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Zinc

Succinct Arguments with Small Arithmetization Overheads from IOPs of Proximity to the Integers

Albert Garreta, Hendrik Waldner, Katerina Hristova, Luca Dall'ava

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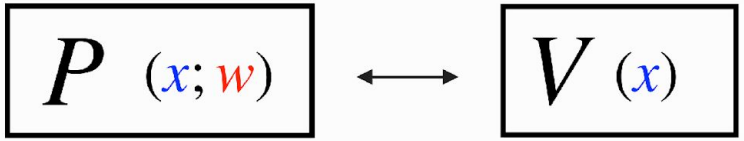

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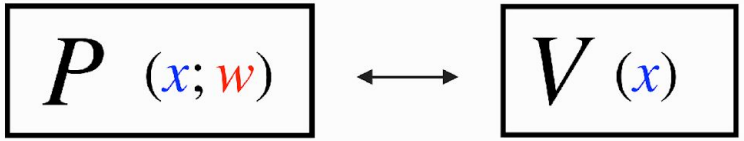

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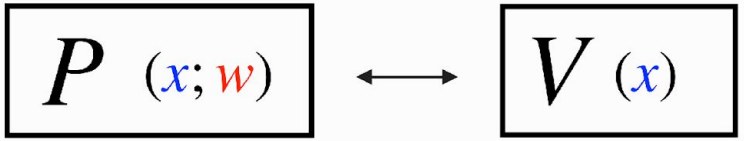

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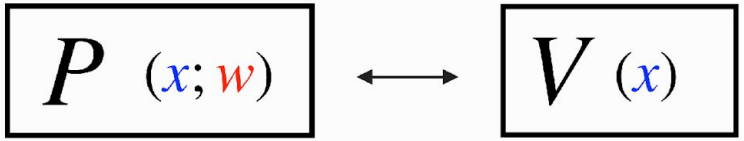

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   Then P can use Π to prove it knows a witness for the transformed instance.

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We believe that reducing this overhead is key to improving SNARK efficiency.

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Our ideas can be extended to other rings (WIP)

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$Az \circ Bz = Cz + u \circ d$ ← $d \in \mathbb{Z}^{m+k+1}$ different moduli per constraint

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$$R_{\text{R1CS}\ell,\mathbb{Z},B} = \left\{ (x; w, u) \;\middle|\; \begin{array}{l} x \in \mathbb{Z}_B^m,\ w \in \mathbb{Z}_B^k,\ u \in \mathbb{Z}_B^{m+k+1} \\ \cancel{Az \circ Bz = Cz + u \circ n} \ \text{ over } \mathbb{Z}, \quad z = (w, x, 1) \end{array} \right\}$$

$$Az \circ Bz = Cz + u \circ d \quad \leftarrow \quad d \in \mathbb{Z}^{m+k+1} \text{ different moduli per constraint}$$

$\mathbb{Z}_B$ is the set of integers with bit-size less than B.

Naïve attempt: A succinct argument for $R_{\mathsf{R1CS}\ell,\mathbb{Z},B}$

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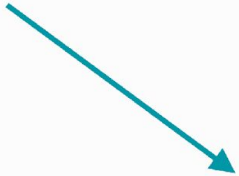

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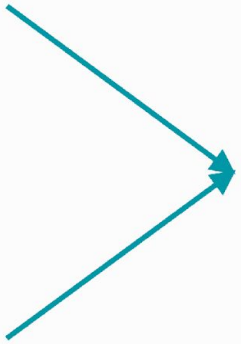

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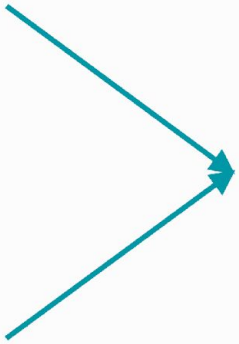

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NETHERMIND RESEARCH

Designing a succinct argument for $R_{\text{R1CS}\ell,\mathbb{Z},B}$

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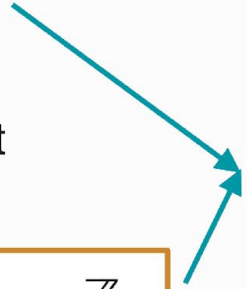

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Let's modify our naïve attempt so as to address these issues.

