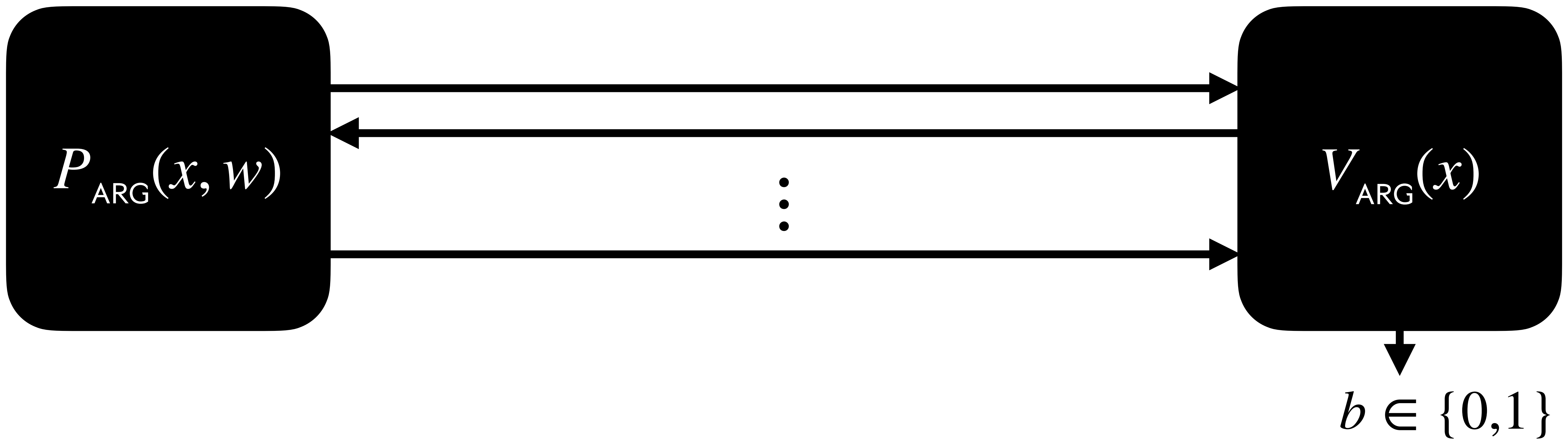
On the Fiat–Shamir Security of Succinct Arguments from Functional Commitments

Alessandro Chiesa, Ziyi Guan, Christian Knabenhans, Zihan Yu

EPFL

Succinct interactive arguments



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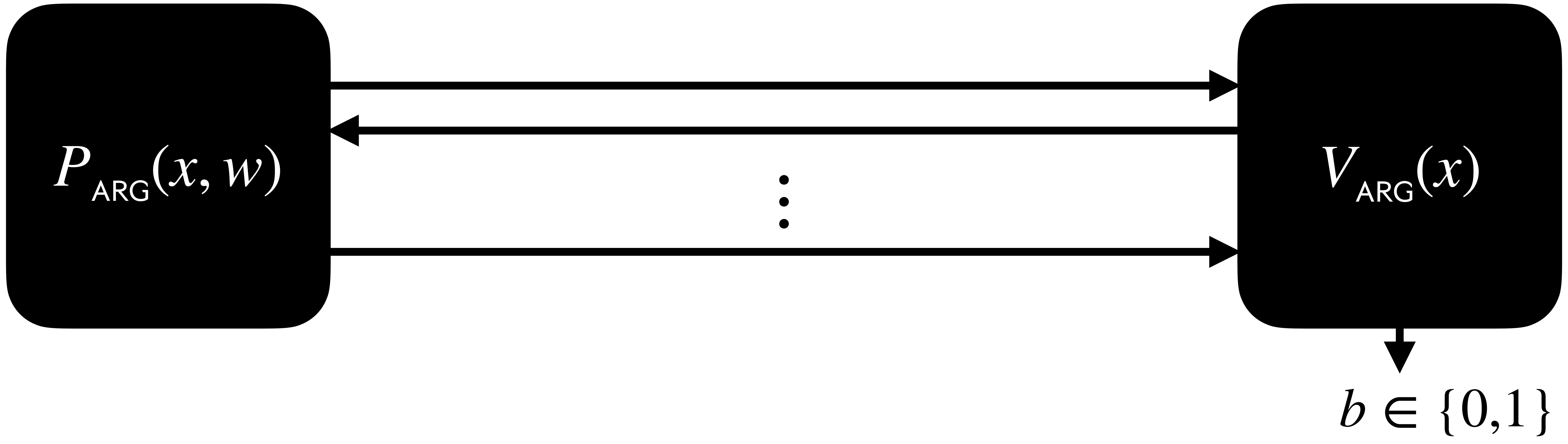
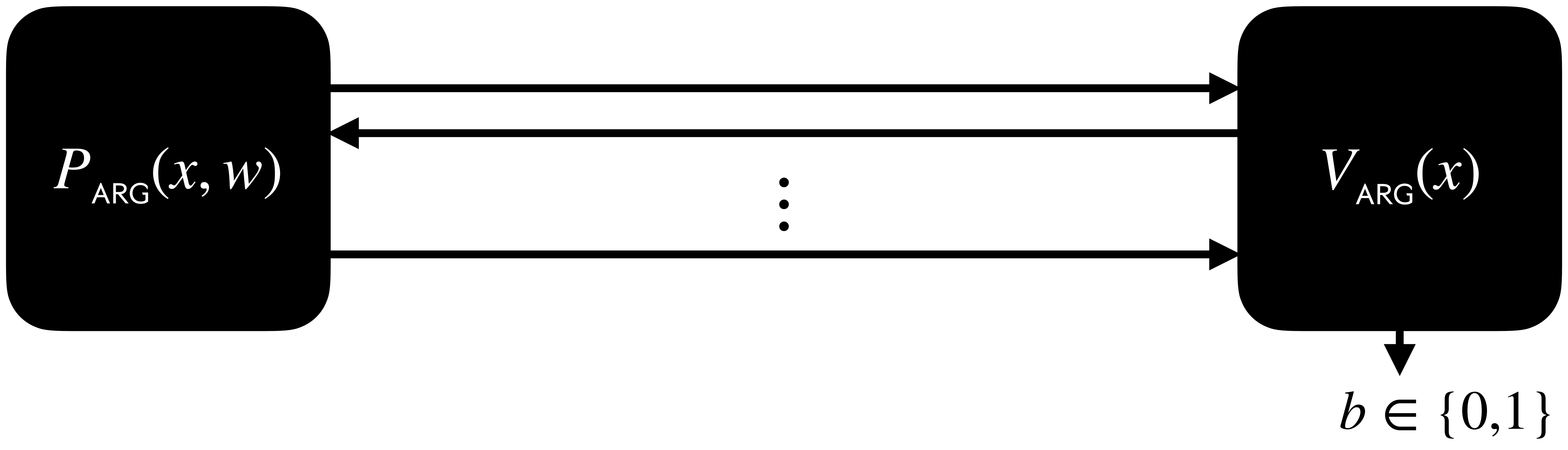


Goal: succinctness $|\text{communication}| \ll |w|$

Succinct interactive arguments



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Applications: SNARGs in the ROM (via Fiat–Shamir in the ROM),
zero-knowledge with non-black-box simulation,
malicious MPC, etc.

How to build succinct interactive arguments?

Some approaches…	Proof string	Query class	Answer

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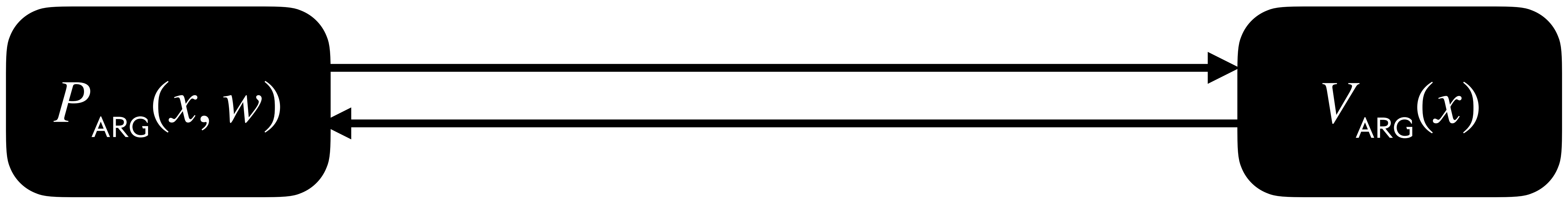
PIOP*+PC* [GWC19]	$\Pi \in (\mathbb{F}[X]^{\leq \mathrm{D}})^{m+n}$ $= (f_1, \ldots, f_m, g_1, \ldots, g_n)$	evaluation queries on structured polys $\mathbf{Q}_{\text{poly}*}$	$\beta = \sum_{k\in[n]} h_k(f_1(\alpha), \cdots, f_m(\alpha)) \cdot g_k(\alpha)$

and more: Bulletproofs (and other sumcheck-based arguments), linear-only encodings [BCIOP13, GGPR13, Groth16], …

How to build succinct interactive arguments?

	Proof string	Query class	Answer
FIOP+FC	$\Pi \in \Sigma^{\ell}$	$\mathbf{Q}$ $= \{\alpha : \Sigma^{\ell} \to \mathbb{D}\}$	$\beta = \alpha(\Pi) \in \mathbb{D}$ for $\alpha \in \mathbf{Q}$

What security for interactive arguments?



soundness

$$\Pr\left[\begin{array}{c} x \notin L(R) \\ \wedge\ b = 1 \end{array} \middle|\ \begin{array}{l} \mathsf{pp} \leftarrow \mathsf{Gen}(1^\lambda) \\ x \leftarrow \tilde{P}_{\text{ARG}}(\mathsf{pp}) \\ b \leftarrow \left\langle \tilde{P}_{\text{ARG}}, V_{\text{ARG}}(\mathsf{pp}, x) \right\rangle \end{array}\right]$$

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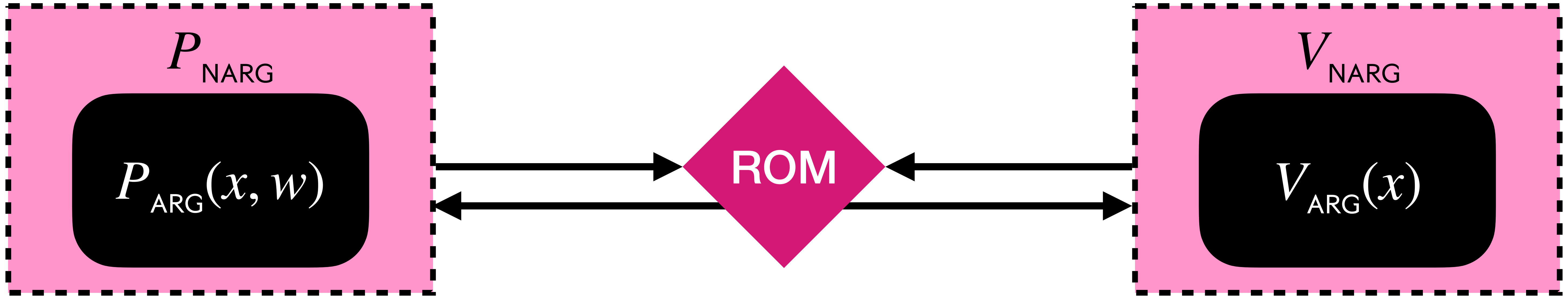

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state-restoration soundness

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captures Fiat–Shamir security in the ROM

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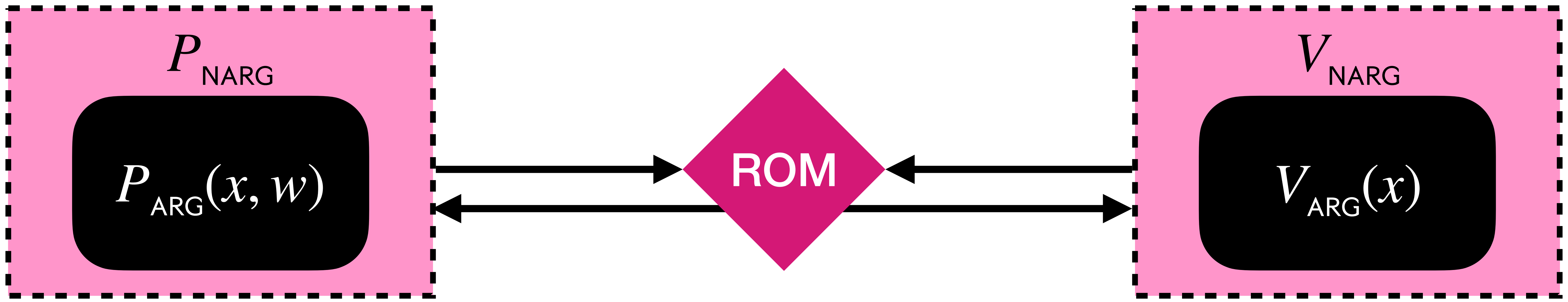

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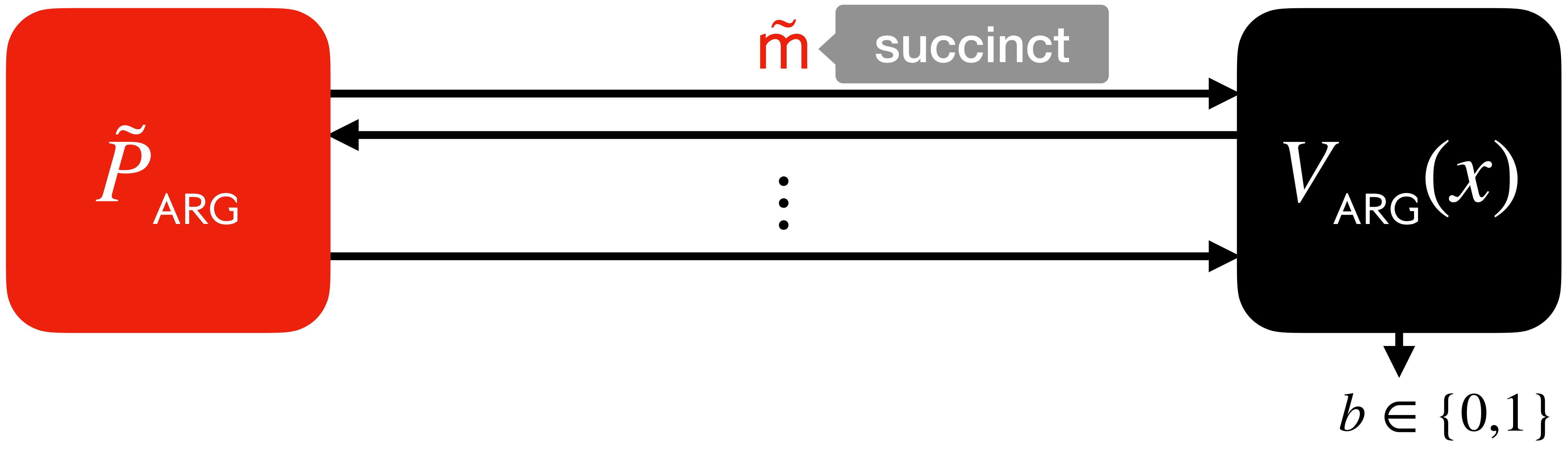
We only discuss the soundness case in this talk, but all results apply to knowledge soundness as well.

captures Fiat–Shamir security in the ROM

How to analyze succinct interactive arguments?



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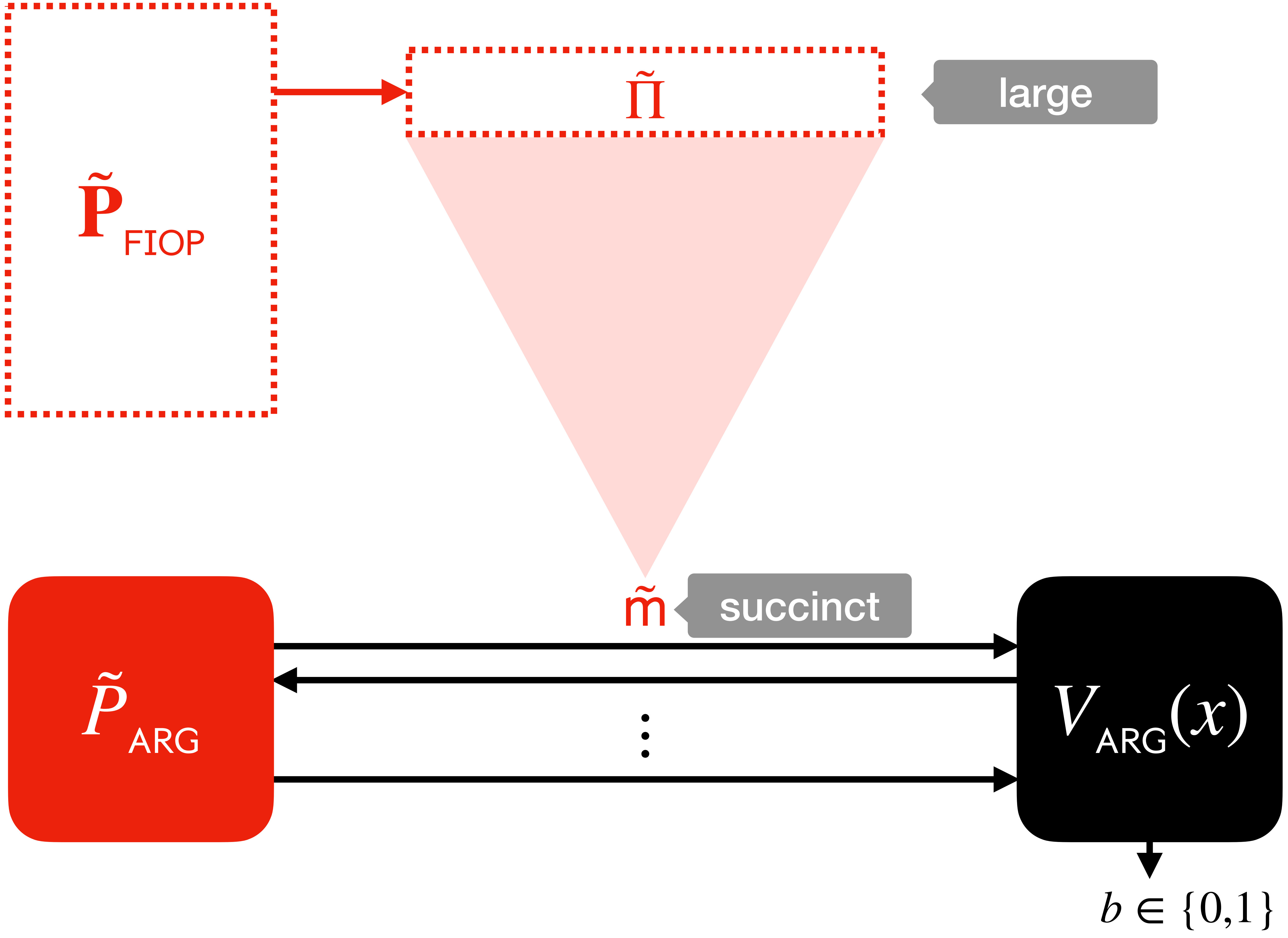

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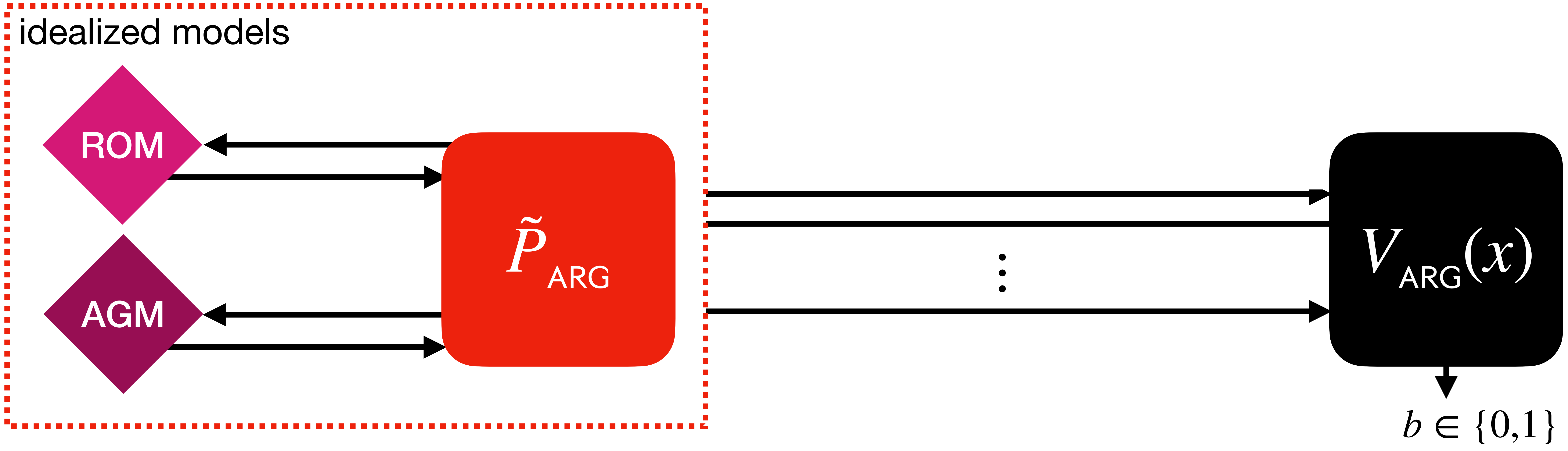

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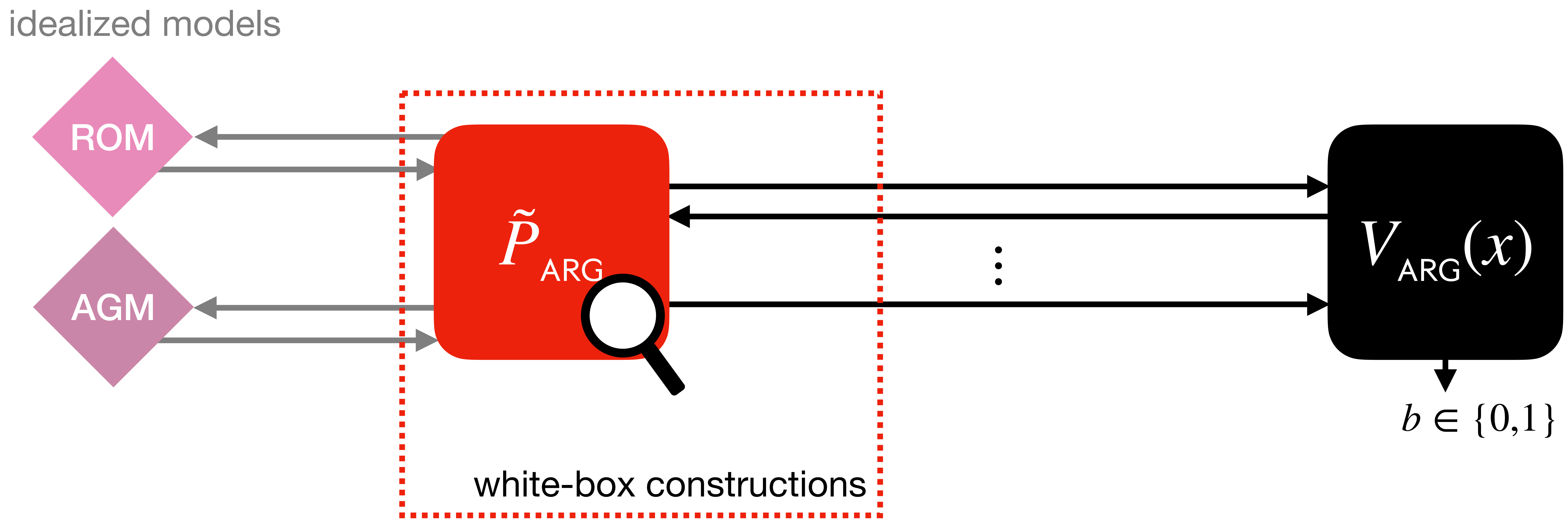

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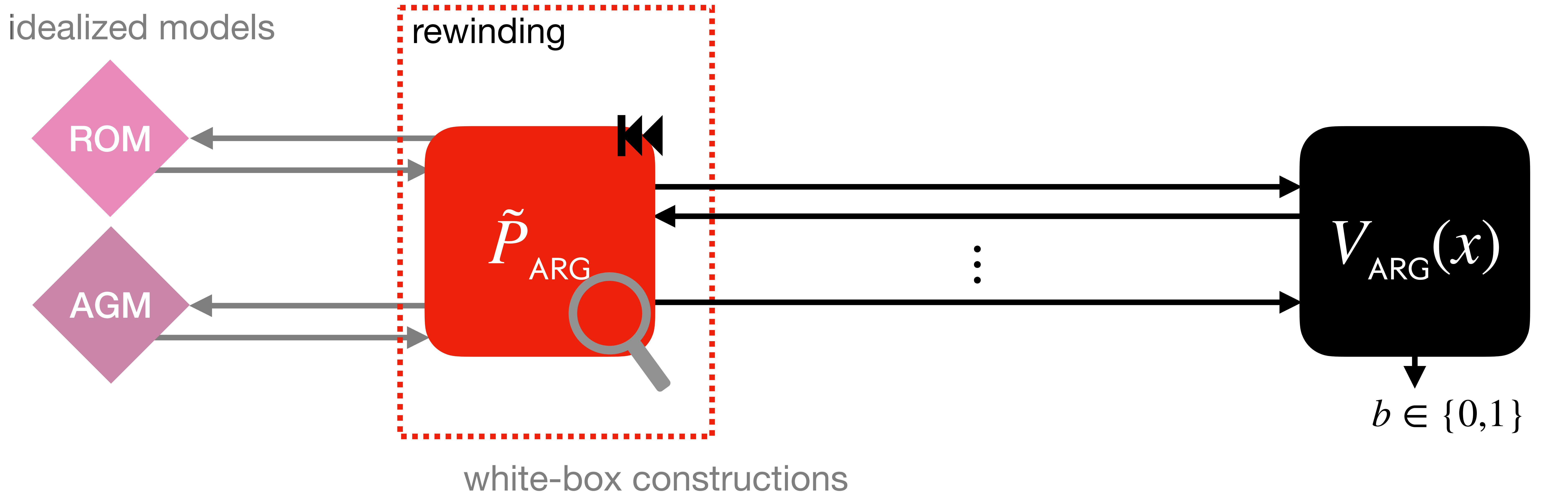

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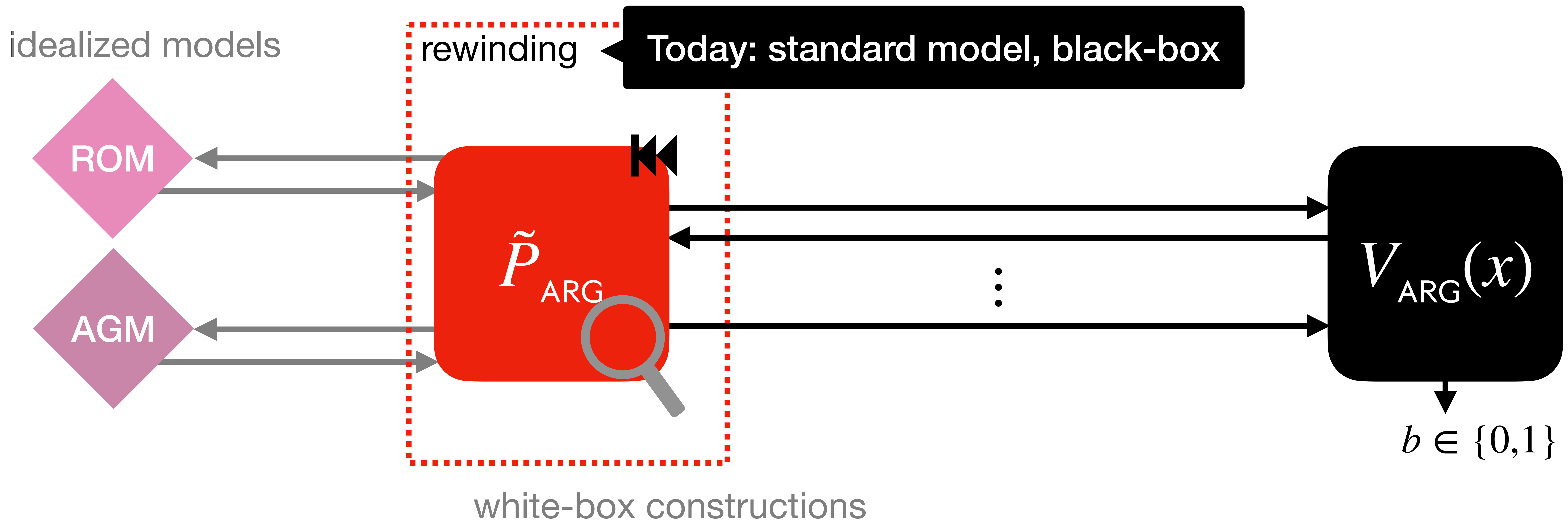

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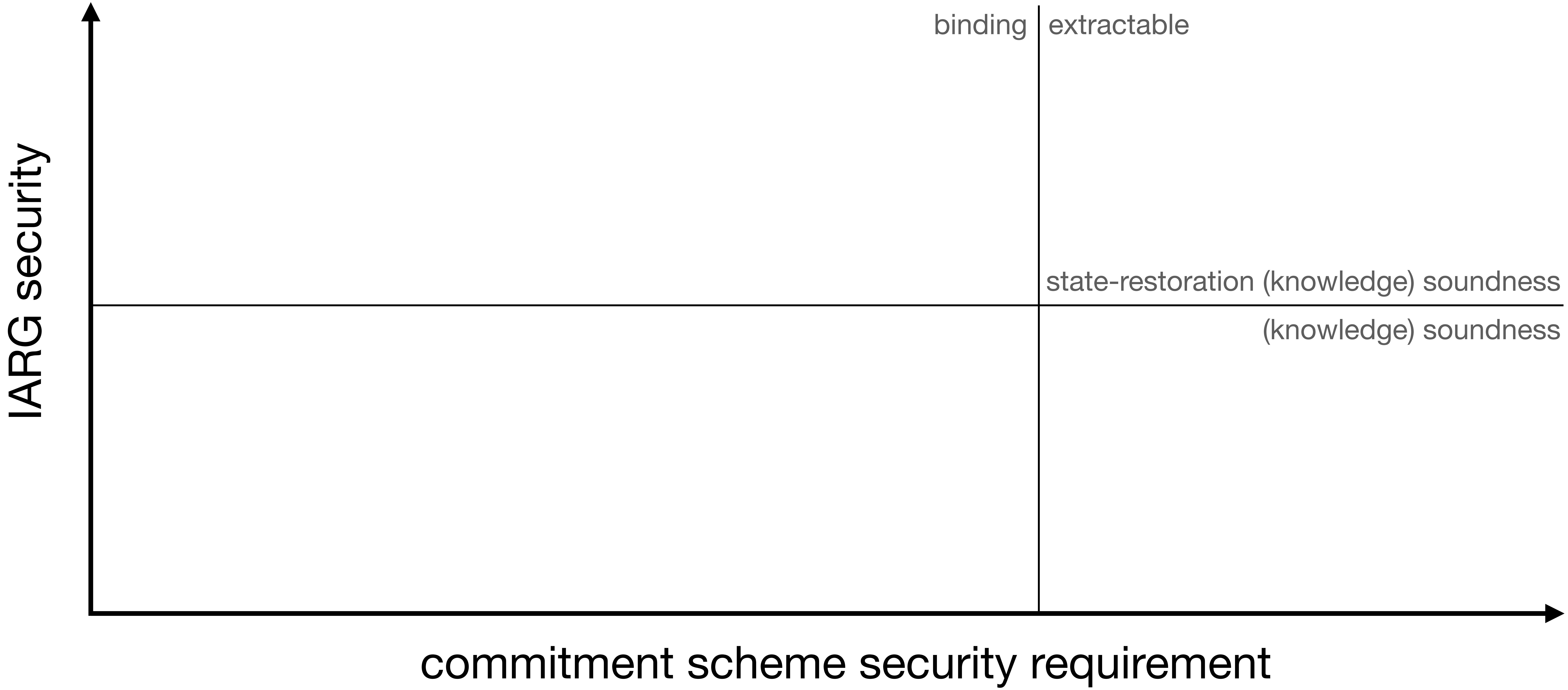

