Simulation extractability of zkSNARKs

or

What Makes Fiat–Shamir zkSNARKs (Updatable) Simulation Extractable?

Chaya Ganesh Hamidreza Khoshakhlagh Markulf Kohlweiss Anca Nitulescu **Michał Zając**

Agenda

Our **contribution**

NIZKs definitions

The **problem** with SRS

Notions of **soundness** and what they give

Simulation extractability

How to **build an extractor** for zkSNARKs?

Contribution

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Define **updatable simulation extractability** for NIZKs

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Analyze three properties **needed** for the NIZK to be simulation extractable

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Show that a wide class (Plonk, Marlin, Lunar, Sonic, Vampire) of **zkSNARKs** is simulation-extractable **out of the box**

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Snarky signature in the **updatable** setting

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If verifier $\mathscr{V}$ accepts a proof for a statement x then the prover $\mathscr{A}(p, srs; r)$ **knows** w such that $R(x, w)$

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Zero knowledge
Verifiers learns nothing besides the validity of the statement

$$\mathcal{S}(p, srs, td, x) \approx \mathcal{P}(p, srs, x, w)$$

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Succinctness
The proof π is short

$$|\pi| = O(\log(|x| + |w|))$$

All the fuss with the SRS

$$td, srs \leftarrow \mathsf{KGen}(p)$$

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Updatable SRS
Everybody can take an existing SRS srs and **update** it to get an SRS srs'
One honest update is enough for the proof system to be sound

$$\mathsf{Update}(srs) \rightarrow srs', \pi_{upd}$$

How are zkSNARK-s built?

srs, x, w

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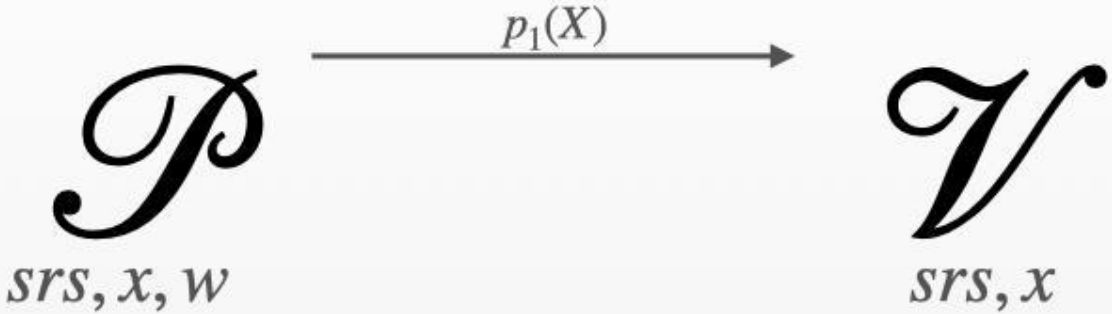

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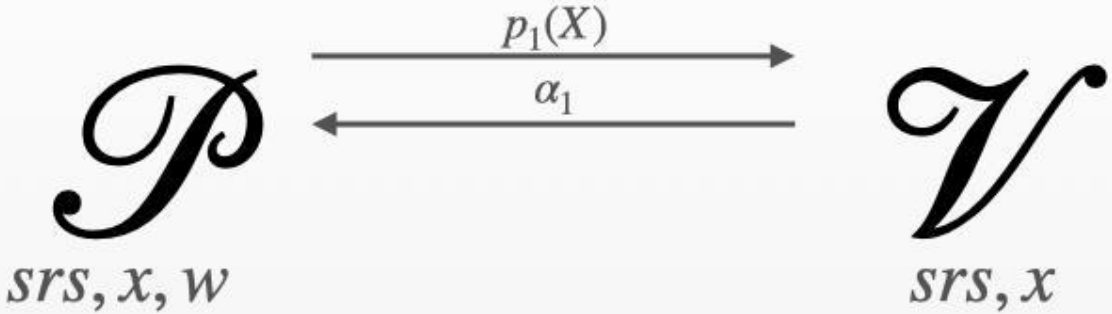

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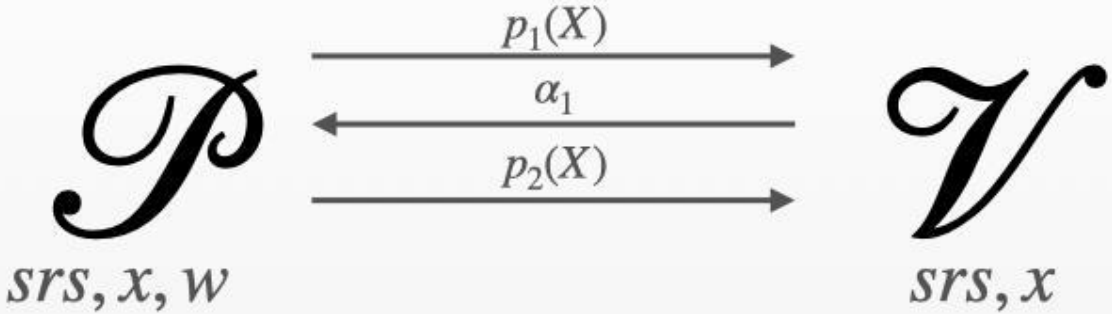

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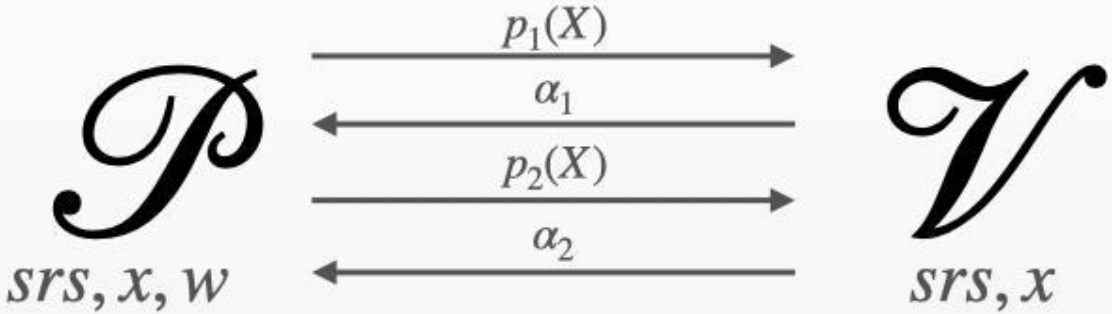

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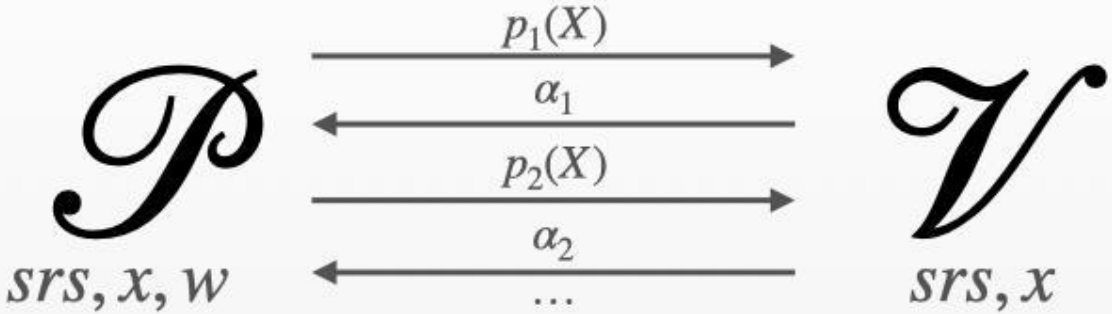

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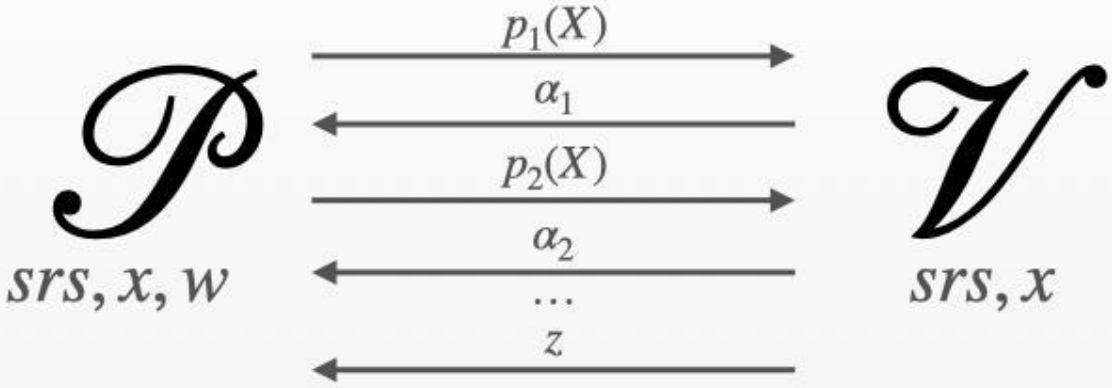

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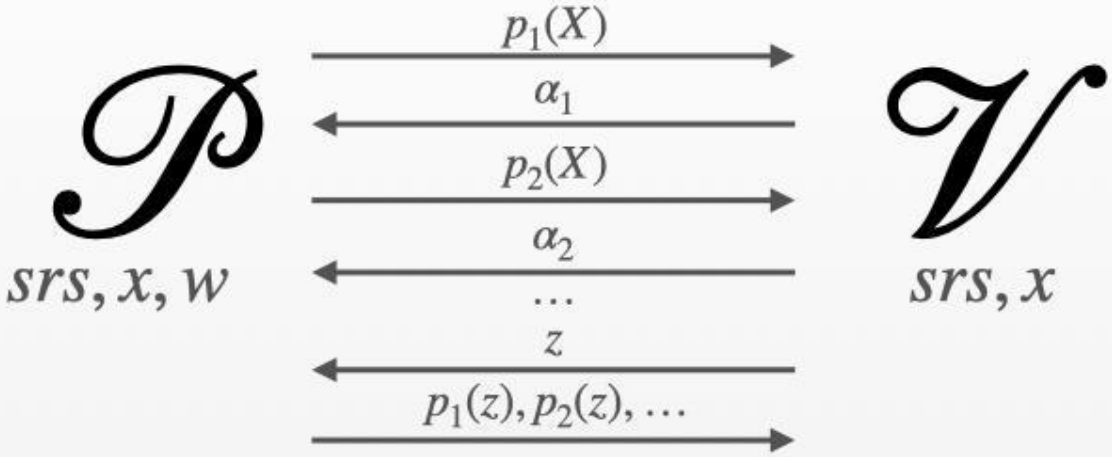

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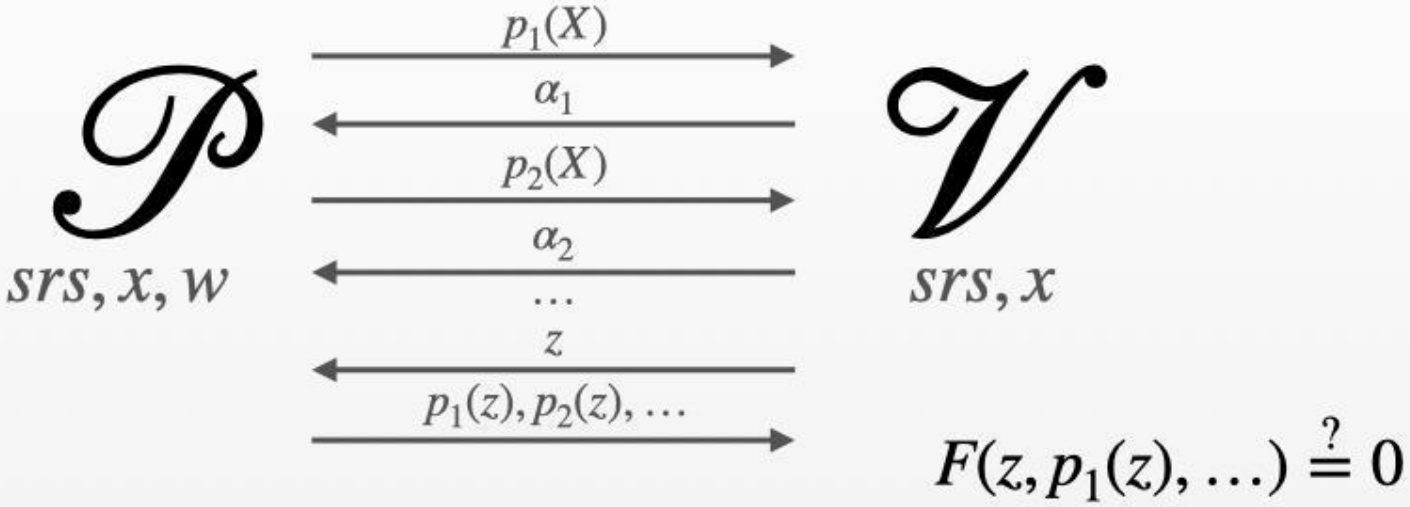

From interactive to non interactive: Fiat–Shamir transformation

srs, x, w

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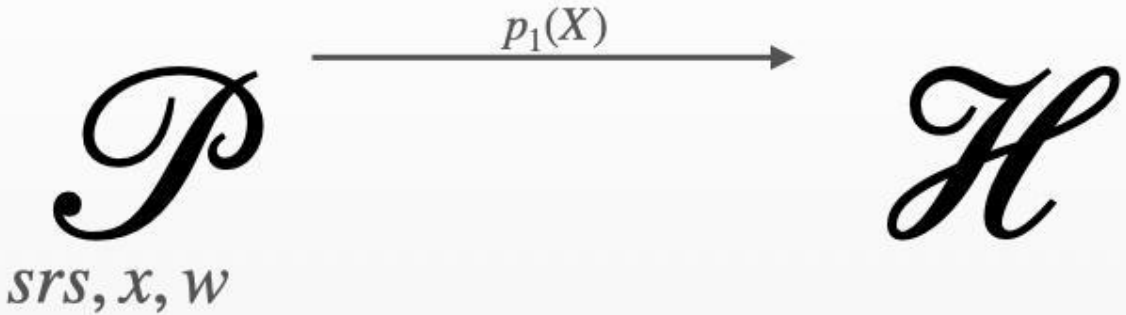

