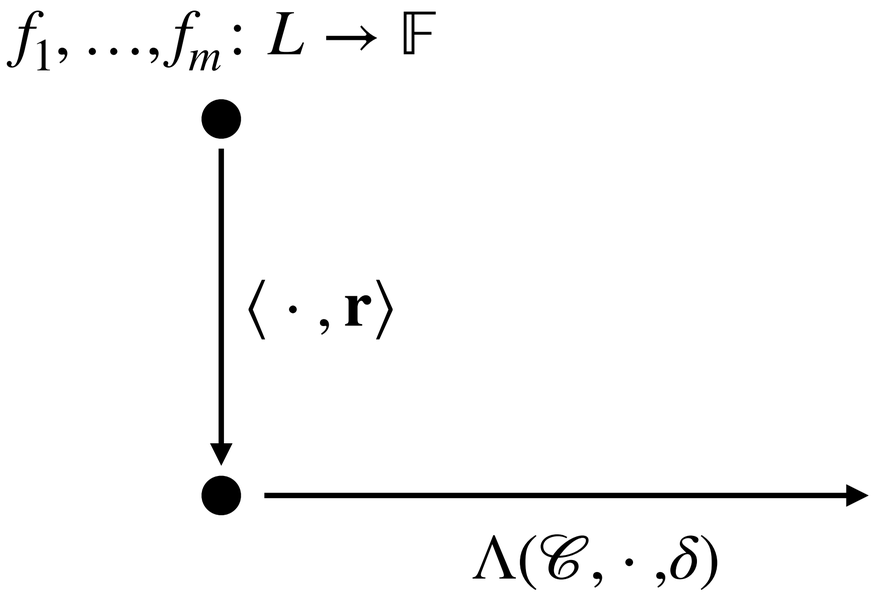

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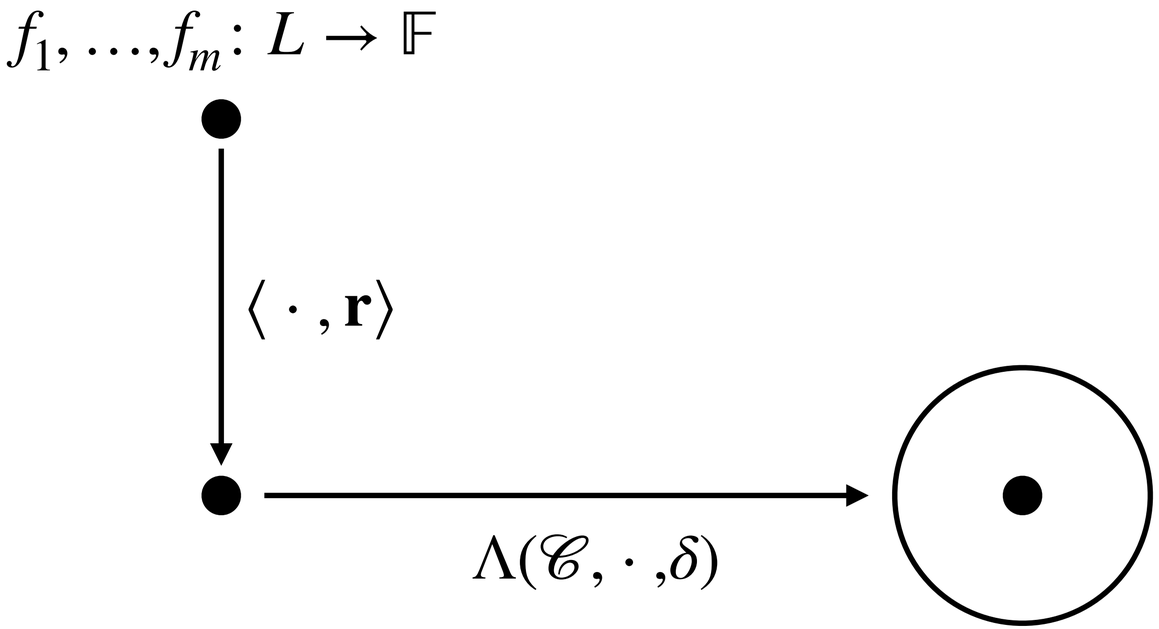

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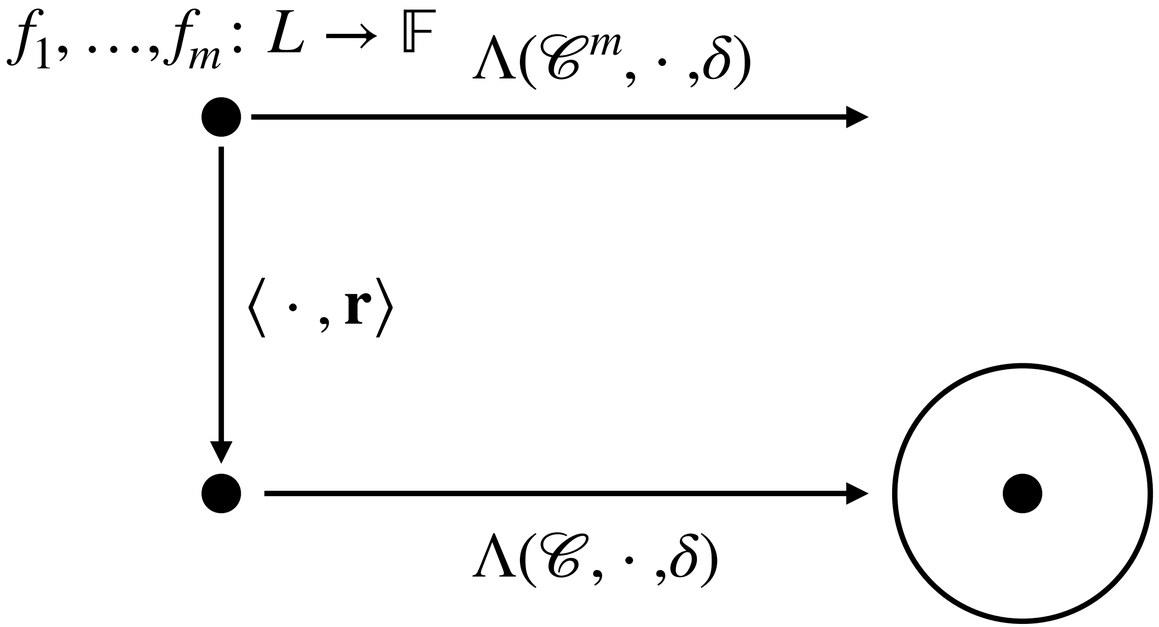

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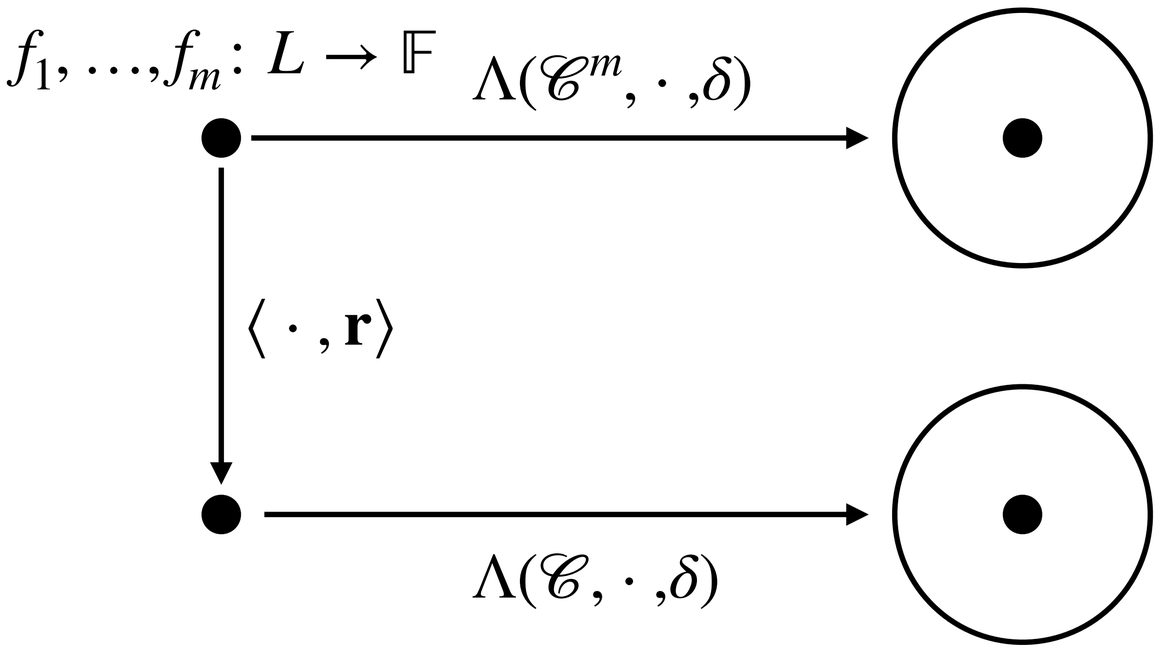

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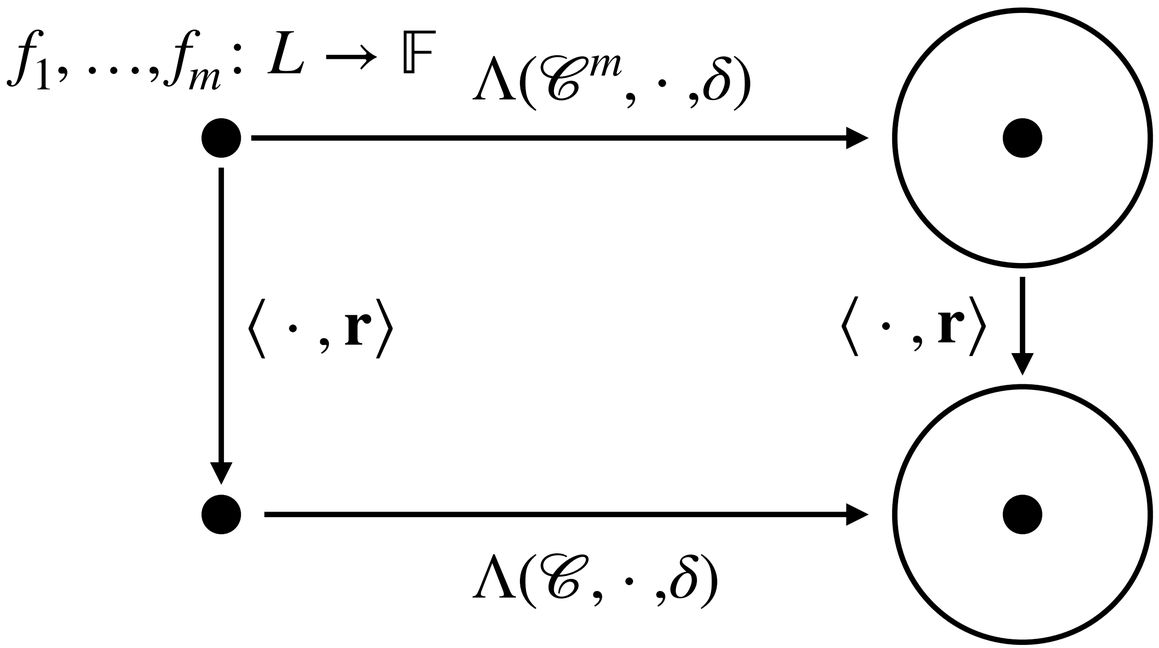

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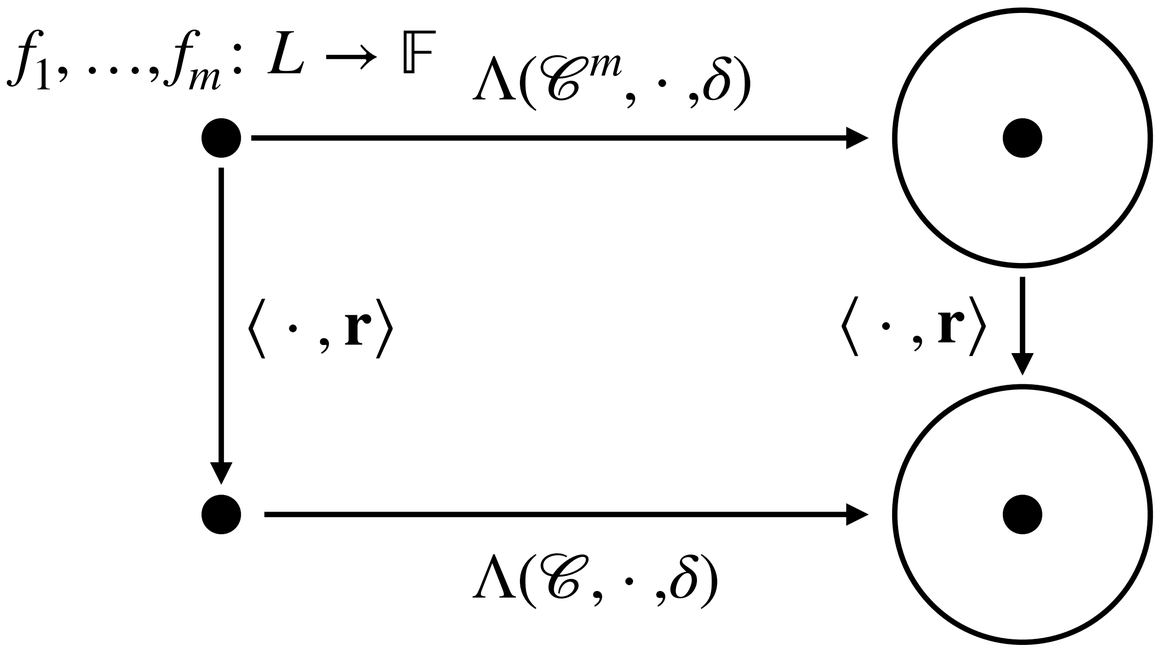

Stronger than what is required for STIR's soundness

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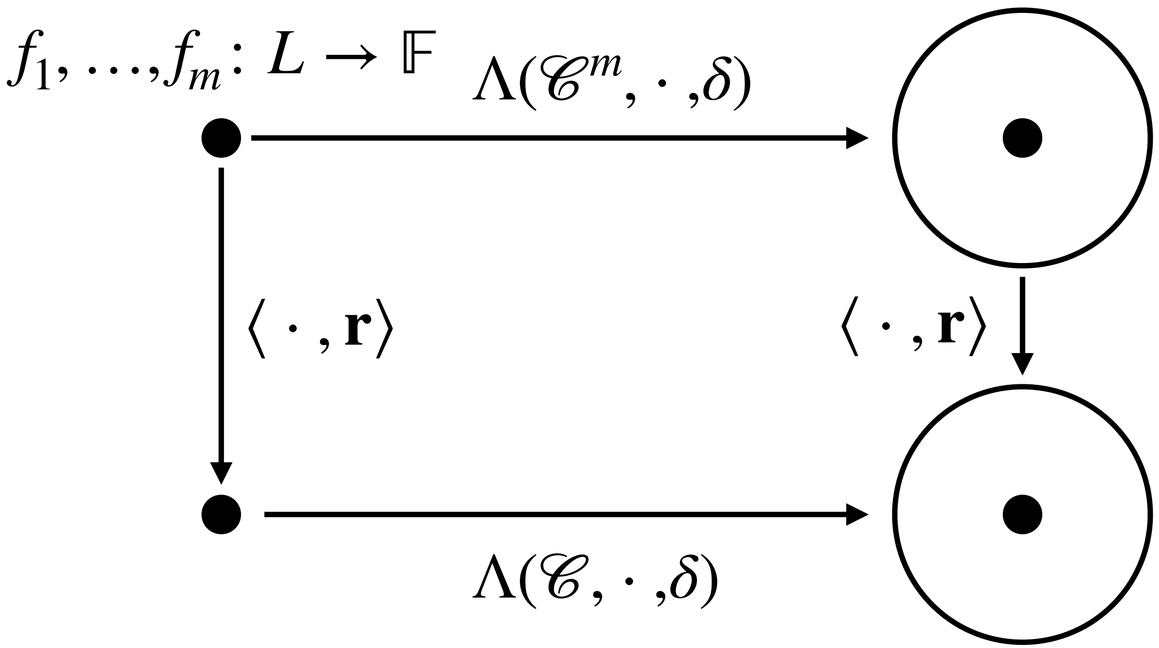

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w.h.p. over $\mathbf{r}$:

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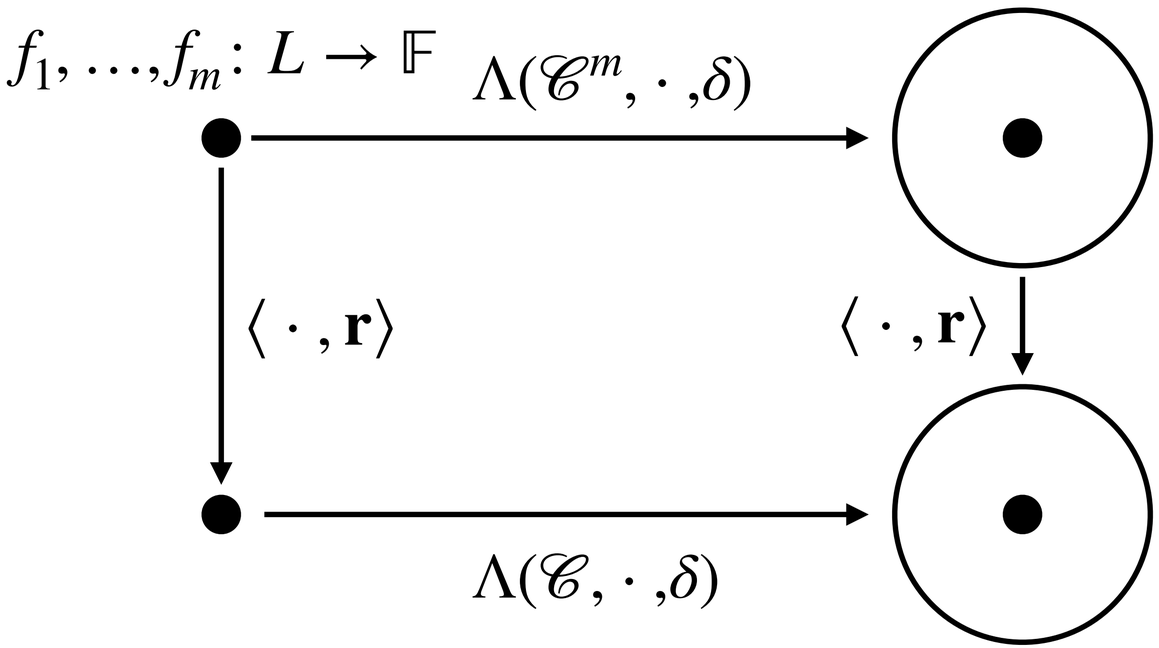

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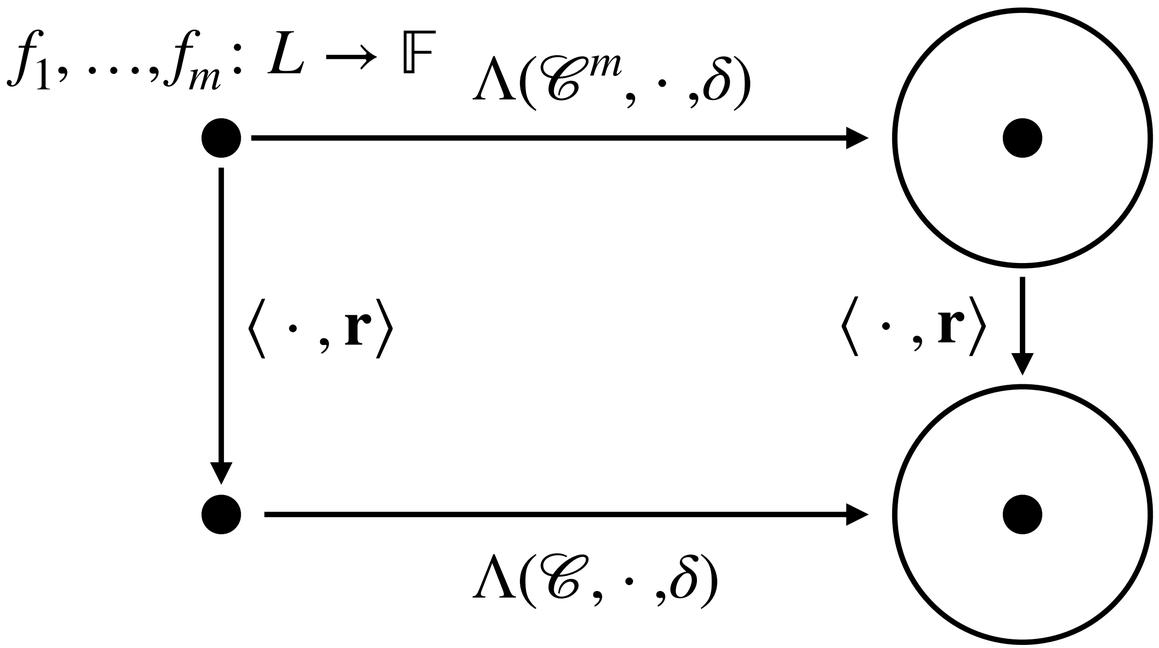

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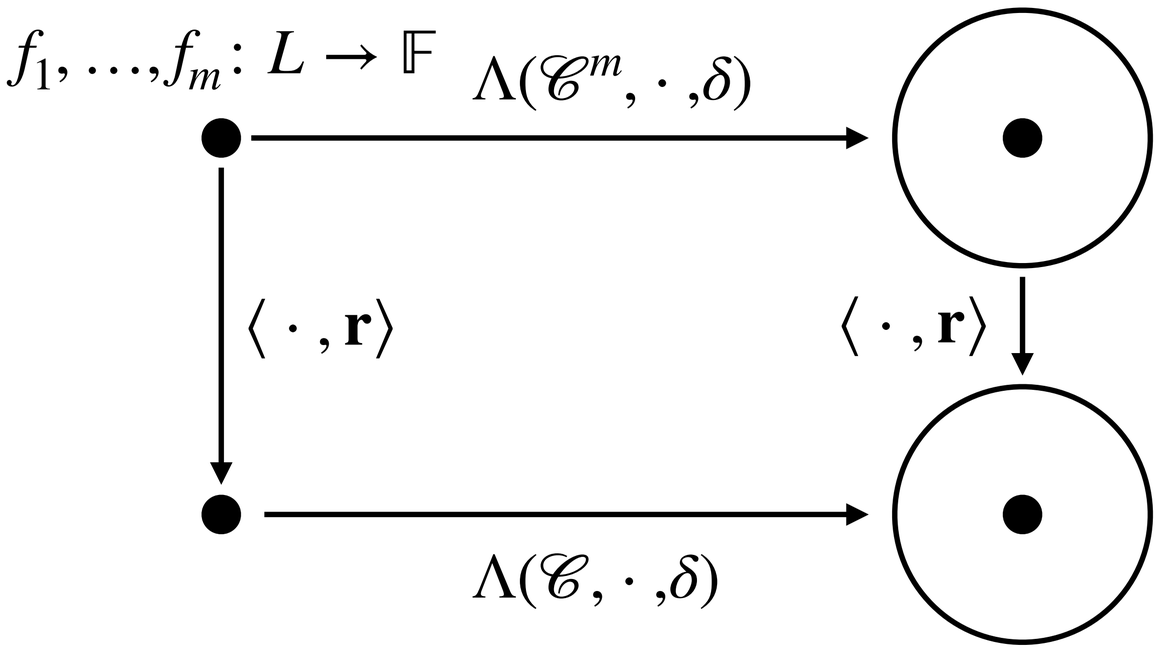

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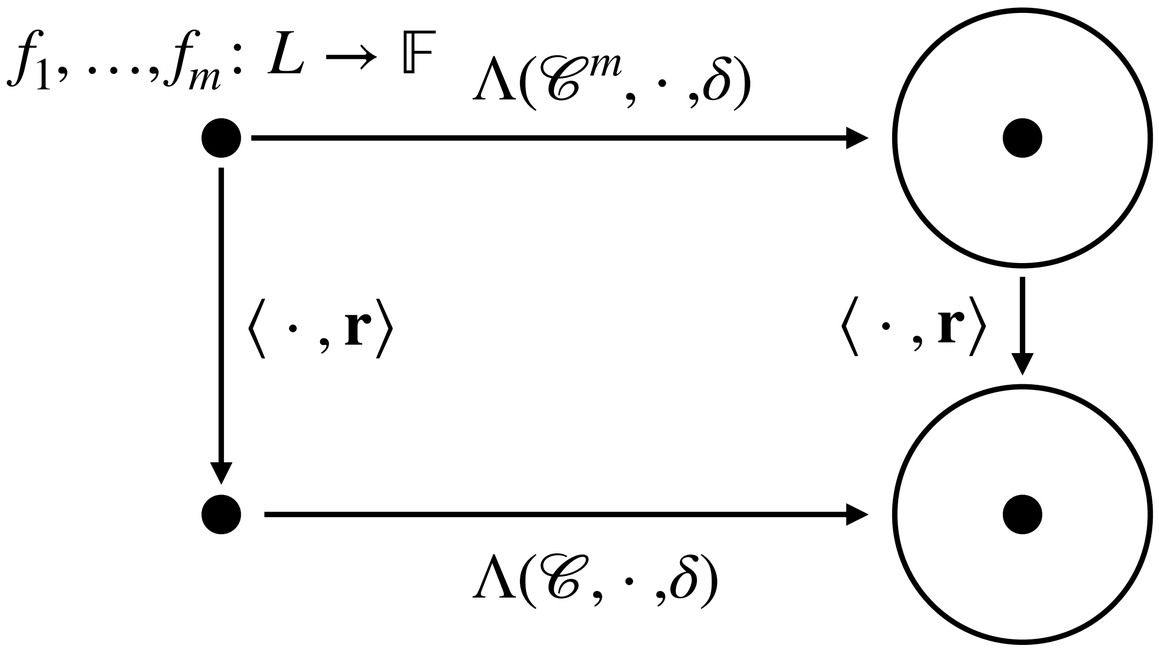

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Recent results show that mutual correlated agreement holds up to 1.5 Johnson for general linear codes!

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Reduce CRS$[n, m, \rho, \hat{w}, \sigma]$ **to** CRS$[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

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P

V

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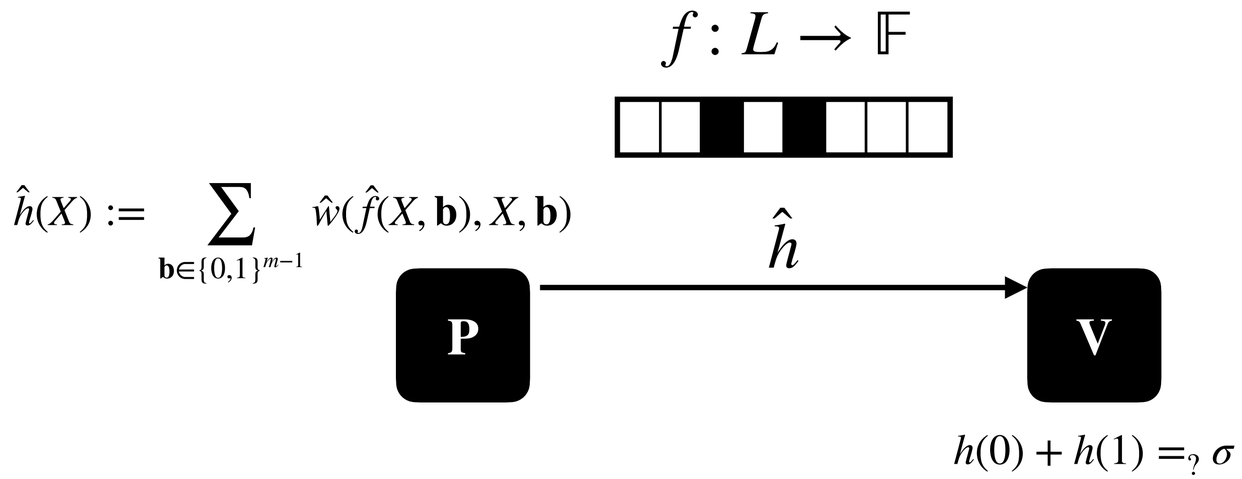

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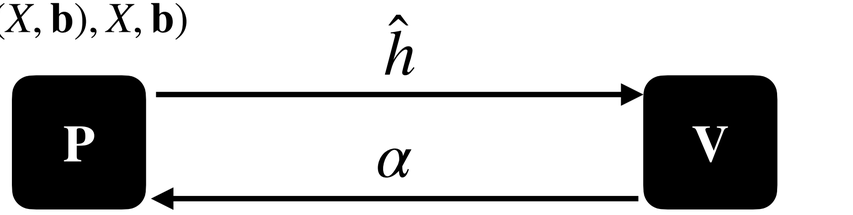
$$f : L \to \mathbb{F}$$

$$\hat{h}(X) := \sum_{\mathbf{b} \in \{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$$



$$h(0) + h(1) =_? \sigma$$

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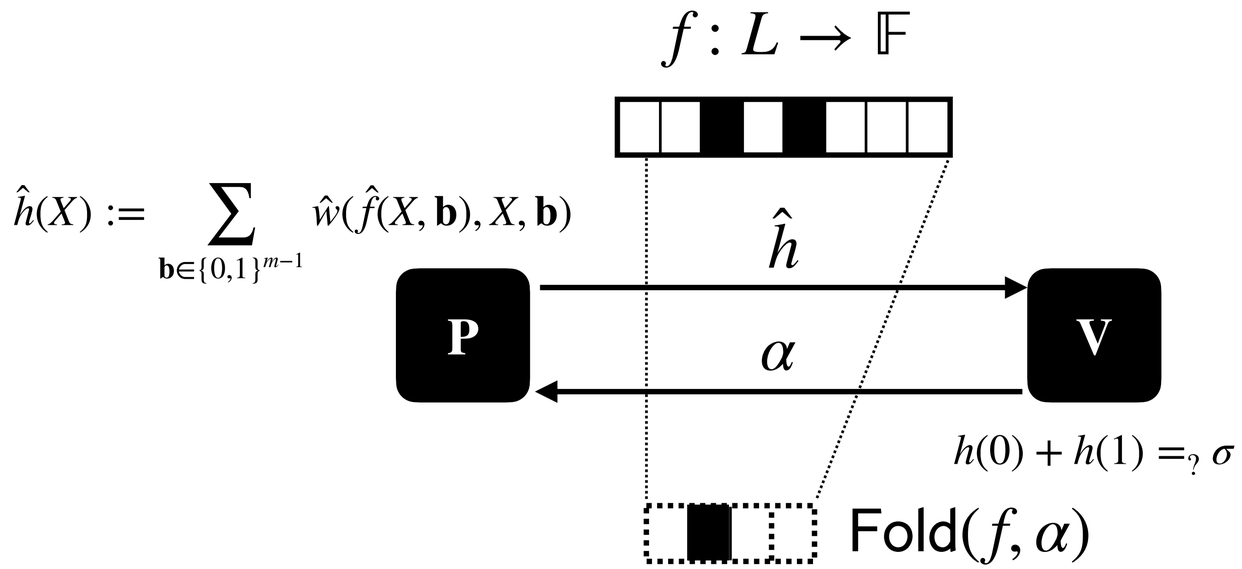

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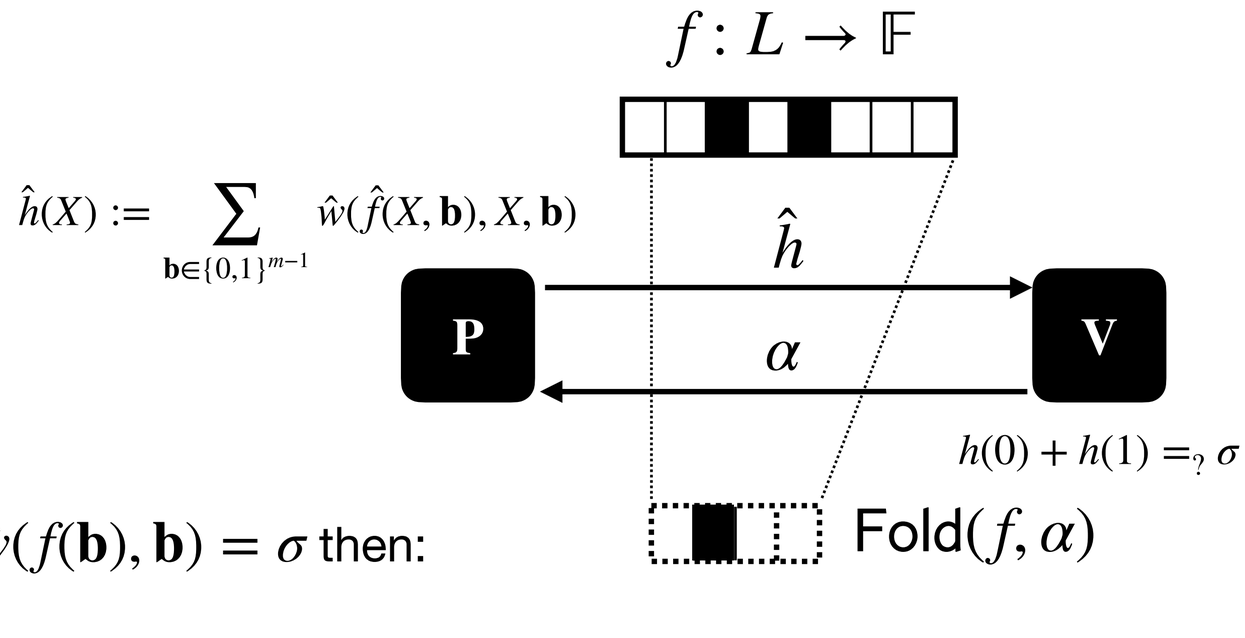

Completeness: $\sum_{\mathbf{b}} \hat{w}(f(\mathbf{b}), \mathbf{b}) = \sigma$ then:

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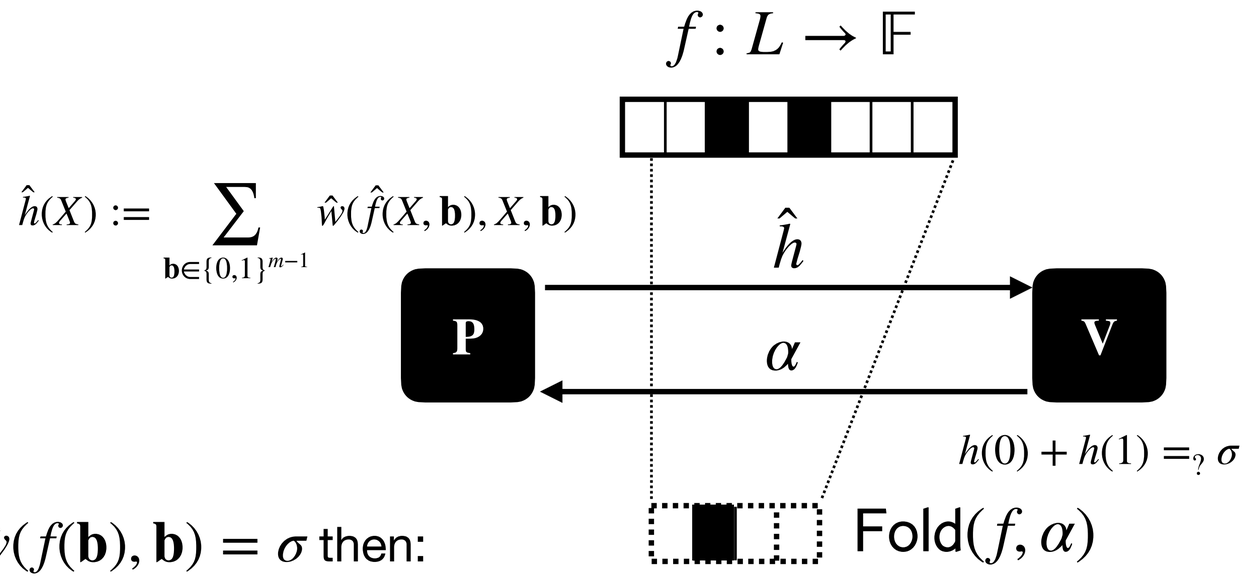

Completeness: $\sum_{\mathbf{b}} \hat{w}(f(\mathbf{b}), \mathbf{b}) = \sigma$ then:

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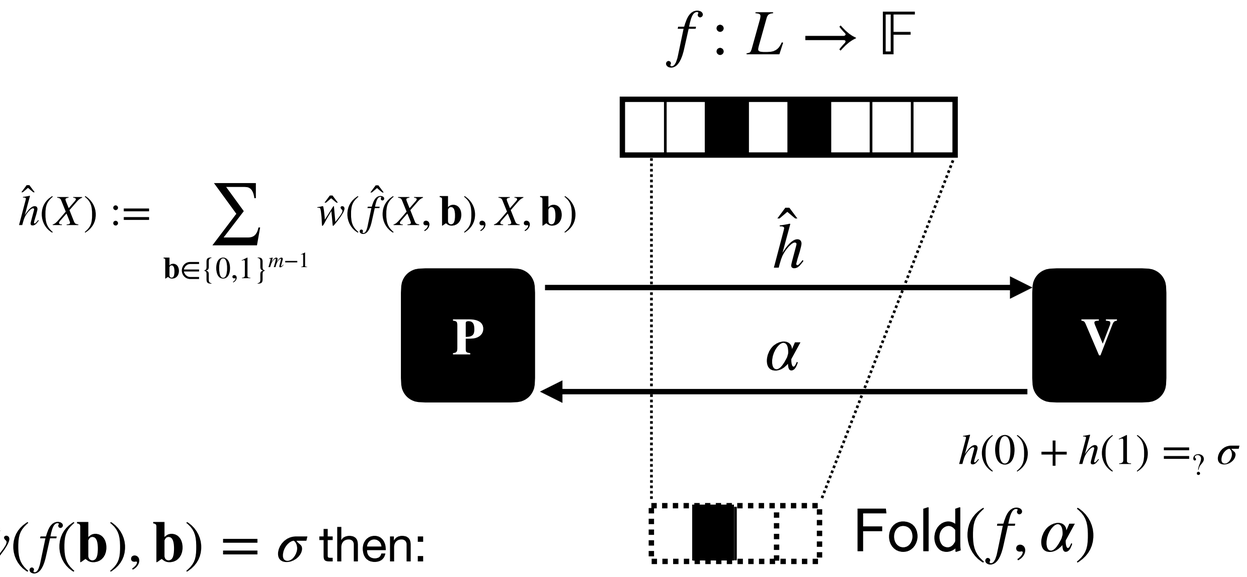

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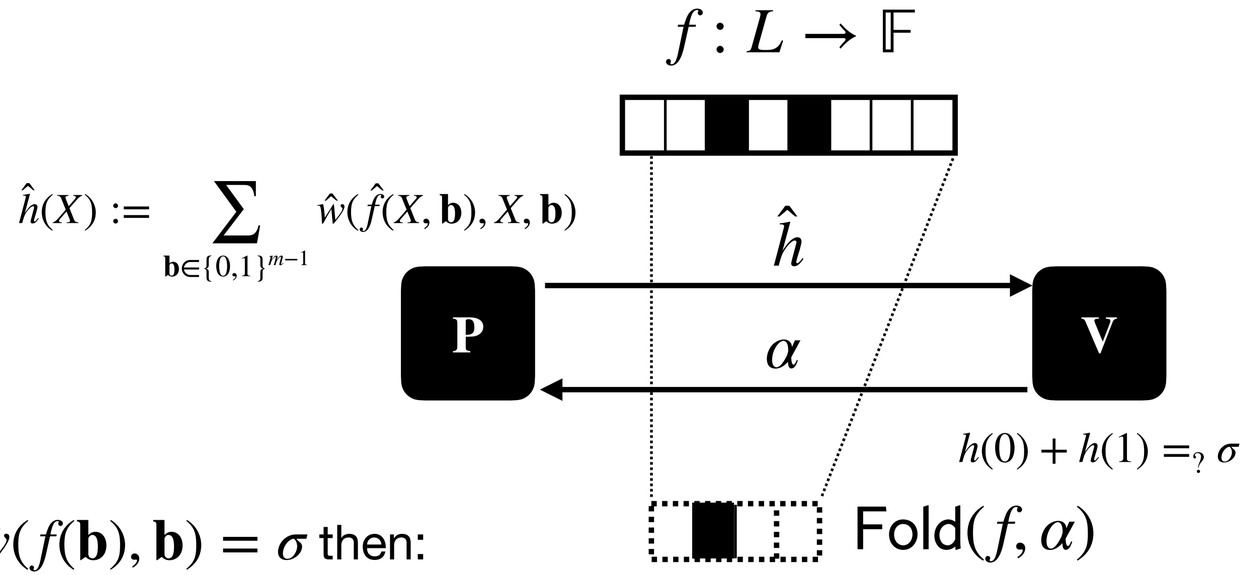