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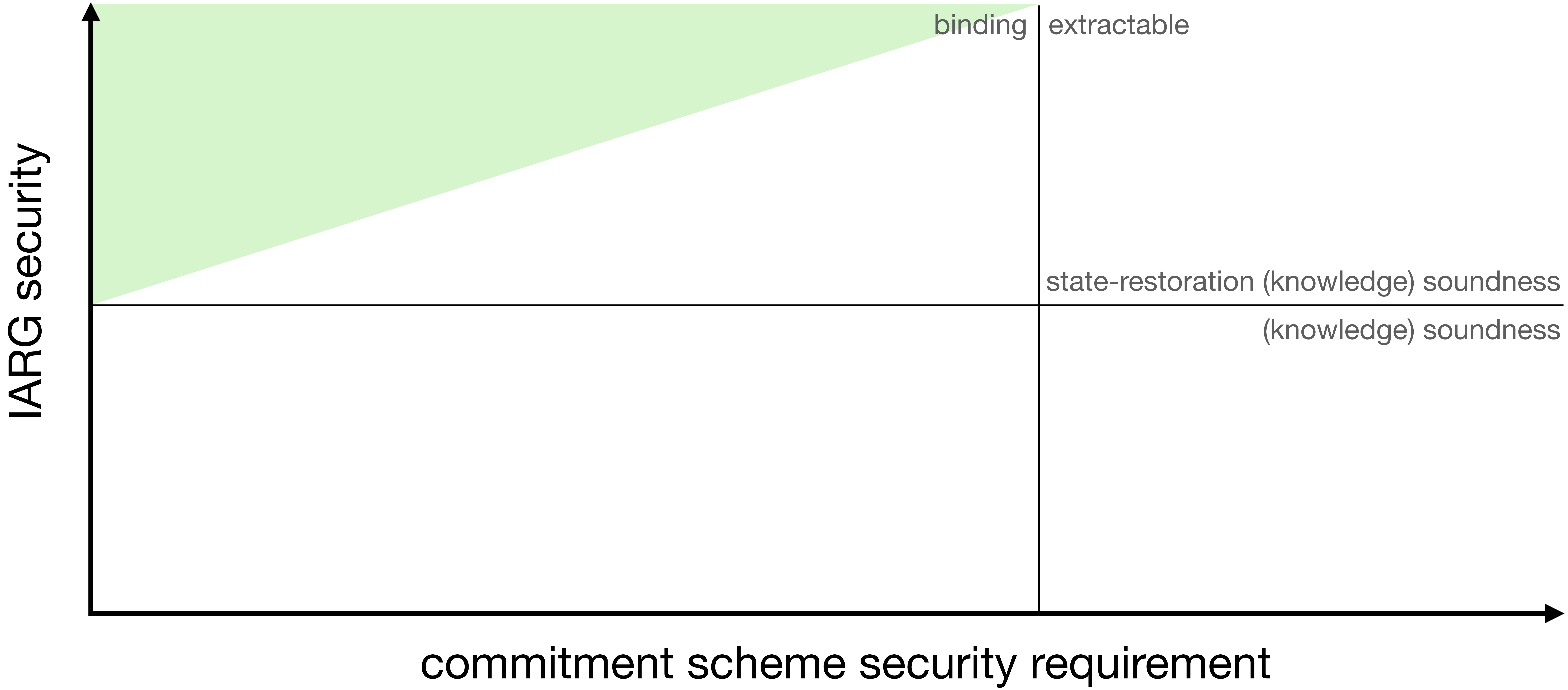

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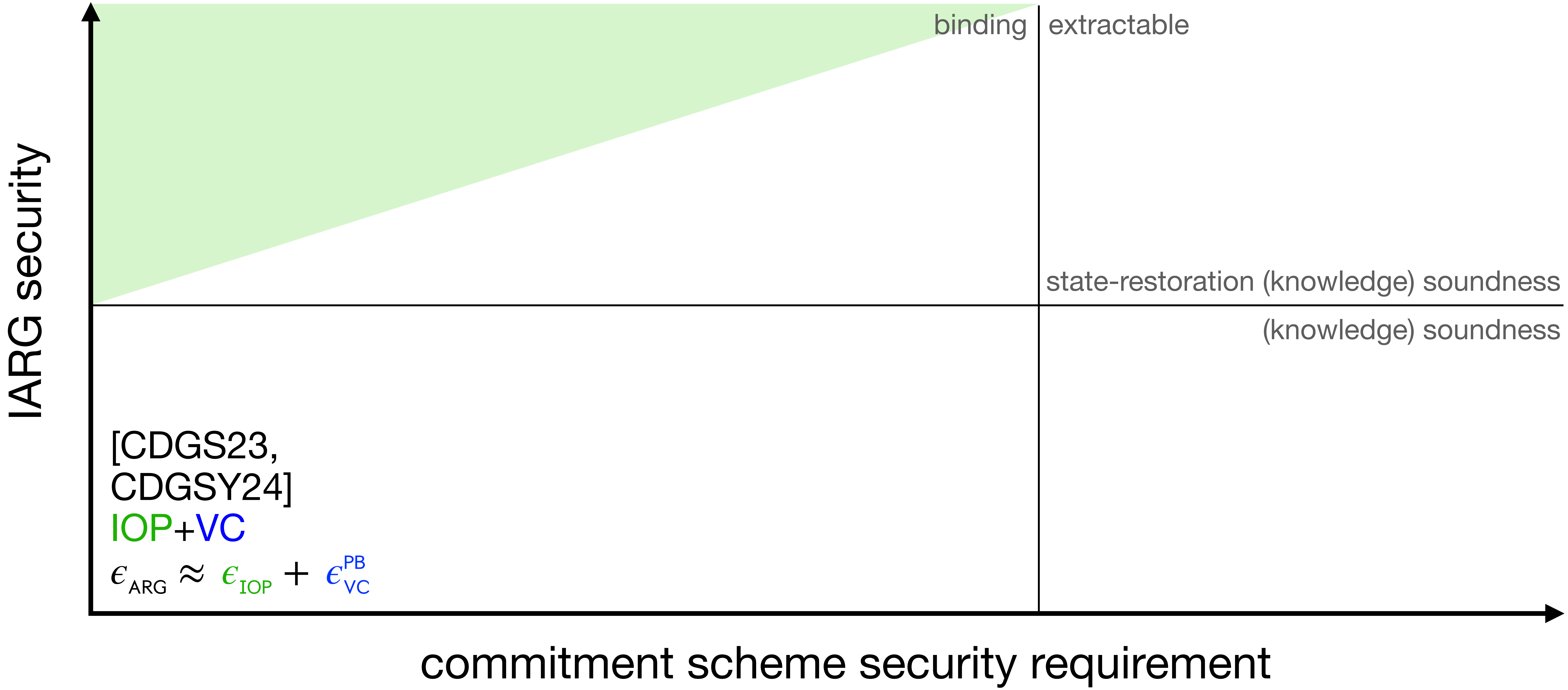

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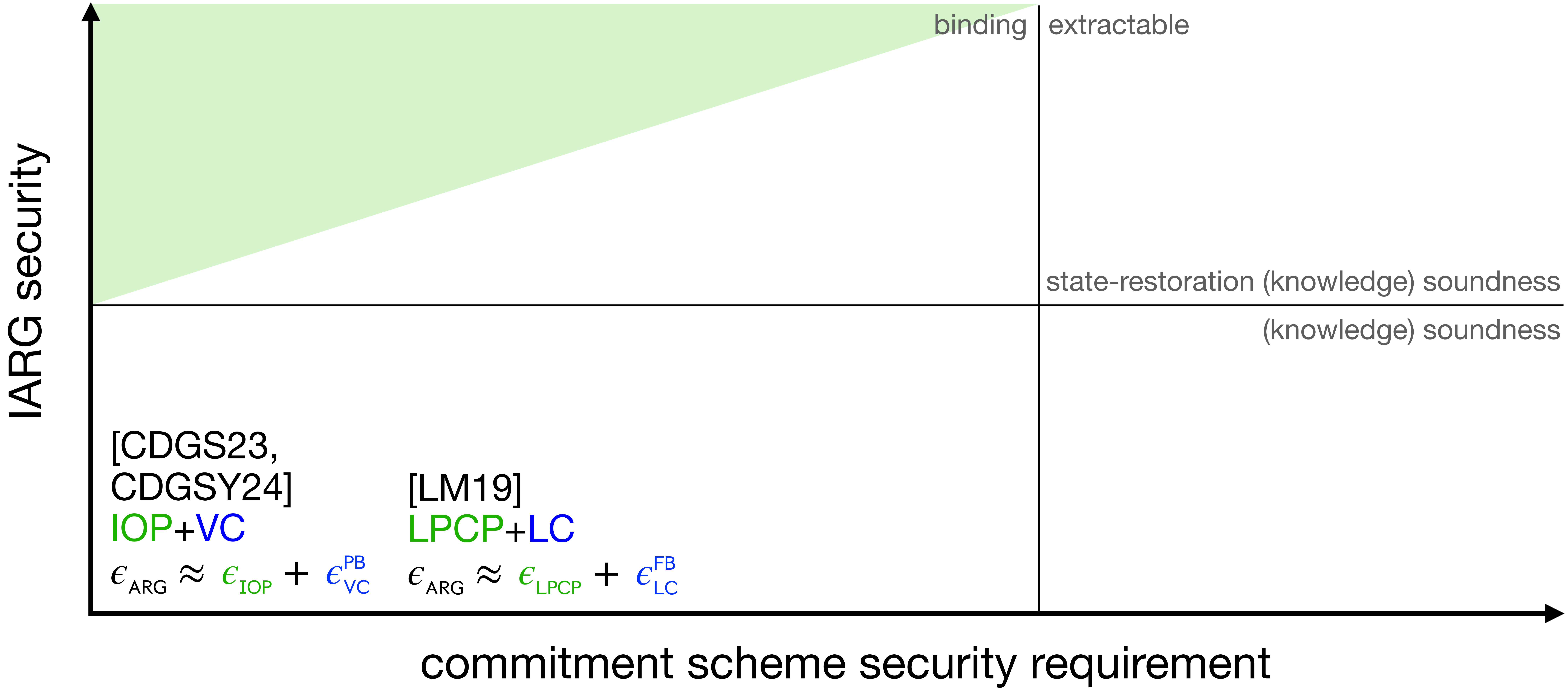

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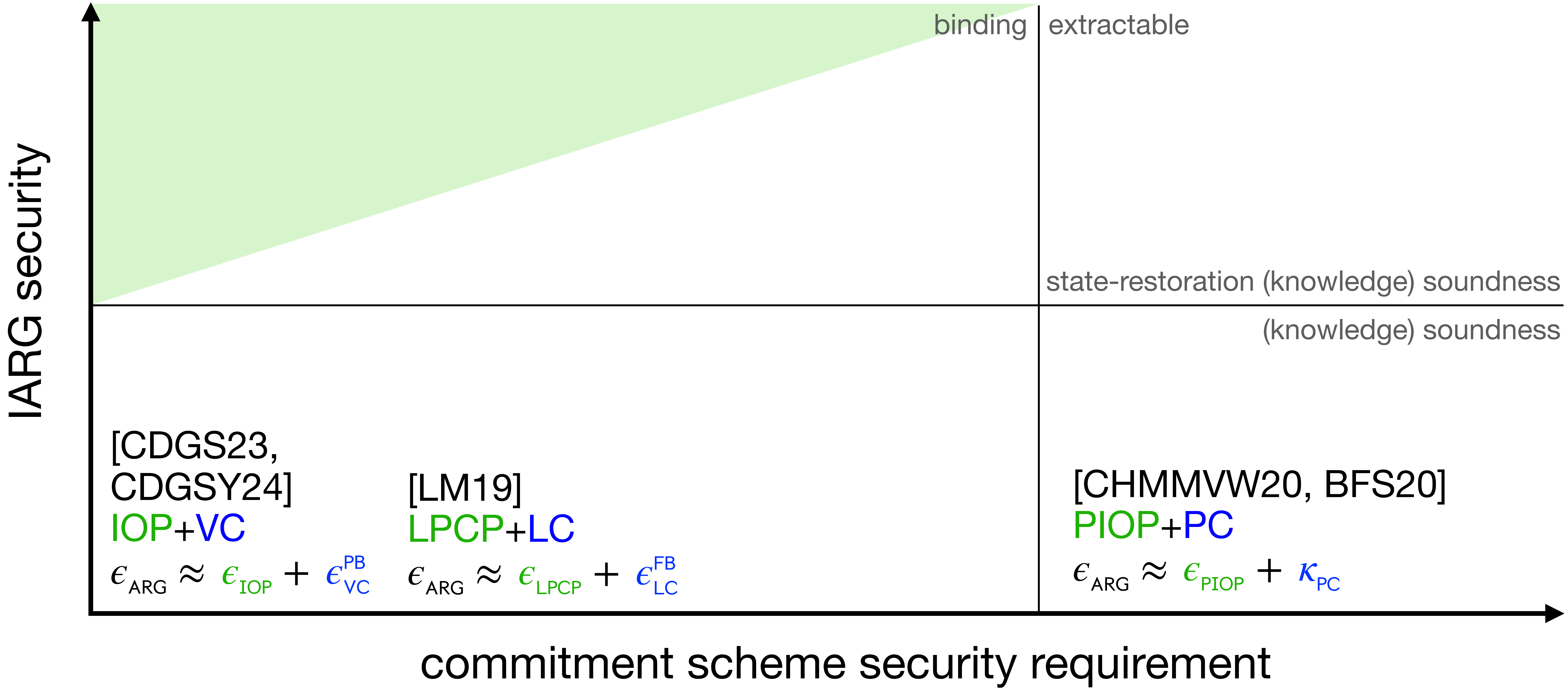

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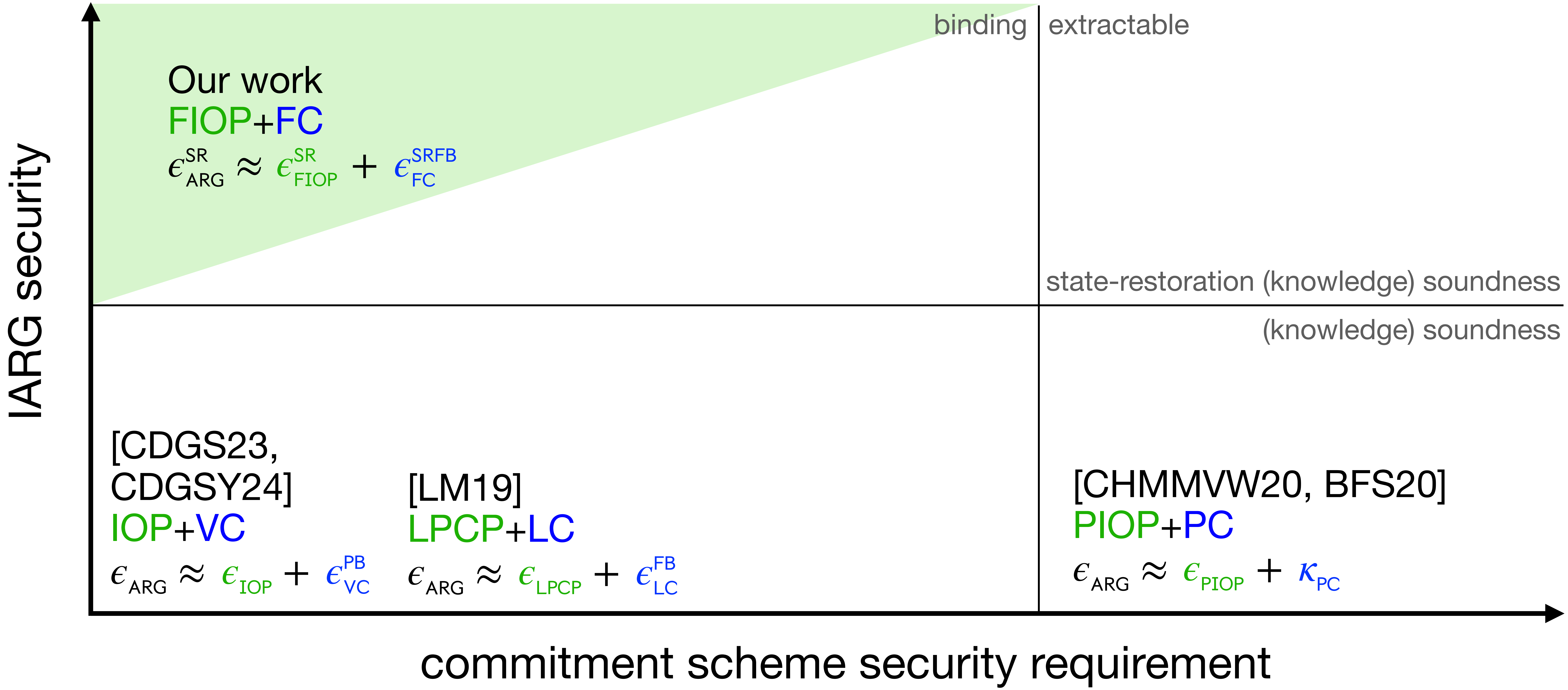

Our results

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FIOP is (knowledge) sound

\+ FC is function binding

$\Rightarrow$ ARG = Funky[FIOP, FC] is (knowledge) sound

$$\epsilon_{\mathsf{ARG}}(\mathsf{k}, \ell, t) \leq \epsilon_{\mathsf{FIOP}}(\mathsf{k}, \ell) + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{k} \cdot \mathsf{N} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}(\ell, \mathsf{q}, \mathsf{N})$$

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ARG = Funky[FIOP, FC] for a query class $\mathbf{Q}$

FIOP is state-restoration (knowledge) sound

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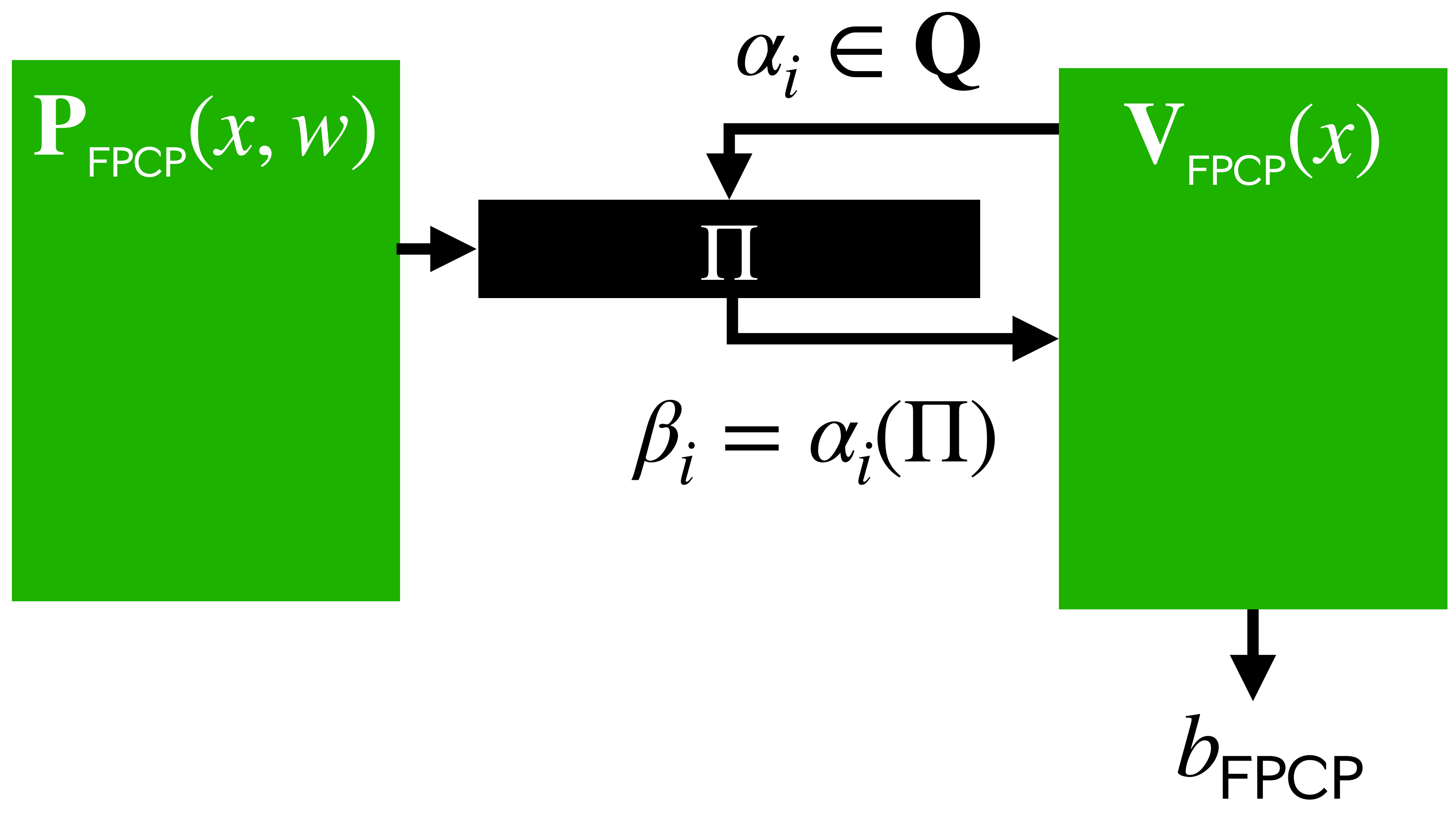
Application: Plonk = FS[Funky[PlonkIOP, linearized KZG]]
is a SNARK in the ROM
from the SDH assumption (previously: from ARSDH+SplitRSDH)

Warm-up:
the Funky protocol (for FPCPs and non-interactive FCs)
and its security

Building blocks

Functional
Probabilistically Checkable Proof
for $\mathbf{Q}$



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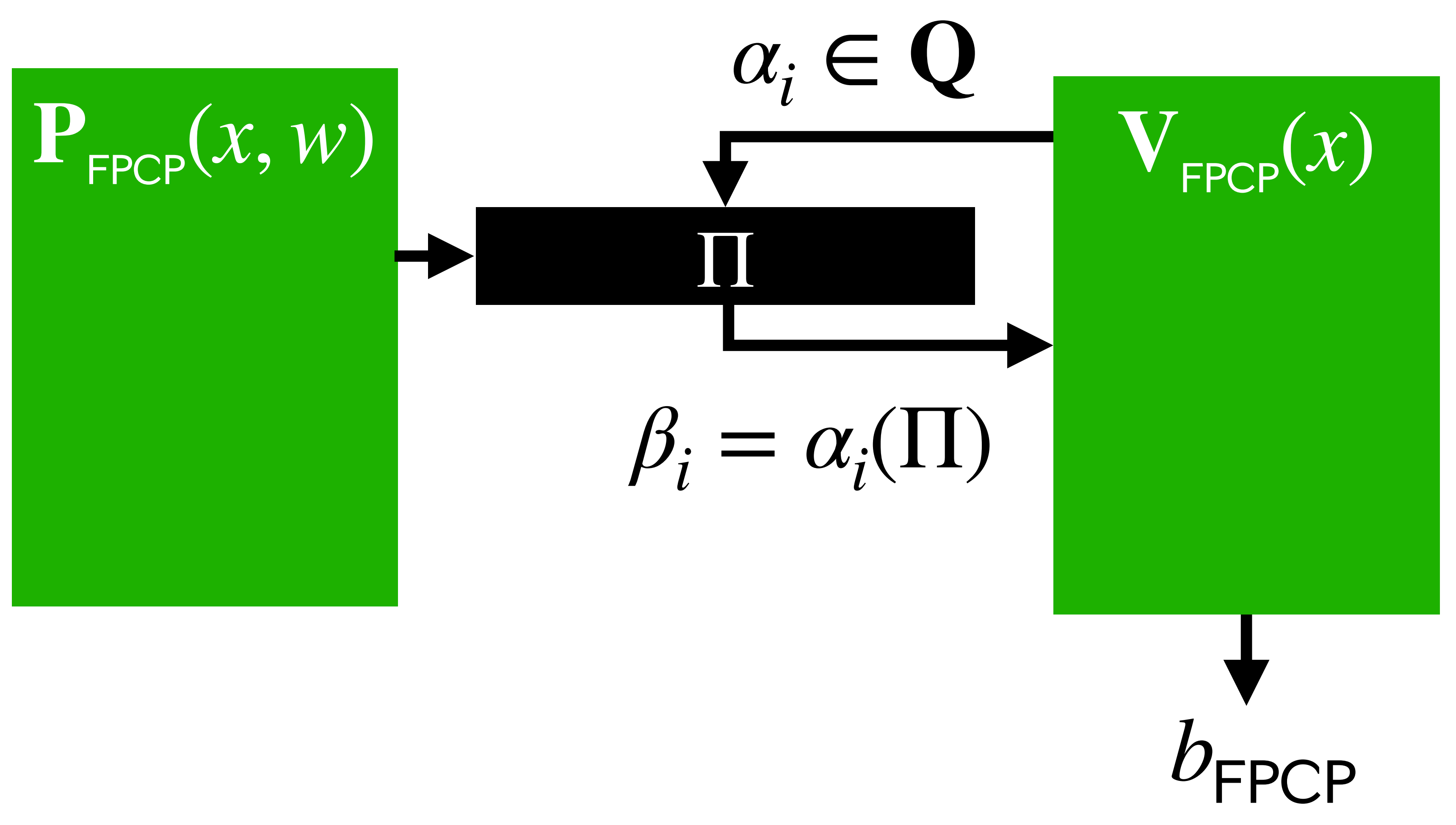

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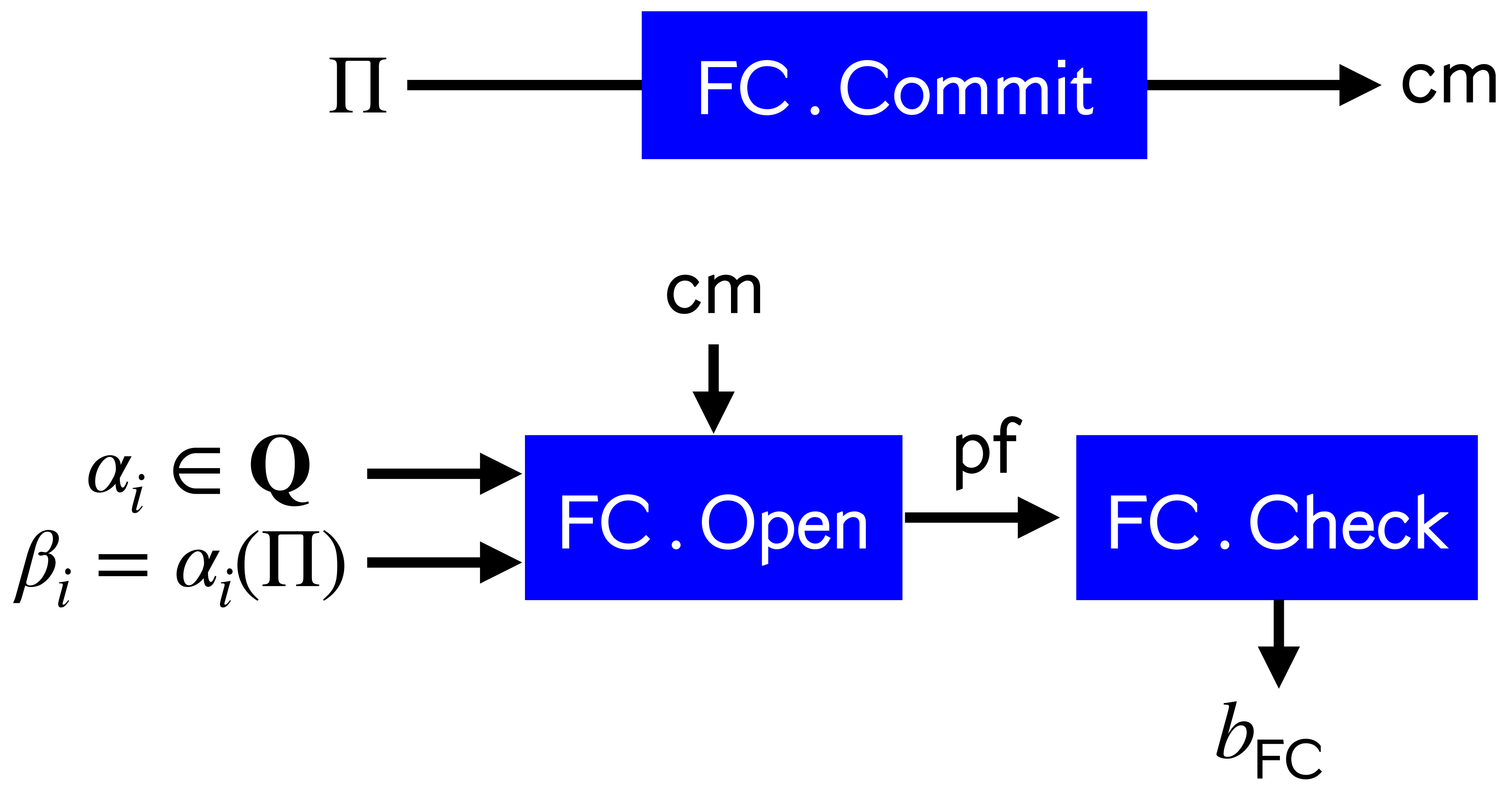

Building blocks

Security: (knowledge) soundness

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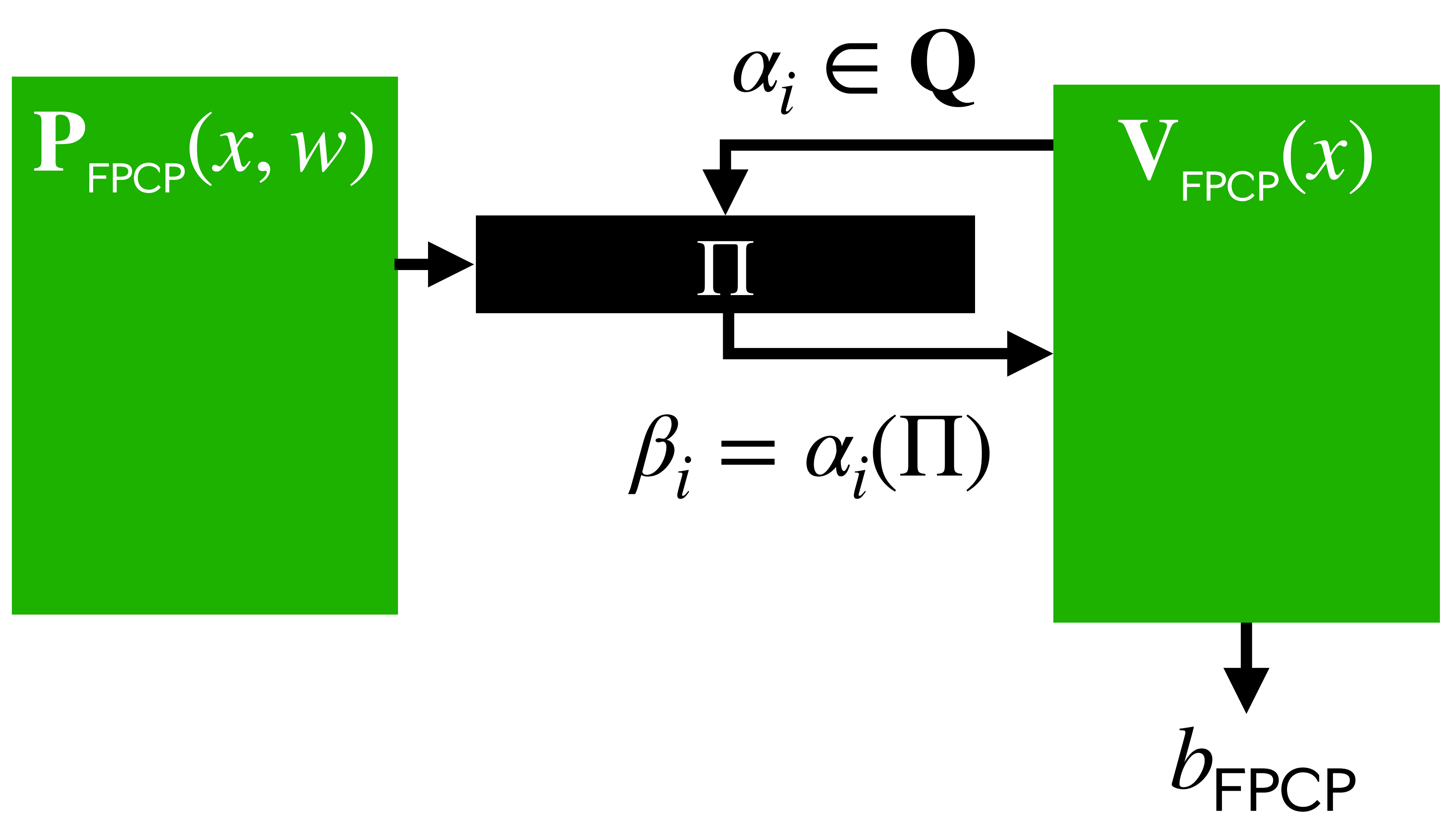

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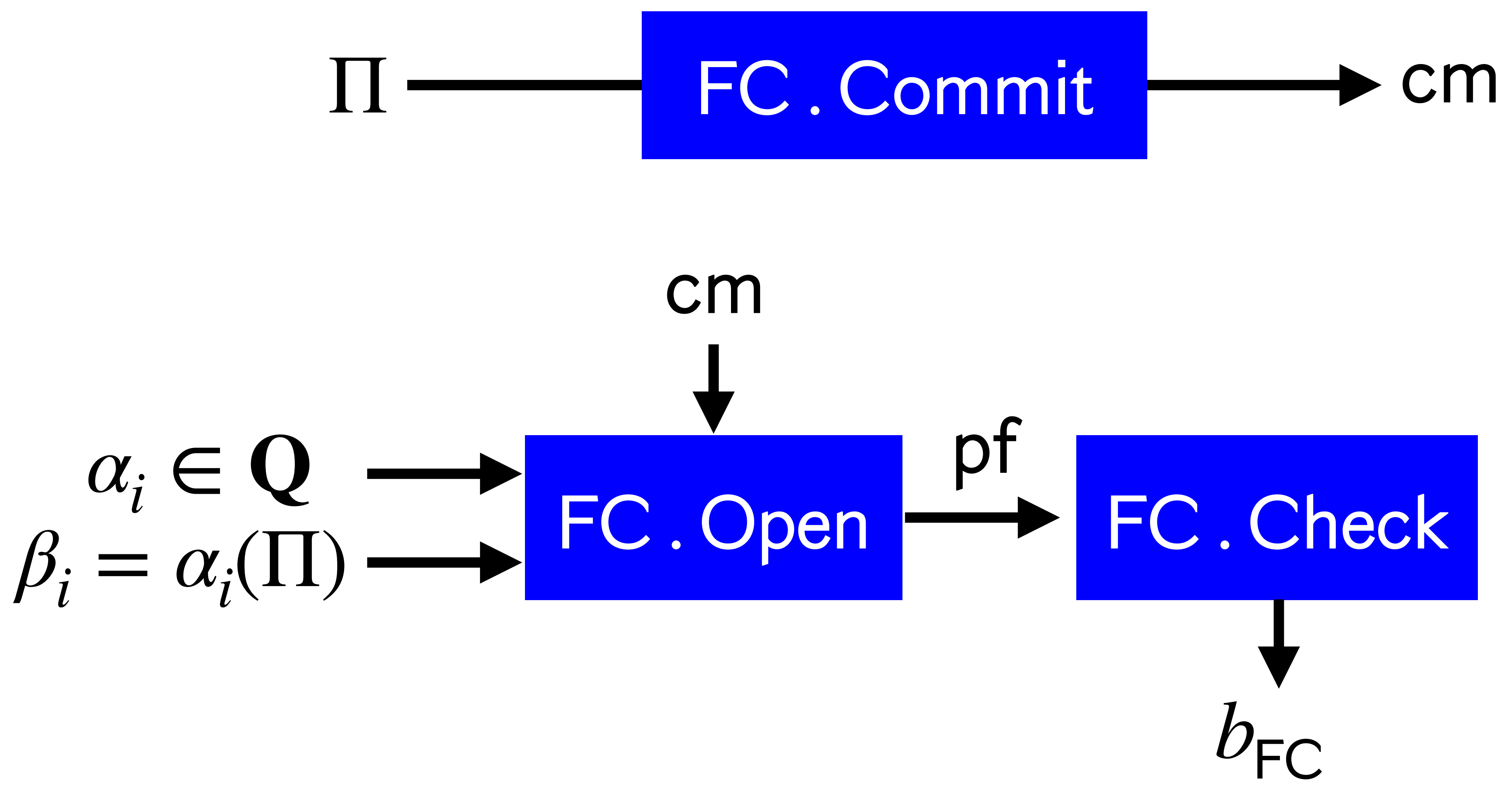

