Building Succinct Arguments From Ideal Hash Functions

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EPFL
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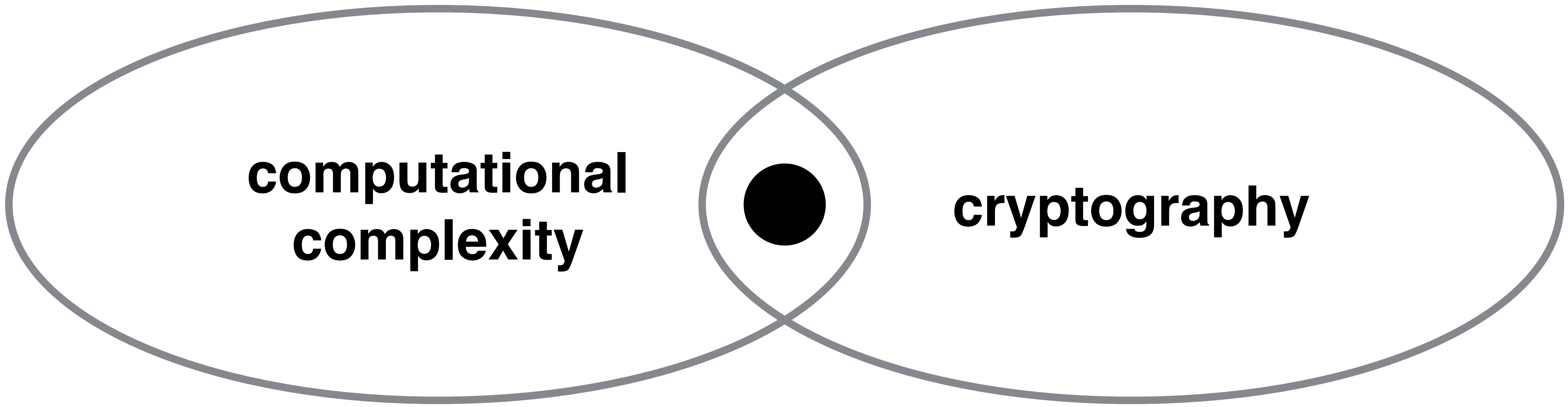

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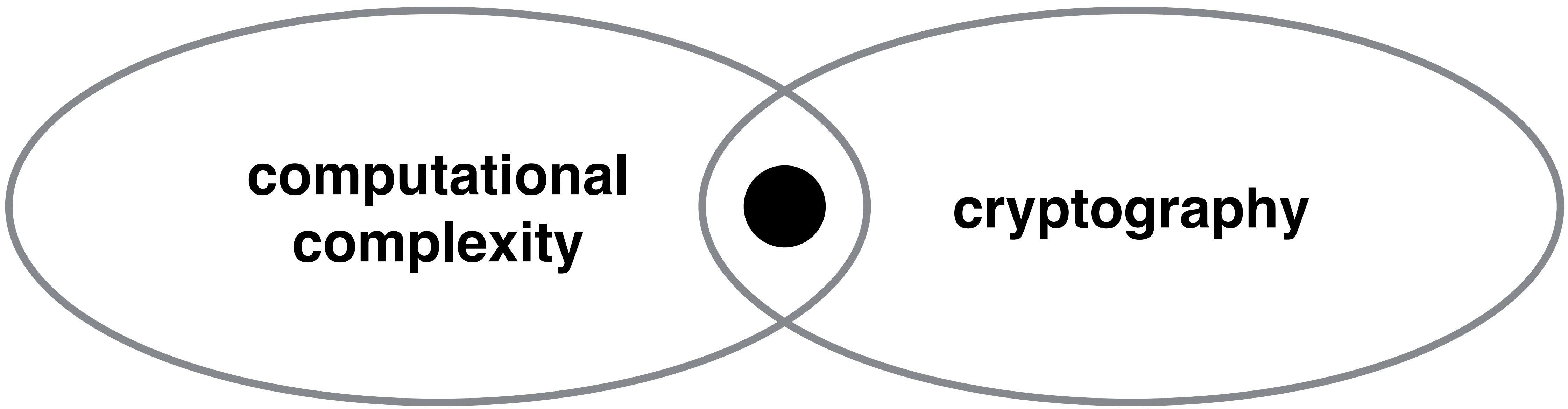

In last 15 years, extraordinary progress in:

- cryptographic foundations
- efficient constructions
- implementations
- applications
- ...

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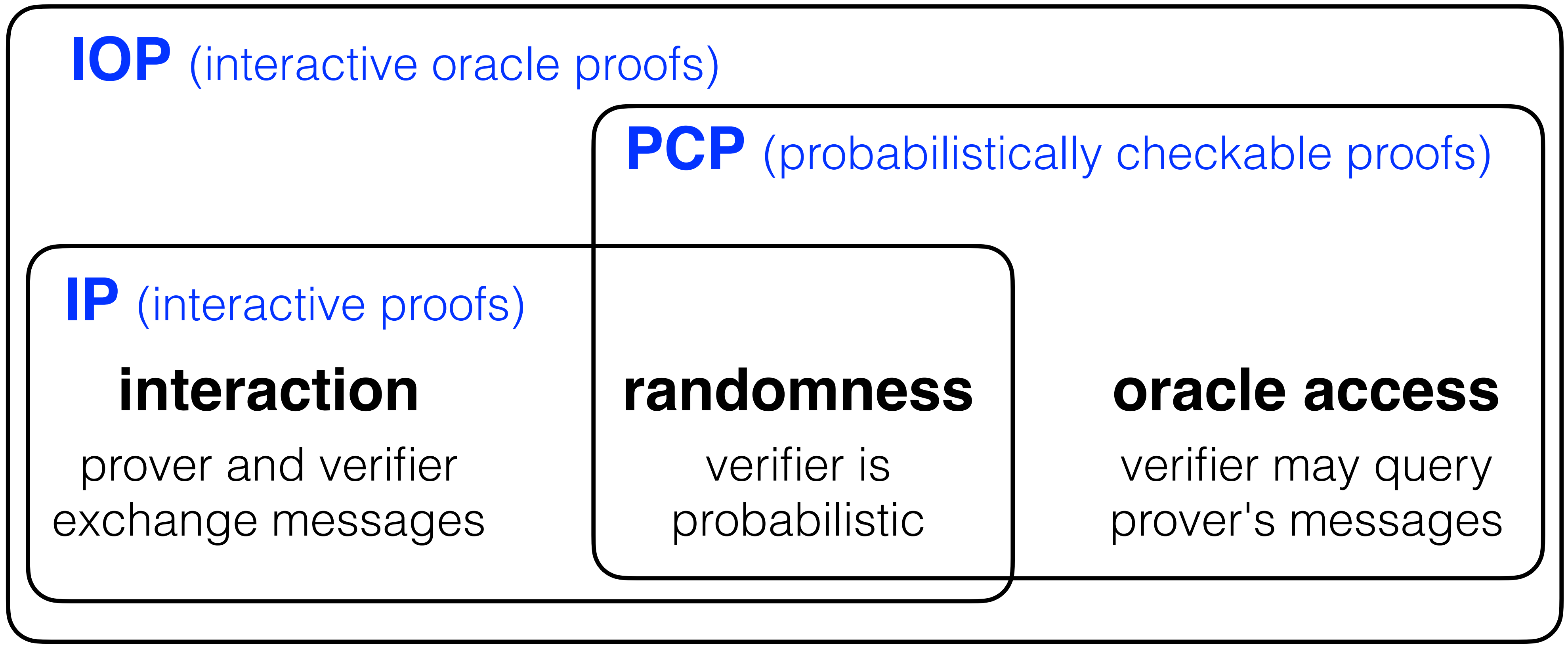

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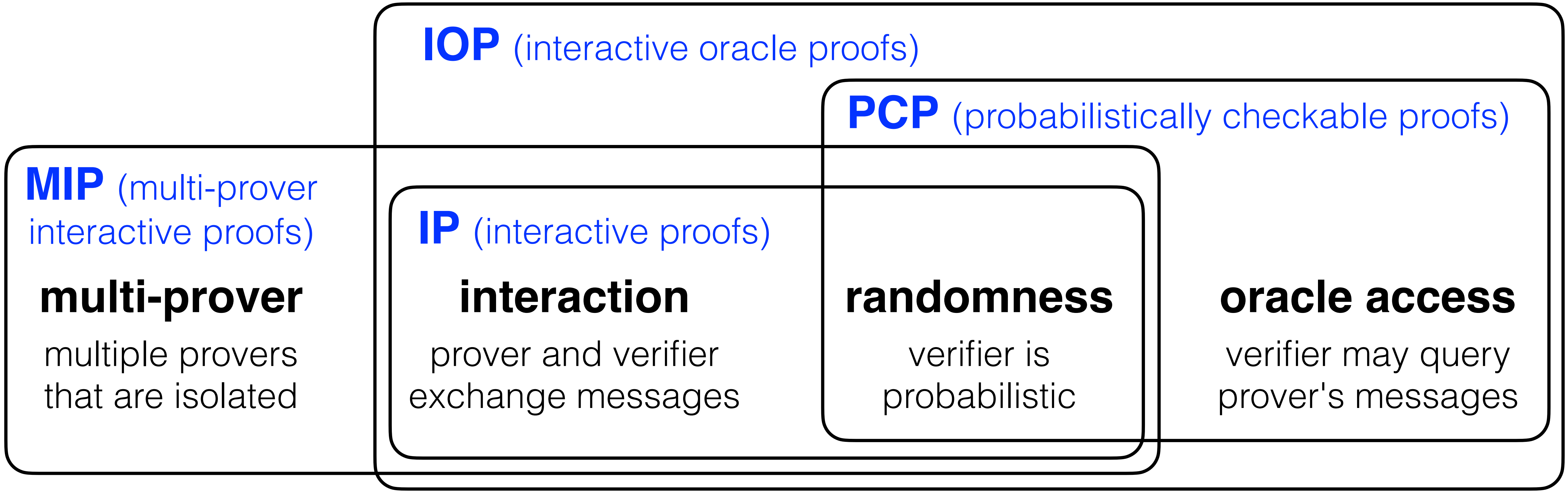

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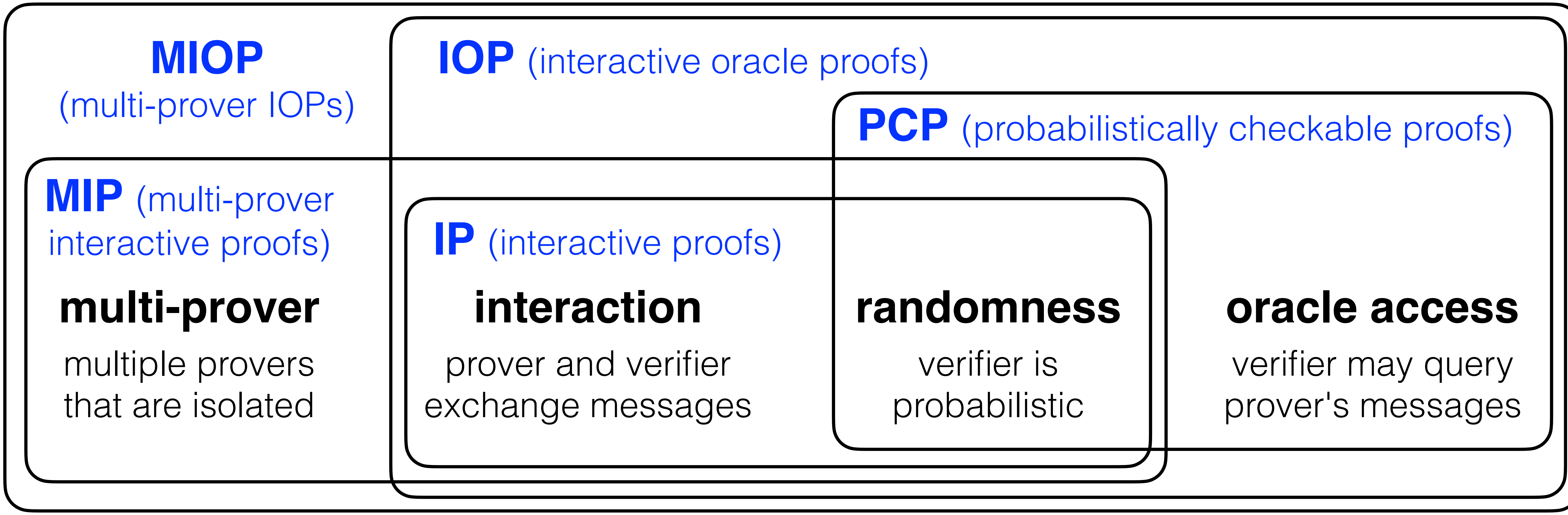

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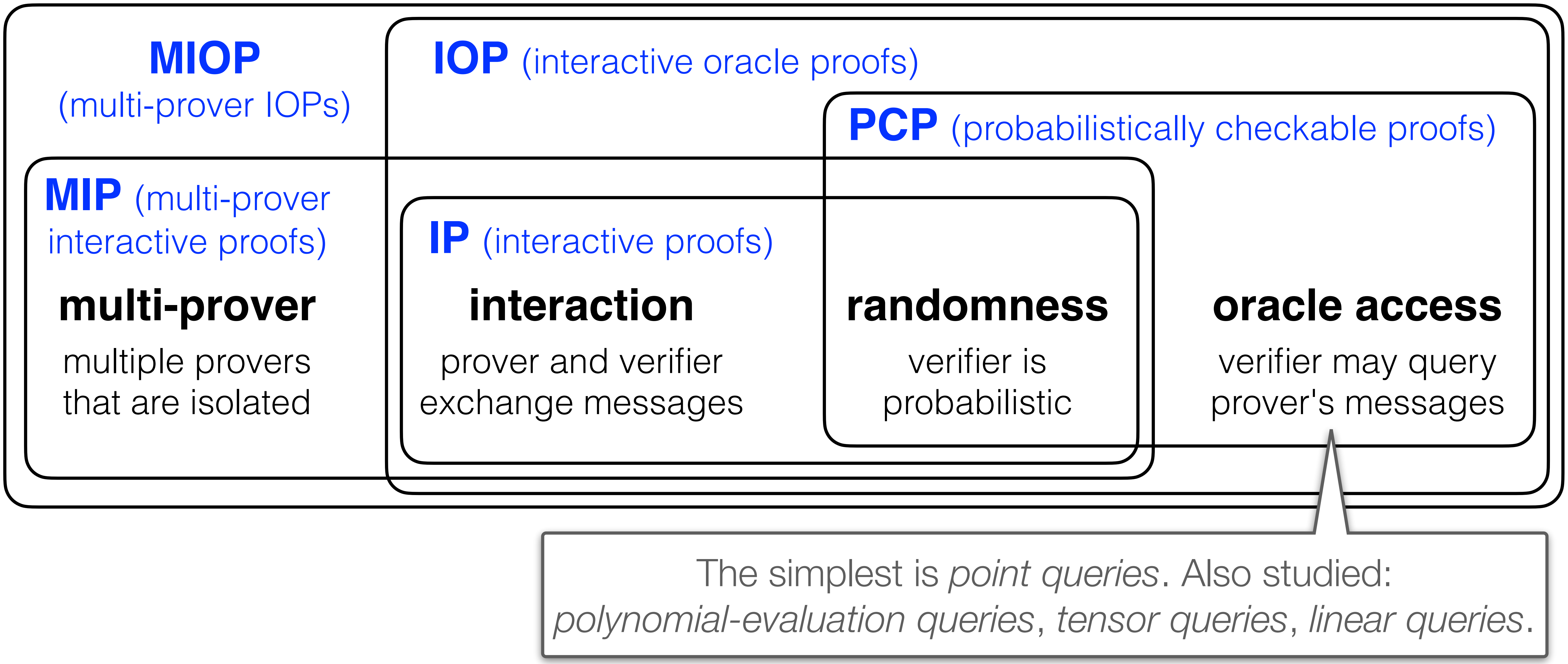

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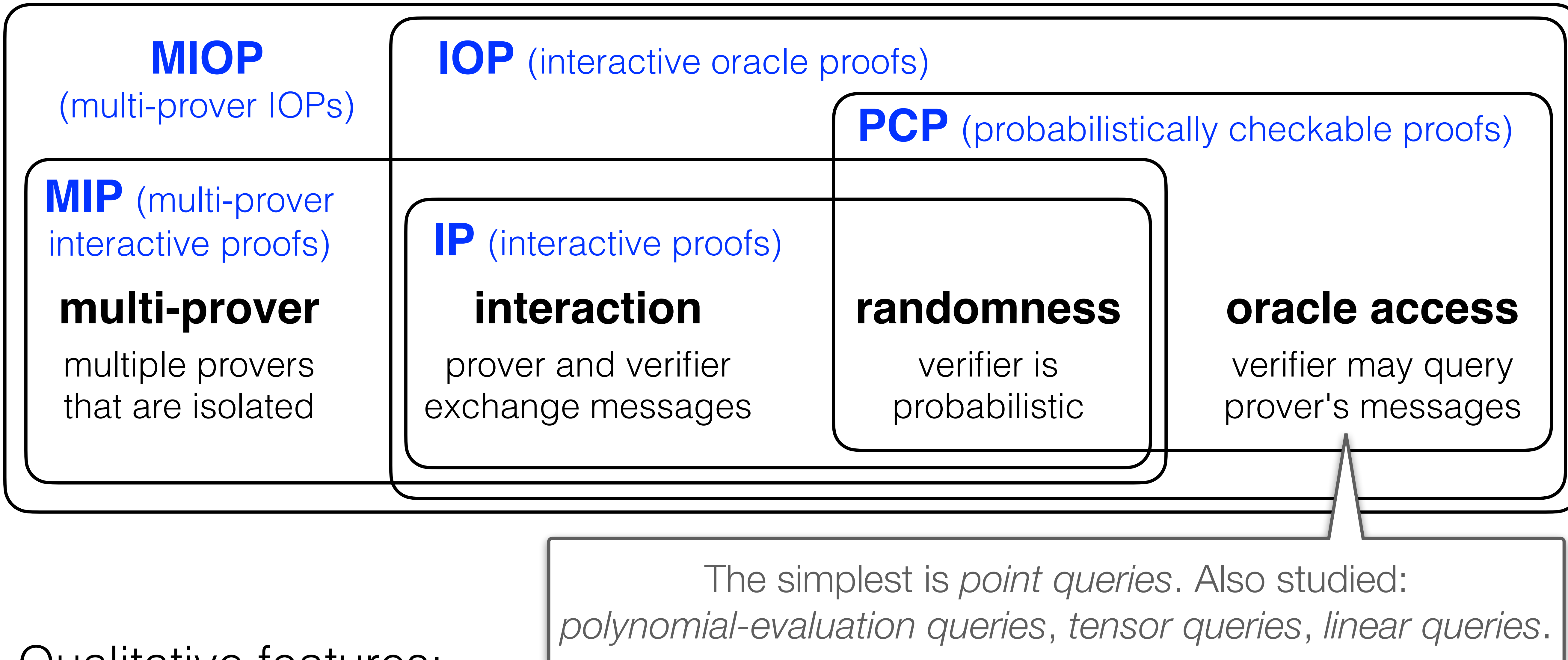

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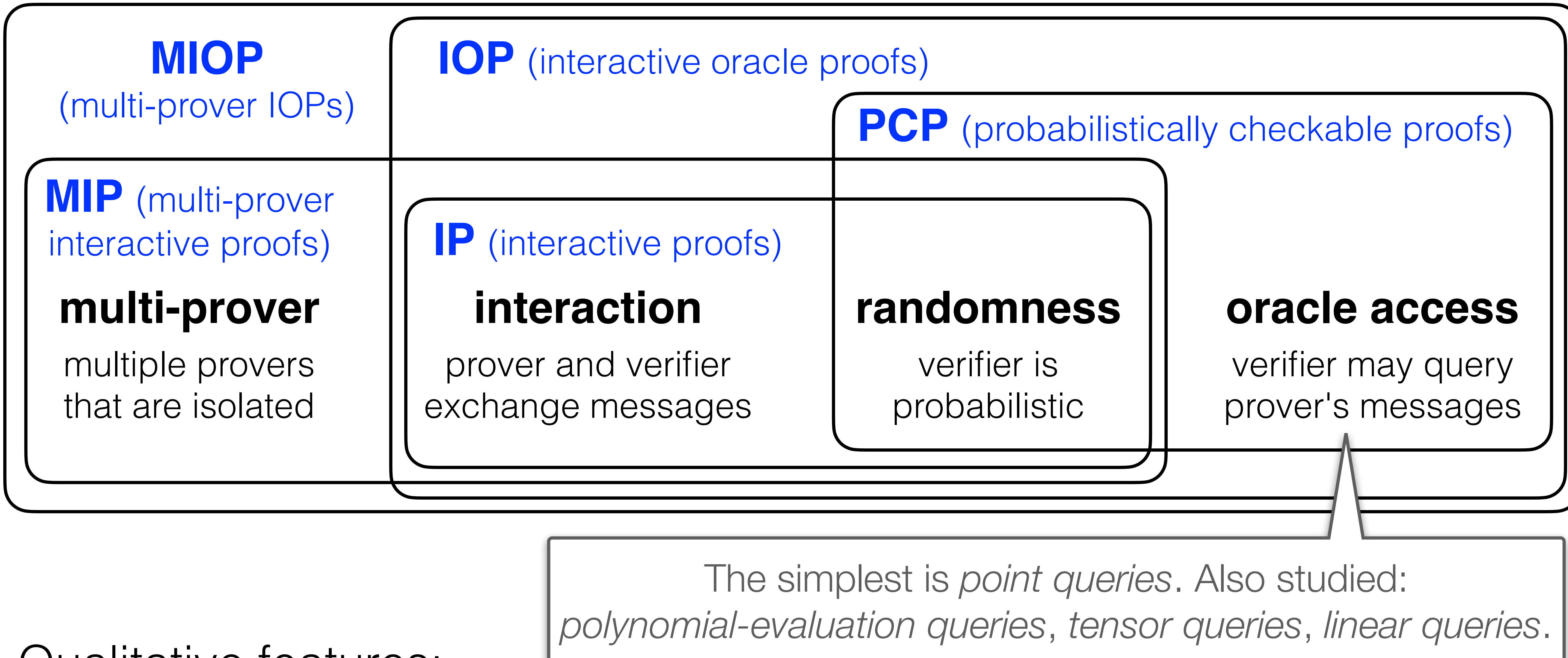

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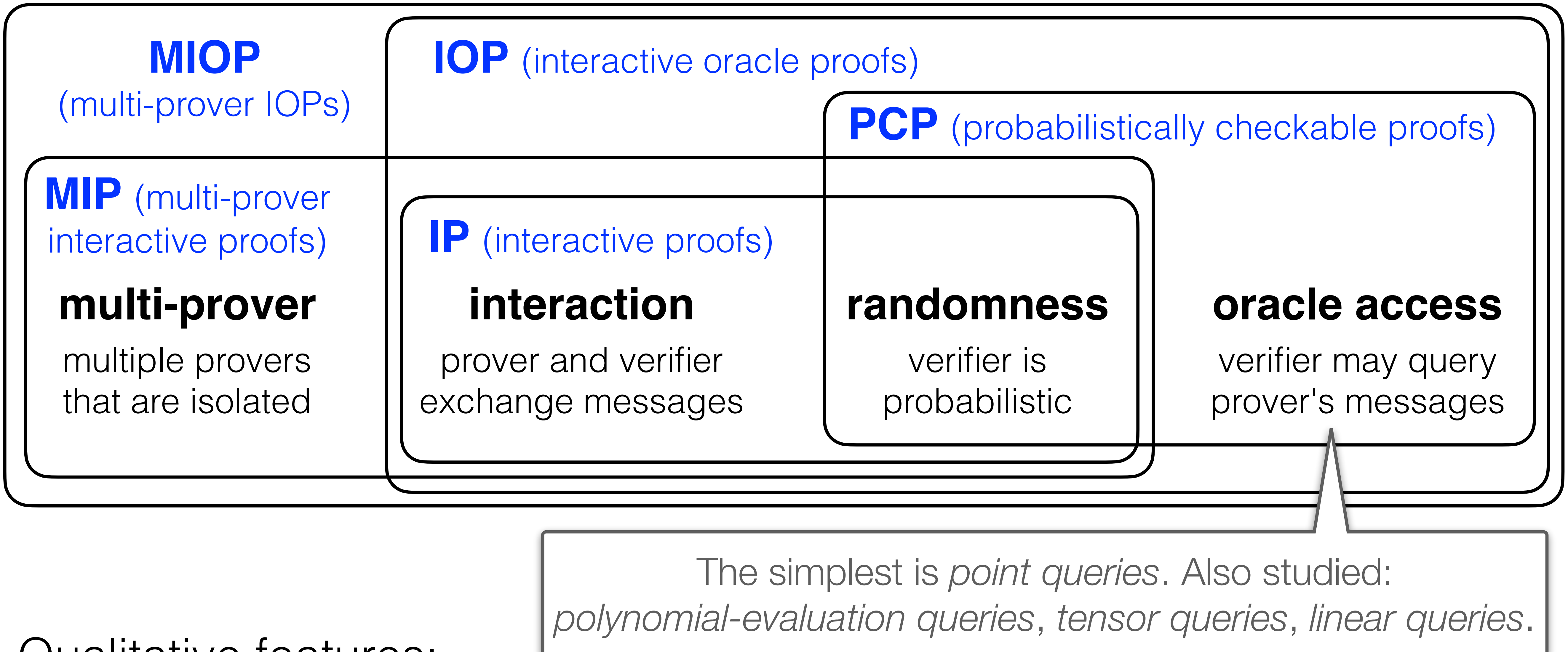

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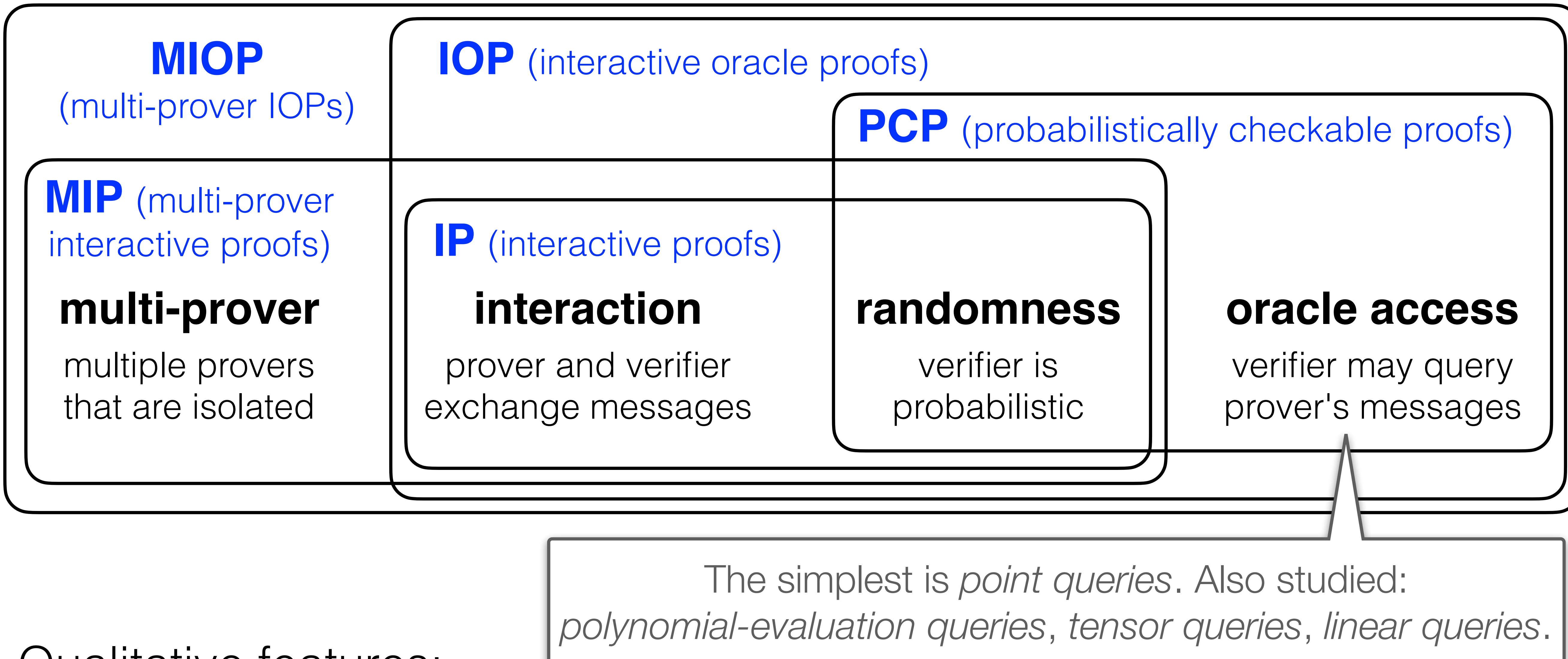

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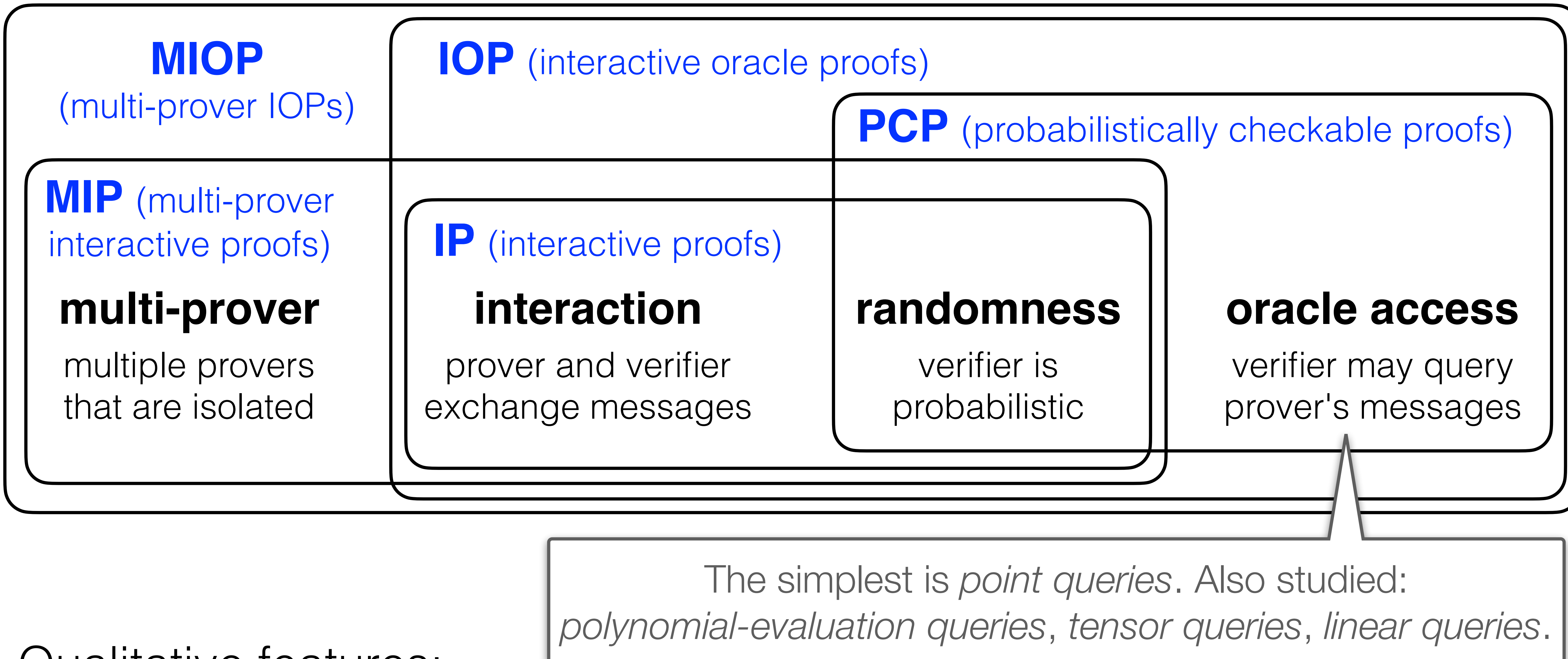

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- **IOP**: underlie most efficient succinct arguments

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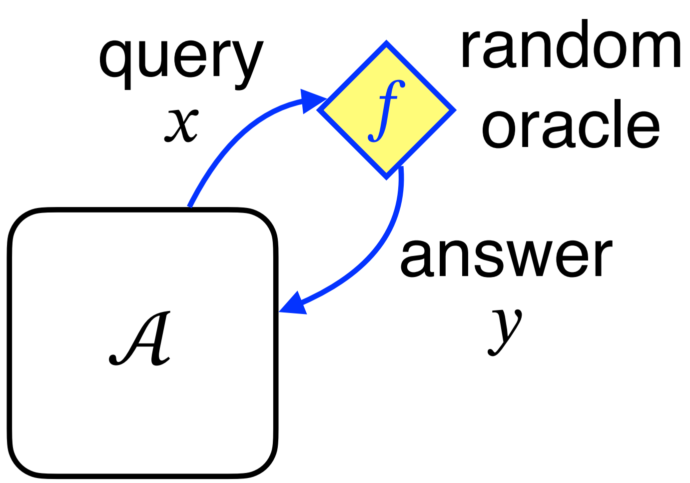

Warm-Up:
Succinct Arguments from PCPs

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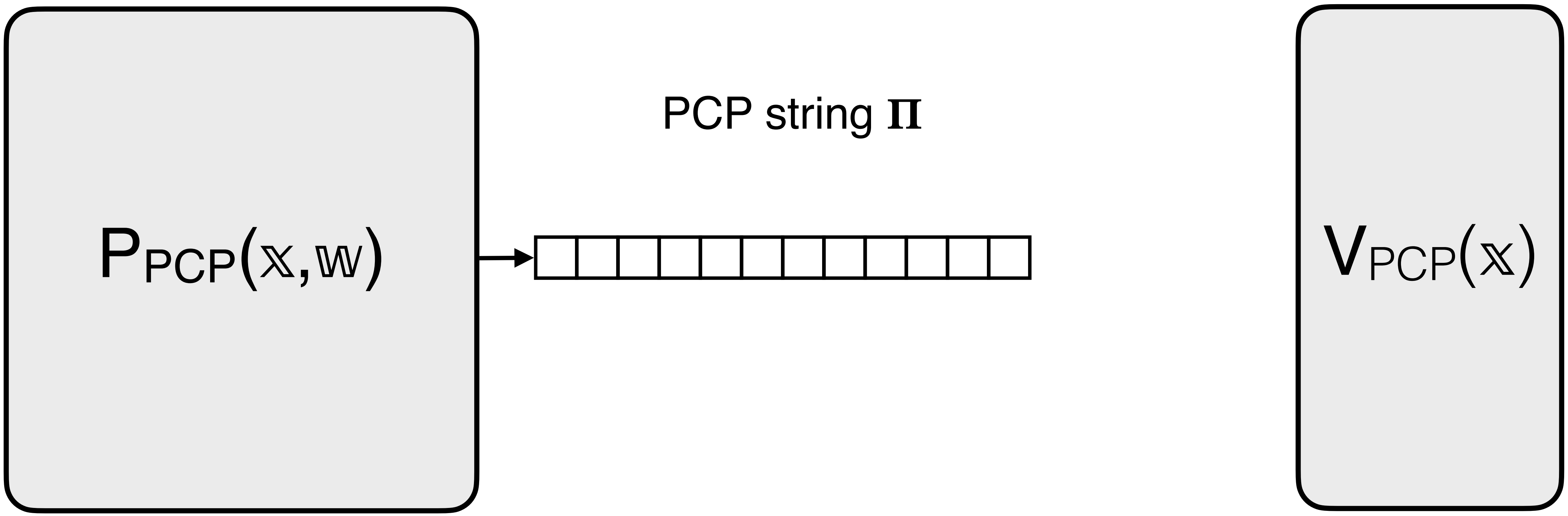

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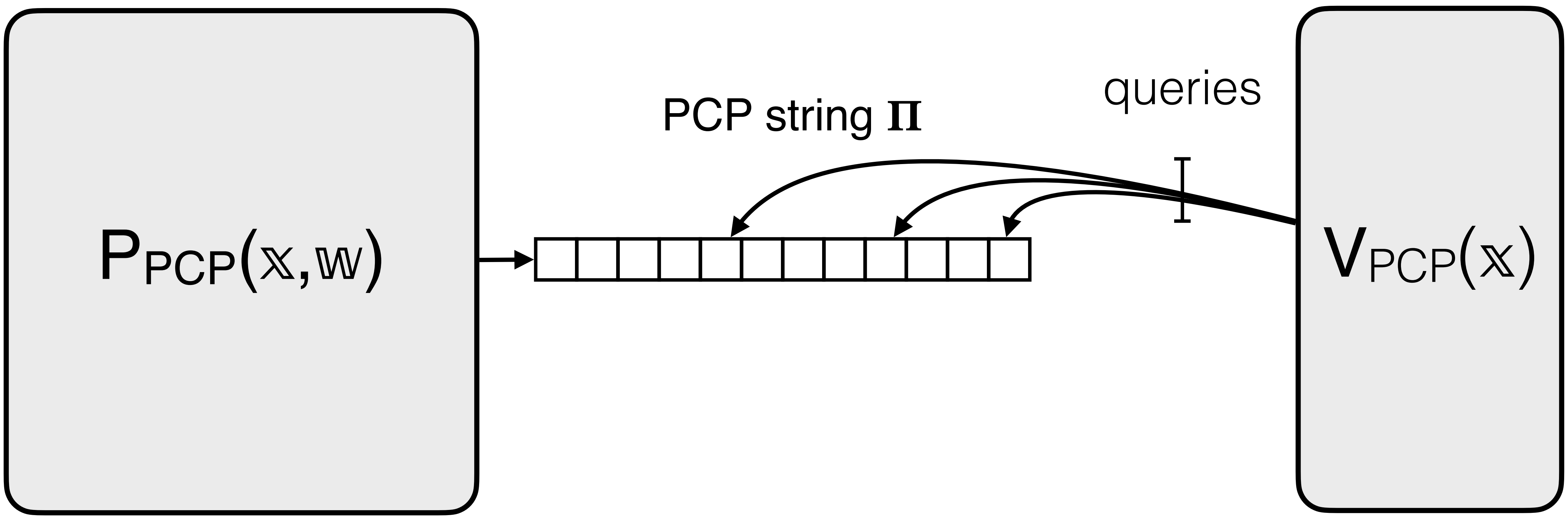

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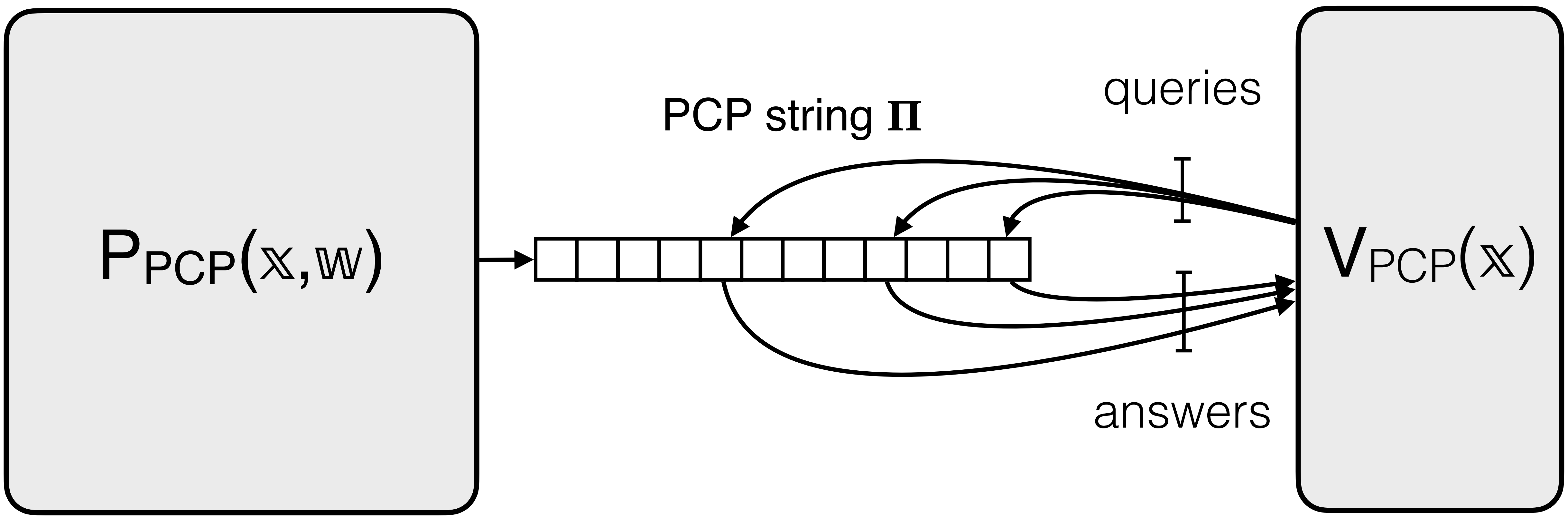

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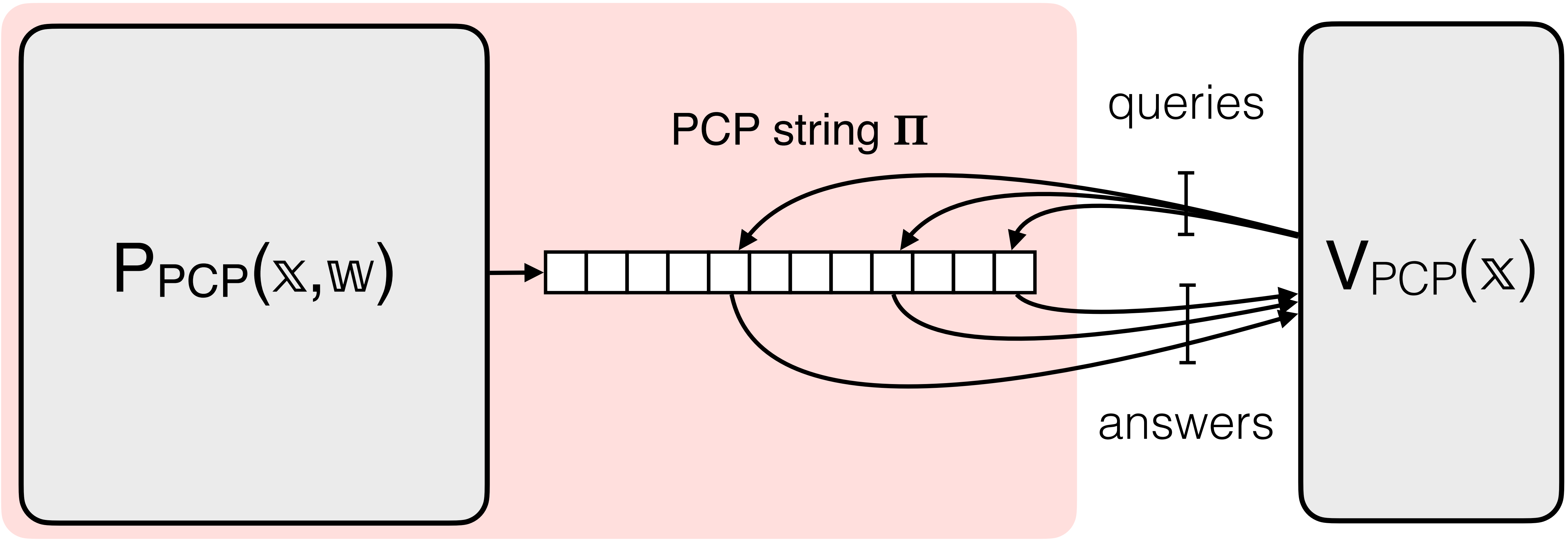

Note: PCP ≠ Succinct Argument!
It is insecure for the verifier to ask the prover to answer queries.