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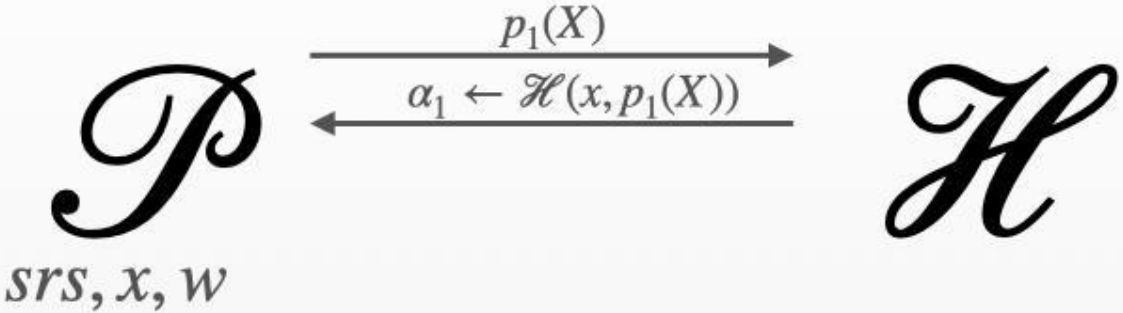

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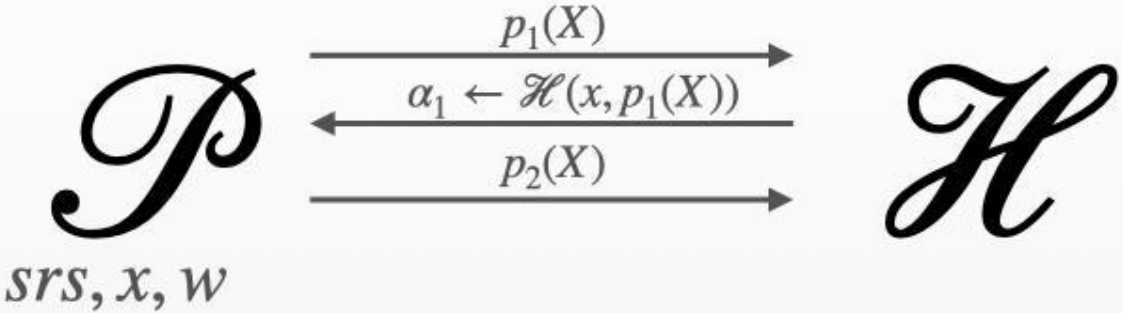

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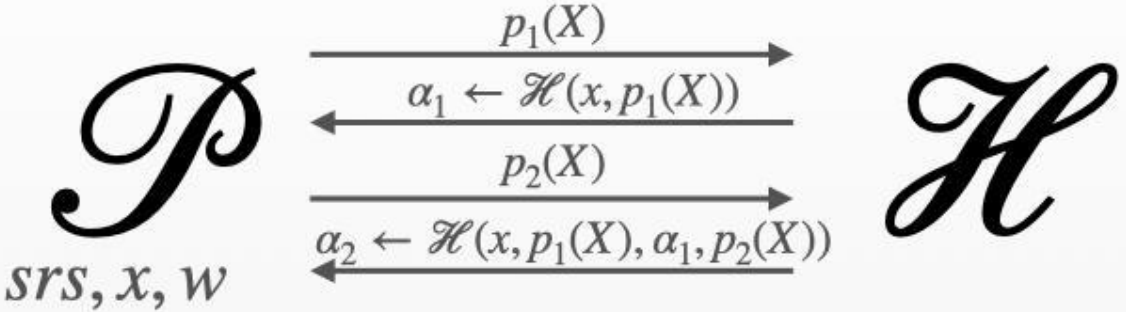

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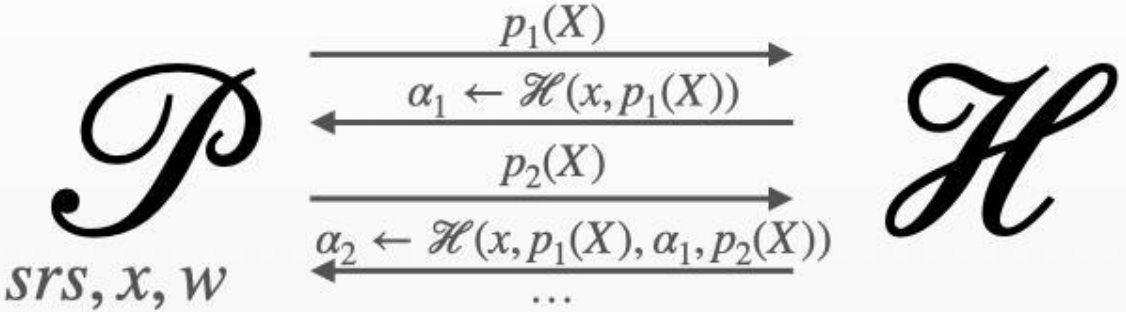

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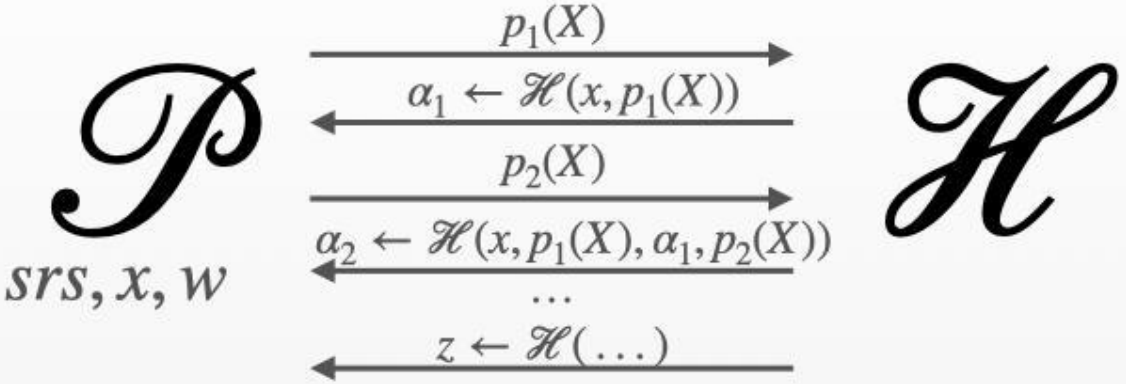

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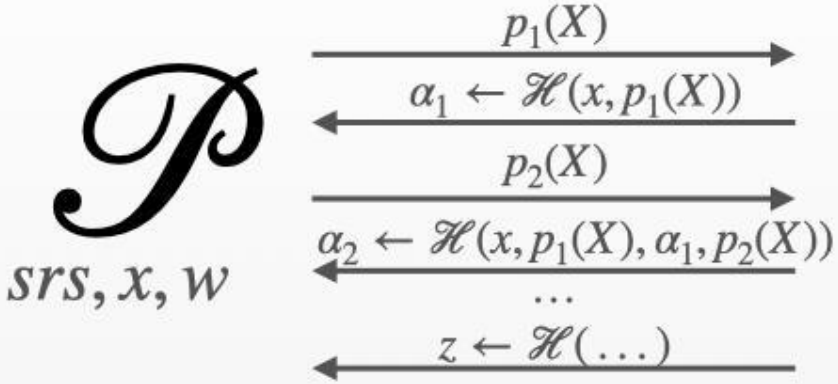

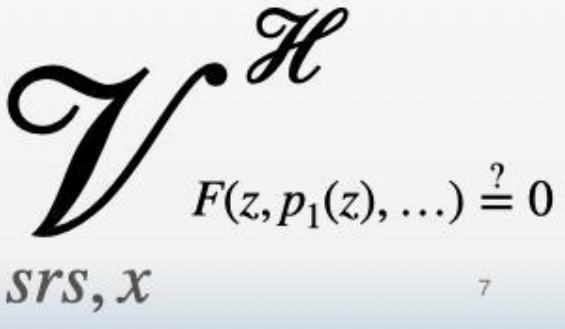

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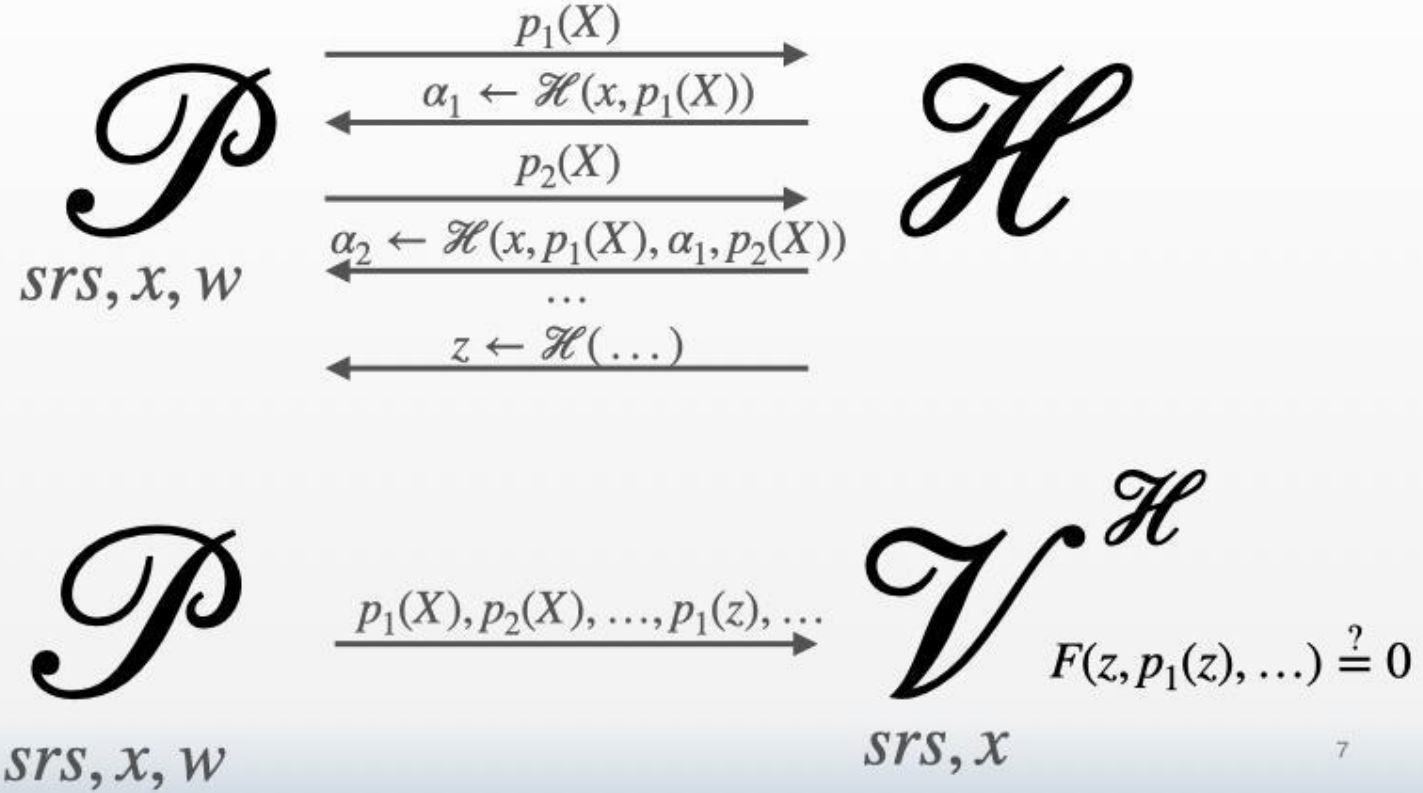

The importance of non-malleability

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Knowledge soundness: $\mathcal{A}^{\mathcal{H}}(p, srs; r)$
cannot convince the verifier $\mathcal{V}$ on x
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Extraction: $\mathcal{E}_{\mathcal{A}}(p, srs, r, Q_{\mathcal{H}}) = w$

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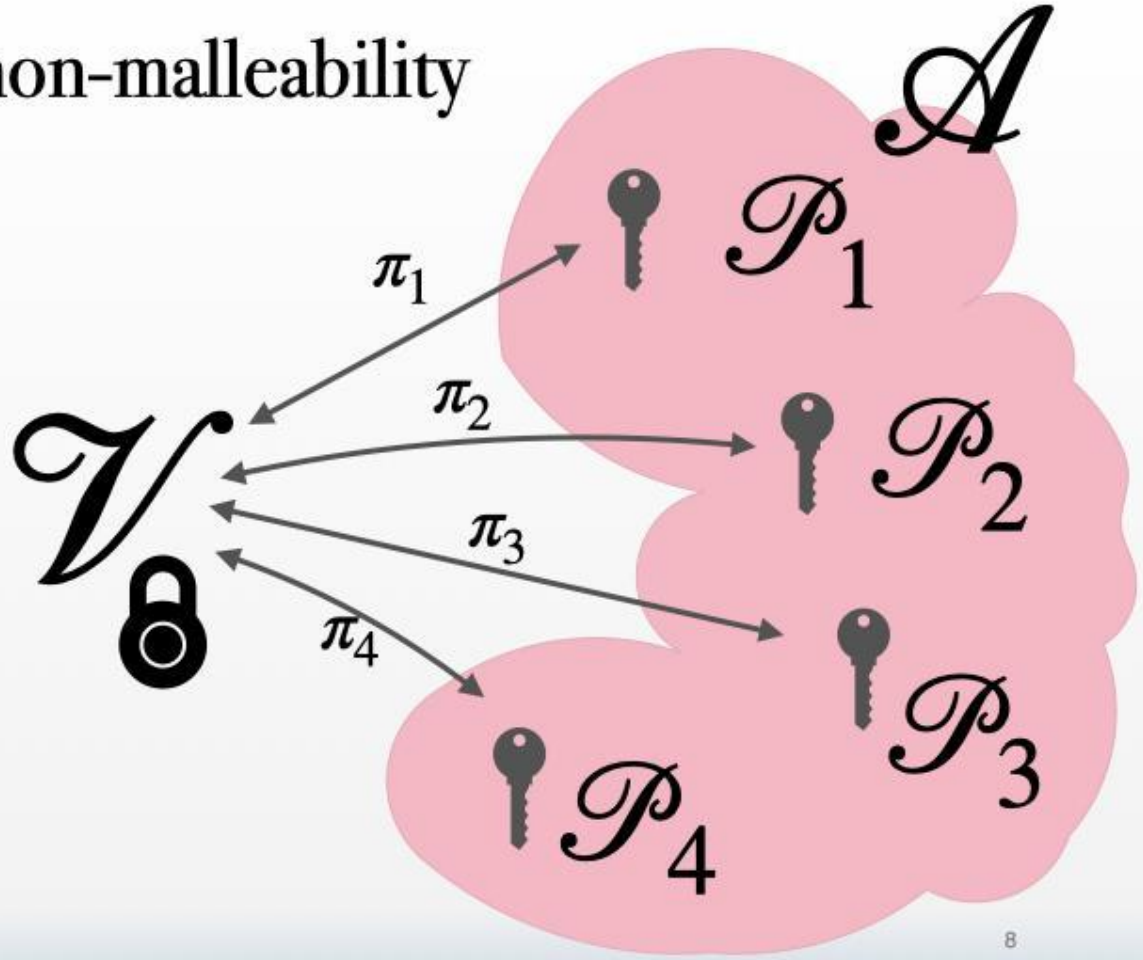