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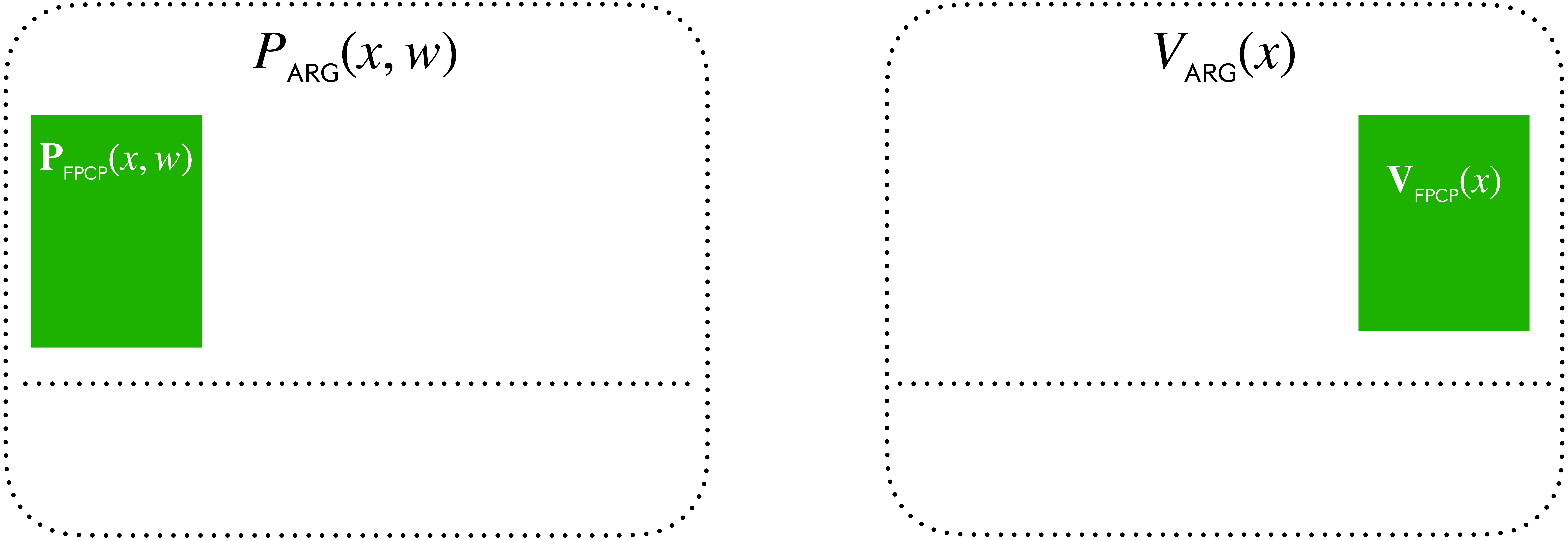
The Funky protocol

For FPCPs and non-interactive FCs



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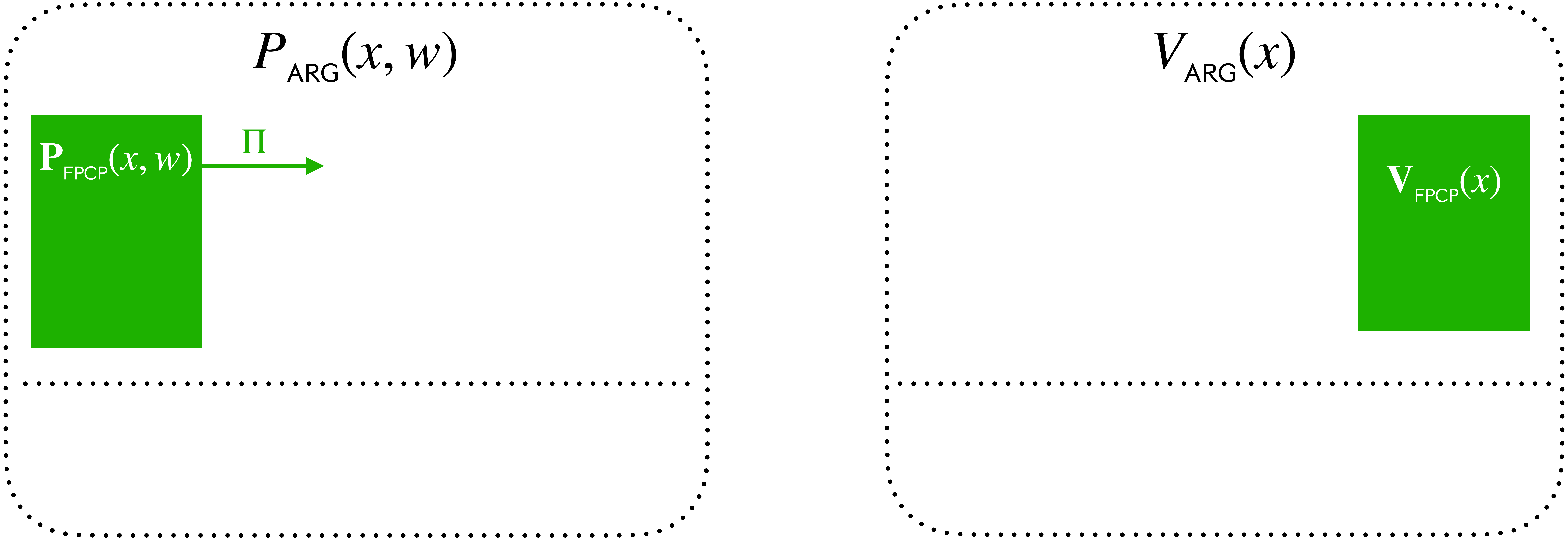

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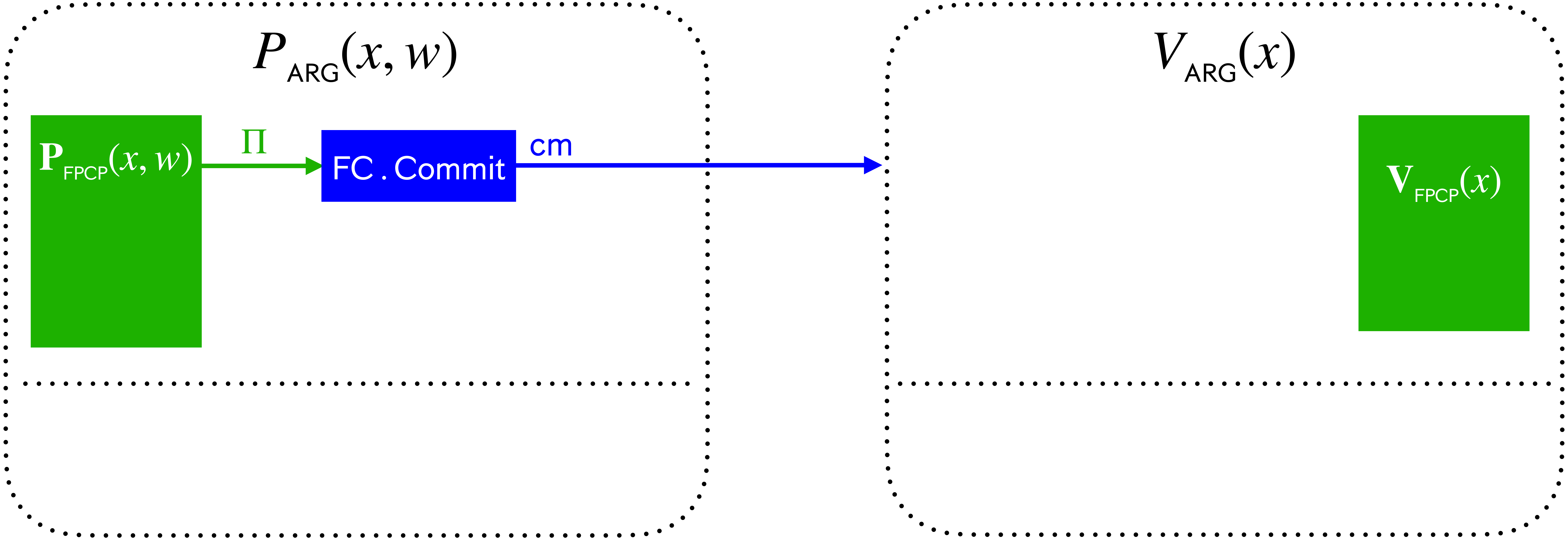

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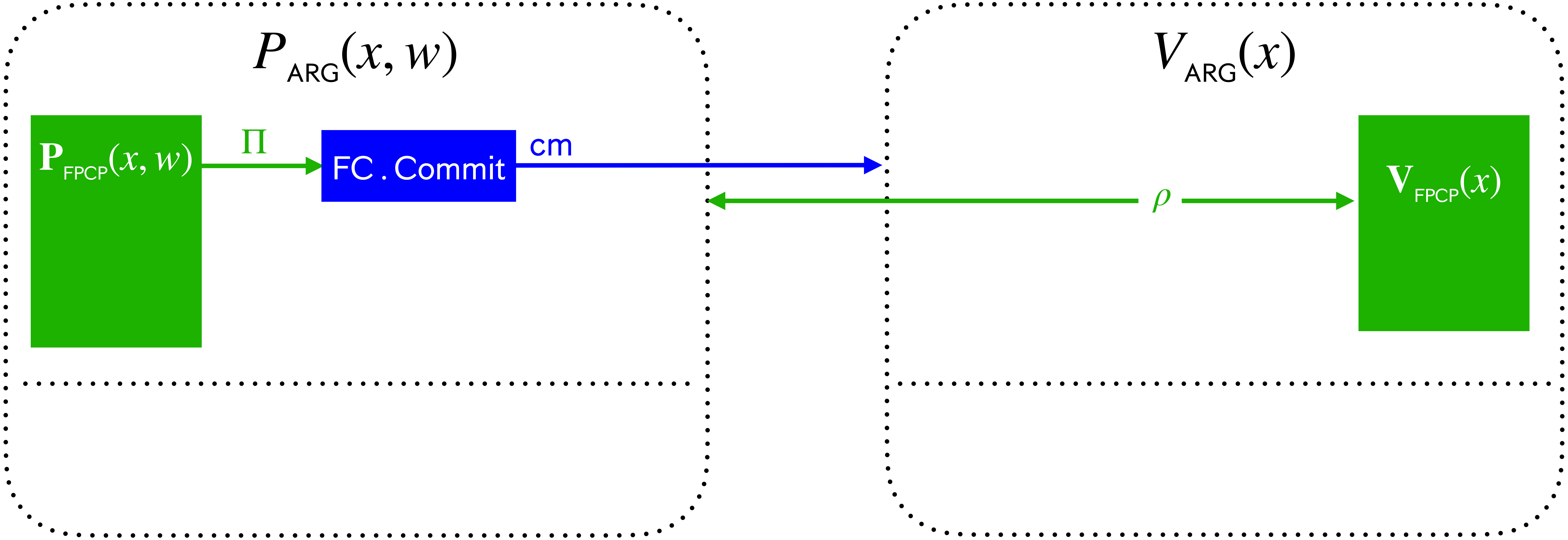

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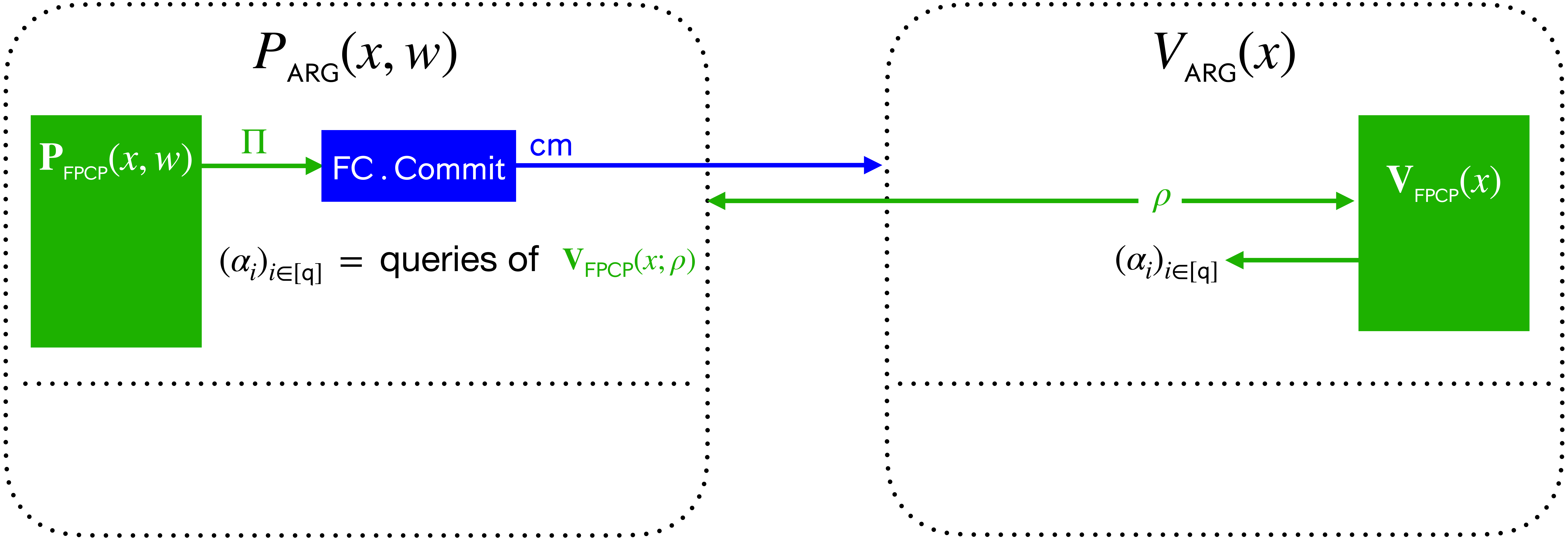

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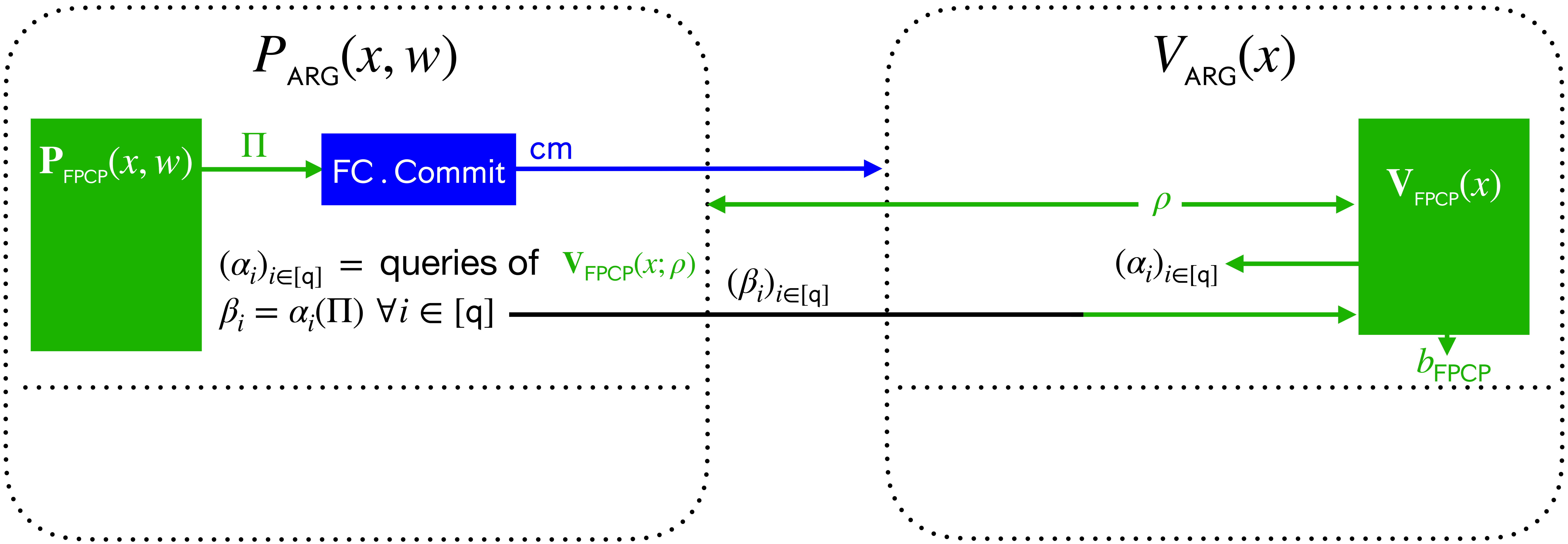

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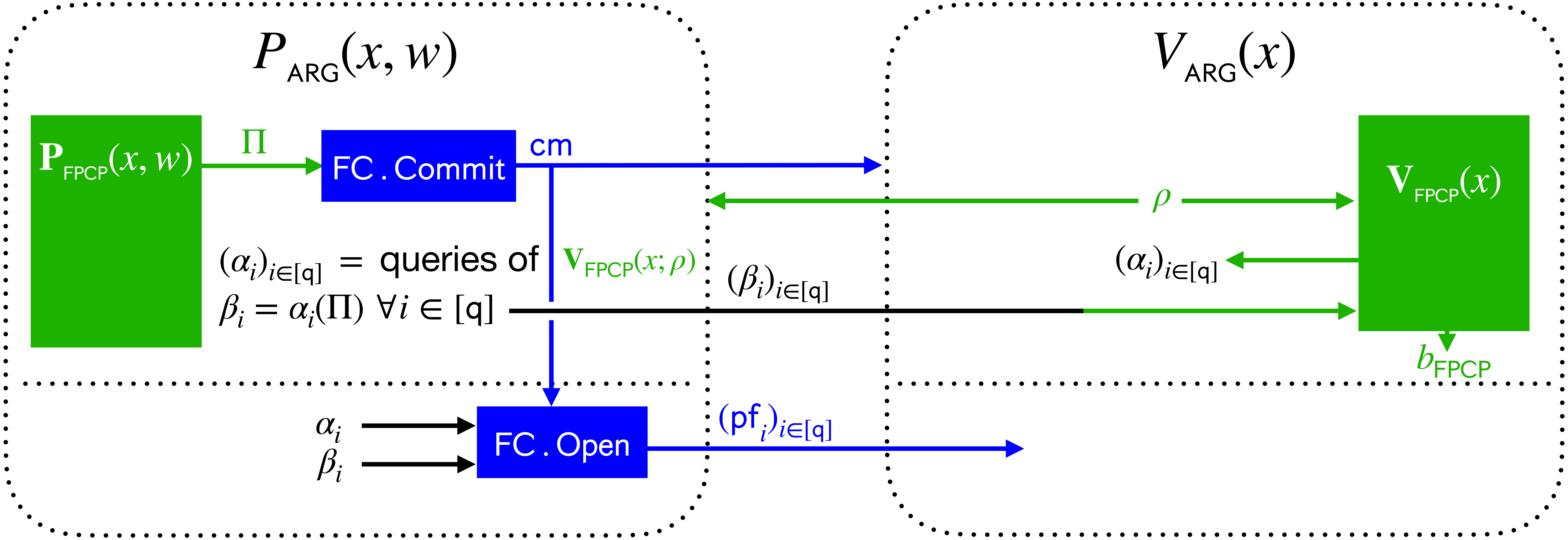

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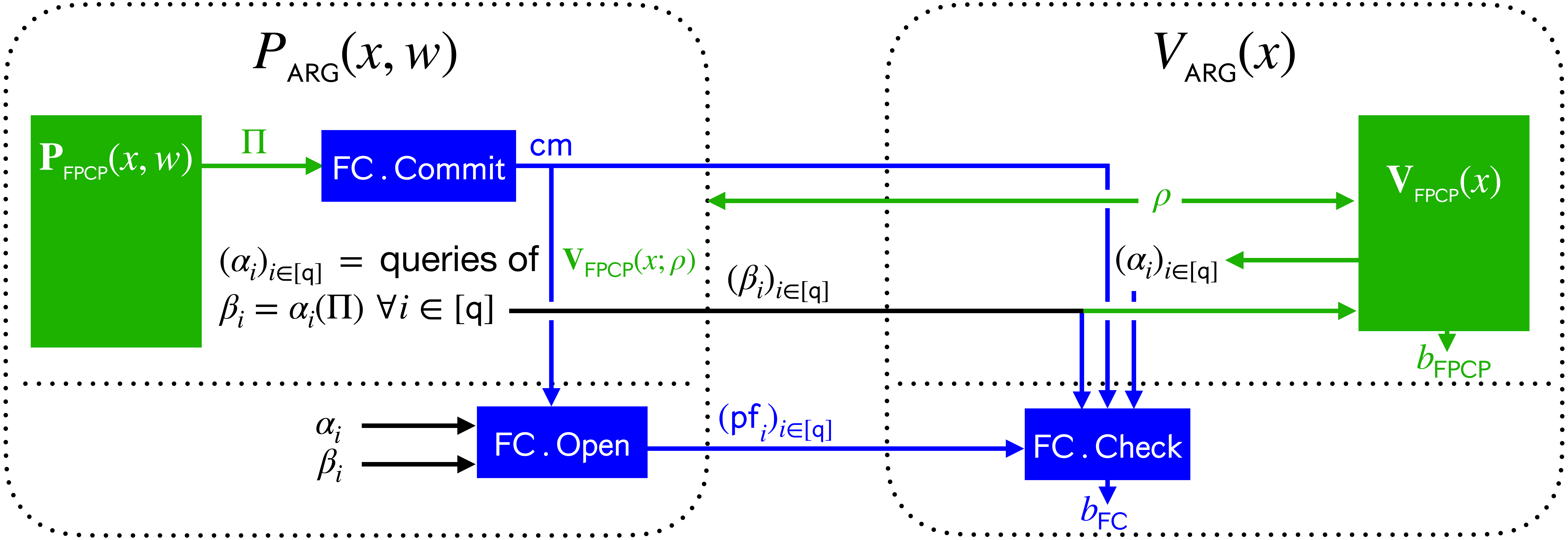

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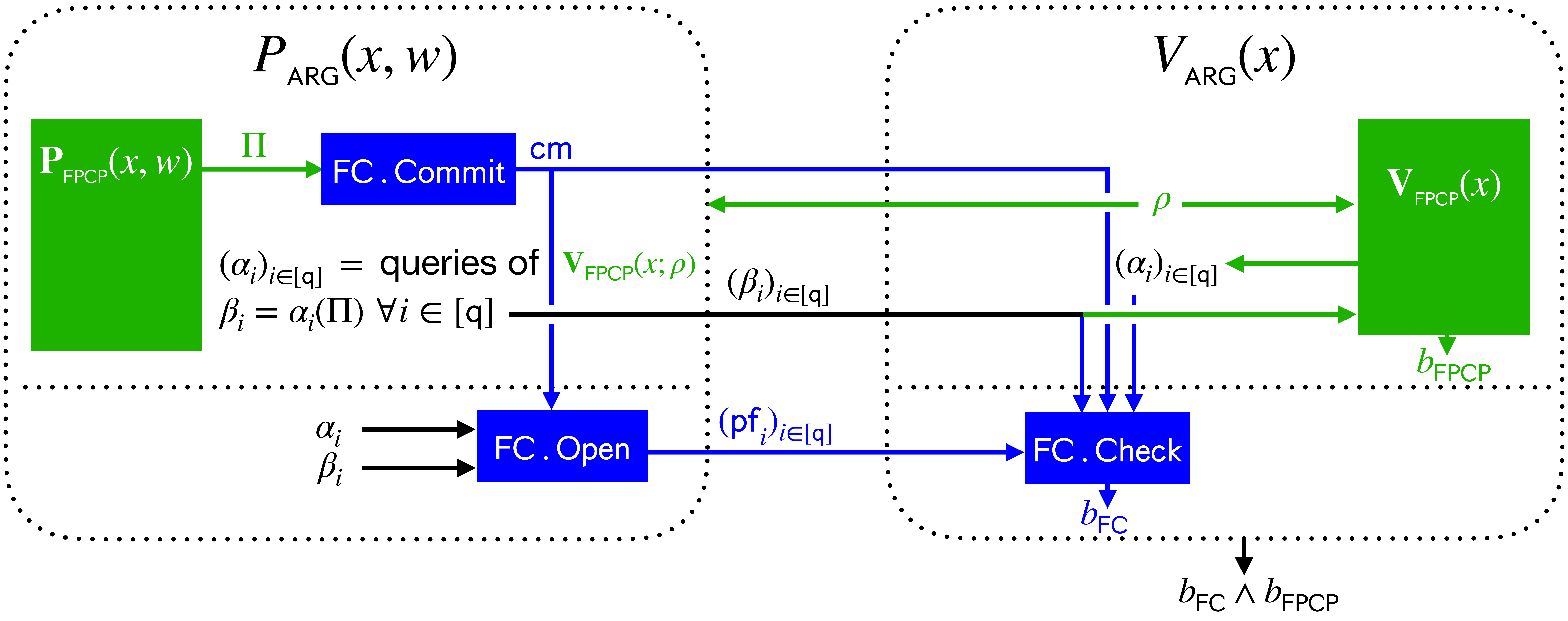

Security reduction for Funky[FPCP, FC]

Goal:

(for FPCPs and non-interactive FCs)

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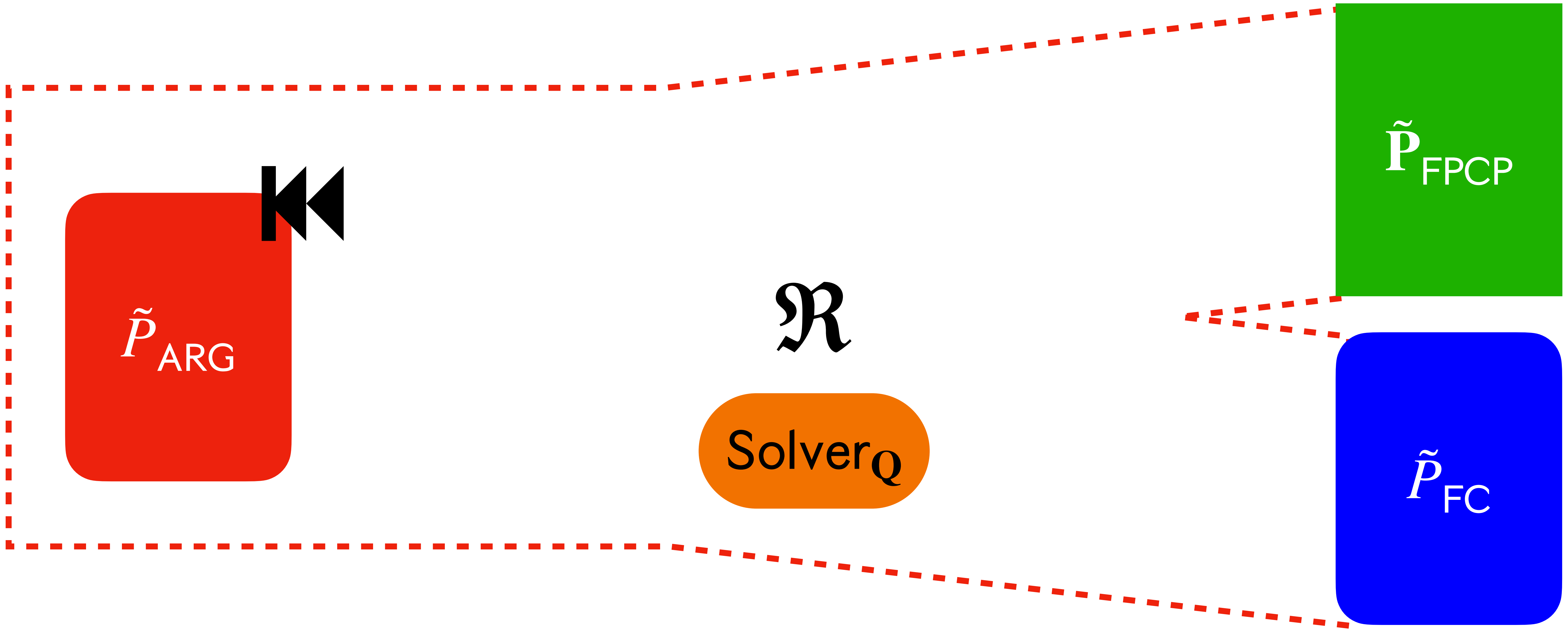
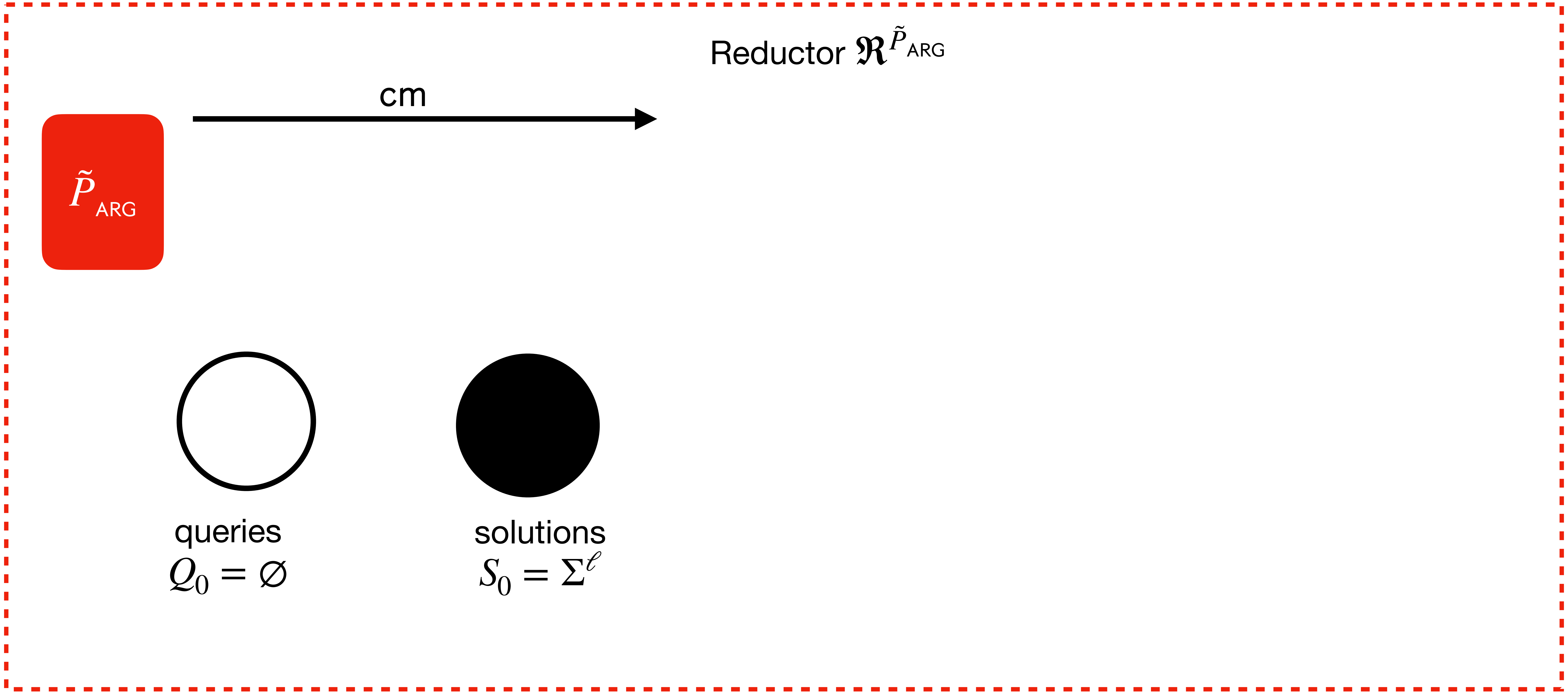
Goal:

(for FPCPs and non-interactive FCs)

