Polymath: Groth16 Is Not The Limit

Helger Lipmaa, University of Tartu, Estonia

ZKProof 7, Sofia, 24.03.25 (paper appeared at Crypto 2024)

ZK-SNARKs

Computation: f
Public input (statement) x
Private input (witness) w

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srs

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Proof π that $f(\mathrm{x}, \mathrm{w}) = 1$

- Completeness
- Knowledge-soundness
- Zero-knowledge
- Succinct arguments

Landscape

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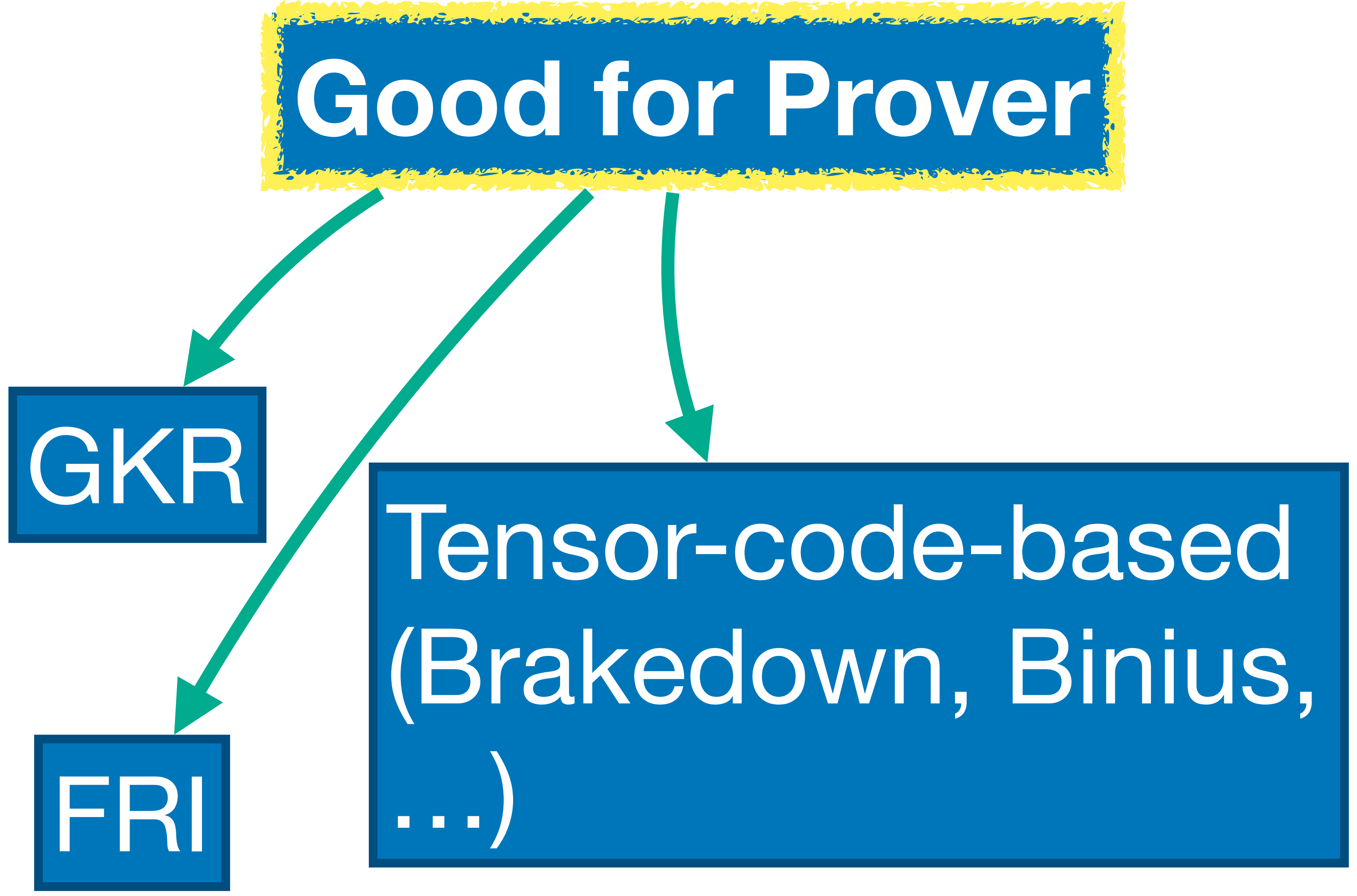

Landscape


Good for Prover
GKR
FRI
Tensor-code-based (Brakedown, Binius, …)
Good for Verifier
…
Plonk
Groth16

Landscape

- **Good for Prover**
  - GKR
  - FRI
  - Tensor-code-based (Brakedown, Binius, …)
- **Good for Both**
  - Composition:
    - GKR + **Groth16**
    - Brakedown + **Groth16**
    - FRI + **Groth16**
    - …
- **Good for Verifier**
  - …
  - Plonk
  - Groth16

Landscape

Huge progress in zk-SNARK land in last 5 years
In 2024,Groth16 still landed supreme after 8 years
- Shortest argument
- Fastest verifier

Good for Prover

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Tensor-code-based (Brakedown, Binius, …)

