SLAP 

Succinct Lattice-Based Polynomial Commitment Schemes from Standard Assumptions

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Joint work with:
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Ngoc Khanh Nguyen

King's College London

Oleksandra Lapiha

Royal Holloway University of London

Motivation

SNARKs

SNARKs

(Succinct Non-Interactive ARguments of Knowledge)

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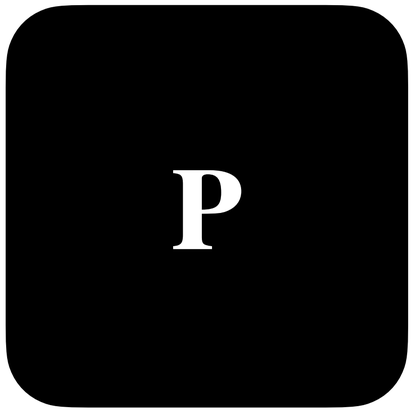

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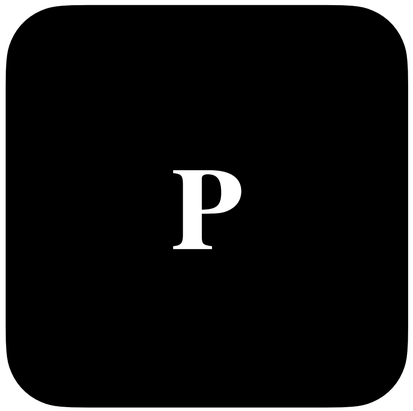

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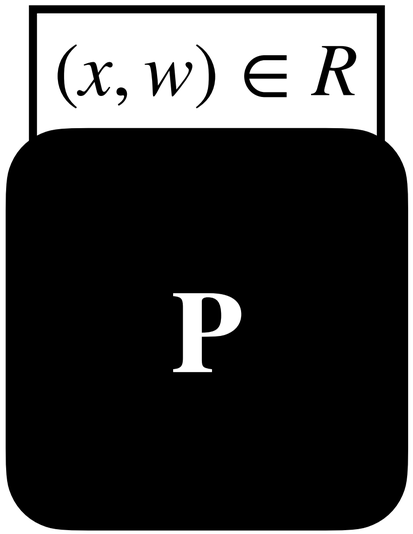

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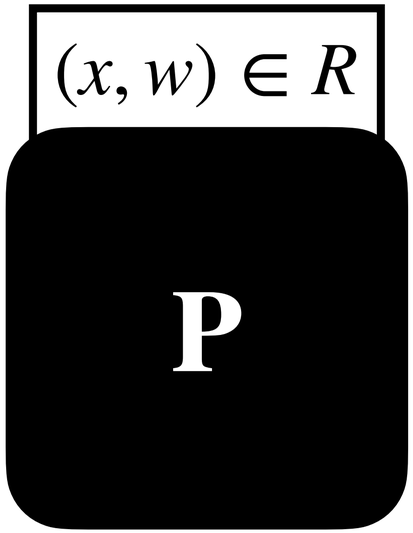

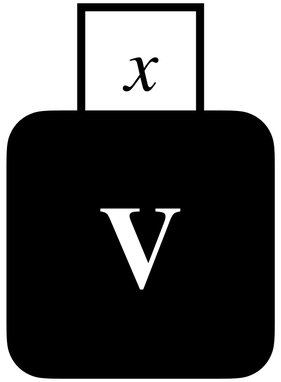

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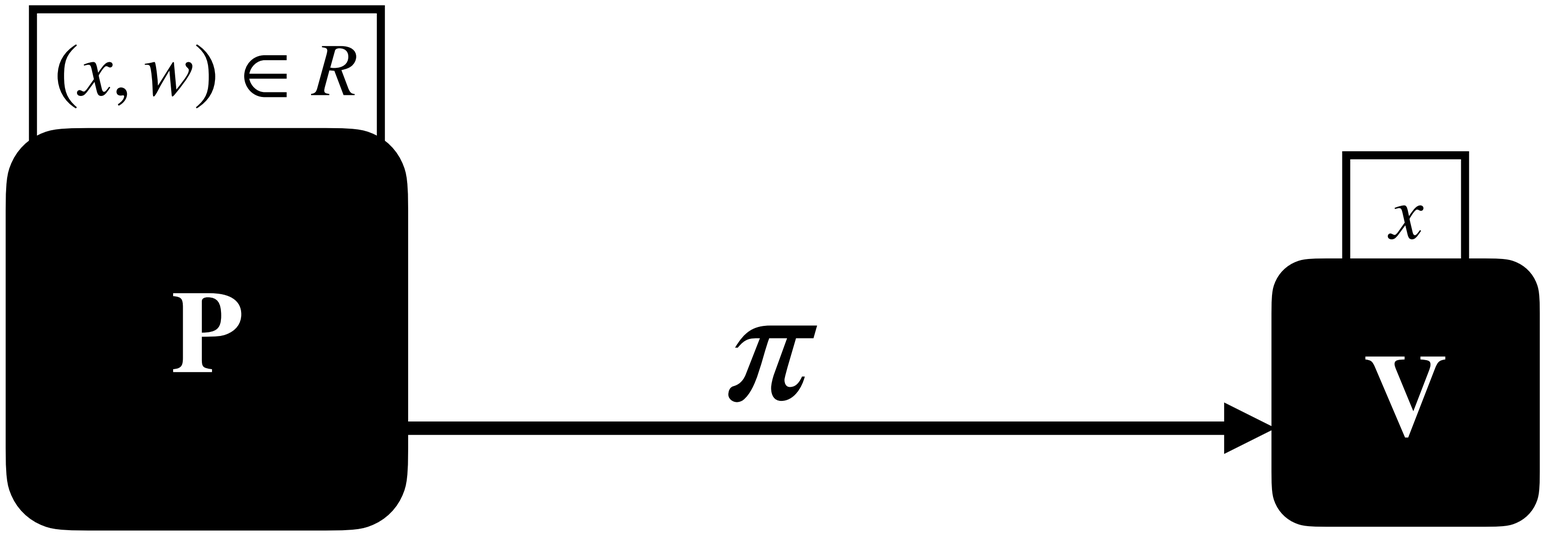

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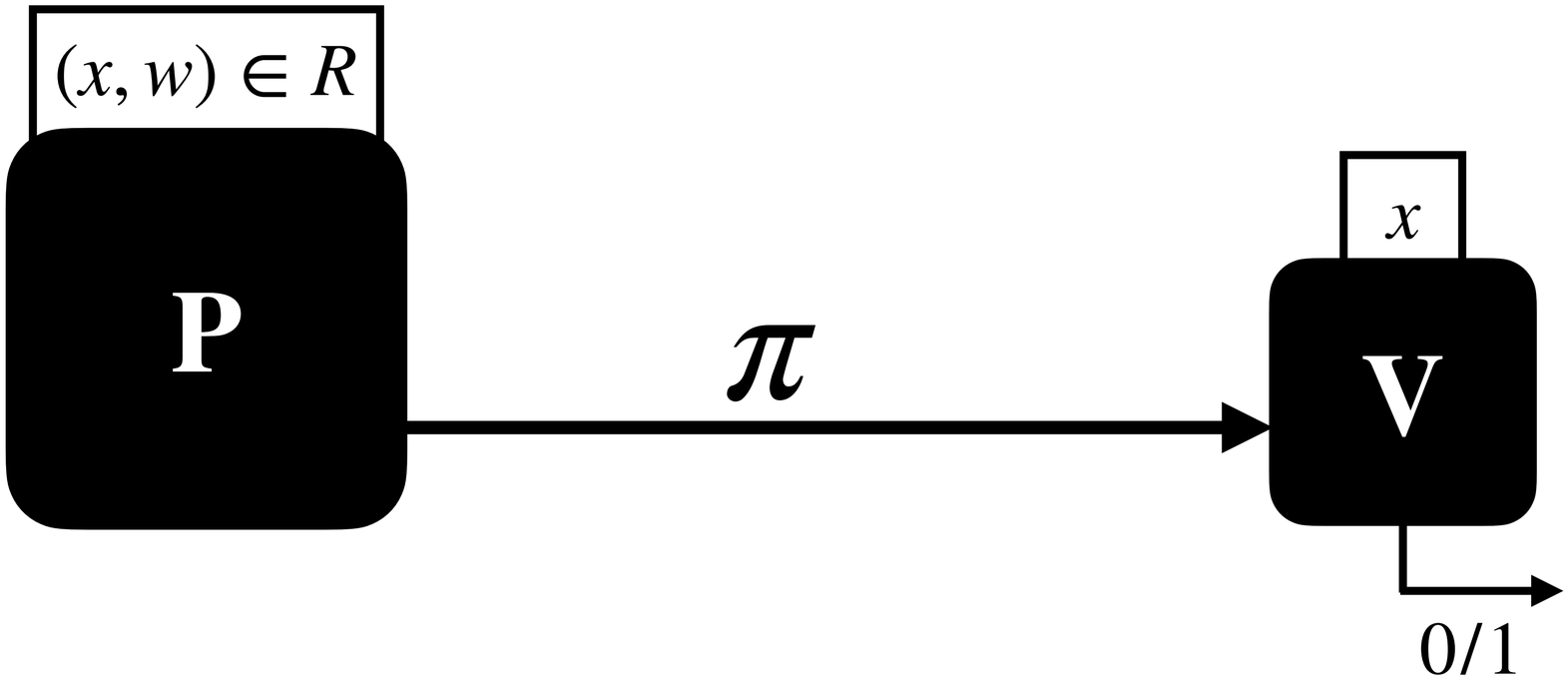

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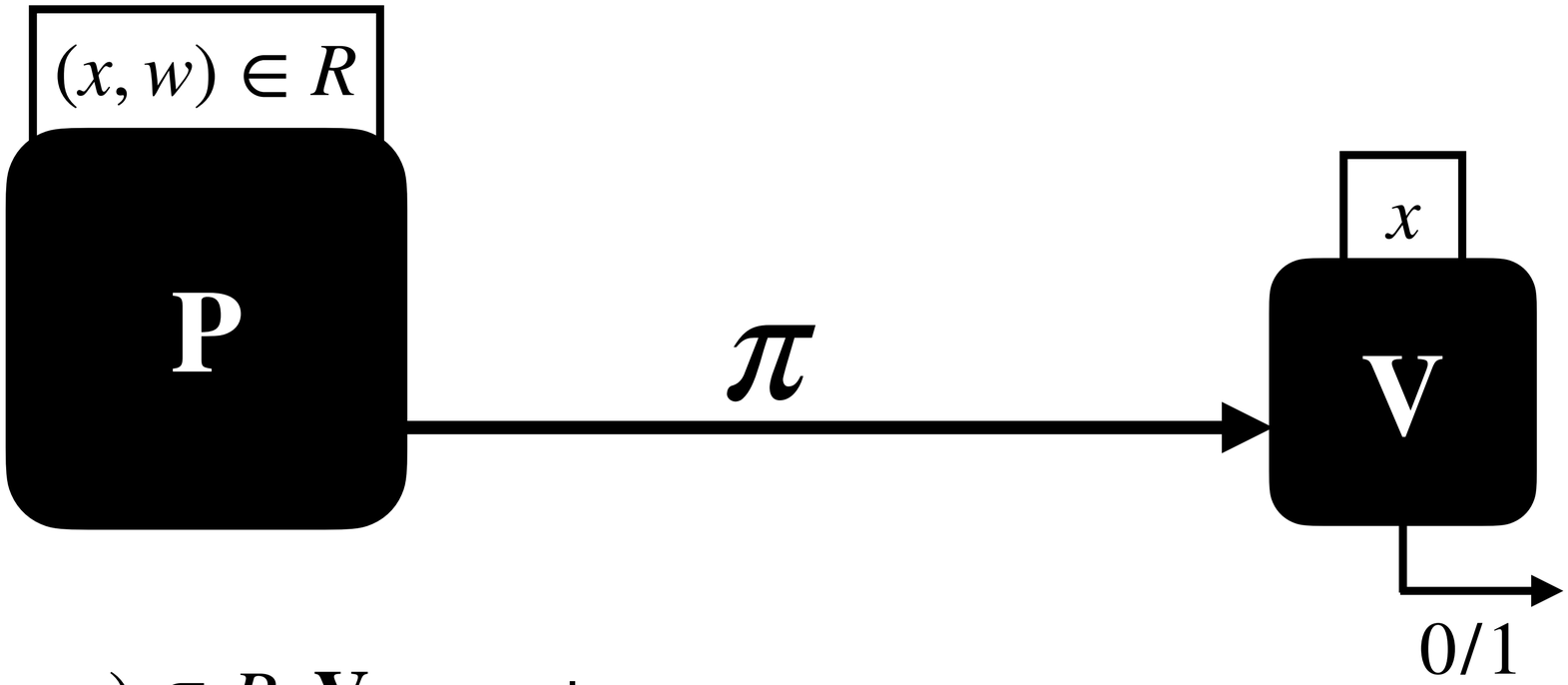

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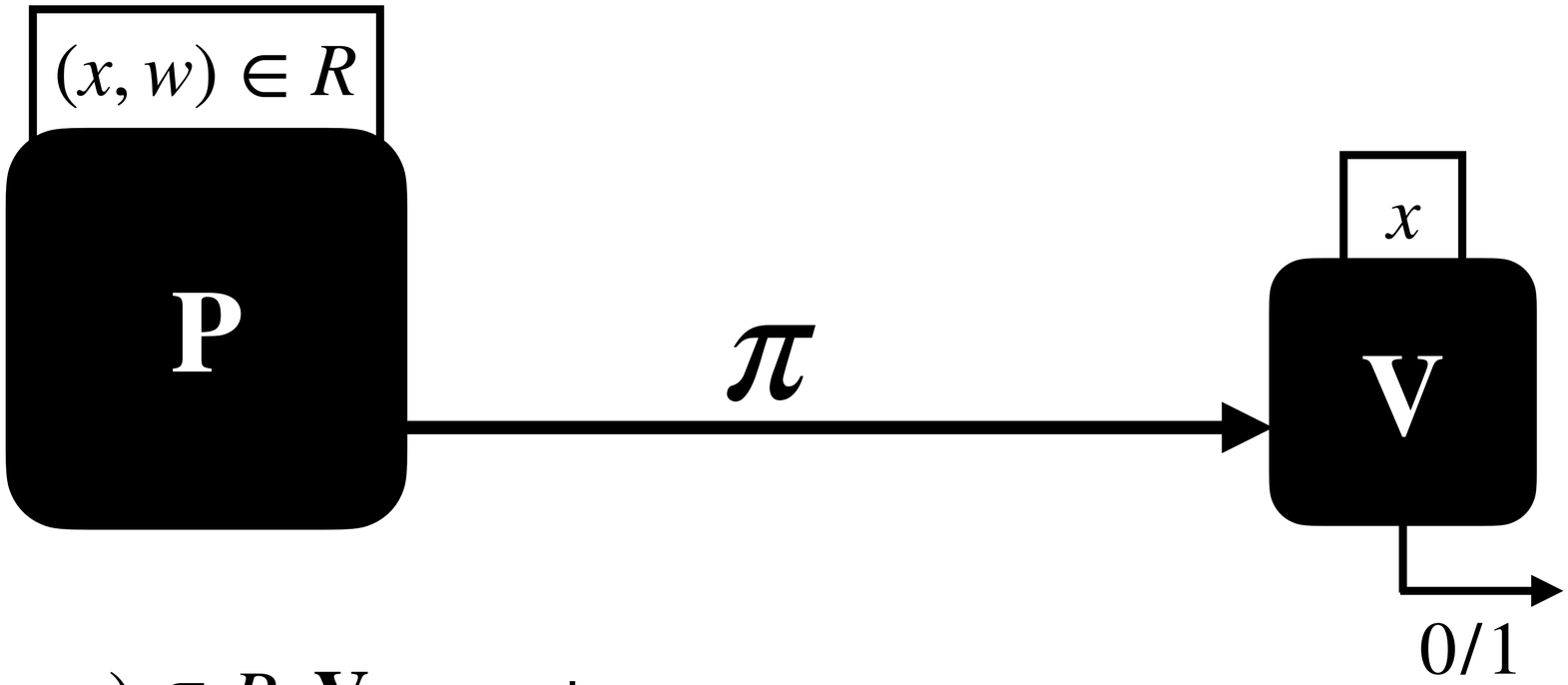

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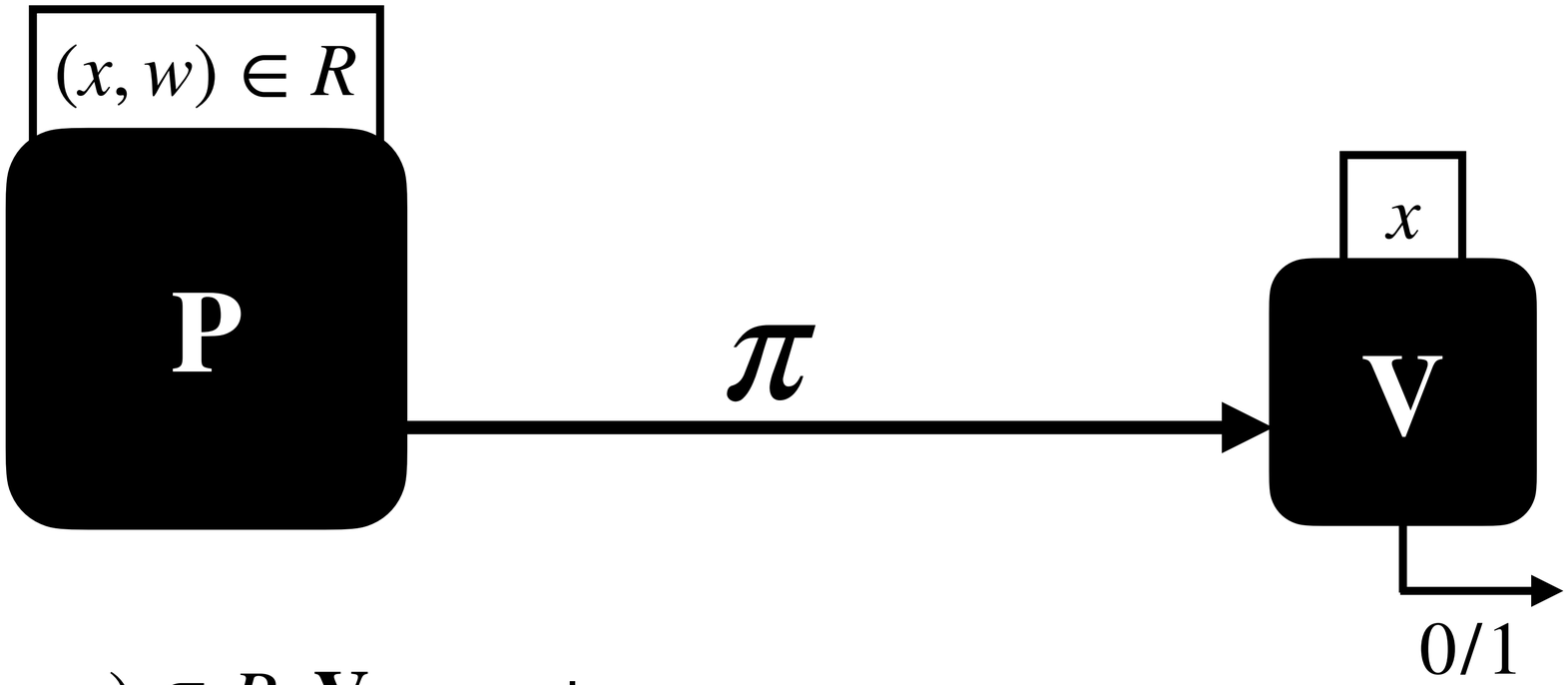

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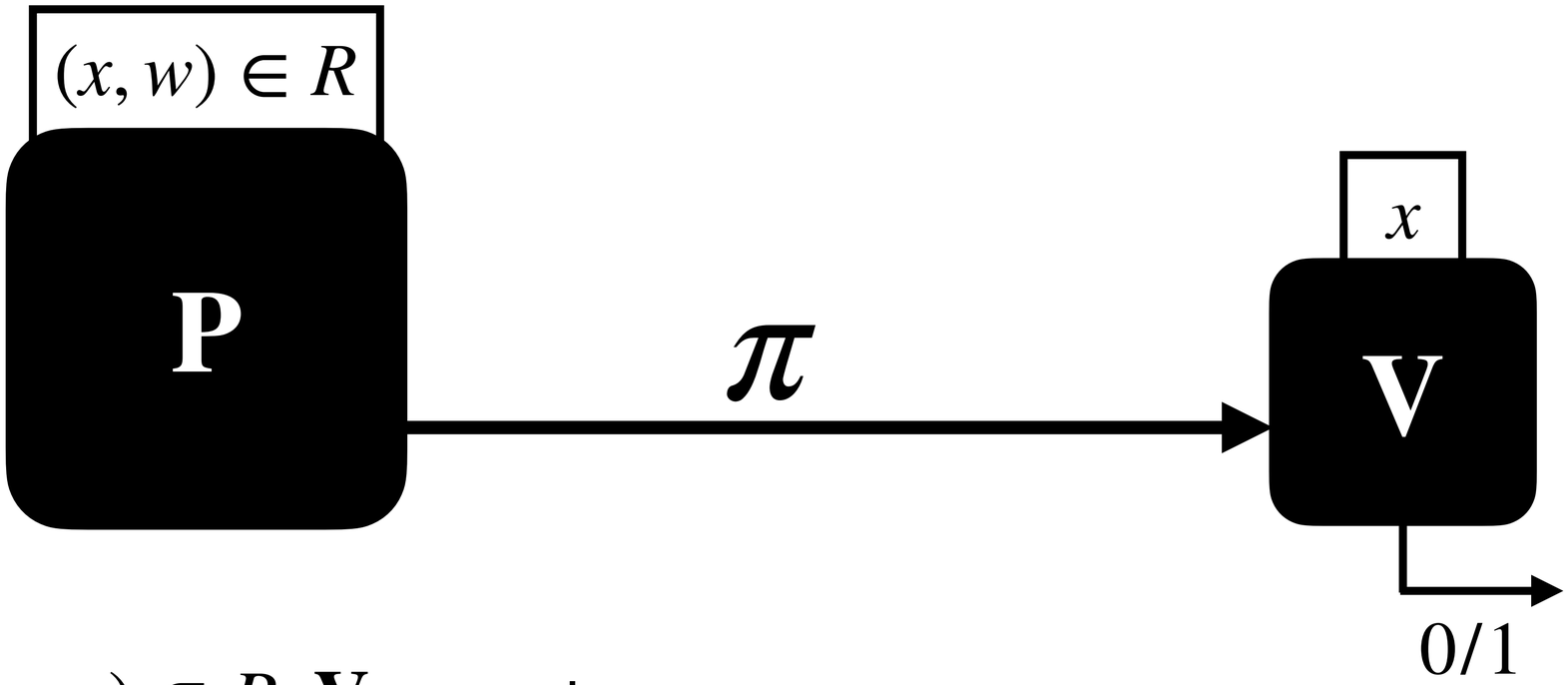

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Knowledge Sound: if $\mathbf{V}(x, \pi) = 1$, can extract w such that $(x, w) \in R$

Constructing SNARKs

The modular way™

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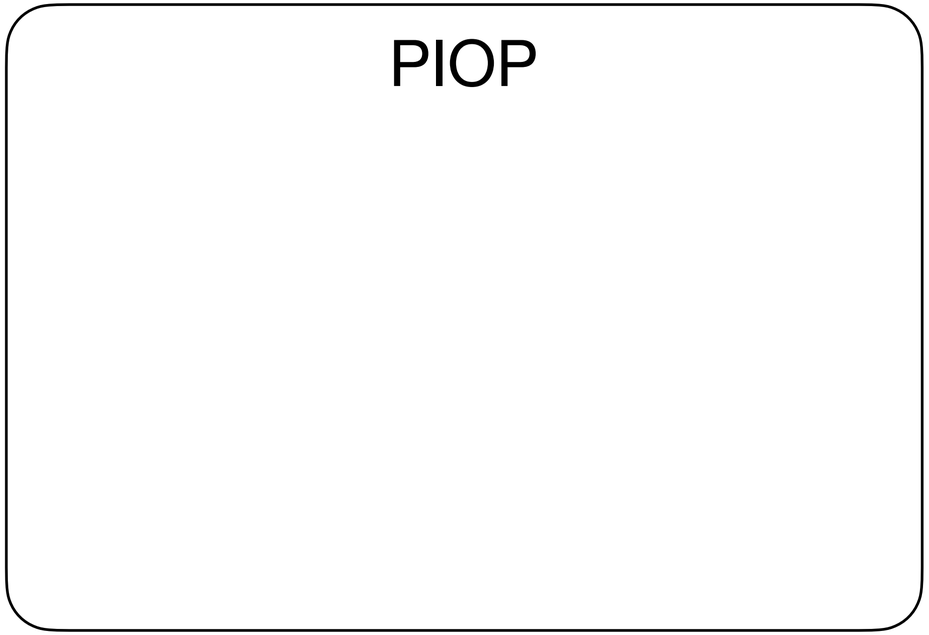

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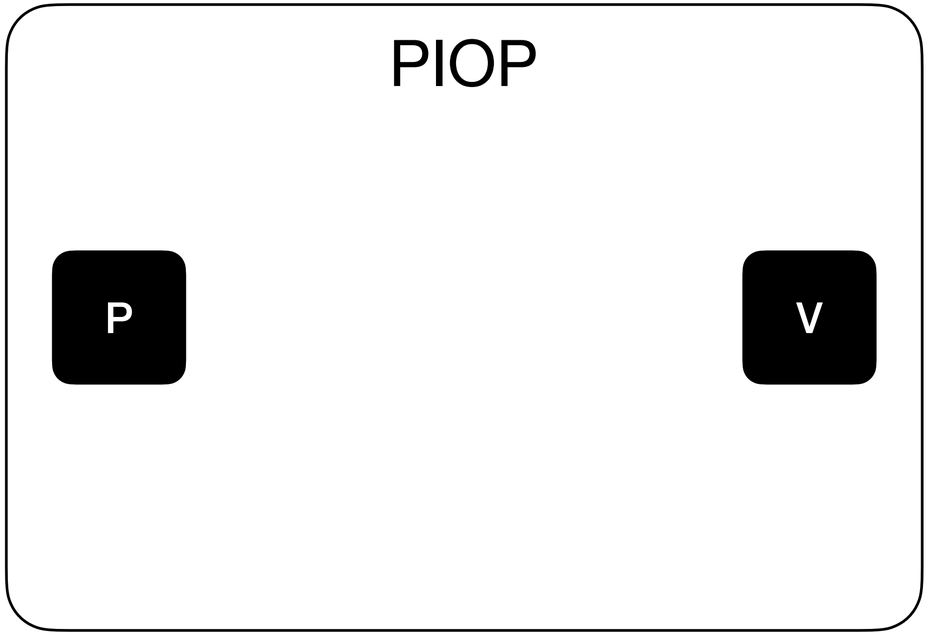