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[CH2024] compile mod-PIOPs with (2) into a succinct argument for $R_{\text{R1CS}\ell,\mathbb{Z},B}$

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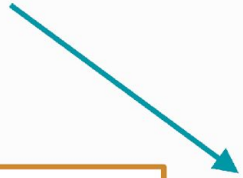

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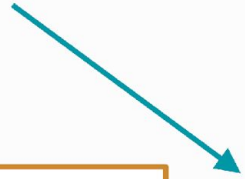

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(Coming from solving systems of equations over $\mathbb{Z}$)

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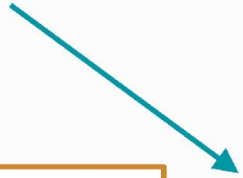

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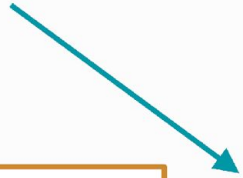

Compile into succinct argument for $R_{\text{R1CS}\ell,\mathbb{Z},B}$

We make the following key design choice:

Designing a succinct argument for $R_{\mathsf{R1CS}\ell,\mathbb{Z},B}$

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We work over $\mathbb{Q}_B$ instead of $\mathbb{Z}_B$.

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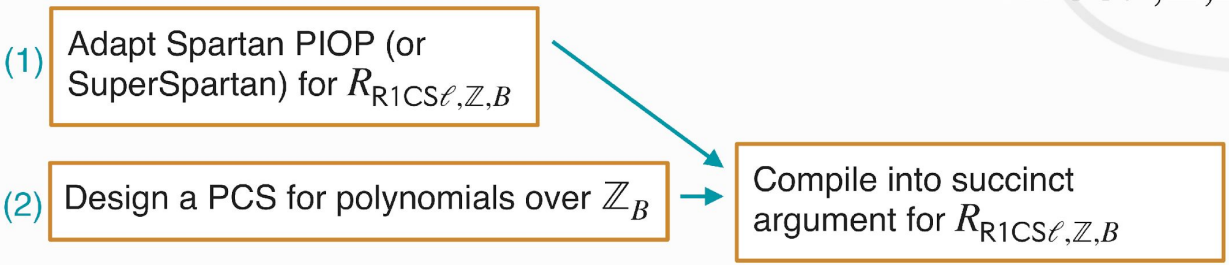

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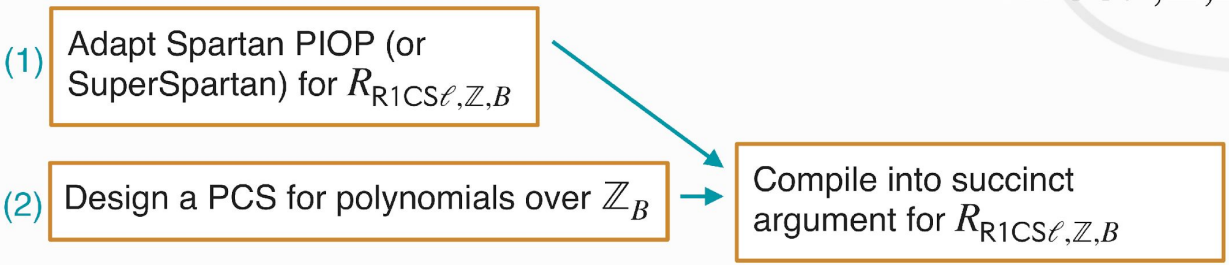

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Add a lookup argument $\mathbb{Q}_B$ to ensure witnesses are in $\mathbb{Z}_B$

The mod-PIOP technique over the rationals

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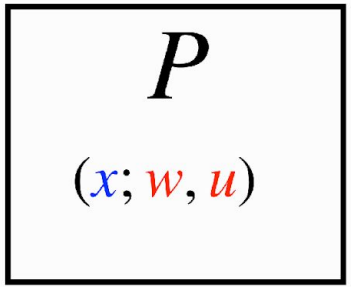