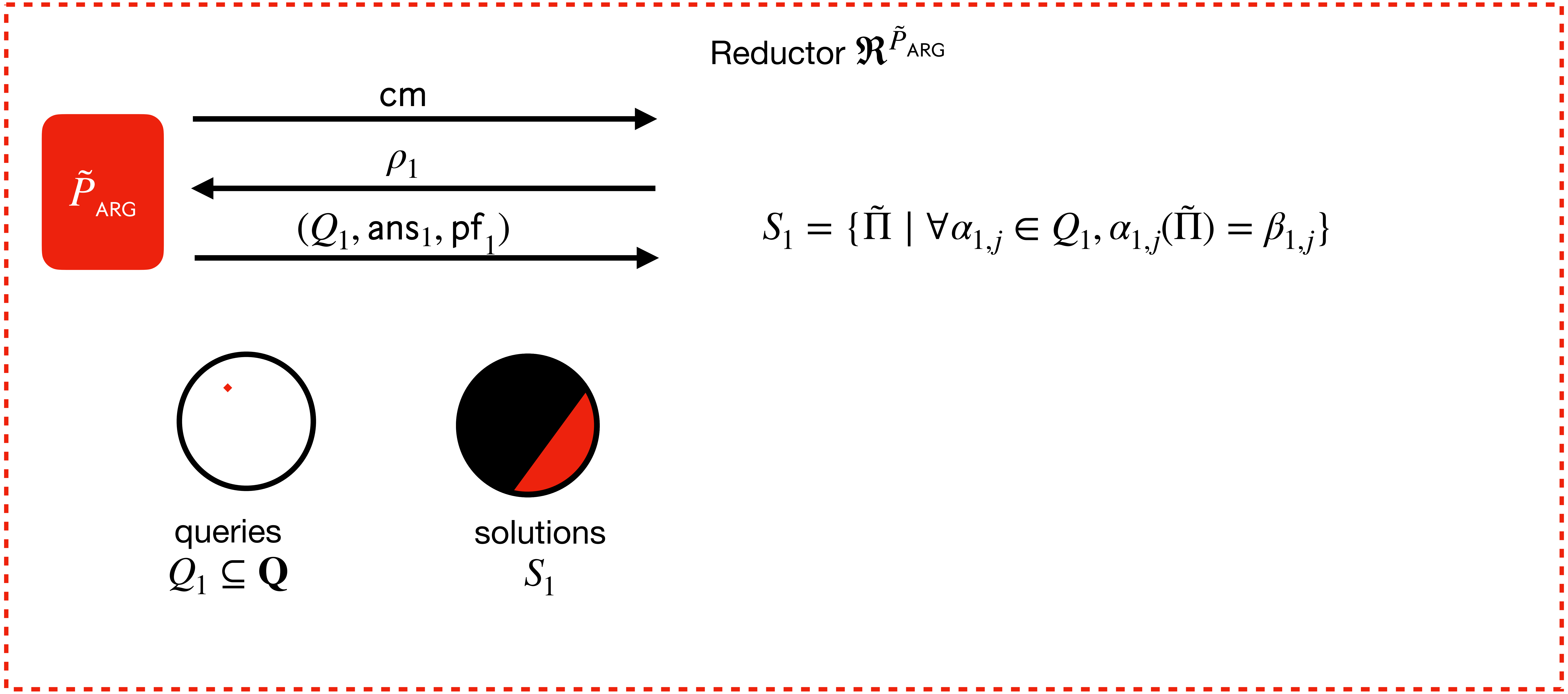
$$\epsilon_{\mathsf{ARG}}(\lambda, \ell, \mathsf{q}, t) \leq \epsilon_{\mathsf{FPCP}}(\ell, \mathsf{q}) + \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t \cdot \mathsf{N} + t_{\mathbf{Q}}) + \epsilon_{\mathbf{Q}}(\ell, \mathsf{N})$$



Security reduction for Funky[FPCP, FC]

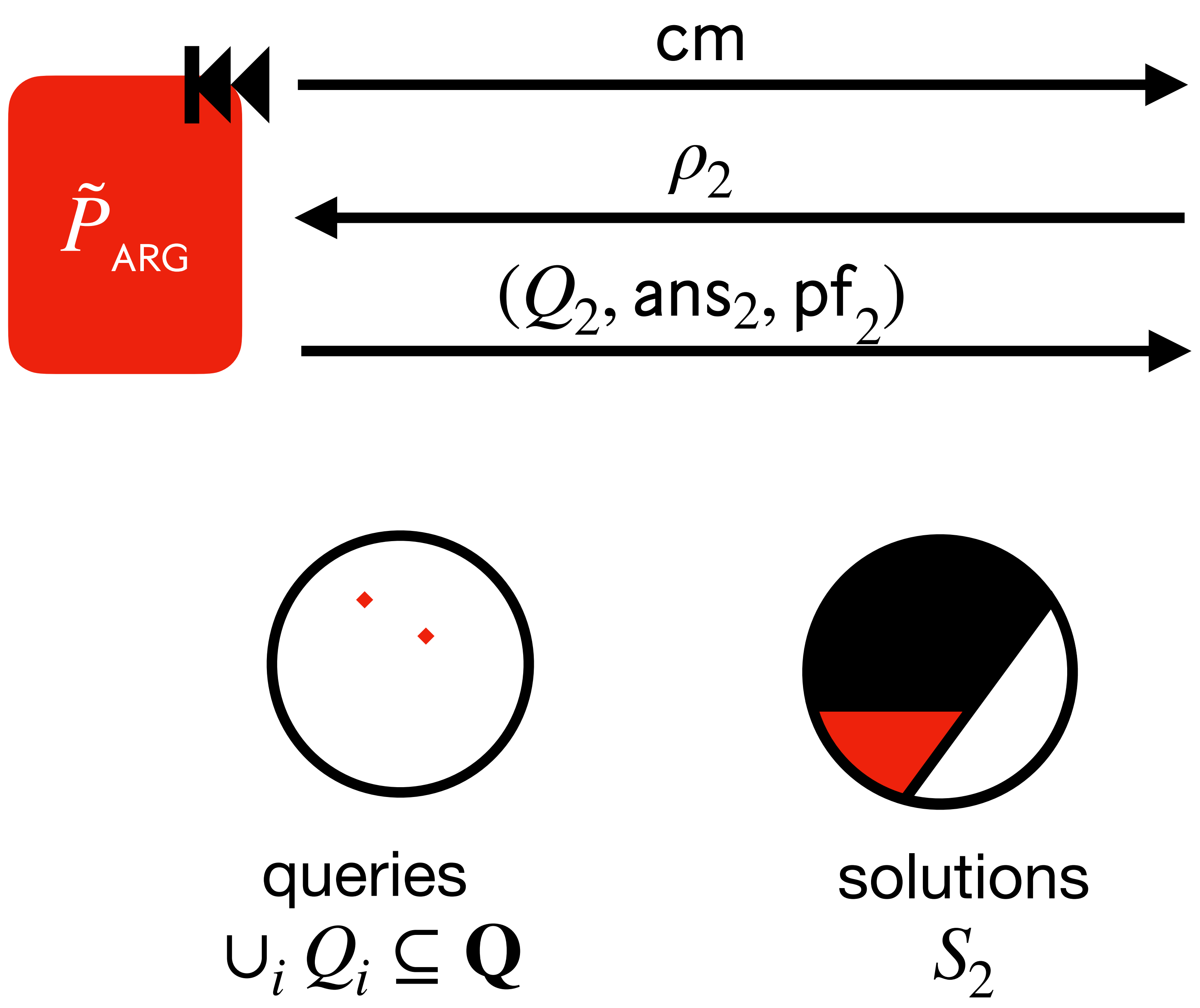


Security reduction for Funky[FPCP, FC]



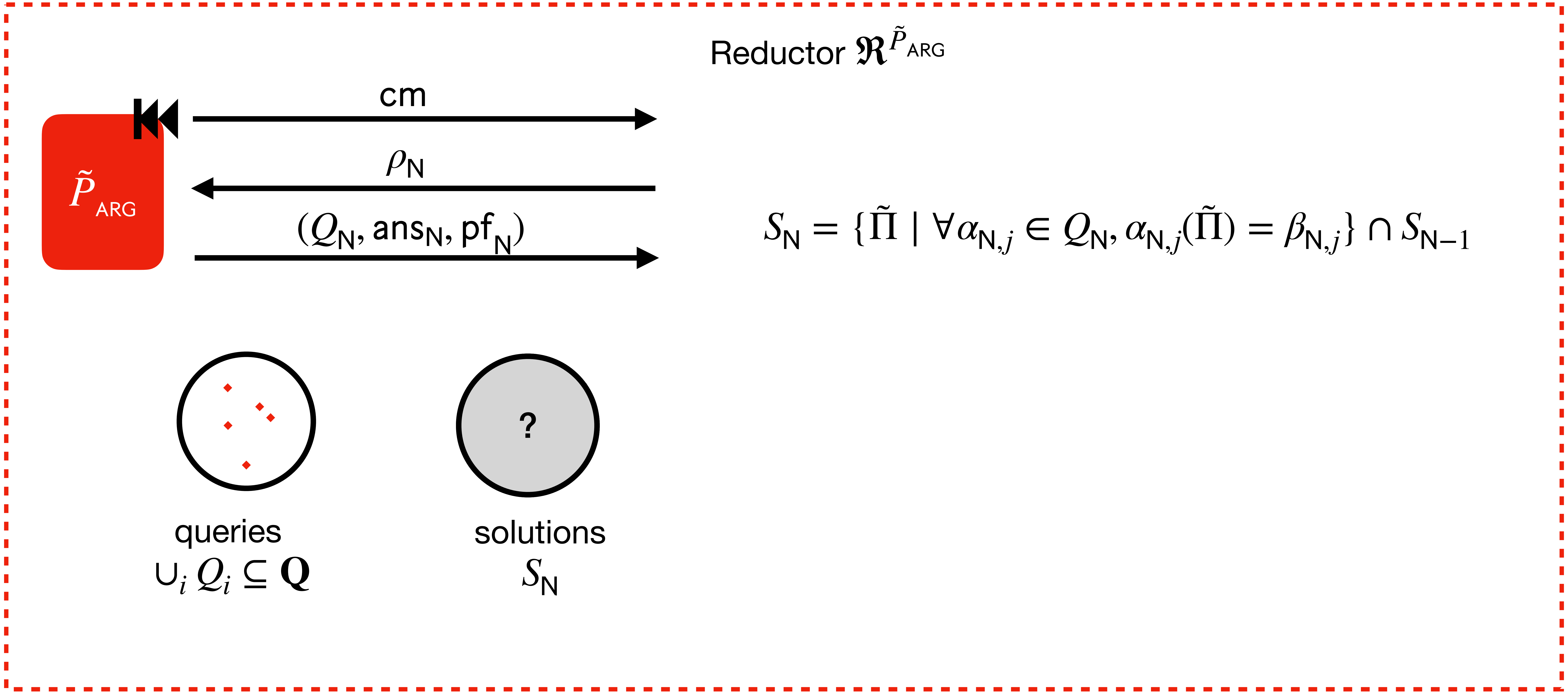
Security reduction for Funky[FPCP, FC]

Reductor $\mathfrak{R}^{\tilde{P}_{\text{ARG}}}$



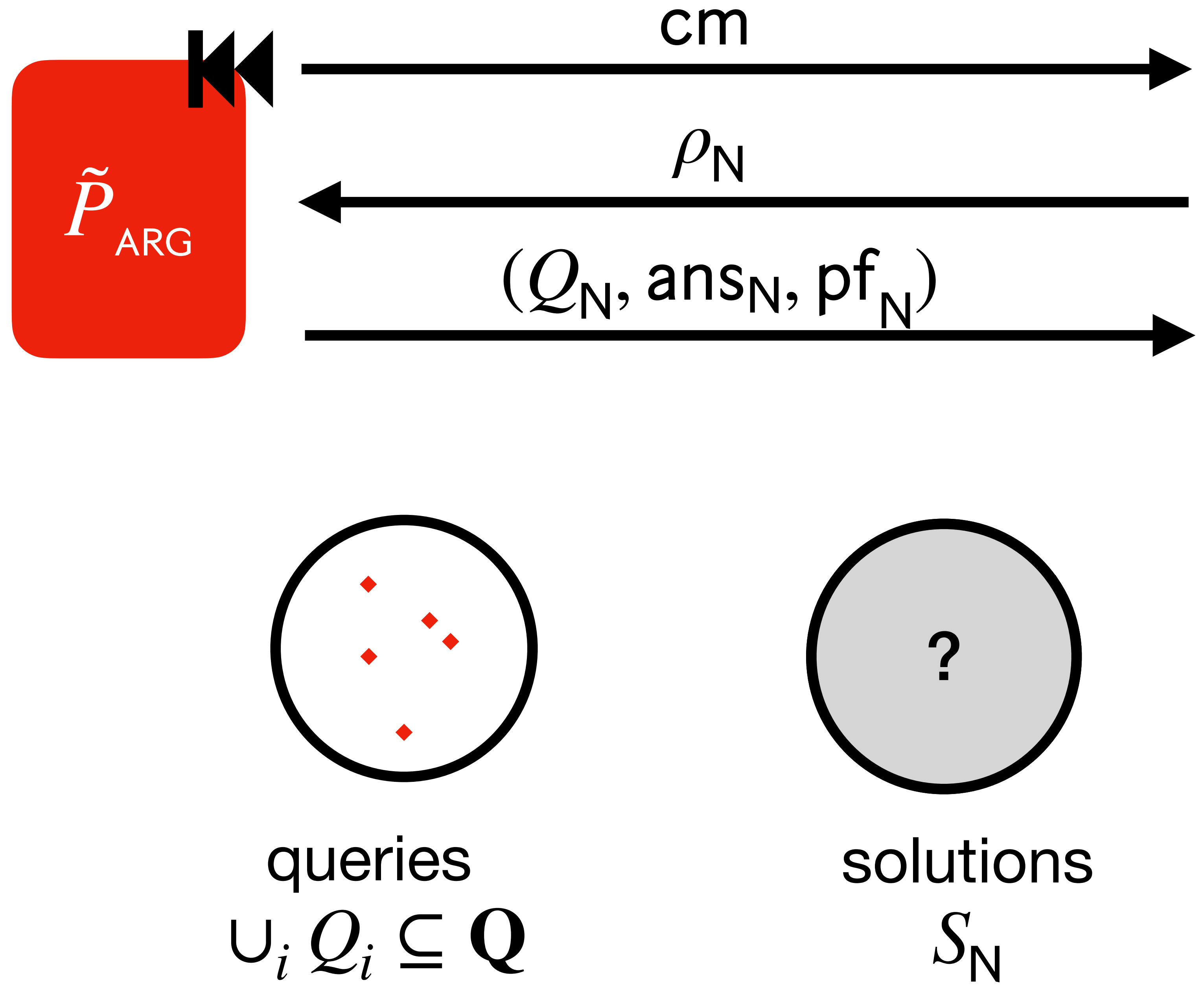
$$S_2 = \{\tilde{\Pi} \mid \forall \alpha_{2,j} \in Q_2, \alpha_{2,j}(\tilde{\Pi}) = \beta_{2,j}\} \cap S_1$$

Security reduction for Funky[FPCP, FC]



Security reduction for Funky[FPCP, FC]

Reductor $\mathfrak{R}^{\tilde{P}_{\mathsf{ARG}}}$



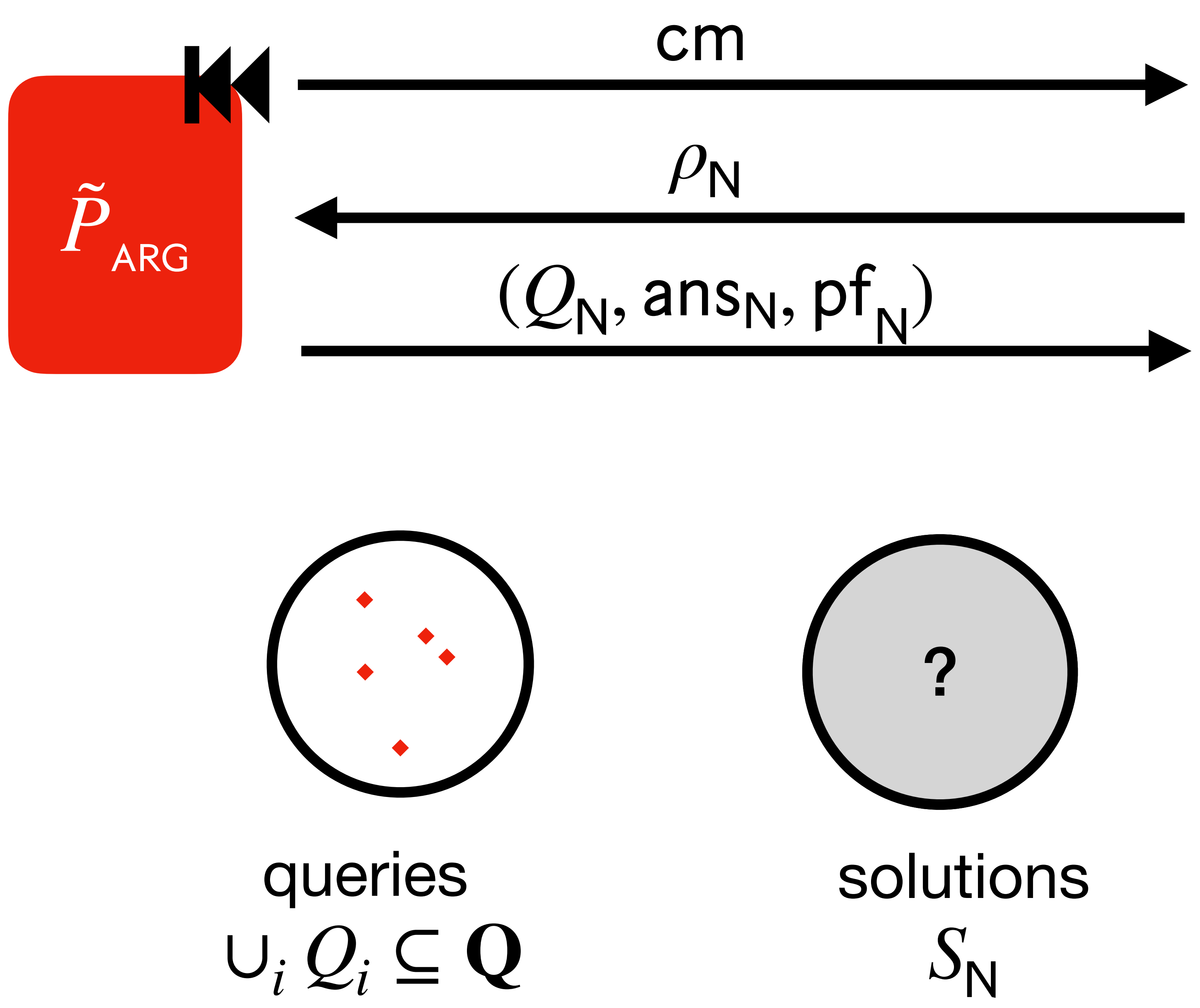
$S_{\mathsf{N}} = \{\tilde{\Pi} \mid \forall \alpha_{\mathsf{N},j} \in Q_{\mathsf{N}}, \alpha_{\mathsf{N},j}(\tilde{\Pi}) = \beta_{\mathsf{N},j}\} \cap S_{\mathsf{N}-1}$

Output $\tilde{\Pi} \leftarrow \mathsf{Solver}_{\mathbf{Q}}((\alpha_{i,j}, \beta_{i,j})_{i\in[\mathsf{N}], j\in[\mathsf{q}]})$

Outputs $\tilde{\Pi} \in S_{\mathsf{N}}$

Security reduction for Funky[FPCP, FC]

Reductor $\mathfrak{R}^{\tilde{P}_{\mathsf{ARG}}}$



$S_{\mathsf{N}} = \{\tilde{\Pi} \mid \forall \alpha_{\mathsf{N},j} \in Q_{\mathsf{N}}, \alpha_{\mathsf{N},j}(\tilde{\Pi}) = \beta_{\mathsf{N},j}\} \cap S_{\mathsf{N}-1}$

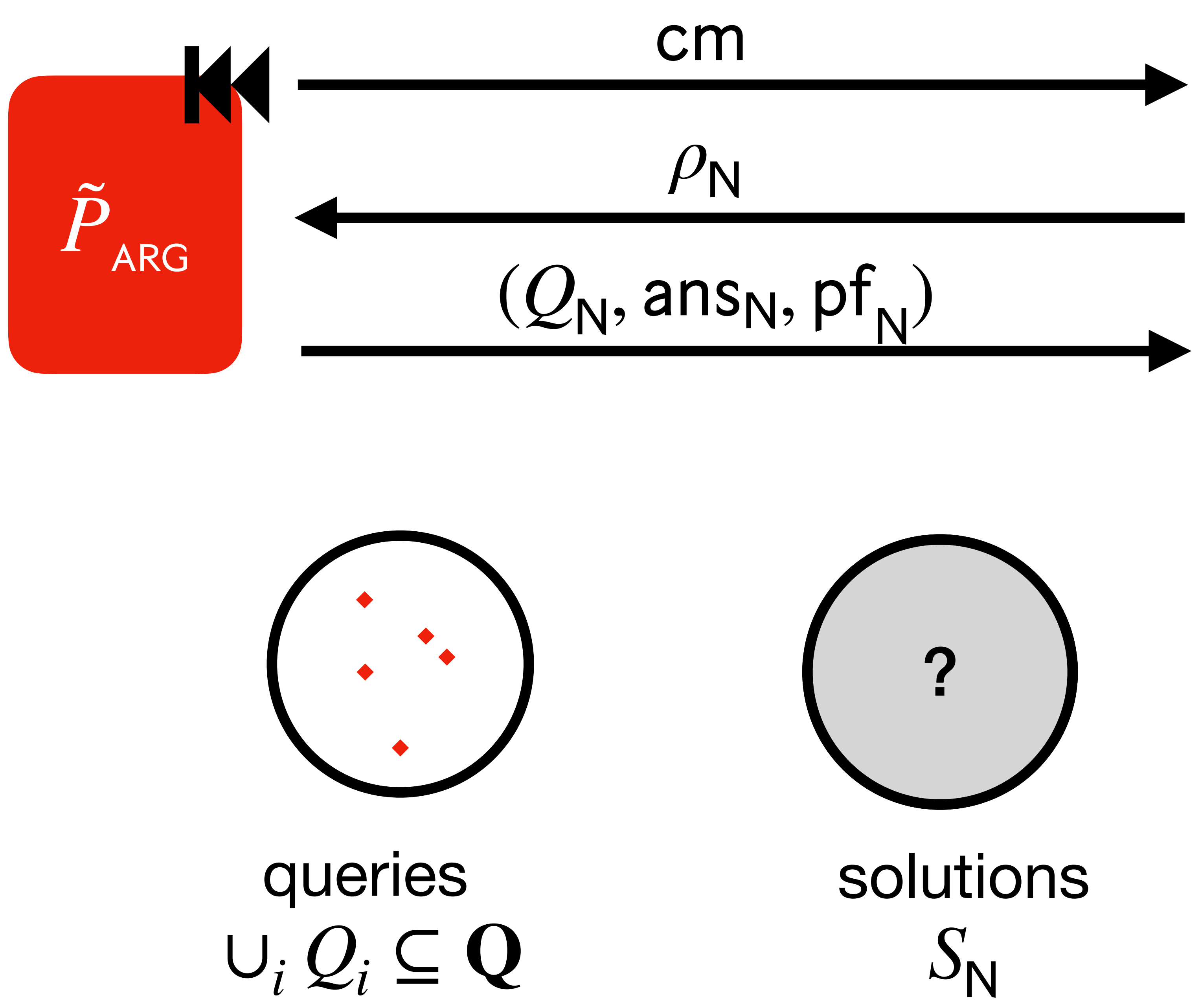
Output $\tilde{\Pi} \leftarrow \mathsf{Solver}_{\mathbf{Q}}((\alpha_{i,j}, \beta_{i,j})_{i \in [\mathsf{N}], j \in [\mathsf{q}]})$

Outputs $\tilde{\Pi} \in S_{\mathsf{N}}$

$S_{\mathsf{N}+1} \neq \emptyset \wedge S_{\mathsf{N}+1} = S_{\mathsf{N}}$ (some solutions)
→ FPCP verifier accepts $\tilde{\Pi}$

Security reduction for Funky[FPCP, FC]

Reductor $\mathfrak{R}^{\tilde{P}_{\mathsf{ARG}}}$



$S_{\mathsf{N}} = \{\tilde{\Pi} \mid \forall \alpha_{\mathsf{N},j} \in Q_{\mathsf{N}}, \alpha_{\mathsf{N},j}(\tilde{\Pi}) = \beta_{\mathsf{N},j}\} \cap S_{\mathsf{N}-1}$

Output $\tilde{\Pi} \leftarrow \mathsf{Solver}_{\mathbf{Q}}((\alpha_{i,j}, \beta_{i,j})_{i \in [\mathsf{N}], j \in [\mathsf{q}]})$

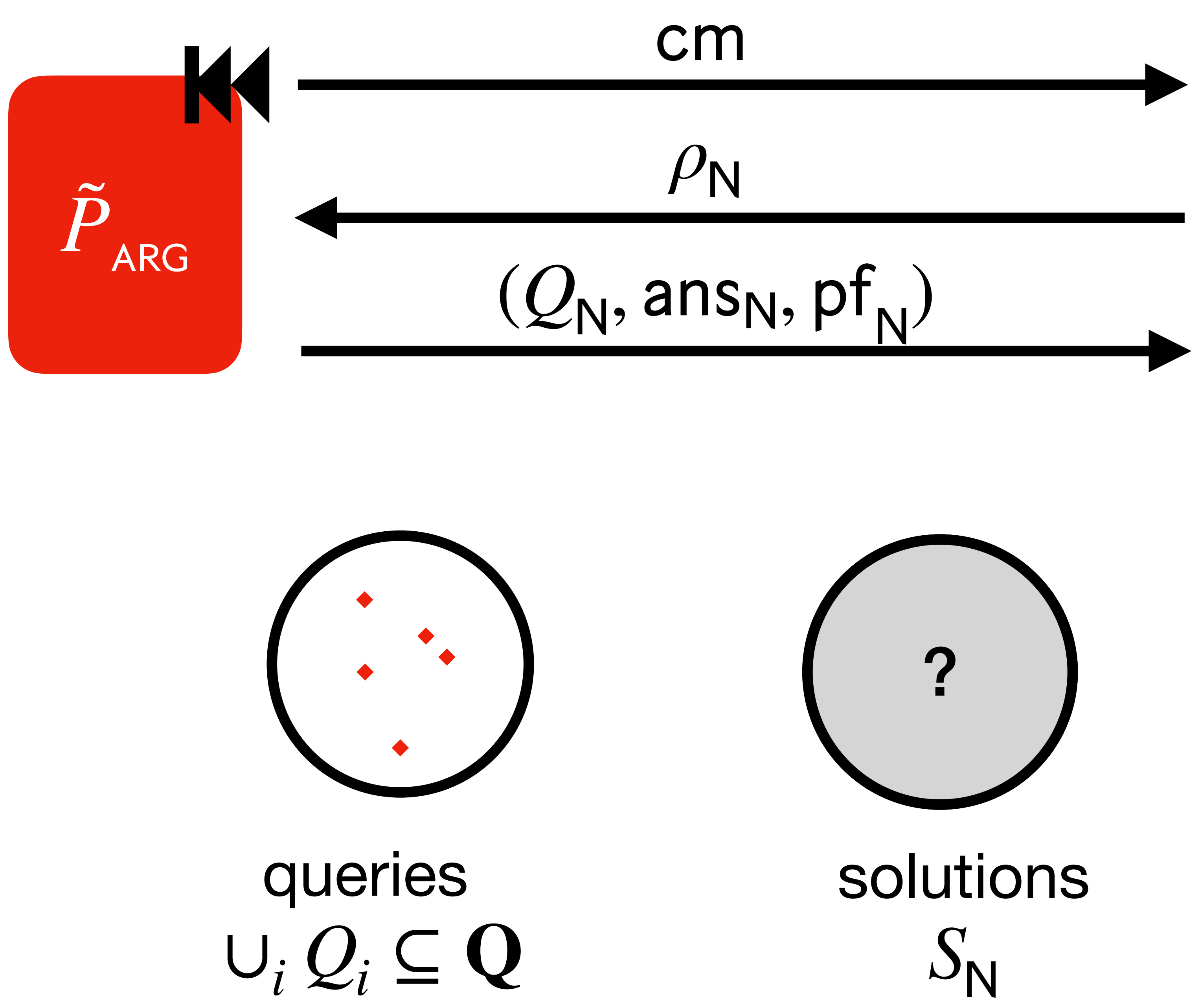
Outputs $\tilde{\Pi} \in S_{\mathsf{N}}$

$S_{\mathsf{N}+1} \neq \emptyset \wedge S_{\mathsf{N}+1} = S_{\mathsf{N}}$ (some solutions)
→ FPCP verifier accepts $\tilde{\Pi}$

$S_{\mathsf{N}+1} = \emptyset$ (no solution)
→ FC verifier accepts all FC proofs

Security reduction for Funky[FPCP, FC]

Reductor $\mathfrak{R}^{\tilde{P}_{\text{ARG}}}$



$S_{\text{N}} = \{\tilde{\Pi} \mid \forall \alpha_{\text{N},j} \in Q_{\text{N}}, \alpha_{\text{N},j}(\tilde{\Pi}) = \beta_{\text{N},j}\} \cap S_{\text{N}-1}$

Output $\tilde{\Pi} \leftarrow \text{Solver}_{\mathbf{Q}}((\alpha_{i,j}, \beta_{i,j})_{i\in[\text{N}],j\in[\text{q}]})$

Outputs $\tilde{\Pi} \in S_{\text{N}}$

$S_{\text{N}+1} \neq \varnothing \wedge S_{\text{N}+1} = S_{\text{N}}$ (some solutions)
→ FPCP verifier accepts $\tilde{\Pi}$

$S_{\text{N}+1} = \varnothing$ (no solution)
→ FC verifier accepts all FC proofs

$S_{\text{N}+1} \neq \varnothing \wedge S_{\text{N}+1} \neq S_{\text{N}}$
→ *tail error* of the query class $\mathbf{Q}$

Security reduction for Funky[FPCP, FC]

$\forall \tilde{P} : \forall x \notin L(R)$:

$\Pr\left[\langle \tilde{P}, V(x) \rangle = 1\right]$

Security reduction for Funky[FPCP, FC]

$\forall \tilde{P} : \forall x \notin L(R)$:

Produced by the reductor $\mathfrak{R}^{\tilde{P}_{\text{ARG}}}$

$$\Pr\left[\langle \tilde{P}, V(x)\rangle = 1\right] \le \Pr\left[\begin{array}{l}\text{Sample } \rho \\ \text{FPCP verifier accepts: } \mathbf{V}^{\tilde{\Pi}}(x;\rho) = 1 \\ \text{ARG verifier accepts: } V(x;\rho;Q,\text{ans},\text{pf})\rangle = 1\end{array}\right]$$

Produced by a t_{ARG}-time adversary $\tilde{P}_{\text{ARG}}$ given ρ

$$+ \Pr\left[\begin{array}{l}\text{Sample } \rho \\ \text{FPCP verifier rejects: } \mathbf{V}^{\tilde{\Pi}}(x;\rho) \neq 1 \\ \text{ARG verifier accepts: } V(x;\rho;Q,\text{ans},\text{pf})\rangle = 1\end{array}\right]$$

Security reduction for Funky[FPCP, FC]

$\forall \tilde{P} : \forall x \notin L(R)$:

$$\Pr\left[\langle \tilde{P}, V(x)\rangle = 1\right] \leq \Pr\begin{bmatrix}\text{Sample } \rho \\ \text{FPCP verifier accepts: } \mathbf{V}^{\tilde{\Pi}}(x;\rho) = 1 \\ \text{ARG verifier accepts: } V(x;\rho;Q,\text{ans},\text{pf}))\rangle = 1\end{bmatrix}$$

FPCP soundness

$\leq \epsilon_{\text{FPCP}}(\ell, \text{q})$

$$+ \Pr\begin{bmatrix}\text{Sample } \rho \\ \text{FPCP verifier rejects: } \mathbf{V}^{\tilde{\Pi}}(x;\rho) \neq 1 \\ \text{ARG verifier accepts: } V(x;\rho;Q,\text{ans},\text{pf}))\rangle = 1\end{bmatrix}$$

Security reduction for Funky[FPCP, FC]

$\forall \tilde{P} : \forall x \notin L(R)$:

$$\Pr\left[\langle \tilde{P}, V(x)\rangle = 1\right] \le \Pr\left[\begin{array}{l}\text{Sample } \rho \\ \text{FPCP verifier accepts: } \mathbf{V}^{\tilde{\Pi}}(x;\rho) = 1 \\ \text{ARG verifier accepts: } V(x;\rho;Q,\text{ans},\text{pf})\rangle = 1\end{array}\right]$$

FPCP soundness
$\le \epsilon_{\mathsf{FPCP}}(\ell, \mathsf{q})$

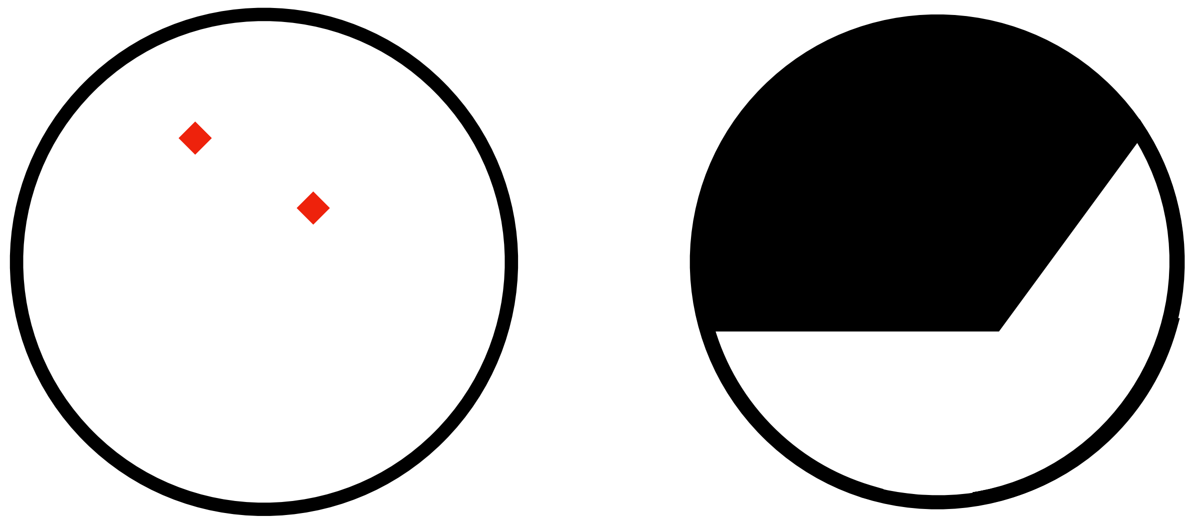
$$+ \Pr\left[\begin{array}{l}\text{Sample } \rho \\ \text{FPCP verifier rejects: } \mathbf{V}^{\tilde{\Pi}}(x;\rho) \neq 1 \\ \text{ARG verifier accepts: } V(x;\rho;Q,\text{ans},\text{pf})\rangle = 1\end{array}\right]$$