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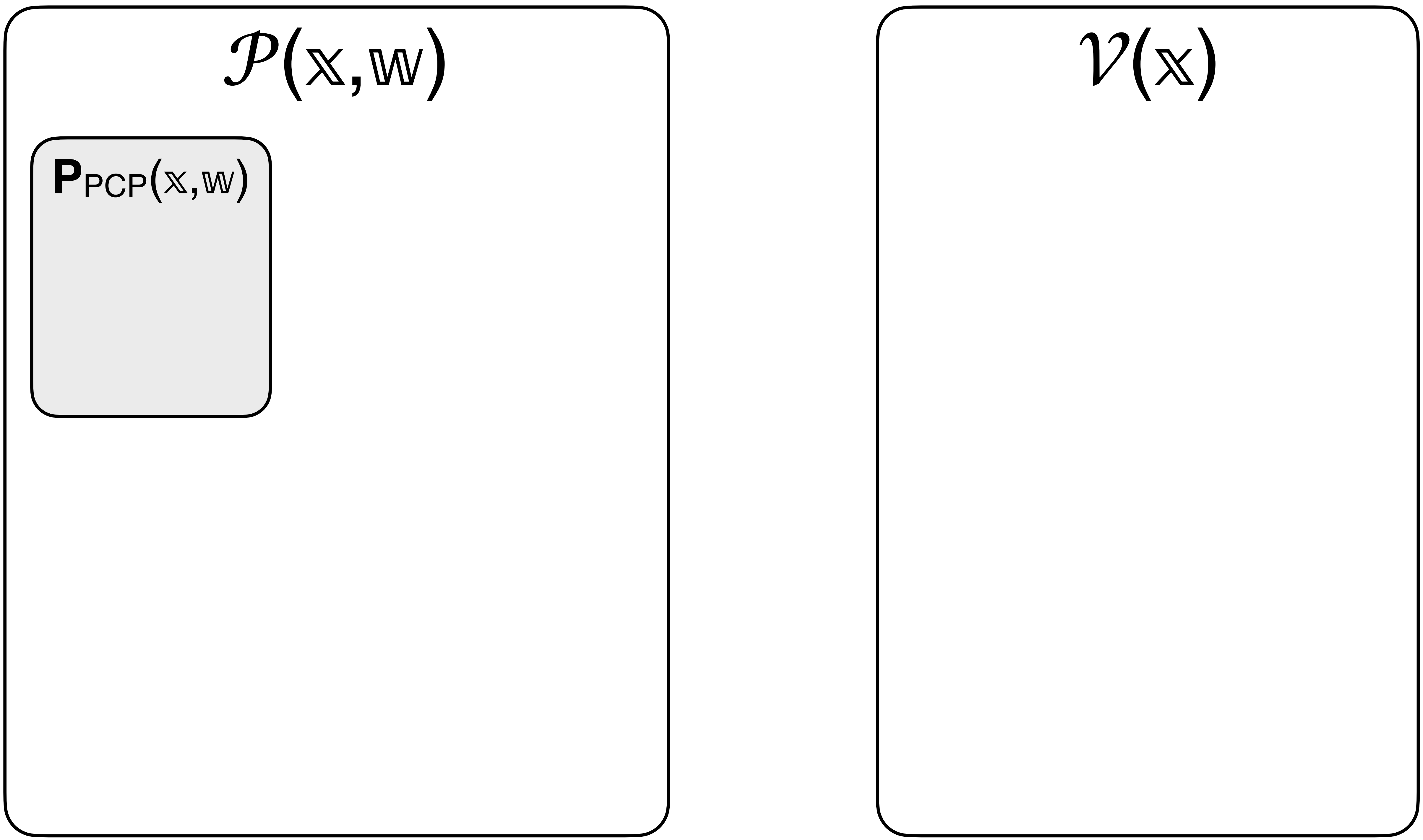

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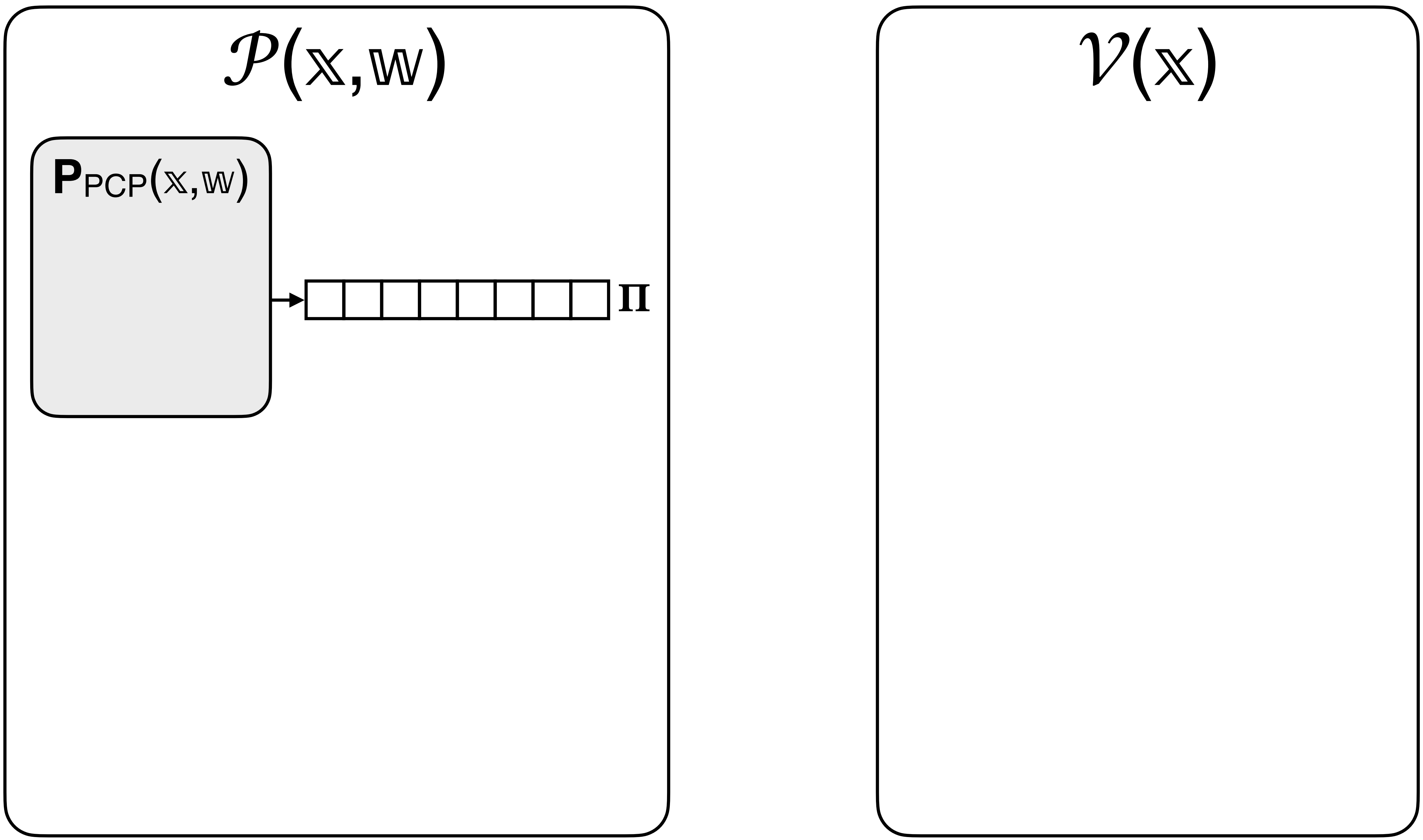

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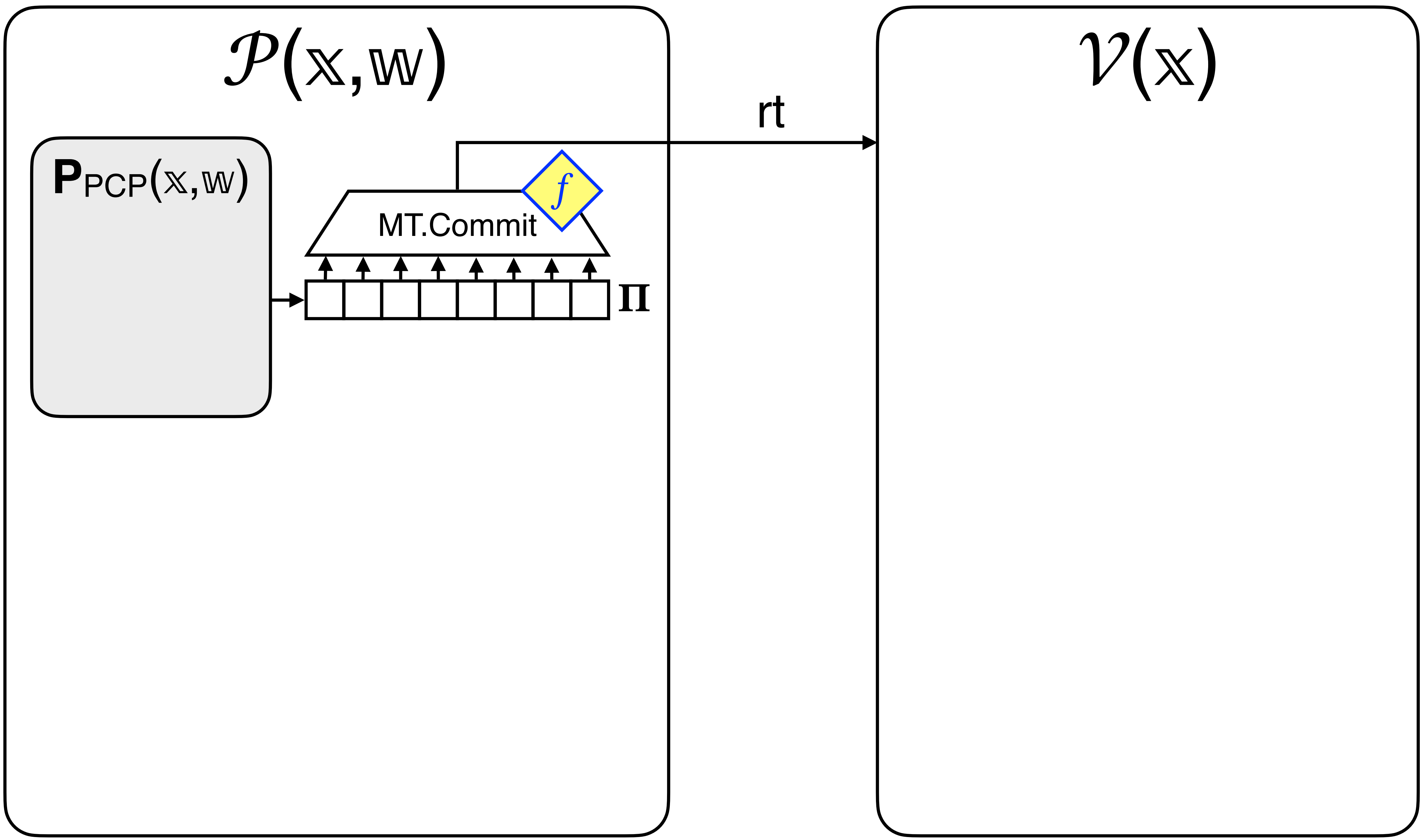

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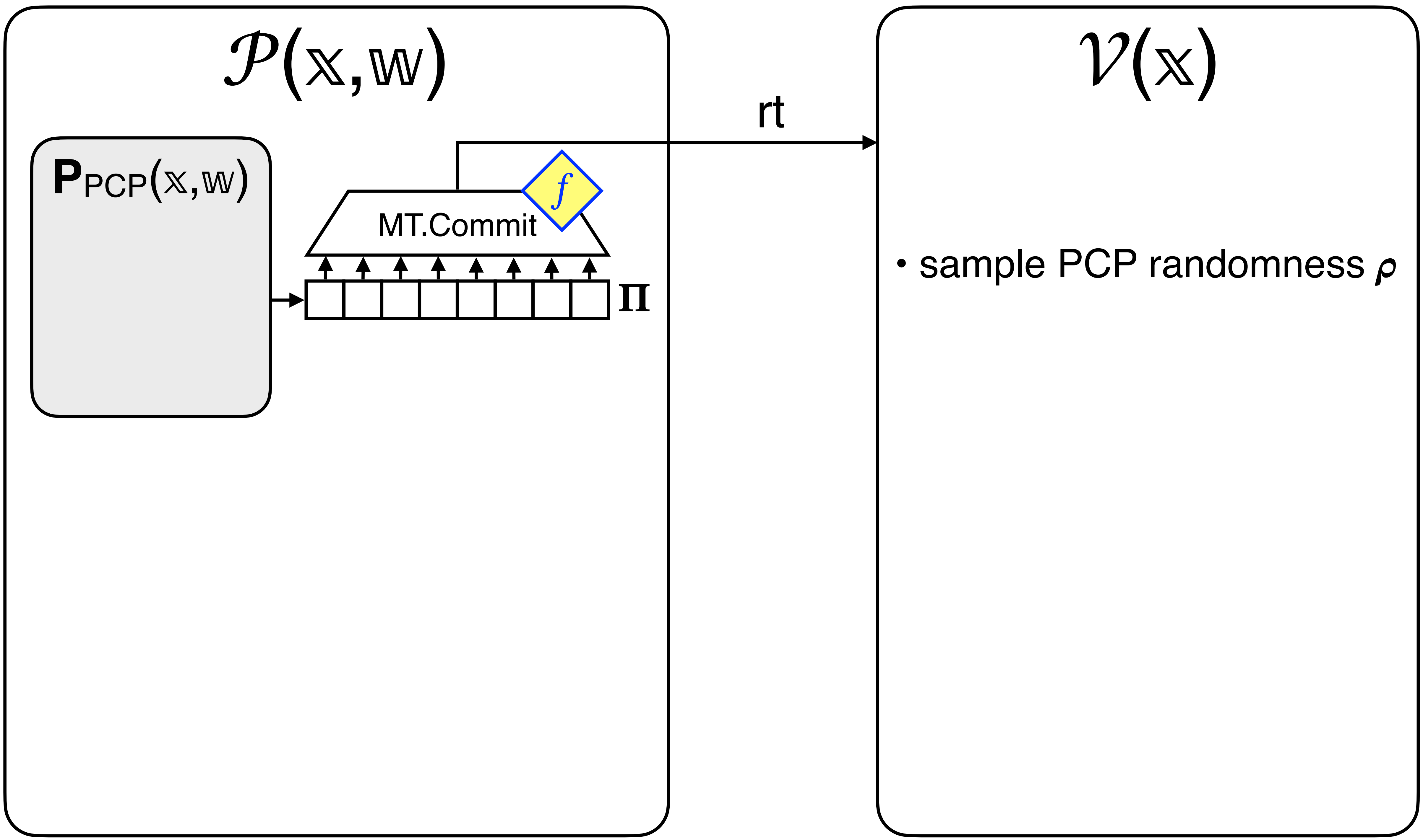

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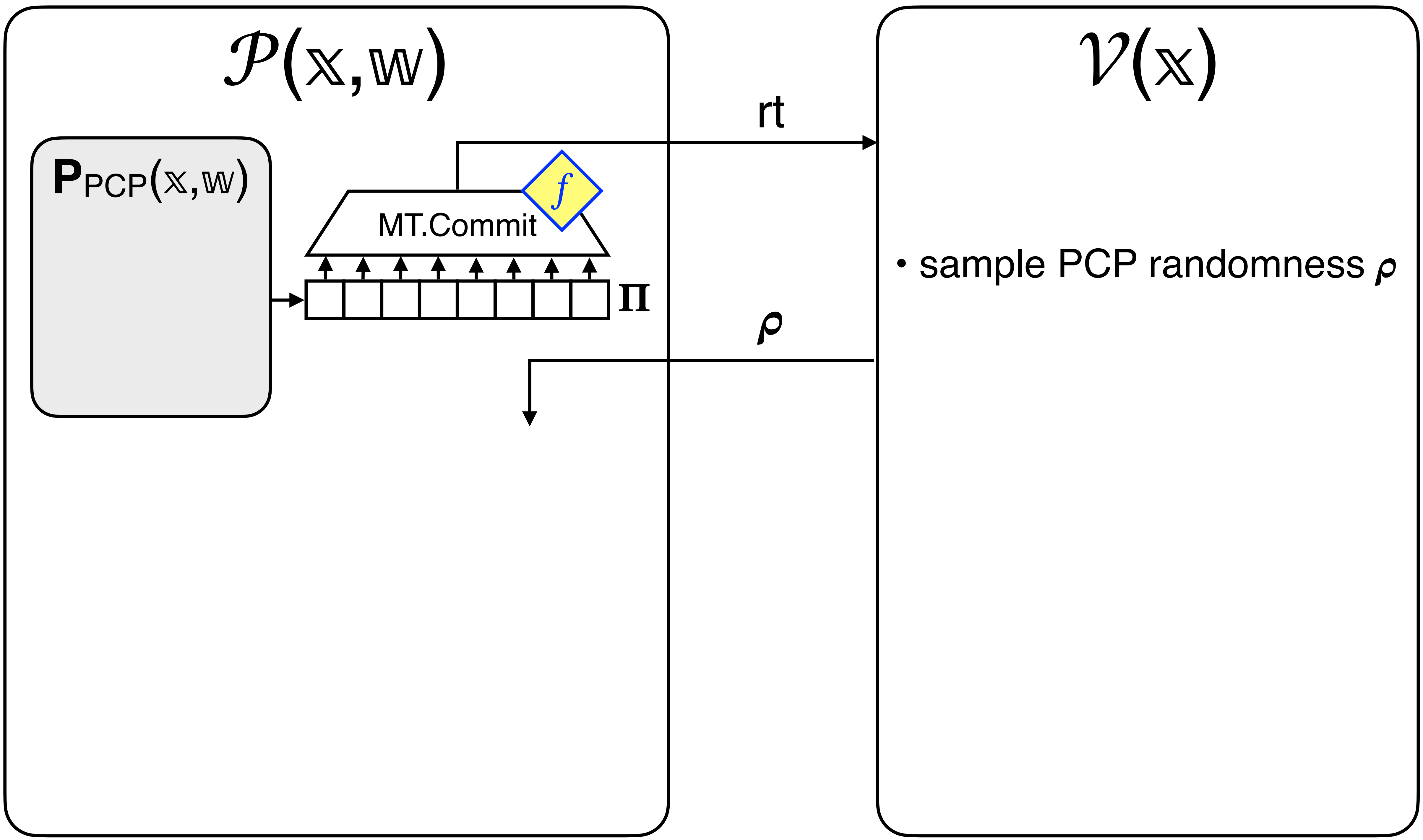

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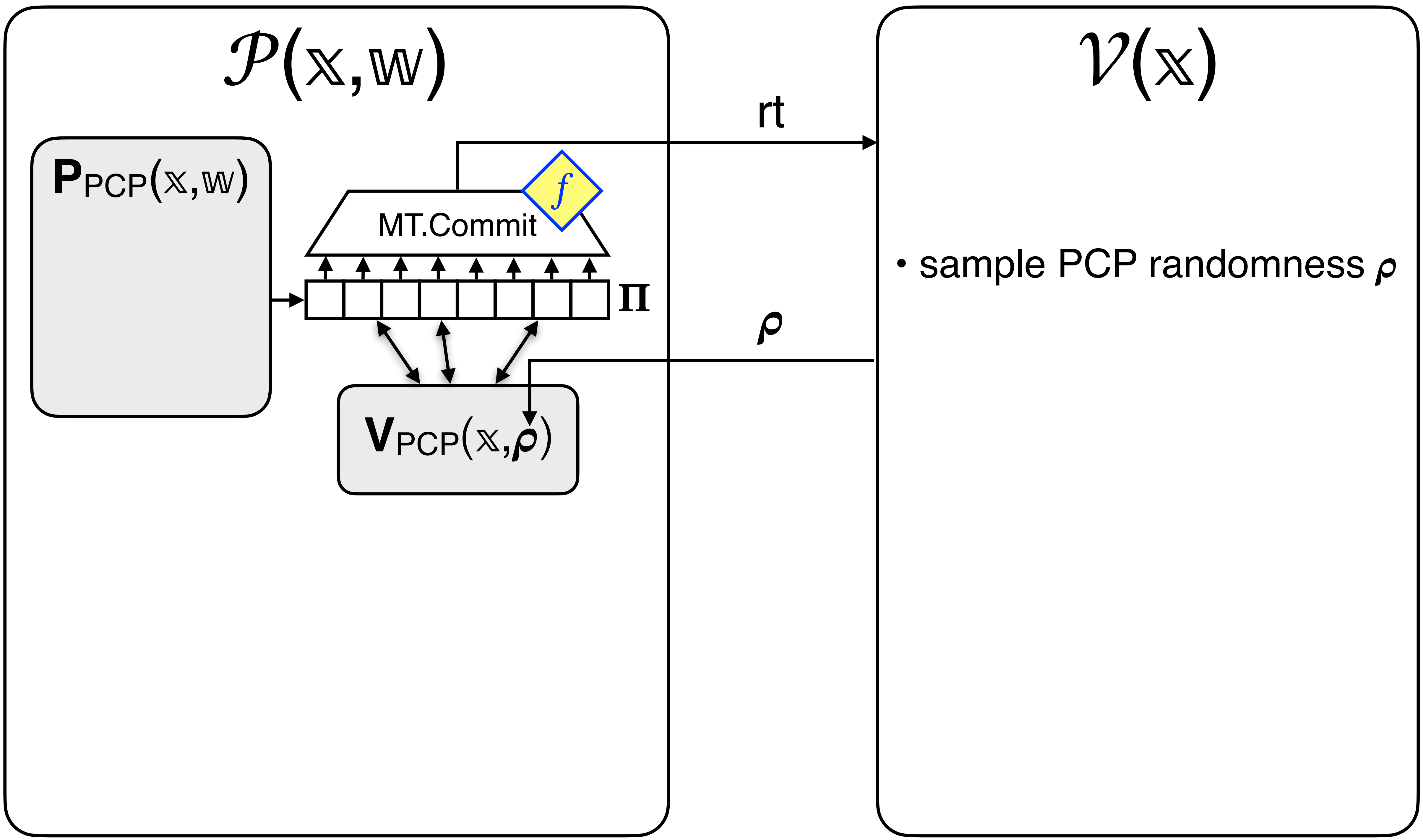

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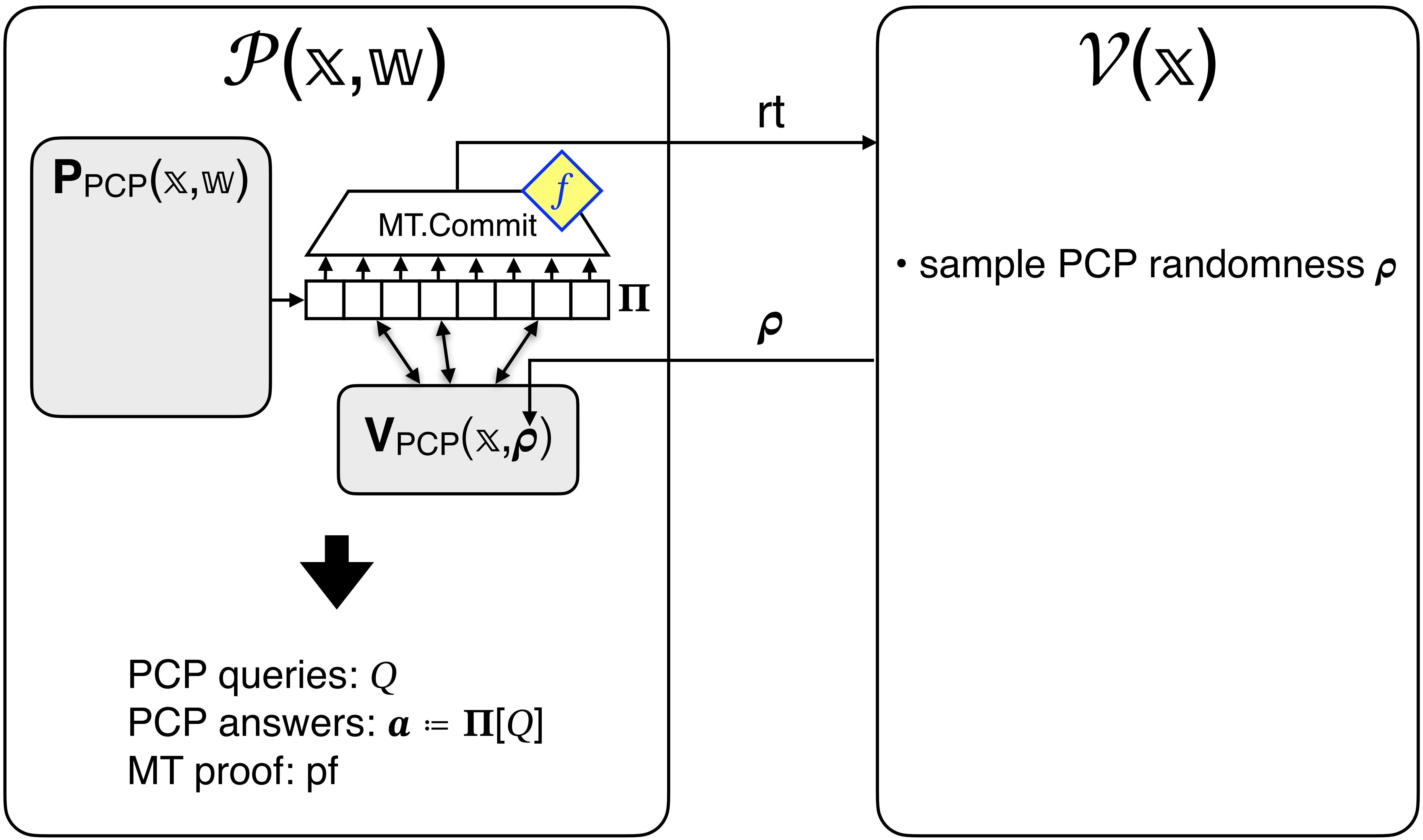

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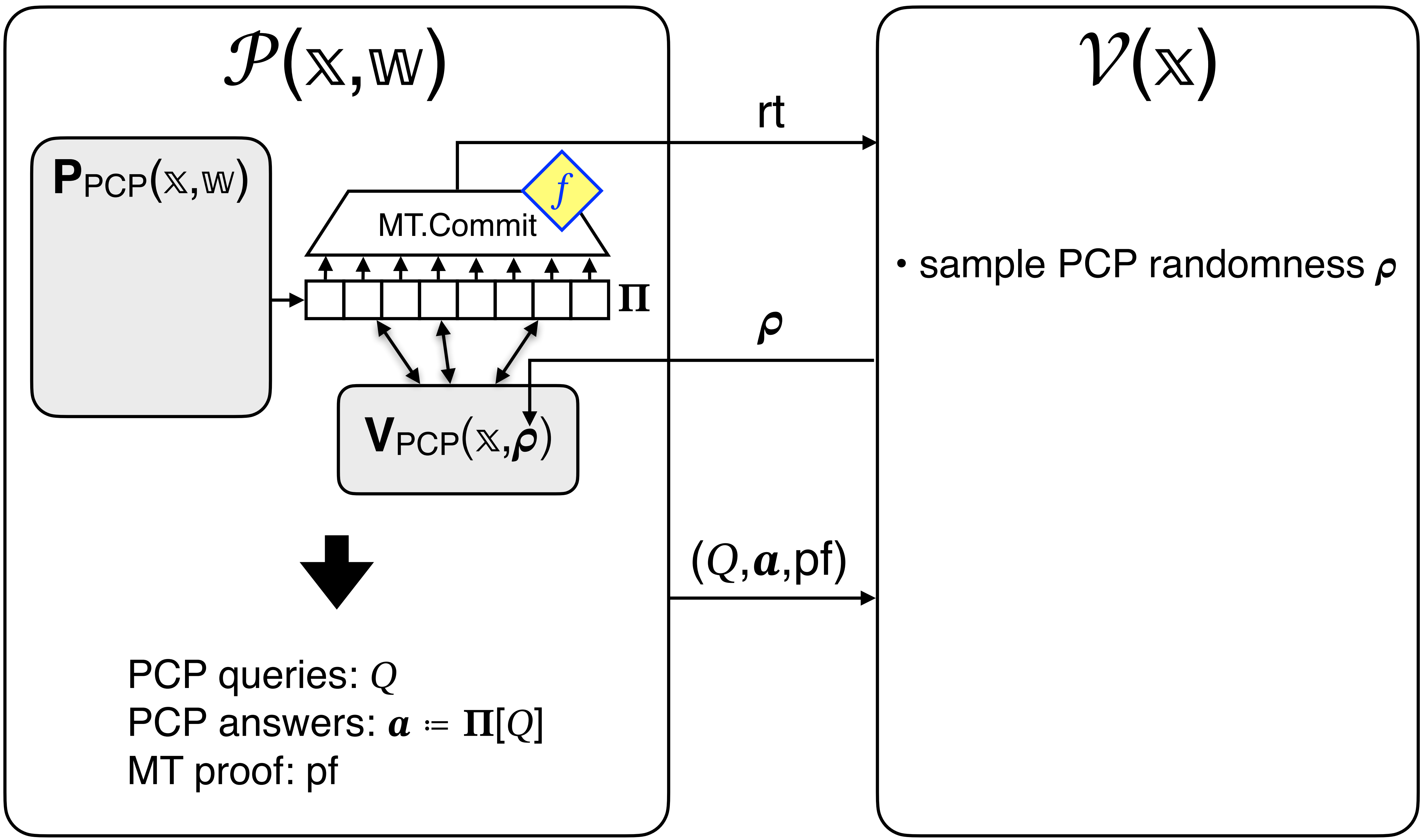

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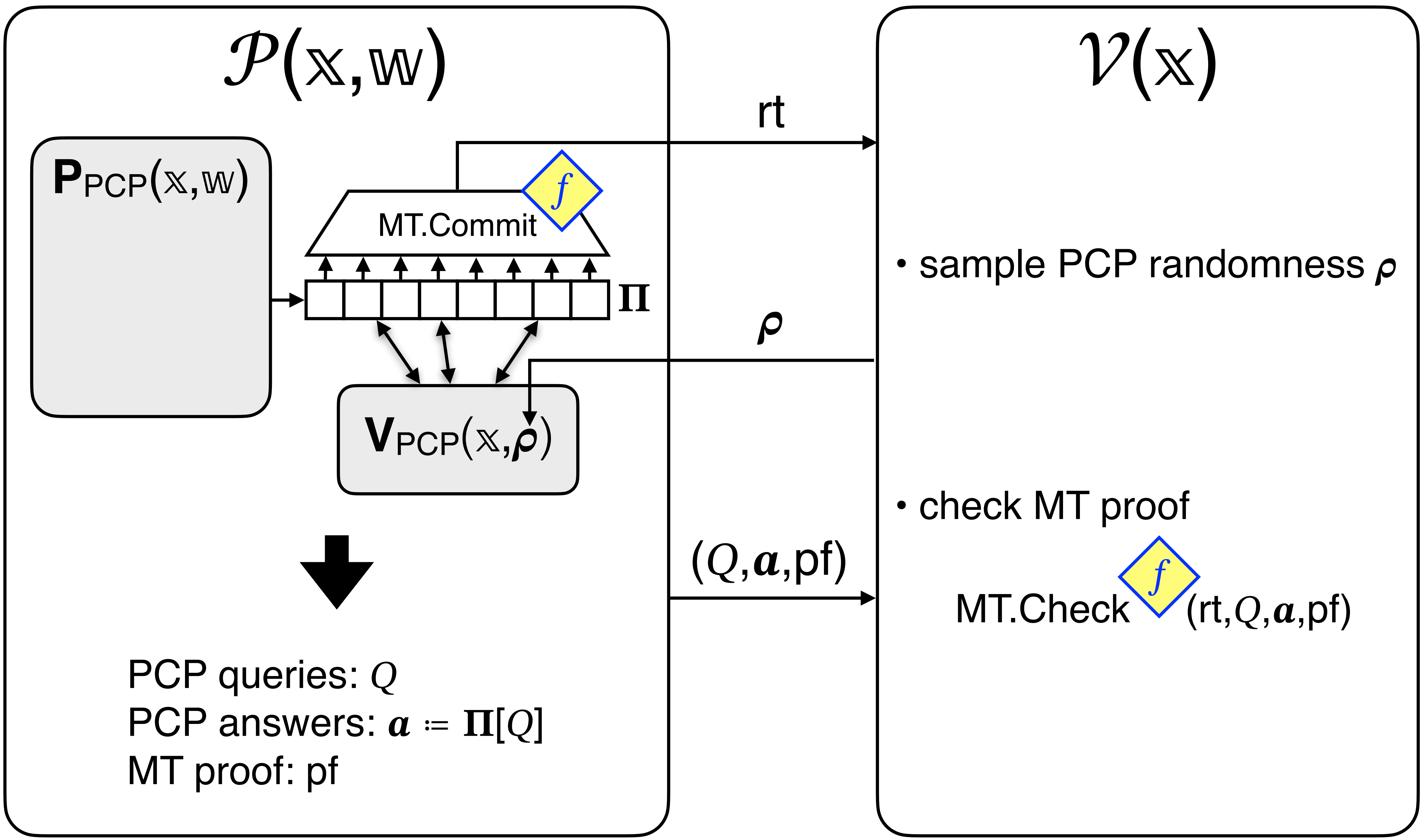

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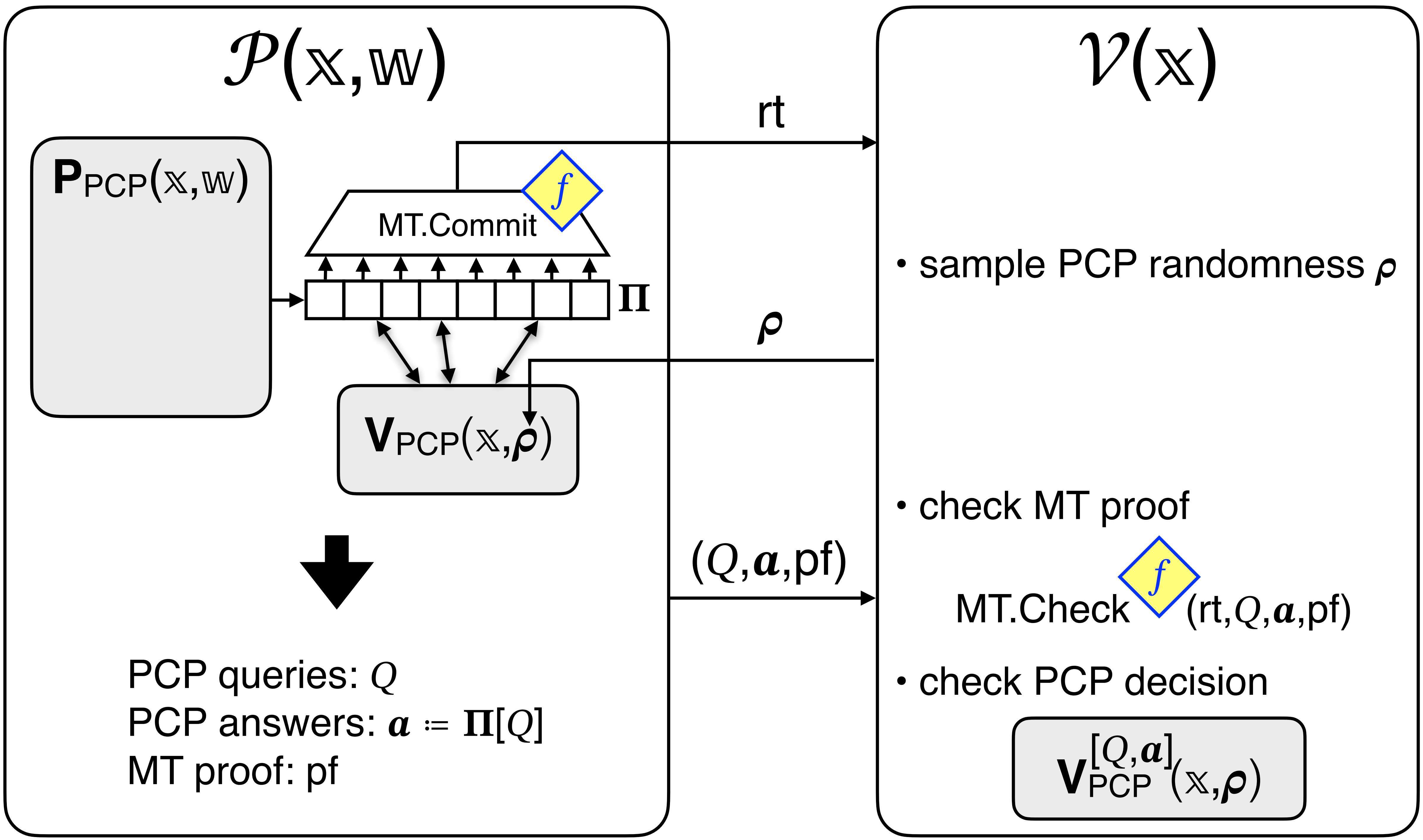

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Example:
Set ε_{PCP} =1/4 and λ=256+2=258.

Every 2^{128}-query adversary breaks Kilian w.p. ≤1/4+1/4=1/2.

