STIR 🥣

Reed-Solomon Proximity Testing with Fewer Queries

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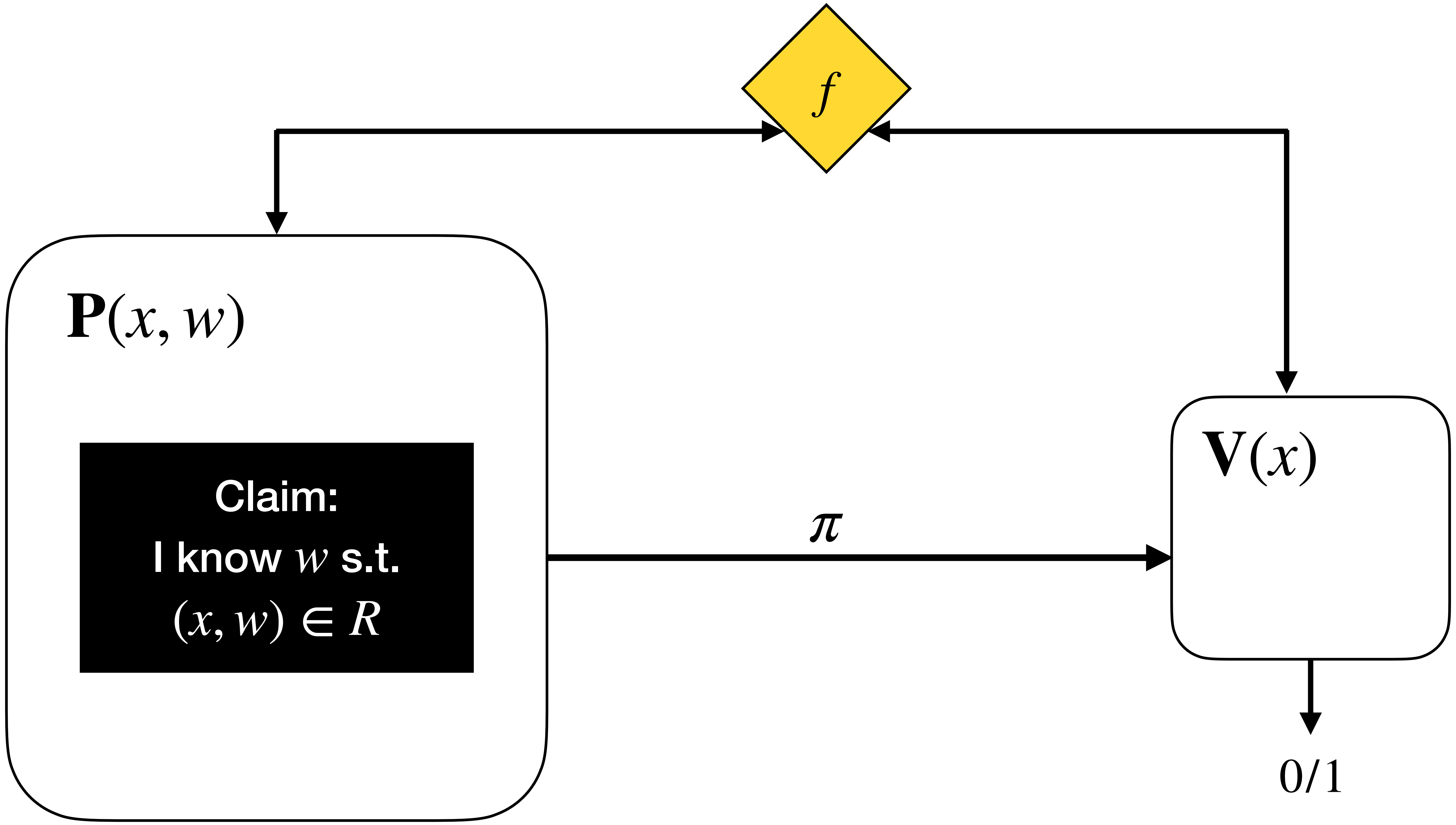
Eylon Yogev

SNARKs in the ROM



- Succinct
  - $|\pi| \ll |w|$
- Non-interactive
- Argument of Knowledge
  - Straightline extractor

SNARKs in Practice

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- ROM instantiated using a cryptographic hash function

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Rollups: STARKWARE polygon zkSync

zkVMs: Succinct RISC ZERO **And more…**

BCS Transformation

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IOP

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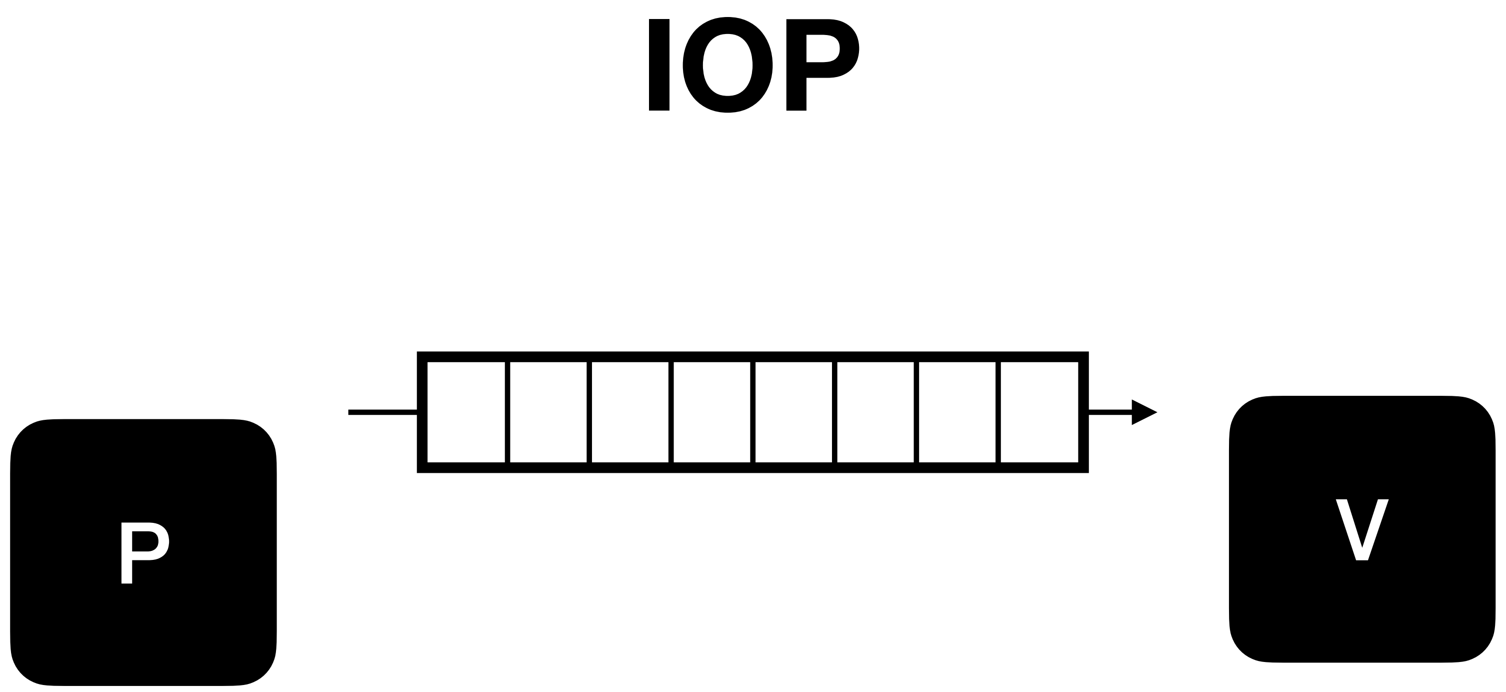
IOP

P

V

BCS Transformation



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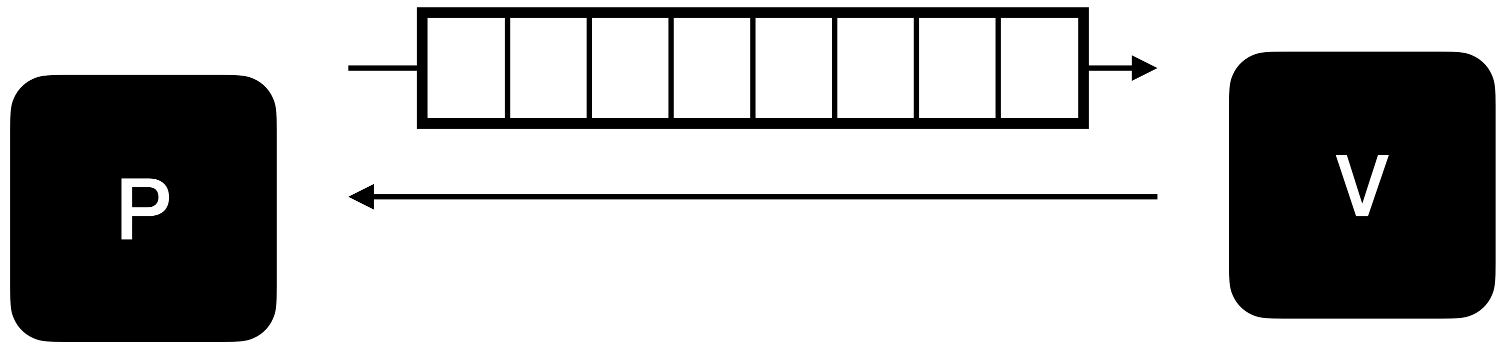

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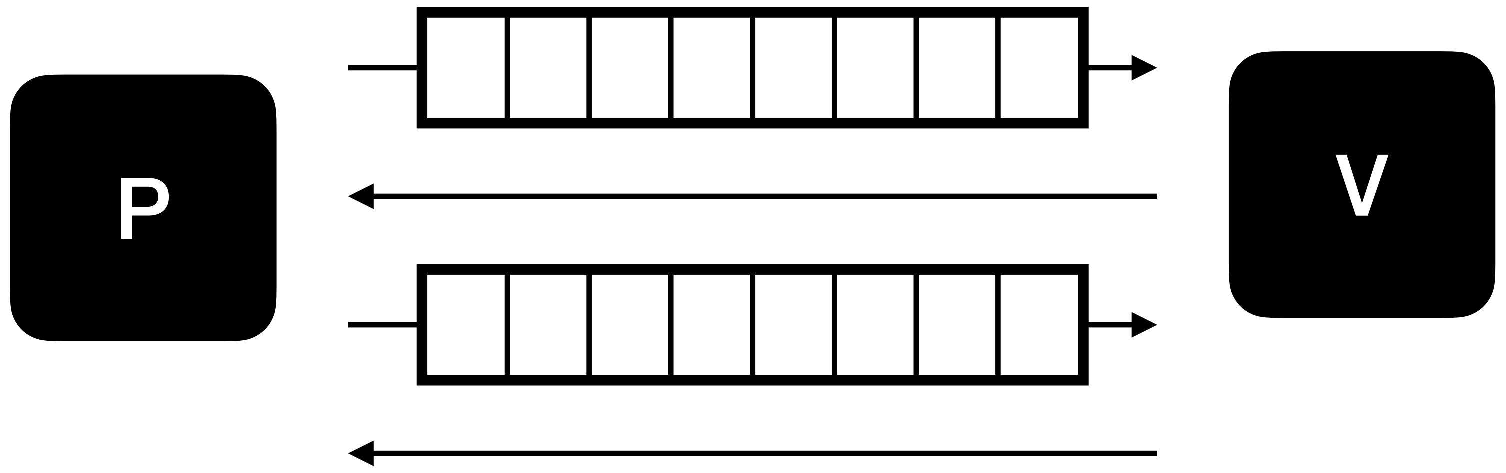

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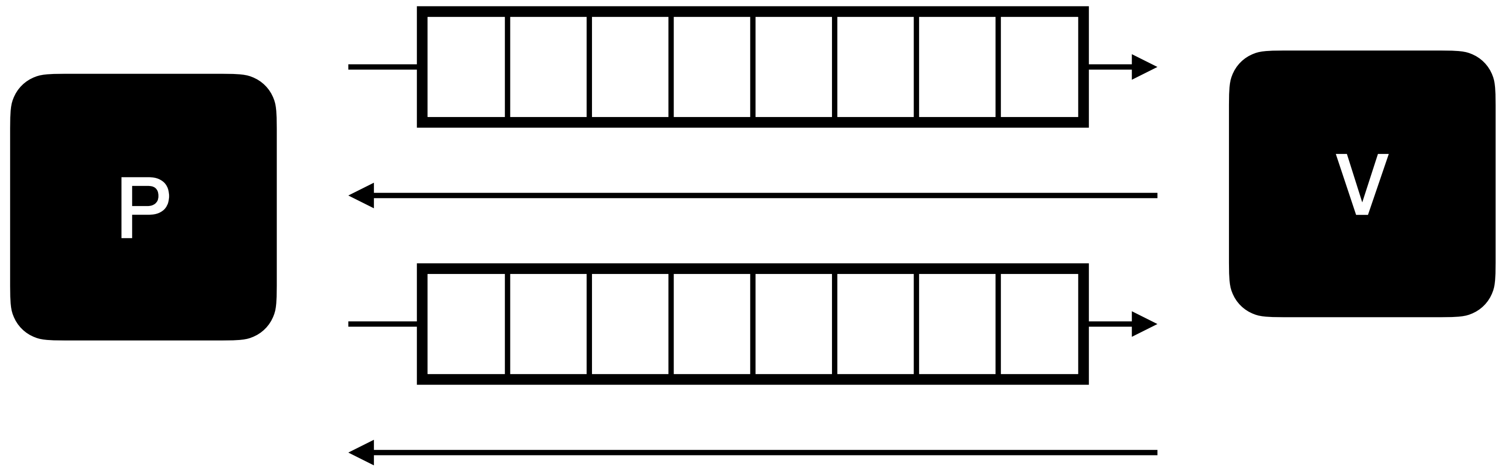

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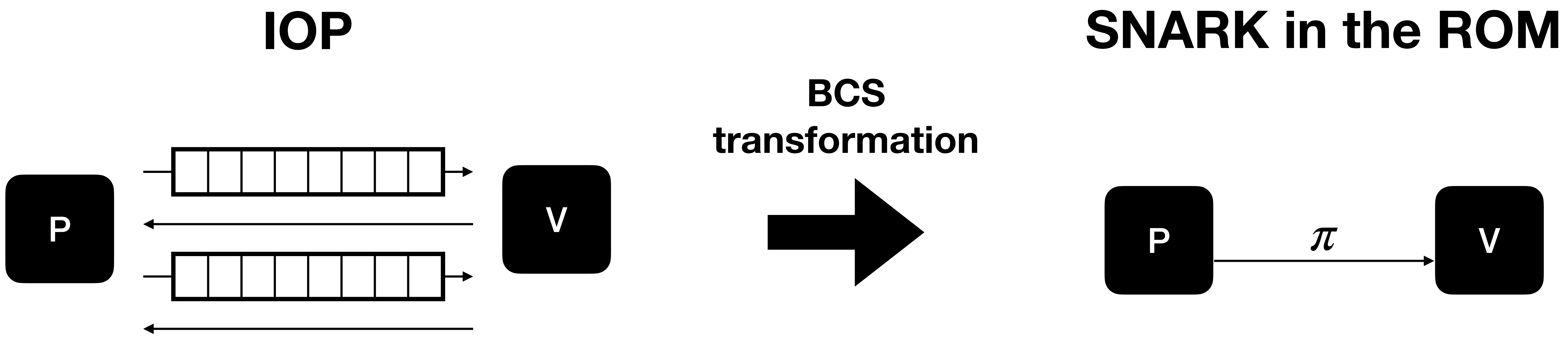

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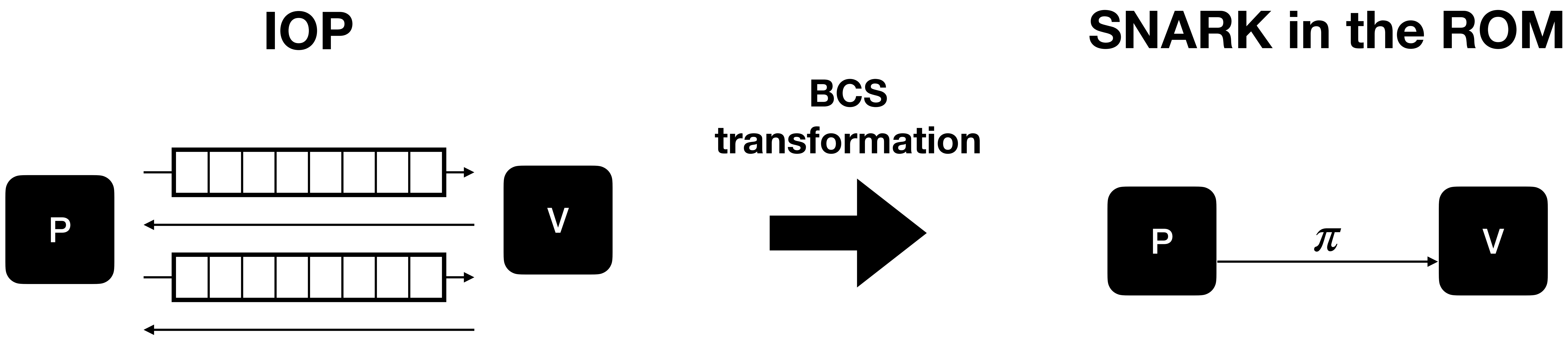

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Queries: q

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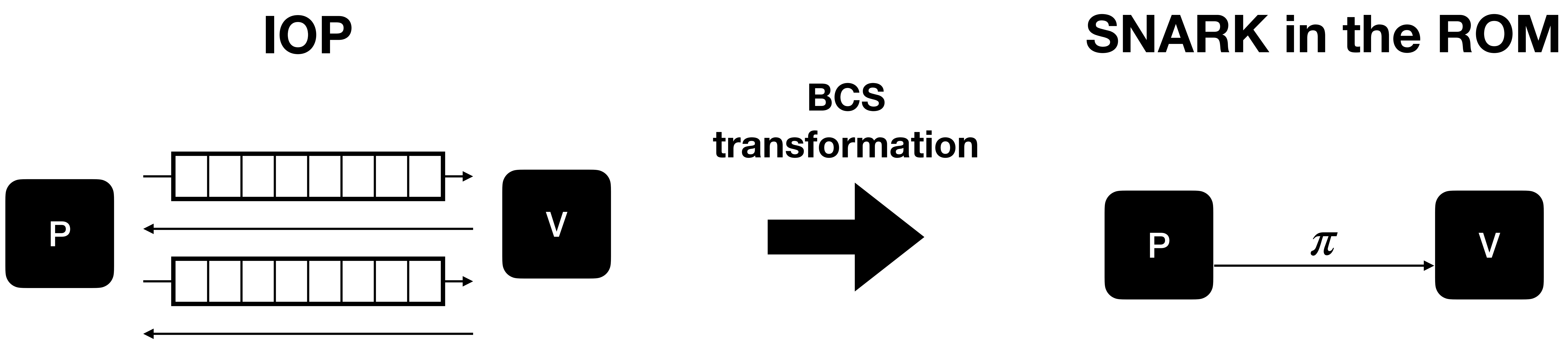
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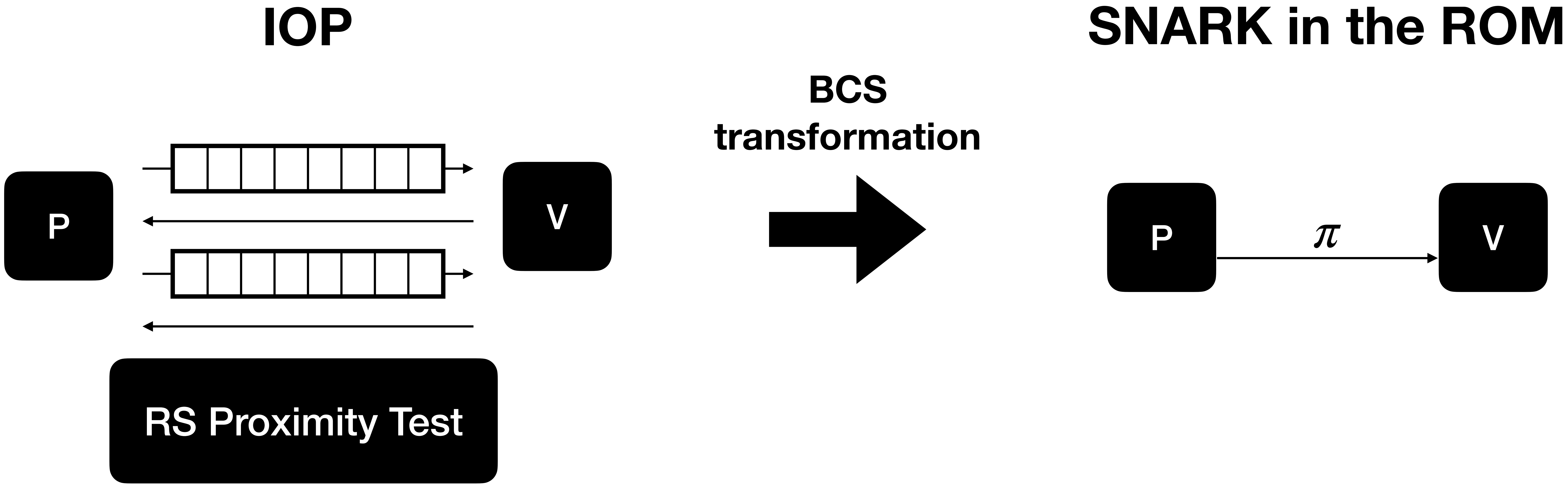

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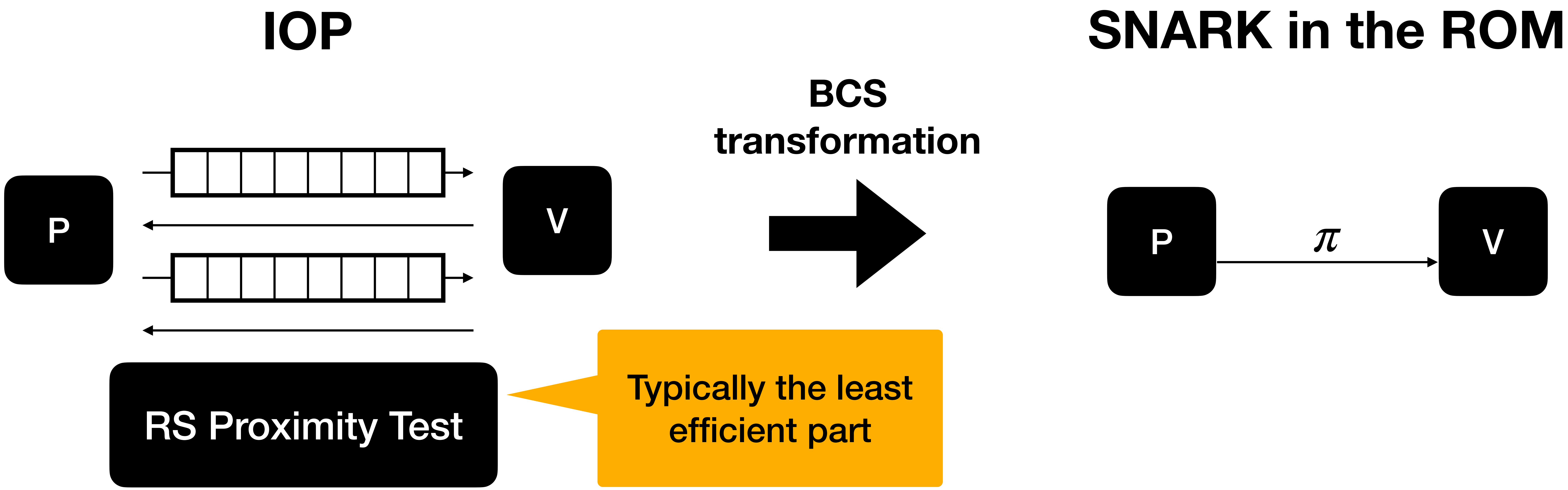

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Field

RS Codes and IOPPs

$$\mathrm{RS}[\underset{\substack{\nearrow\\ \text{Field}}}{\mathbb{F}}, \underset{\substack{\uparrow\\ \text{Evaluation}\\ \text{Domain}}}{L}, d]$$

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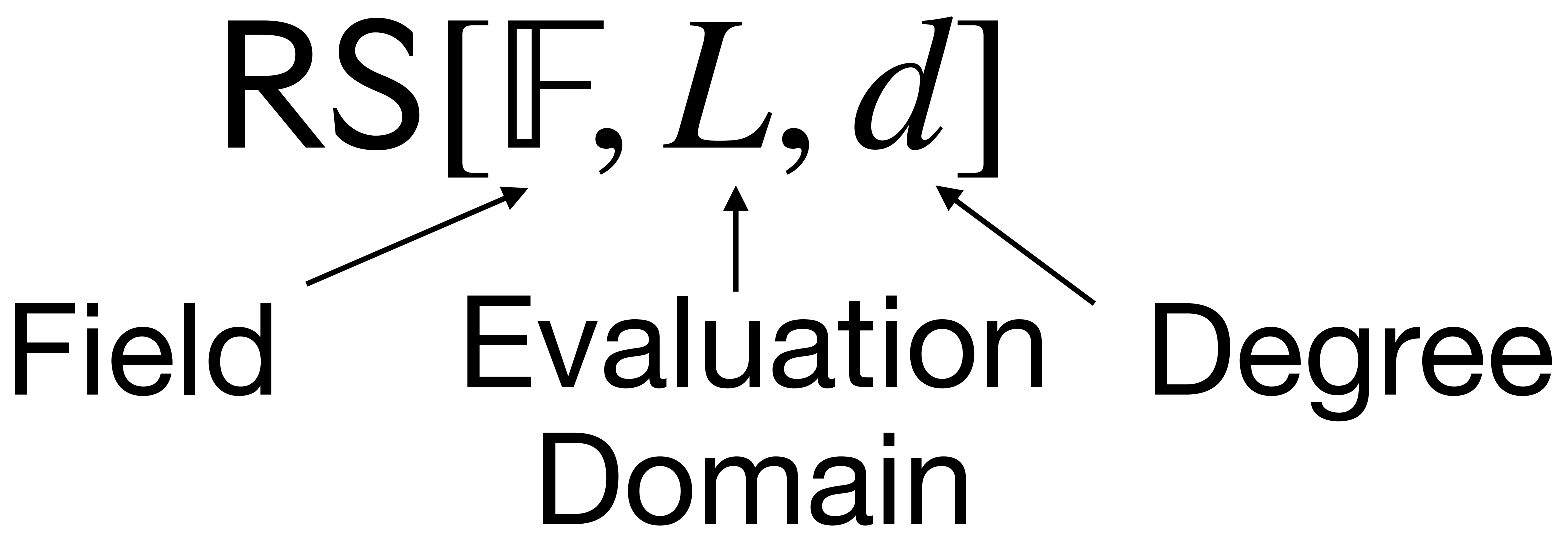

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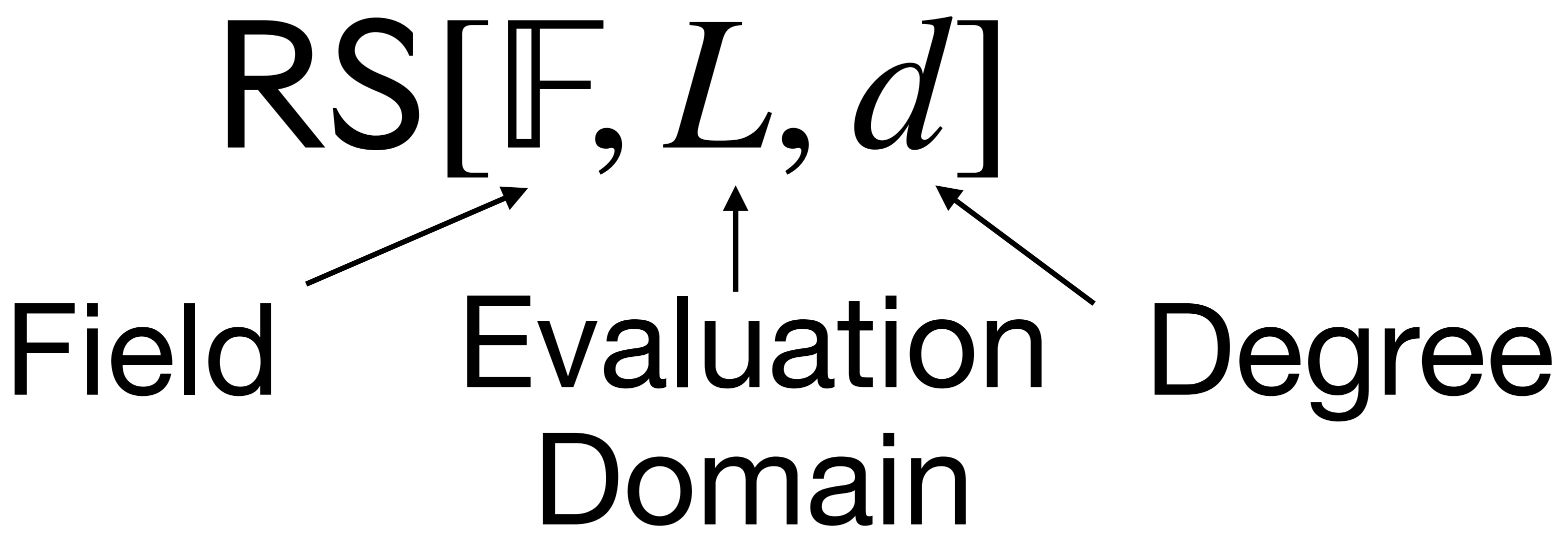

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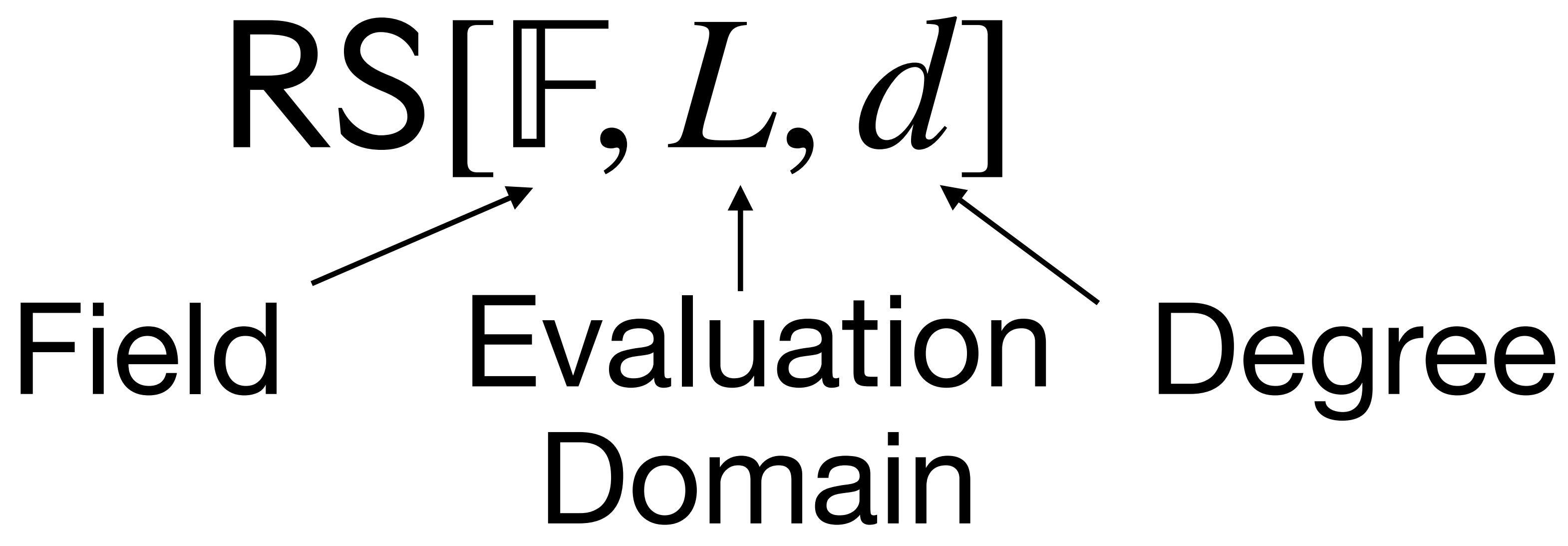

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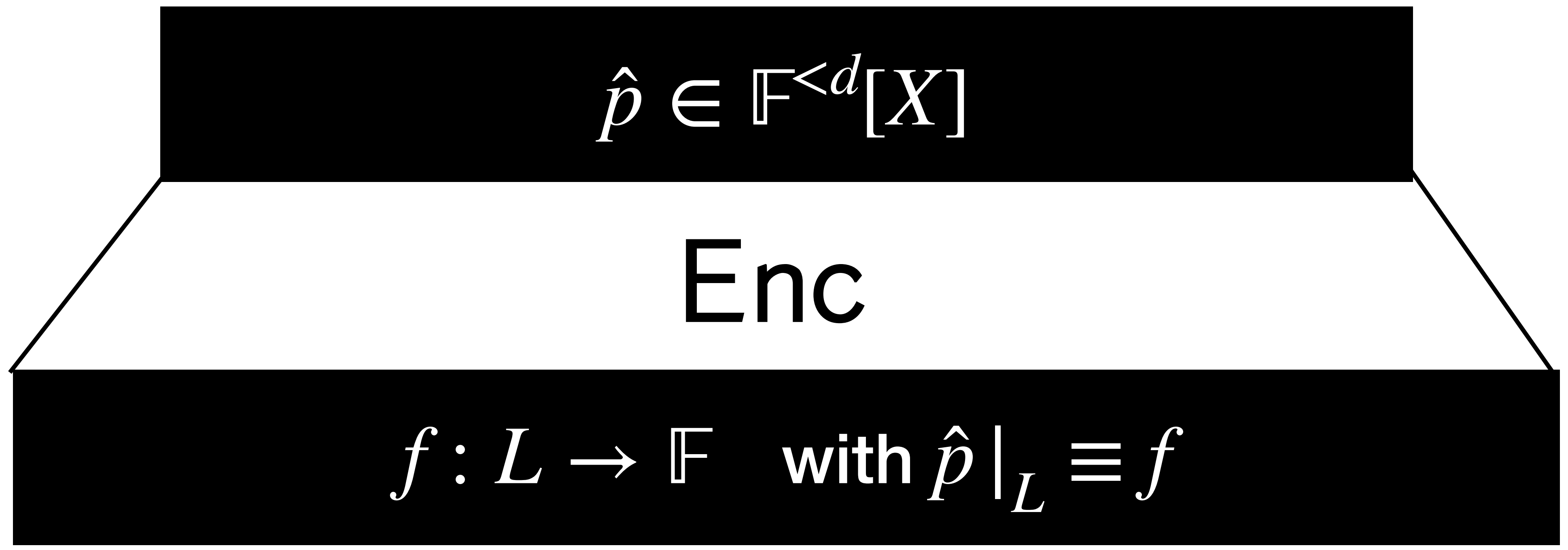

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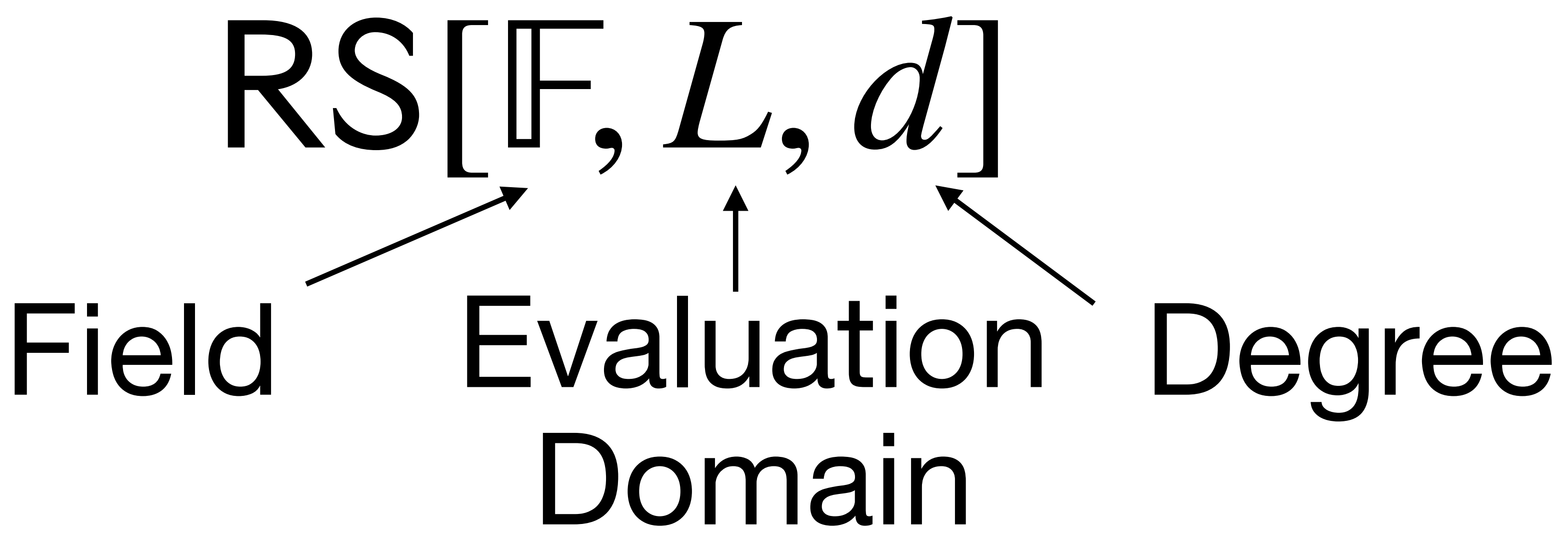

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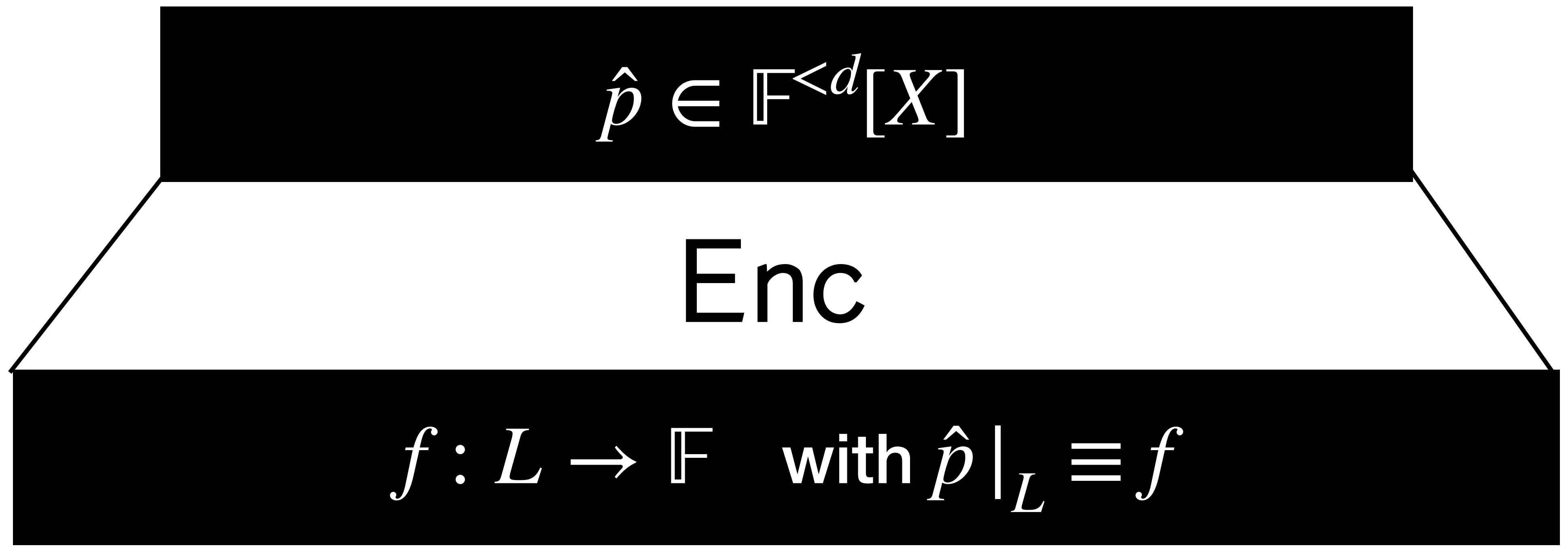

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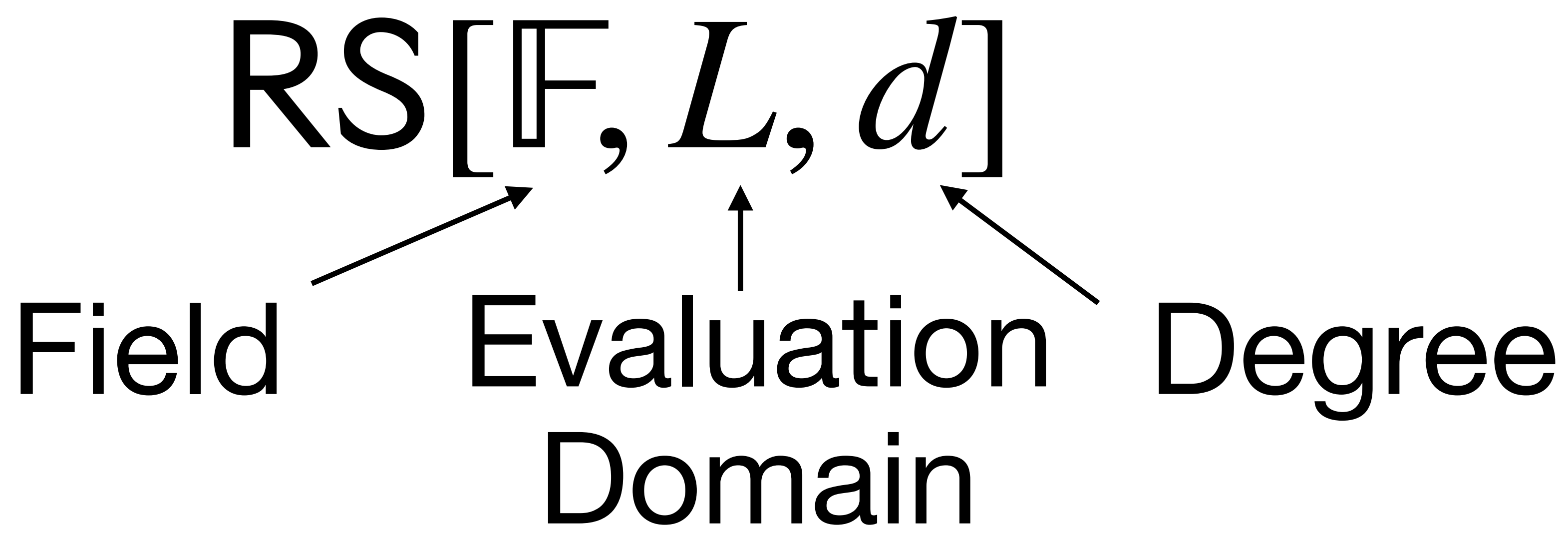

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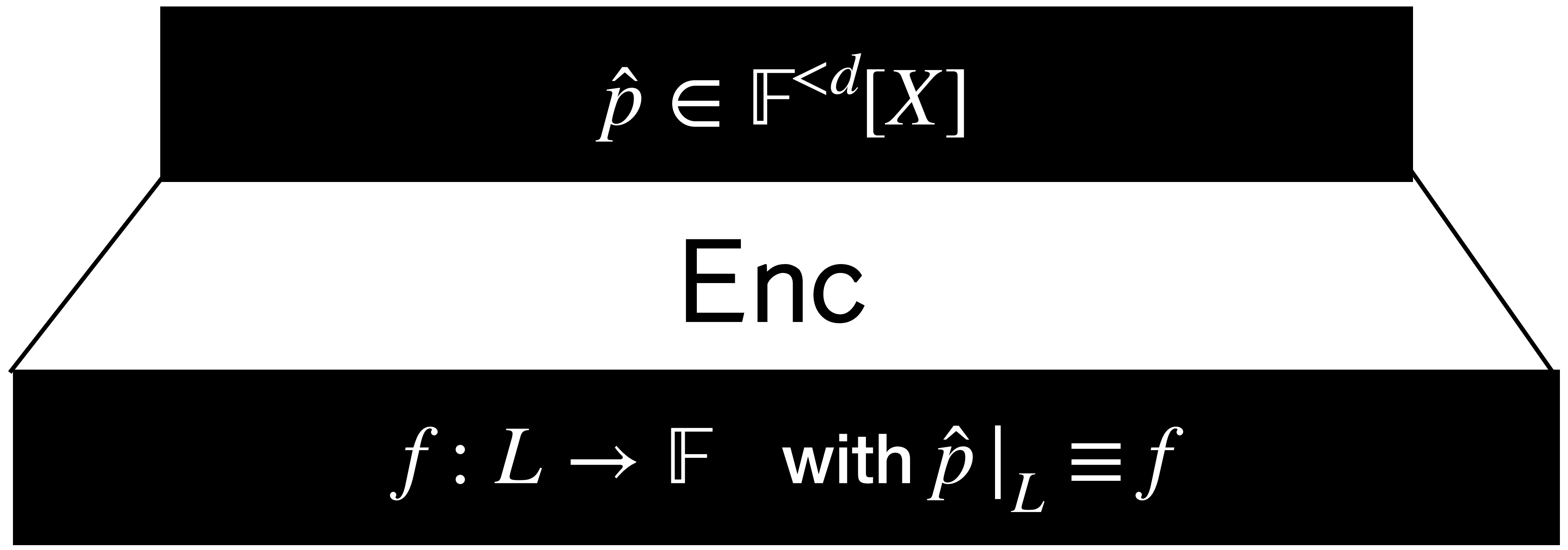

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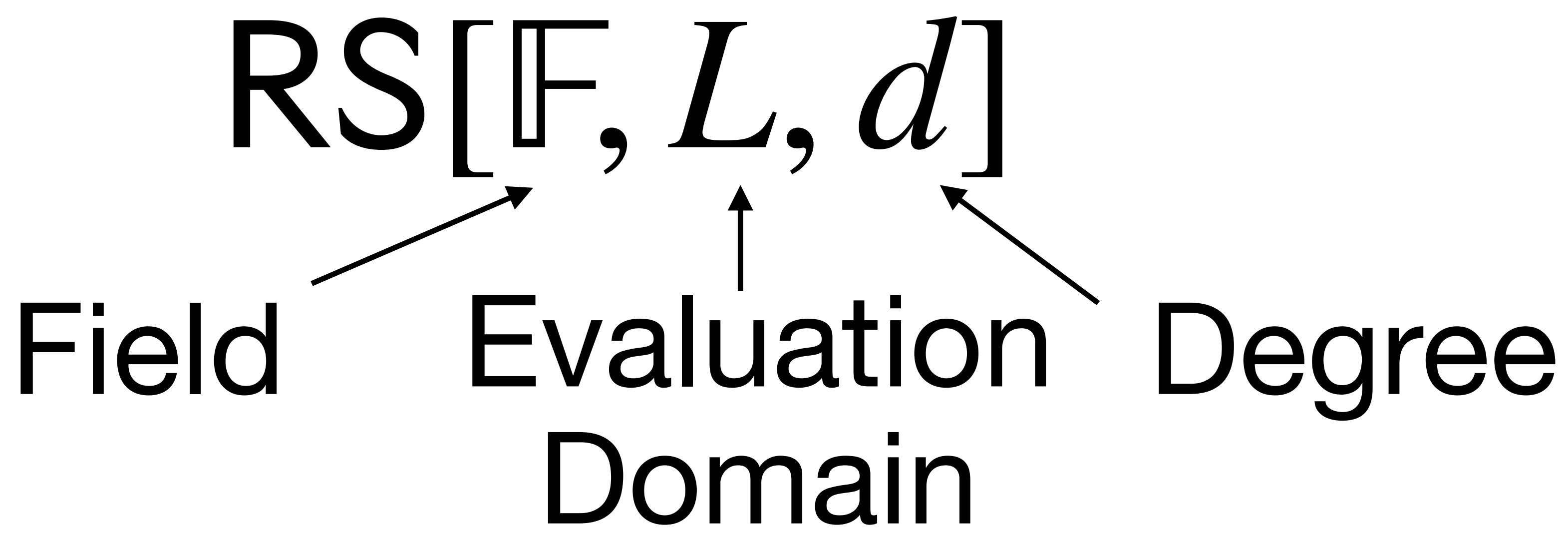

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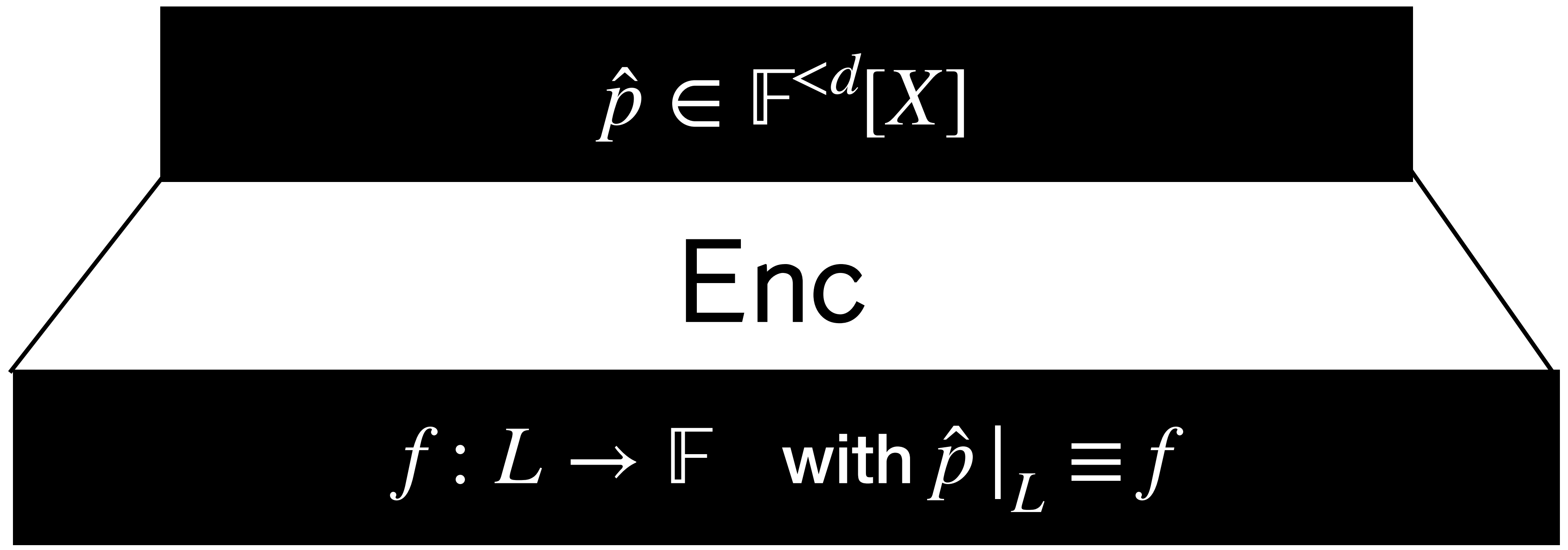

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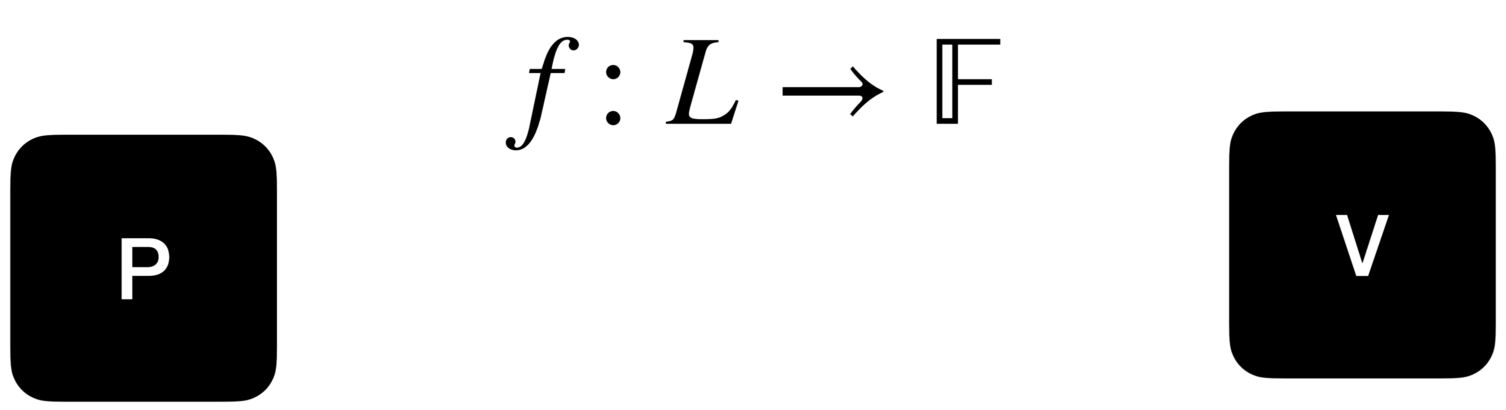

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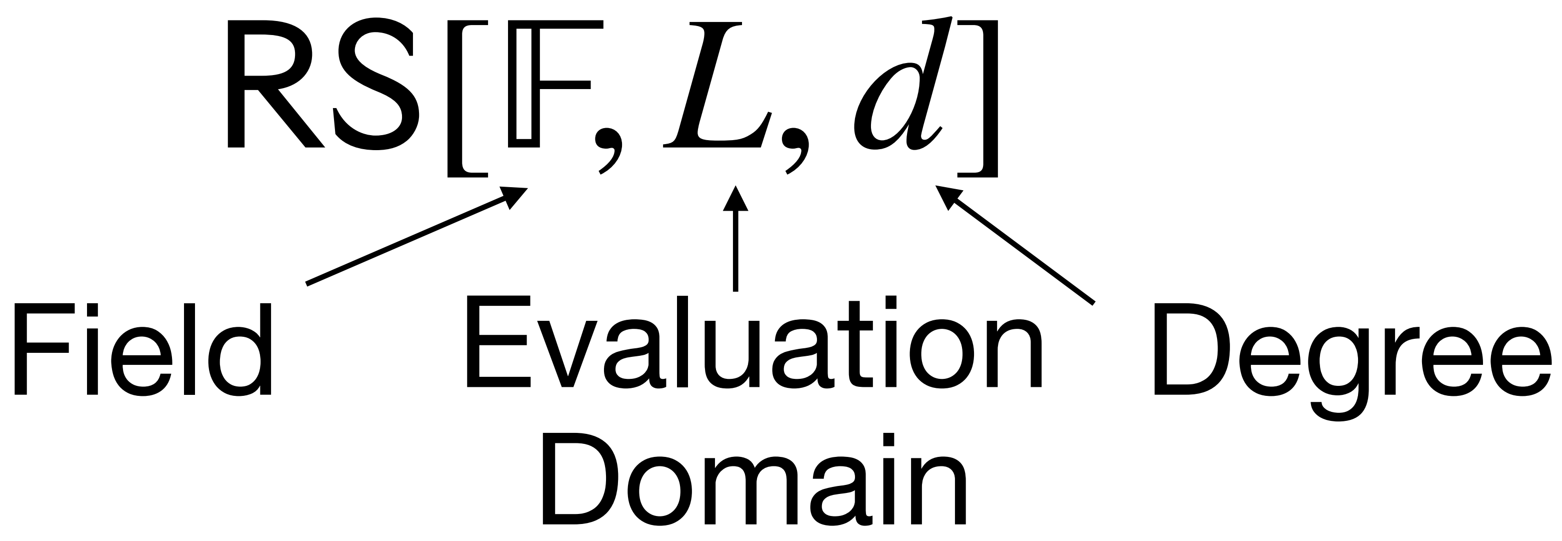

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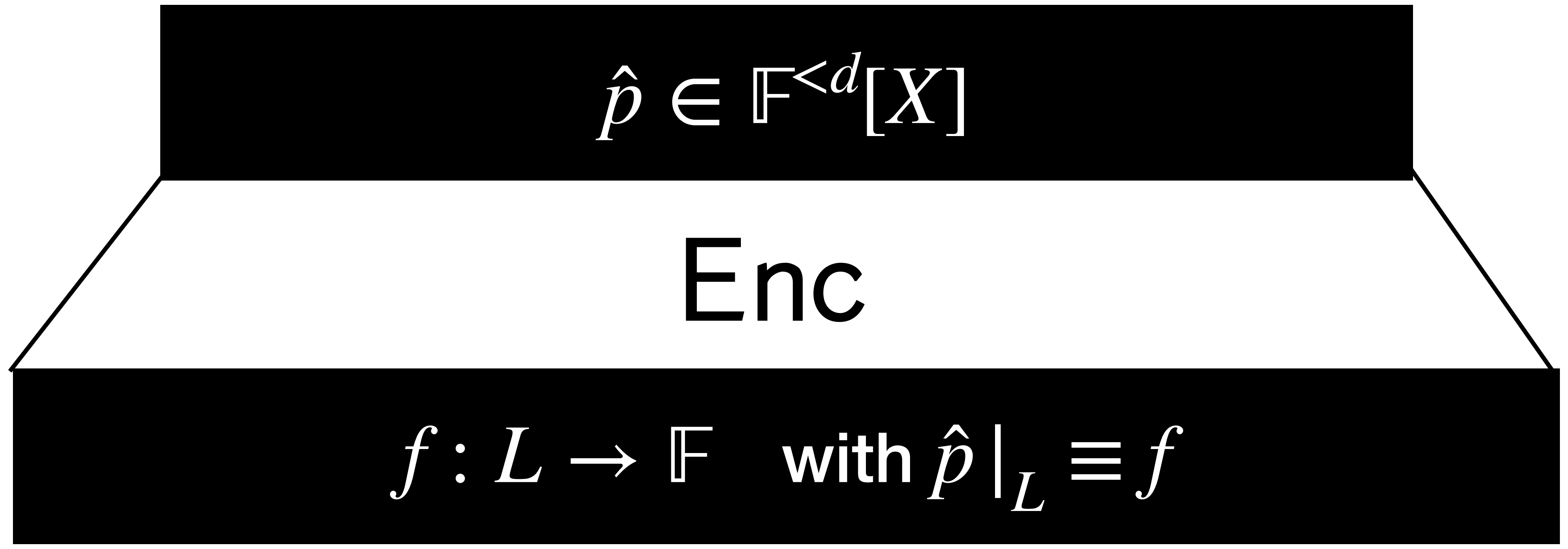

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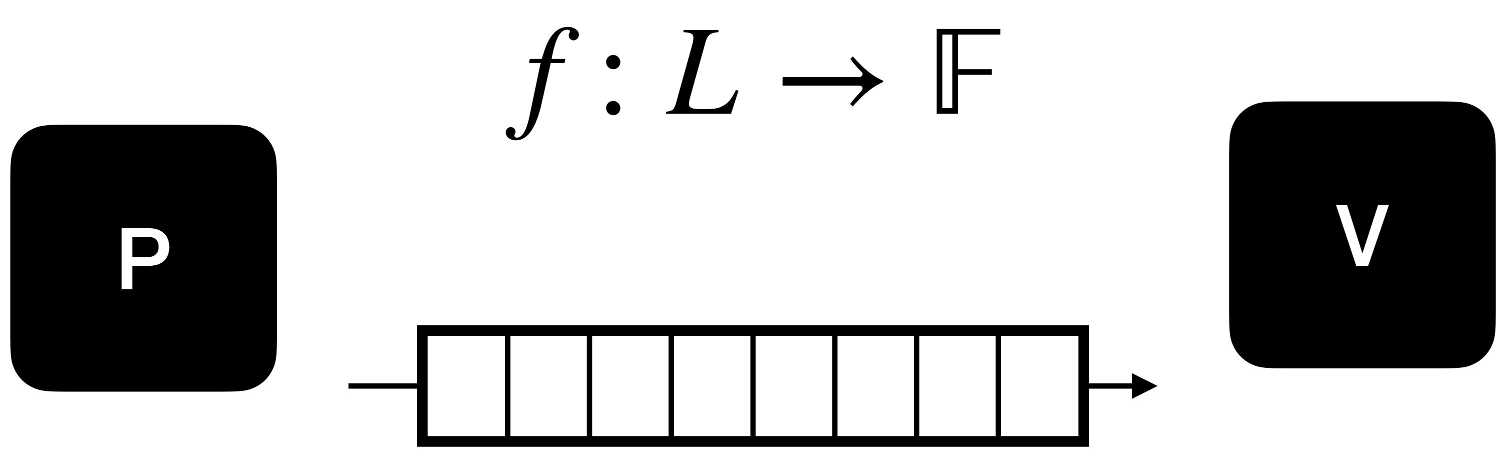