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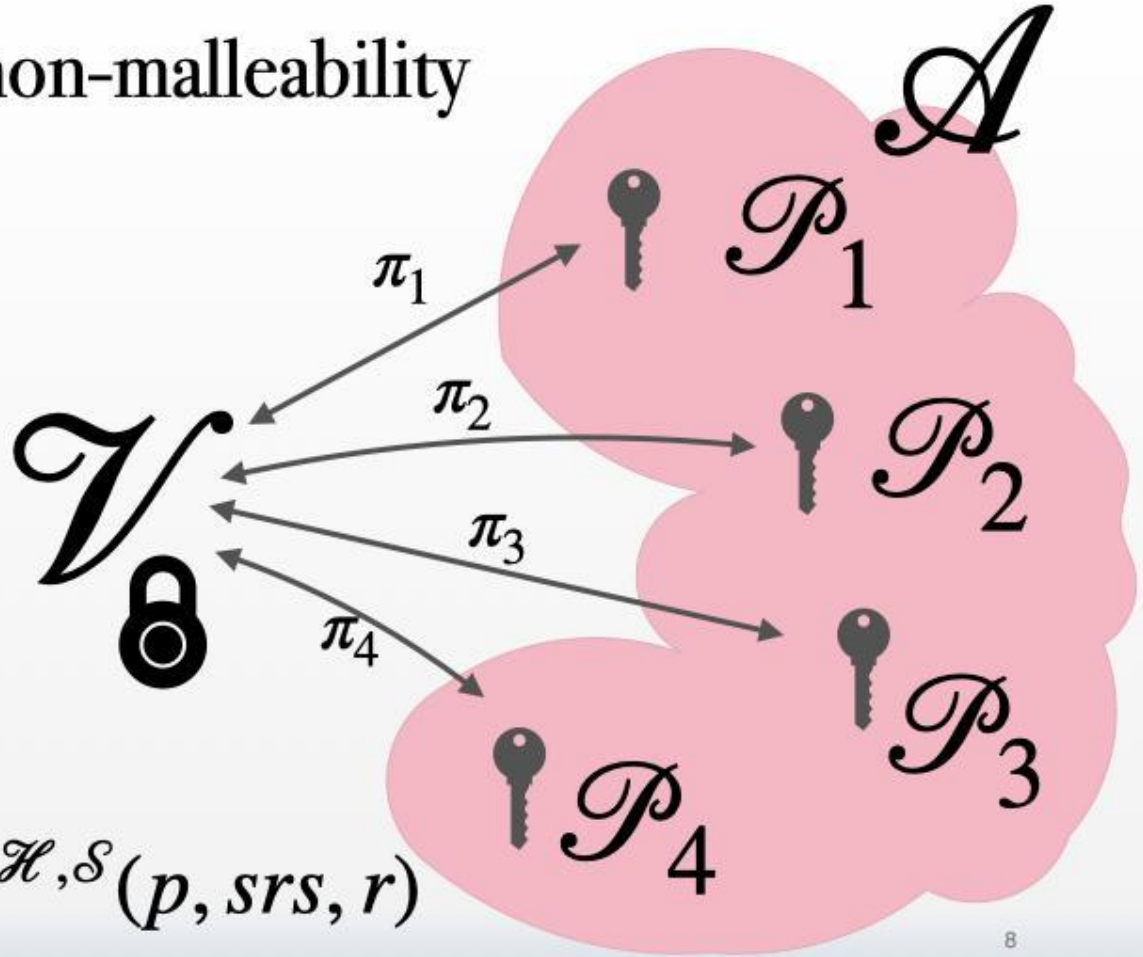
Simulation-extractability: $\mathcal{A}^{\mathcal{H},\mathcal{S}}(p, srs; r)$ cannot convince the verifier $\mathcal{V}$ on x unless it knows w such that $R(x, w)$ **given access to simulated proofs**

Extraction: $\mathcal{E}_{\mathcal{A}}(p, srs, r, Q_{\mathcal{H}}, Q_{\mathcal{S}}) = w$



$$\mathcal{A}^{\mathcal{H}}(p, srs, r) \mapsto \mathcal{A}^{\mathcal{H},\mathcal{S}}(p, srs, r)$$

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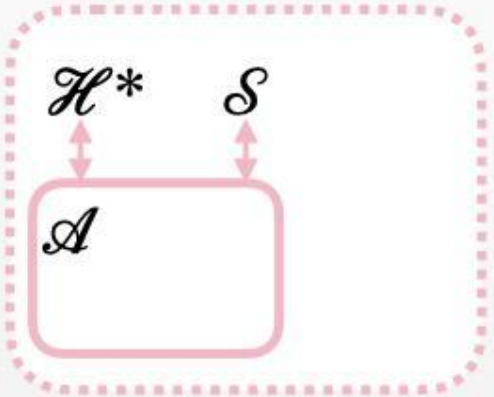

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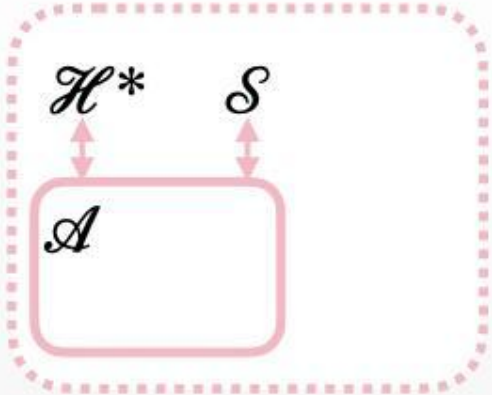

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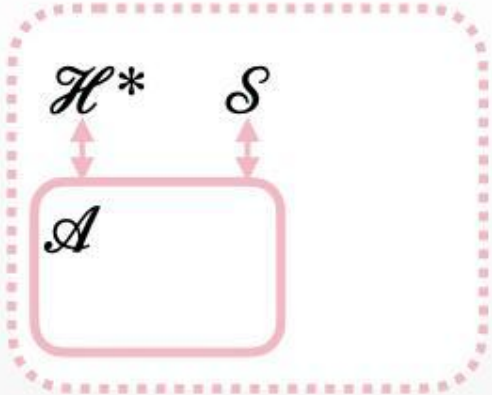

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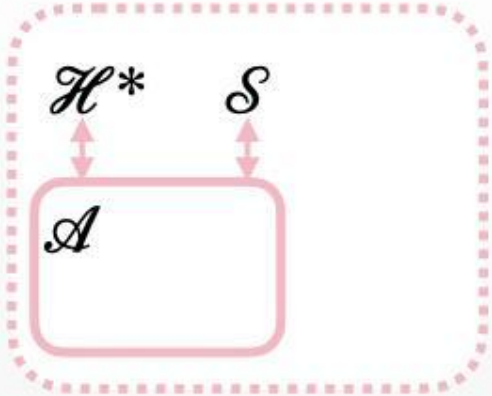

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Unique response property: It is infeasible to provide two transcripts that
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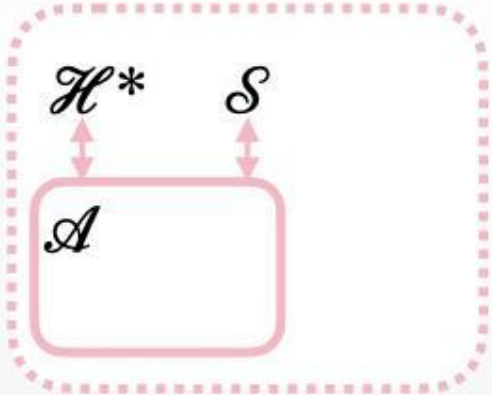

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$\mathscr{H}$

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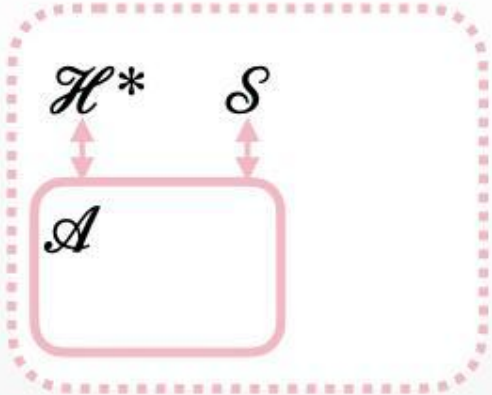

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REQUIREMENT 2
Trapdoor-less zero-knowledge: there exists a simulator that provides valid proofs without using SRS's trapdoor

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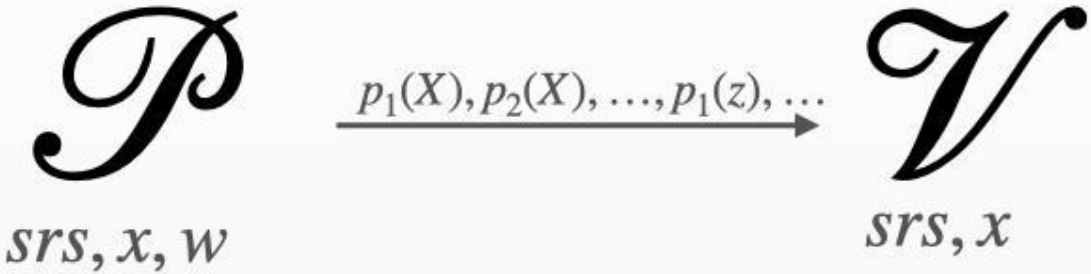

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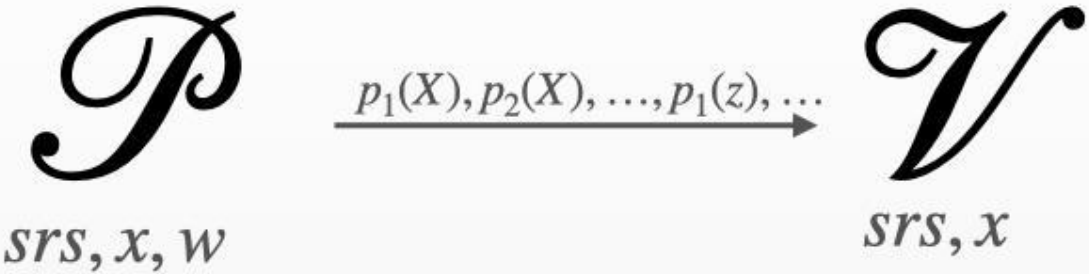

Plonk:

$w = (w_1, \ldots, w_{3n})$

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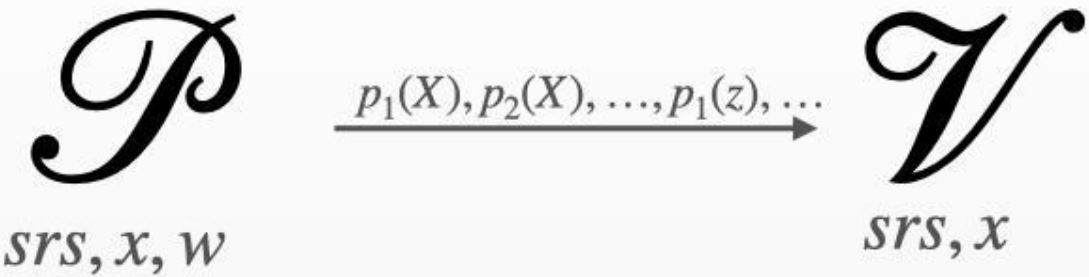

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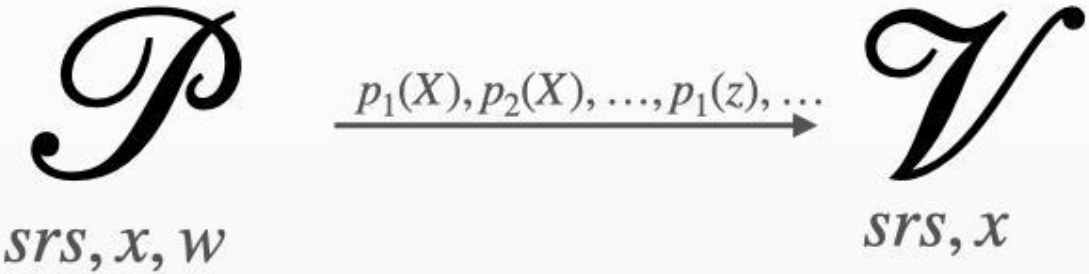

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$\text{Decode}(p_1(X), \ldots) = w$

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$p_1(X) = w_1 + w_2 X + \ldots w_n X^{n-1} + \ldots$

$p_2(X) = w_{n+1} + w_{n+2} X + \ldots w_{2n} X^{n-1} + \ldots$

$p_3(X) = w_{2n+1} + w_{2n+2} X + \ldots w_{3n} X^{n-1} + \ldots$

Observation:

$\deg(p_1) = \deg(p_2) = \deg(p_3) = n + 2$

If we evaluate $p_1(X), p_2(X), p_3(X)$ at $n + 3$ points then we can reveal their coefficients

How to extract?