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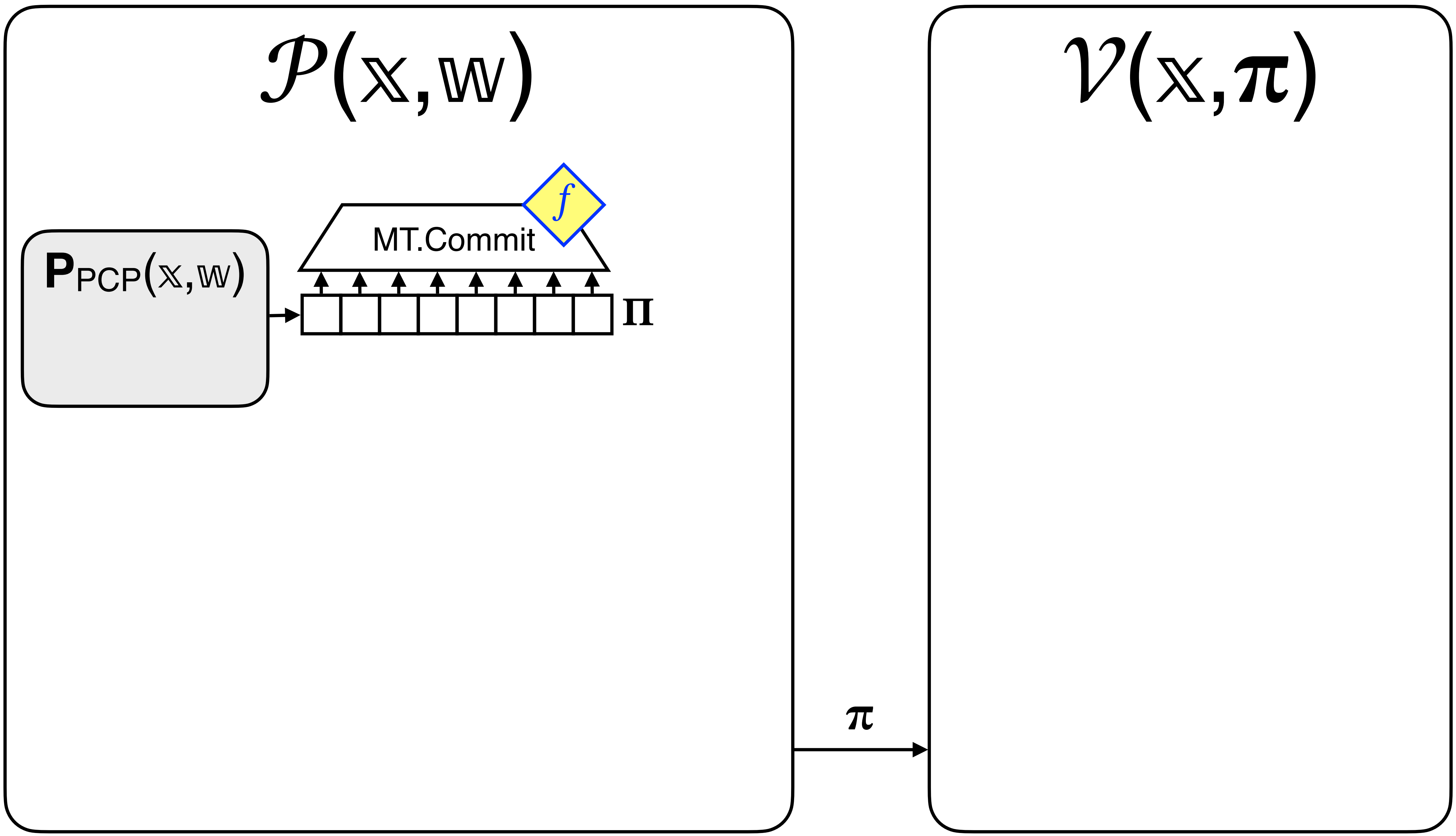

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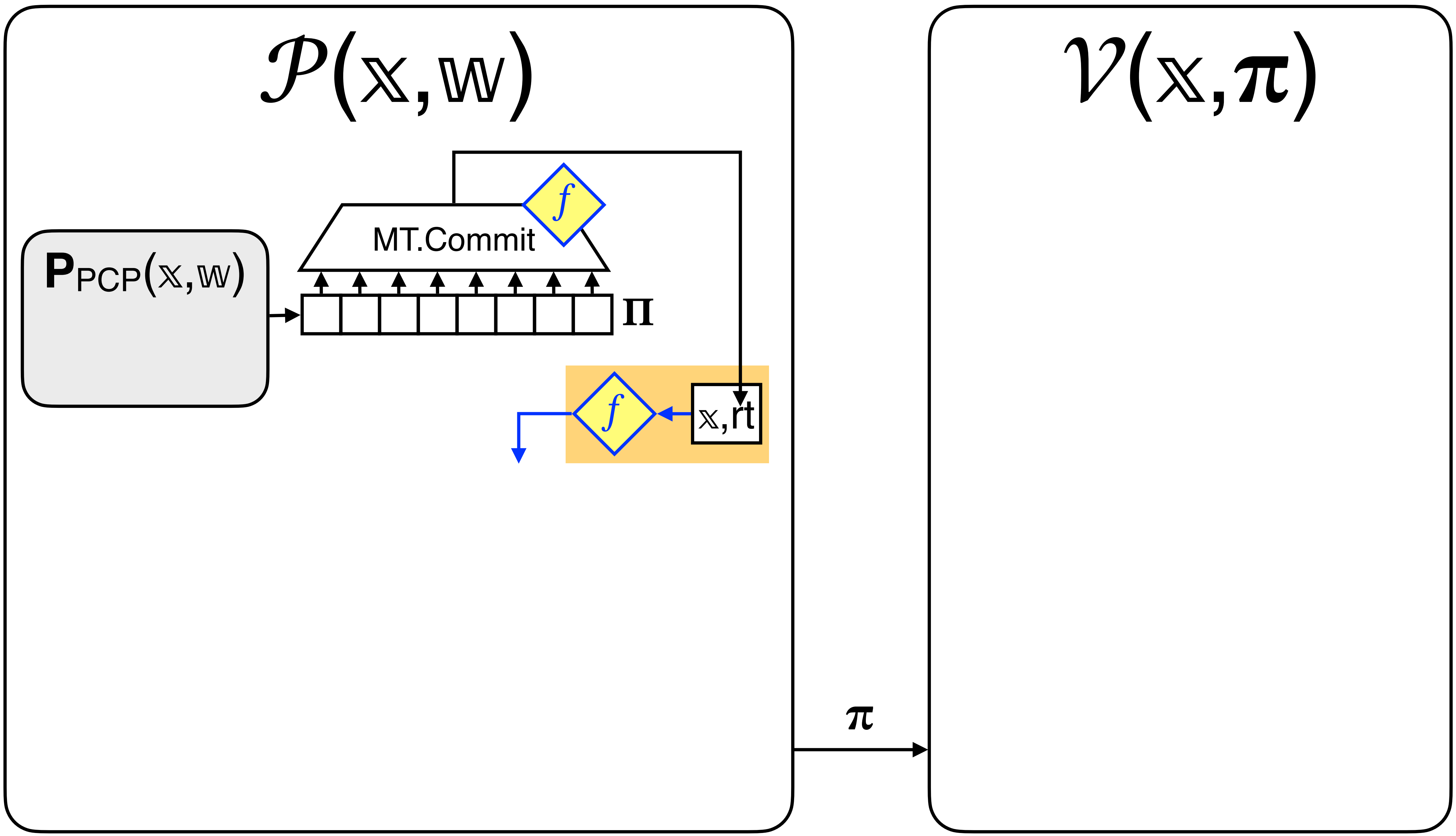

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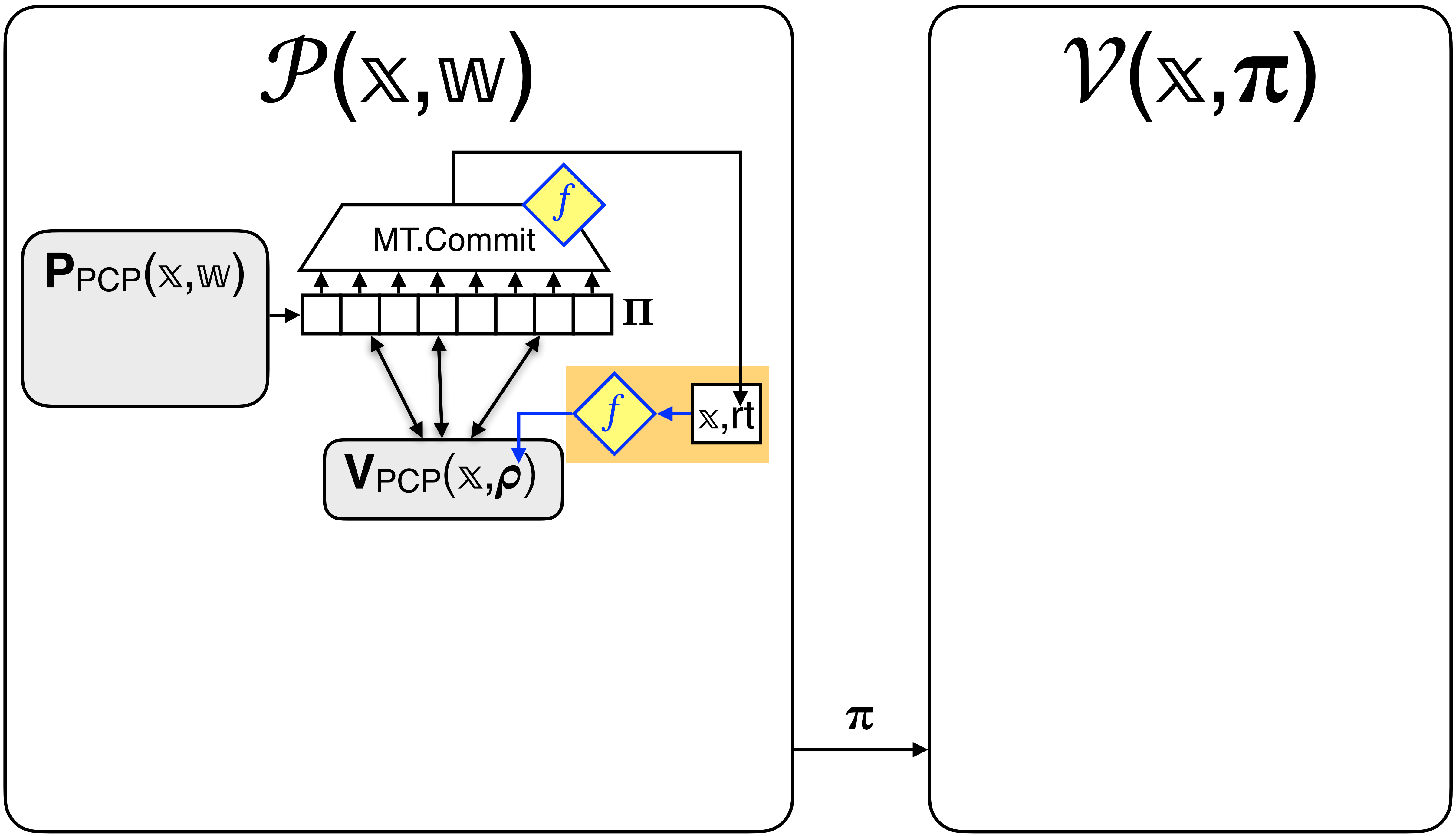

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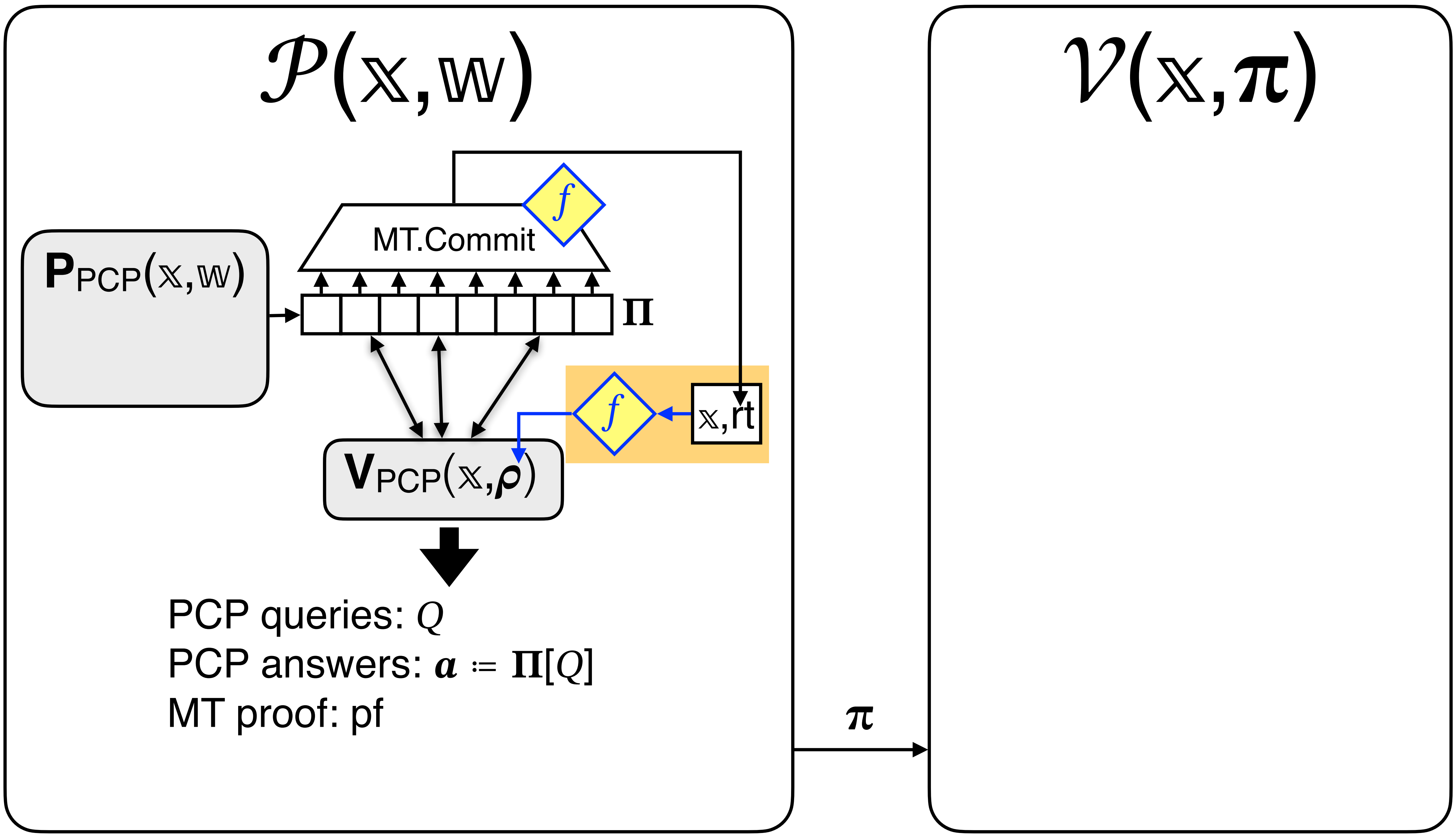

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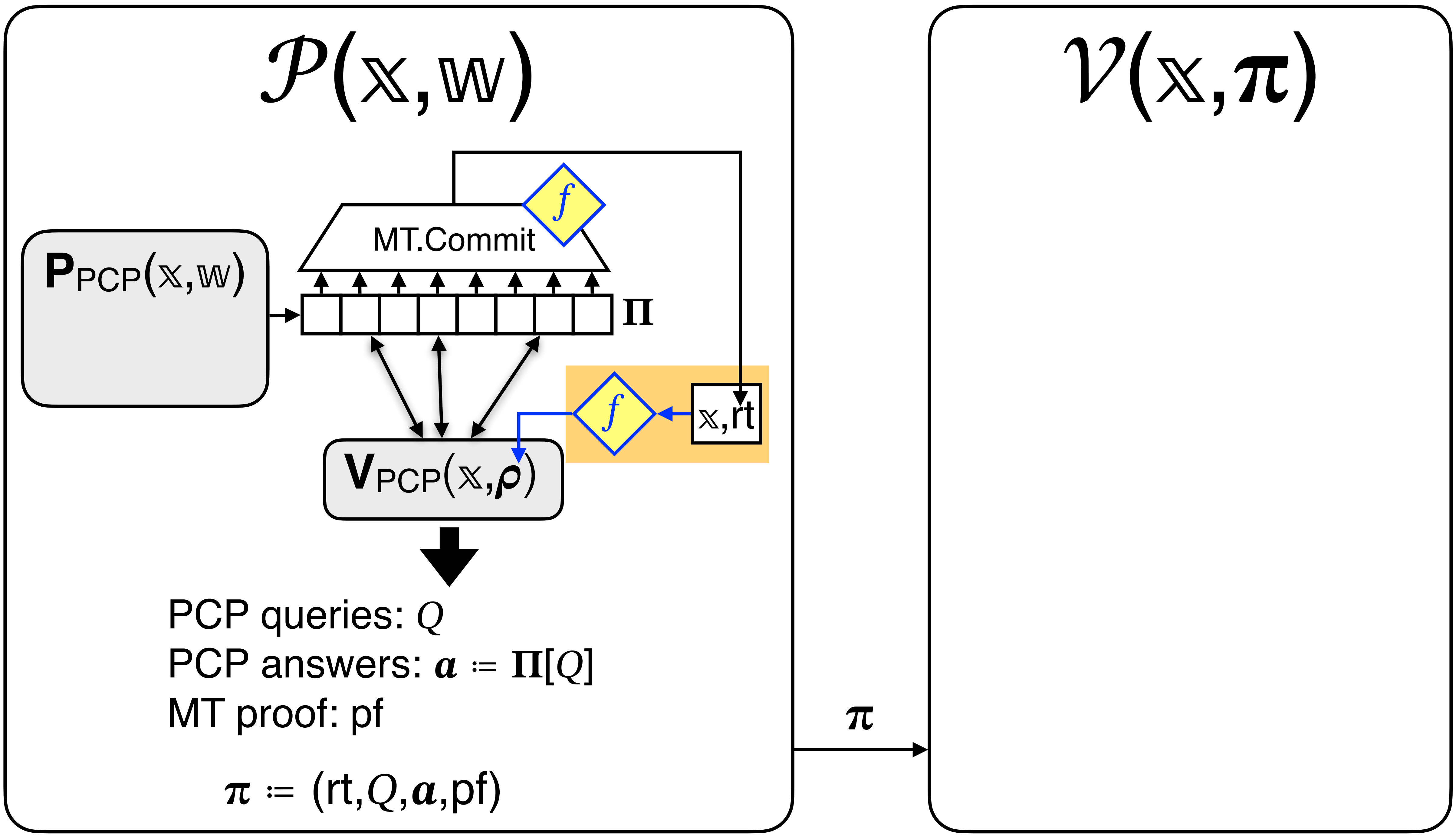

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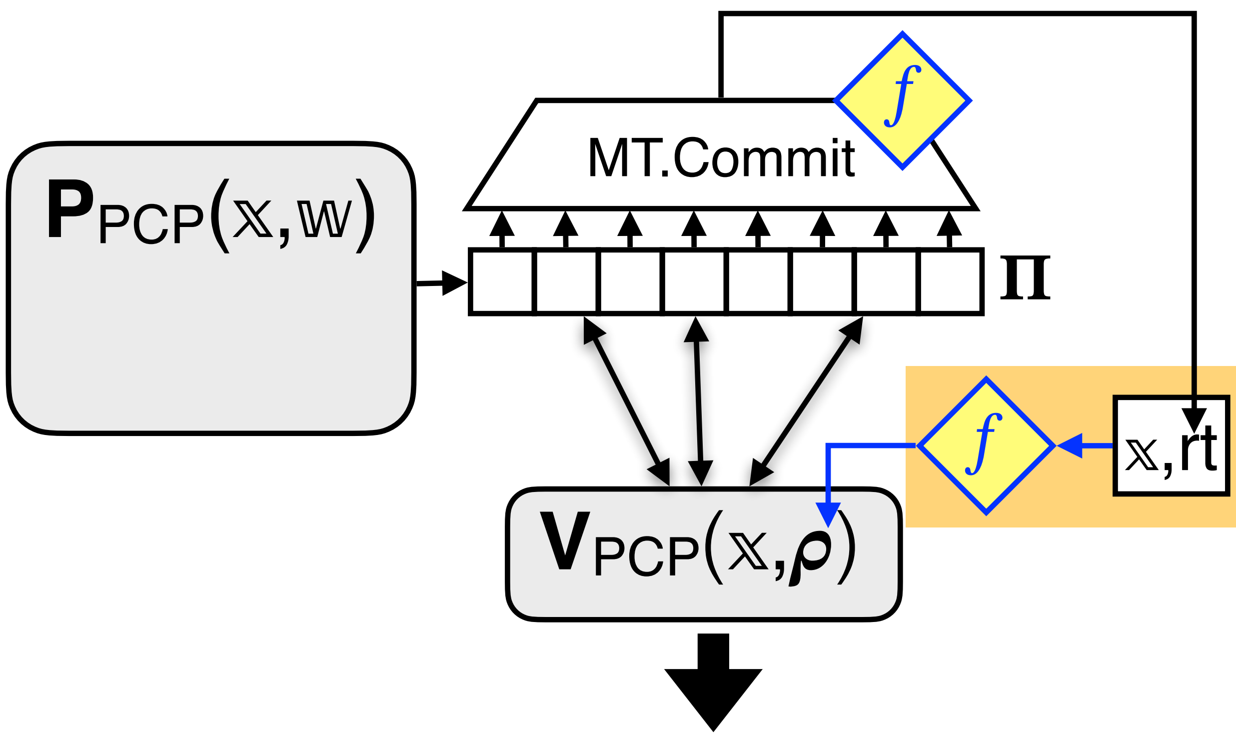

PCP queries: Q

PCP answers: $\boldsymbol{a} \coloneqq \boldsymbol{\Pi}[Q]$

MT proof: pf

$\boldsymbol{\pi} \coloneqq (\mathrm{rt}, Q, \boldsymbol{a}, \mathrm{pf})$

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- parse $\boldsymbol{\pi}$ as $(\mathrm{rt}, Q, \boldsymbol{a}, \mathrm{pf})$

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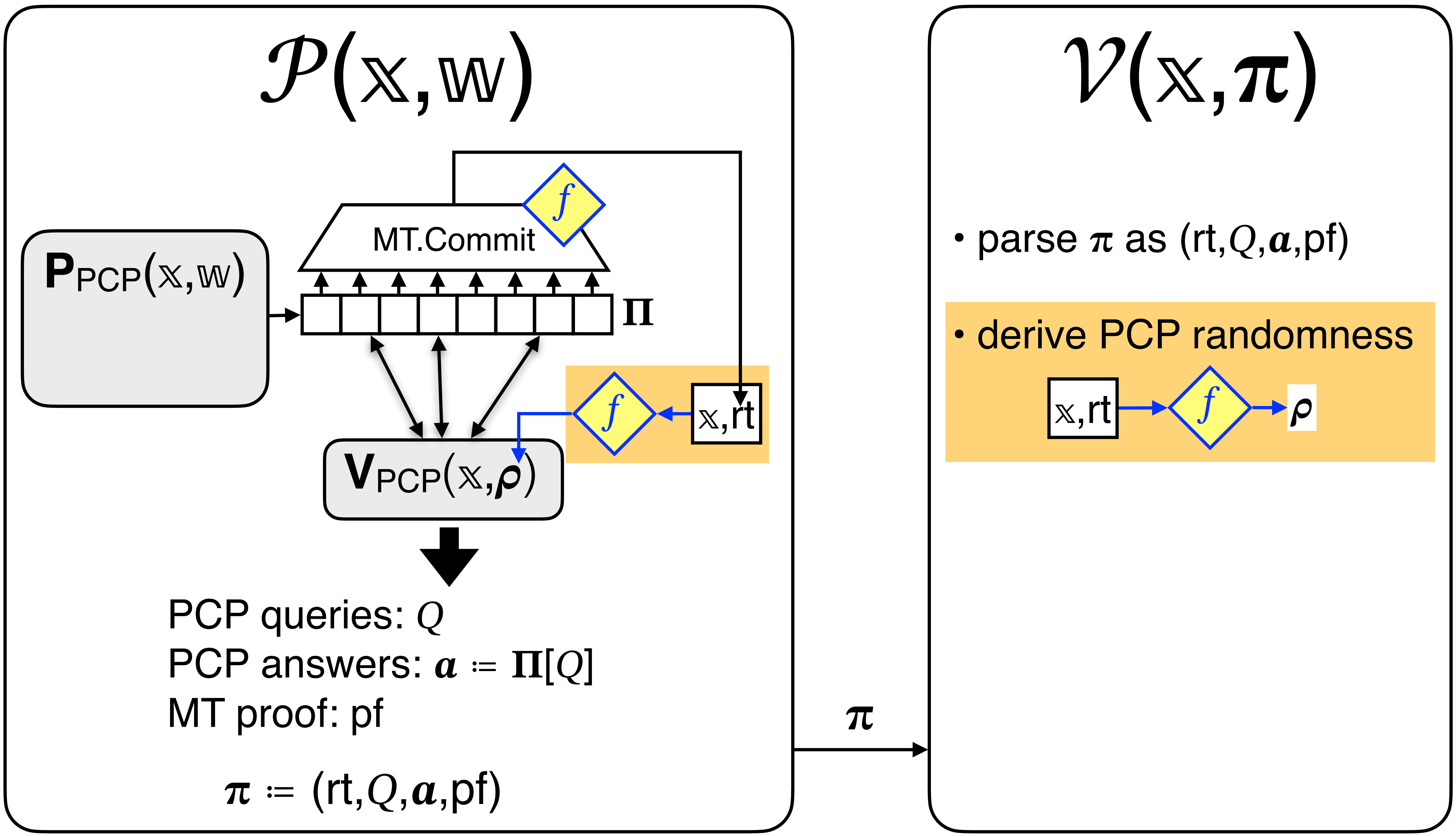

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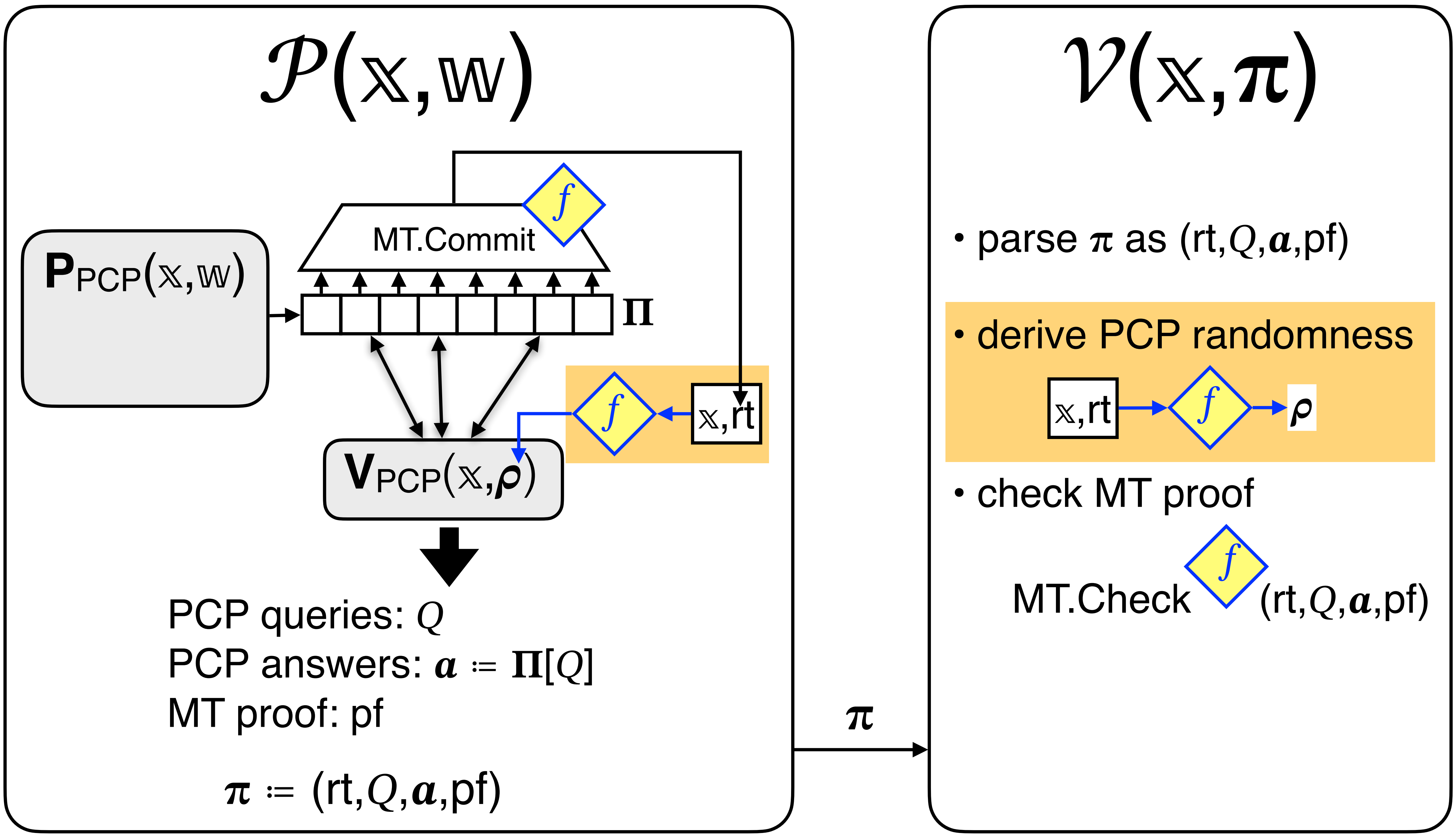

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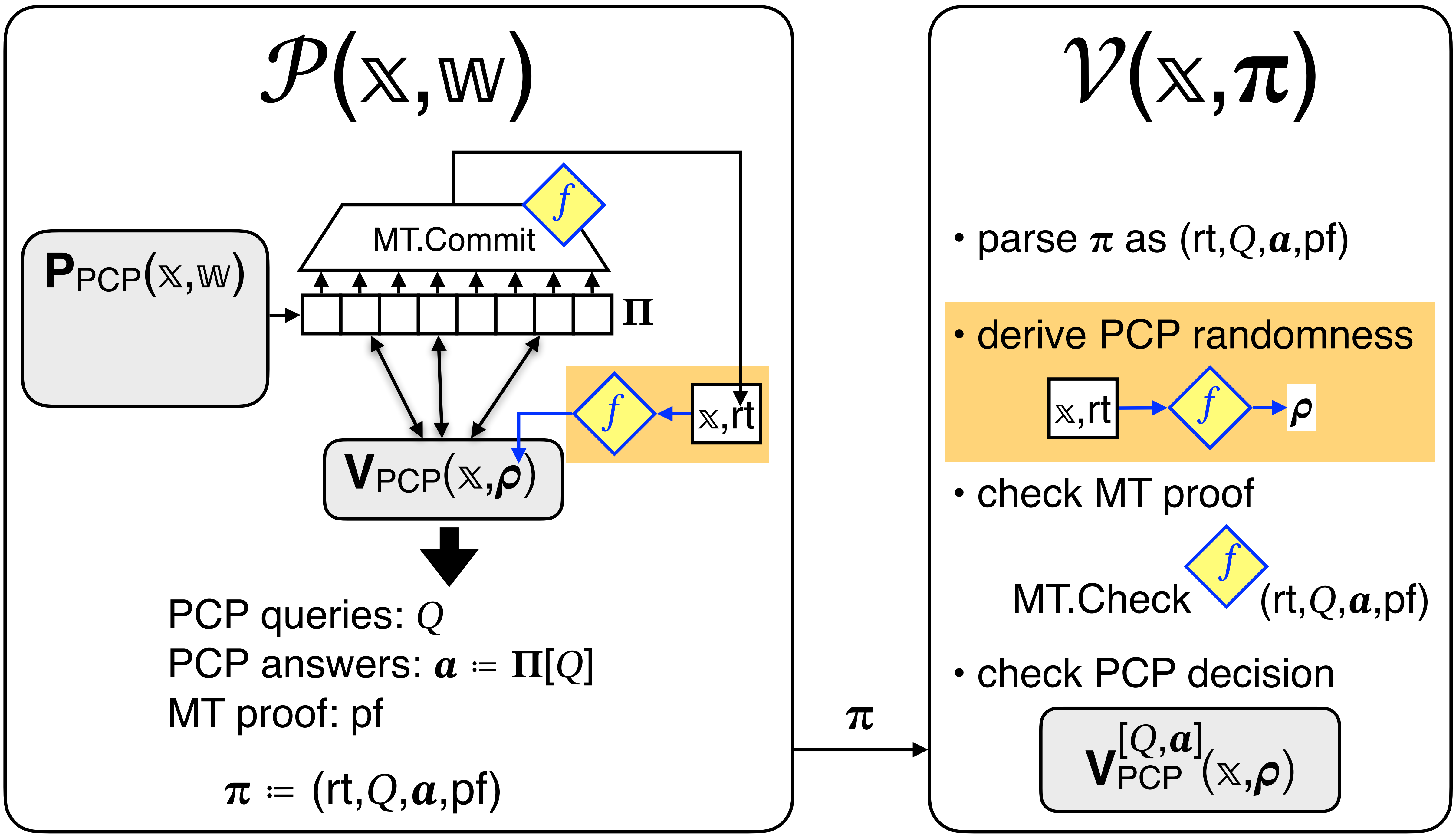

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Known PCPs are inefficient, so the Micali protocol is not used.

In Practice:
Succinct Arguments from IOPs

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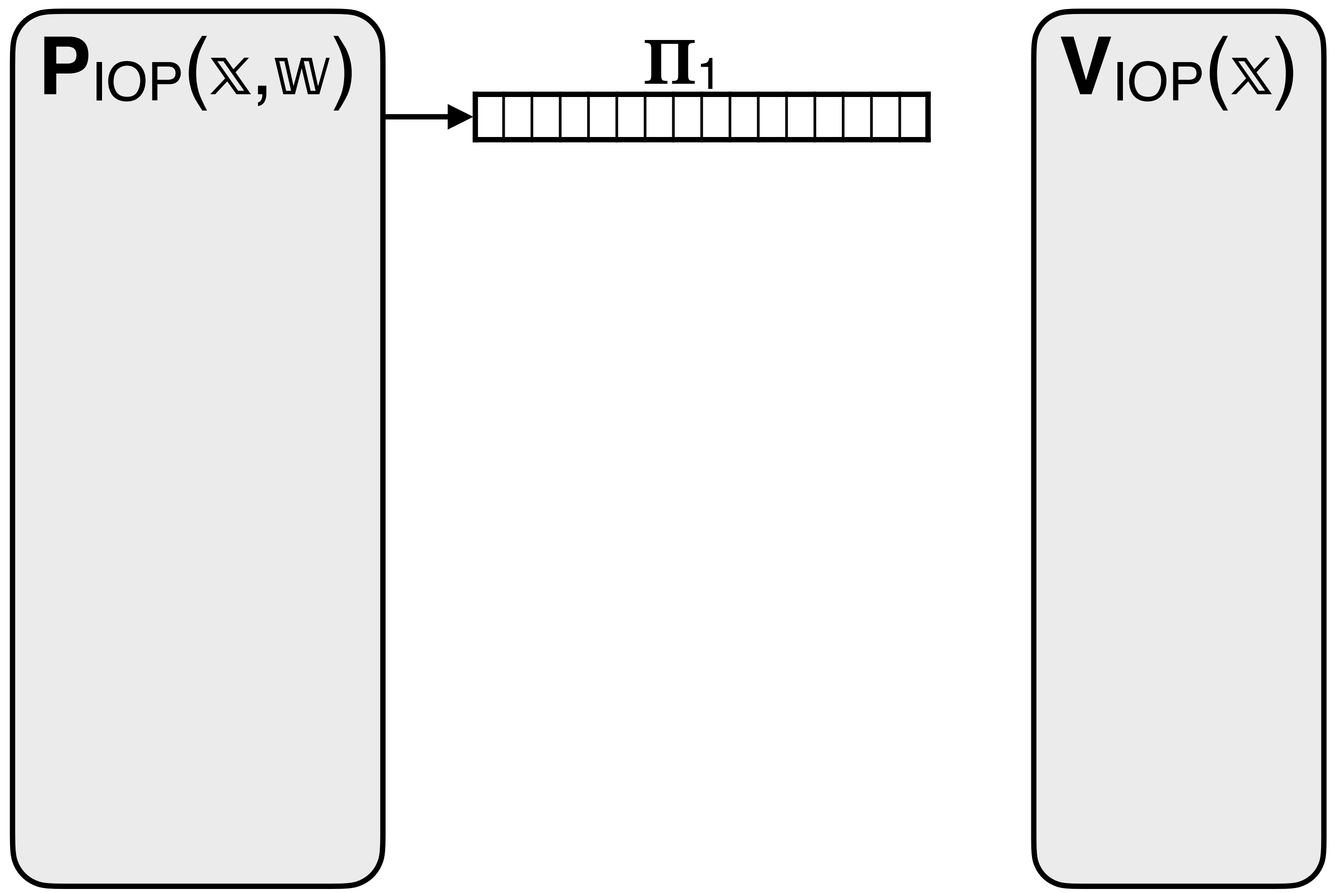

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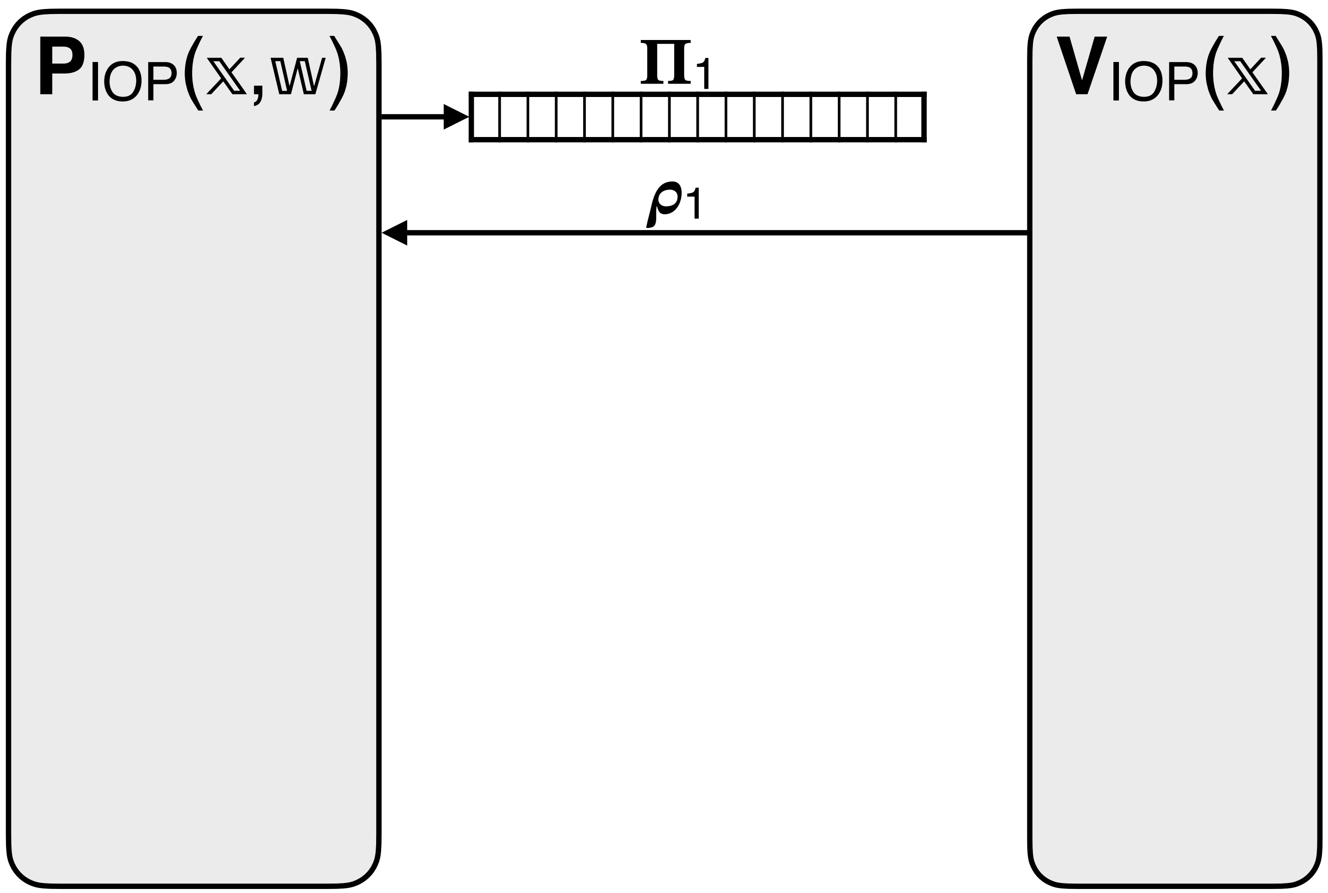