Security reduction lemma
$\le \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t_{\mathsf{FC}}) + \epsilon_{\mathbf{Q}}(\ell, \mathsf{q}, \mathsf{N})$

Security reduction lemma

queries $\cup_i Q_i$ solutions S_i

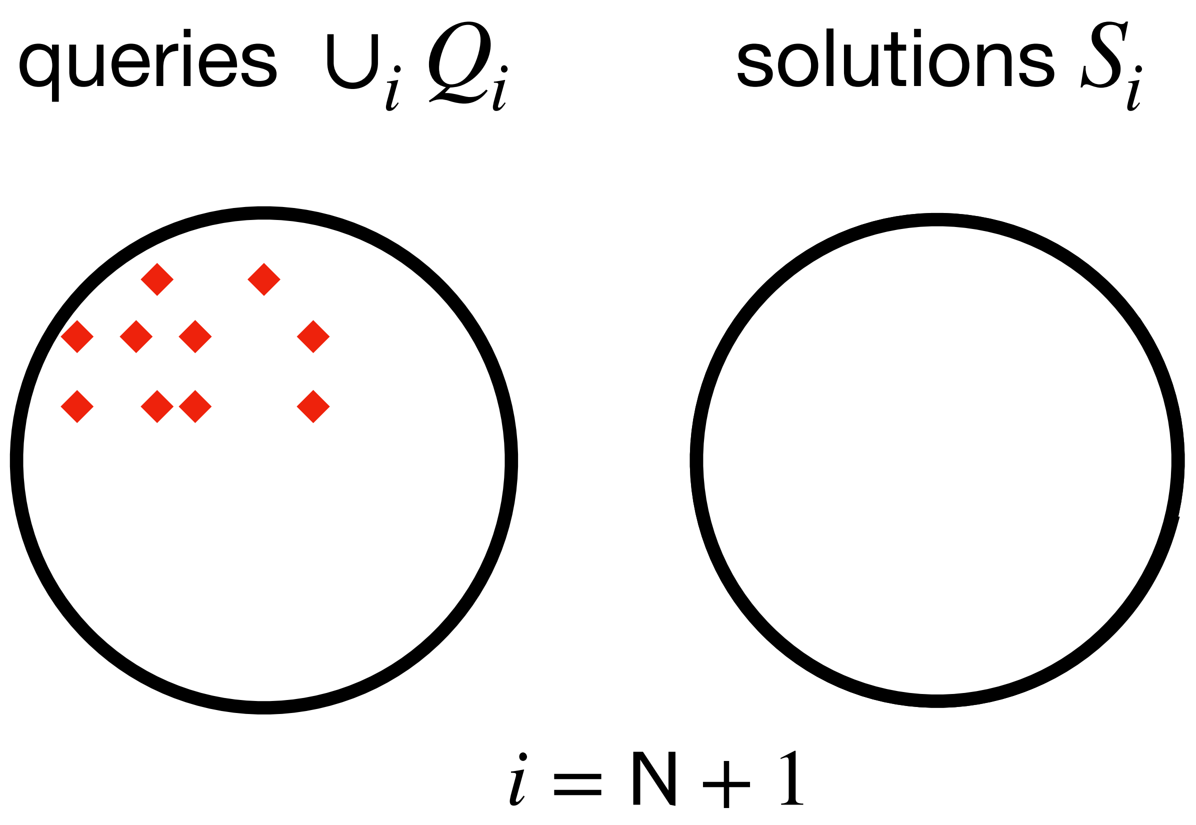
Case 1:



$i = 2$

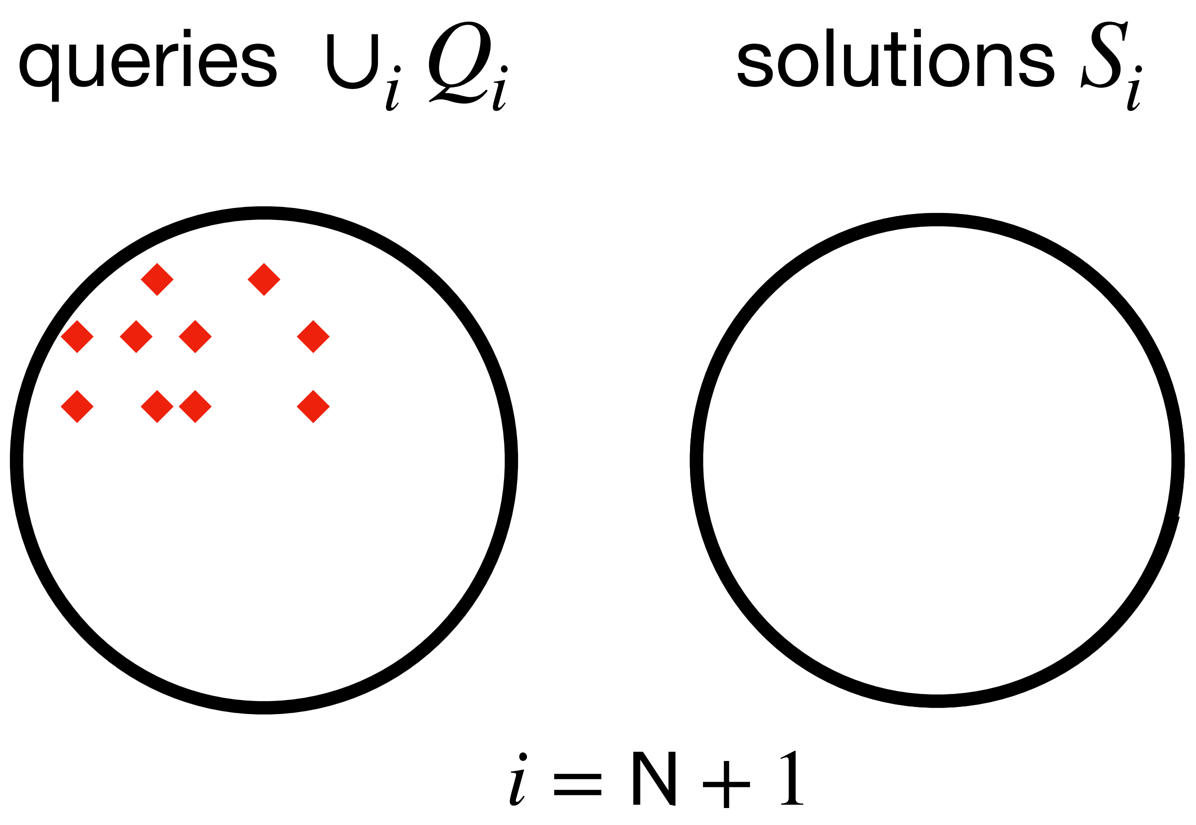
Security reduction lemma

Case 1:



Security reduction lemma

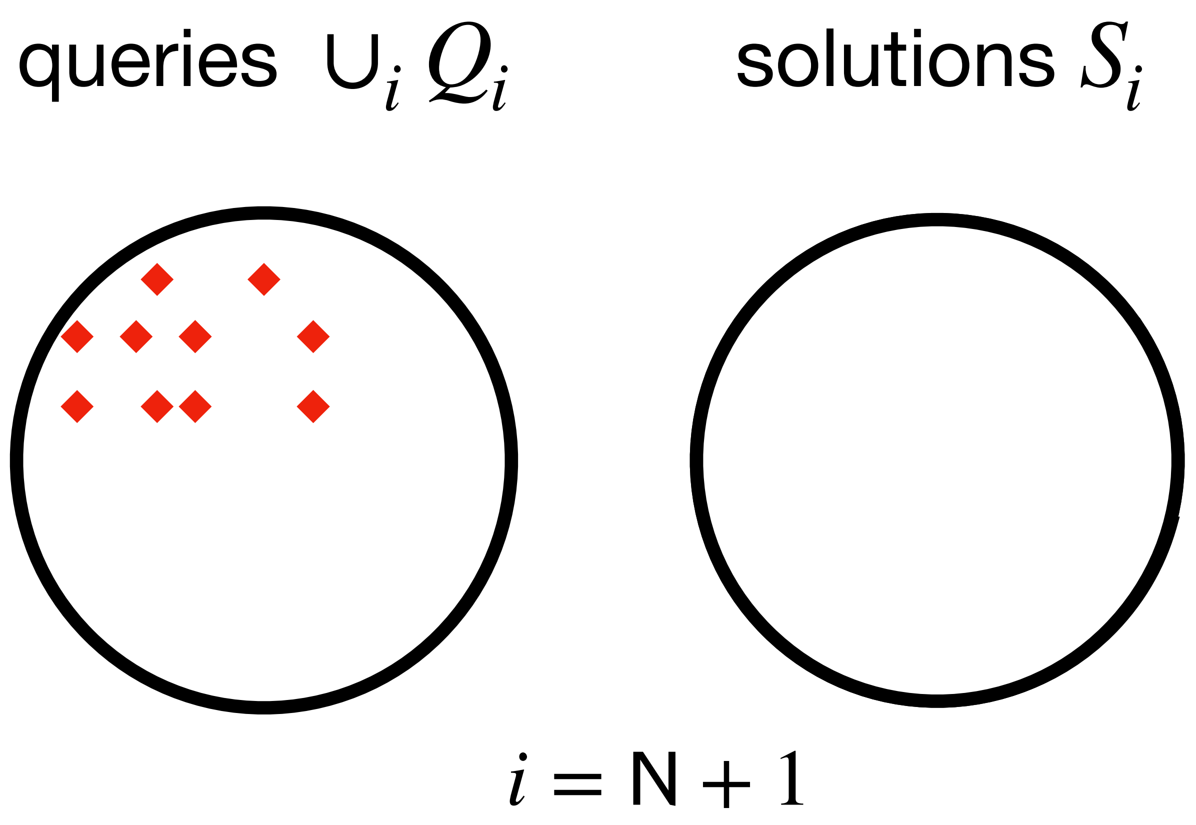
Case 1:



$$\left[\begin{array}{l} \nexists \tilde{\Pi} : \forall \alpha \in \cup_i Q_i : \alpha(\tilde{\Pi}) = \mathsf{ans}[\alpha] \\ \text{Yet all FC checks pass} \end{array}\right]$$

Security reduction lemma

Case 1:



$$\Pr\left[\begin{array}{l}\nexists \tilde{\Pi} : \forall \alpha \in \cup_i Q_i : \alpha(\tilde{\Pi}) = \mathsf{ans}[\alpha] \\ \text{Yet all FC checks pass}\end{array}\right] \leq \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t_{\mathsf{FC}})$$

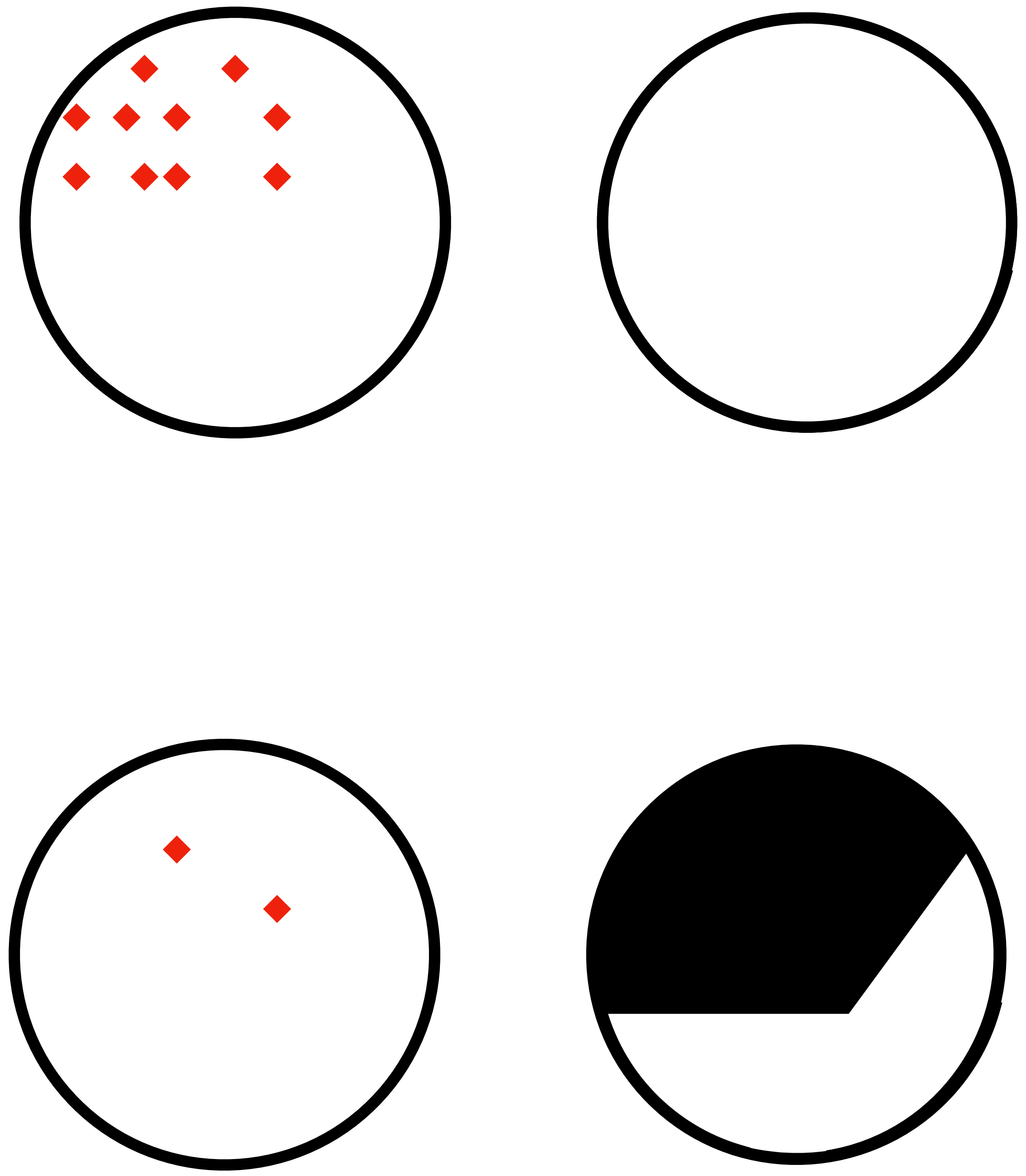
Function binding

Security reduction lemma

queries $\cup_i Q_i$ solutions S_i

Case 1:

Case 2:



$i = 2$

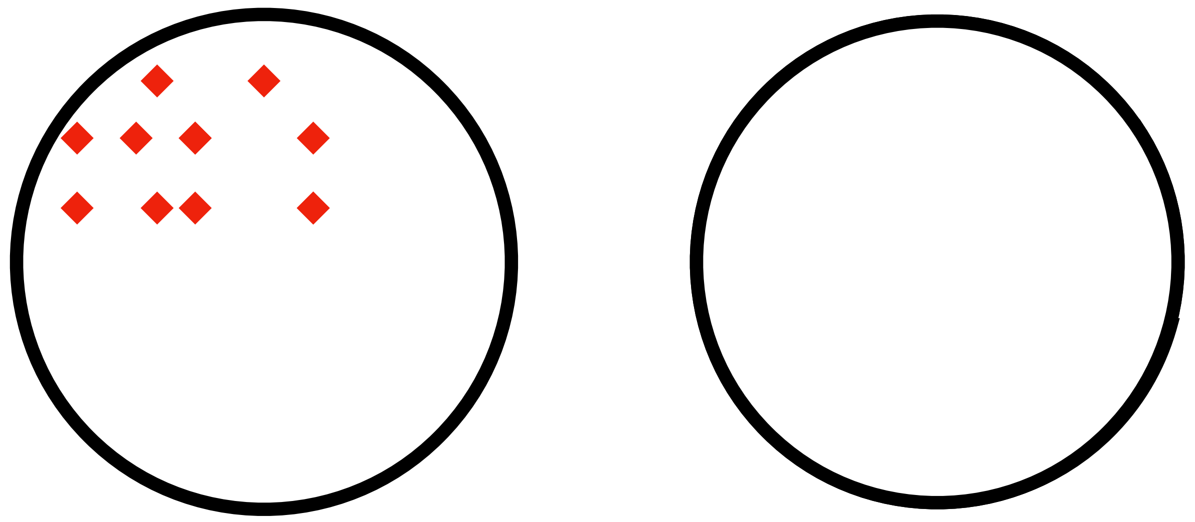
$$\Pr\left[\begin{array}{l}\nexists \tilde{\Pi} : \forall \alpha \in \cup_i Q_i : \alpha(\tilde{\Pi}) = \mathsf{ans}[\alpha] \\ \text{Yet all FC checks pass}\end{array}\right] \leq \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t_{\mathsf{FC}})$$

Function binding

Security reduction lemma

queries $\cup_i Q_i$ solutions S_i

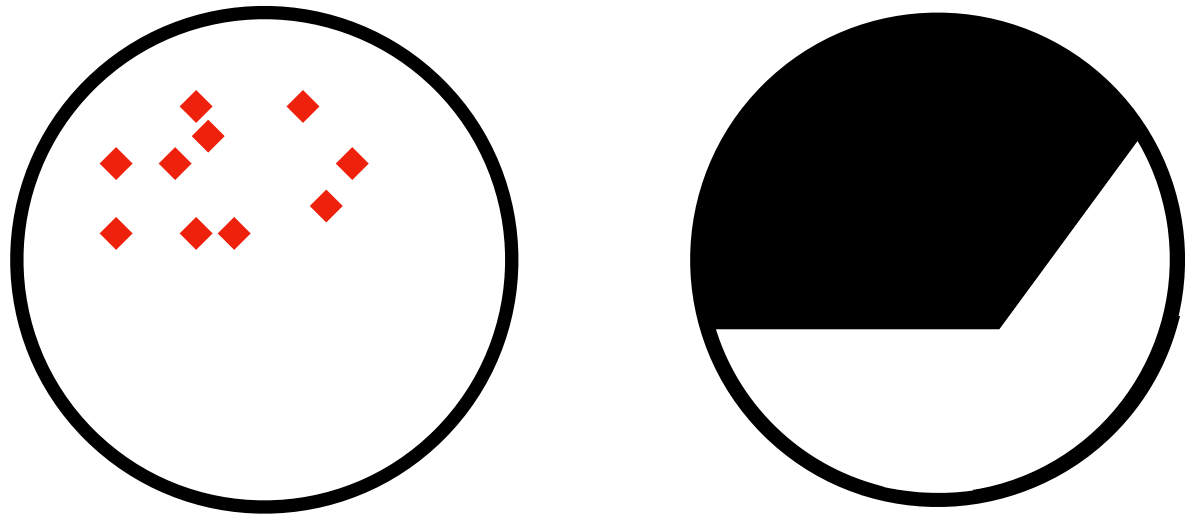
Case 1:



$$\Pr\left[\begin{array}{l}\nexists \tilde{\Pi} : \forall \alpha \in \cup_i Q_i : \alpha(\tilde{\Pi}) = \mathsf{ans}[\alpha] \\ \text{Yet all FC checks pass}\end{array}\right] \leq \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t_{\mathsf{FC}})$$

Function binding

Case 2:

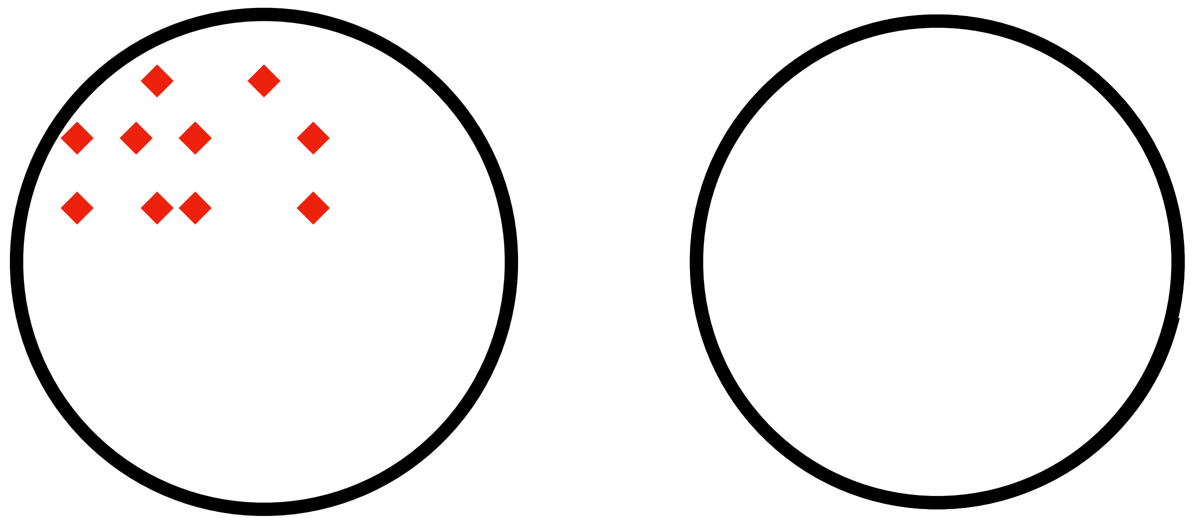


$i = \mathsf{N}$

Security reduction lemma

queries $\cup_i Q_i$ solutions S_i

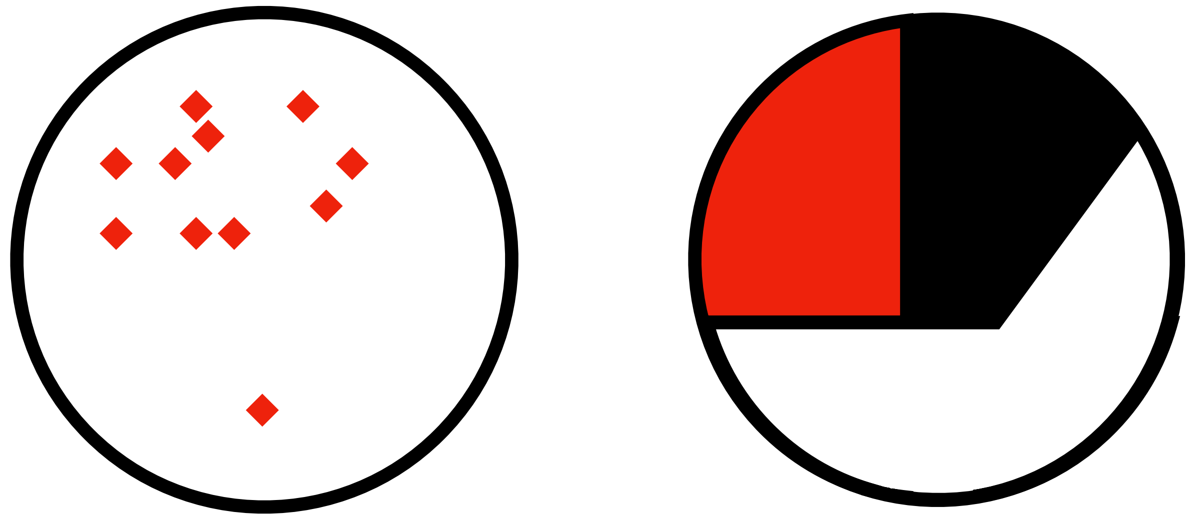
Case 1:



$$\Pr\left[\begin{array}{l}\nexists\tilde{\Pi} : \forall\alpha \in \cup_i Q_i : \alpha(\tilde{\Pi}) = \mathsf{ans}[\alpha] \\ \text{Yet all FC checks pass}\end{array}\right] \leq \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t_{\mathsf{FC}})$$

Function binding

Case 2:



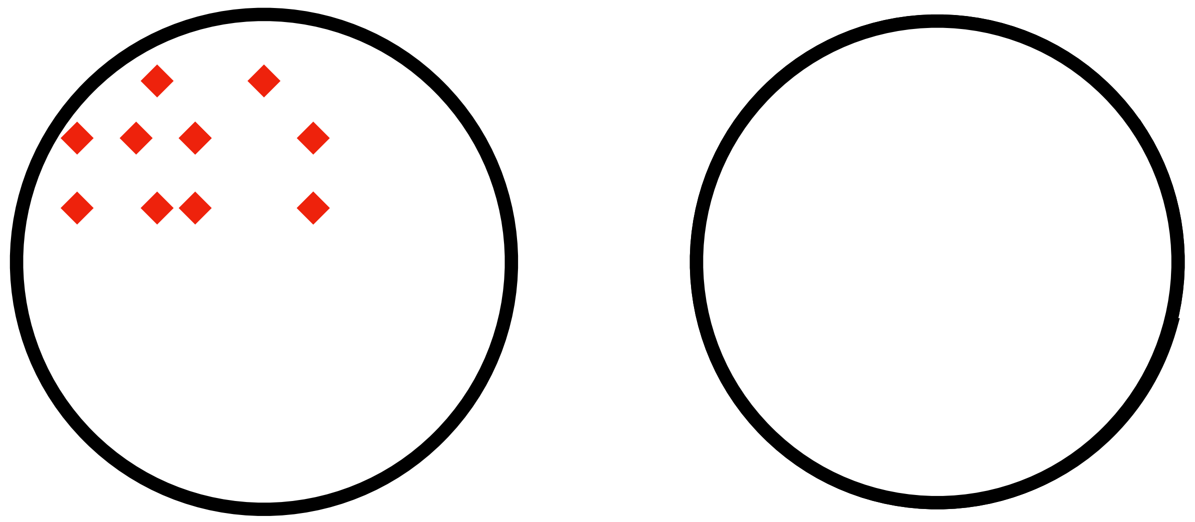
$i = \mathsf{N} + 1$

$$[S_{\mathsf{N}+1} \neq \emptyset \wedge S_{\mathsf{N}+1} \neq S_{\mathsf{N}}]$$

Security reduction lemma

queries $\cup_i Q_i$ solutions S_i

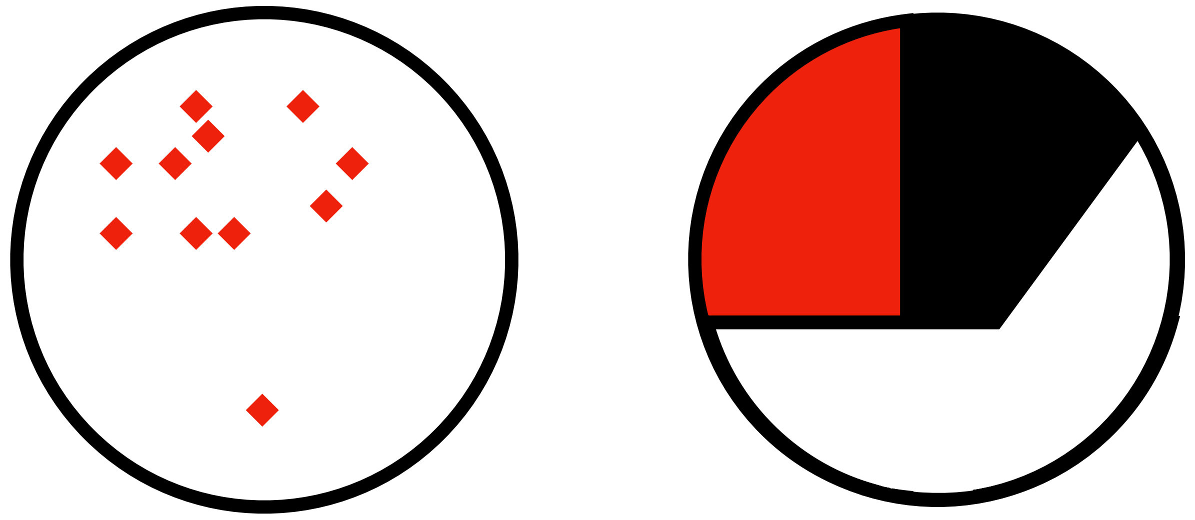
Case 1:



$$\Pr\left[\begin{array}{l} \nexists \tilde{\Pi} : \forall \alpha \in \cup_i Q_i : \alpha(\tilde{\Pi}) = \mathsf{ans}[\alpha] \\ \text{Yet all FC checks pass} \end{array}\right] \le \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t_{\mathsf{FC}})$$

Function binding

Case 2:



$i = \mathsf{N} + 1$

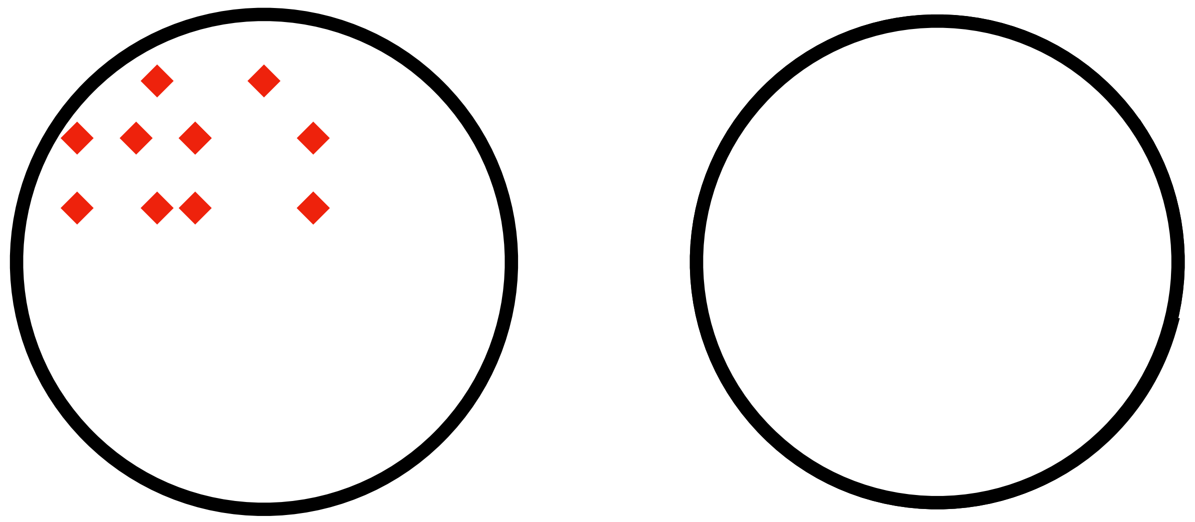
$$\Pr\left[S_{\mathsf{N}+1} \neq \emptyset \wedge S_{\mathsf{N}+1} \neq S_{\mathsf{N}}\right] \le \epsilon_{\mathbf{Q}}(\ell, \mathsf{N})$$

Tail error

Security reduction lemma

queries $\cup_i Q_i$ solutions S_i

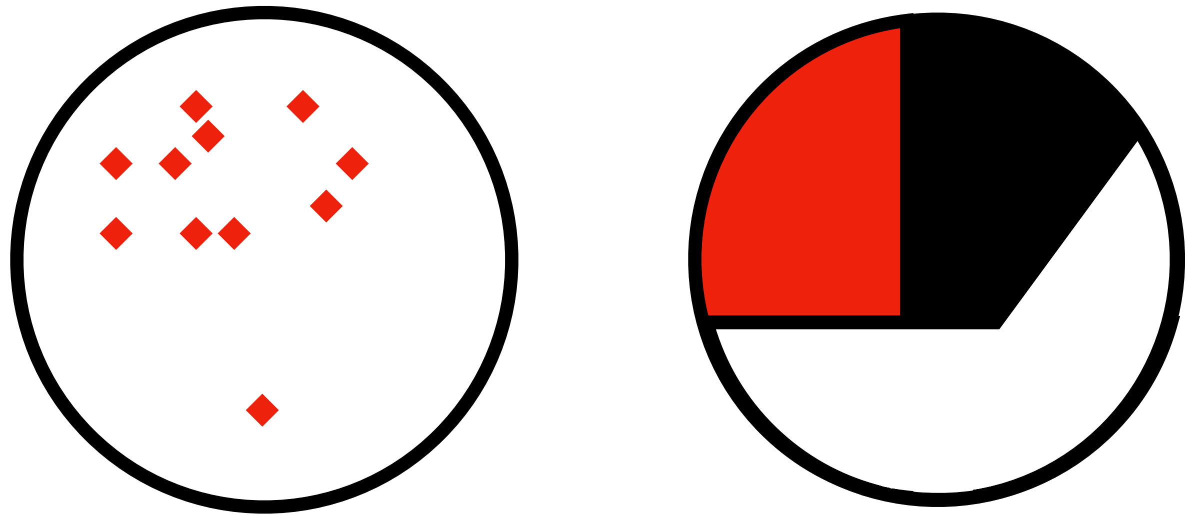
Case 1:



$$\Pr\left[\begin{array}{l}\nexists \tilde{\Pi} : \forall \alpha \in \cup_i Q_i : \alpha(\tilde{\Pi}) = \mathsf{ans}[\alpha] \\ \text{Yet all FC checks pass}\end{array}\right] \leq \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t_{\mathsf{FC}})$$

Function binding

Case 2:



$i = \mathsf{N} + 1$

depends on query class, but independent of FC!

$$\Pr\left[S_{\mathsf{N}+1} \neq \emptyset \wedge S_{\mathsf{N}+1} \neq S_{\mathsf{N}}\right] \leq \epsilon_{\mathbf{Q}}(\ell, \mathsf{N})$$