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Let the extractor learn **multiple evaluations** of polynomials

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Lagrange **interpolation**

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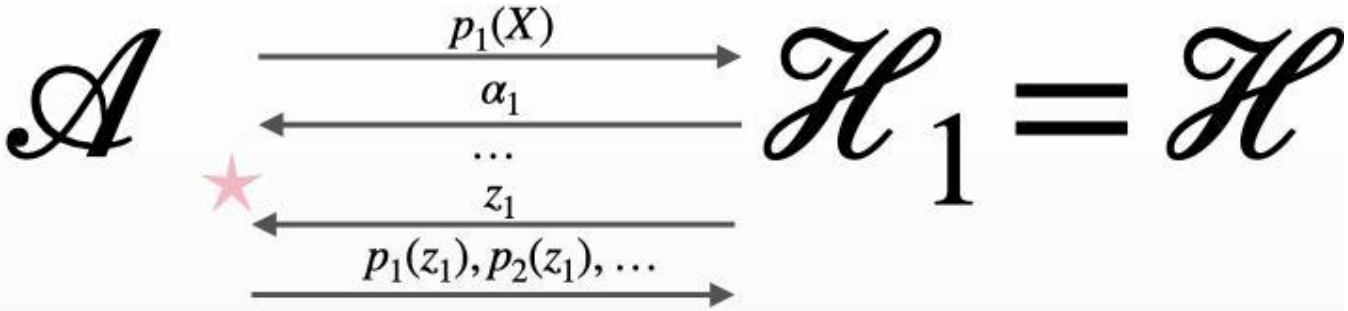
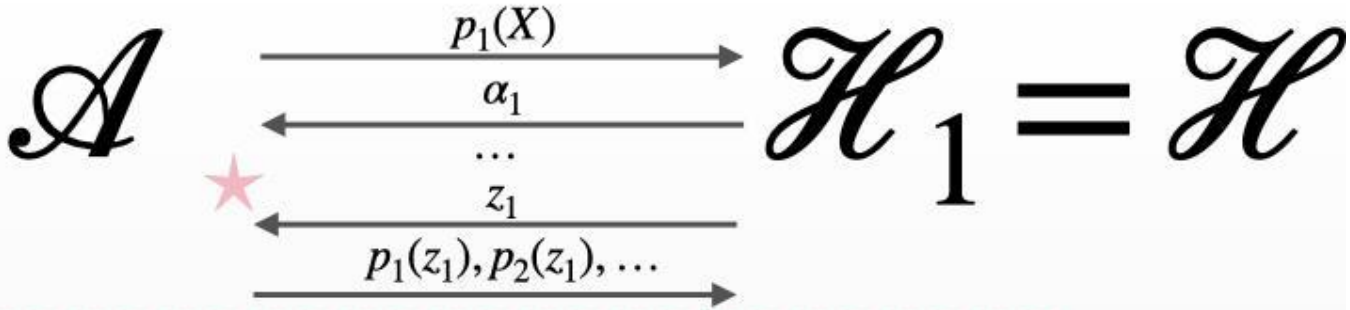
Lagrange **interpolation**

$$p_1(X)$$

Rewinding

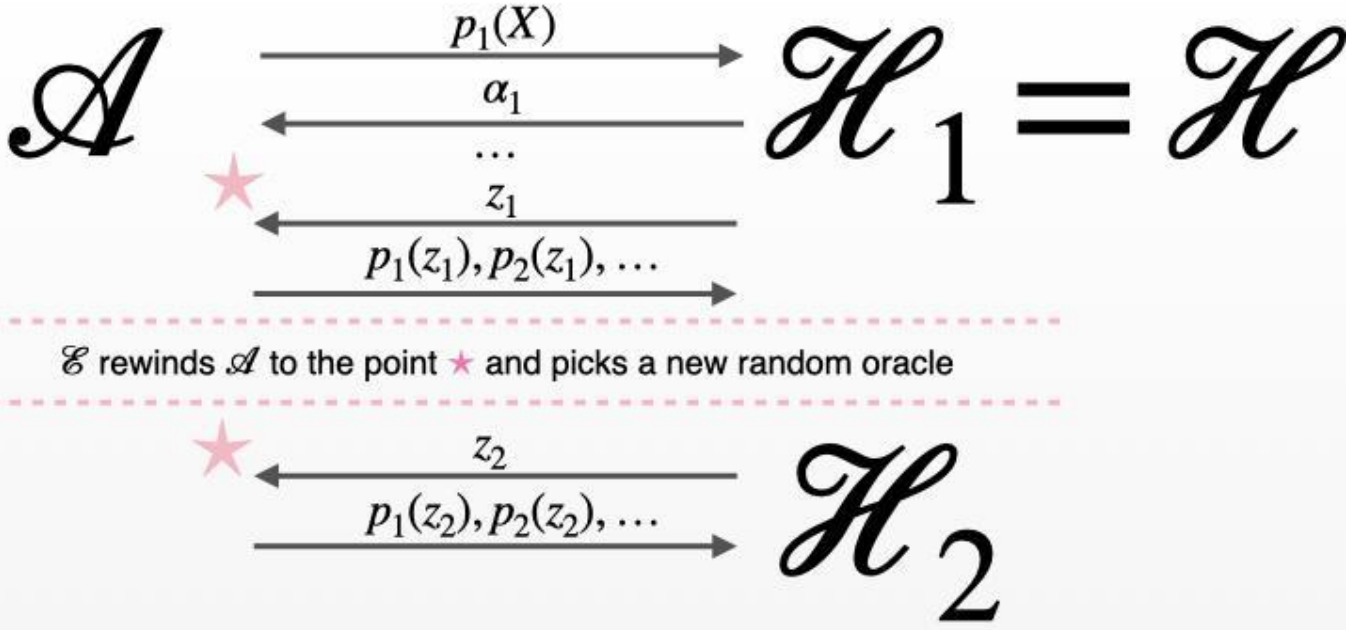


Rewinding

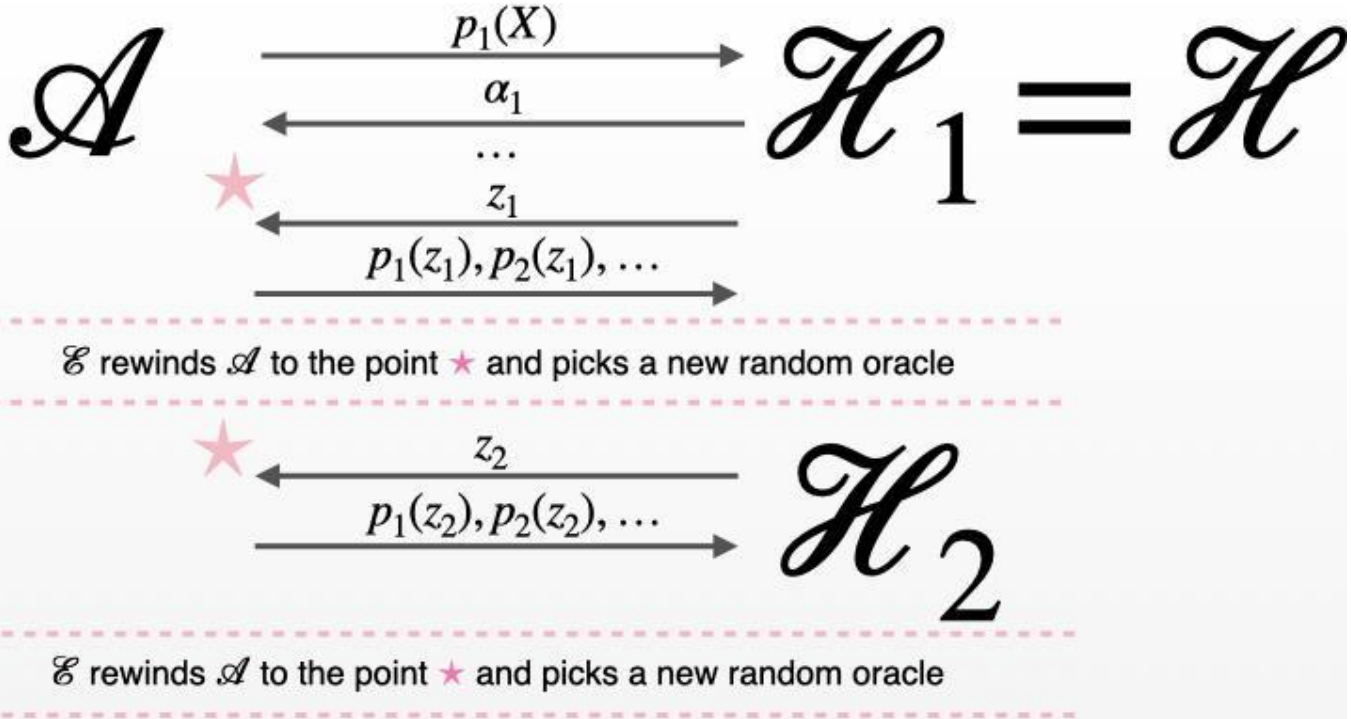


$\mathscr{E}$ rewinds $\mathscr{A}$ to the point ★ and picks a new random oracle

Rewinding


$\mathscr{A}$
$p_1(X)$
α_1
…
z_1
$p_1(z_1), p_2(z_1), \ldots$
$\mathscr{H}_1 = \mathscr{H}$
$\mathscr{E}$ rewinds $\mathscr{A}$ to the point ★ and picks a new random oracle
z_2
$p_1(z_2), p_2(z_2), \ldots$
$\mathscr{H}_2$

Rewinding


$\mathscr{A}$
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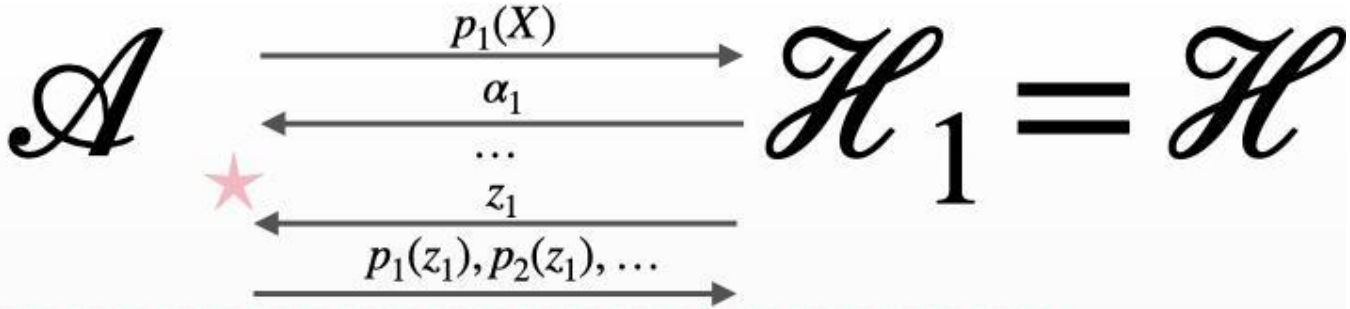

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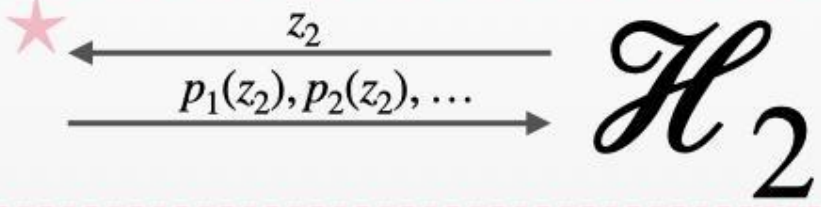
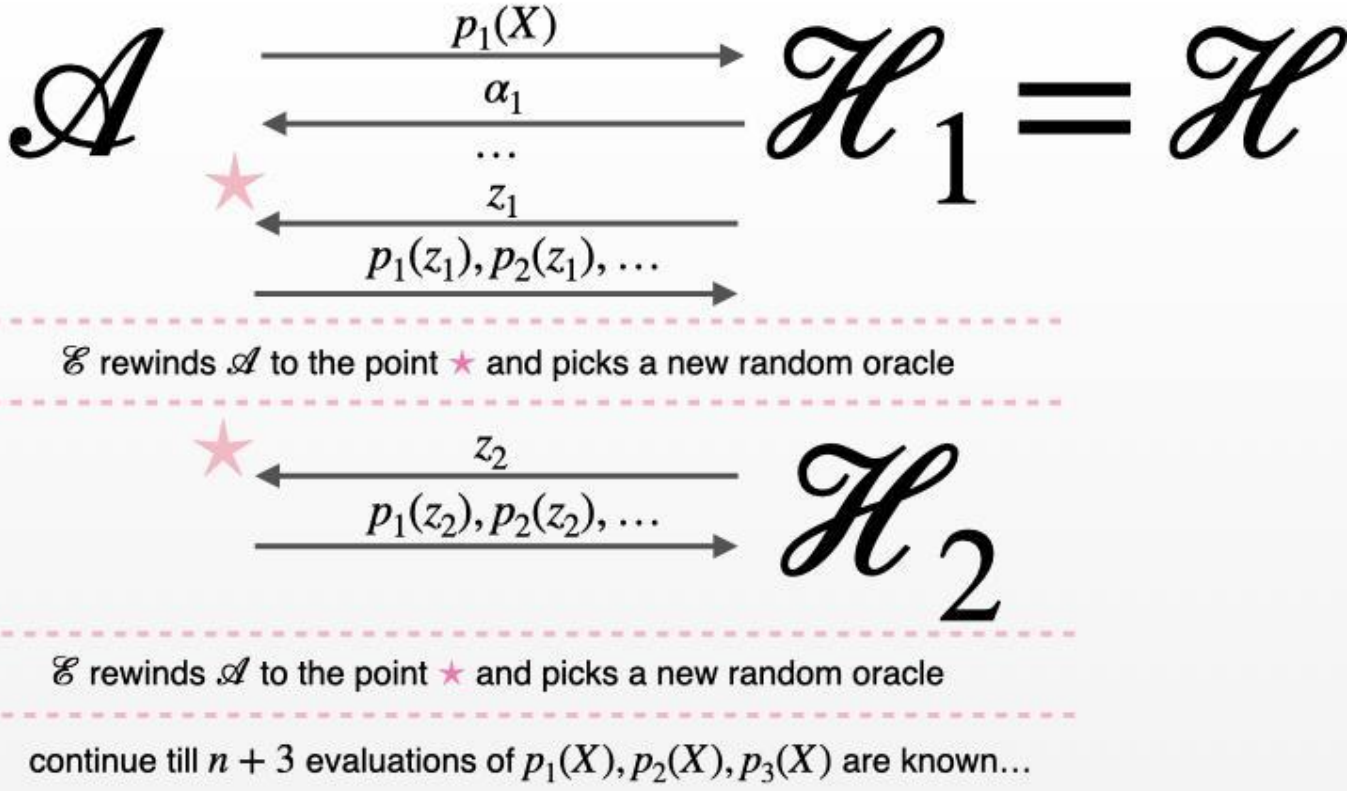