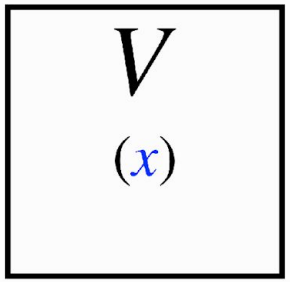

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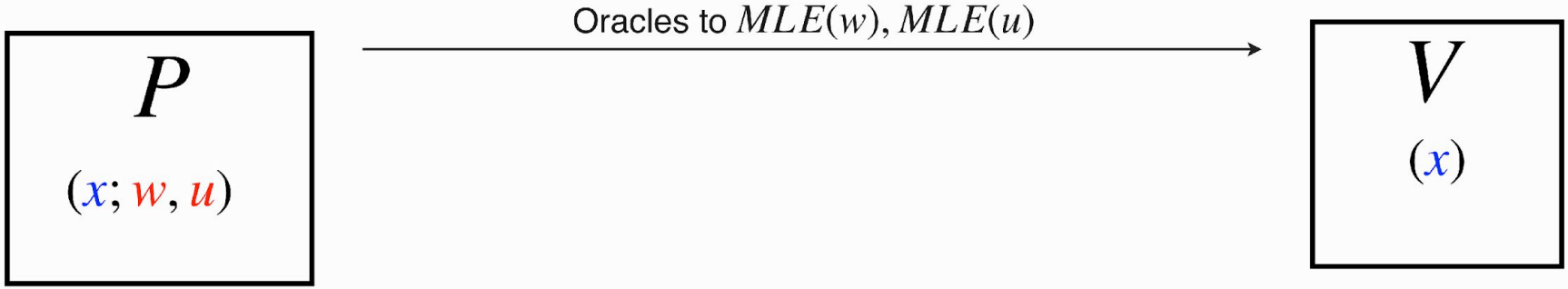

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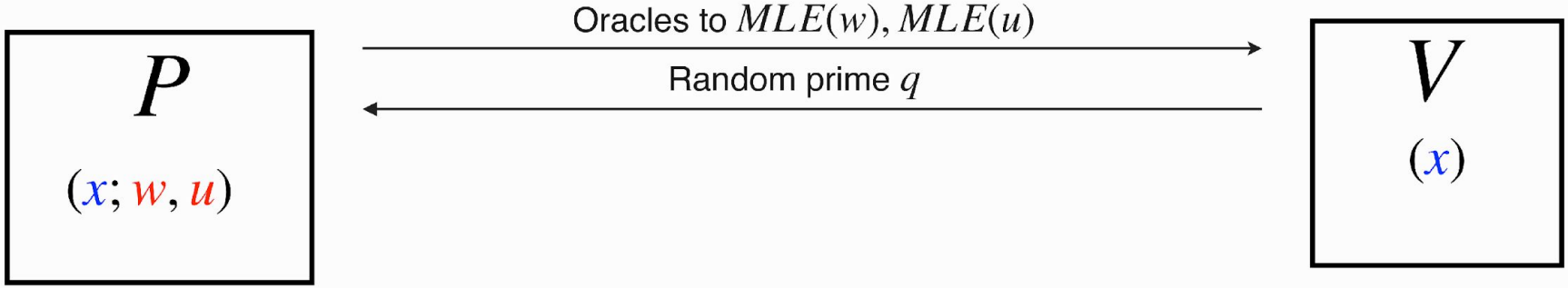

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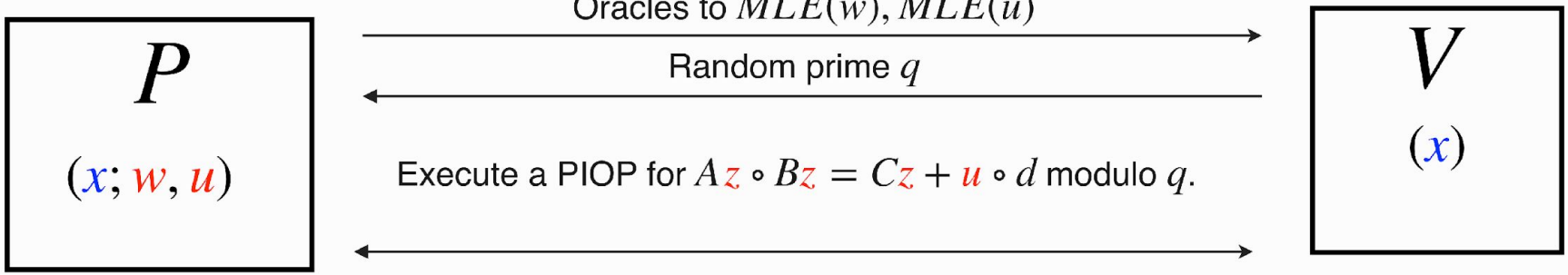