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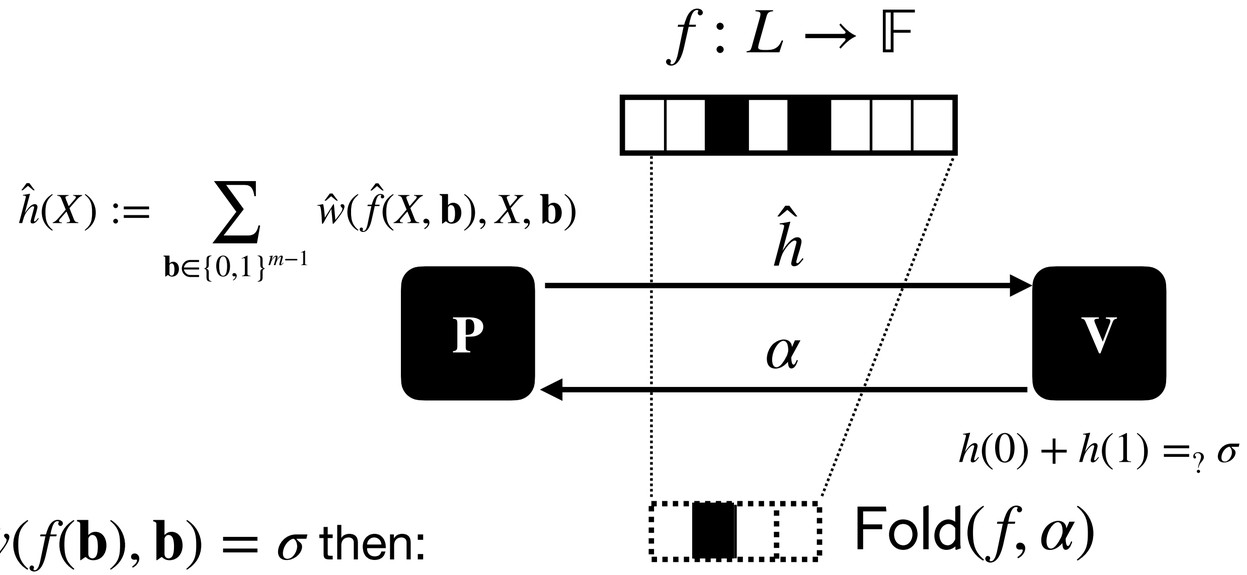

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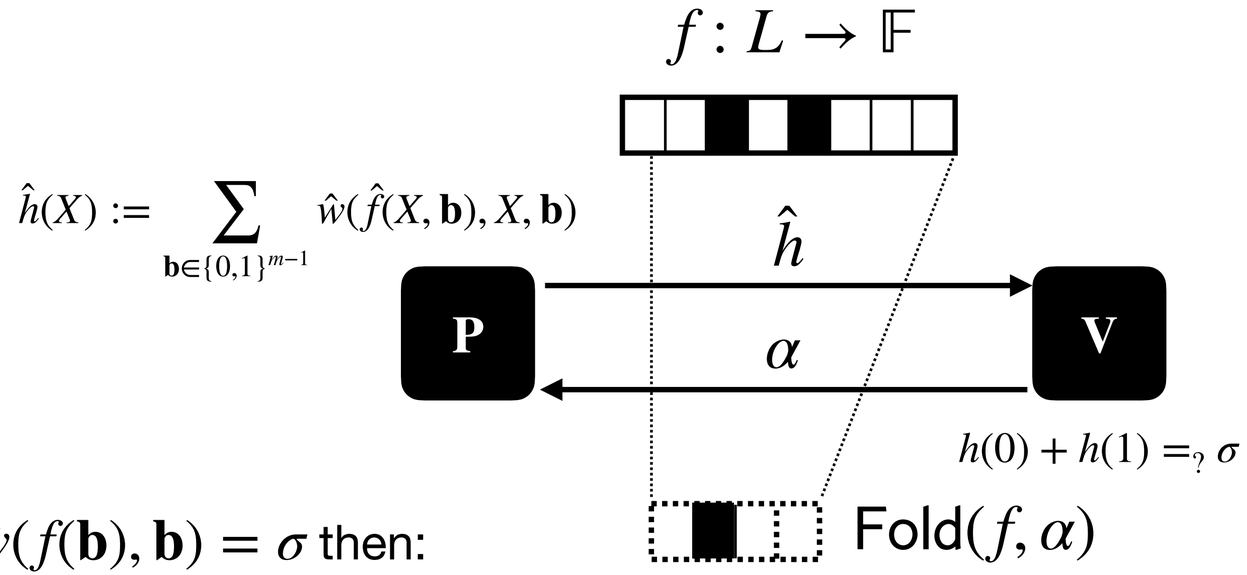

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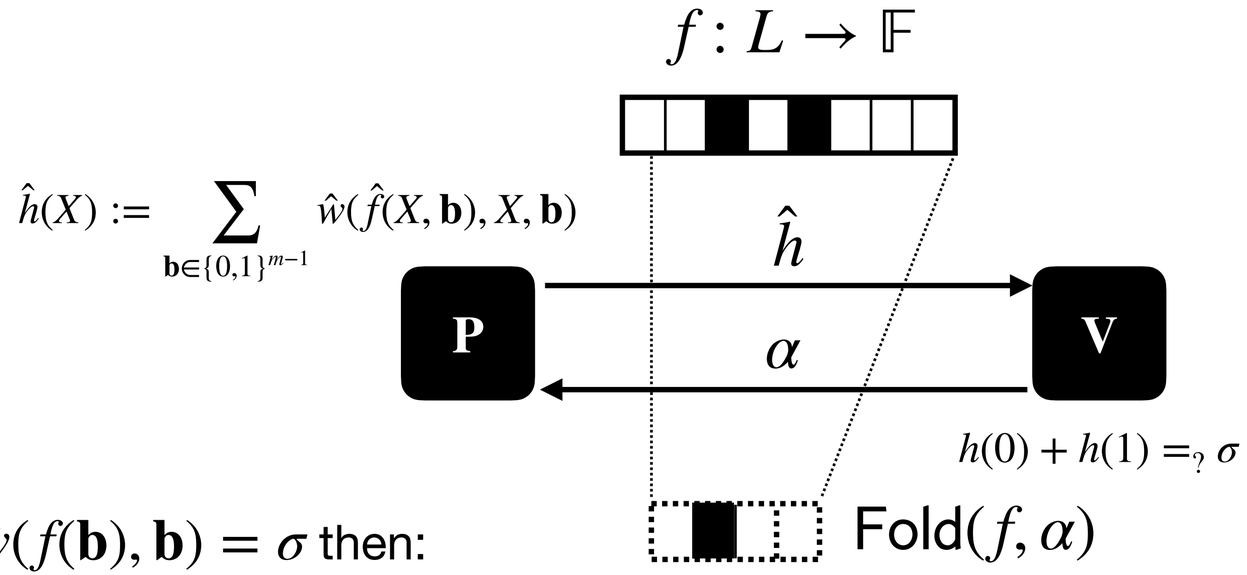

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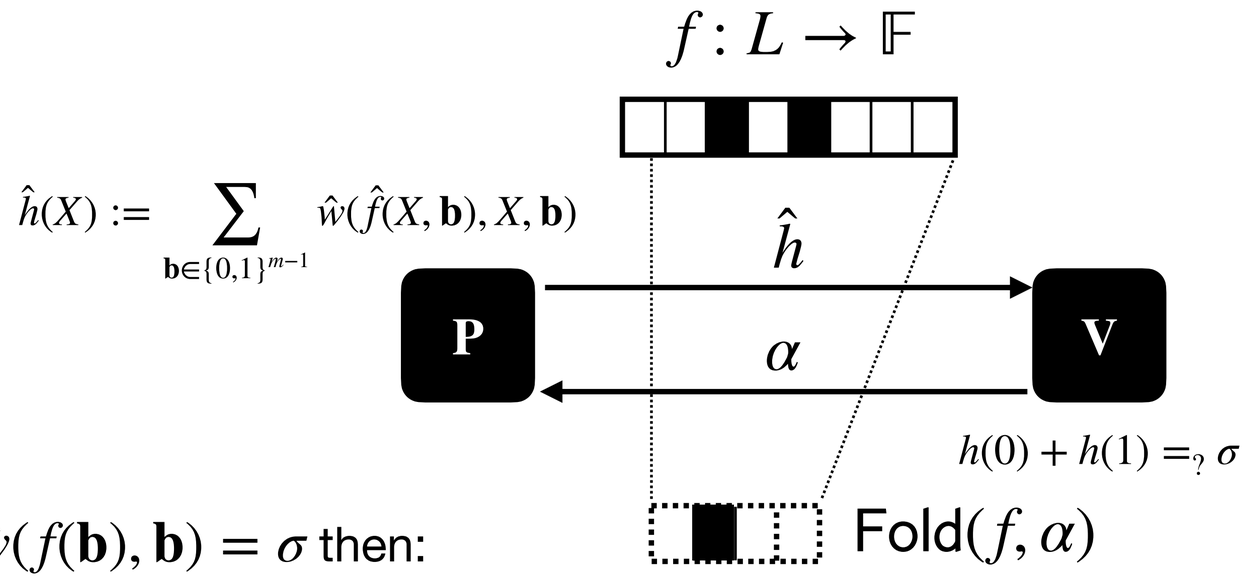

In the full protocol, we fold by 2-by-2 k times. Can also fold by 2^k at a time (nice for first round!)

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WHIR iteration

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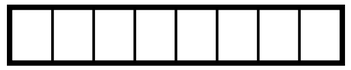
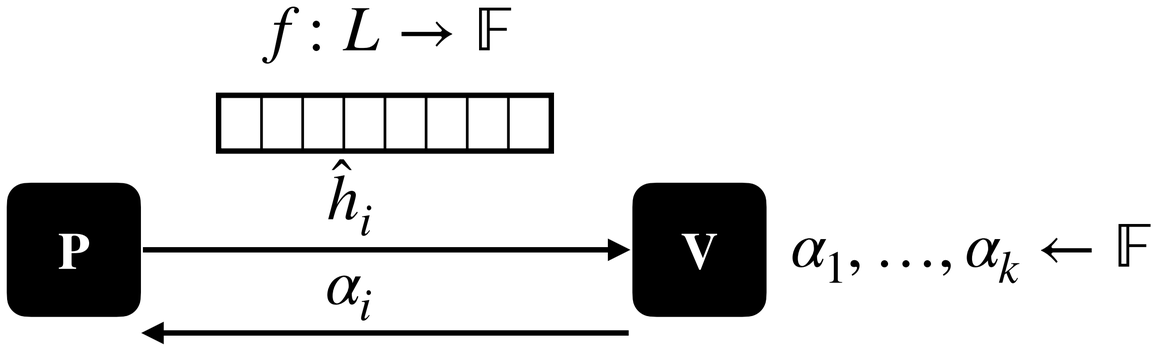


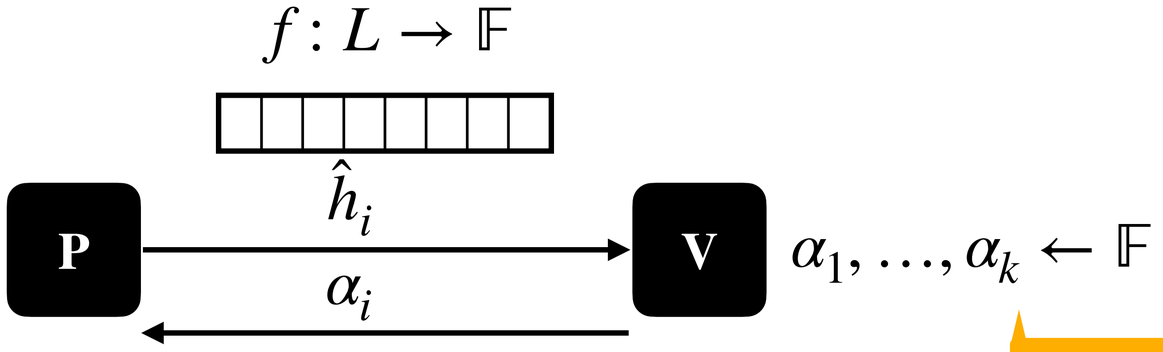
P

V

WHIR iteration



WHIR iteration



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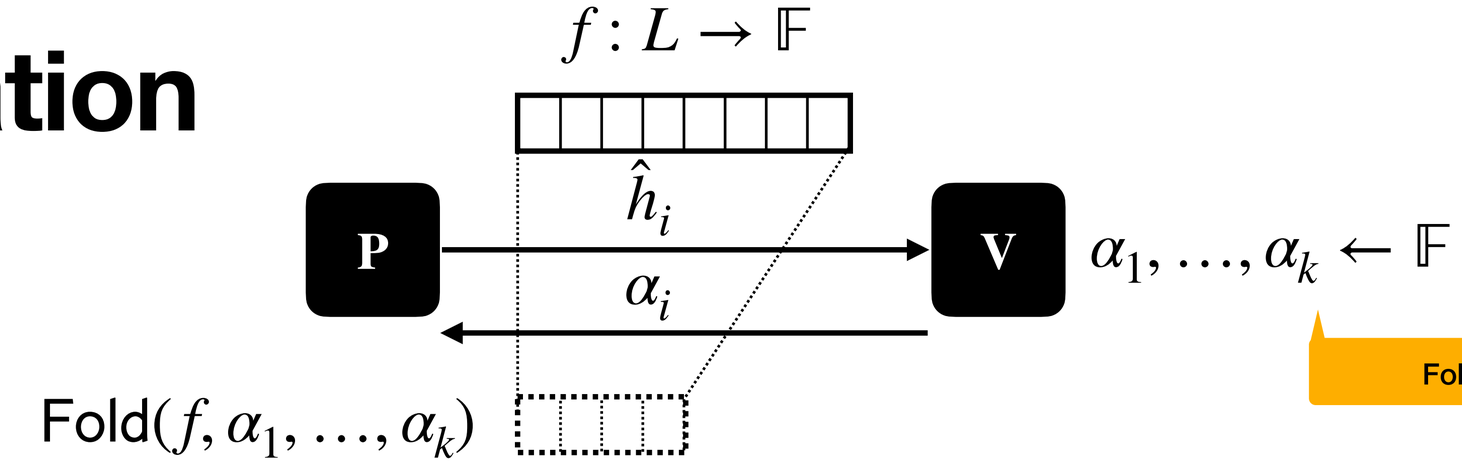
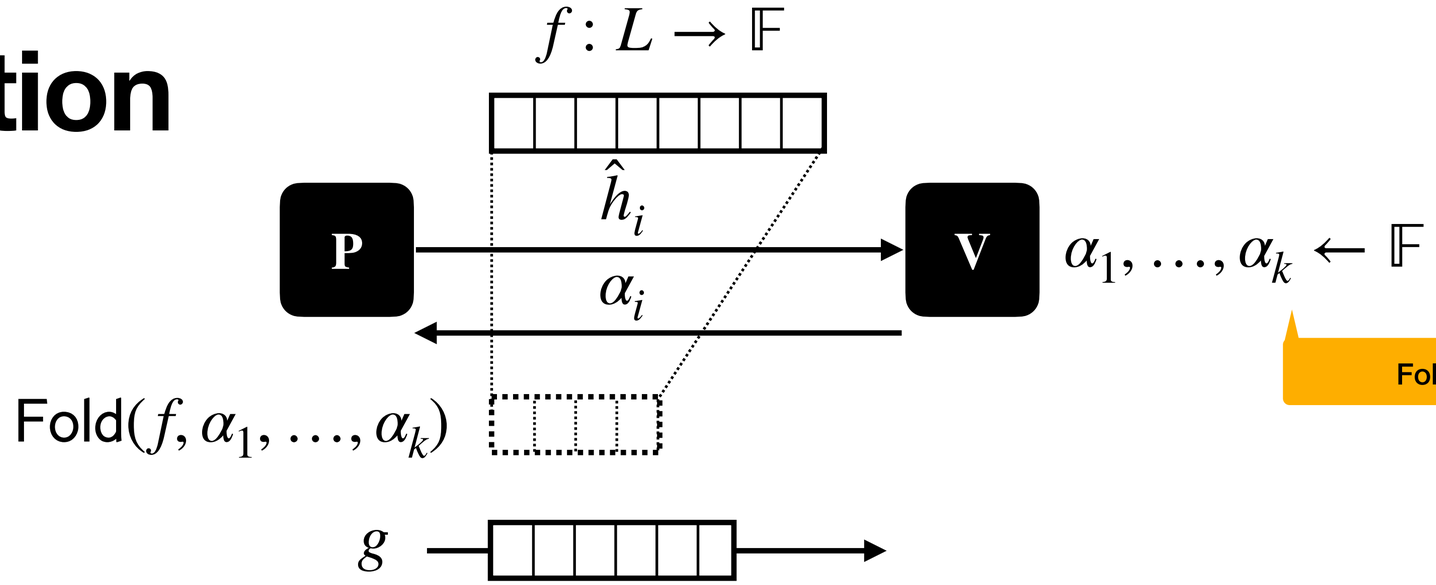


Folding k times by 2

WHIR iteration



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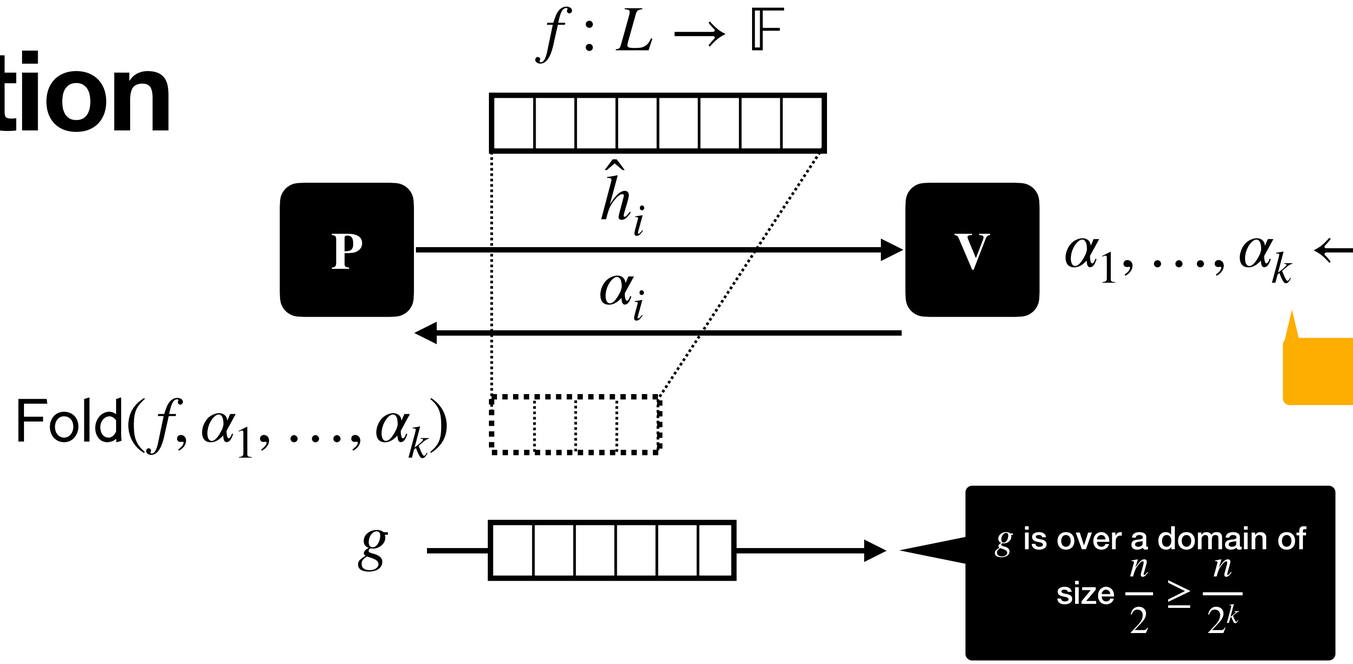

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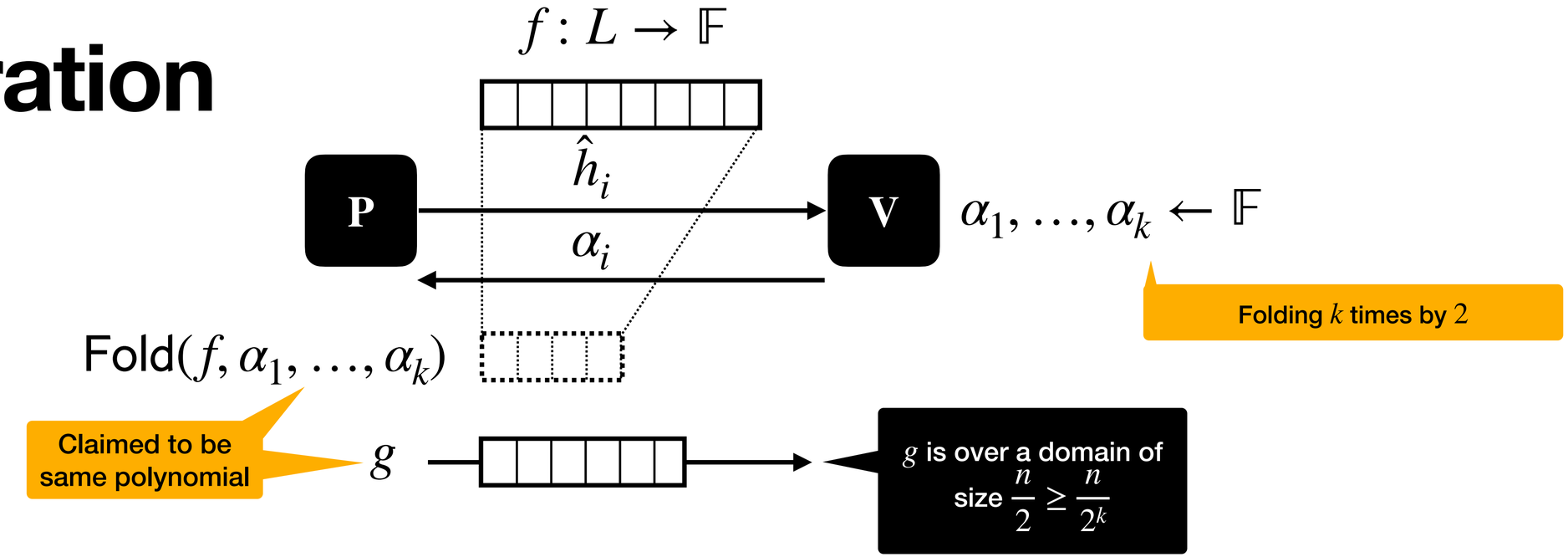

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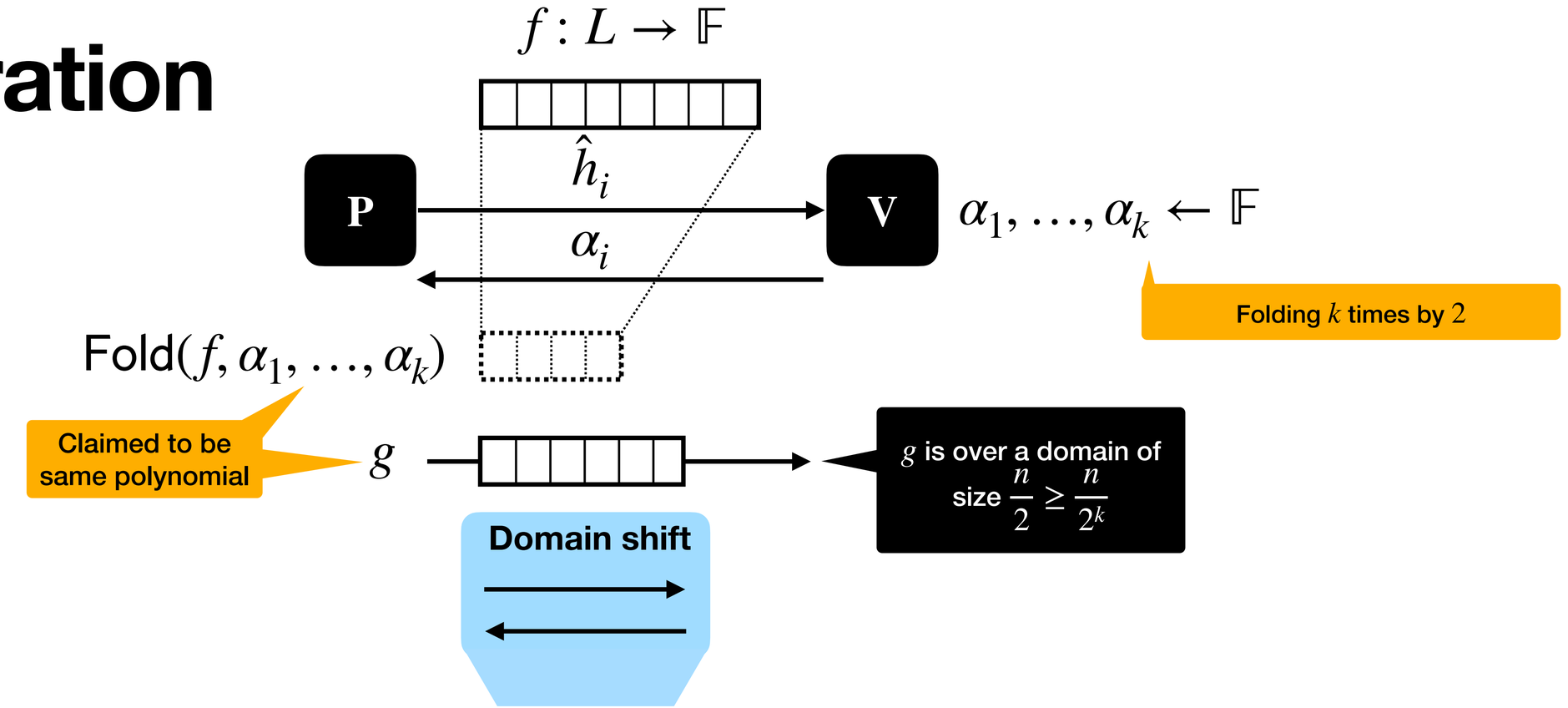

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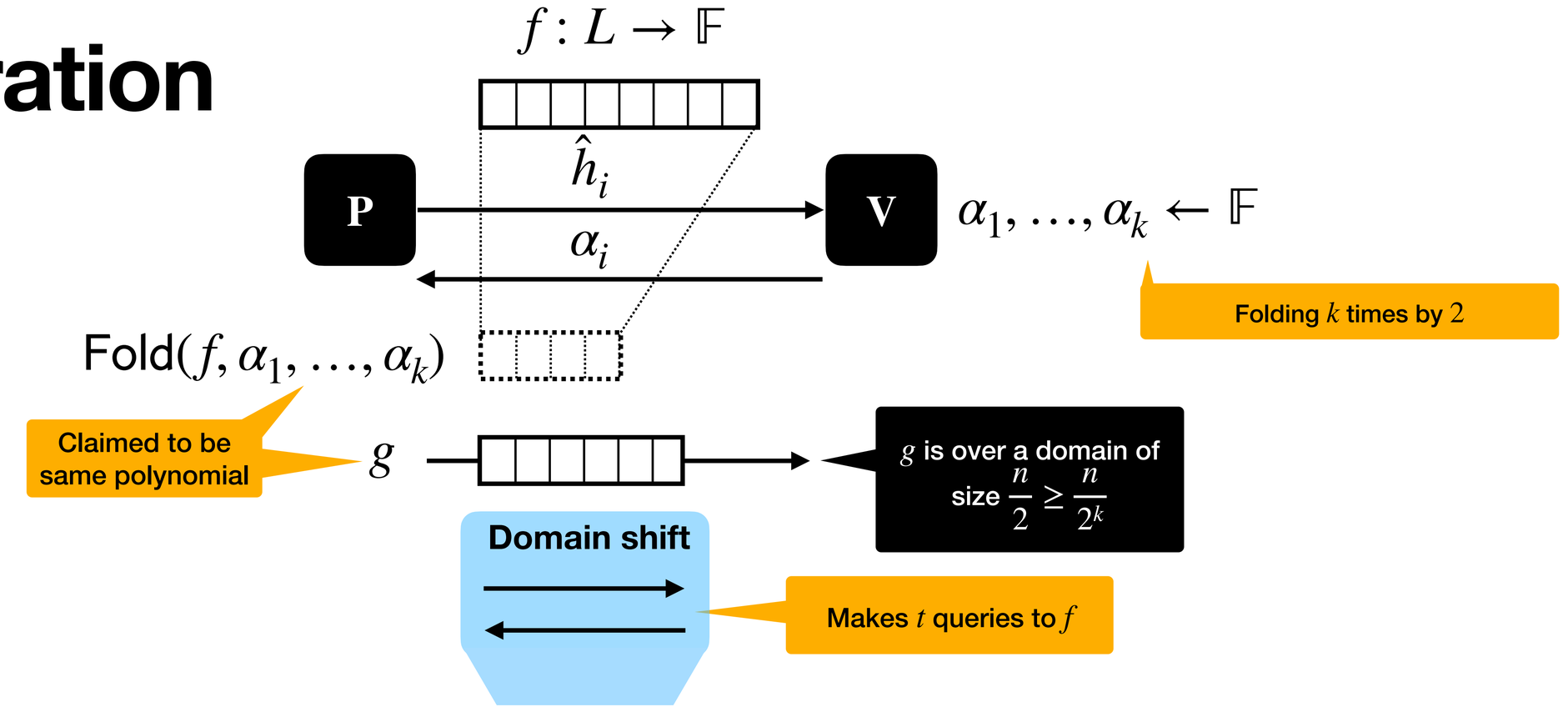

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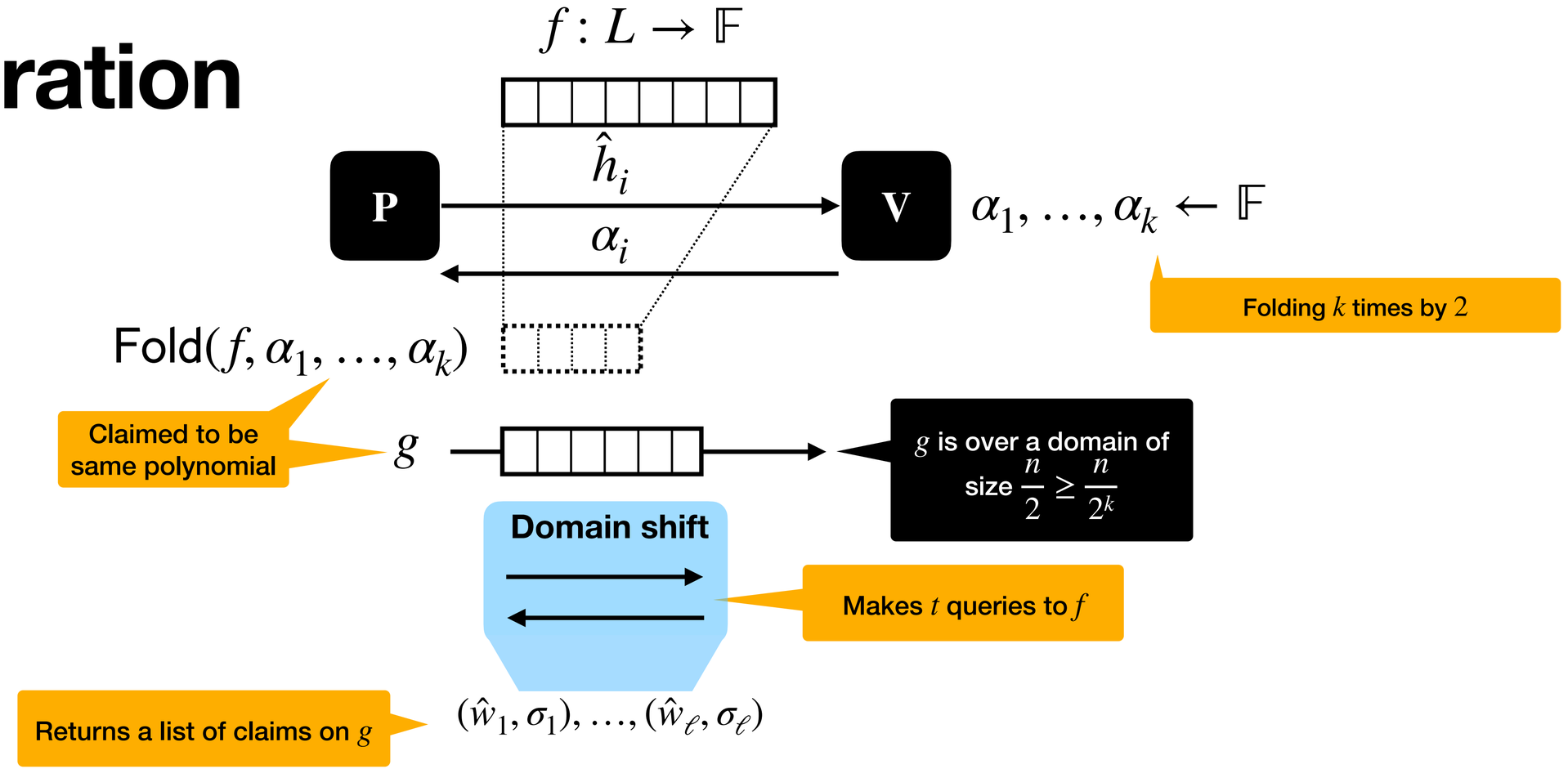

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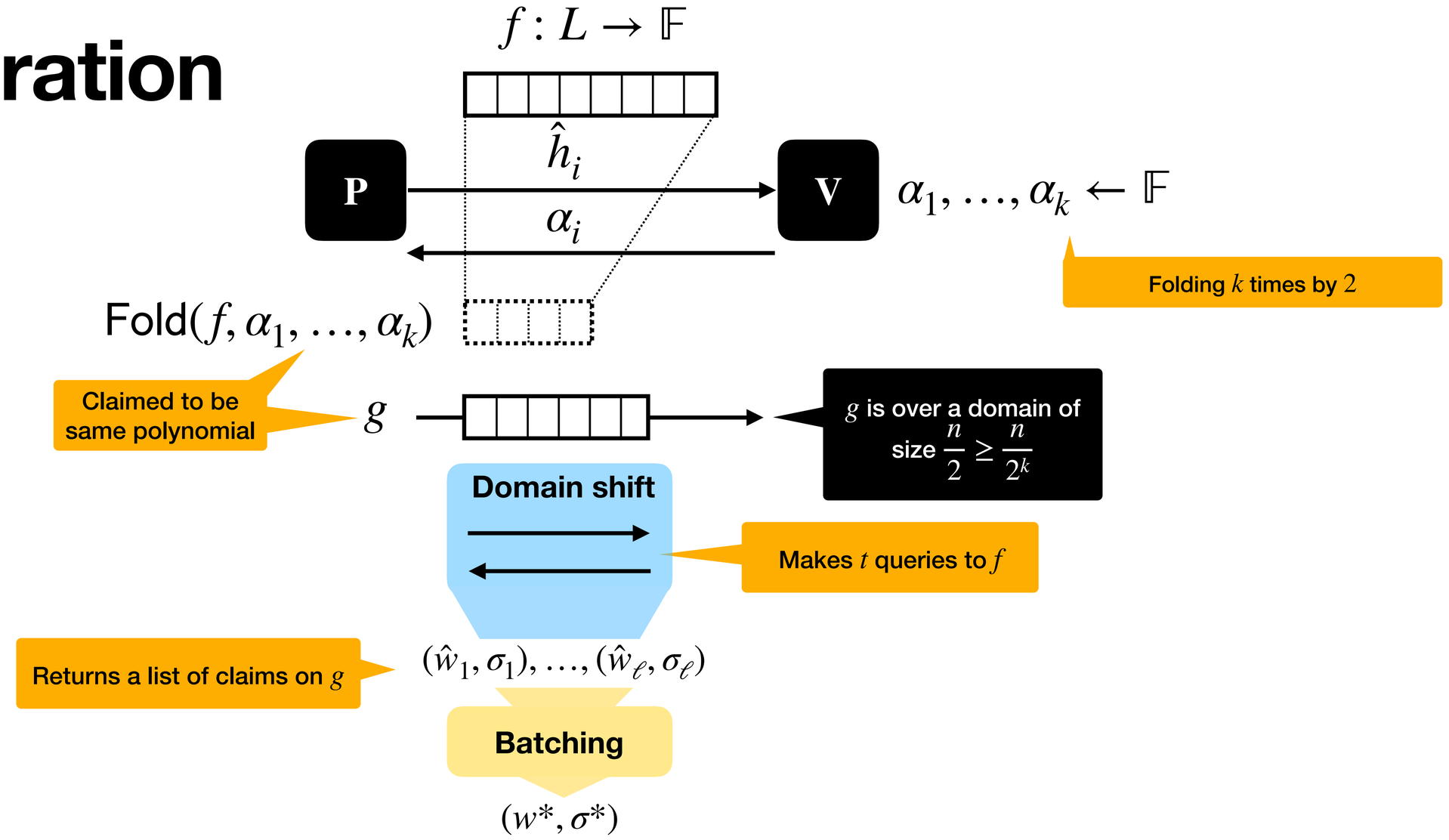

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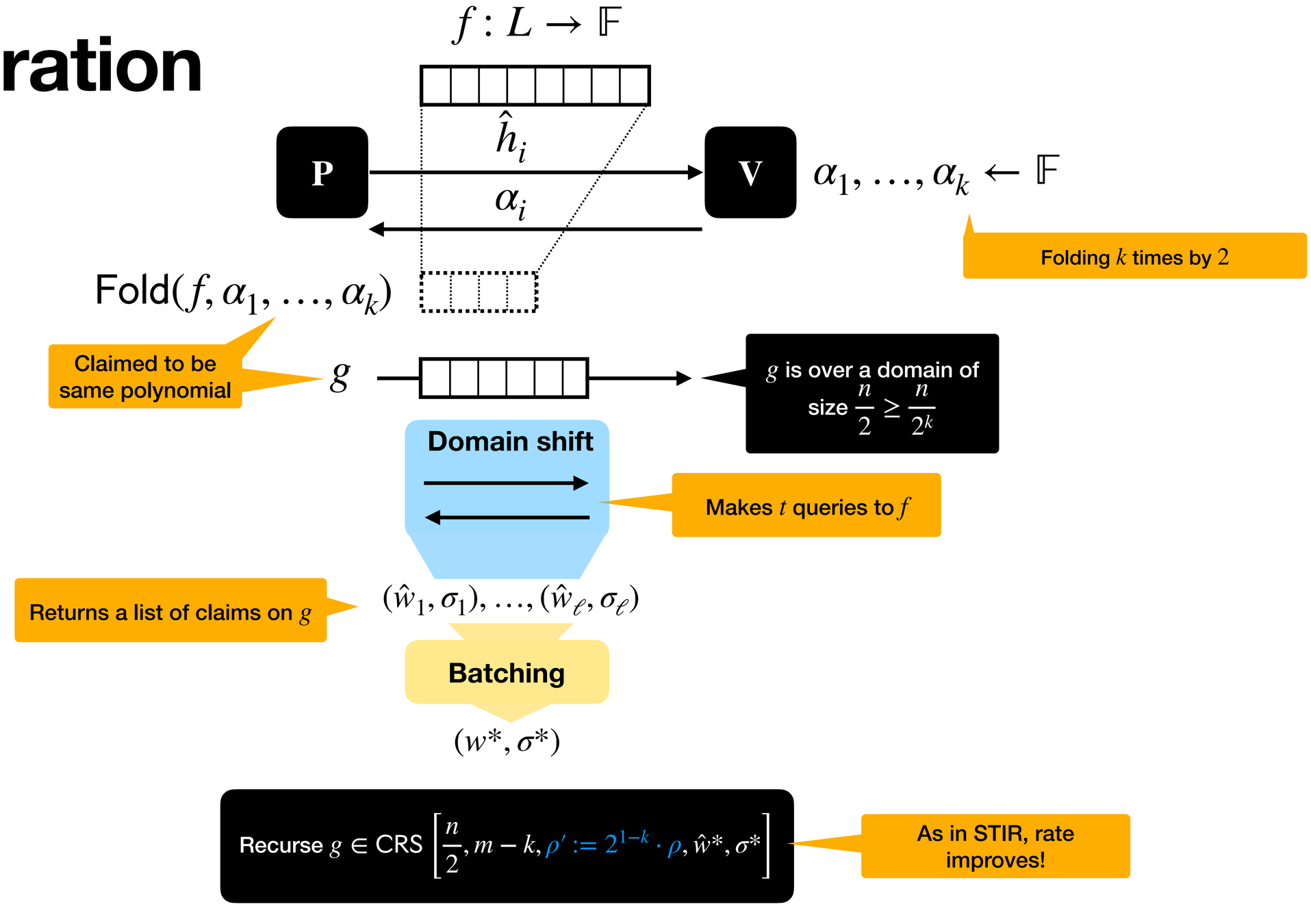

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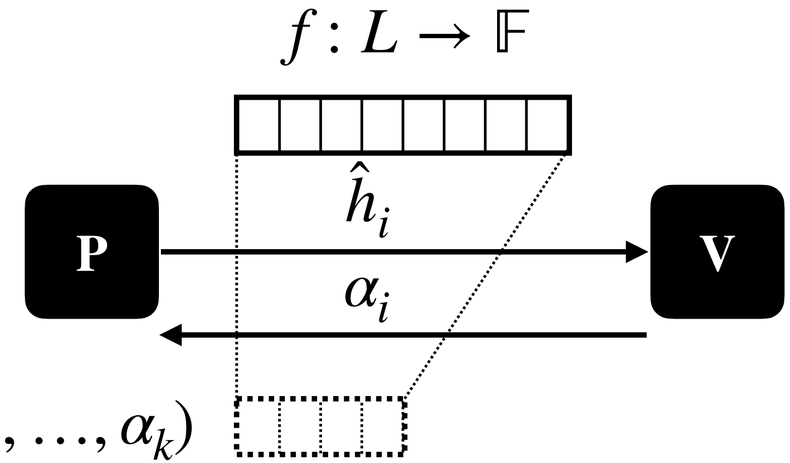


$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$

Folding k times by 2

$\text{Fold}(f, \alpha_1, \ldots, \alpha_k)$

Claimed to be same polynomial

g

g is over a domain of size $\frac{n}{2} \geq \frac{n}{2^k}$

Domain shift

Makes t queries to f

Returns a list of claims on g

$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

Batching

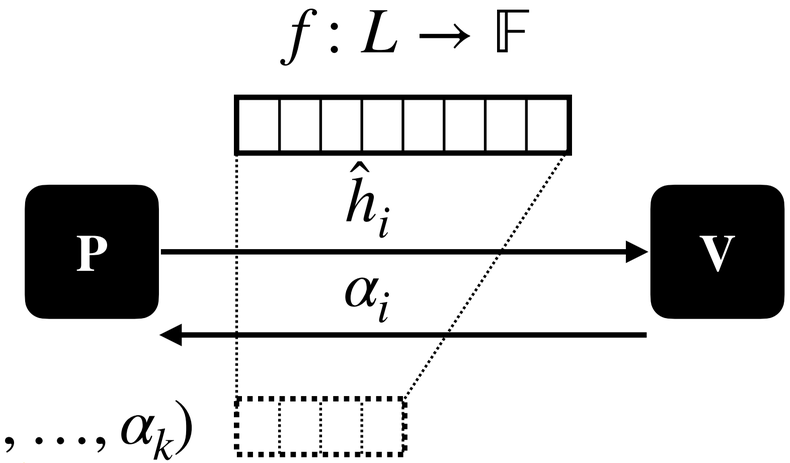
(w^*, σ^*)

Recurse $g \in \text{CRS}\left[\frac{n}{2}, m-k, \rho' := 2^{1-k} \cdot \rho, \hat{w}^*, \sigma^*\right]$

As in STIR, rate improves!