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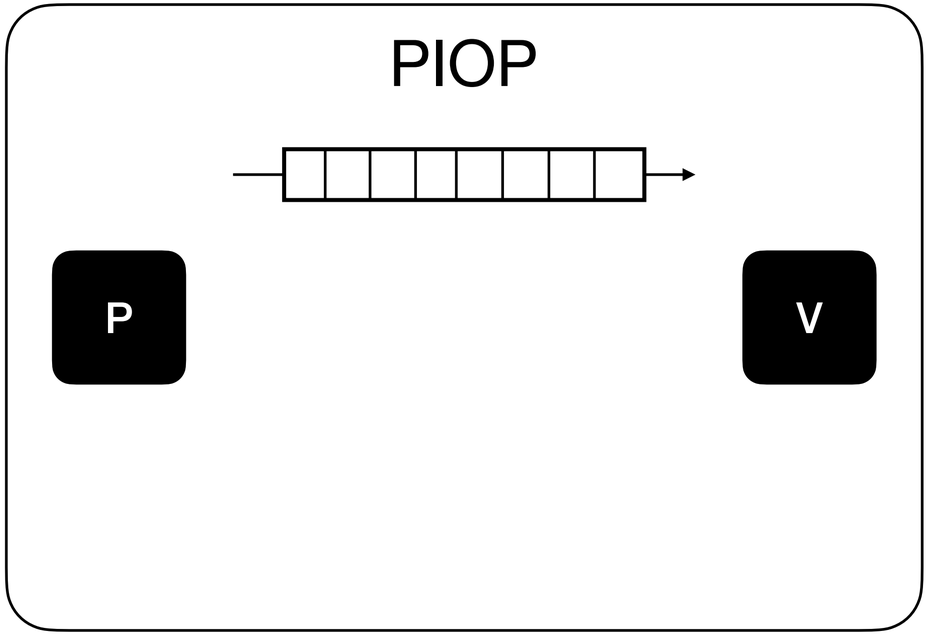

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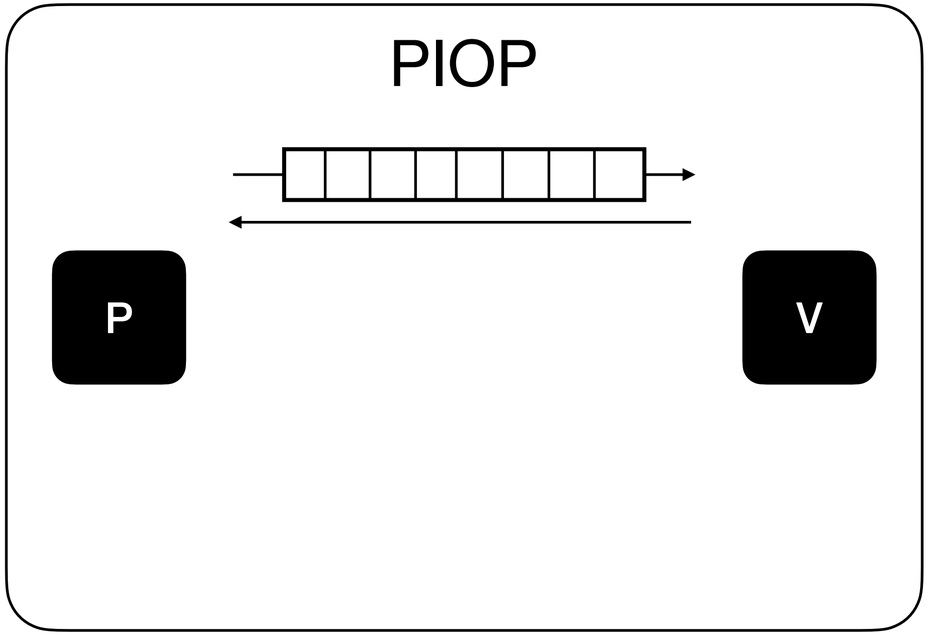

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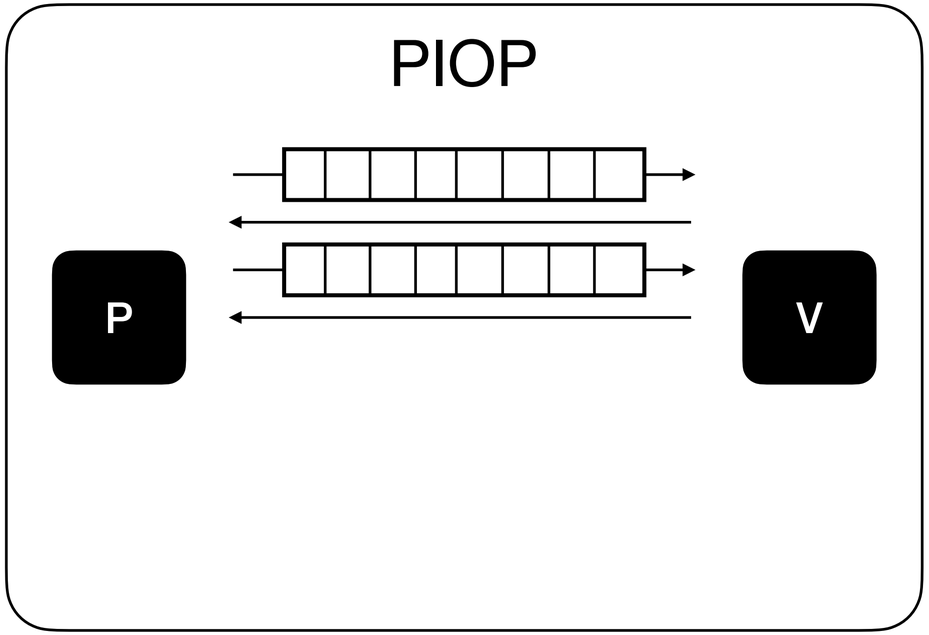

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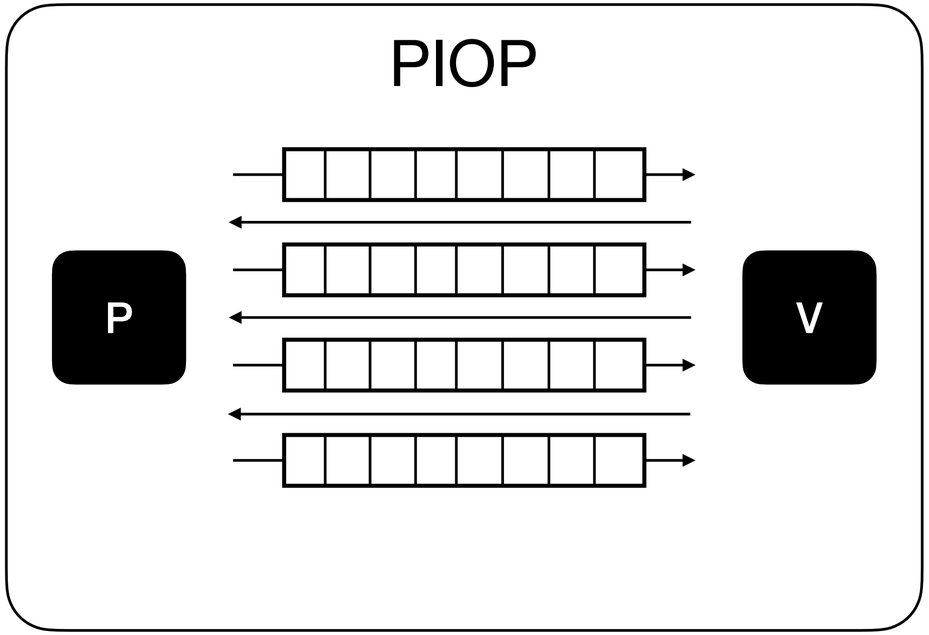

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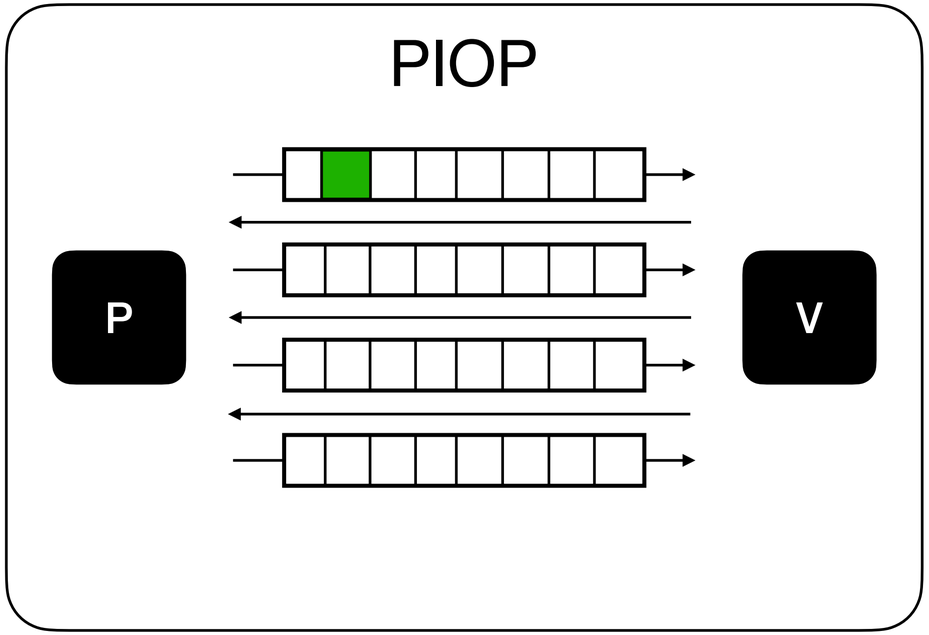

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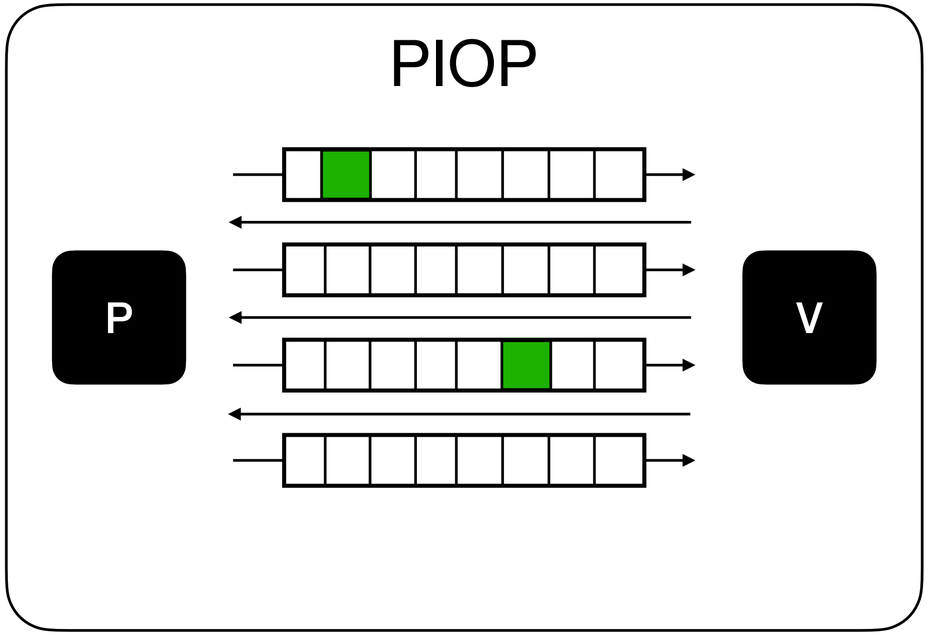

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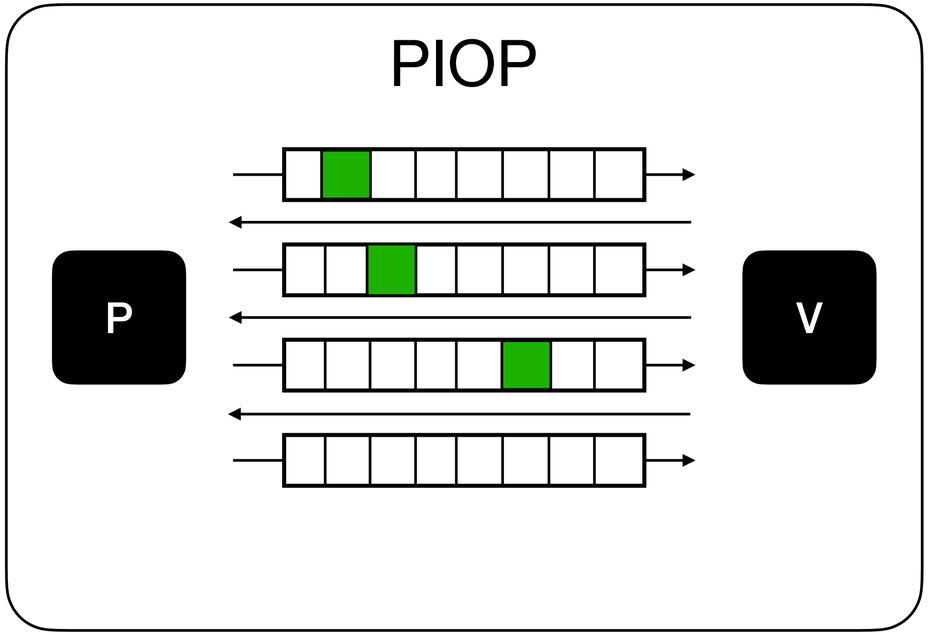

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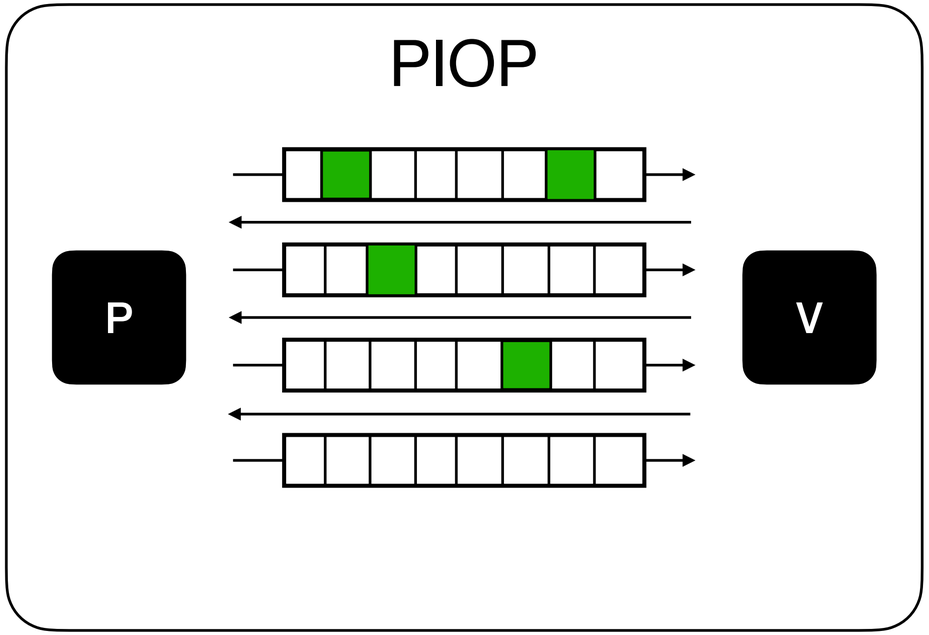

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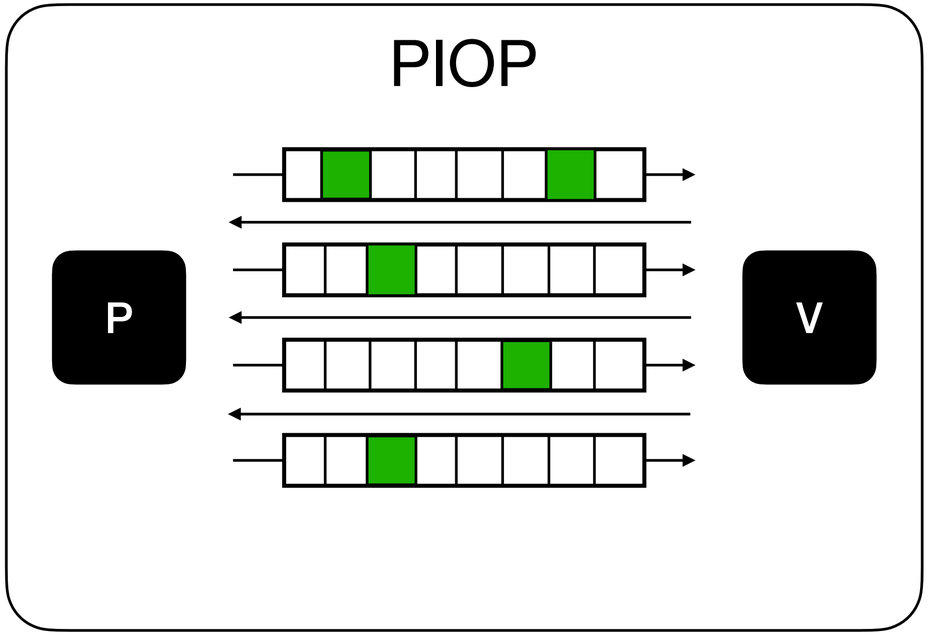

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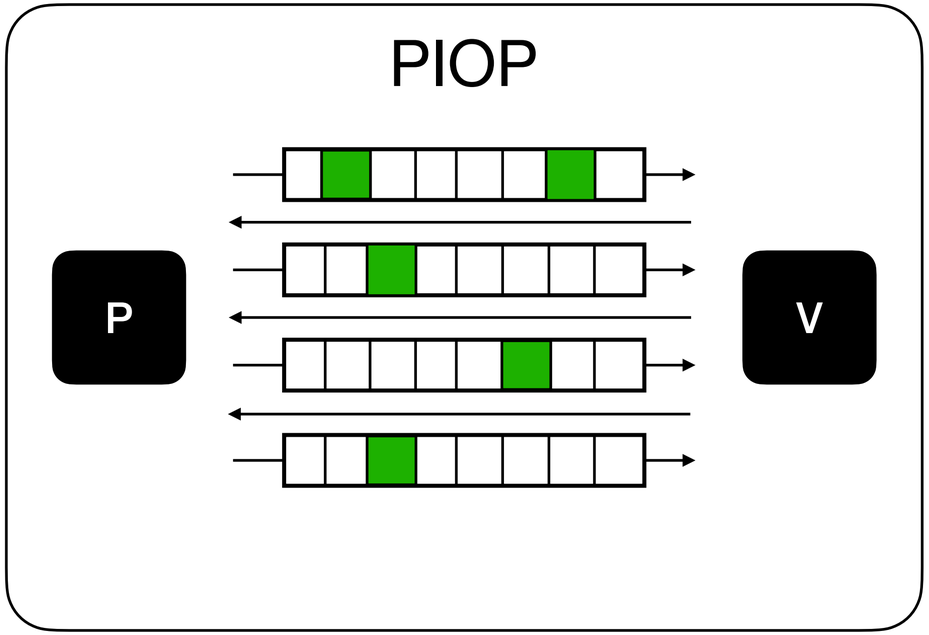

- Oracles are polynomials

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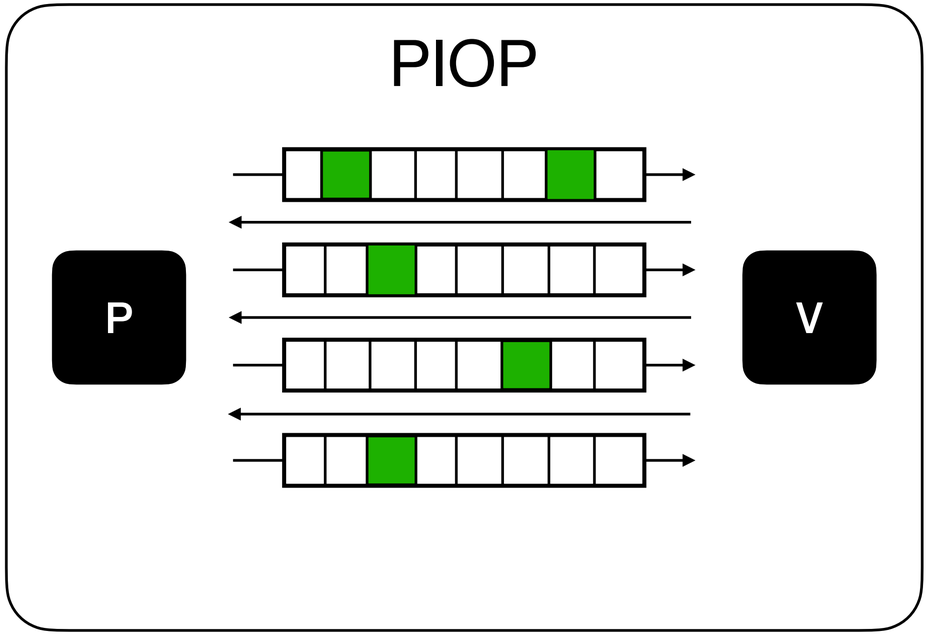

- Oracles are polynomials
- Security is information-theoretical

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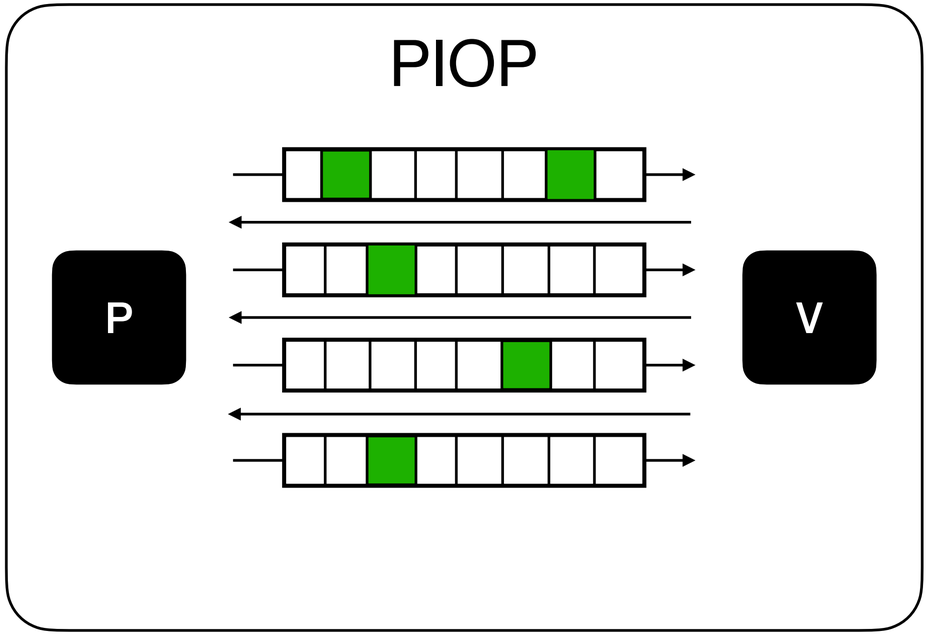

- Oracles are polynomials
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- Proof length is $\Omega(n)$ (not succinct)

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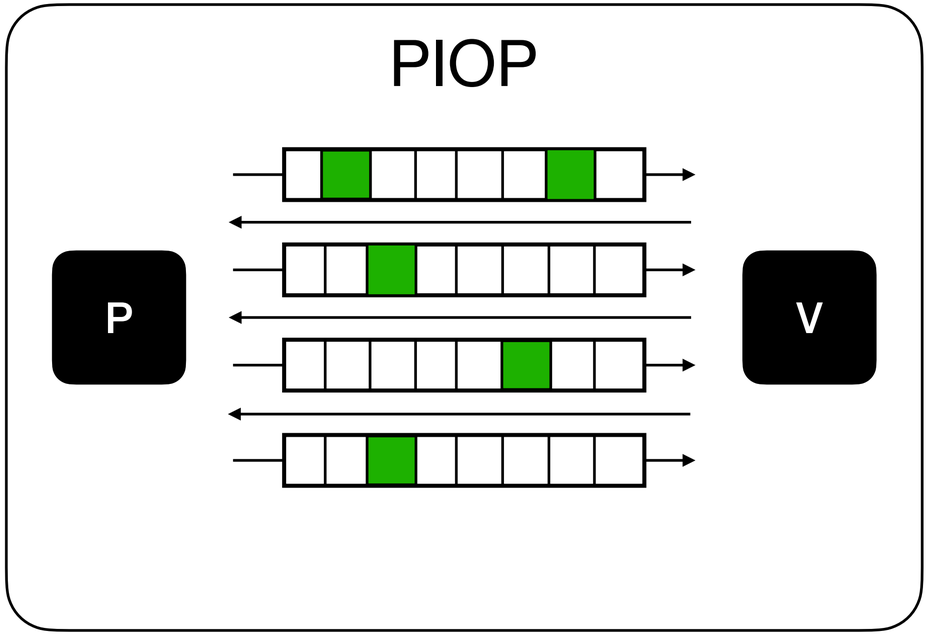

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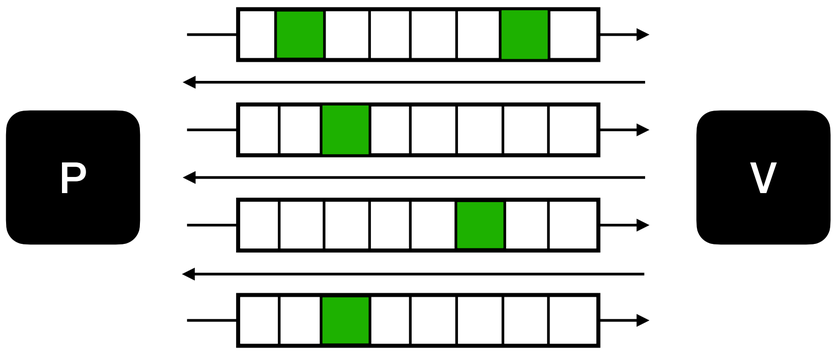
PIOP