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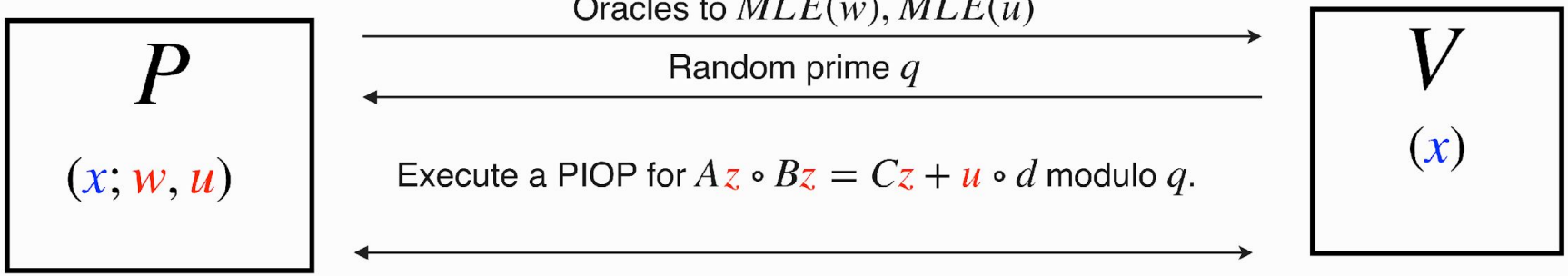

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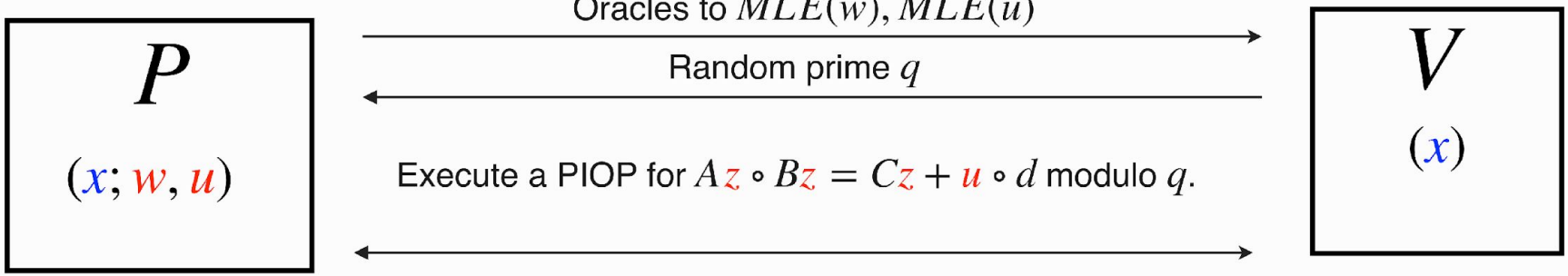

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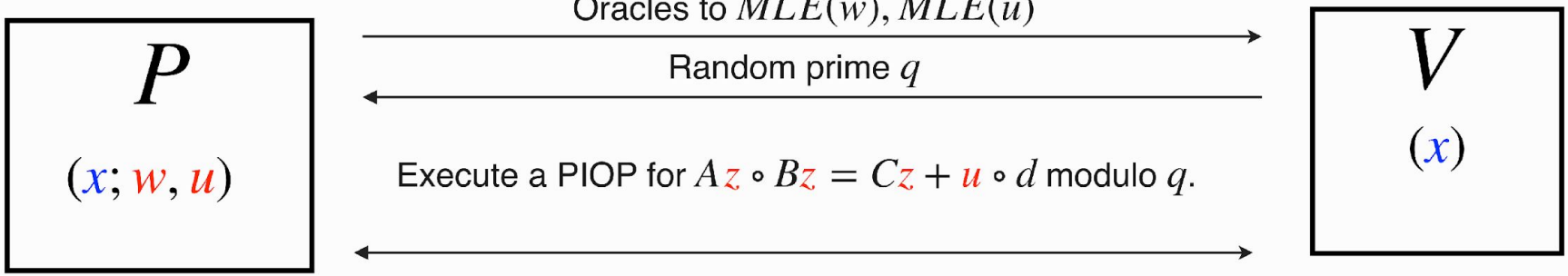

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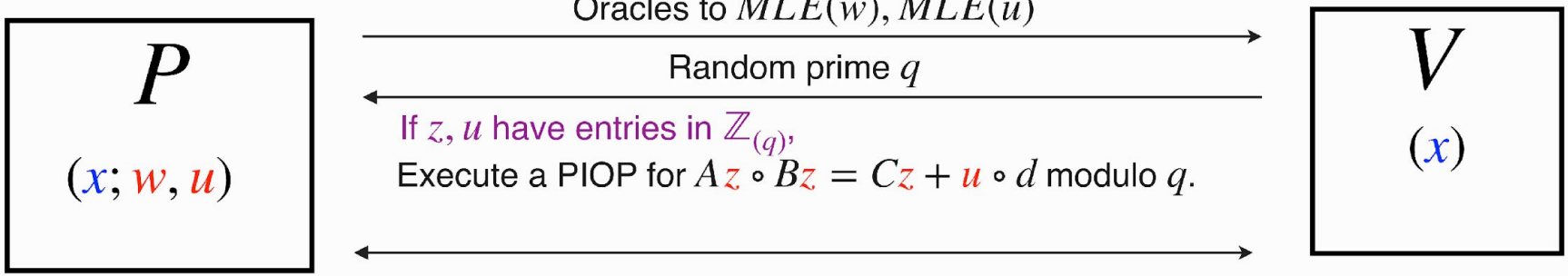