Similar structure to STIR! Multilinear structure **forbids** using quotients: we need new ideas to domain shift!

WHIR iteration



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Domain shifting

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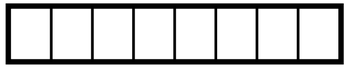
Claim on f: $(\hat{w}, \sigma)$

$$f : L \to \mathbb{F}$$



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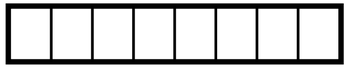

$g : L^* \to \mathbb{F}$

Domain shifting

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Output claims on g:
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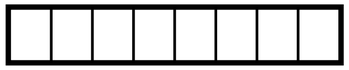

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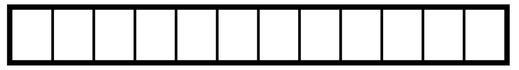

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f and g claimed to be evaluations of same polynomial. Want to output **claims** on g.

Goal: If f is $\left(1 - \sqrt{\rho}\right)$-far from CRS$[|L|, m, \rho, \hat{w}, \sigma]$, w.h.p. g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[|L^*|, m, \rho', \hat{w}_i, \sigma_i]$ for at least one $i \in [\ell]$

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Assume there is unique polynomial $\hat{p}$ that is $\left(1 - \sqrt{\rho'}\right)$-close to g.

OOD subprotocol (next)

Domain shifting

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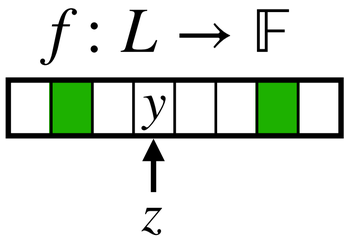
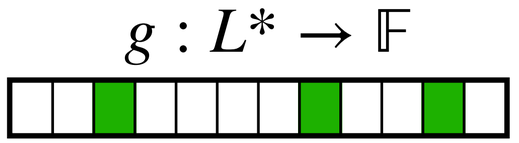
Then, if $\hat{p}$ satisfies the $(\hat{w}, \sigma)$-constraint f must be be $\left(1 - \sqrt{\rho}\right)$-far from it.

Domain shifting

Claim on f: $(\hat{w}, \sigma)$





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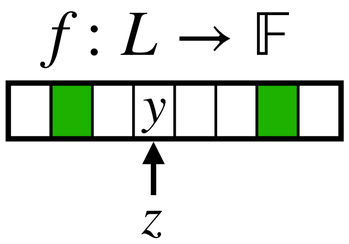

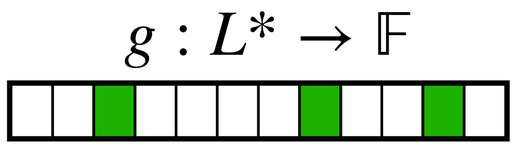

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OOD subprotocol (next)

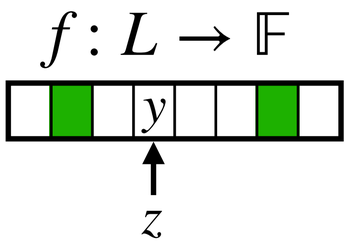
Then, if $\hat{p}$ satisfies the $(\hat{w}, \sigma)$-constraint f must be be $\left(1 - \sqrt{\rho}\right)$-far from it.

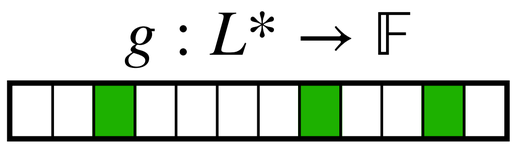
New constraints: (i) original constraint $(\hat{w}, \sigma)$ (ii) $\hat{p}(z) = y$ for some random point z.

Just an evaluation constraint which we know how to handle!

Domain shifting

Claim on f: $(\hat{w}, \sigma)$





Output claims on g:
$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

f and g claimed to be evaluations of same polynomial. Want to output **claims** on g.

Goal: If f is $\left(1 - \sqrt{\rho}\right)$-far from CRS$[|L|, m, \rho, \hat{w}, \sigma]$, w.h.p. g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[|L^*|, m, \rho', \hat{w}_i, \sigma_i]$ for at least one $i \in [\ell]$

Assume there is unique polynomial $\hat{p}$ that is $\left(1 - \sqrt{\rho'}\right)$-close to g.

OOD subprotocol (next)

Then, if $\hat{p}$ satisfies the $(\hat{w}, \sigma)$-constraint f must be be $\left(1 - \sqrt{\rho}\right)$-far from it.

New constraints: (i) original constraint $(\hat{w}, \sigma)$ (ii) $\hat{p}(z) = y$ for some random point z.

Just an evaluation constraint which we know how to handle!

So, except with probability $\sqrt{\rho}$, g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[|L^*|, m, \rho', (\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)]$.

Can amplify to $\sqrt{\rho}^t$

Out Of Domain

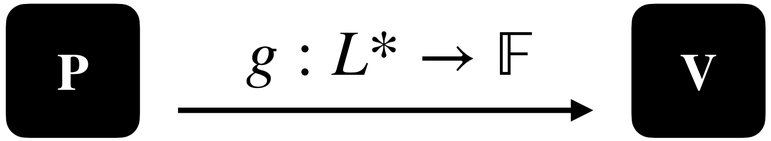
Subprotocol to force unique

Out Of Domain

Subprotocol to force unique

Out Of Domain

Subprotocol to force unique



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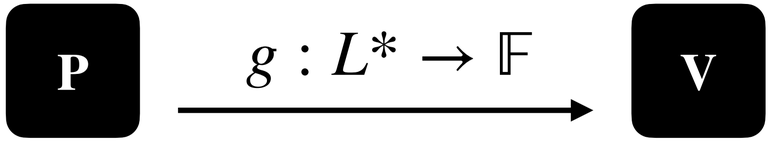

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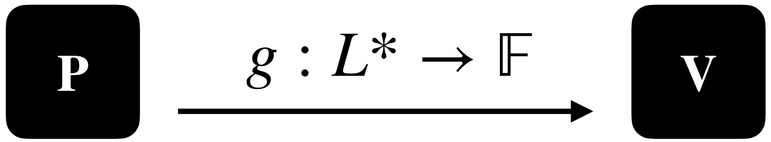

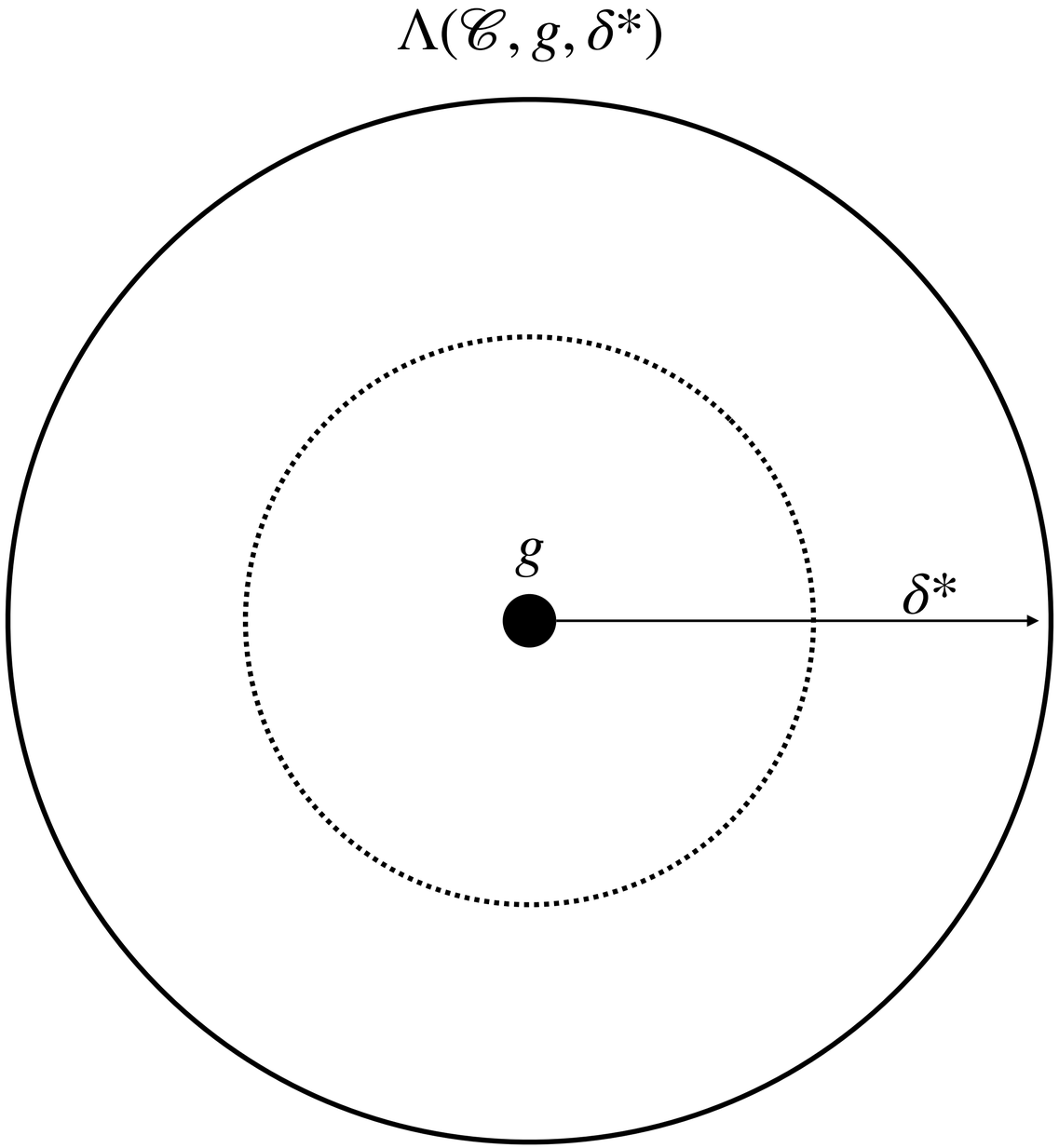

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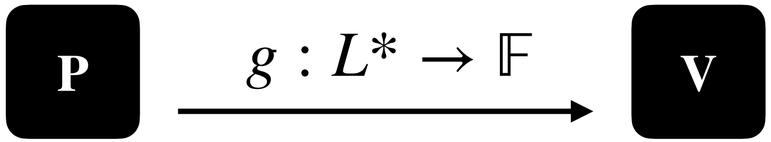

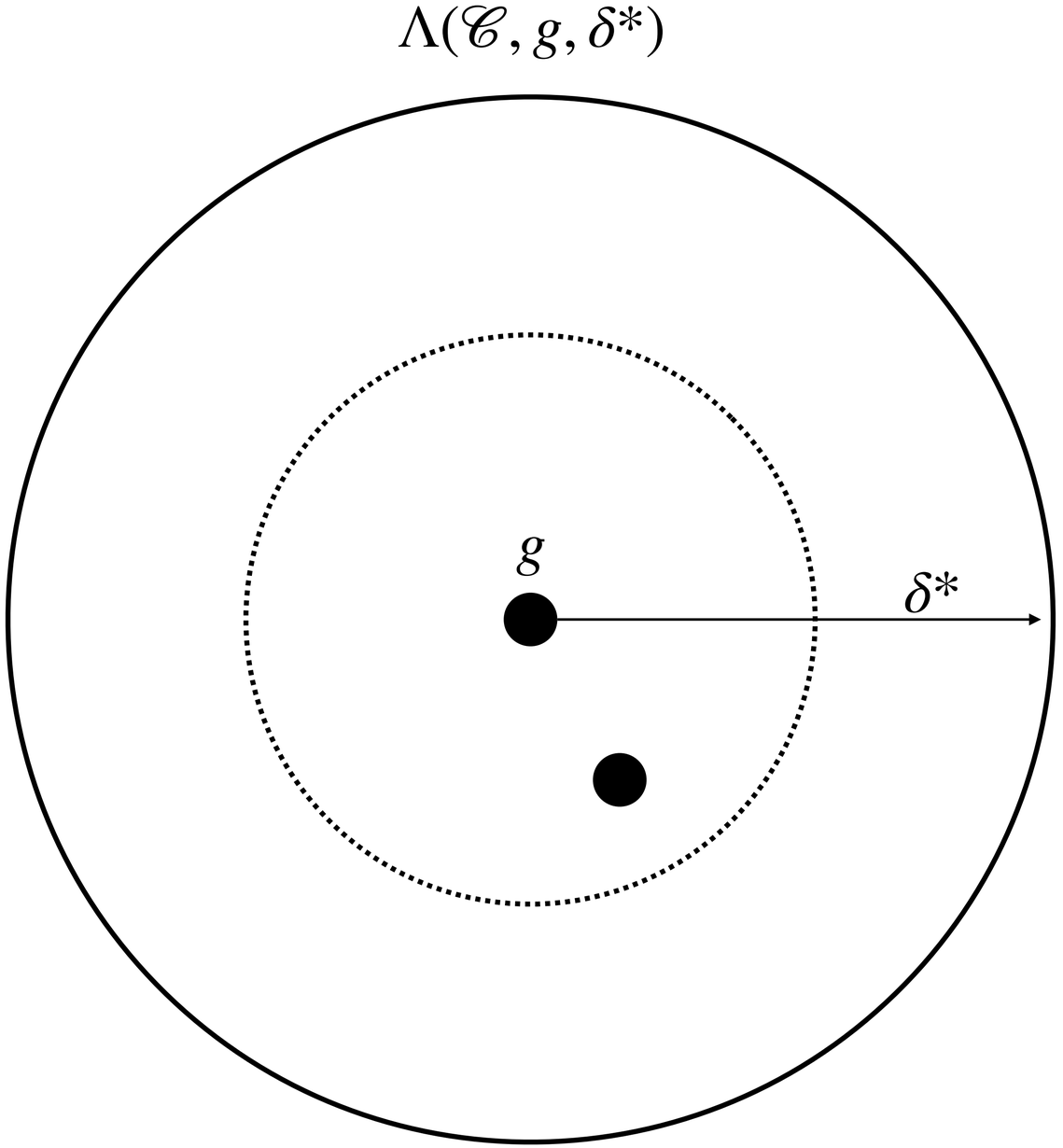

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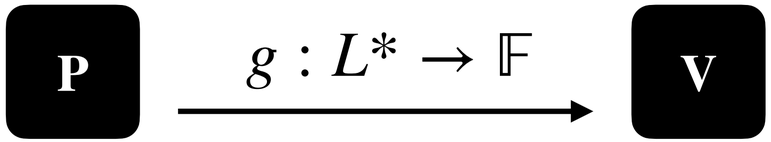

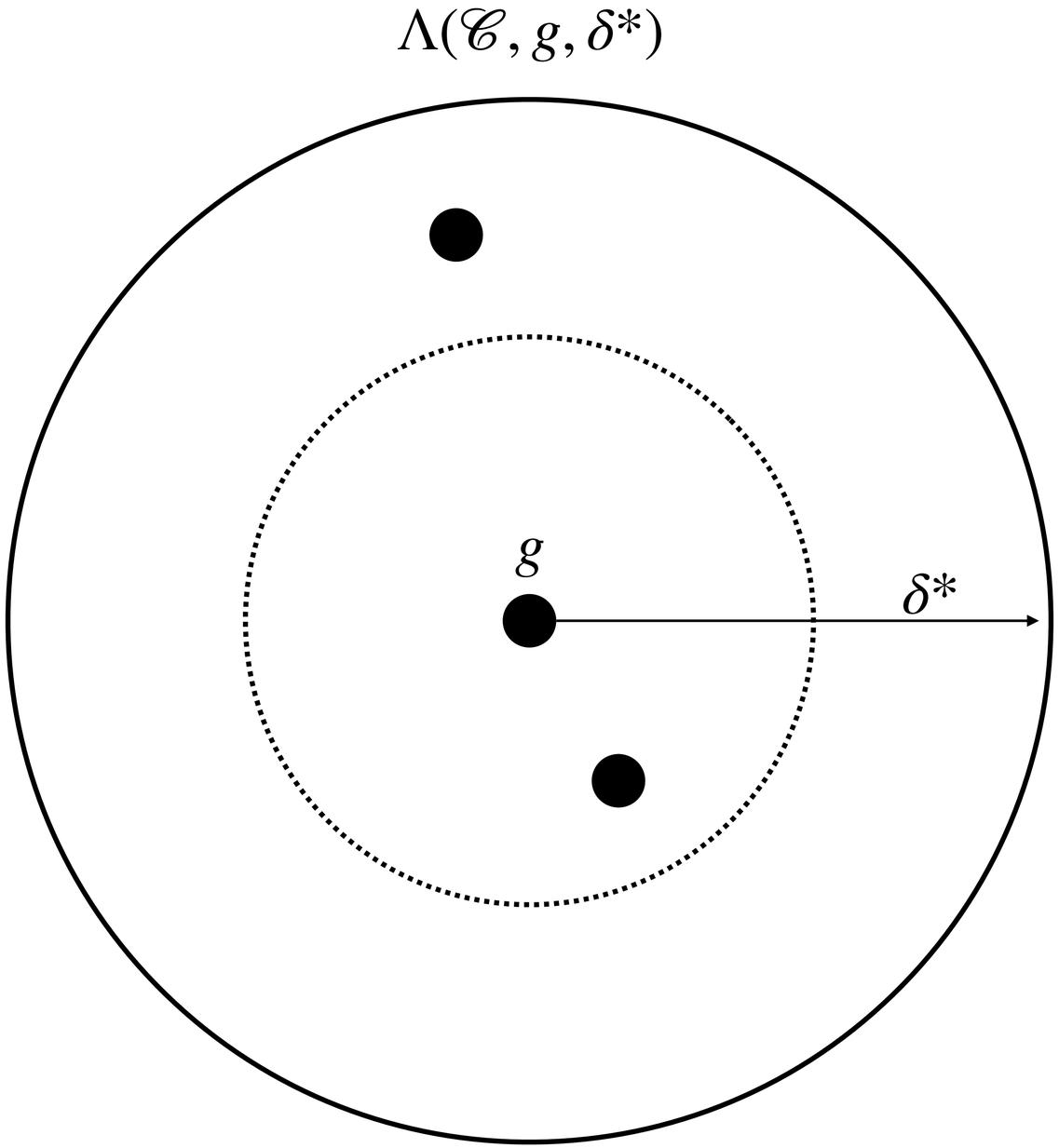

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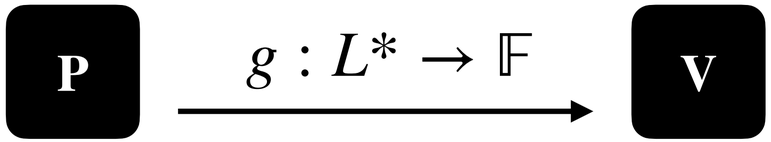

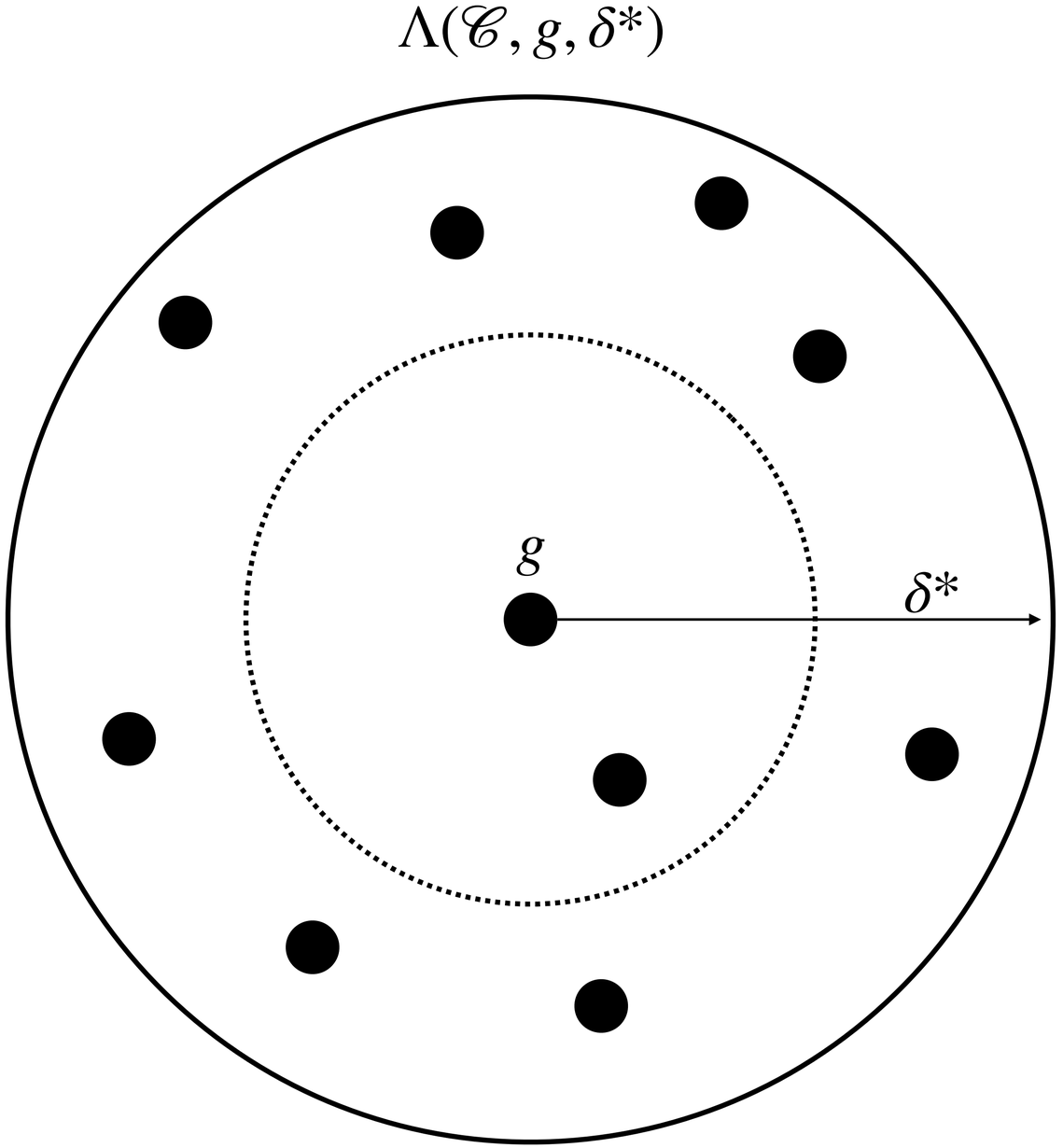

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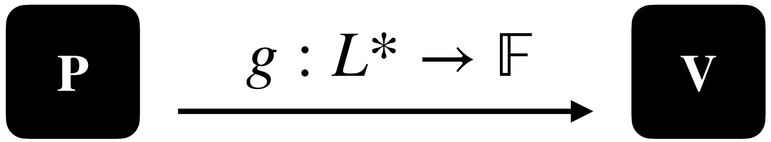

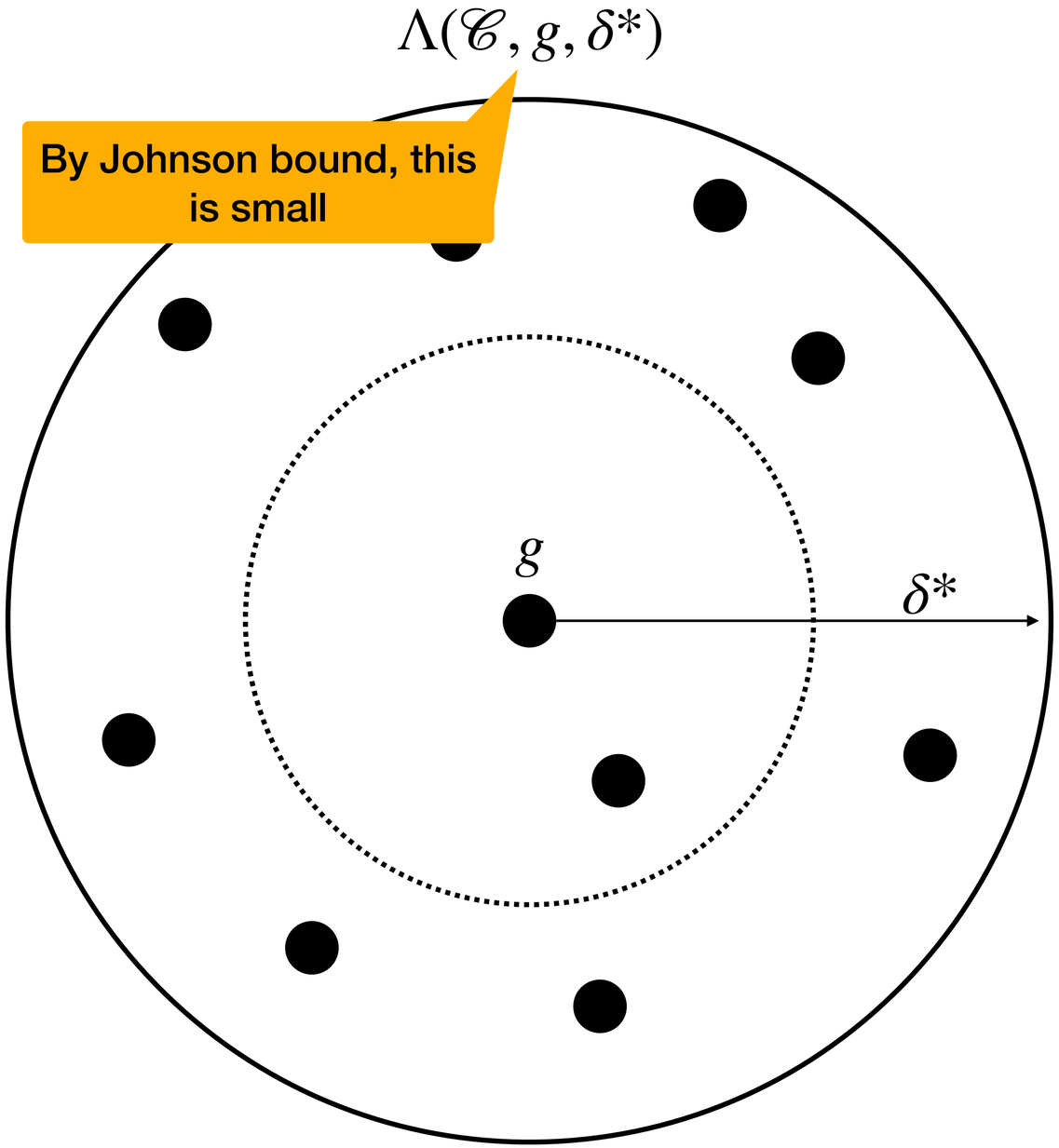

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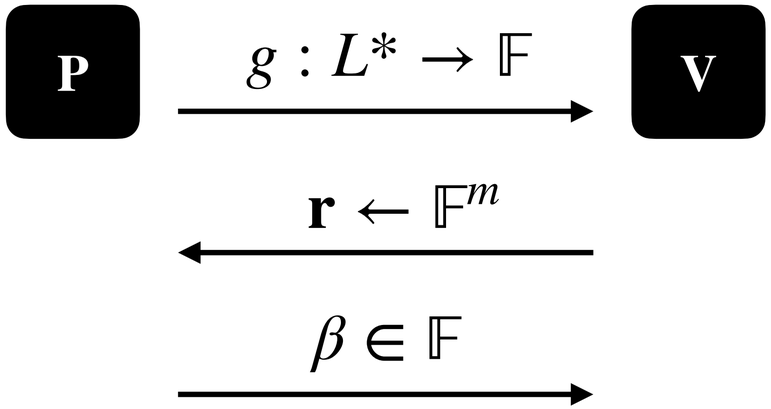

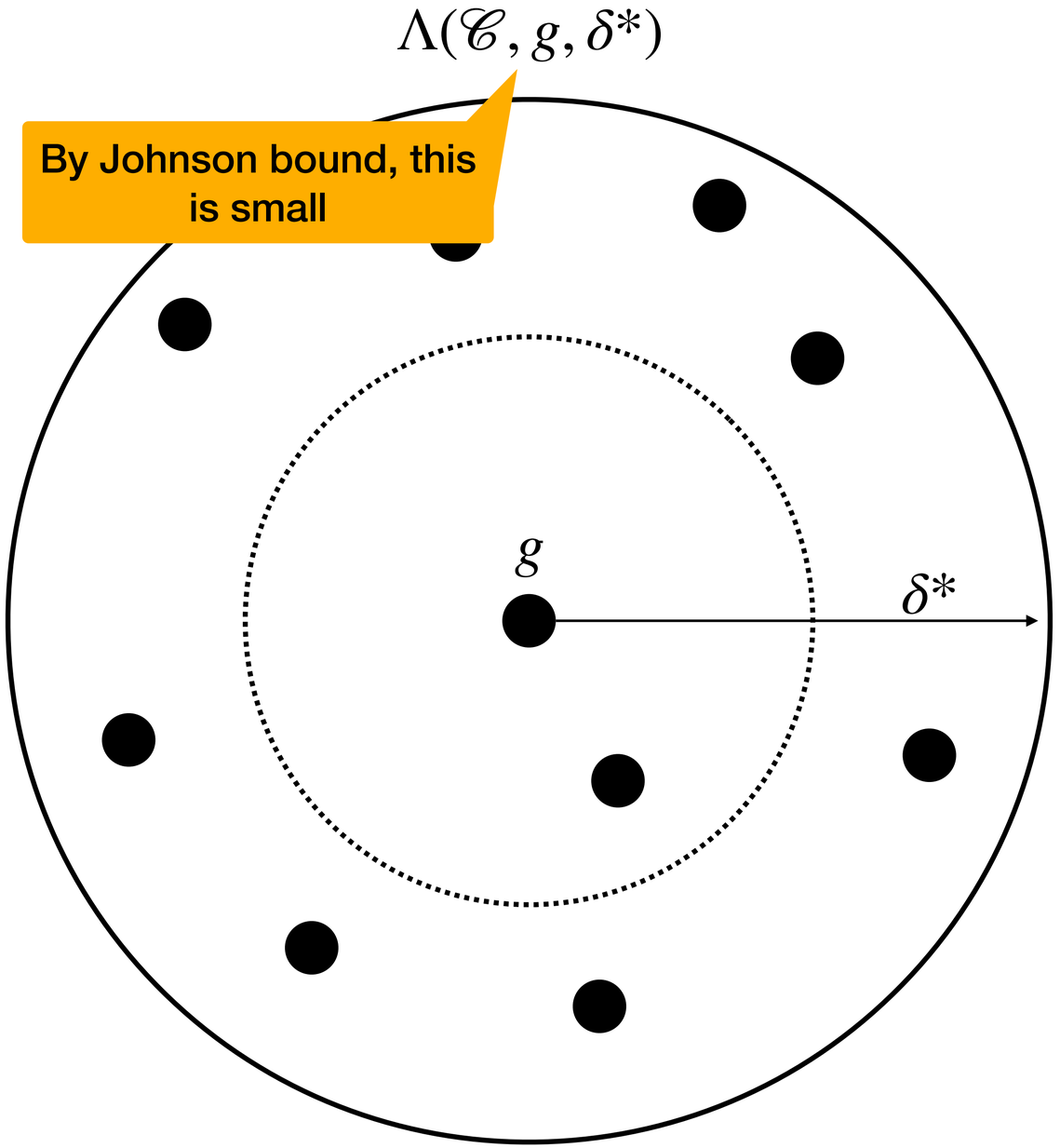

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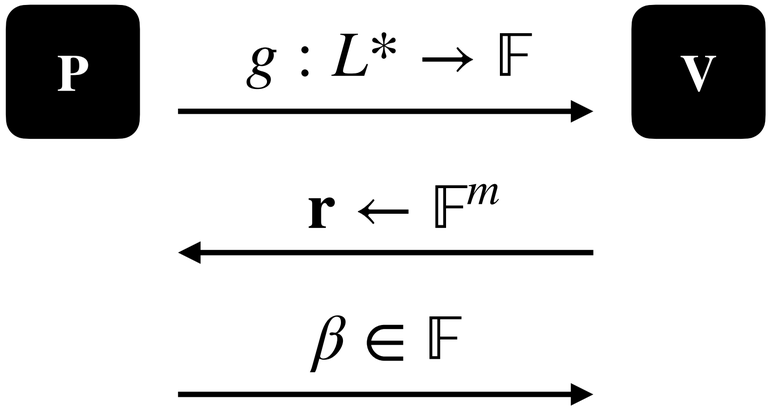
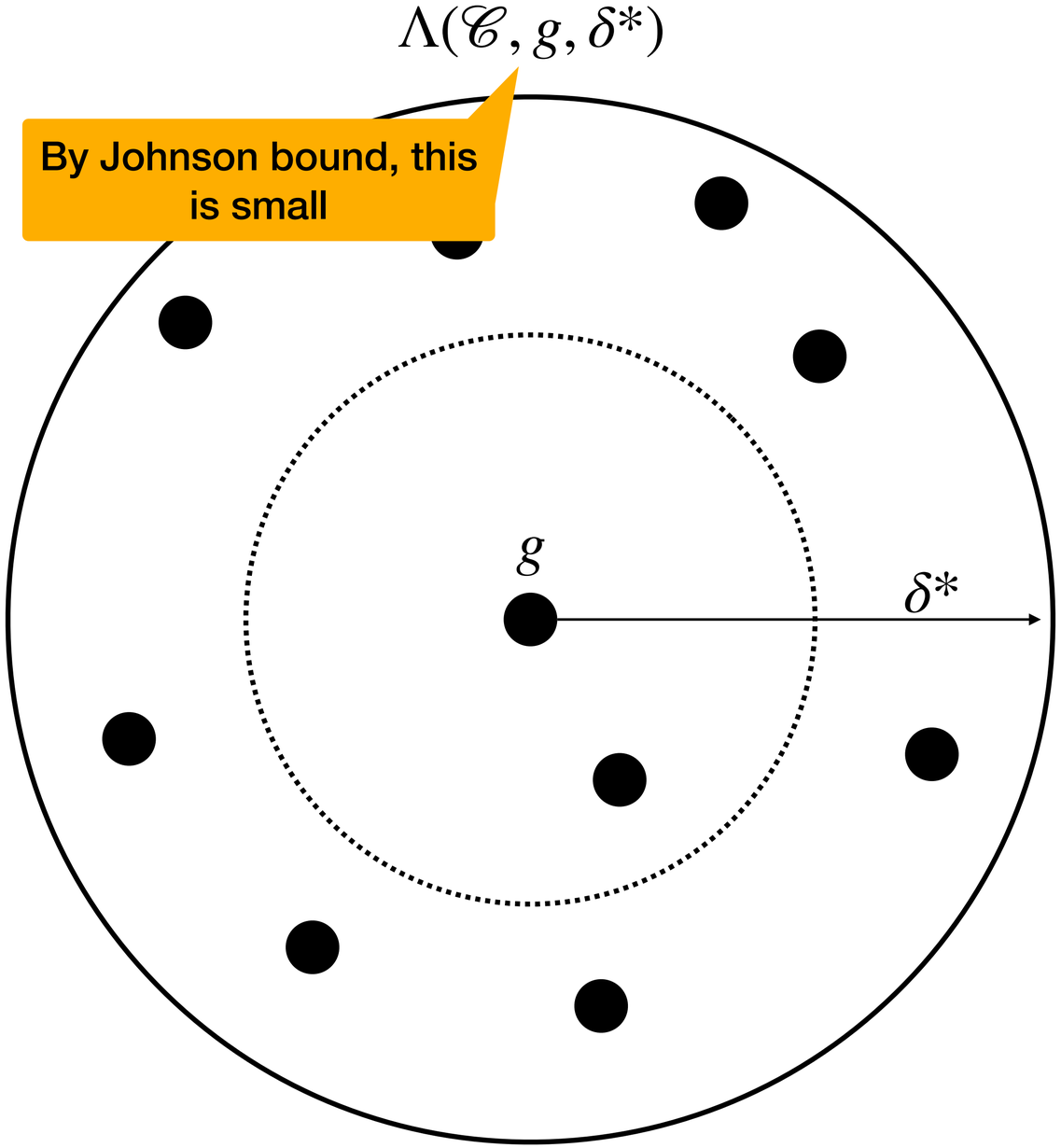


- By fundamental theorem of algebra of w.h.p. no pair $\hat{u}, \hat{v}$ with $\hat{u}(\mathbf{r}) = \hat{v}(\mathbf{r})$
- Prover "chooses" which codeword $\hat{u}$ it "commits" to



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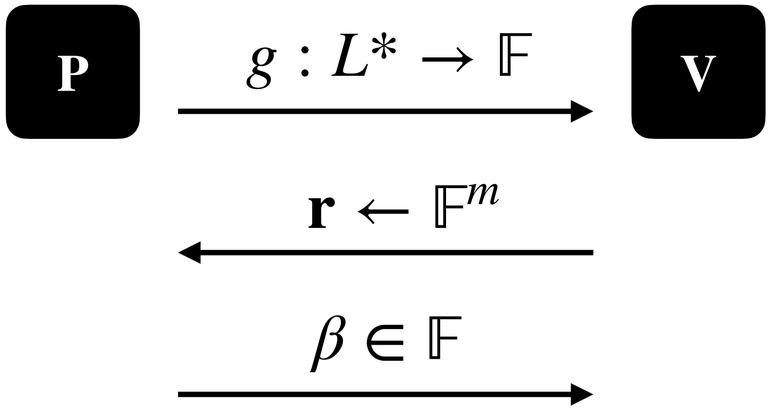

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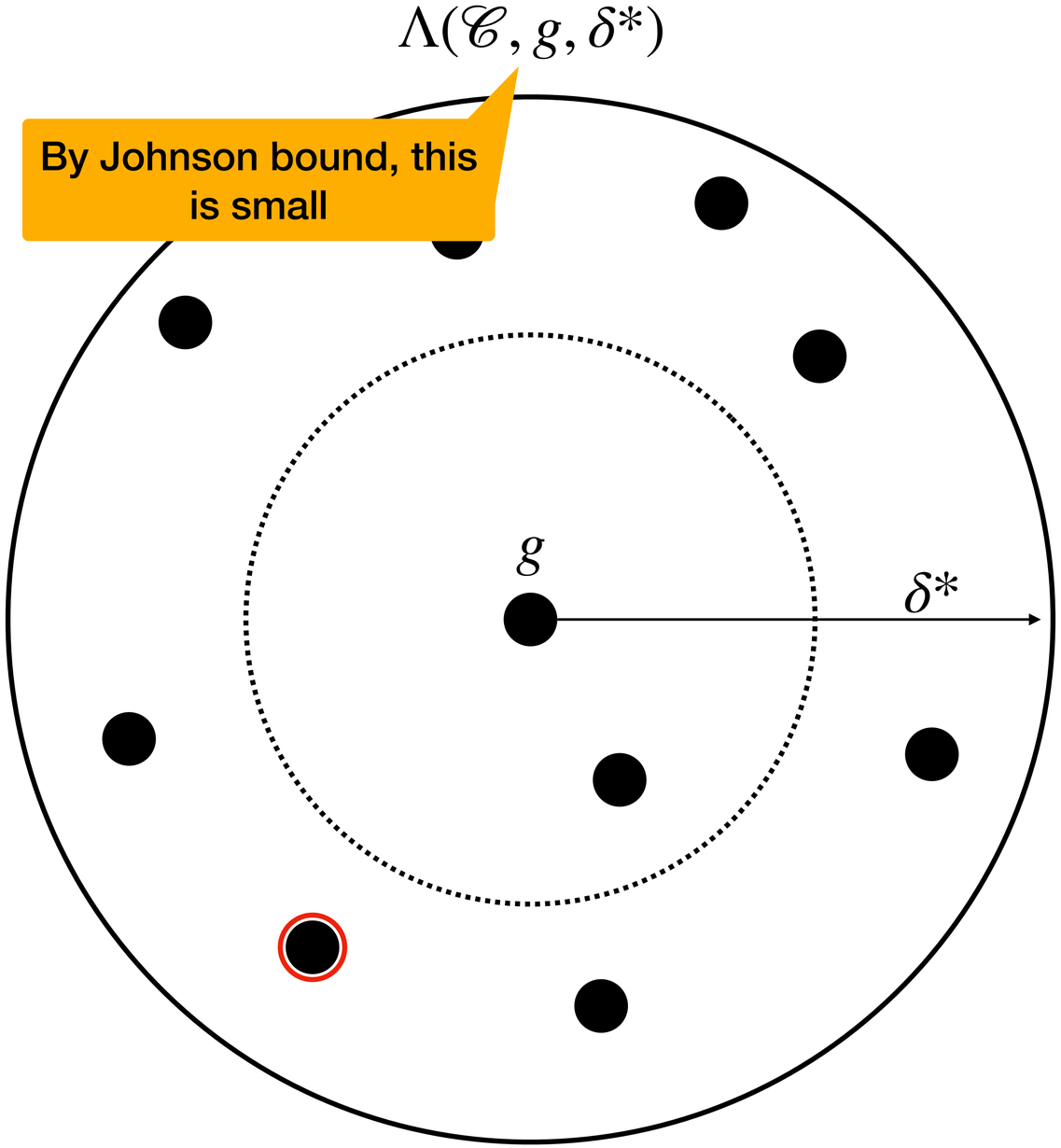