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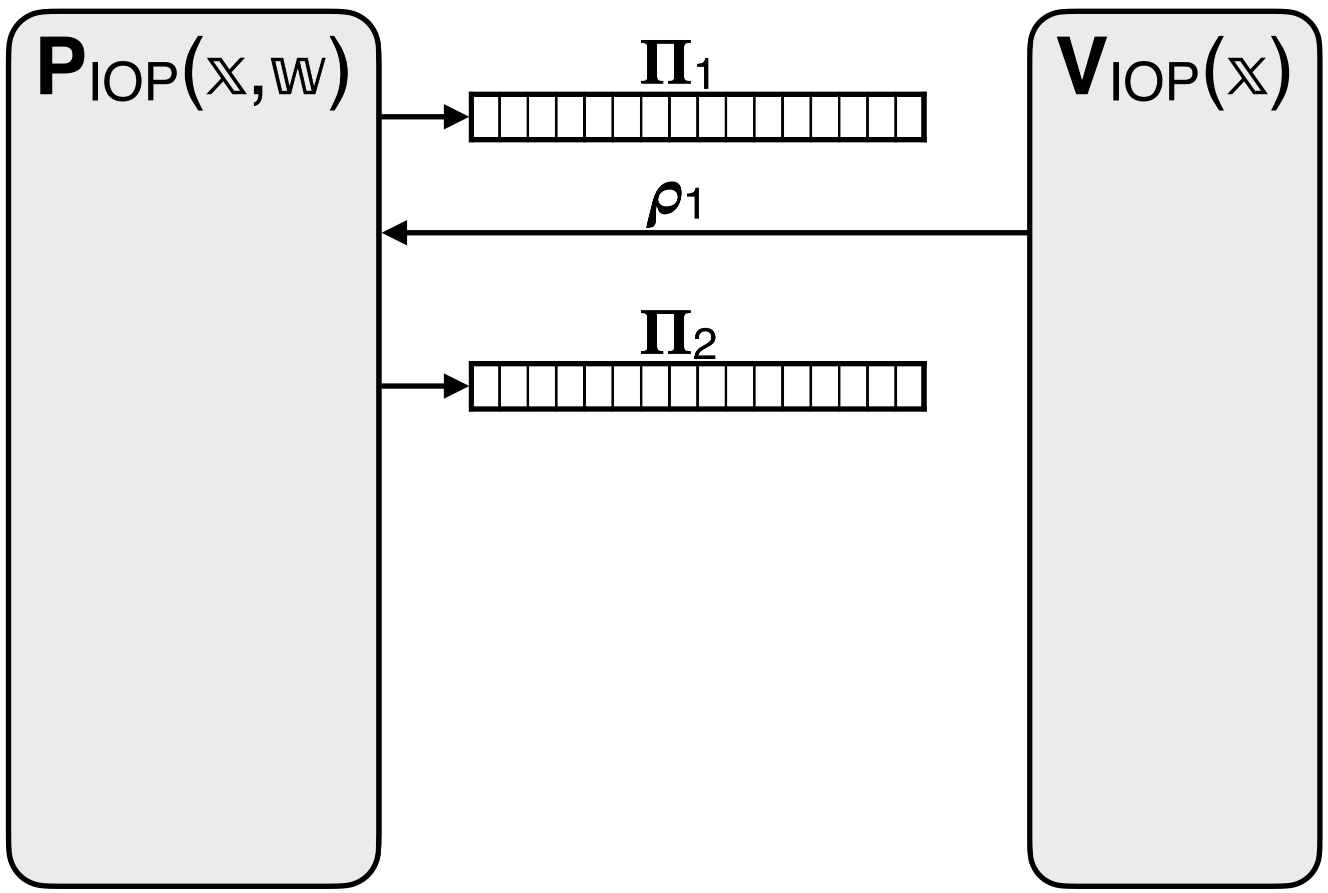

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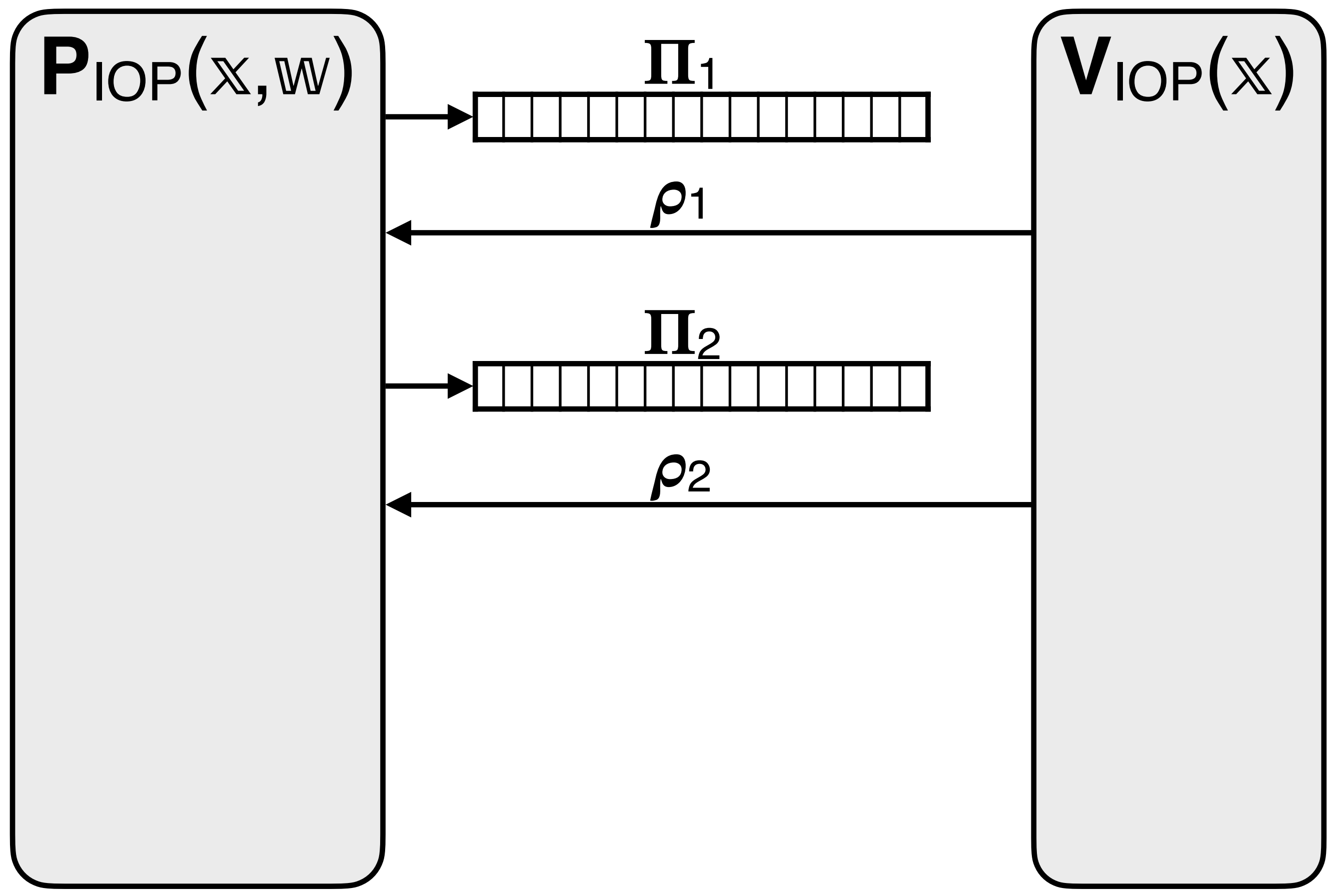

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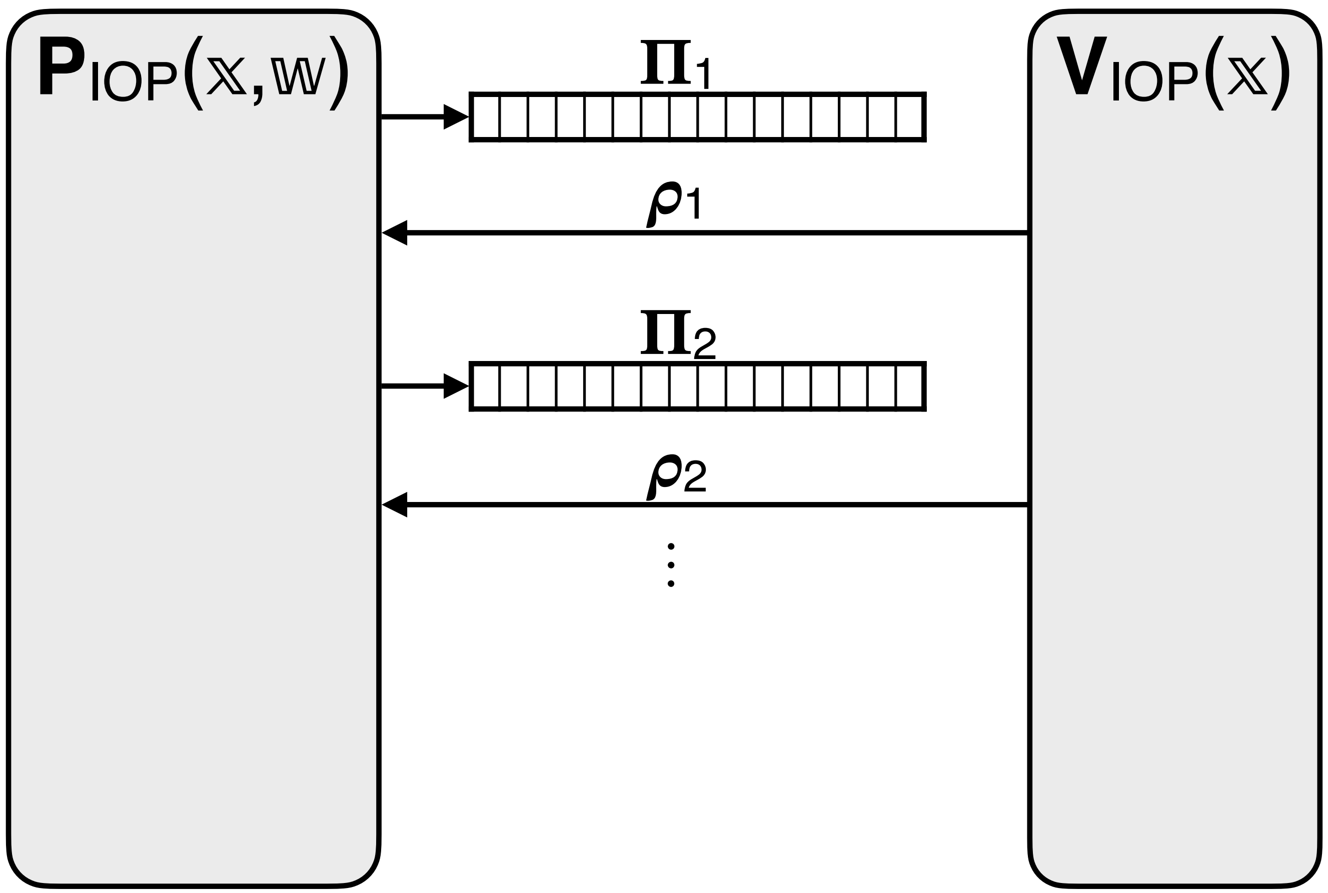

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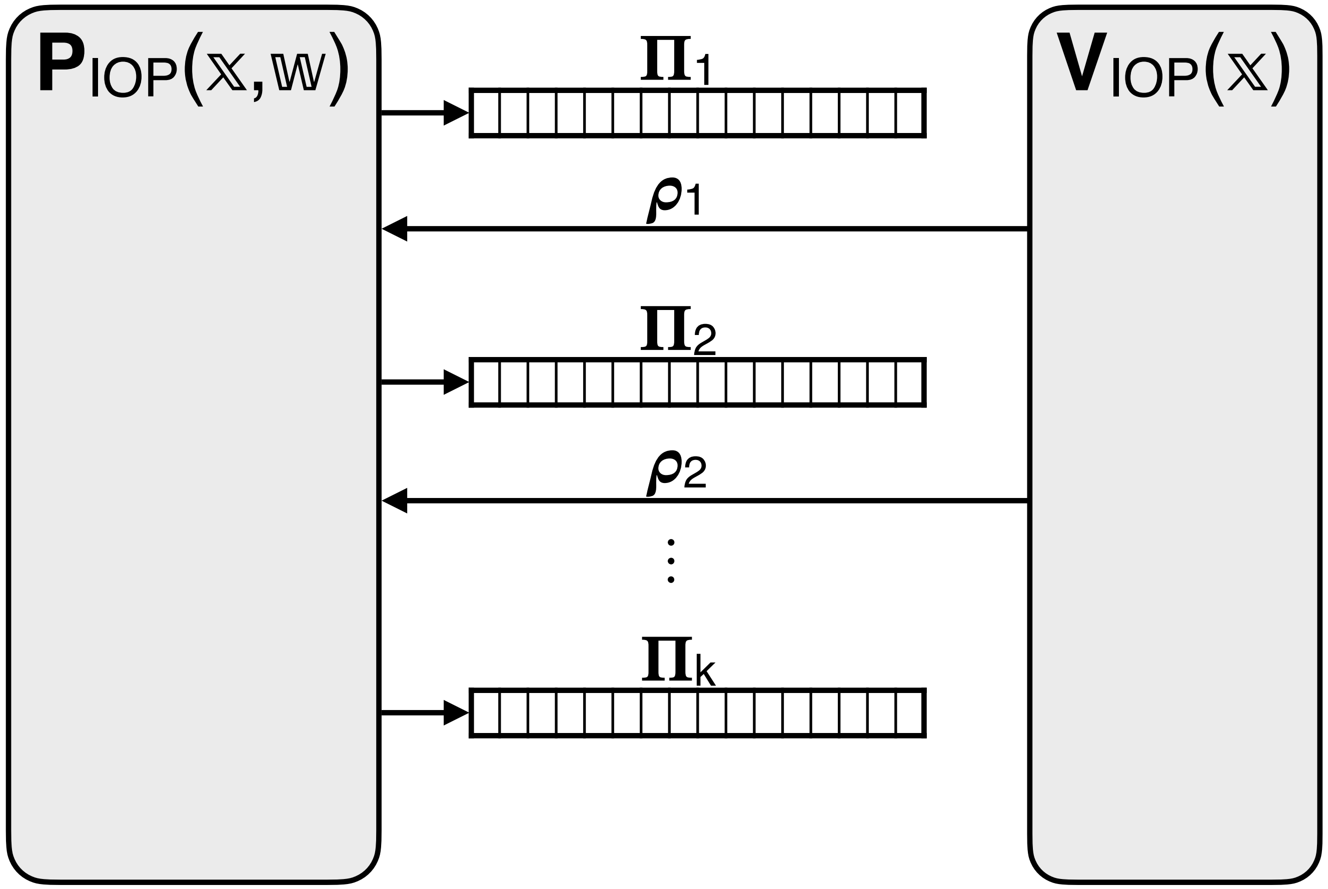

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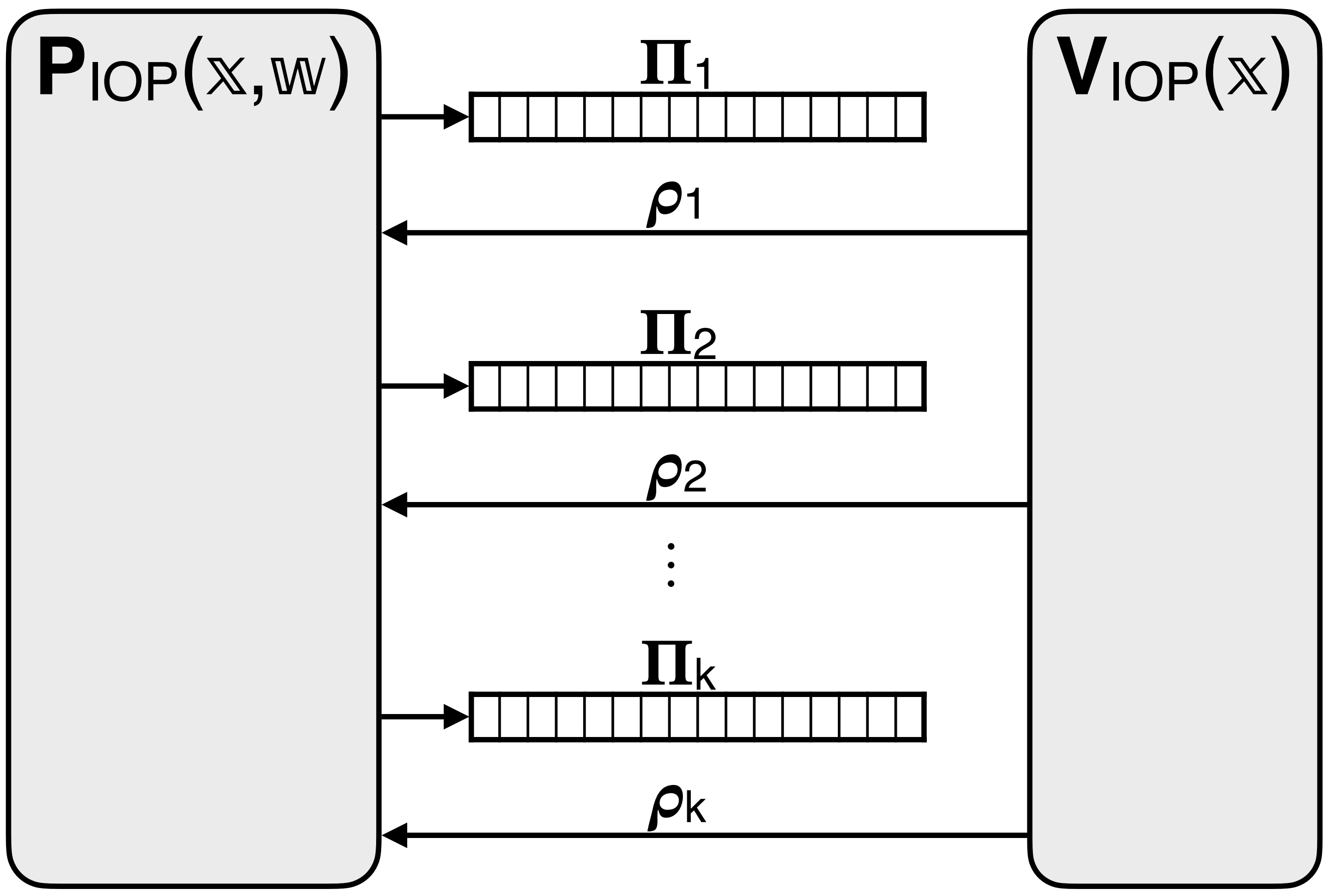

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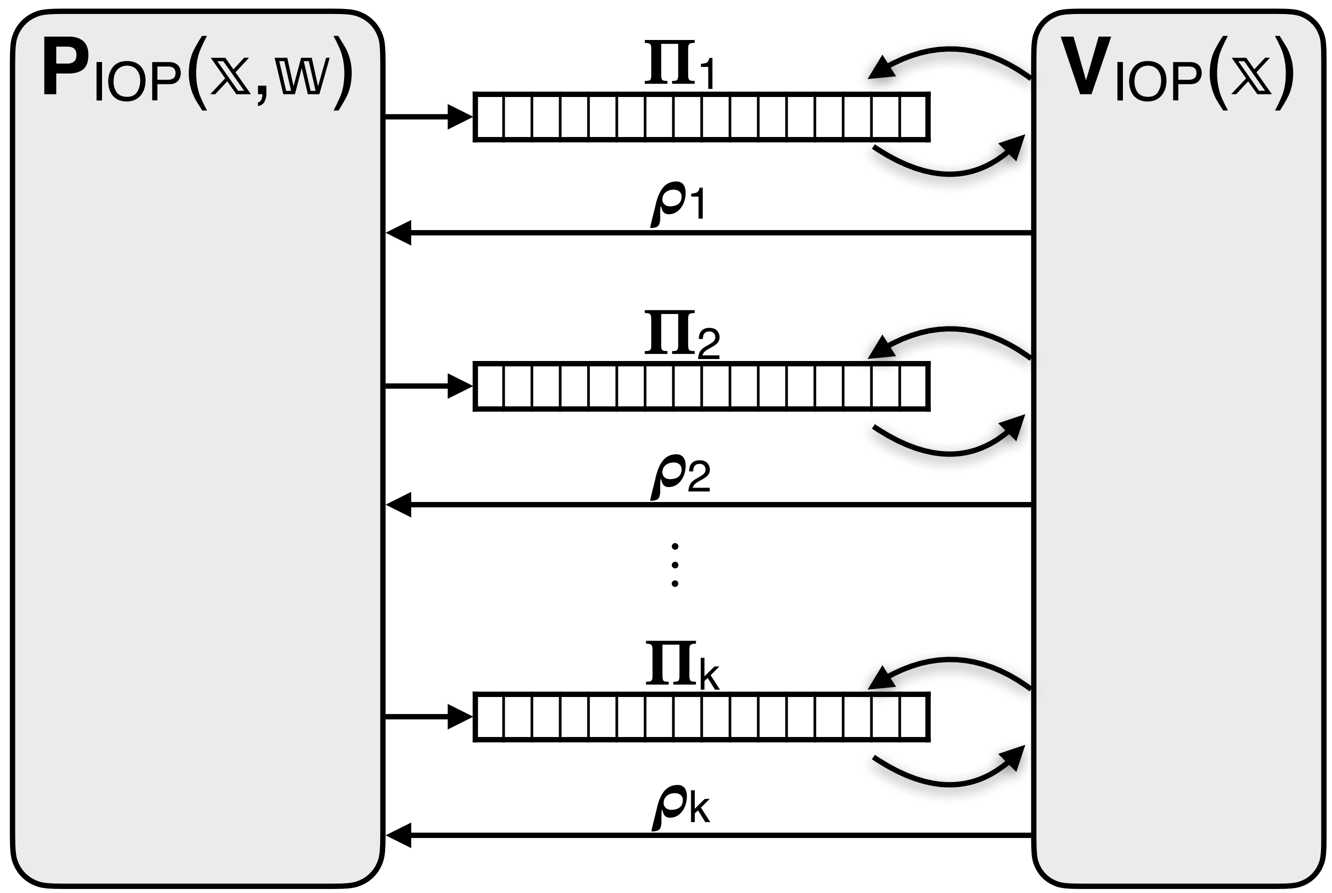

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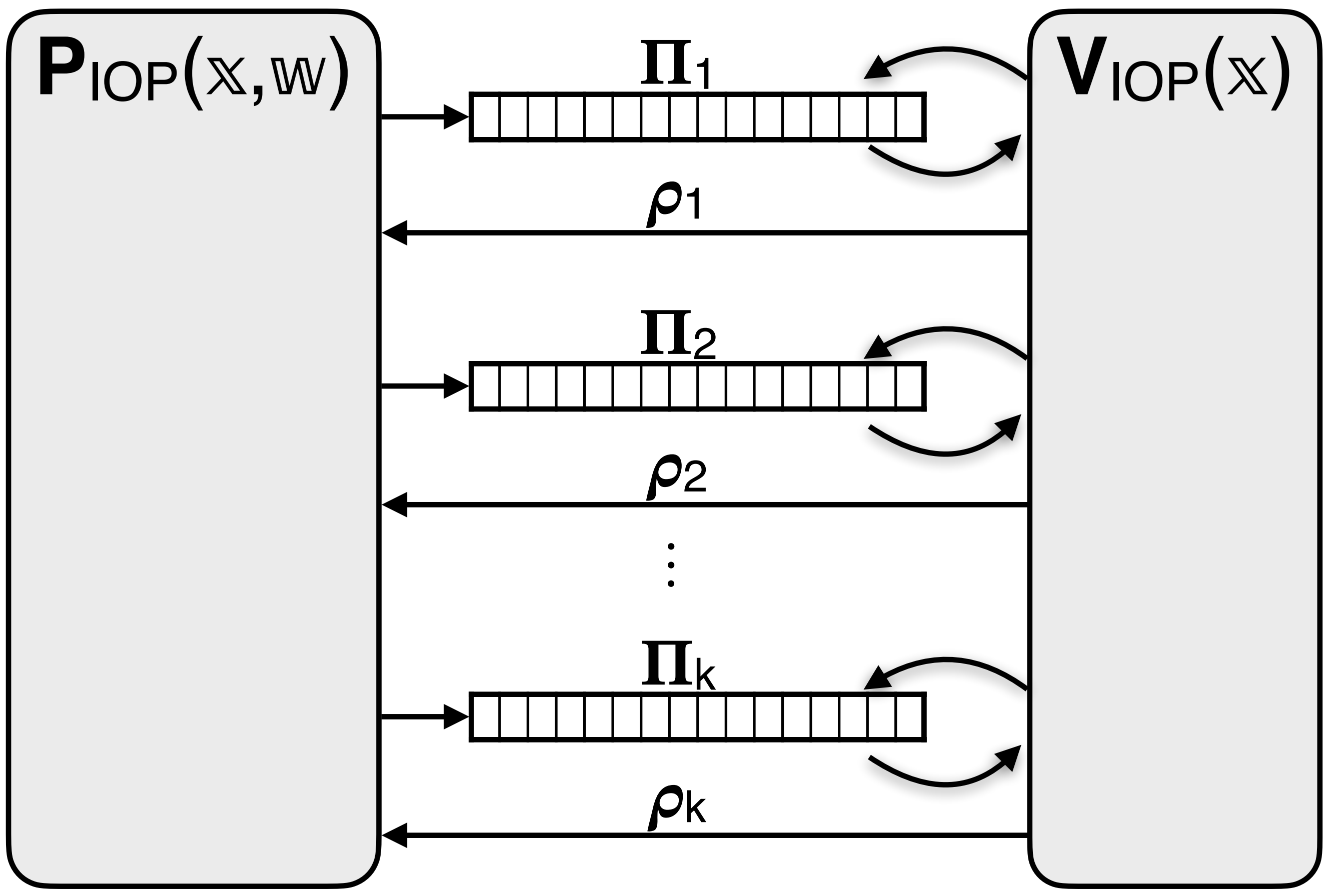

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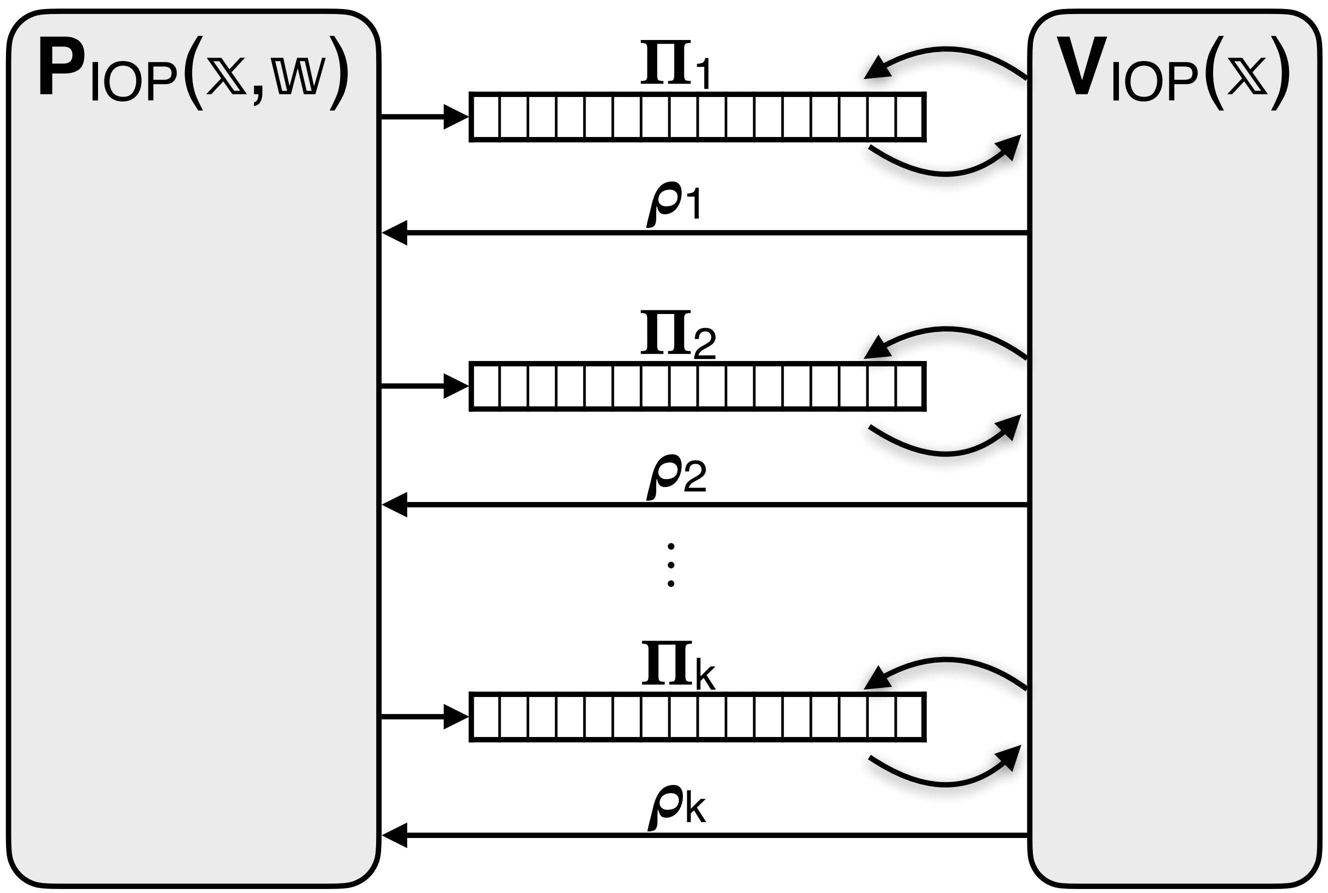

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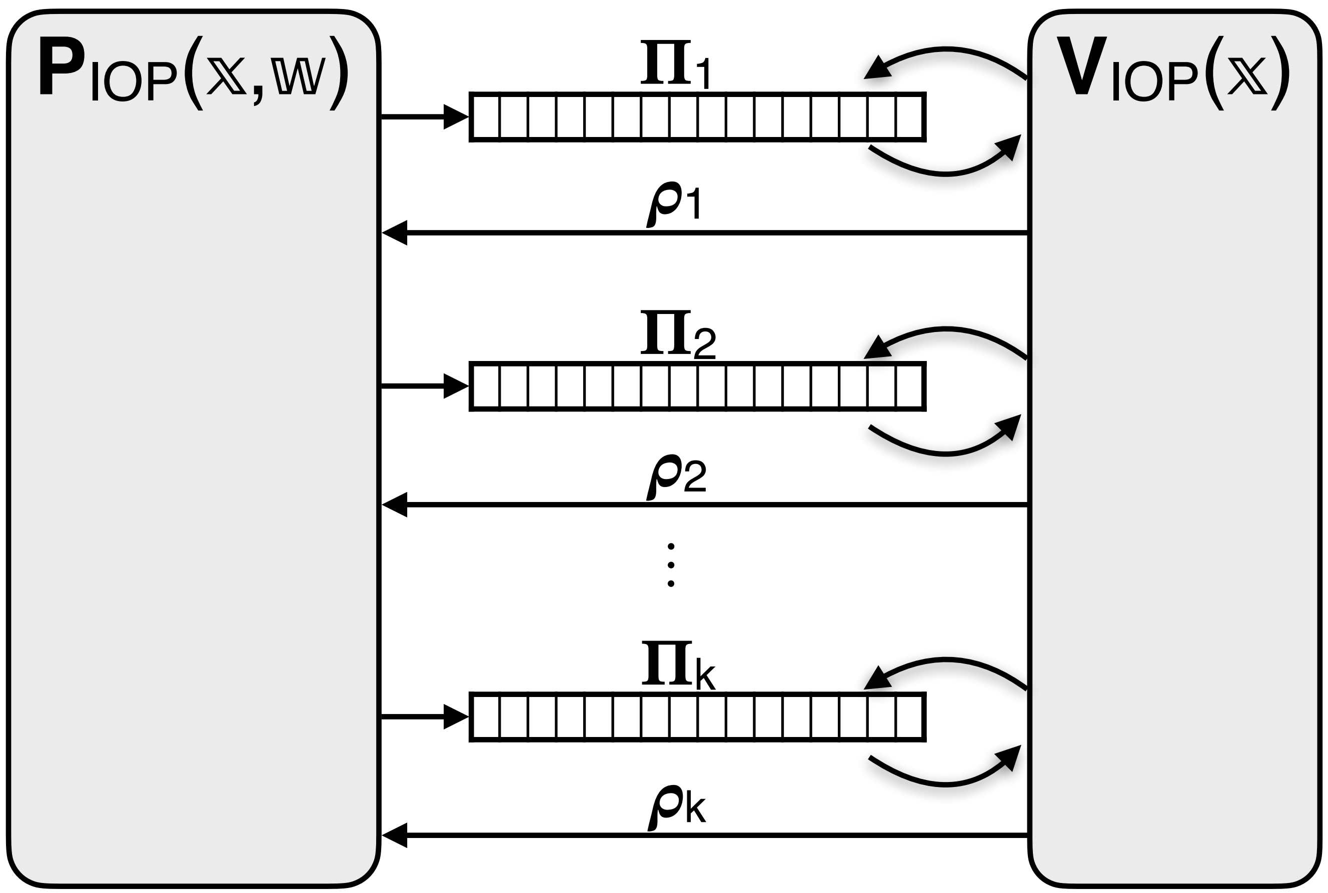

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IOPs underlie essentially all modern succinct arguments.

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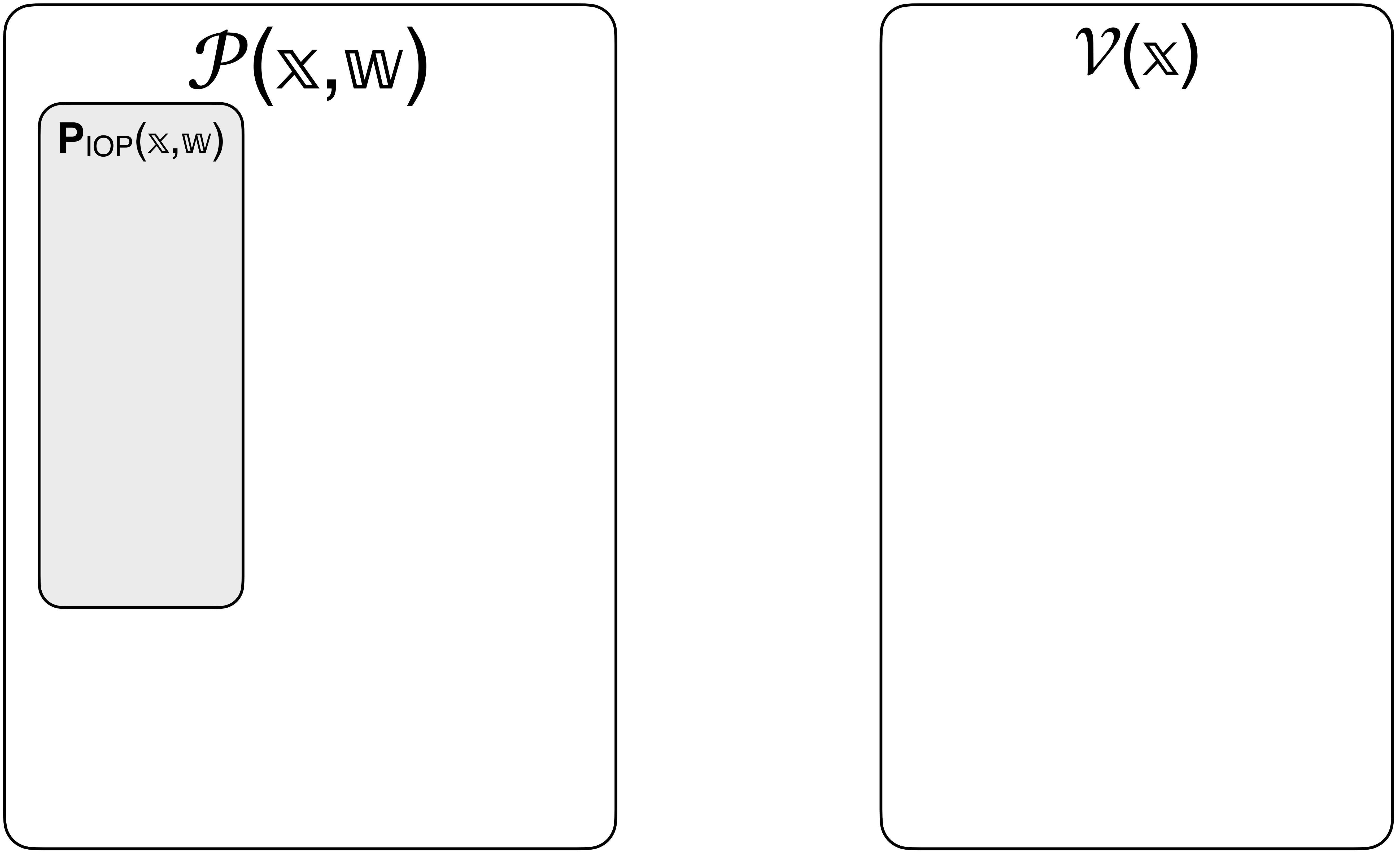

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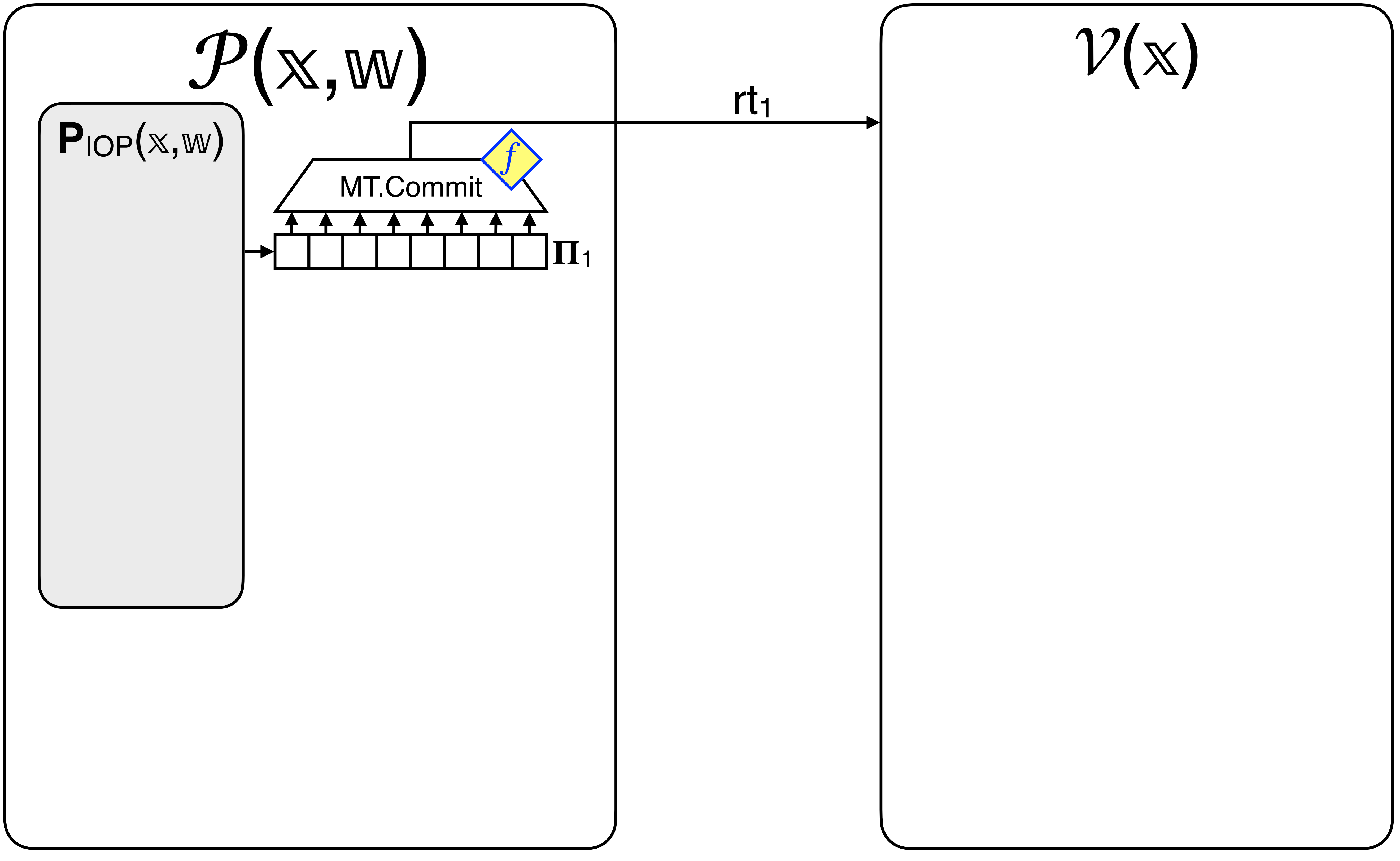