Tail error

Security reduction for Funky[FPCP, FC]

$$\epsilon_{\mathsf{ARG}}(\lambda, \ell, \mathsf{q}, t) \leq \epsilon_{\mathsf{FPCP}}(\ell, \mathsf{q}) + \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t \cdot \mathsf{N} + t_{\mathbf{Q}}) + \epsilon_{\mathbf{Q}}(\ell, \mathsf{N})$$

Security reduction for Funky[FPCP, FC]

$\leq \ell / \mathsf{N}$
for $\mathbf{Q}$ of interest

$$\epsilon_{\mathsf{ARG}}(\lambda, \ell, \mathsf{q}, t) \leq \epsilon_{\mathsf{FPCP}}(\ell, \mathsf{q}) + \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t \cdot \mathsf{N} + t_{\mathbf{Q}}) + \epsilon_{\mathbf{Q}}(\ell, \mathsf{N})$$

Security reduction for Funky[FPCP, FC]

poly$(\ell, \mathsf{q}, \mathsf{N})$
for $\mathbf{Q}$ of interest

$\leq \ell/\mathsf{N}$
for $\mathbf{Q}$ of interest

$$\epsilon_{\mathsf{ARG}}(\lambda, \ell, \mathsf{q}, t) \leq \epsilon_{\mathsf{FPCP}}(\ell, \mathsf{q}) + \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t \cdot \mathsf{N} + t_{\mathbf{Q}}) + \epsilon_{\mathbf{Q}}(\ell, \mathsf{N})$$

Security reduction for Funky[FPCP, FC]

poly$(\ell, \mathsf{q}, \mathsf{N})$ for $\mathbf{Q}$ of interest

$\leq \ell/\mathsf{N}$ for $\mathbf{Q}$ of interest

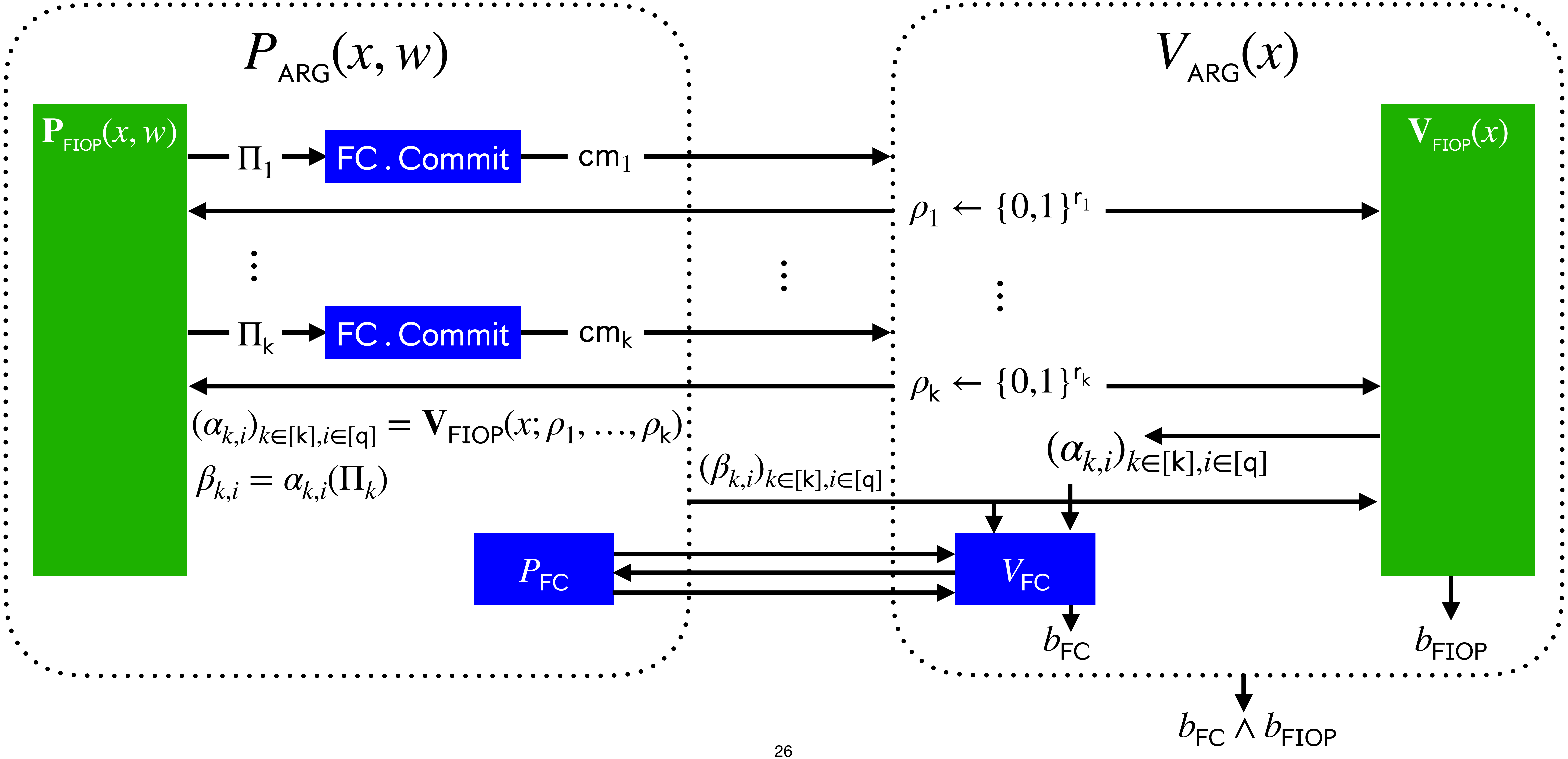
$$\epsilon_{\mathsf{ARG}}(\lambda, \ell, \mathsf{q}, t) \leq \epsilon_{\mathsf{FPCP}}(\ell, \mathsf{q}) + \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t \cdot \mathsf{N} + t_{\mathbf{Q}}) + \epsilon_{\mathbf{Q}}(\ell, \mathsf{N})$$

For $\mathbf{Q}$ of interest: $\leq \epsilon_{\mathsf{FPCP}}(\ell, \mathsf{q}) + \epsilon_{\mathsf{FC}}(\lambda, \ell, \mathsf{q}, t \cdot \ell/\epsilon + t_{\mathbf{Q}}) + \epsilon$

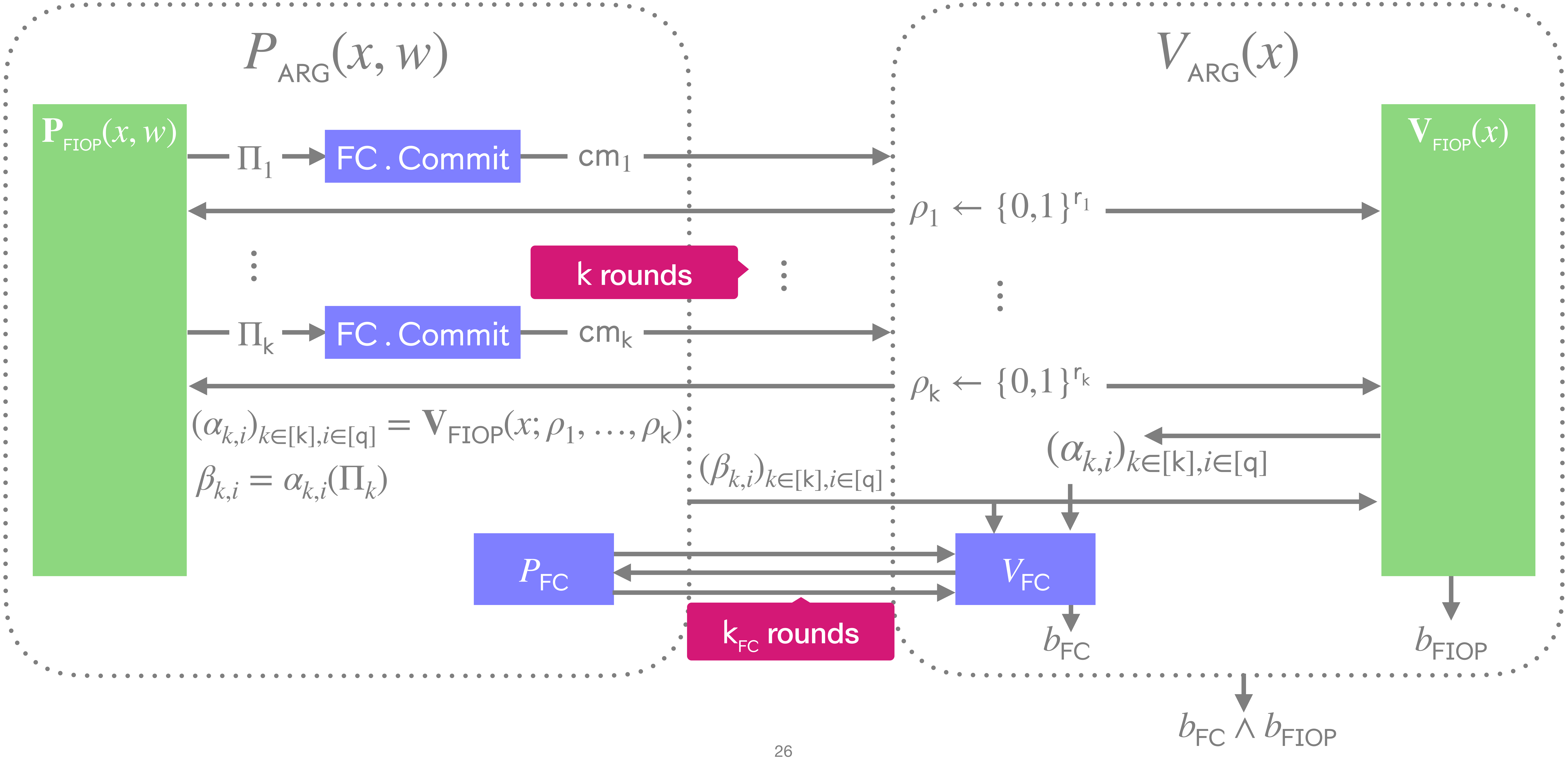
$\mathsf{N} := \ell/\epsilon$

Fiat–Shamir security of Funky in the ROM

The Funky protocol



The Funky protocol



State-restoration security of Funky[FIOP, FC]

State-restoration security of Funky[FIOP, FC]

Analogous for FIOP:

$$\epsilon_{\mathsf{ARG}}(t) \leq \epsilon_{\mathsf{FIOP}} + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}$$

State-restoration security of Funky[FIOP, FC]

Analogous for FIOP:

$$\epsilon_{\mathsf{ARG}}(t) \leq \epsilon_{\mathsf{FIOP}} + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}$$

RO queries

SR moves

Generically: $\epsilon_{\mathsf{NARG}}(m, t) \leq \epsilon^{\mathsf{SR}}_{\mathsf{ARG}}(m, t)$

State-restoration security of Funky[FIOP, FC]

Analogous for FIOP:

$$\epsilon_{\mathsf{ARG}}(t) \leq \epsilon_{\mathsf{FIOP}} + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}$$

RO queries

SR moves

Generically:

$$\epsilon_{\mathsf{NARG}}(m, t) \leq \epsilon^{\mathsf{SR}}_{\mathsf{ARG}}(m, t)$$
$$\leq (\epsilon_{\mathsf{FIOP}} + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}) \cdot (m + 1)^{\mathsf{k}}$$

State-restoration security of Funky[FIOP, FC]

Analogous for FIOP:

$$\epsilon_{\mathsf{ARG}}(t) \leq \epsilon_{\mathsf{FIOP}} + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}$$

RO queries

SR moves

k might be superconstant!

Generically:
$$\epsilon_{\mathsf{NARG}}(m, t) \leq \epsilon^{\mathsf{SR}}_{\mathsf{ARG}}(m, t)$$
$$\leq (\epsilon_{\mathsf{FIOP}} + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}) \cdot (m + 1)^{\mathsf{k}}$$

State-restoration security of Funky[FIOP, FC]

Analogous for FIOP:

$$\epsilon_{\mathsf{ARG}}(t) \leq \epsilon_{\mathsf{FIOP}} + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}$$

RO queries

SR moves

k might be superconstant!

Generically:

$$\begin{aligned}\epsilon_{\mathsf{NARG}}(m, t) &\leq \epsilon^{\mathsf{SR}}_{\mathsf{ARG}}(m, t) \\ &\leq (\epsilon_{\mathsf{FIOP}} + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}) \cdot (m + 1)^{\mathsf{k}}\end{aligned}$$

Theorem:

$$\epsilon^{\mathsf{SR}}_{\mathsf{ARG}}(m, t) \leq \epsilon^{\mathsf{SR}}_{\mathsf{FIOP}}(m + \mathsf{k}) + \epsilon^{\mathsf{SR}}_{\mathsf{FC}}(m \cdot \mathsf{kN}, t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}$$

State-restoration security of Funky[FIOP, FC]

Analogous for FIOP:

$$\epsilon_{\mathsf{ARG}}(t) \leq \epsilon_{\mathsf{FIOP}} + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}$$

RO queries

SR moves

k might be superconstant!

Generically:

$$\epsilon_{\mathsf{NARG}}(m, t) \leq \epsilon^{\mathsf{SR}}_{\mathsf{ARG}}(m, t)$$
$$\leq (\epsilon_{\mathsf{FIOP}} + \epsilon_{\mathsf{FC}}(t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}) \cdot (m + 1)^{\mathsf{k}}$$

Theorem:

$$\epsilon^{\mathsf{SR}}_{\mathsf{ARG}}(m, t) \leq \epsilon^{\mathsf{SR}}_{\mathsf{FIOP}}(m + \mathsf{k}) + \epsilon^{\mathsf{SR}}_{\mathsf{FC}}(m \cdot \mathsf{kN}, t \cdot \mathsf{kN} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}$$

$\leq \epsilon_{\mathsf{FIOP}} \cdot \mathsf{poly}(m + \mathsf{k})$
for FIOPs of interest

$\leq \epsilon_{\mathsf{FC}} \cdot \mathsf{poly}(m \cdot \mathsf{kN})$
for FCs of interest

Application:
Plonk is a SNARK in the ROM from the SDH assumption

Plonk’s security

iPlonk = Funky[PlonkIOP, linKZG]
Plonk = FS[iPlonk]

Plonk's security

constant-round FIOP

iPlonk = Funky[PlonkIOP, linKZG]

Plonk = FS[iPlonk]

Plonk's security

constant-round FIOP

one-round FC

iPlonk = Funky[PlonkIOP, linKZG]

Plonk = FS[iPlonk]

Plonk's security

Query class: evaluations of structured polynomials $\mathbf{Q}_{\text{poly}}{}^{*}$

constant-round FIOP

one-round FC

iPlonk = Funky[PlonkIOP, linKZG]

Plonk = FS[iPlonk]

Plonk’s security

Query class: evaluations of structured polynomials $\mathbb{Q}_{\mathrm{poly}}{}^{*}$

constant-round FIOP

one-round FC

iPlonk = Funky[PlonkIOP, linKZG]

Plonk = FS[iPlonk]

Not special sound!
(assuming DLog [LPS24b])

Plonk's security

Query class: evaluations of structured polynomials $\mathbf{Q}_{\text{poly}}{}^{*}$

constant-round FIOP

one-round FC

iPlonk = Funky[PlonkIOP, linKZG]

Plonk = FS[iPlonk]

Plonk's security

Query class: evaluations of structured polynomials $\mathbf{Q}_{\text{poly}}{}^{*}$

constant-round FIOP

one-round FC

iPlonk = Funky[PlonkIOP, linKZG]

Security in idealized models

Knowledge-sound in AGM [GWC19]

Plonk's security

Query class: evaluations of structured polynomials $\mathbf{Q}_{\text{poly}}{}^{*}$

constant-round FIOP

one-round FC

iPlonk = Funky[PlonkIOP, linKZG]

Security in idealized models

Knowledge-sound in AGM [GWC19]

Security in standard model

Special-sound from new ARSDH and SplitRSDH assumptions [LPS24b]

Plonk's security

Query class: evaluations of structured polynomials $\mathbf{Q}_{\text{poly}}{}^{*}$

constant-round FIOP

one-round FC

iPlonk = Funky[PlonkIOP, linKZG]

Security in idealized models

Knowledge-sound in AGM [GWC19]

Security in standard model

Special-sound from new ARSDH and SplitRSDH assumptions [LPS24b]

Corollary:

iPlonk is state-restoration (knowledge) sound
assuming Strong Diffie-Hellman

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Lemma:

PlonkIOP is state-restoration (knowledge) sound

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constant-round FIOP

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Corollary:

iPlonk is state-restoration (knowledge) sound *assuming SDH*

Lemma:

PlonkIOP is state-restoration (knowledge) sound

Lemma:

linKZG is state-restoration function binding *assuming SDH*

Plonk's security

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$\mathbf{Q}_{\text{poly}^*}$ has good tail error and solver time

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linKZG is state-restoration function binding

linKZG = lin[batchKZG] is state-restoration function binding

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$\boxed{\text{Lemma: lin[bPC] SR-FB} \Leftarrow \text{bPC SR-FB for any homomorphic bPC}}$

$\Leftarrow$ batchKZG = batch[KZG] is state-restoration function binding

linKZG is state-restoration function binding

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$\Leftarrow$ batchKZG = batch[KZG] is state-restoration function binding

Lemma: batch[PC] SR-FB $\Leftarrow$ PC SR-FB for any homomorphic PC

$\Leftarrow$ KZG is state-restoration function binding

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Lemma: batch[PC] SR-FB $\Leftarrow$ PC SR-FB for any homomorphic PC

$\Leftarrow$ KZG is state-restoration function binding

$\Leftarrow$ KZG is function binding (KZG is non-interactive)

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Lemma: lin[bPC] SR-FB ⇐ bPC SR-FB for any homomorphic bPC

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Lemma: batch[PC] SR-FB ⇐ PC SR-FB for any homomorphic PC

⇐ KZG is state-restoration function binding

⇐ KZG is function binding (KZG is non-interactive)

Lemma: PC FB ⇐ PC EB for any homomorphic PC

⇐ KZG is evaluation binding

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$\boxed{\text{Lemma: batch[PC] SR-FB} \Leftarrow \text{PC SR-FB for any homomorphic PC}}$

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$\boxed{\text{Lemma: PC FB} \Leftarrow \text{PC EB for any homomorphic PC}}$

$\Leftarrow$ KZG is evaluation binding

$\Leftarrow$ *Strong Diffie-Hellman* [KZG10]

Our results

ARG = Funky[FIOP, FC] for a query class $\mathbf{Q}$

FIOP is state-restoration (knowledge) sound

+ FC is state-restoration function binding

$\Rightarrow$ ARG = Funky[FIOP, FC] is state-restoration (knowledge) sound

$$\epsilon^{\mathsf{SR}}_{\mathsf{ARG}}(\mathsf{k}, \ell, t) \leq \epsilon^{\mathsf{SR}}_{\mathsf{FIOP}}(\mathsf{k}, \ell) + \epsilon^{\mathsf{SR}}_{\mathsf{FC}}(t \cdot \mathsf{k} \cdot \mathsf{N} + t_{\mathbf{Q}} \cdot \mathsf{k}) + \mathsf{k} \cdot \epsilon_{\mathbf{Q}}(\ell, \mathsf{q}, \mathsf{N})$$

Application: Plonk = FS[Funky[PlonkIOP, linearized KZG]]

is a SNARK in the ROM

from the SDH assumption (previously: from ARSDH+SplitRSDH)

On the Fiat–Shamir Security of Succinct Arguments from Functional Commitments

Alessandro Chiesa, Ziyi Guan, Christian Knabenhans, Zihan Yu

EPFL

Tail error and solving time

	Messages	Answers		Tail error $\epsilon_{\mathbf{Q}}(\ell, \mathrm{N})$	Solving time $t_{\mathbf{Q}}(\ell, \mathrm{q}, \mathrm{N})$

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Point queries	$\Pi \in \mathbb{F}^{\ell}$	$\beta = \Pi[\alpha]$	fill	ℓ / N	$O(\mathrm{qN})$

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Evaluation queries on polynomials	$\Pi \in \mathbb{F}[X]^{<\ell}$	$\beta = \Pi(\alpha)$	interpolation	ℓ/N	$O(\ell^2 + qN\ell)$

Tail error and solving time

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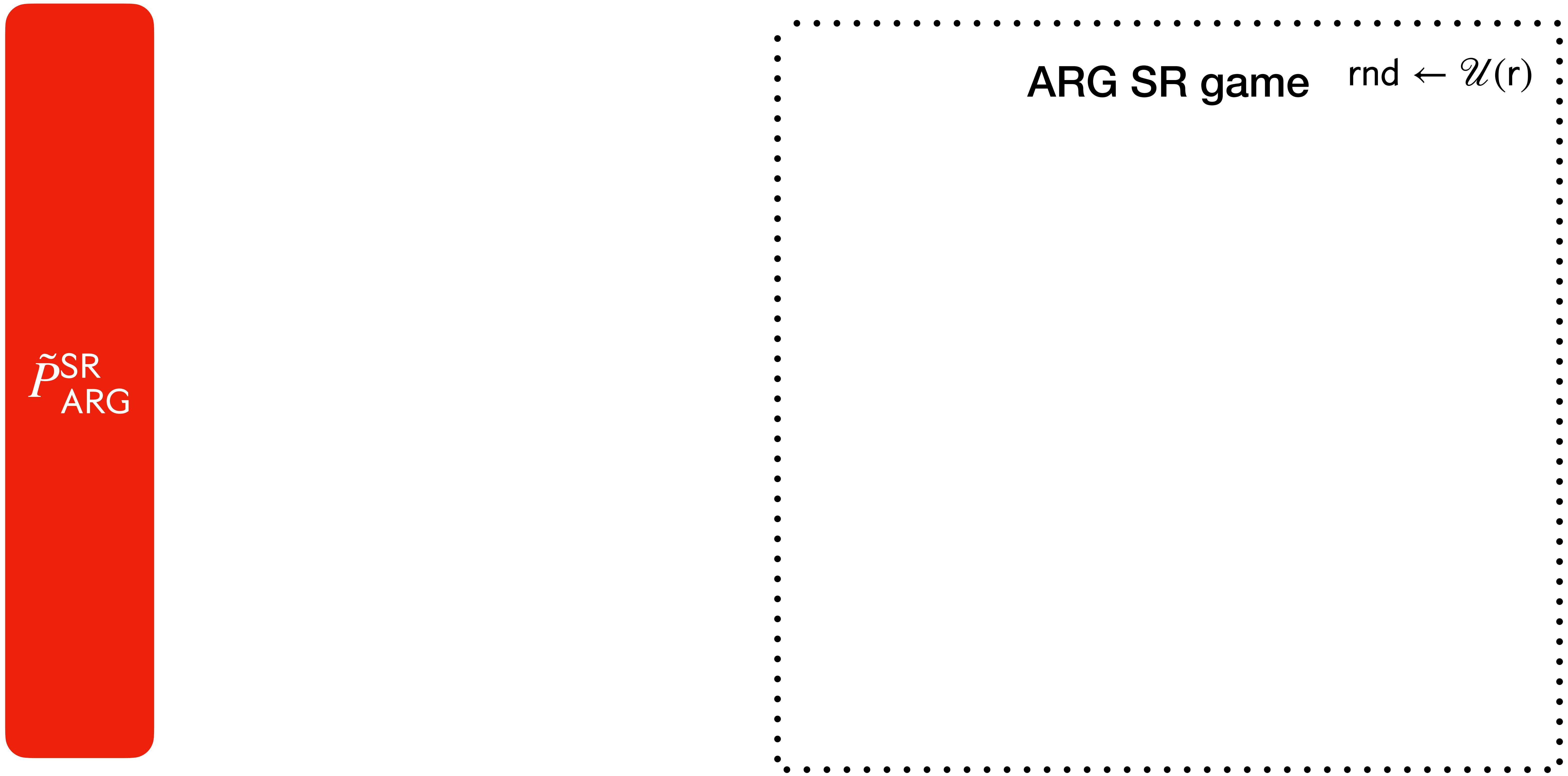
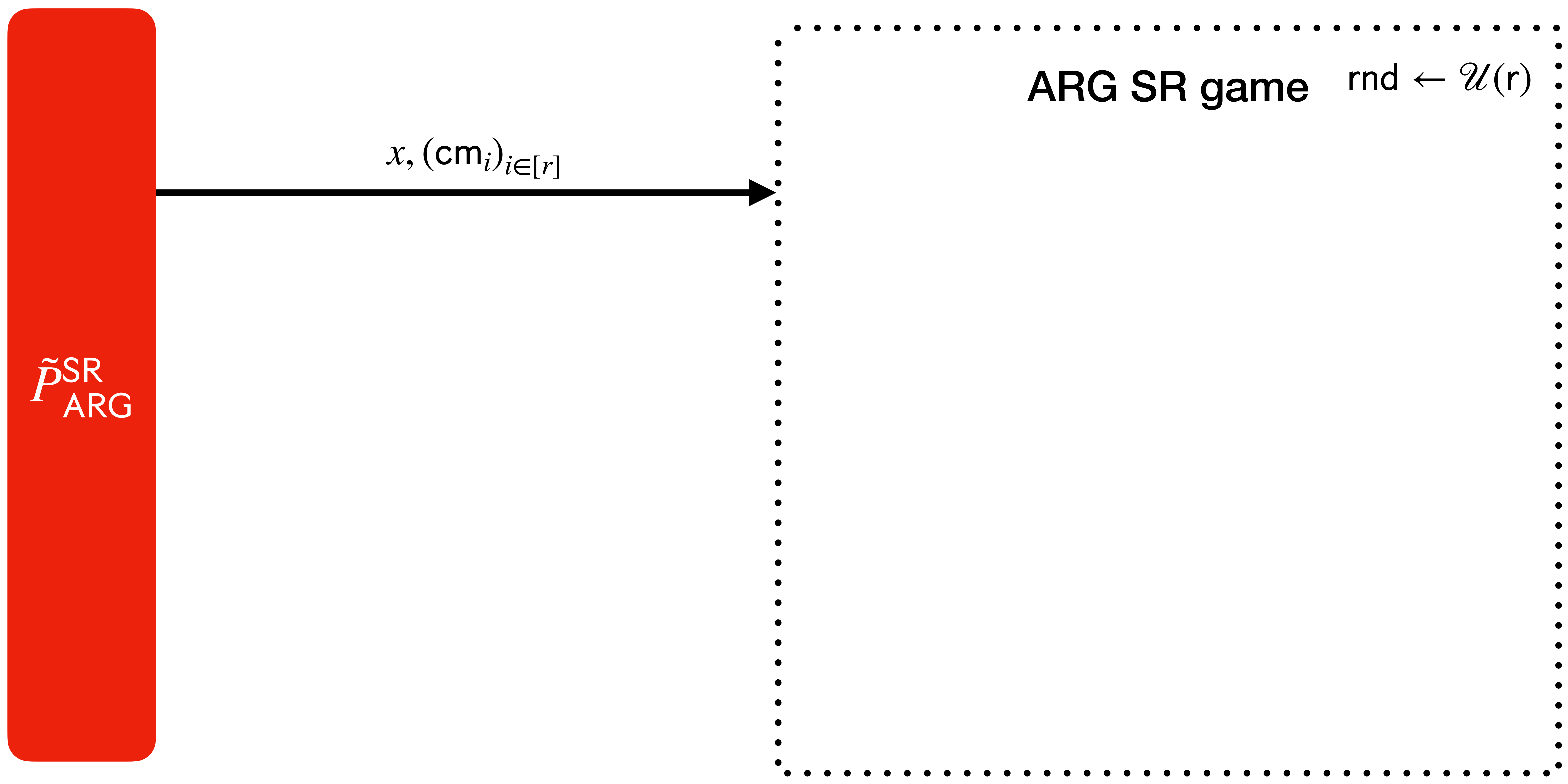
Linear queries	$\Pi \in \mathbb{F}^{\ell}$	$\beta = \sum_{i\in[\ell]} \Pi[i] \cdot \alpha[i]$	Gaussian elimination	ℓ/N	$O(\mathsf{qN}\ell^2)$
Evaluation queries on polynomials	$\Pi \in \mathbb{F}[X]^{<\ell}$	$\beta = \Pi(\alpha)$	interpolation	ℓ/N	$O(\ell^2 + \mathsf{qN}\ell)$
Evaluation queries on structured polynomials	$\Pi \in (\mathbb{F}[X]^{\leq \mathsf{D}})^{m+n} = (f_1, \ldots, f_m, g_1, \ldots, g_n)$	$\beta = \sum_{k\in[n]} h_k(f_1(\alpha), \cdots, f_m(\alpha)) \cdot g_k(\alpha)$		ℓ/N $\ell = (m+n)(\mathsf{D}+1)$	$O(\mathsf{qN}\ell^2)$

Security reduction for Funky[FIOP, FC]



Security reduction for Funky[FIOP, FC]



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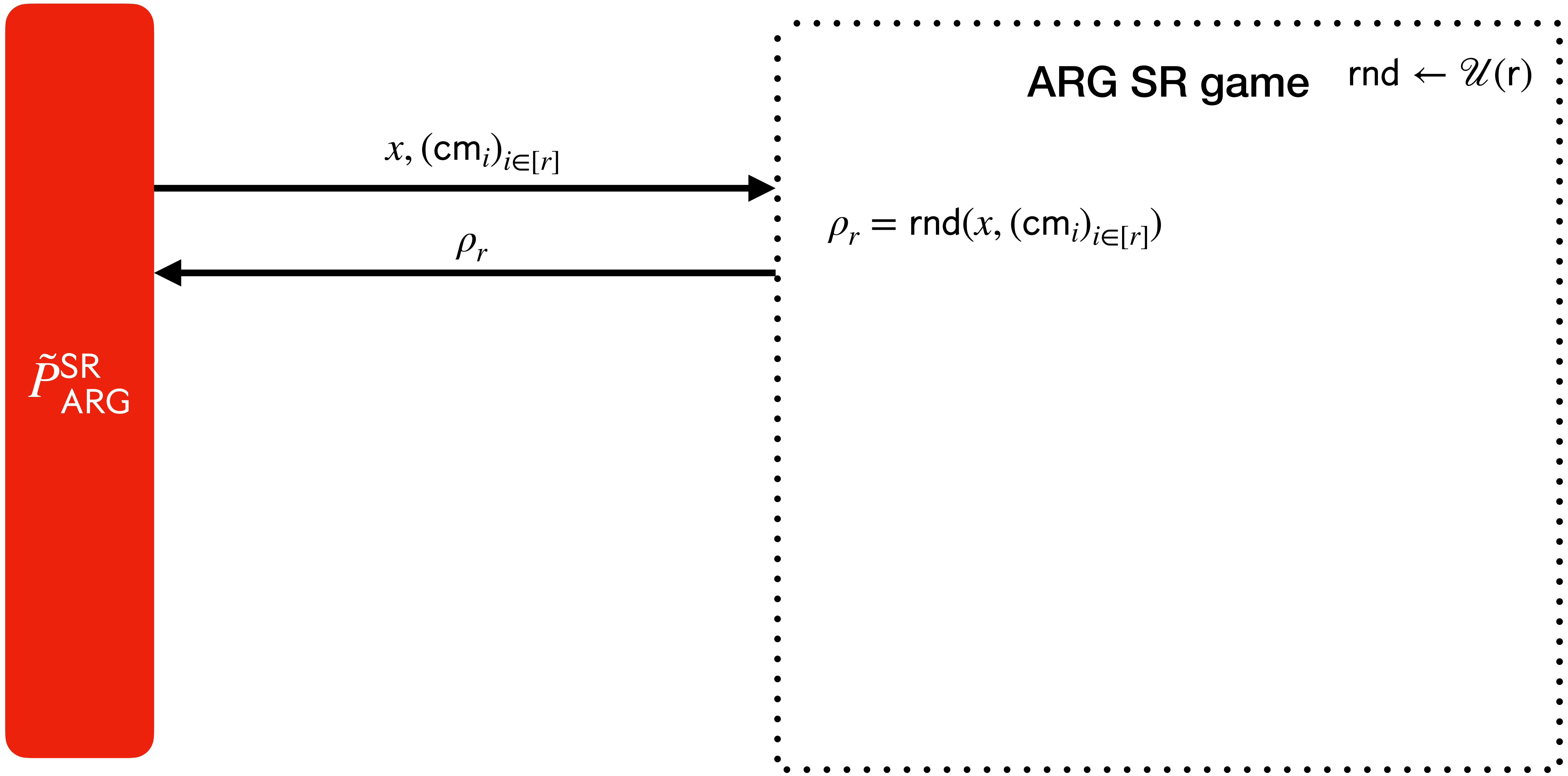