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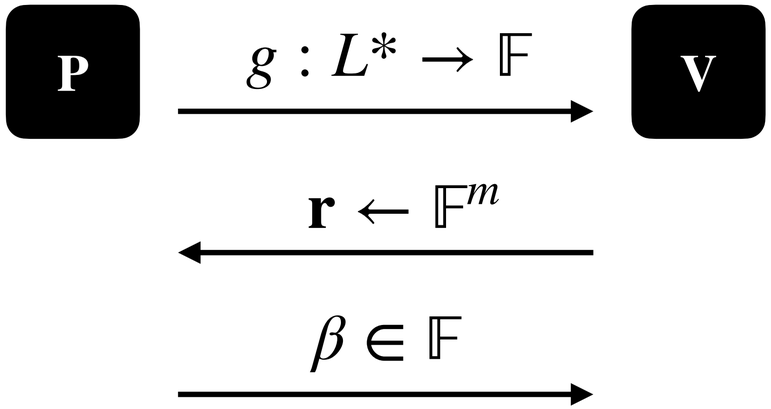

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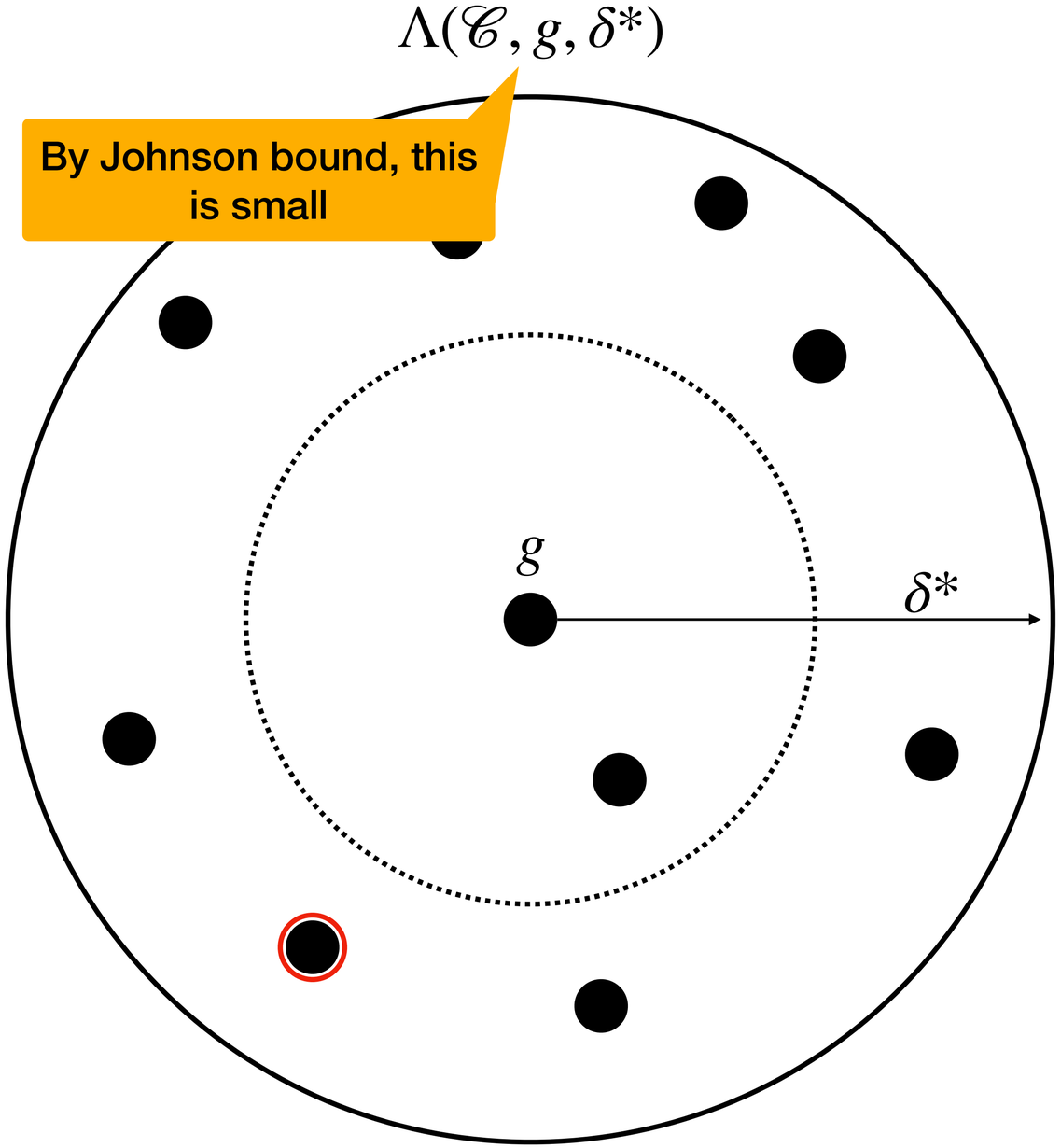
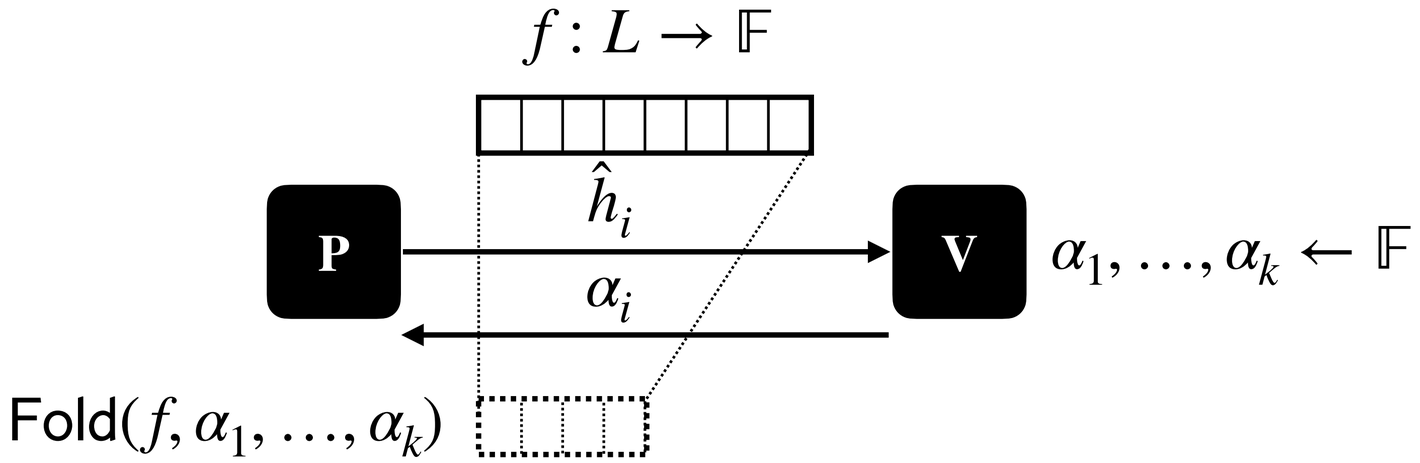


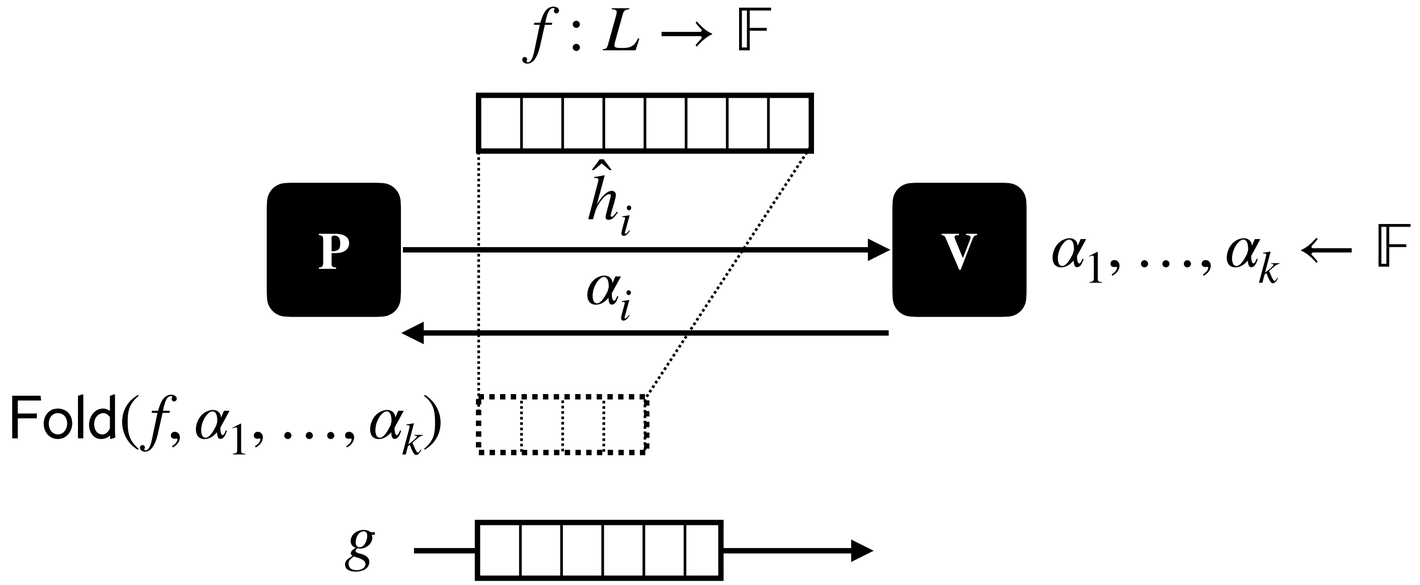
Add to list of constraints to **enforce**!

WHIR 

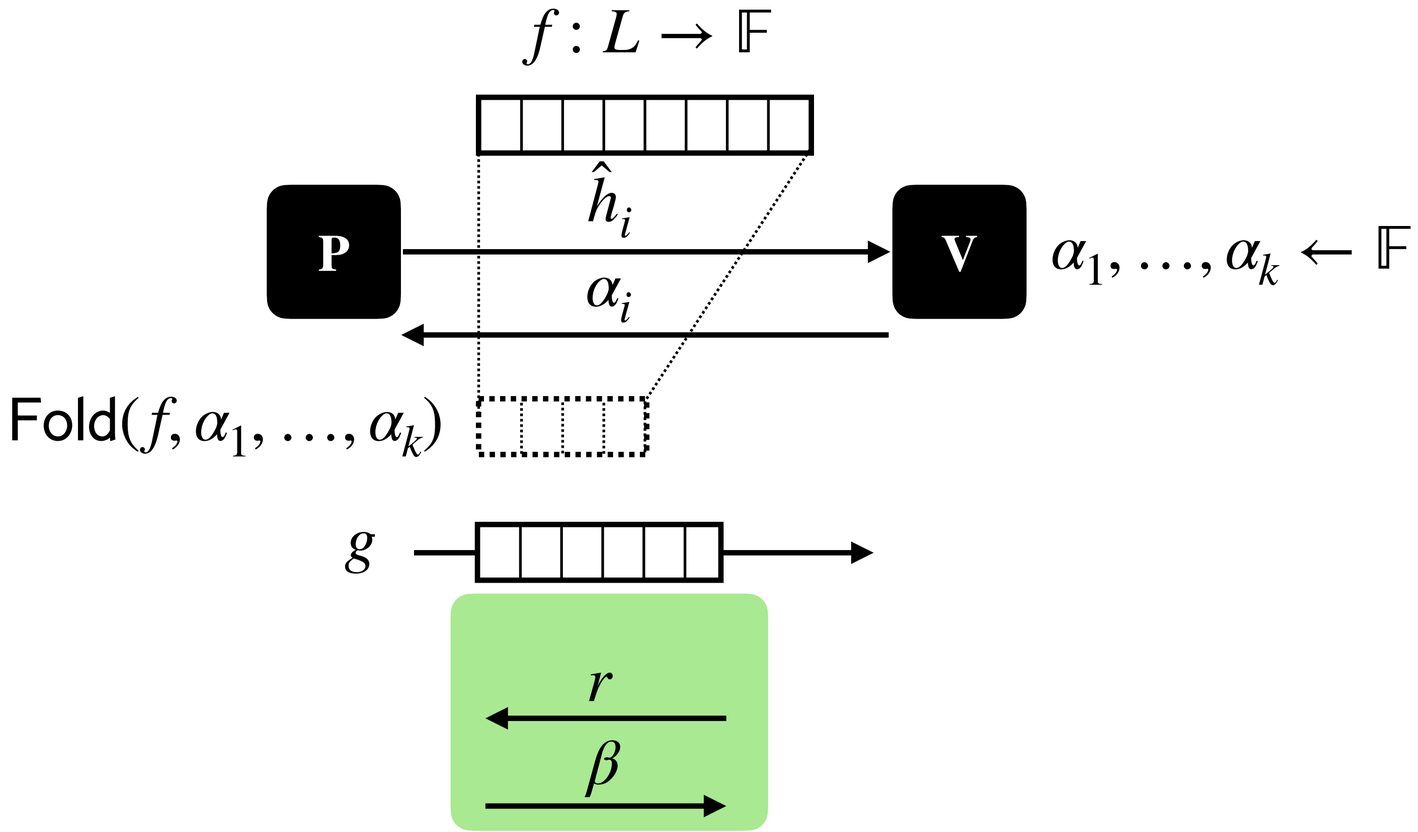
WHIR


$f : L \to \mathbb{F}$
$\hat{h}_i$
P
V
α_i
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
Fold$(f, \alpha_1, \ldots, \alpha_k)$

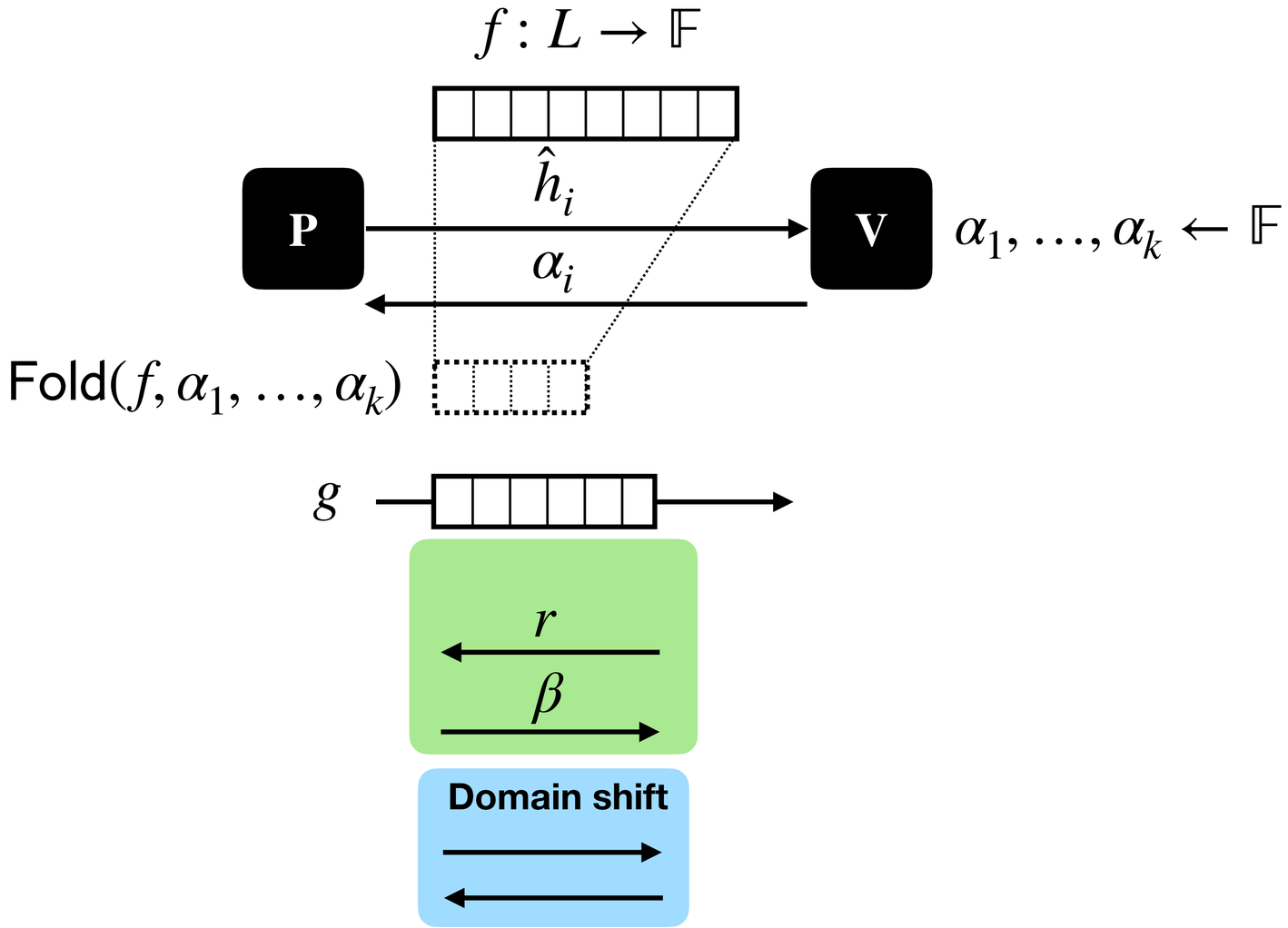
WHIR


$f : L \to \mathbb{F}$
$\hat{h}_i$
P
V
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
α_i
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g

WHIR


$f : L \to \mathbb{F}$
$\hat{h}_i$
P
V
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
α_i
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g
r
β

WHIR


$f : L \to \mathbb{F}$
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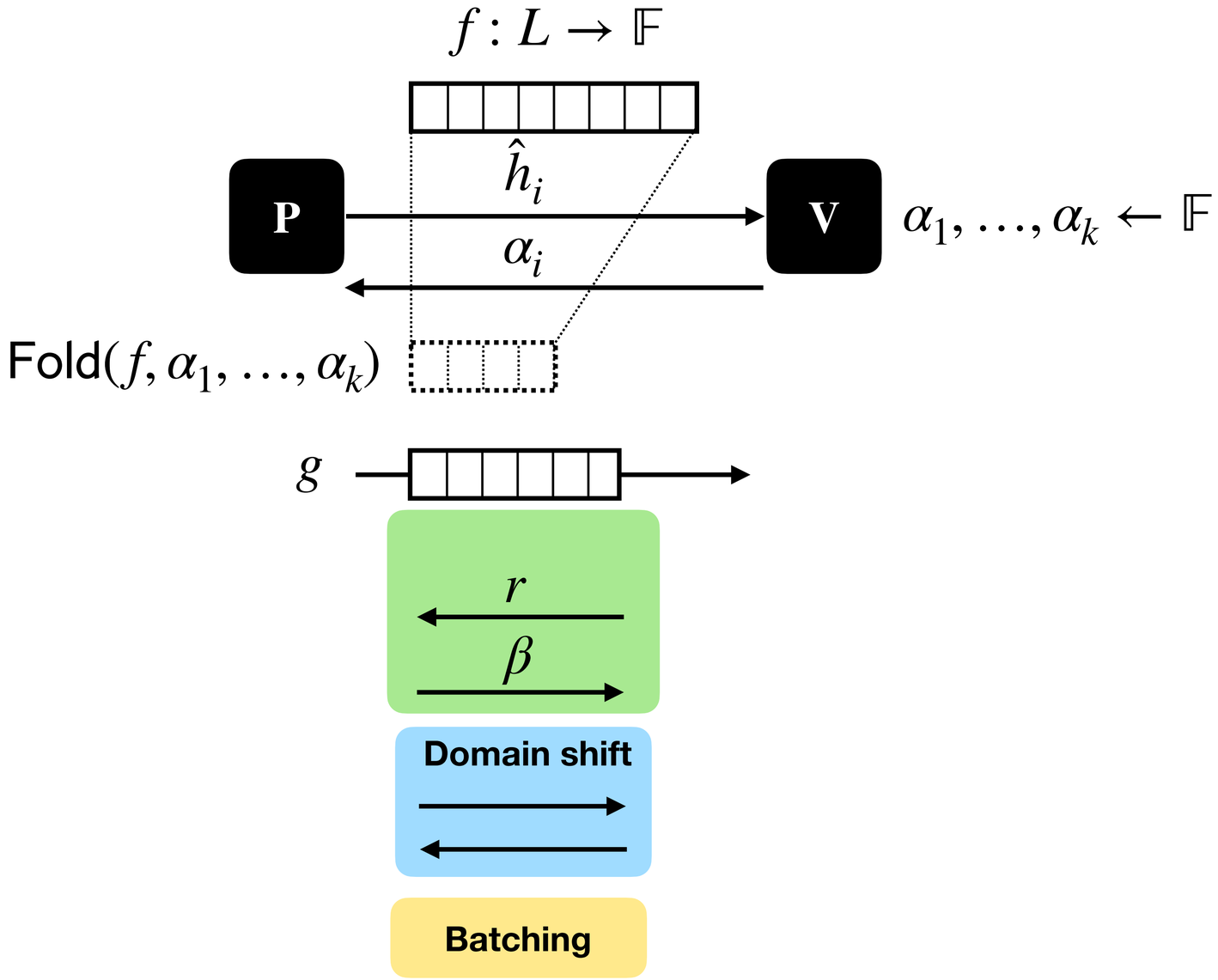
P
V
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
α_i
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g
r
β
Domain shift

WHIR


$f : L \to \mathbb{F}$
$\hat{h}_i$
P
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$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
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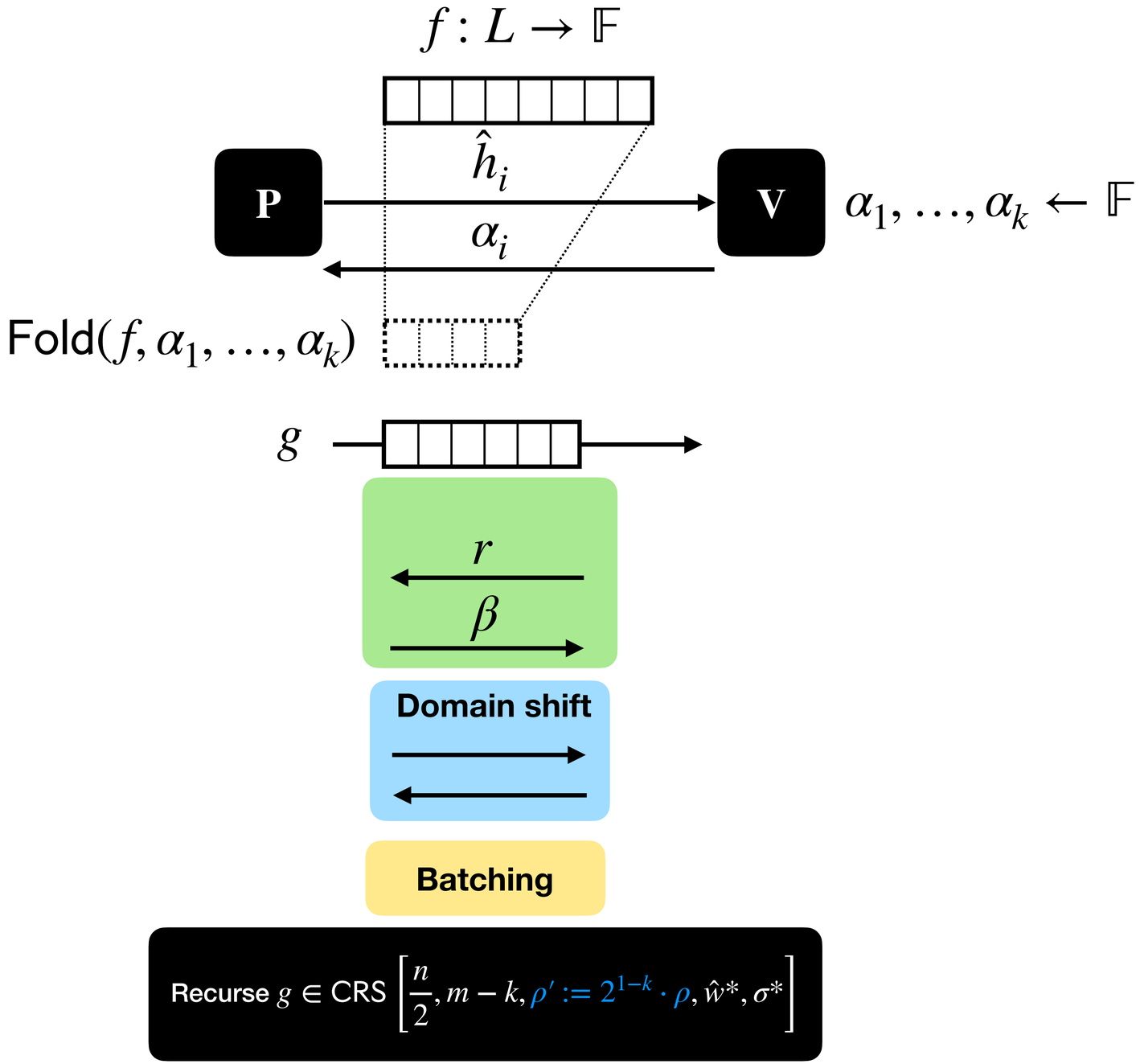
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g
r
β
Domain shift
Batching

WHIR


$f : L \to \mathbb{F}$
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P
V
α_i
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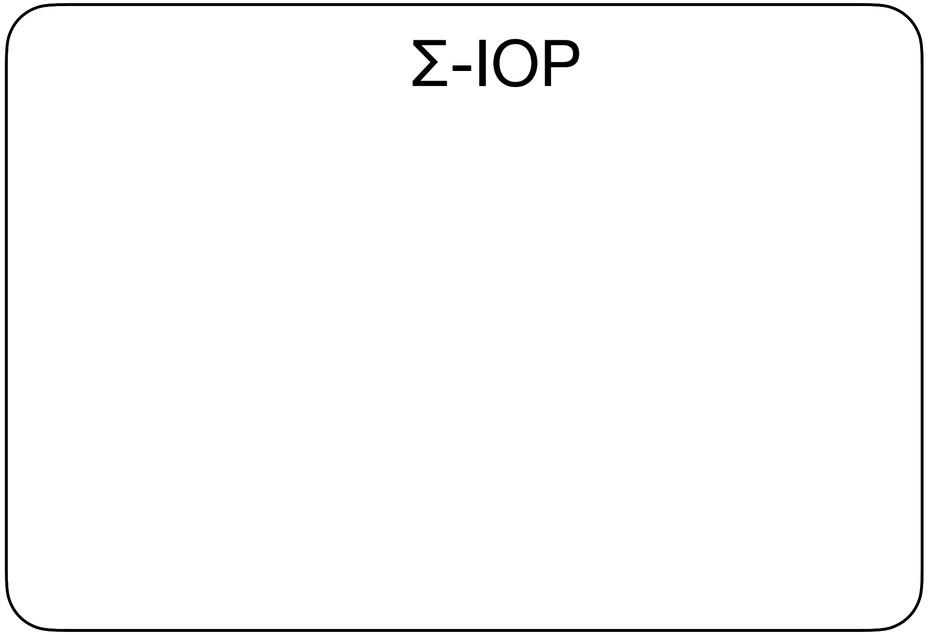
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g
r
β
Domain shift
Batching
Recurse $g \in$ CRS $\left[\frac{n}{2}, m-k, \rho' := 2^{1-k} \cdot \rho, \hat{w}*, \sigma*\right]$

Application: Σ-IOP

High soundness compilation using constrained codes

Application: Σ-IOP

High soundness compilation using constrained codes



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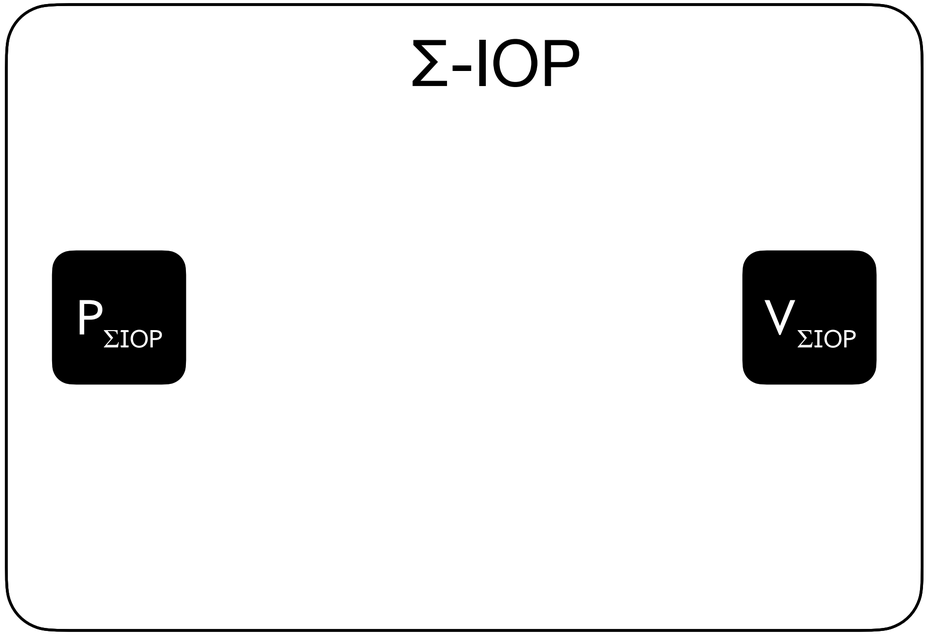

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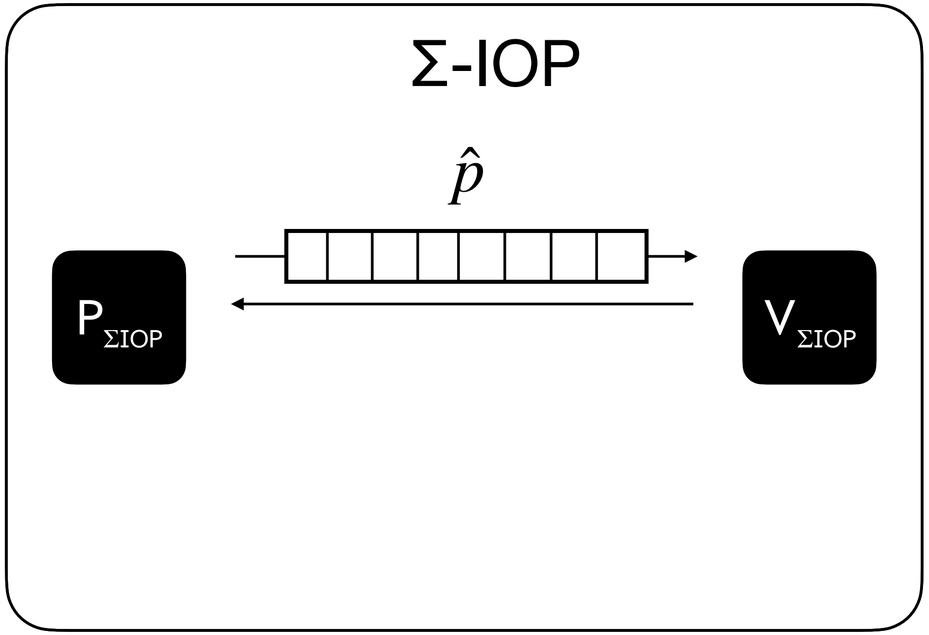

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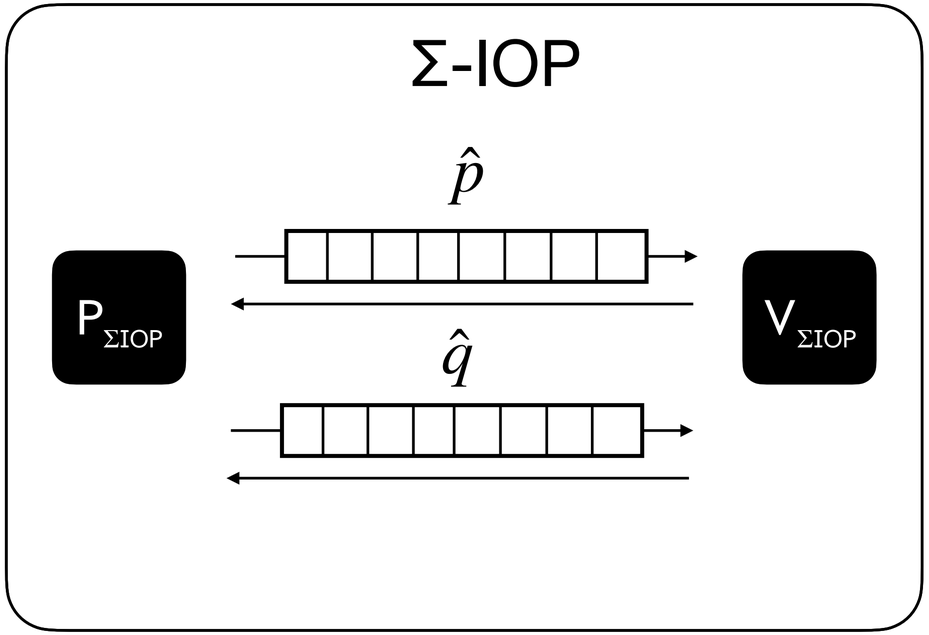
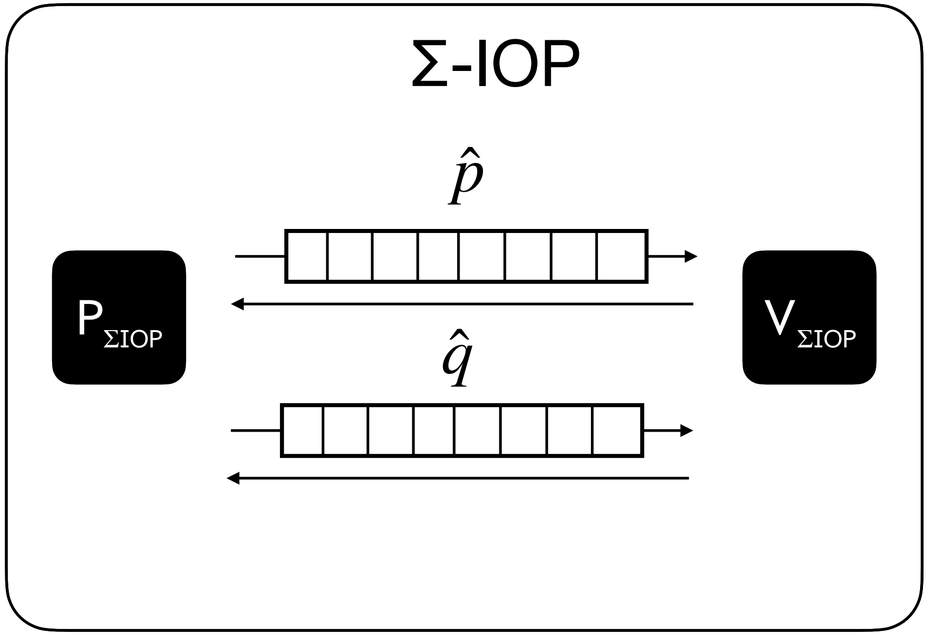


Application: Σ-IOP

High soundness compilation using constrained codes

Generalizes univariate and multilinear PIOPs at no extra cost!