$\mathscr{E}$ rewinds $\mathscr{A}$ to the point ★ and picks a new random oracle

continue till $n + 3$ evaluations of $p_1(X), p_2(X), p_3(X)$ are known…

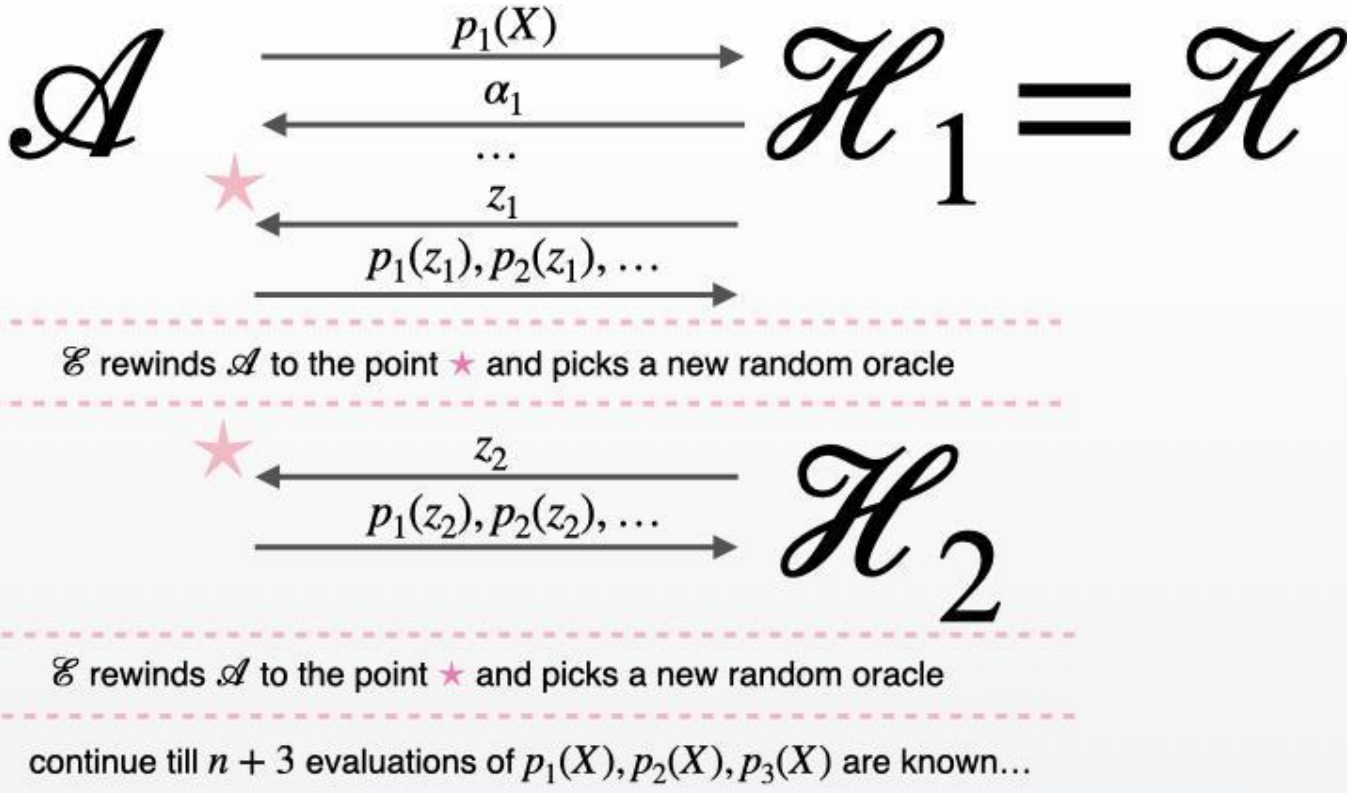
Rewinding



QUESTION 3

How do we know that rewinding reveals the witness? Or the adversary doesn't break if it sees z_2?

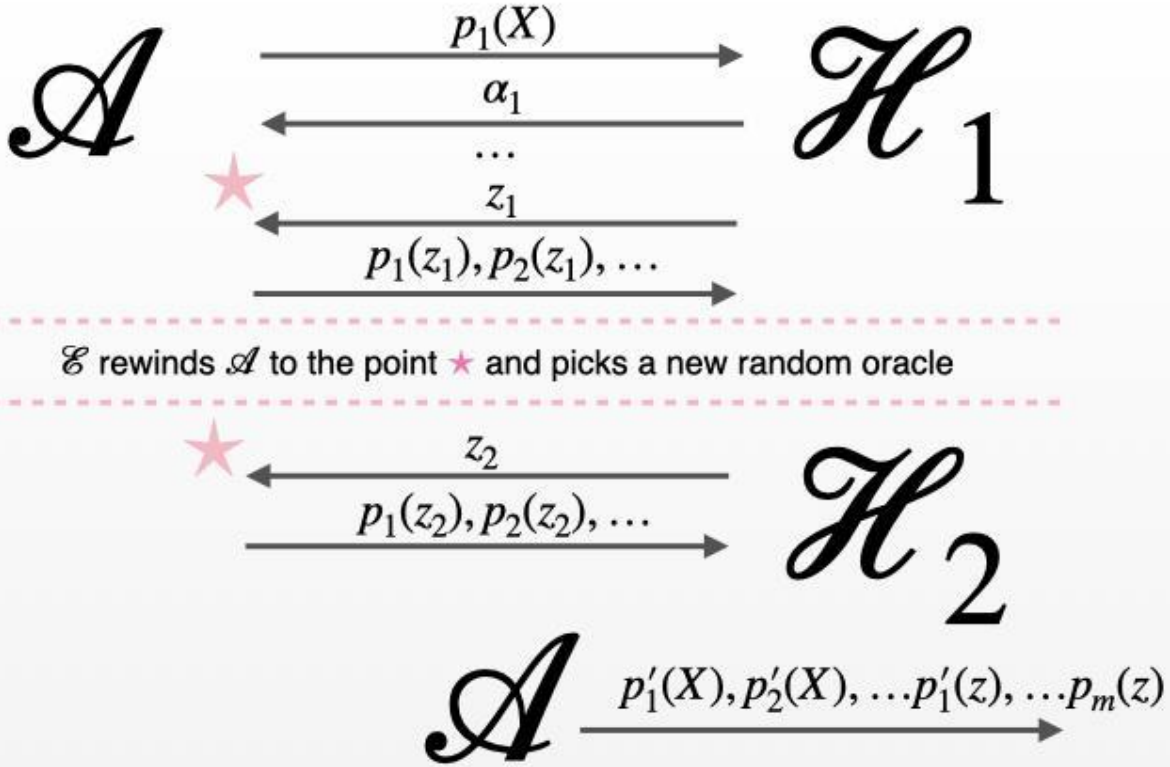
Rewinding



QUESTION 3
How do we know that rewinding reveals the witness? Or the adversary doesn't break if it sees z_2?

REQUIREMENT 3
Rewinding-based knowledge soundness: It is feasible to extract the witness from the polynomials sent by the prover

If k too big


$\mathscr{A}$
$p_1(X)$
α_1
…
z_1
$p_1(z_1), p_2(z_1), \ldots$
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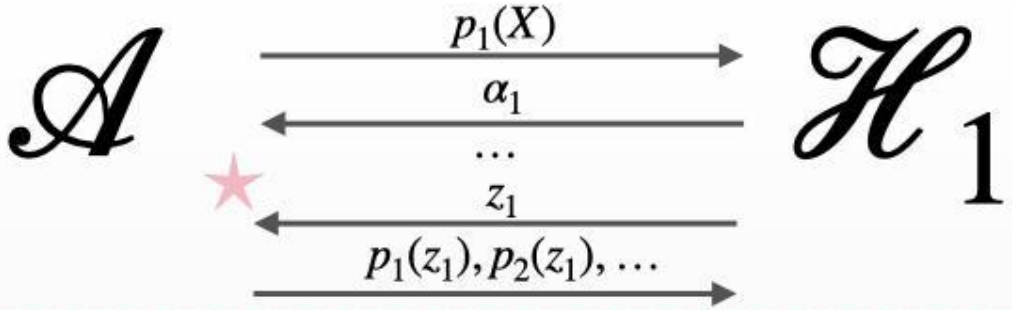
z_2
$p_1(z_2), p_2(z_2), \ldots$
$\mathscr{H}_2$
$\mathscr{A}$
$p'_1(X), p'_2(X), \ldots p'_1(z), \ldots p_m(z)$

If k too big



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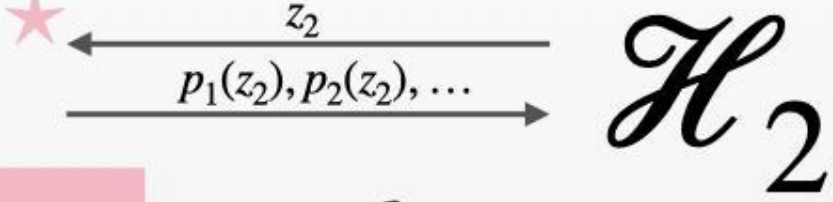
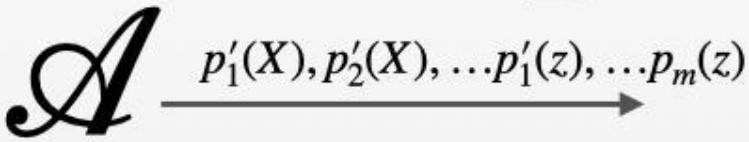
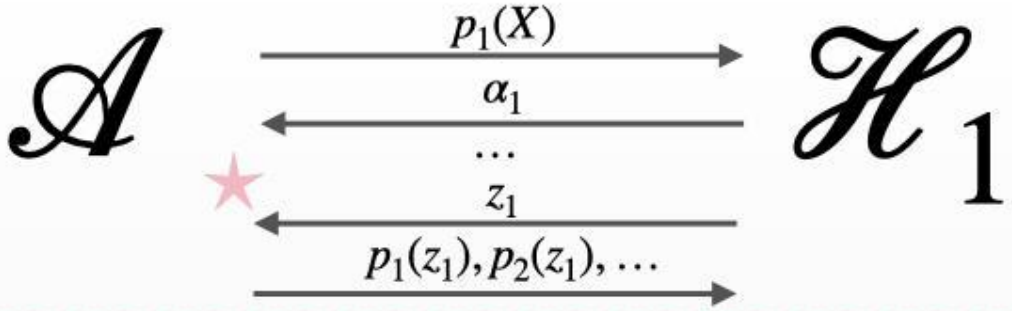


REQUIREMENT 1
Unique response property: It is infeasible to provide two transcripts that
- are different, and
- match on the first k messages, and
- are both acceptable



If k too big



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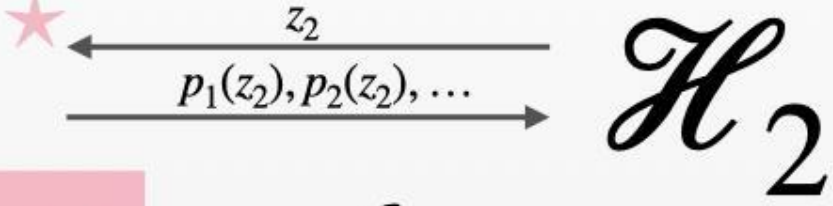

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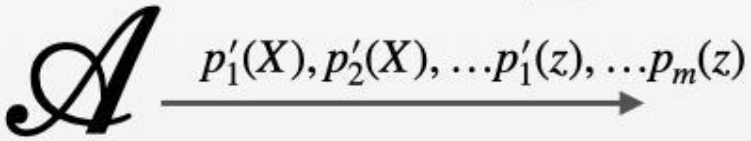
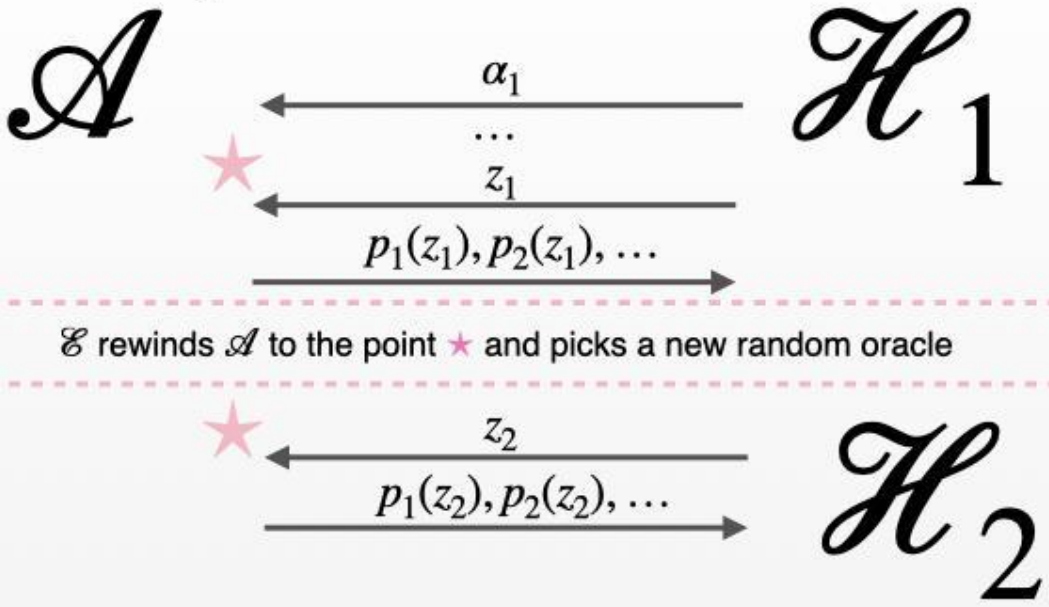
(challenge z should come after round k)