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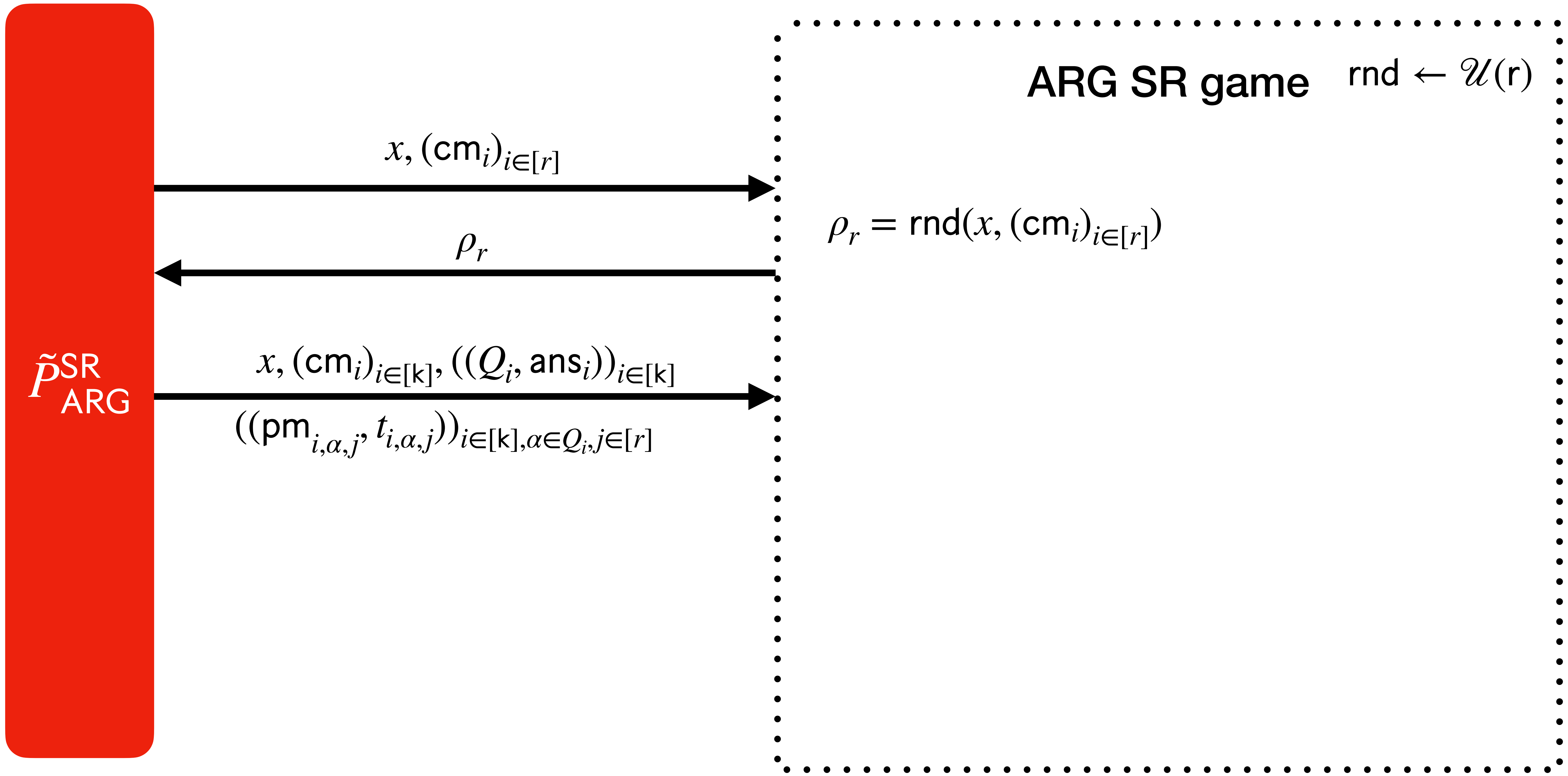

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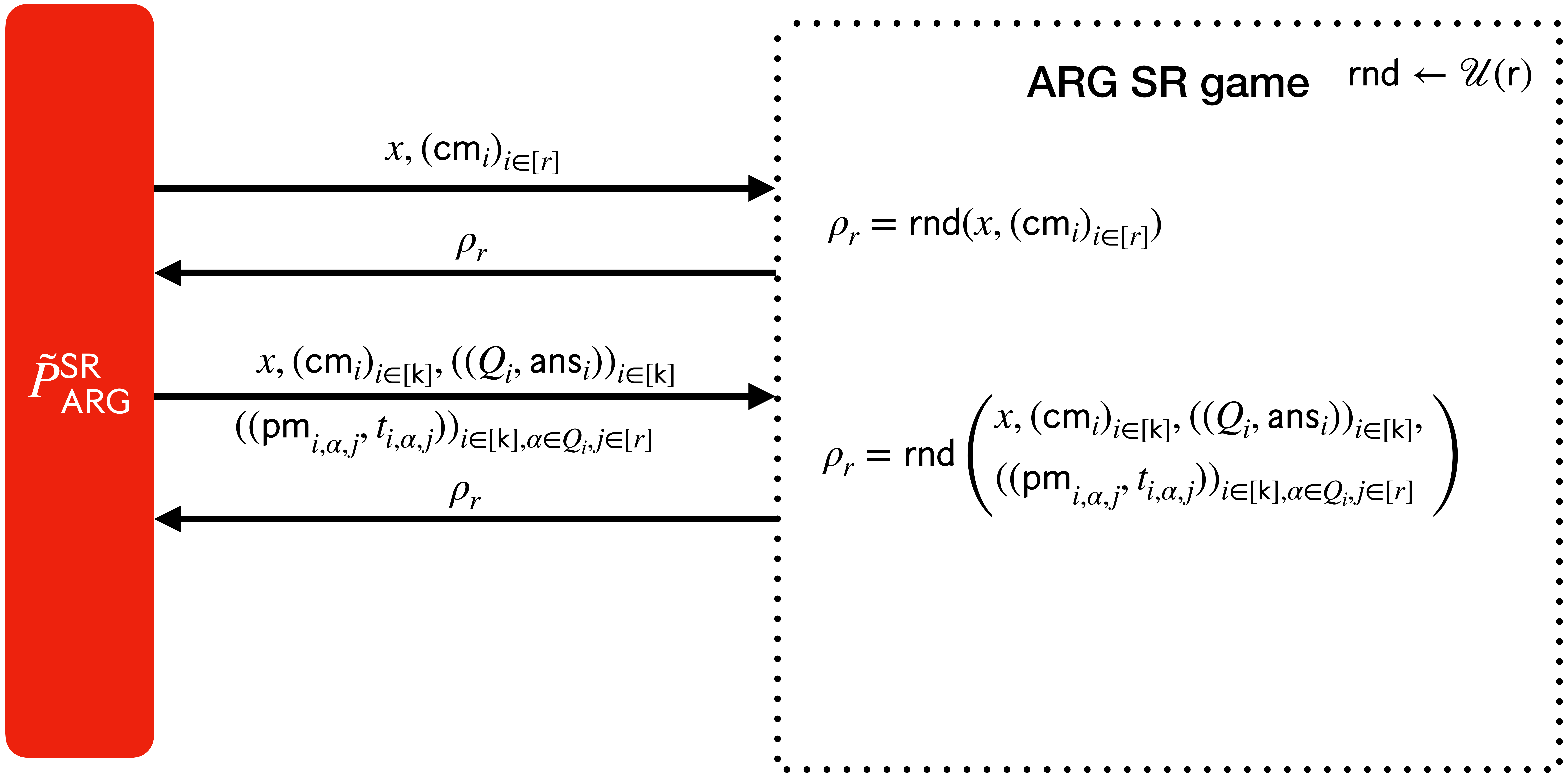

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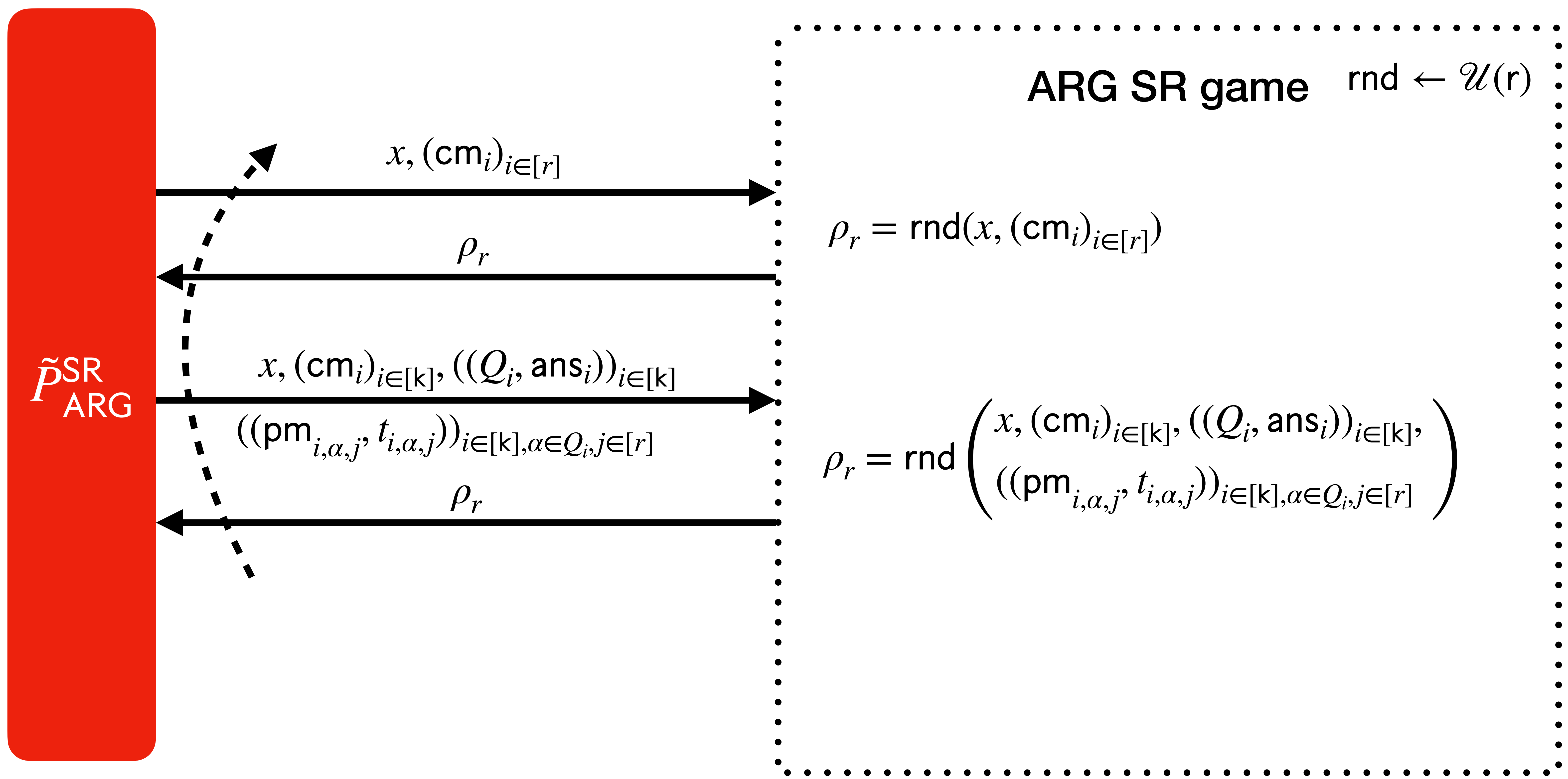

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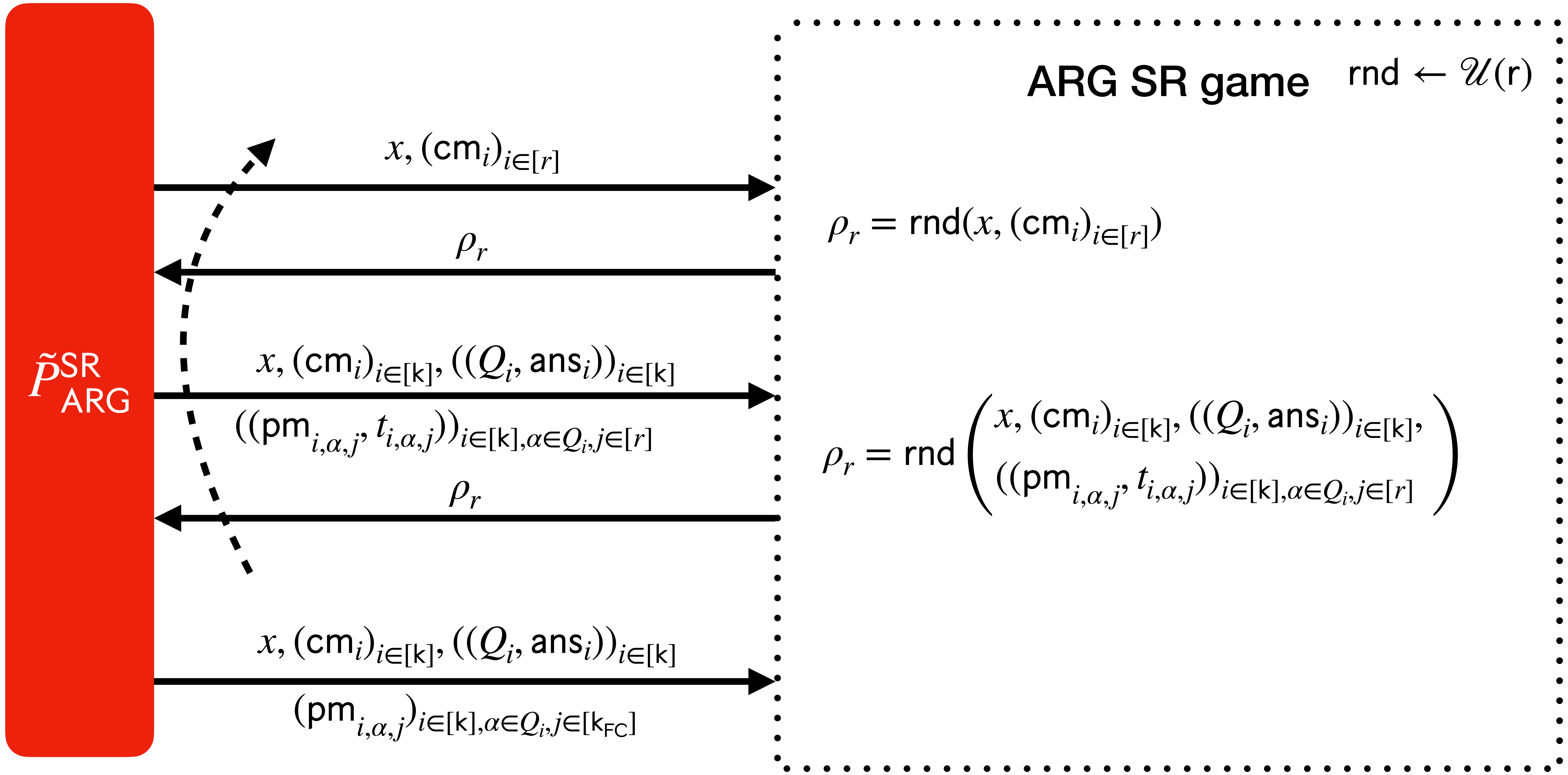

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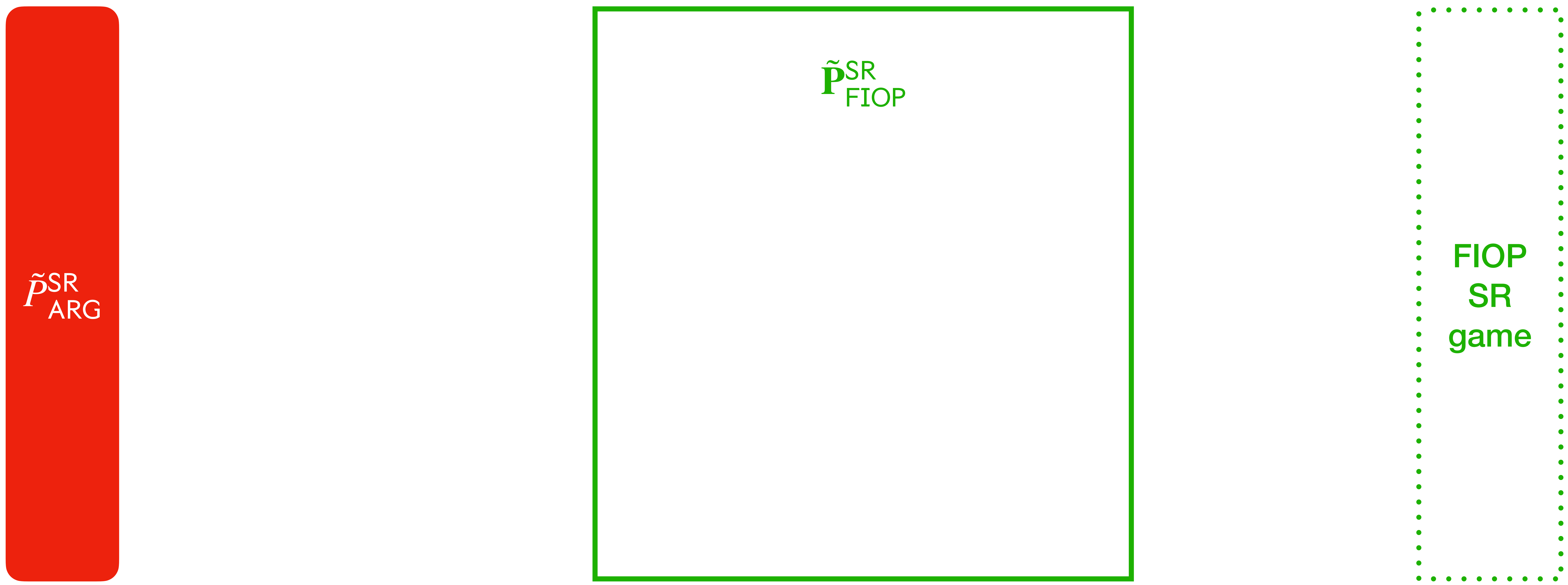

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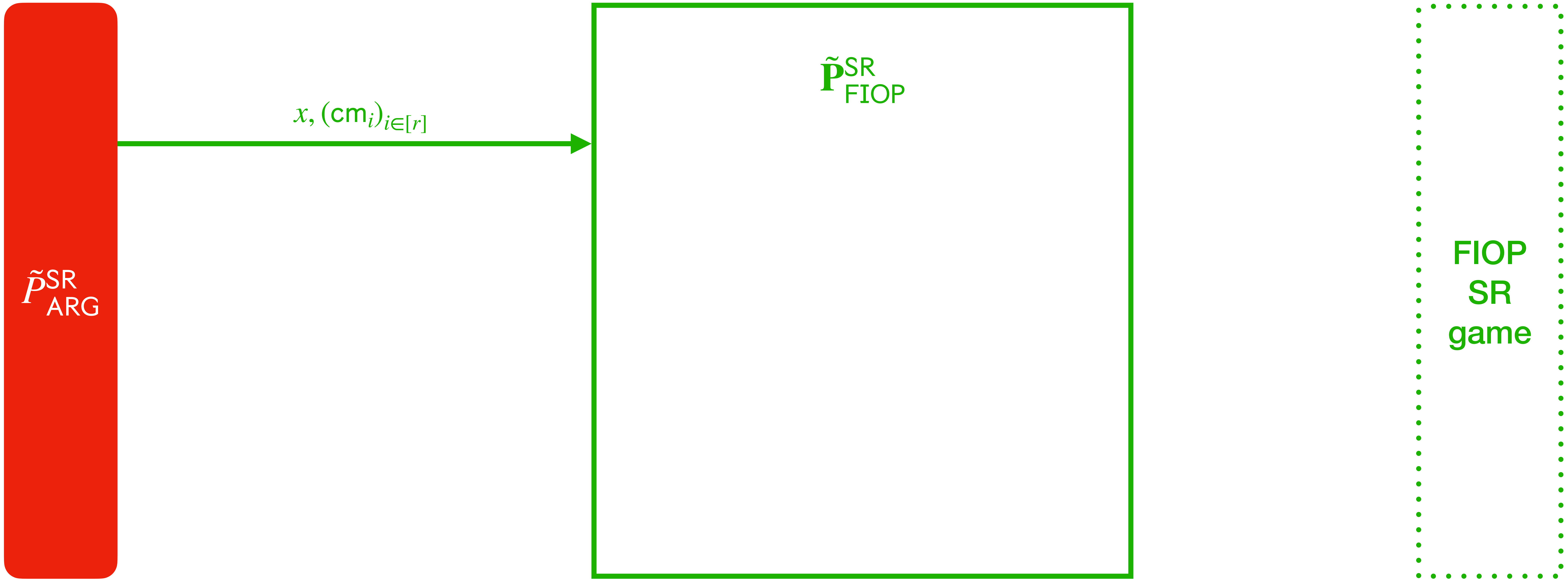

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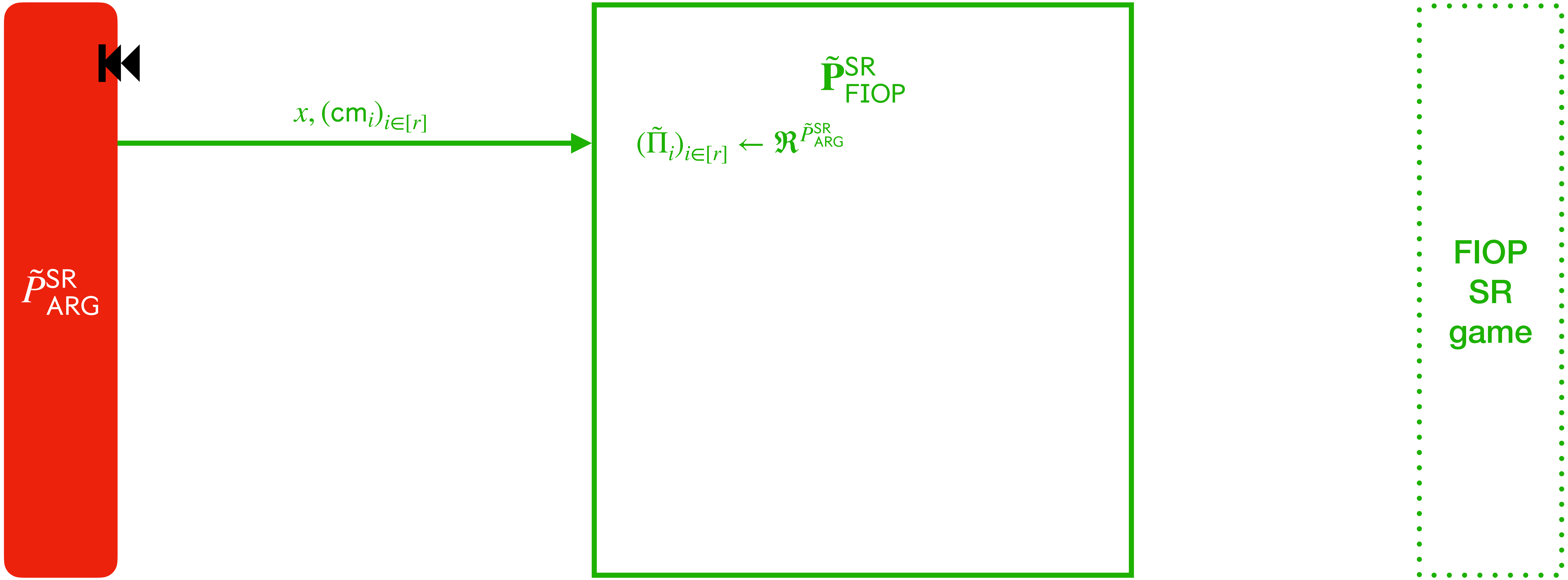

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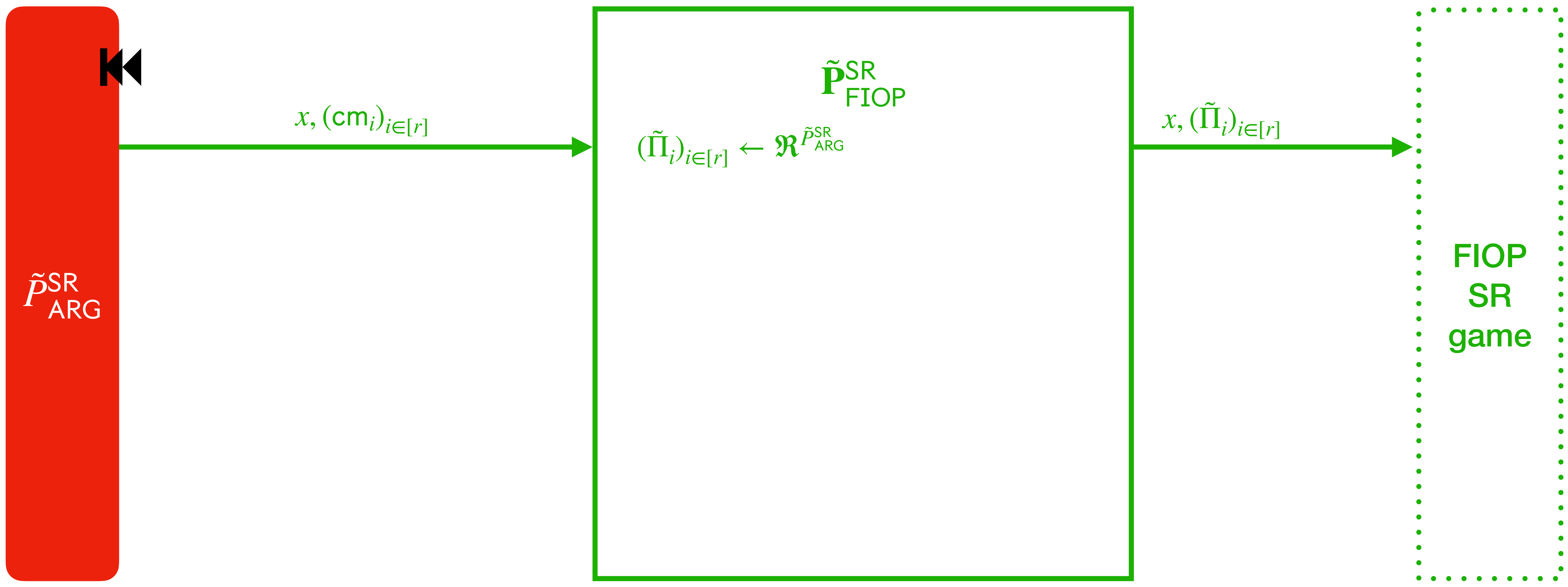

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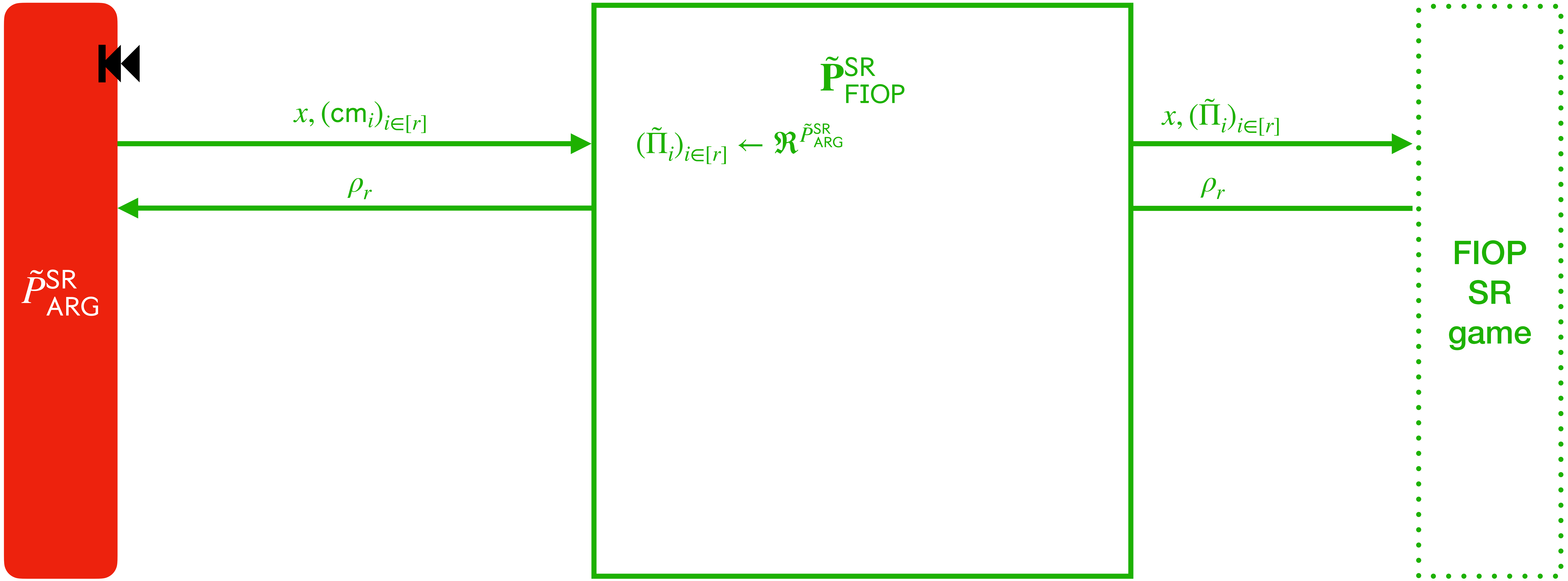

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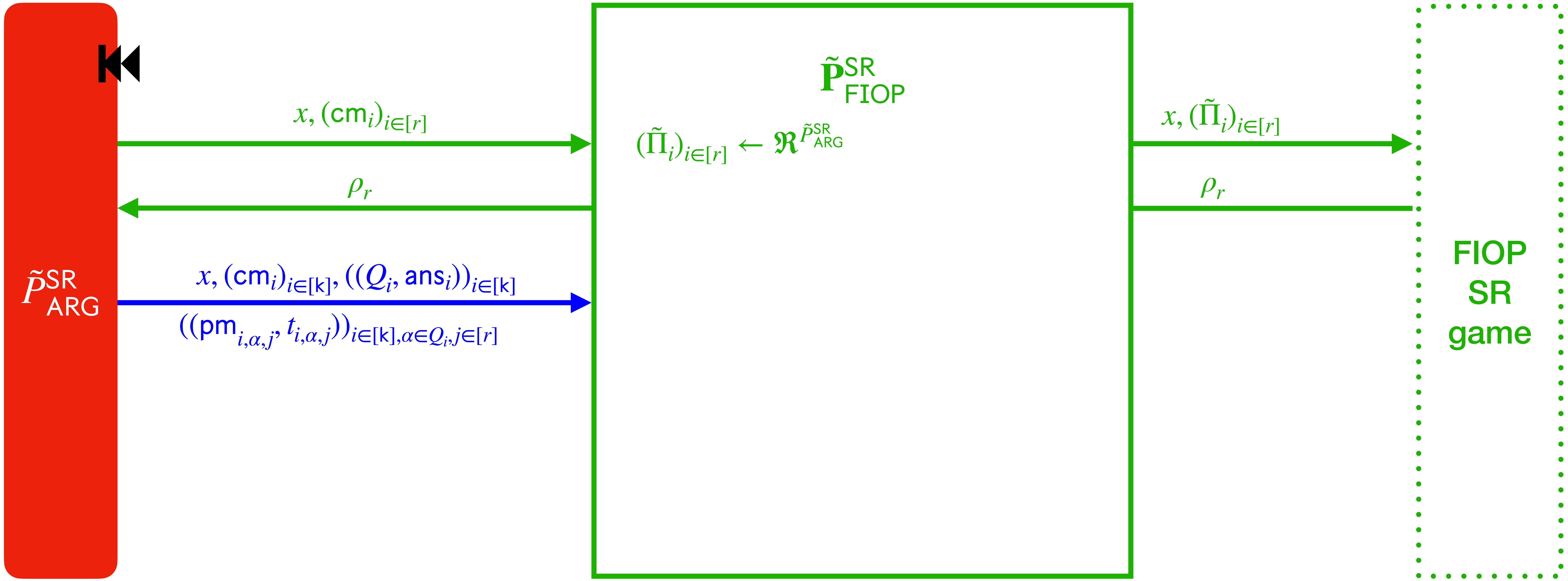

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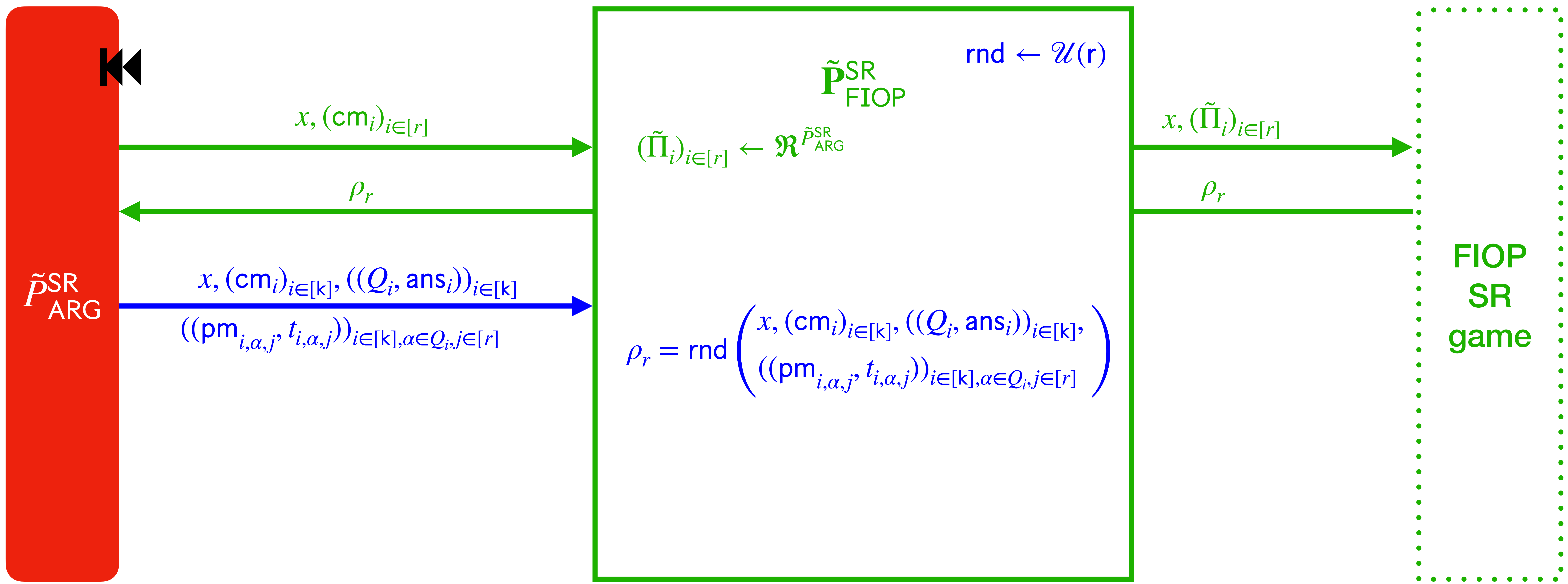