Verifier can ask **sumcheck queries**

i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

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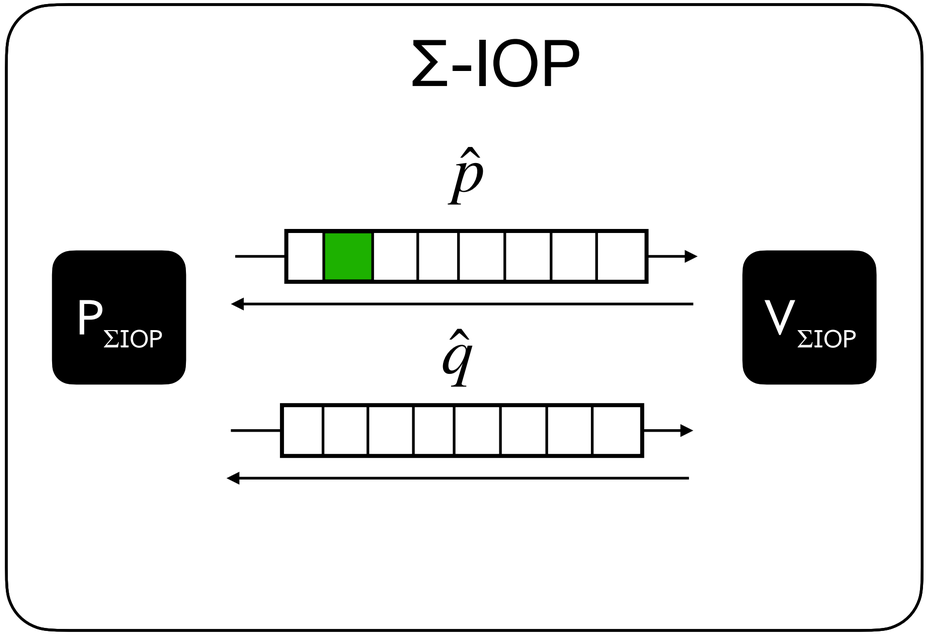

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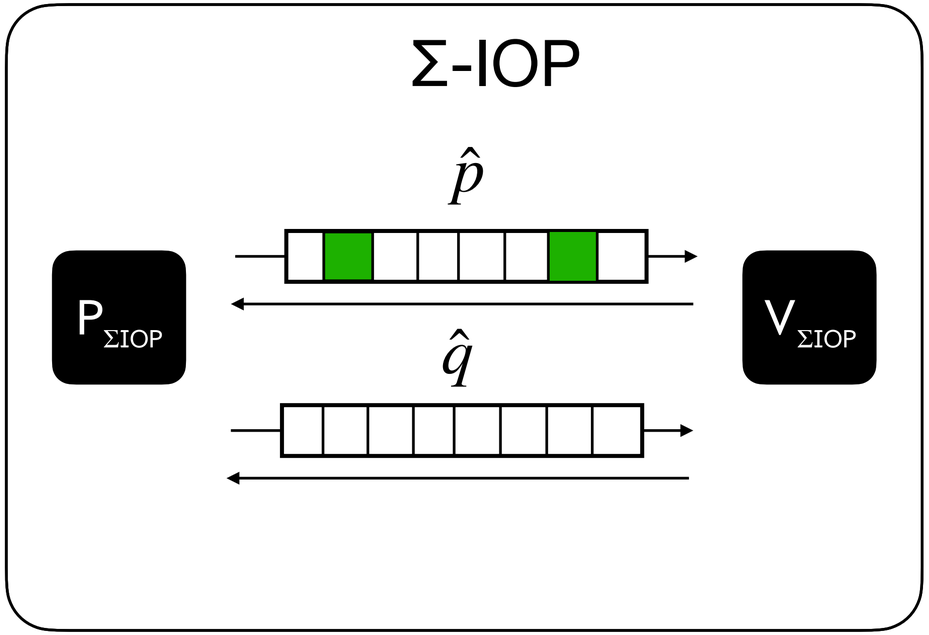

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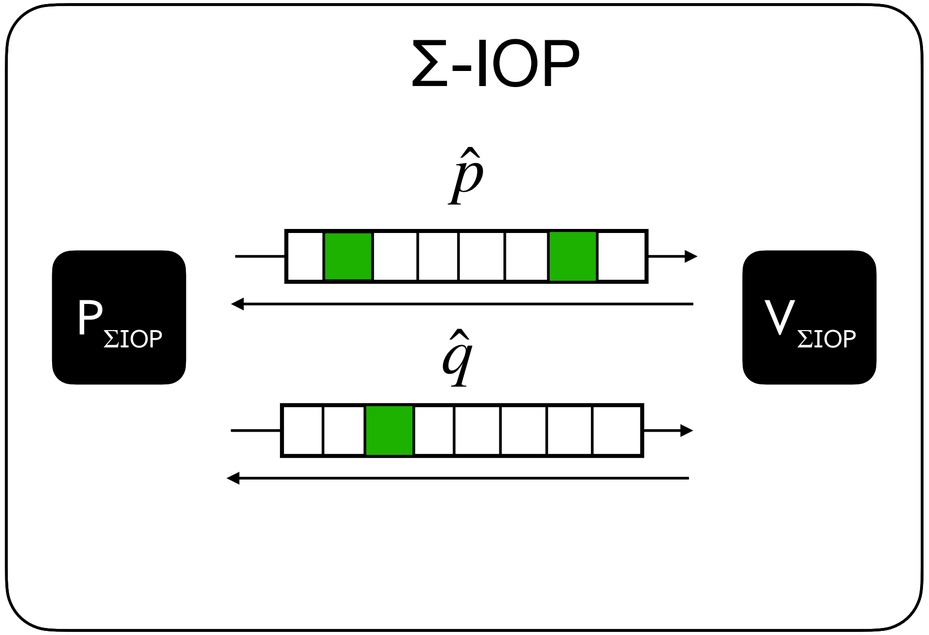

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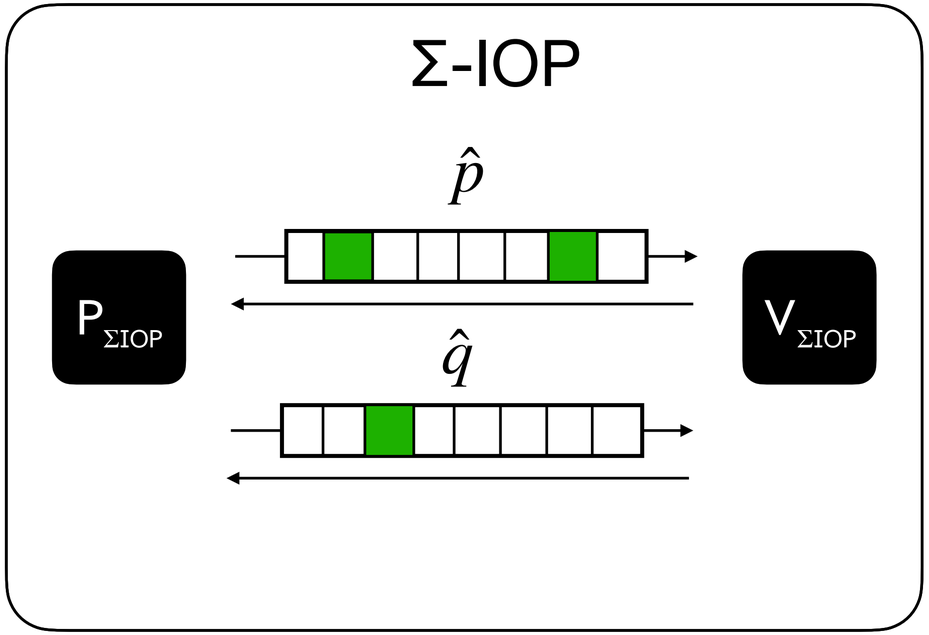

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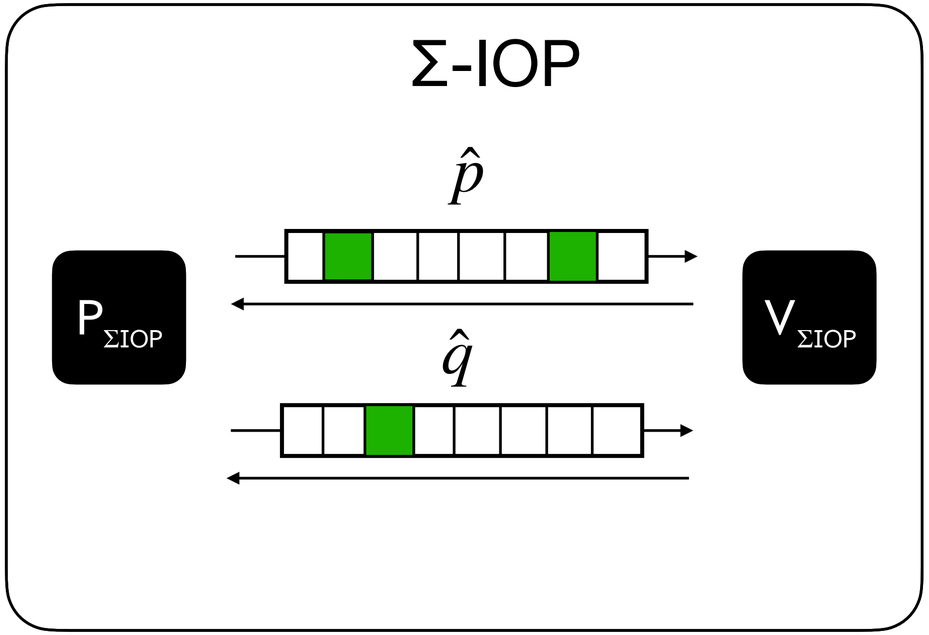

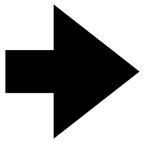

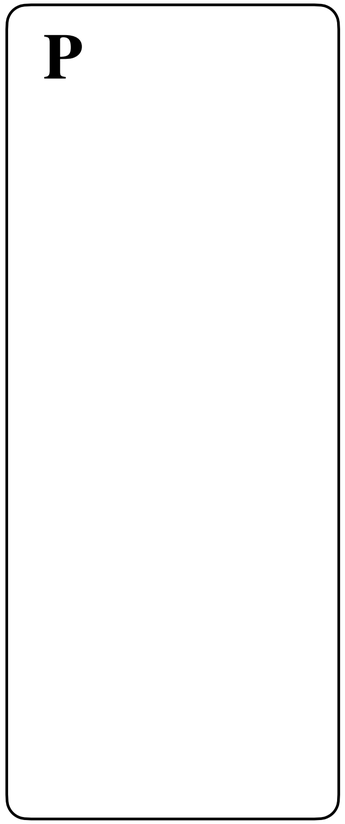

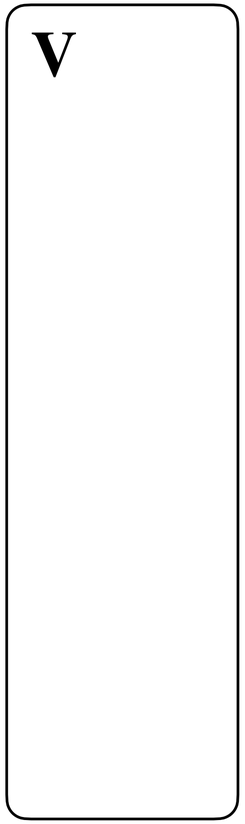

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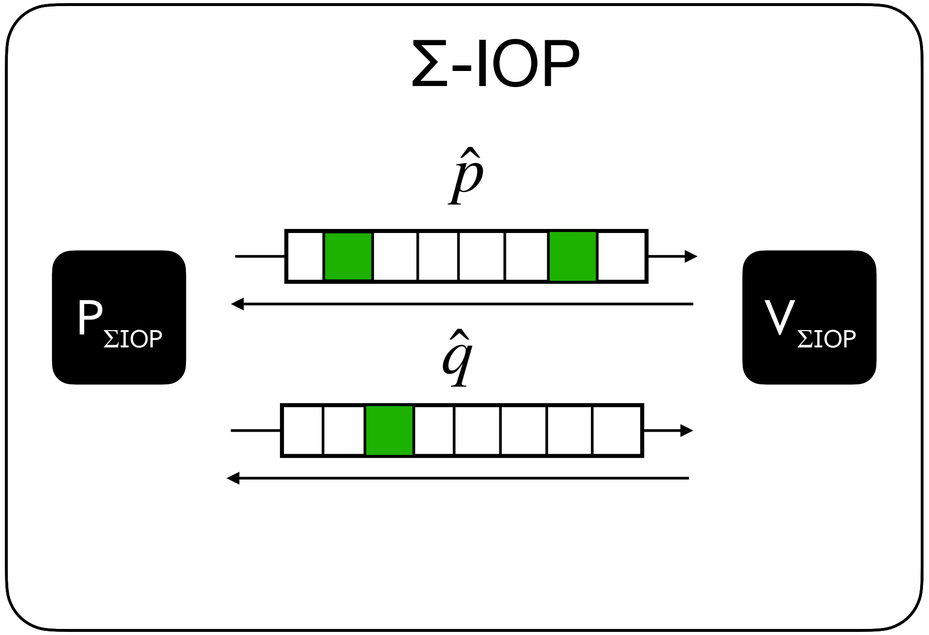

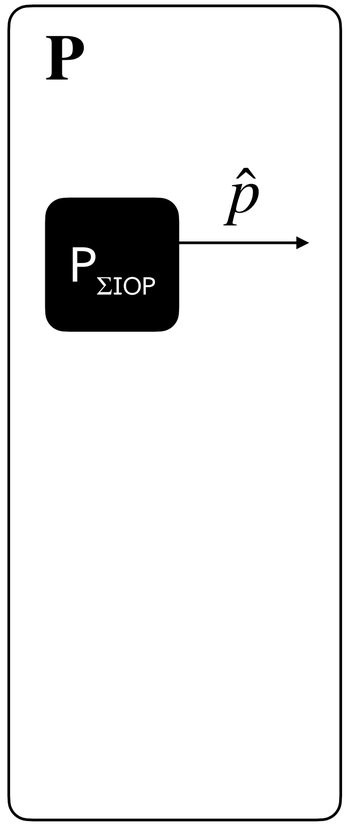

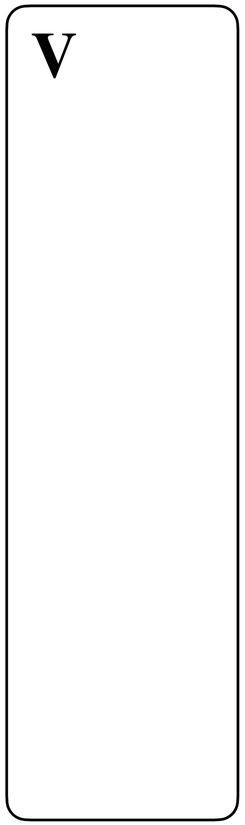

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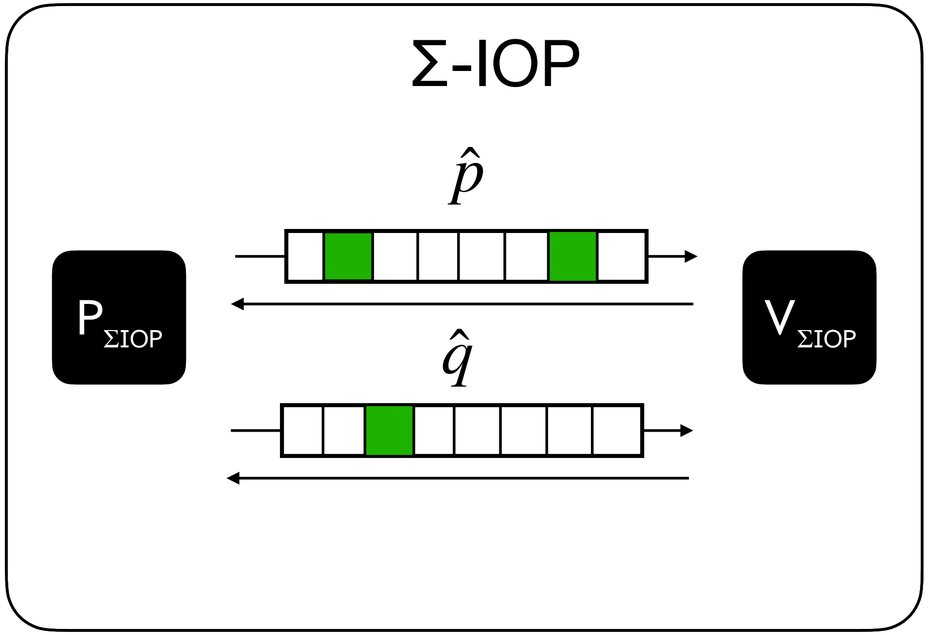

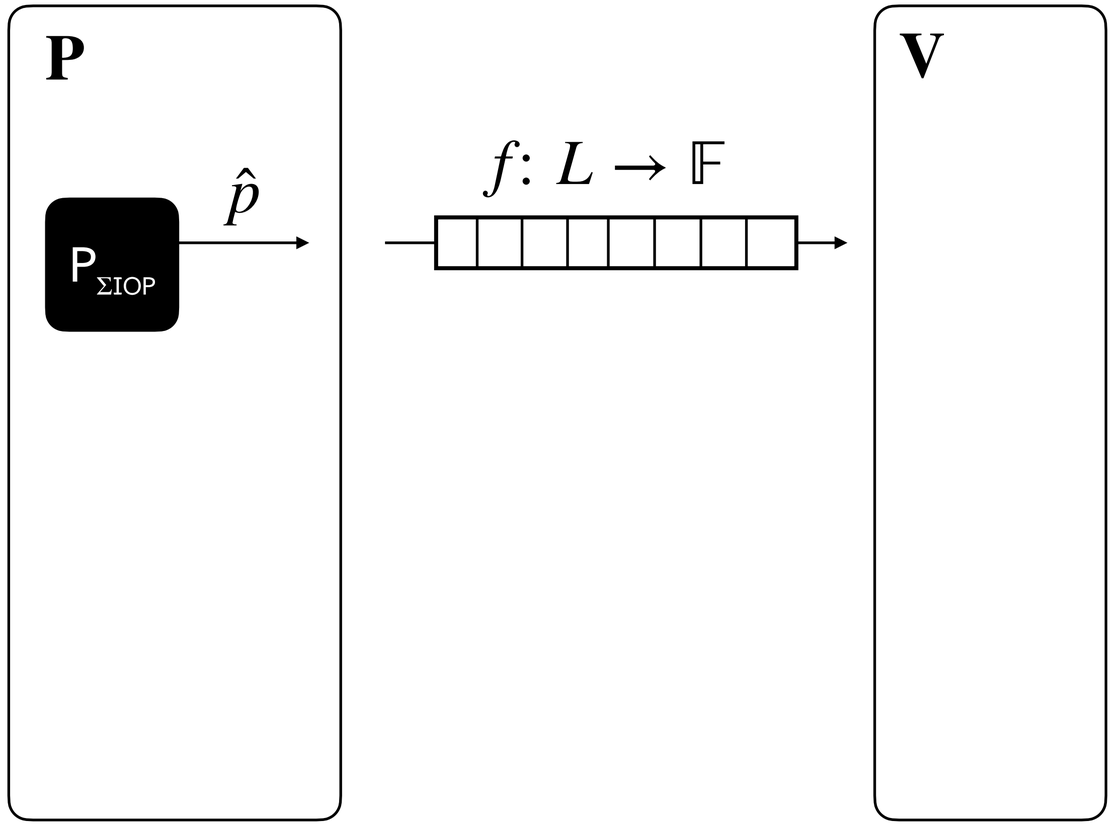

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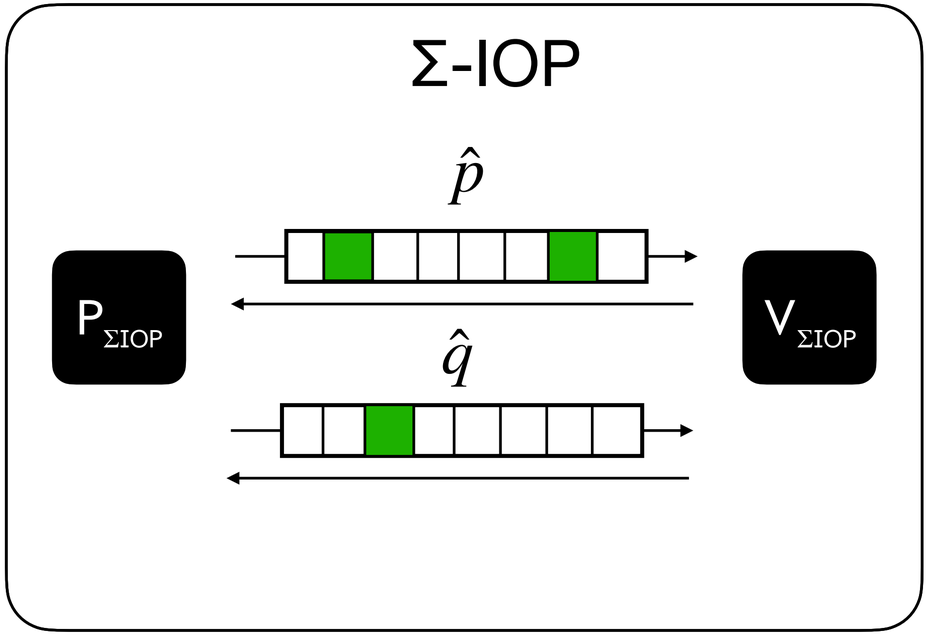

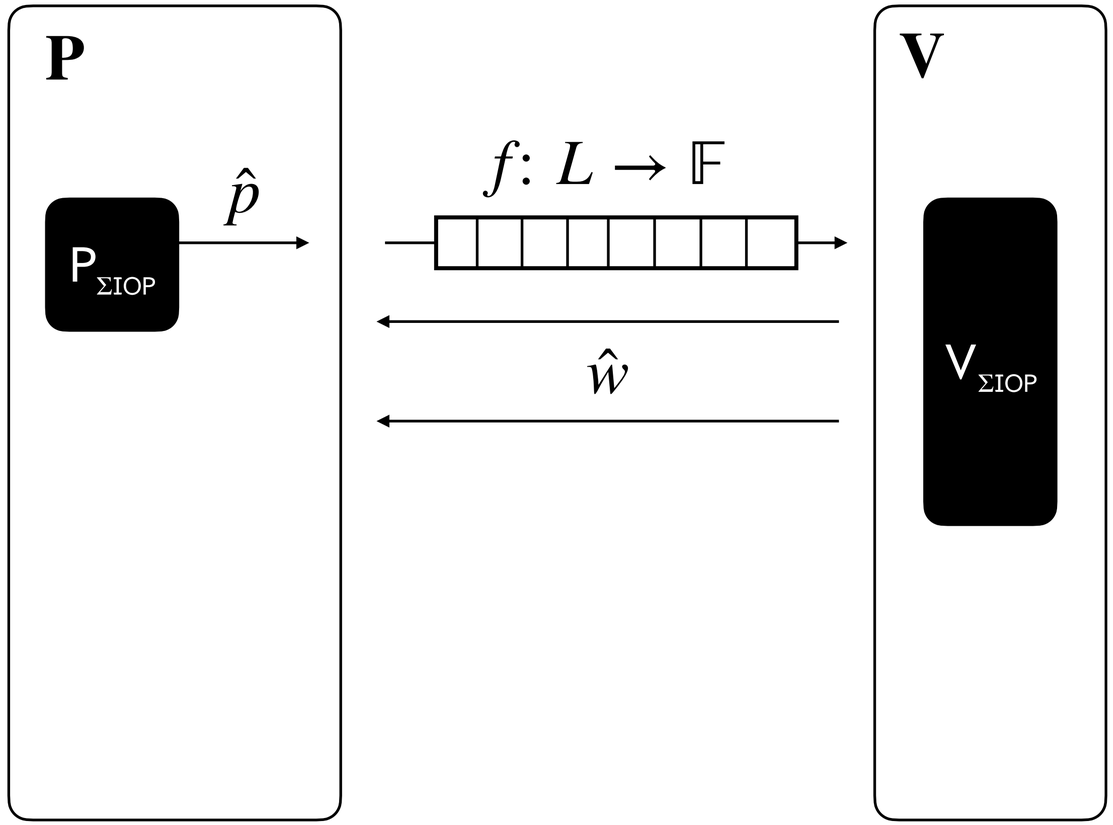

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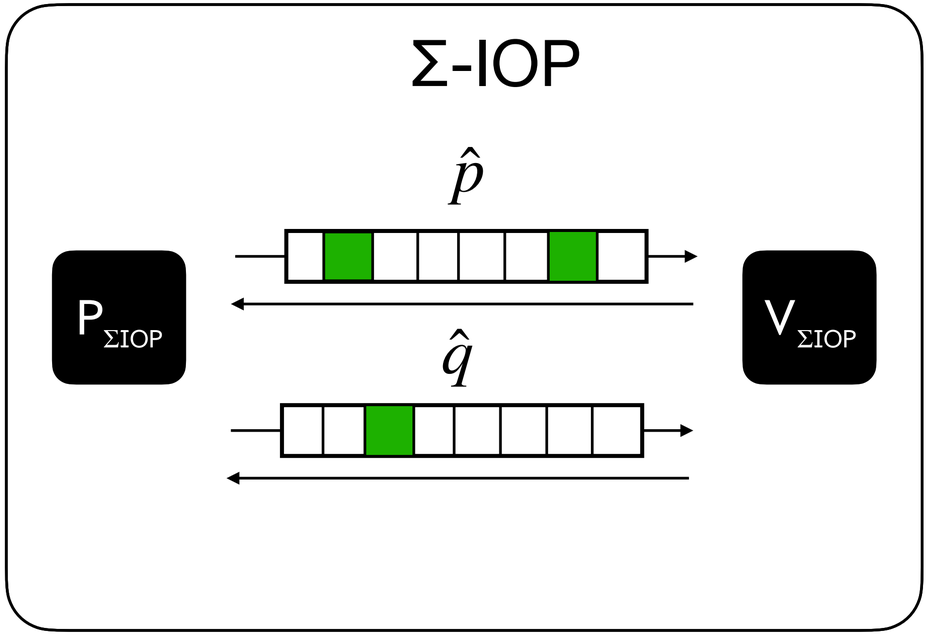

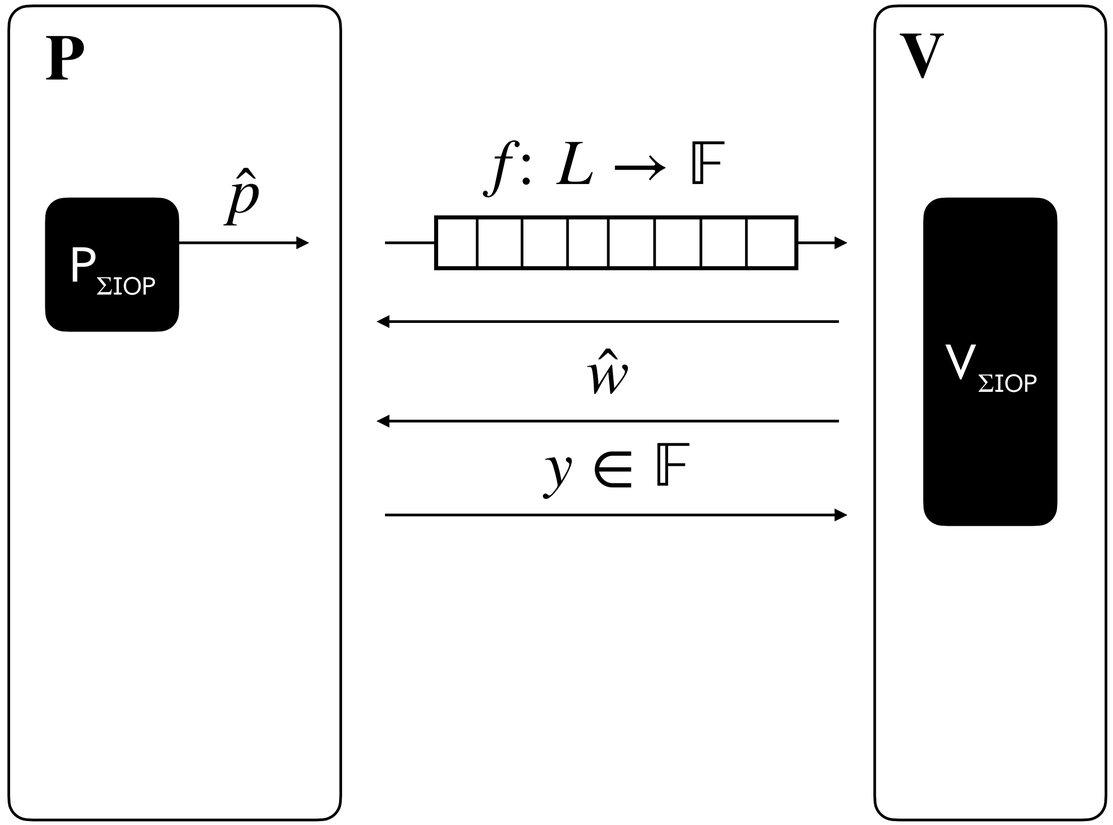

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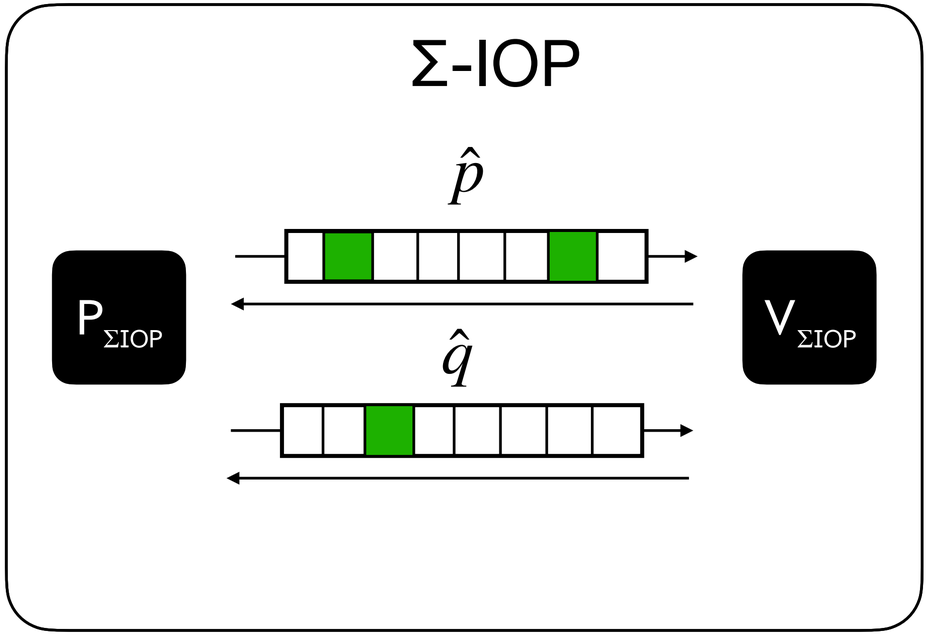

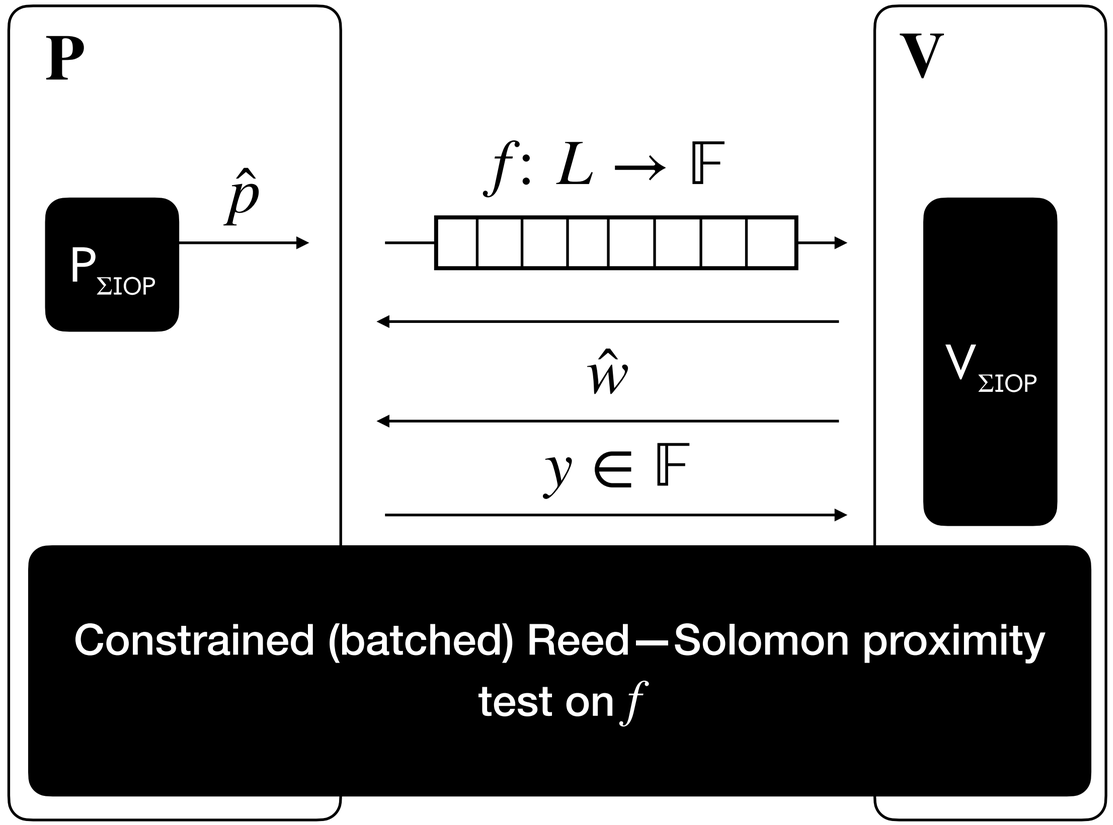

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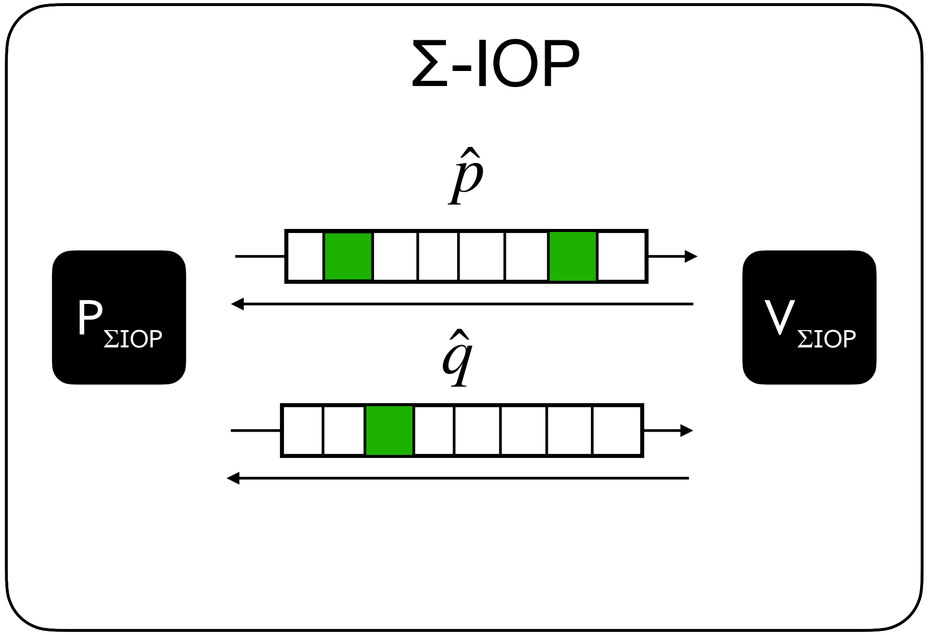
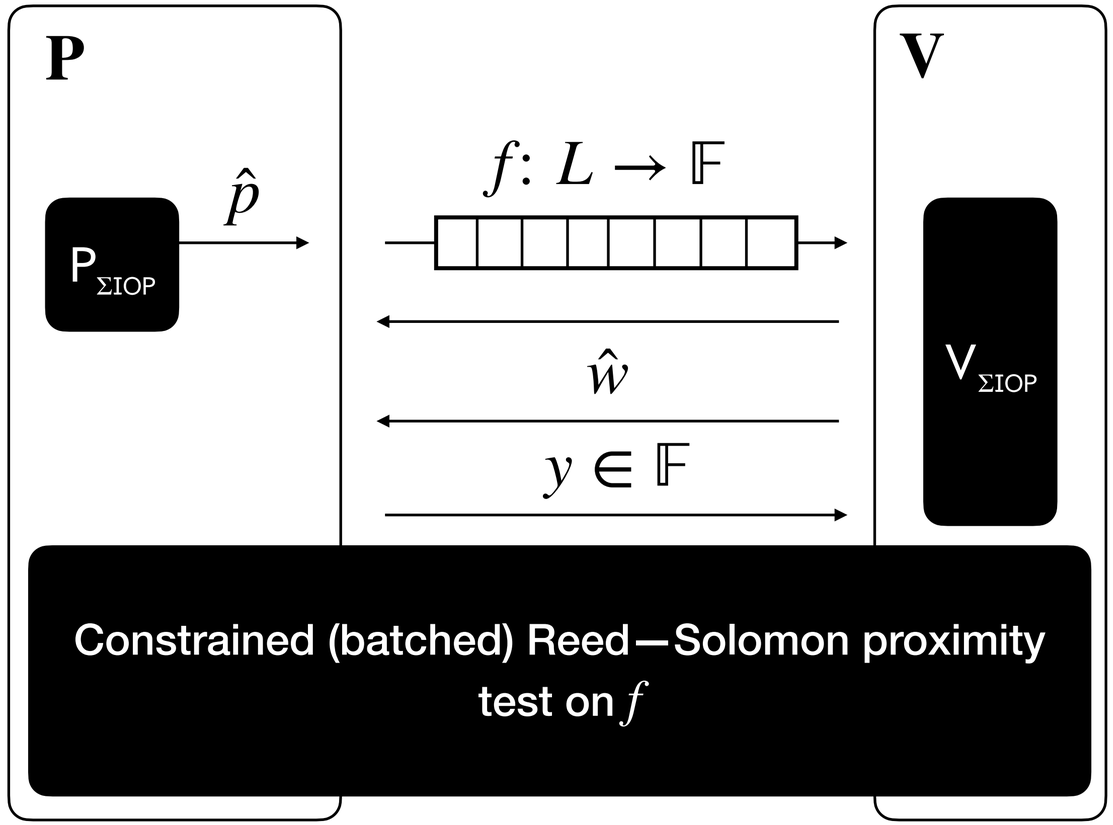
Q: Can we use this to do more **efficient** arithmetizations?





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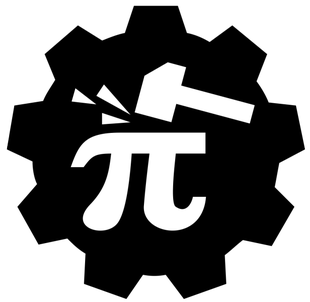
- Enables high-soundness compilation for **Σ-IOP**

Extra slides

Implementation



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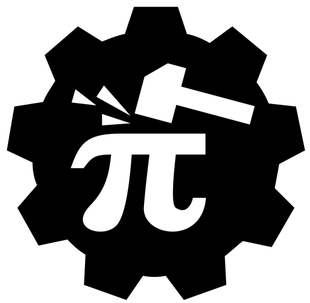

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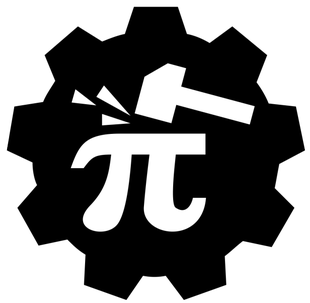

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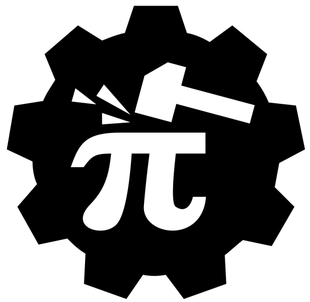

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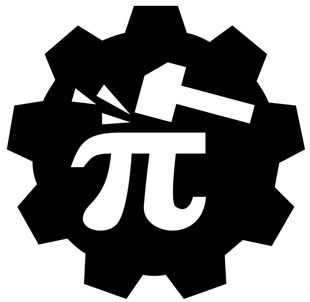

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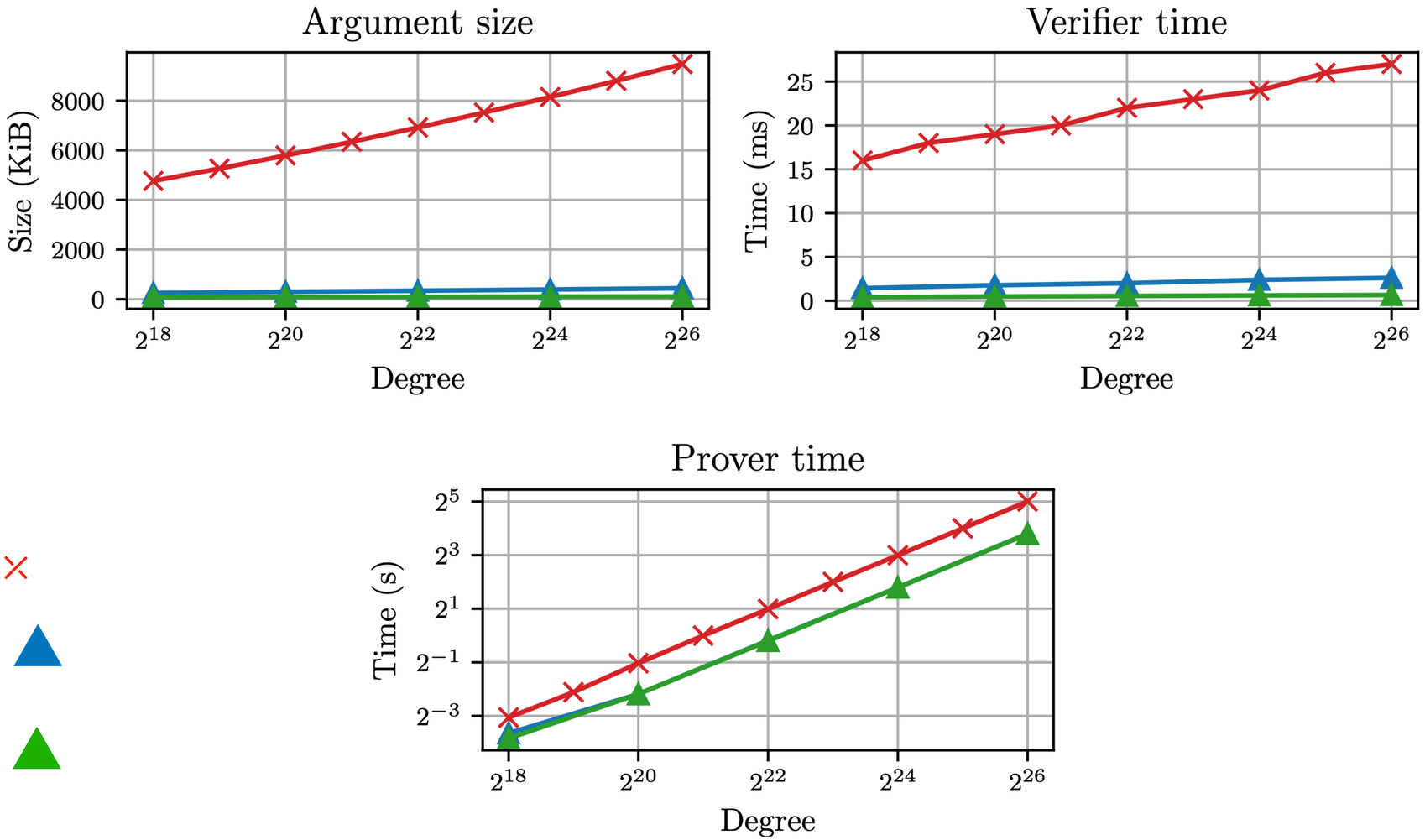

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WHIR-UD: ▲

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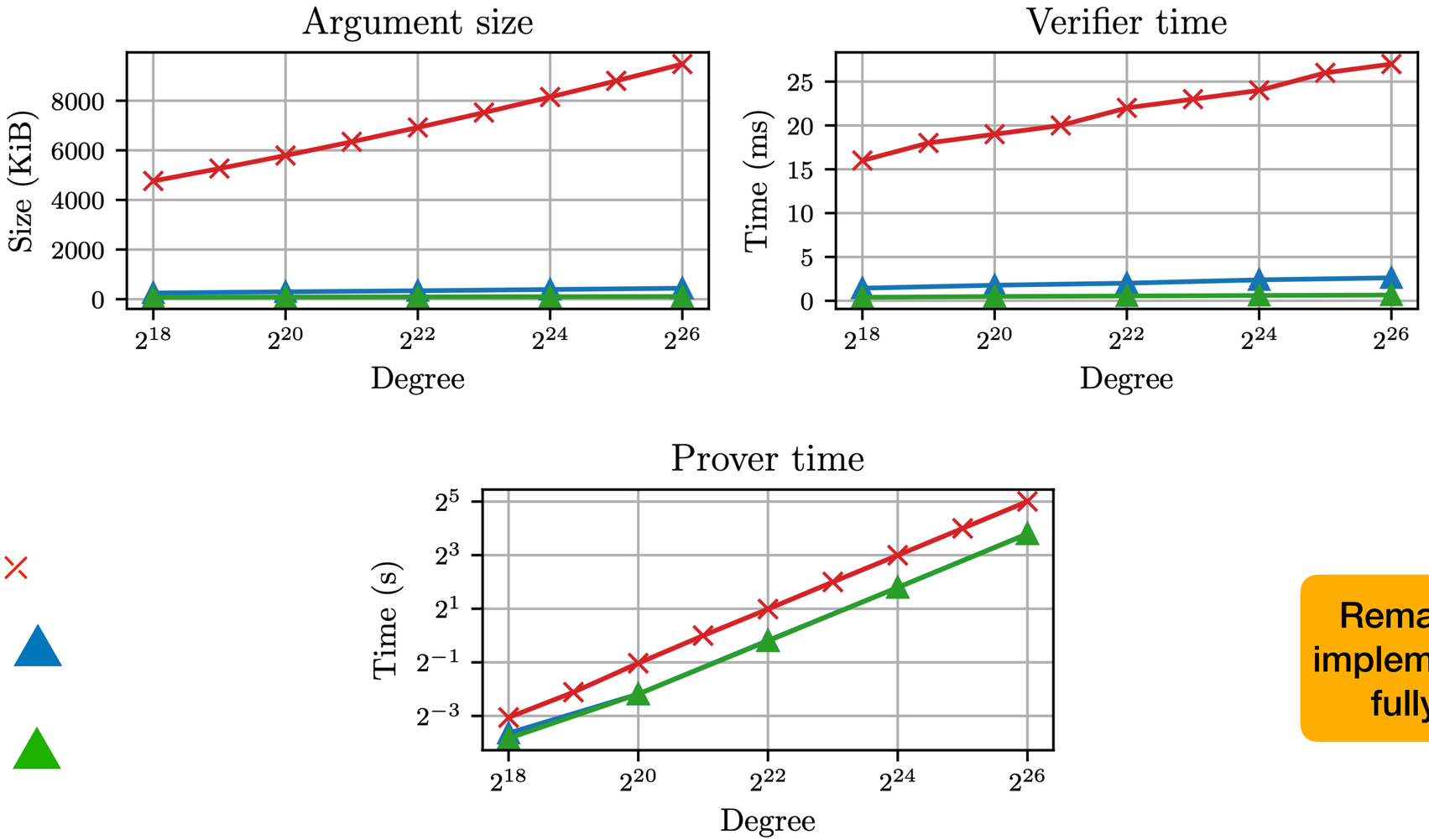

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Remark: BaseFold implementation is not fully optimised

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128-bits security level.