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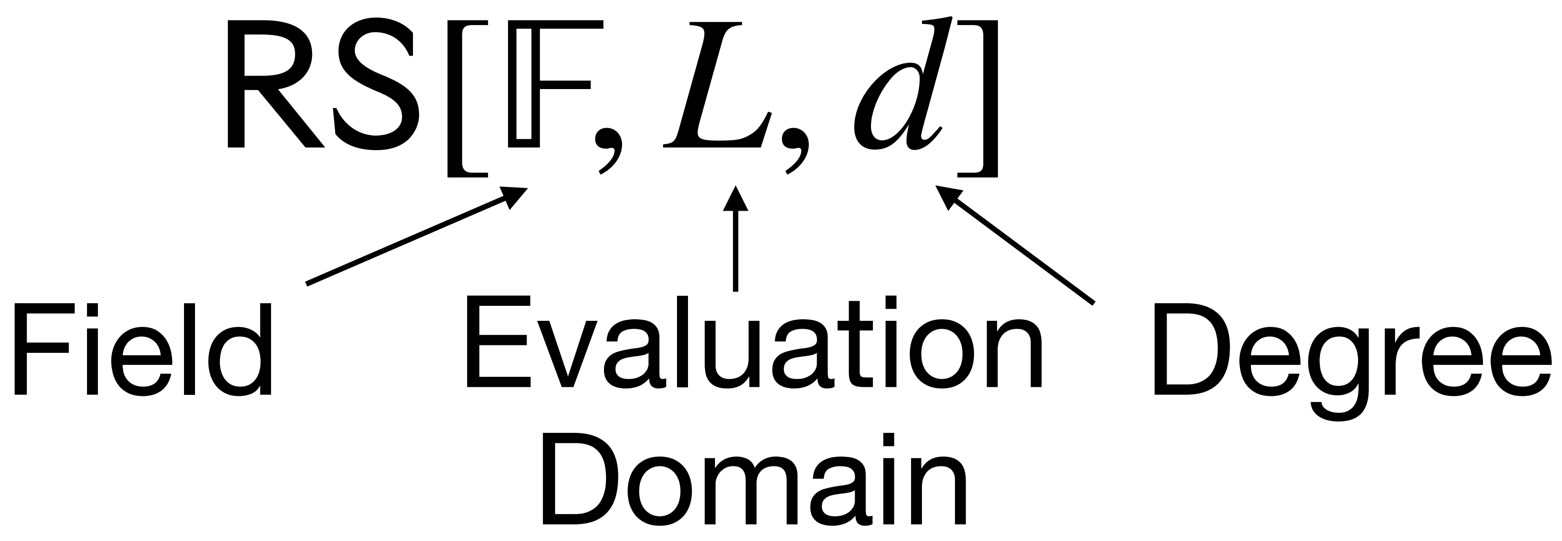

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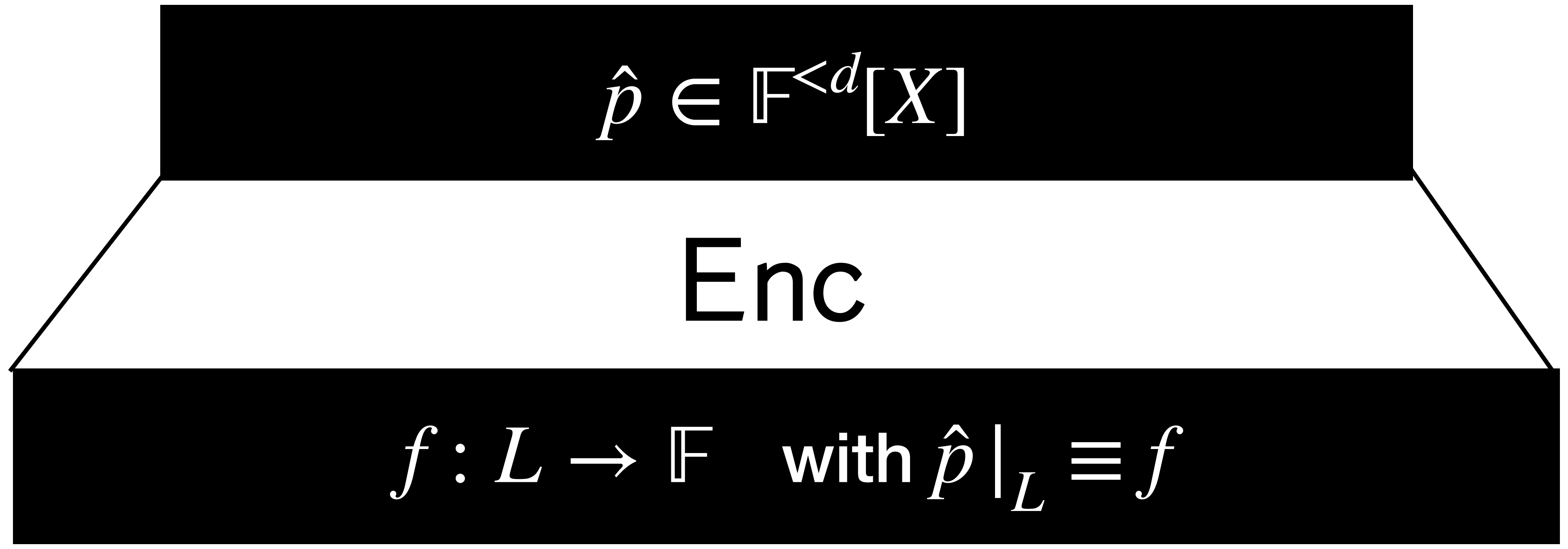

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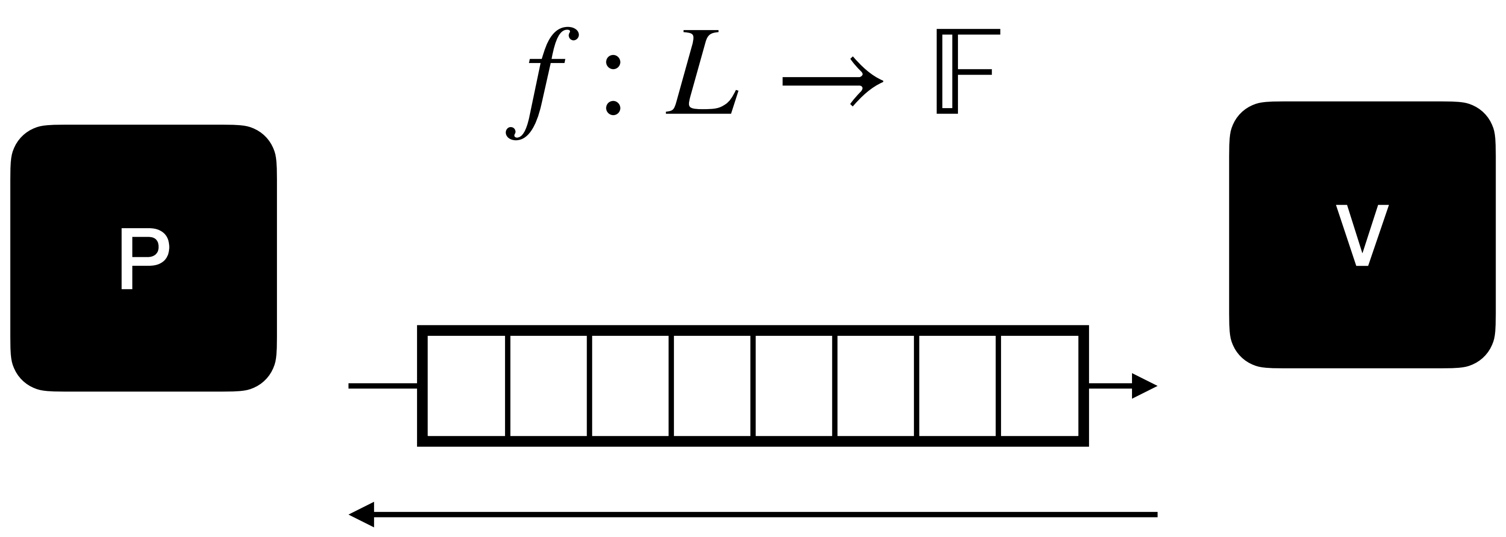

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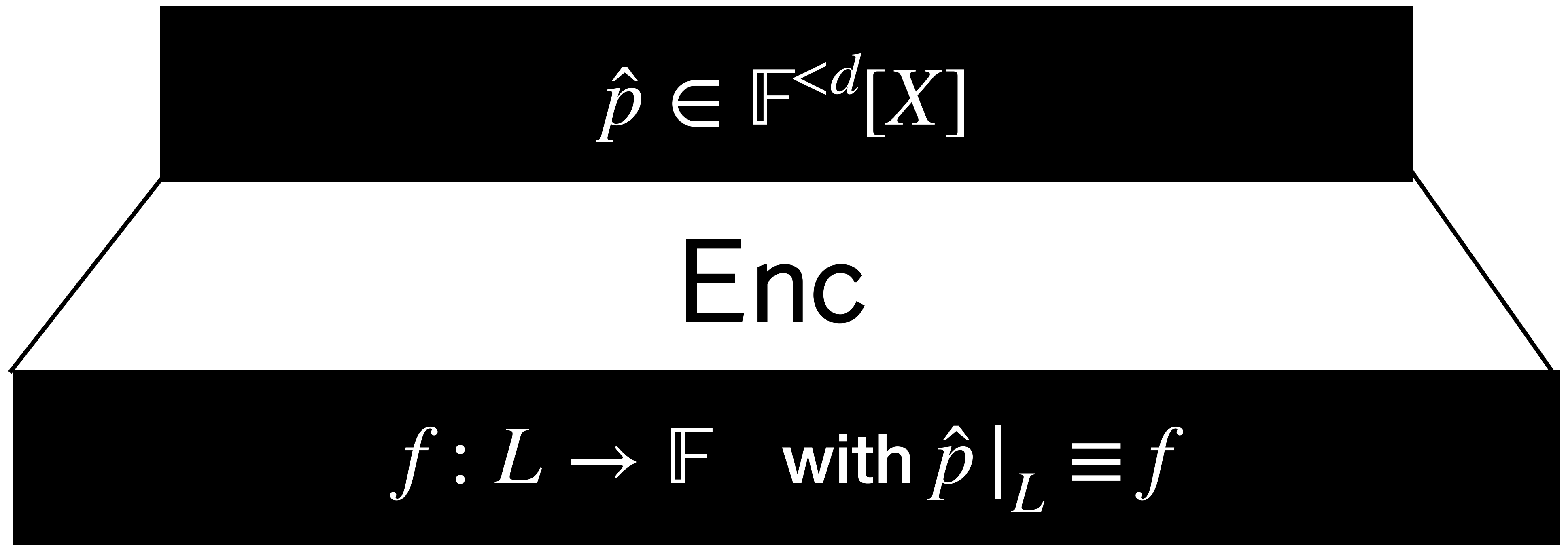

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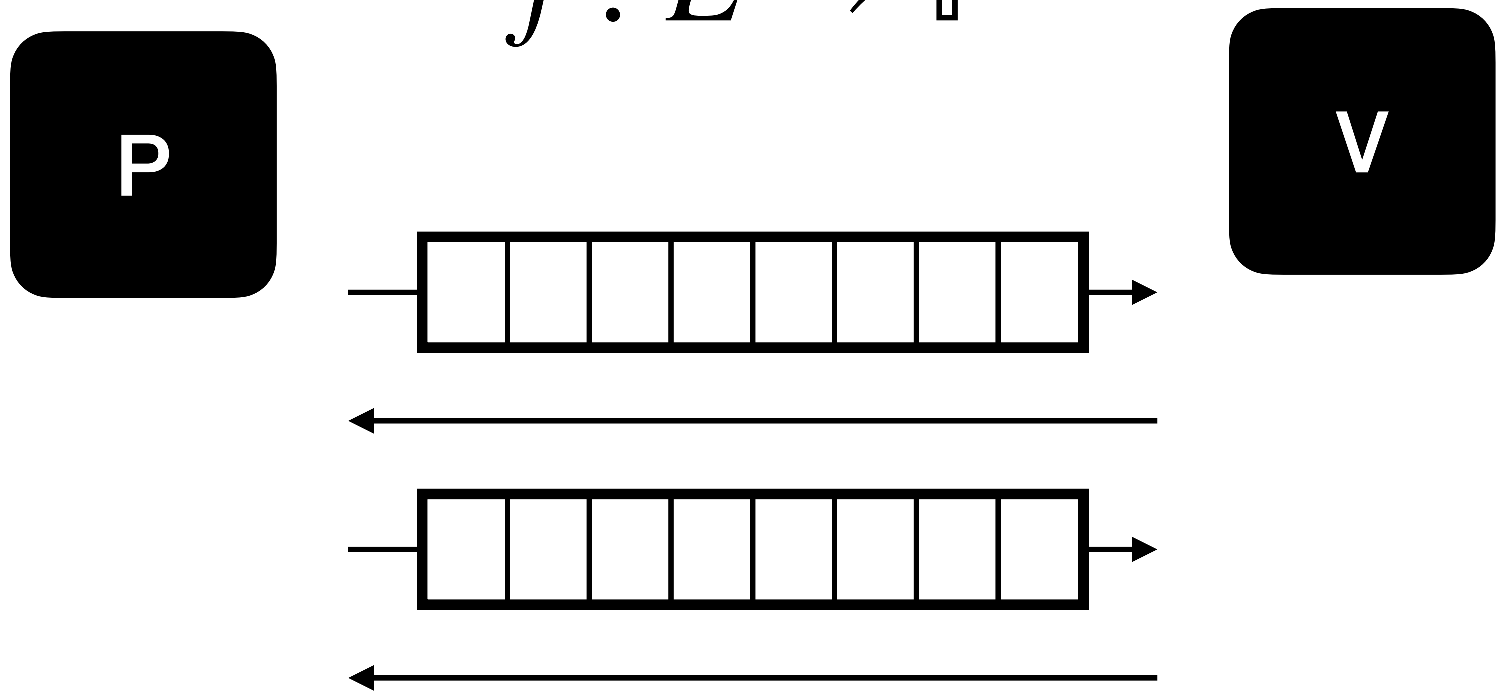

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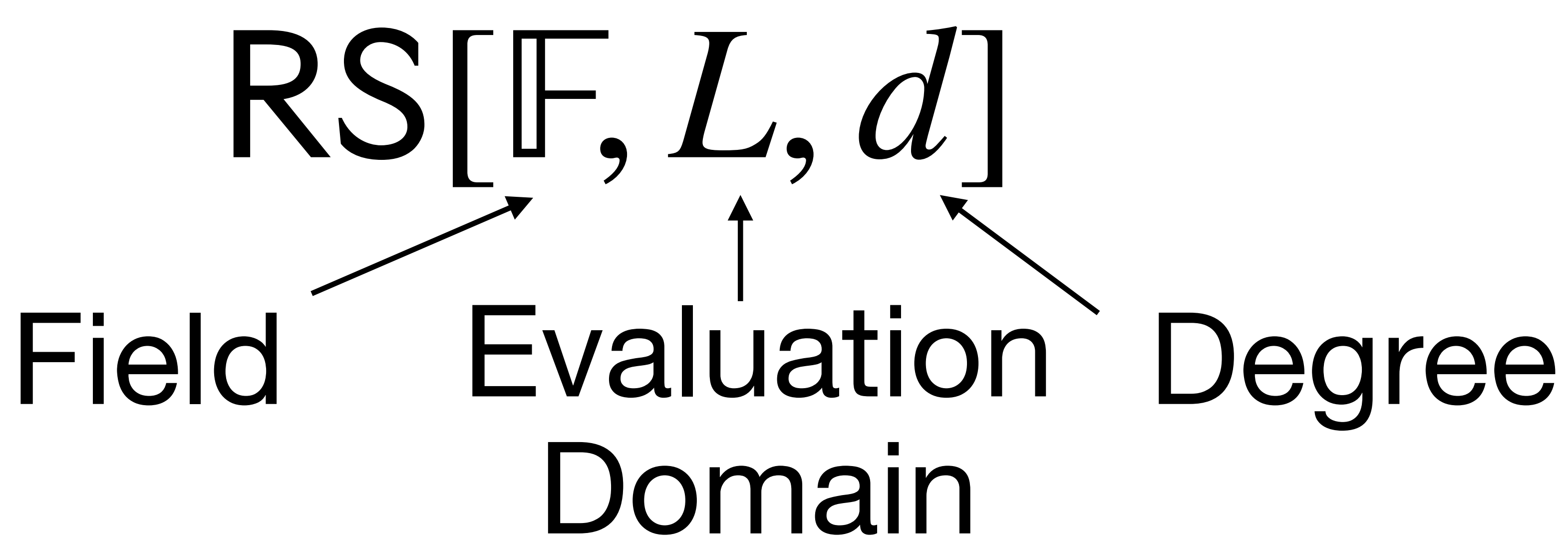

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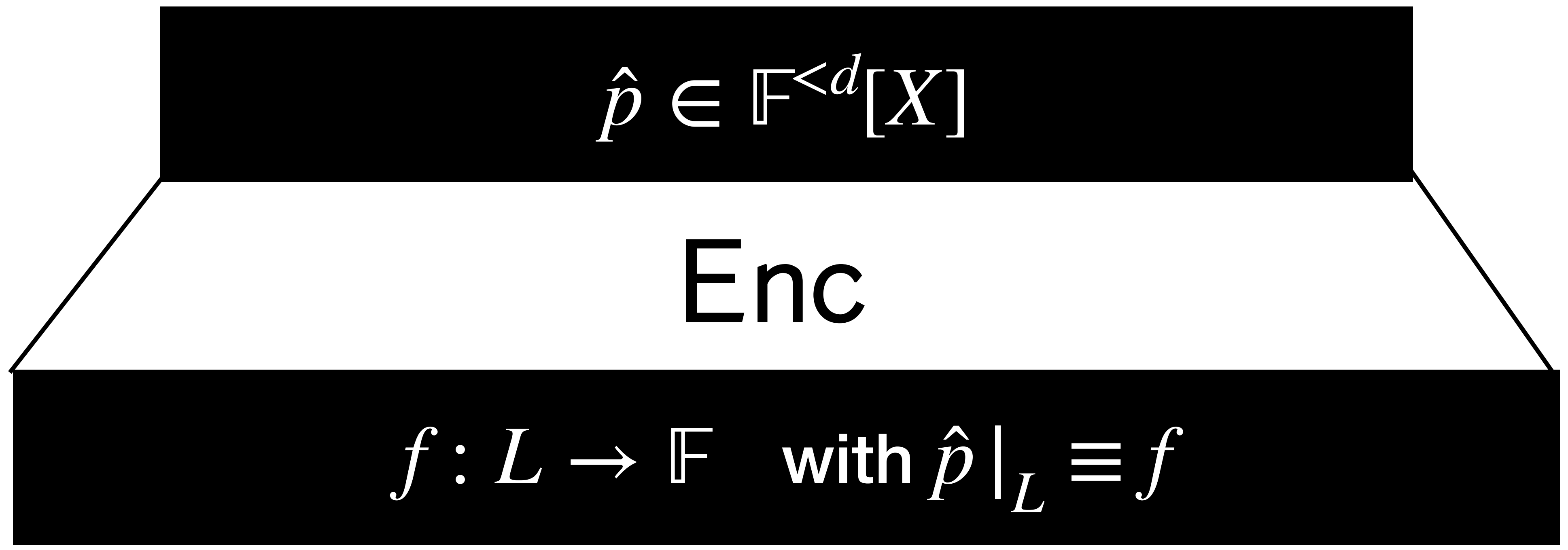

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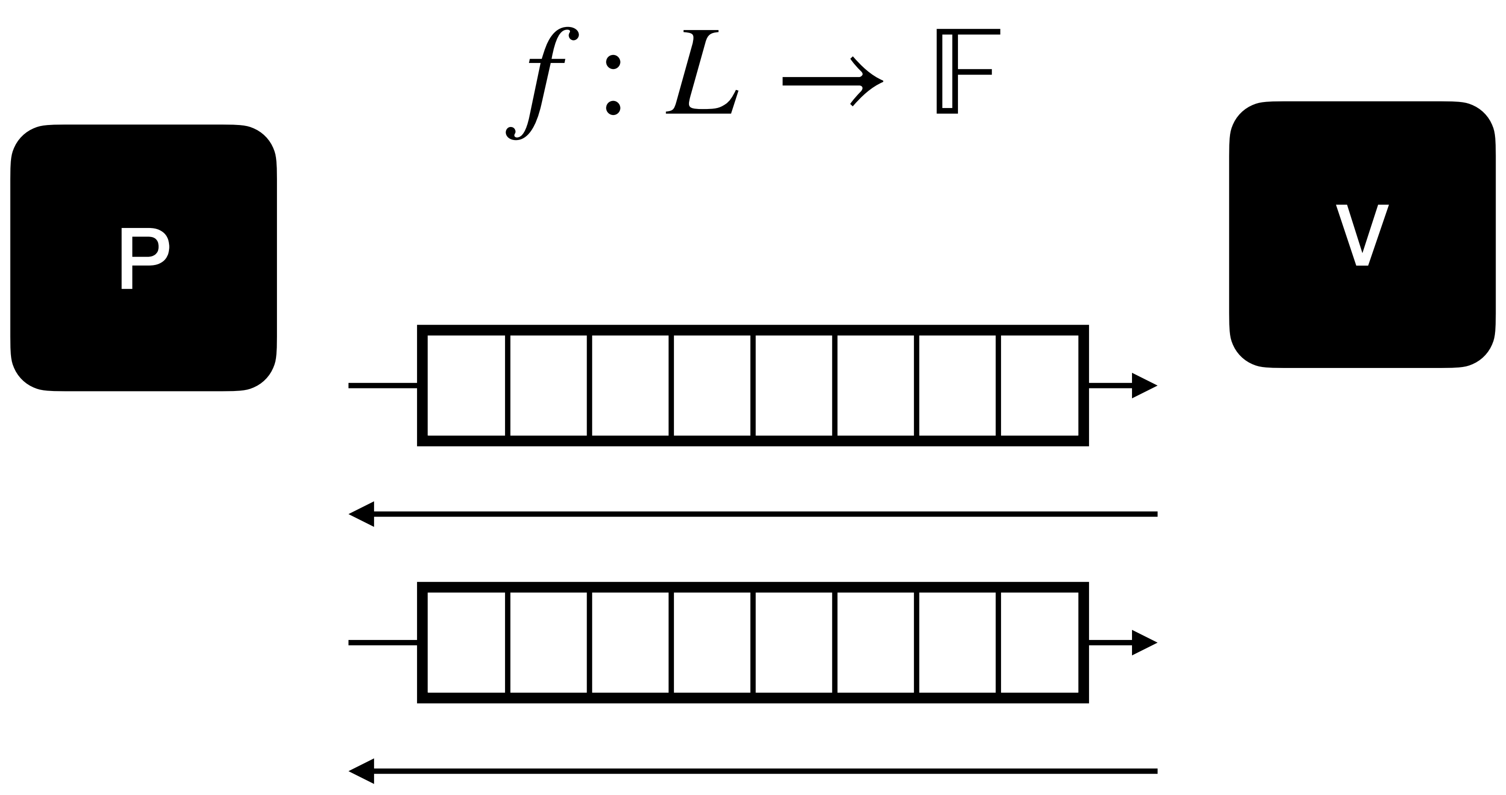

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V

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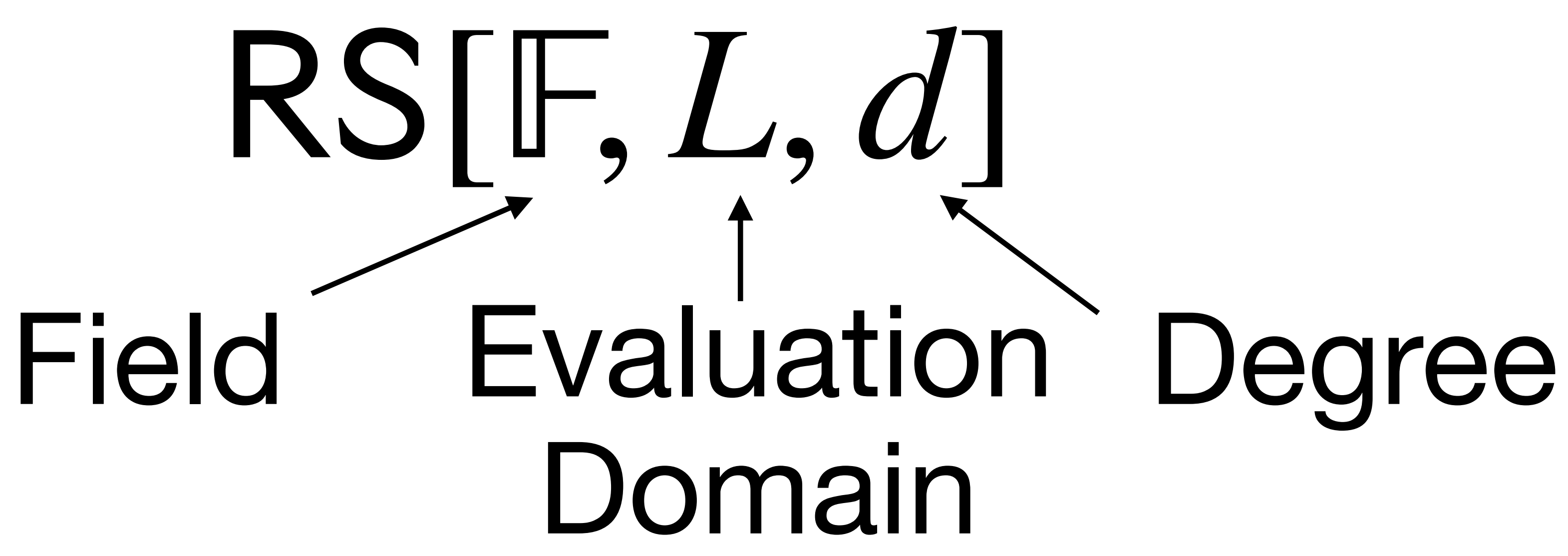

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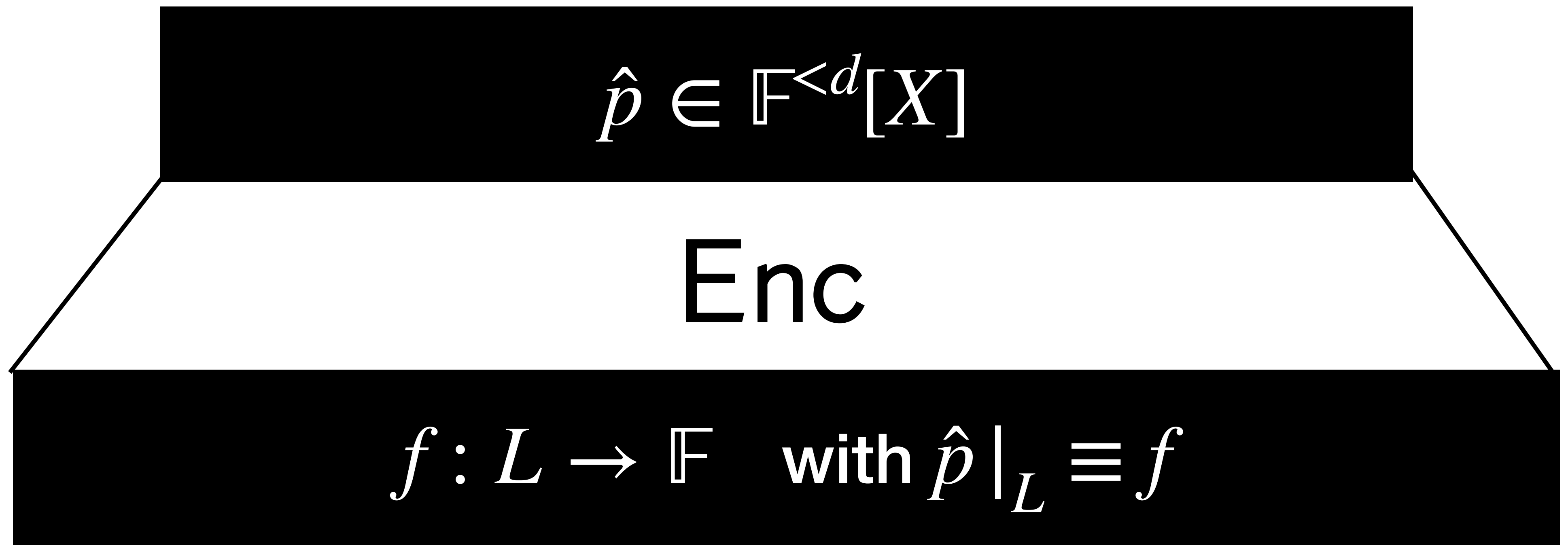

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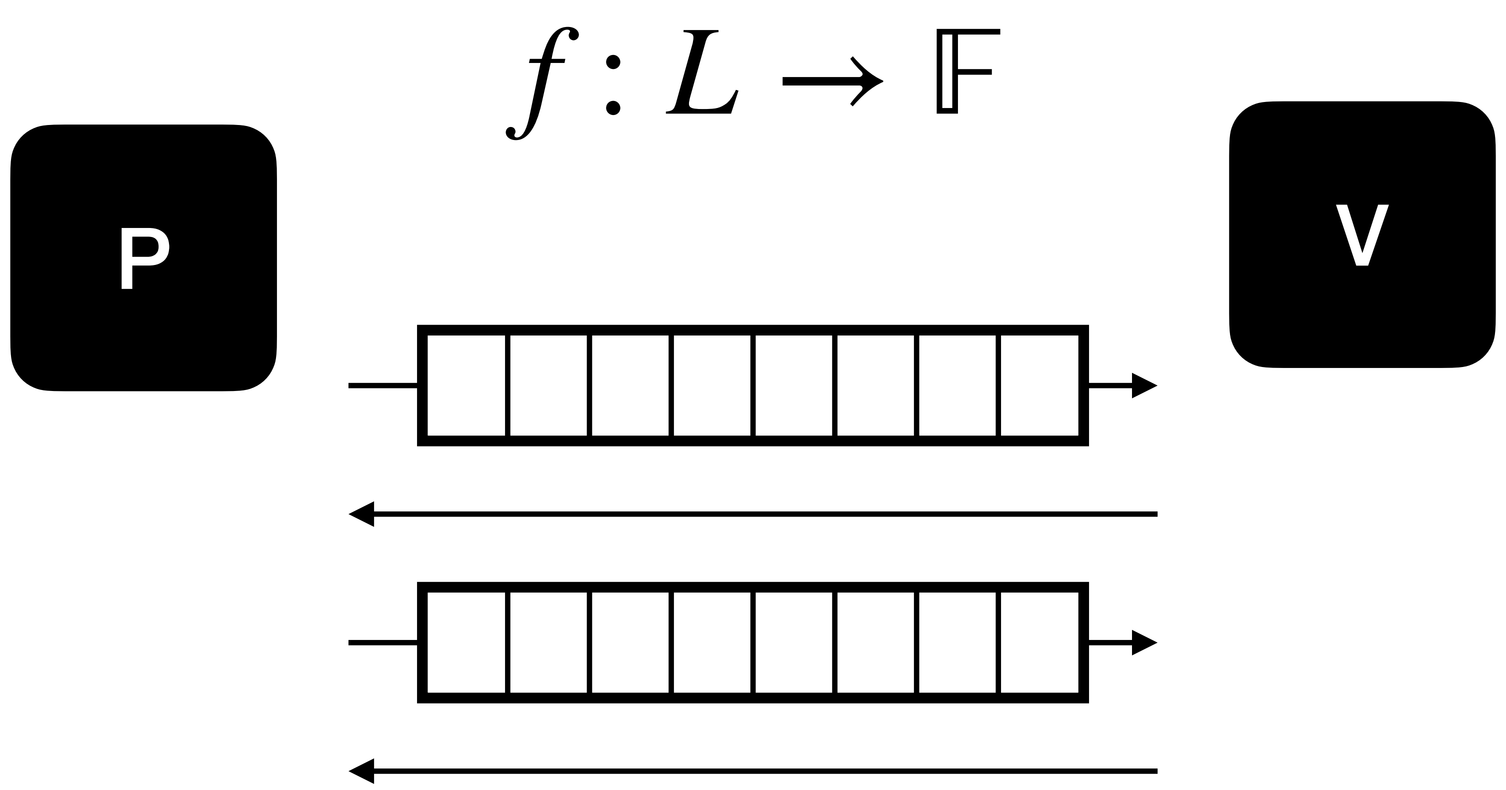

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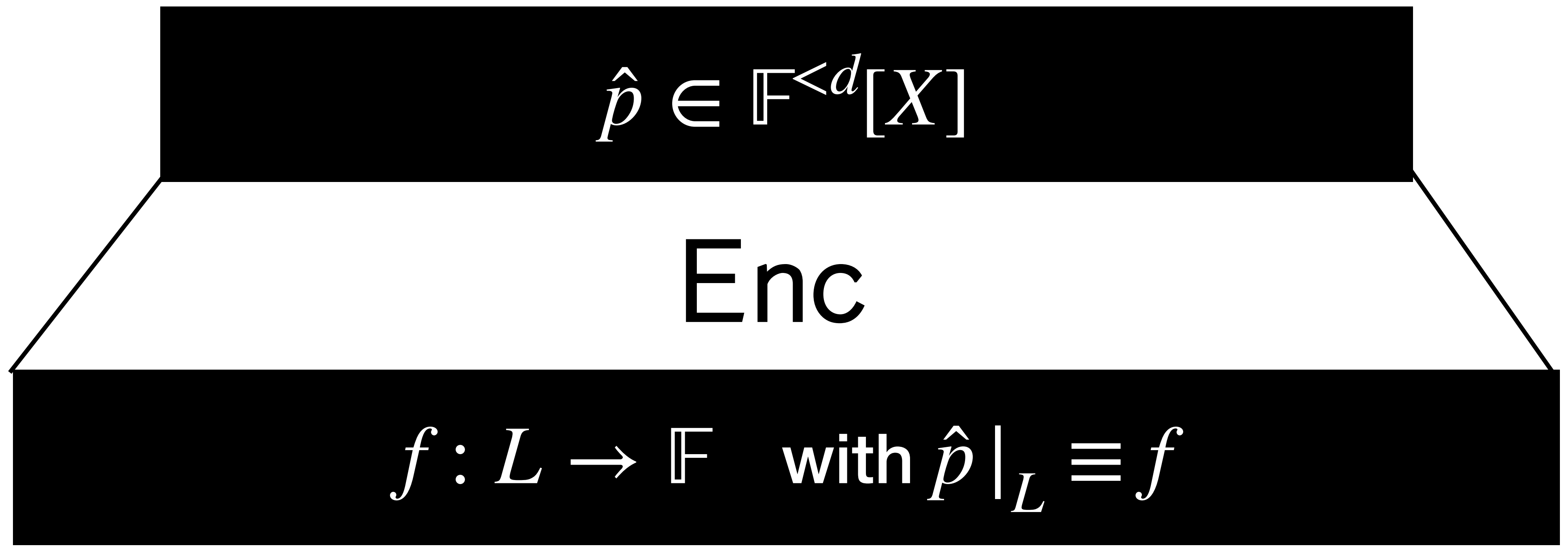
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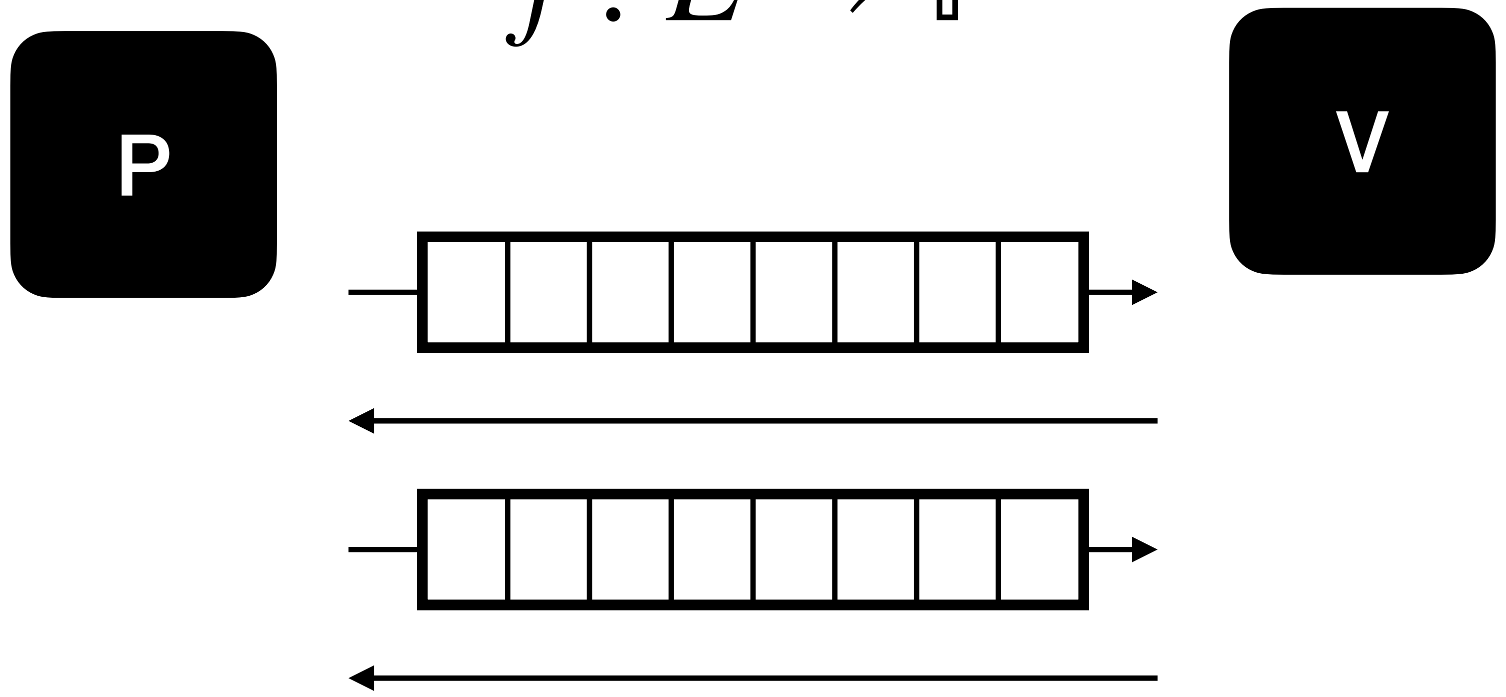

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V makes “few queries” to f and proof oracles

Our results

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Rounds: $O(\log d)$

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- Implemented both FRI and STIR
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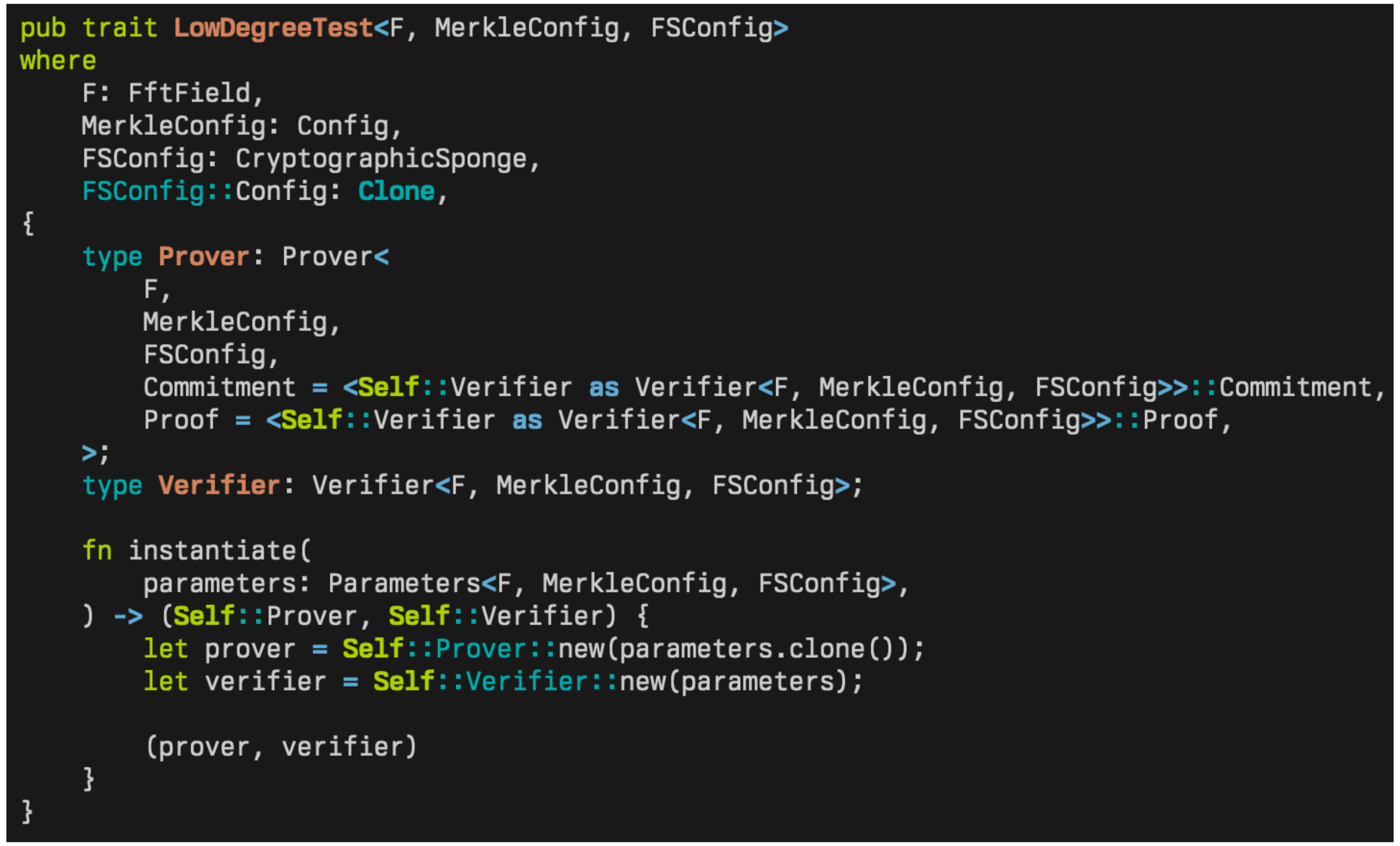

```
pub trait LowDegreeTest<F, MerkleConfig, FSConfig>
where
    F: FftField,
    MerkleConfig: Config,
    FSConfig: CryptographicSponge,
    FSConfig::Config: Clone,
{
    type Prover: Prover<
        F,
        MerkleConfig,
        FSConfig,
        Commitment = <Self::Verifier as Verifier<F, MerkleConfig, FSConfig>>::Commitment,
        Proof = <Self::Verifier as Verifier<F, MerkleConfig, FSConfig>>::Proof,
    >;
    type Verifier: Verifier<F, MerkleConfig, FSConfig>;

    fn instantiate(
        parameters: Parameters<F, MerkleConfig, FSConfig>,
    ) -> (Self::Prover, Self::Verifier) {
        let prover = Self::Prover::new(parameters.clone());
        let verifier = Self::Verifier::new(parameters);

        (prover, verifier)
    }
}
```

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  - FRI: ~ **400** queries vs STIR: ~ **200** queries

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  - FRI: ~ **400** queries vs STIR: ~ **200** queries

can increase by PoW

- Similar prover runtime (bottleneck is initial function evaluation)

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Assuming conjecture

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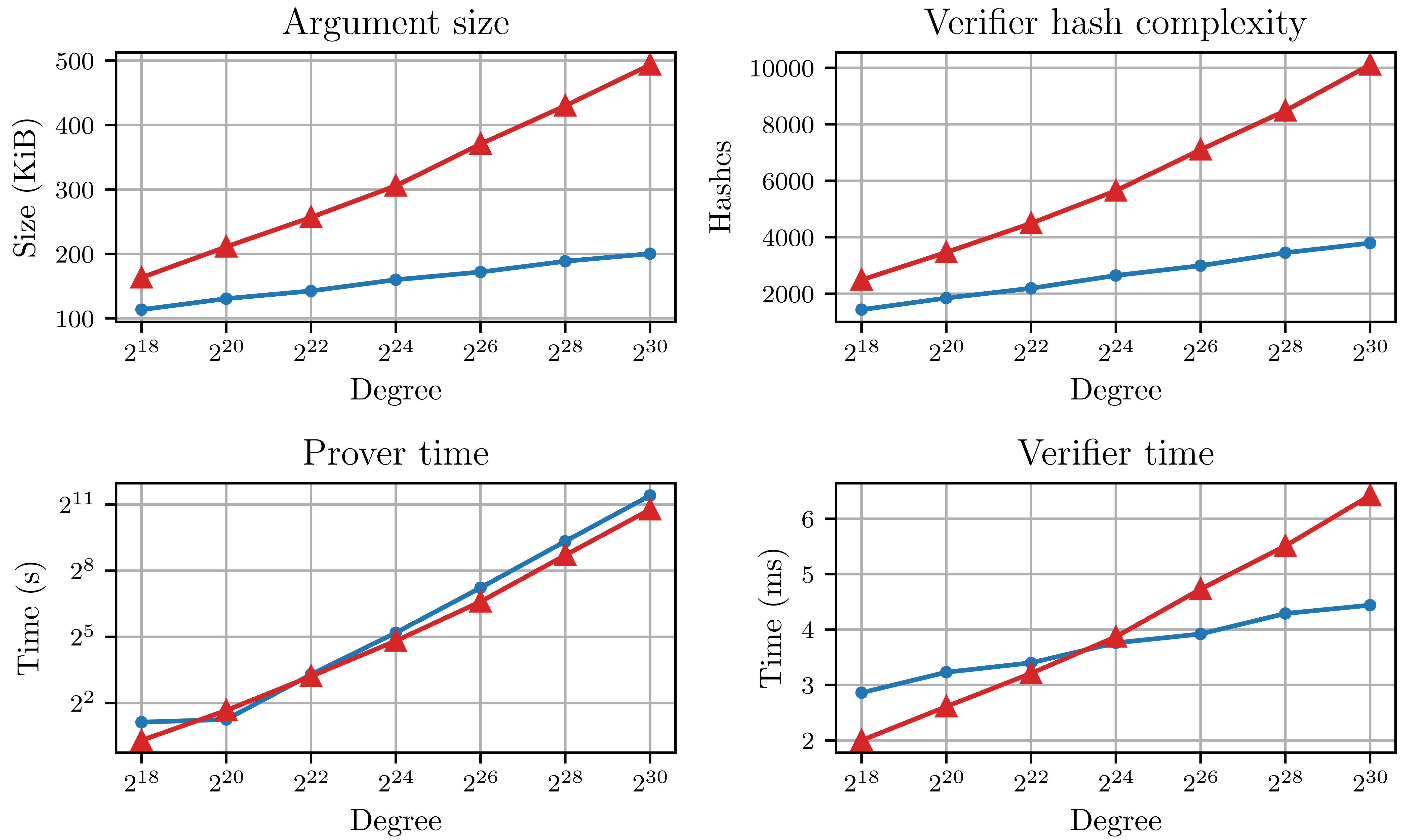

What about the conjecture?

FRI and STIR benefit in roughly the same way

- Conjecture on list-decoding up to distance $1-\rho$ (instead of $1-\sqrt{\rho}$)
- STIR queries: $$O\left(\lambda \cdot \log\left(\frac{\log d}{-\log \rho}\right) + \log d\right)$$
- FRI queries: $$O\left(\lambda \cdot \frac{\log d}{-\log \rho}\right)$$

In both, for $\delta = 1-\rho$,

reduces queries by ~2x

Techniques

Folding

Reduce $\mathsf{RS}[\mathbb{F}, L, d]$ **to** $\mathsf{RS}[\mathbb{F}, L^k, d/k]$

Folding

$L^k = \{x^k : x \in L\}$

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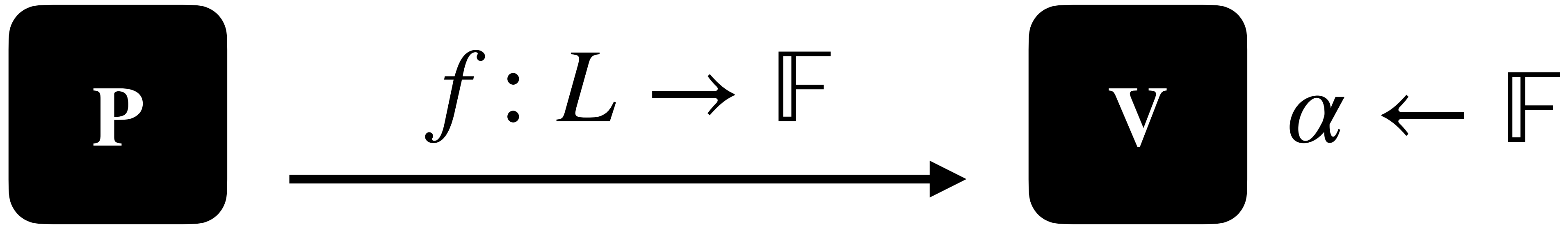

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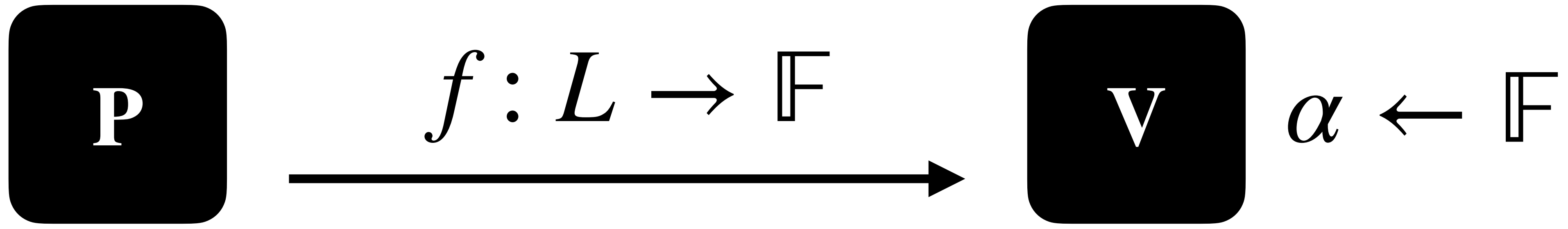

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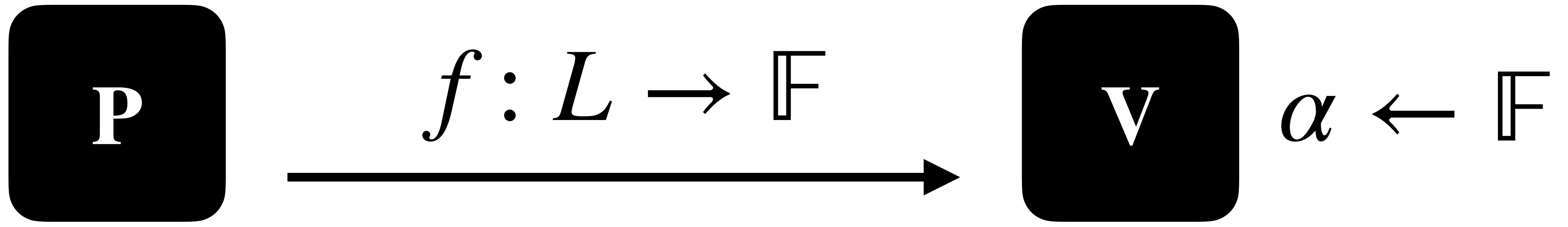

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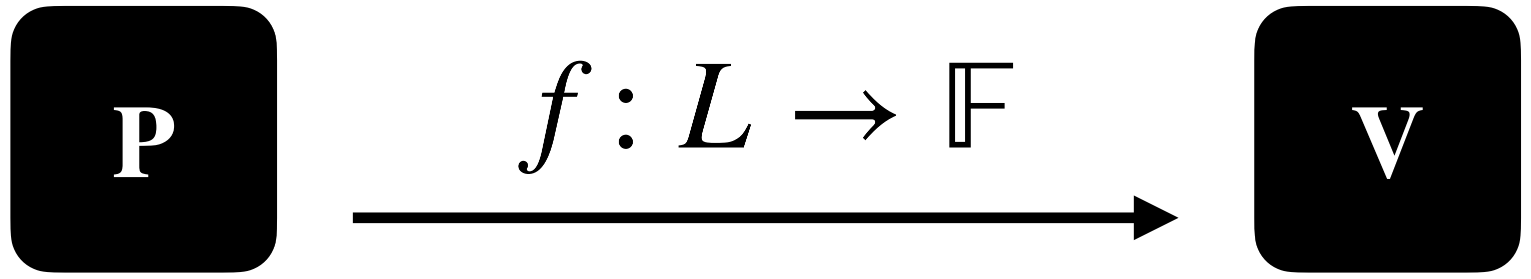

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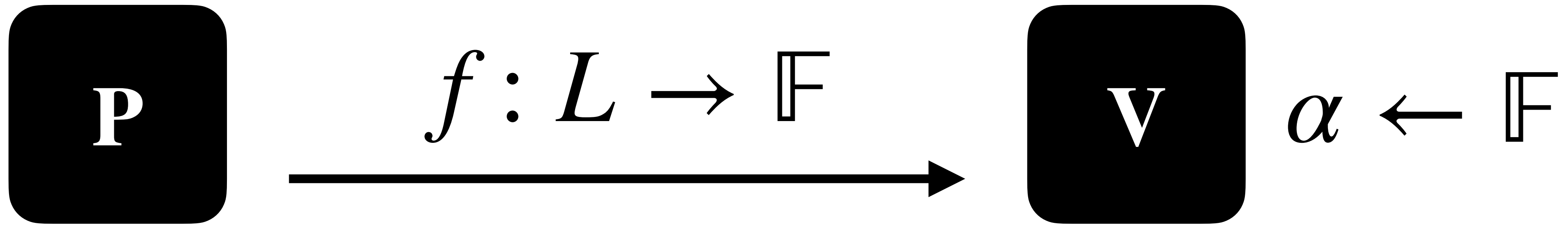

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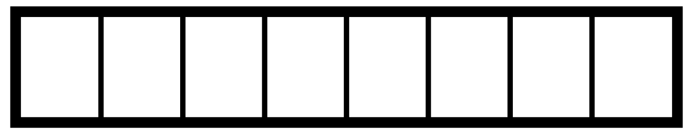

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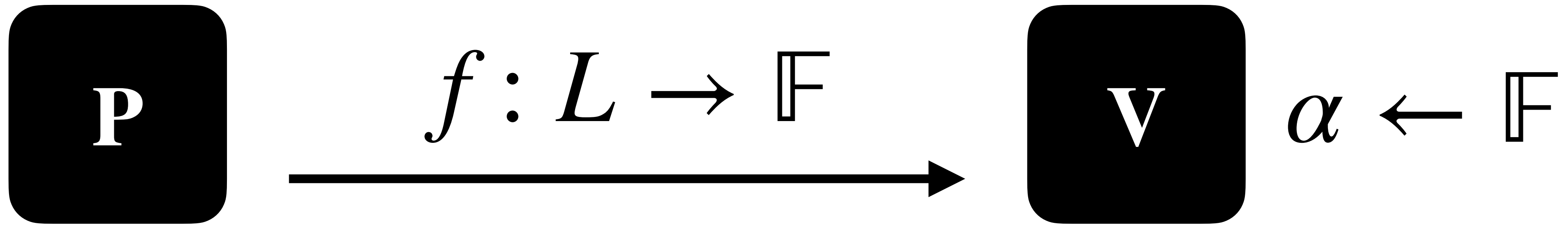

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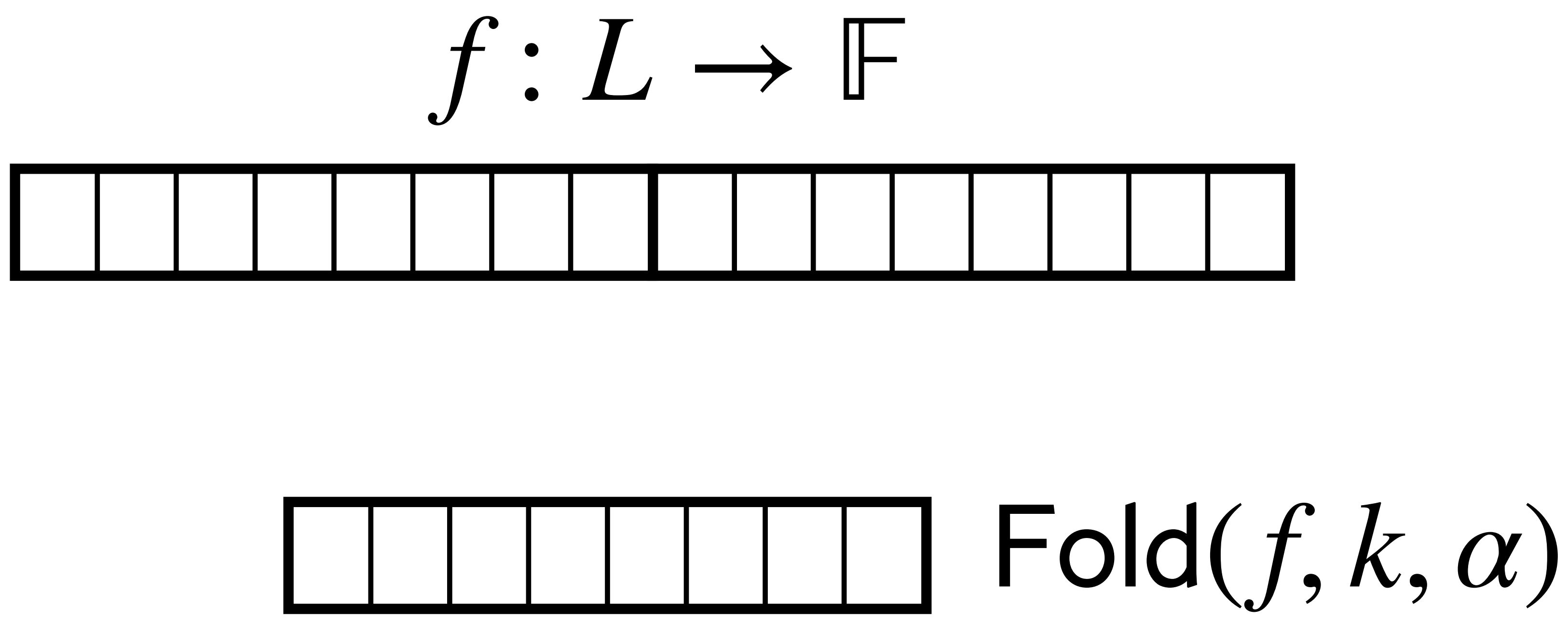