PCS

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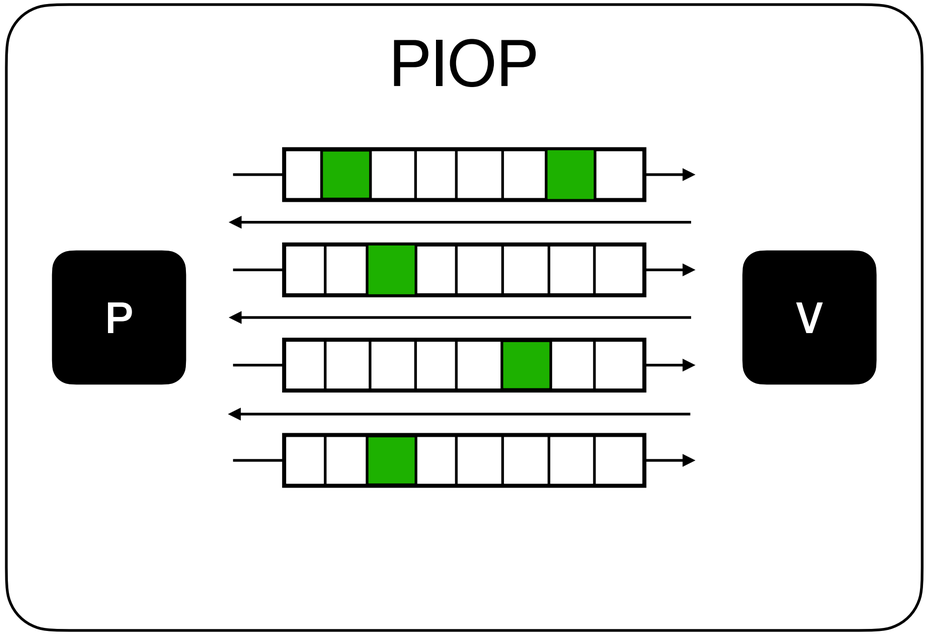

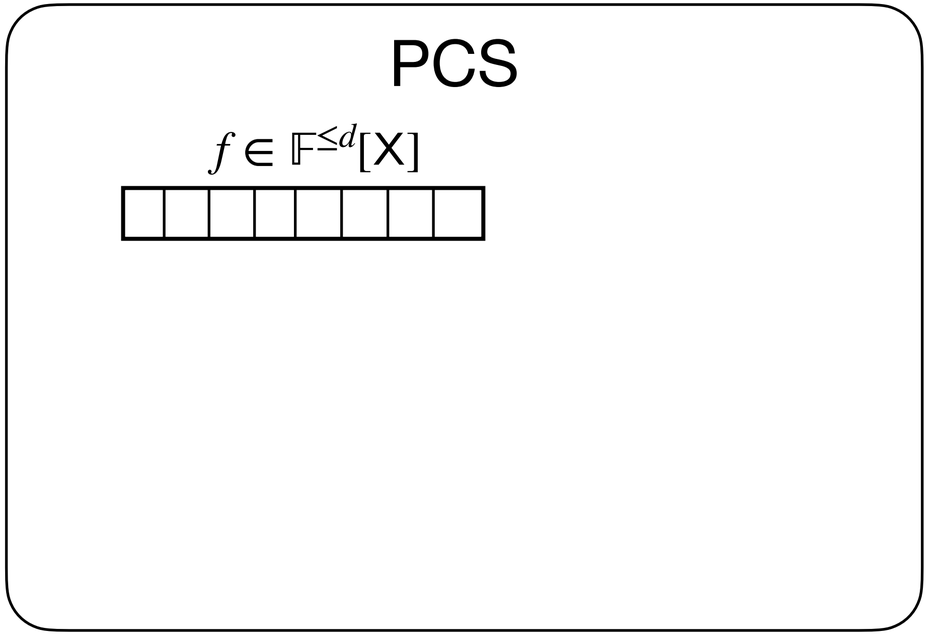

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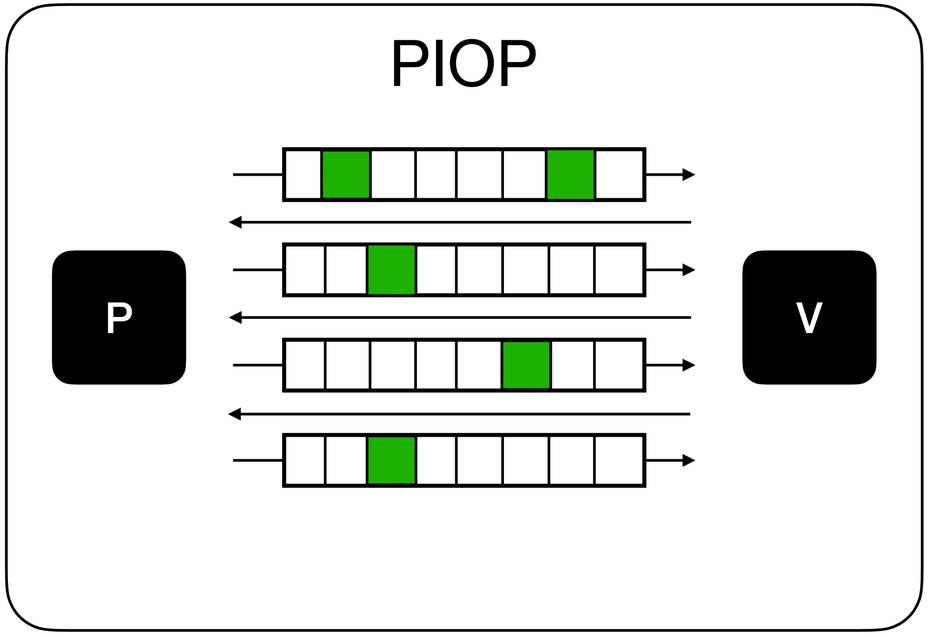

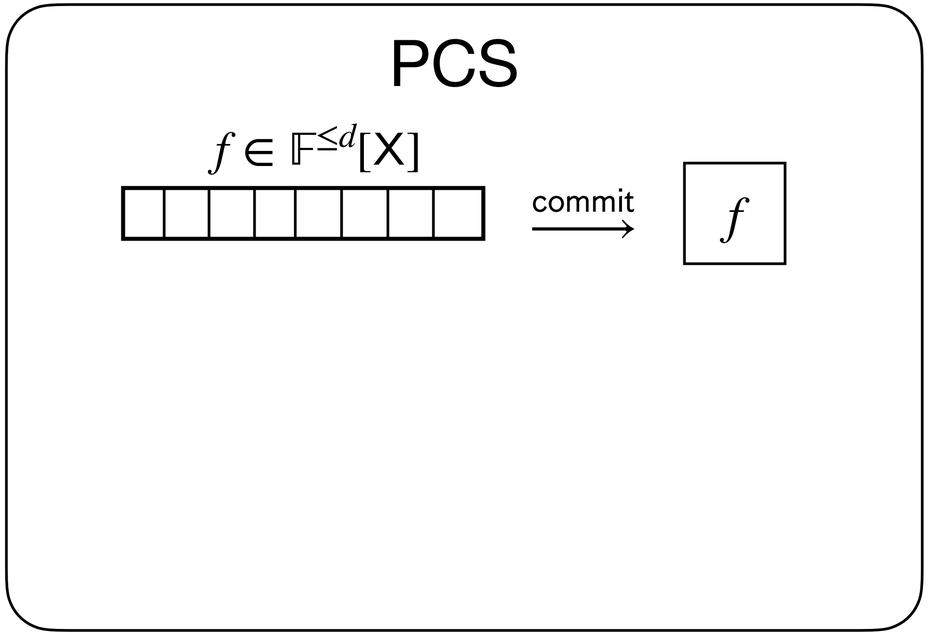

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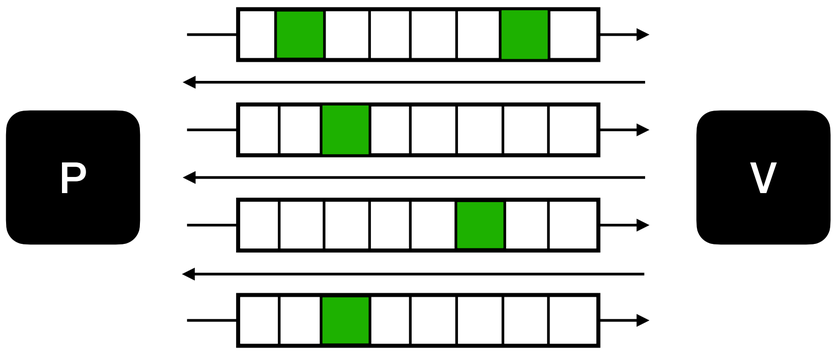

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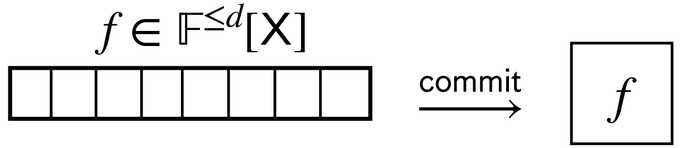

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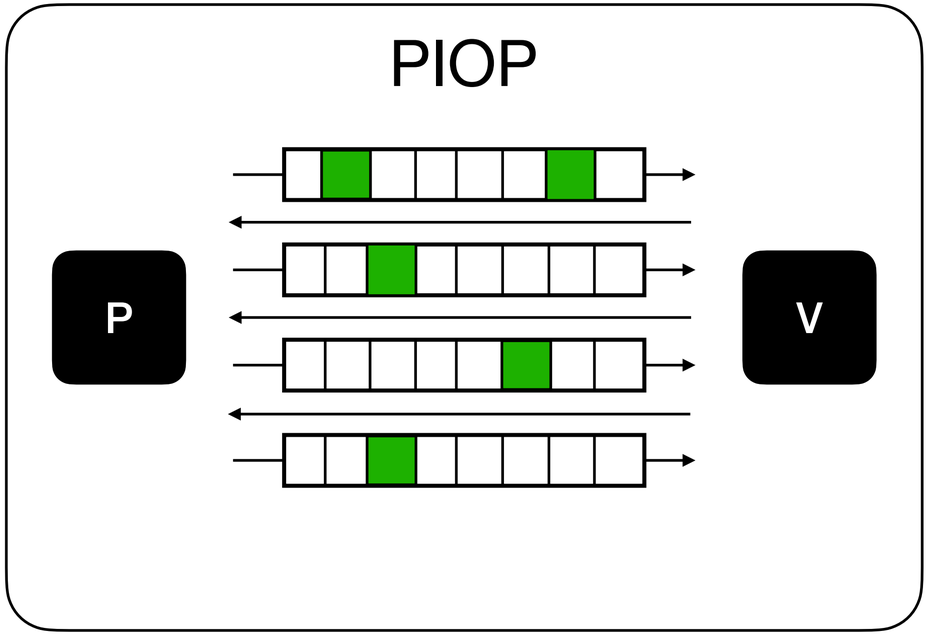

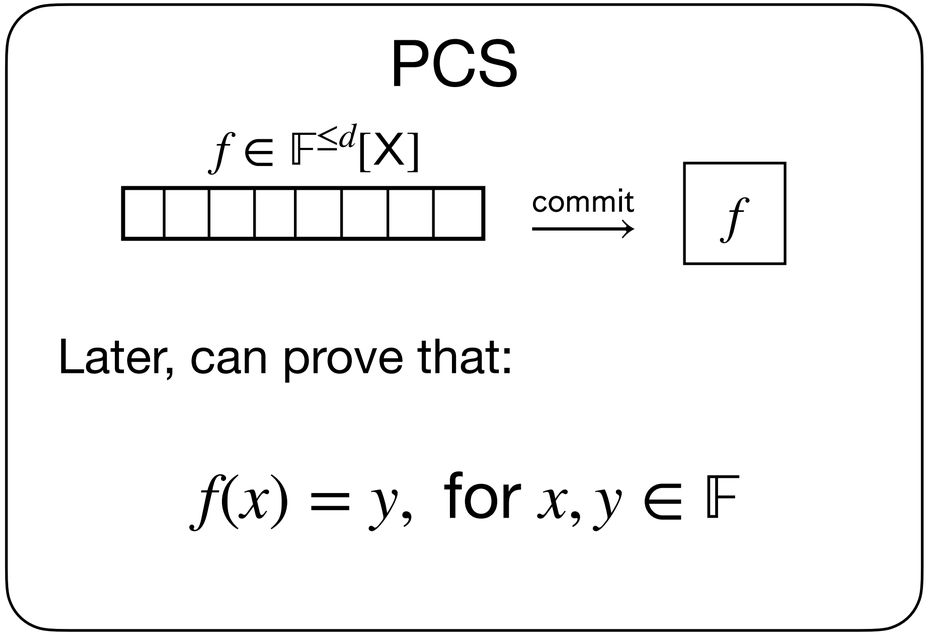

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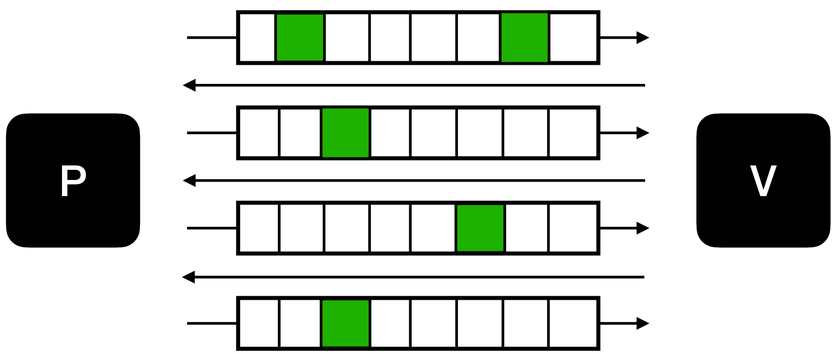

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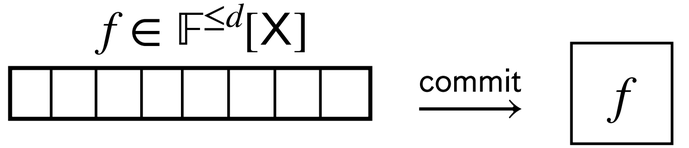

Later, can prove that:

$$f(x) = y, \text{ for } x, y \in \mathbb{F}$$

- Cryptography goes here!

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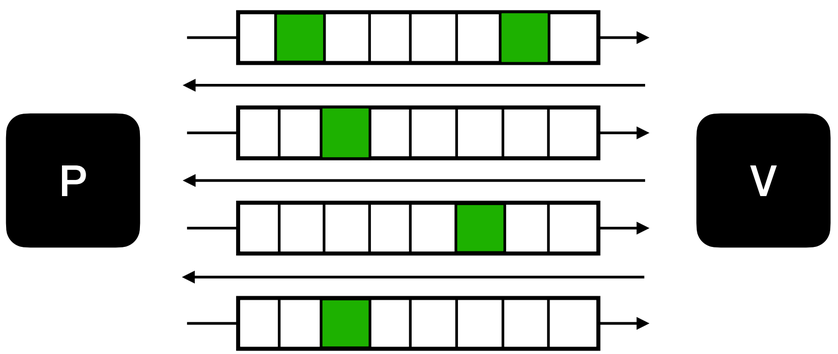

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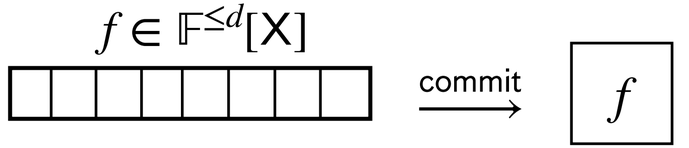

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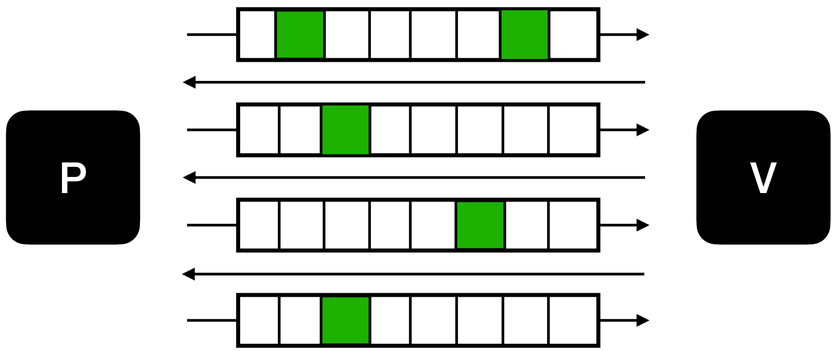

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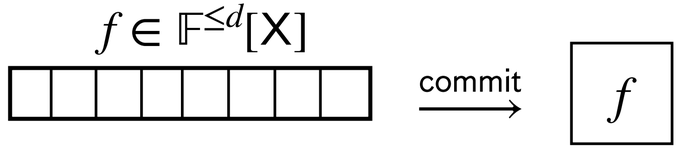

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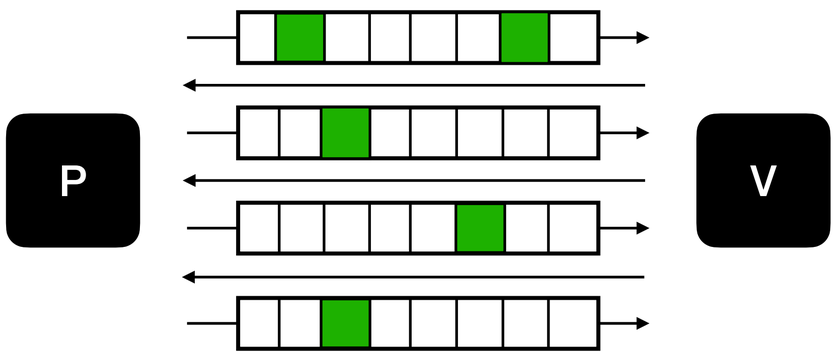

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FS

PCS

$f \in \mathbb{F}^{\leq d}[X]$

commit →

f

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We focus on this!

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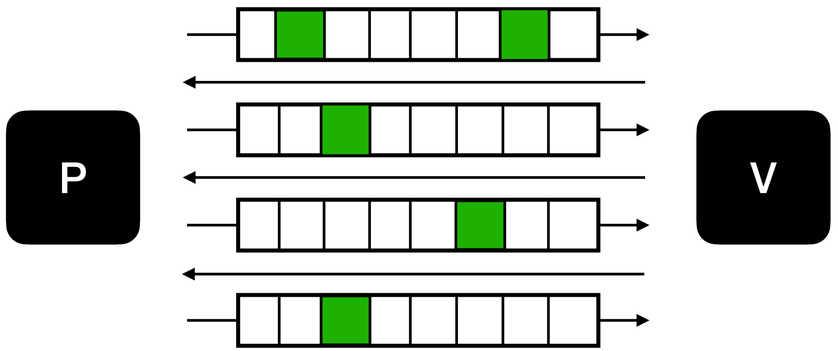

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Zoo of Polynomial Commitments

A very incomplete list…

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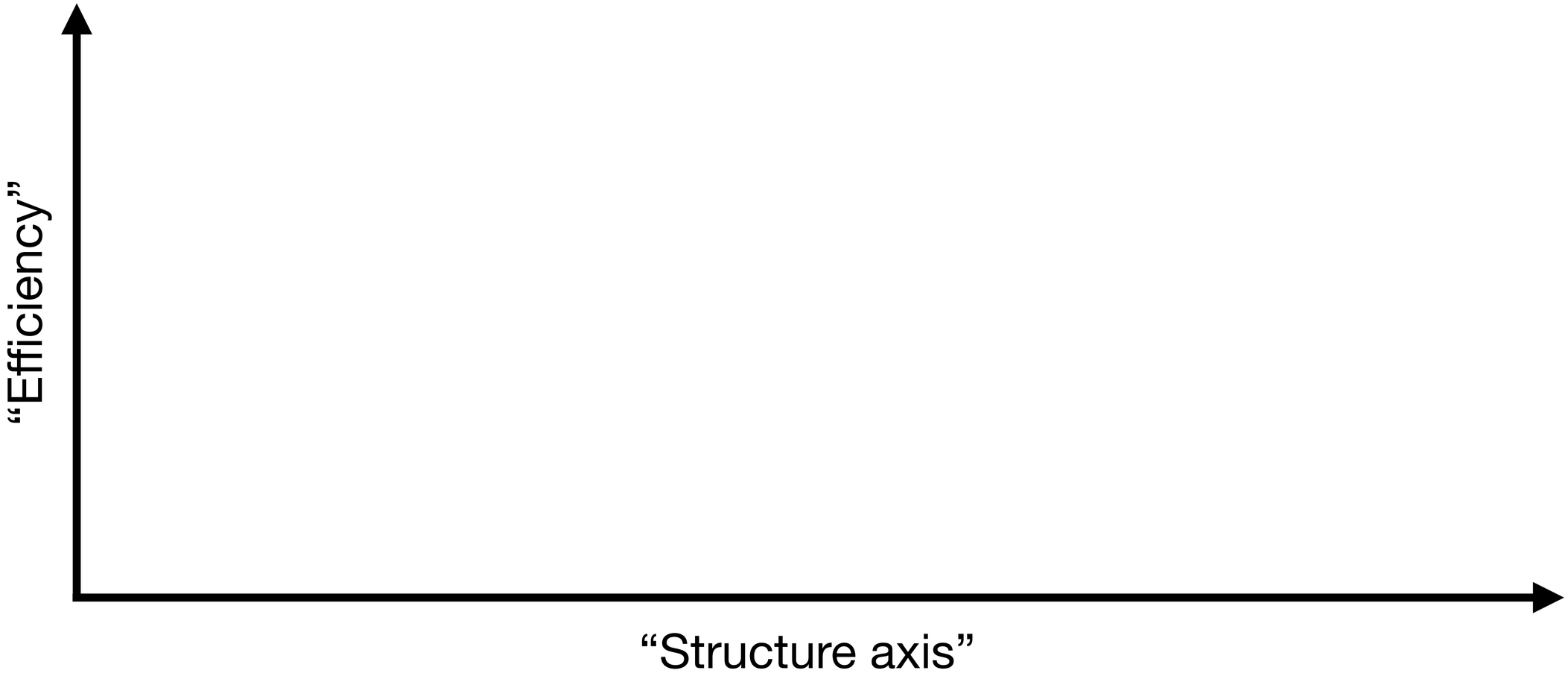

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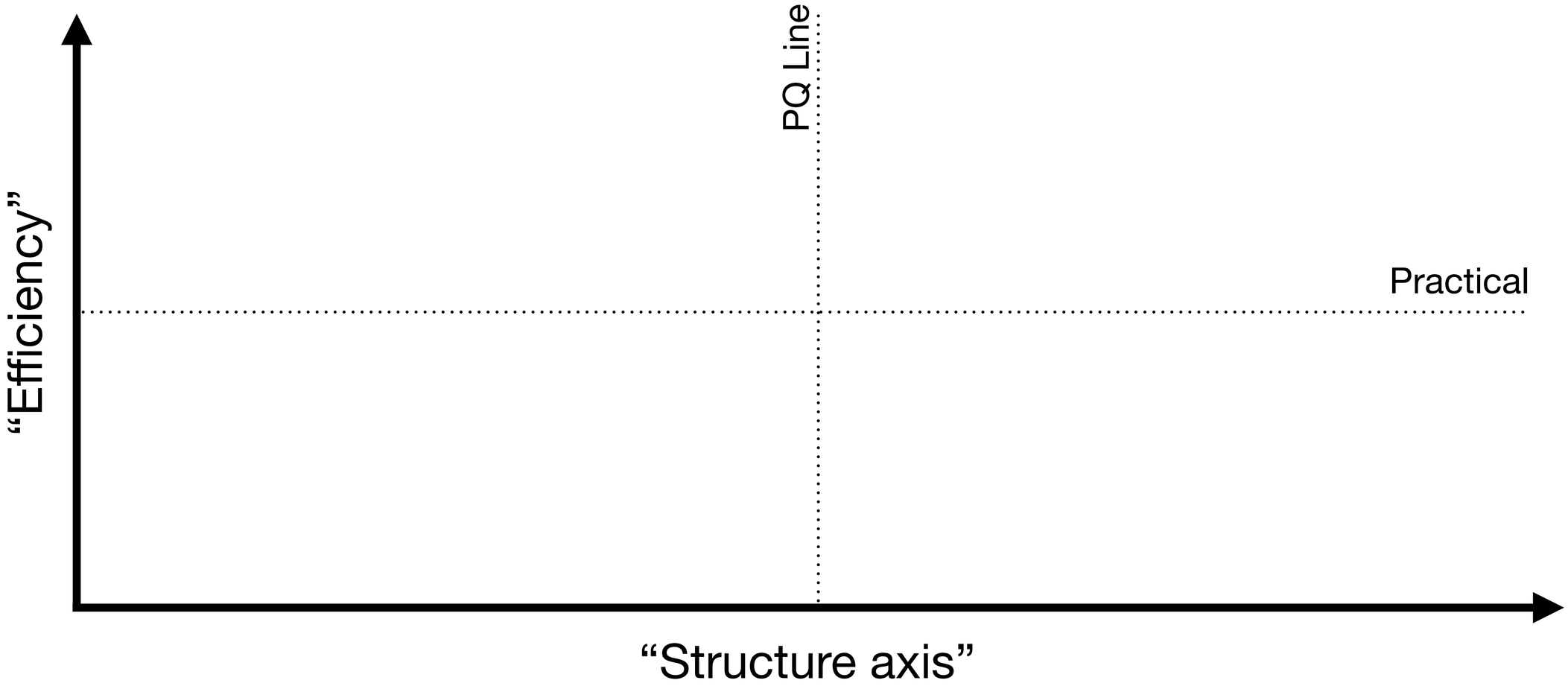