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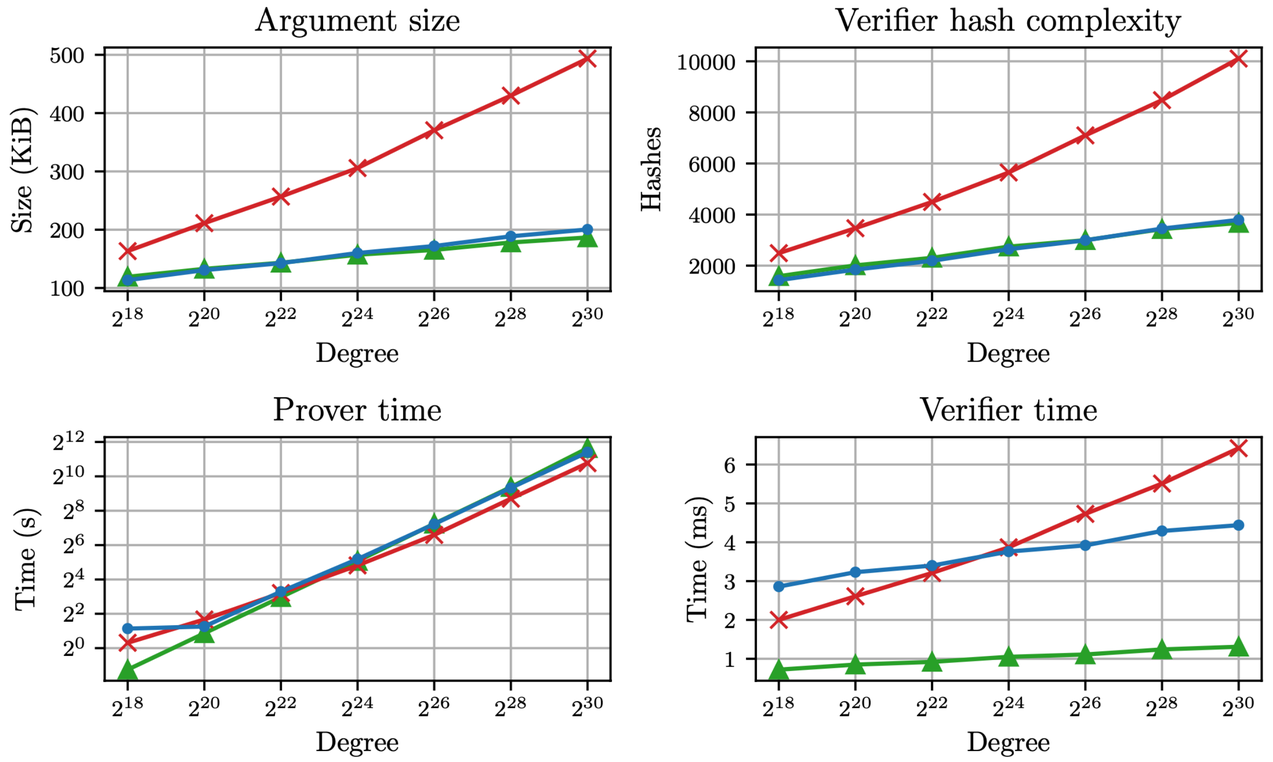

Figure 2: Comparison of FRI, STIR and WHIR for $\rho = 1/2$. FRI: ×, STIR: ●, WHIR-CB: ▲. Pro
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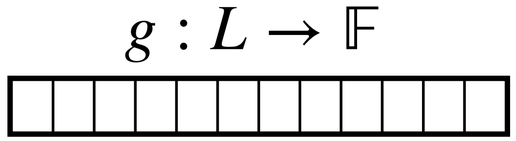
Batching

Pick your favourite sumcheck batching

Sumcheck claims on g:
$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

Batching



Sumcheck claim on g: $(\hat{w}^*, \sigma^*)$

Many ways this can be done: **we chose random linear combination.**

SNARKs

Succinct Non-interactive Arguments of Knowledge

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- Want to show “knowledge” of w s.t. $(x, w) \in R$

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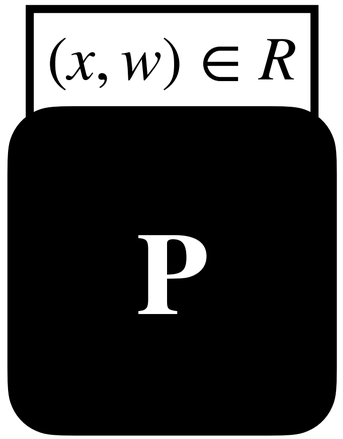

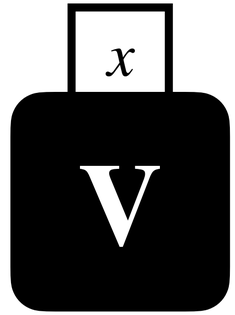

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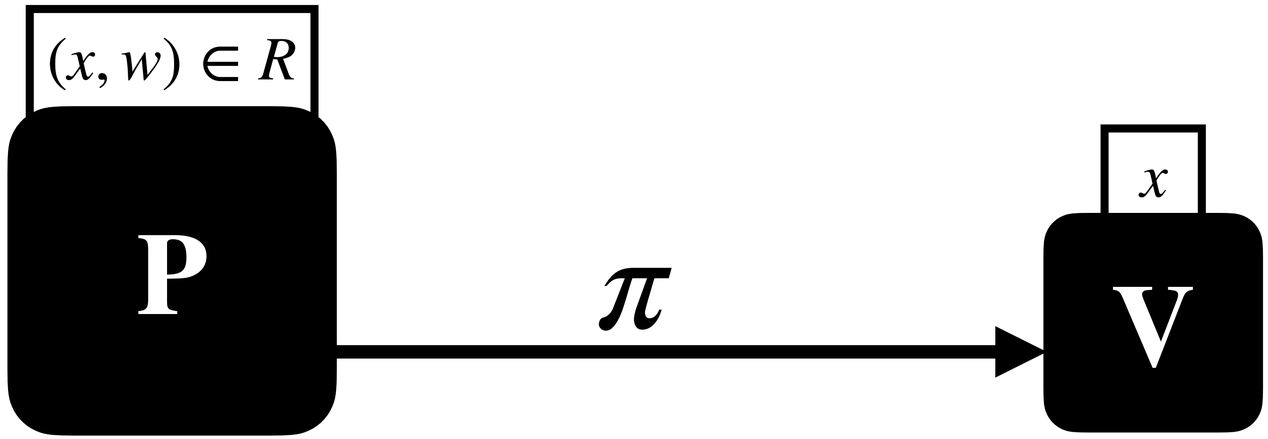

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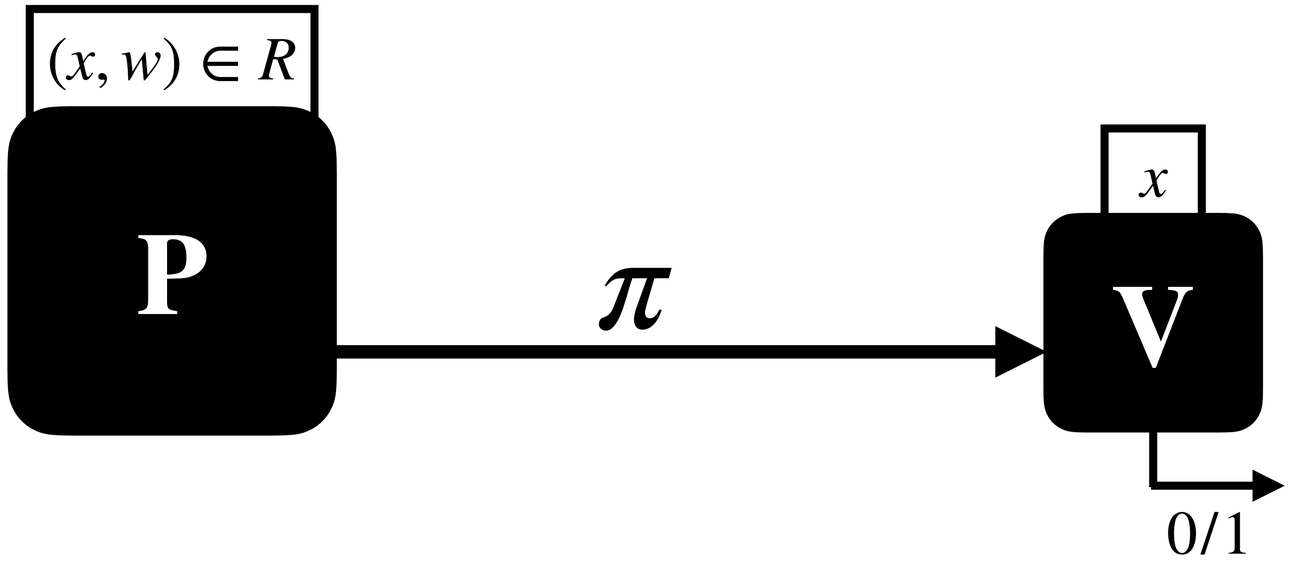

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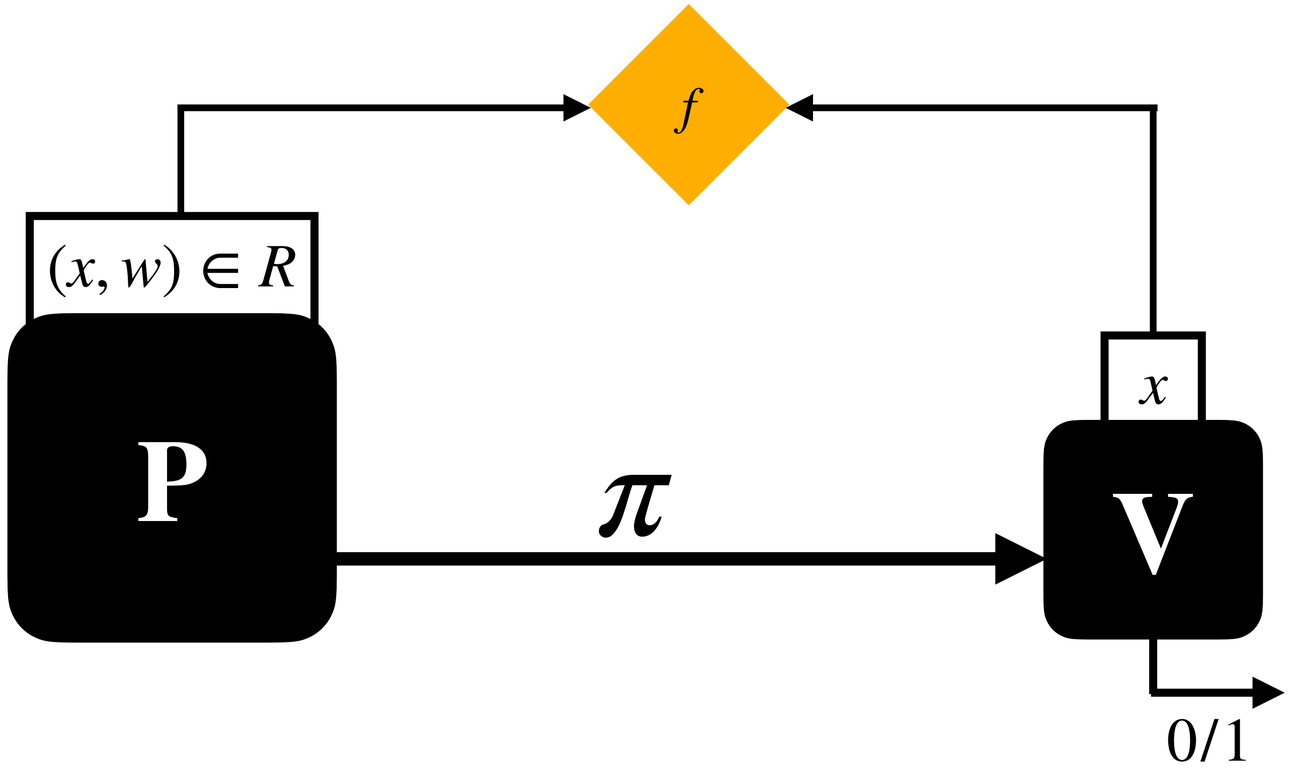

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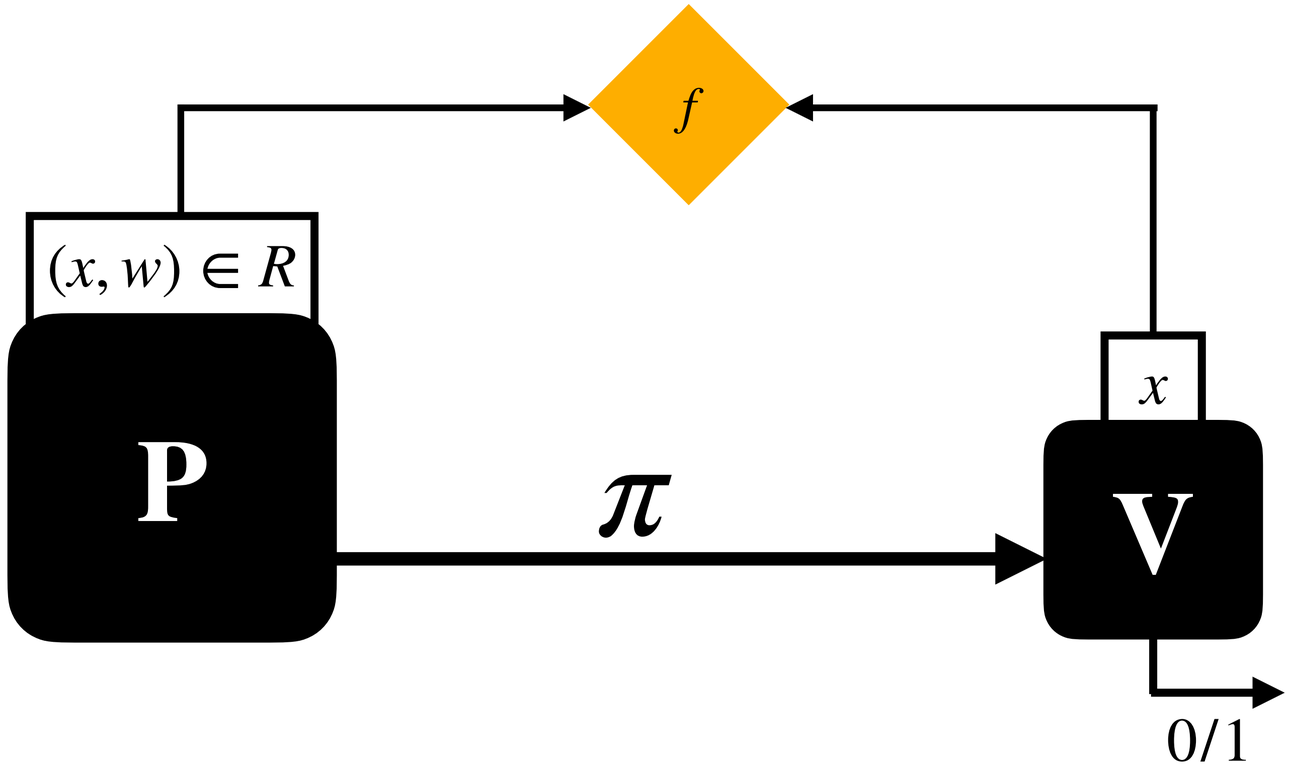

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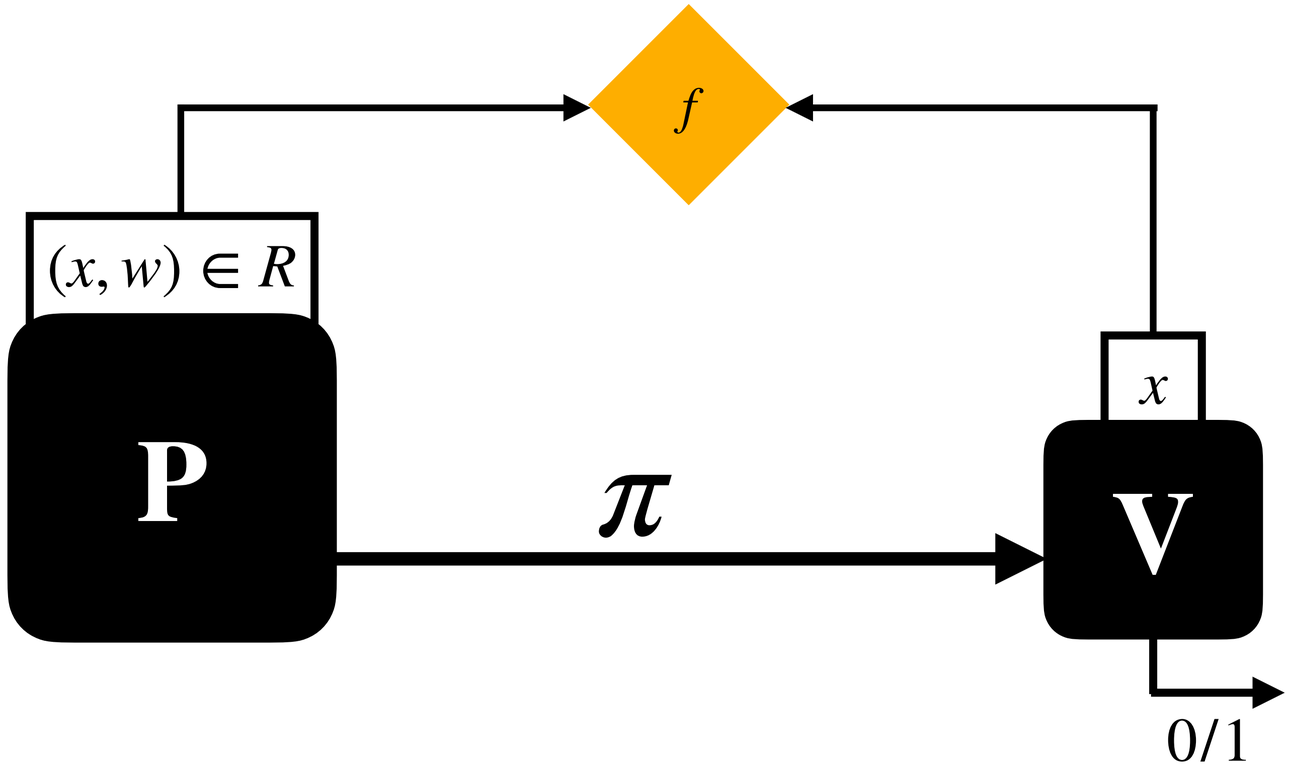

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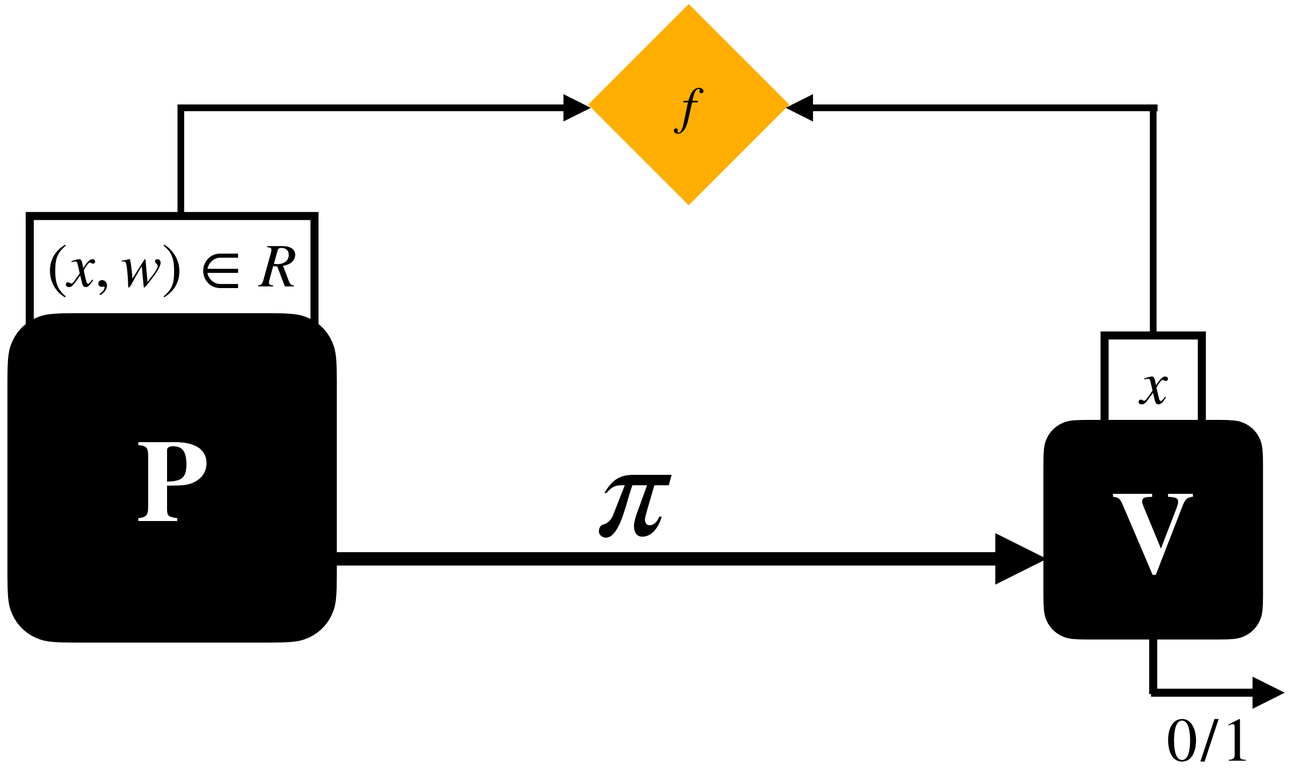
- Will call these **hash-based SNARKs**.



Hash-based SNARKs

In practice

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STARKWARE

polygon

Irreducible

LAMBDA CLASS

zkSync

dYdX

Matter Labs

RISC ZERO

Succinct

And many more…

Constructing SNARKs

[BCS16] Construction

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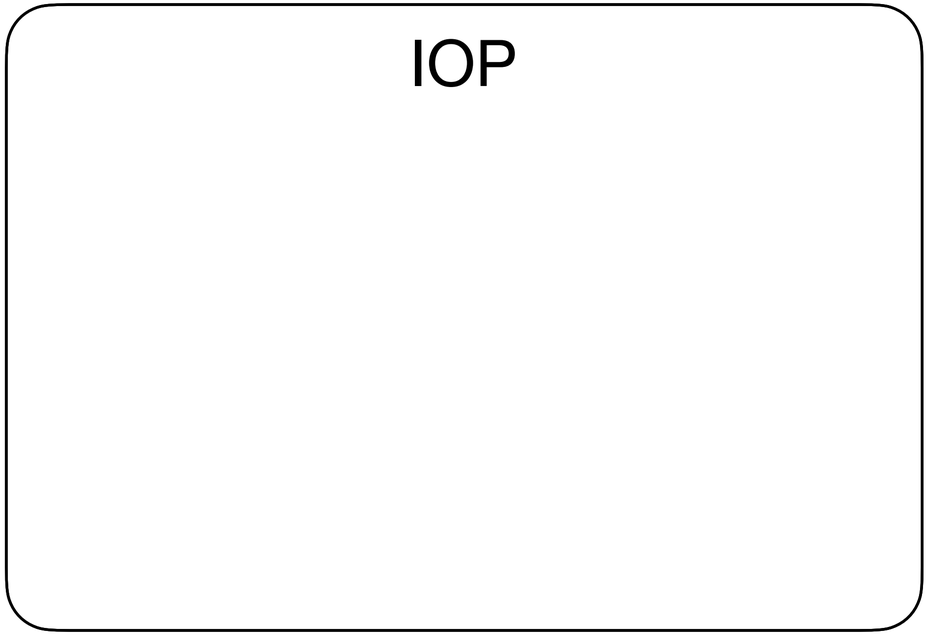

Constructing SNARKs

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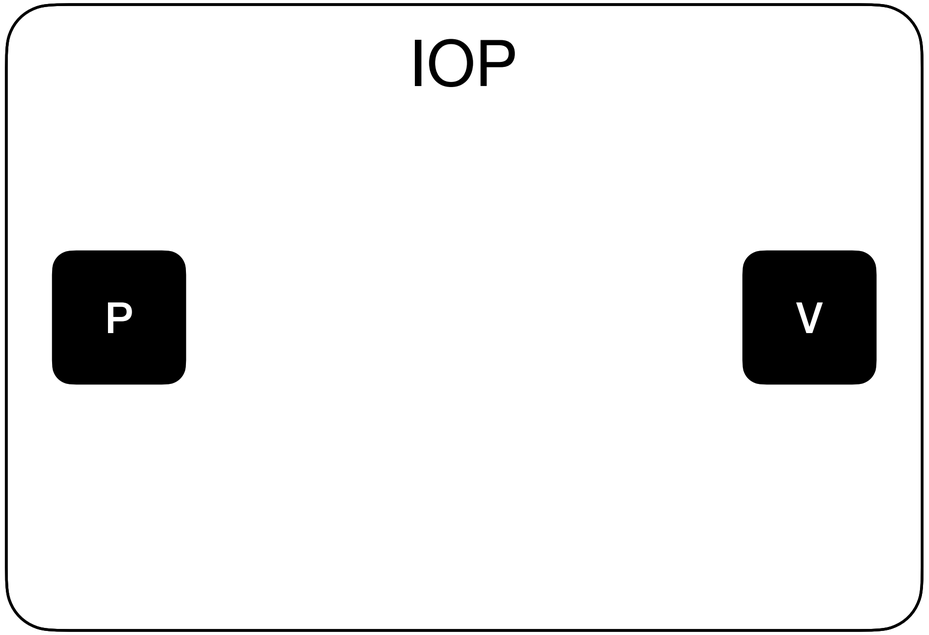

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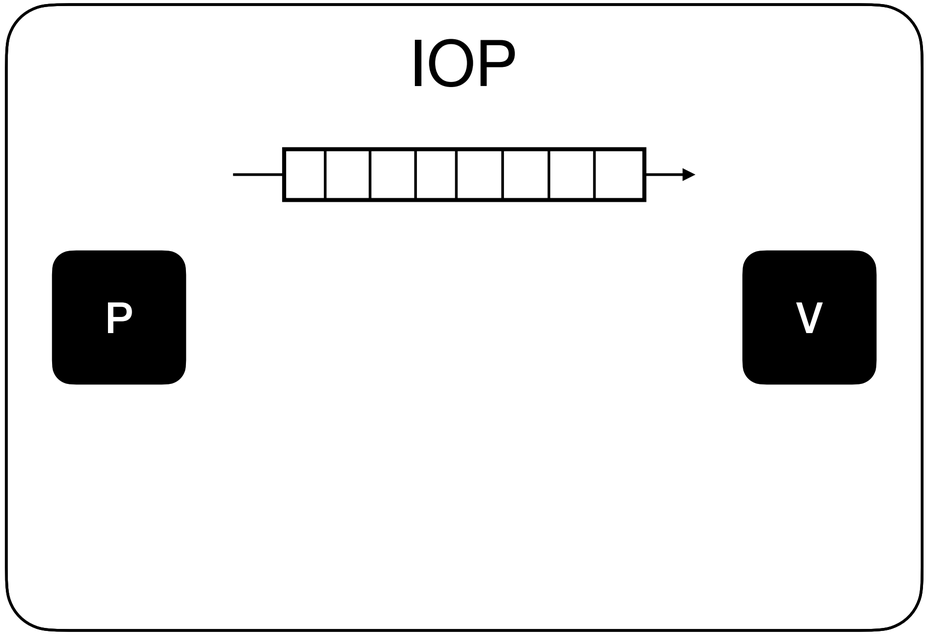

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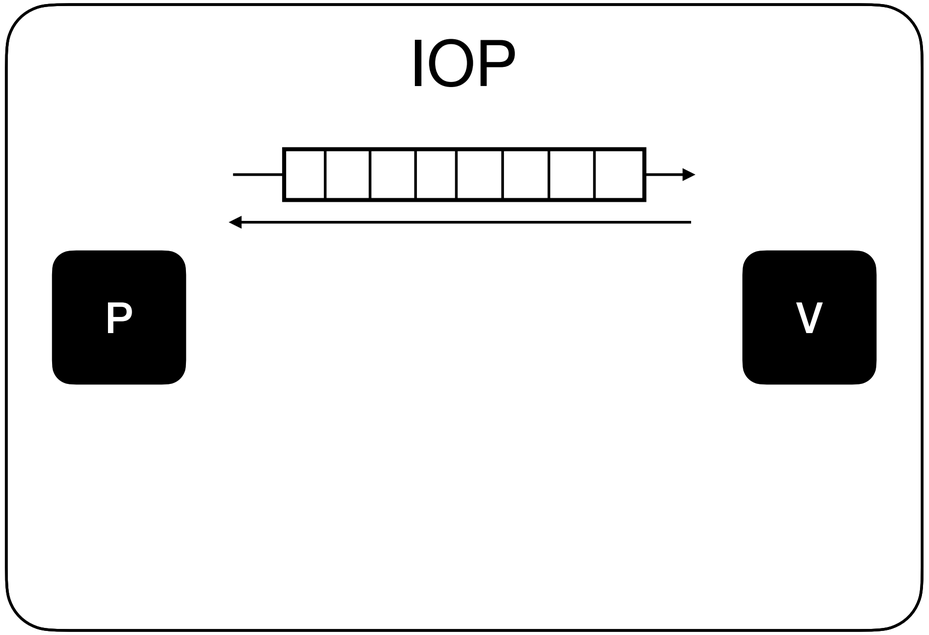

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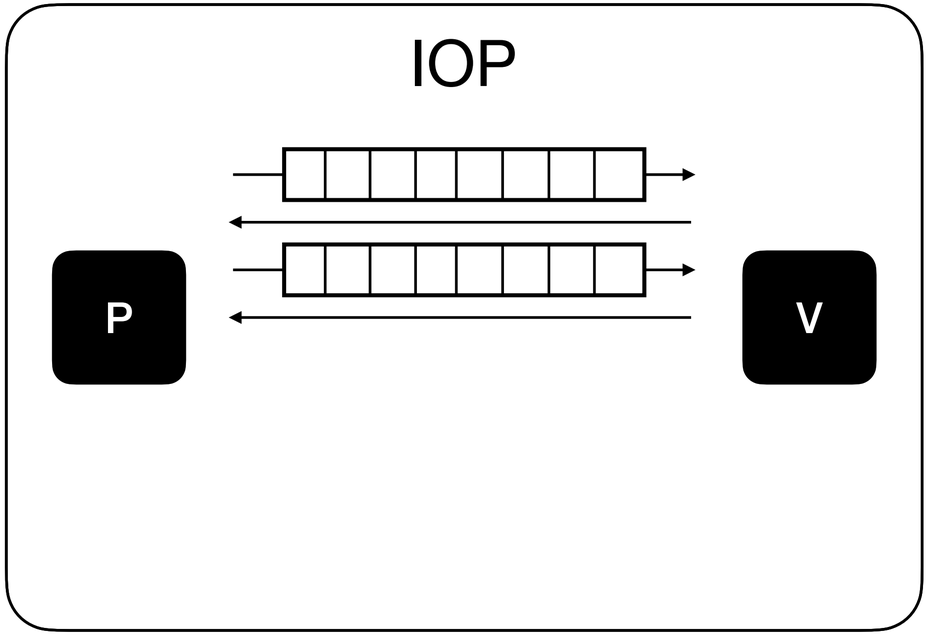

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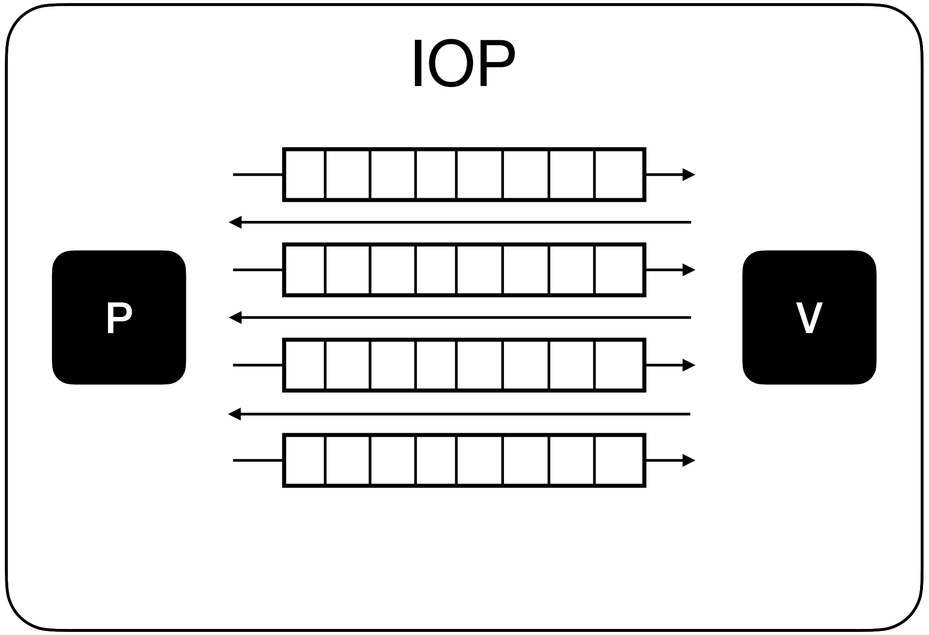

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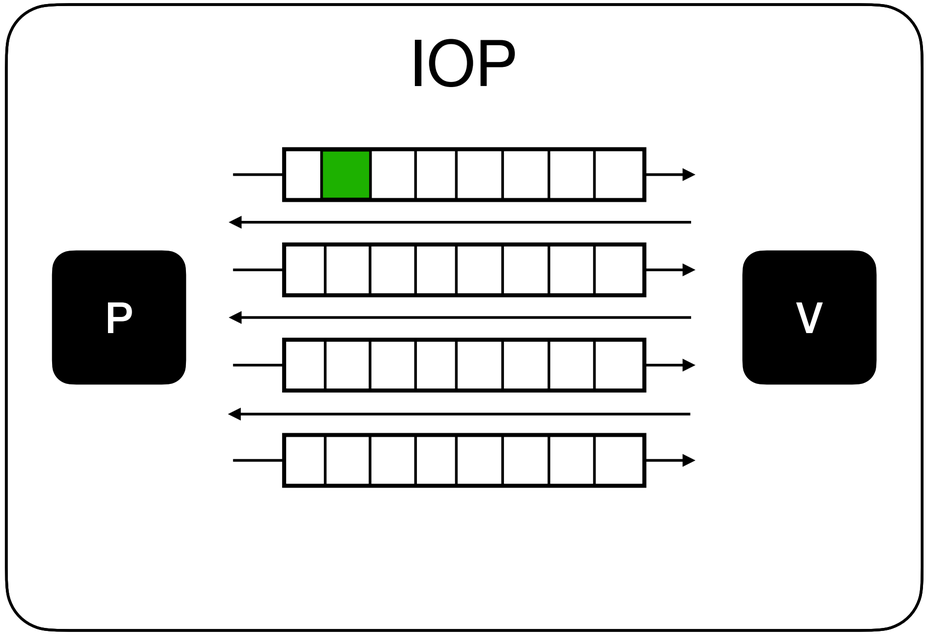

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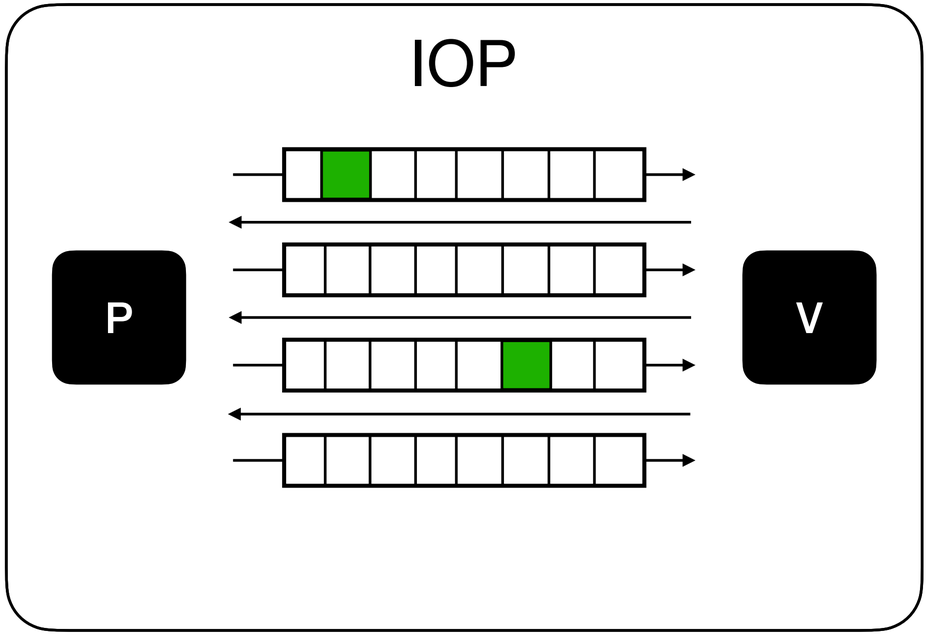

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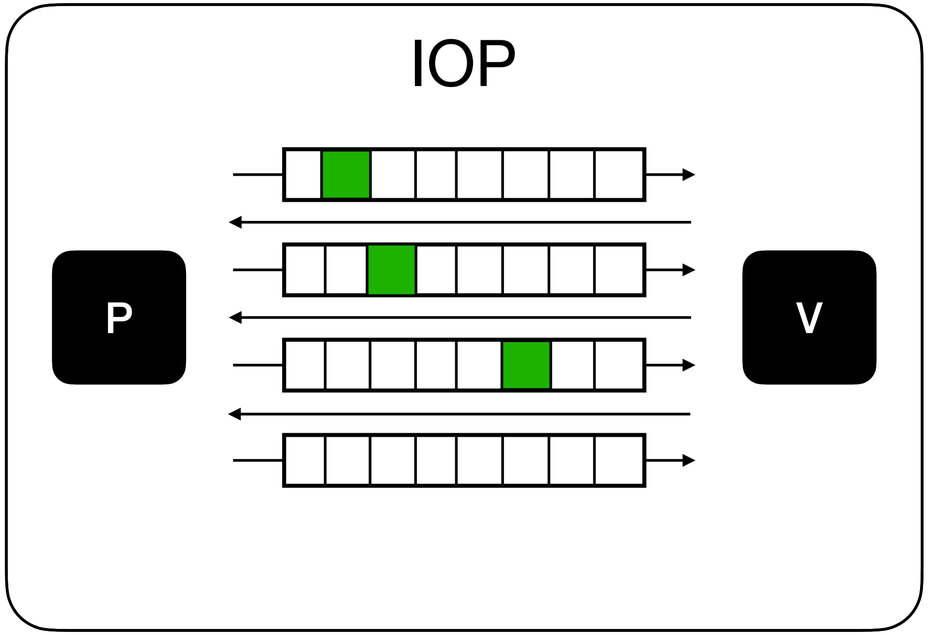

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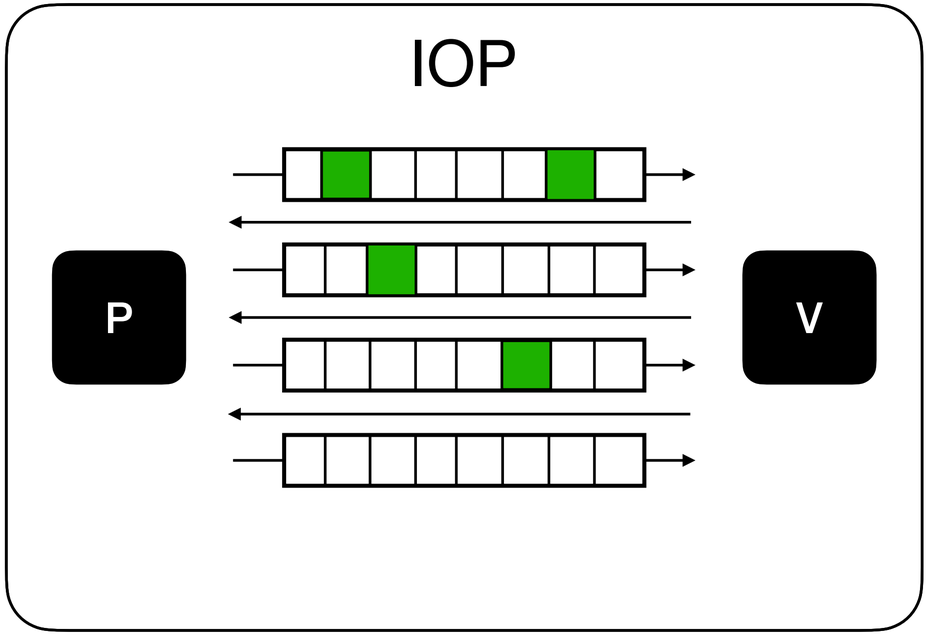