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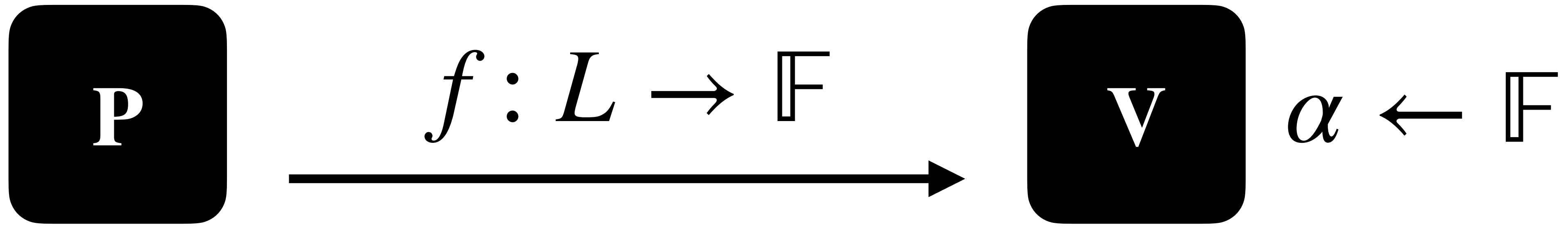

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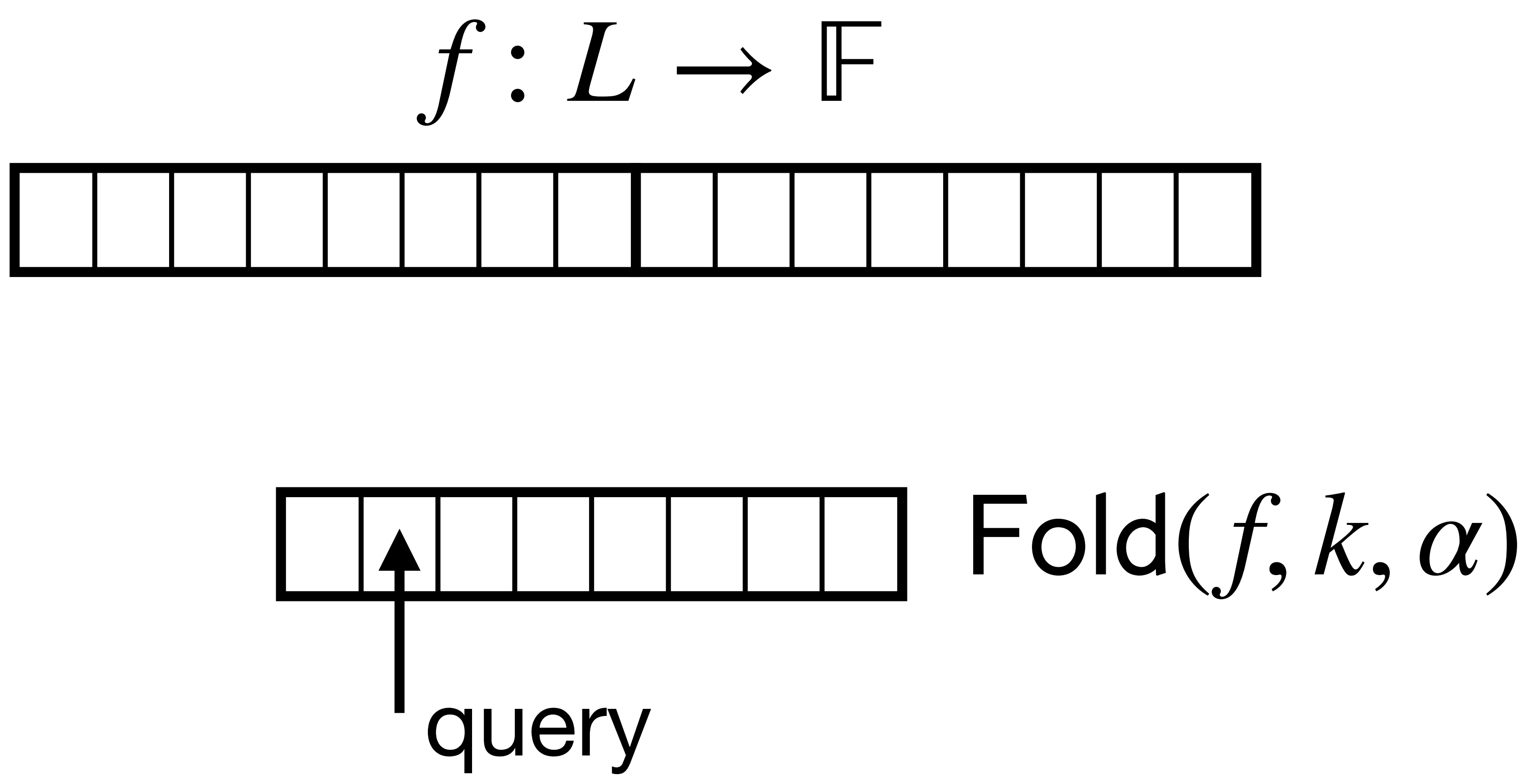

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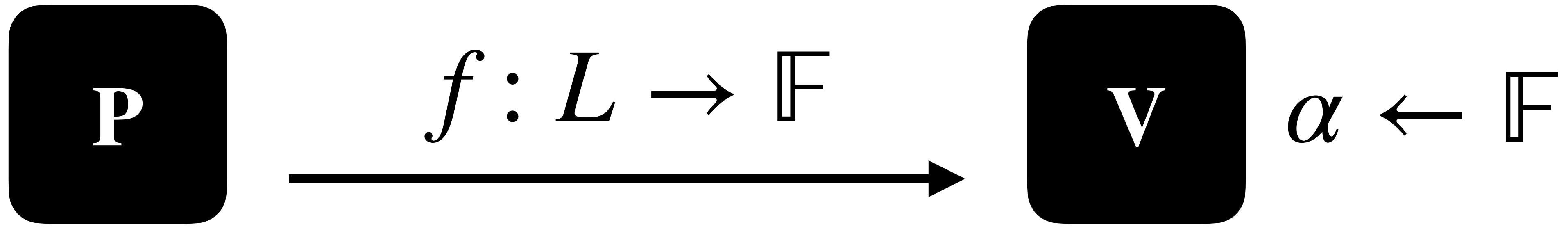

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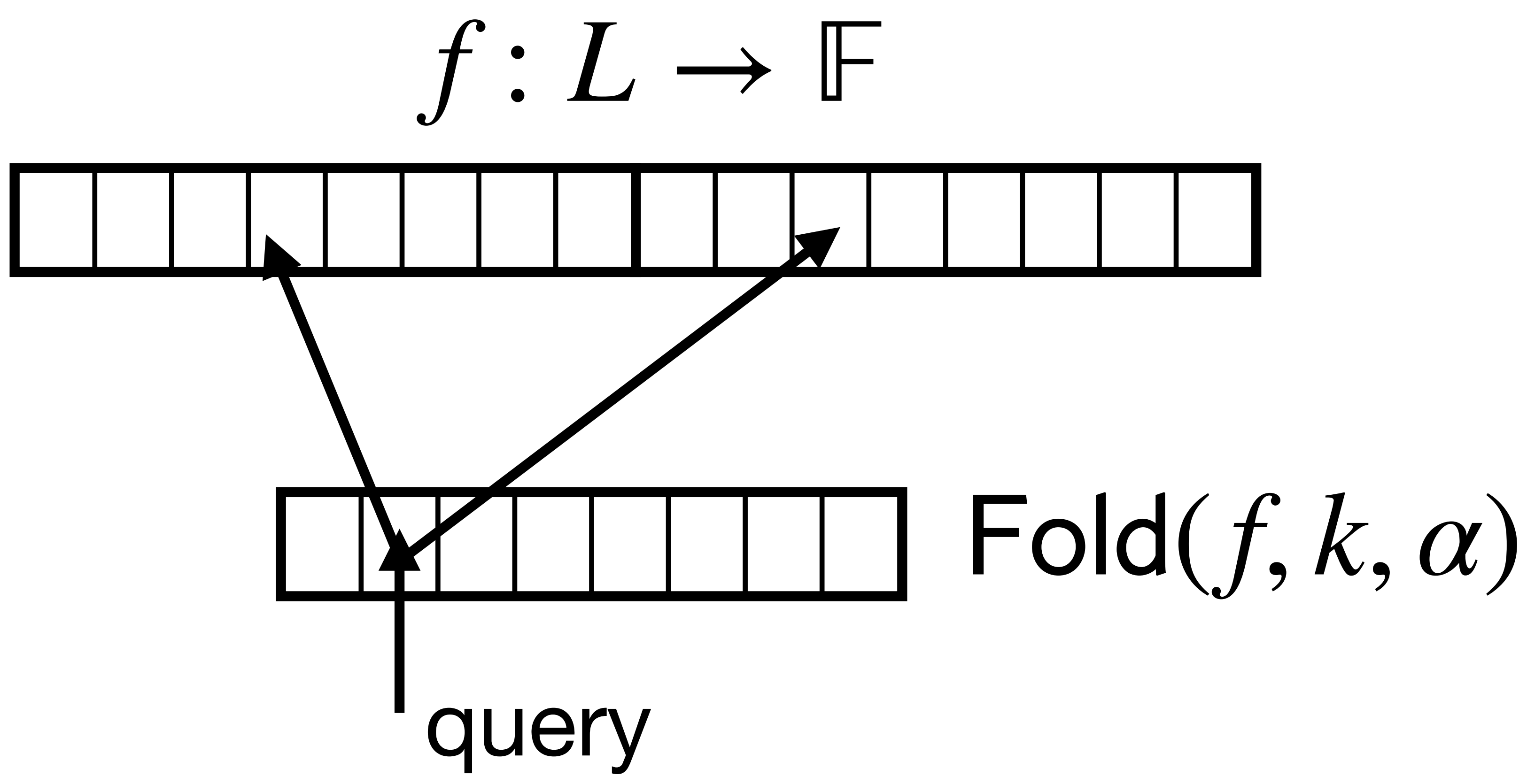

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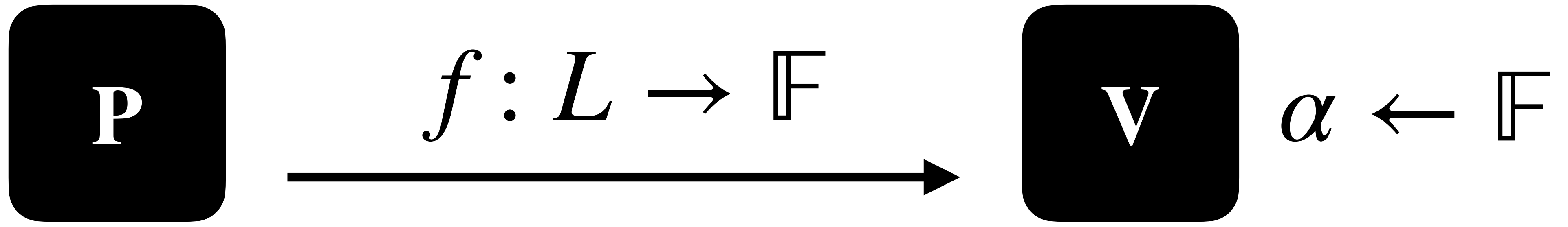

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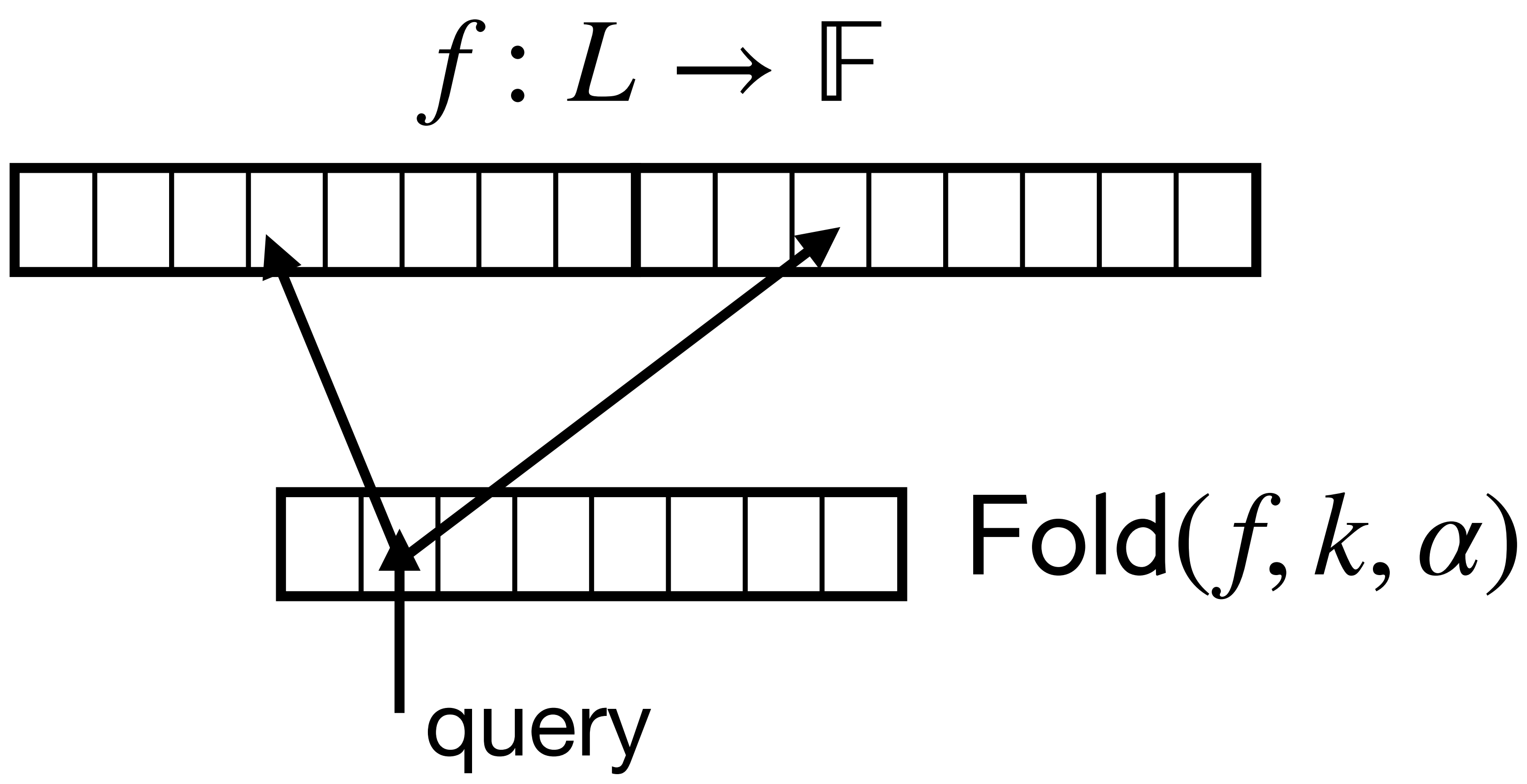

Distance Preserving

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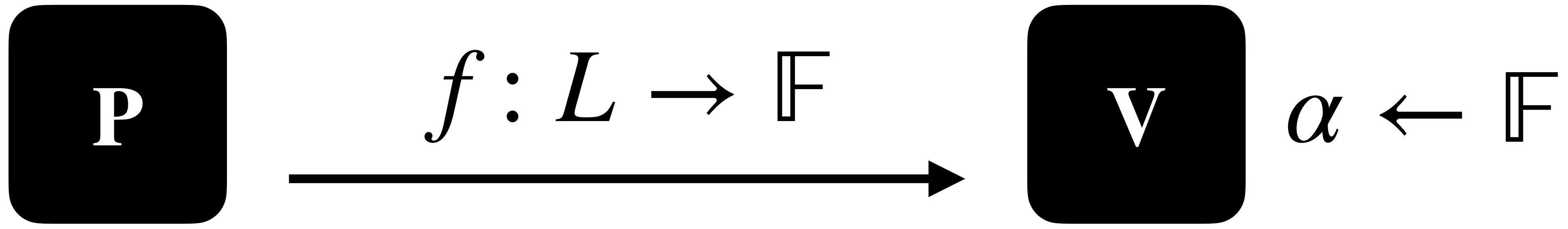

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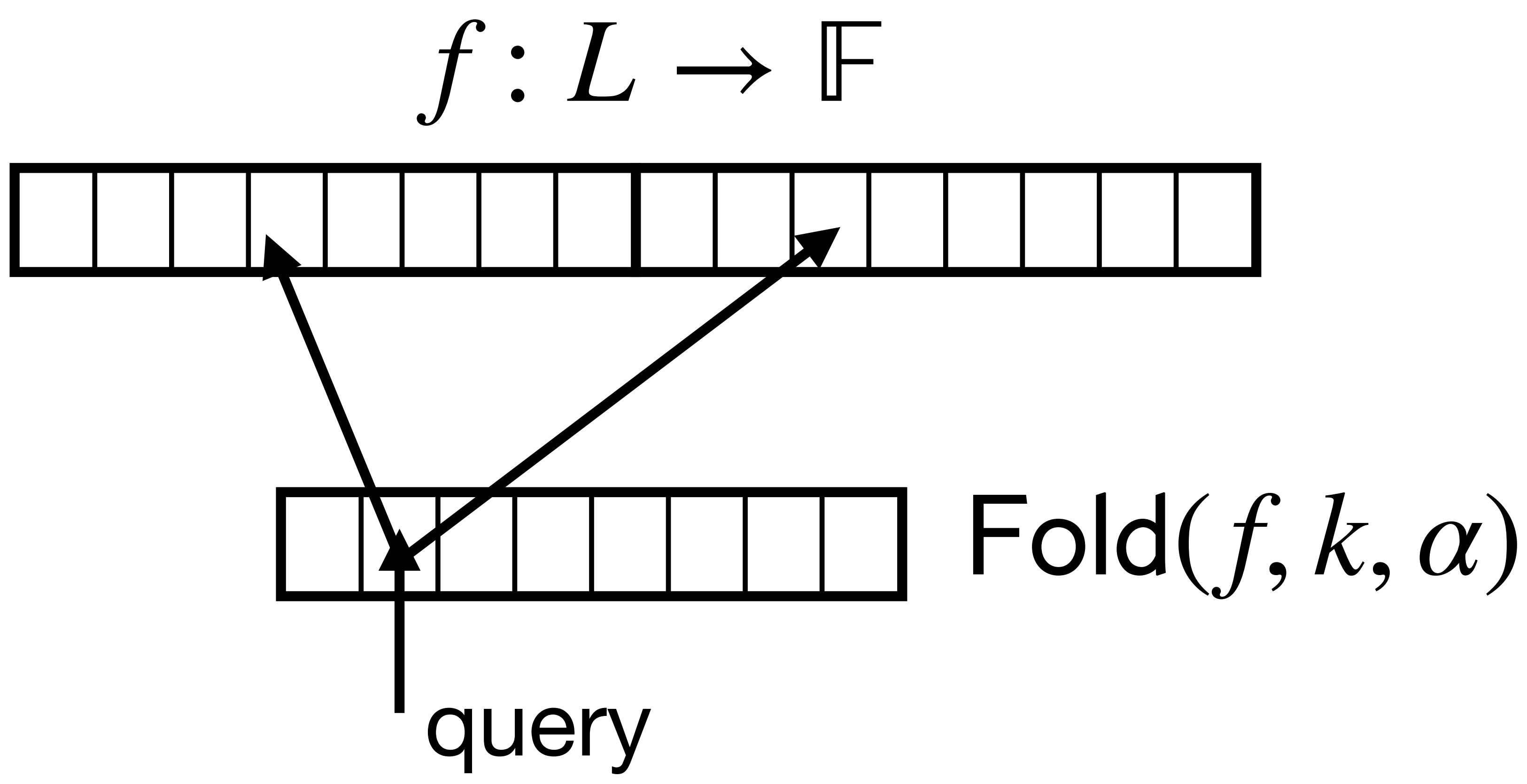

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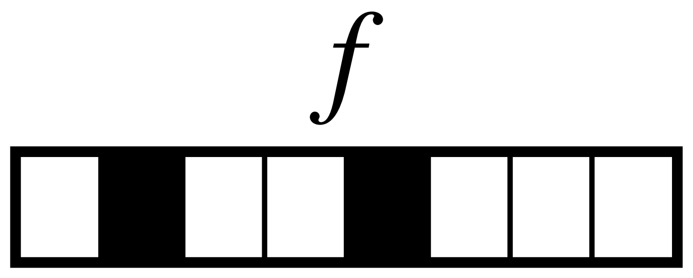
Fold(f, k, α)

query

Distance Preserving



f

f is δ-far from RS

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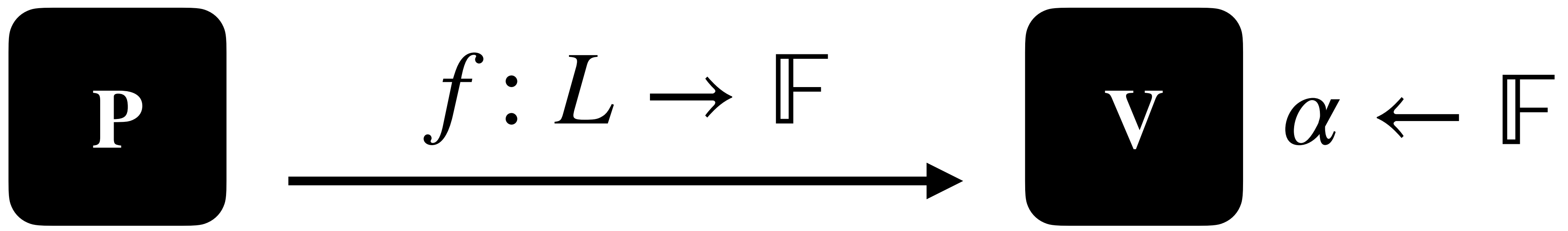

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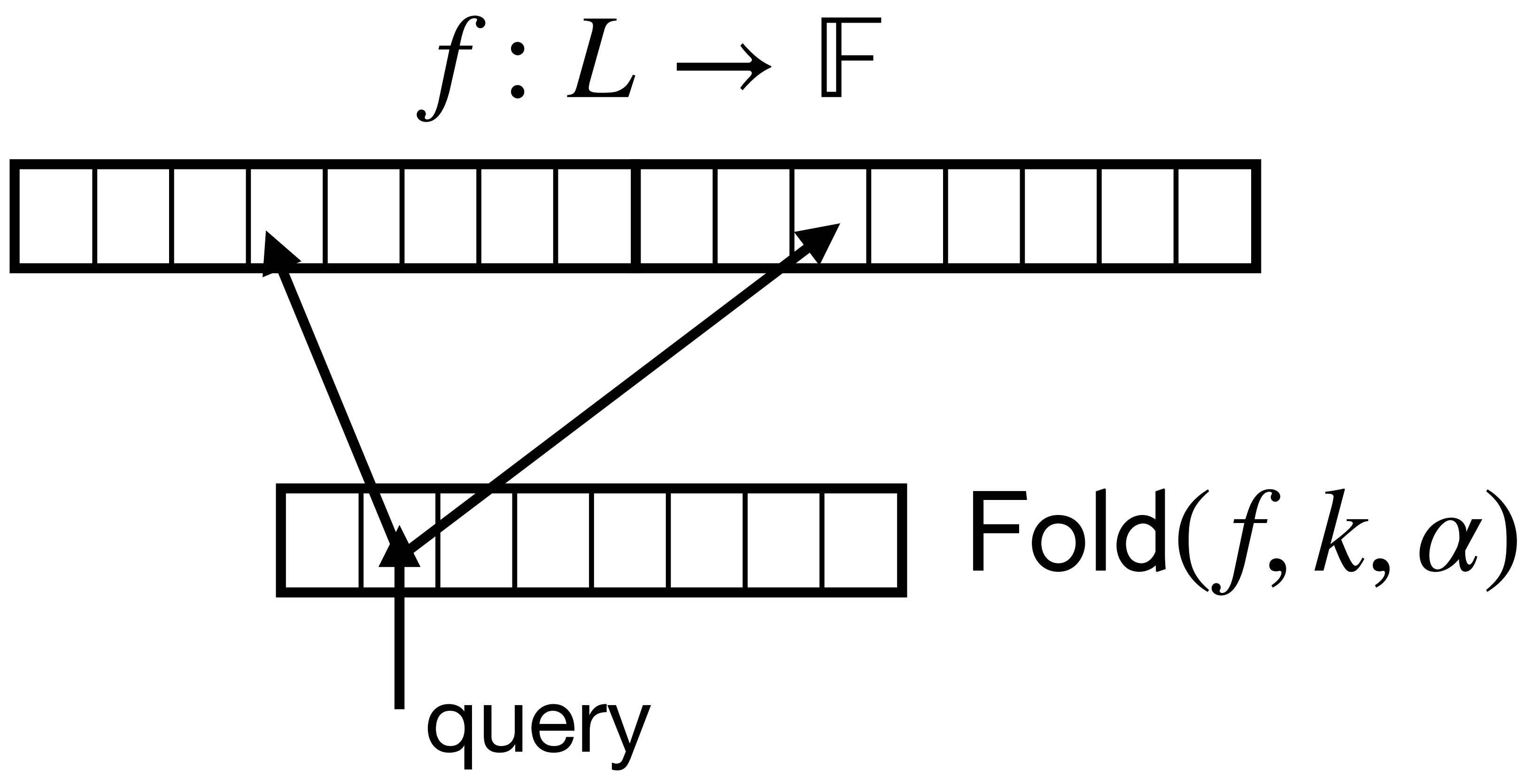

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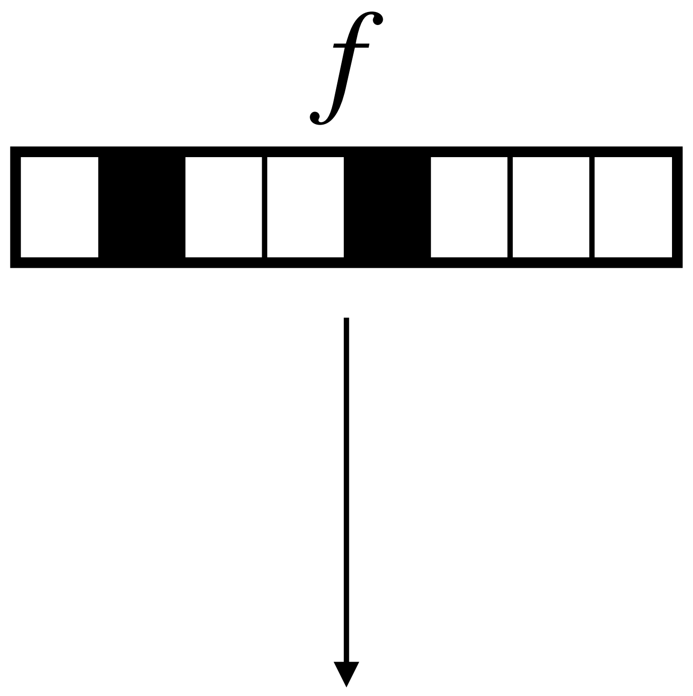

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w.h.p. over α

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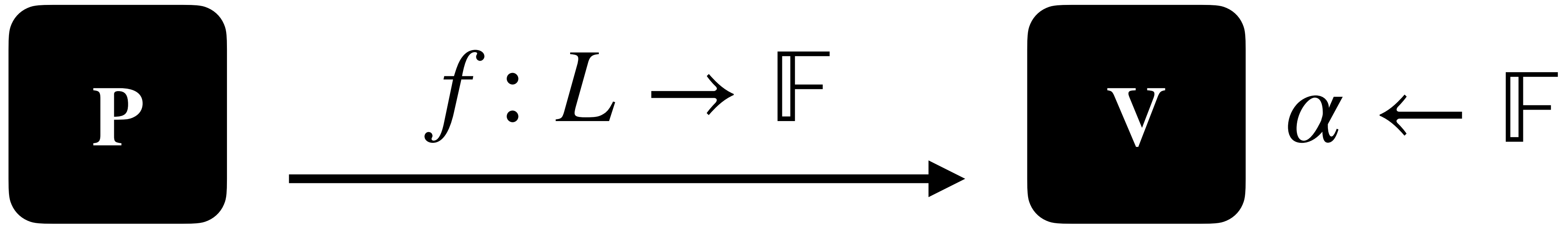

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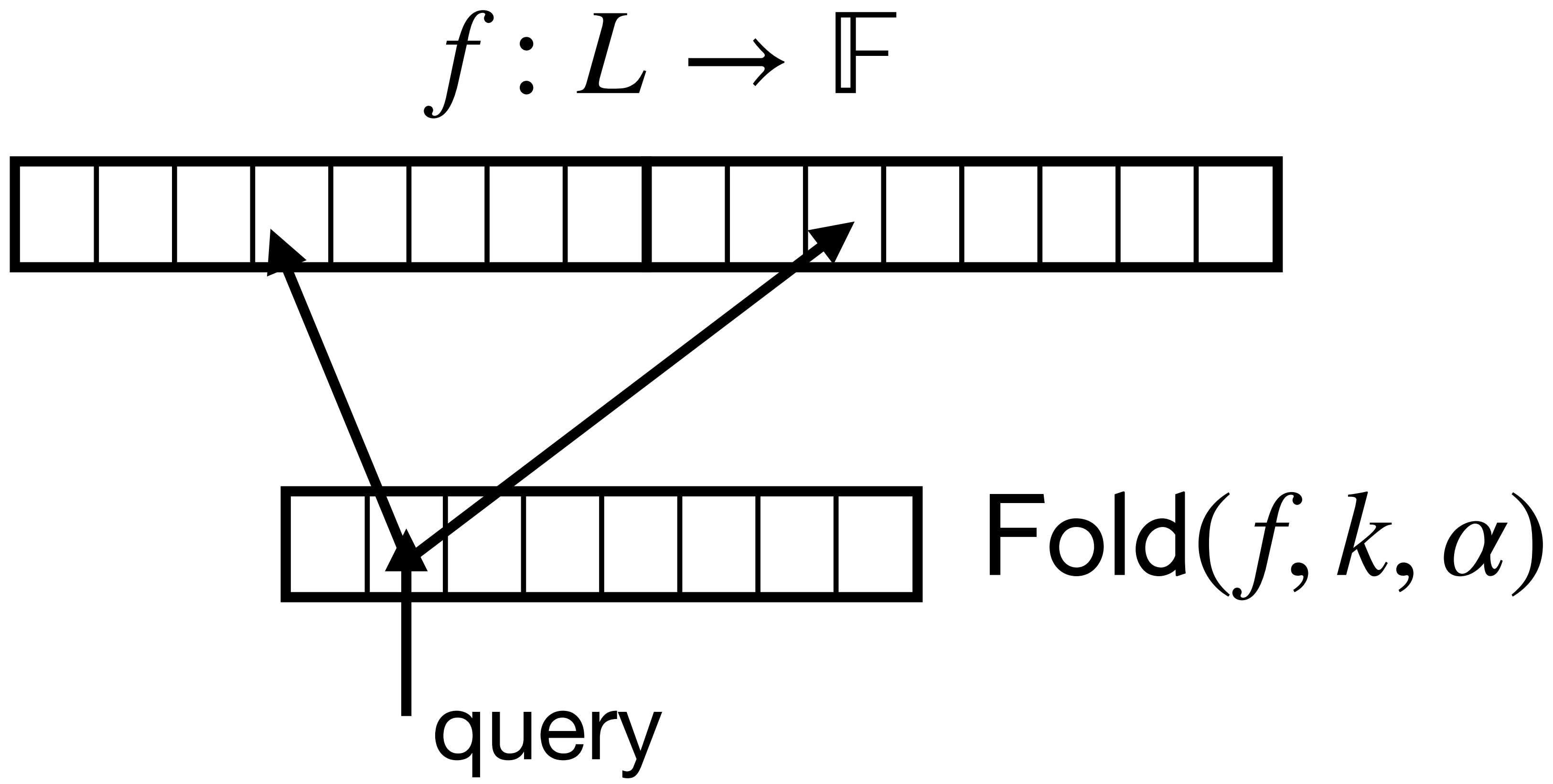

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query

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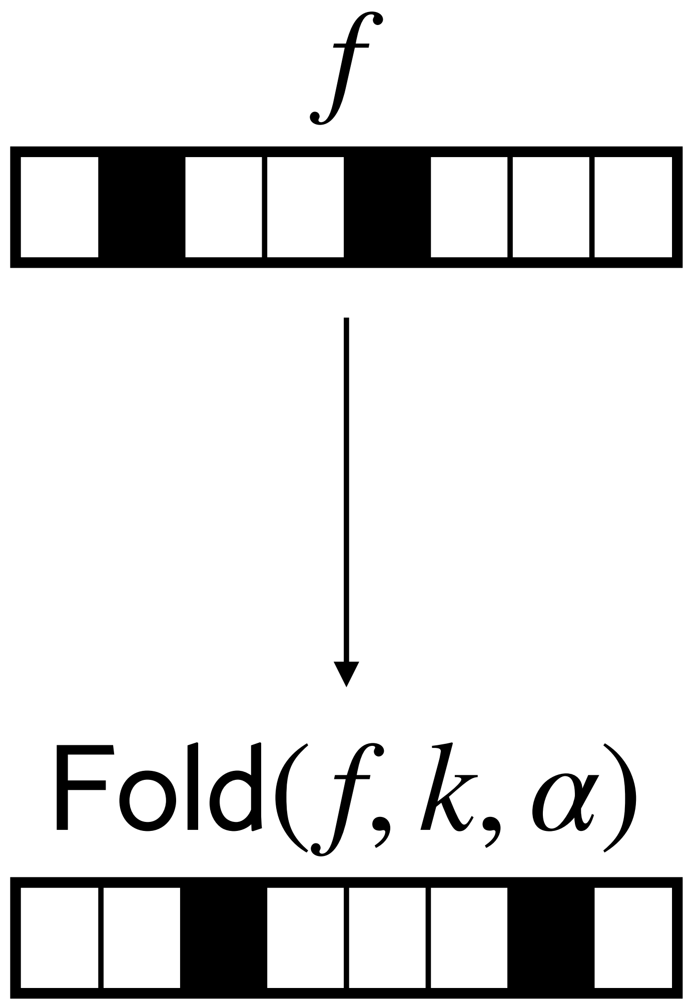

f

Fold(f, k, α)

f is δ-far from RS

w.h.p. over α

Fold is δ-far from RS

FRI Protocol

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$$f_0 : L \to \mathbb{F}$$

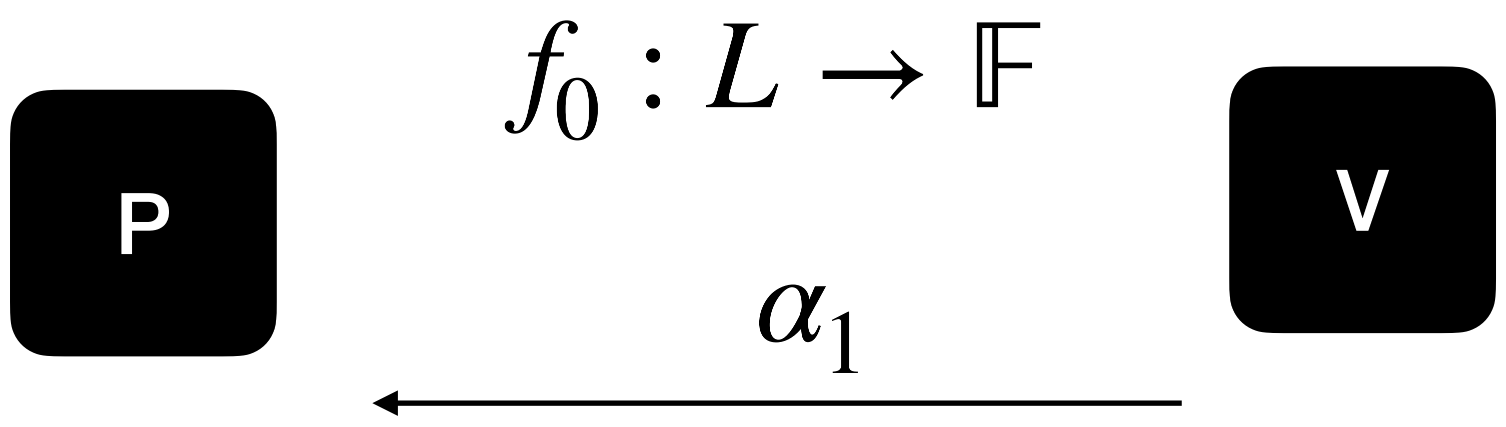
FRI Protocol

P

$f_0 : L \to \mathbb{F}$

V

FRI Protocol



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α_1

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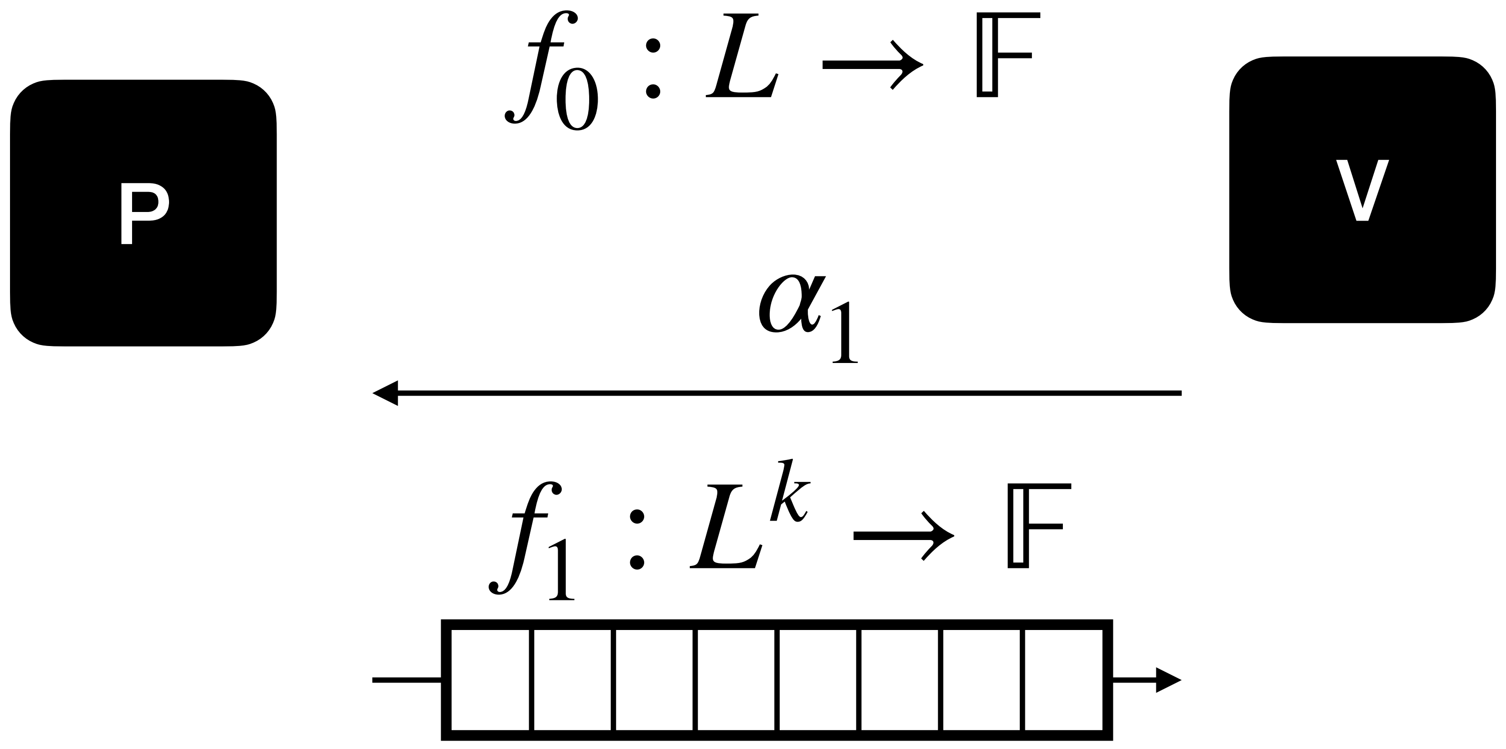

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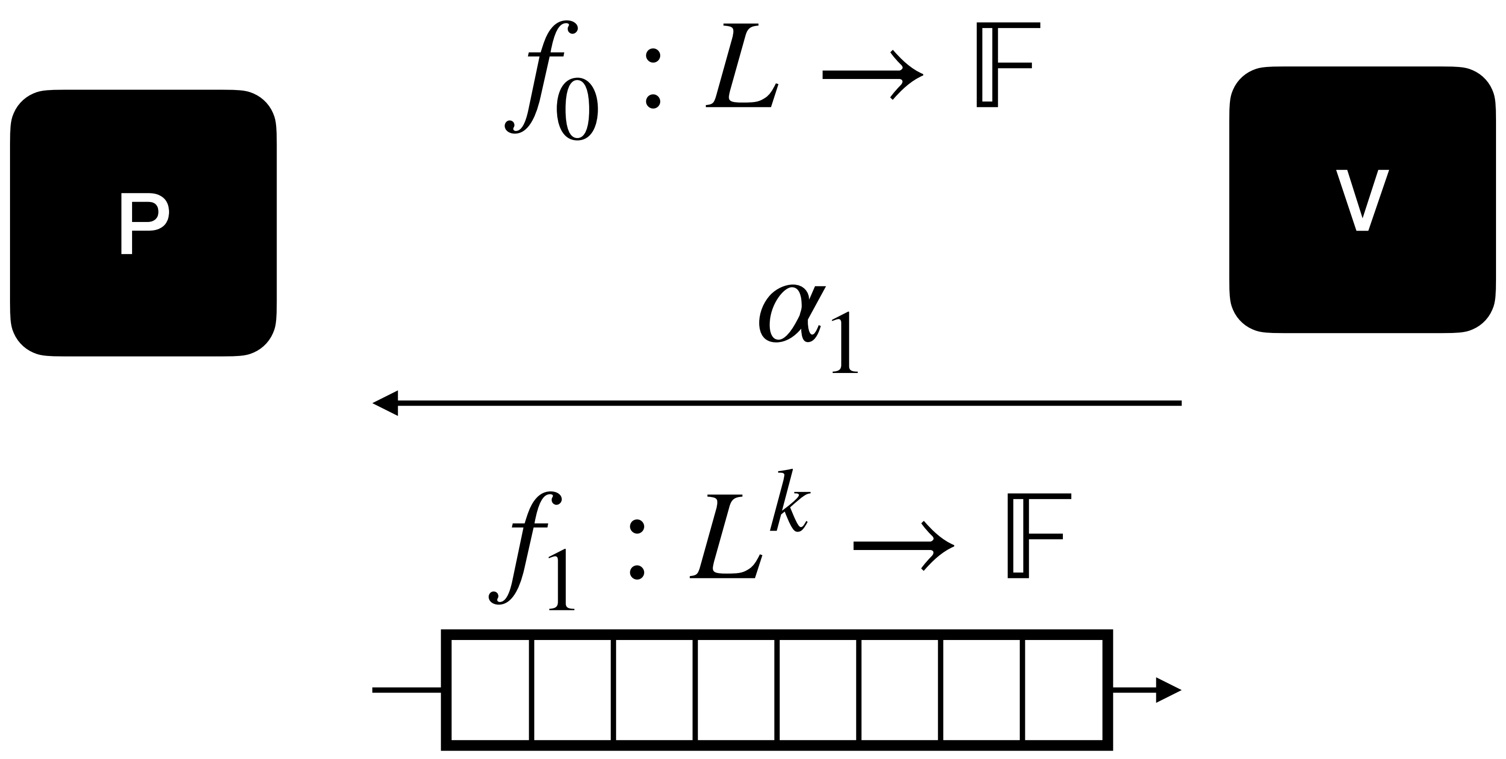

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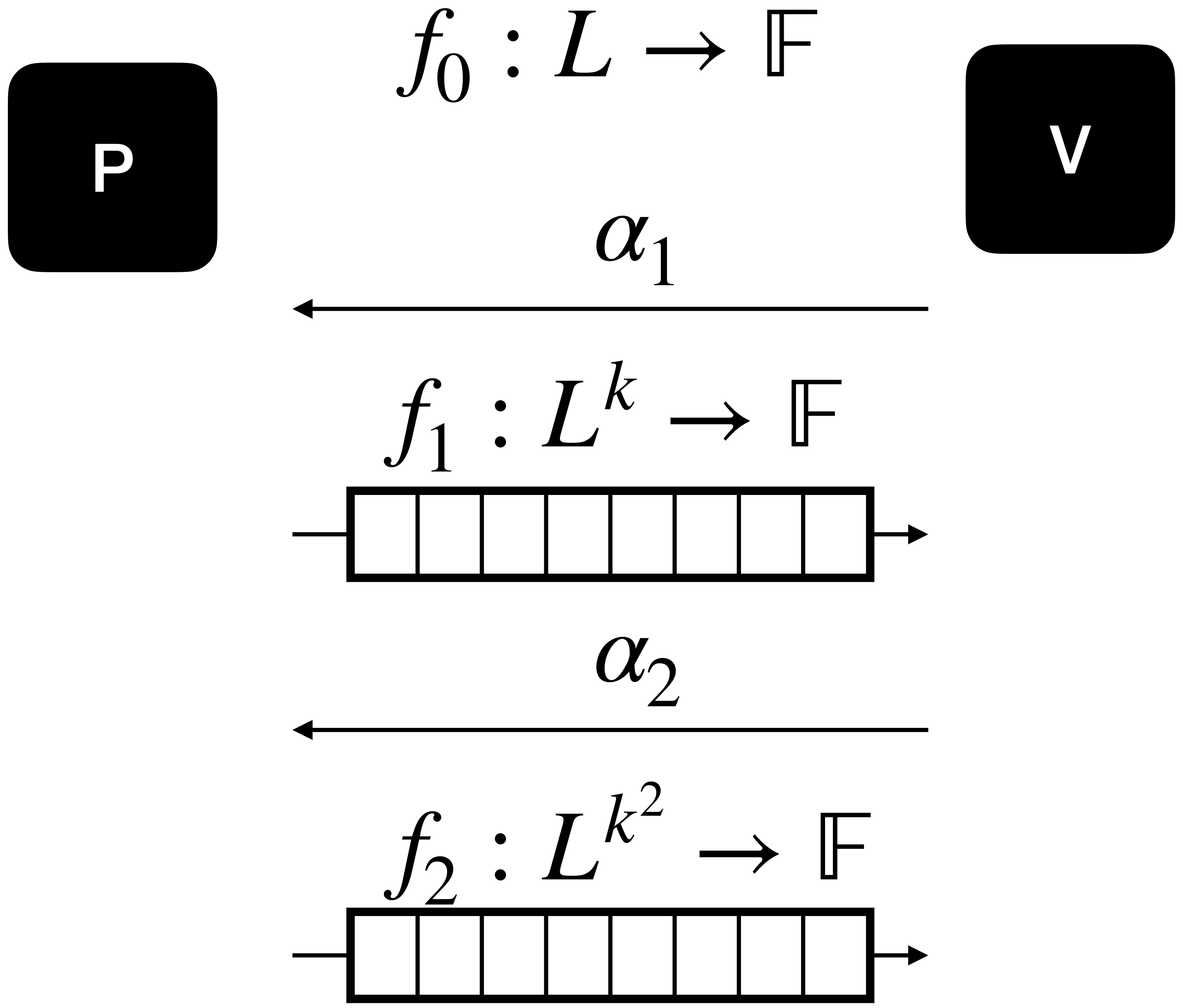

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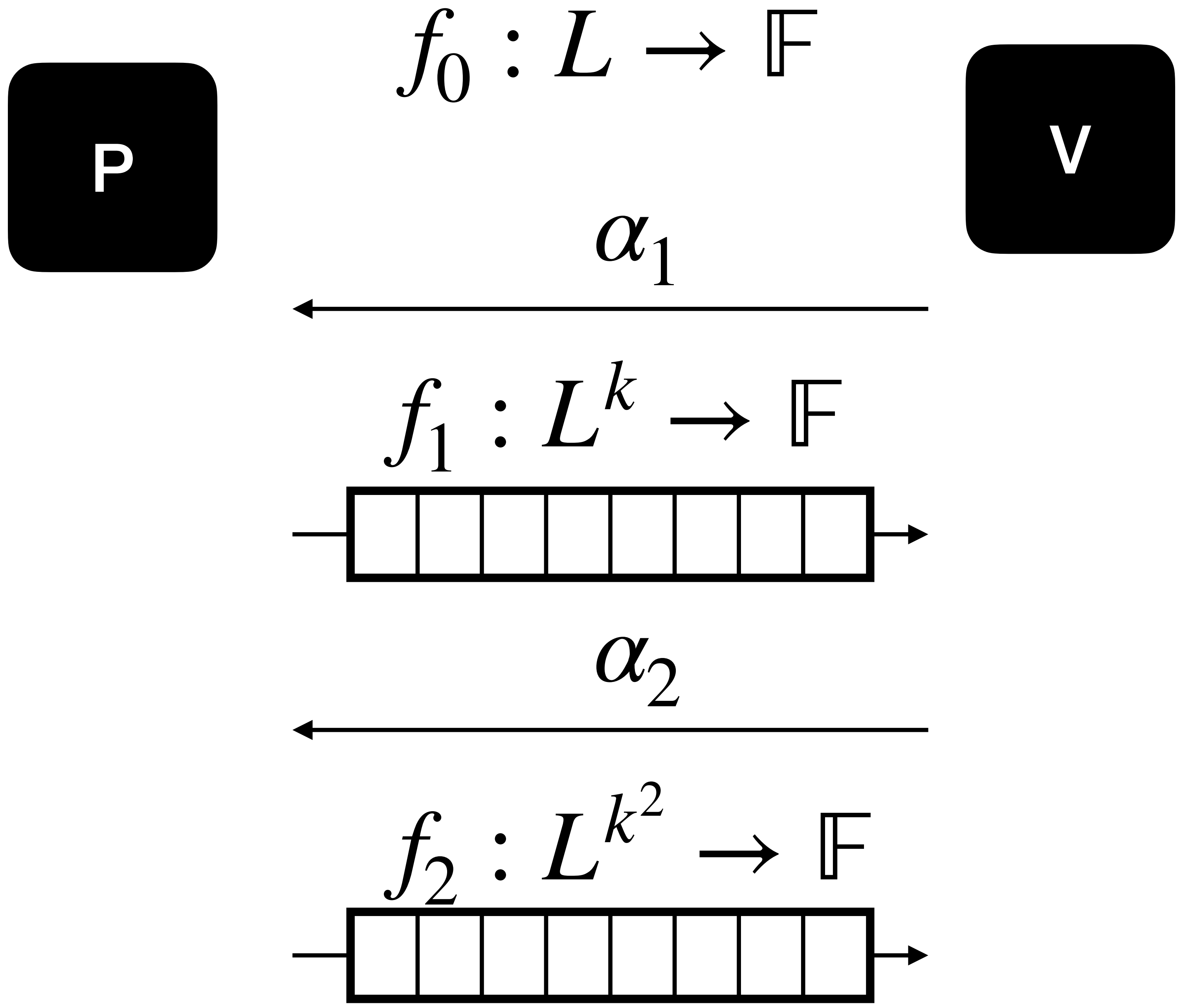

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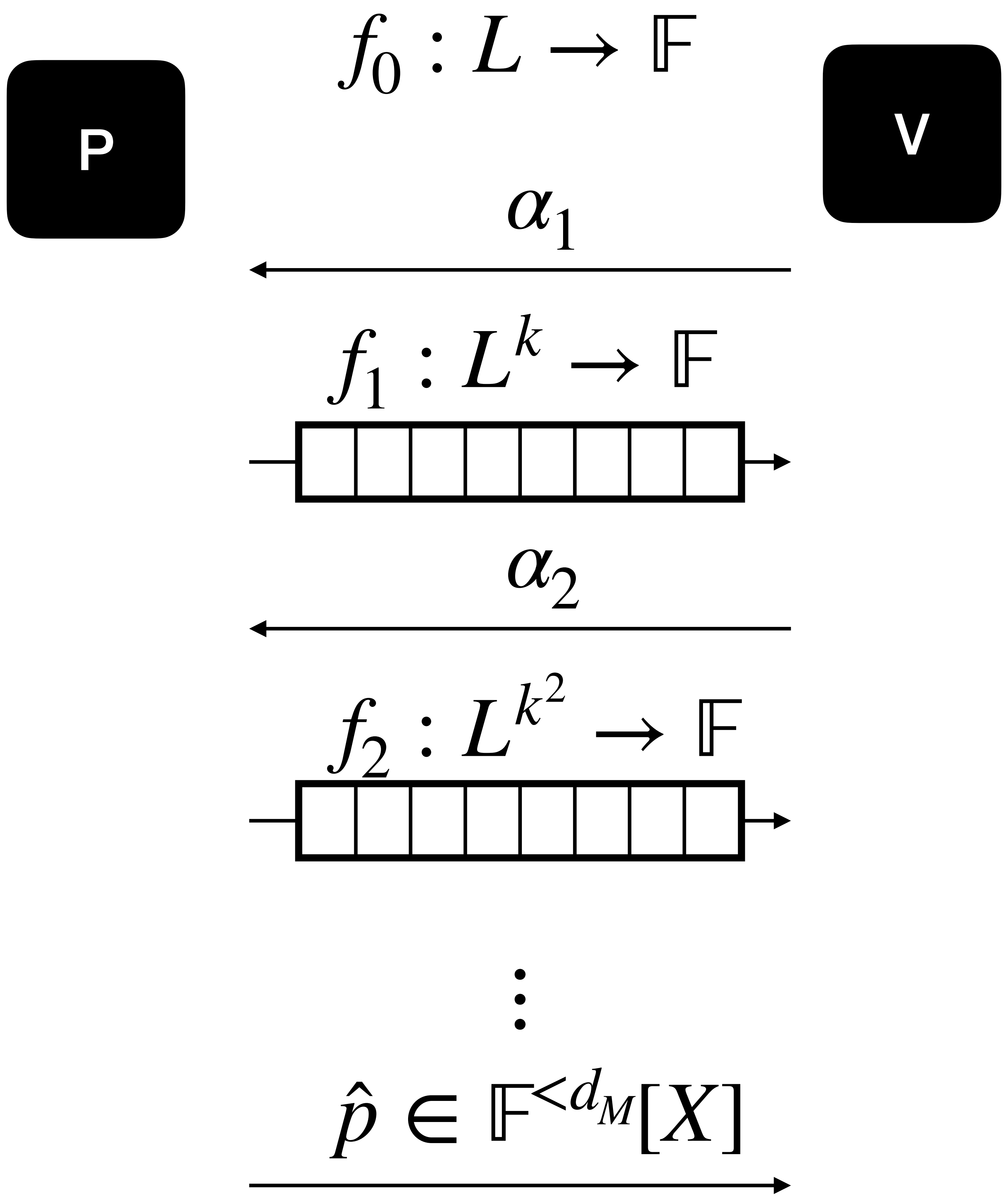

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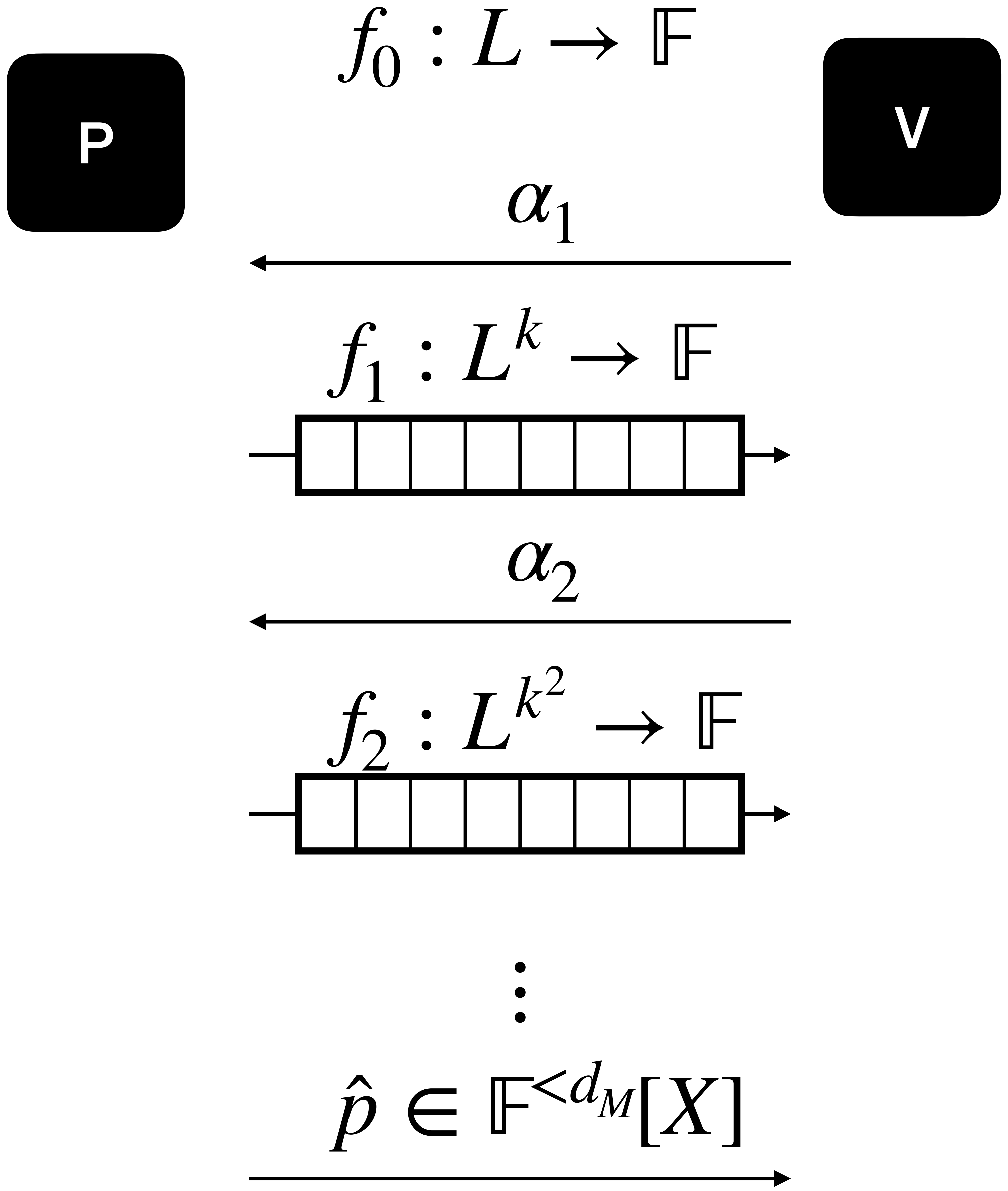

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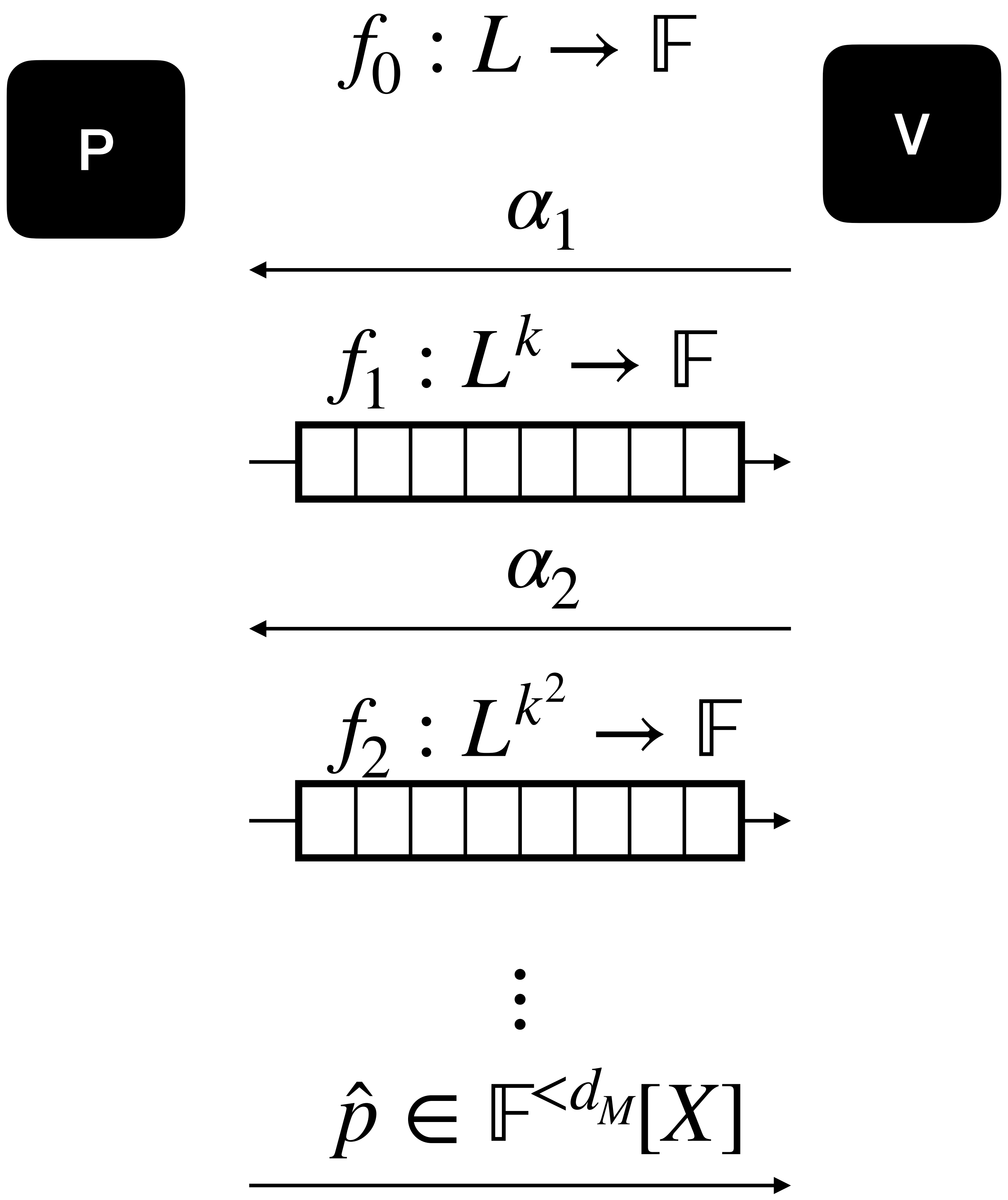

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Soundness error: $\rho^{\frac{t}{2}}$

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- Example $k = 16$, $\rho_0 = 1/2$, 100 bits of security:

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- Round-by-round errors: $\rho_0^{\frac{t_1}{2}}, \ldots, \rho_M^{\frac{t_M}{2}}$ $\quad t_i = \dfrac{\lambda}{-\log(\sqrt{\rho_i})}$ ← Less queries each round!
- Example $k = 16$, $\rho_0 = 1/2$, 100 bits of security:

  $$\rho_1 = 1/16, \rho_2 = 1/128, \ \ldots$$

STIR: Shift-To-Improve-Rate

- Main idea to reduce:

$$\mathsf{RS}[\mathbb{F}, L, d] \quad \Rightarrow \quad \mathsf{RS}[\mathbb{F}, L^2, d/k]$$

  where $|L^2| = |L|/2$.