We cannot change the **simulated** proofs the adversary has seen

If $\mathscr{A}$ keeps to ask questions…



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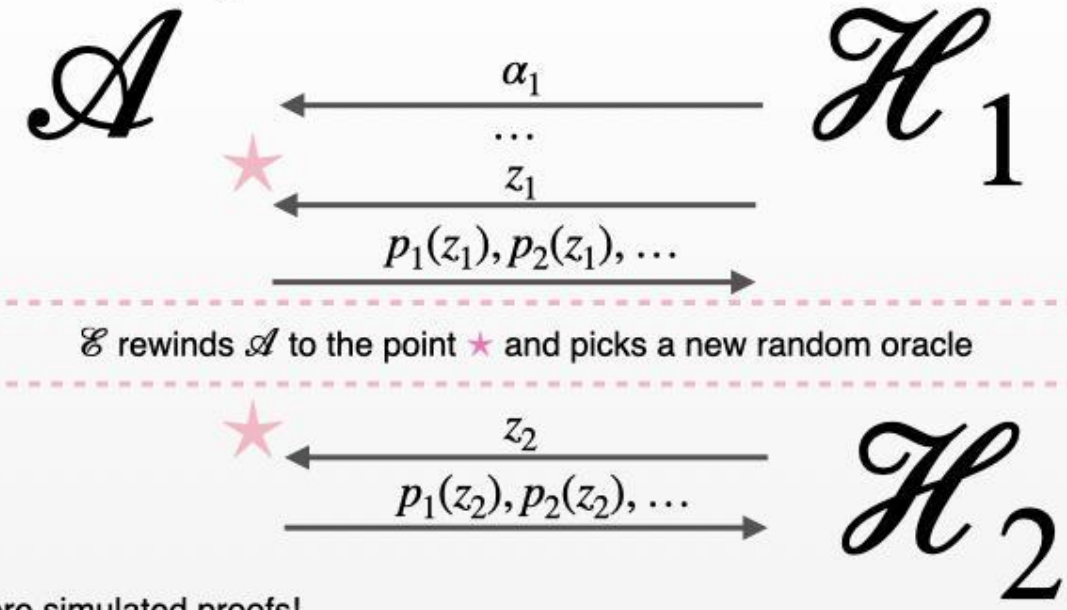

Give me more simulated proofs!

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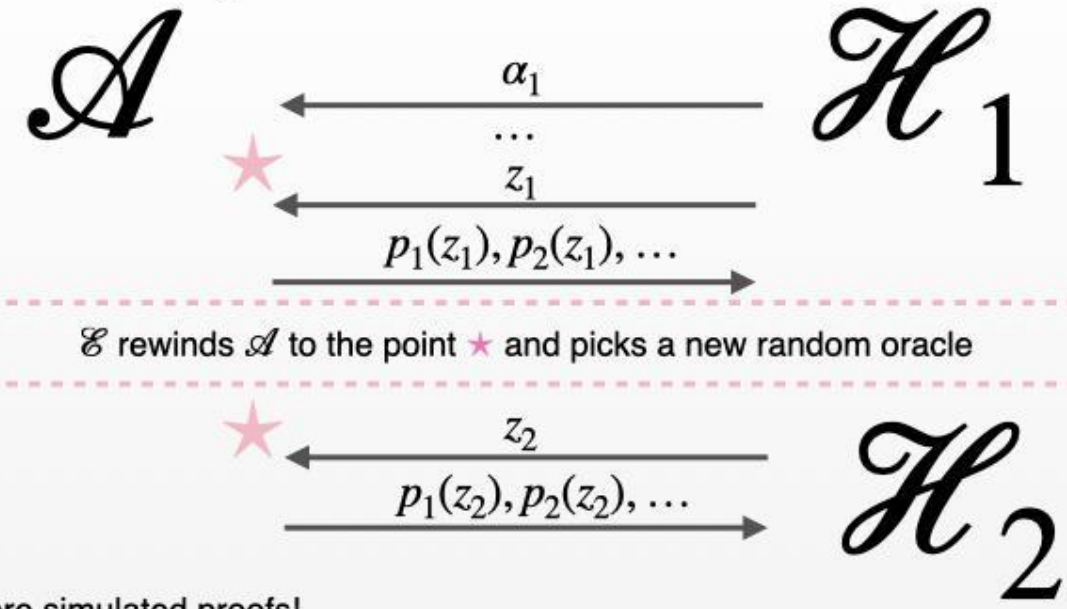

Give me more simulated proofs!

$$\mathscr{C}_{\mathscr{A}}(p, srs, r, Q_{\mathscr{H}}, Q_{\mathscr{S}})$$

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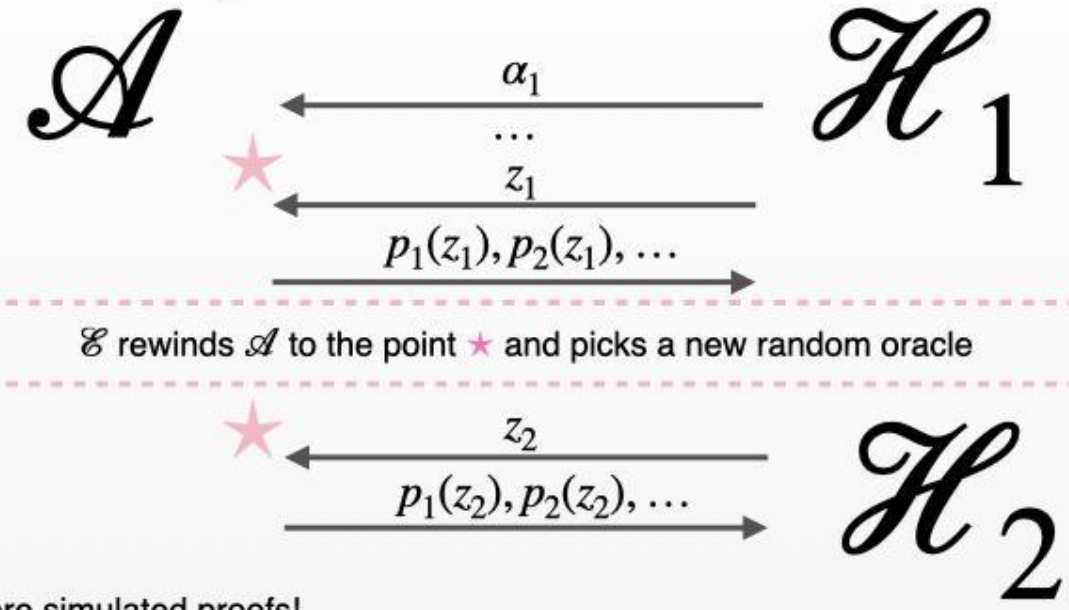

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$$\mathscr{C}_{\mathscr{A}}(p, srs, r, Q_{\mathscr{H}}, Q_{\mathcal{S}})$$

REQUIREMENT 2
Trapdoor-less zero-knowledge: there exists a simulator that provides valid proofs without using SRS's trapdoor

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https://eprint.iacr.org/2021/511

Additional slides

Snarky Signatures (aka succinct signatures of knowledge)

$R(x, w)$
Signing of m is possible only if w known
Verification done by using x

Updatable SE

Definition 2 (Updatable Simulation Extractability). *Let* $\Psi_{\mathsf{NI}} = (\mathsf{SRS}, \mathsf{P}, \mathsf{V}, \mathsf{Sim})$ *be a NIZK proof system with an updatable SRS setup. We say that* Ψ_{NI} *is* updatable simulation-extractable *with security loss* $\varepsilon_{\mathsf{se}}(\lambda, \mathsf{acc}, q)$ *if for any* PPT *adversary* $\mathcal{A}$ *that is given oracle access to setup oracle* UpdO *and simulation oracle* SimO *and that produces an accepting proof for* Ψ_{NI} *with probability* acc*, where*

$$\mathsf{acc} = \Pr\left[\begin{array}{c|c} \mathsf{V}(\mathsf{srs}, \mathsf{x}, \pi) = 1 \\ \wedge (\mathsf{x}, \pi) \notin Q \end{array} \middle| \begin{array}{c} r \leftarrow\$ \mathsf{R}(\mathcal{A}) \\ (\mathsf{x}, \pi) \leftarrow \mathcal{A}^{\mathsf{UpdO}, \mathsf{SimO}.\mathcal{H}, \mathsf{SimO}.\mathsf{P}'}(1^\lambda; r) \end{array}\right]$$

there exists an expected PPT extractor $\mathsf{Ext}_{\mathsf{se}}$ *such that*

$$\Pr\left[\begin{array}{c|c} \mathsf{V}(\mathsf{srs}, \mathsf{x}, \pi) = 1, \\ (\mathsf{x}, \pi) \notin Q, \\ \mathbf{R}(\mathsf{x}, \mathsf{w}) = 0 \end{array} \middle| \begin{array}{c} r \leftarrow\$ \mathsf{R}(\mathcal{A}), (\mathsf{x}, \pi) \leftarrow \mathcal{A}^{\mathsf{UpdO}, \mathsf{SimO}.\mathcal{H}, \mathsf{SimO}.\mathsf{P}'}(1^\lambda; r) \\ \mathsf{w} \leftarrow \mathsf{Ext}_{\mathsf{se}}(\mathsf{srs}, \mathcal{A}, r, Q_{\mathsf{srs}}, Q_{\mathcal{H}}, Q) \end{array}\right] \leq \varepsilon_{\mathsf{se}}(\lambda, \mathsf{acc}, q)$$

Here, srs *is the finalized SRS. List* Q_{srs} *contains all* (srs, ρ) *of update SRSs and their proofs, list* $Q_{\mathcal{H}}$ *contains all* $\mathcal{A}$*'s queries to* $\mathsf{SimO}.\mathcal{H}$ *and the (simulated) random oracle's answers,* $|Q_{\mathcal{H}}| \leq q$*, and list* Q *contains all* (x, π) *pairs where* x *is an instance queried to* $\mathsf{SimO}.\mathsf{P}'$ *by the adversary and* π *is the simulator's answer .*

Contribution

Define **updatable simulation extractability** for NIZKs

Analyze three properties **needed** for the NIZK to be simulation extractable

Show that a wide class (Plonk, Marlin, Lunar, Sonic) of **zkSNARKs** is simulation-extractable **out of the box**

no changes in the protocol needed $\mapsto$ **no efficiency loss**!

Snarky signature in the **updatable** setting

zkSNARKs properties

$$\mathcal{P}(srs, x, w) \xrightarrow{\pi} \mathcal{V}(srs, x)$$

$$R(x, w)$$

Completeness
Honest verifier always accepts a proof from an honest prover $\mathcal{P}$

Knowledge soundness
If verifier $\mathcal{V}$ accepts a proof for a statement x then the prover $\mathcal{A}(p, srs; r)$ **knows** w such that $R(x, w)$

$$\mathcal{E}_{\mathcal{A}}(p, srs, r) = w$$

Zero knowledge
Verifiers learns nothing besides the validity of the statement

$$\mathcal{S}(p, srs, td, x) \approx \mathcal{P}(p, srs, x, w)$$

Succinctness
The proof π is short

$$|\pi| = O(\log(|x| + |w|))$$

All the fuss with the SRS

$$td, srs \leftarrow \mathsf{KGen}(p)$$

Trusted party
Party that creates the SRS can easily provide valid proofs for false statements