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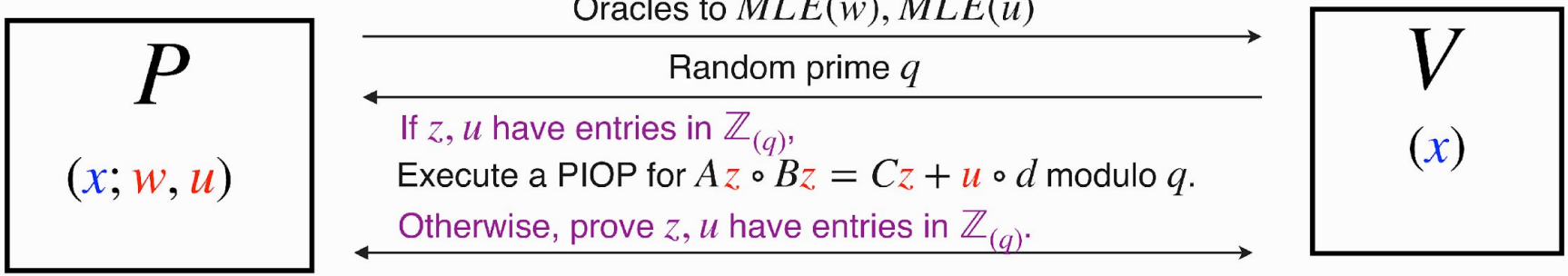

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NETHERMIND RESEARCH

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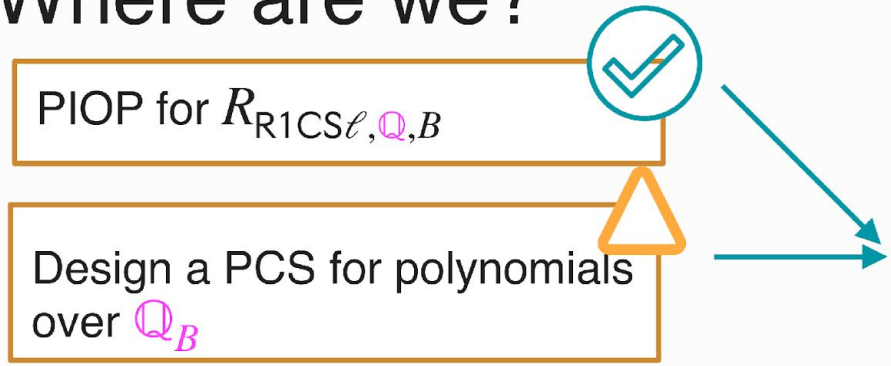

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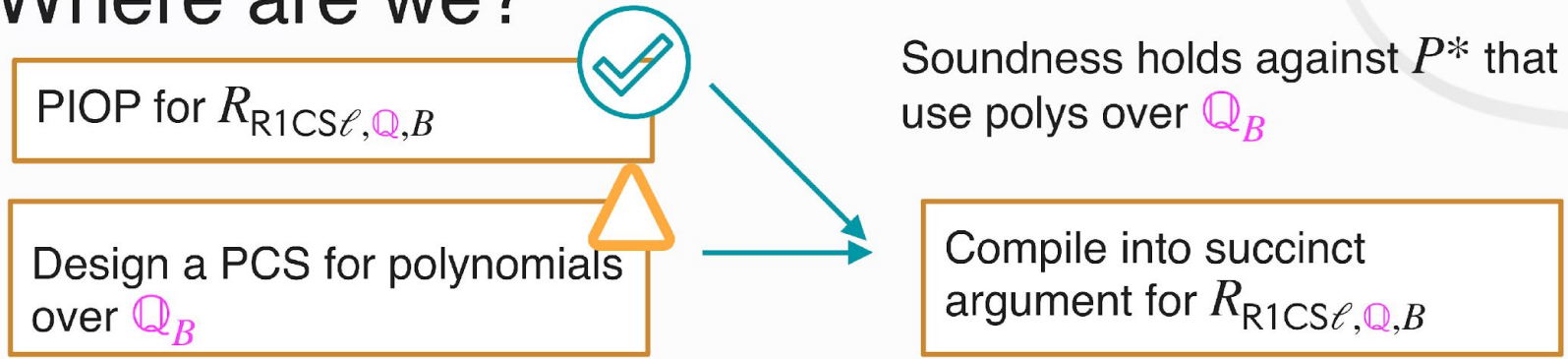