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- Example $k = 16$, $\rho_0 = 1/2$, 100 bits of security:

$$\rho_1 = 1/16,\ \rho_2 = 1/128,\ \ldots$$

$$t_0 = 200,\ t_1 = 50,\ t_2 = 29,\ \ldots$$

Domain shifting

Given test for $\mathrm{RS}[\mathbb{F}, L*, d]$, test $\mathrm{RS}[\mathbb{F}, L, d]$

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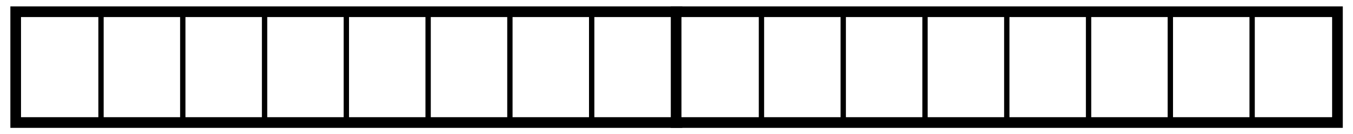

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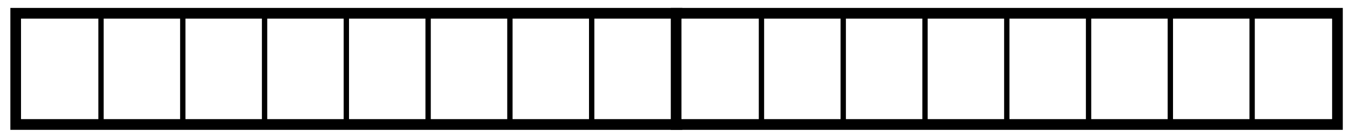

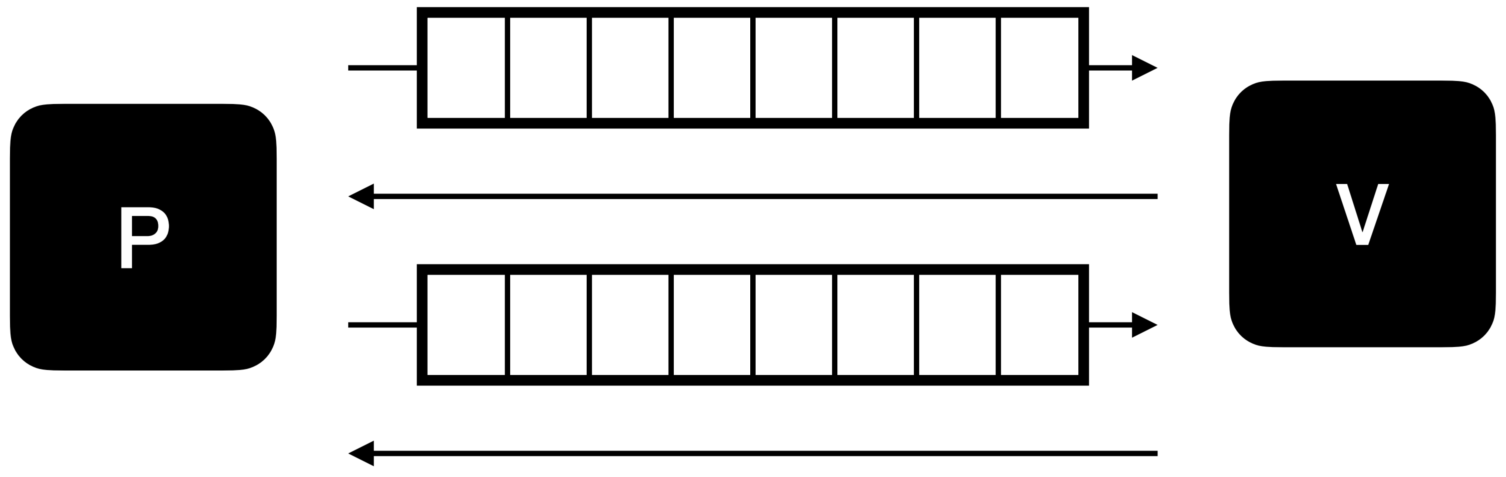

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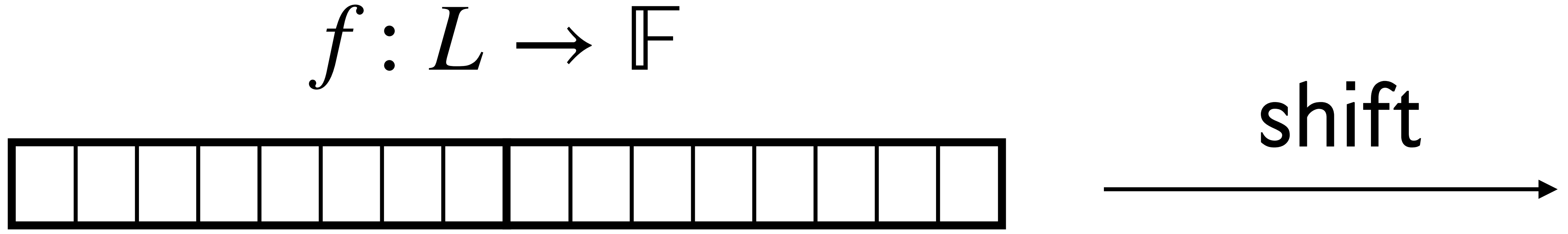

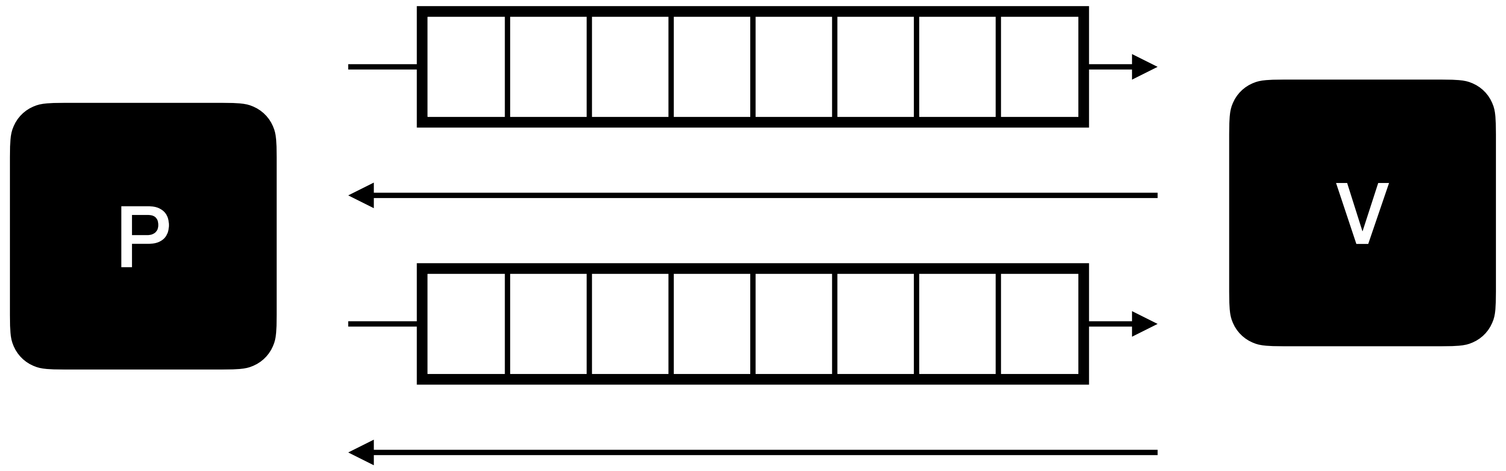

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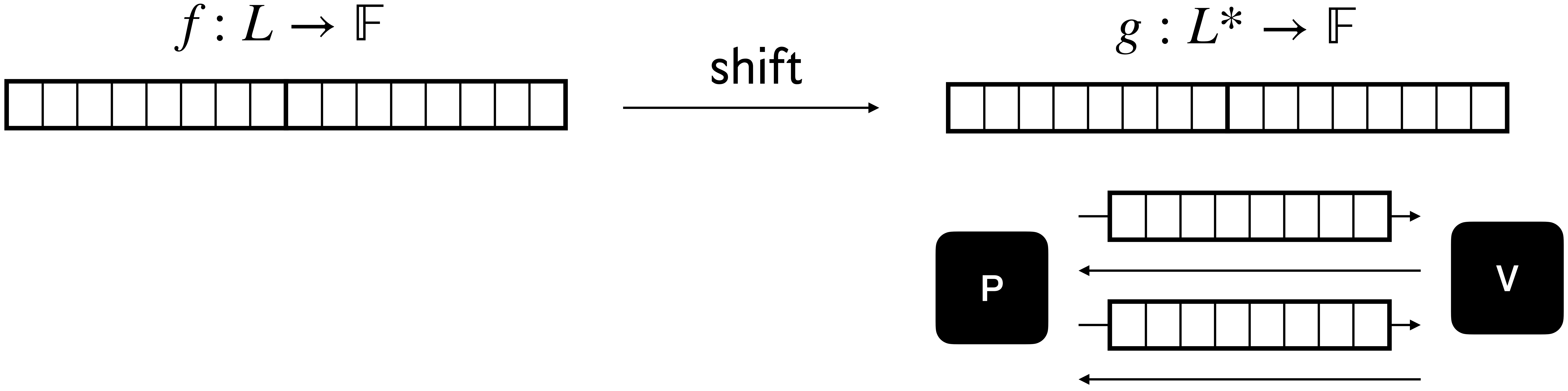

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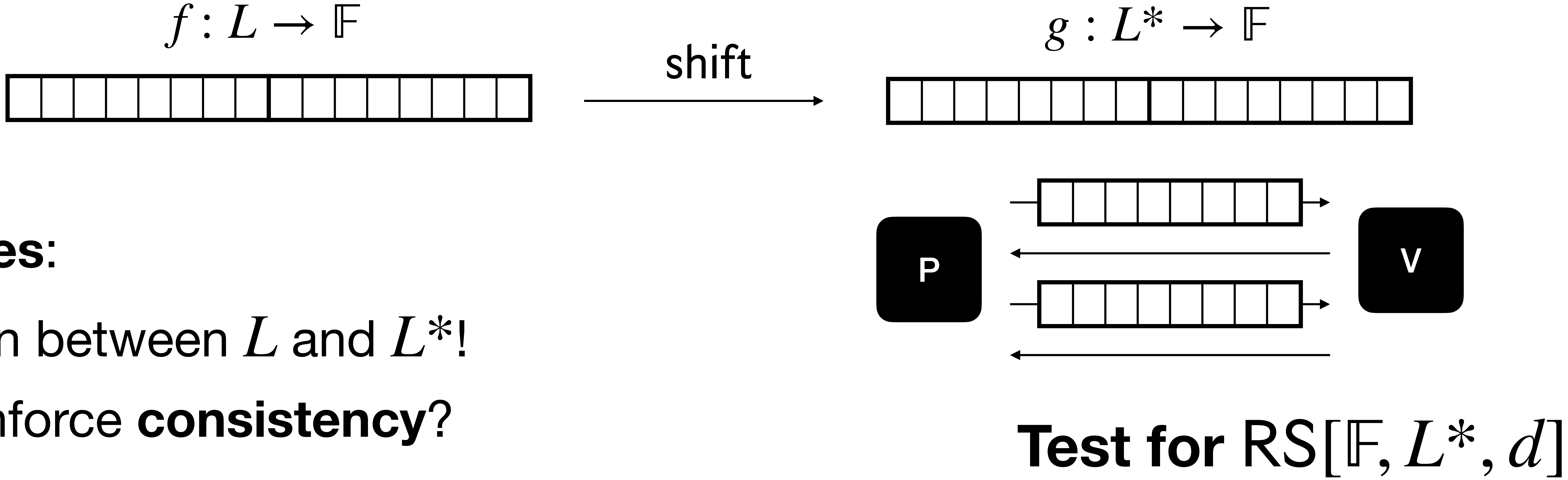

Test for $\mathsf{RS}[\mathbb{F}, L^*, d]$

Challenges:

No relation between L and L^*!

How to enforce **consistency**?

Quotienting

Enforce constraints on f or amplify distance

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Let:

$f : L \to \mathbb{F}$ be a function

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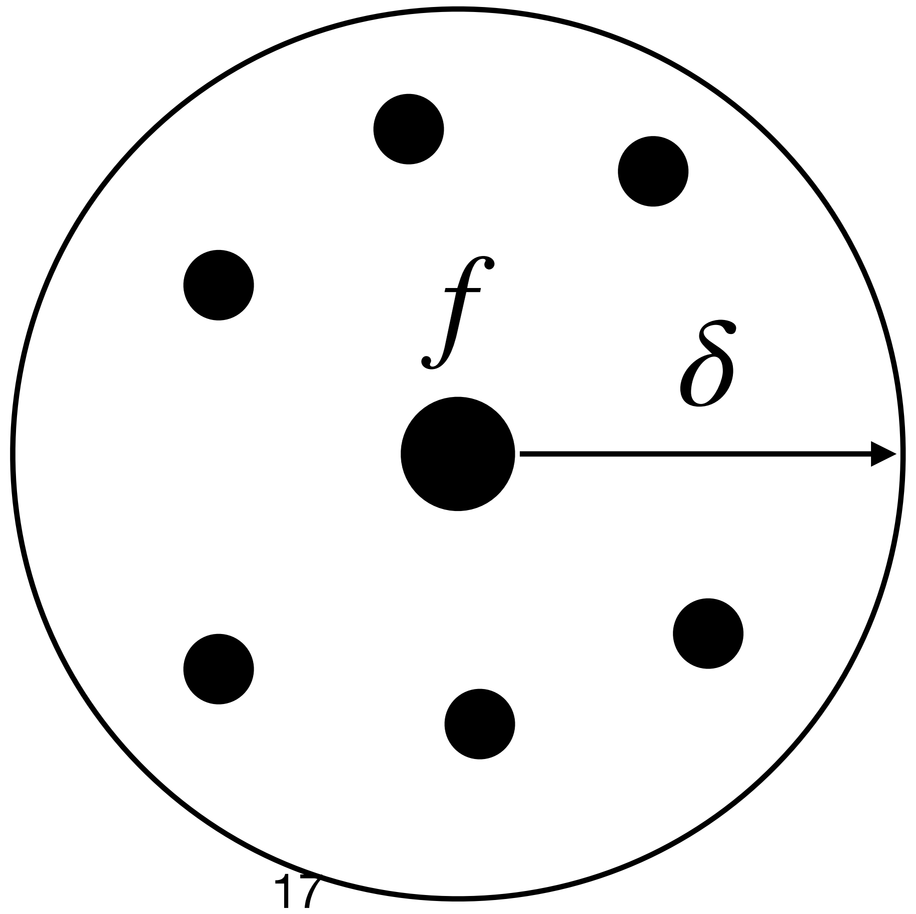

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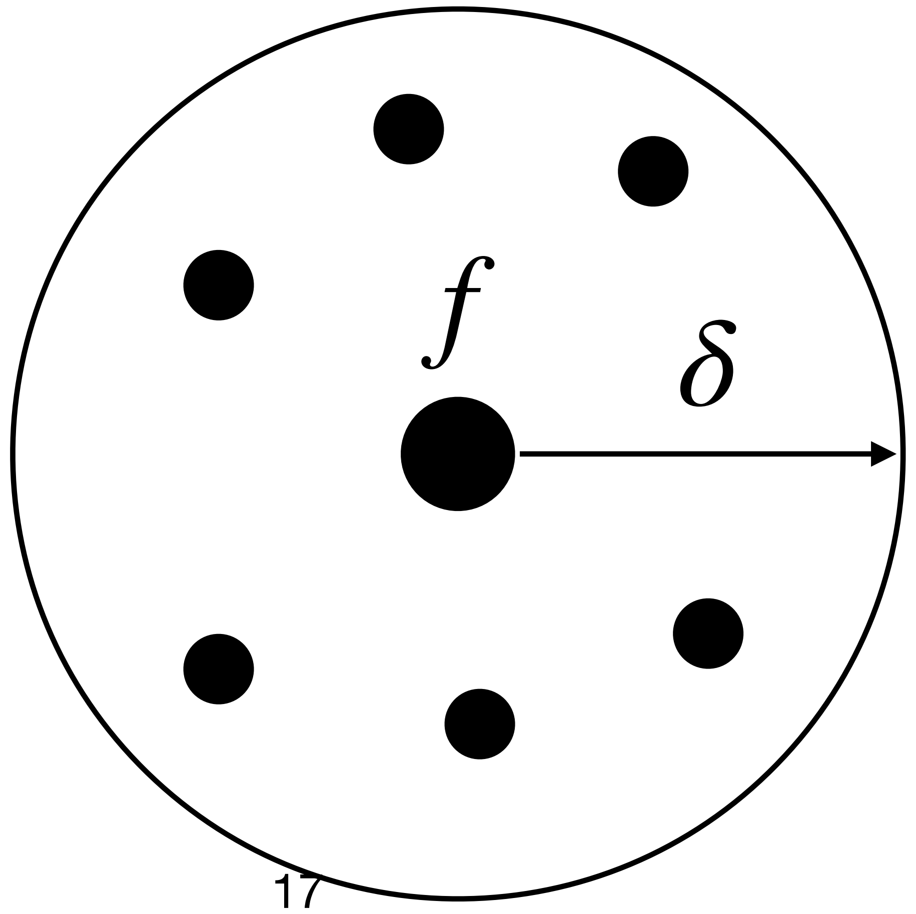

If every $\hat{v} \in \mathsf{List}(f, d, \delta)$ has $\hat{v}\,|_S \not\equiv \mathsf{Ans}$ then $\mathsf{Quotient}(f, \mathsf{Ans})$ is δ-far from RS

Domain shifting in unique decoding

Let $\delta^* := \dfrac{1-\rho^*}{2}, L \cap L^* = \varnothing$.

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At this distance, **at most one** codeword is close.

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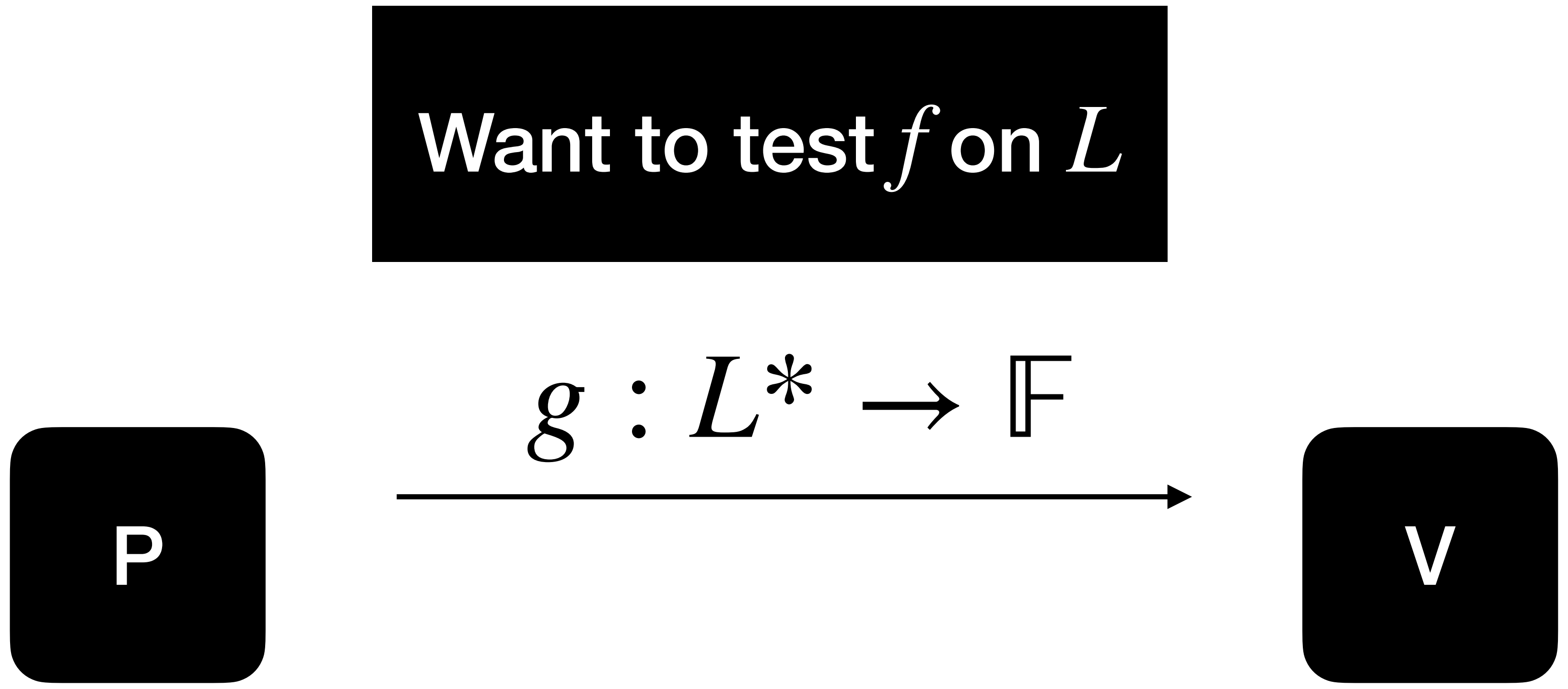

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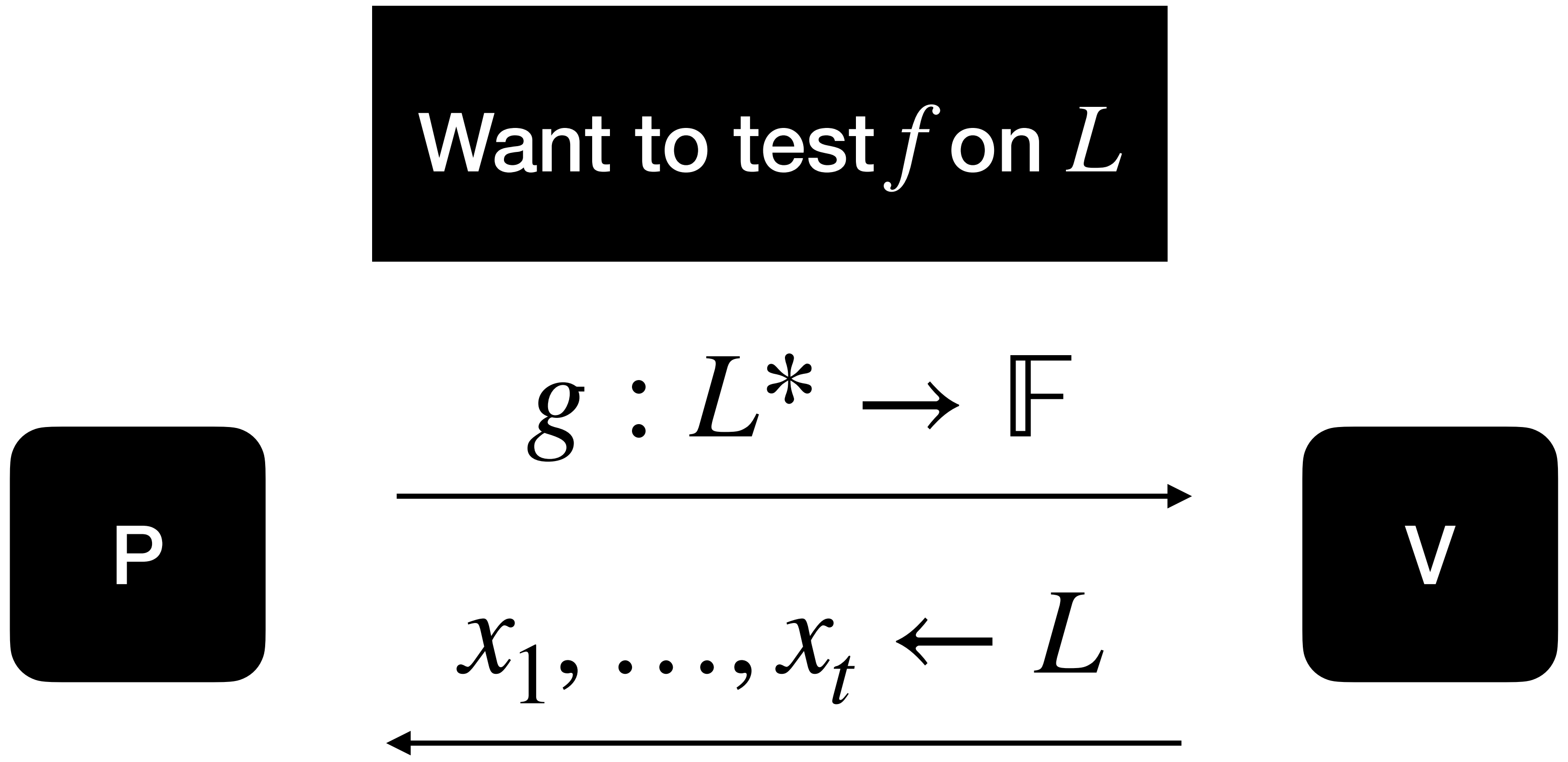

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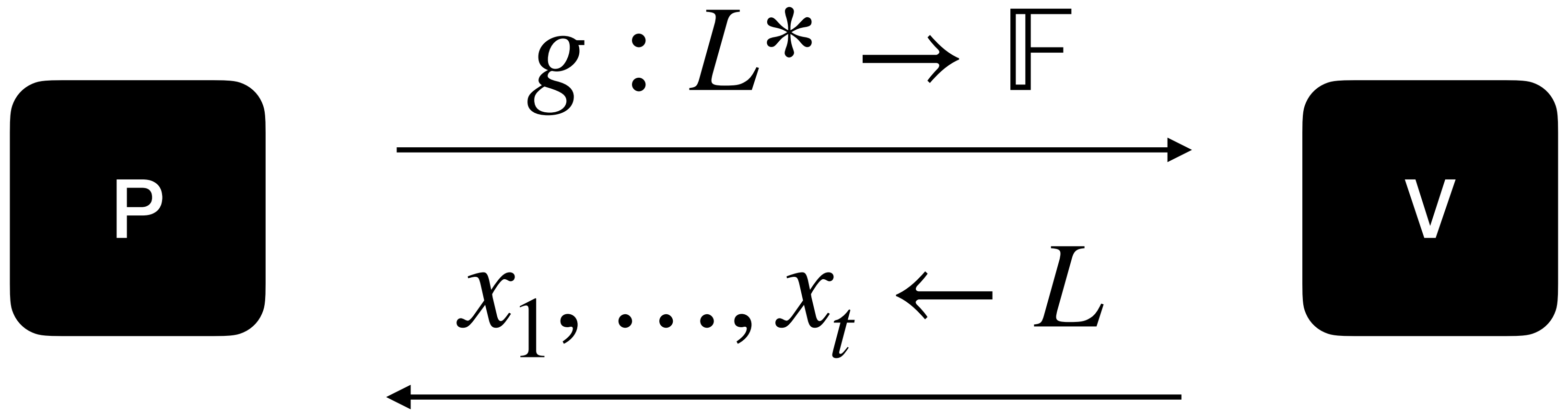

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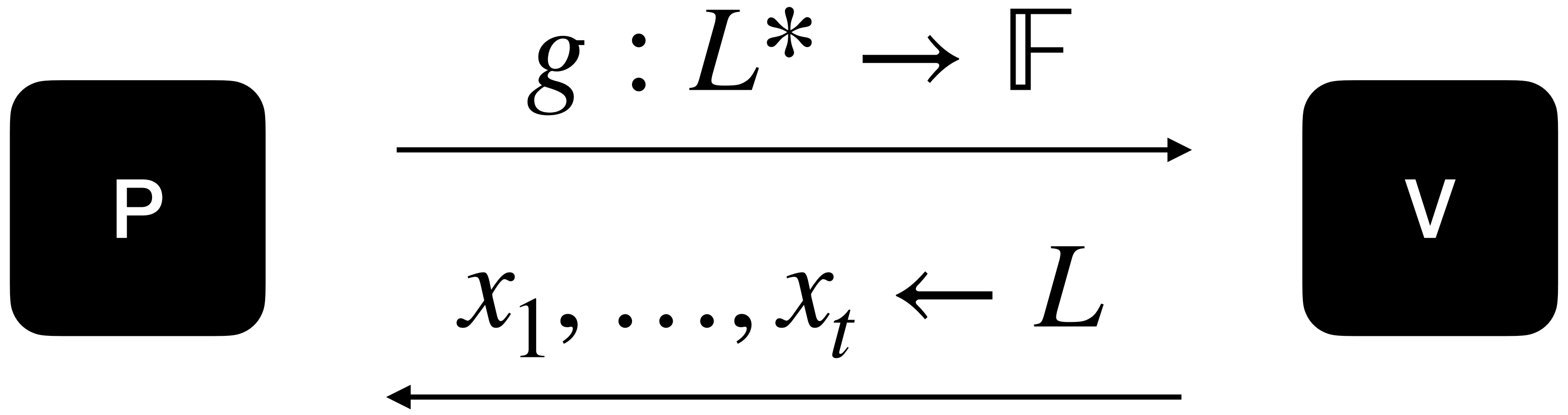

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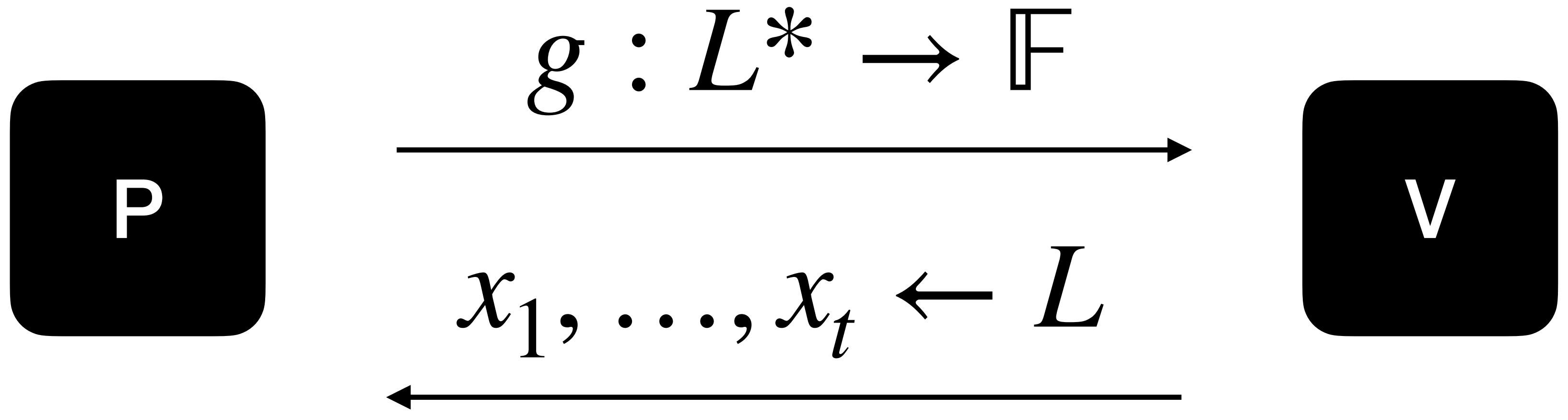

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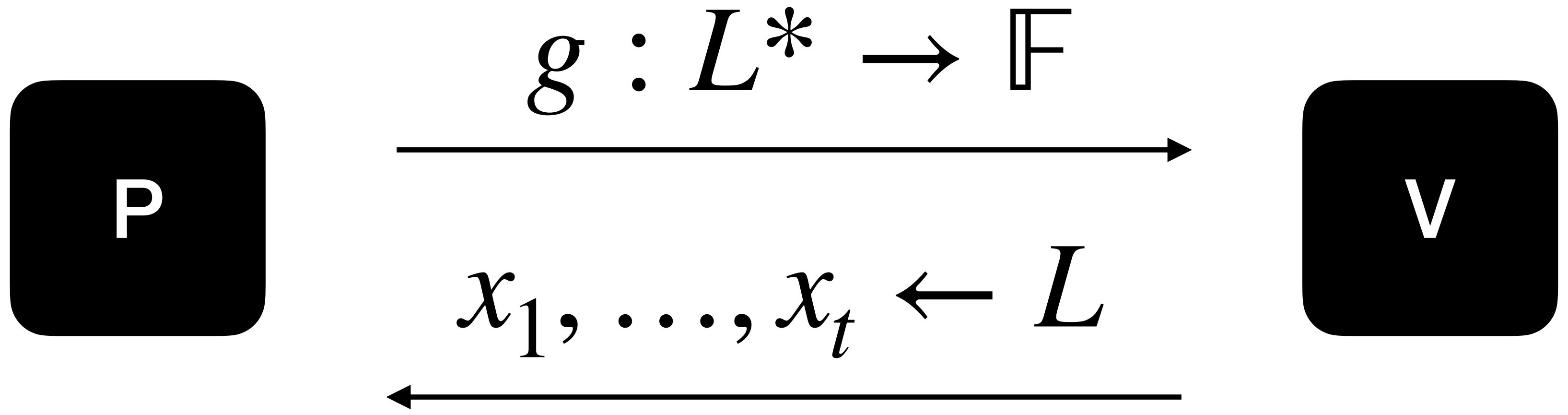

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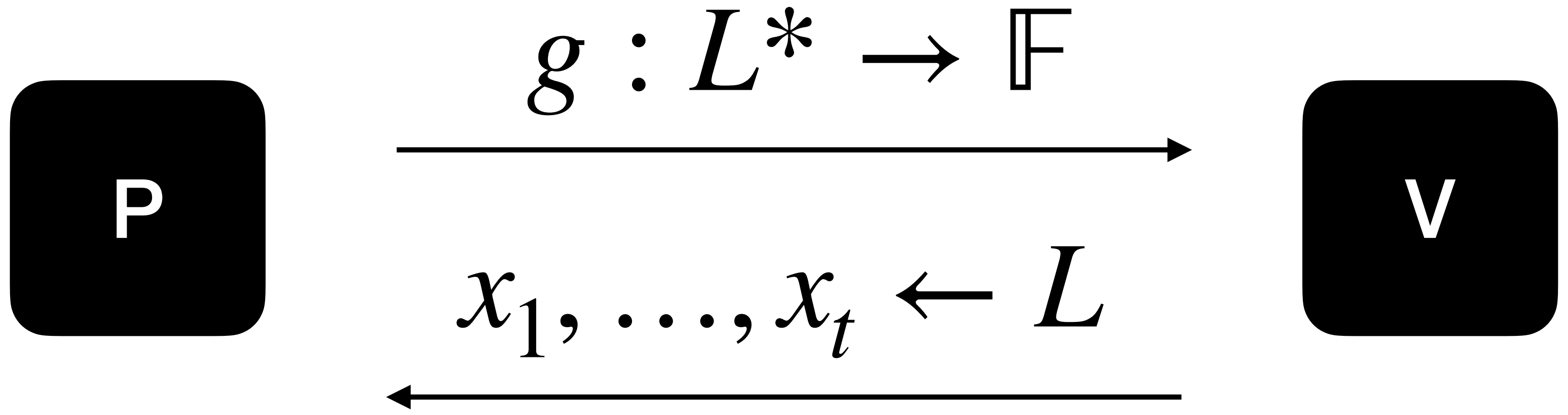

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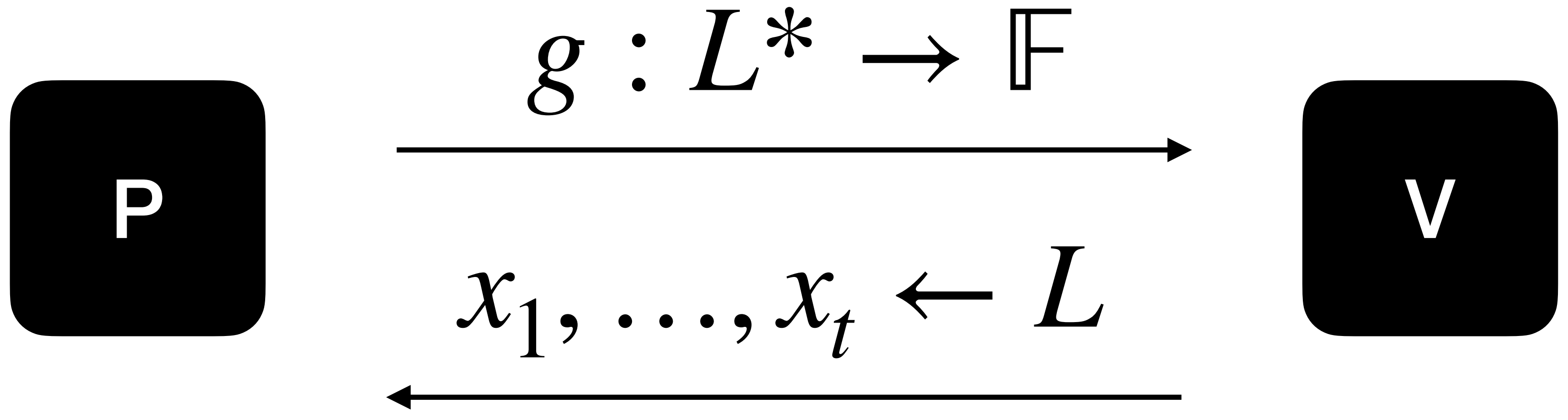

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Since $\Delta(f, \hat{v}\,|_L) > \delta$, then:

$$\begin{aligned}
&\Pr\left[g^* \text{ is } \delta^* \text{ close }\right] \\
&\leq \Pr\left[\forall i,\ \hat{v}(x_i) = y_i\right] \\
&= \Pr\left[\forall i,\ \hat{v}(x_i) = f(x_i)\right] \\
&\leq (1-\delta)^t
\end{aligned}$$

Out Of Domain sampling

Move to unique decoding range

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P

V

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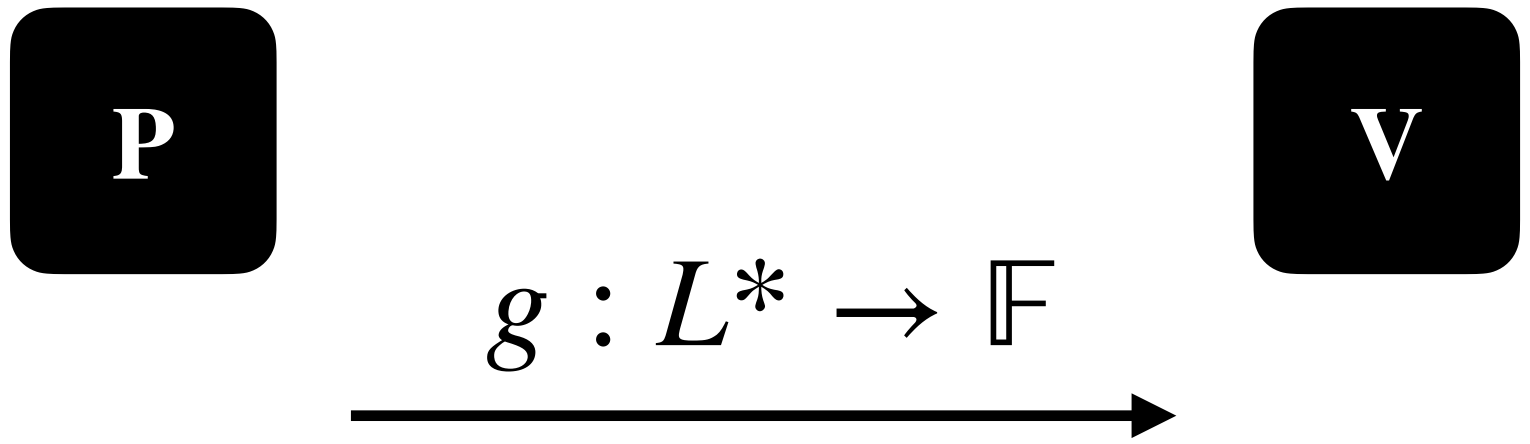

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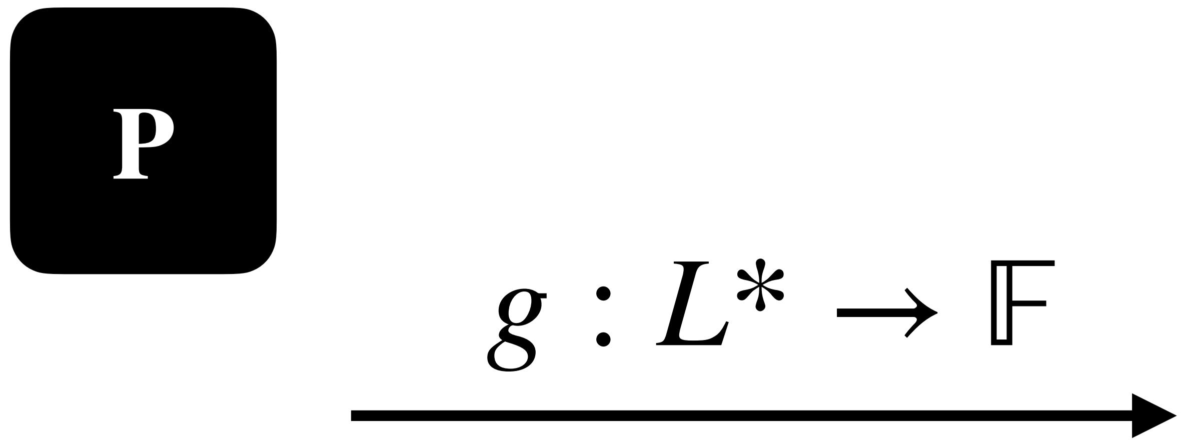

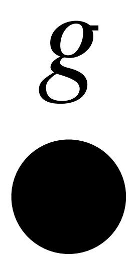

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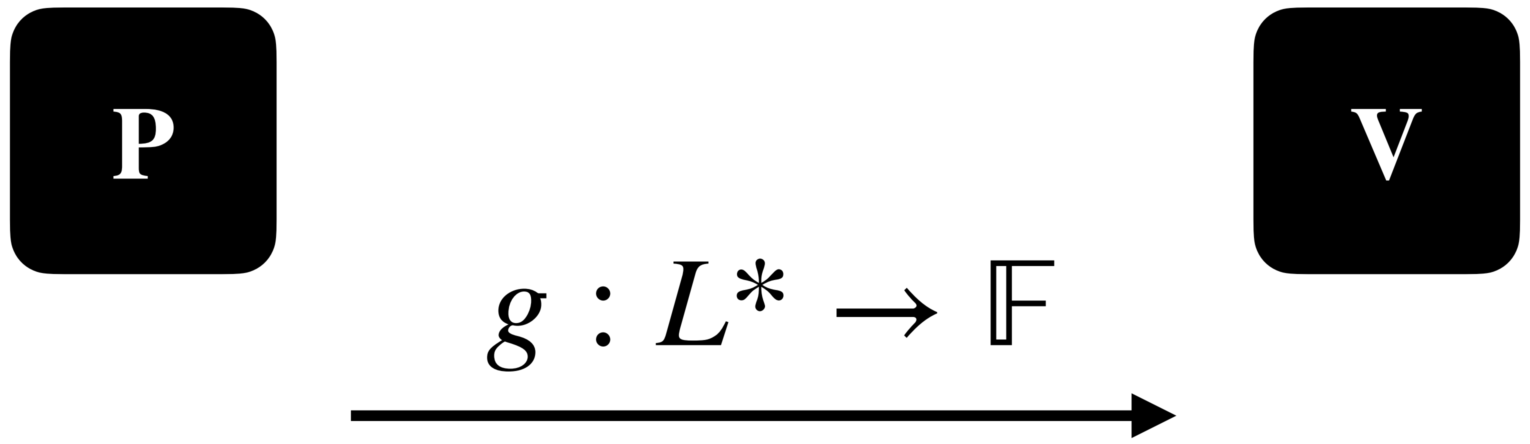

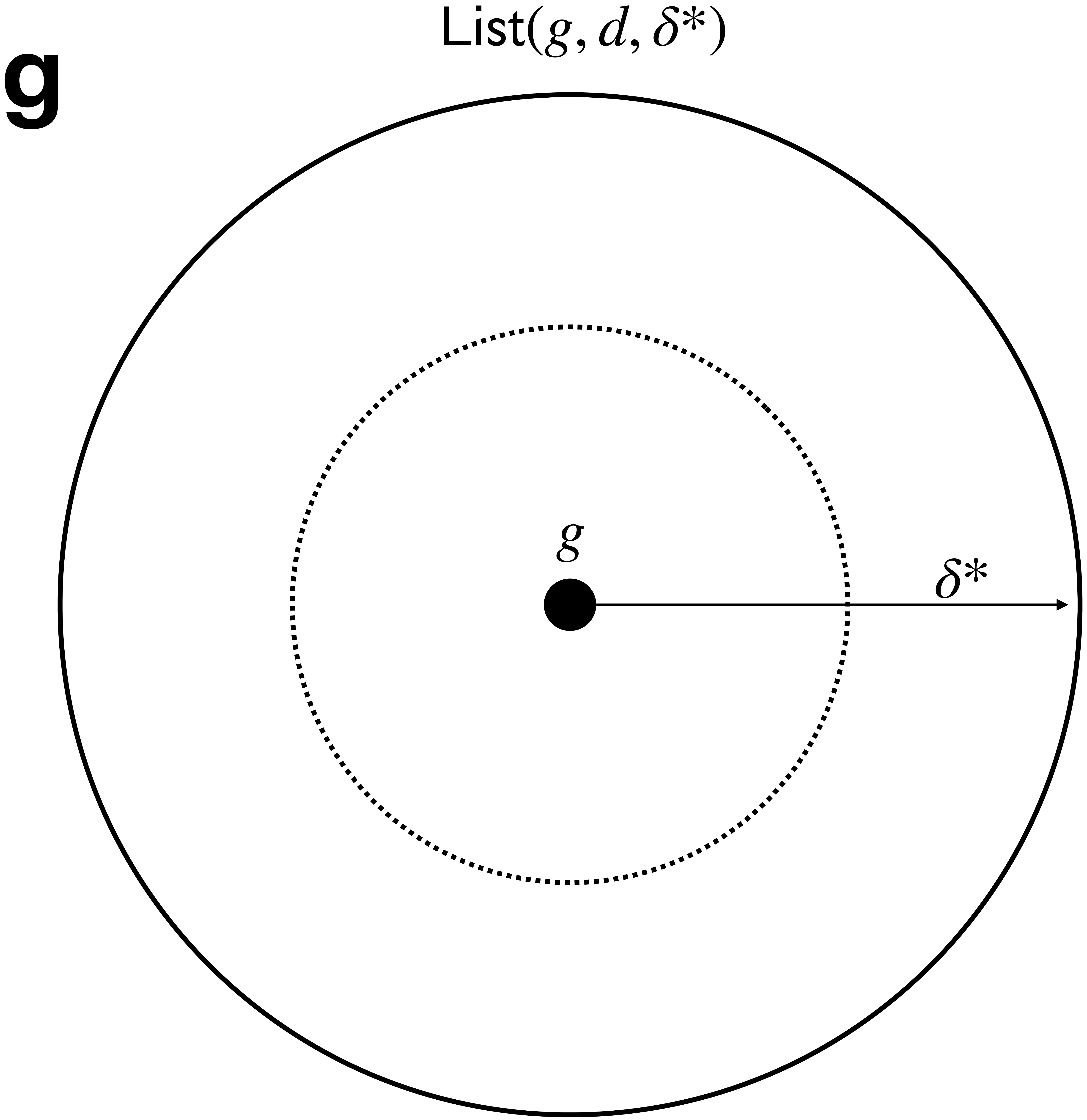

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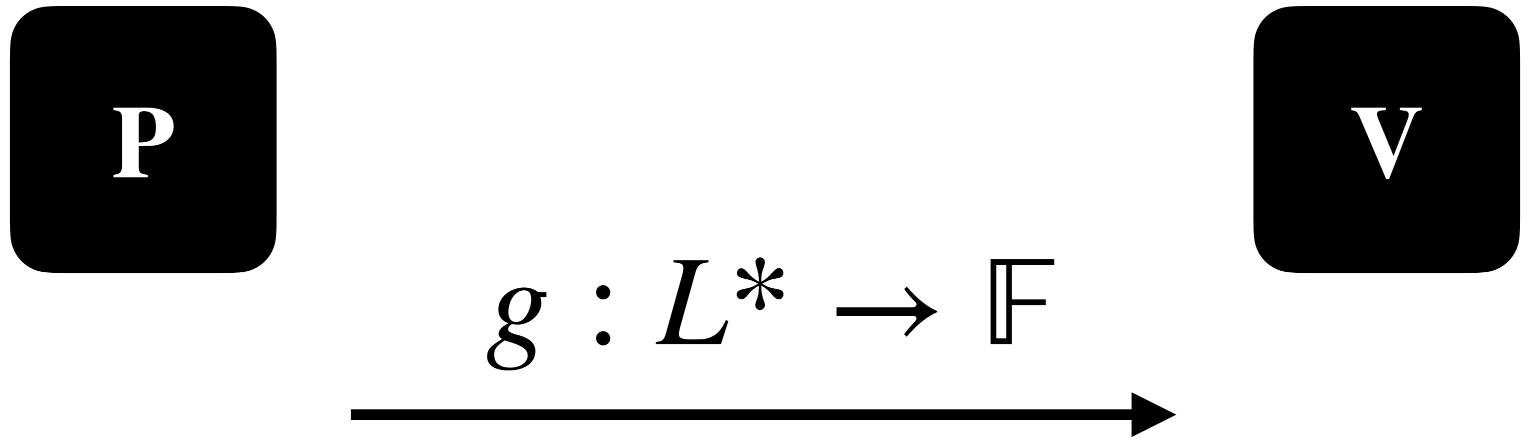

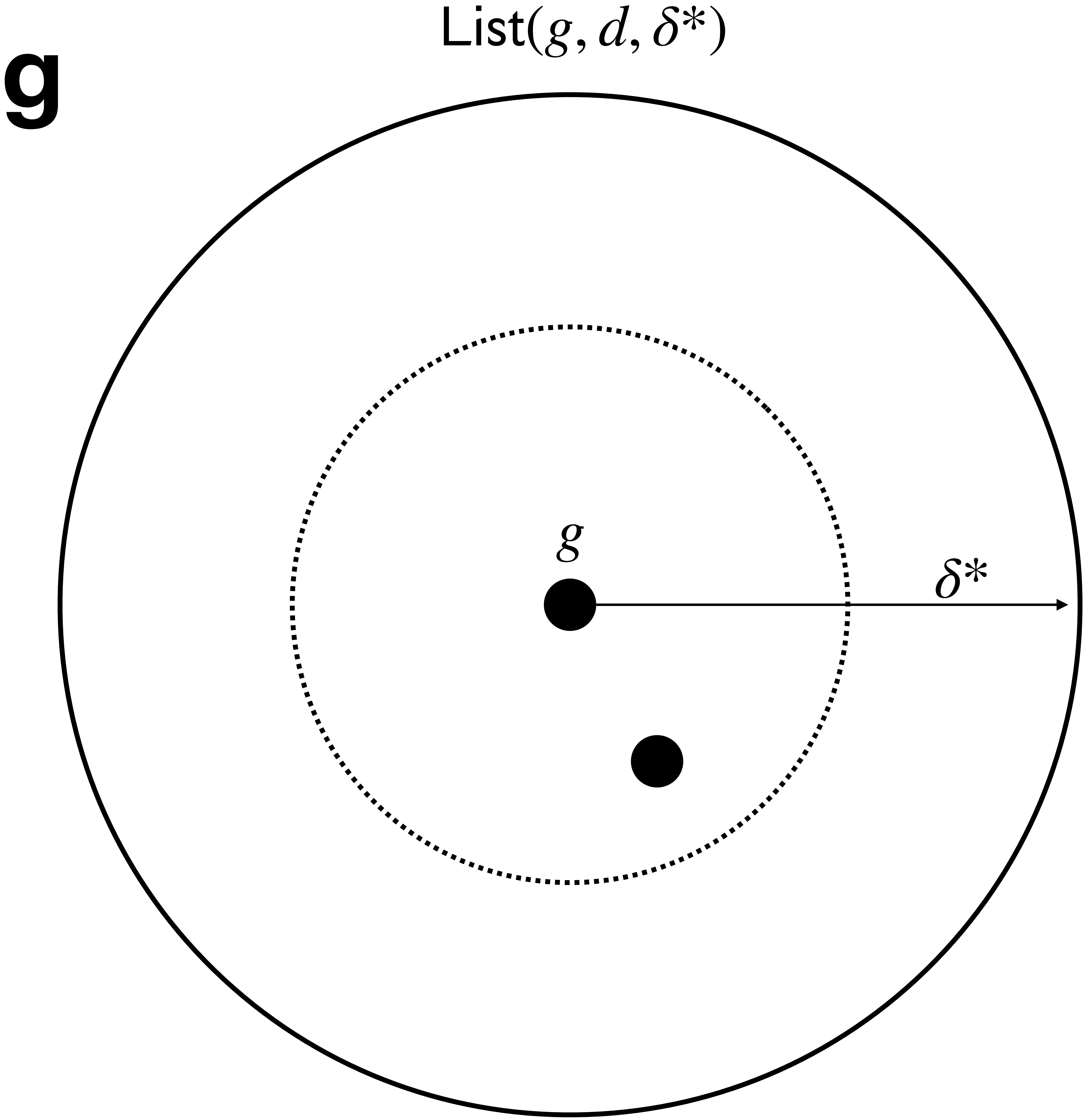

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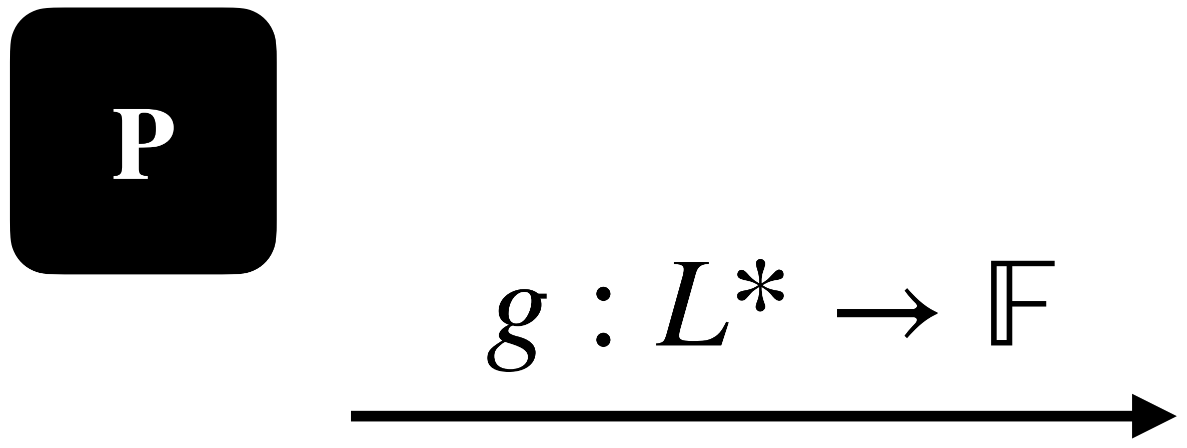