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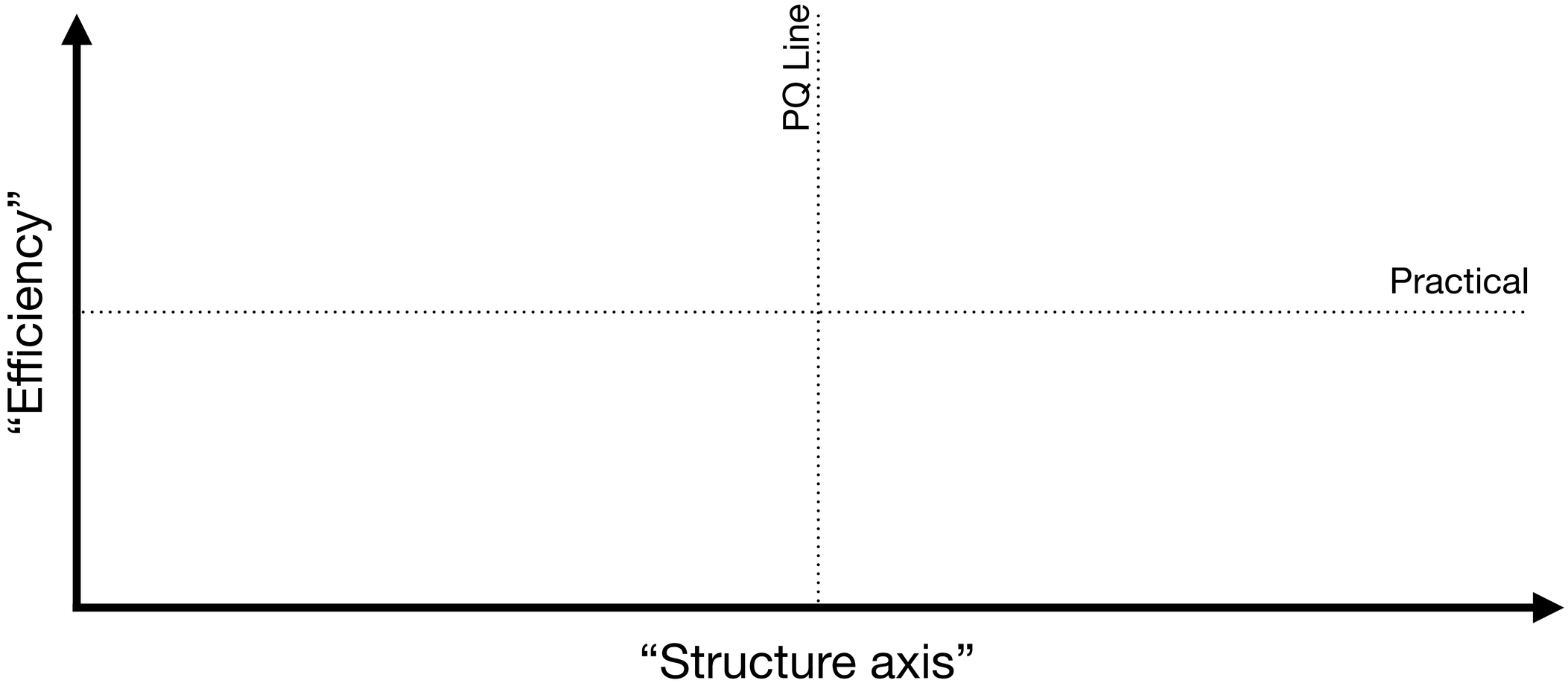

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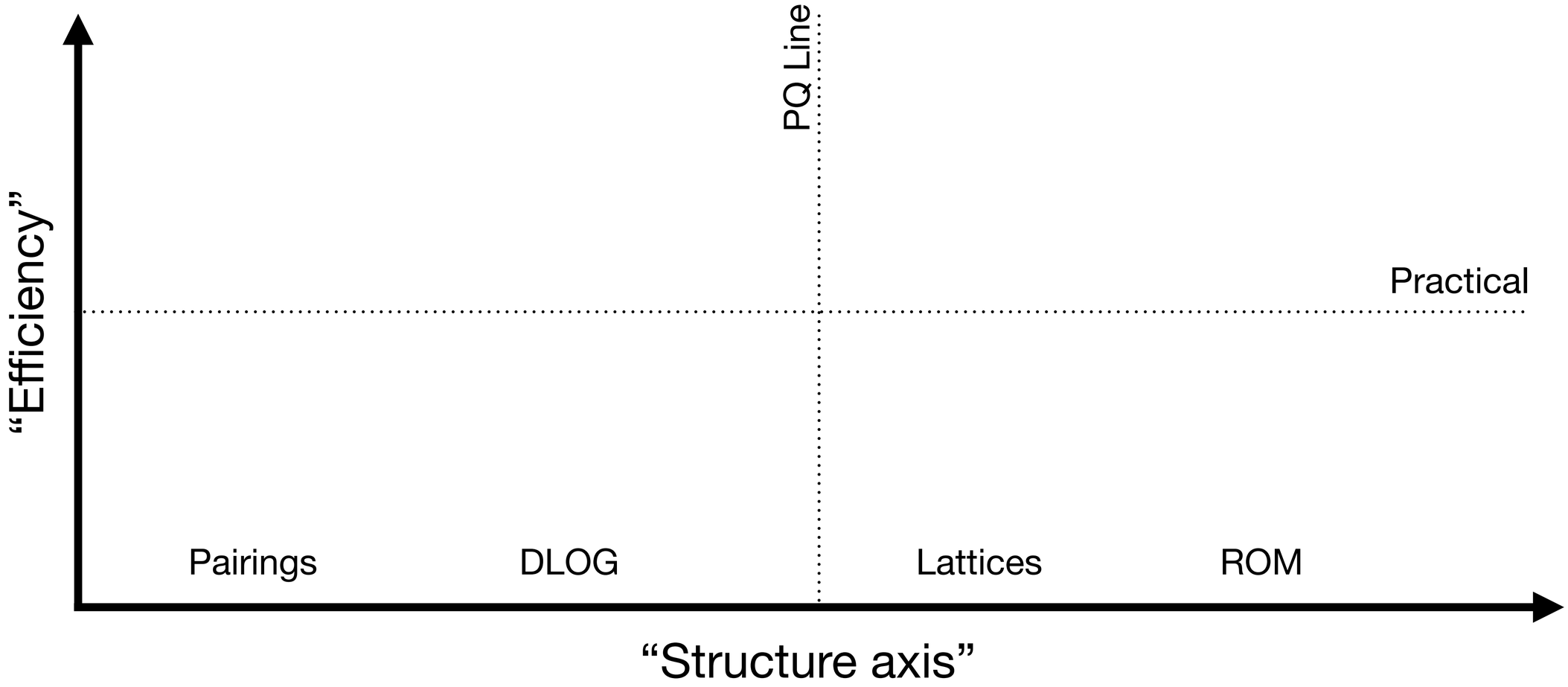

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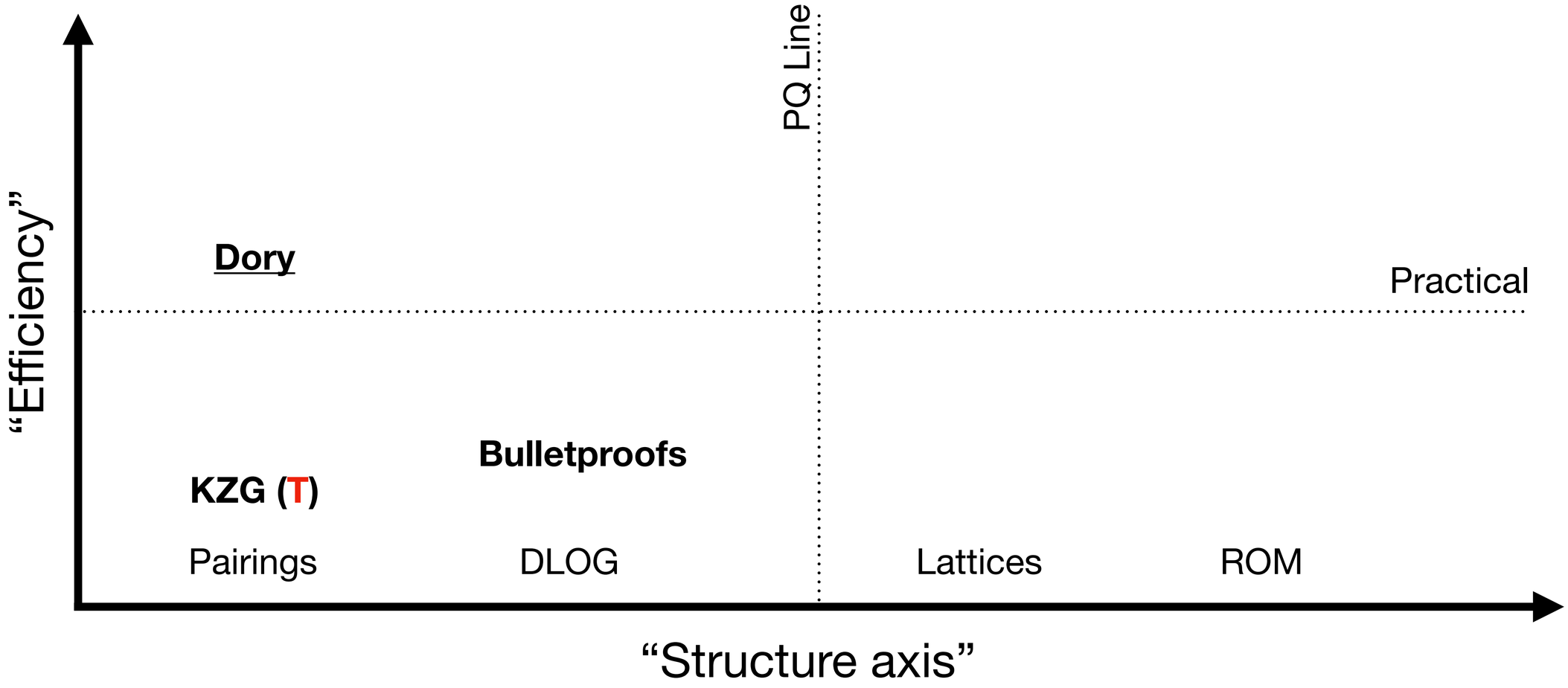

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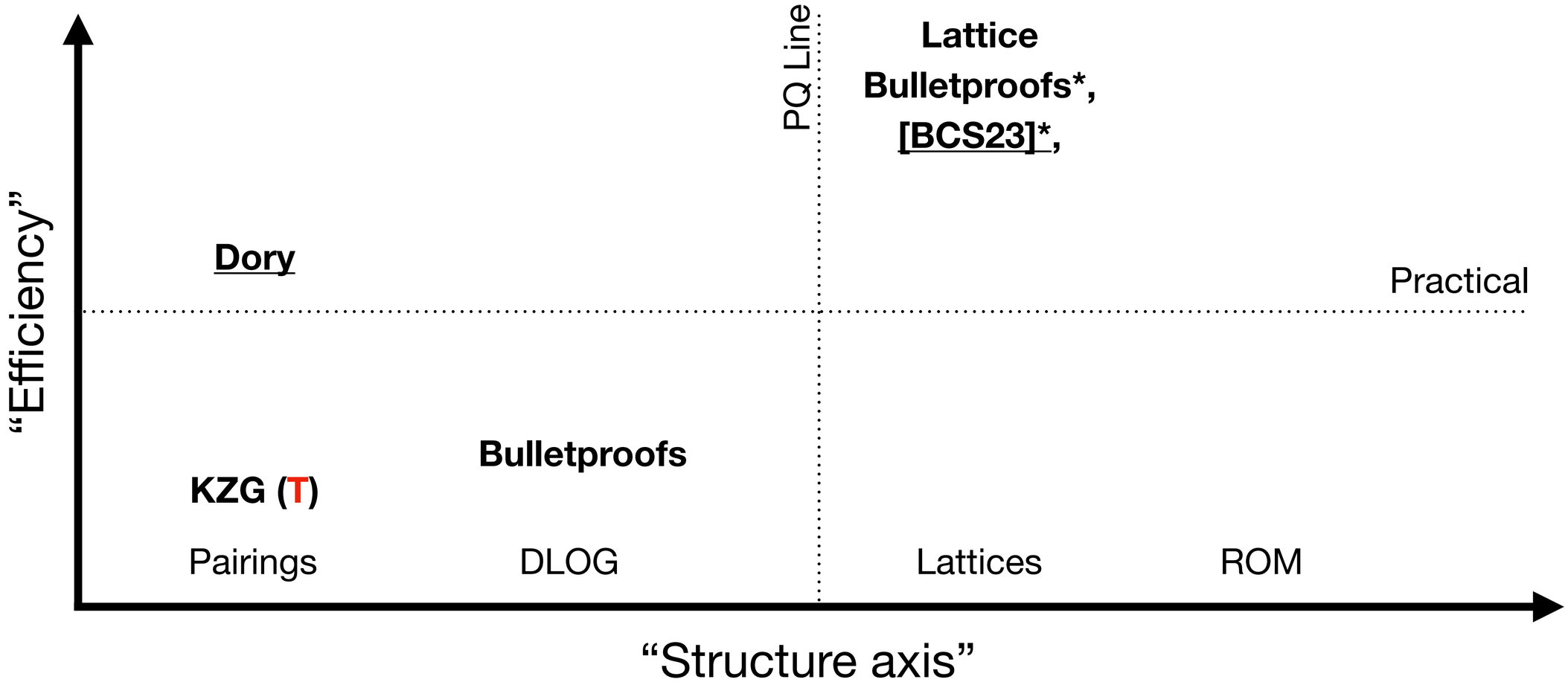

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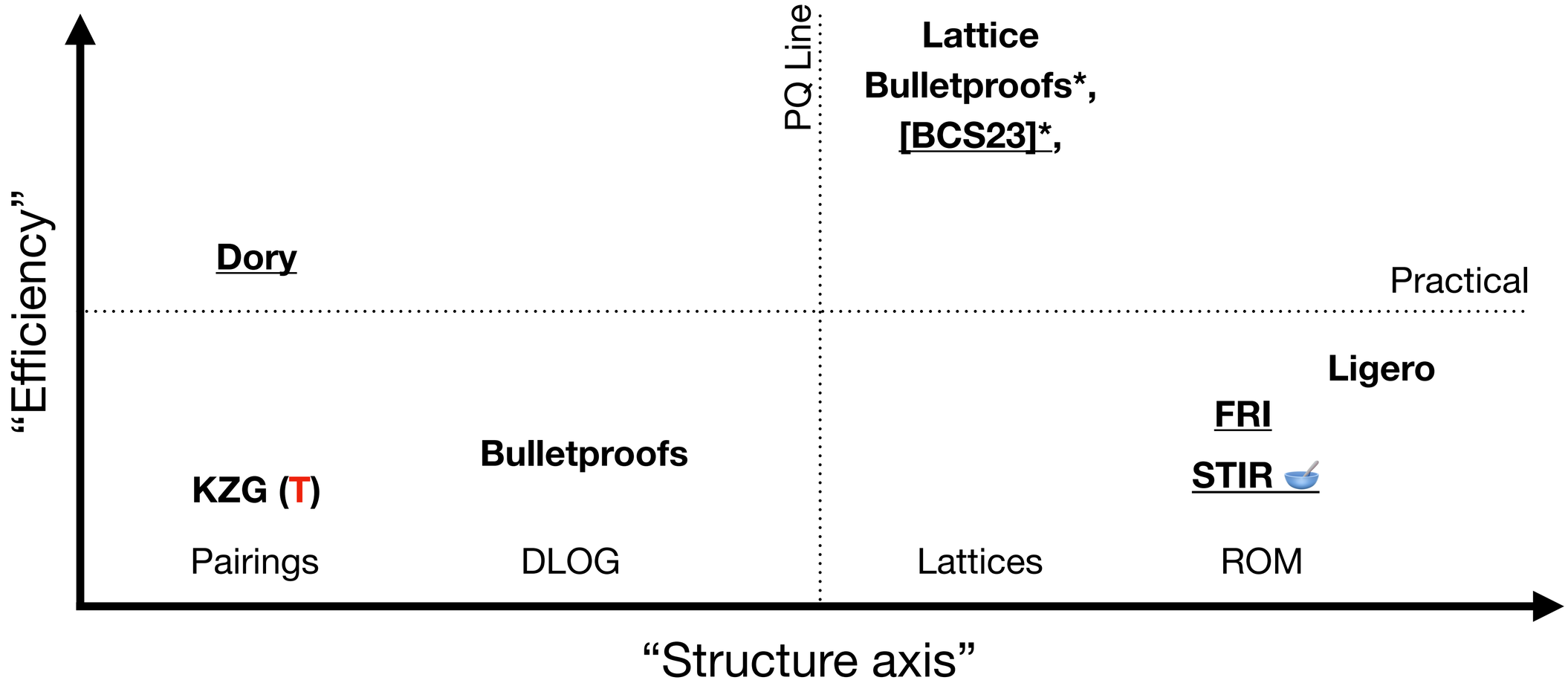

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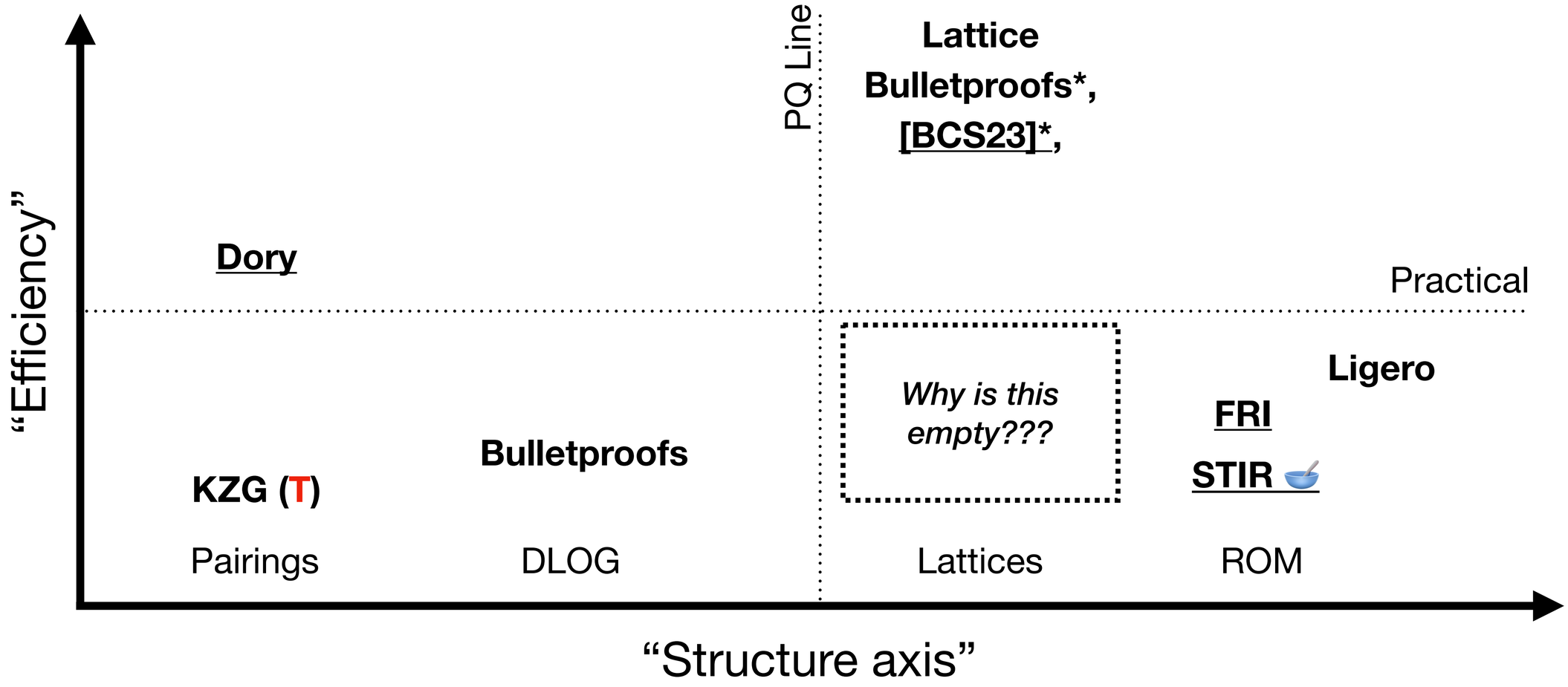

Our Results

SLAP: Succinct Lattice-Based Polynomial Commitments from Standard Assumptions

Martin R. Albrecht
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We construct a non-interactive lattice-based polynomial commitment with:

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We construct a non-interactive lattice-based polynomial commitment with:

1. Succinct proofs
2. Succinct verification time
3. Binding under (M)SIS

Techniques

Lattice-Based SNARKs

How to get around [GW11]?

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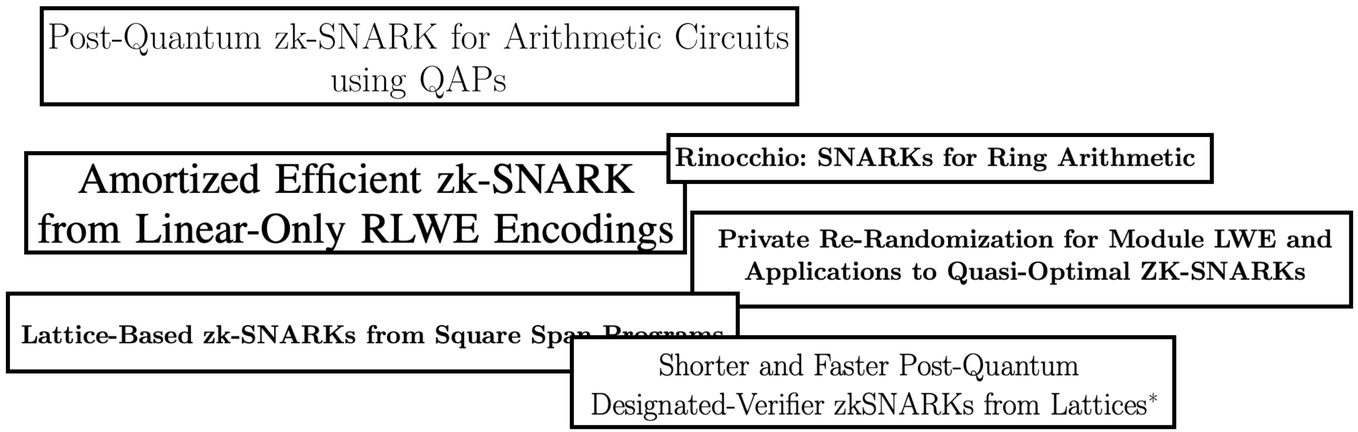

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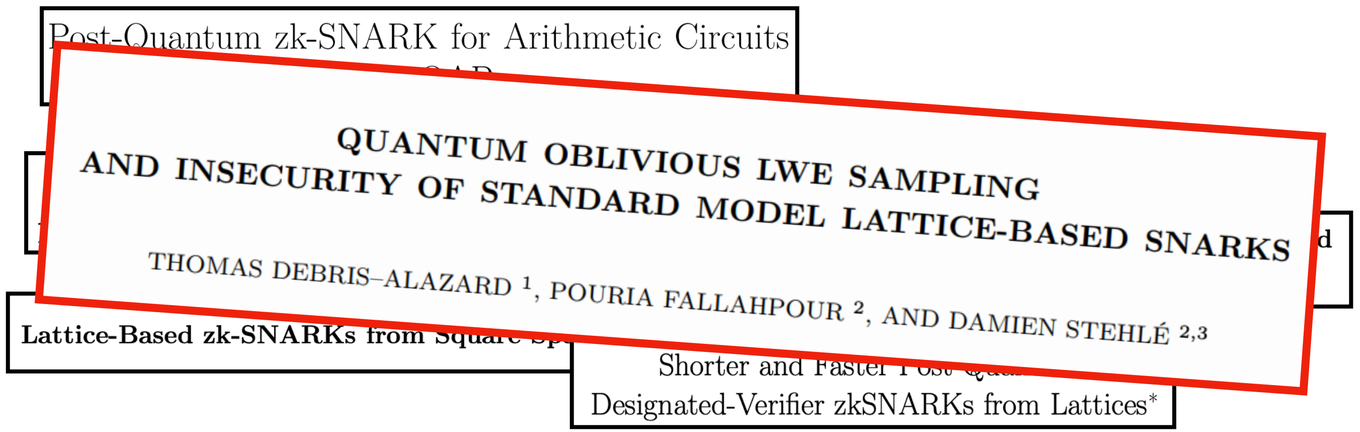

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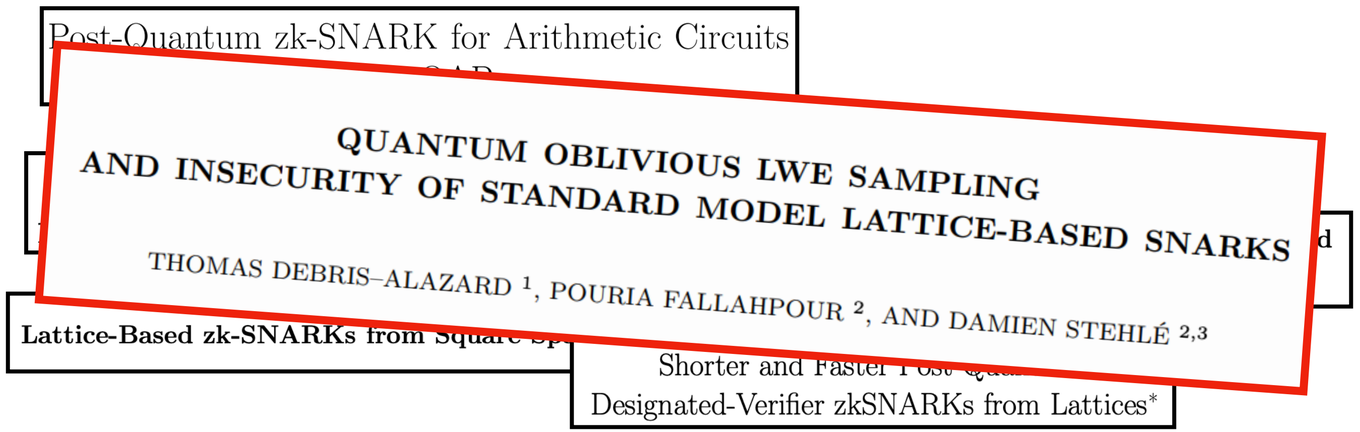