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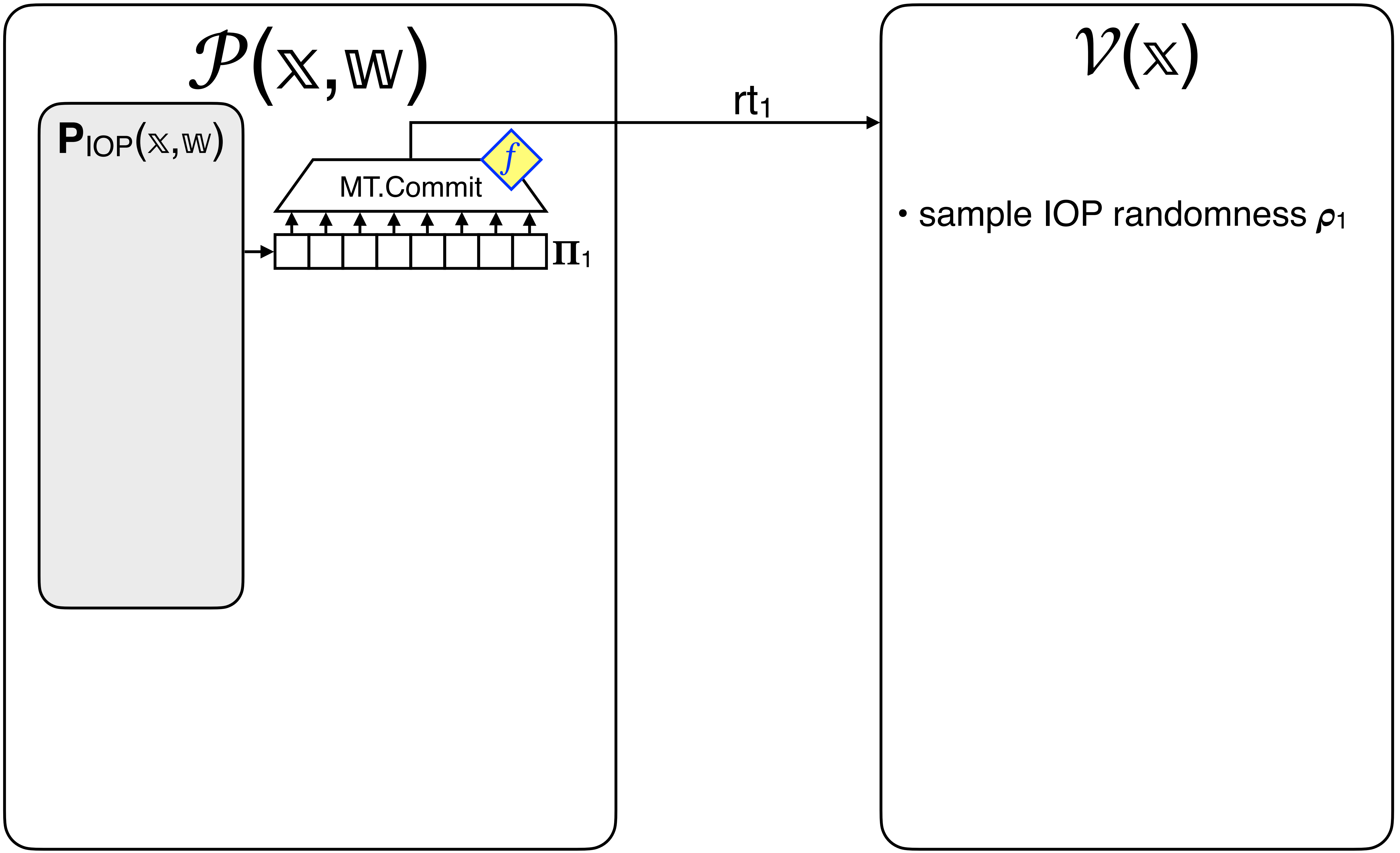

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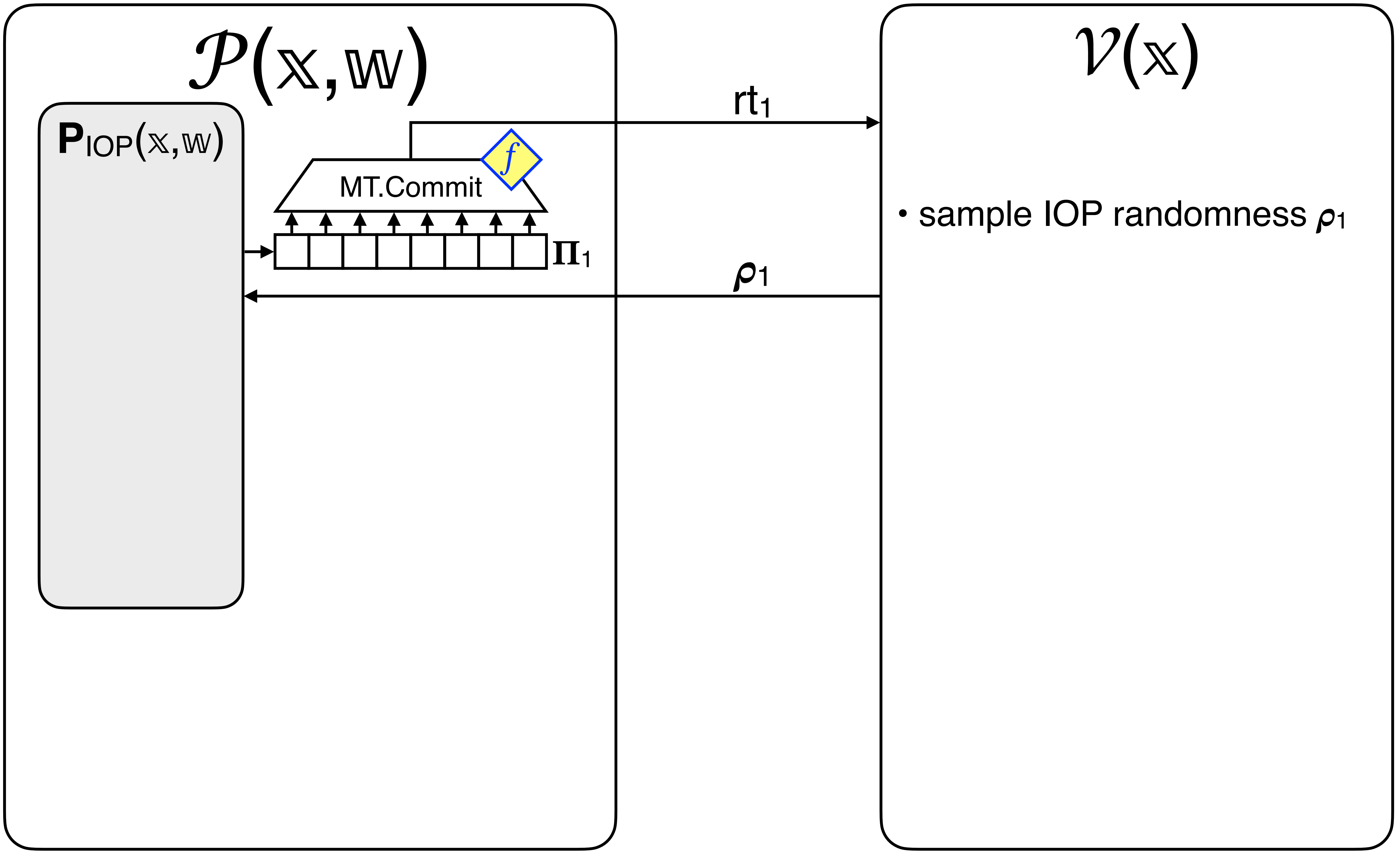

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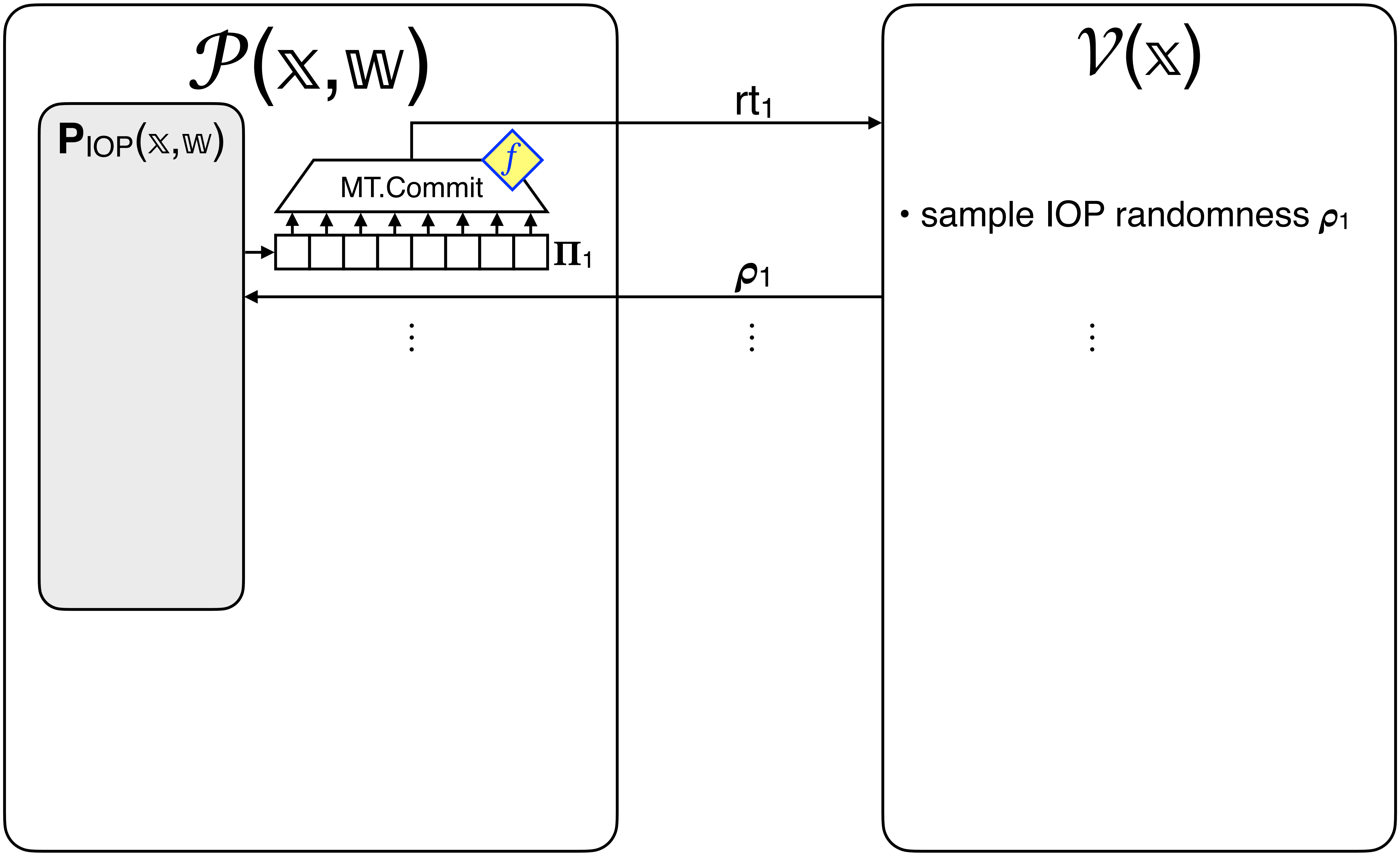

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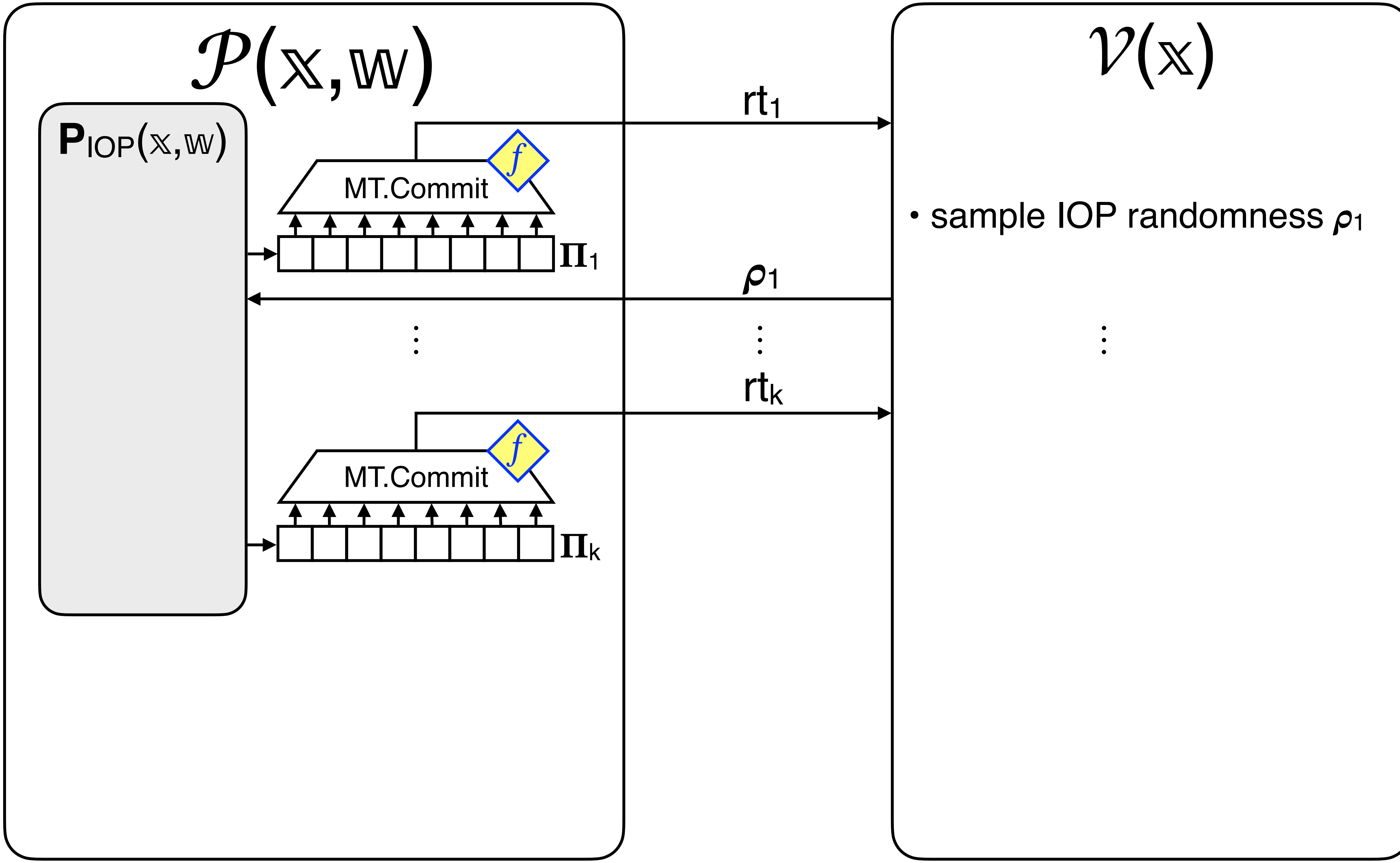

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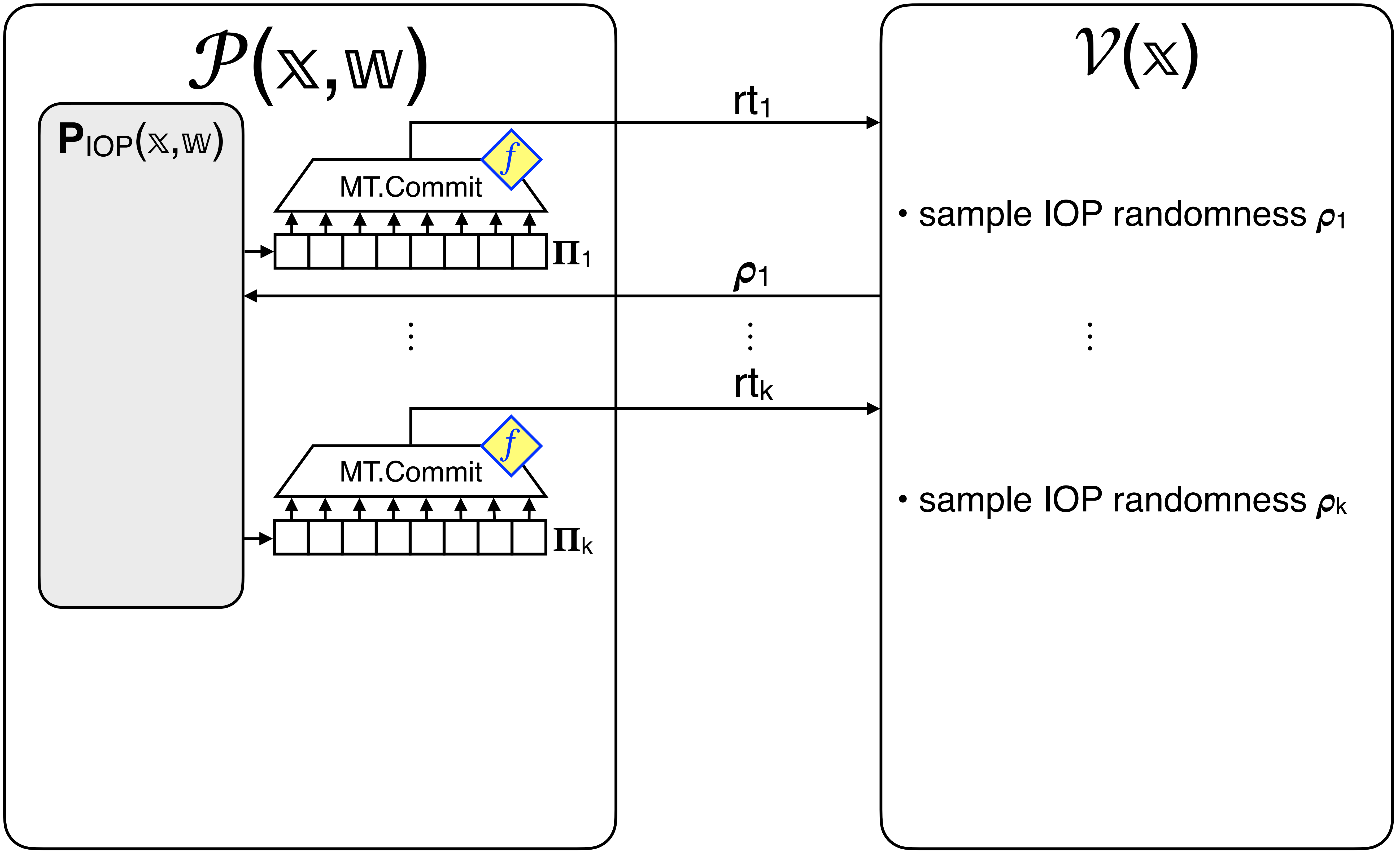

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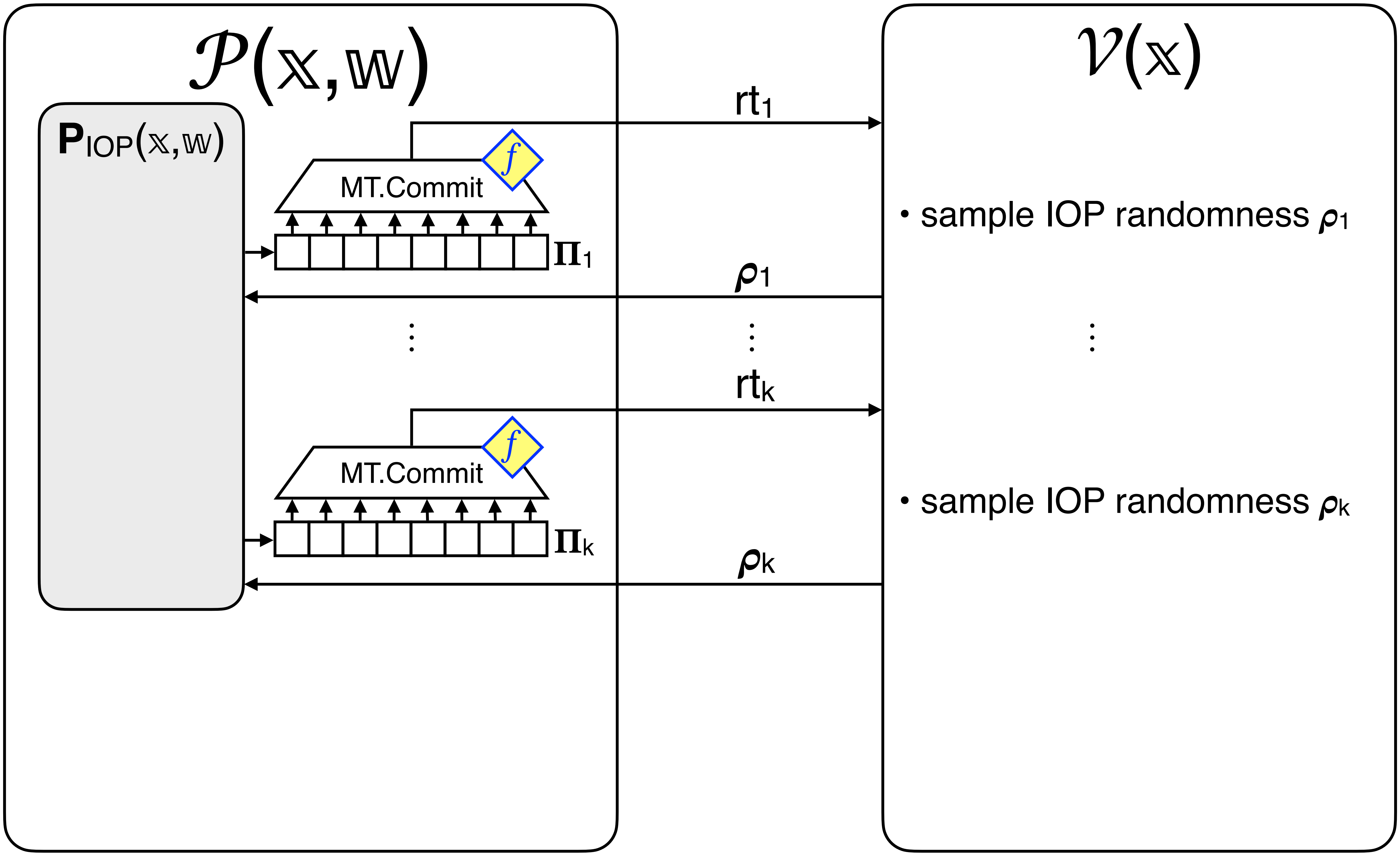

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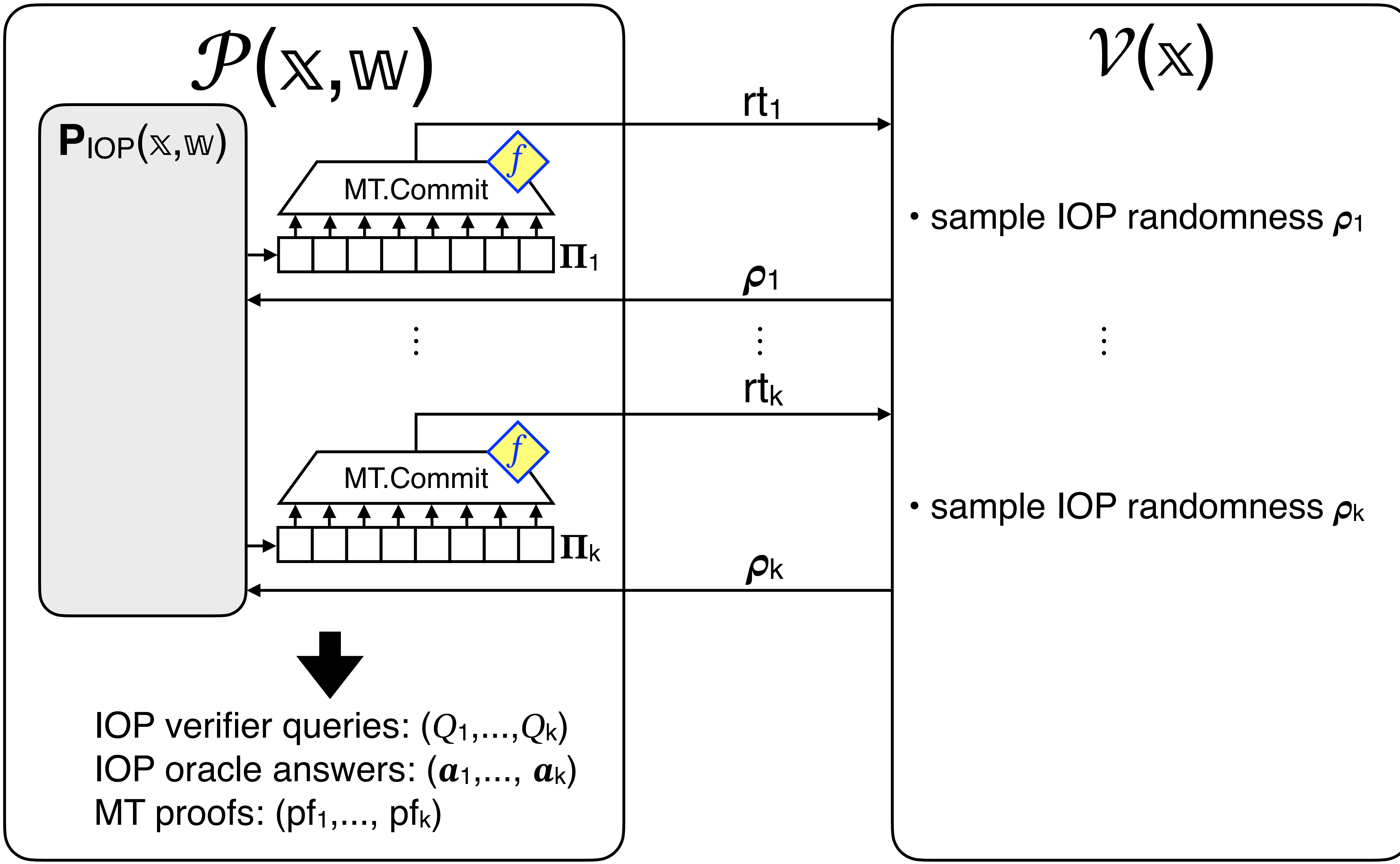

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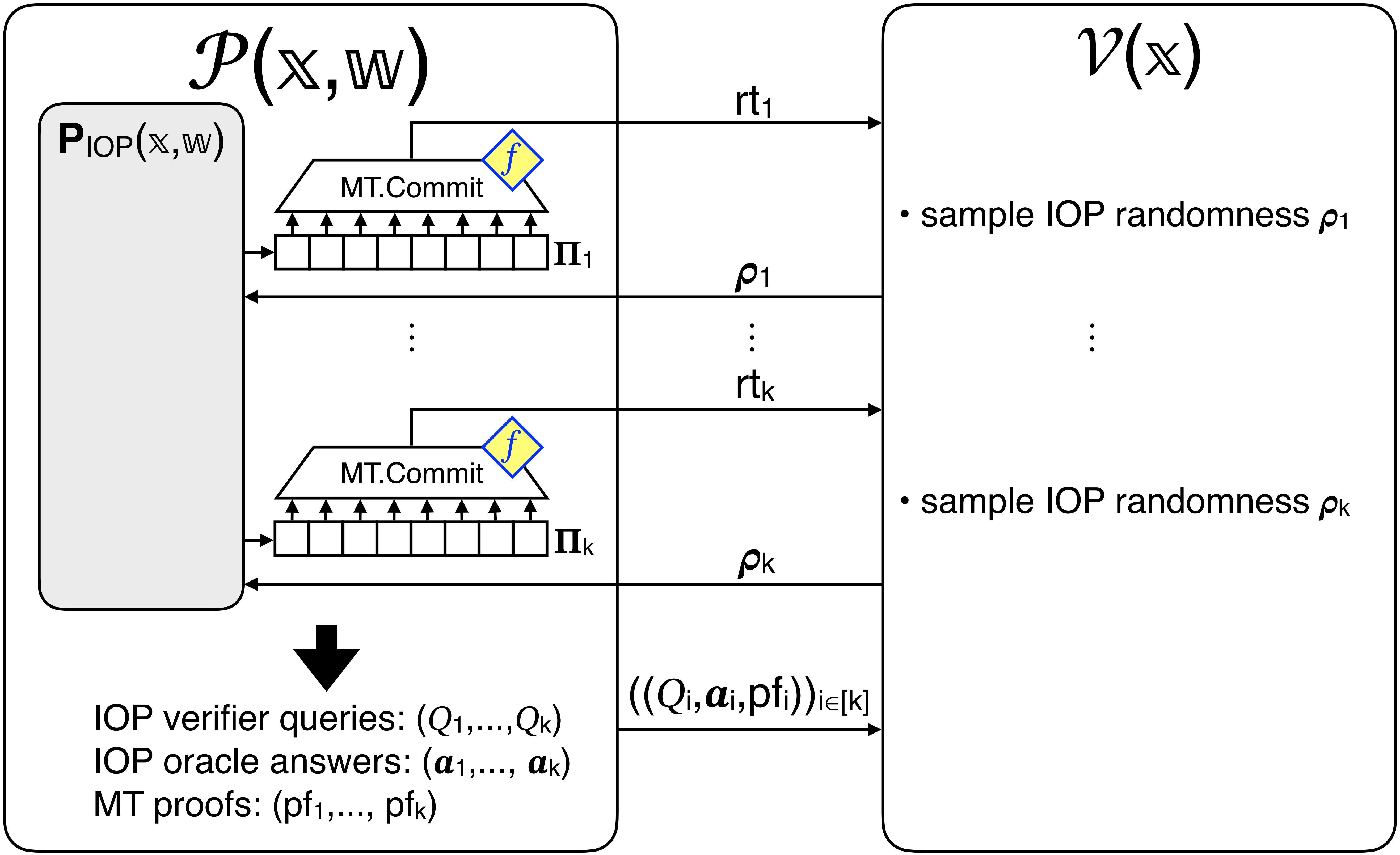

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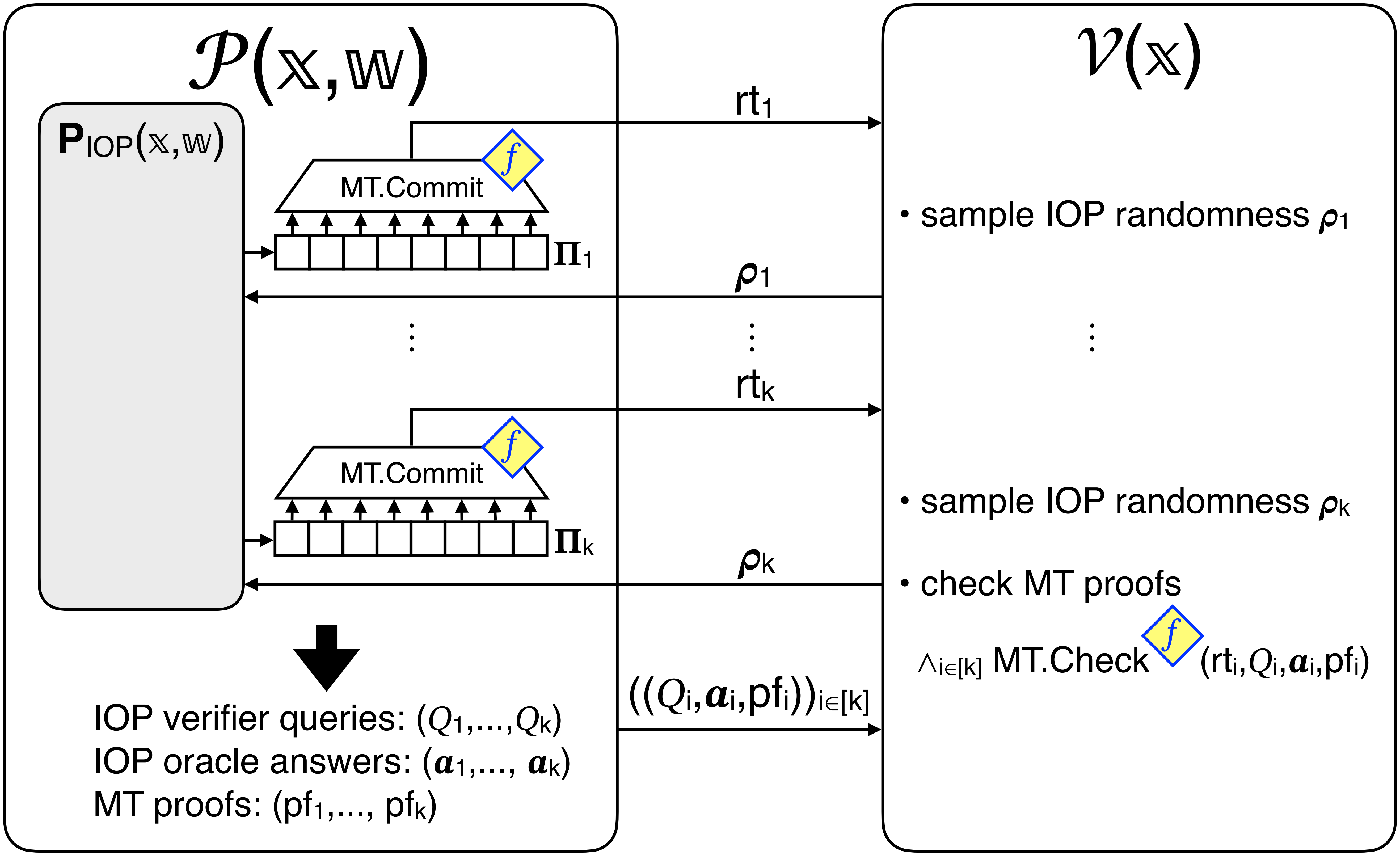

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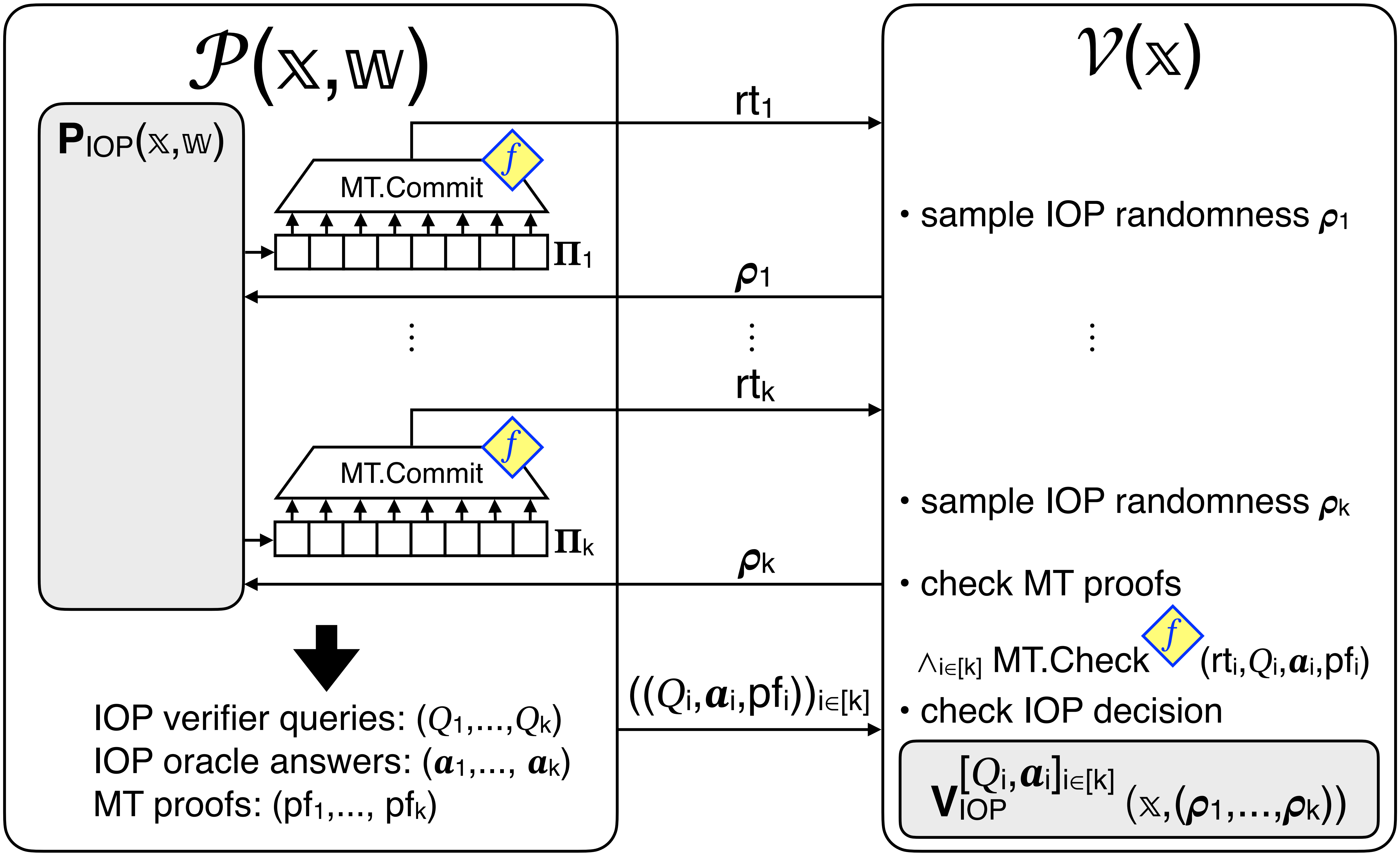

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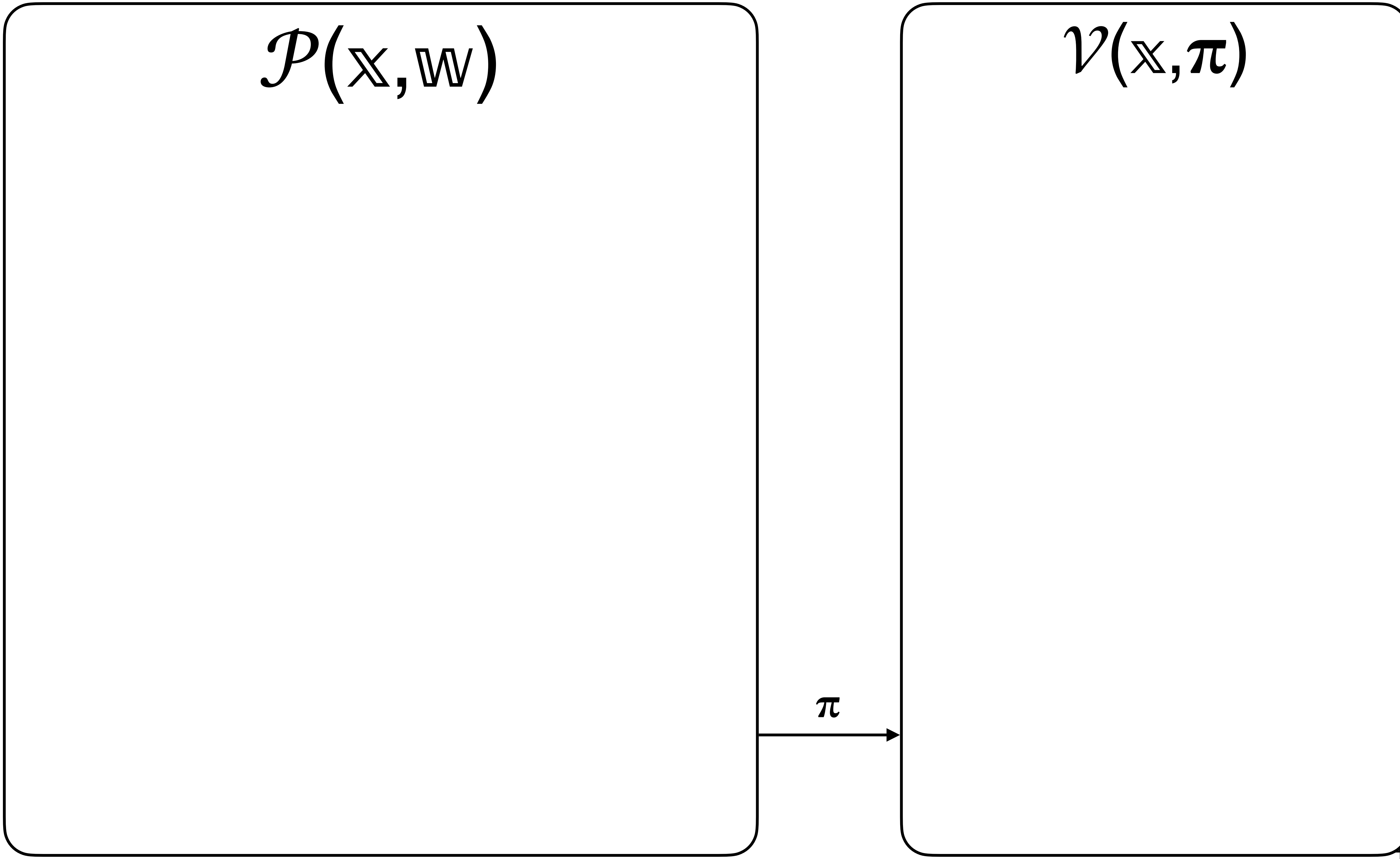