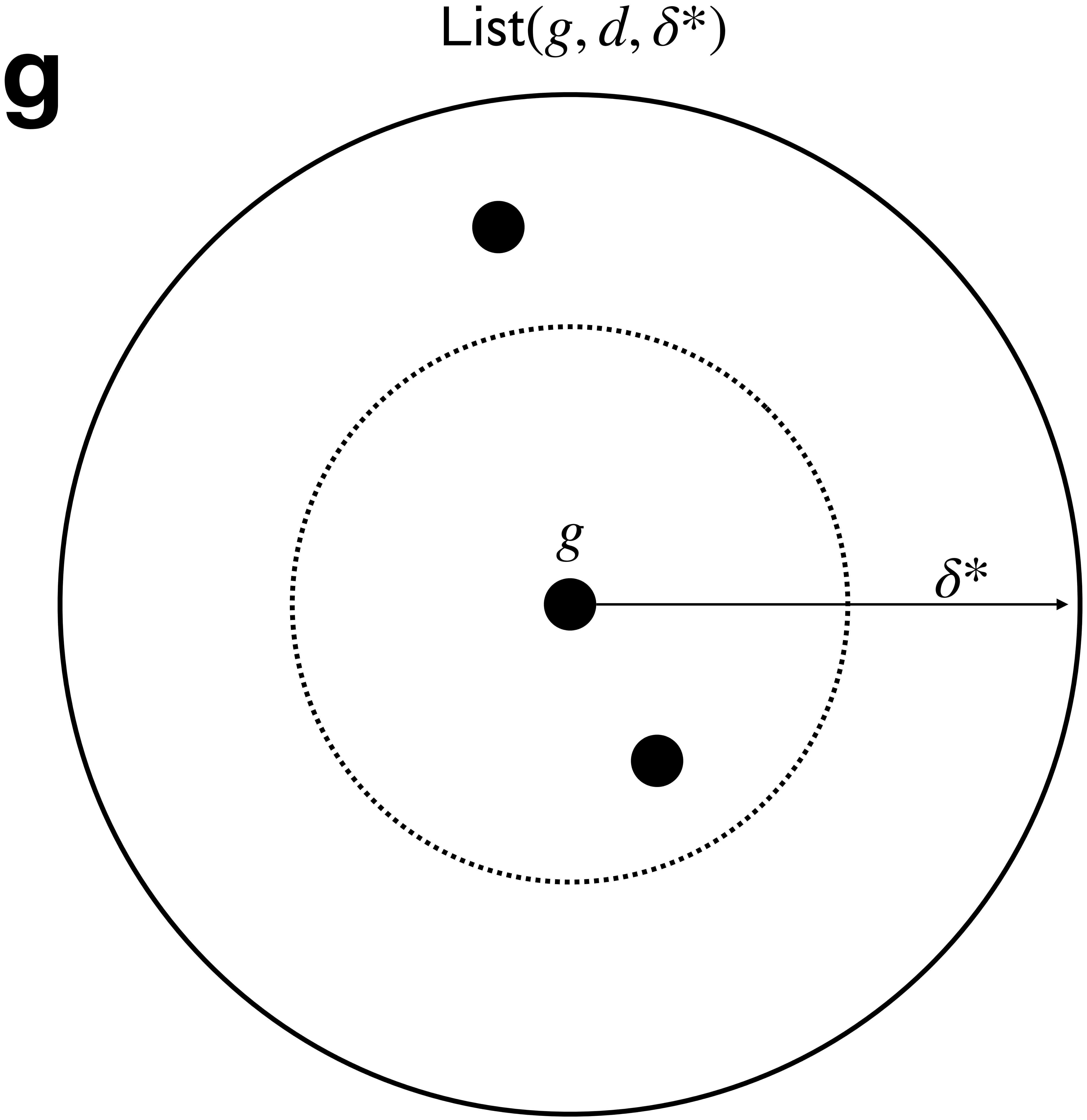

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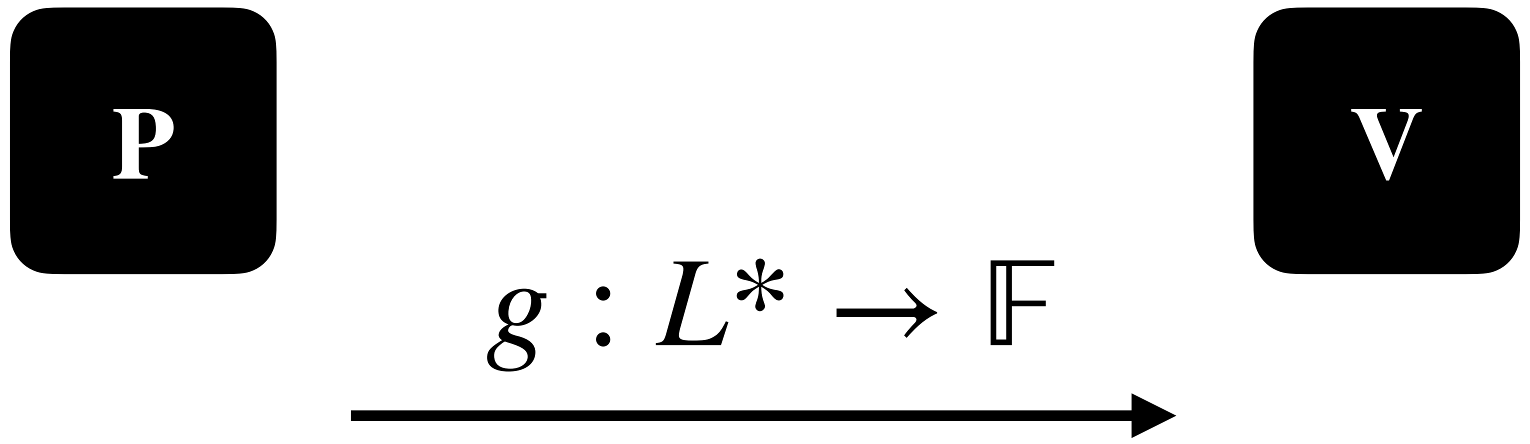
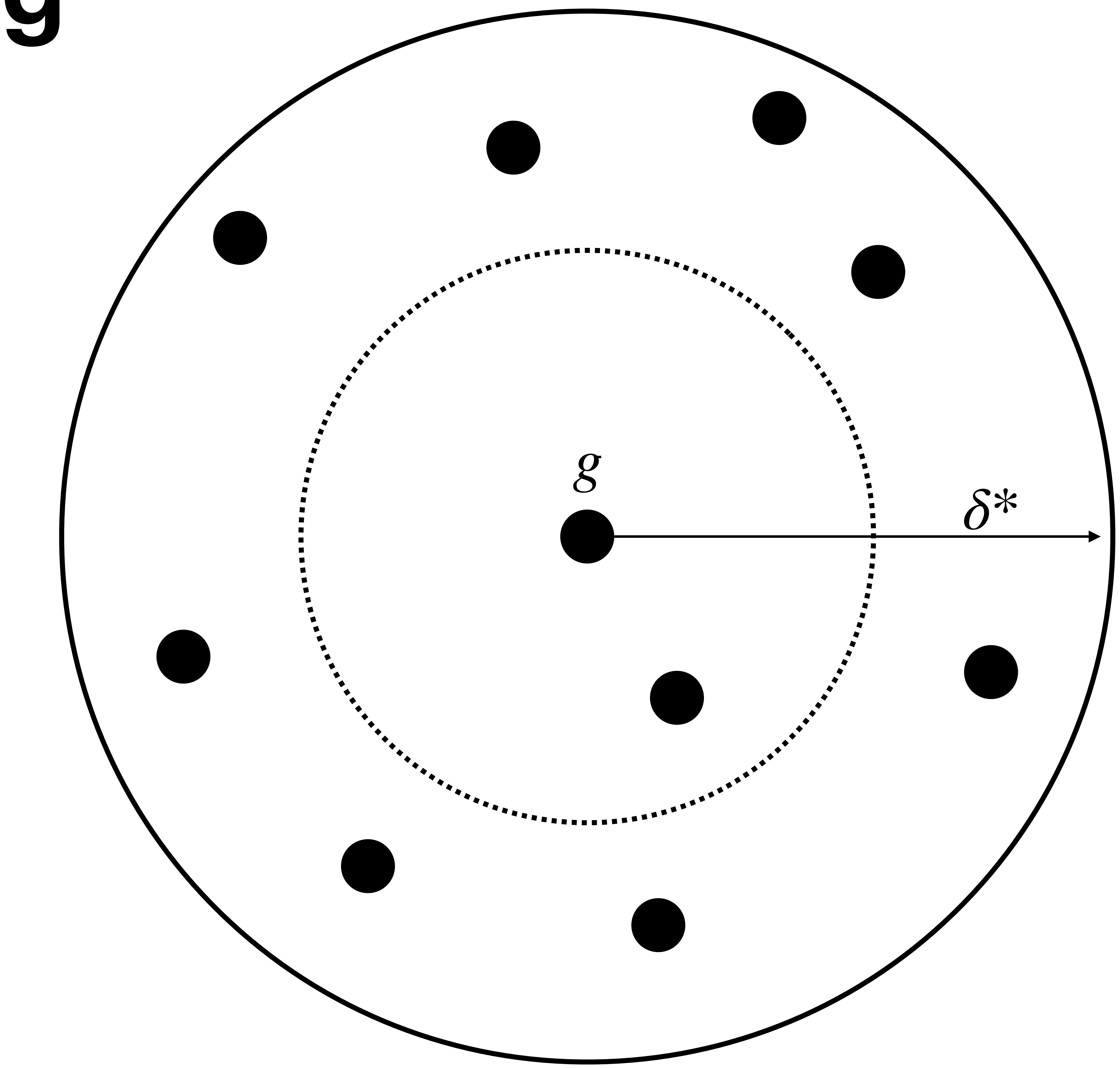


$\mathsf{List}(g, d, \delta^*)$



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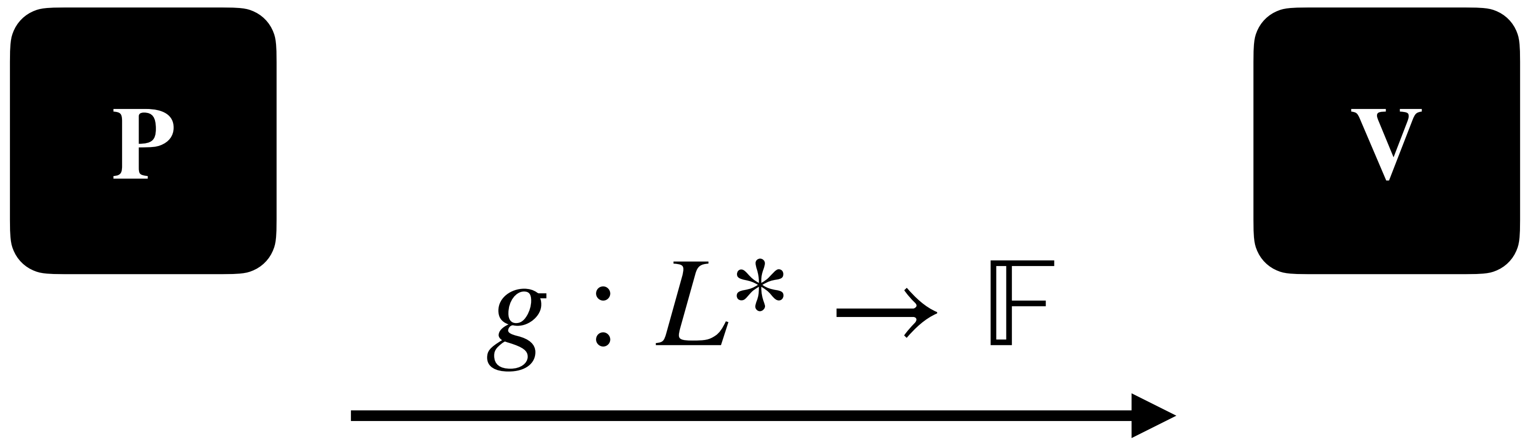

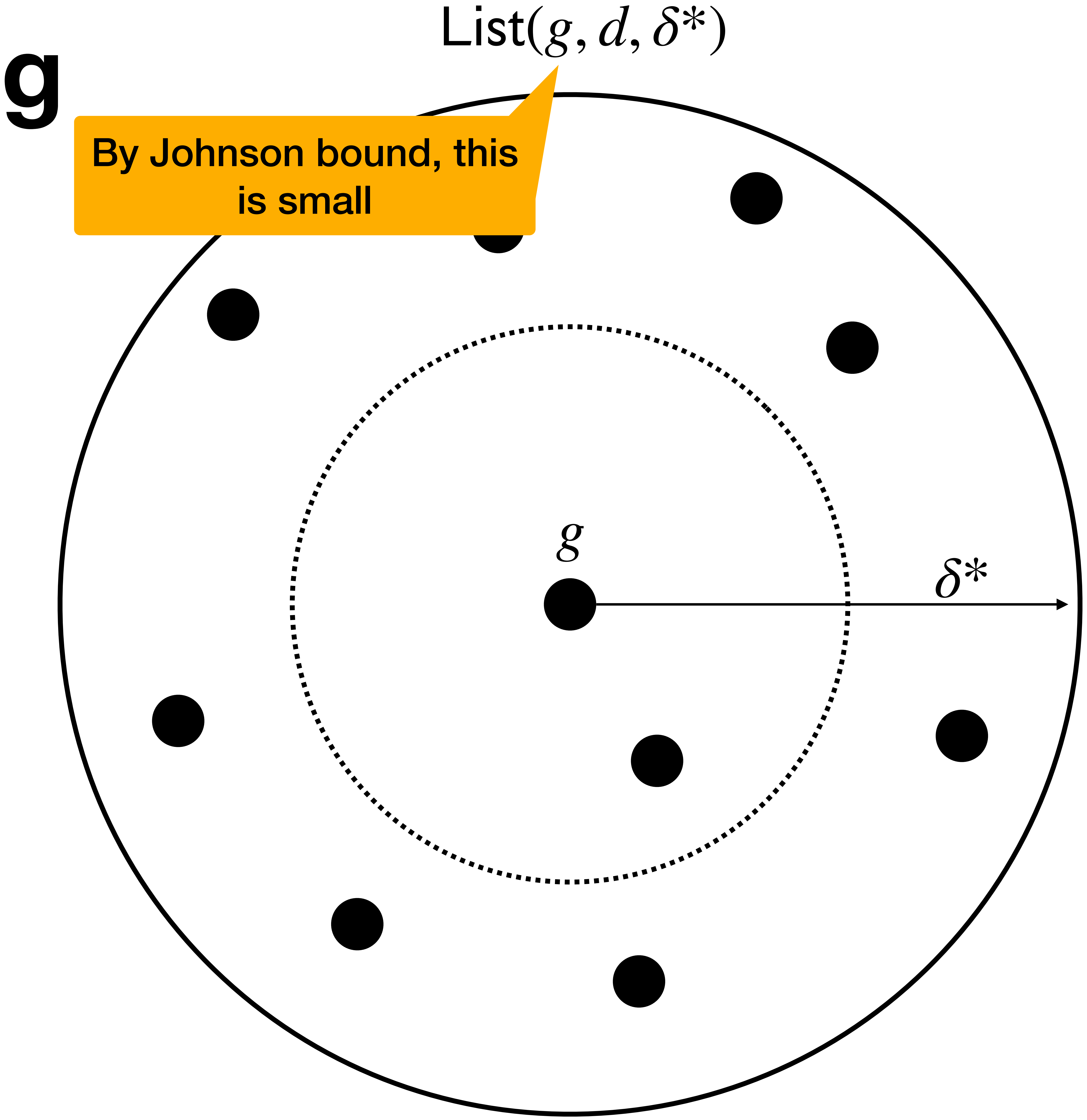

By Johnson bound, this is small

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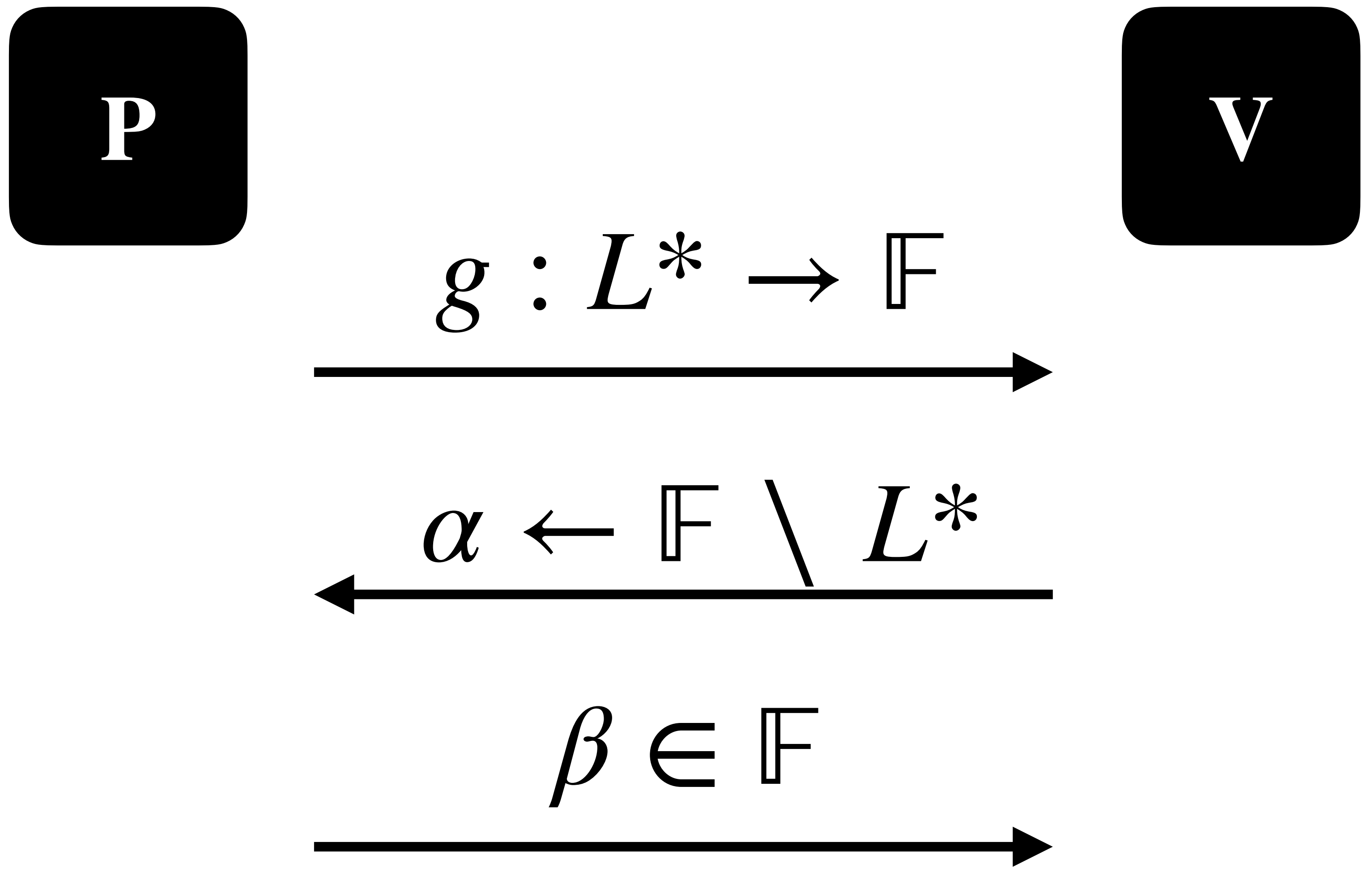
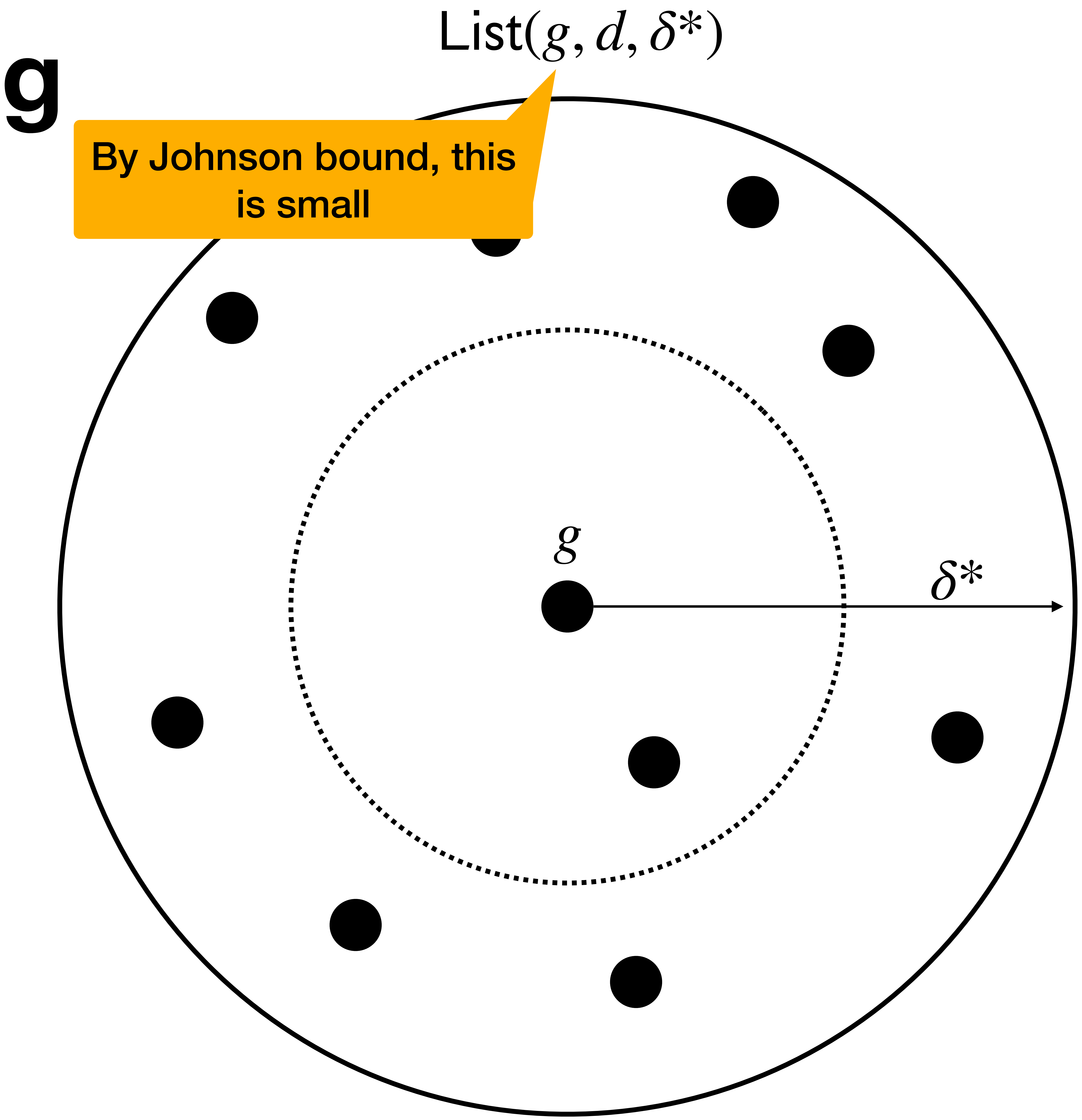


$g : L^* \to \mathbb{F}$

$\alpha \leftarrow \mathbb{F} \setminus L^*$

$\beta \in \mathbb{F}$



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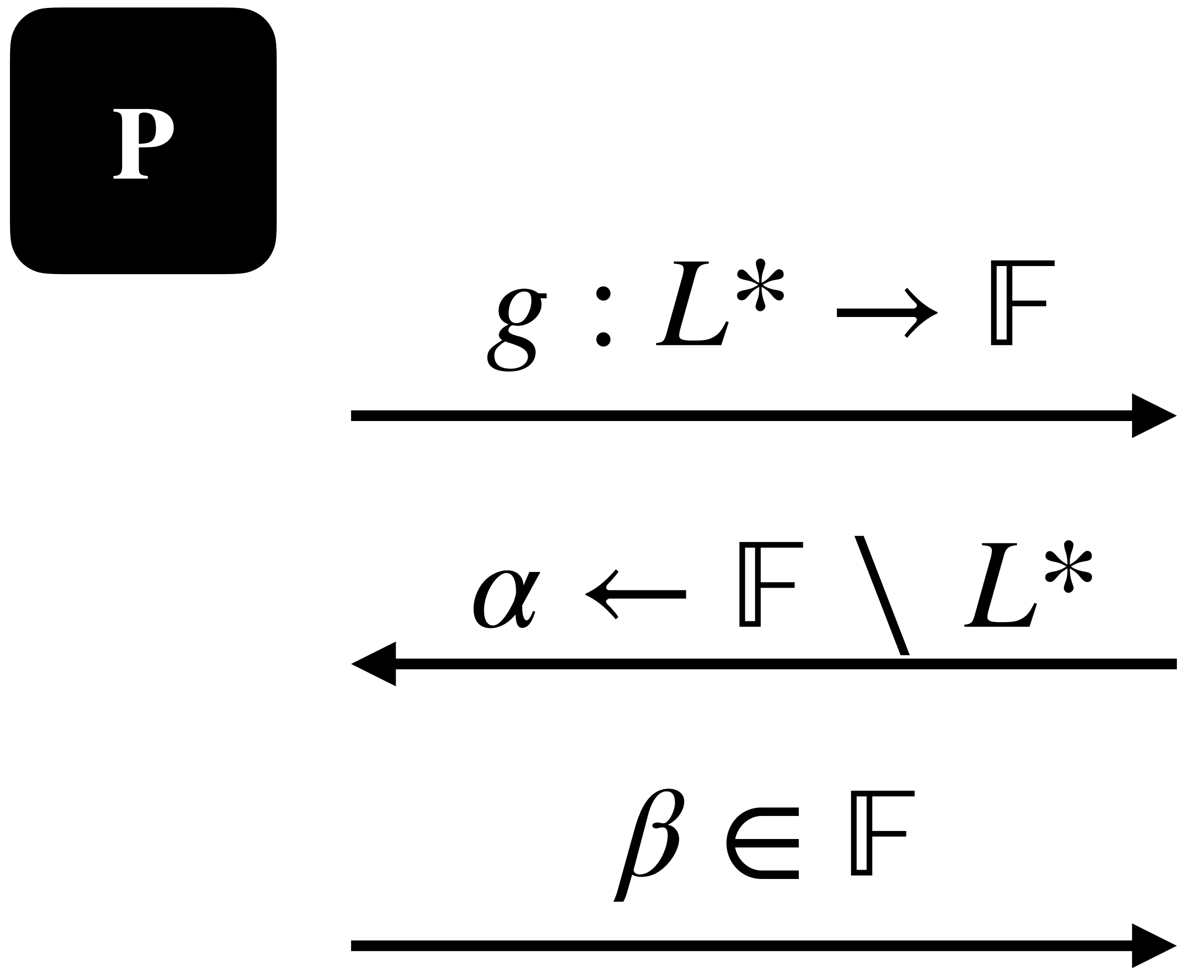
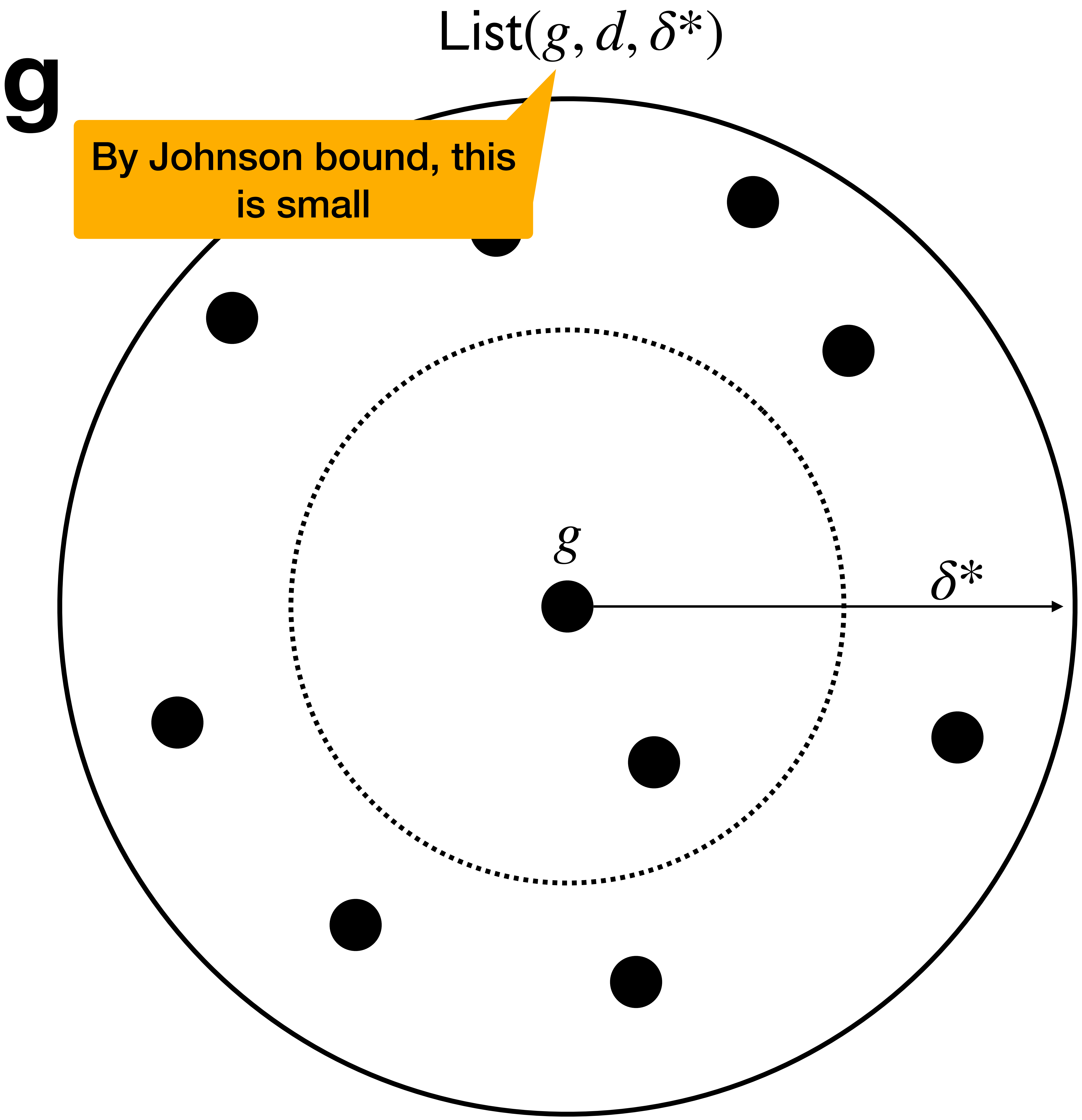


- By fundamental theorem of algebra of w.h.p. no pair $\hat{u}, \hat{v}$ with $\hat{u}(\alpha) = \hat{v}(\alpha)$
- Prover "chooses" which codeword $\hat{u}$ it "commits" to



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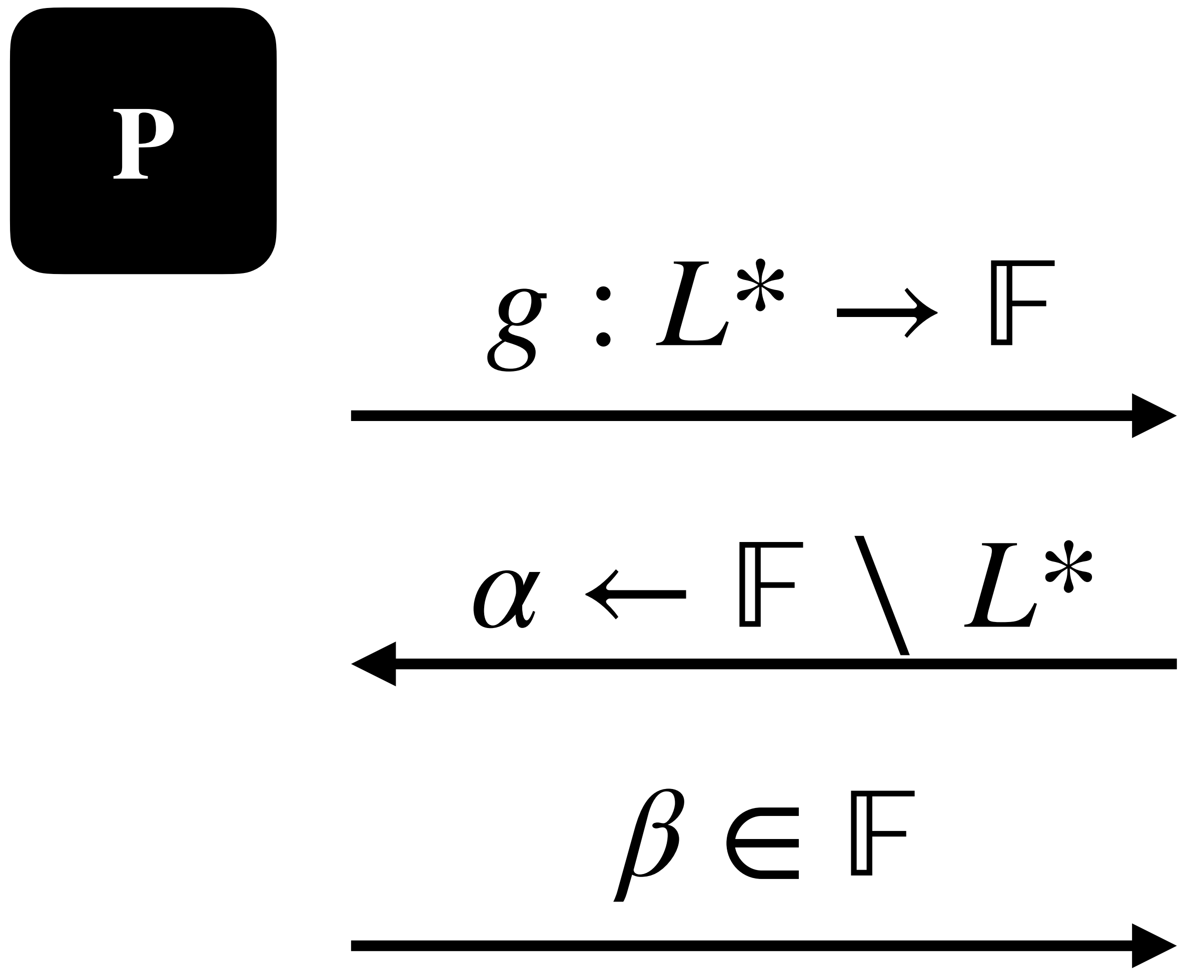

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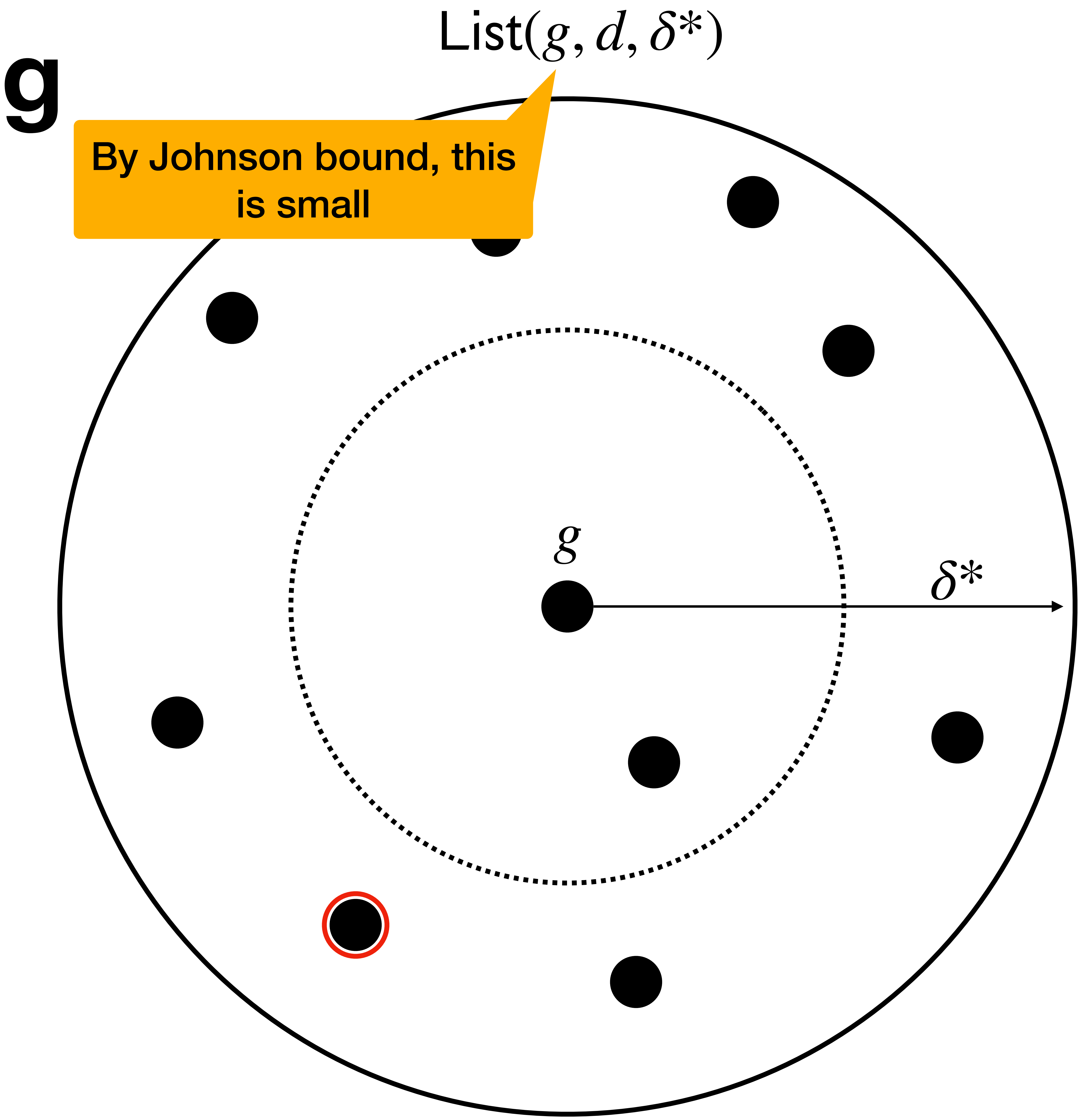

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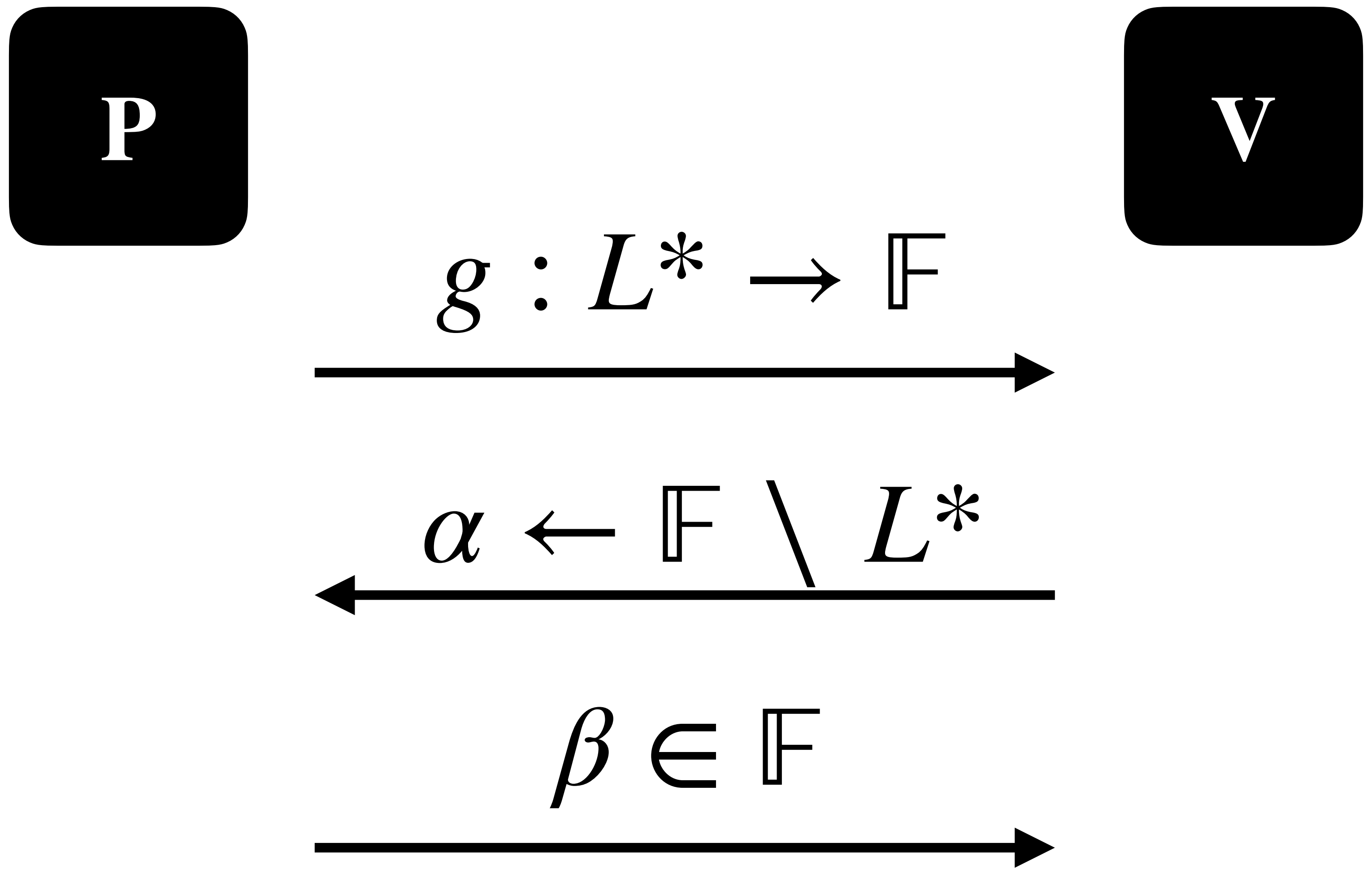

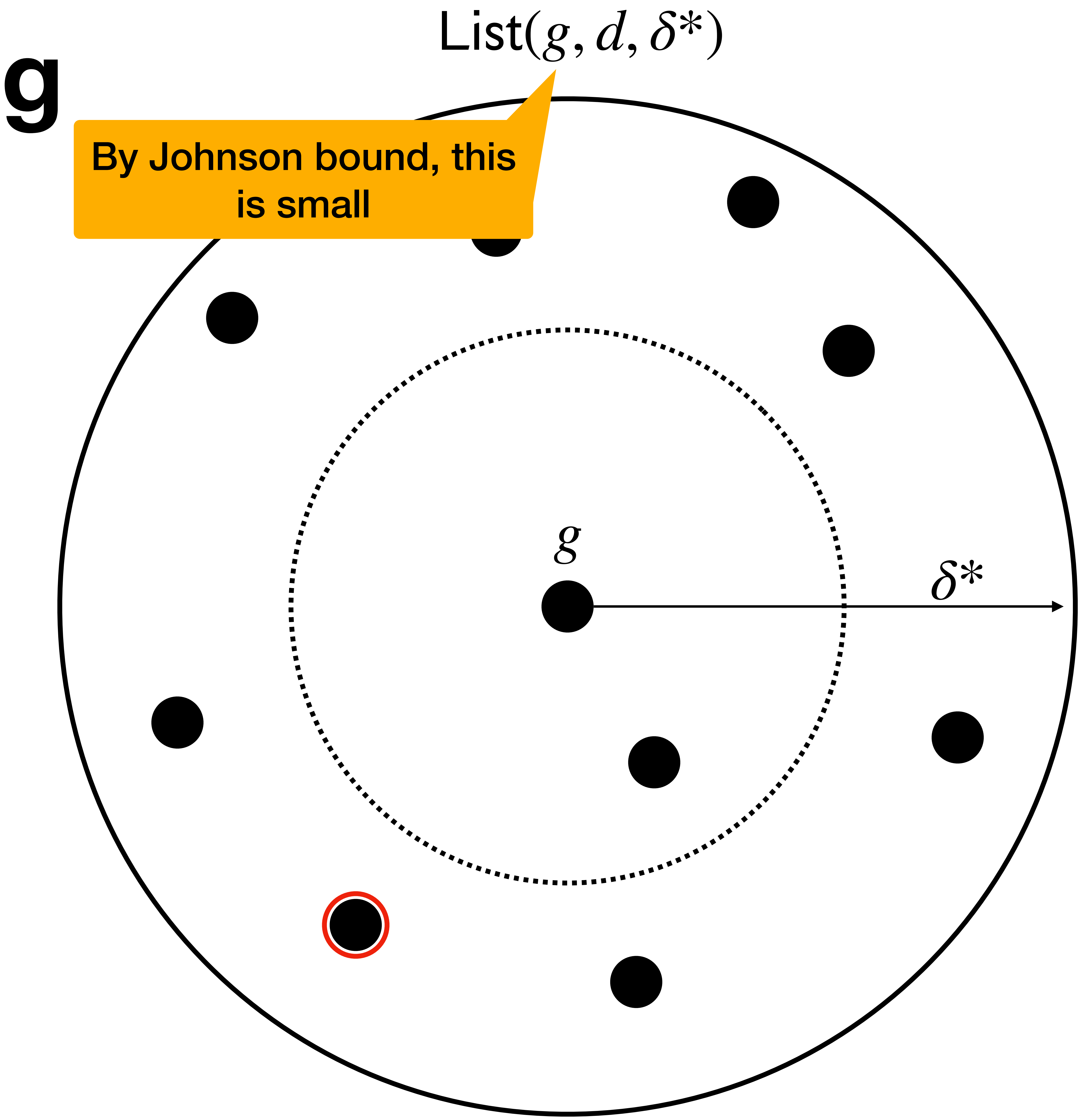

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- Prover "chooses" which codeword $\hat{u}$ it "commits" to

Use Quotient$(g, \alpha \mapsto \beta)$ to enforce the constraint

Domain shifting in list decoding

Let $\delta^* := 1 - \sqrt{\rho^*}, L \cap L^* = \varnothing$.

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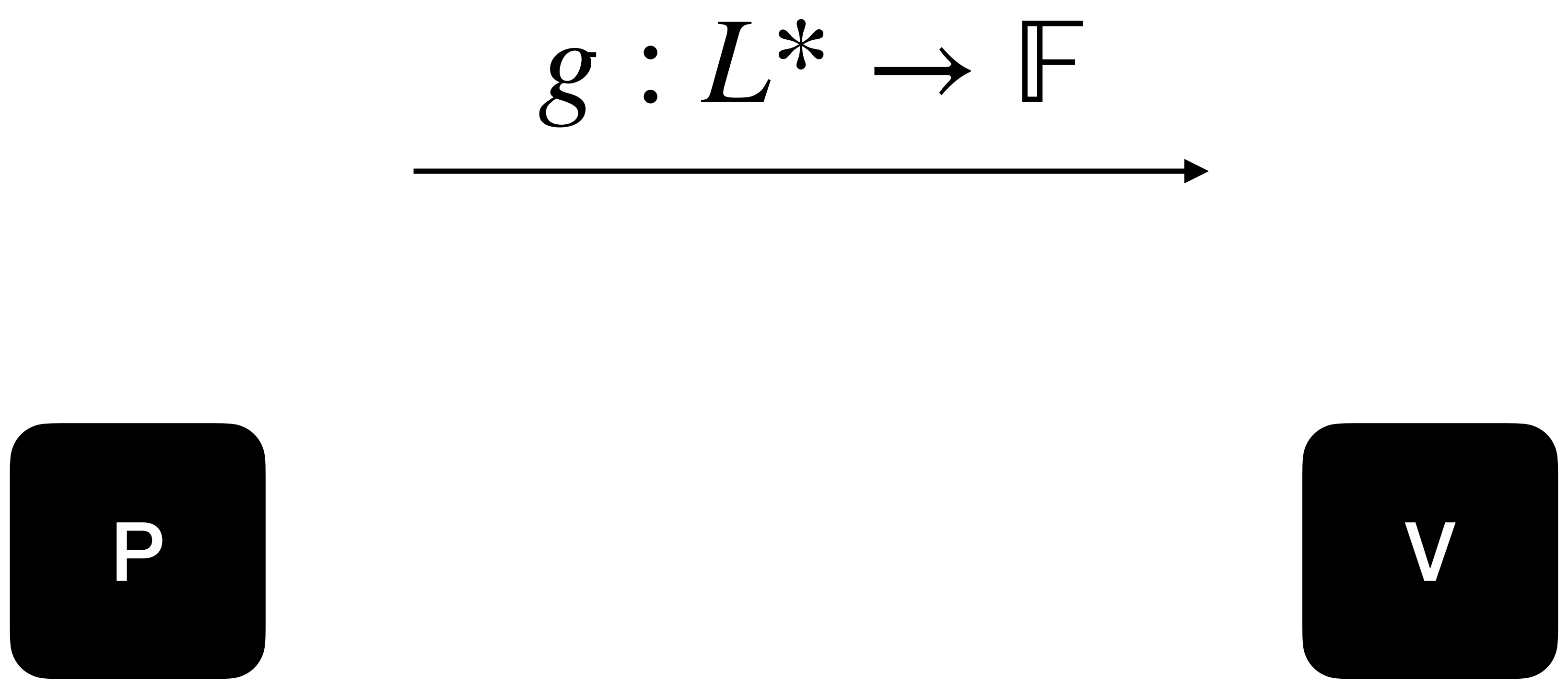

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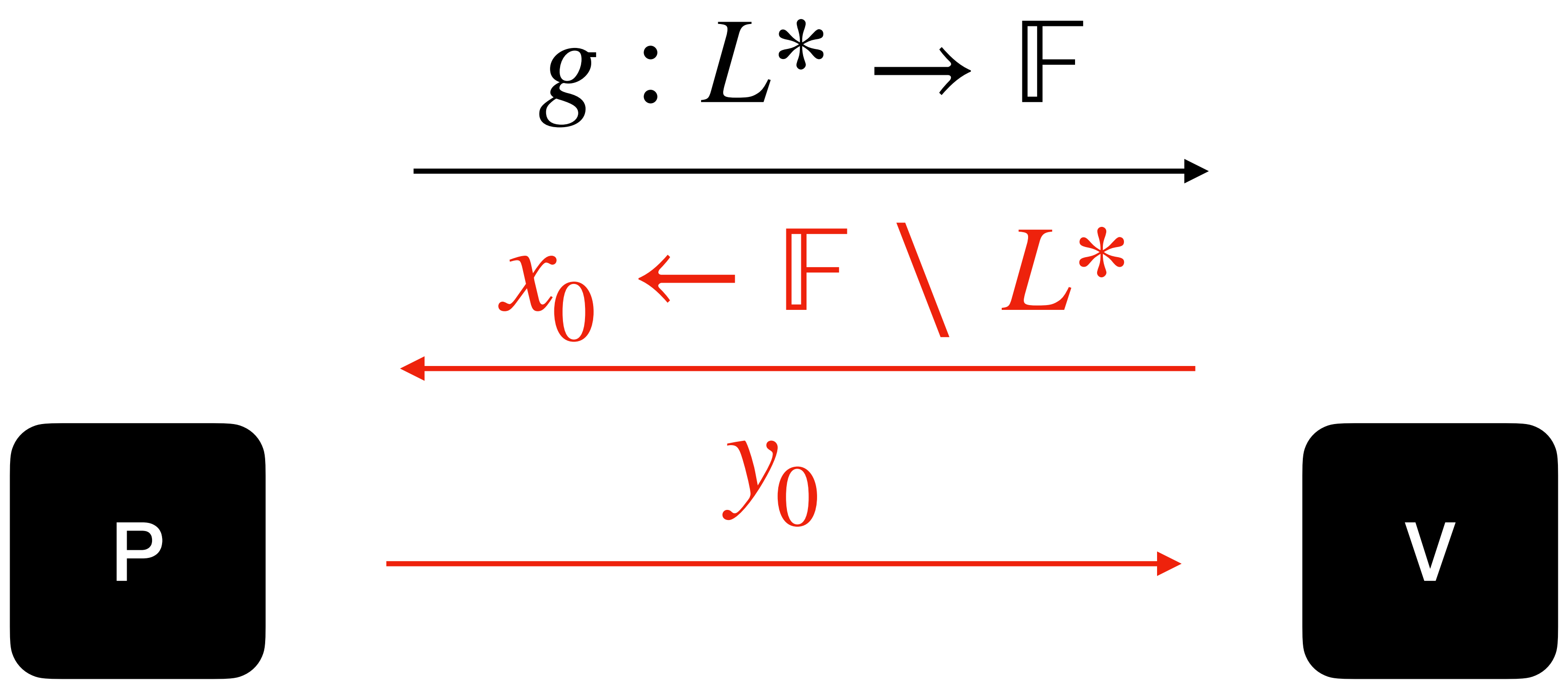

P

$g : L^* \to \mathbb{F}$

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y_0

V

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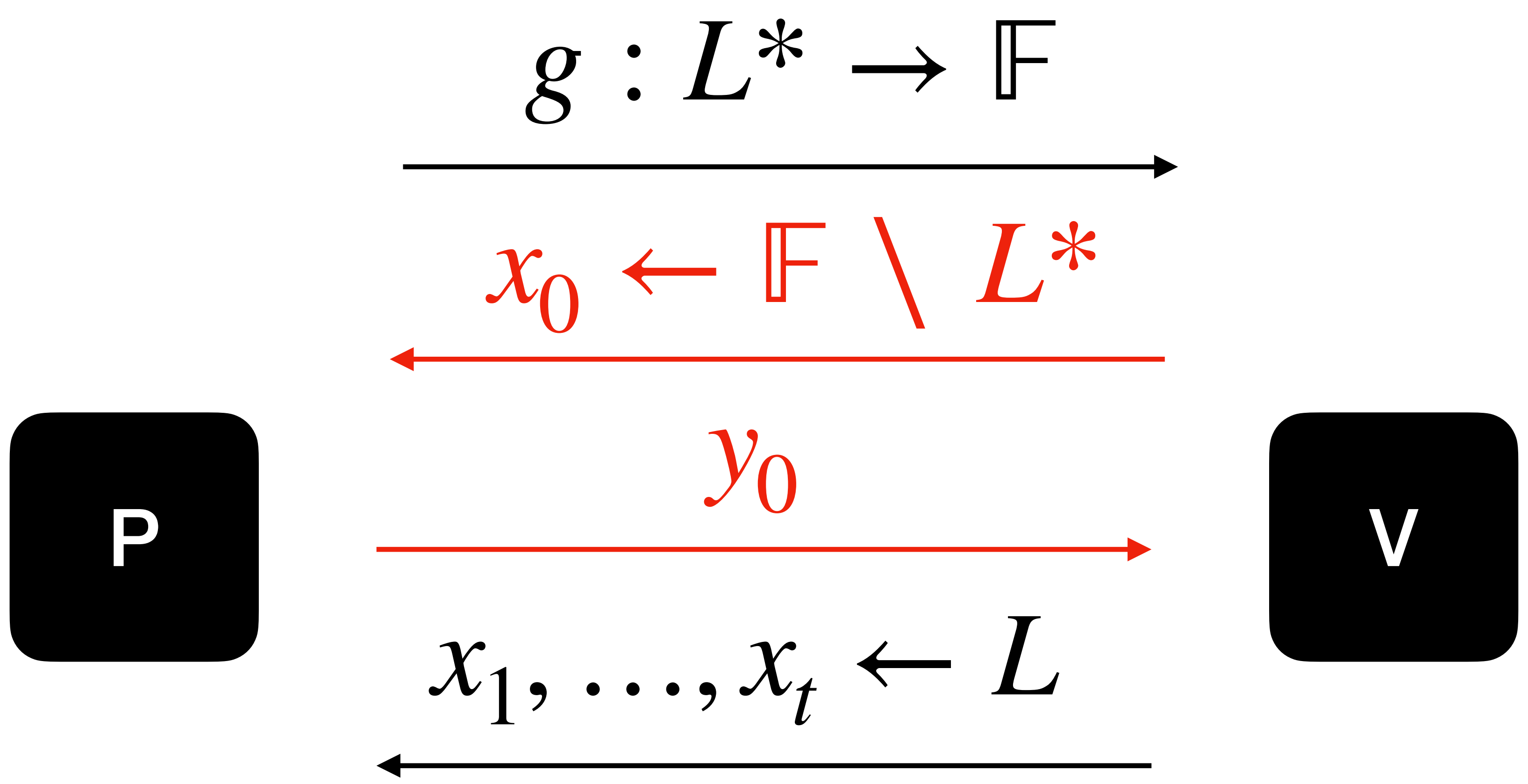

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Domain shifting in list decoding

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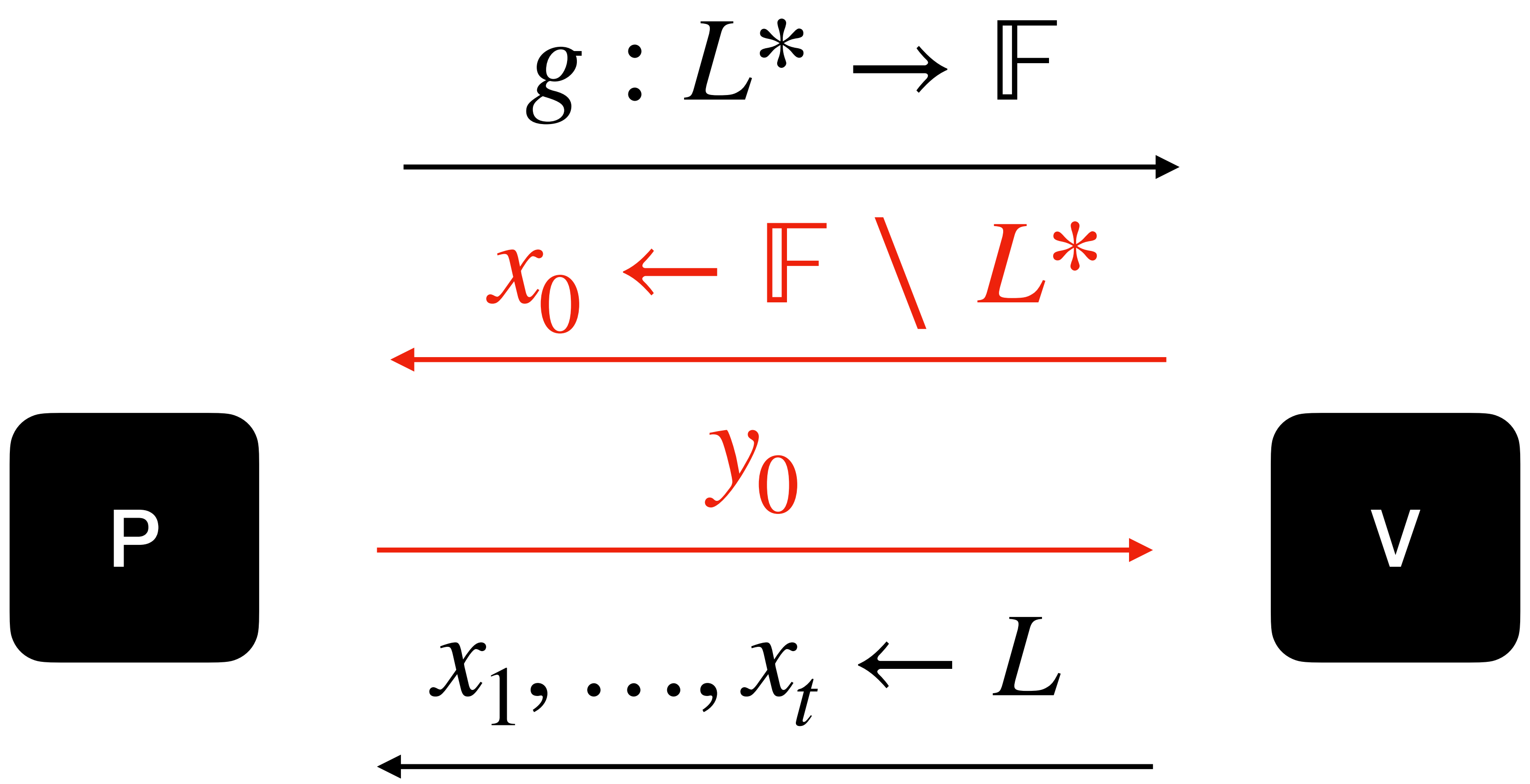