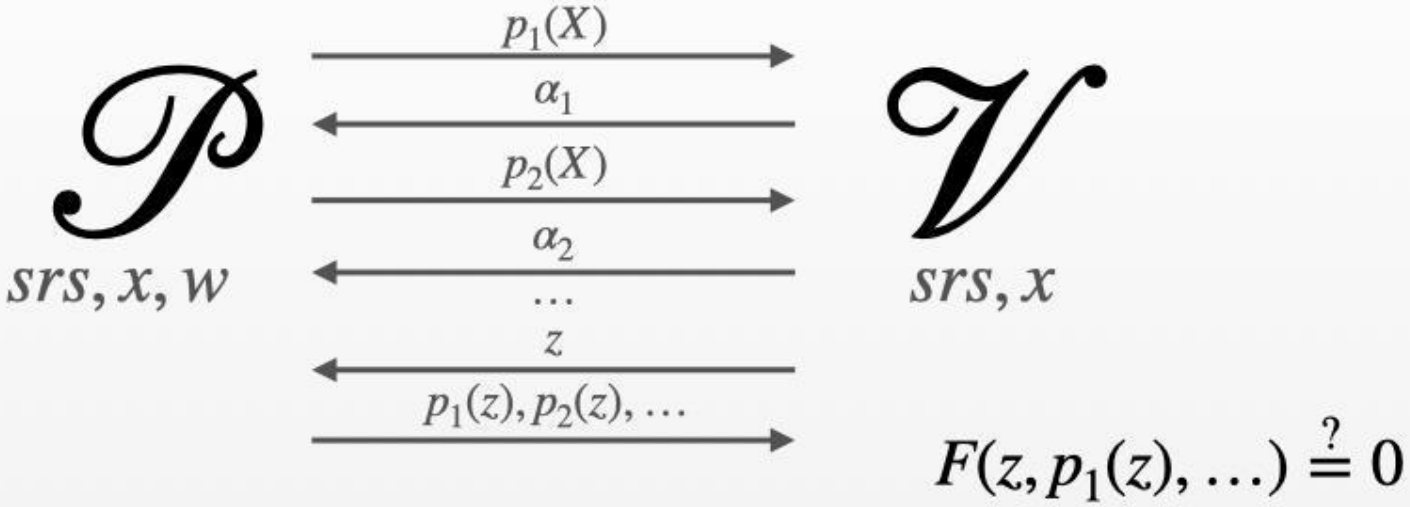
$$\mathcal{S}(p, srs, td, x)$$

Real life instantiation
MPC ceremonies to generate the SRS

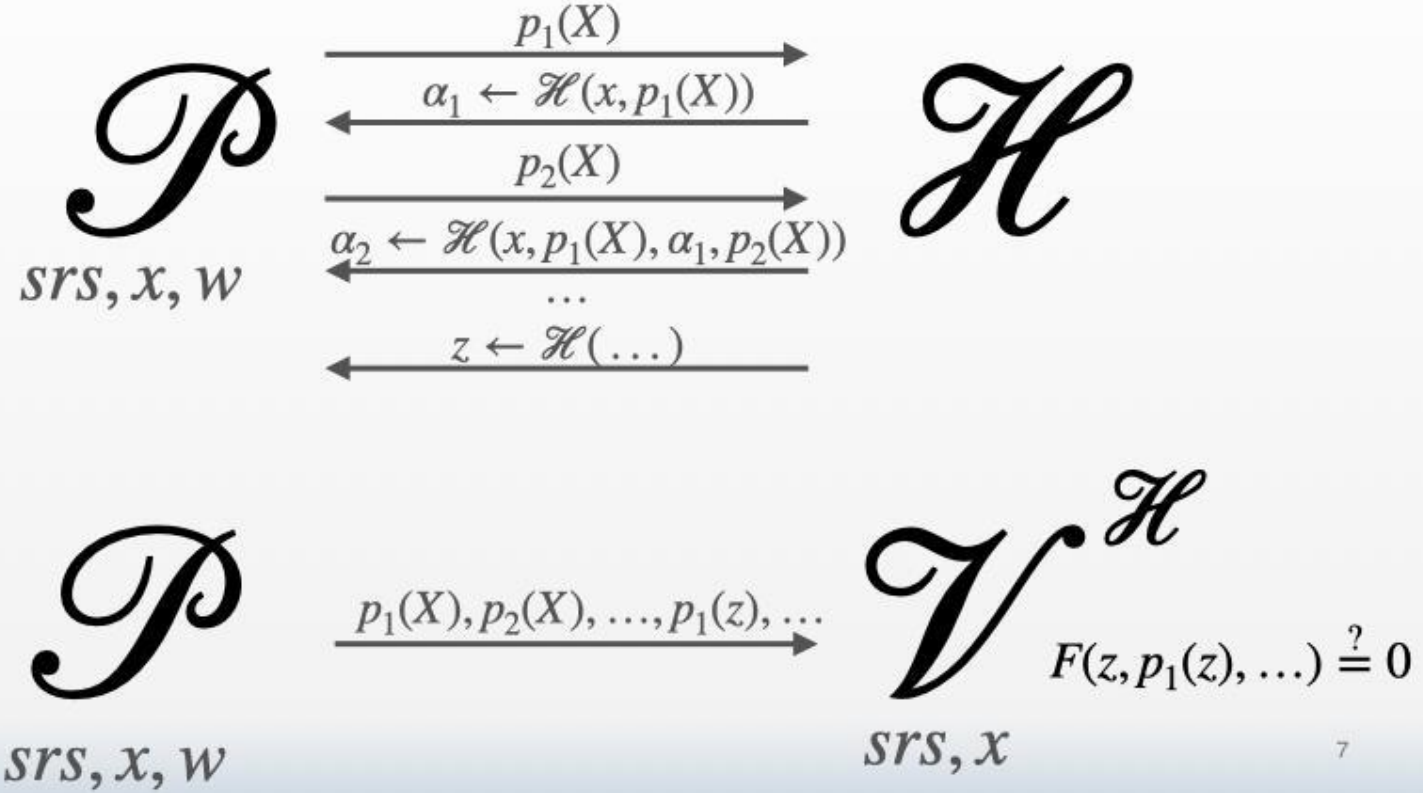
Updatable SRS
Everybody can take an existing SRS srs and **update** it to get an SRS srs'
One honest update is enough for the proof system to be sound

$$\mathsf{Update}(srs) \rightarrow srs', \pi_{upd}$$

How are zkSNARK-s built?



From interactive to non interactive: Fiat–Shamir transformation



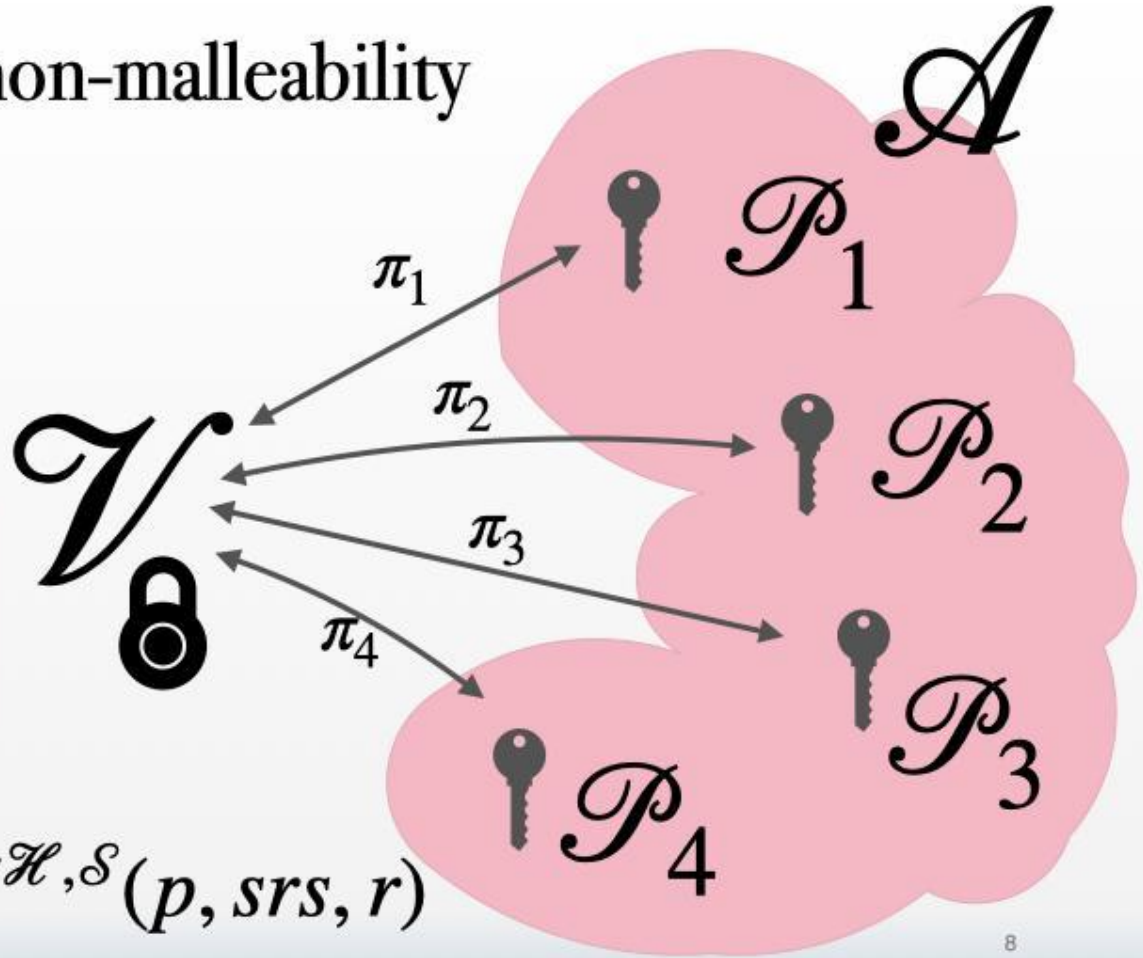
The importance of non-malleability

Knowledge soundness: $\mathcal{A}^{\mathcal{H}}(p, srs; r)$ cannot convince the verifier $\mathcal{V}$ on x unless it knows w such that $R(x, w)$

Extraction: $\mathcal{E}_{\mathcal{A}}(p, srs, r, Q_{\mathcal{H}}) = w$

Simulation-extractability: $\mathcal{A}^{\mathcal{H},\mathcal{S}}(p, srs; r)$ cannot convince the verifier $\mathcal{V}$ on x unless it knows w such that $R(x, w)$ **given access to simulated proofs**

Extraction: $\mathcal{E}_{\mathcal{A}}(p, srs, r, Q_{\mathcal{H}}, Q_{\mathcal{S}}) = w$



$$\mathcal{A}^{\mathcal{H}}(p, srs, r) \mapsto \mathcal{A}^{\mathcal{H},\mathcal{S}}(p, srs, r)$$

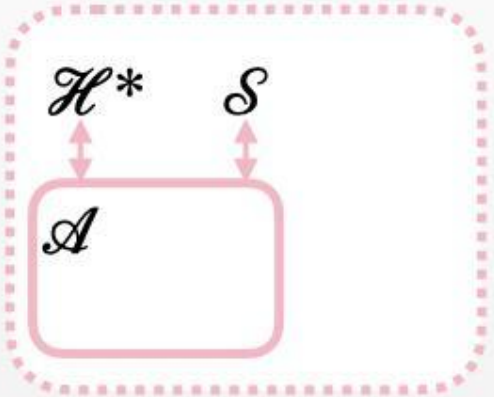
Simulation-extractability

$$\mathcal{A}^{\mathcal{H},\mathcal{S}}(p, srs, r) = x, \pi$$

$\mathcal{S}(x)$ responds with a simulated proof for x

When adversary wins?
$\mathcal{A}$ returns an instance—proof pair (x, π) such that
- π is not a simulated proof
- $\mathcal{A}$ doesn't know the witness

$$\mathcal{E}_{\mathcal{A}}(p, srs, r, Q_{\mathcal{H}}, Q_{\mathcal{S}}) = w \qquad \mathcal{H}$$

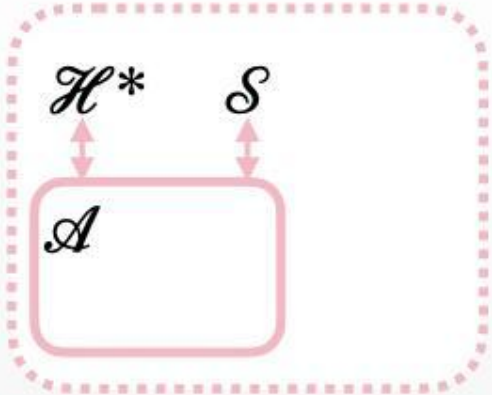


Some issues with extraction

$\mathscr{E}_{\mathscr{A}}(p, srs, r, Q_{\mathscr{H}}, Q_{\mathcal{S}}) = w$

$\mathscr{H}$

$\mathscr{A}^{\mathscr{H},\mathcal{S}}(p, srs, r) = x, \pi$



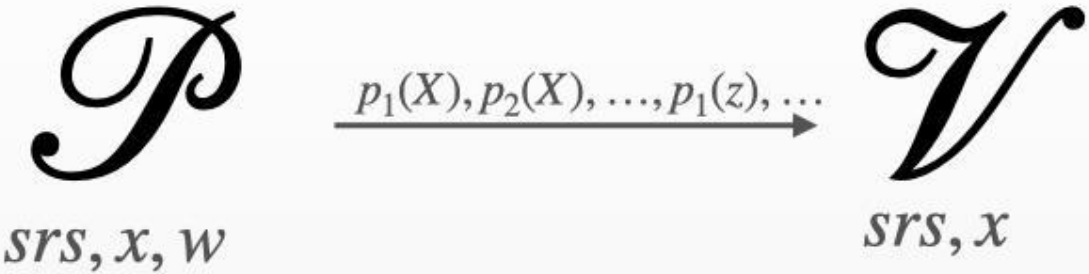
QUESTION 1
What if $\mathscr{A}$ outputs a re-randomized simulated proof?

QUESTION 2
How can $\mathscr{E}$ provide simulated proofs?

REQUIREMENT 1
Unique response property: It is infeasible to provide two transcripts that
- are different, and
- match on the first k messages, and
- are both acceptable

REQUIREMENT 2
Trapdoor-less zero-knowledge: there exists a simulator that provides valid proofs without using SRS's trapdoor

How are zkSNARK-s built?



Plonk:

$w = (w_1, \ldots, w_{3n})$

$p_1(X) = w_1 + w_2 X + \ldots w_n X^{n-1} + \ldots$

$p_2(X) = w_{n+1} + w_{n+2} X + \ldots w_{2n} X^{n-1} + \ldots$

$p_3(X) = w_{2n+1} + w_{2n+2} X + \ldots w_{3n} X^{n-1} + \ldots$

Observation:
Some of the polynomials $p_i(X)$ **encode** the witness

$\text{Decode}(p_1(X), \ldots) = w$

How to extract?