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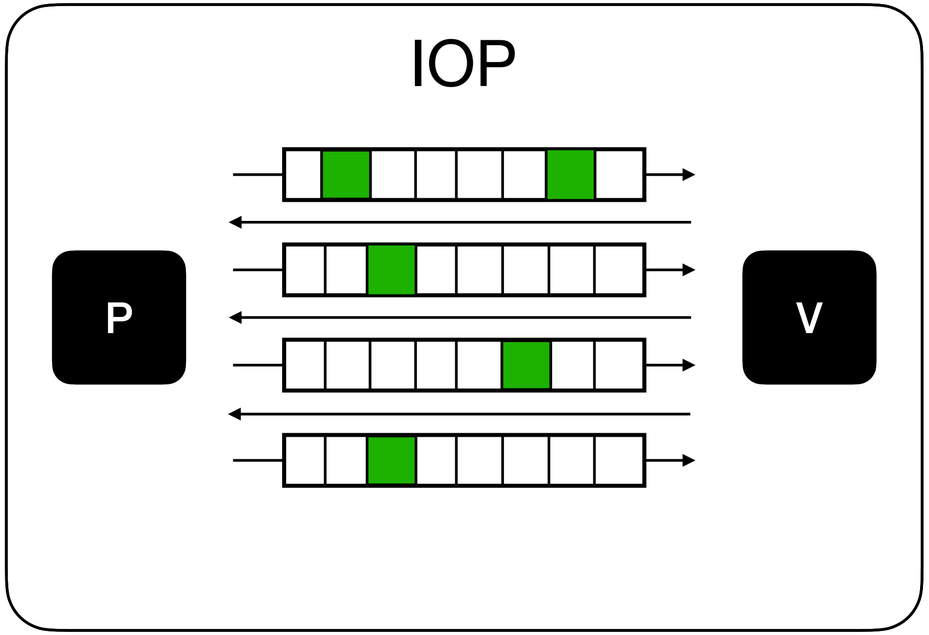

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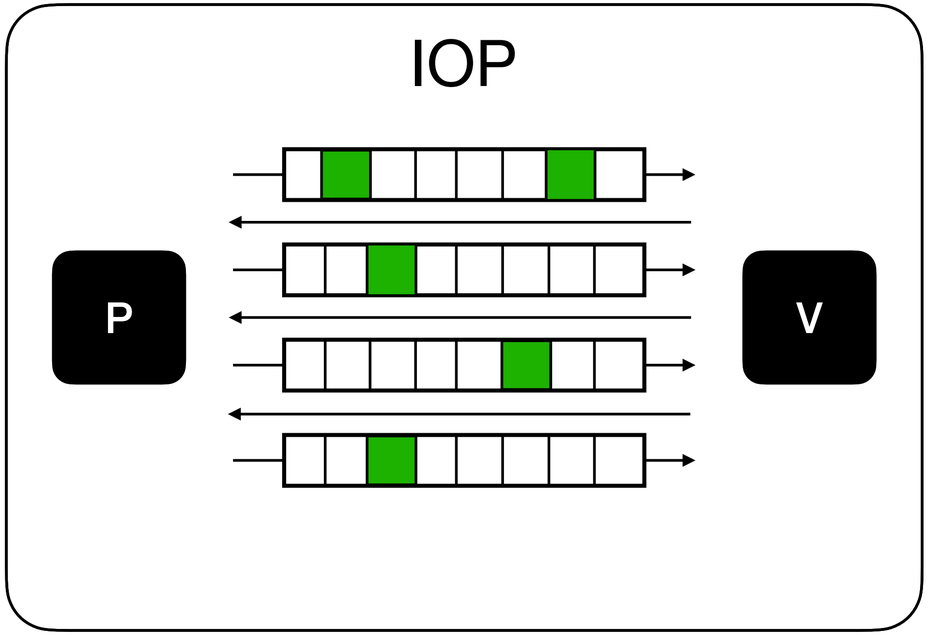
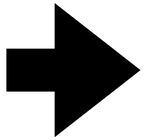


BCS



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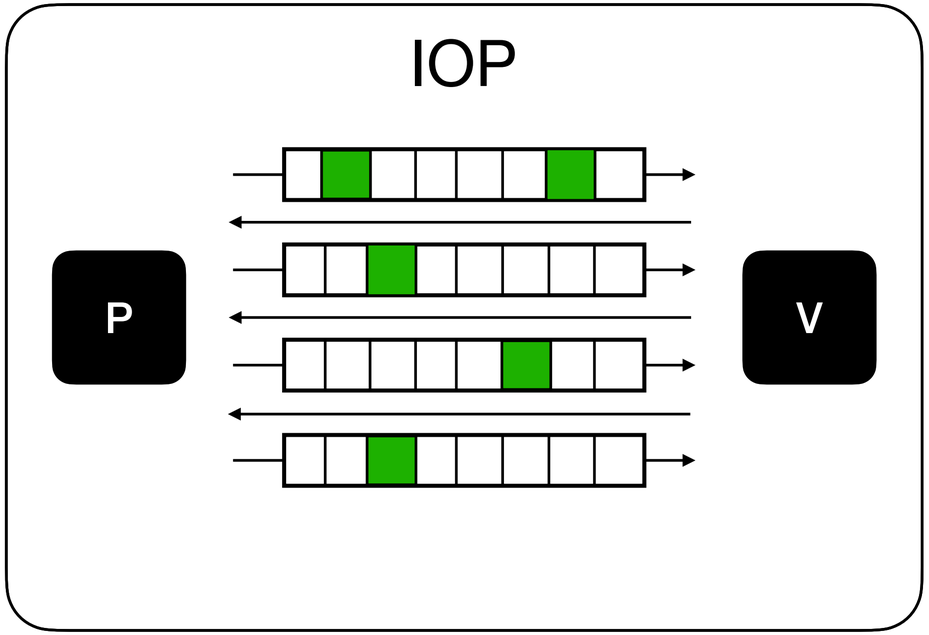

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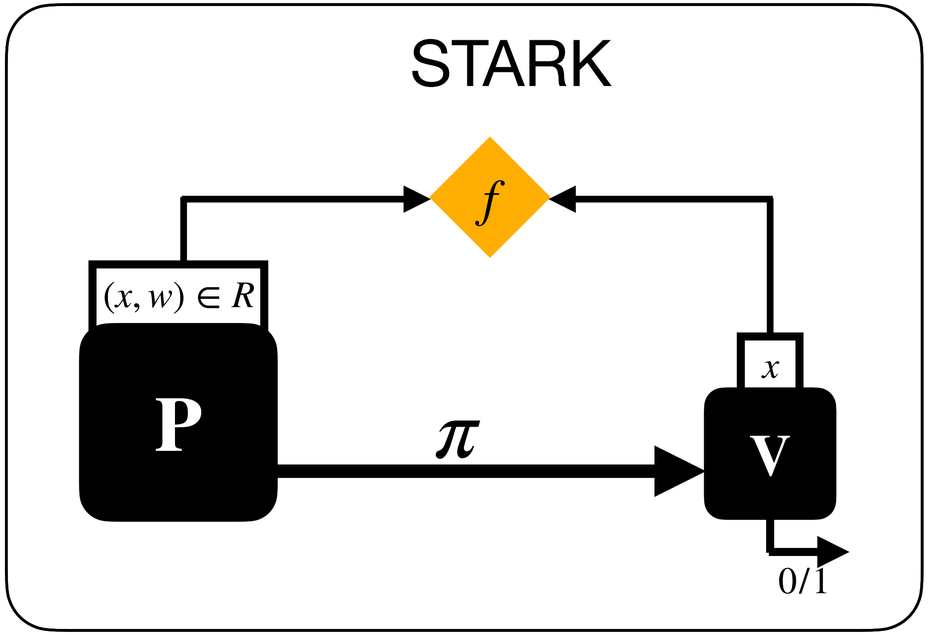

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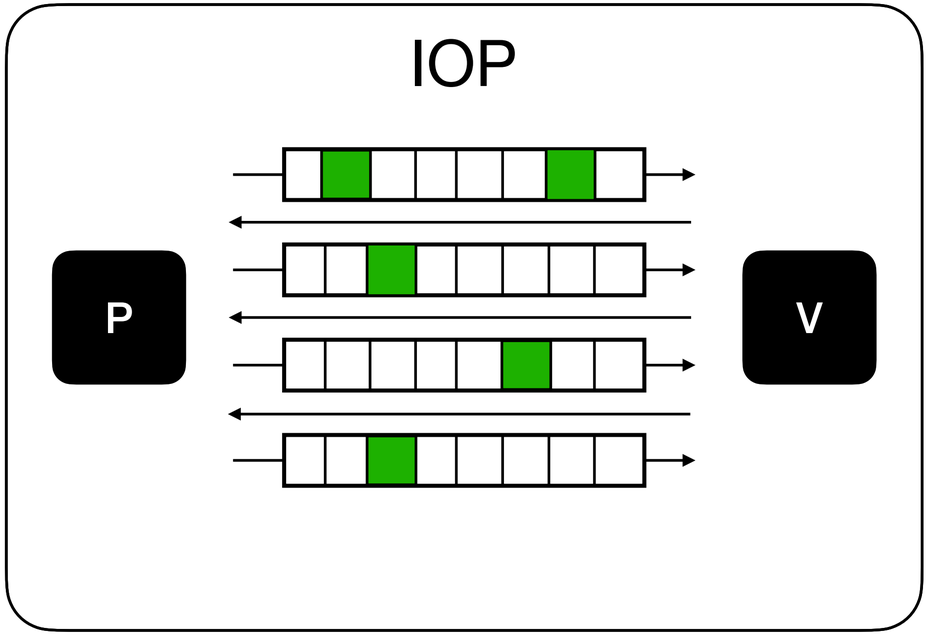

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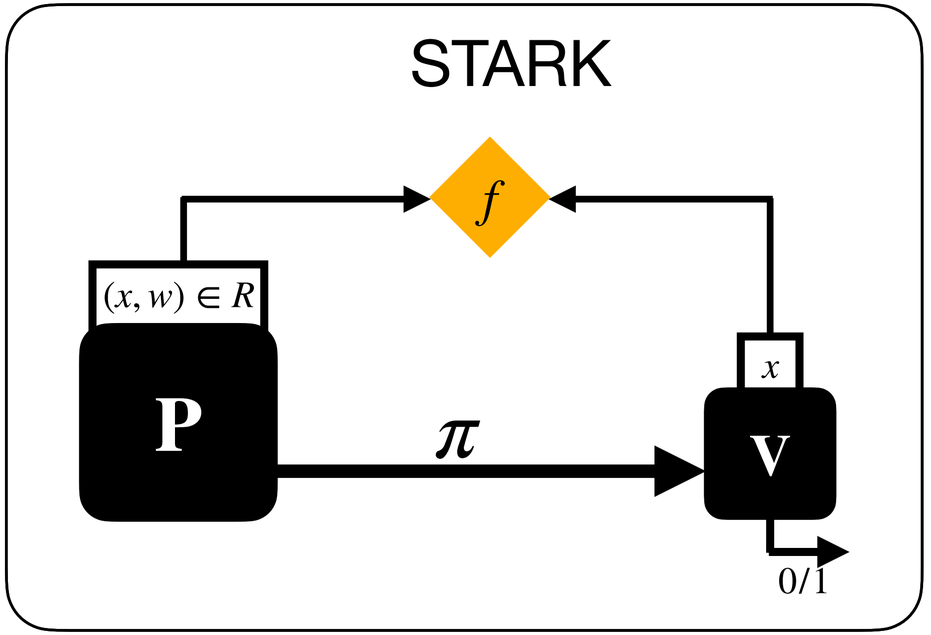

Proof length l $\approx O(n)$

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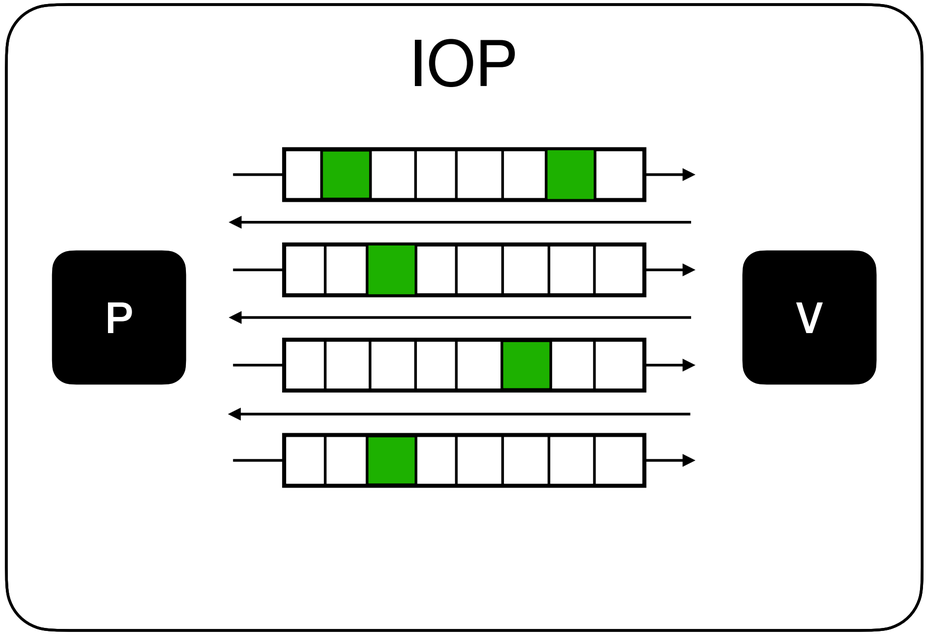

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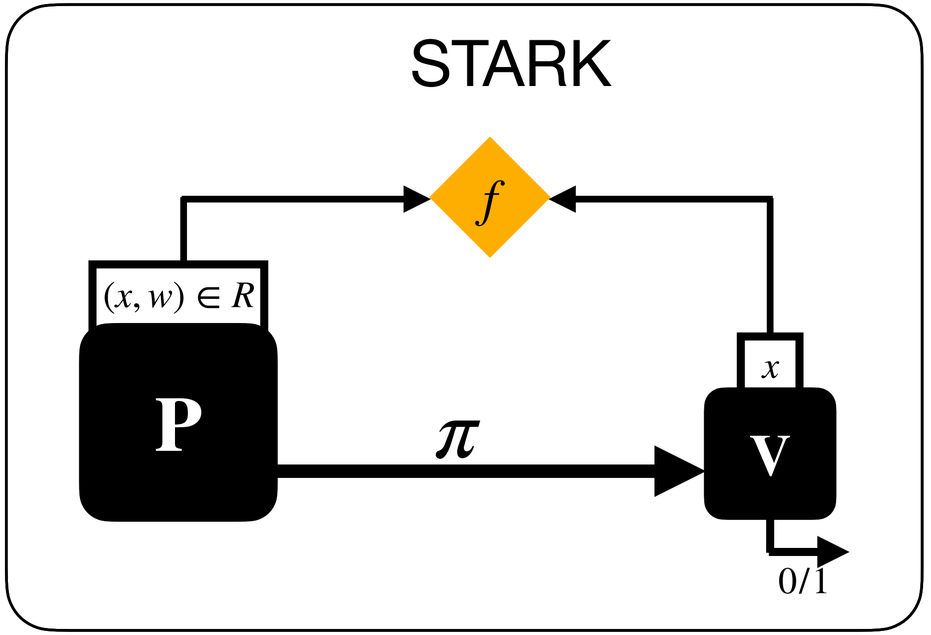

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Constructing SNARKs

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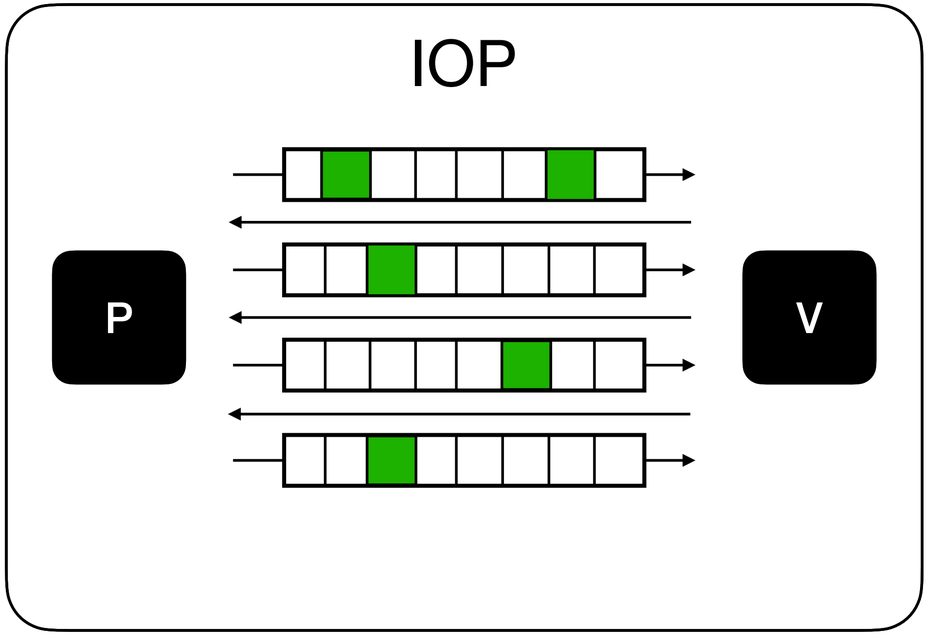

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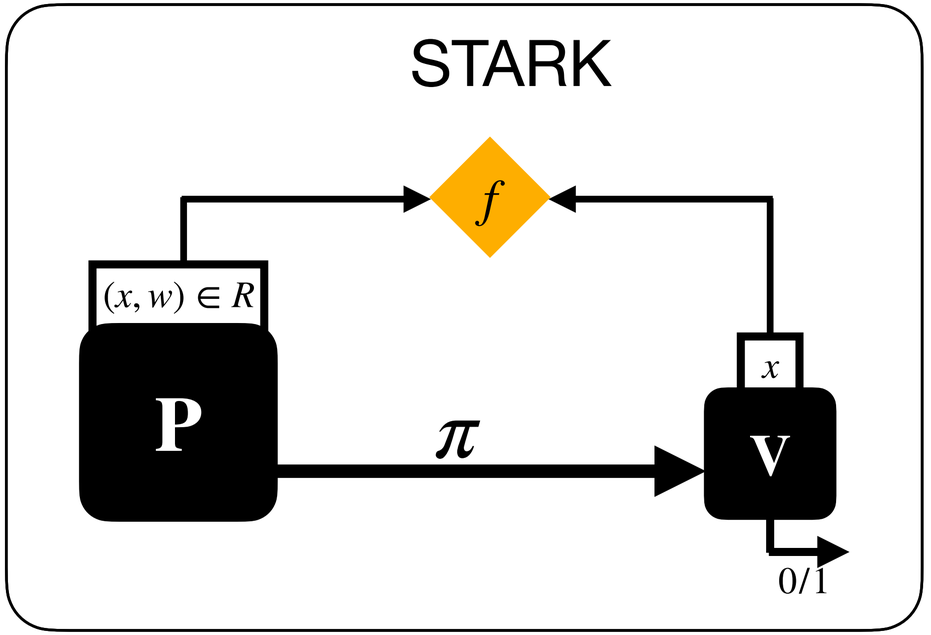

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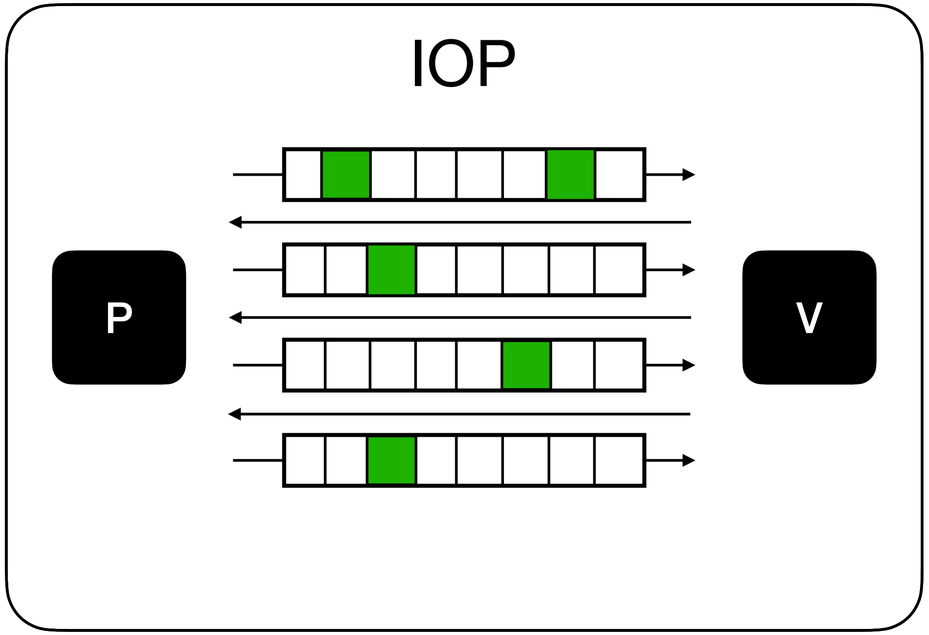

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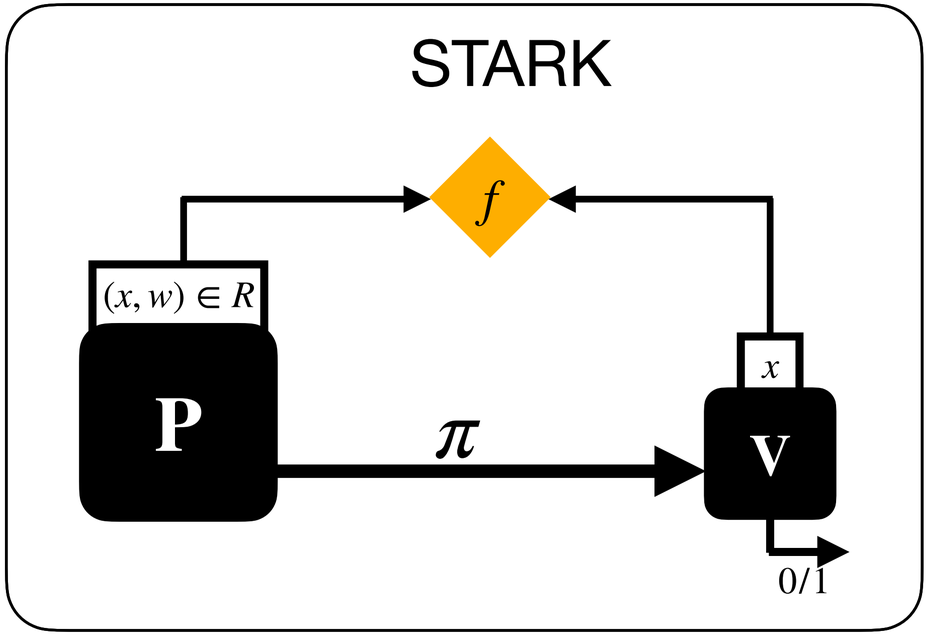

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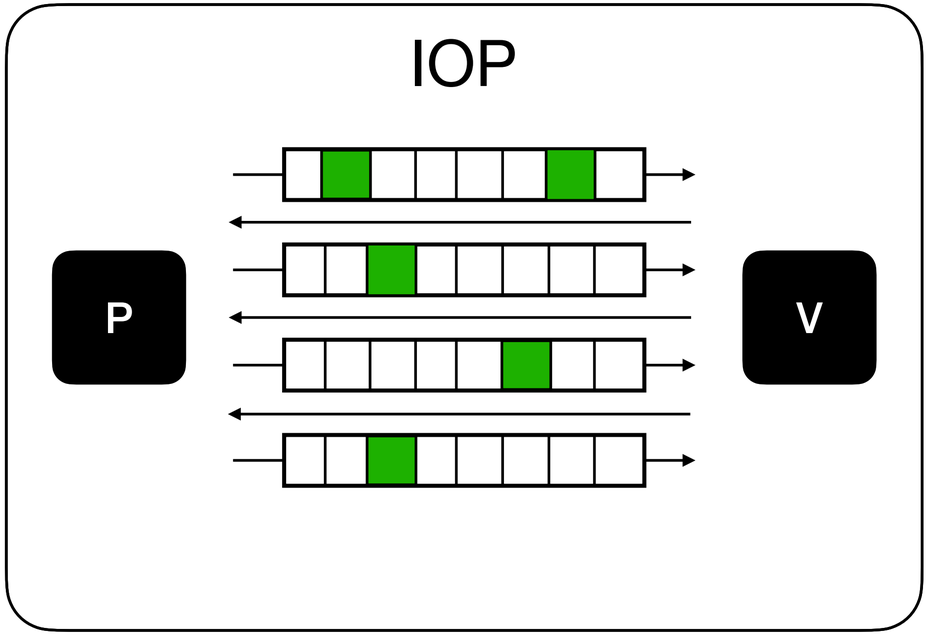
Constructing SNARKs

[BCS16] Construction

In this talk, we focus on the IOP!



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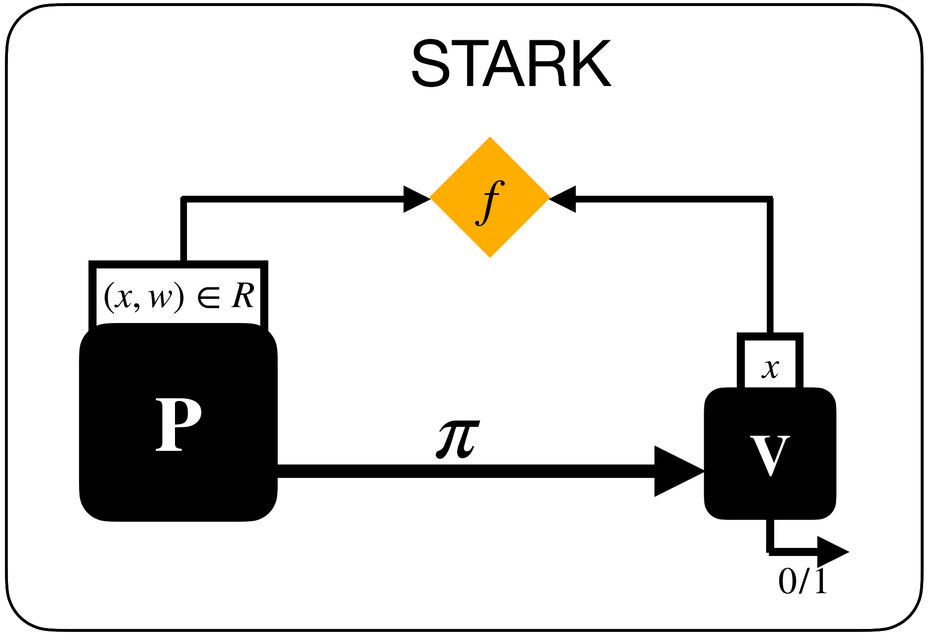

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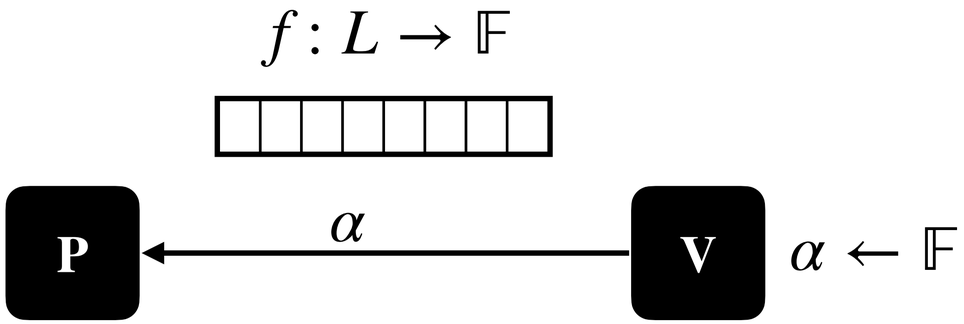
$$f : L \to \mathbb{F}$$

P

V

Review: FRI iteration



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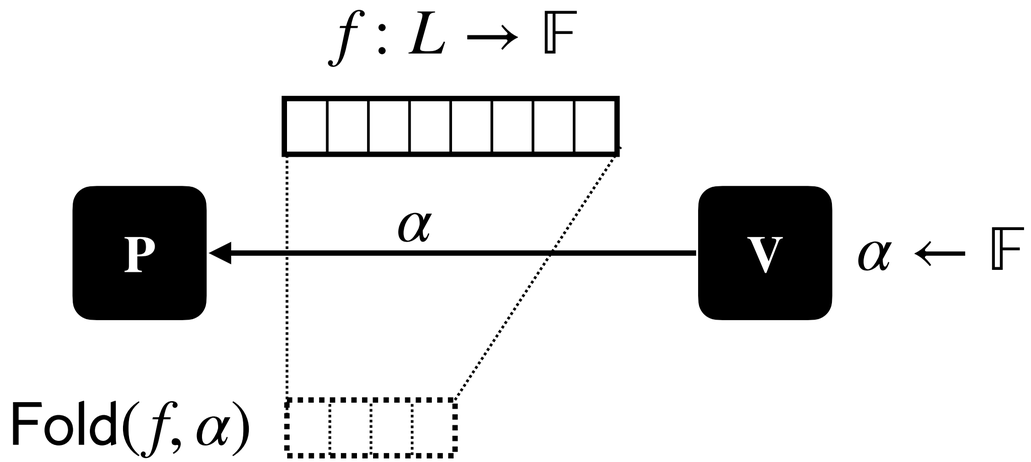

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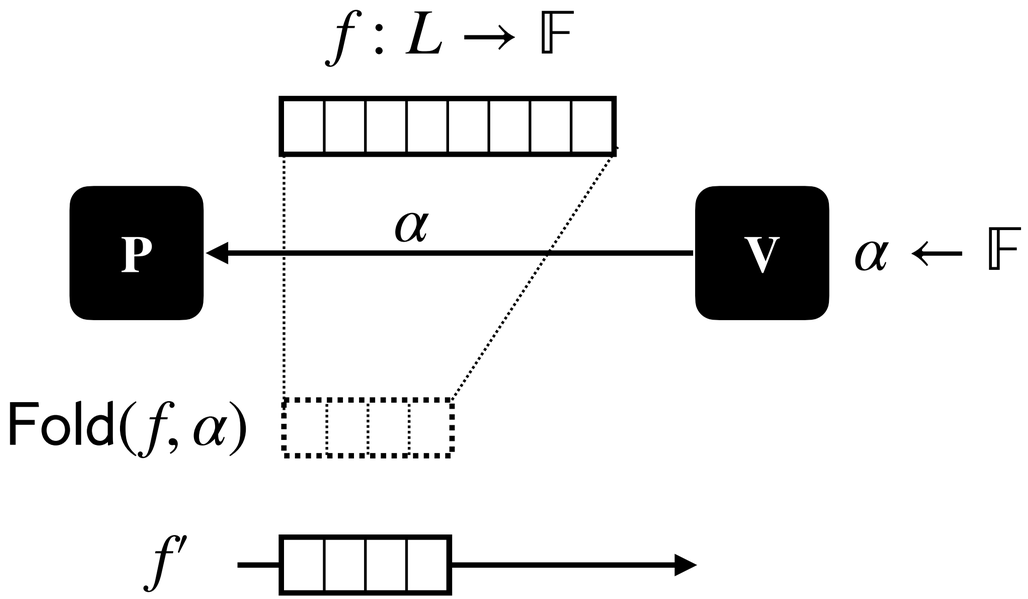

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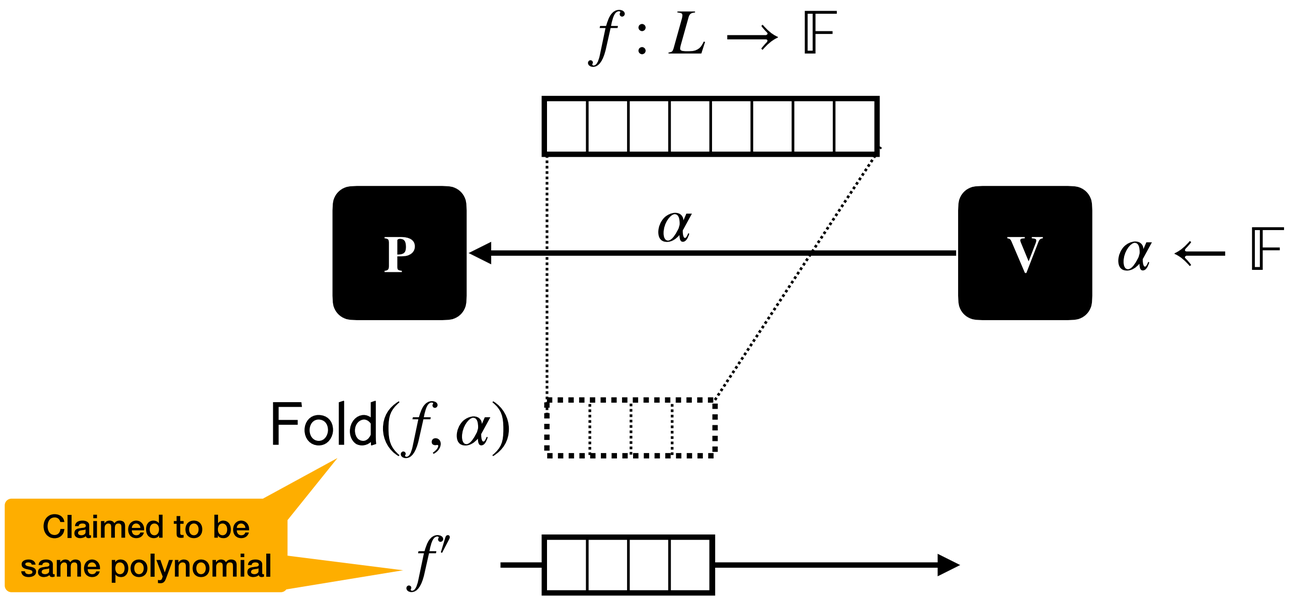

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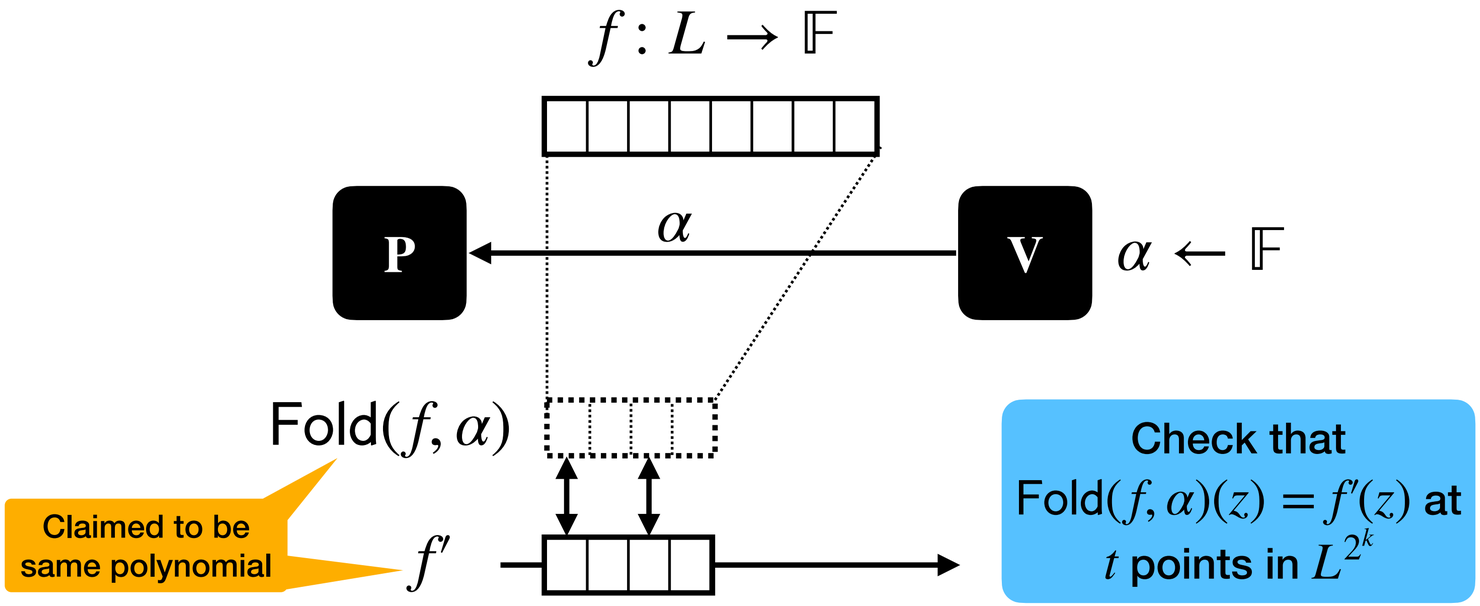

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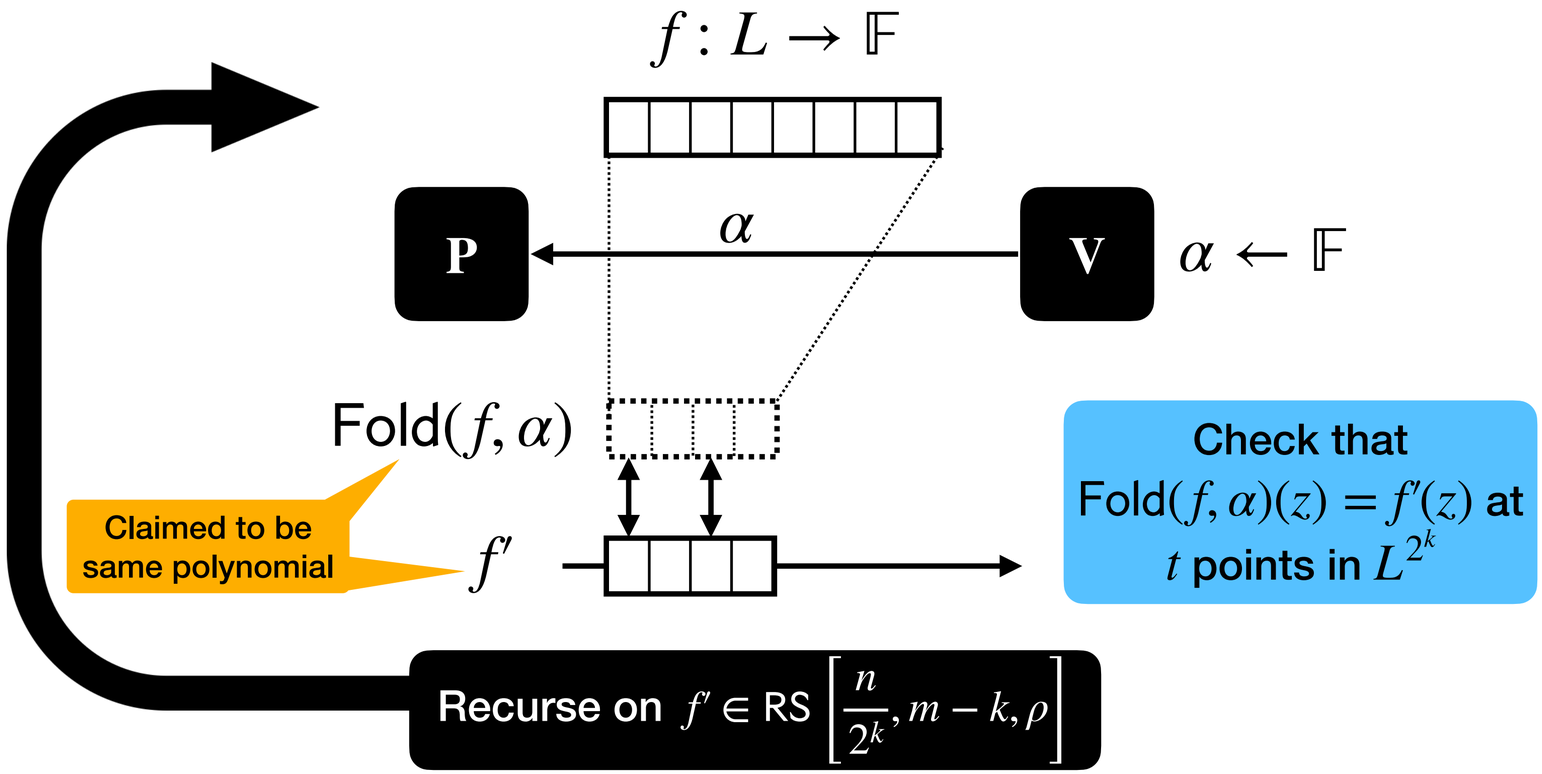