P

$g : L^* \to \mathbb{F}$

$x_0 \leftarrow \mathbb{F} \setminus L^*$

y_0

$x_1, \ldots, x_t \leftarrow L$

V

Query $y_i = f(x_i)$

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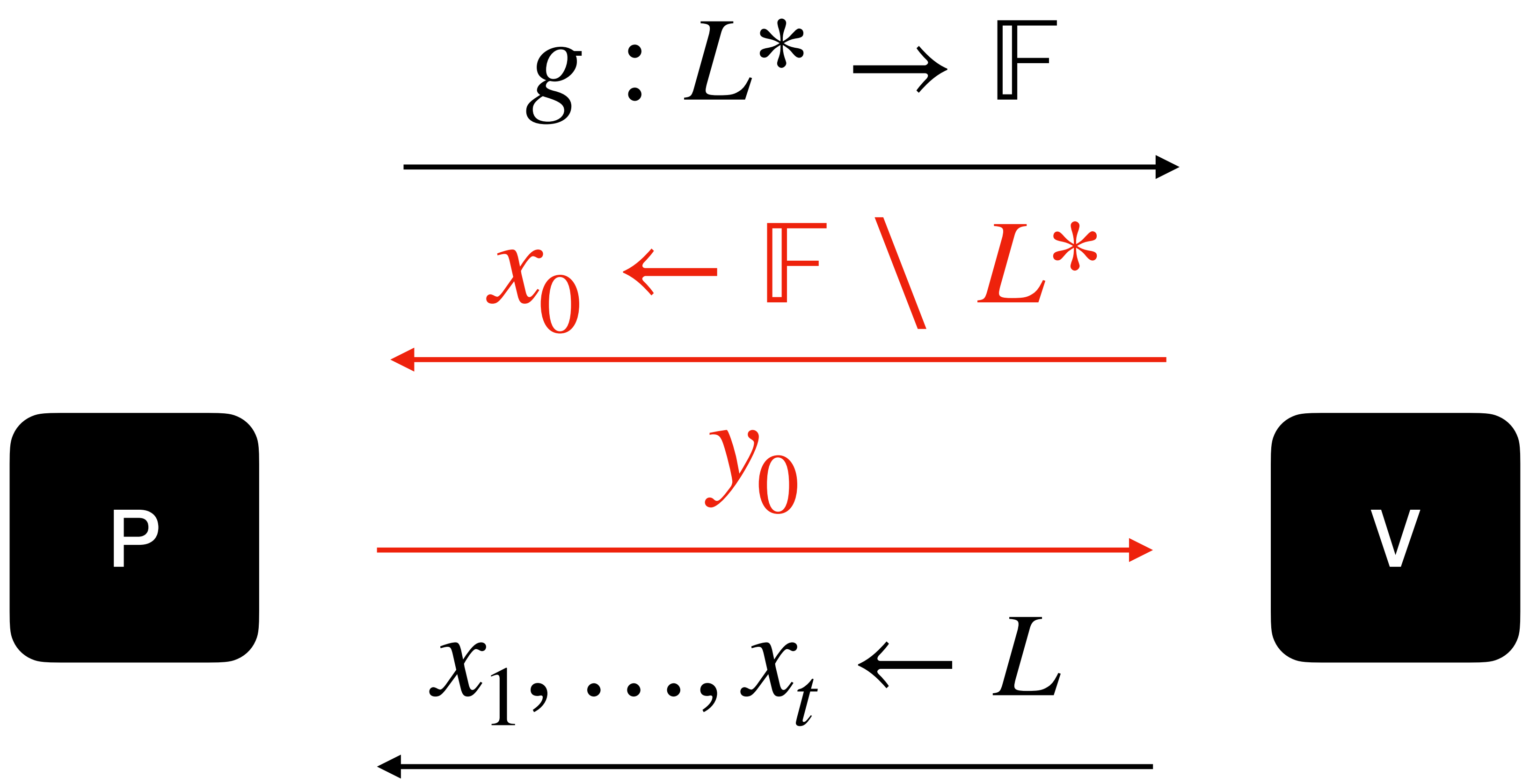

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Test g^* on L^*

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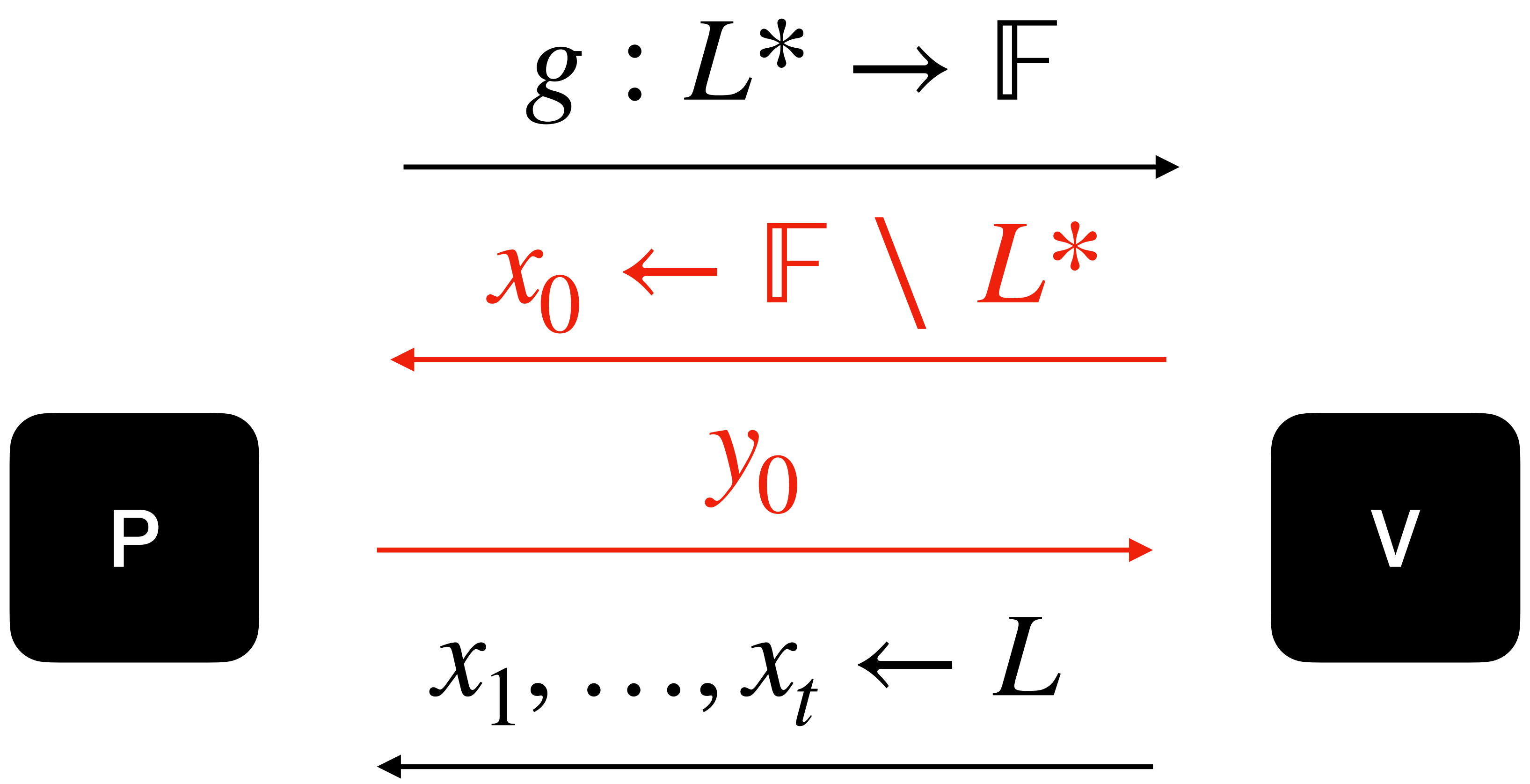

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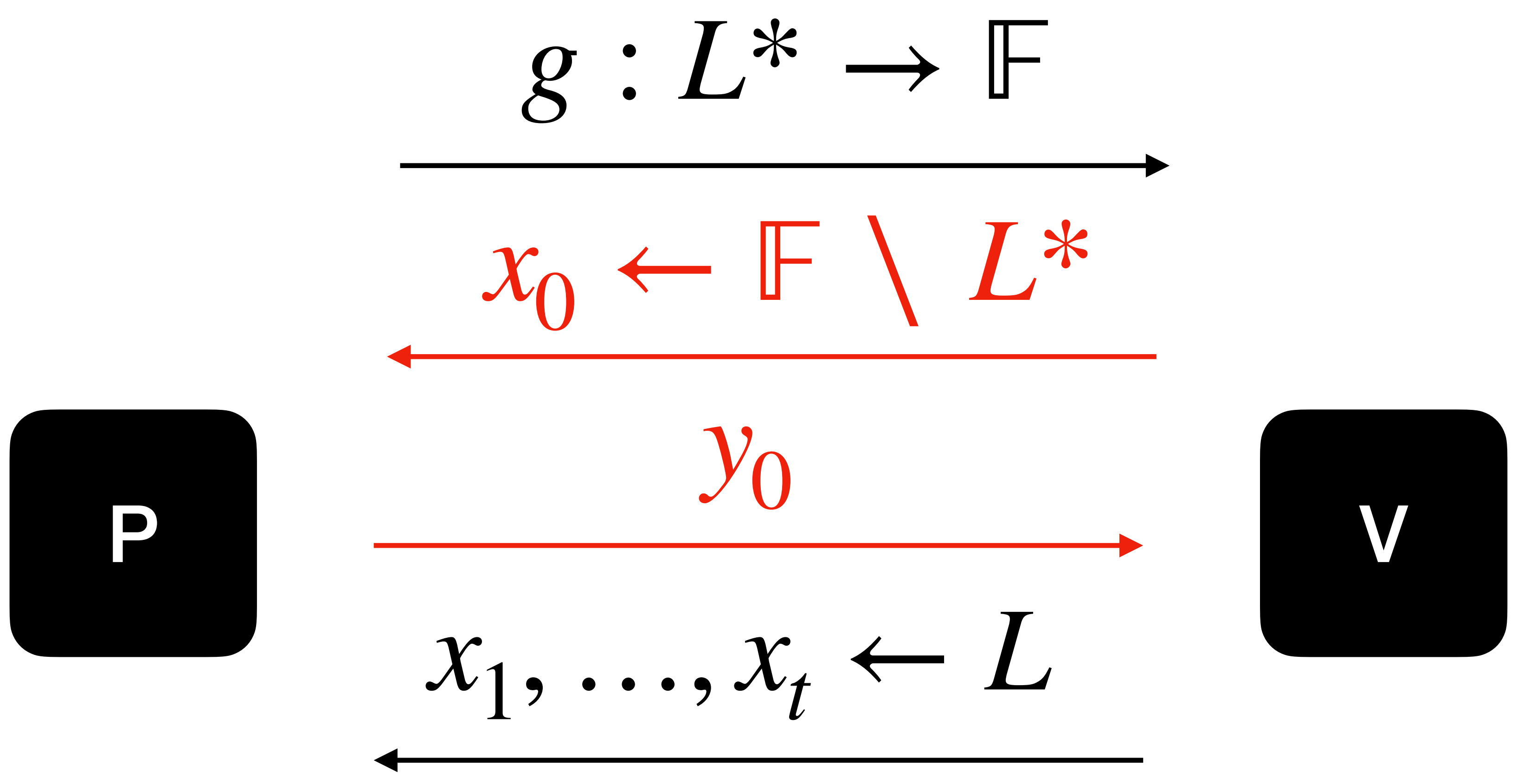

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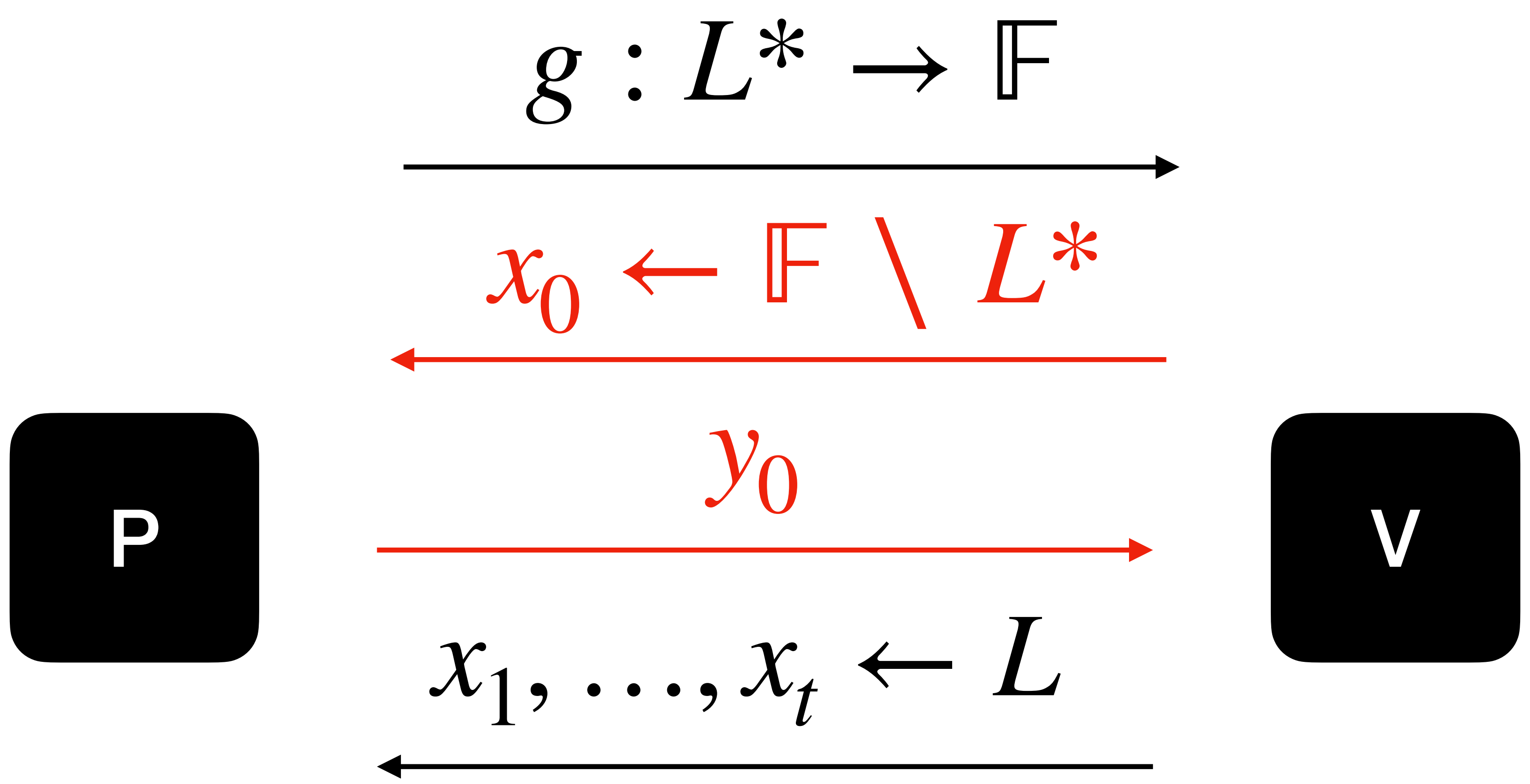

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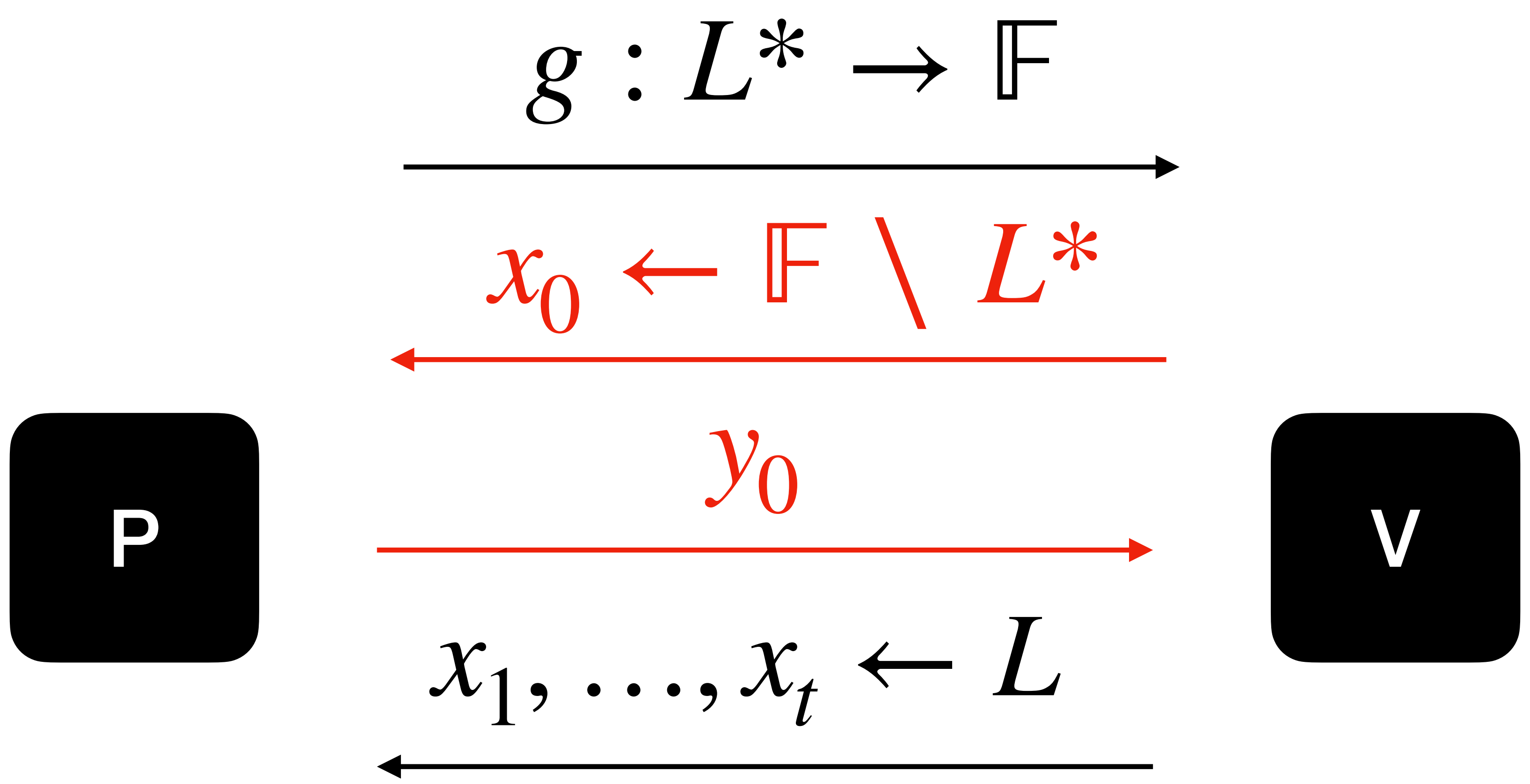

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Since $\Delta(f, \hat{v}|_L) > \delta$, then:

$$\Pr\left[g^* \text{ is } \delta^* \text{ close }\right]$$

$$\leq \Pr\left[\forall i,\ \hat{v}(x_i) = y_i\right]$$

$$= \Pr\left[\forall i,\ \hat{v}(x_i) = f(x_i)\right]$$

$$\leq (1 - \delta)^t$$

Final Protocol

STIR: domain shifting of fold

STIR

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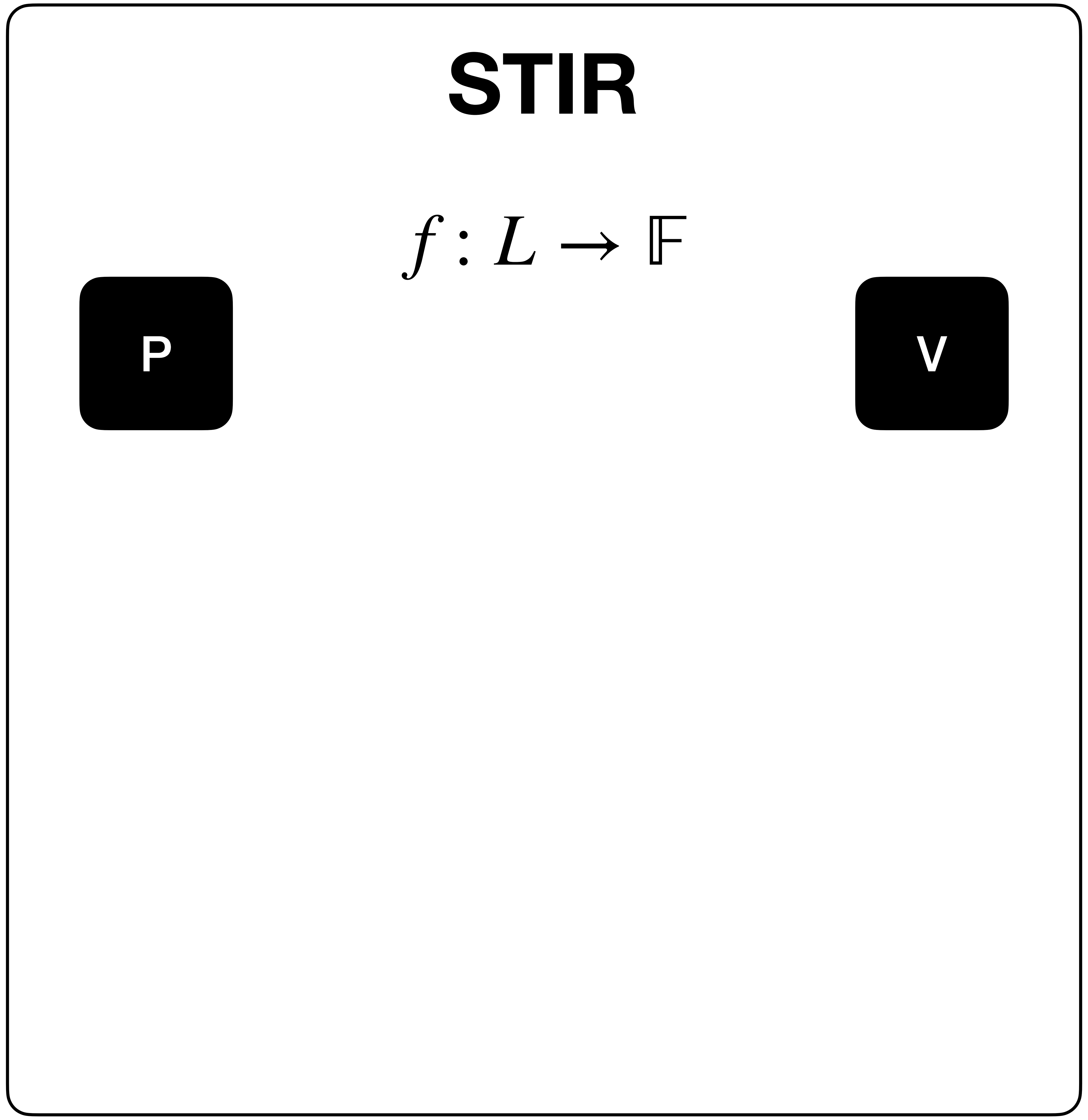
STIR: domain shifting of fold



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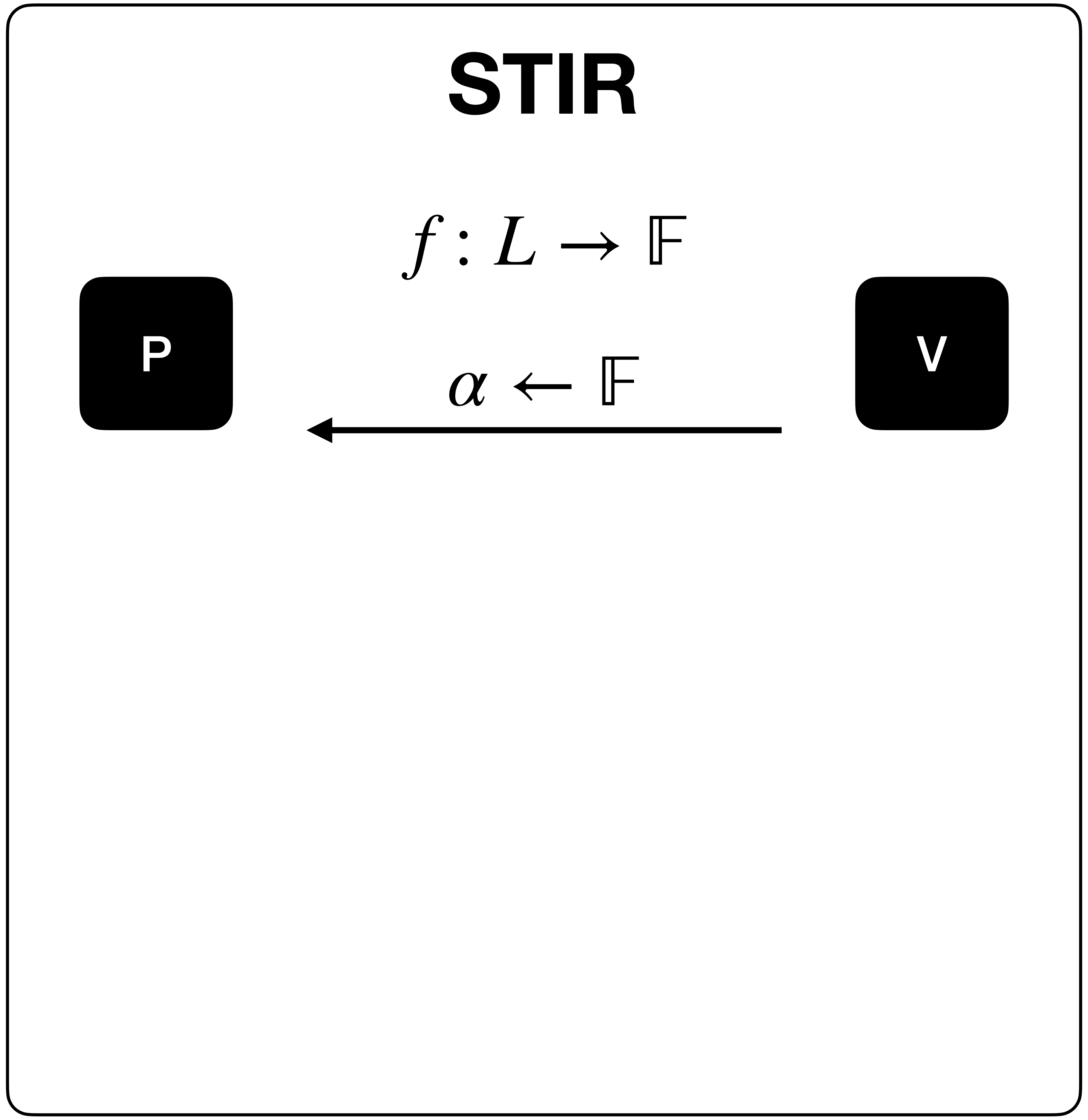

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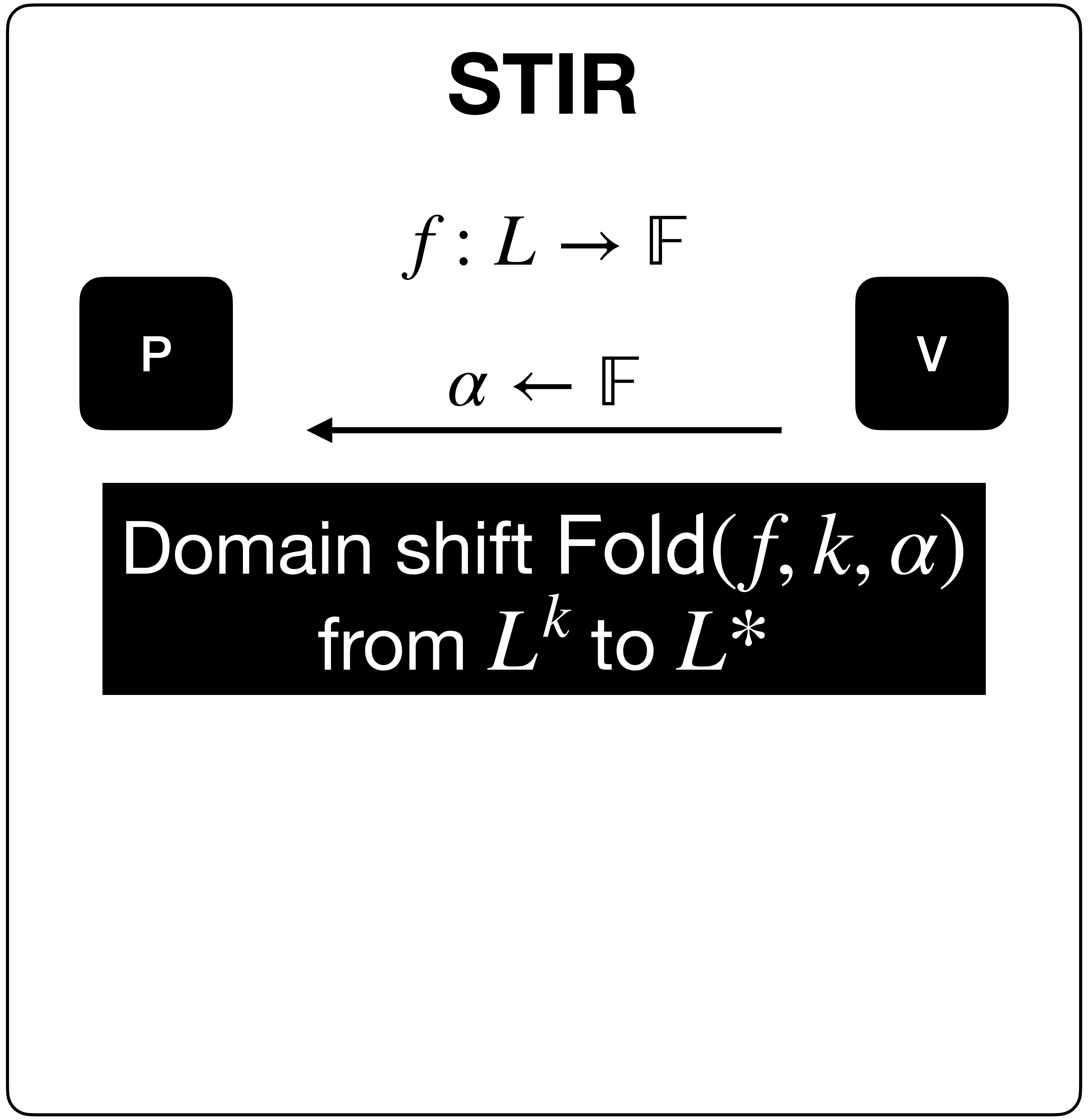

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Domain shift Fold(f, k, α) from L^k to L^*

Final Protocol

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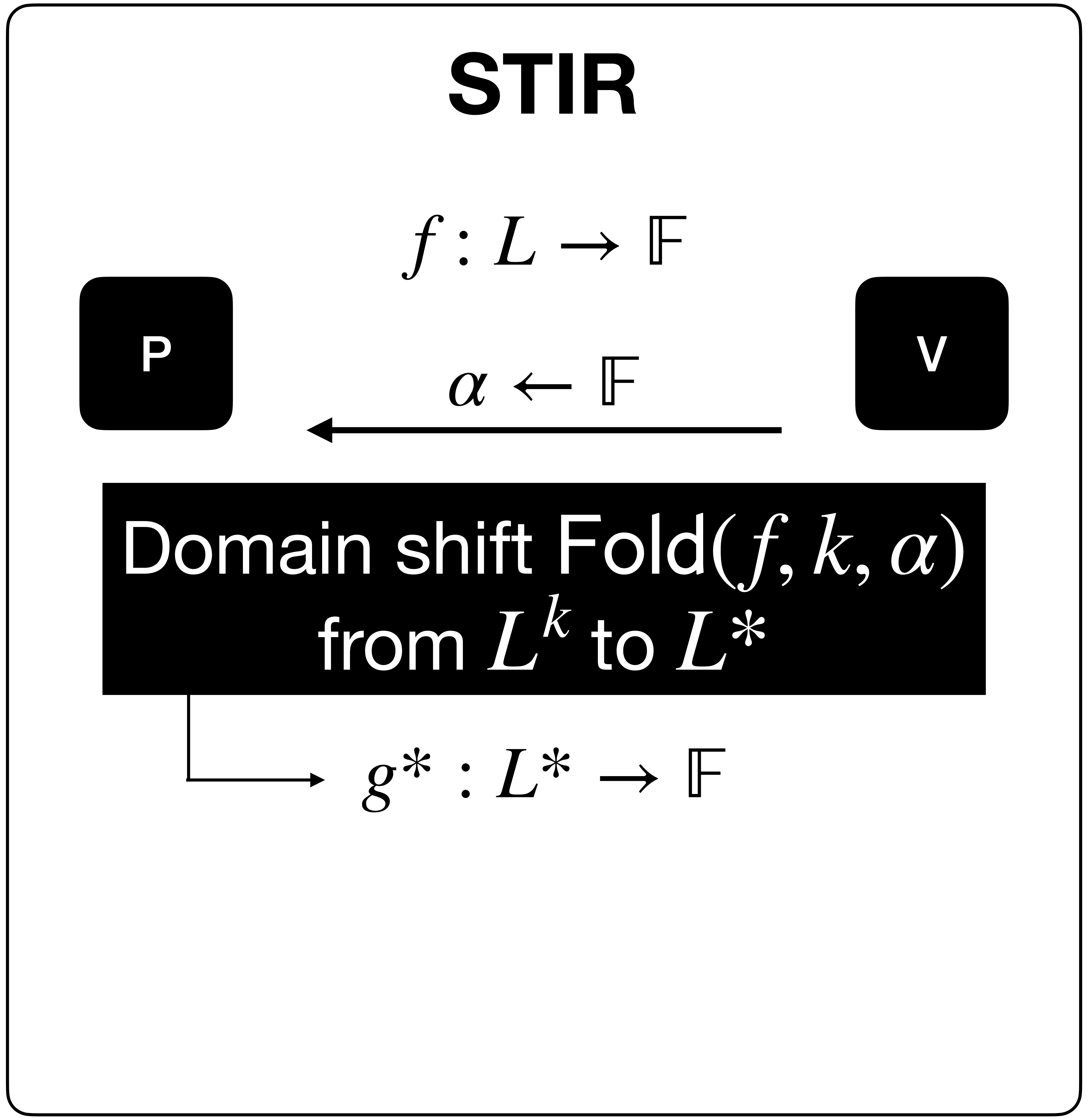

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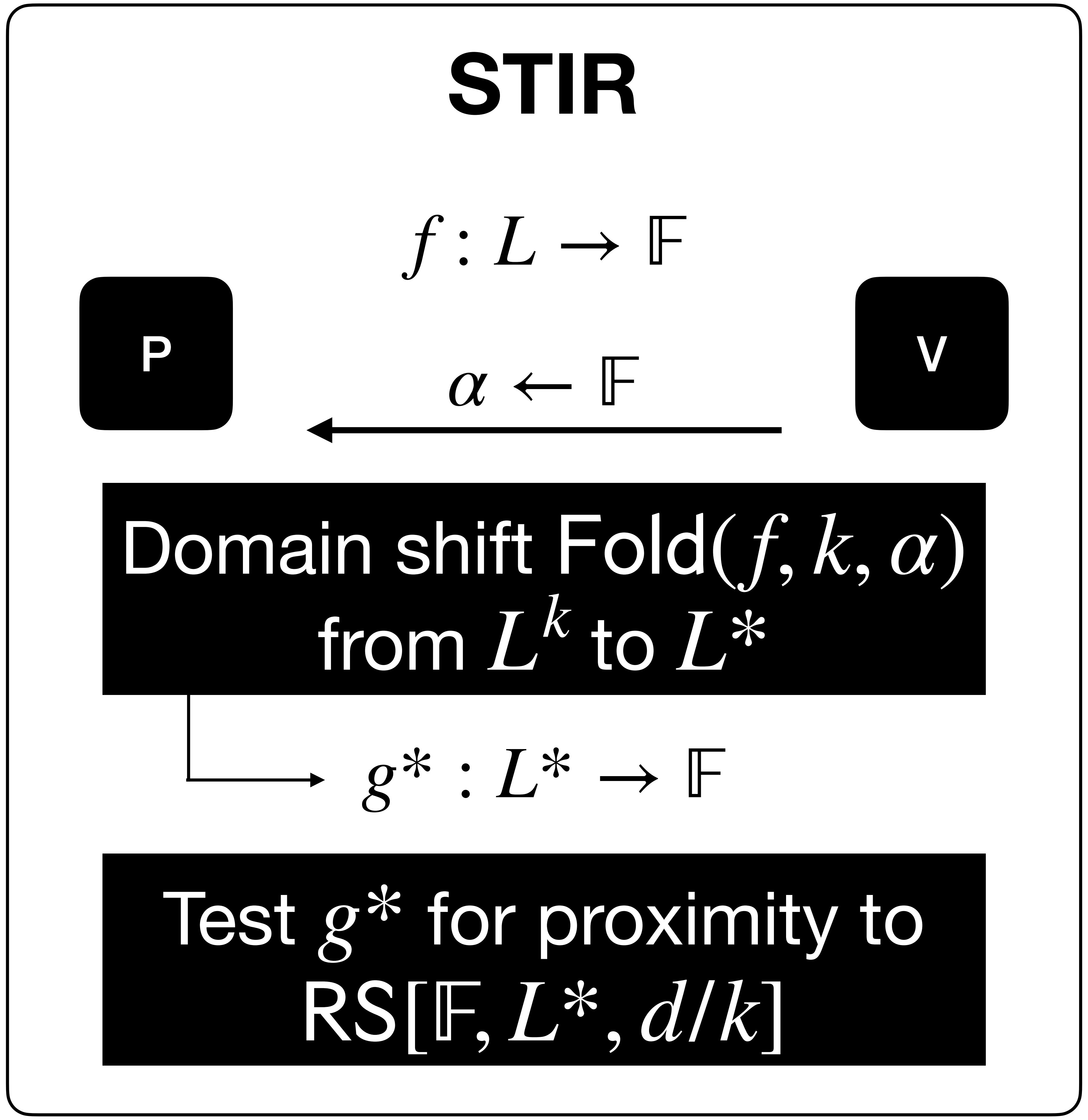

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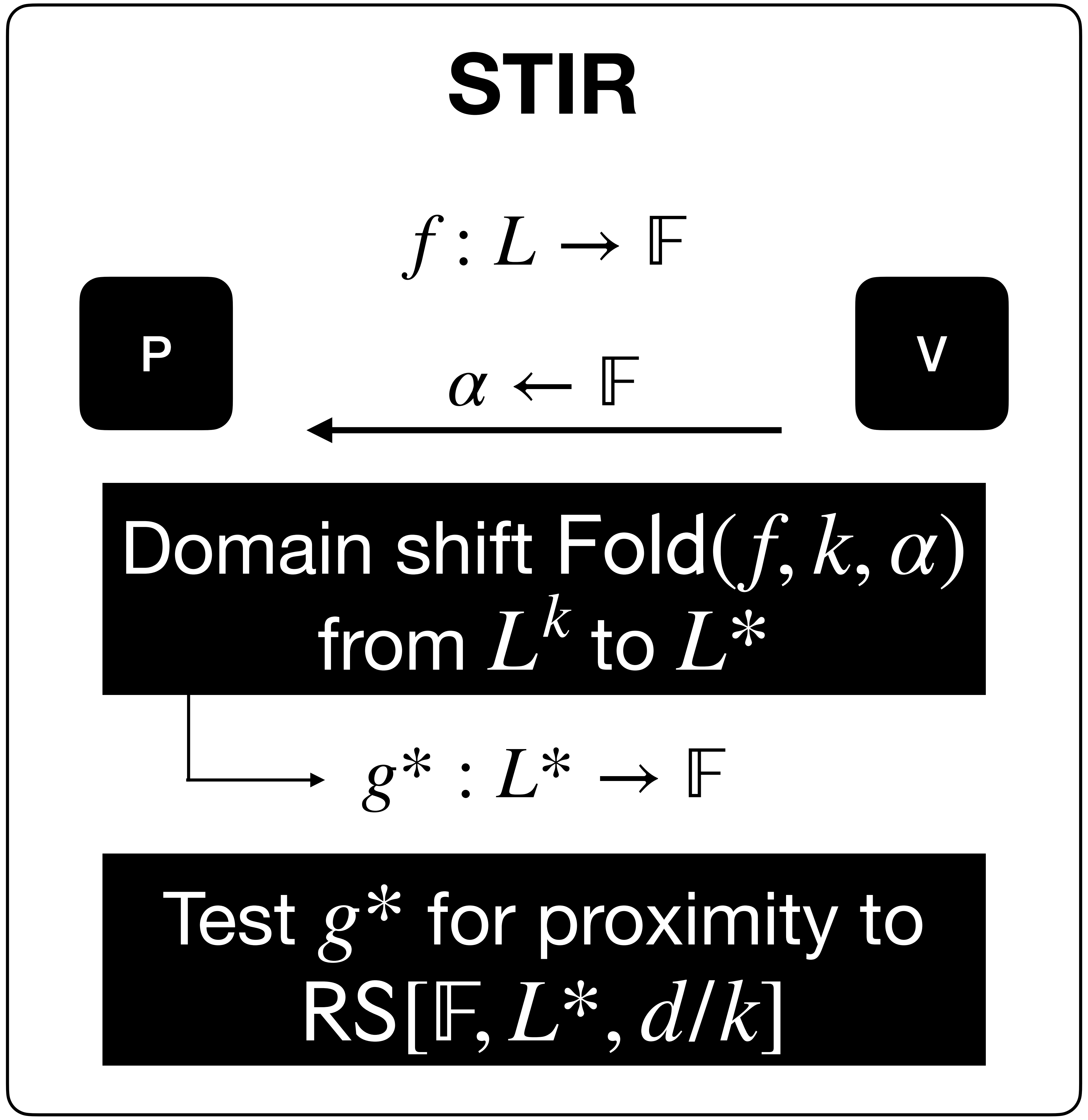

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Soundness

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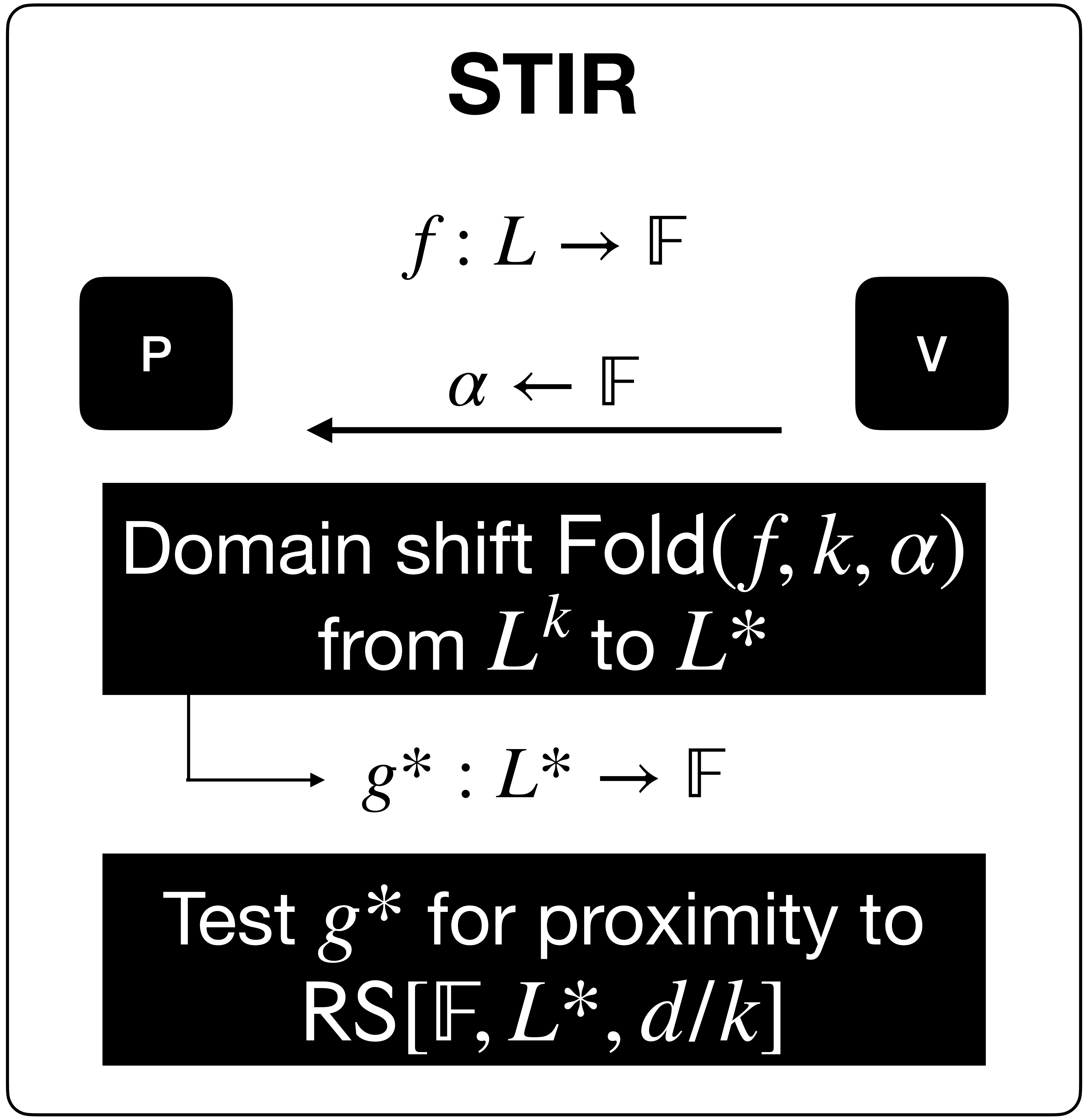

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Soundness

By distance-preservation of **folding**, $\Delta($Fold$(f, k, \alpha),$ RS$[\mathbb{F}, L^k, d/k]) > \delta$ w.h.p

Final Protocol

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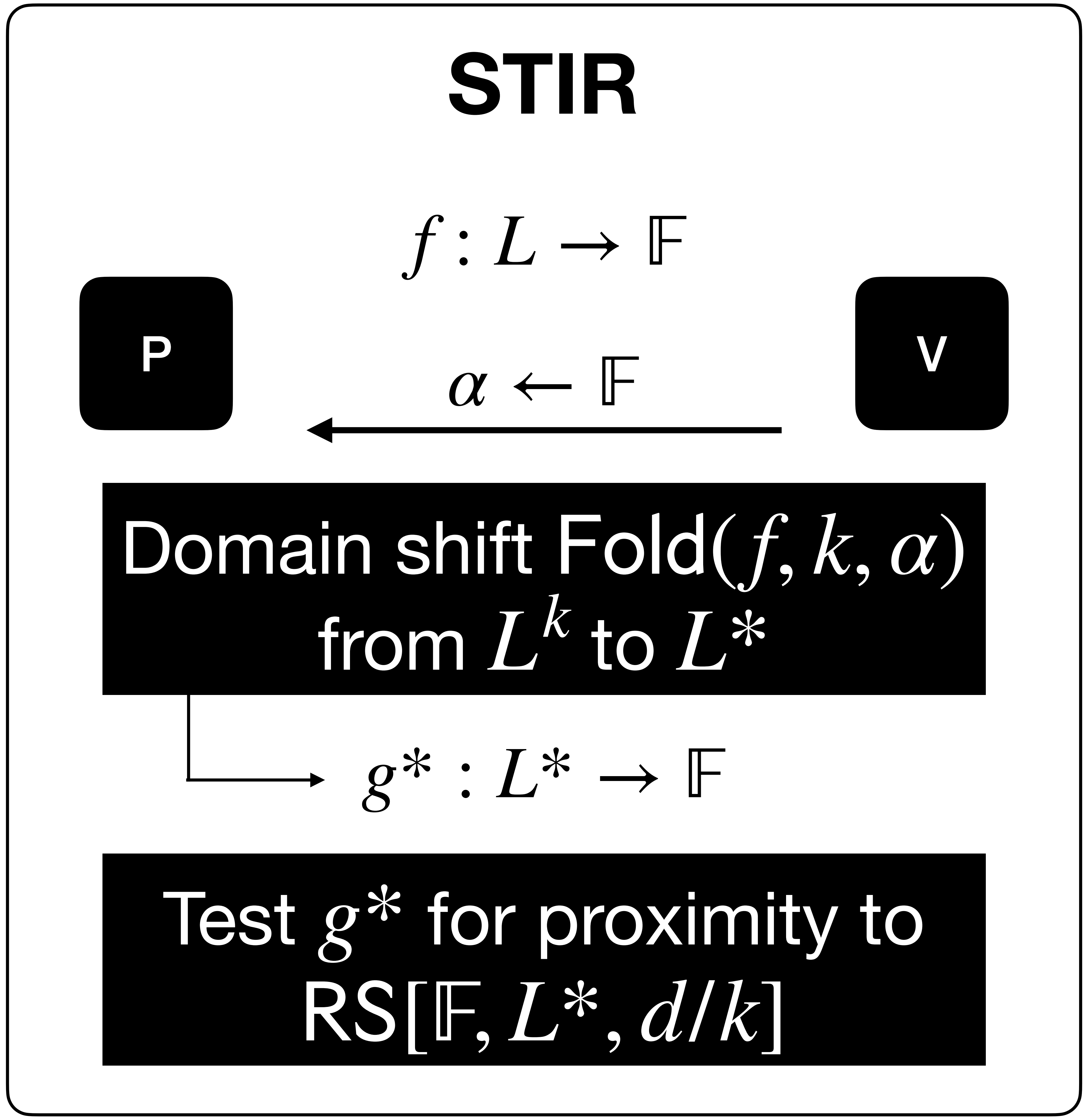

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$\Delta(\text{Fold}(f, k, \alpha), \text{RS}[\mathbb{F}, L^k, d/k]) > \delta$
w.h.p

By **domain-shifting**,
$\Delta(g^*, \text{RS}[\mathbb{F}, L^*, d/k]) > 1 - \sqrt{\rho^*}$
unless w.p. $(1 - \delta)^t$

Final Protocol

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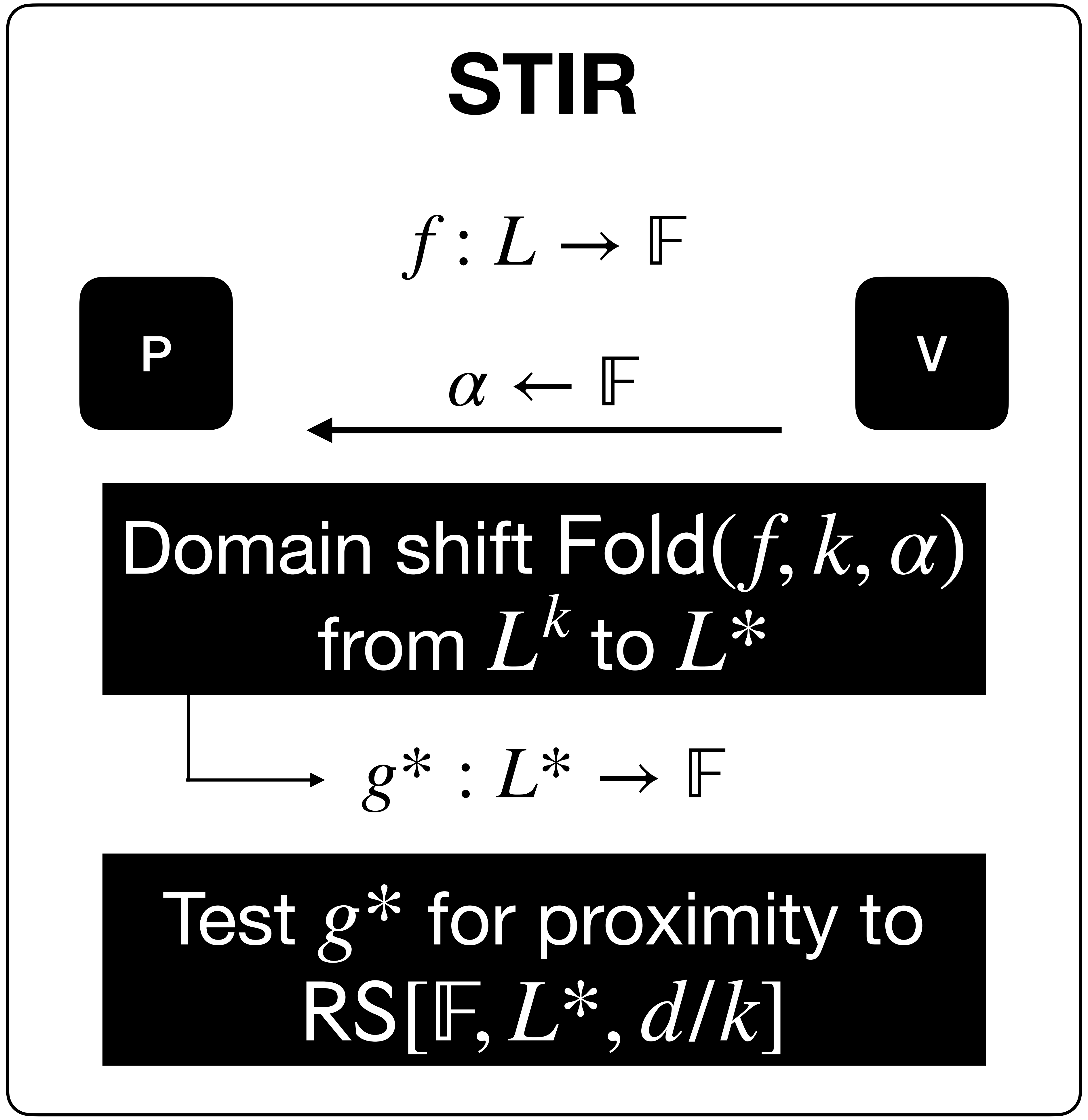

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Recursing, yields **STIR** 🥣

Conclusion

What we saw

What we did have time to talk about

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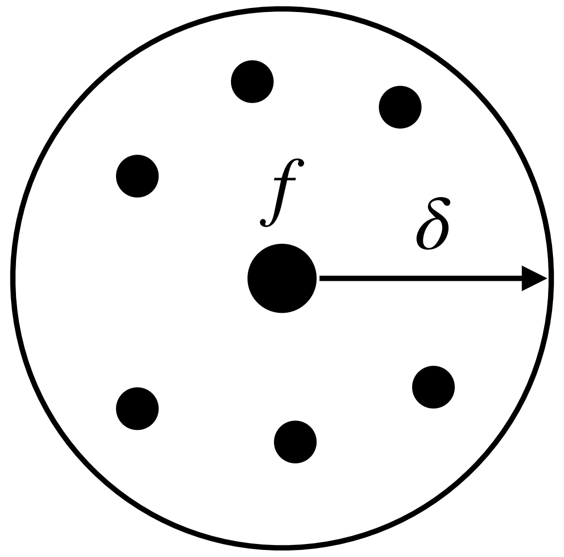
- Domain shifting:

What we saw

What we did have time to talk about

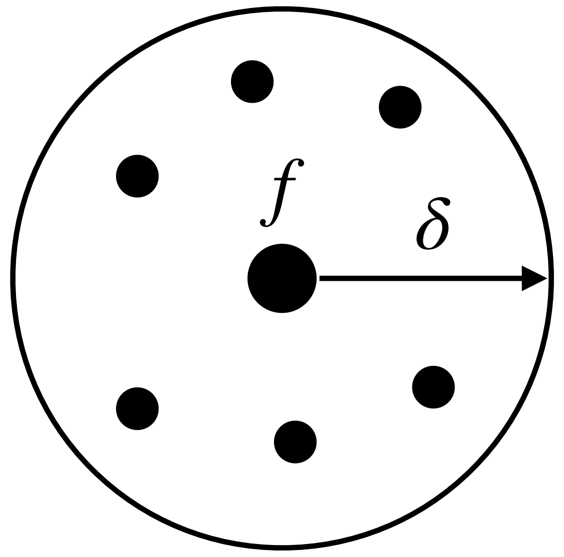
- Domain shifting:
  - Quotienting and its properties

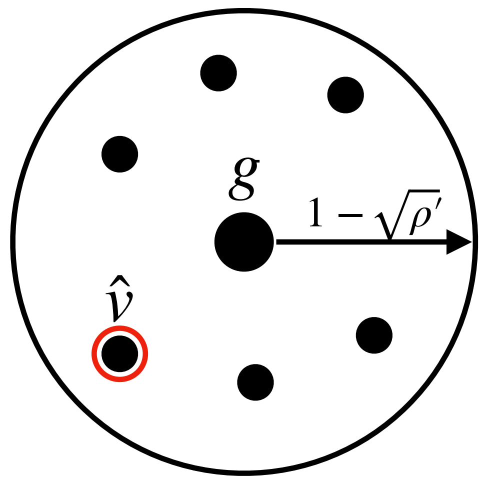


What we saw

What we did have time to talk about

- Domain shifting:
    - Quotienting and its properties
    - Out-Of-Domain sampling





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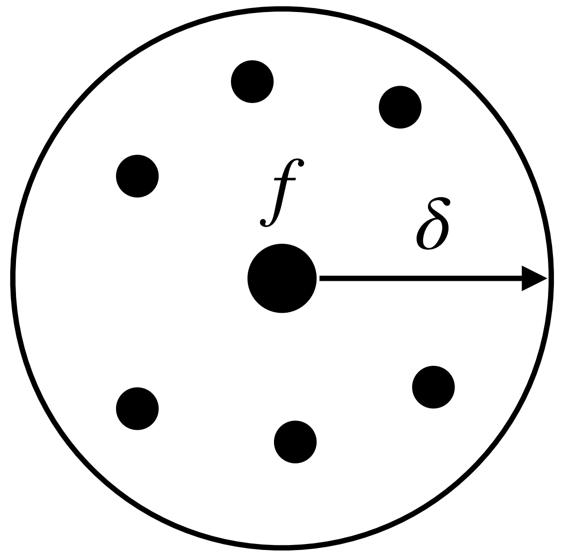