Plonk:

$w = (w_1, \ldots, w_{3n})$

$p_1(X) = w_1 + w_2 X + \ldots w_n X^{n-1} + \ldots$

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$p_3(X) = w_{2n+1} + w_{2n+2} X + \ldots w_{3n} X^{n-1} + \ldots$

Observation:

$\deg(p_1) = \deg(p_2) = \deg(p_3) = n + 2$

If we evaluate $p_1(X), p_2(X), p_3(X)$ at $n + 3$ points then we can reveal their coefficients

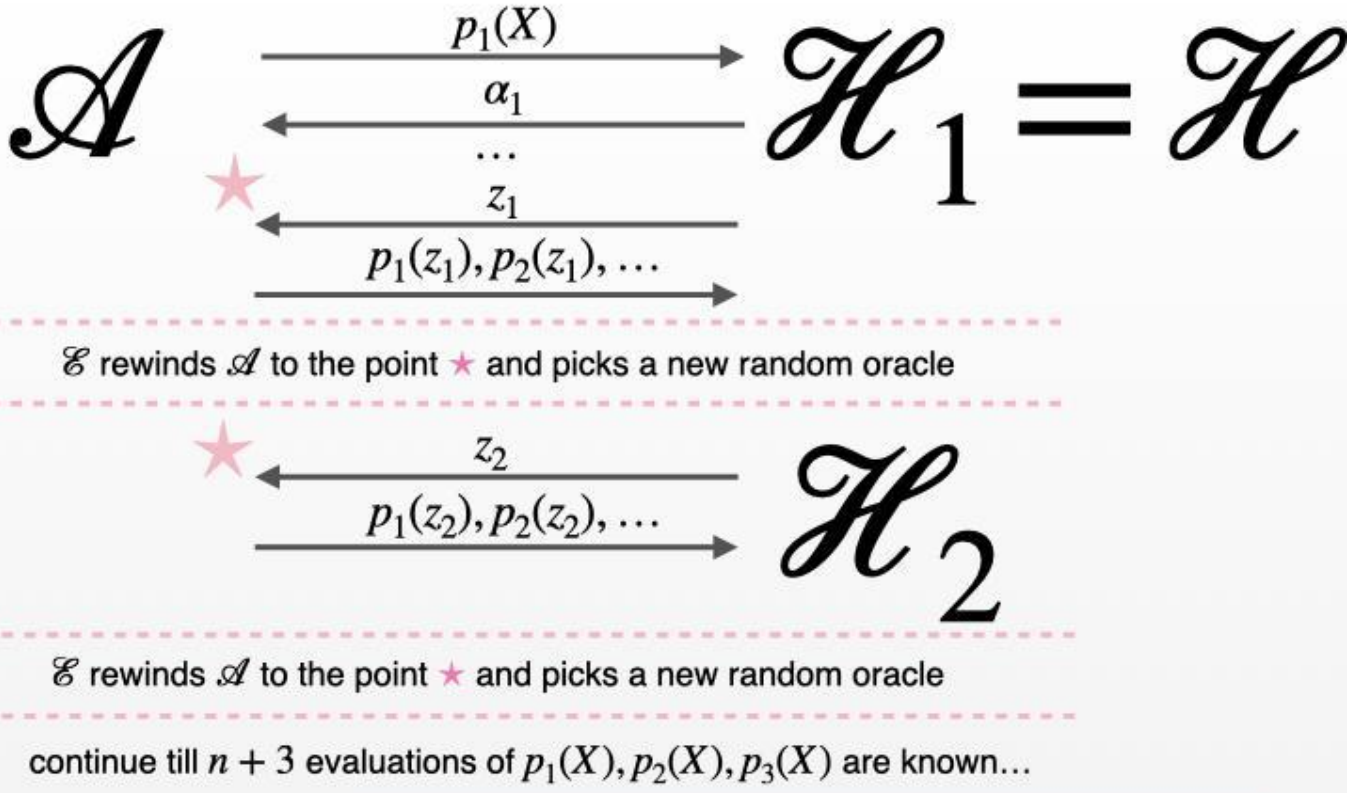
Idea:
Let the extractor learn **multiple evaluations** of polynomials

$$(z_1, p_1(z_1)), \ldots, (z_{n+3}, p_1(z_{n+3}))$$

Lagrange **interpolation**

$$p_1(X)$$

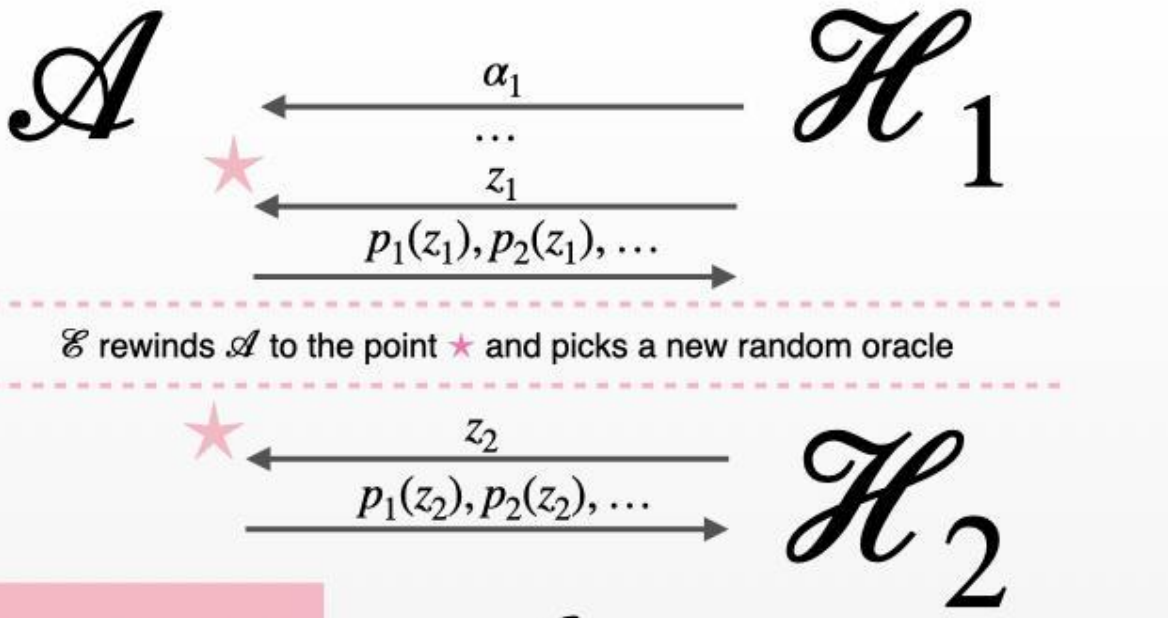
Rewinding



QUESTION 3
How do we know that rewinding reveals the witness? Or the adversary doesn't break if it sees z_2?

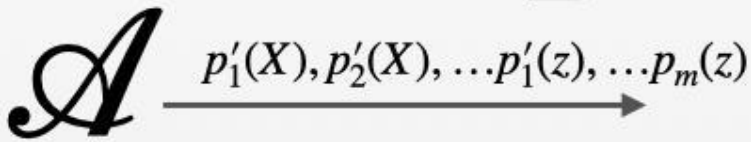
REQUIREMENT 3
Rewinding-based knowledge soundness: It is feasible to extract the witness from the polynomials sent by the prover

If k too big



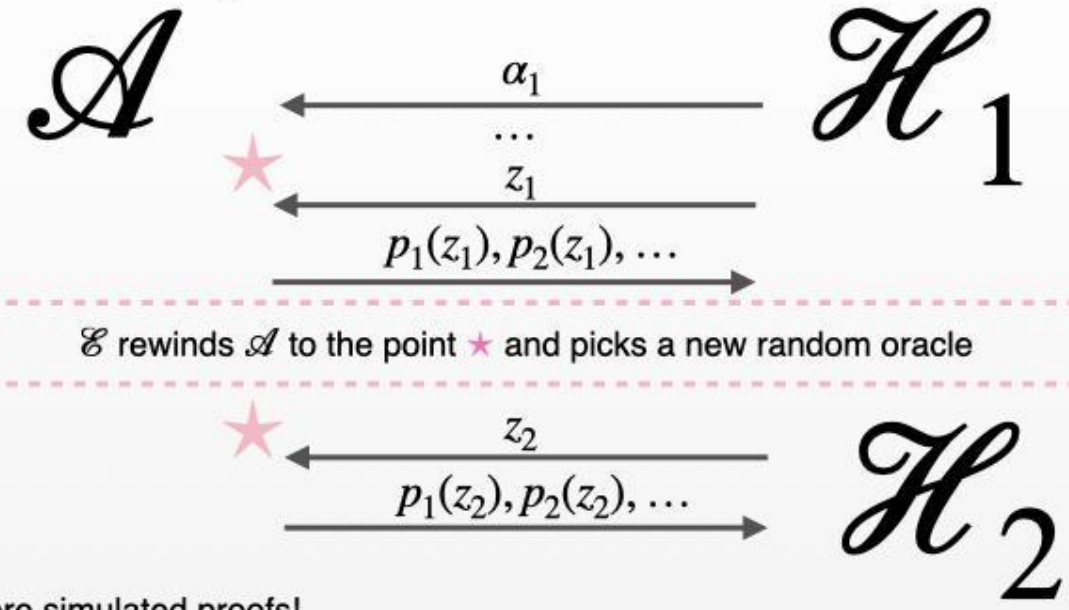
REQUIREMENT 1
Unique response property: It is infeasible to provide two transcripts that
- are different, and
- match on the first k messages, and
- are both acceptable

(challenge z should come after round k)



We cannot change the **simulated** proofs the adversary has seen

If $\mathscr{A}$ keeps to ask questions…



Give me more simulated proofs!

$$\mathscr{C}_{\mathscr{A}}(p, srs, r, Q_{\mathscr{H}}, Q_{\mathcal{S}})$$

REQUIREMENT 2
Trapdoor-less zero-knowledge: there exists a simulator that provides valid proofs without using SRS's trapdoor

What Makes Fiat–Shamir zkSNARKs (Updatable SRS) Simulation Extractable?

How extractor can provide simulated proofs?

Trapdoor-less zero-knowledge: there exists a simulator that provides valid proofs without using SRS's trapdoor

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Additional slides

Snarky Signatures (aka succinct signatures of knowledge)

$R(x, w)$
Signing of m is possible only if w known
Verification done by using x

Updatable SE

Definition 2 (Updatable Simulation Extractability). *Let* $\Psi_{\mathsf{NI}} = (\mathsf{SRS}, \mathsf{P}, \mathsf{V}, \mathsf{Sim})$ *be a NIZK proof system with an updatable SRS setup. We say that* Ψ_{NI} *is* updatable simulation-extractable *with security loss* $\varepsilon_{\mathsf{se}}(\lambda, \mathsf{acc}, q)$ *if for any* PPT *adversary* $\mathcal{A}$ *that is given oracle access to setup oracle* UpdO *and simulation oracle* SimO *and that produces an accepting proof for* Ψ_{NI} *with probability* acc*, where*

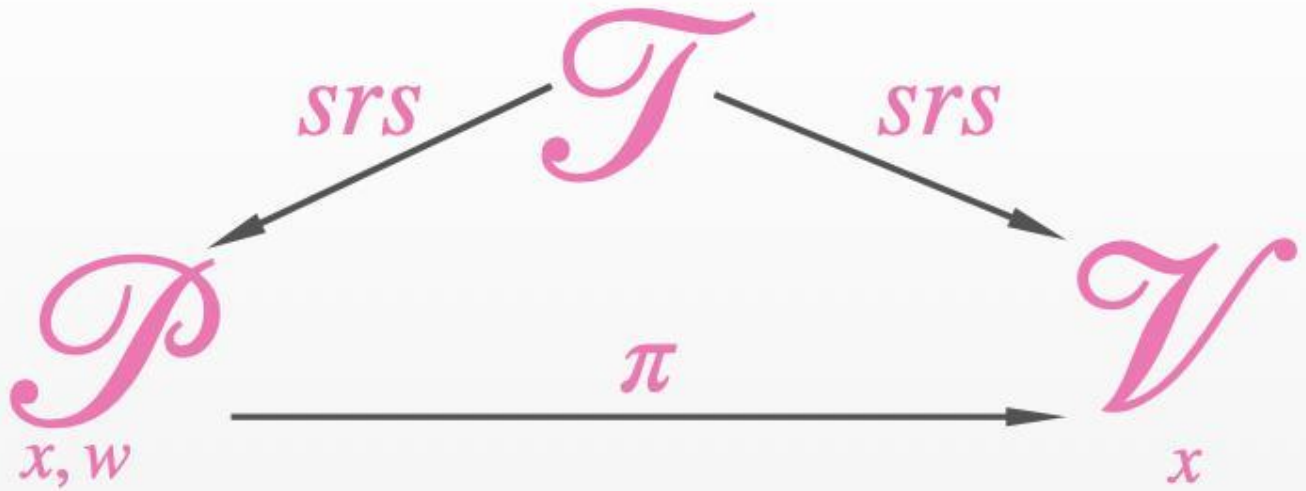
$$\mathsf{acc} = \Pr\left[\begin{array}{c|c} \mathsf{V}(\mathsf{srs}, \mathsf{x}, \pi) = 1 \\ \wedge (\mathsf{x}, \pi) \notin Q \end{array} \middle| \begin{array}{c} r \leftarrow\$ \mathsf{R}(\mathcal{A}) \\ (\mathsf{x}, \pi) \leftarrow \mathcal{A}^{\mathsf{UpdO}, \mathsf{SimO}.\mathcal{H}, \mathsf{SimO}.\mathsf{P}'}(1^\lambda; r) \end{array}\right]$$

there exists an expected PPT extractor $\mathsf{Ext}_{\mathsf{se}}$ *such that*

$$\Pr\left[\begin{array}{c|c} \mathsf{V}(\mathsf{srs}, \mathsf{x}, \pi) = 1, \\ (\mathsf{x}, \pi) \notin Q, \\ \mathbf{R}(\mathsf{x}, \mathsf{w}) = 0 \end{array} \middle| \begin{array}{c} r \leftarrow\$ \mathsf{R}(\mathcal{A}), (\mathsf{x}, \pi) \leftarrow \mathcal{A}^{\mathsf{UpdO}, \mathsf{SimO}.\mathcal{H}, \mathsf{SimO}.\mathsf{P}'}(1^\lambda; r) \\ \mathsf{w} \leftarrow \mathsf{Ext}_{\mathsf{se}}(\mathsf{srs}, \mathcal{A}, r, Q_{\mathsf{srs}}, Q_{\mathcal{H}}, Q) \end{array}\right] \leq \varepsilon_{\mathsf{se}}(\lambda, \mathsf{acc}, q)$$

Here, srs *is the finalized SRS. List* Q_{srs} *contains all* (srs, ρ) *of update SRSs and their proofs, list* $Q_{\mathcal{H}}$ *contains all* $\mathcal{A}$*'s queries to* $\mathsf{SimO}.\mathcal{H}$ *and the (simulated) random oracle's answers,* $|Q_{\mathcal{H}}| \leq q$*, and list* Q *contains all* (x, π) *pairs where* x *is an instance queried to* $\mathsf{SimO}.\mathsf{P}'$ *by the adversary and* π *is the simulator's answer .*

Zero-knowledge proofs



$$\langle \mathcal{S}(p, srs, td, x), \mathcal{V}(p, srs, x) \rangle \approx \langle \mathcal{P}(p, srs, x, w), \mathcal{V}(p, srs, x) \rangle$$