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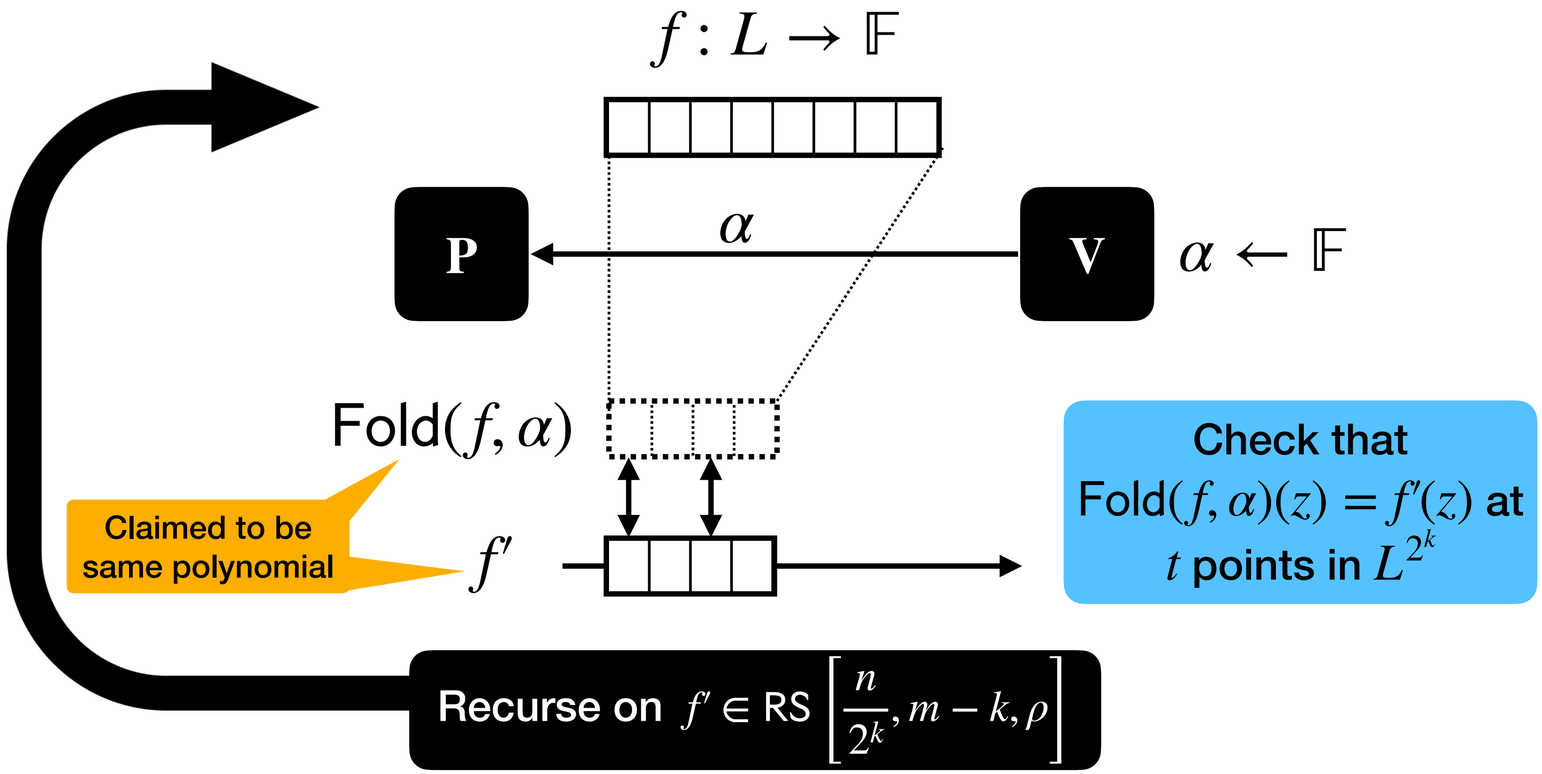
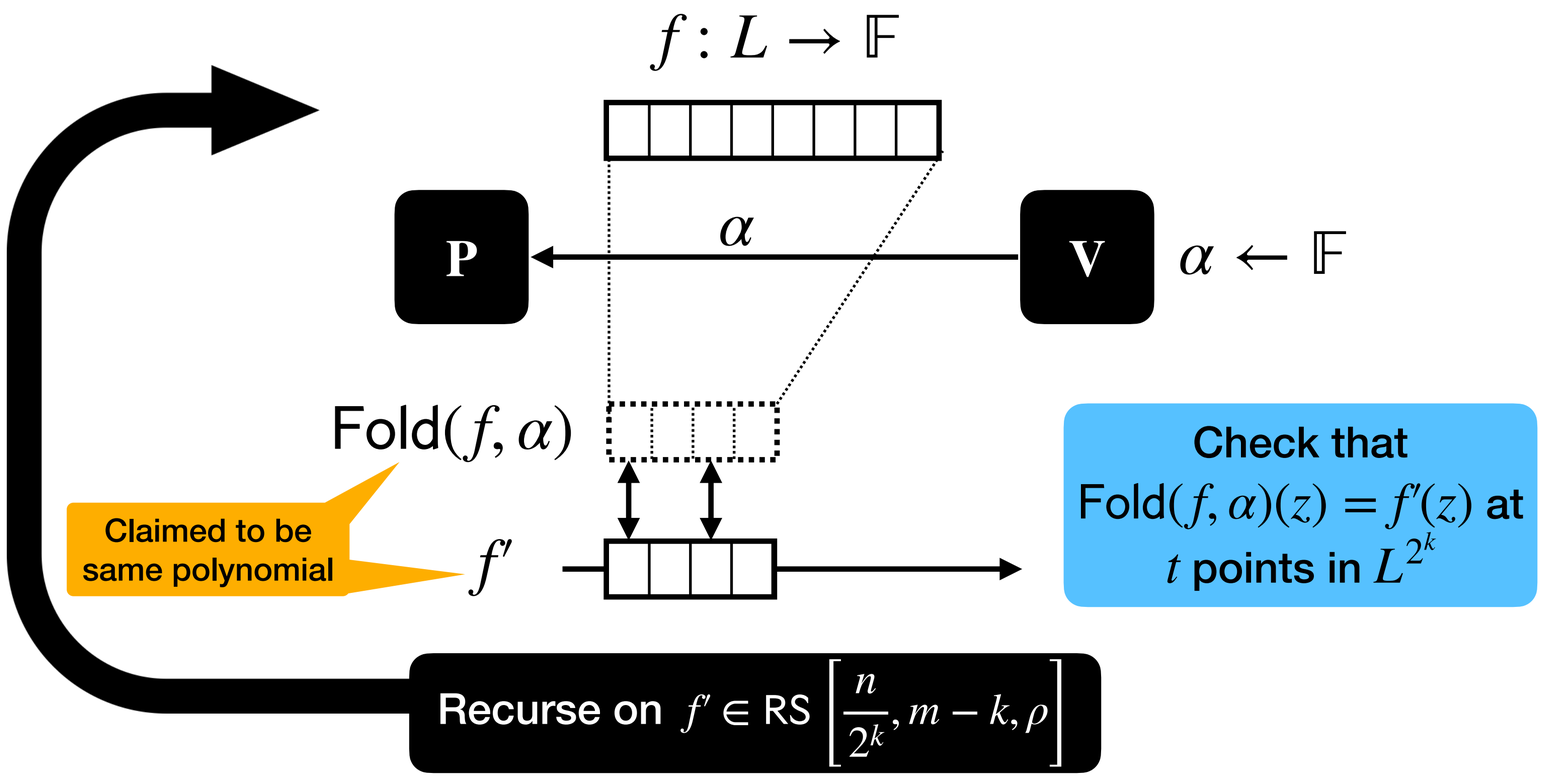


Disclaimer: in full FRI consistency checks are correlated between rounds.

Review: FRI iteration



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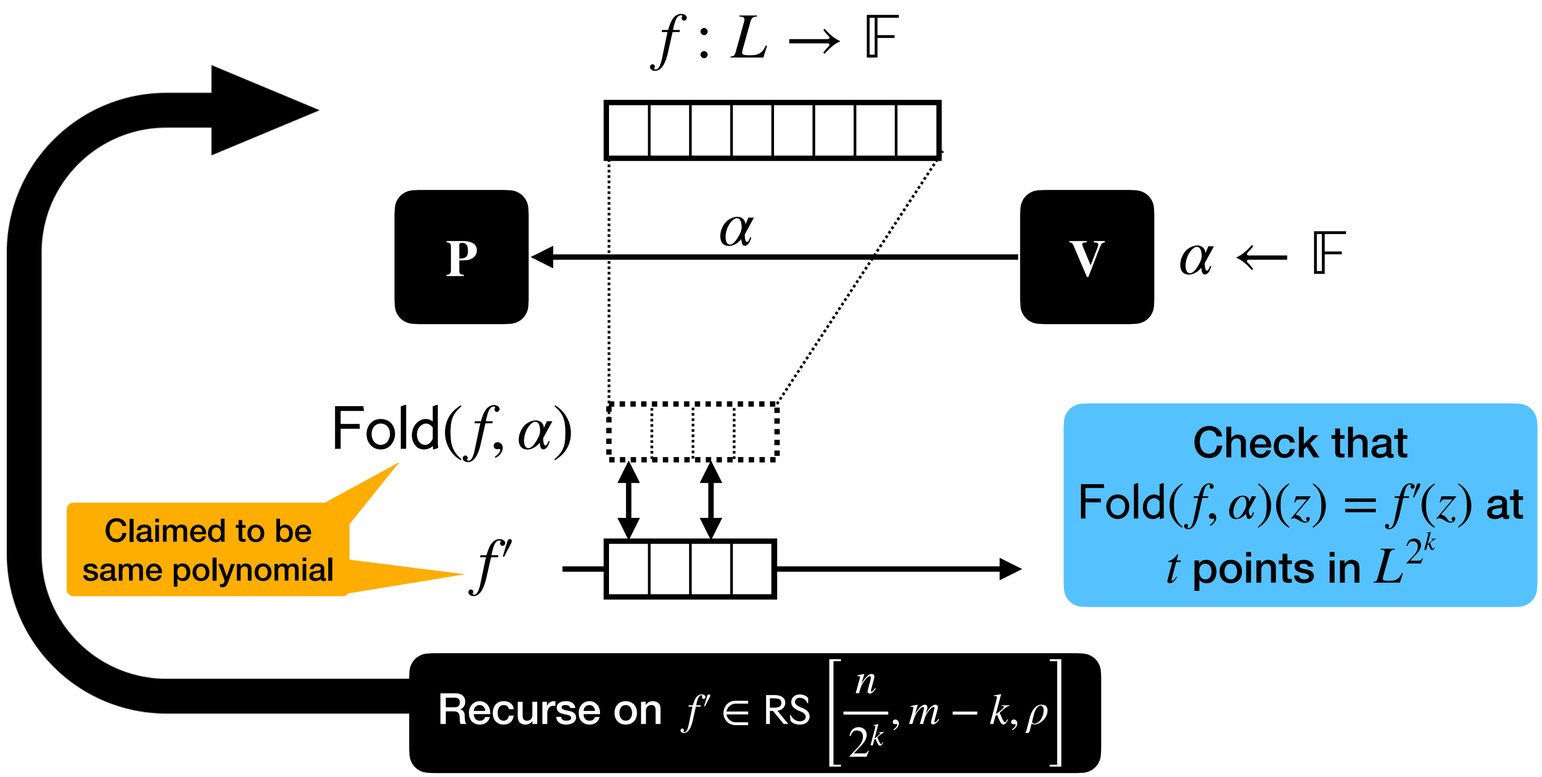

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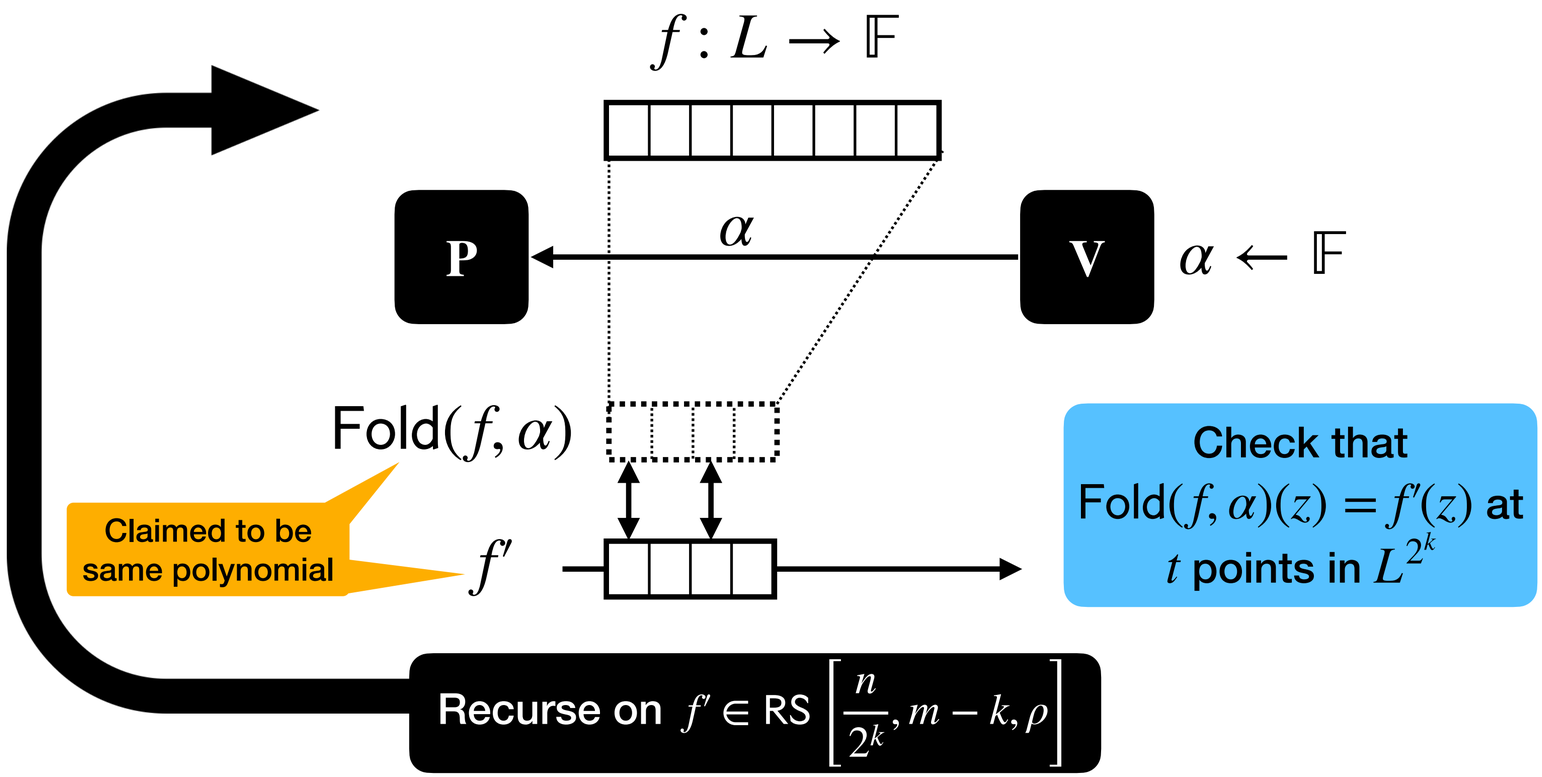

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To get soundness error $\varepsilon_{\mathrm{RBR}} \leq 2^{-\lambda}$:

set $\delta := 1 - \sqrt{\rho}$ and $t := \dfrac{\lambda}{-\log\sqrt{\rho}}$

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Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

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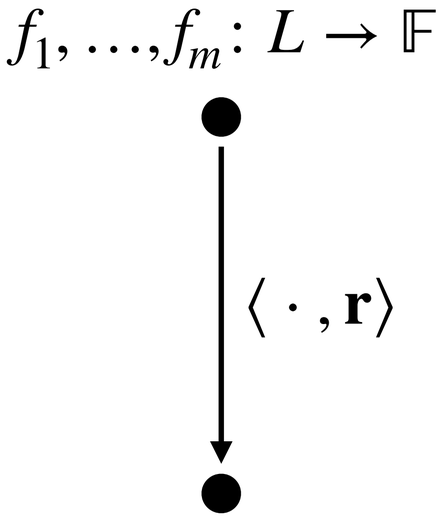

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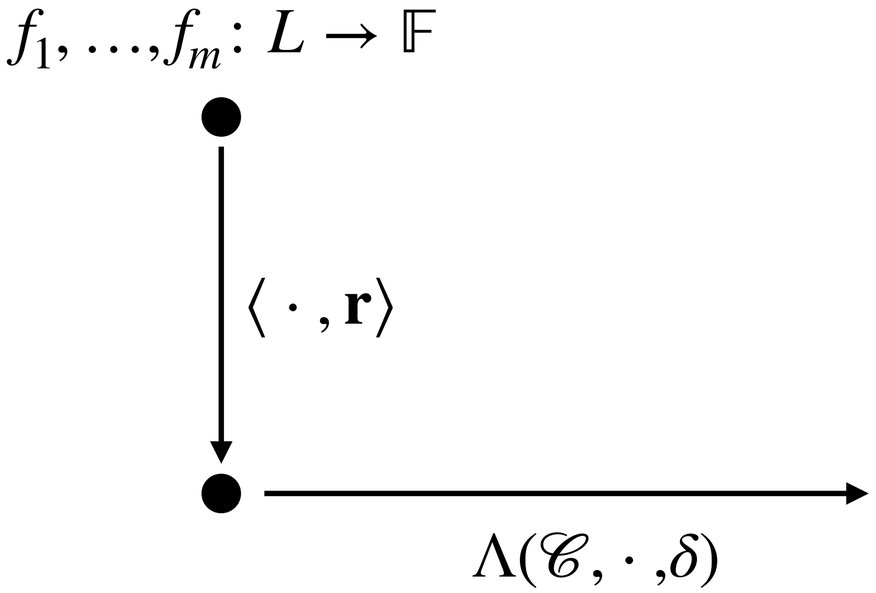

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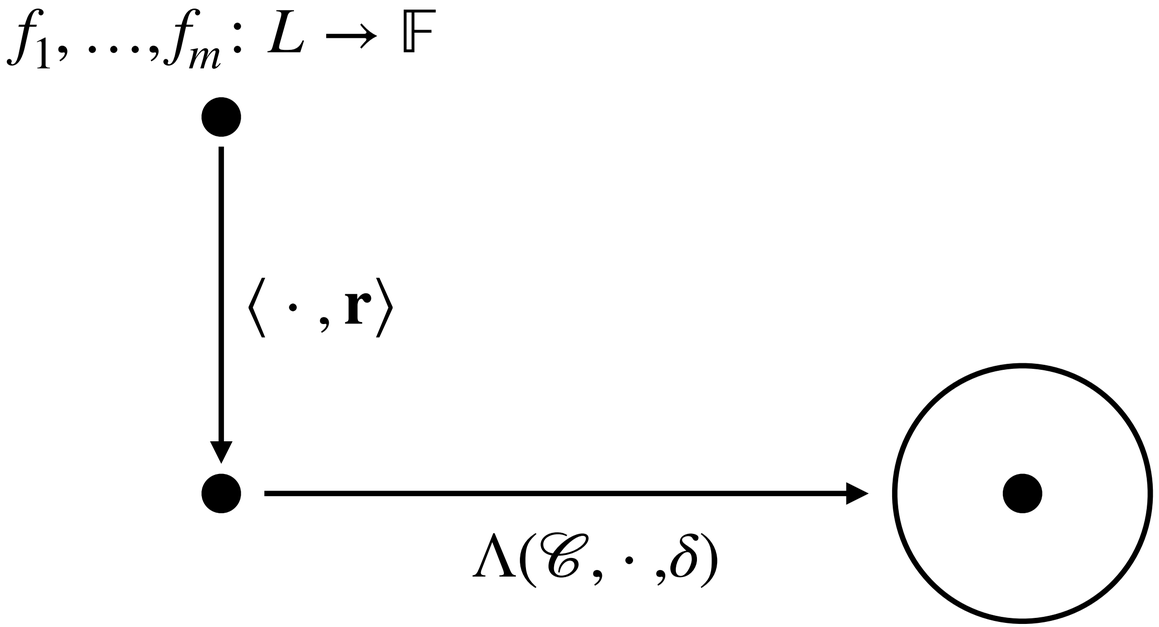

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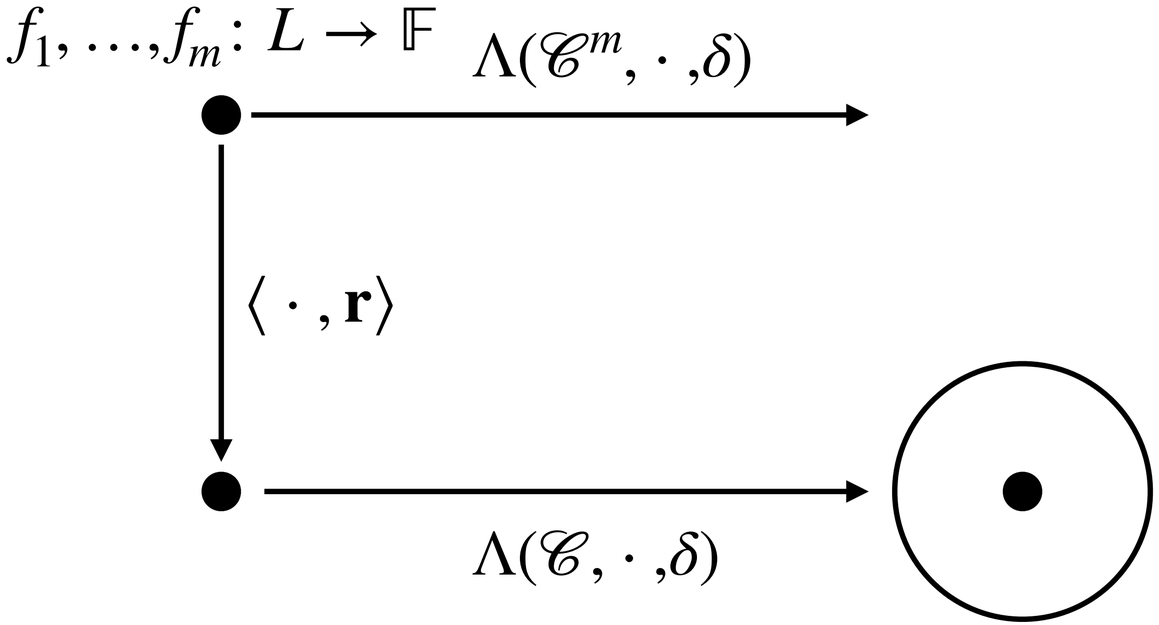

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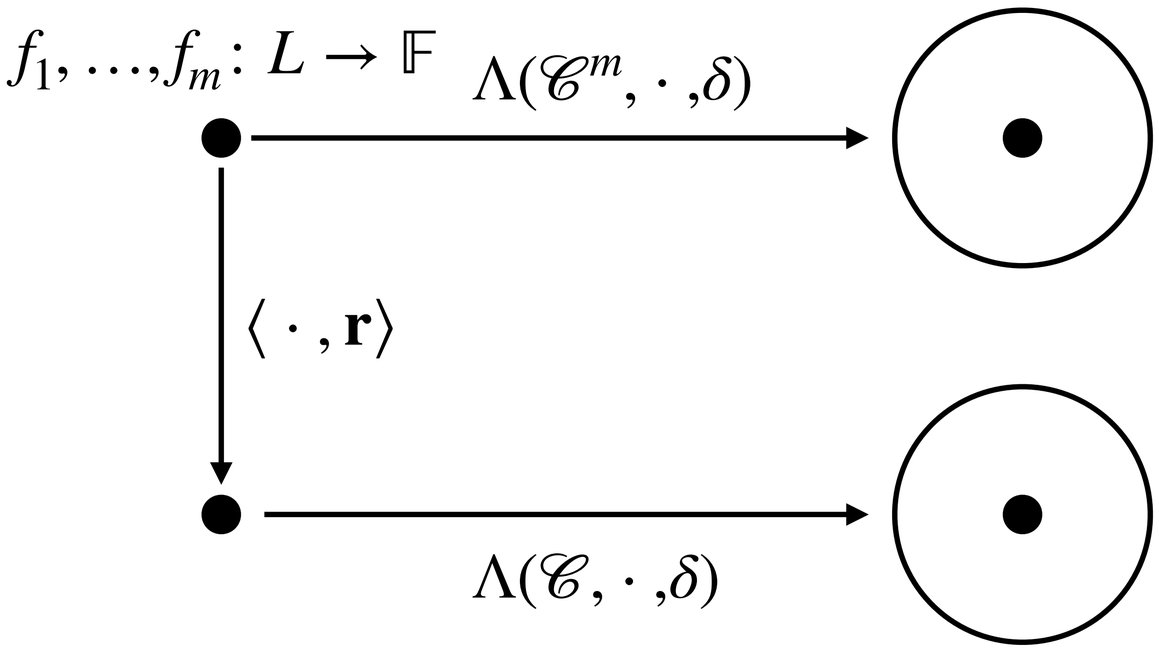

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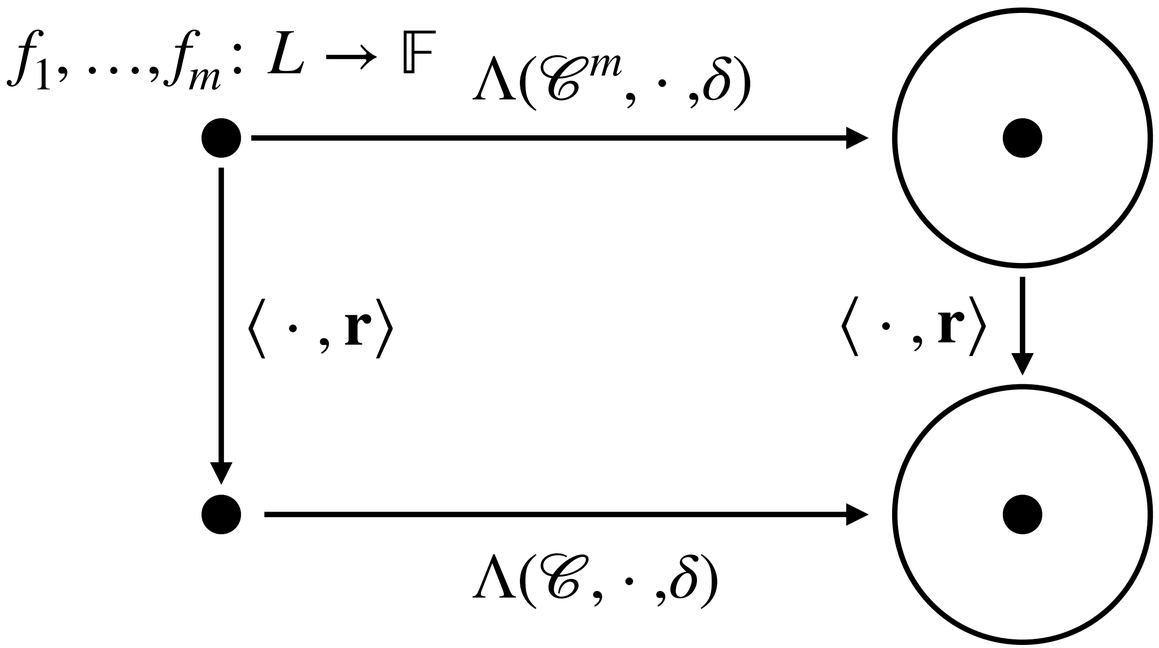

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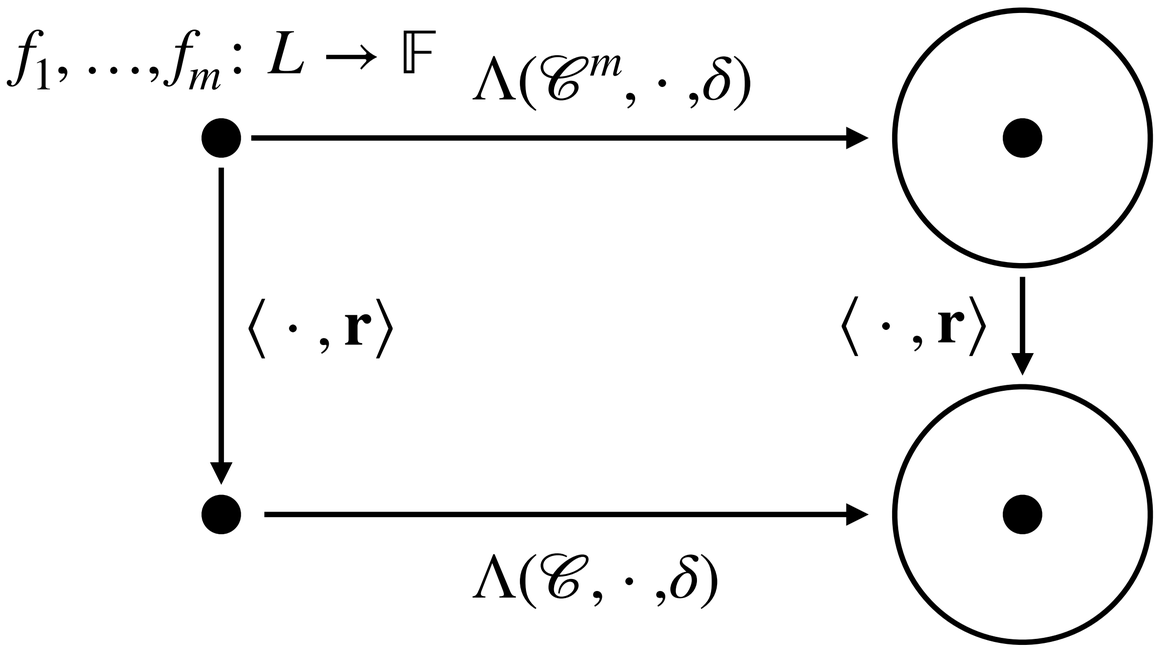

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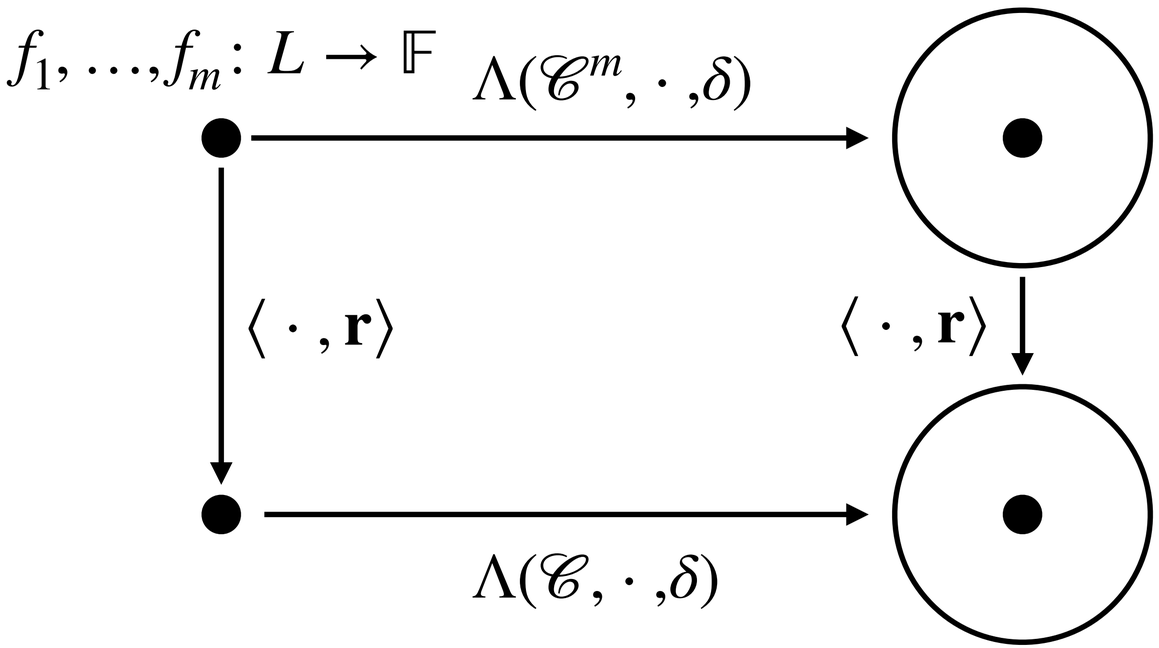

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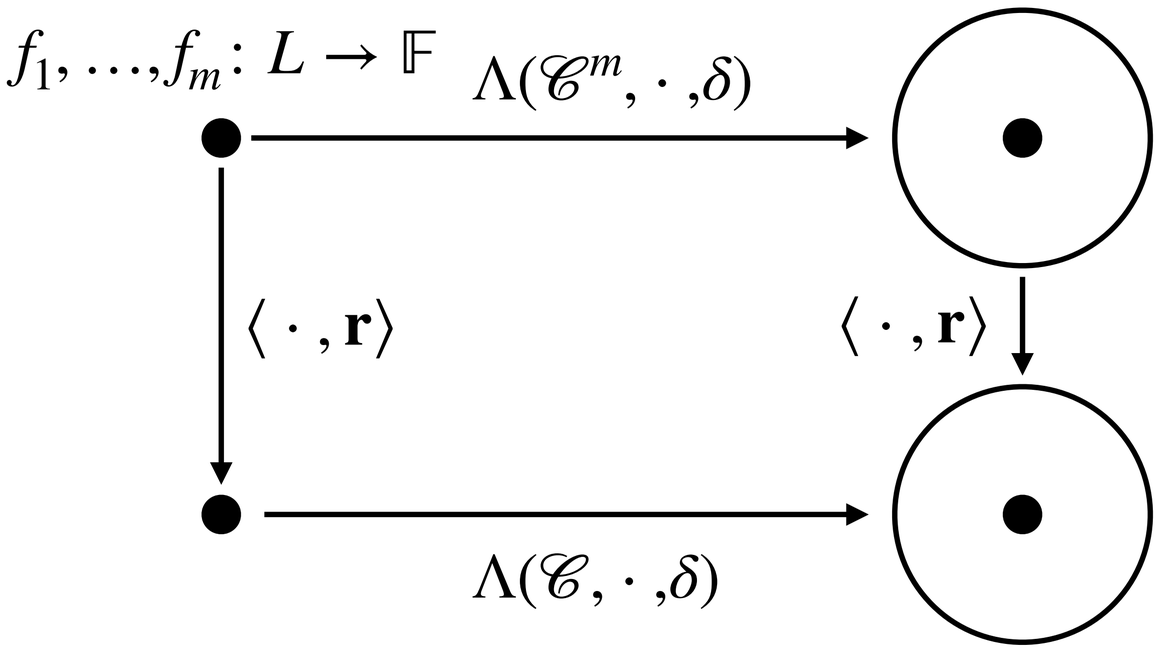

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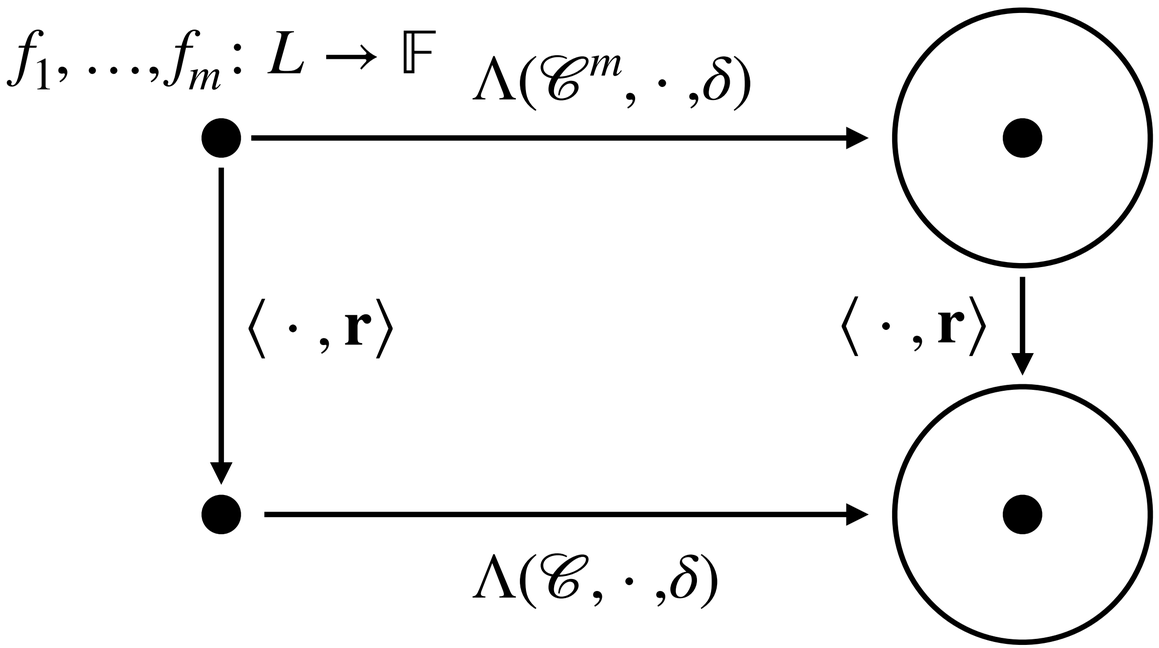

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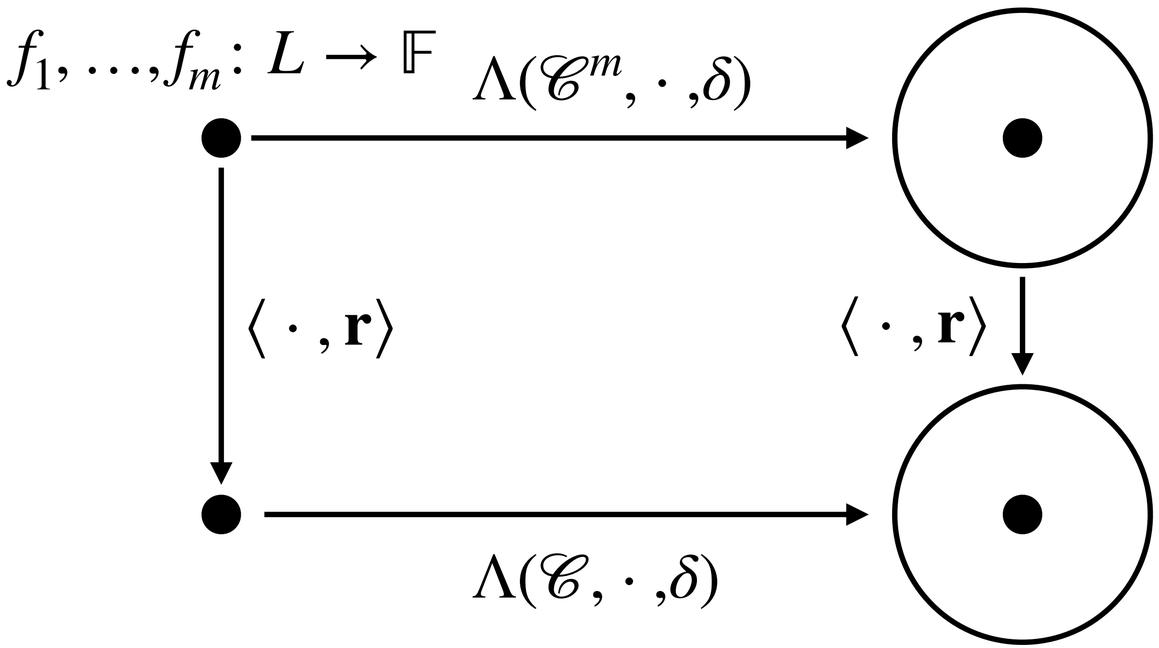

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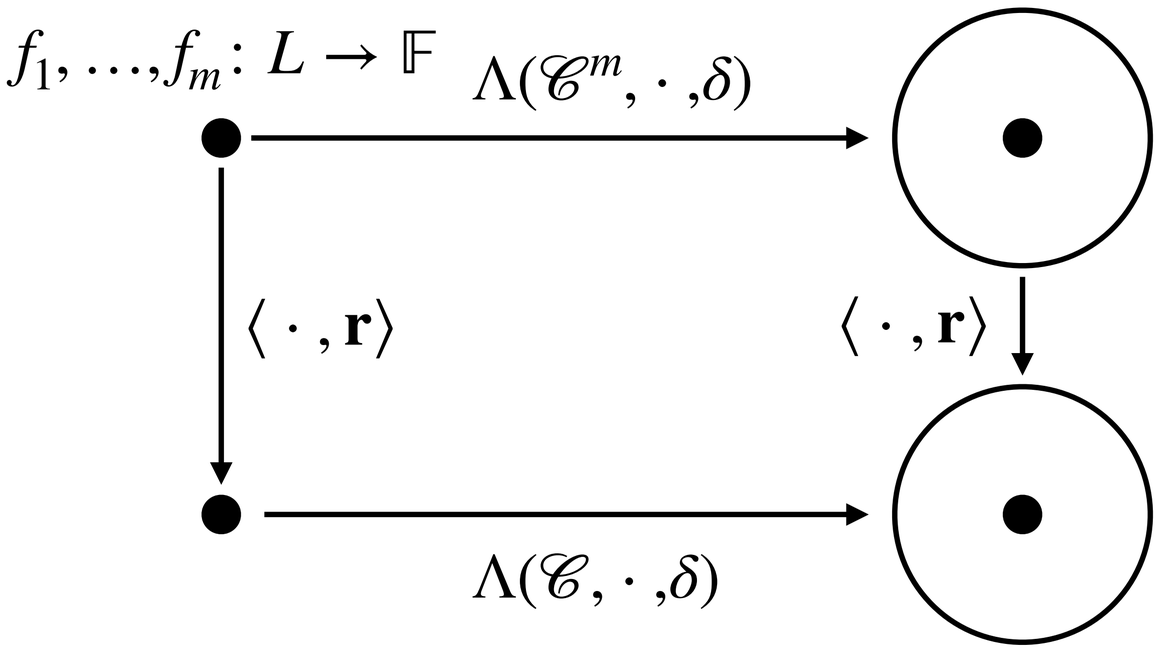

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Recent results show it holds up to 1.5 Johnson for general linear codes!