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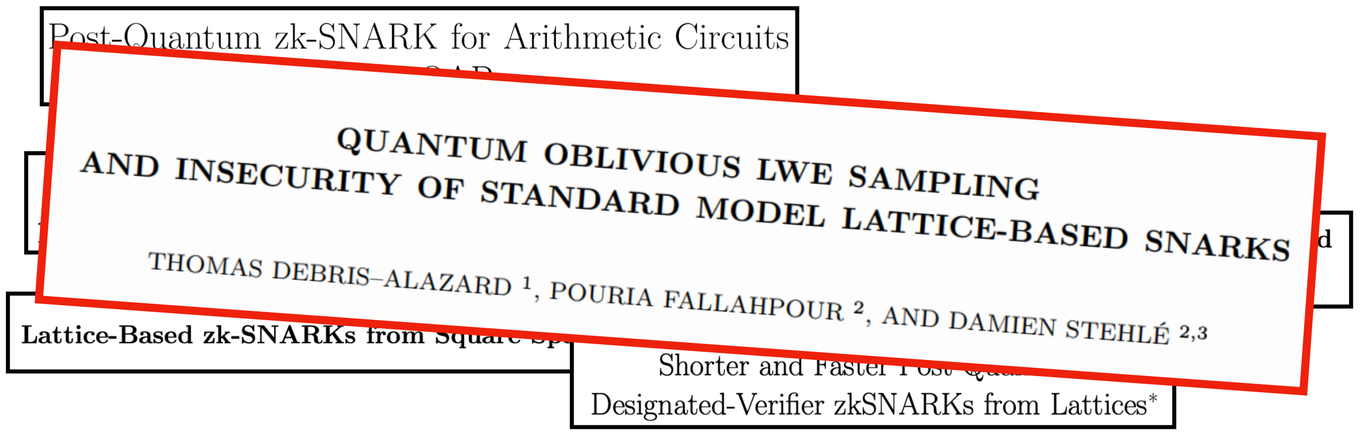

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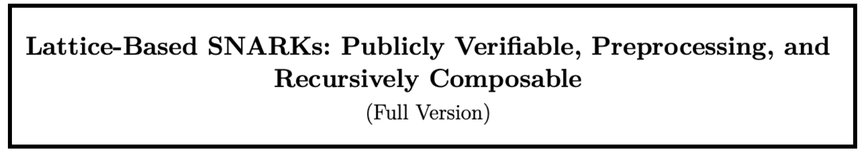

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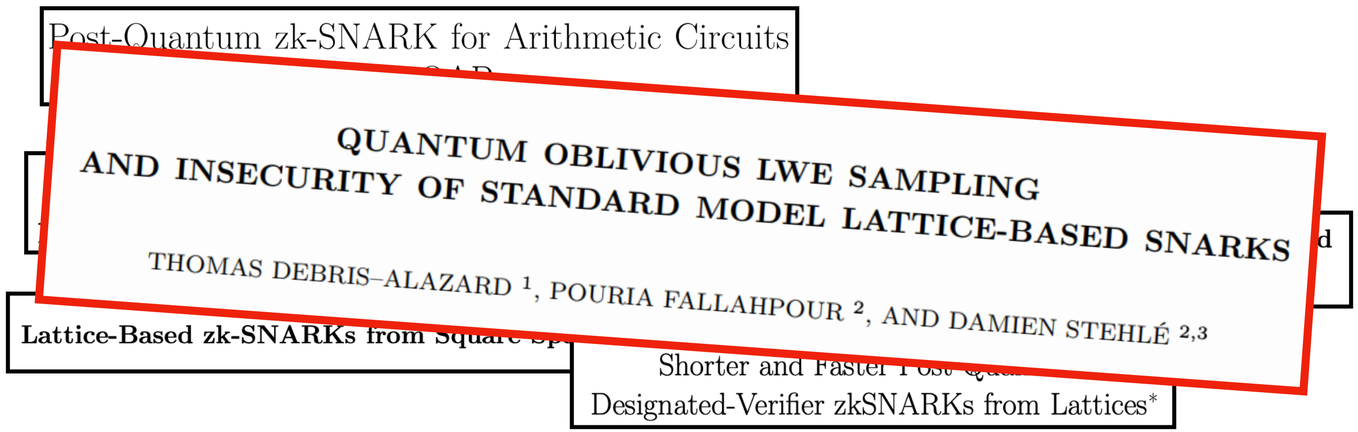

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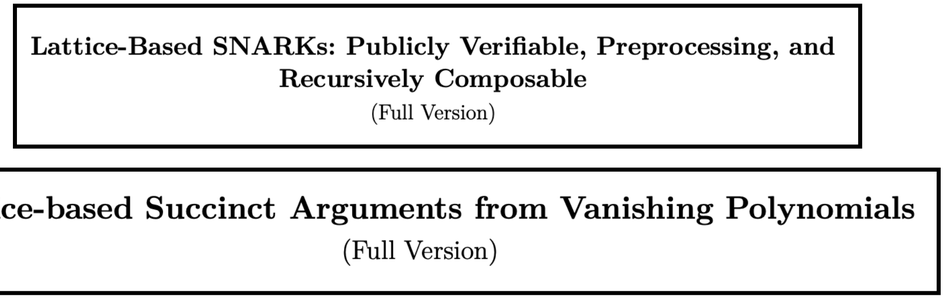

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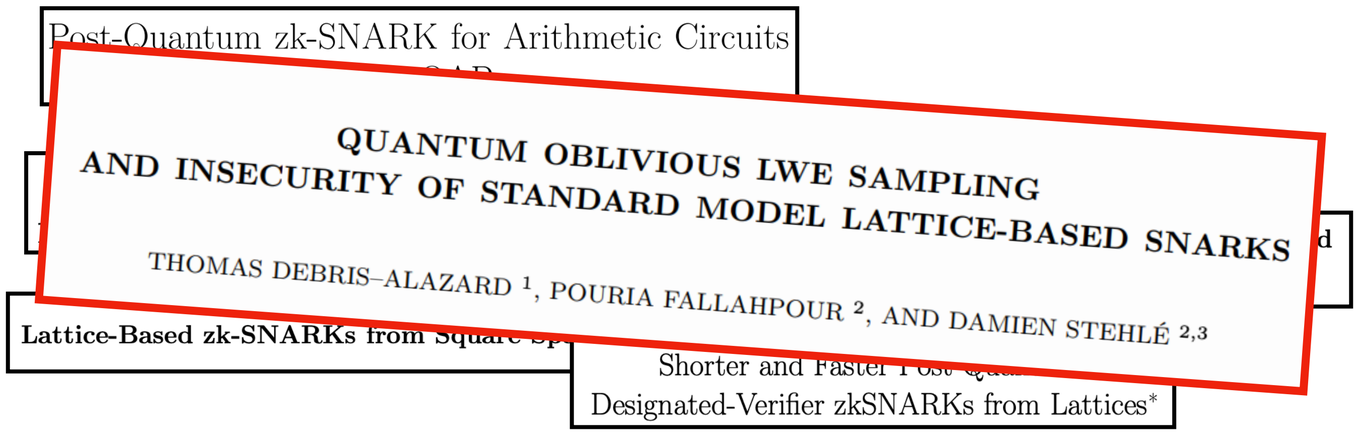

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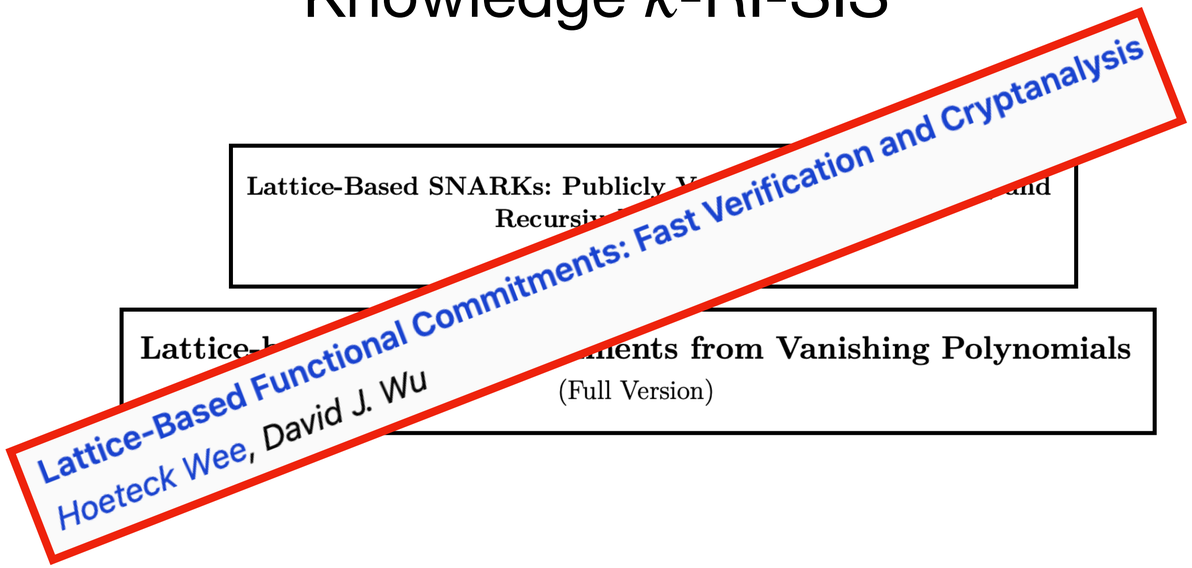

Lattice Assumptions ❤️ ROM

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Special Sound
Interactive Protocol

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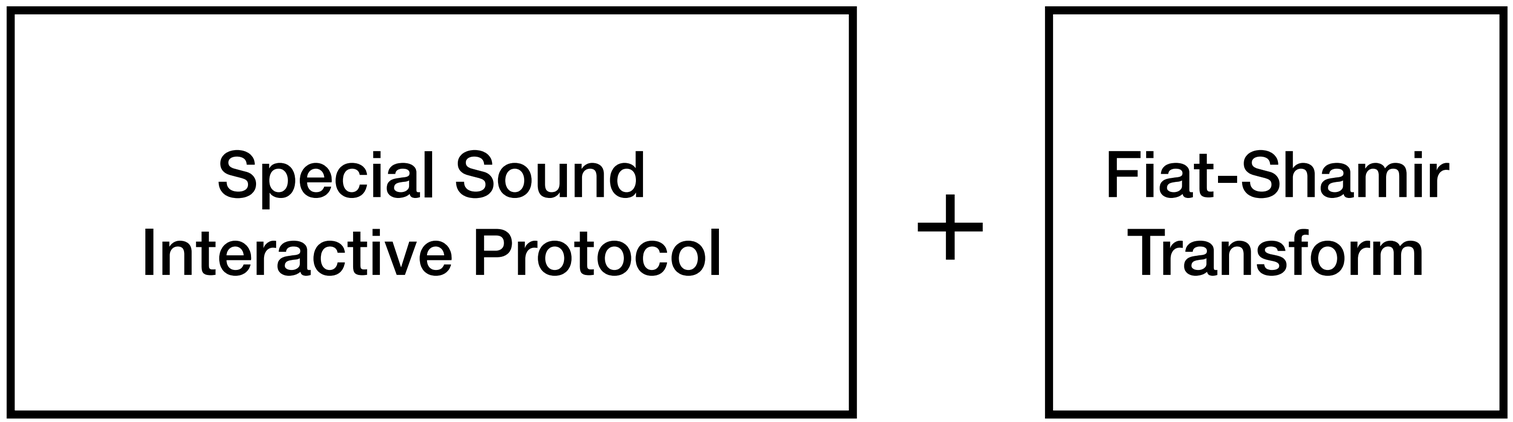

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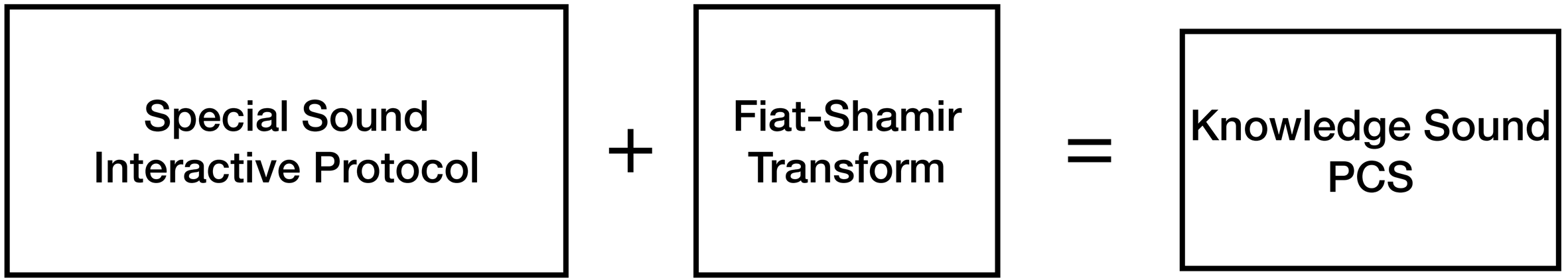

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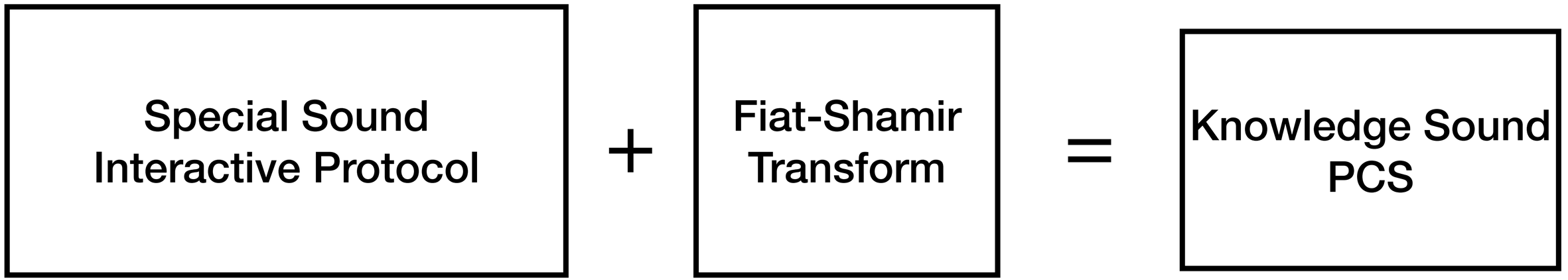

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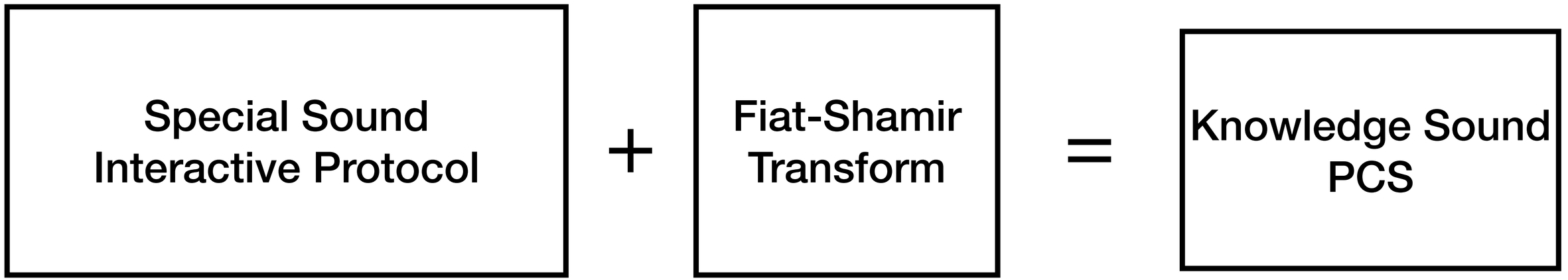

- Use lattices to get succinctness in the interactive protocol.
- **Open Question**: ROM alone is sufficient for efficient PCS (e.g. STIR), what do we gain by using lattices?

Building succinct PCS

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Commitment Scheme

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- Commit to a vector $\mathbf{f} \in \mathscr{R}_q^d$

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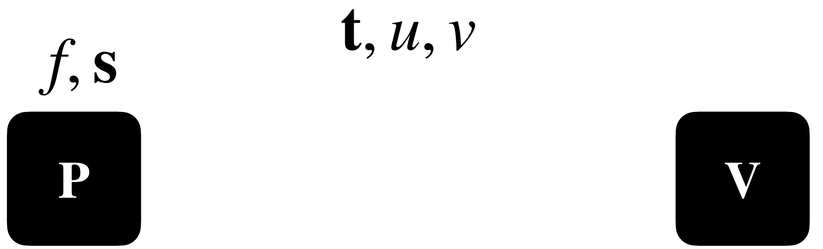

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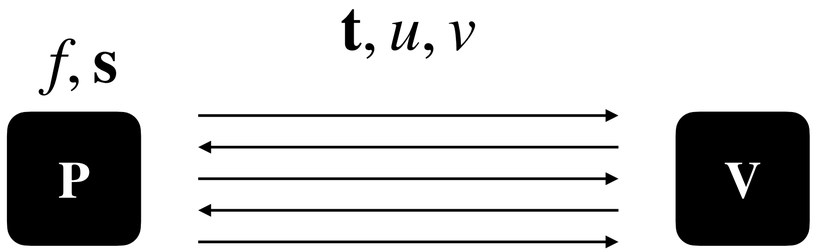

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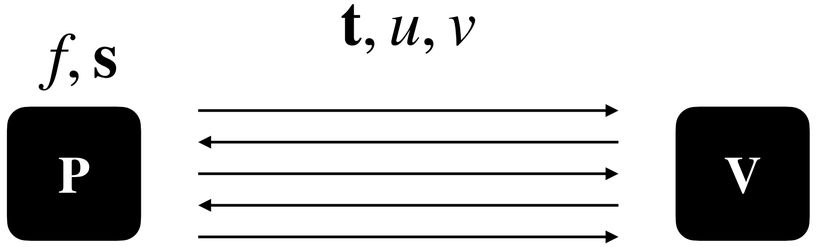

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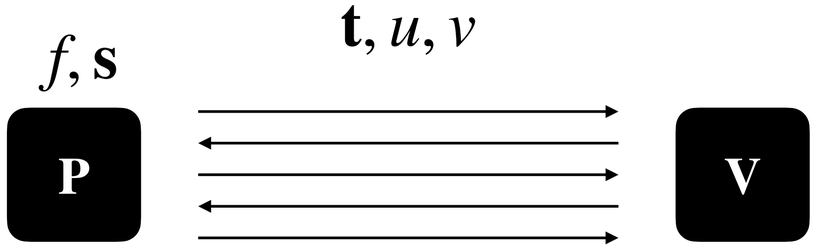

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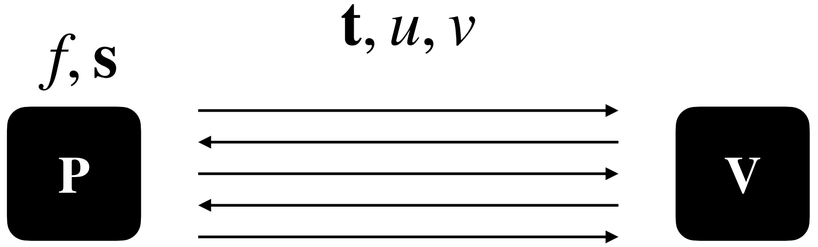

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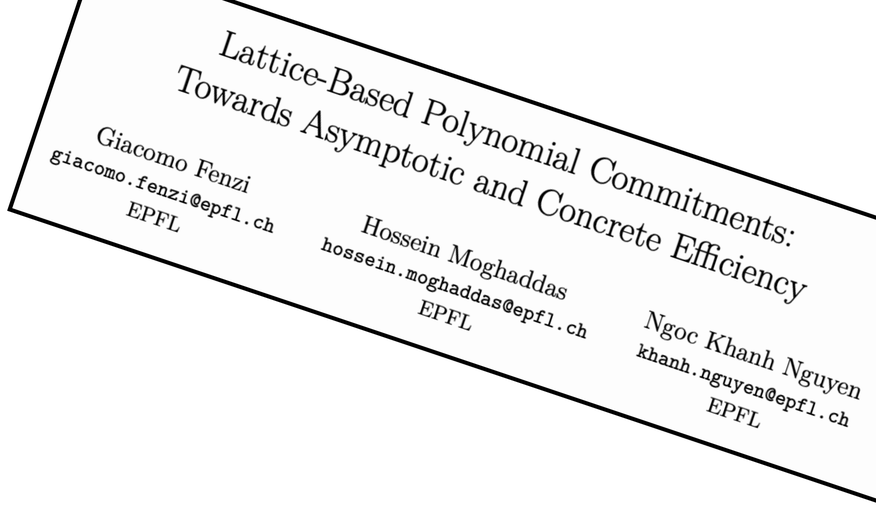
PRISIS Commitments I

A starting point [FMN23]



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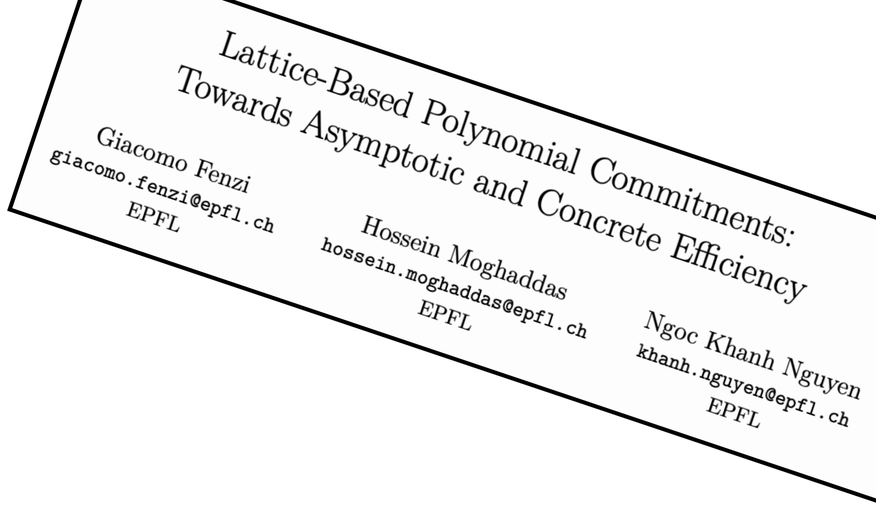
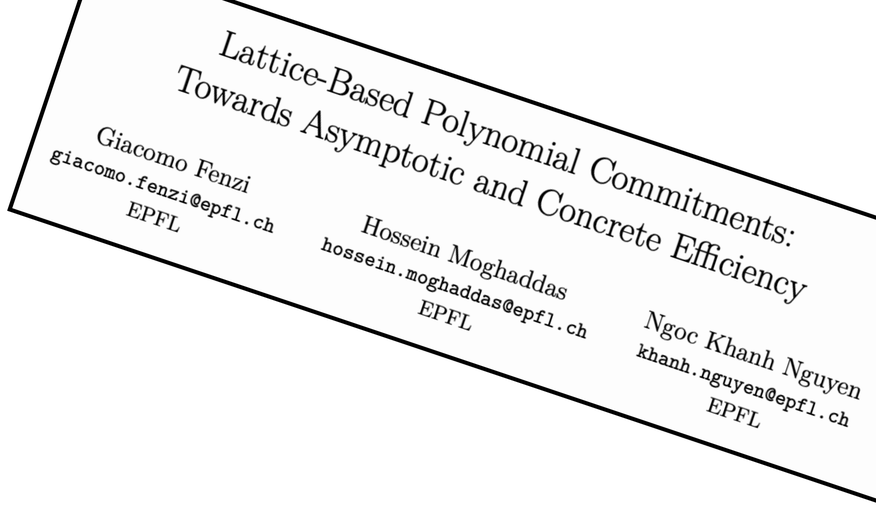


Given $\mathbf{B} := \begin{bmatrix} w^0\mathbf{A} & \dots & -\mathbf{G} \\ & \ddots & \\ & \dots \quad w^{\ell-1}\mathbf{A} & -\mathbf{G} \end{bmatrix}$ and trapdoor $\mathbf{T}$ for $\mathbf{B}$

PRISIS Commitments I

A starting point [FMN23]



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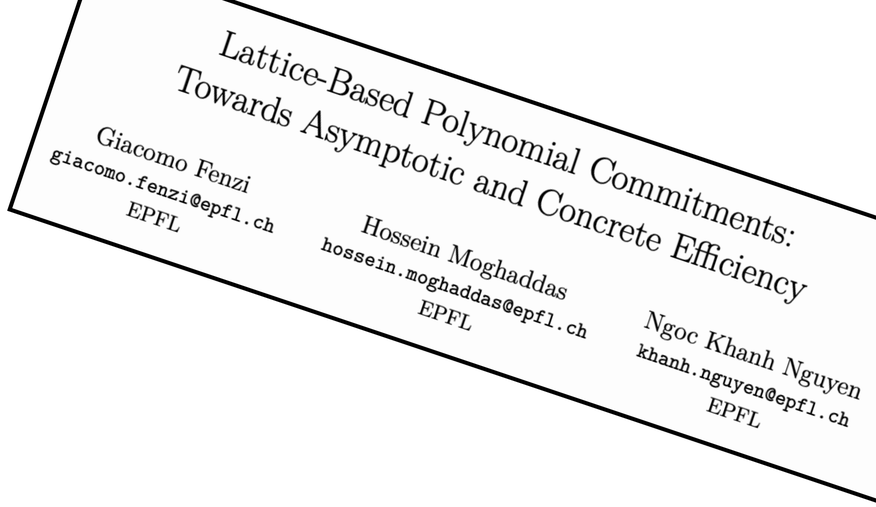