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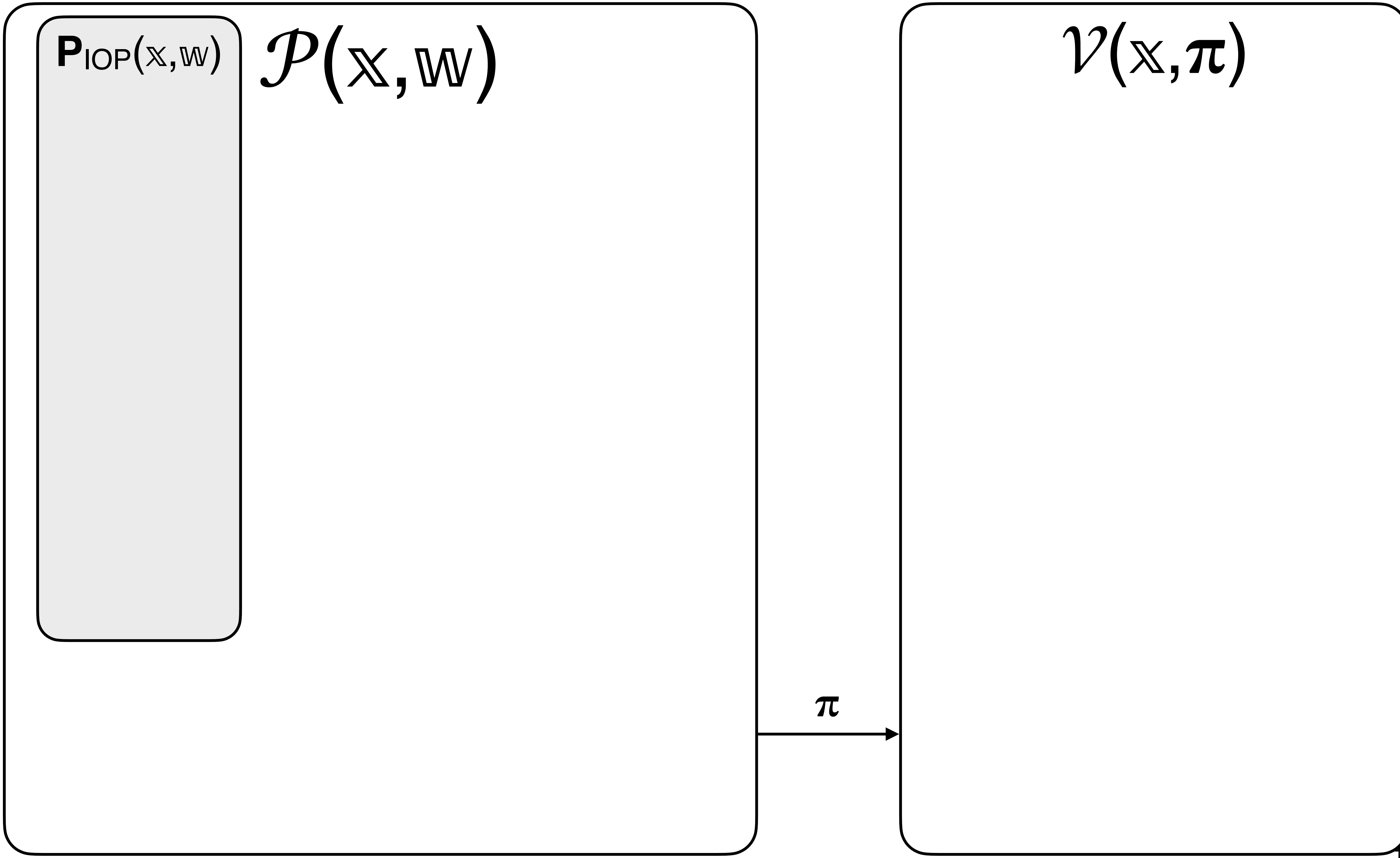

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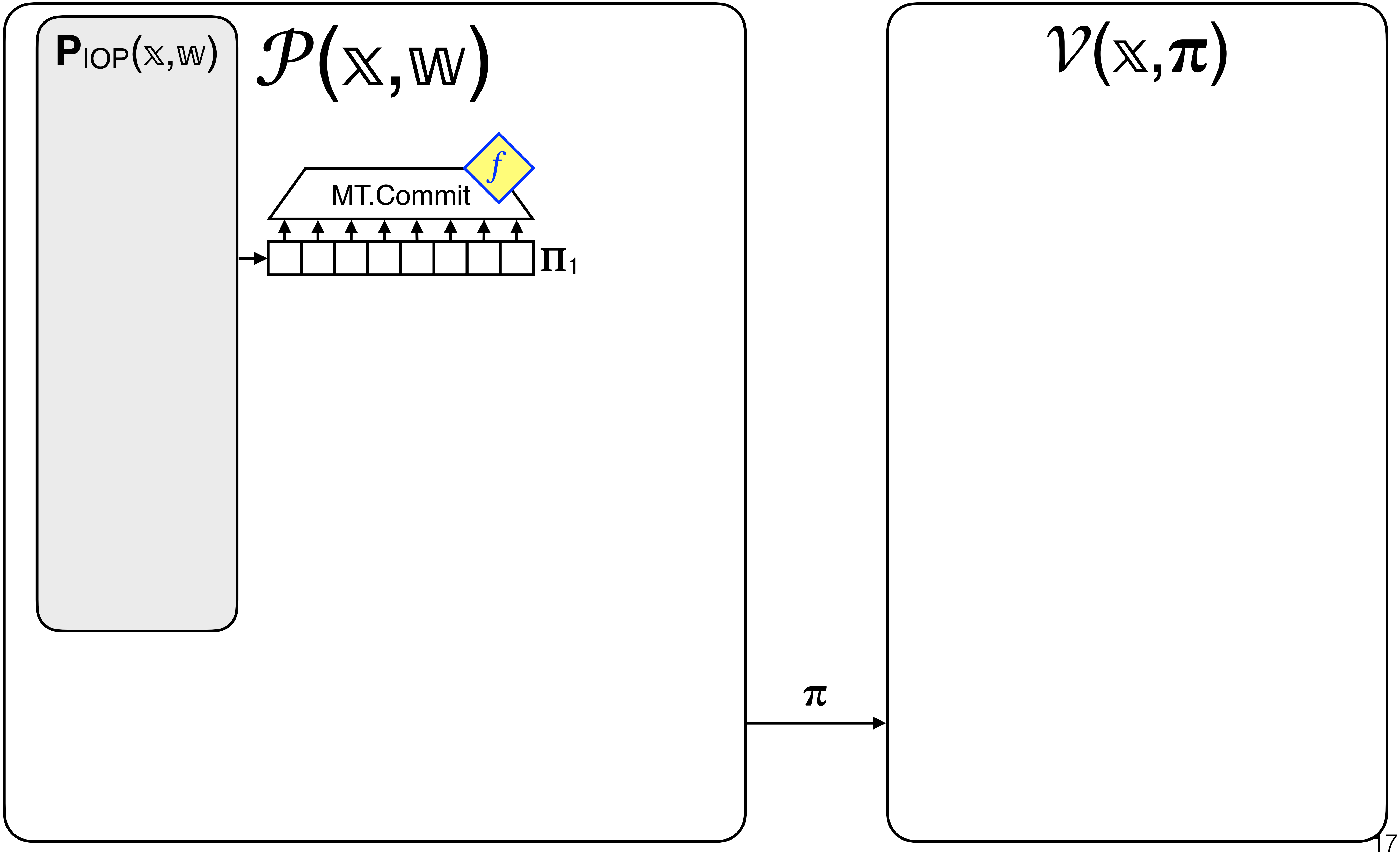

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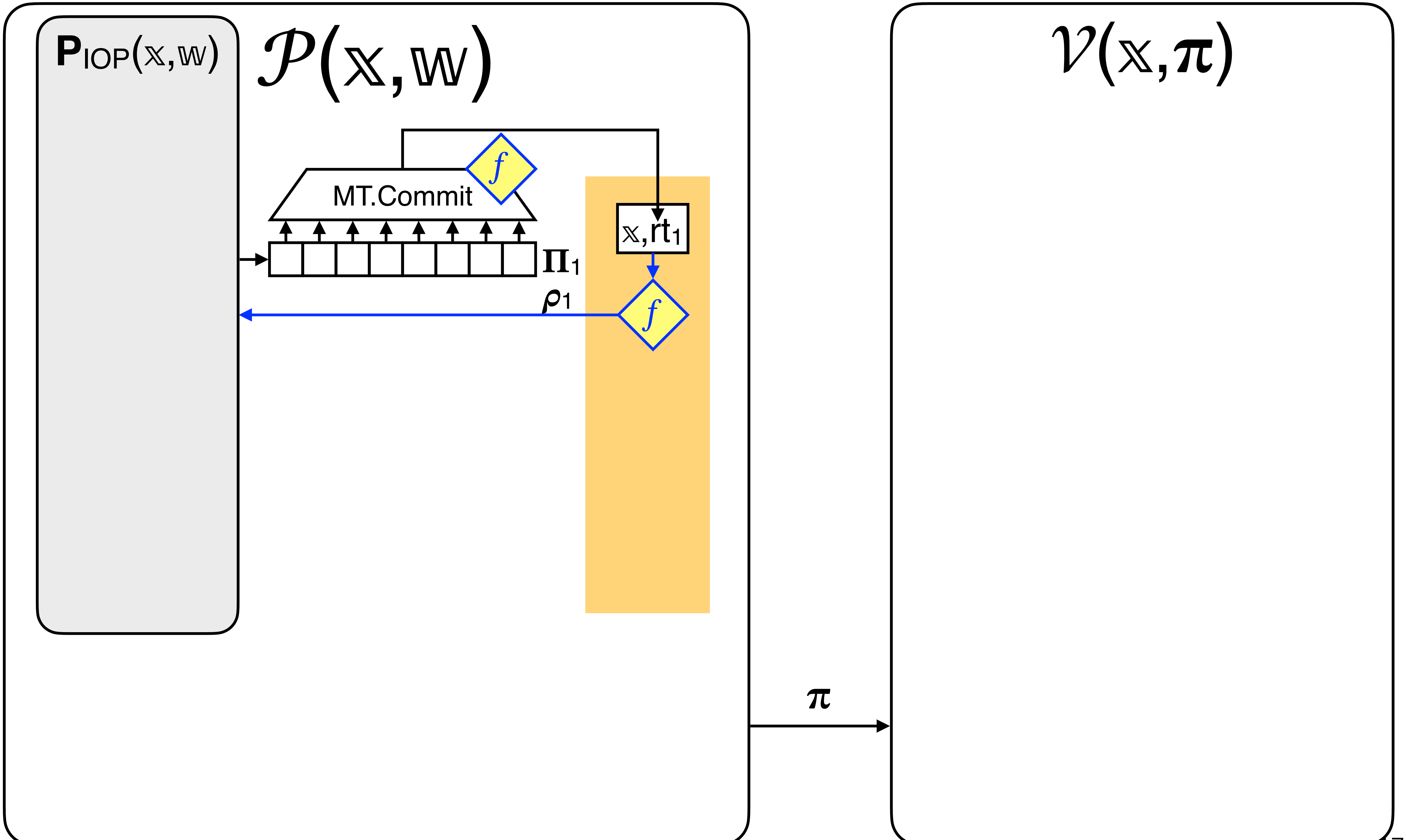

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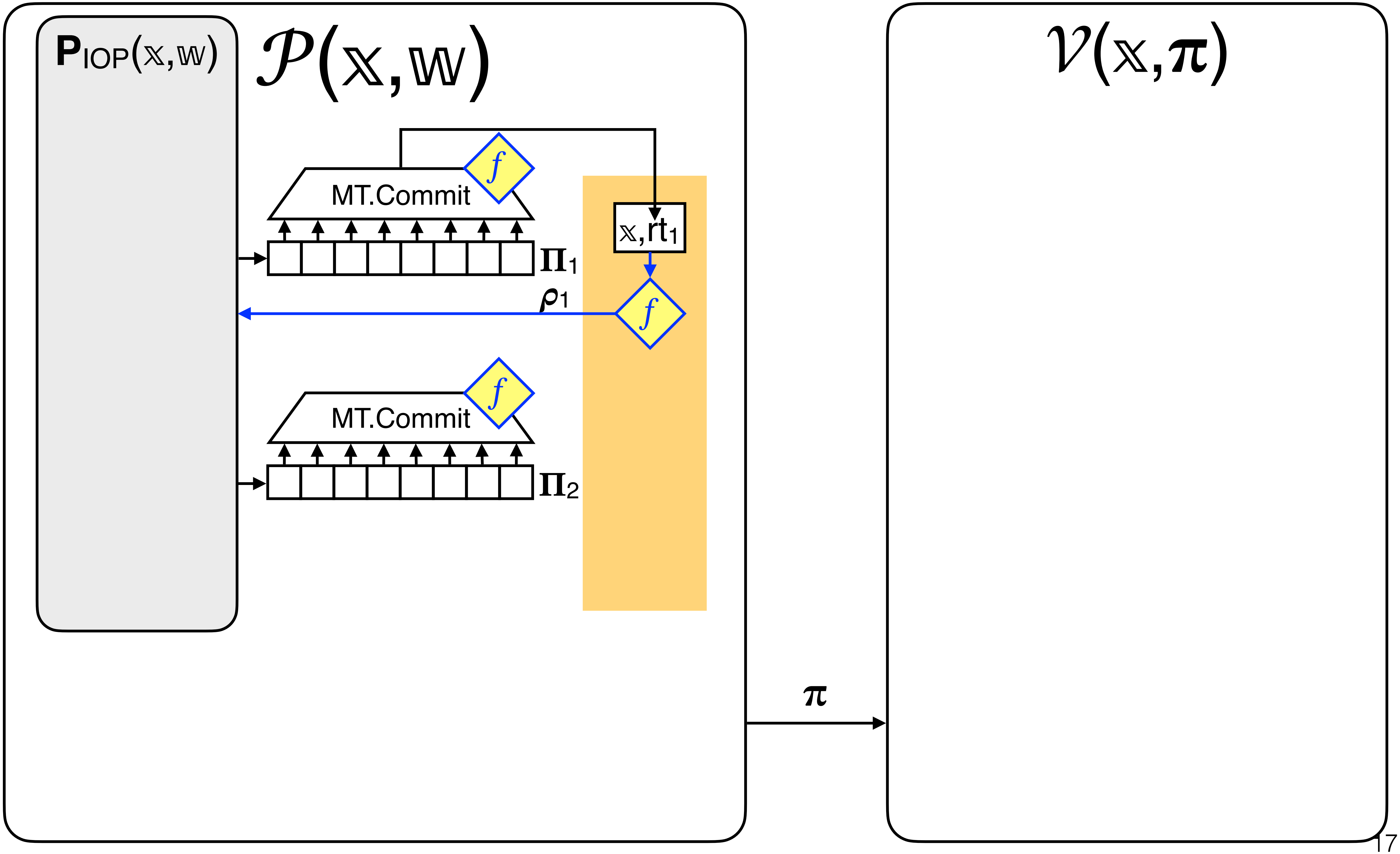

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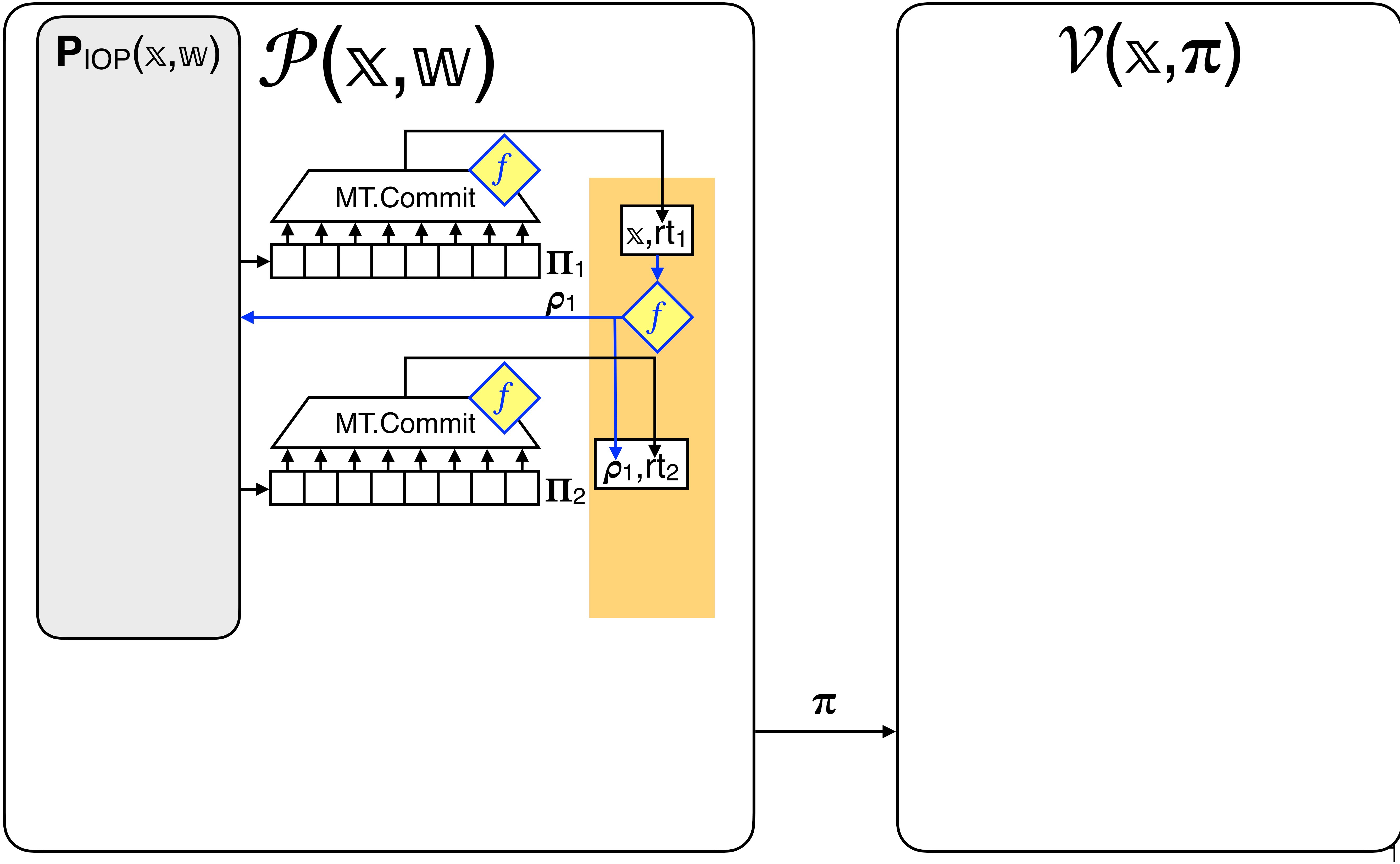

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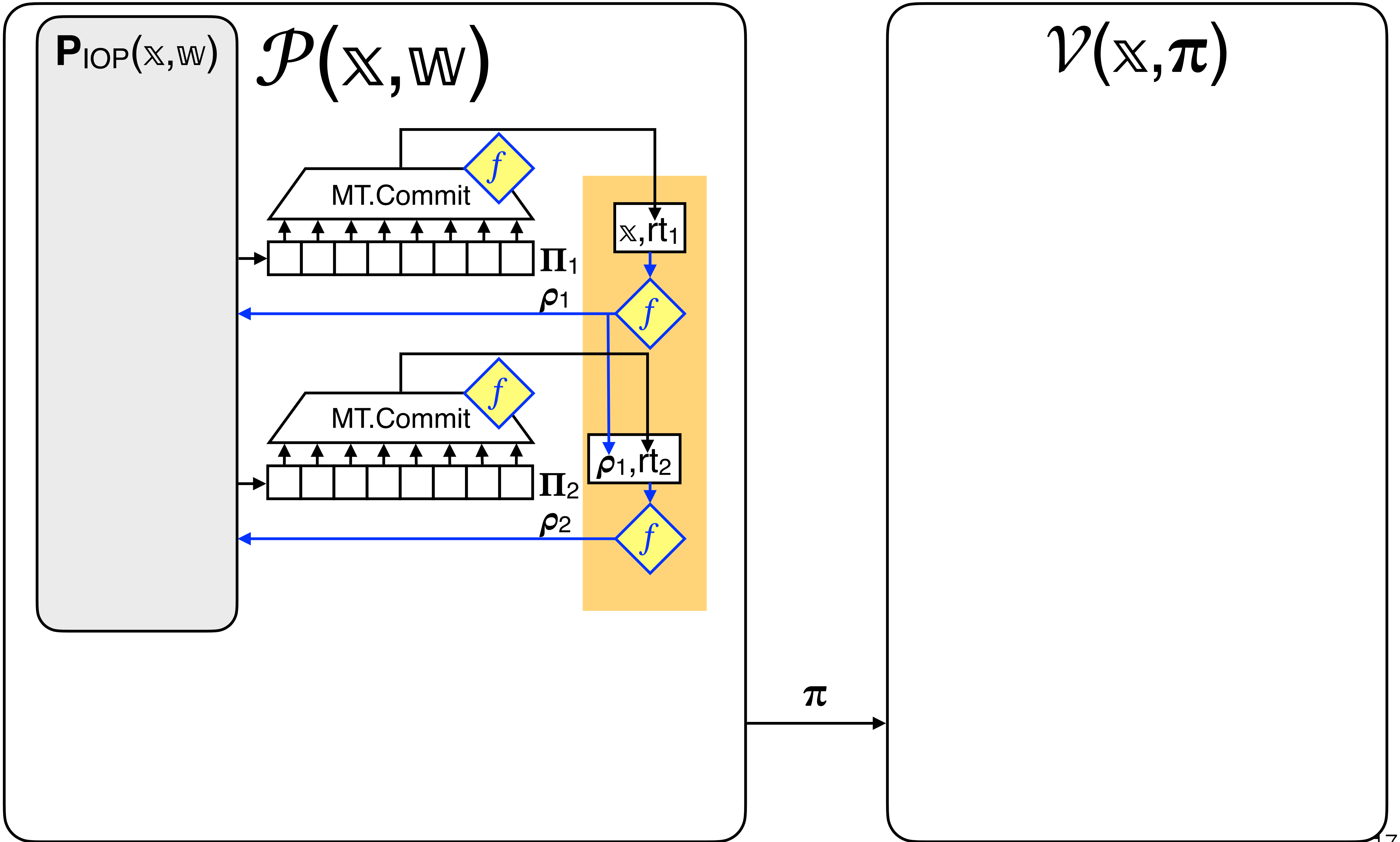

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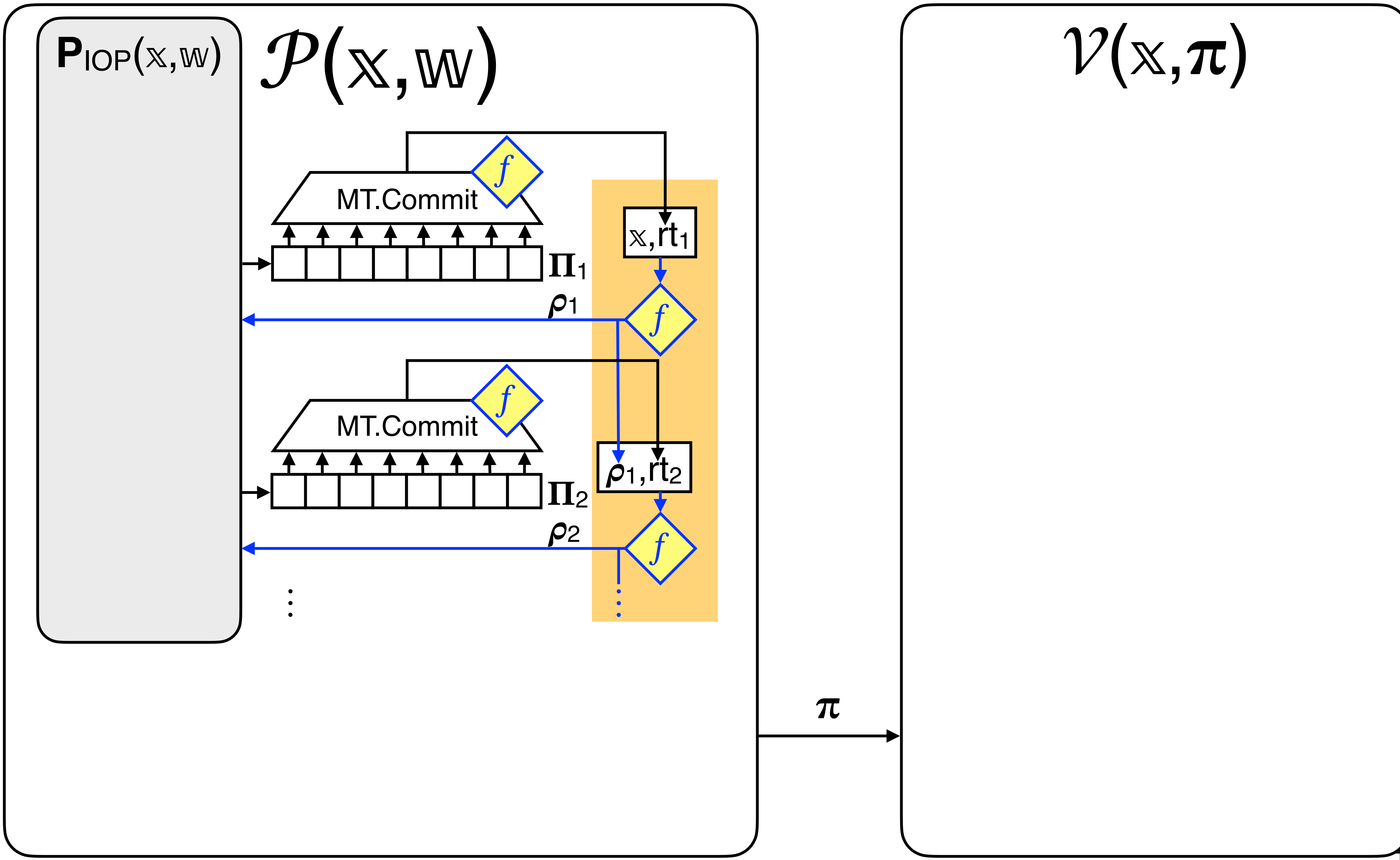

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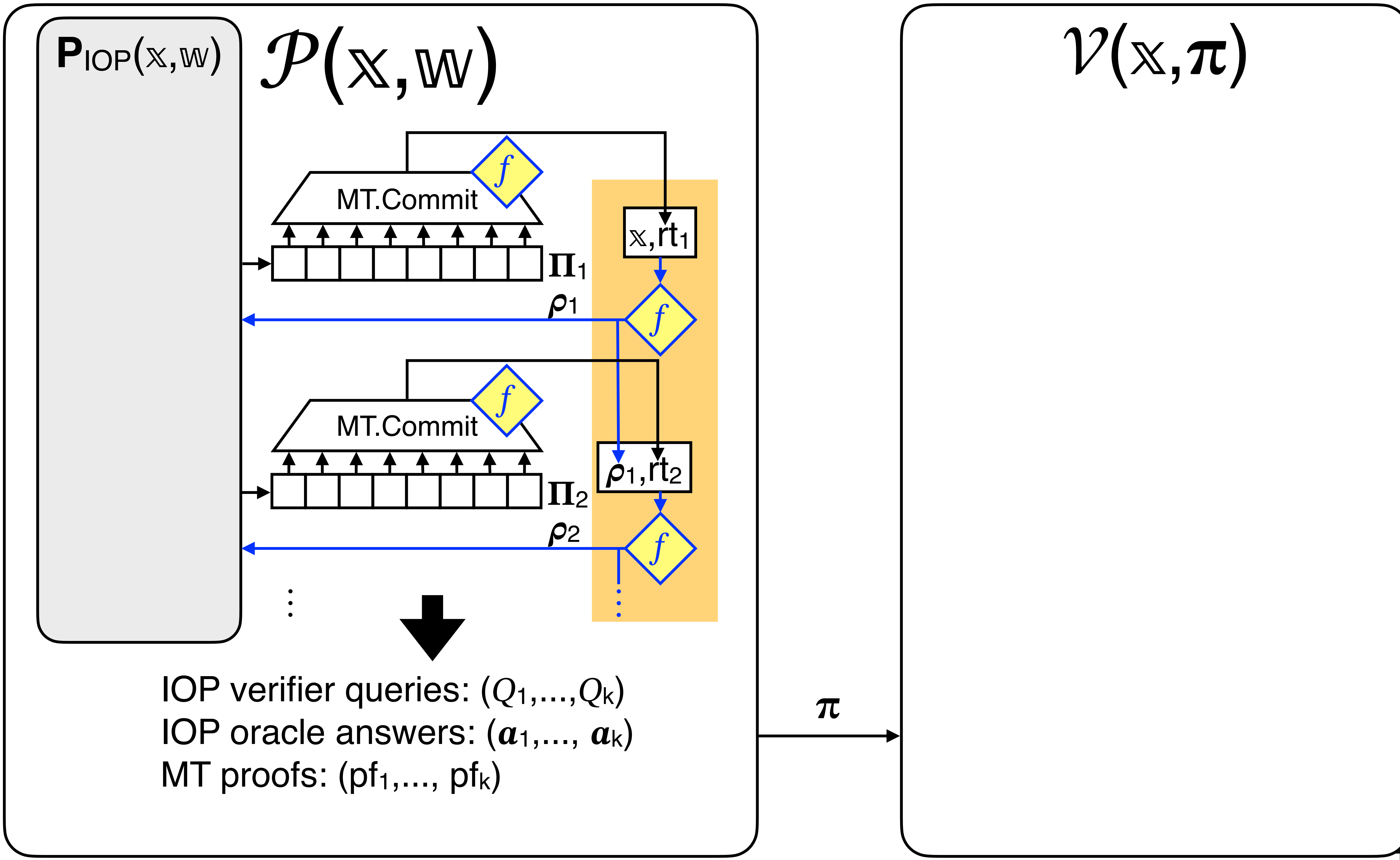

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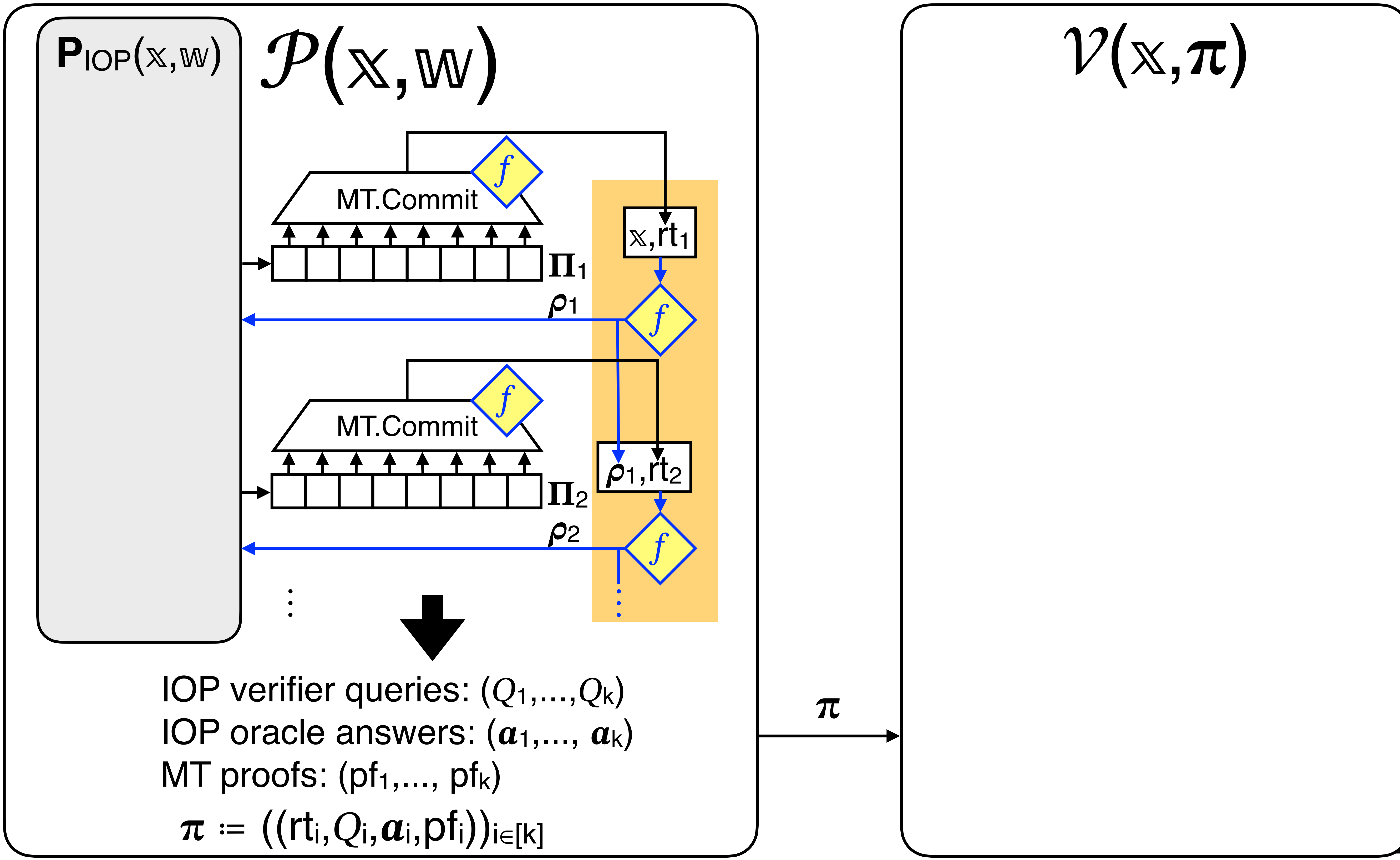

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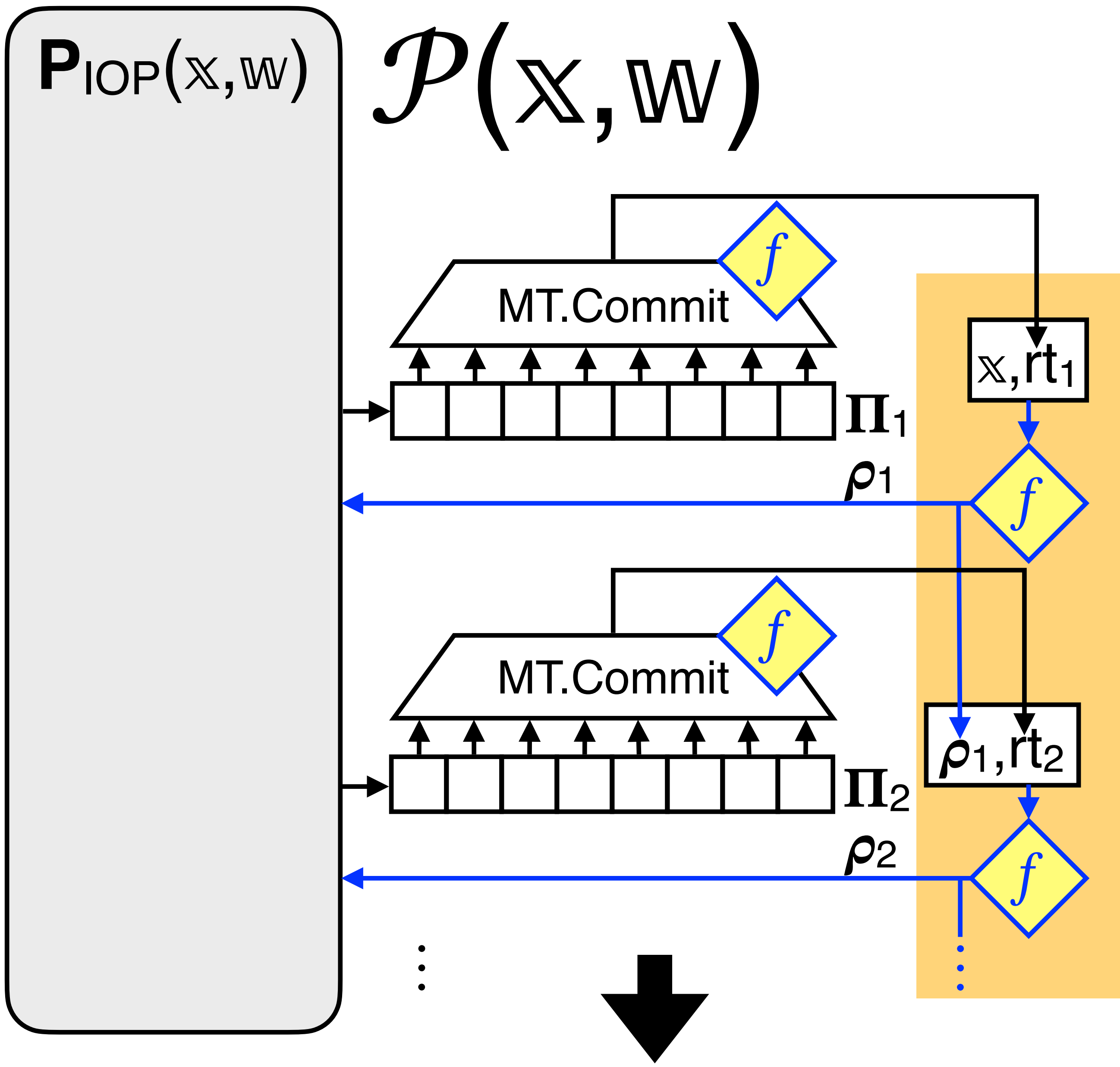

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$\boldsymbol{\pi} := ((\mathrm{rt}_i,Q_i,\boldsymbol{a}_i,\mathrm{pf}_i))_{i\in[k]}$

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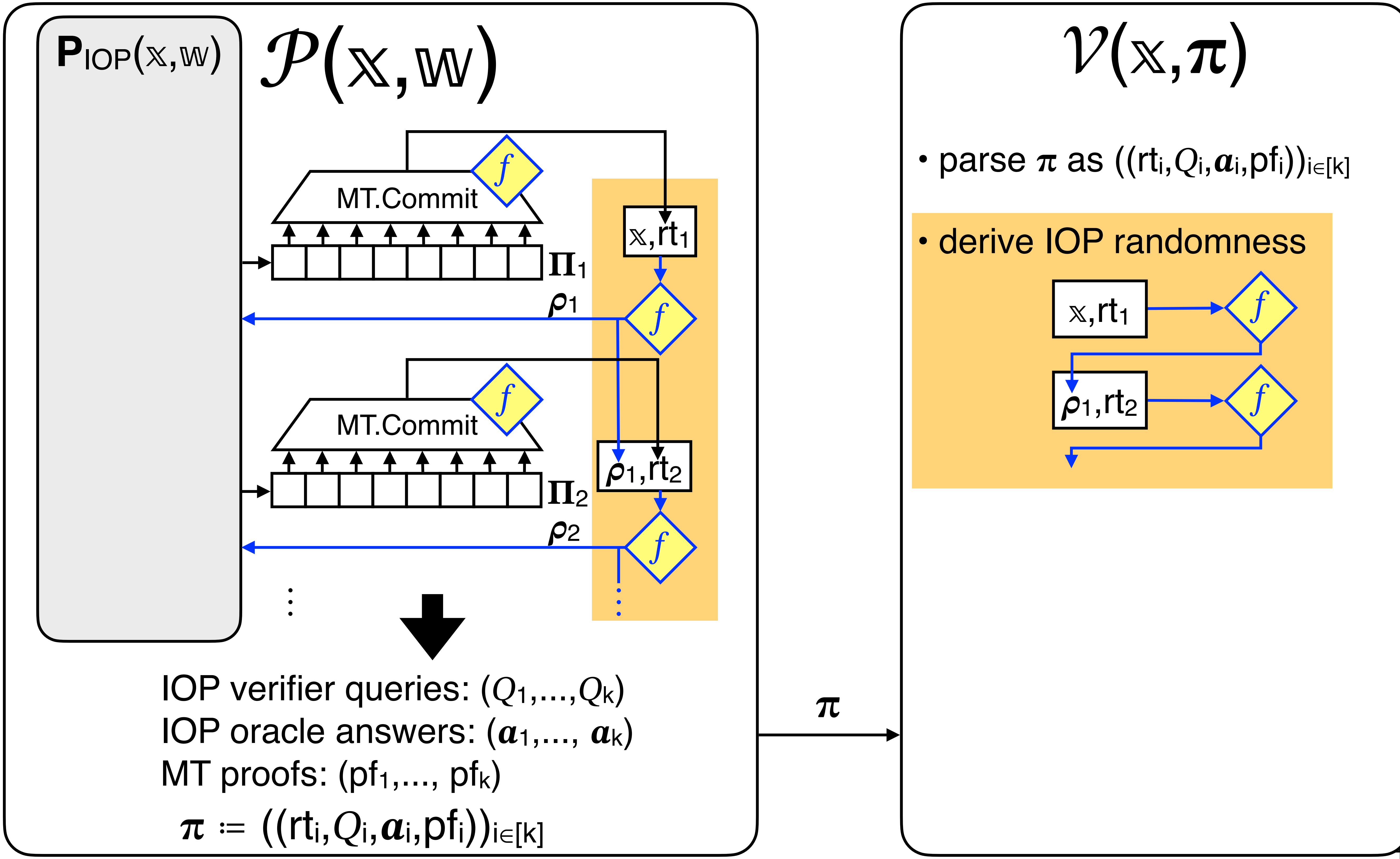