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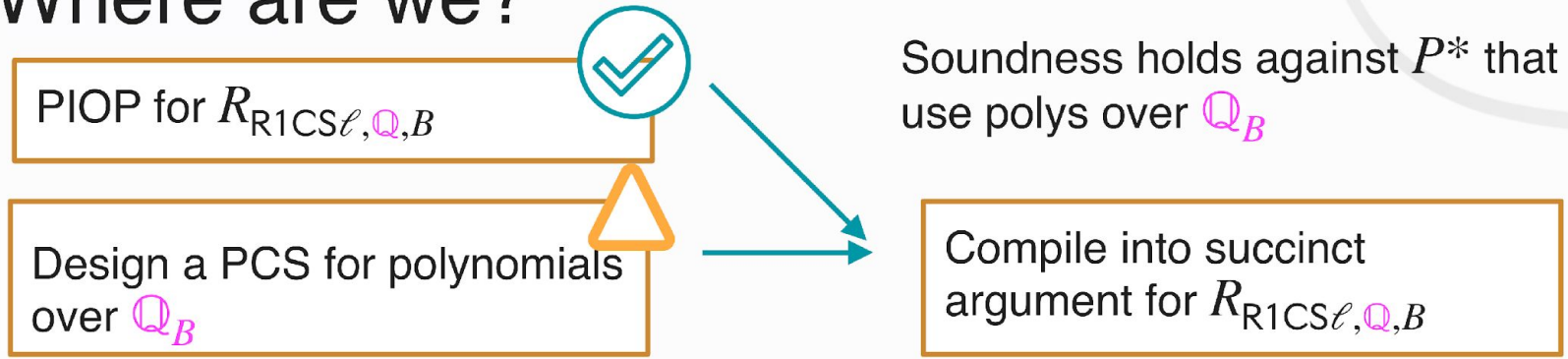

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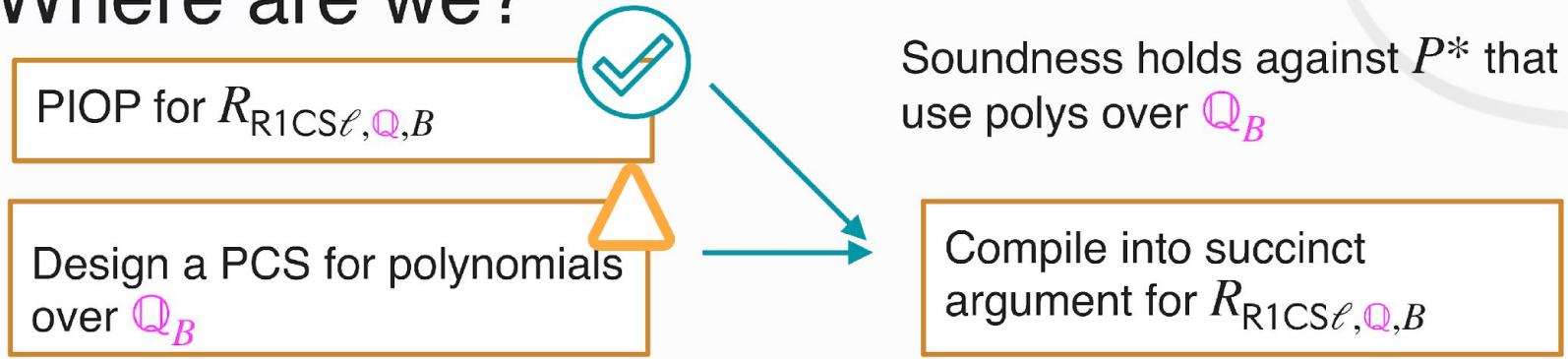