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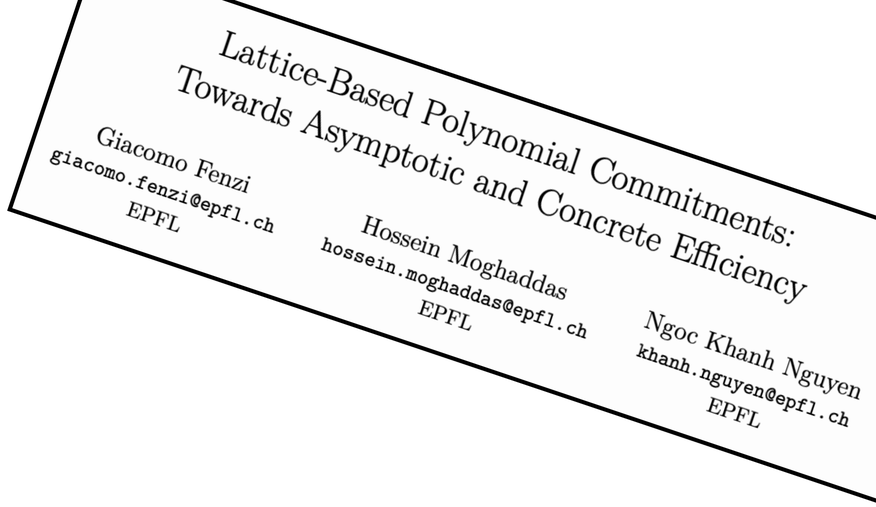

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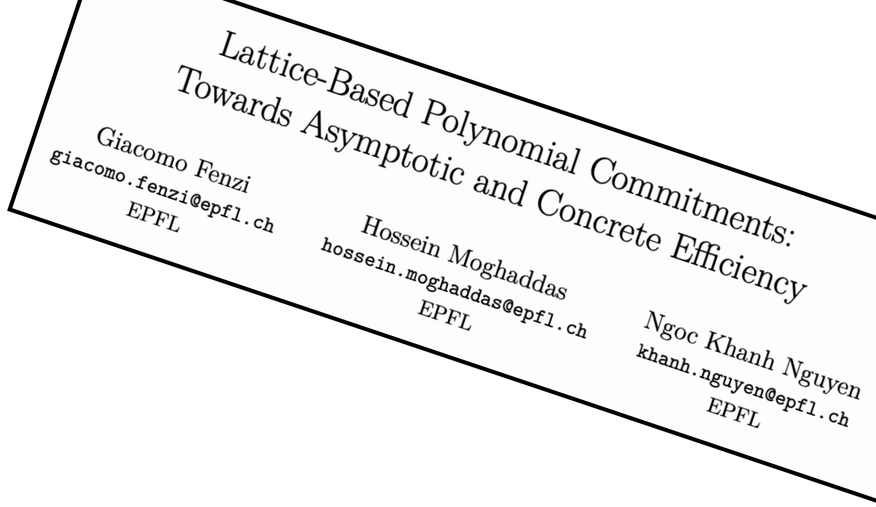

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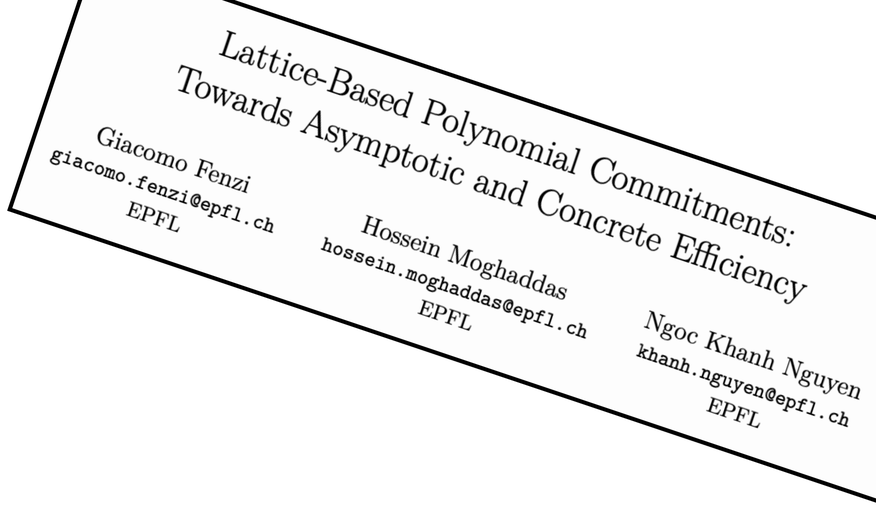

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$\mathbf{A}\mathbf{s}_i + f_i\mathbf{e}_1 = w^{-i}\mathbf{t}$ and $\mathbf{s}_i$ short

BASIS-👨‍👩‍👧‍👧 [WW23]

BASIS-👨‍👩‍👧‍👧 [WW23]

BASIS Game

BASIS-👨‍👩‍👧‍👧 [WW23]

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$$\mathbf{A}^\star \leftarrow \mathscr{R}_q^{m \times n}$$

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BASIS-👨‍👩‍👧‍👧 [WW23]

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Can we do better?

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$$\sigma \geq \delta \sqrt{tN \cdot (N^2 \mathfrak{s}^2 m + 2t)} \cdot \omega(\sqrt{N \log nN}).$$

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Small-Dimension PRISIS

[FMN23]: $\ell = 2$ reduces to MSIS

Lemma 3.6 (PRISIS $\implies$ MSIS)**.** *Let $n > 0, m \geq n$ and denote $t = (n+1)\tilde{q}$. Let $q = \omega(N)$. Take $\epsilon \in (0, 1/3)$ and $\mathfrak{s} \geq \max(\sqrt{N \ln(8Nq)} \cdot q^{1/2+\epsilon}, \omega(N^{3/2} \ln^{3/2} N))$ such that $2^{10N} q^{-\lfloor \epsilon N \rfloor}$ is negligible. Let*

$$\sigma \geq \delta \sqrt{tN \cdot (N^2 \mathfrak{s}^2 m + 2t)} \cdot \omega(\sqrt{N \log nN}).$$

Then, $\mathsf{PRISIS}_{n,m,N,q,2,\sigma,\beta}$ *is hard under the* $\mathsf{MSIS}_{n,m,N,q,\beta}$ *assumption.*

Multi-Instance BASIS

h-instance BASIS Game

$\mathbf{A}_1^\star, \ldots, \mathbf{A}_h^\star \leftarrow \mathscr{R}_q^{m \times n}$

$\mathsf{aux}_i \leftarrow \mathsf{Samp}(\mathbf{A}_i^\star)$ for $i \in [h]$

return $((\mathbf{A}_i^\star, \mathsf{aux}_i)_i)$ to $\mathscr{A}$

$\mathscr{A}$ wins if it finds $\mathbf{x}$:

- $[\mathbf{A}_1^\star, \ldots, \mathbf{A}_h^\star] \cdot \mathbf{x} = 0$
- $0 < |\mathbf{x}| \leq \beta$

For $\ell = O(1)$, if PRISIS_ℓ is hard so is h-PRISIS_ℓ!

Merkle-PRISIS I

Example with $d = 8$

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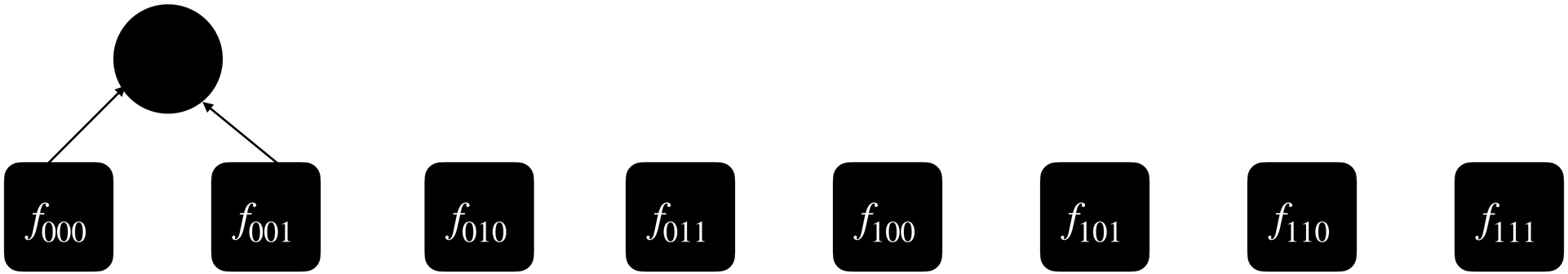

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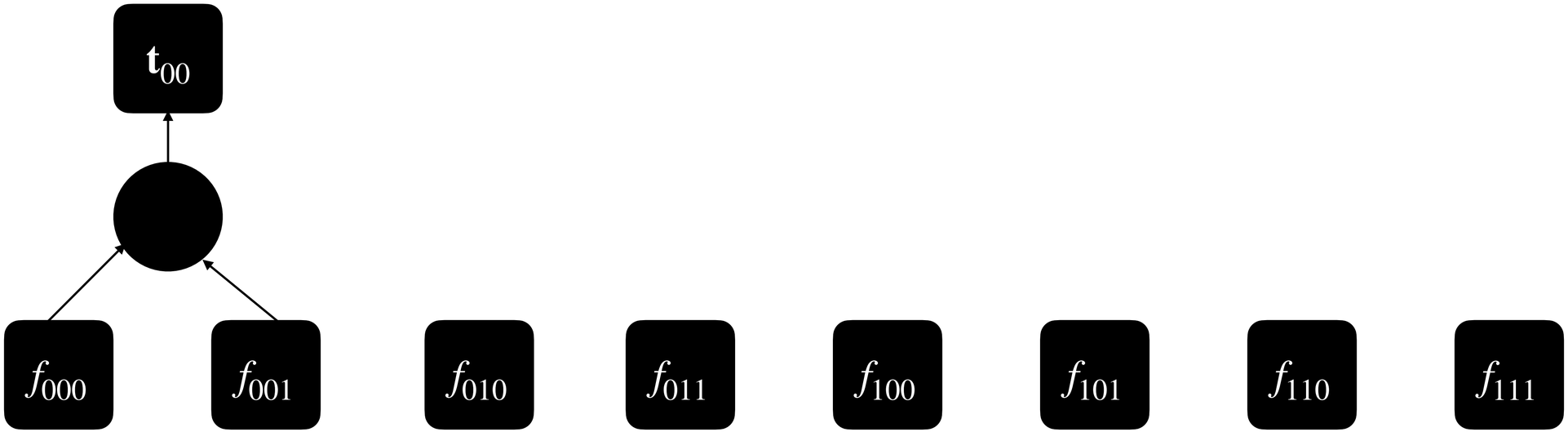

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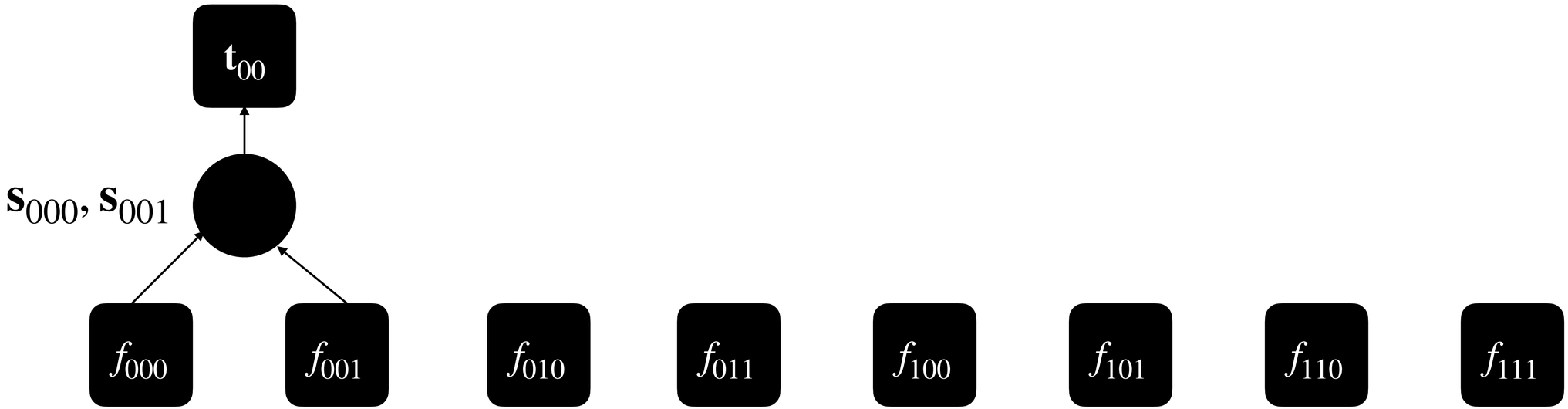

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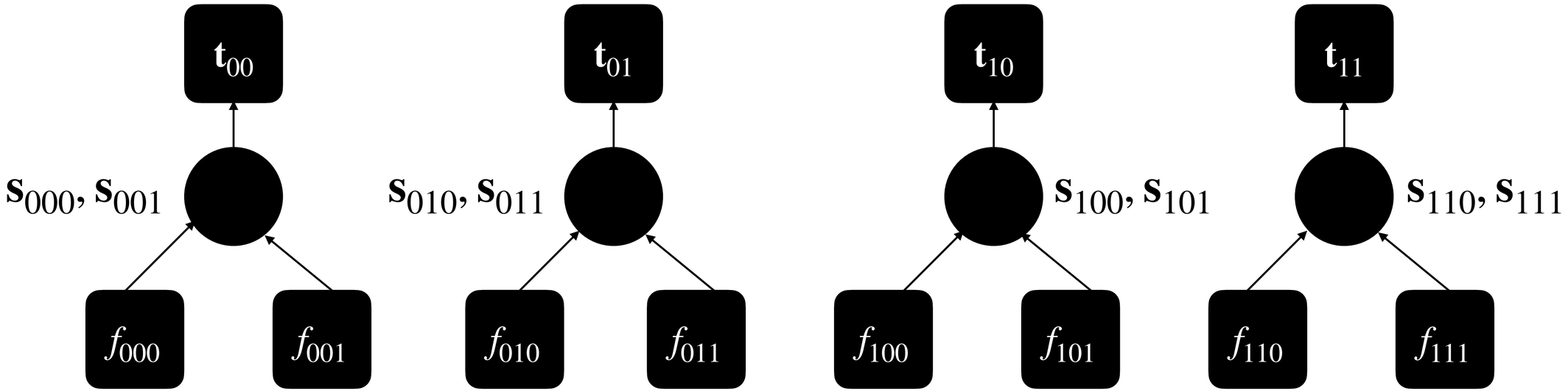

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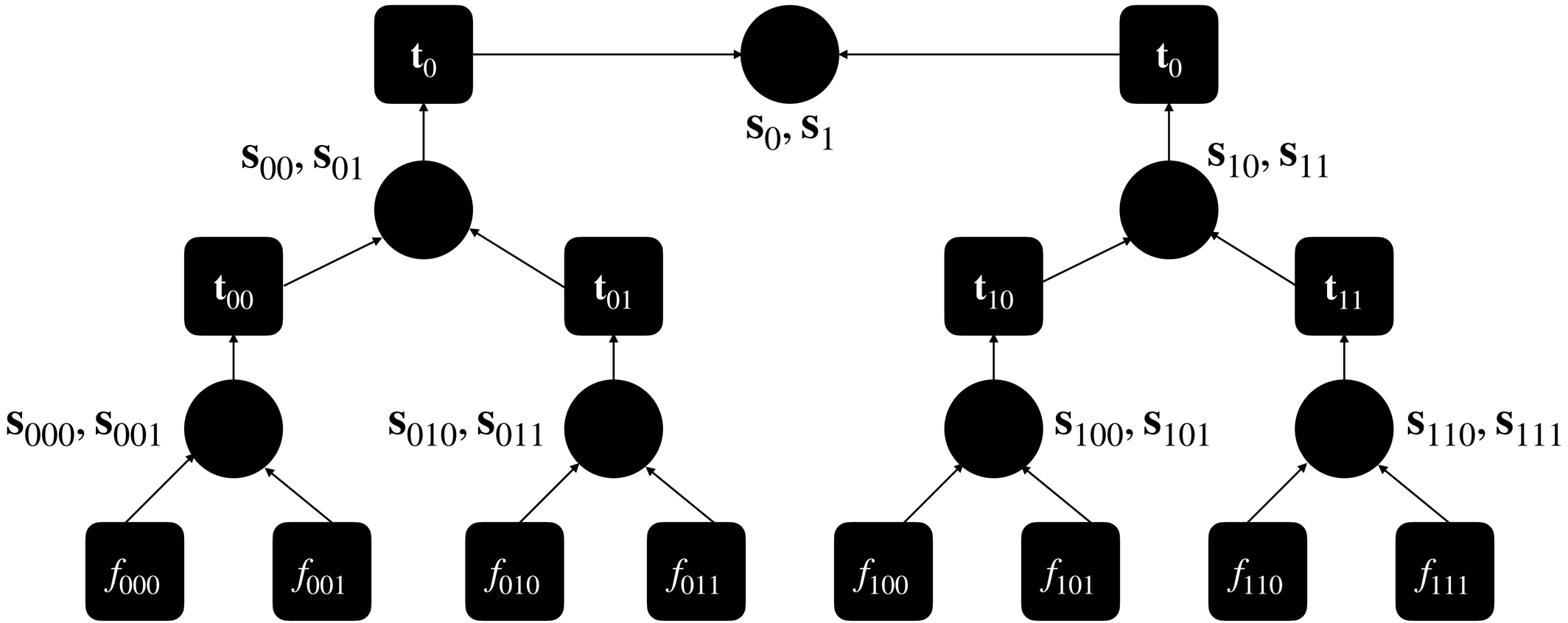

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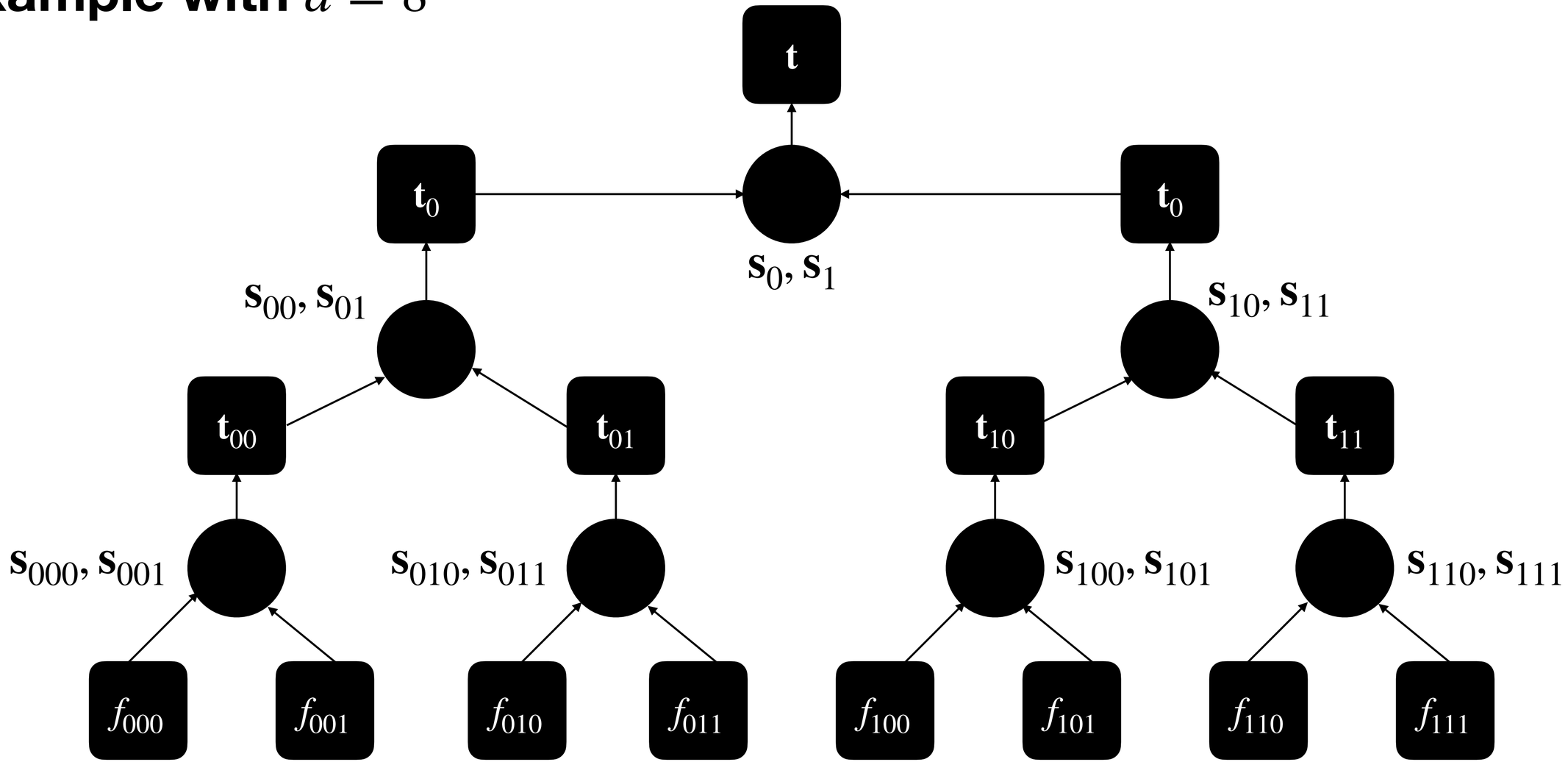

Merkle-PRISIS II

How to check an opening

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  reduces to h-PRISIS$_\ell$ i.e. **MSIS**!

Merkle-PRISIS III

Pros ✅ and Cons ❌

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- Commitment is **succinct**.

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Can we do an efficient evaluation protocol?

Evaluation Protocol I

Strategy

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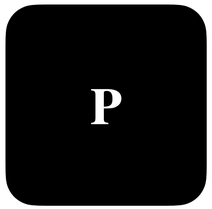

Evaluation Protocol I

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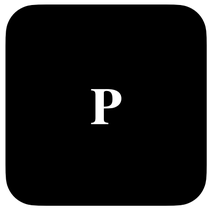

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Prover knows:

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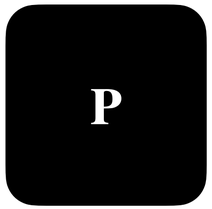

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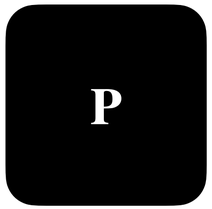

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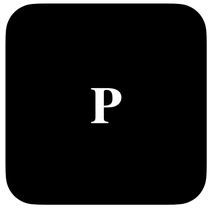

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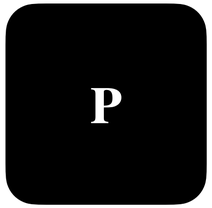

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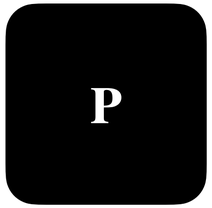

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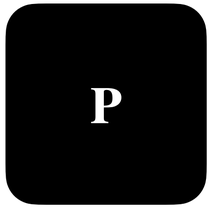

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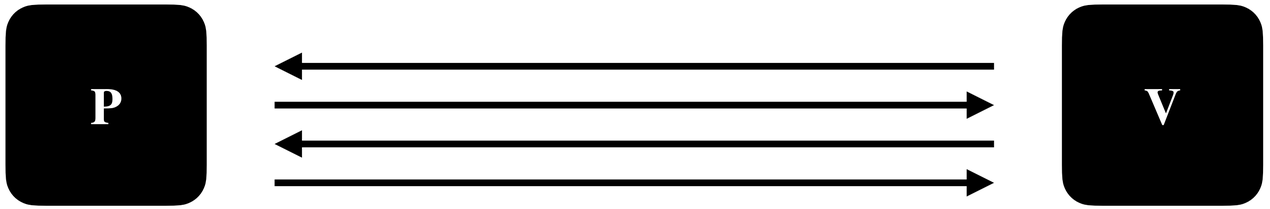

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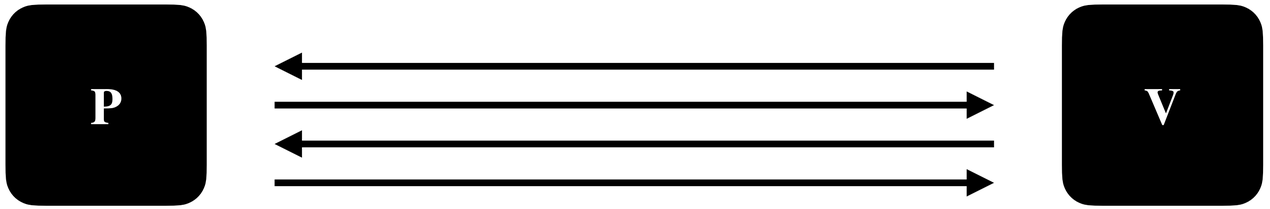

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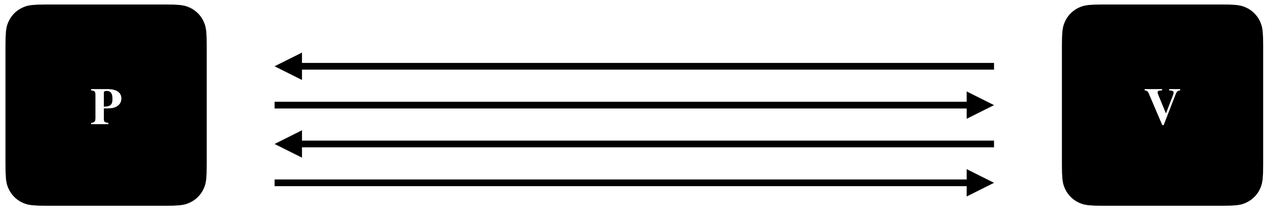

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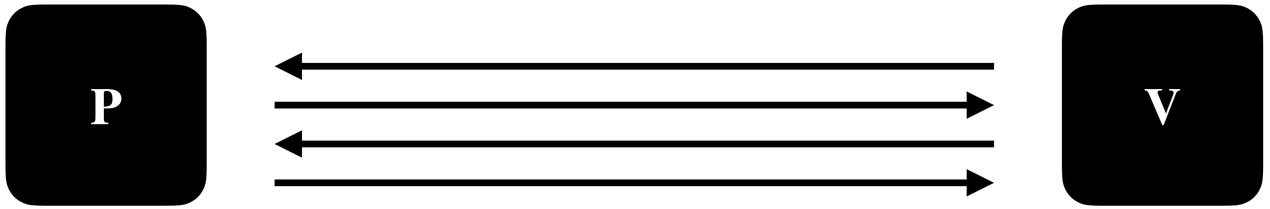