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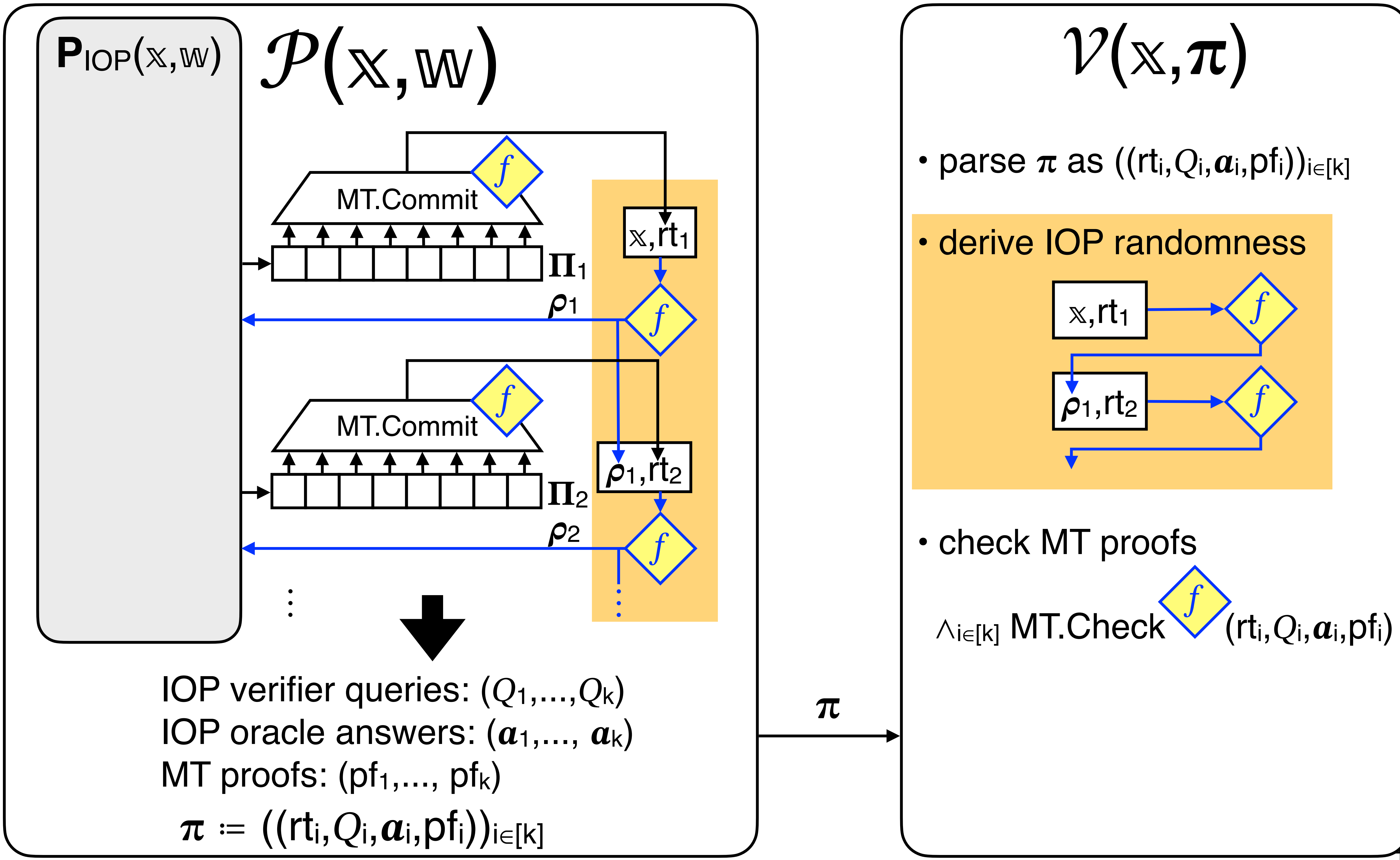

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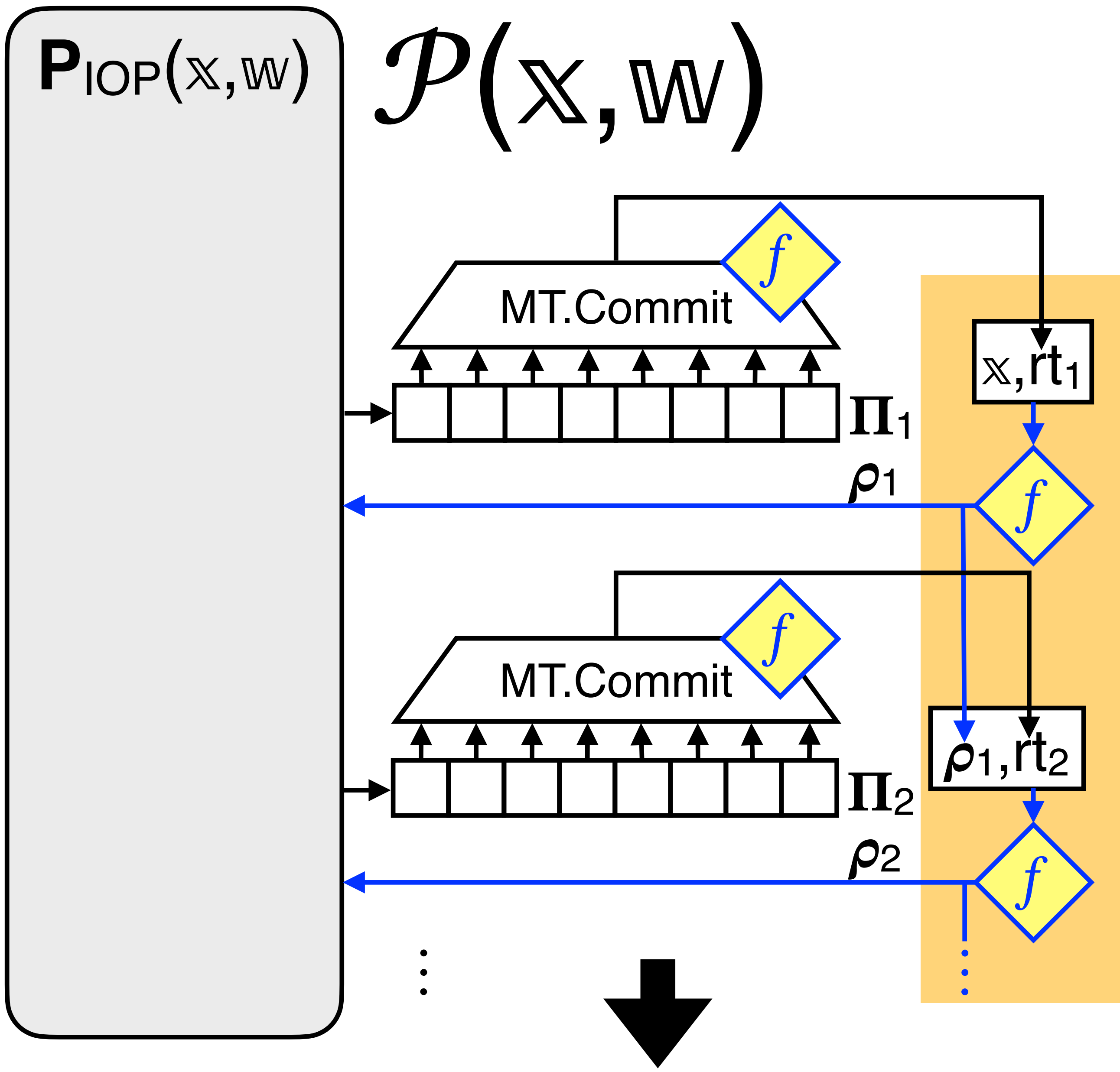

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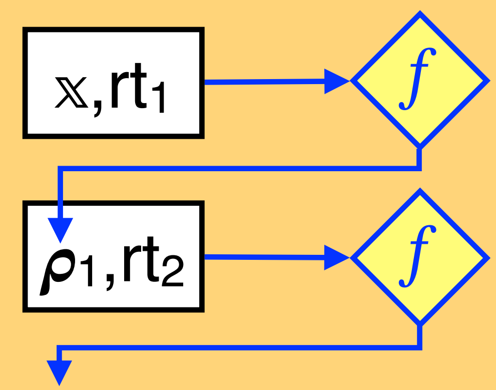

- check MT proofs

$\wedge_{i\in[k]}$ MT.Check^{f}(rt$_i$,Q_i,$\boldsymbol{a}_i$,pf$_i$)

- check IOP decision

$$V_{IOP}^{[Q_i,\boldsymbol{a}_i]_{i\in[k]}}(\mathbb{x},(\rho_1,\ldots,\rho_k))$$

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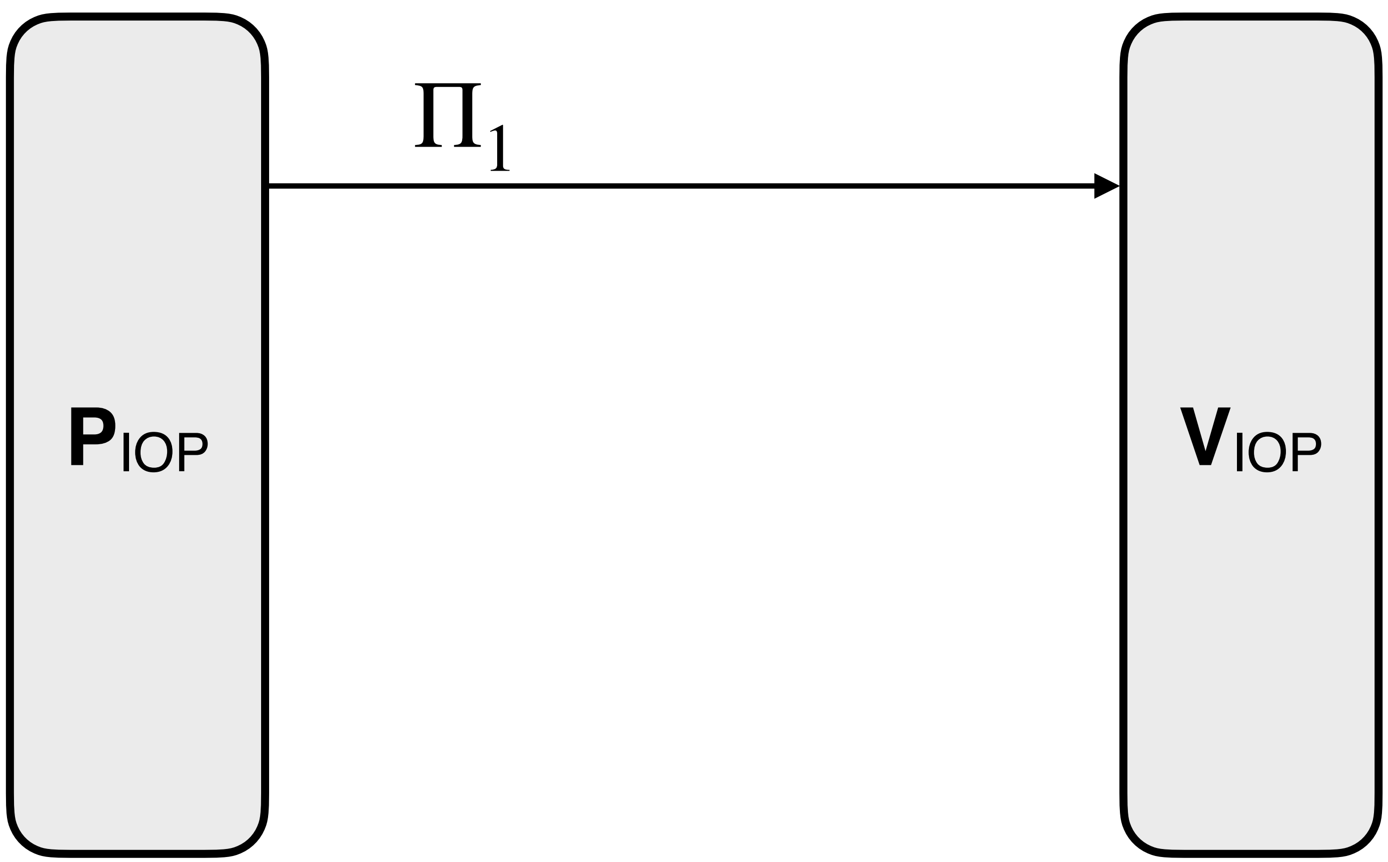

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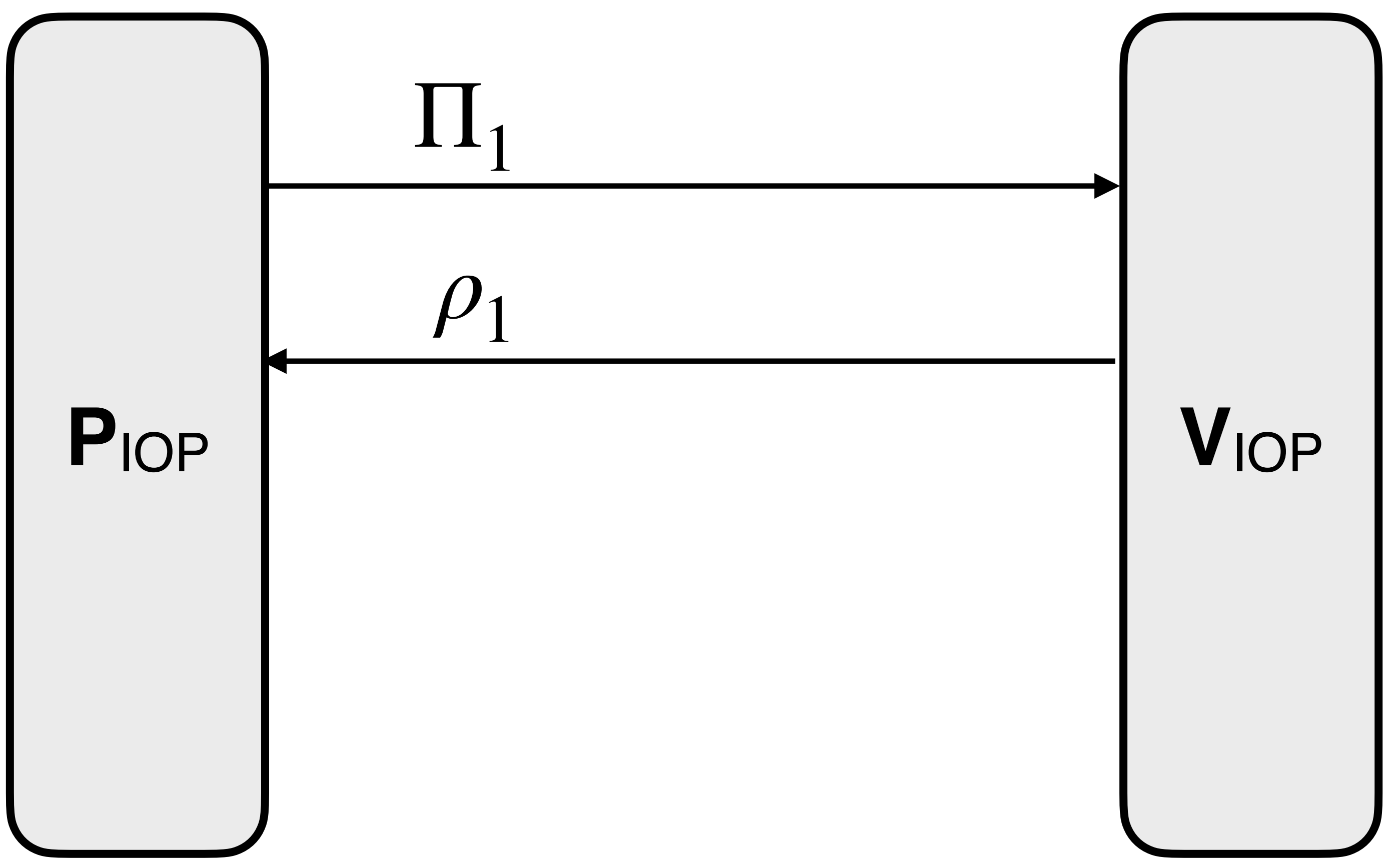

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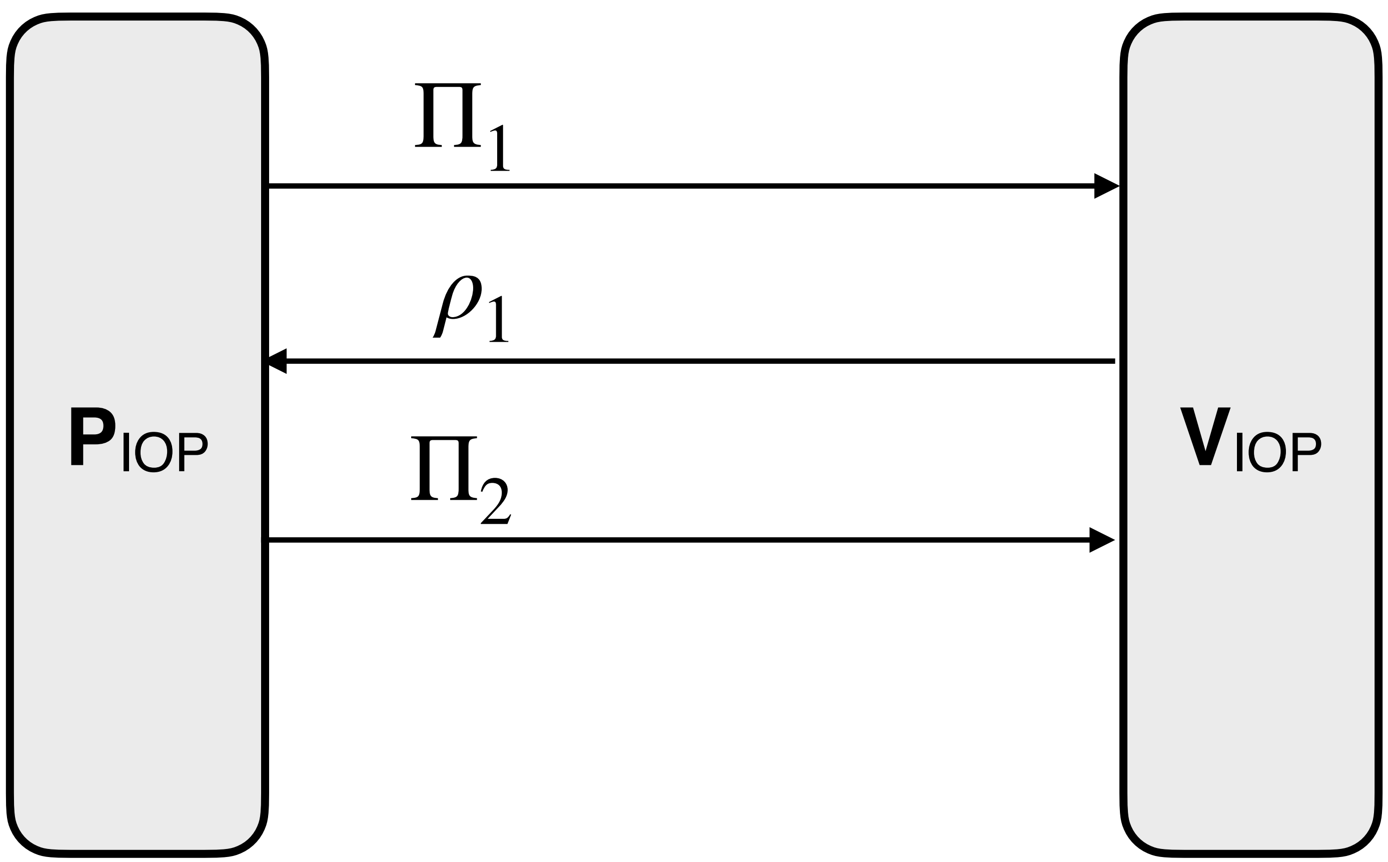

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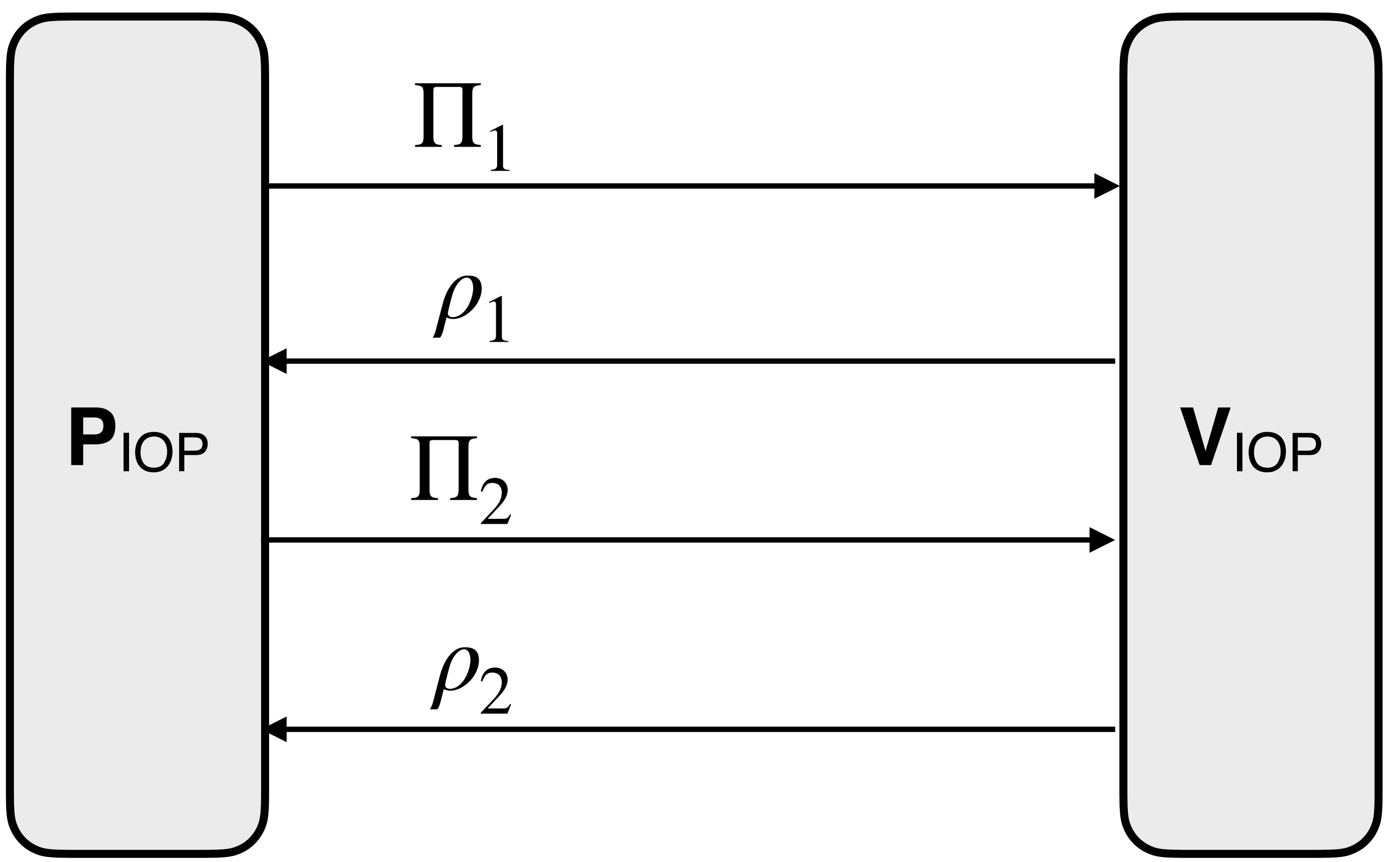

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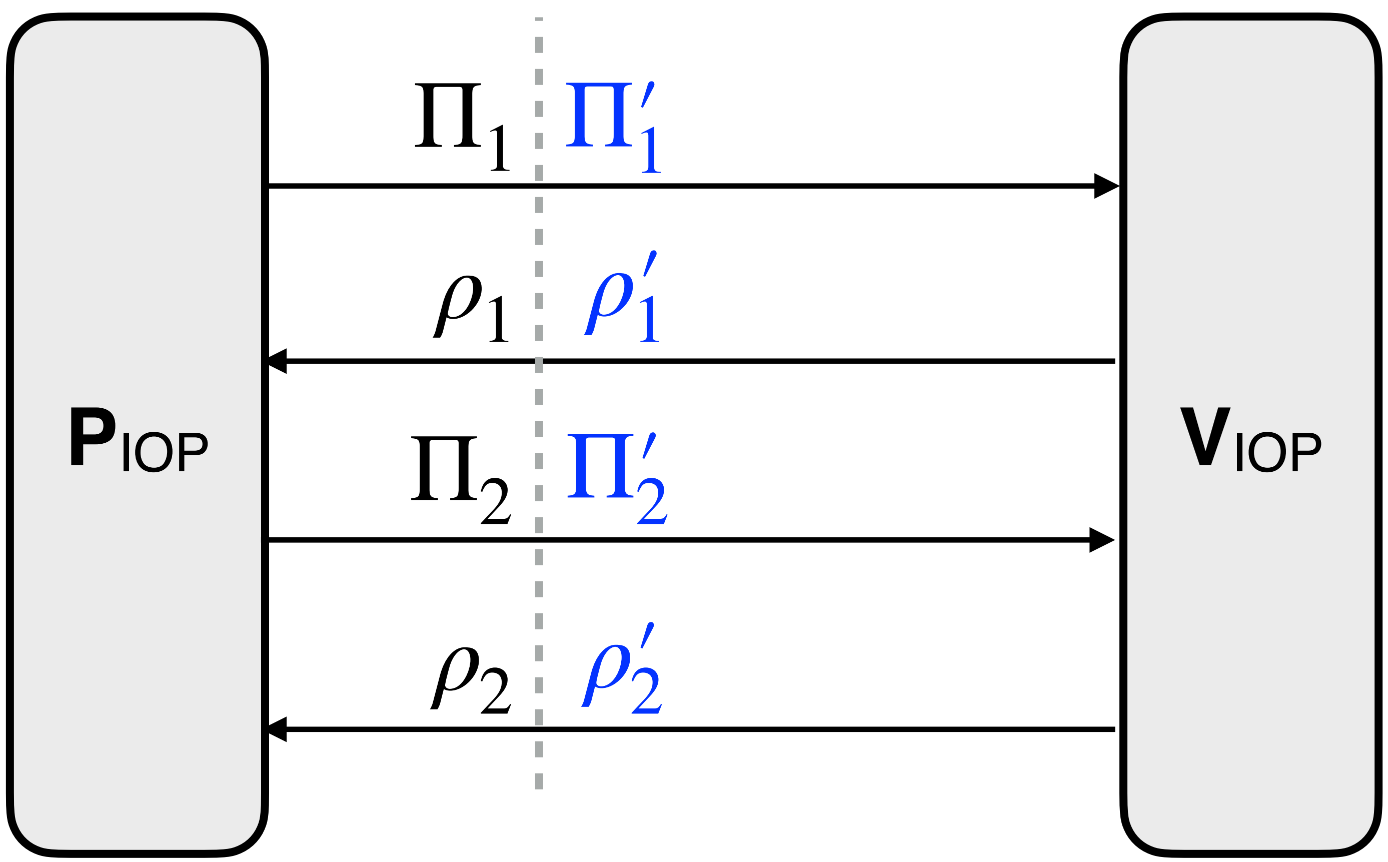

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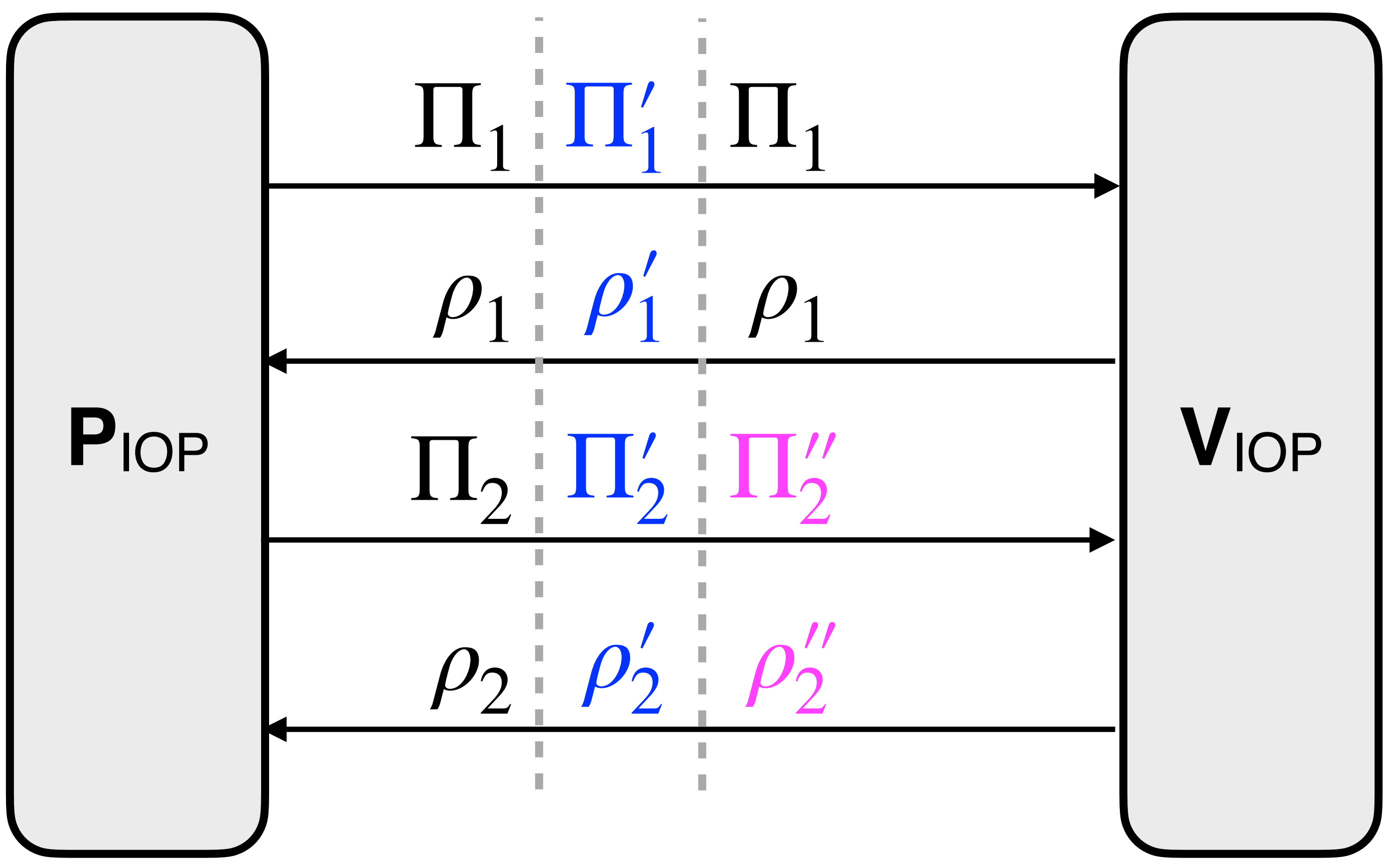

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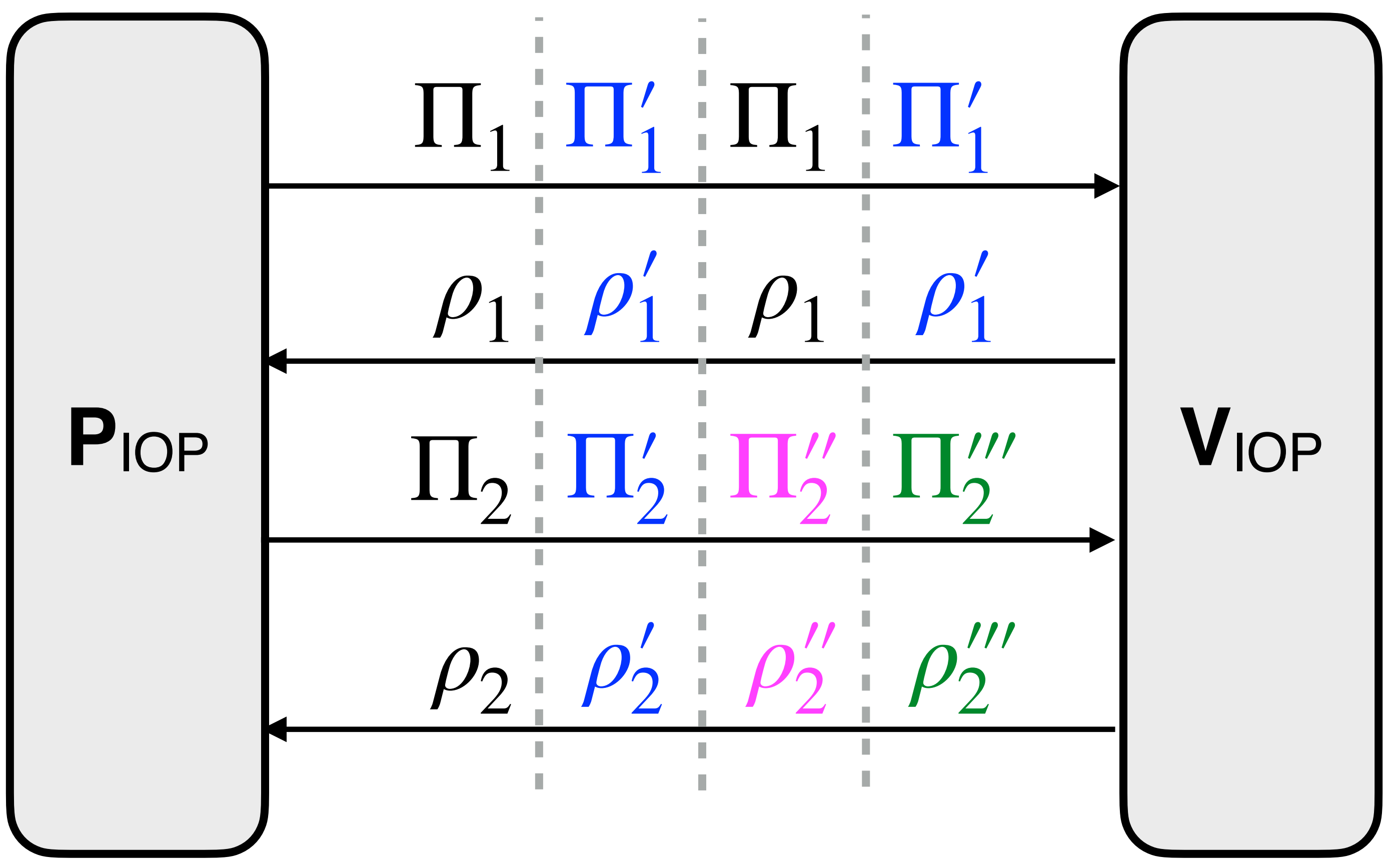

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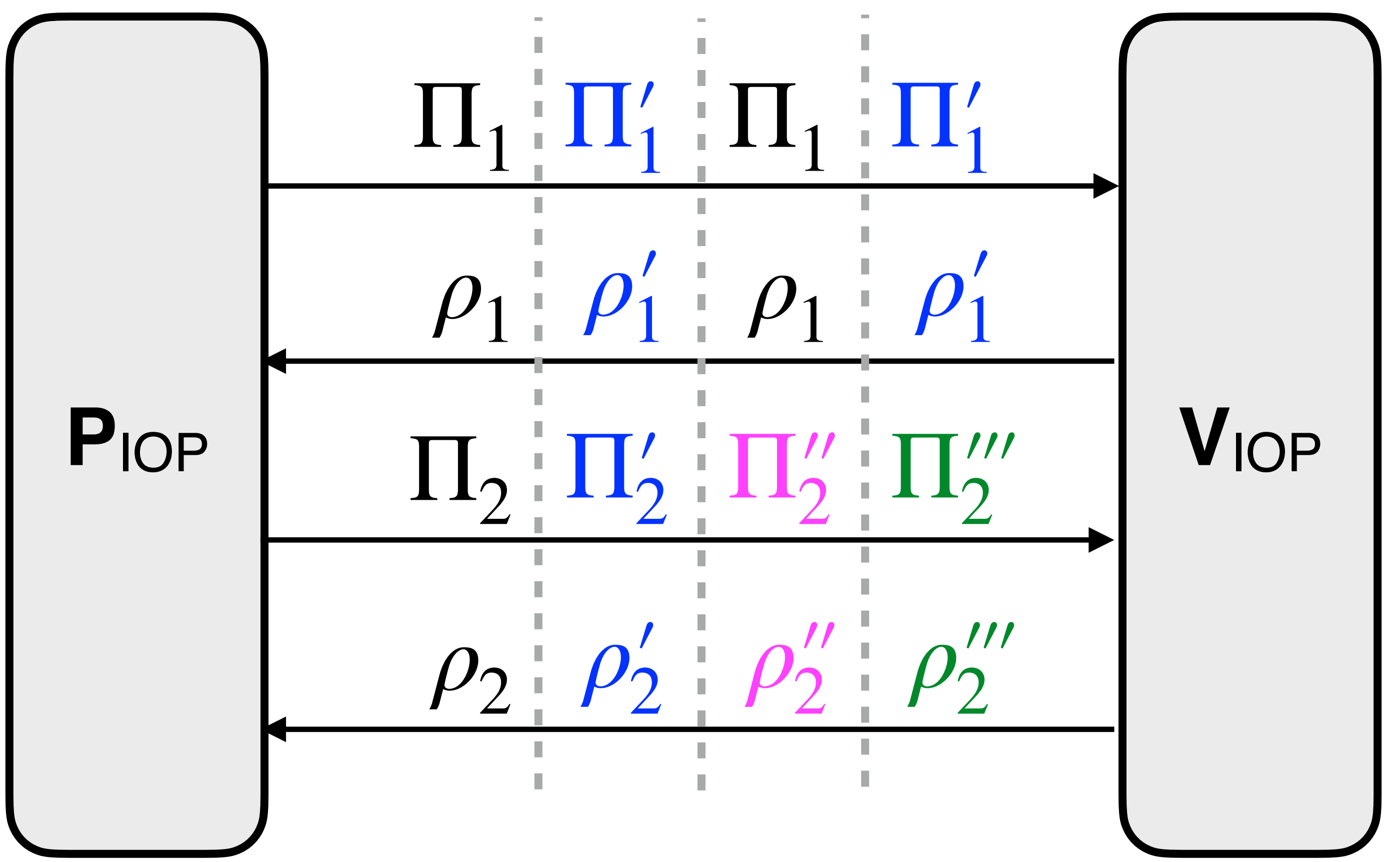

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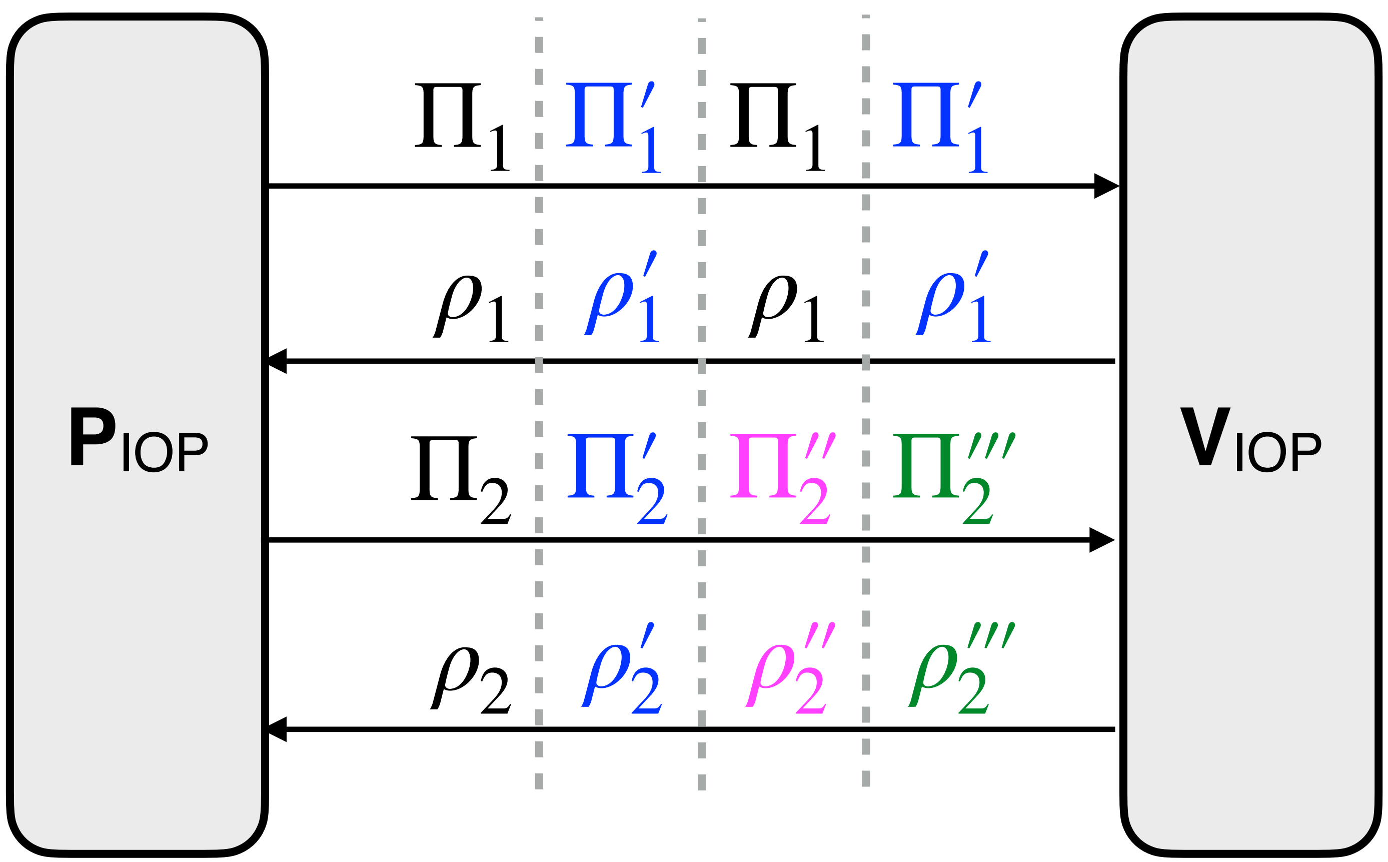

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This is known as a **state-restoration attack**.