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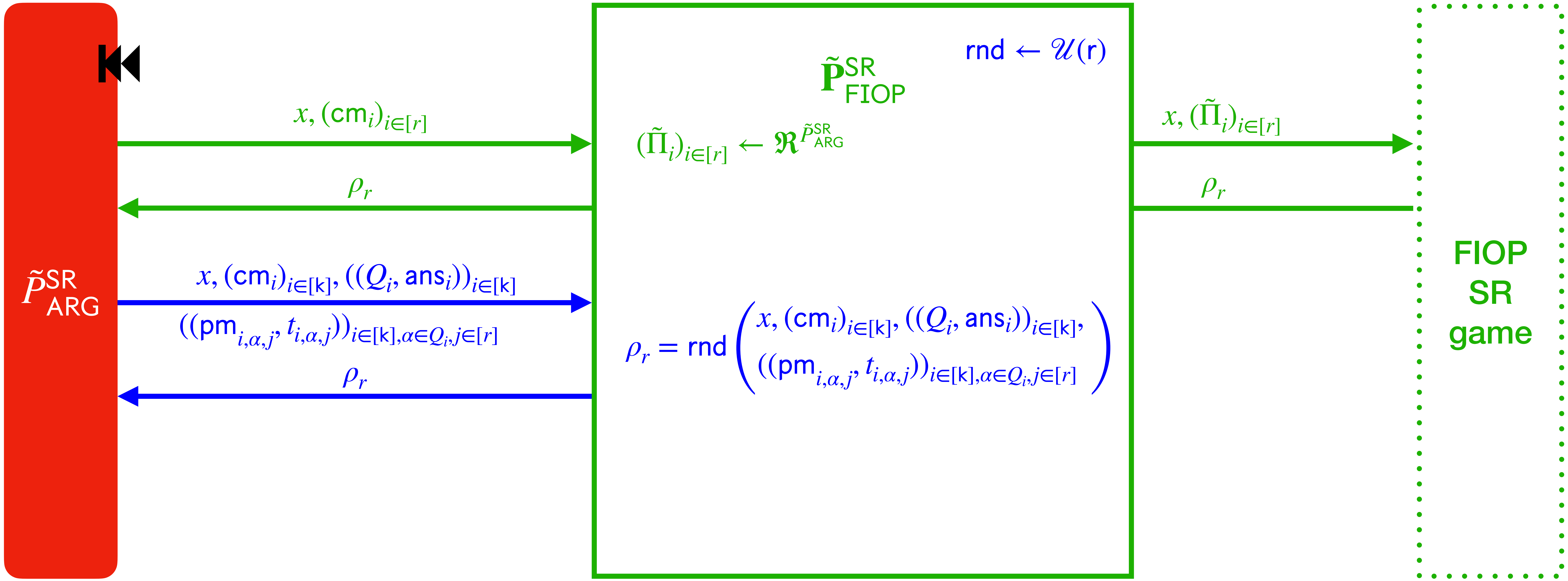

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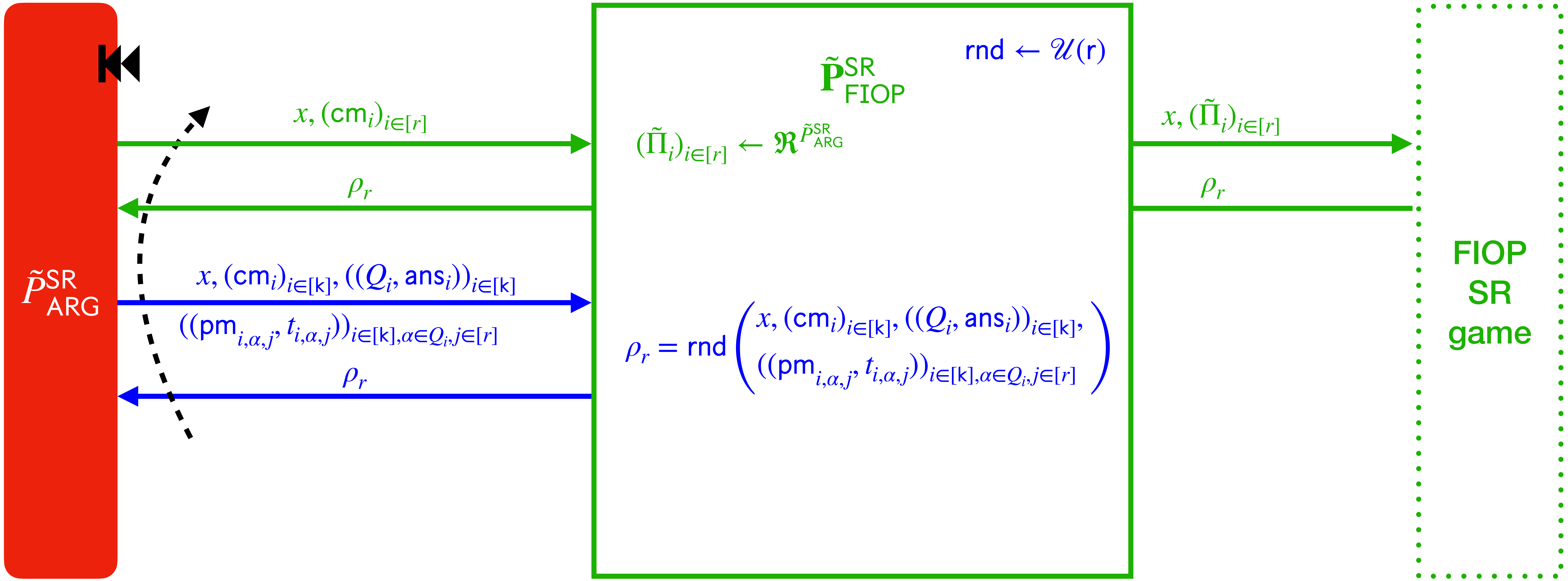

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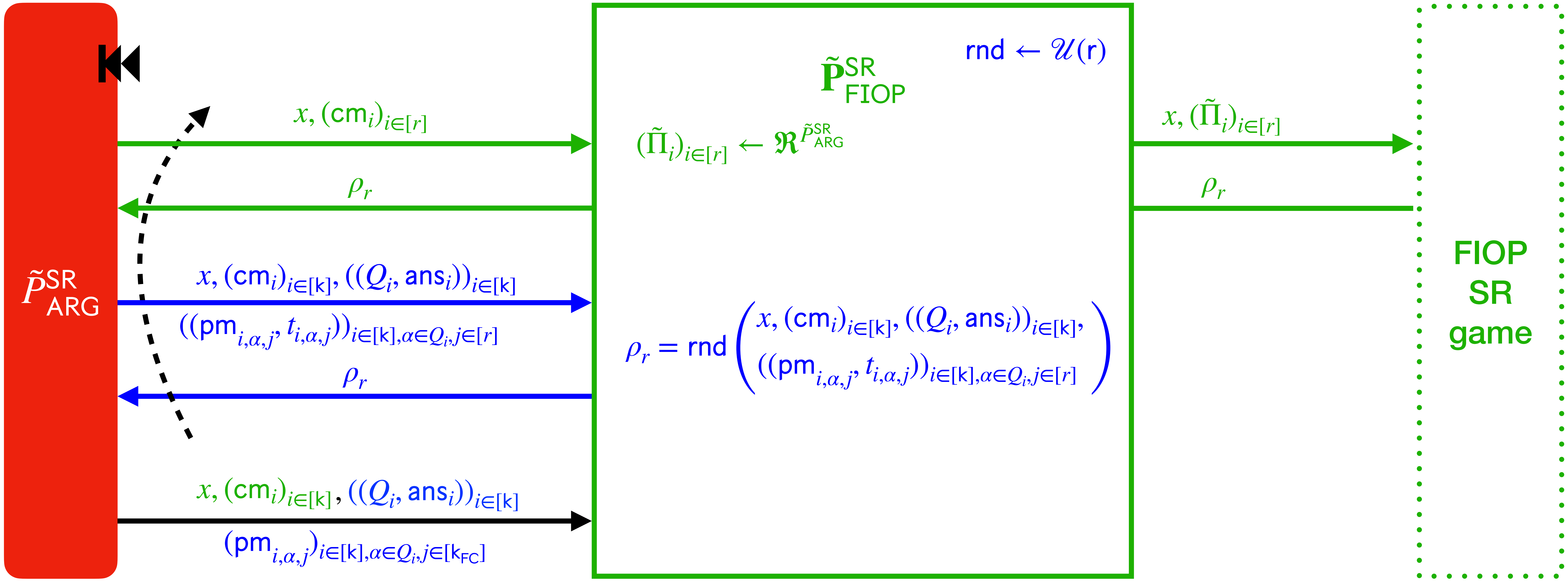

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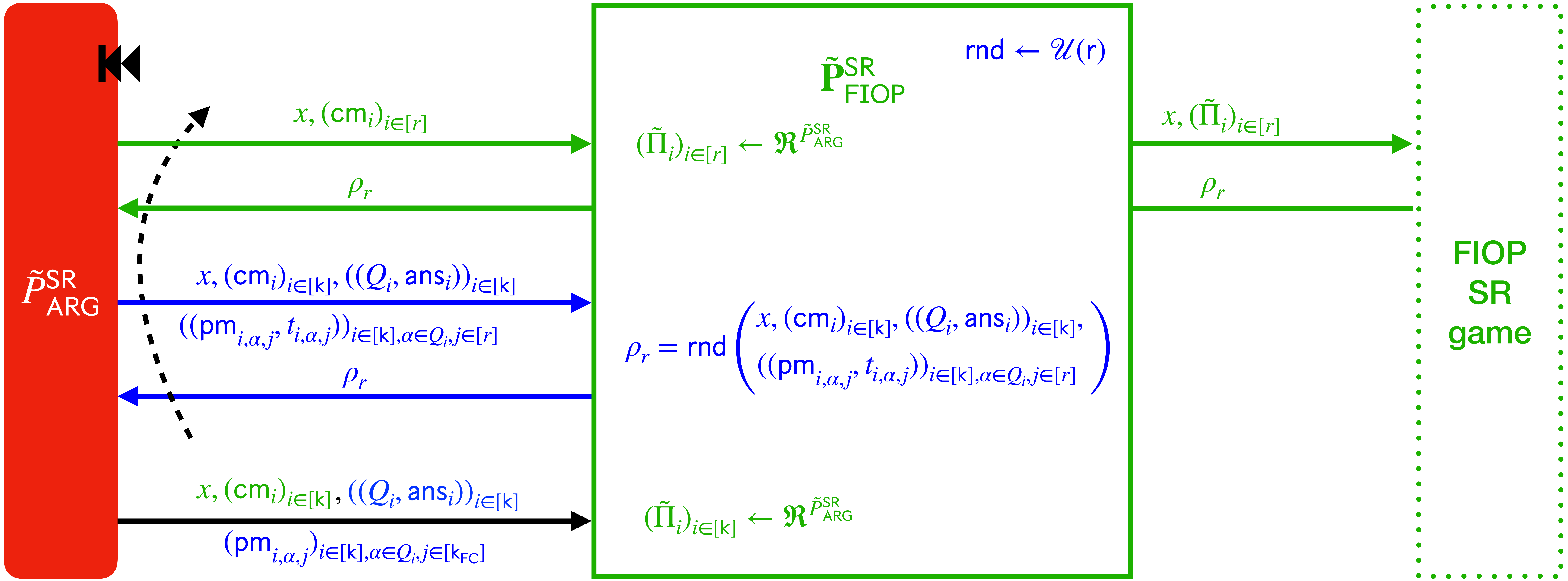

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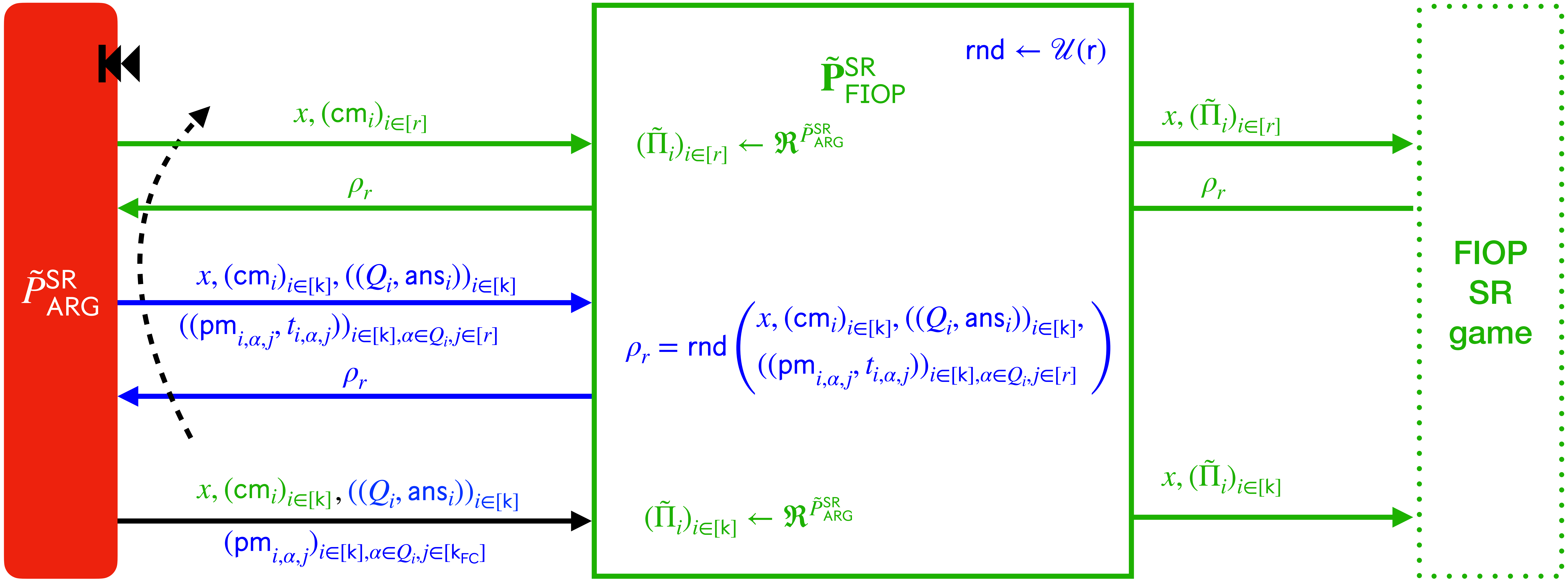

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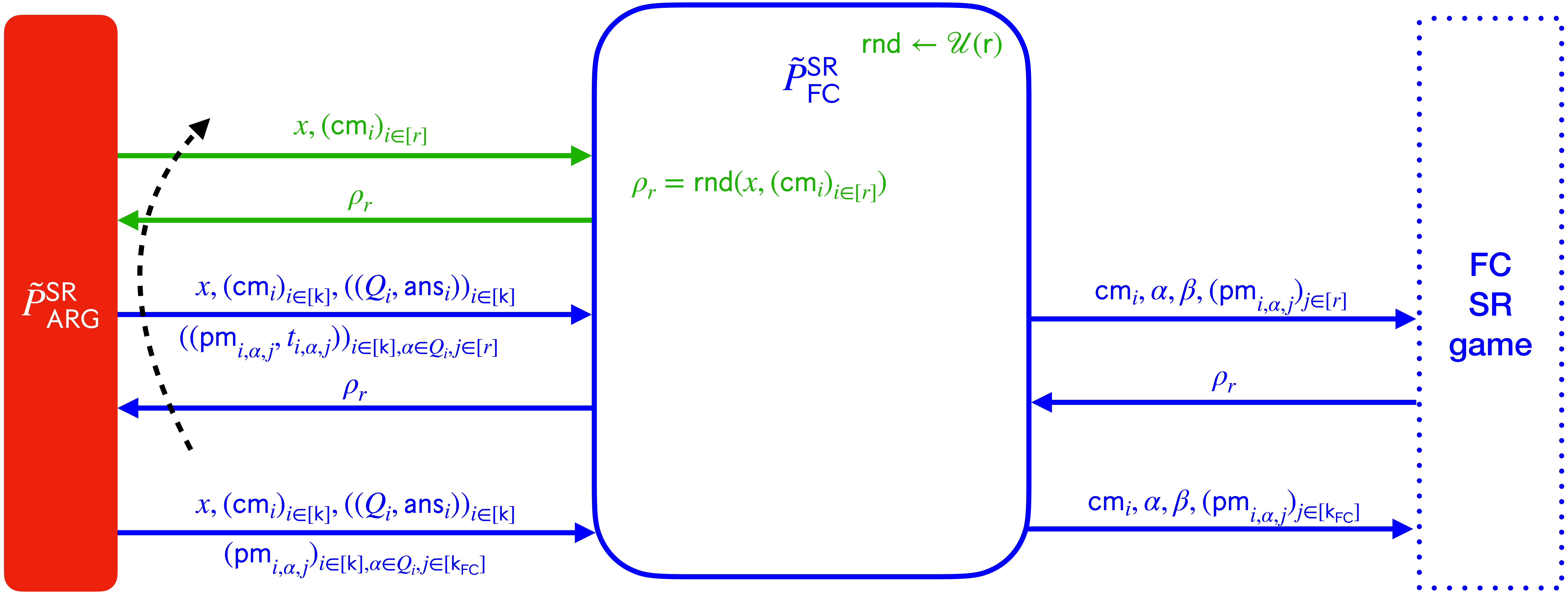

linKZG = Lin[batchKZG]

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public

$\in \mathbb{F}[X]^{\leq D}$

$\in \mathbb{F}[X]^{\leq D}$

$$\Pi(X) = \sum_{k \in [n]} h_k(f_1(X), \ldots, f_m(X)) \cdot g_k(X)$$

linKZG = Lin[batchKZG]

public $\in \mathbb{F}[X]^{\leq D}$ $\in \mathbb{F}[X]^{\leq D}$

$$\Pi(X) = \sum_{k\in[n]} h_k(f_1(X), \ldots, f_m(X)) \cdot g_k(X)$$

$$\mathsf{Commit}(\Pi) = \mathsf{cm}_{f_1}, \ldots, \mathsf{cm}_{f_m}, \mathsf{cm}_{g_1}, \ldots, \mathsf{cm}_{g_n}$$

linKZG = Lin[batchKZG]

public $\quad \in \mathbb{F}[X]^{\leq D} \quad \in \mathbb{F}[X]^{\leq D}$

$$\Pi(X) = \sum_{k\in[n]} h_k(f_1(X), \ldots, f_m(X)) \cdot g_k(X)$$

$$\text{Commit}(\Pi) = \text{cm}_{f_1}, \ldots, \text{cm}_{f_m}, \text{cm}_{g_1}, \ldots, \text{cm}_{g_n}$$

$$P_{\text{lin}}(\text{cm}, \alpha, \beta; \Pi)$$

$$P_{\text{batch}}$$

$$\left(\begin{bmatrix} \text{cm}_{f_1} \\ \vdots \\ \text{cm}_{f_m} \\ \sum_k h_k(f_1(\alpha), \ldots, f_m(\alpha)) \cdot \text{cm}_{g_k} \end{bmatrix}, \alpha, \begin{bmatrix} f_1(\alpha) \\ \vdots \\ f_m(\alpha) \\ \beta \end{bmatrix}, \begin{bmatrix} f_1 \\ \vdots \\ f_m \\ \sum_{k\in[m]} h_k(f_1(\alpha), \ldots, f_m(\alpha)) \cdot g_k(X) \end{bmatrix} \right)$$

linKZG = Lin[batchKZG]

public $\quad \in \mathbb{F}[X]^{\leq D} \quad \in \mathbb{F}[X]^{\leq D}$

$$\Pi(X) = \sum_{k \in [n]} h_k(f_1(X), \ldots, f_m(X)) \cdot g_k(X)$$

$$\text{Commit}(\Pi) = \mathsf{cm}_{f_1}, \ldots, \mathsf{cm}_{f_m}, \mathsf{cm}_{g_1}, \ldots, \mathsf{cm}_{g_n}$$

$$P_{\text{lin}}(\mathsf{cm}, \alpha, \beta; \Pi)$$

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constant $\in \mathbb{F}$ $\quad$ Linearized polynomial $\in \mathbb{F}[X]^{\leq D}$

linKZG = Lin[batchKZG]

public $\quad \in \mathbb{F}[X]^{\leq D} \quad \in \mathbb{F}[X]^{\leq D}$

$$\Pi(X) = \sum_{k\in[n]} h_k(f_1(X), \ldots, f_m(X)) \cdot g_k(X)$$

$$\text{Commit}(\Pi) = \mathsf{cm}_{f_1}, \ldots, \mathsf{cm}_{f_m}, \mathsf{cm}_{g_1}, \ldots, \mathsf{cm}_{g_n}$$

$P_{\text{lin}}(\mathsf{cm}, \alpha, \beta; \Pi)$

P_{batch}

$$\left(\begin{bmatrix} \mathsf{cm}_{f_1} \\ \vdots \\ \mathsf{cm}_{f_m} \\ \sum_k h_k(f_1(\alpha), \ldots, f_m(\alpha)) \cdot \mathsf{cm}_{g_k} \end{bmatrix}, \alpha, \begin{bmatrix} f_1(\alpha) \\ \vdots \\ f_m(\alpha) \\ \beta \end{bmatrix}, \begin{bmatrix} f_1 \\ \vdots \\ f_m \\ \sum_{k\in[m]} h_k(f_1(\alpha), \ldots, f_m(\alpha)) \cdot g_k(X) \end{bmatrix} \right)$$

constant $\in \mathbb{F}$

Linearized polynomial $\in \mathbb{F}[X]^{\leq D}$

$V_{\text{lin}}(\mathsf{cm}, \alpha, \beta)$

V_{batch}

$$\left(\begin{bmatrix} \mathsf{cm}_{f_1} \\ \vdots \\ \mathsf{cm}_{f_m} \\ \sum_k h_k(f_1(\alpha), \ldots, f_m(\alpha)) \cdot \mathsf{cm}_{g_k} \end{bmatrix}, \alpha, \begin{bmatrix} f_1(\alpha) \\ \vdots \\ f_m(\alpha) \\ \beta \end{bmatrix} \right)$$

linKZG = Lin[batchKZG]

public $\quad \in \mathbb{F}[X]^{\leq \mathsf{D}} \quad \in \mathbb{F}[X]^{\leq \mathsf{D}}$

$$\Pi(X) = \sum_{k\in[n]} h_k(f_1(X), \ldots, f_m(X)) \cdot g_k(X)$$

Degree: $\mathsf{D}_h \cdot \mathsf{D} + \mathsf{D}$

$$\mathsf{Commit}(\Pi) = \mathsf{cm}_{f_1}, \ldots, \mathsf{cm}_{f_m}, \mathsf{cm}_{g_1}, \ldots, \mathsf{cm}_{g_n}$$

$P_{\text{lin}}(\mathsf{cm}, \alpha, \beta; \Pi)$

Batch size: $m+1$

Degree: D

P_{batch}

$$\left(\begin{bmatrix} \mathsf{cm}_{f_1} \\ \vdots \\ \mathsf{cm}_{f_m} \\ \sum_k \underbrace{h_k(f_1(\alpha), \ldots, f_m(\alpha))}_{\text{constant } \in \mathbb{F}} \cdot \mathsf{cm}_{g_k}) \end{bmatrix}, \alpha, \begin{bmatrix} f_1(\alpha) \\ \vdots \\ f_m(\alpha) \\ \beta \end{bmatrix}, \begin{bmatrix} f_1 \\ \vdots \\ f_m \\ \underbrace{\sum_{k\in[m]} h_k(f_1(\alpha), \ldots, f_m(\alpha)) \cdot g_k(X)}_{\text{Linearized polynomial } \in \mathbb{F}[X]^{\leq \mathsf{D}}} \end{bmatrix}\right)$$

$V_{\text{lin}}(\mathsf{cm}, \alpha, \beta)$

V_{batch}

$$\left(\begin{bmatrix} \mathsf{cm}_{f_1} \\ \vdots \\ \mathsf{cm}_{f_m} \\ \sum_k h_k(f_1(\alpha), \ldots, f_m(\alpha)) \cdot \mathsf{cm}_{g_k}) \end{bmatrix}, \alpha, \begin{bmatrix} f_1(\alpha) \\ \vdots \\ f_m(\alpha) \\ \beta \end{bmatrix}\right)$$

constant $\in \mathbb{F}$

Linearized polynomial $\in \mathbb{F}[X]^{\leq \mathsf{D}}$

KZG function binding $\Leftarrow$ evaluation binding

Goal: given A_{FB}, construct A_{EB}

KZG function binding $\Leftarrow$ evaluation binding

Goal: given A_{FB}, construct A_{EB}

Proof sketch:

$\mathsf{cm}, ((\alpha_i, \beta_i, \mathsf{pf}_i))_{i \in [n]} \leftarrow A_{\mathsf{FB}}$ s.t. $\begin{cases} \forall i : \mathsf{Check}(\mathsf{cm}, \alpha_i, \beta_i, \mathsf{pf}_i) = 1 & (1) \\ \nexists \Pi \in \mathbb{F}[X]^{\leq \mathsf{D}} : \forall i : \Pi(\alpha_i) = \beta_i & (2) \end{cases}$

KZG function binding $\Leftarrow$ evaluation binding

Goal: given A_{FB}, construct A_{EB}

Proof sketch:

$$\mathsf{cm}, ((\alpha_i, \beta_i, \mathsf{pf}_i))_{i\in[n]} \leftarrow A_{\mathsf{FB}} \text{ s.t. } \begin{cases} \forall i : \mathsf{Check}(\mathsf{cm}, \alpha_i, \beta_i, \mathsf{pf}_i) = 1 & (1) \\ \nexists \Pi \in \mathbb{F}[X]^{\leq \mathsf{D}} : \forall i : \Pi(\alpha_i) = \beta_i & (2) \end{cases}$$

If $\exists i, j : \alpha_i = \alpha_j \wedge \beta_i \neq \beta_j$: output $(\mathsf{cm}, \alpha_i, \beta_i, \beta_j, \mathsf{pf}_i, \mathsf{pf}_j)$

KZG function binding $\Leftarrow$ evaluation binding

Goal: given A_{FB}, construct A_{EB}

Proof sketch:

$$\mathsf{cm}, ((\alpha_i, \beta_i, \mathsf{pf}_i))_{i \in [n]} \leftarrow A_{\mathsf{FB}} \text{ s.t. } \begin{cases} \forall i : \mathsf{Check}(\mathsf{cm}, \alpha_i, \beta_i, \mathsf{pf}_i) = 1 & (1) \\ \nexists \Pi \in \mathbb{F}[X]^{\leq \mathsf{D}} : \forall i : \Pi(\alpha_i) = \beta_i & (2) \end{cases}$$

If $\exists i, j : \alpha_i = \alpha_j \wedge \beta_i \neq \beta_j$: output $(\mathsf{cm}, \alpha_i, \beta_i, \beta_j, \mathsf{pf}_i, \mathsf{pf}_j)$

Pick any i, choose any $\beta_i^* \neq \beta_i$

$\color{blue}{\text{KZG}}$ function binding $\Leftarrow$ evaluation binding

Goal: given A_{FB}, construct A_{EB}

Proof sketch:

$\mathsf{cm}, ((\alpha_i, \beta_i, \mathsf{pf}_i))_{i \in [n]} \leftarrow A_{\mathsf{FB}}$ s.t. $\begin{cases} \forall i : \mathsf{Check}(\mathsf{cm}, \alpha_i, \beta_i, \mathsf{pf}_i) = 1 & (1) \\ \nexists \Pi \in \mathbb{F}[X]^{\leq \mathsf{D}} : \forall i : \Pi(\alpha_i) = \beta_i & (2) \end{cases}$

If $\exists i, j : \alpha_i = \alpha_j \wedge \beta_i \neq \beta_j$: output $(\mathsf{cm}, \alpha_i, \beta_i, \beta_j, \mathsf{pf}_i, \mathsf{pf}_j)$

Pick any i, choose any $\beta_i^* \neq \beta_i$, find $\vec{r}$ s.t. $\begin{bmatrix} 1 & 1 & 1 \\ \alpha_{i_1} & \alpha_{i_2} & \alpha_{i_3} \\ \beta_{i_1} & \beta_{i_2} & \beta_{i_3} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \alpha_i \\ \beta_i^* \end{bmatrix}$

invertible, as otherwise there is a linear function Π s.t. $\forall i : \Pi(\alpha_i) = \beta_i$

KZG function binding $\Leftarrow$ evaluation binding

Goal: given A_{FB}, construct A_{EB}

Proof sketch:

$\mathsf{cm}, ((\alpha_i, \beta_i, \mathsf{pf}_i))_{i\in[n]} \leftarrow A_{\mathsf{FB}}$ s.t. $\begin{cases} \forall i : \mathsf{Check}(\mathsf{cm}, \alpha_i, \beta_i, \mathsf{pf}_i) = 1 & (1) \\ \nexists \Pi \in \mathbb{F}[X]^{\leq D} : \forall i : \Pi(\alpha_i) = \beta_i & (2) \end{cases}$

If $\exists i, j : \alpha_i = \alpha_j \wedge \beta_i \neq \beta_j$: output $(\mathsf{cm}, \alpha_i, \beta_i, \beta_j, \mathsf{pf}_i, \mathsf{pf}_j)$

Pick any i, choose any $\beta_i^* \neq \beta_i$, find $\vec{r}$ s.t. $\begin{bmatrix} 1 & 1 & 1 \\ \alpha_{i_1} & \alpha_{i_2} & \alpha_{i_3} \\ \beta_{i_1} & \beta_{i_2} & \beta_{i_3} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \alpha_i \\ \beta_i^* \end{bmatrix}$

invertible, as otherwise there is a linear function Π s.t. $\forall i : \Pi(\alpha_i) = \beta_i$

Homomorphism: $(\forall j : \mathsf{Check}(\mathsf{cm}, \alpha_{i_j}, \beta_{i_j}, \mathsf{pf}_{i_j}) = 1) \Rightarrow \mathsf{Check}(\mathsf{cm}, \sum_j r_j \alpha_{i_j}, \sum_j r_j \beta_{i_j}, \sum_j r_j \mathsf{pf}_{i_j}) = 1$

$\textcolor{blue}{\text{KZG}}$ function binding $\Leftarrow$ evaluation binding

Goal: given A_{FB}, construct A_{EB}

Proof sketch:

$$\text{cm}, ((\alpha_i, \beta_i, \text{pf}_i))_{i\in[n]} \leftarrow A_{\text{FB}} \text{ s.t. } \begin{cases} \forall i : \text{Check}(\text{cm}, \alpha_i, \beta_i, \text{pf}_i) = 1 & (1) \\ \nexists \Pi \in \mathbb{F}[X]^{\leq D} : \forall i : \Pi(\alpha_i) = \beta_i & (2) \end{cases}$$

If $\exists i, j : \alpha_i = \alpha_j \wedge \beta_i \neq \beta_j$: output $(\text{cm}, \alpha_i, \beta_i, \beta_j, \text{pf}_i, \text{pf}_j)$

Pick any i, choose any $\beta_i^* \neq \beta_i$, find $\vec{r}$ s.t. $\begin{bmatrix} 1 & 1 & 1 \\ \alpha_{i_1} & \alpha_{i_2} & \alpha_{i_3} \\ \beta_{i_1} & \beta_{i_2} & \beta_{i_3} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \alpha_i \\ \beta_i^* \end{bmatrix}$

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$$\Rightarrow \text{Check}(\text{cm}, \alpha_i, \beta_i^*, \textstyle\sum_j r_j \text{pf}_{i_j}) = \text{Check}(\sum_j r_j \text{cm}, \sum_j r_j \alpha_{i_j}, \sum_j r_j \beta_{i_j}, \sum_j r_j \text{pf}_{i_j}) = 1$$

KZG function binding $\Leftarrow$ evaluation binding

Goal: given A_{FB}, construct A_{EB}

Proof sketch:

$$\mathsf{cm}, ((\alpha_i, \beta_i, \mathsf{pf}_i))_{i\in[n]} \leftarrow A_{\mathsf{FB}} \text{ s.t. } \begin{cases} \forall i : \mathsf{Check}(\mathsf{cm}, \alpha_i, \beta_i, \mathsf{pf}_i) = 1 & (1) \\ \nexists \Pi \in \mathbb{F}[X]^{\leq D} : \forall i : \Pi(\alpha_i) = \beta_i & (2) \end{cases}$$

If $\exists i, j : \alpha_i = \alpha_j \wedge \beta_i \neq \beta_j$: output $(\mathsf{cm}, \alpha_i, \beta_i, \beta_j, \mathsf{pf}_i, \mathsf{pf}_j)$

Pick any i, choose any $\beta_i^* \neq \beta_i$, find $\vec{r}$ s.t. $\begin{bmatrix} 1 & 1 & 1 \\ \alpha_{i_1} & \alpha_{i_2} & \alpha_{i_3} \\ \beta_{i_1} & \beta_{i_2} & \beta_{i_3} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \alpha_i \\ \beta_i^* \end{bmatrix}$

invertible, as otherwise there is a linear function Π s.t. $\forall i : \Pi(\alpha_i) = \beta_i$

Homomorphism: $(\forall j : \mathsf{Check}(\mathsf{cm}, \alpha_{i_j}, \beta_{i_j}, \mathsf{pf}_{i_j}) = 1) \Rightarrow \mathsf{Check}(\mathsf{cm}, \sum_j r_j \alpha_{i_j}, \sum_j r_j \beta_{i_j}, \sum_j r_j \mathsf{pf}_{i_j}) = 1$

$$\Rightarrow \mathsf{Check}(\mathsf{cm}, \alpha_i, \beta_i^*, \underbrace{\textstyle\sum_j r_j \mathsf{pf}_{i_j}}) = \mathsf{Check}(\textstyle\sum_j r_j \mathsf{cm}, \sum_j r_j \alpha_{i_j}, \sum_j r_j \beta_{i_j}, \sum_j r_j \mathsf{pf}_{i_j}) = 1$$

Output $(\mathsf{cm}, \alpha_i, \beta_i, \beta_i^*, \mathsf{pf}_i, \mathsf{pf}_i^*)$