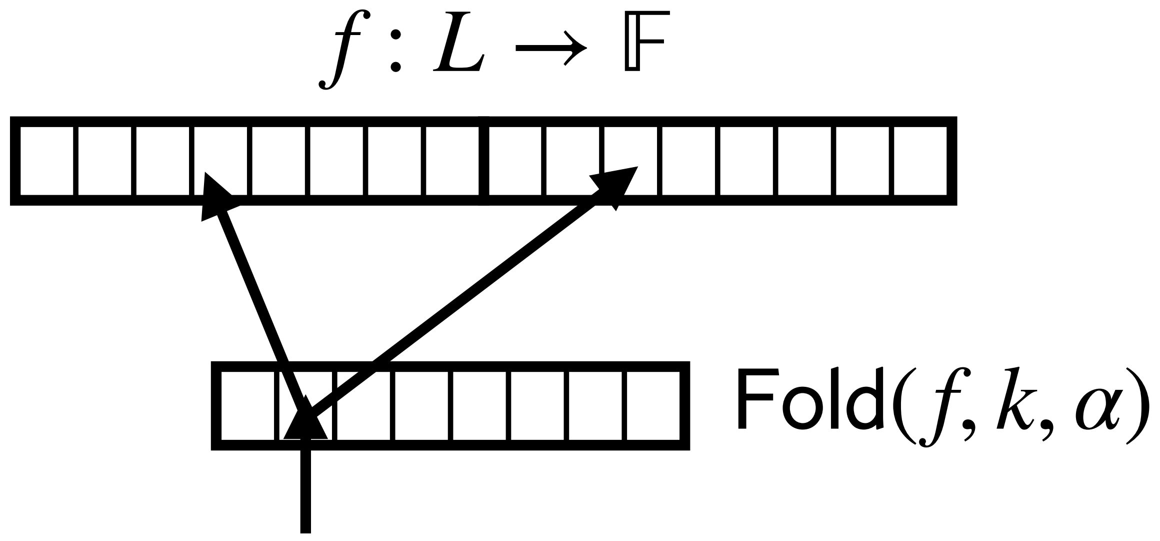

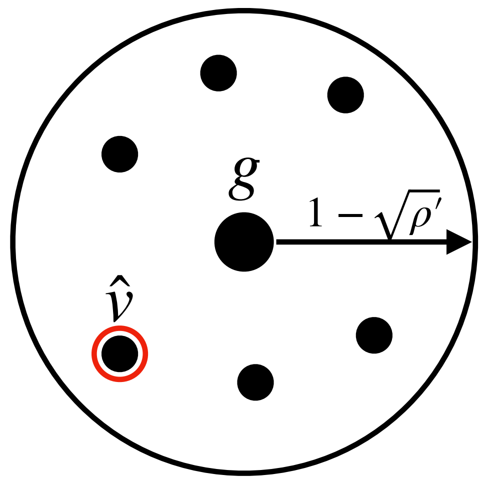

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- Domain shifting:
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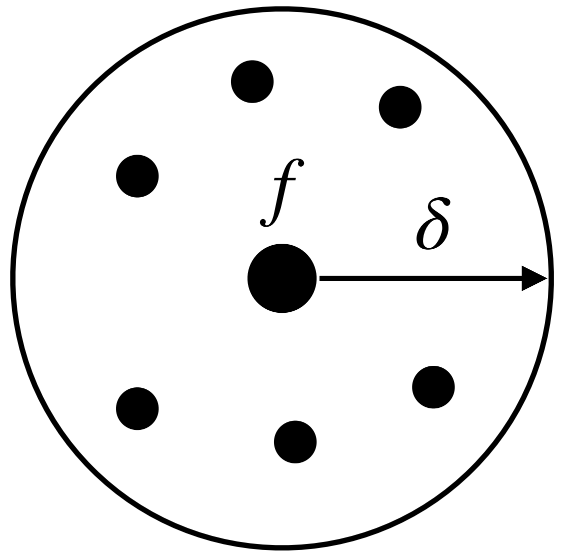
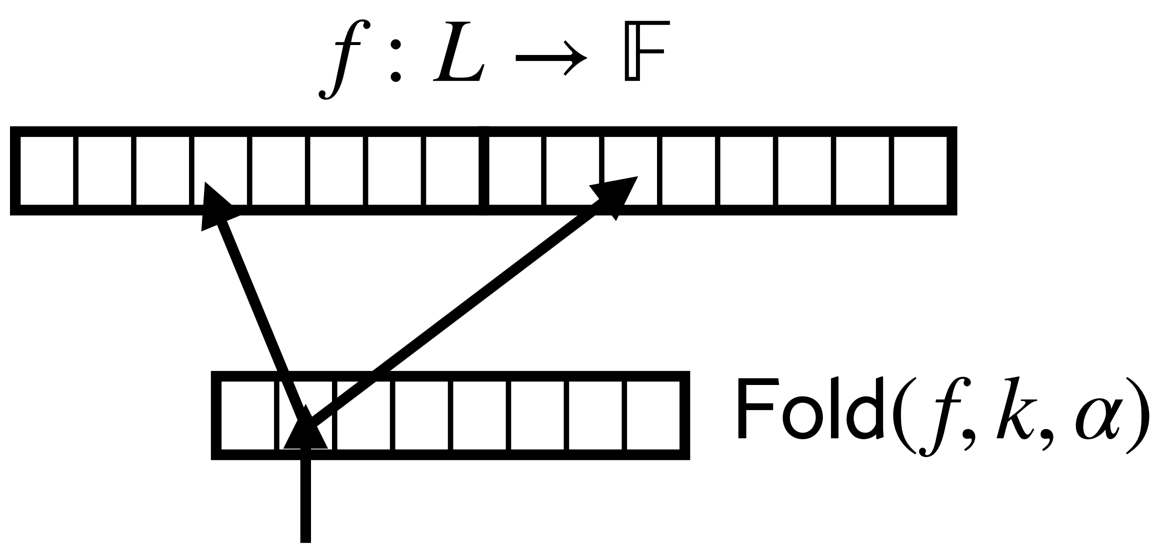
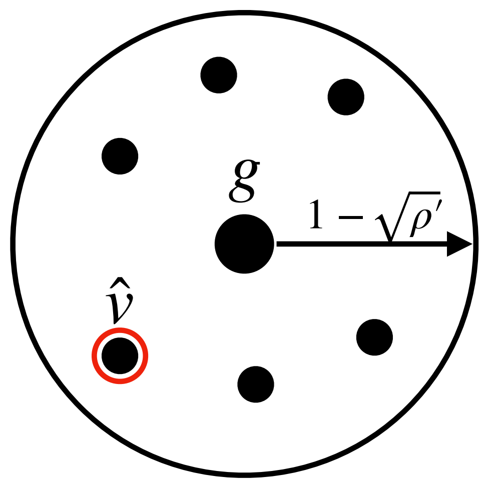
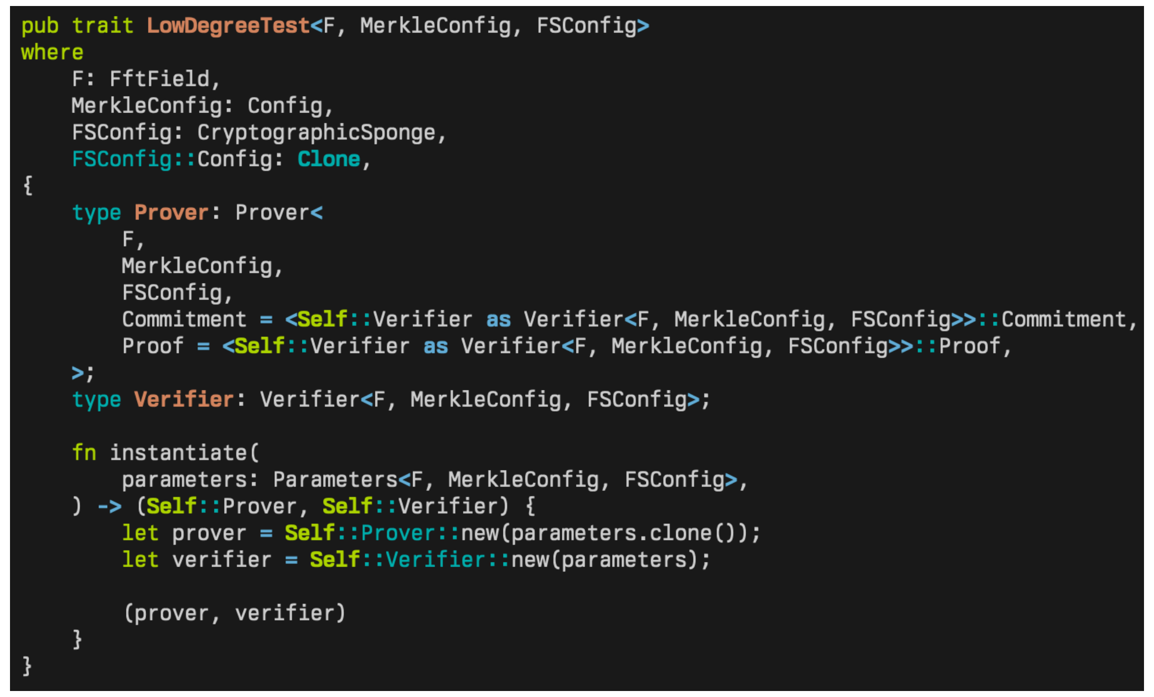
- Folding and its properties
- STIR 🥣

STIR







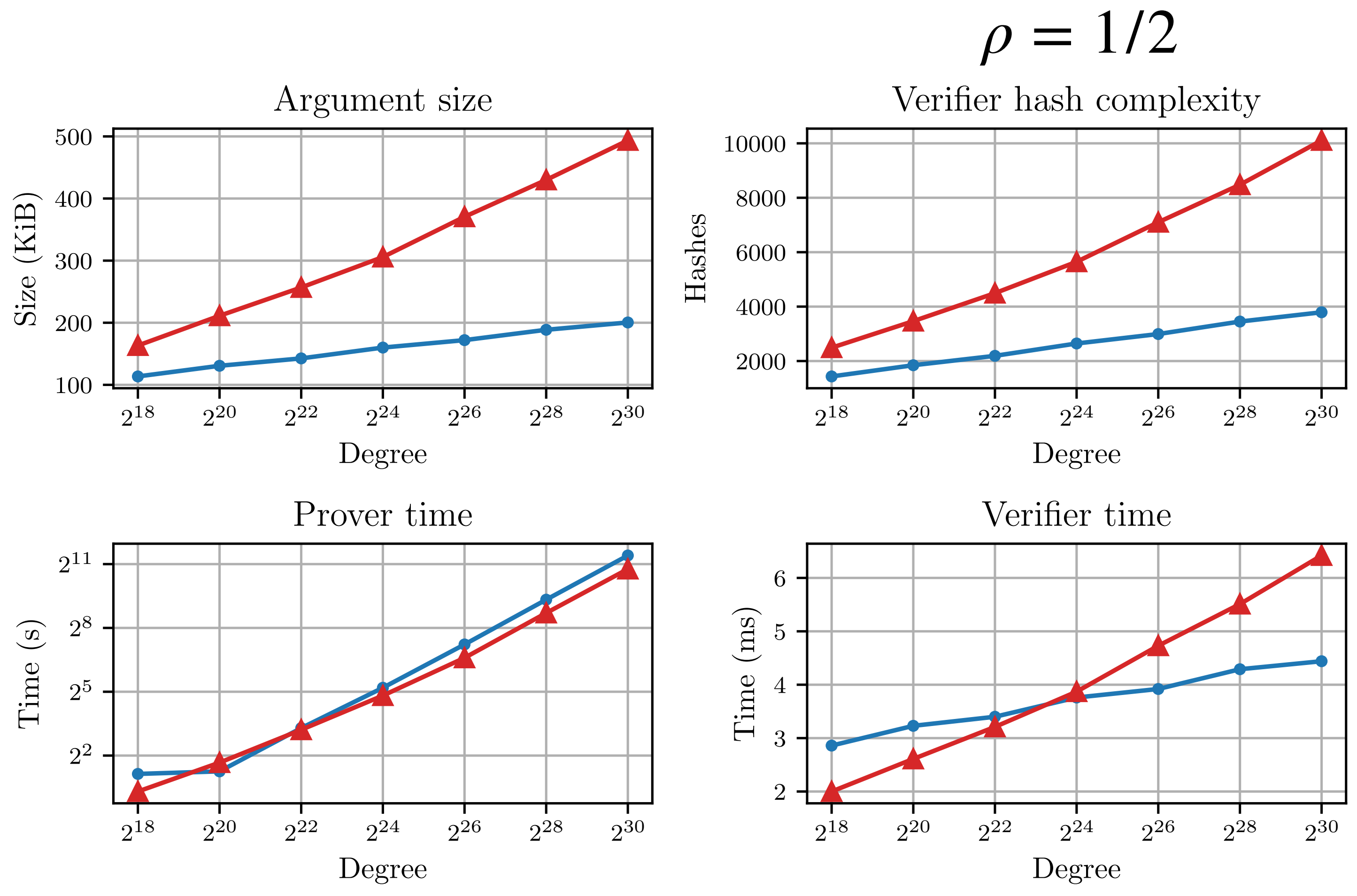


```rust
pub trait LowDegreeTest<F, MerkleConfig, FSConfig>
where
    F: FftField,
    MerkleConfig: Config,
    FSConfig: CryptographicSponge,
    FSConfig::Config: Clone,
{
    type Prover: Prover<
        F,
        MerkleConfig,
        FSConfig,
        Commitment = <Self::Verifier as Verifier<F, MerkleConfig, FSConfig>>::Commitment,
        Proof = <Self::Verifier as Verifier<F, MerkleConfig, FSConfig>>::Proof,
    >;
    type Verifier: Verifier<F, MerkleConfig, FSConfig>;

    fn instantiate(
        parameters: Parameters<F, MerkleConfig, FSConfig>,
    ) -> (Self::Prover, Self::Verifier) {
        let prover = Self::Prover::new(parameters.clone());
        let verifier = Self::Verifier::new(parameters);

        (prover, verifier)
    }
}
```

$\rho = 1/2$



There is more!

What we did not have time to talk about

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Degree corrections

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Degree corrections

- $\text{Quotient}(f, \text{Ans})$ has degree $d - |S|$, how to bump up to d?

There is more!

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High-soundness compiler for Poly-IOPs

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High-soundness compiler for Poly-IOPs

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Round-by-round soundness of STIR $\implies$ secure in non-interactive setting

What’s next?

What we hope to have time to talk about next talk!

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Wh xt?

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-
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USE STIR🥣!!!

Thank you!

See paper:
ia.cr/2024/390

And blog post:
gfenzi.io/papers/stir

Extra slides

Anatomy of an IOP-based SNARK

Reducing to low-degree testing

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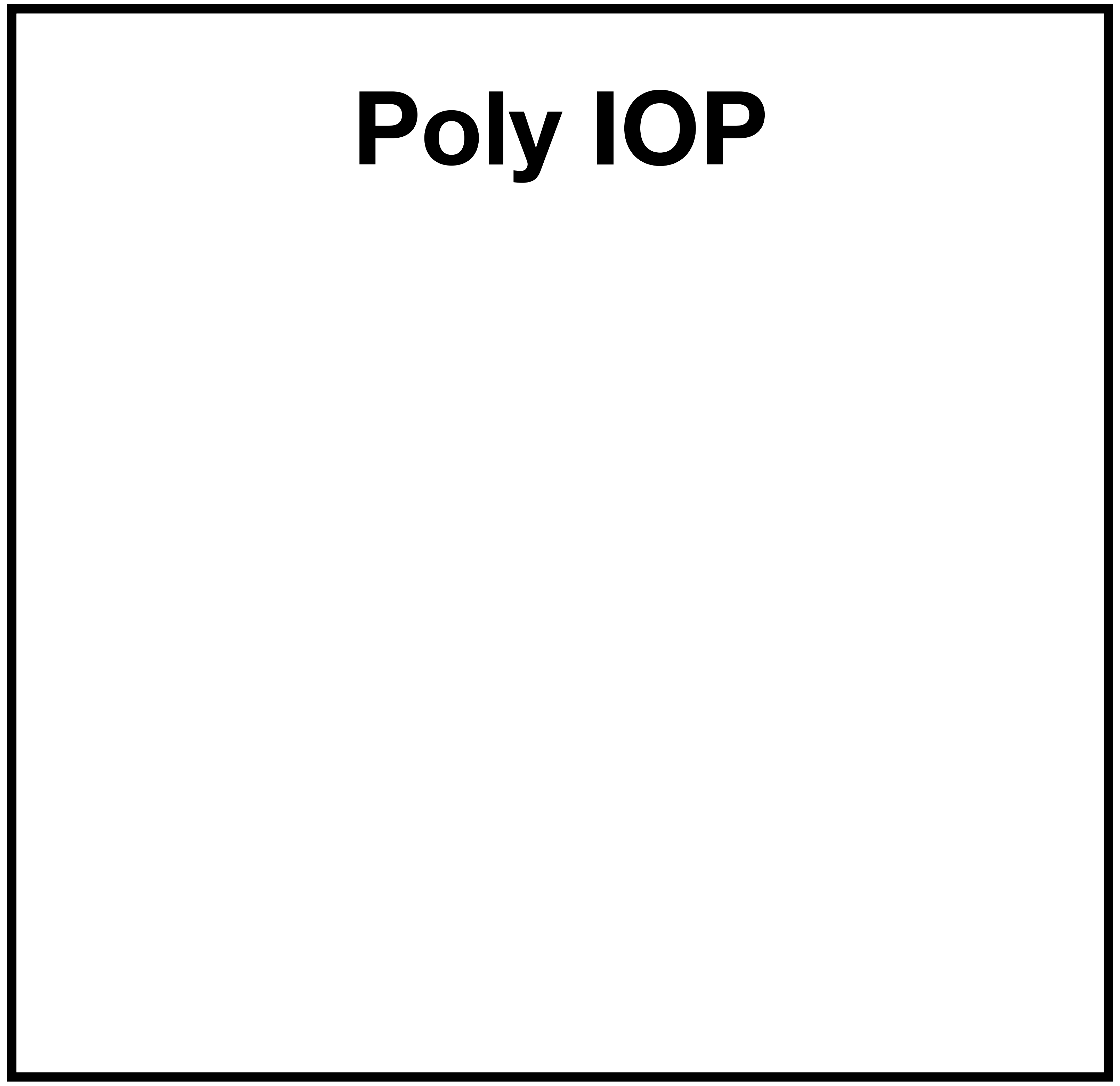
Reducing to low-degree testing



Anatomy of an IOP-based SNARK

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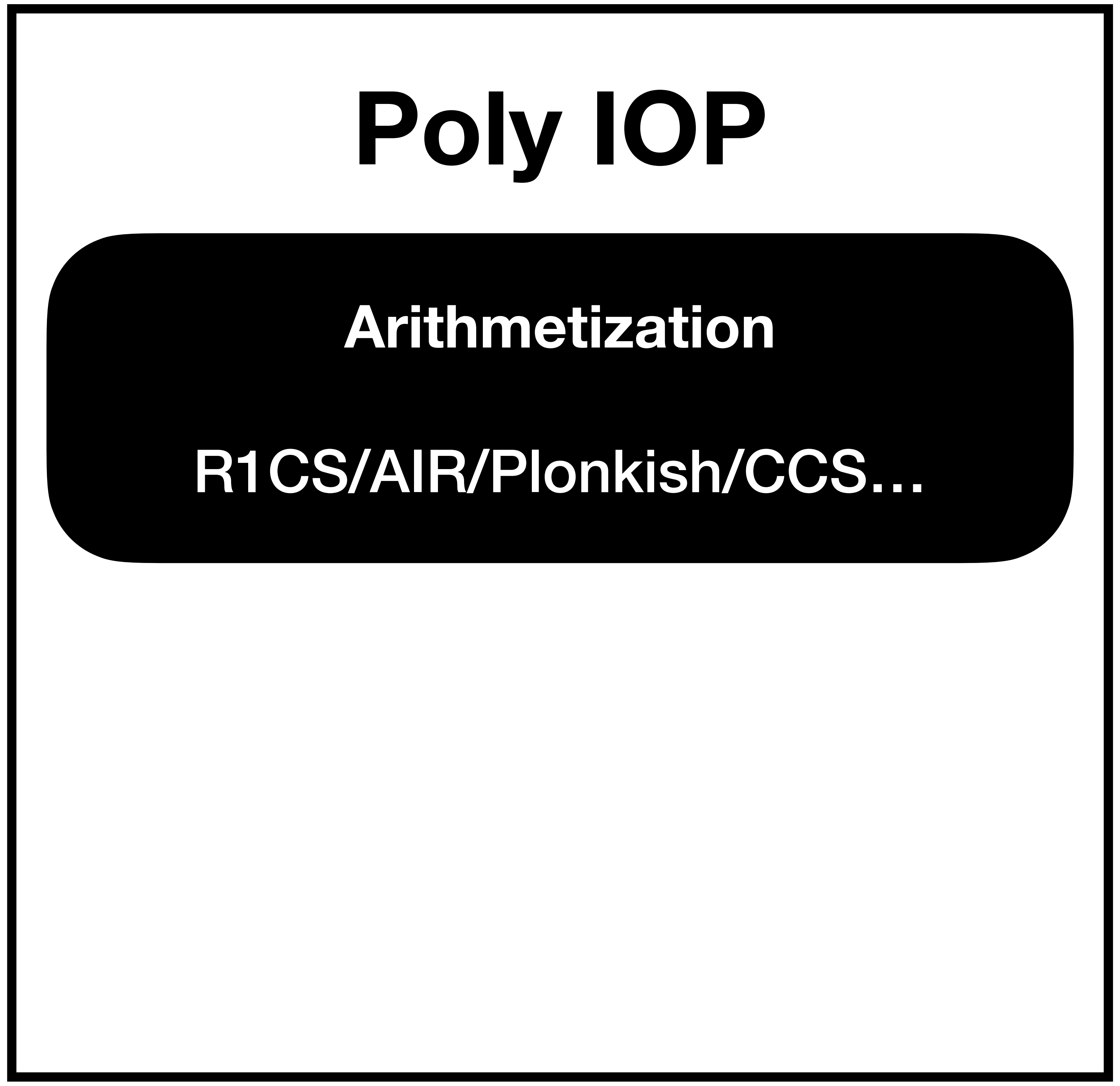

Poly IOP

Arithmetization

R1CS/AIR/Plonkish/CCS…

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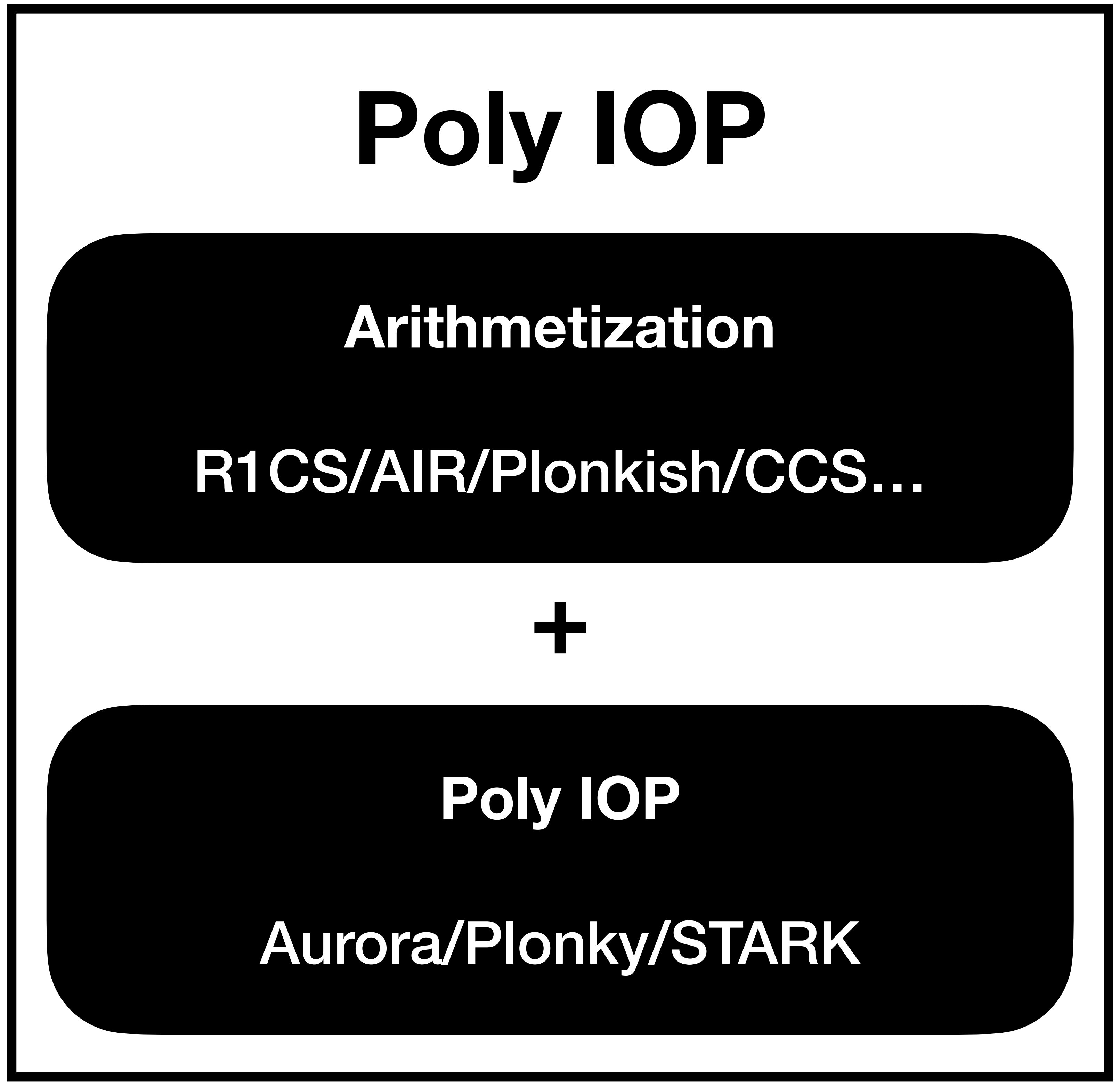

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Aurora/Plonky/STARK

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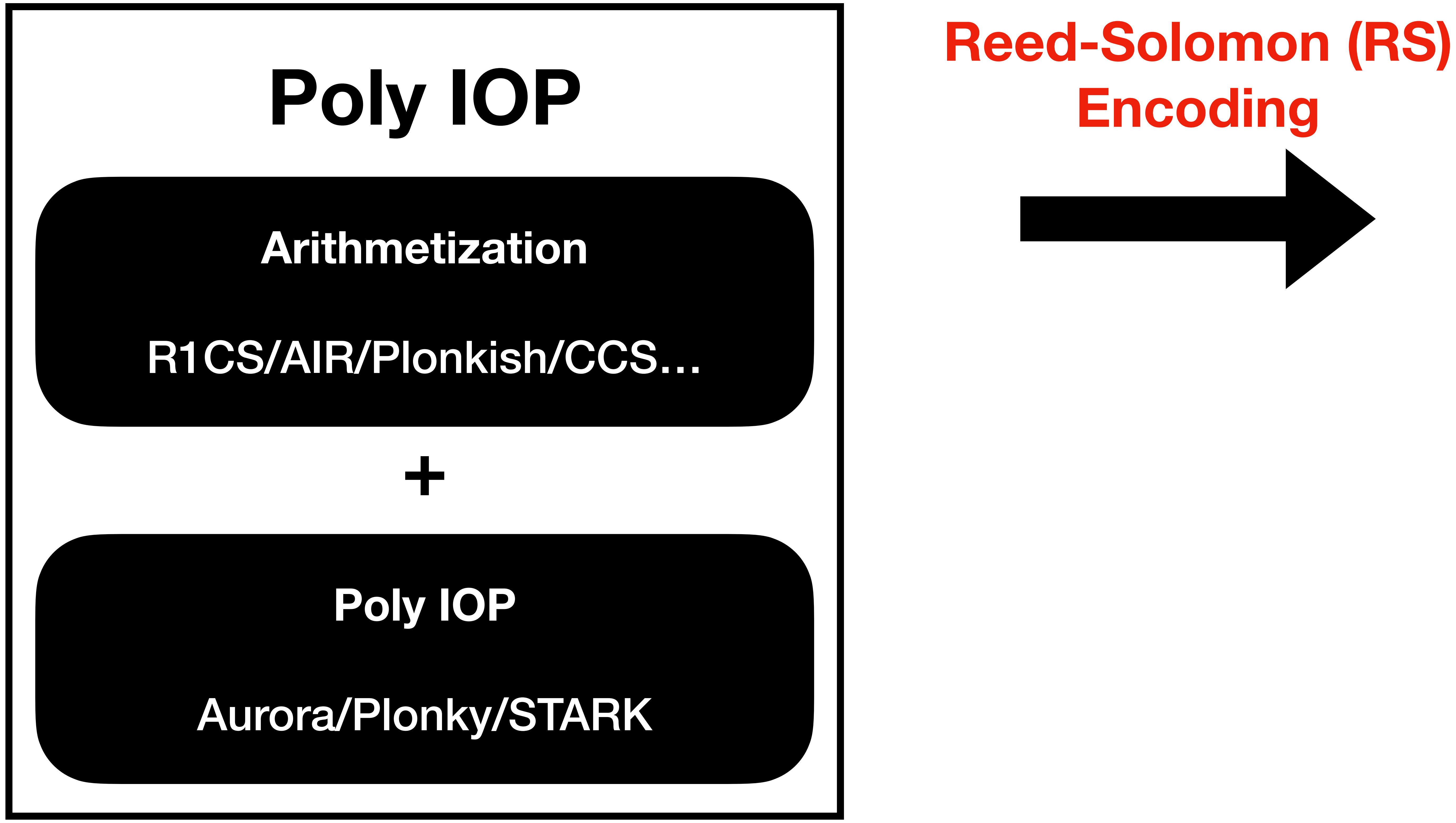

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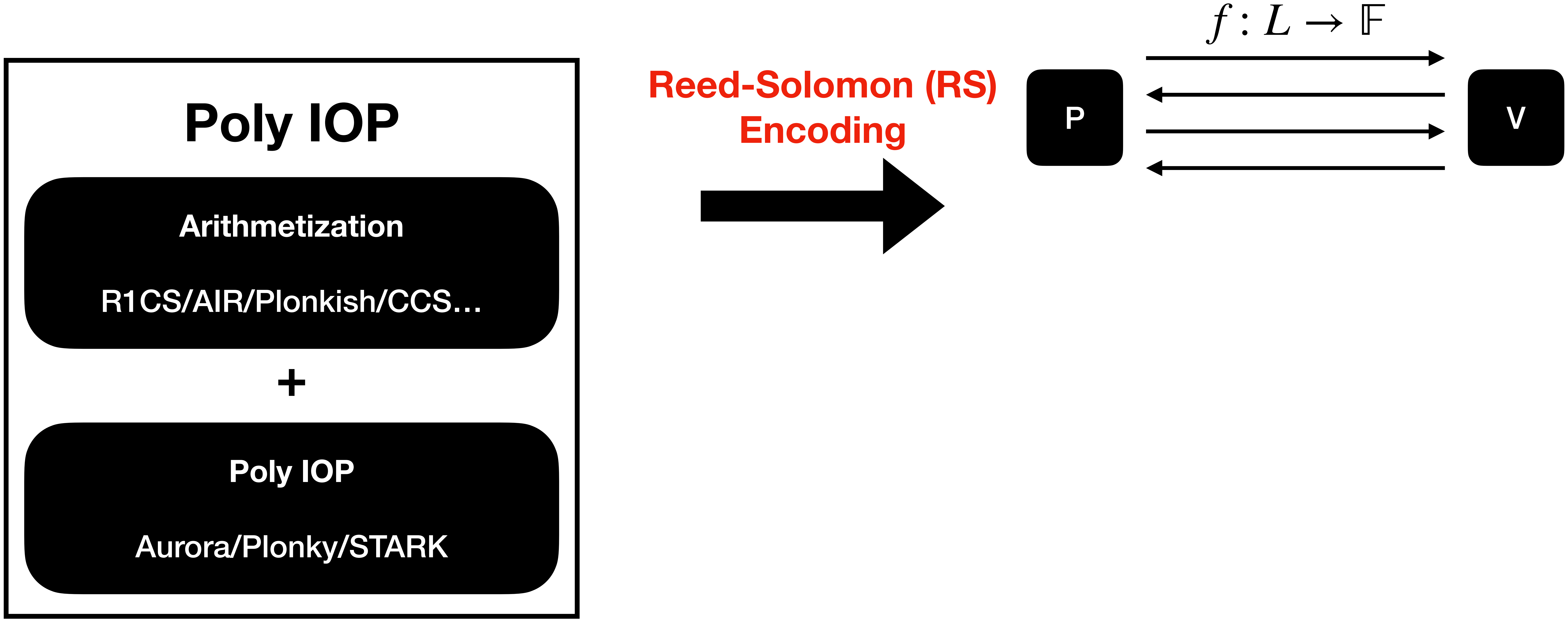

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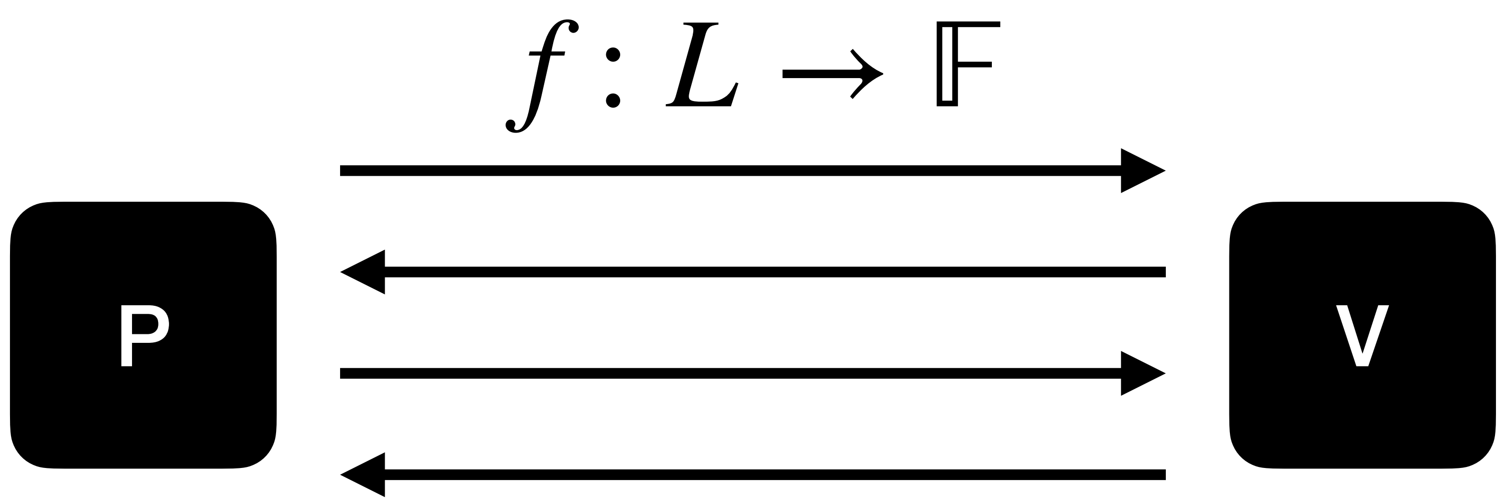
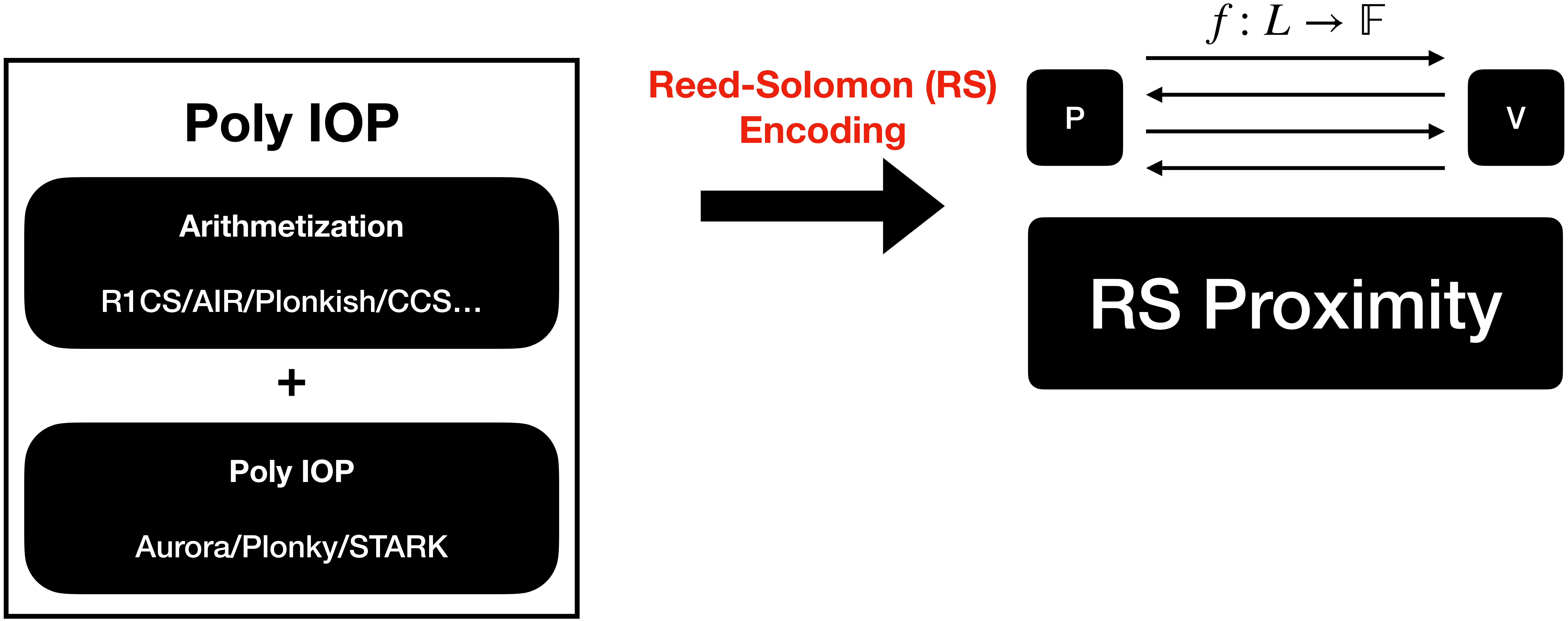


$\mathbf{V}$ needs to test f is close to a low-degree function

Anatomy of an IOP-based SNARK

Reducing to low-degree testing



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R1CS/AIR/Plonkish/CCS…

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Aurora/Plonky/STARK

Reed-Solomon (RS) Encoding

$f : L \to \mathbb{F}$

P

V

RS Proximity

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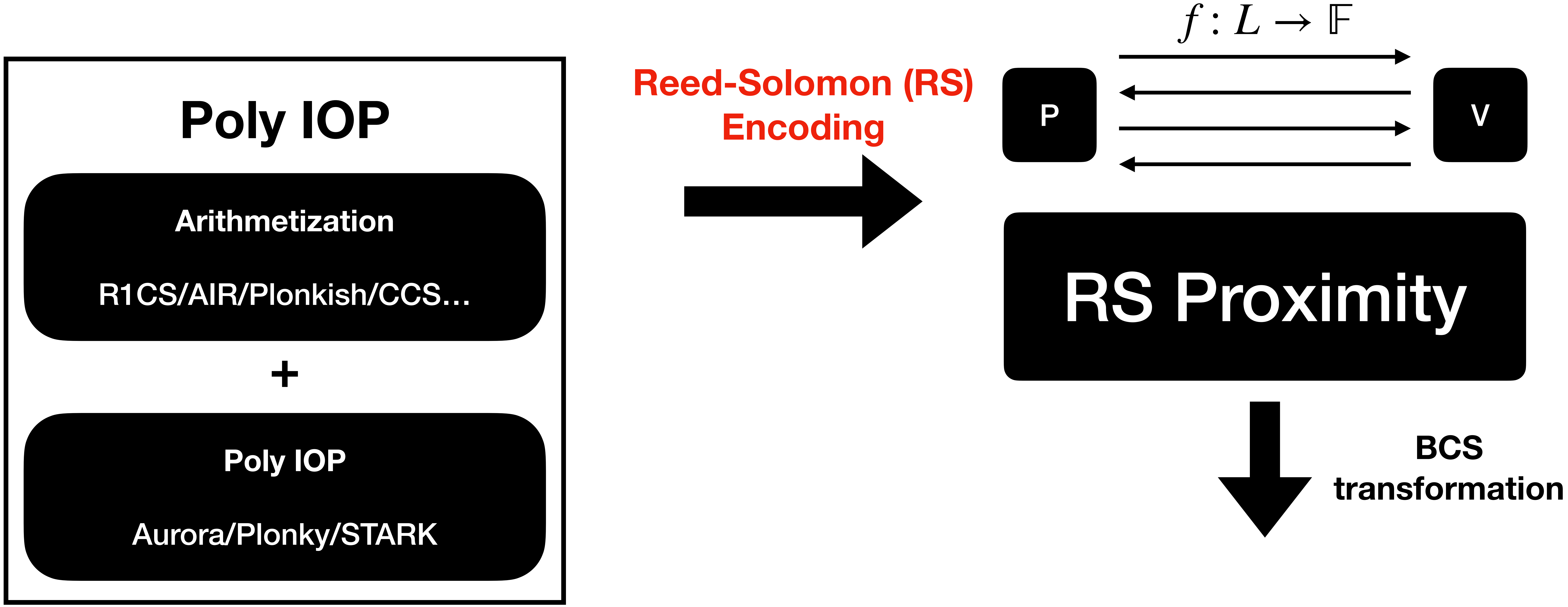

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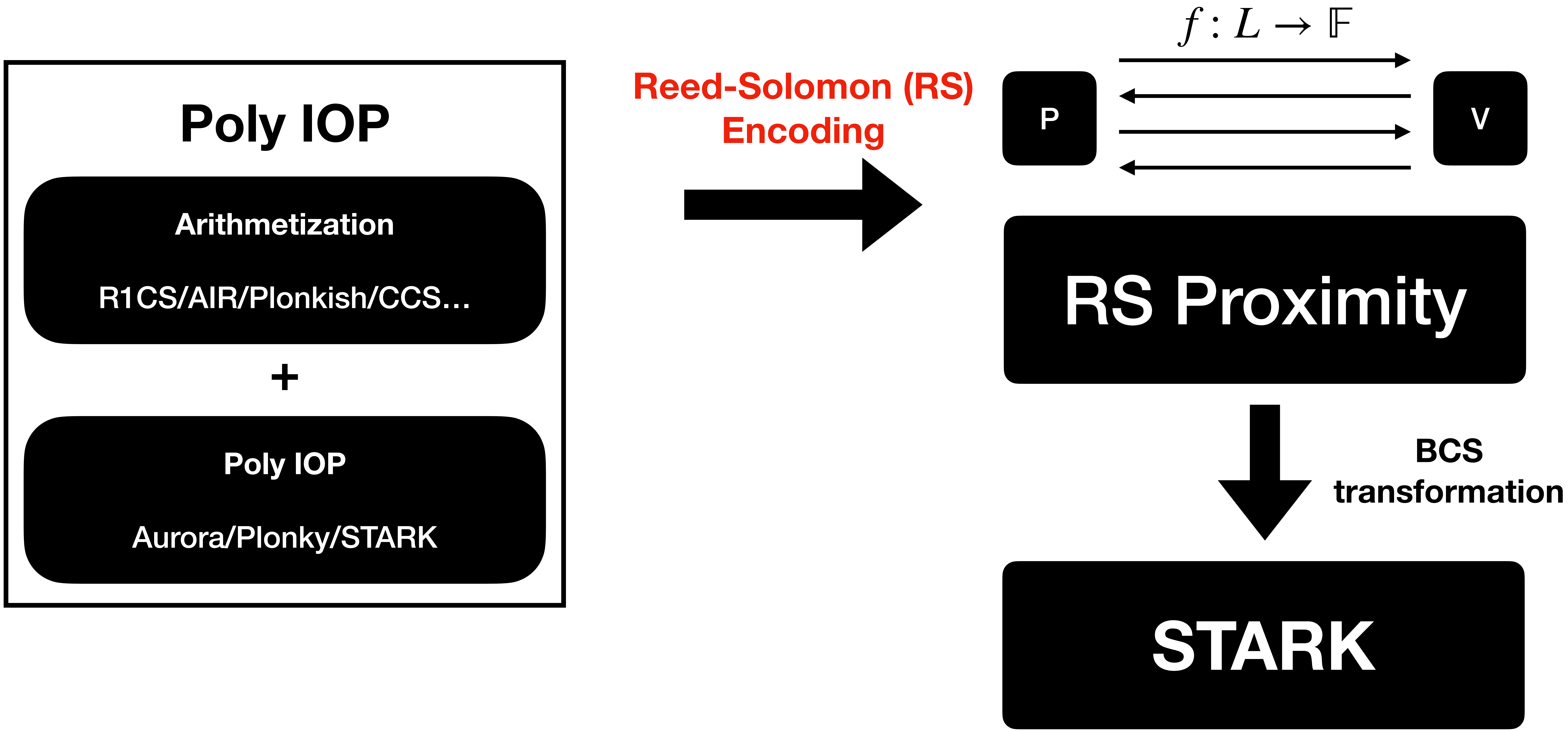

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Reed-Solomon (RS) Encoding

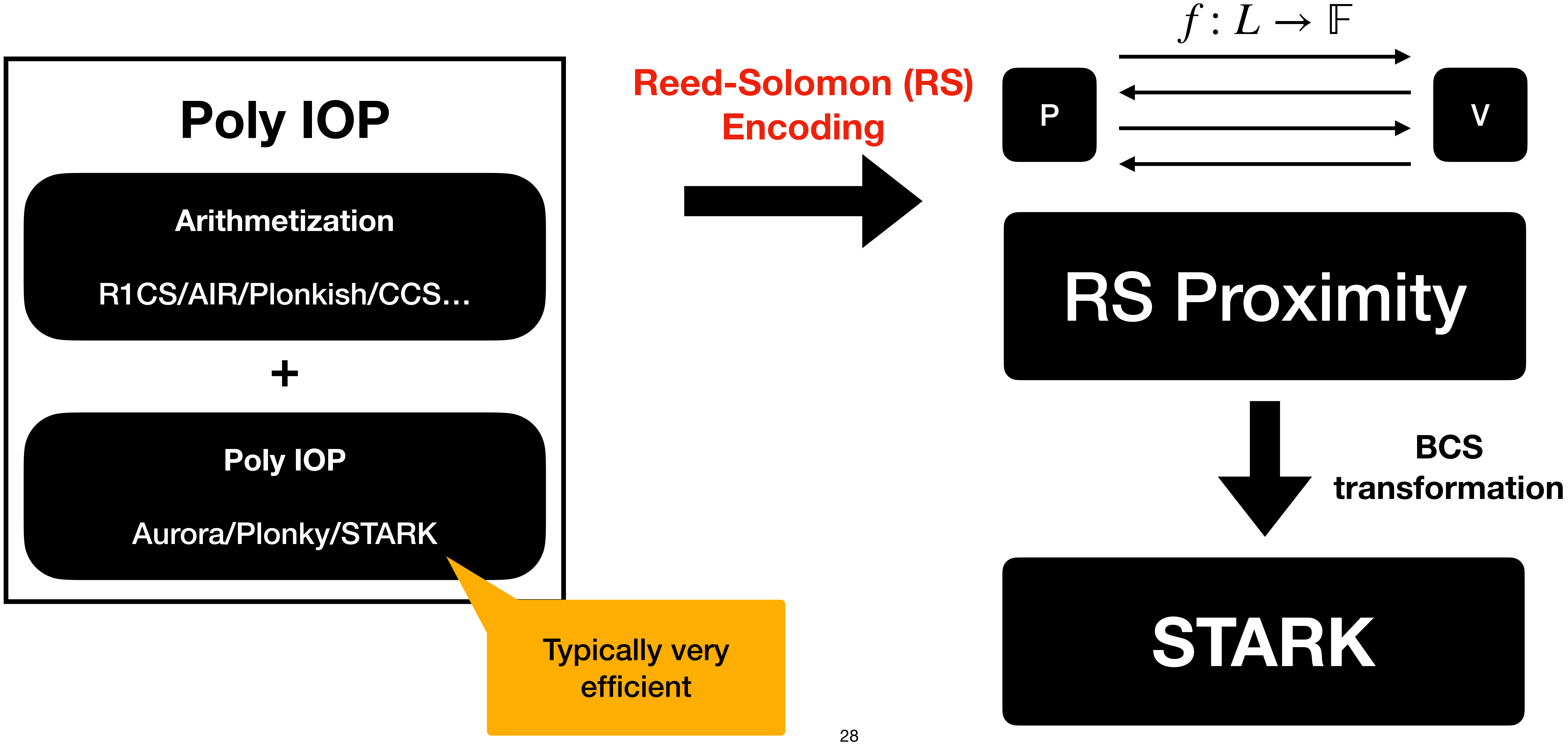
RS Proximity

BCS transformation

STARK

Anatomy of an IOP-based SNARK

Reducing to low-degree testing



Poly IOP

Arithmetization

R1CS/AIR/Plonkish/CCS…

+

Poly IOP

Aurora/Plonky/STARK

Typically very efficient

Reed-Solomon (RS) Encoding

$f : L \to \mathbb{F}$

P

V

RS Proximity

BCS transformation

STARK

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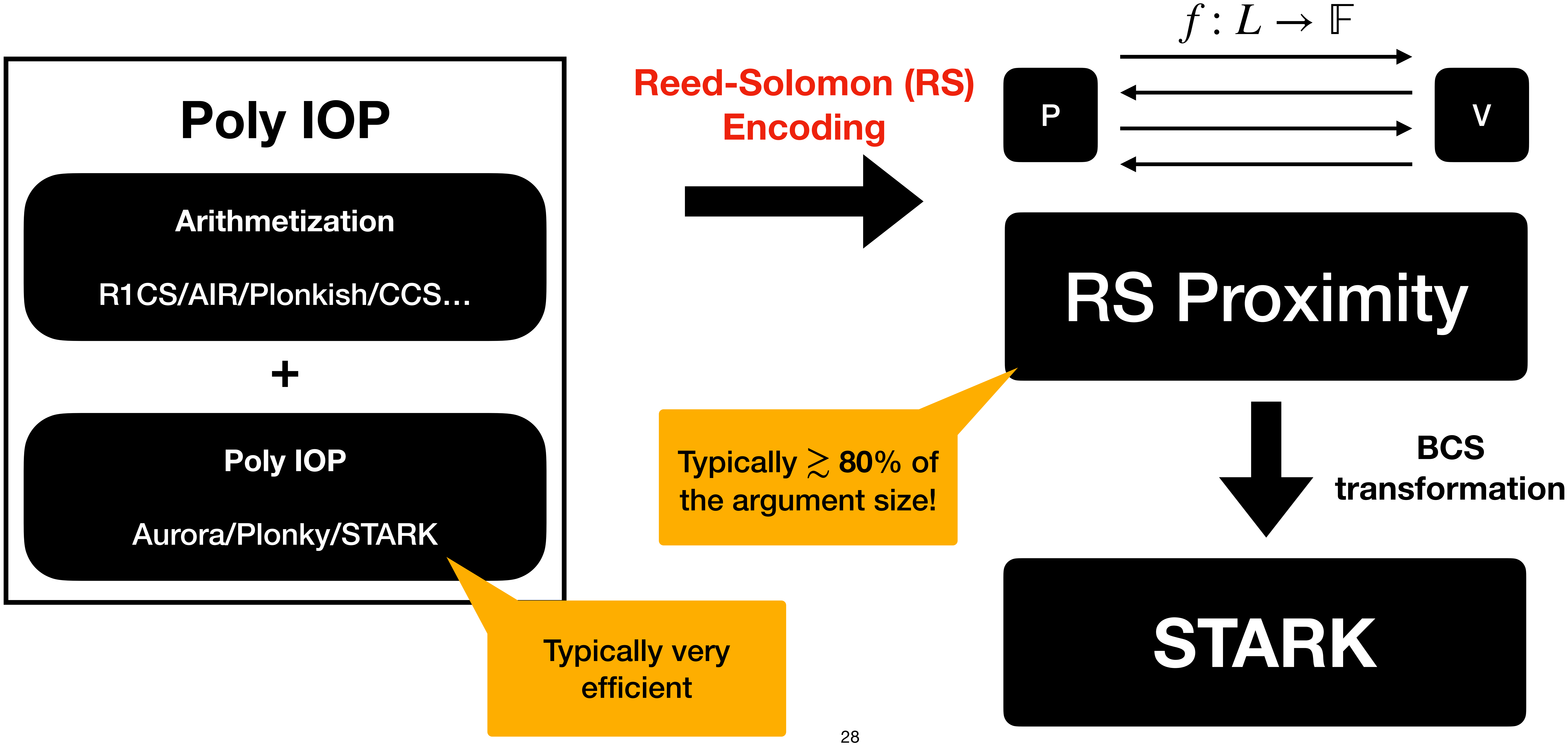

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RS Proximity

Typically $\gtrsim$ 80% of the argument size!

BCS transformation

STARK

STARKs & Friends

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What about the conjecture?

FRI and STIR benefit in roughly the same way

- Conjecture on list-decoding up to distance $1 - \rho$ (instead of $1 - \sqrt{\rho}$)
- FRI queries:

$$O\left(\lambda \cdot \frac{\log d}{-\log \rho}\right)$$

In both, for $\delta = 1 - \rho$,

reduces queries by ~2x

- STIR queries:

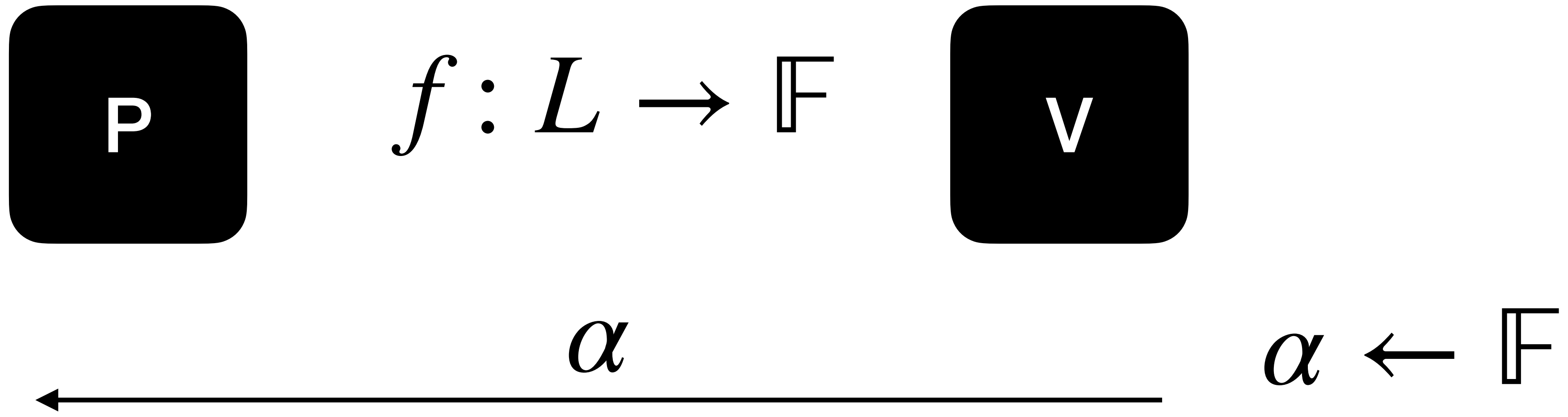
$$O\left(\lambda \cdot \log\left(\frac{\log d}{-\log \rho}\right) + \log d\right)$$

STIR iteration

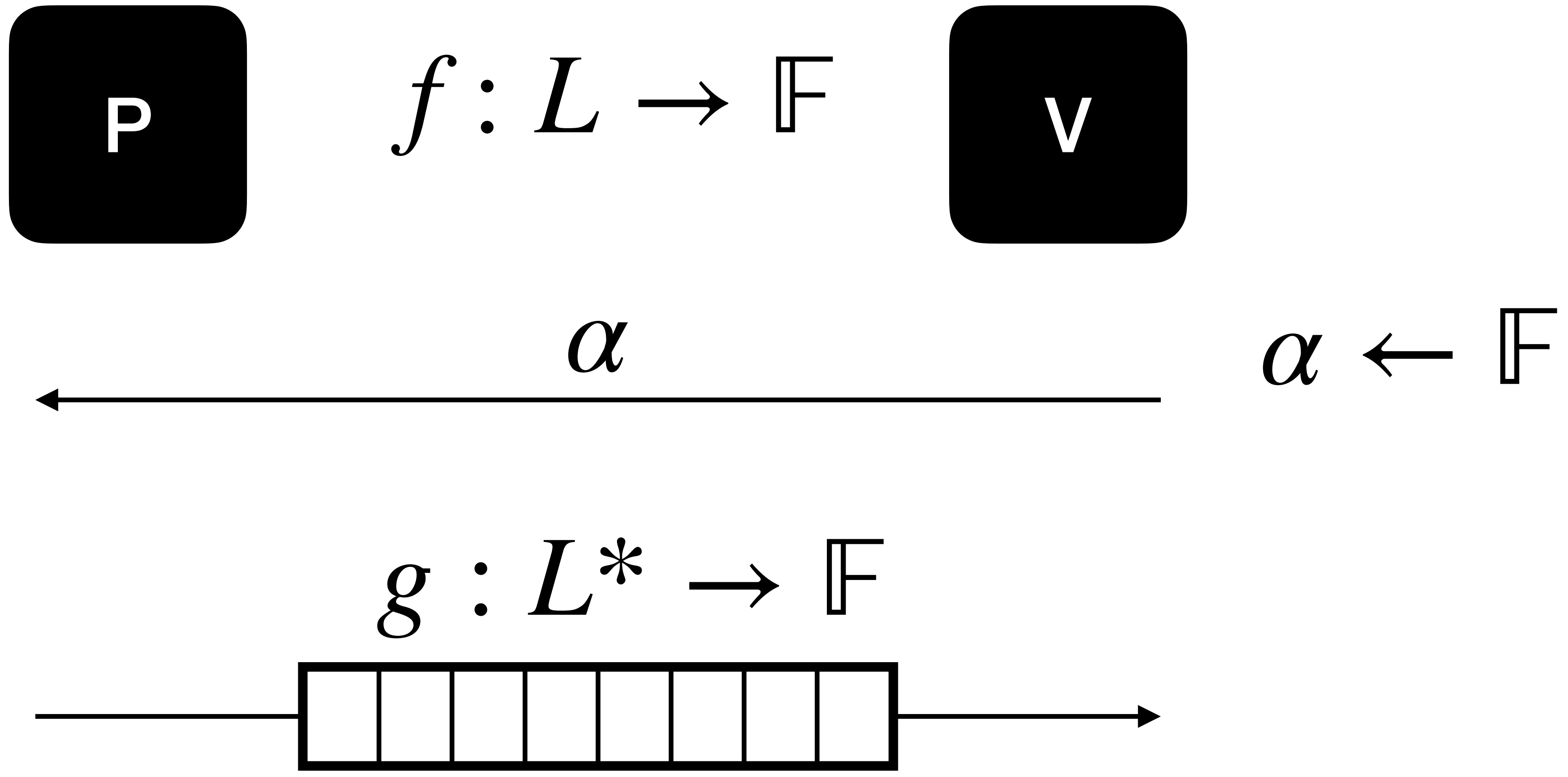
STIR iteration

P $f : L \to \mathbb{F}$ V

STIR iteration



STIR iteration



P

$f : L \to \mathbb{F}$

V

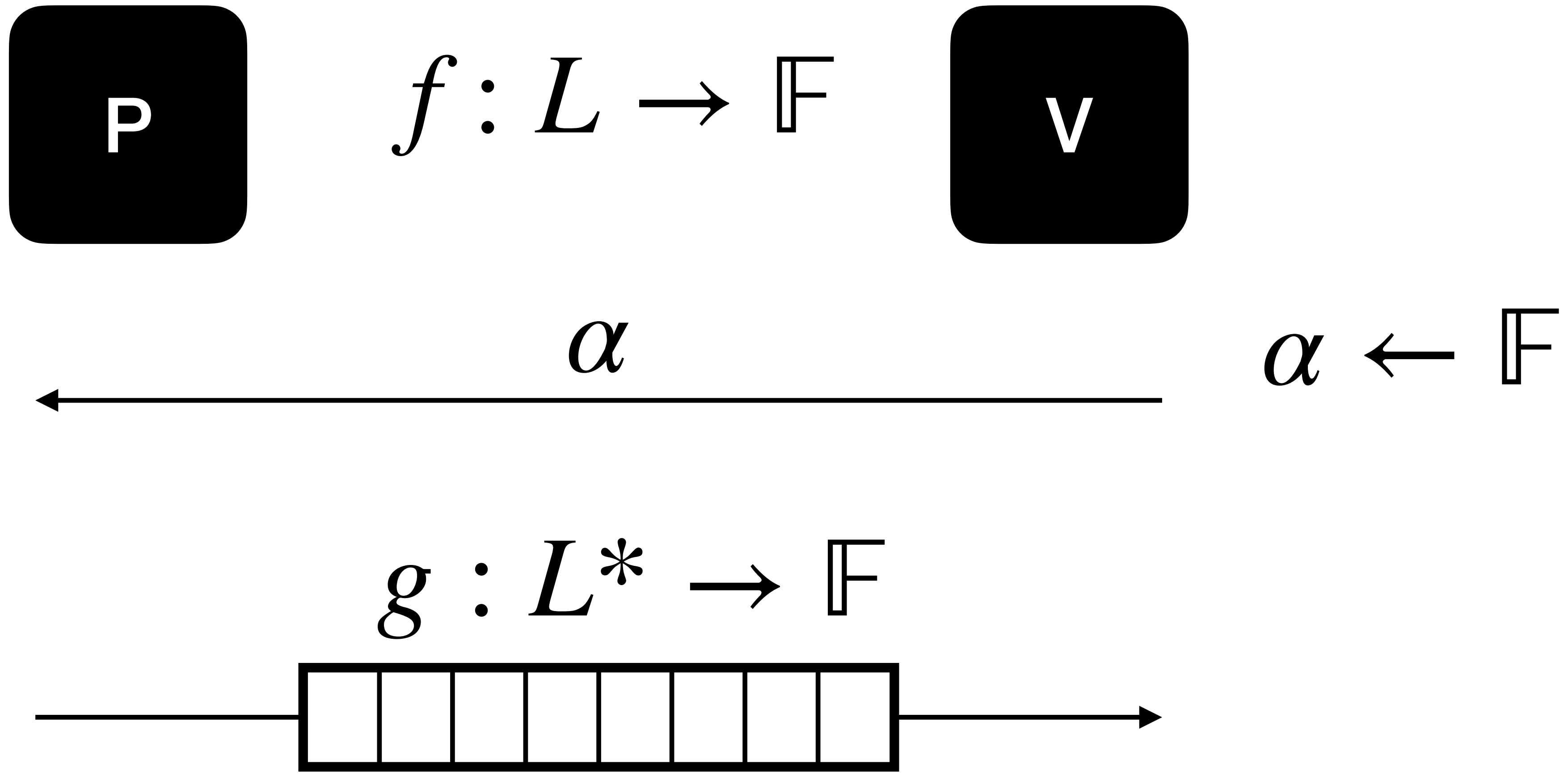
α

$\alpha \leftarrow \mathbb{F}$

$g : L^* \to \mathbb{F}$

STIR iteration

g is *claimed* to be equal to (the extension of) Fold(f, k, α) on L^*



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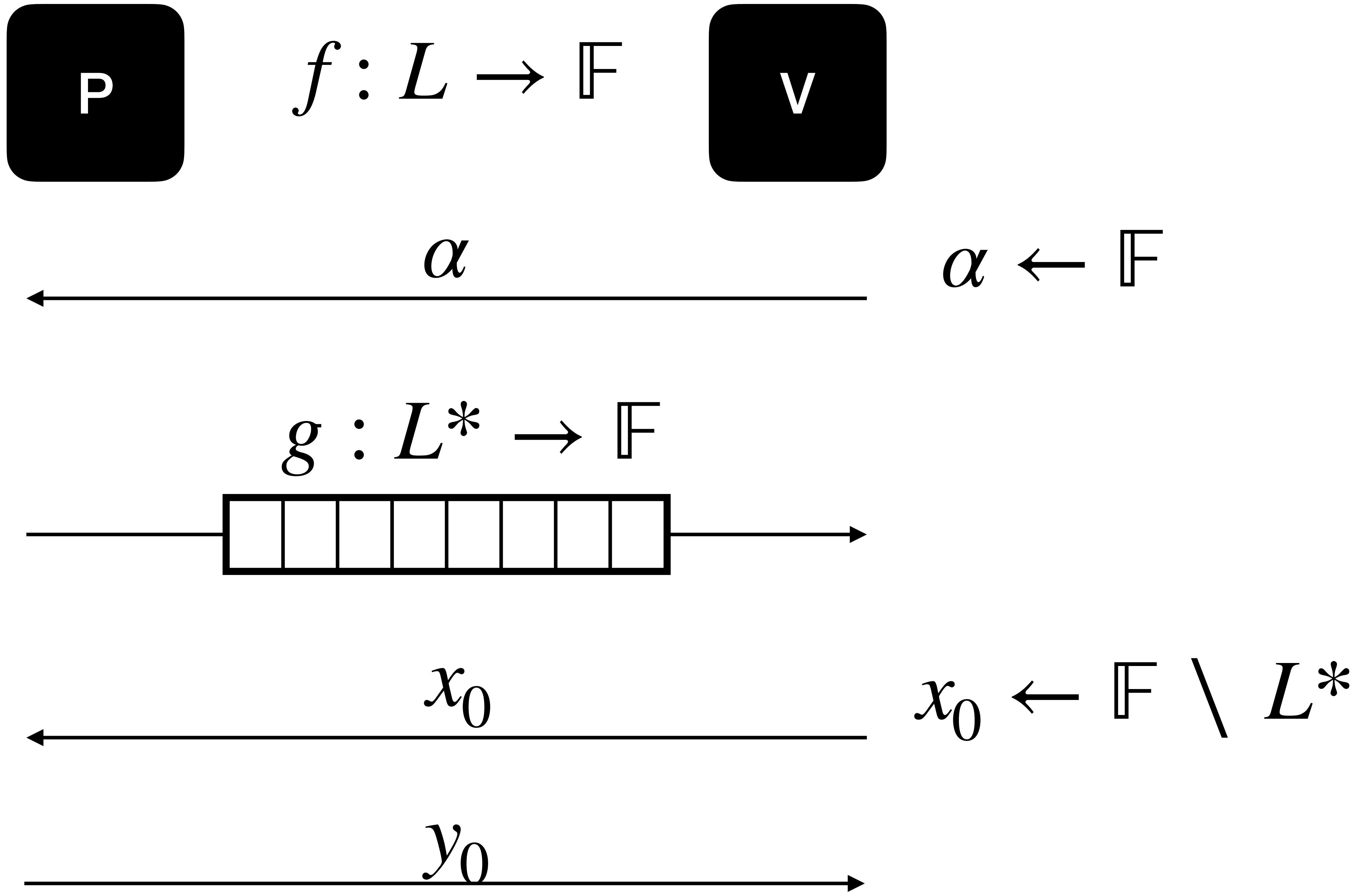

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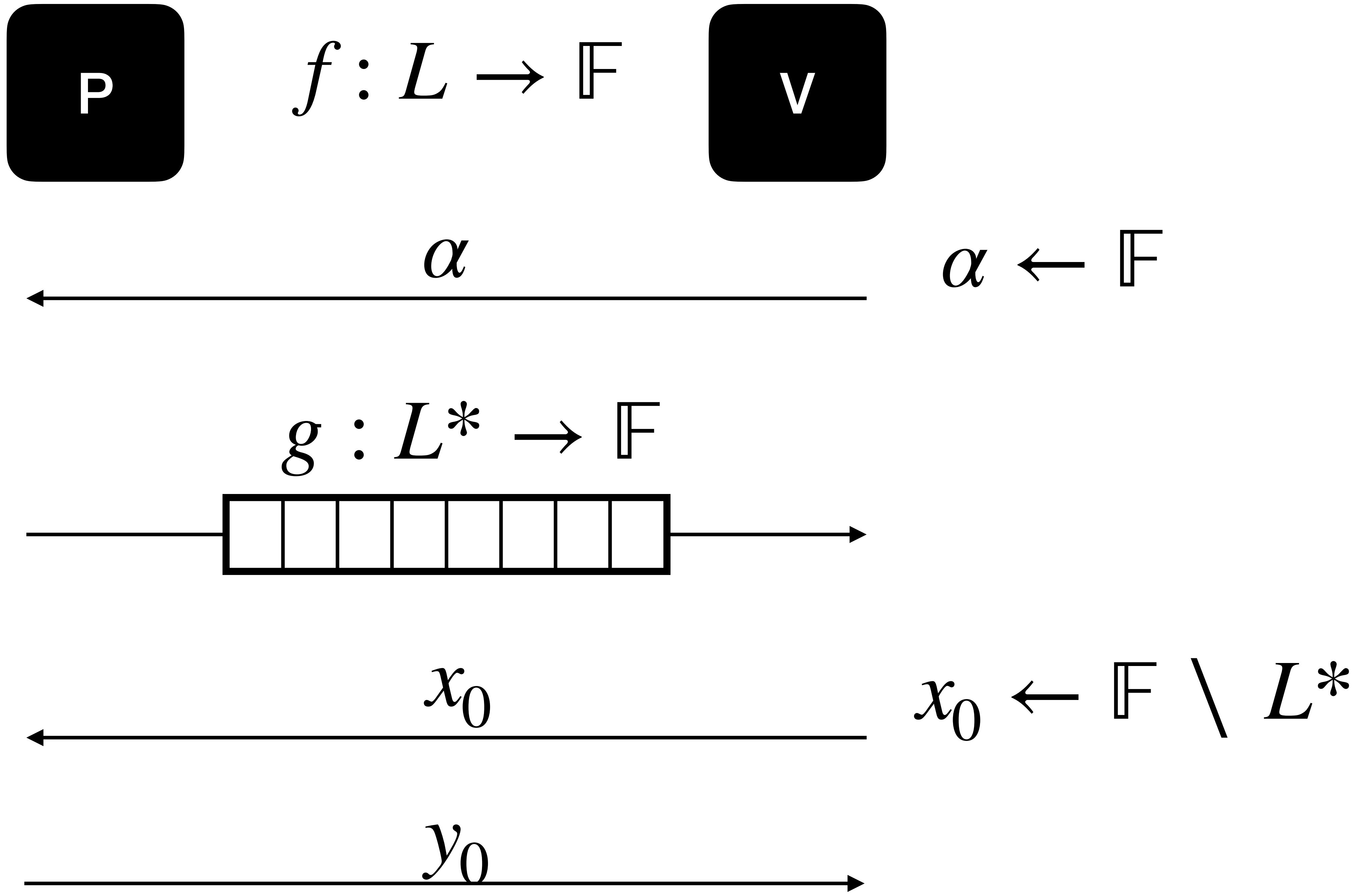
Problem: We can only query Fold(f, k, α) on $L^k \neq L^*$.