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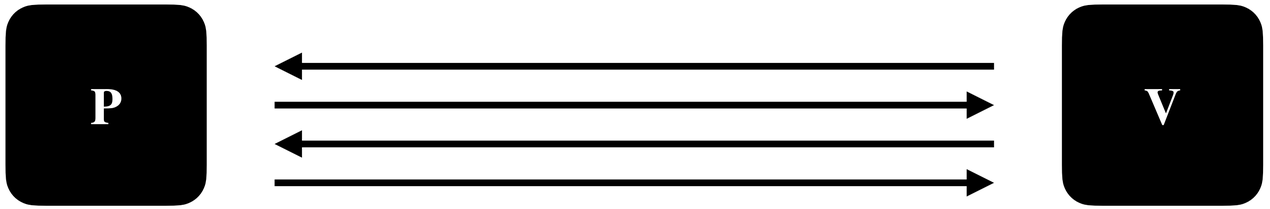

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Verifier now knows:

- Common reference string crs'

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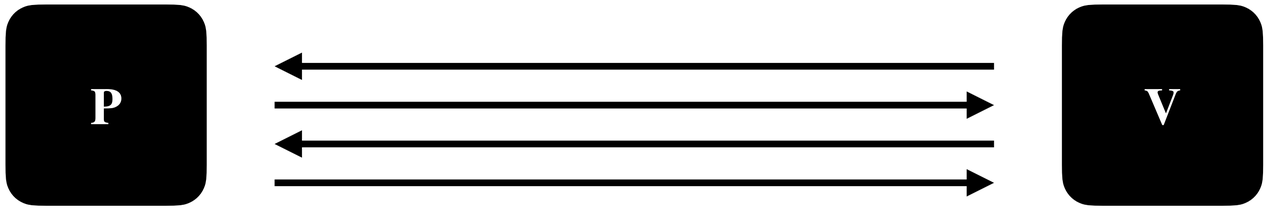

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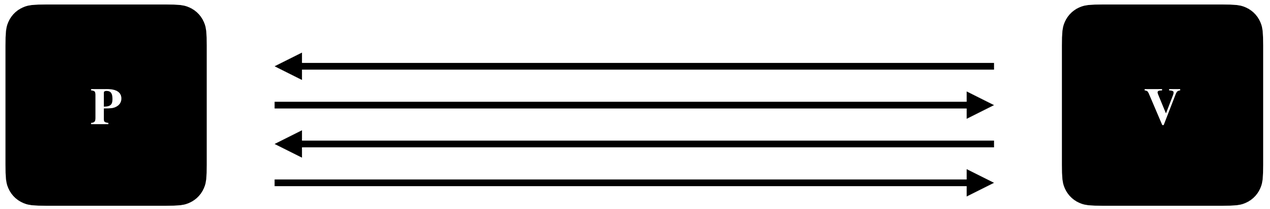

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Verifier now knows:

- Common reference string crs$'$
- Commitment $\mathbf{t}'$
- New claim: $g(u') = v'$ and $\text{Open}(\text{crs}', \mathbf{t}', g, (\mathbf{z_b})_\mathbf{b}) = 1$

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Split and fold (Evaluations)

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Fast Reed-Solomon Interactive Oracle Proofs of Proximity

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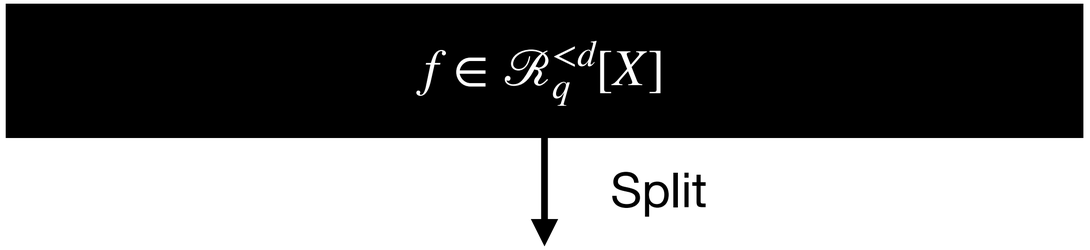

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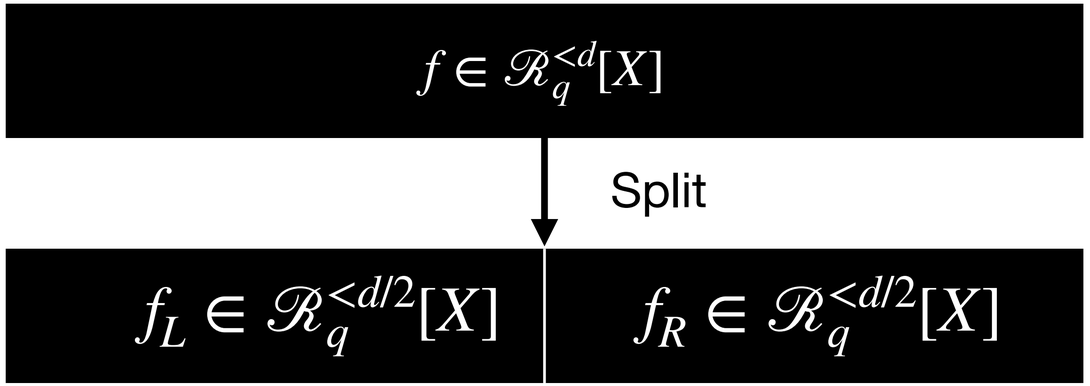

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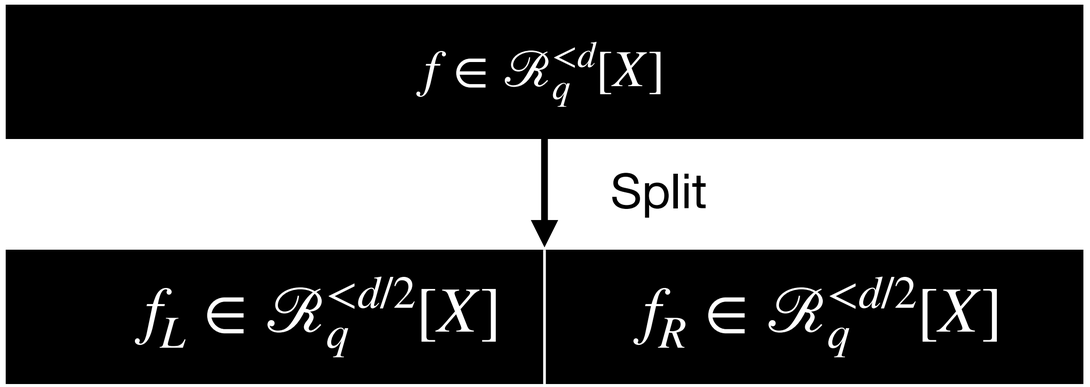

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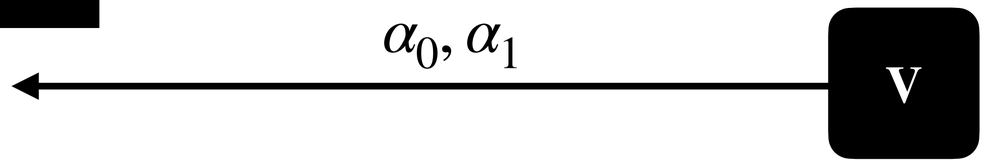

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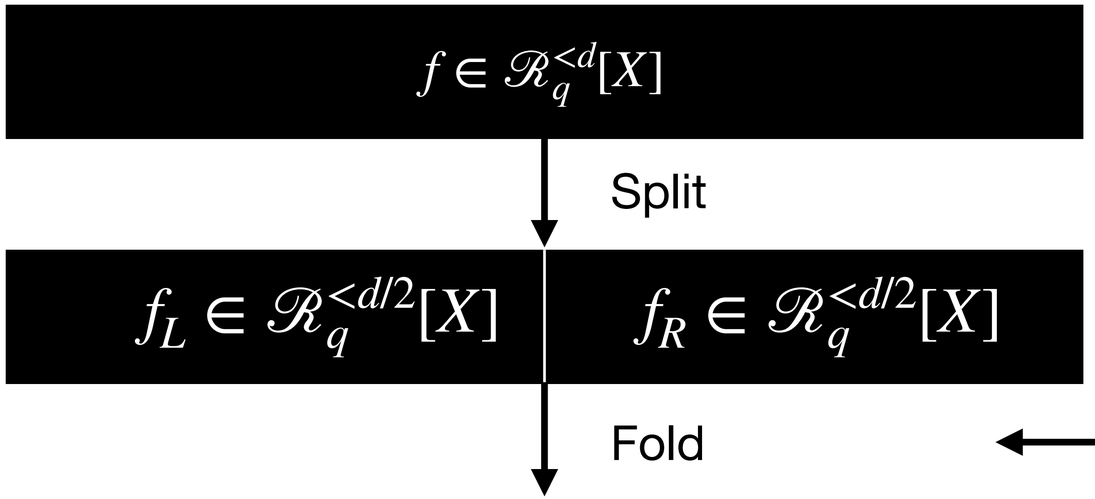
Fast Reed-Solomon Interactive Oracle Proofs of Proximity

Eli Ben-Sasson* Iddo Bentov† Ynon Horesh* Michael Riabzev*



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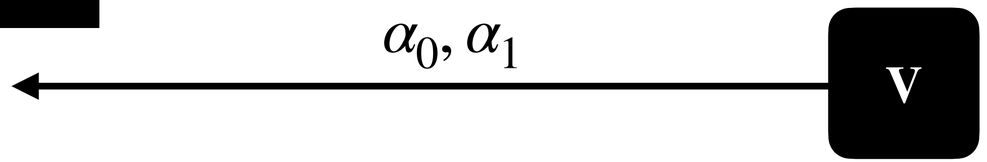

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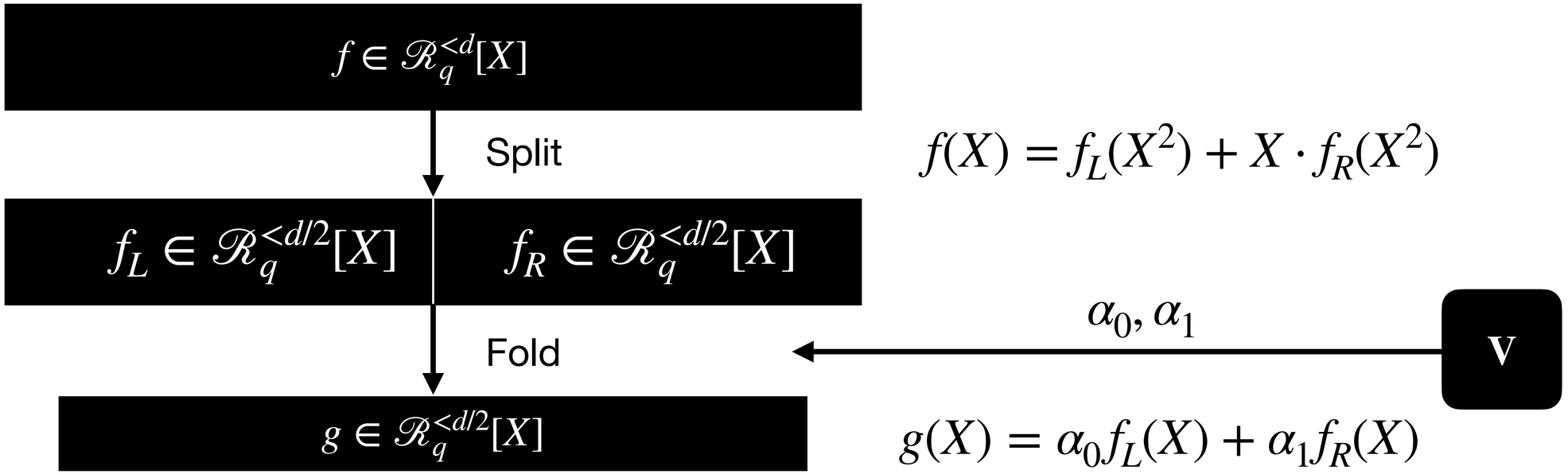

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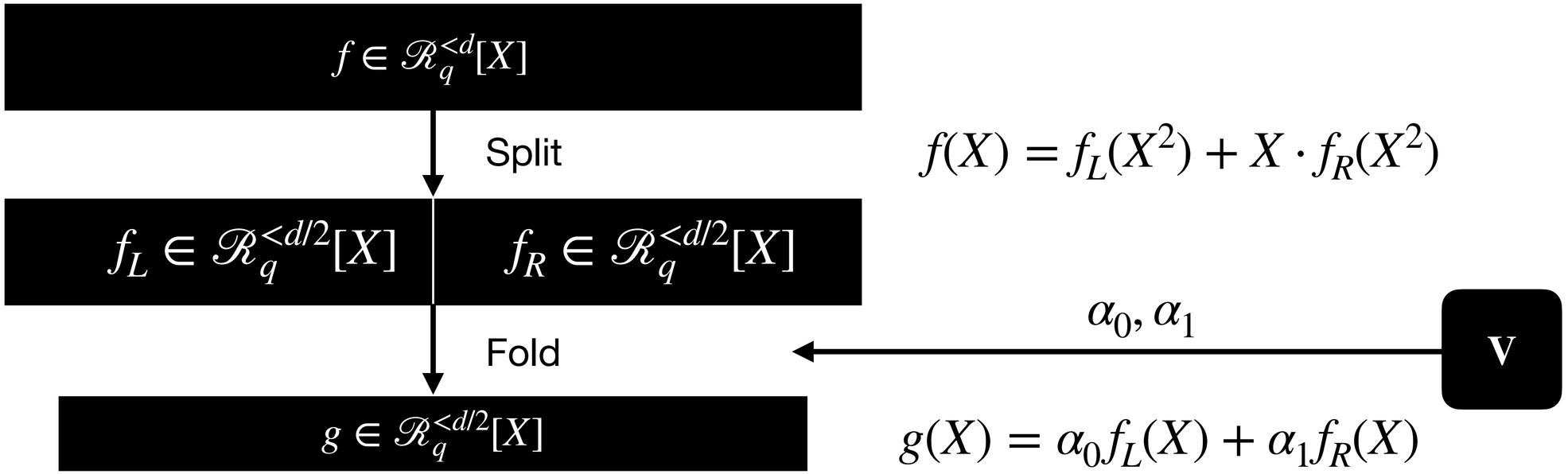

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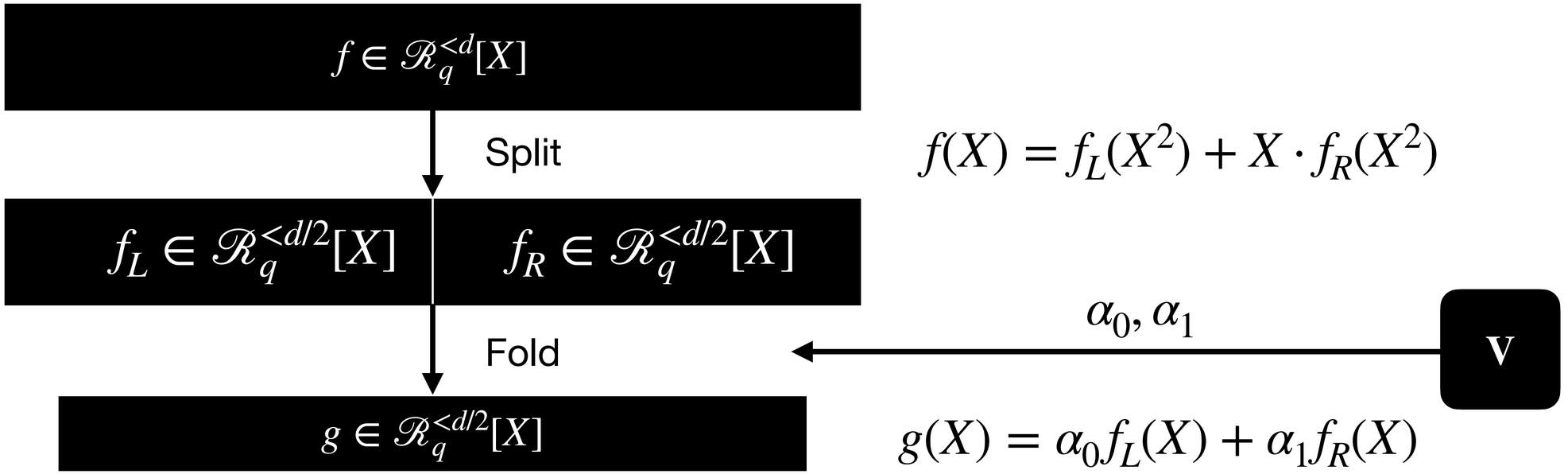

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Split and fold (Openings)

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$f \in \mathscr{R}_q^{<d}[X]$							
f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7

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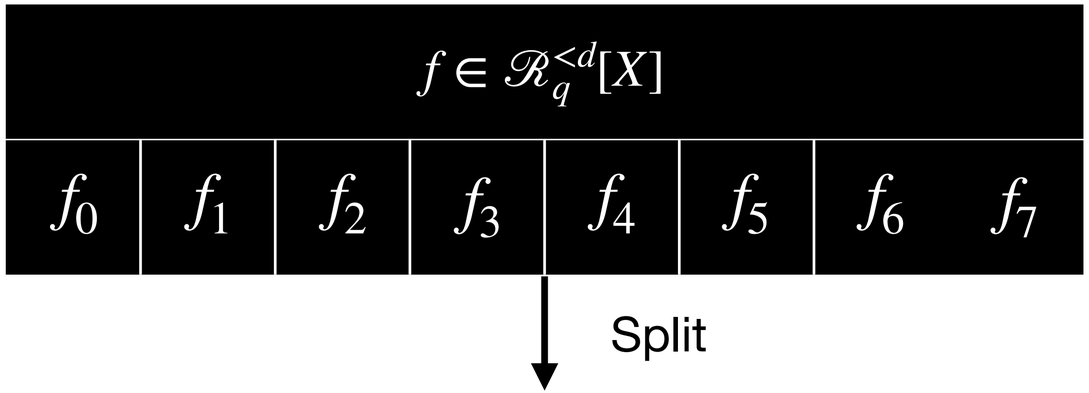

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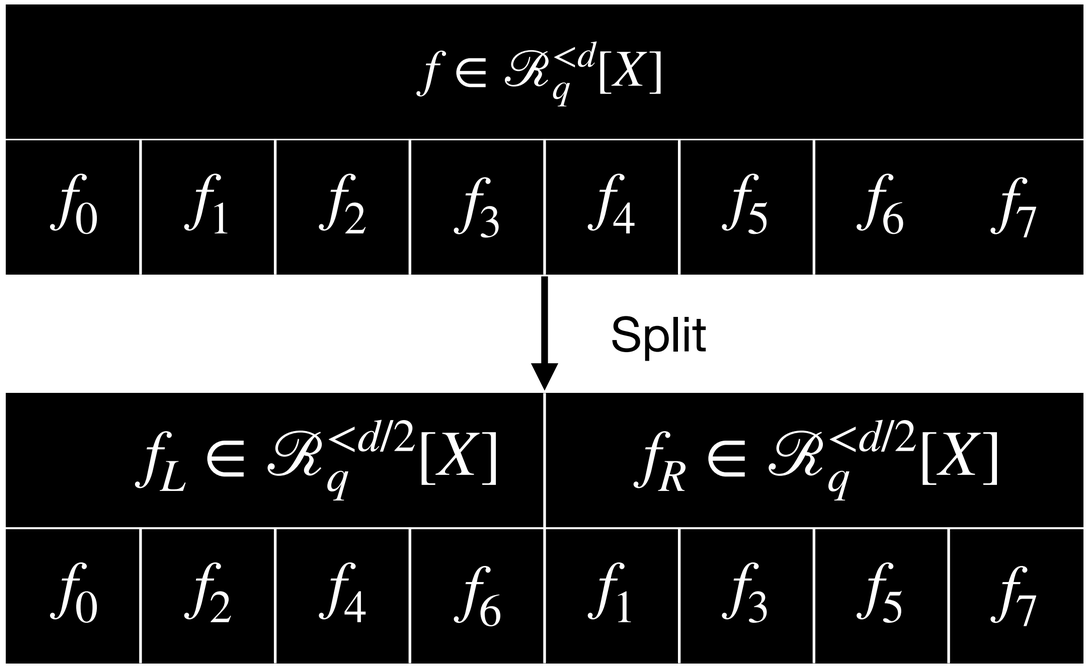

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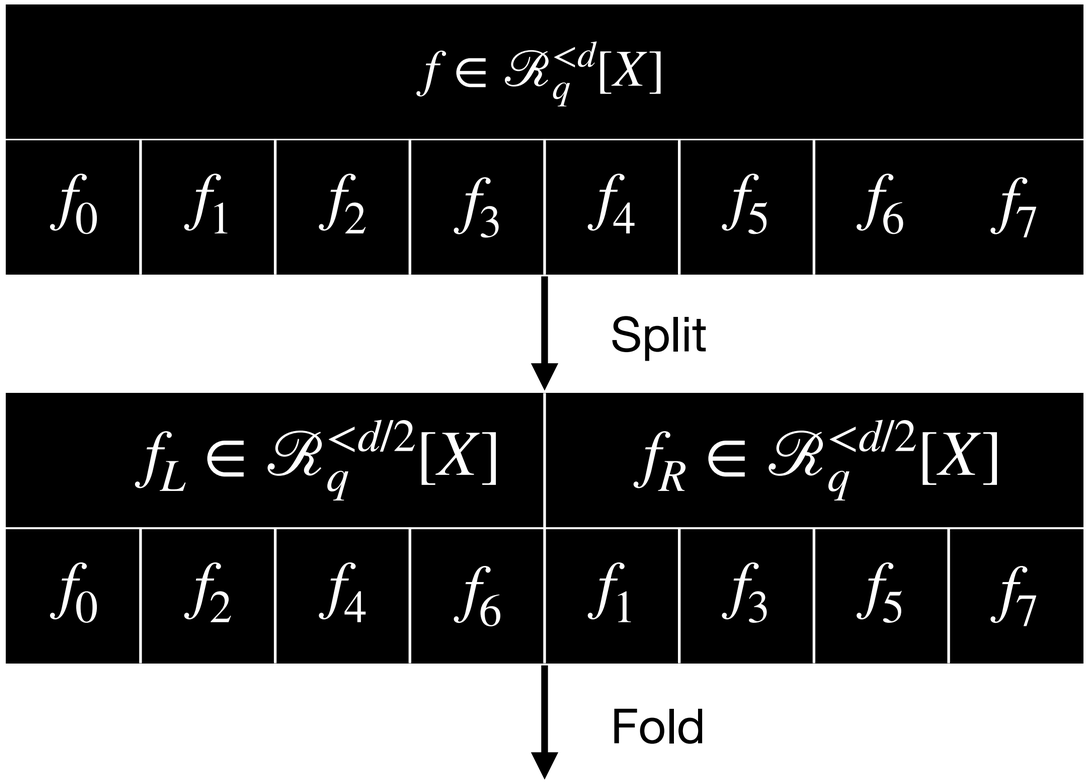

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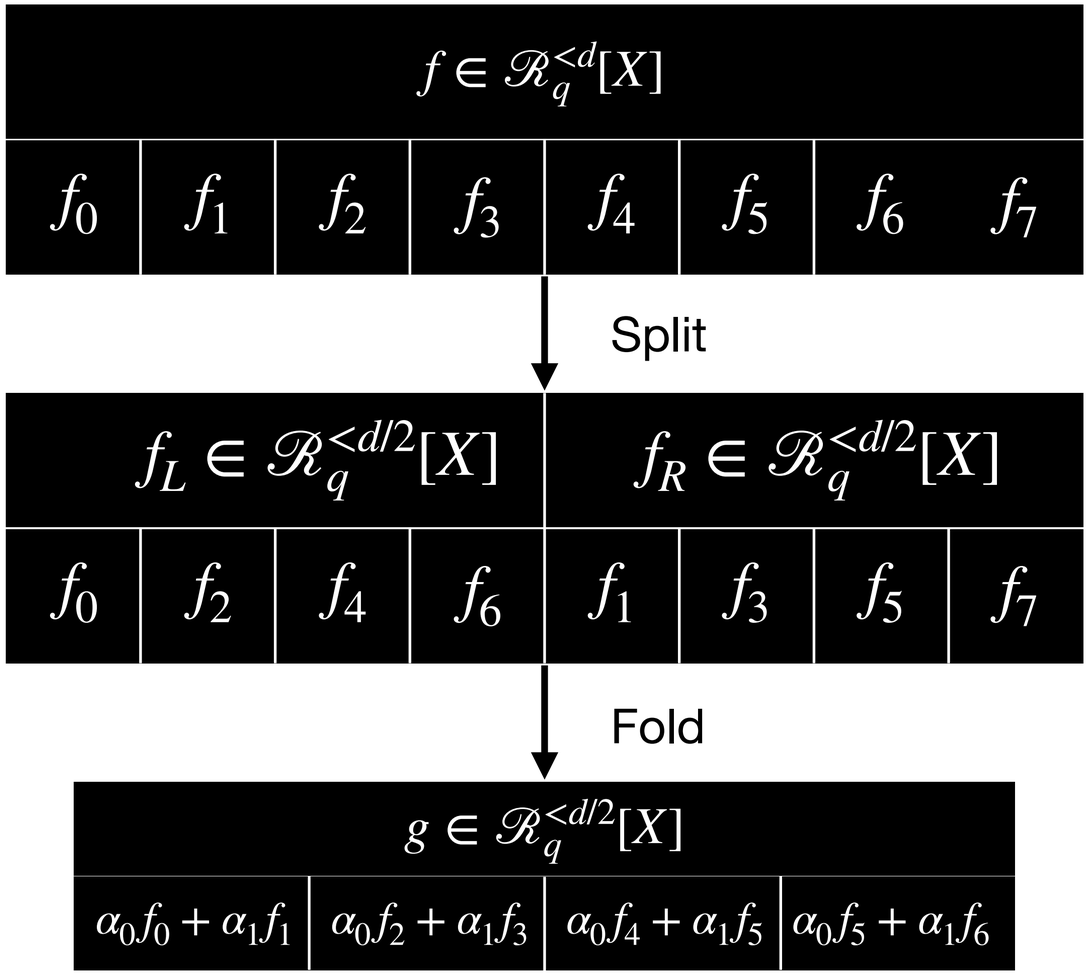