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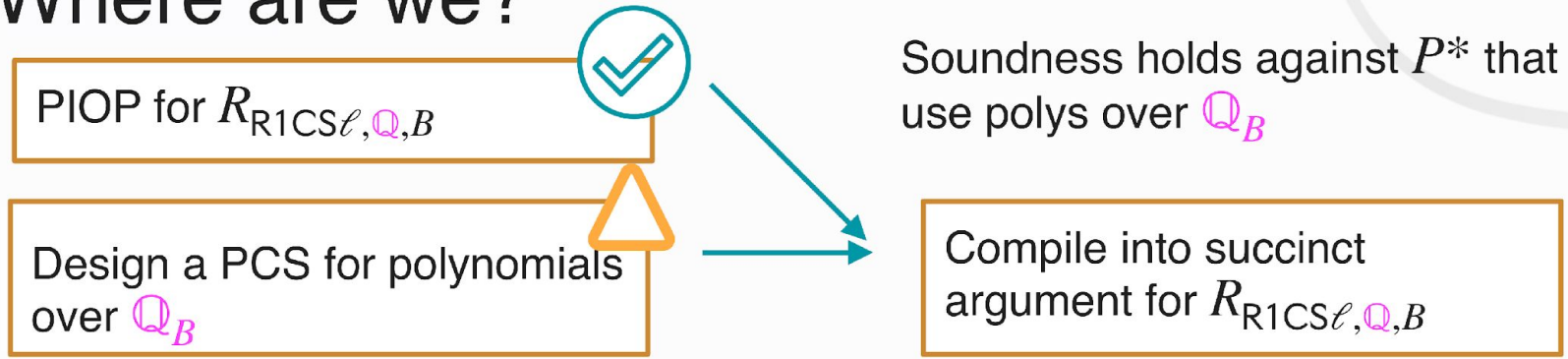

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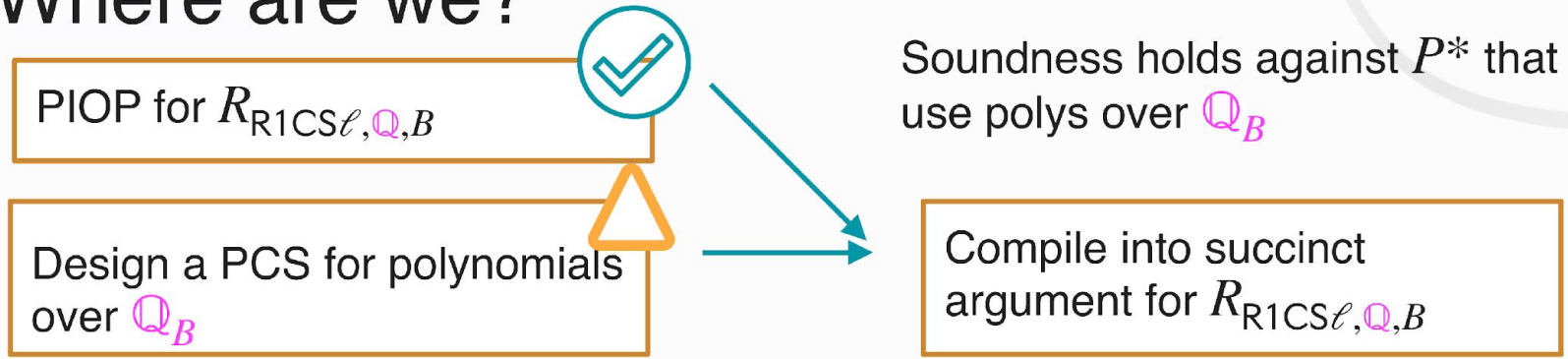

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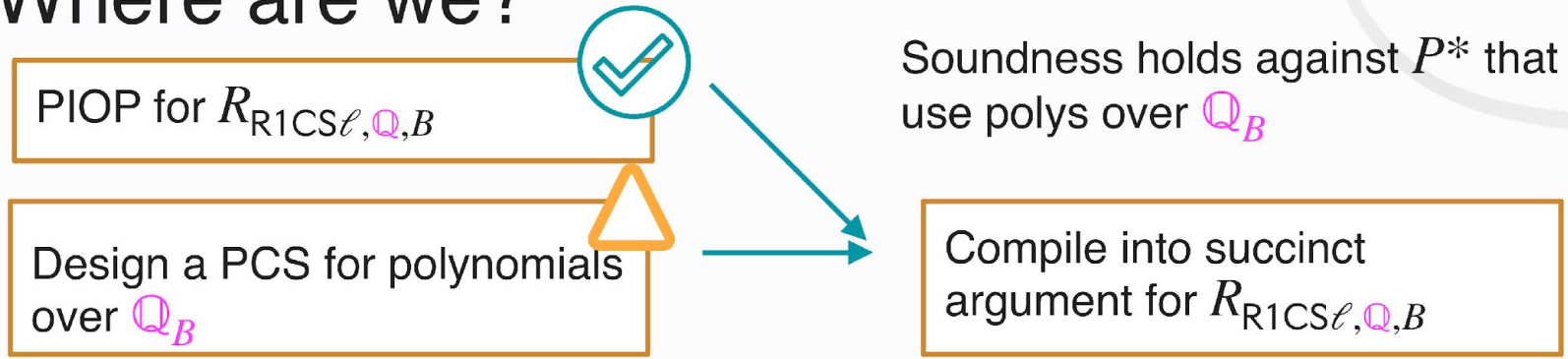