Enforce consistency via Quotient!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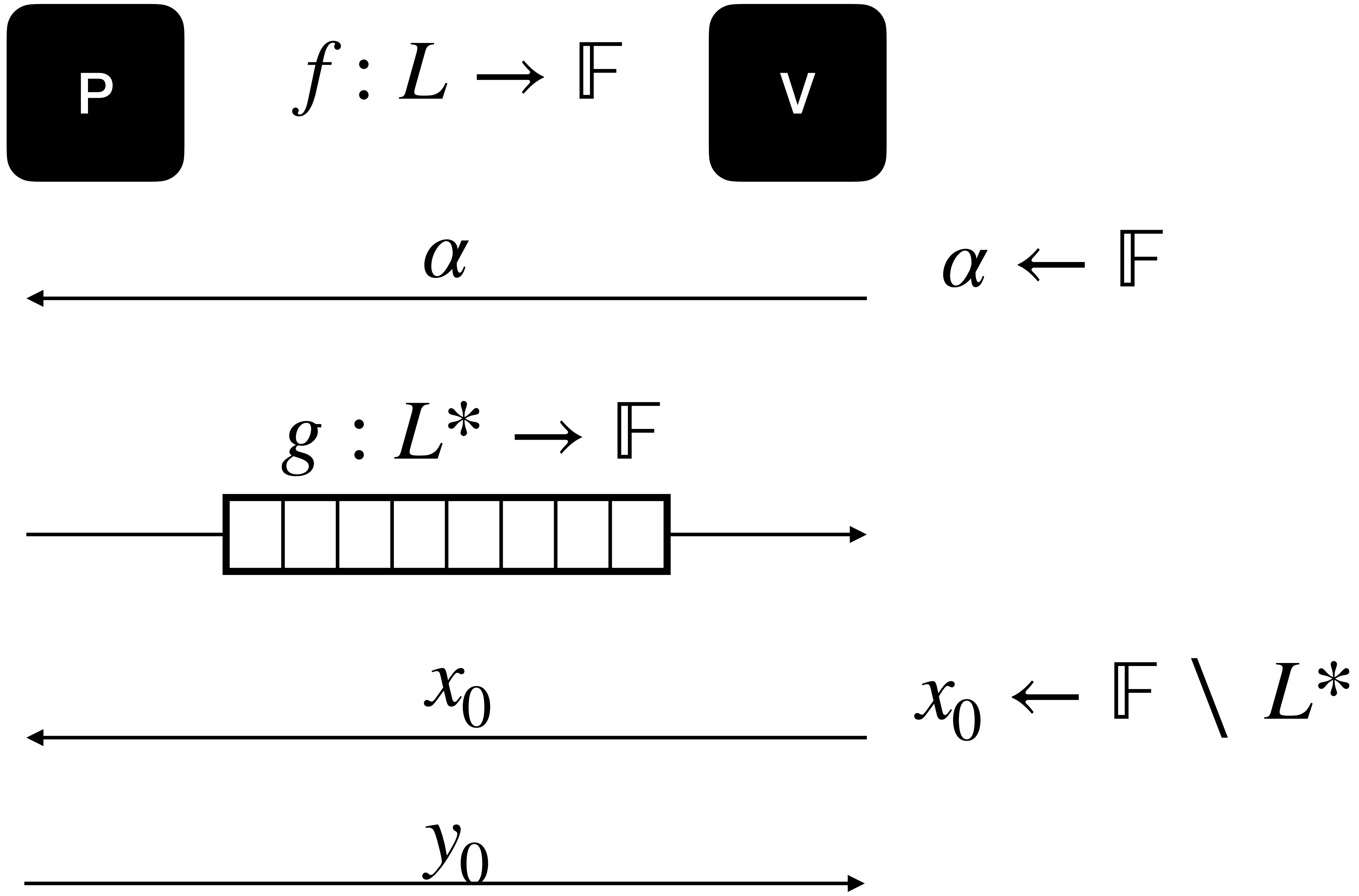
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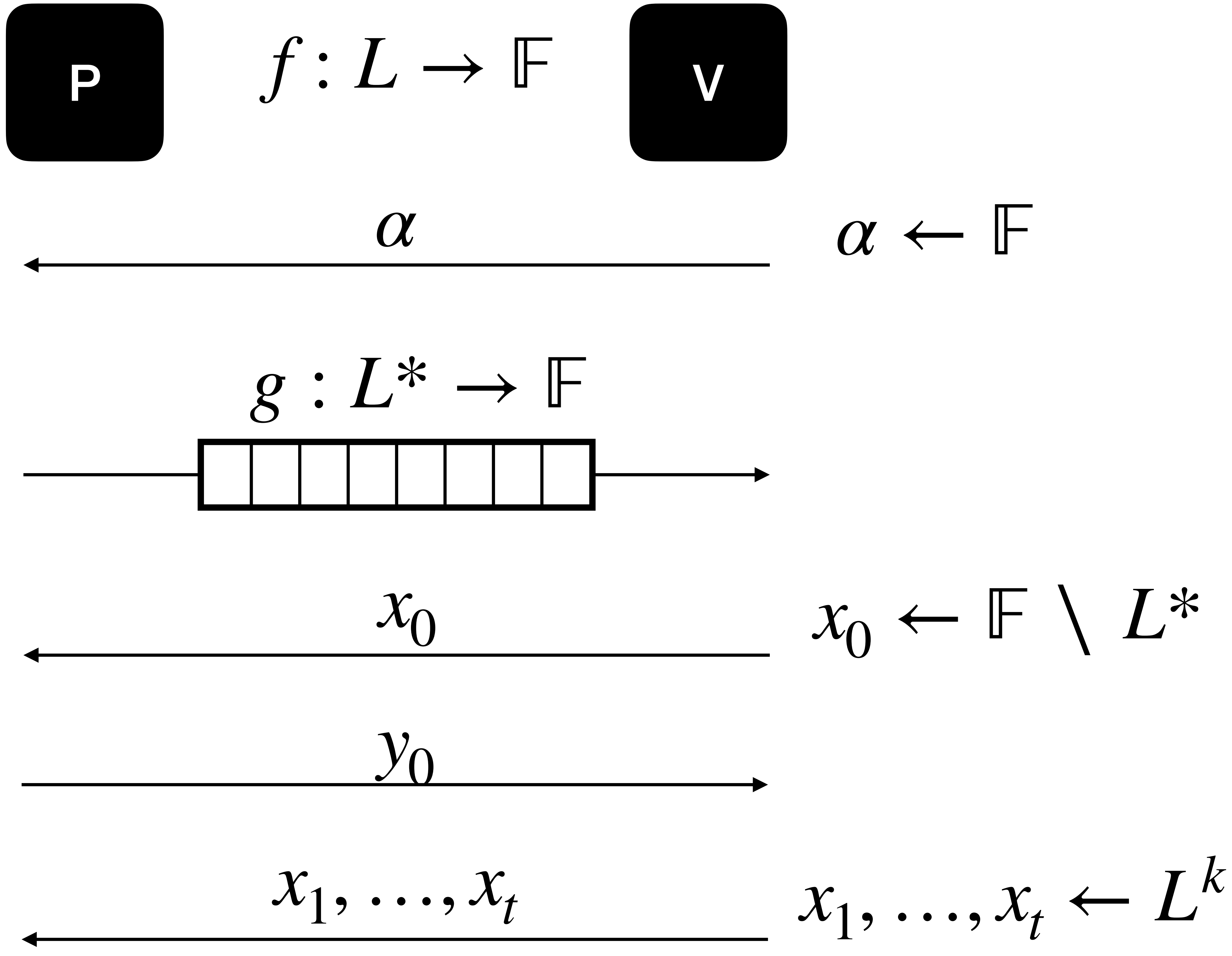
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$\forall\ i$: Ans(x_i) $= y_i$

f' := Quotient(g, Ans)

Query f to get

y_i = Fold(f, k, α)(x_i)

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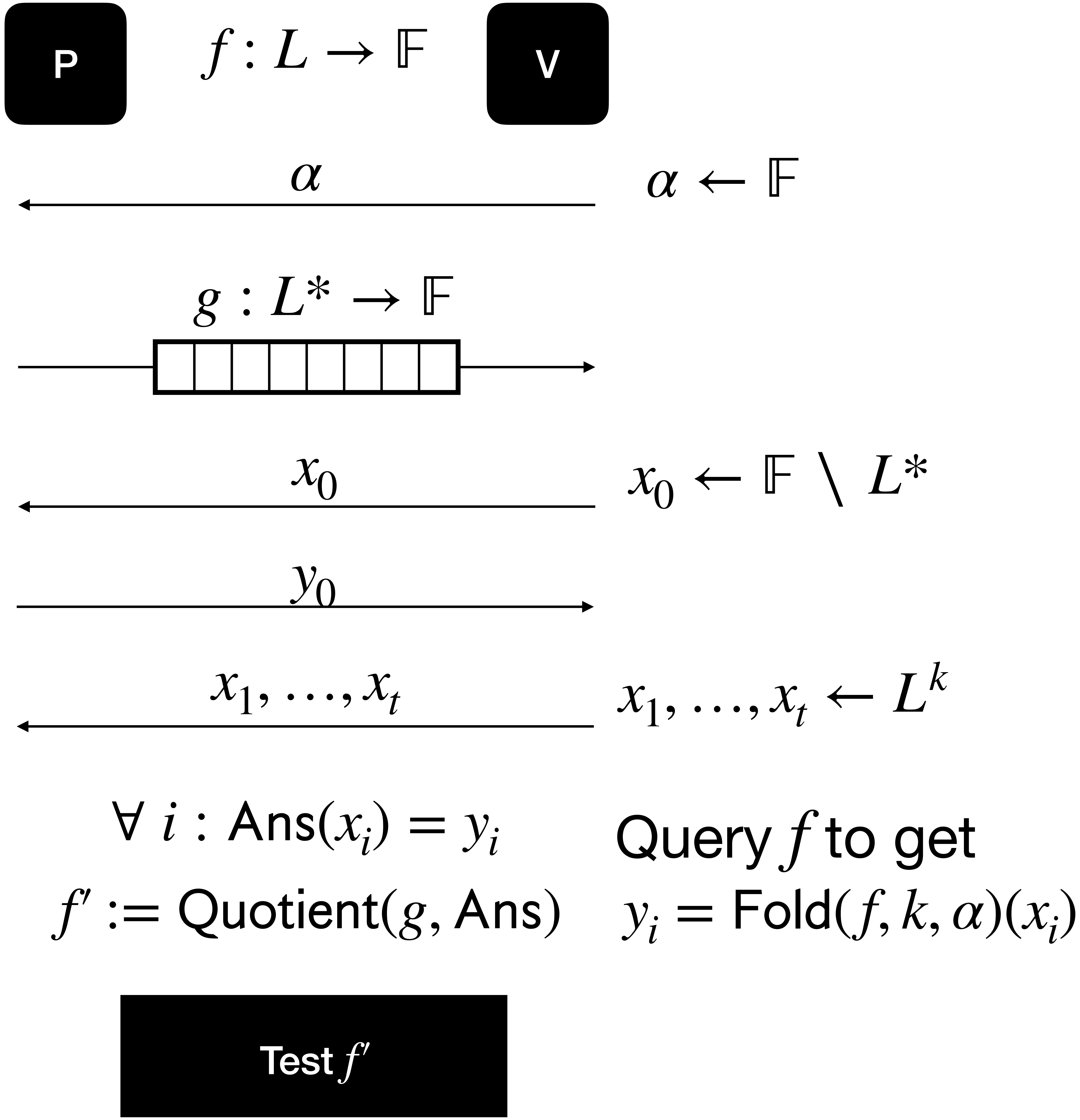

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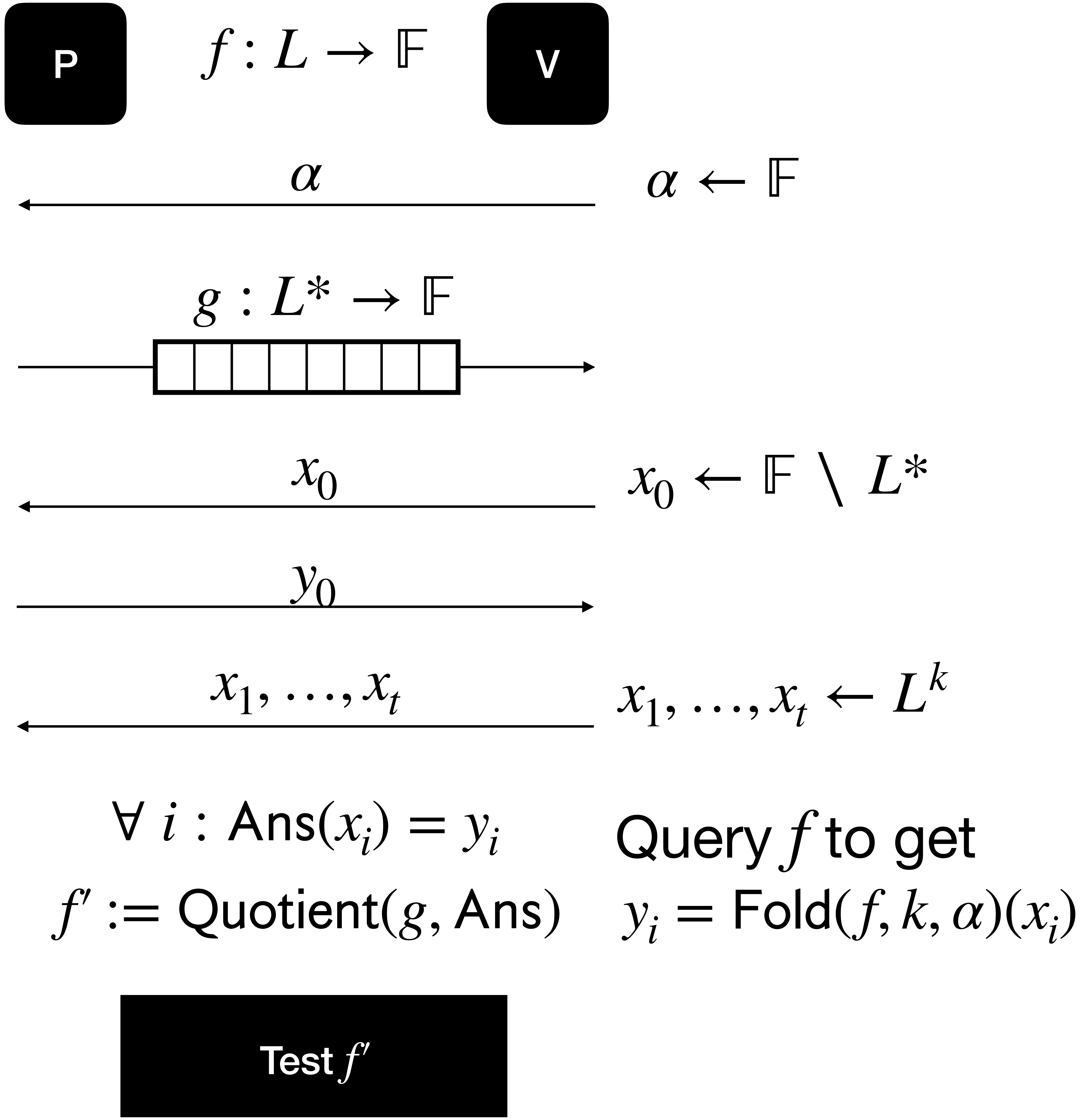
New function is quotient of g w.r.t. to these points + OOD sample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$f' :=$ Quotient$(g,$ Ans$)$
Query f to get
$y_i =$ Fold$(f, k, \alpha)(x_i)$
Test f'

Claim: if f is δ-far from C, unless with probability $\approx (1 - \delta)^t, f'$ is $(1 - \sqrt{\rho'})$ far from C'

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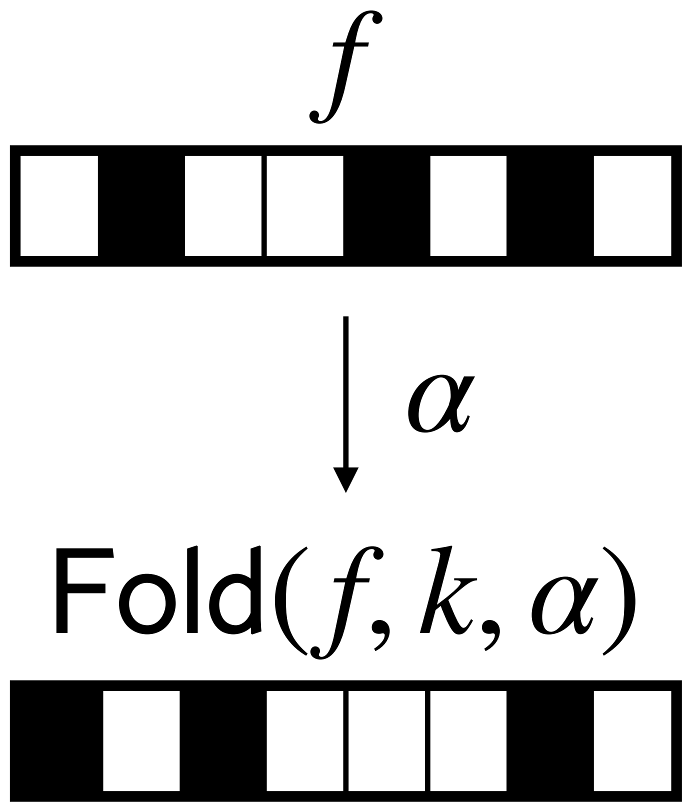
Folding



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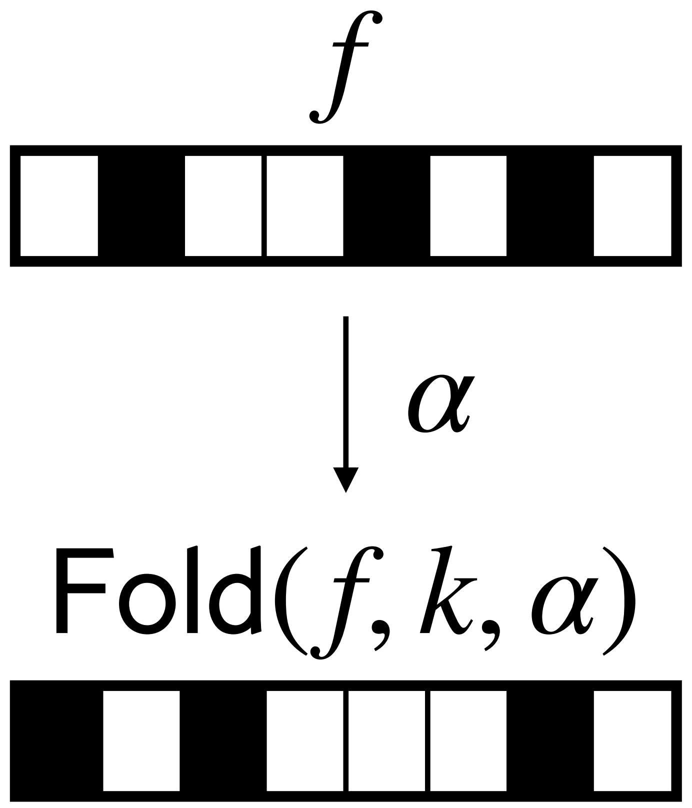
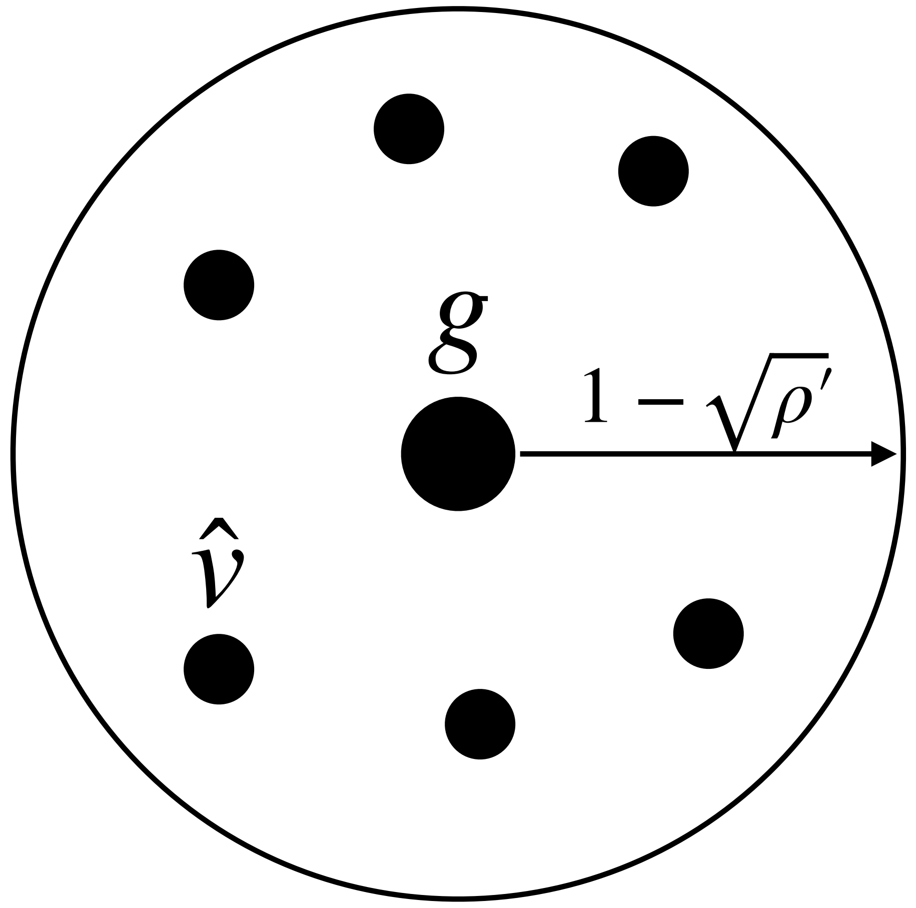


OOD



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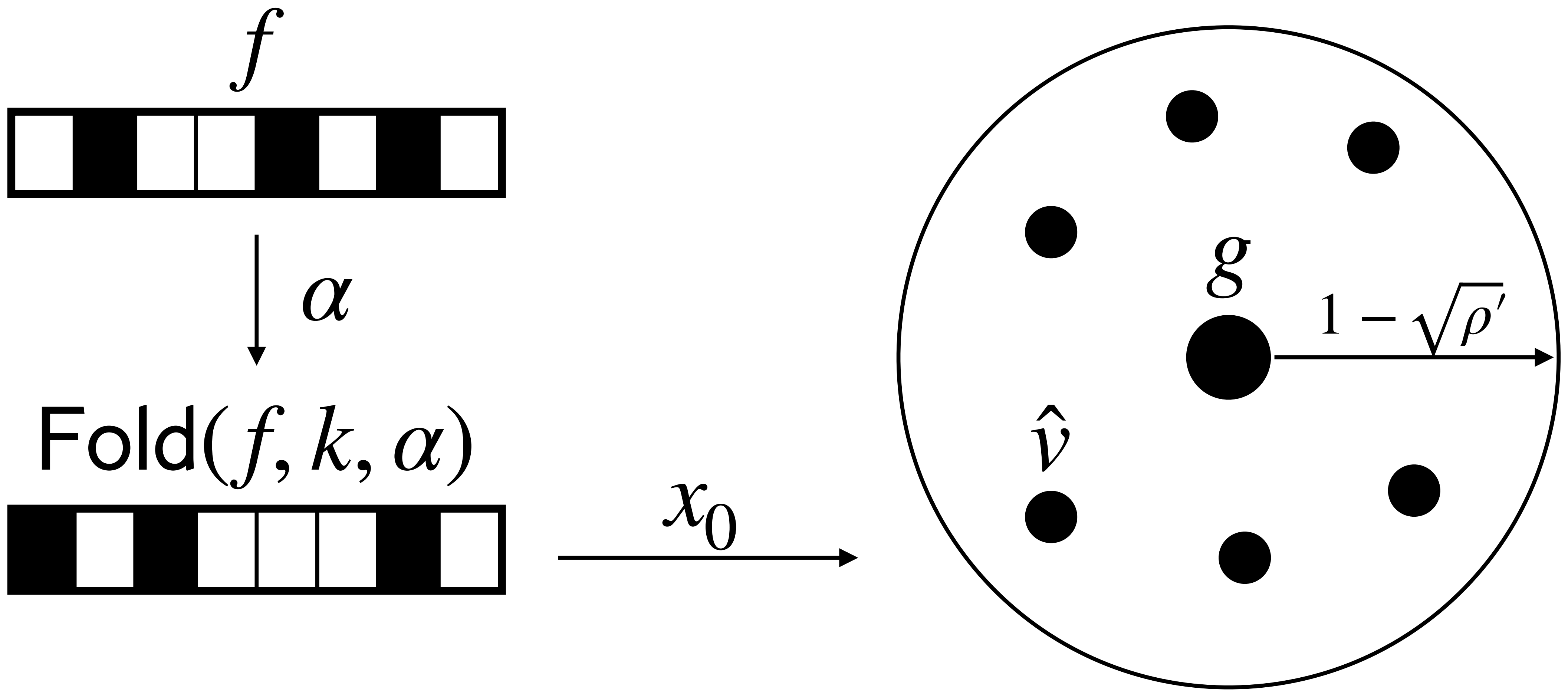

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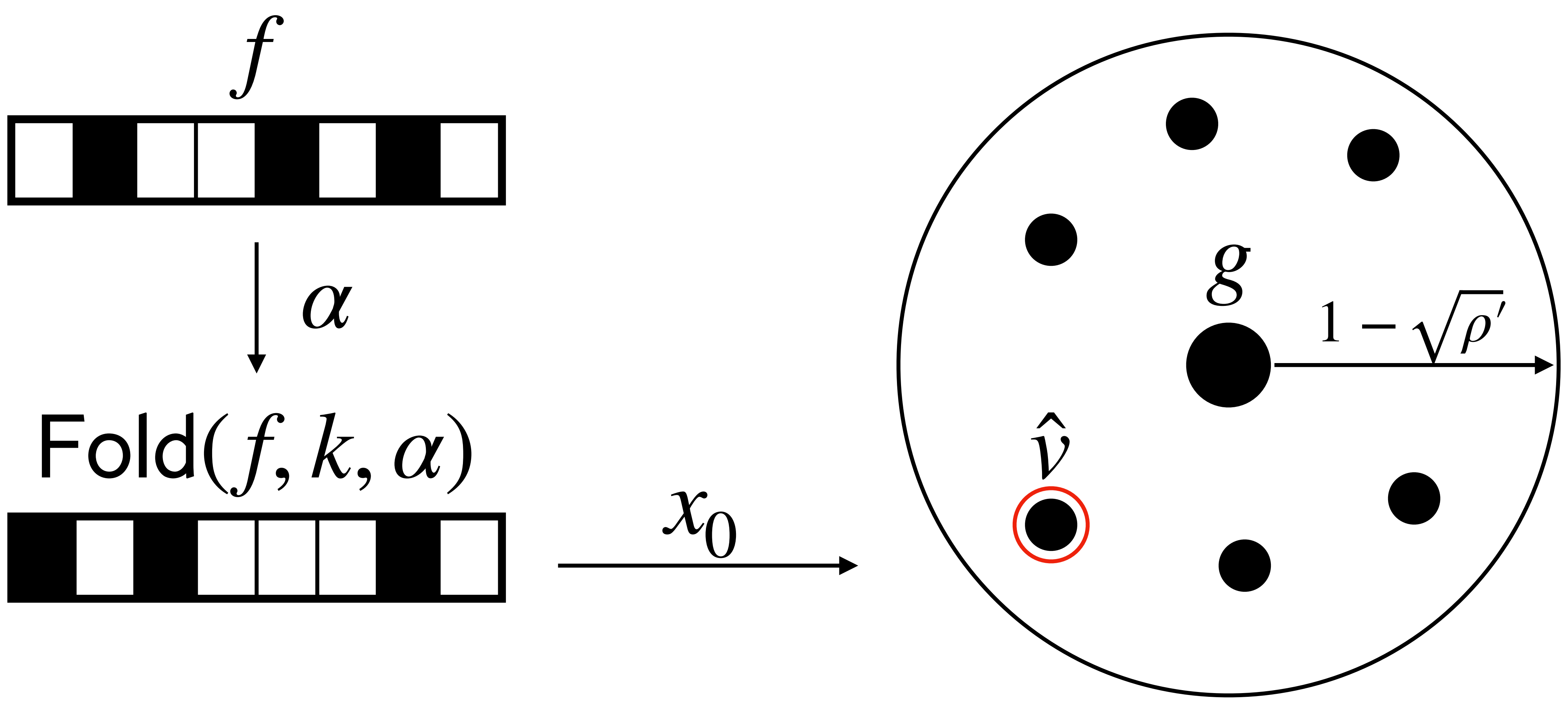

$\hat{v}$ is **unique** close codeword to g with $\hat{v}(x_0) = y_0$

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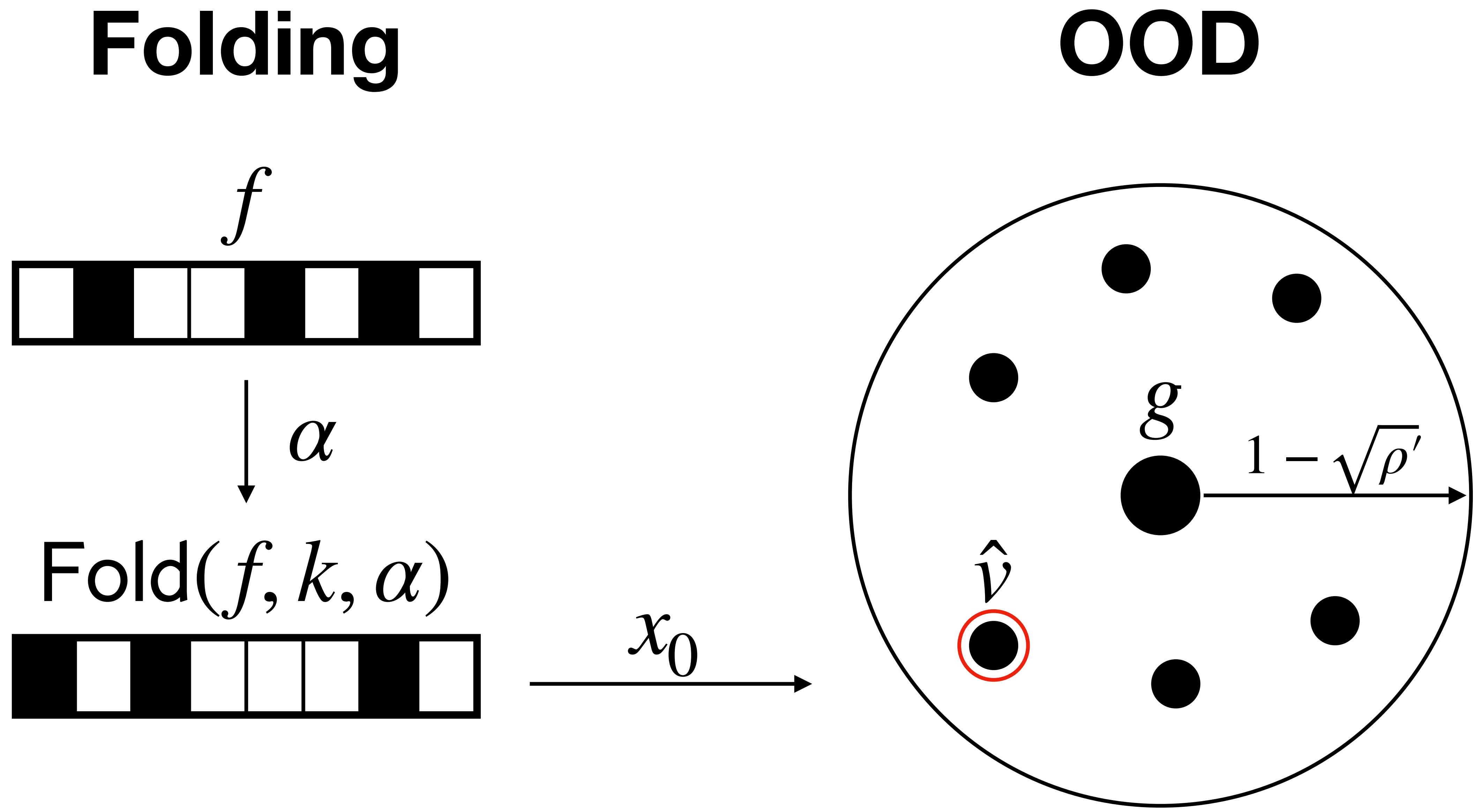

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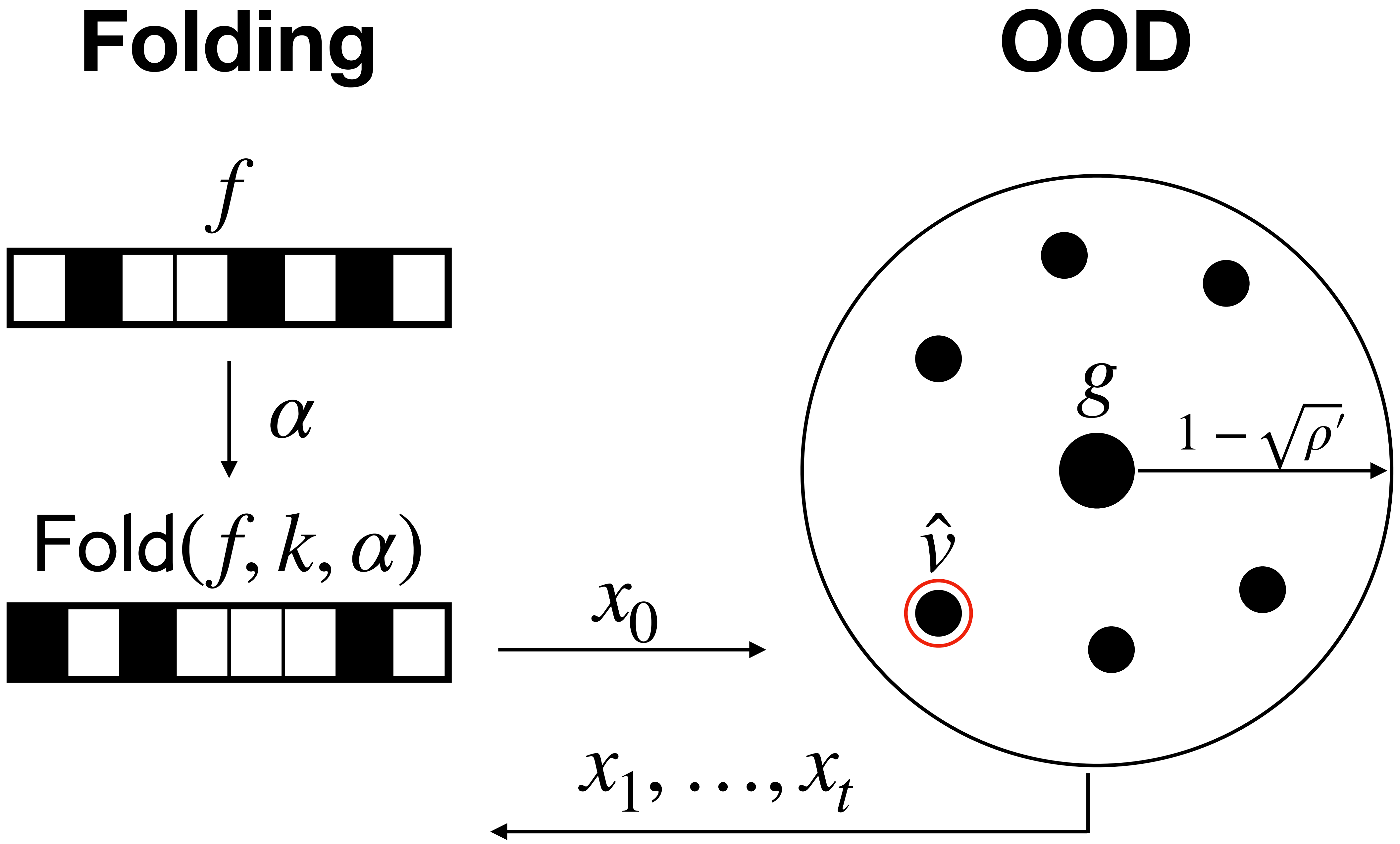

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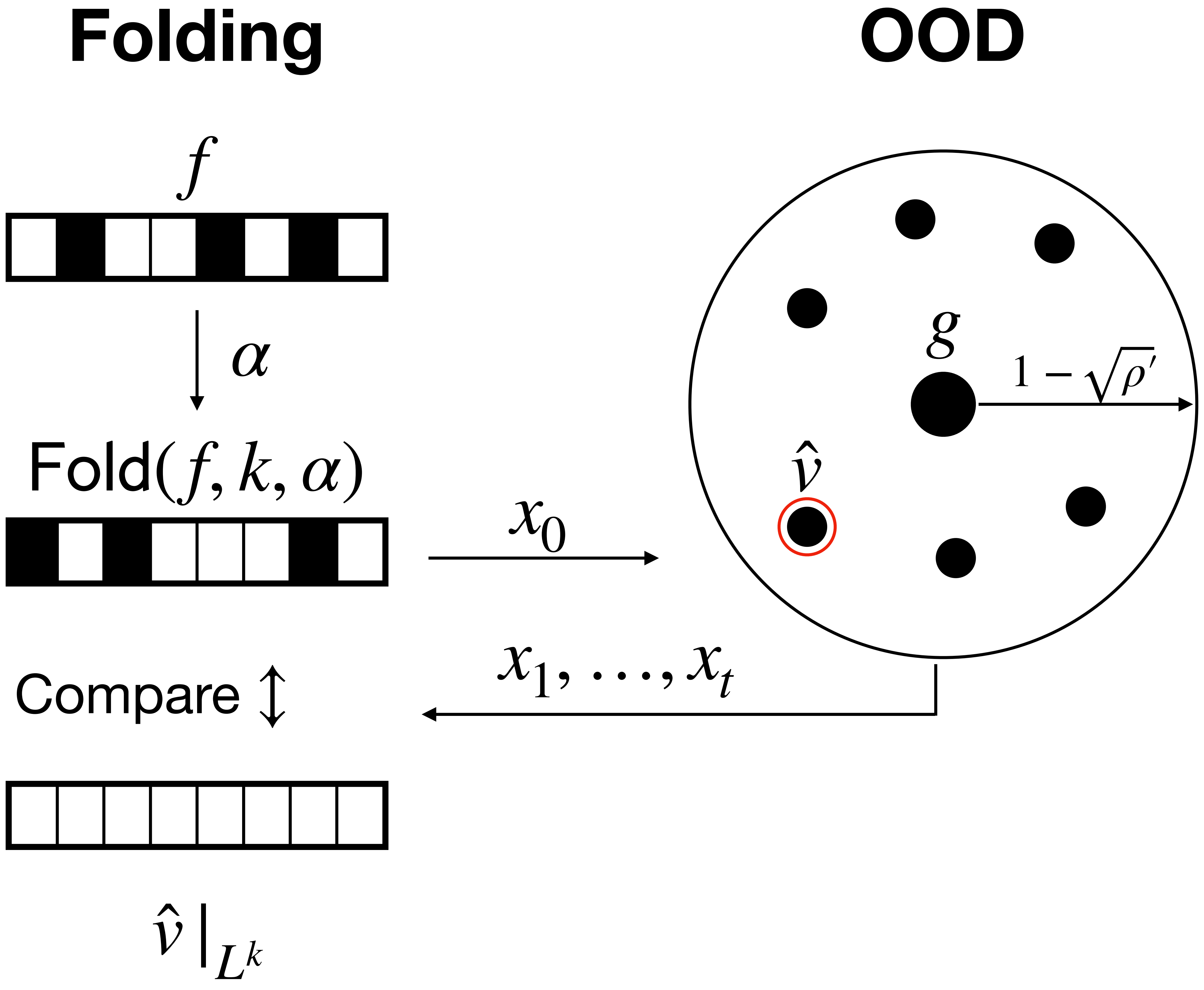

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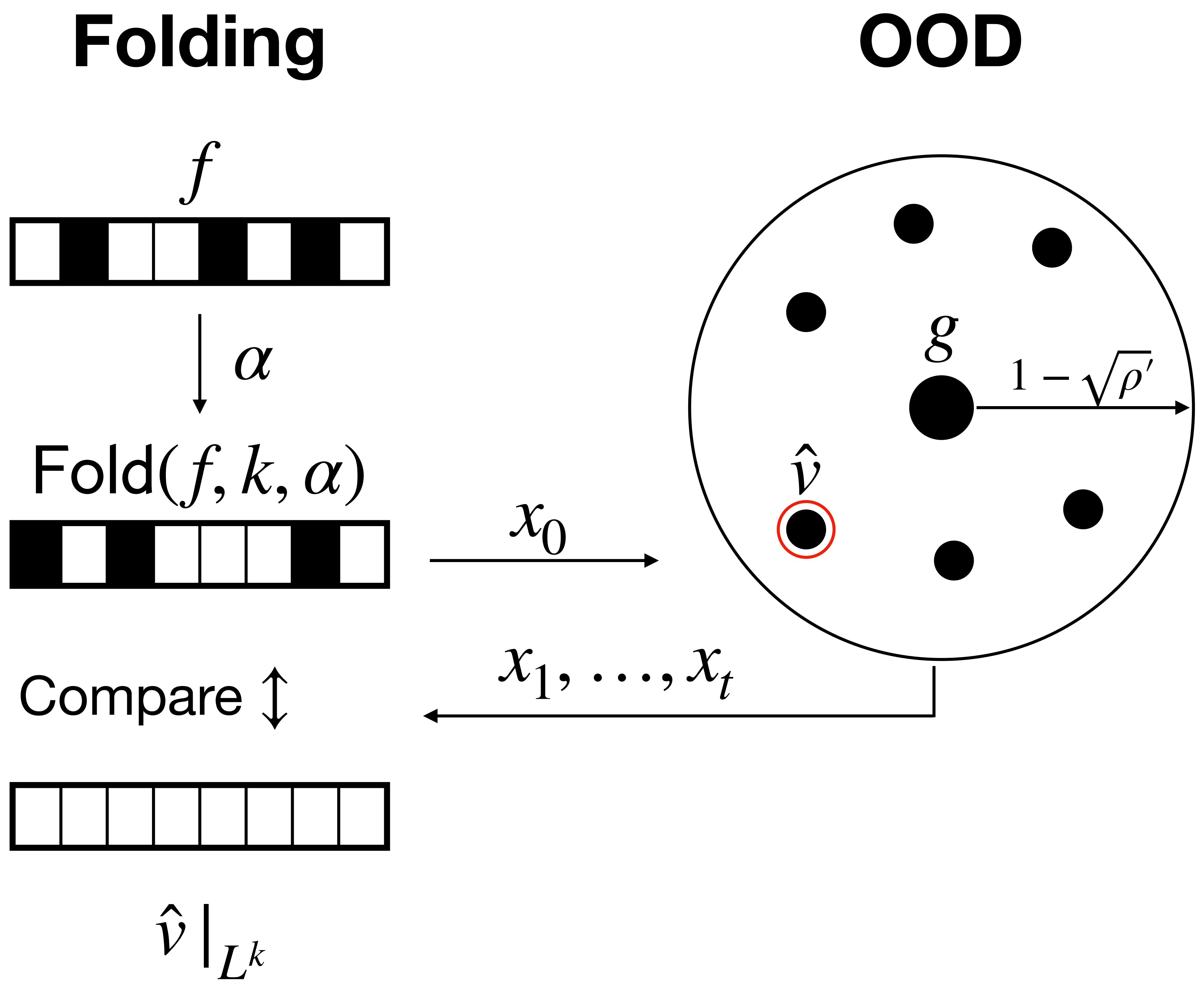

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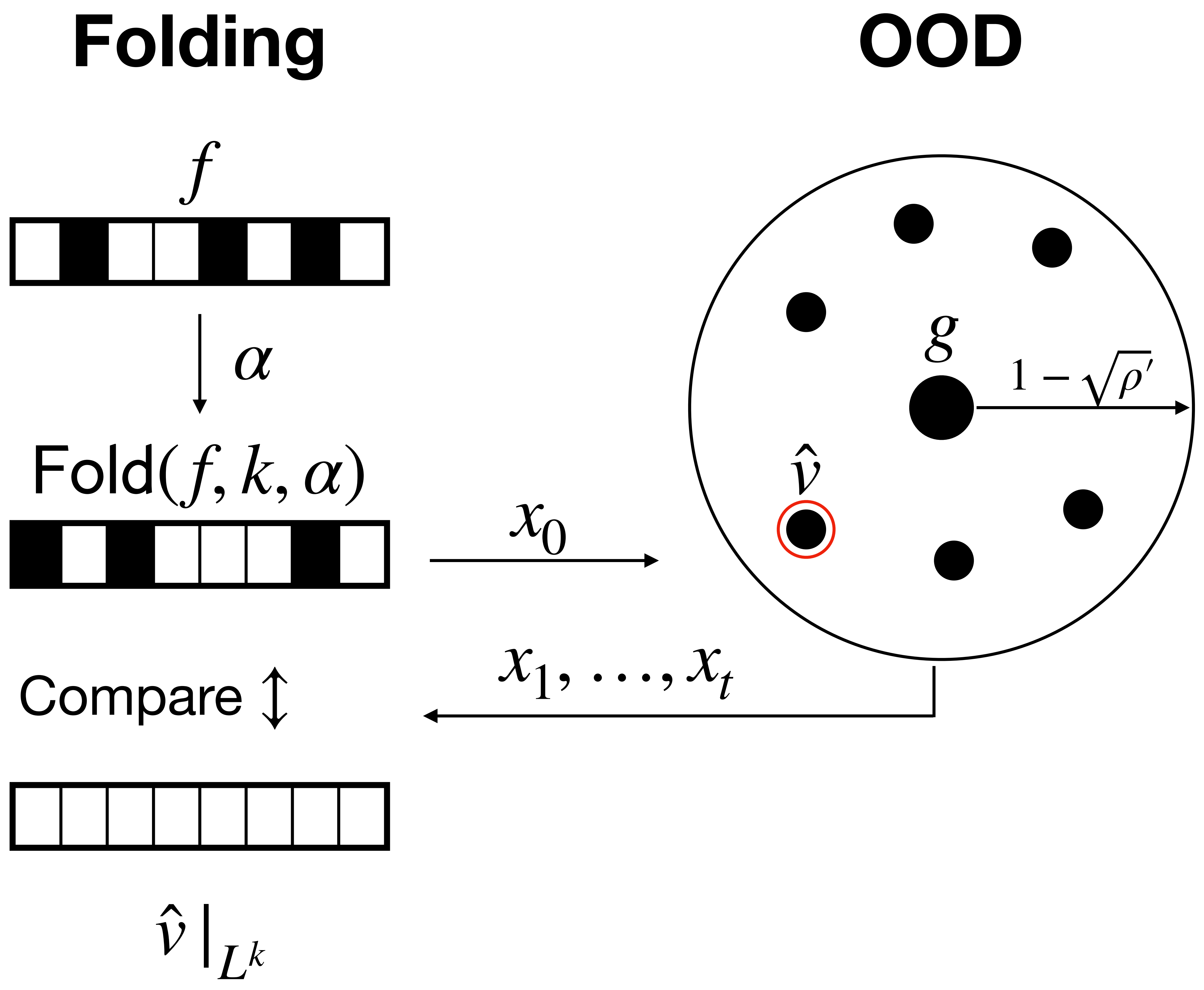

Folding

OOD

f

α

Fold(f, k, α)

x_0

g

$1-\sqrt{\rho'}$

$\hat{v}$

$x_1, \ldots, x_t$

Compare ↕

$\hat{v}\,|_{L^k}$

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Since Fold is δ-far from the code, $\Delta(\hat{v}\,|_{L^k}, \text{Fold}(f, k, \alpha)) > \delta$

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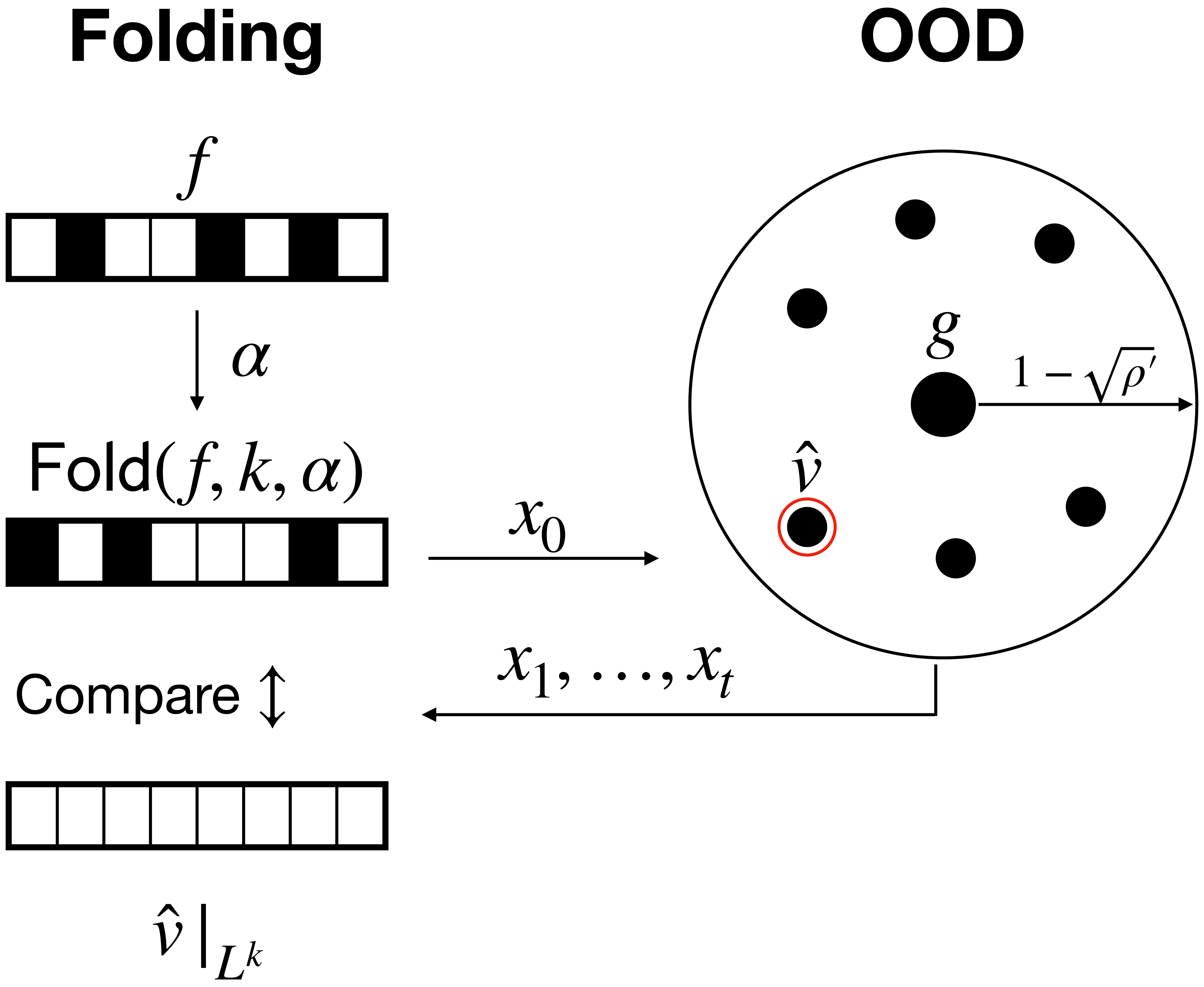

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$$\Pr\left[f' \text{ is } 1-\sqrt{\rho'} \text{ close }\right]$$
$$\leq \Pr\left[\forall i,\ \hat{v}(x_i) = y_i\right]$$
$$= \Pr\left[\forall i,\ \hat{v}(x_i) = \text{Fold}(f, k, \alpha)(x_i)\right]$$
$$\leq (1-\delta)^t$$