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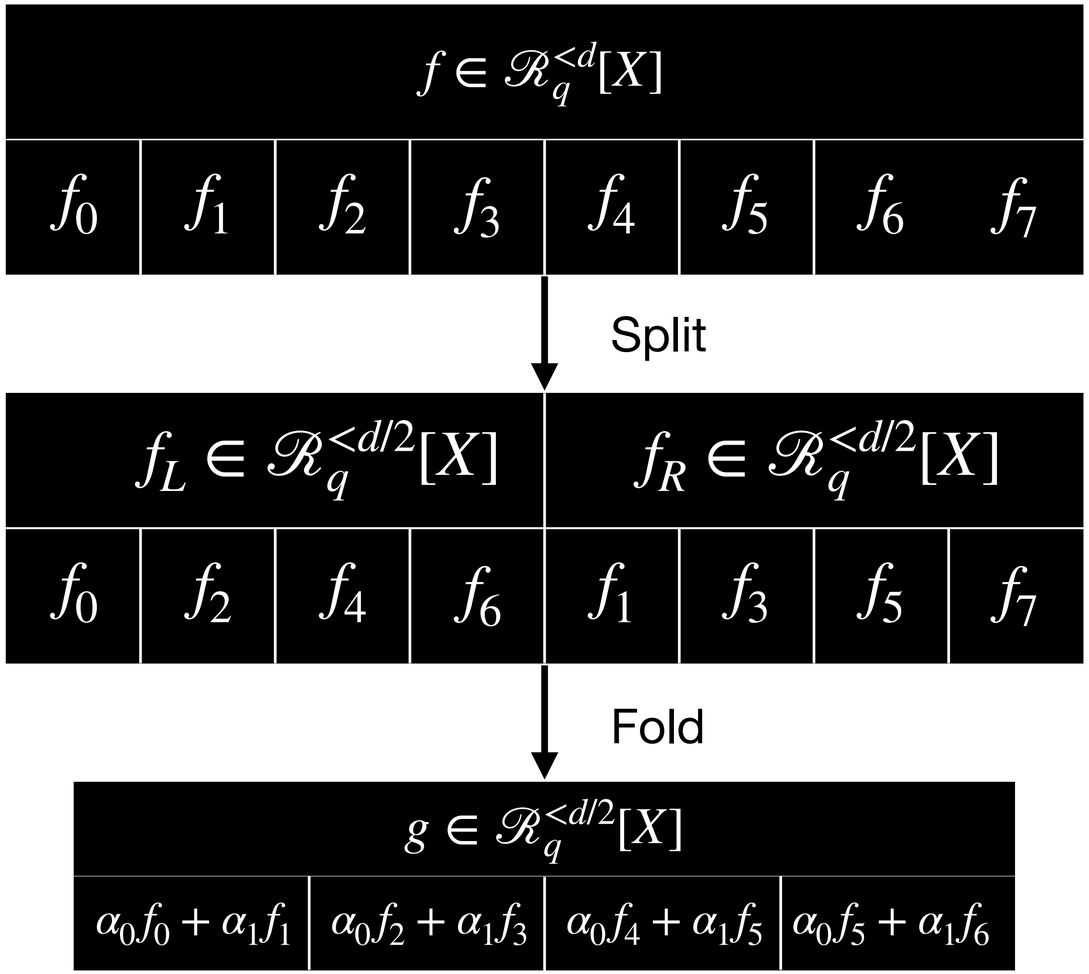

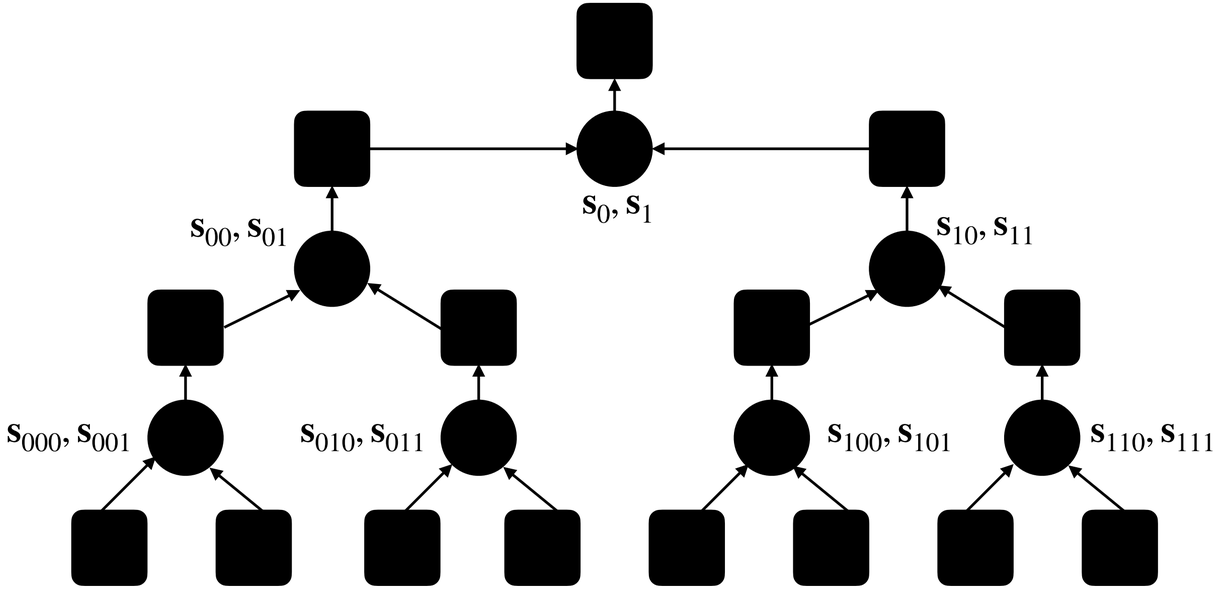

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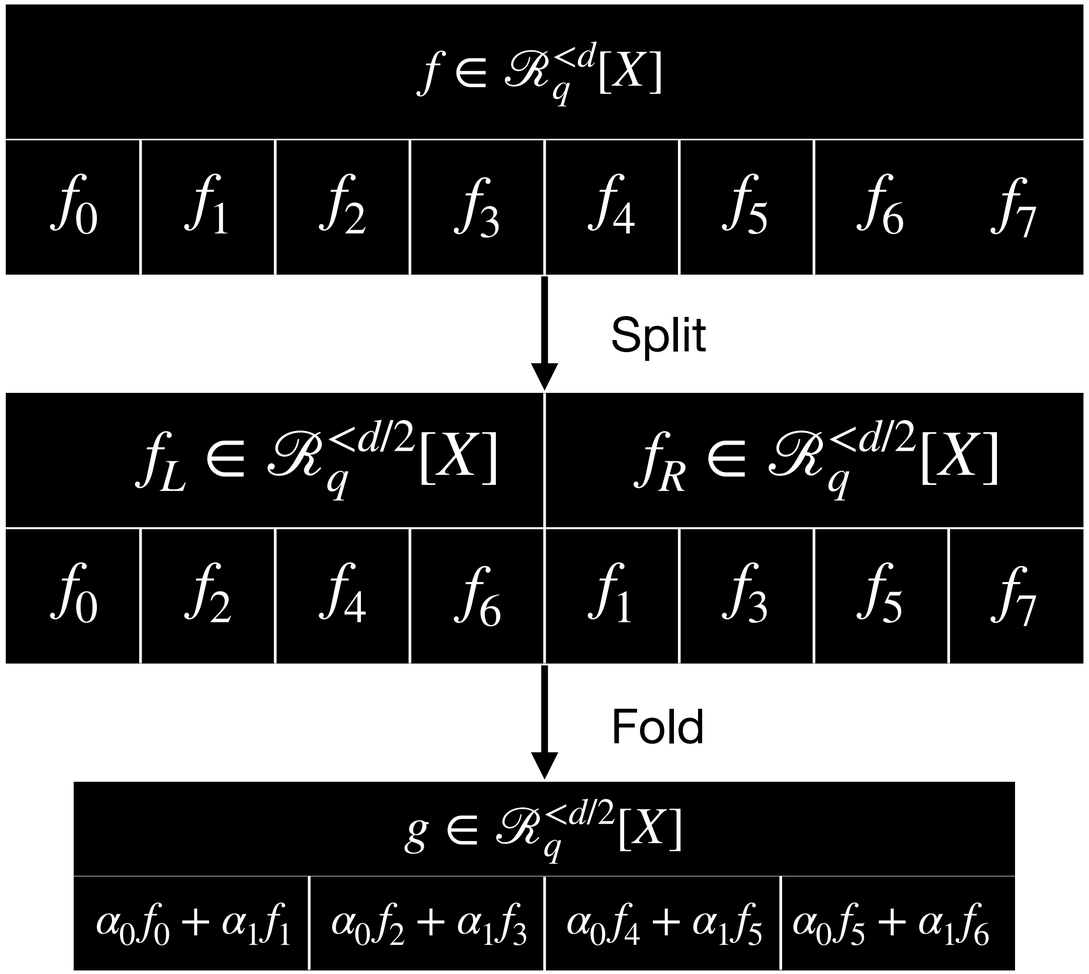

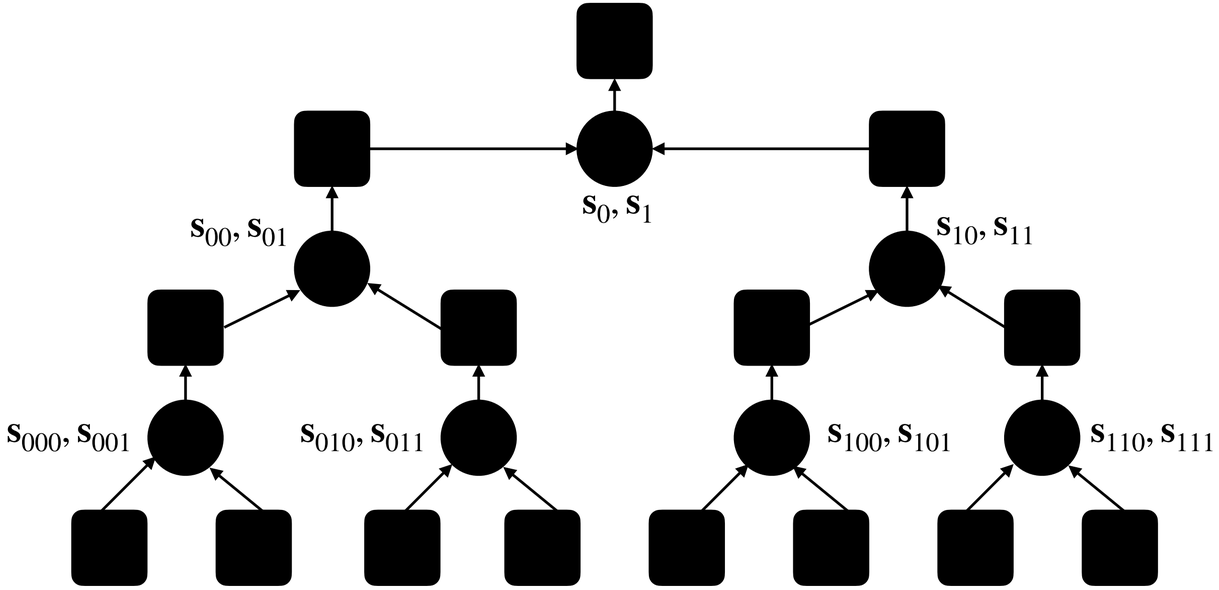

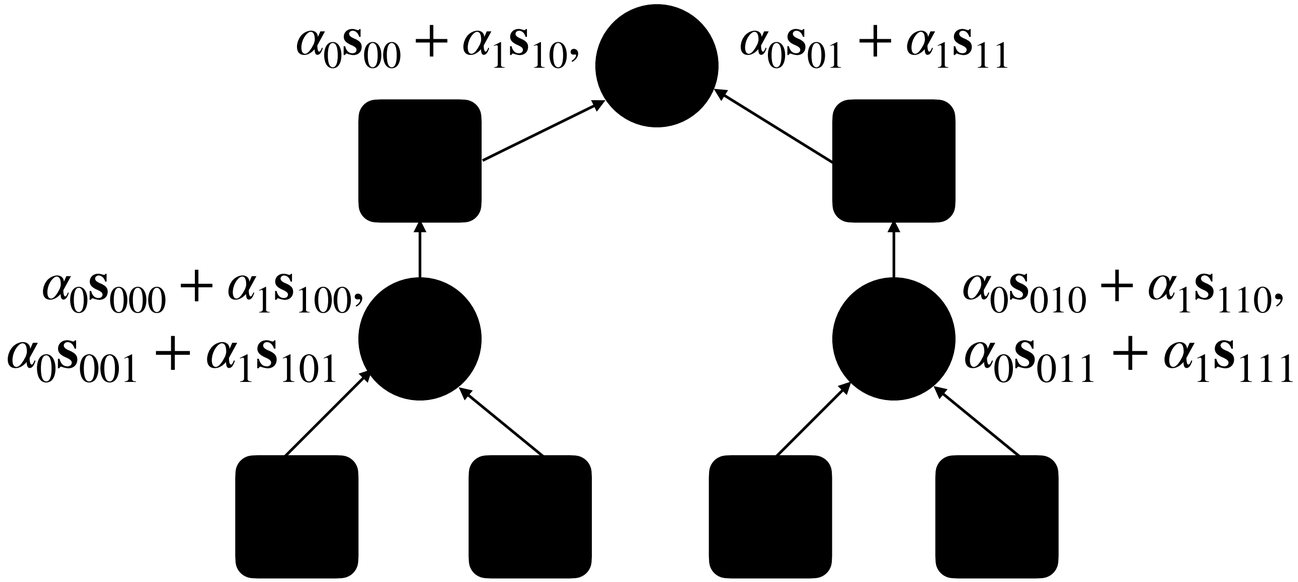

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- Prover reveals $\mathbf{s}_0, \mathbf{s}_1$. Verifier sets RHS as new updated commitment.

Evaluation Protocol V

Putting it all together

Basic Σ-Protocol

Prover		**Verifier**
$f(\mathsf{X}) = f_0(\mathsf{X}^2) + \mathsf{X} f_1(\mathsf{X}^2)$		
$z_i \coloneqq f_i(u^2)$ for $i \in \mathbb{Z}_2$	$\xrightarrow{z_0, z_1, \mathbf{s}_0, \mathbf{s}_1}$	Check: $z_0 + u z_1 =_? z$; Check: $\mathbf{s}_0, \mathbf{s}_1$ short
$g(\mathsf{X}) \coloneqq \alpha_0 f_0(\mathsf{X}) + \alpha_1 f_1(\mathsf{X})$	$\xleftarrow{\alpha_0, \alpha_1}$	$\alpha_0, \alpha_1 \leftarrow \{ X^i : i \in \mathbb{Z} \}$
$\mathbf{z_b} \coloneqq \alpha_0 \mathbf{s_{b,0}} + \alpha_1 \mathbf{s_{b,1}}$ for $\mathbf{b} \in \mathbb{Z}_2^{\leq h-1}$	$\xrightarrow{g, (\mathbf{z_b})_\mathbf{b}}$	$\mathsf{crs}' \coloneqq (\mathbf{A}_{1+t}, w_{1+t}, \mathbf{T}_{1+t})_{t \in [h-1]}$
		$\mathbf{t}' \coloneqq \alpha_0 \cdot \left(\mathbf{t} - w_1^0 \mathbf{A}_1 \mathbf{s}_0\right) + \alpha_1 \cdot \left(\mathbf{t} - w_1^1 \mathbf{A}_1 \mathbf{s}_1\right)$
		$u' \coloneqq u^2; z' \coloneqq \alpha_0 \cdot z_0 + \alpha_1 \cdot z_1$
		Check: $g(u') = z'$
		Check: $\mathsf{Open}(\mathsf{crs}', \mathbf{t}', g, (\mathbf{z_b})_\mathbf{b}) = 1$

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  - Subtractive challenge space => Challenge space of size at most $\text{poly}(\lambda)$ [AL21]

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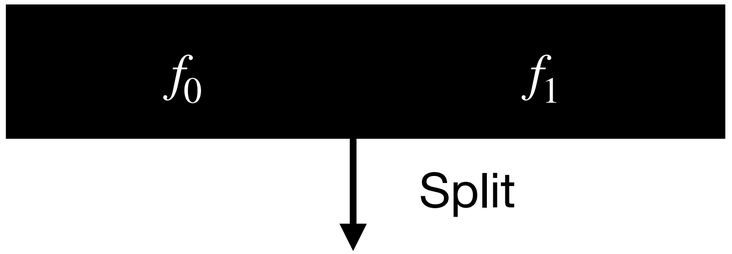

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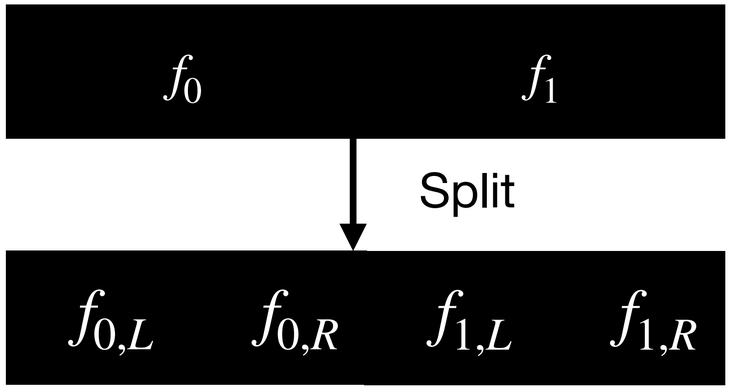

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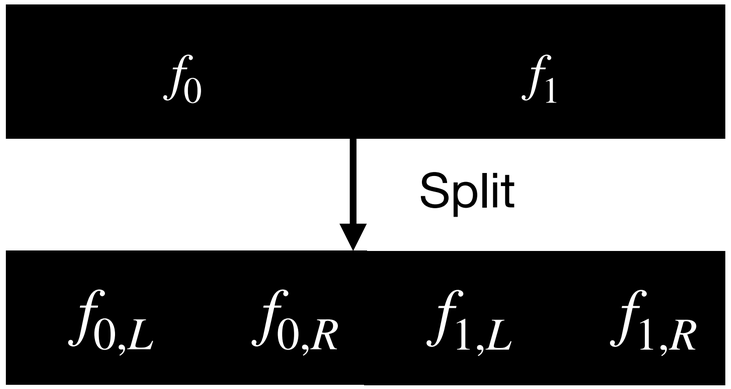

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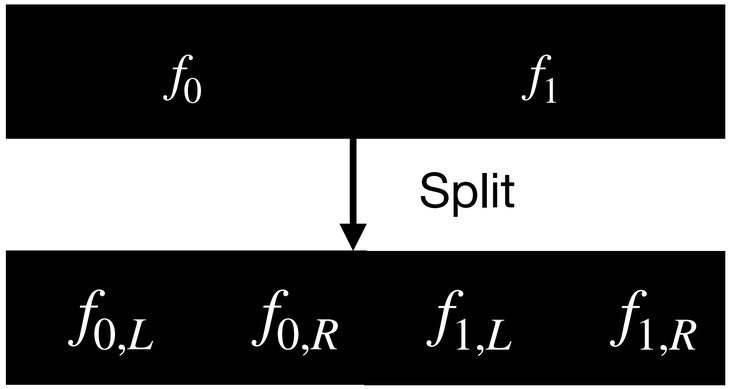

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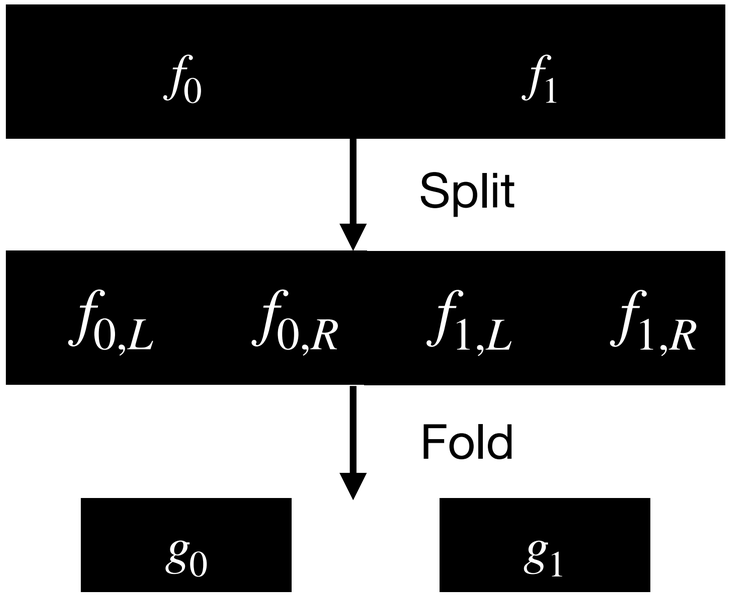

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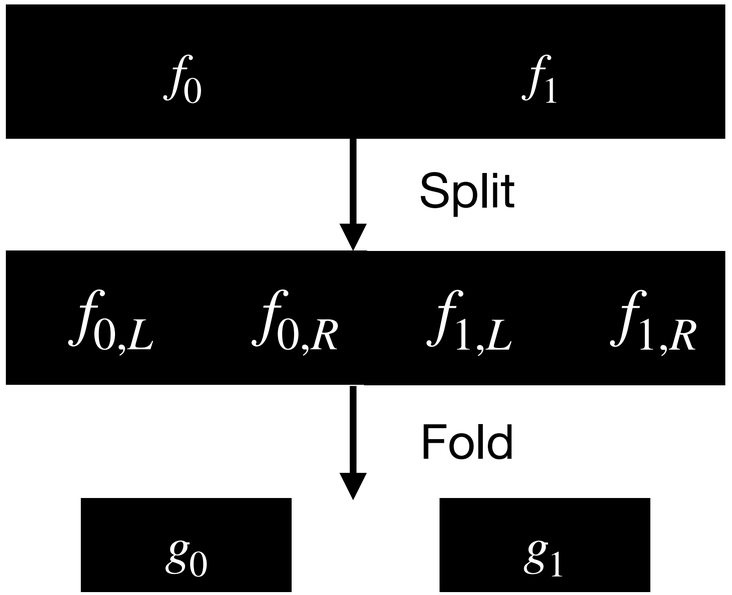

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Folded polynomial:

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- To prove a single claim $f(u) = v$, simply set $f_1, \ldots, f_r = f$ and $v_1, \ldots, v_r = v$.

Conclusion

 - SLAP

A non-interactive lattice-based polynomial commitment with succinct proofs and verification time, from standard lattice assumptions.

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Thank you!

Extra slides

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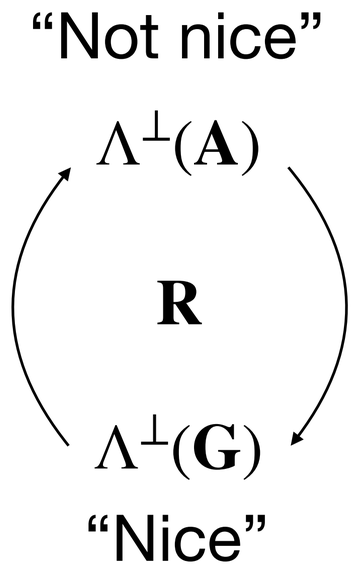

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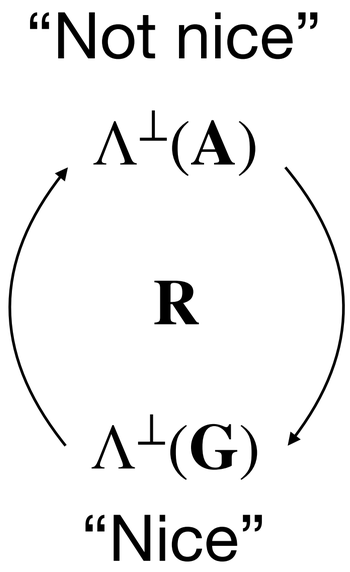

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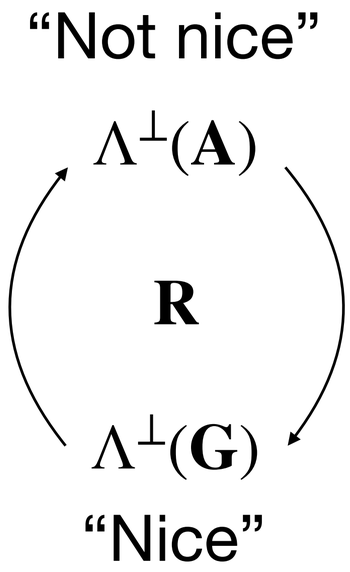

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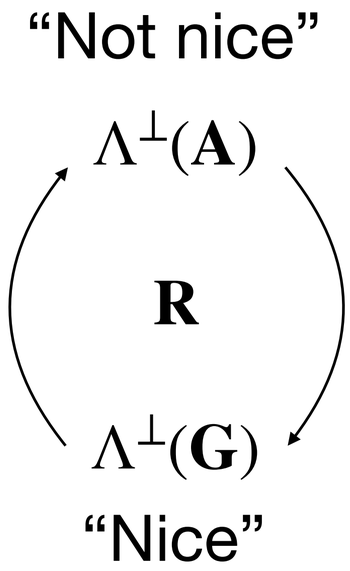

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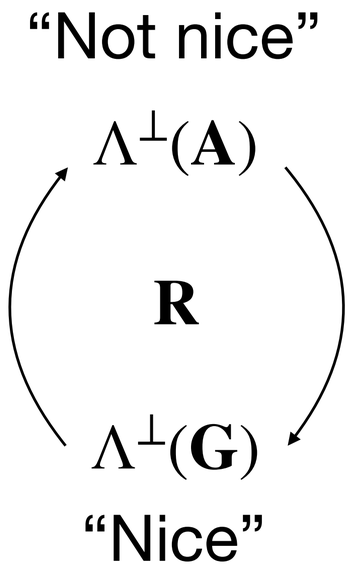

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- BASIS style assumption say:

"Given $\mathbf{A}, \mathbf{B}, \mathbf{T}$, hard to find short $\mathbf{x}$ for $\mathbf{Ax} = \mathbf{0}$"

Recap:

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- Succinct evaluation protocol for Merkle-PRISIS
- Boosting soundness via claim bundling

There is more!

What we did not talk about

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