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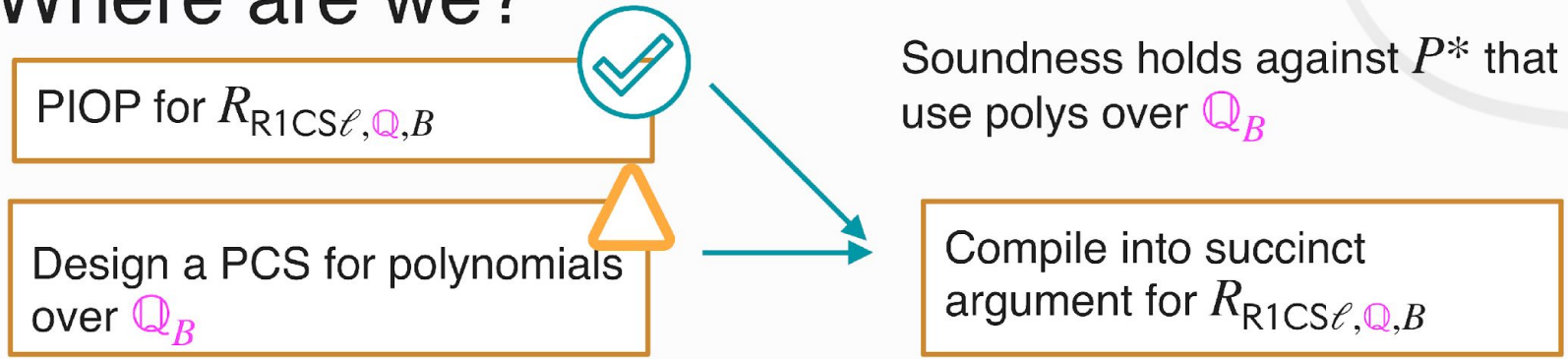

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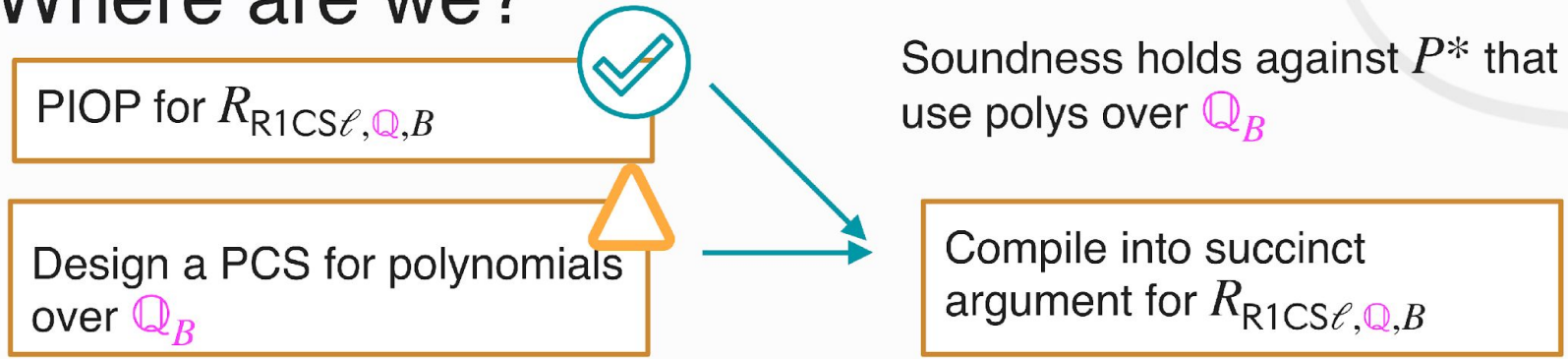

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- IOP of proximity to a linear code ← New!
- IOP of proximity to the integers

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Soundness

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Guaranteed: P committed to

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(We can extend Zip to enable completeness for $f \in \mathbb{Q}_{B'}[X_1, \ldots, X_\mu]$.)

Zip as an IOP of proximity to $\mathbb{Z}$

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Thanks