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Remark: a PCP has state-restoration soundness error $t \cdot \epsilon_{\mathrm{PCP}}$, explaining the error term that appears for the Micali construction.

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(Similar implications hold for knowledge soundness.)

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What about **Non-Malleability**?

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Theorem: Micali and BCS emulate $\mathscr{F}_{\text{ARG}}$ in the $\mathscr{F}_{\text{GROM}}$-hybrid model

zkSNARKs in the ROM with Unconditional UC-Security

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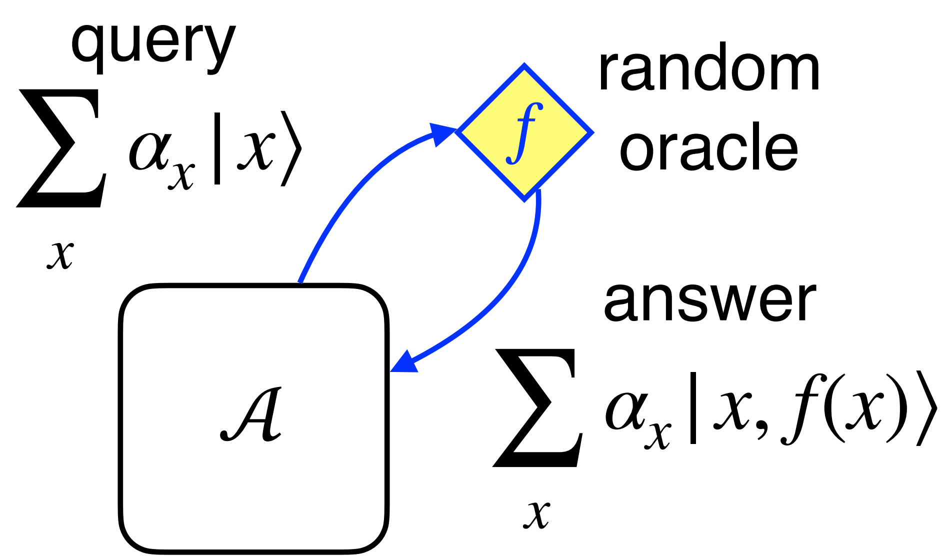

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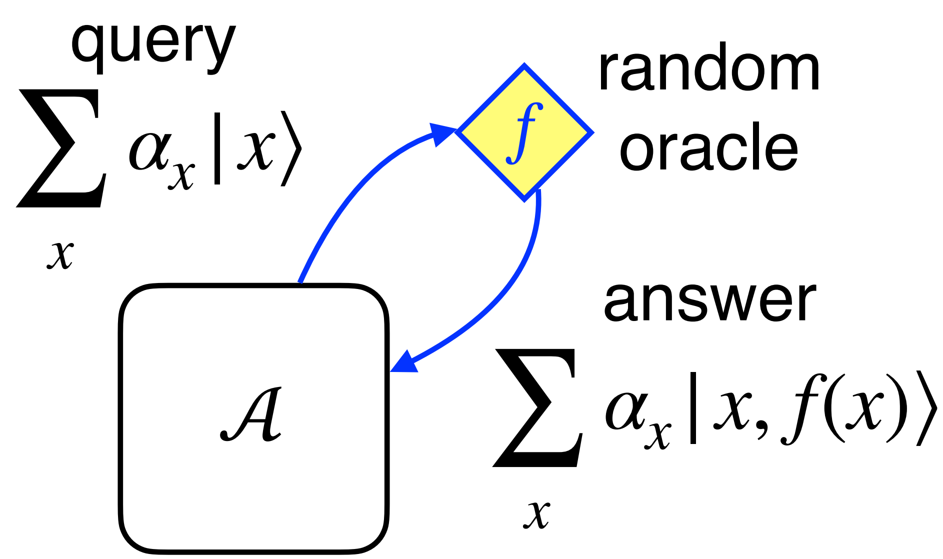

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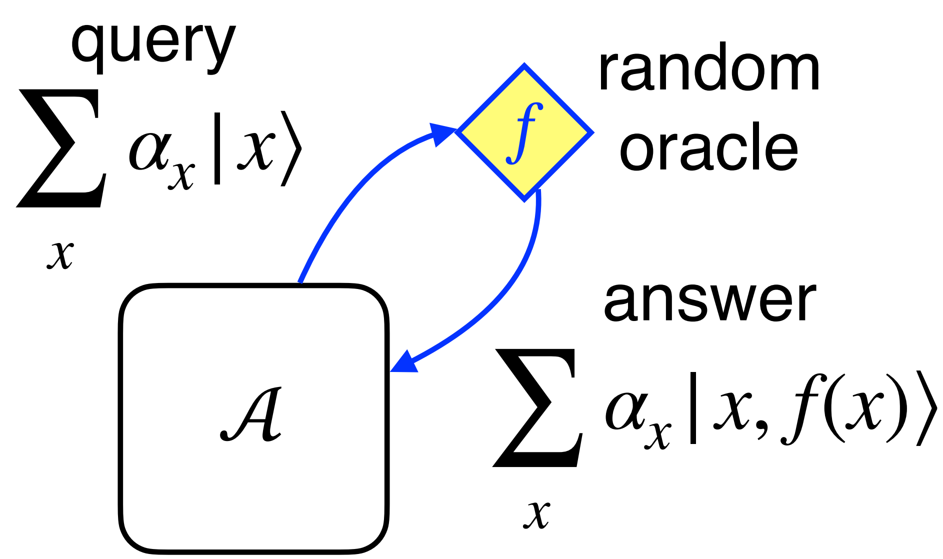

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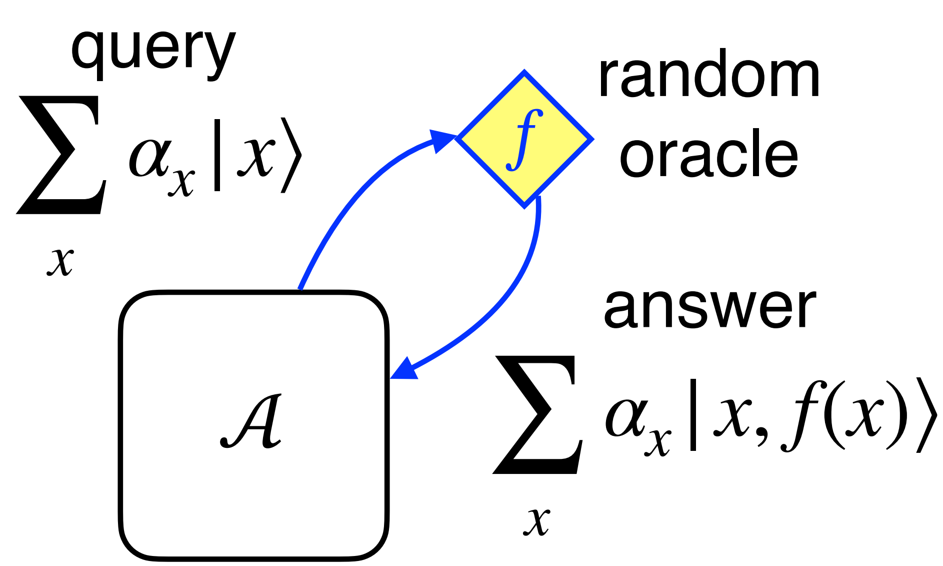

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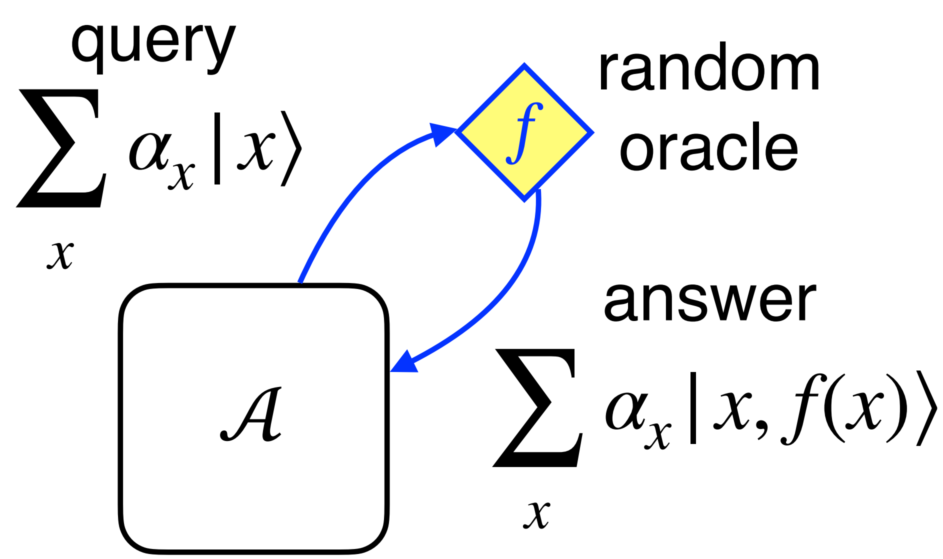

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∃ SNARG that is secure in ROM but not QROM.

Nevertheless, SNARGs of interest **are** secure in the QROM:

(1) ∀ t-query quantum $\mathcal{A}$, Pr[$\mathcal{A}$ breaks Micali] $\leq t^2 \cdot \epsilon_{\mathrm{PCP}} + \frac{t^3}{2^\lambda}$

(2) ∀ t-query quantum $\mathcal{A}$, Pr[$\mathcal{A}$ breaks BCS] $\leq (t+k)^2 \cdot \epsilon_{\mathrm{IOP}}^{\mathrm{RBR}} + \frac{t^3}{2^\lambda}$

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IOPs are inherent: [CY20] shows a transformation **T** such that

succinct argument in the ROM $\longrightarrow \boxed{\mathbf{T}} \longrightarrow$ IOP

Want to Learn More?

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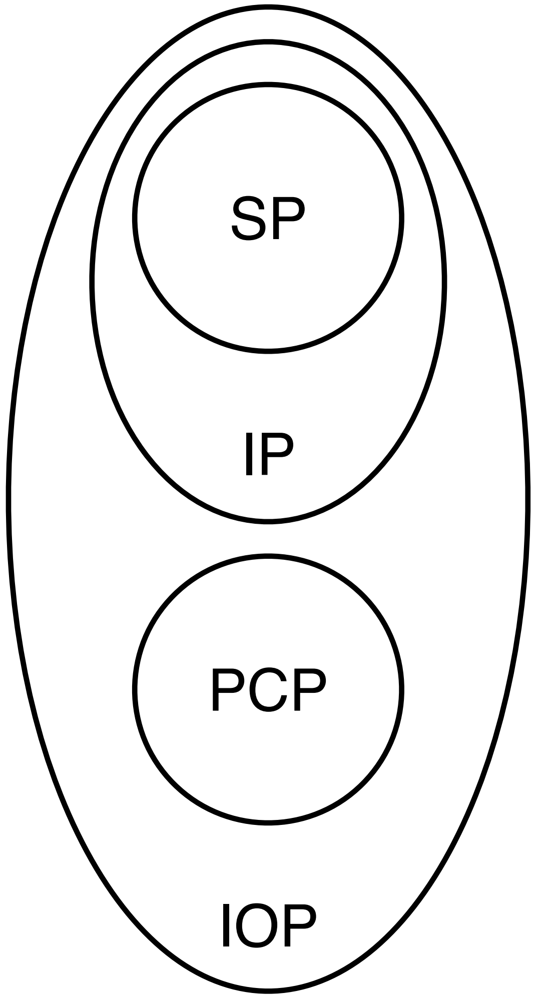

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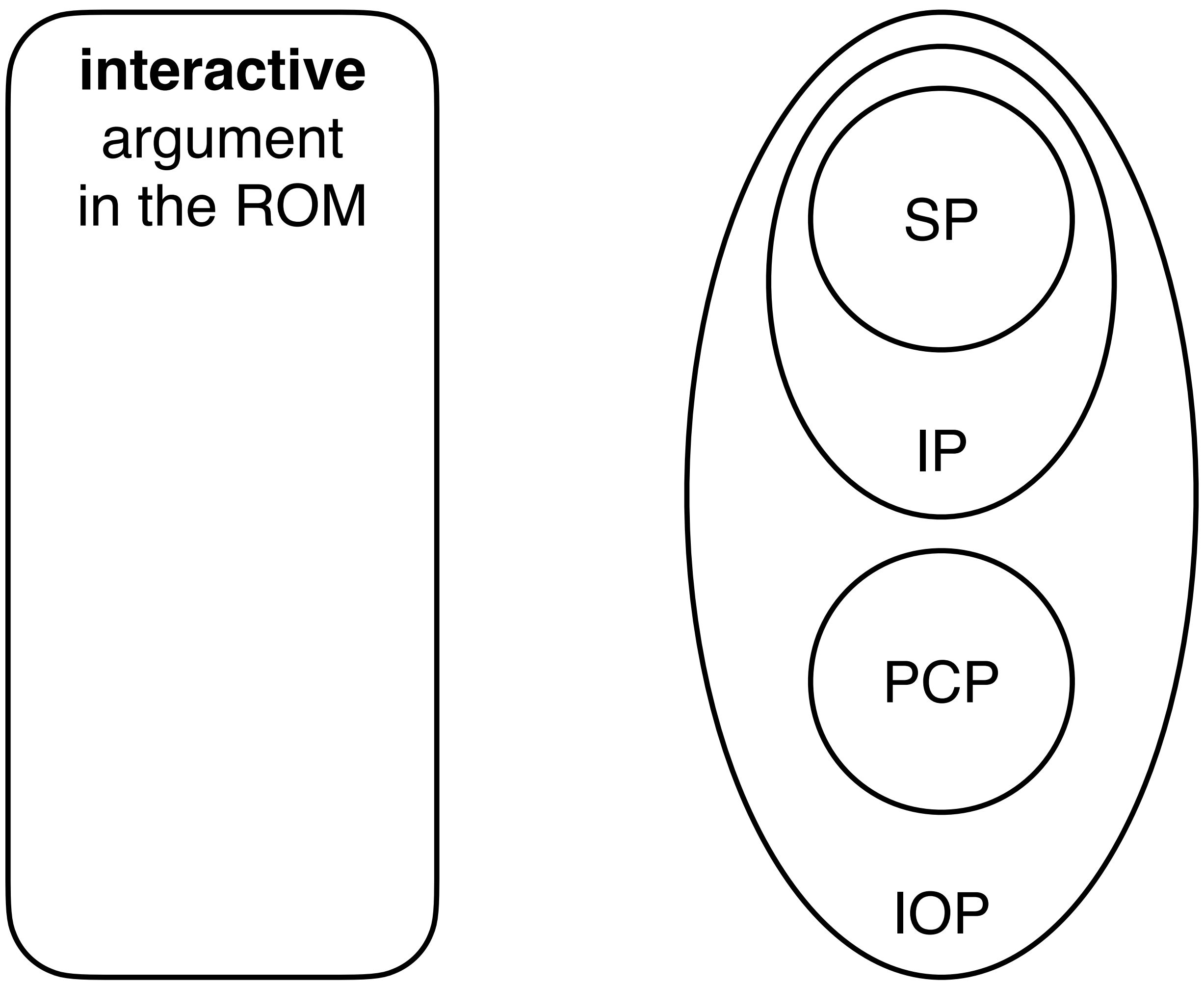

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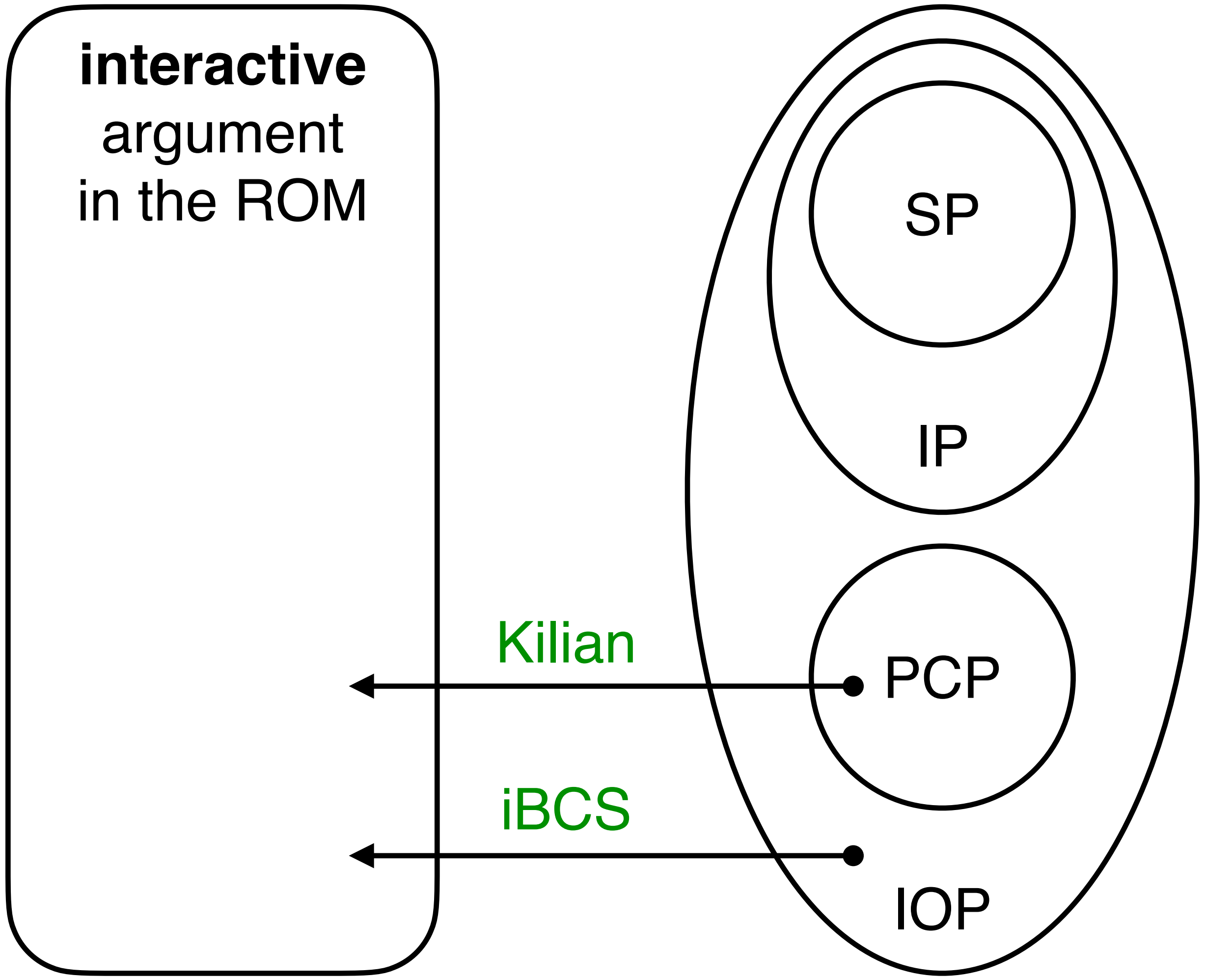

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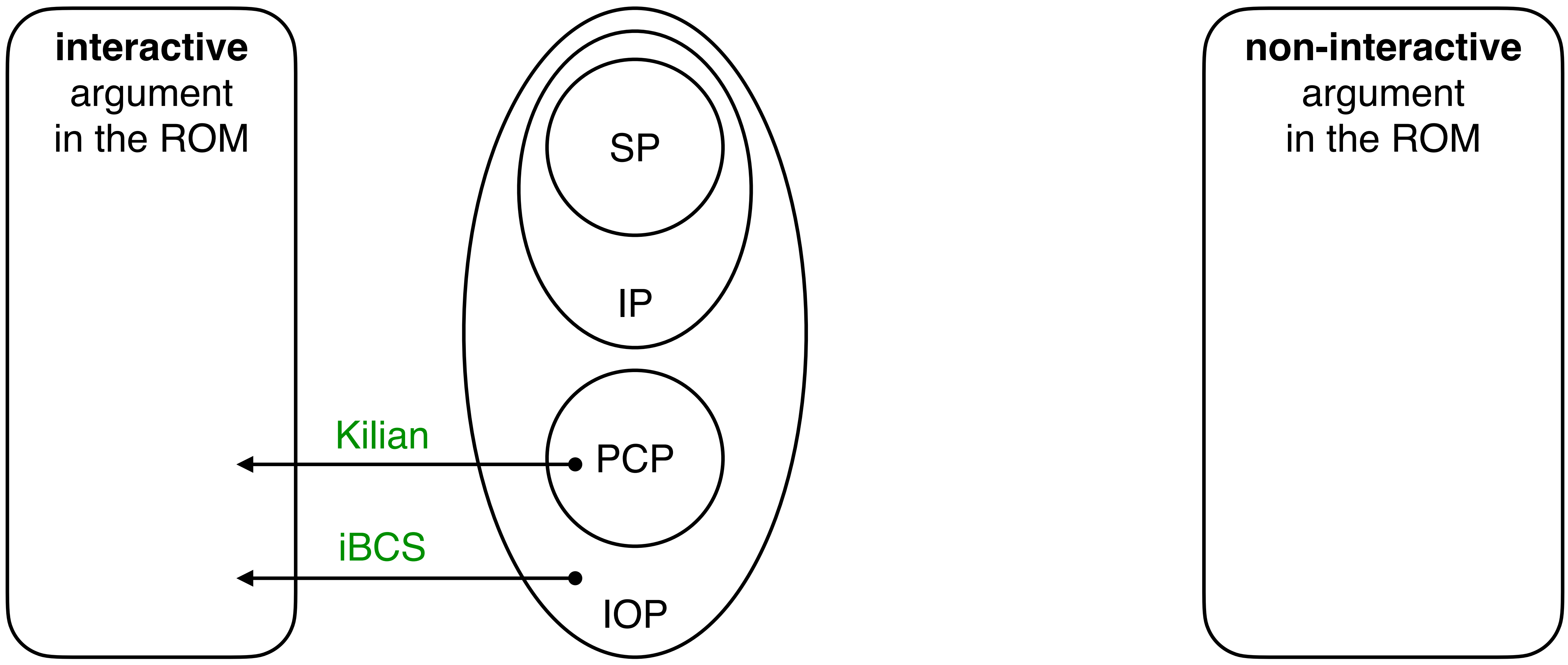

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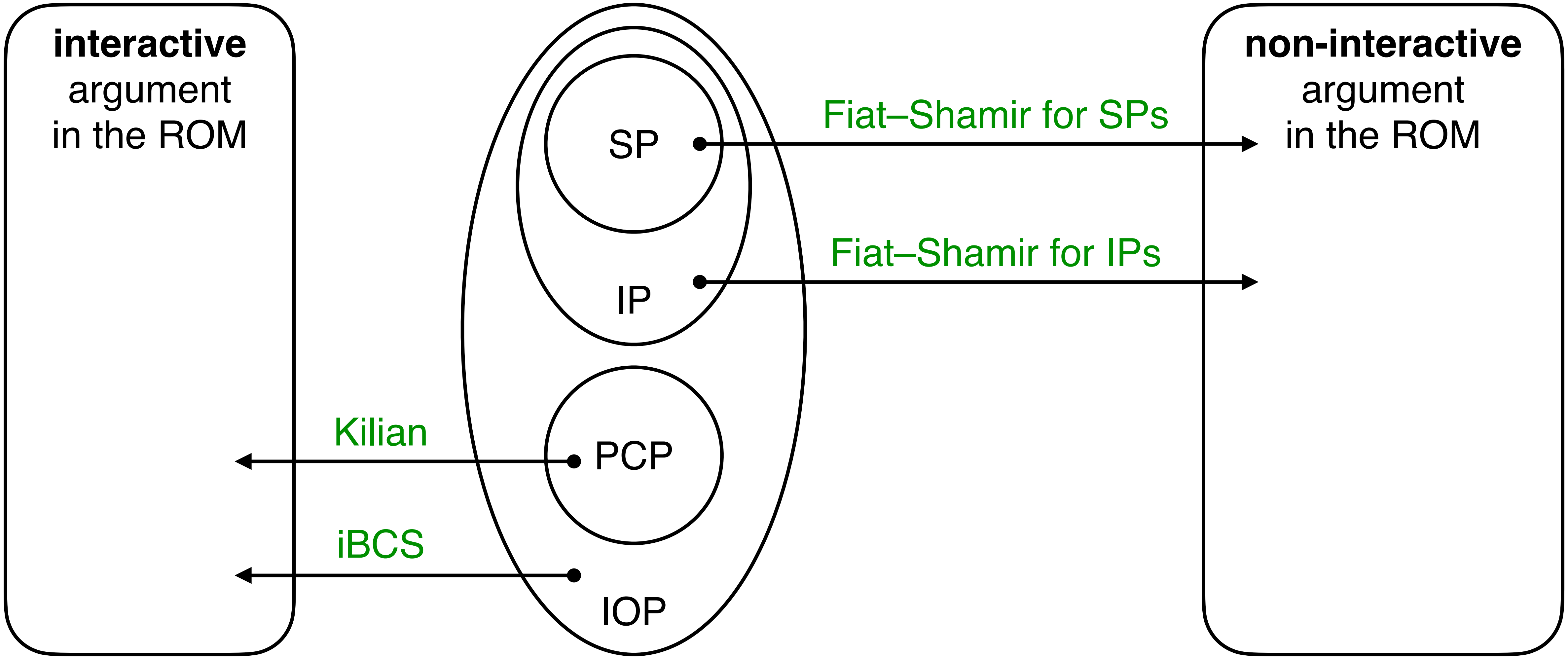

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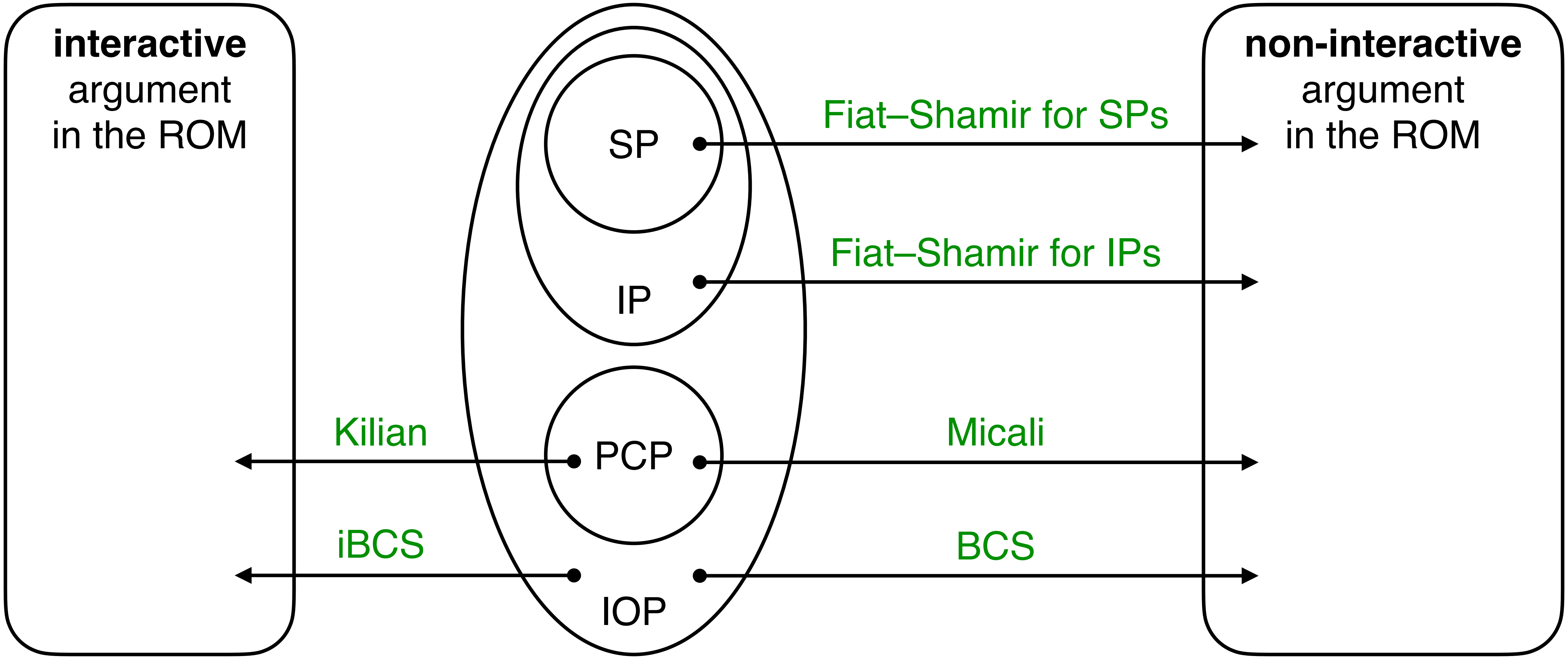

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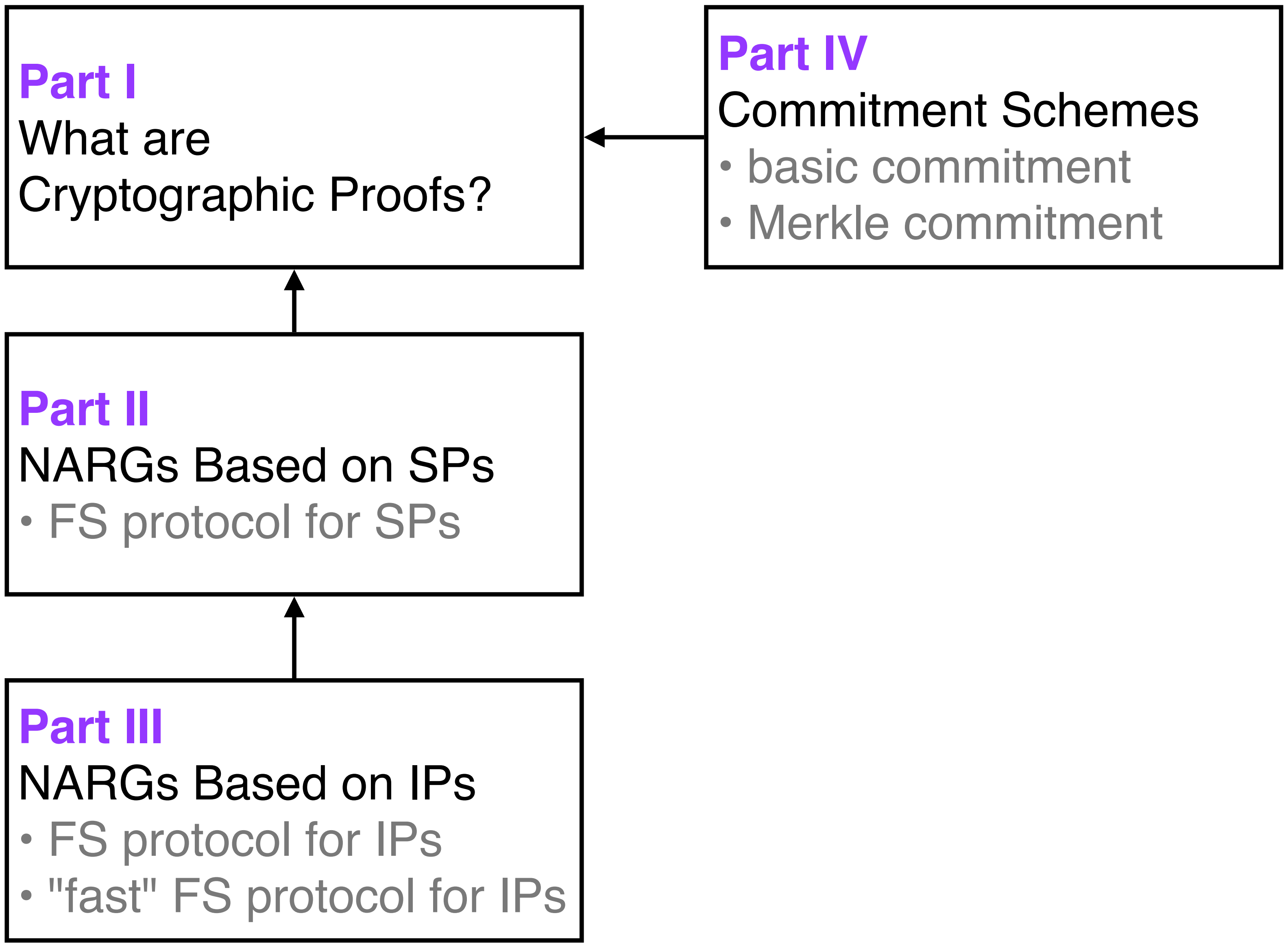

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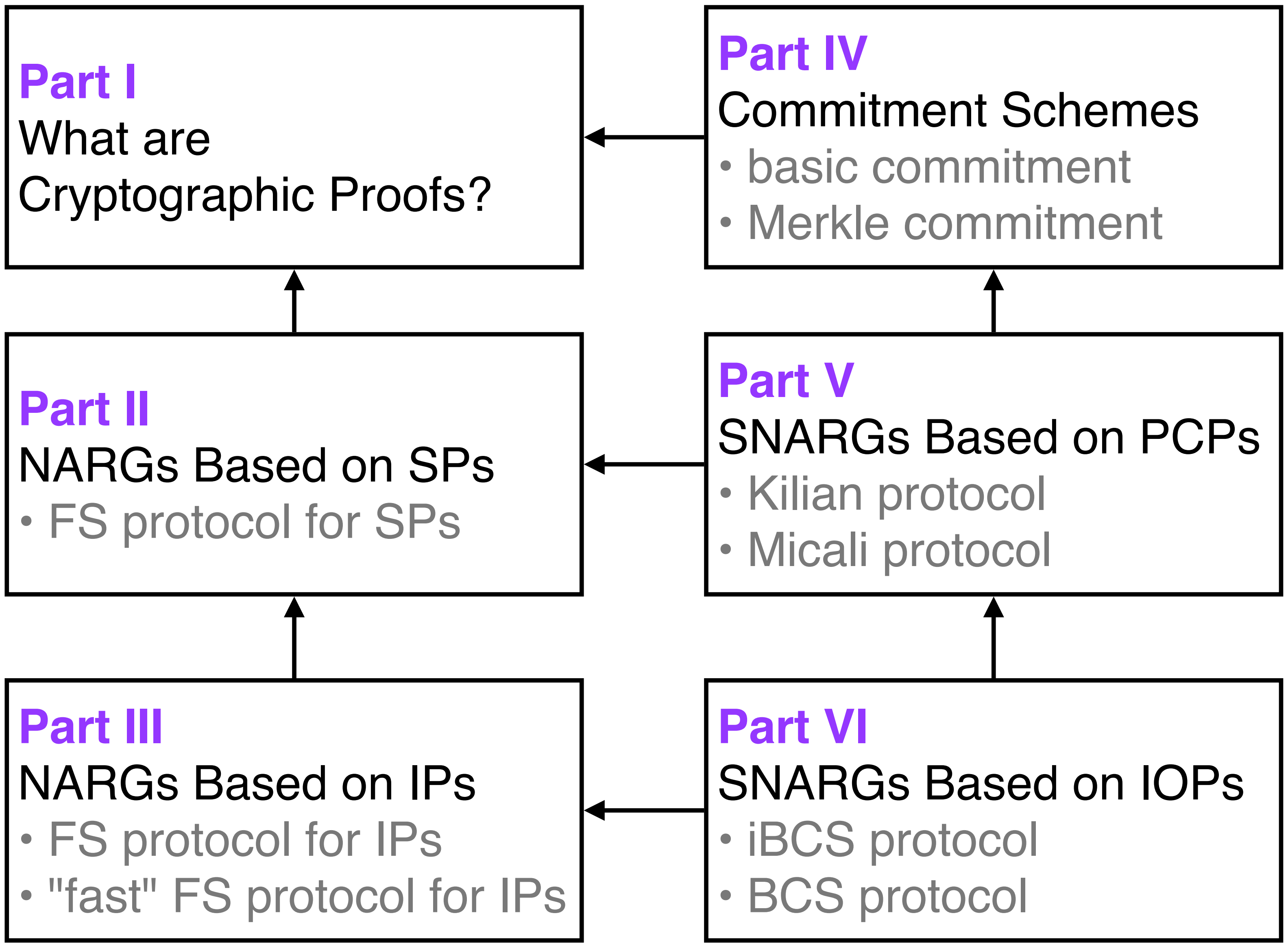

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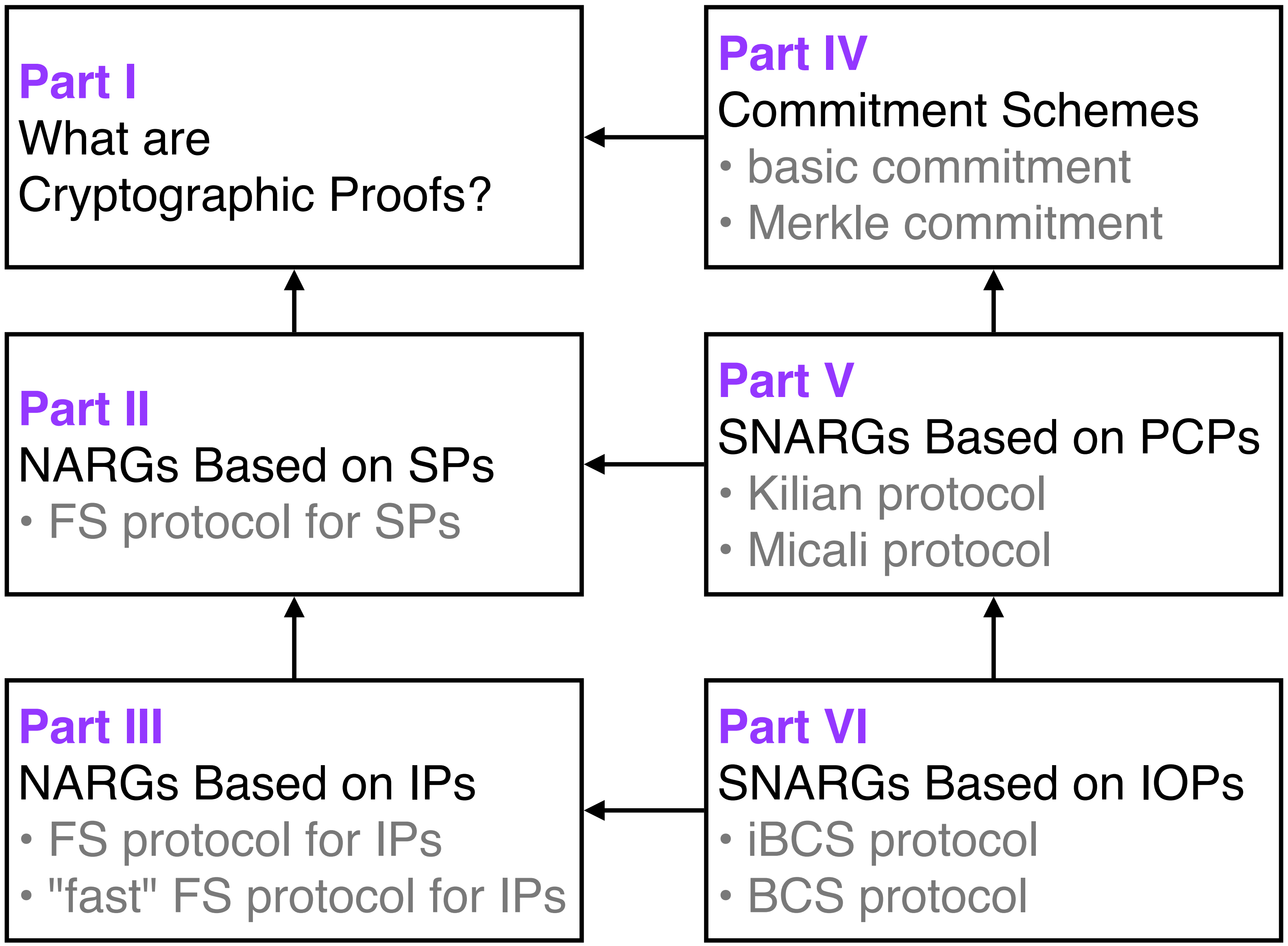

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Thanks!