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- GKR + **Groth16**
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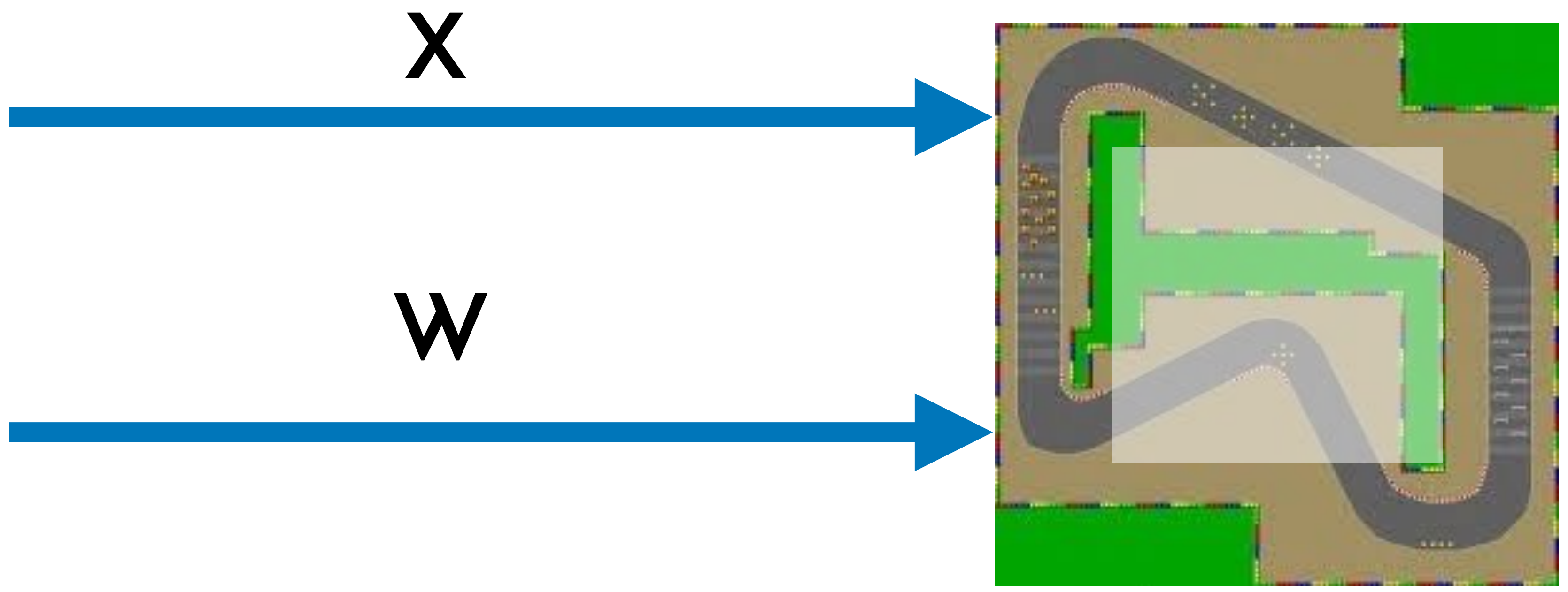
Groth16

Composition (Intuition)

Long

$n = |\mathrm{x}| \approx 2^{26}$



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x

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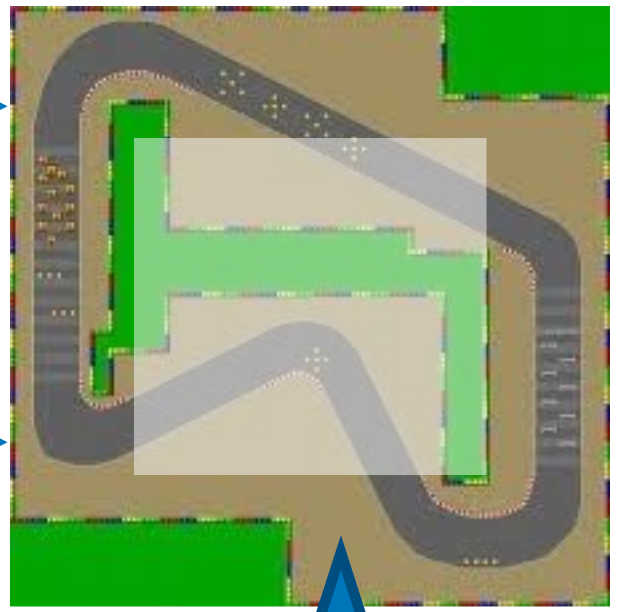

Prove
$(\mathrm{x}, \mathrm{w}) \in \mathscr{R}$

Composition (Intuition)

Long

$n = |\mathrm{x}| \approx 2^{26}$

Semi-long

$\sqrt{n} = \sqrt{|\mathrm{x}|} \approx 2^{13}$?

x

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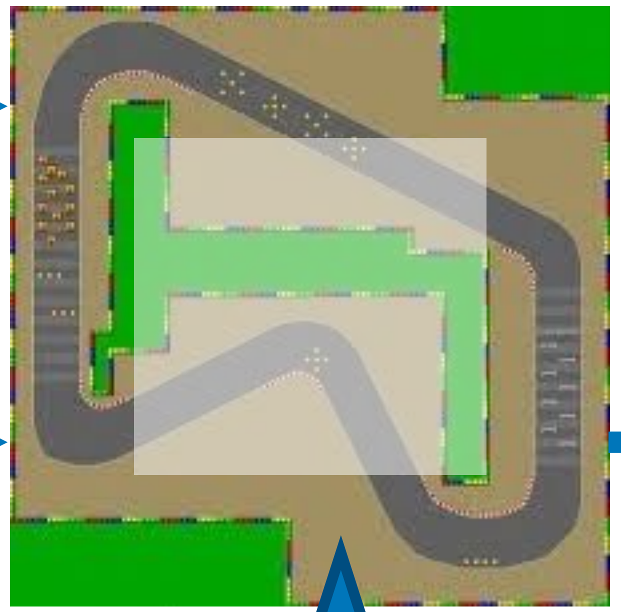

π_{in}

w_{in}

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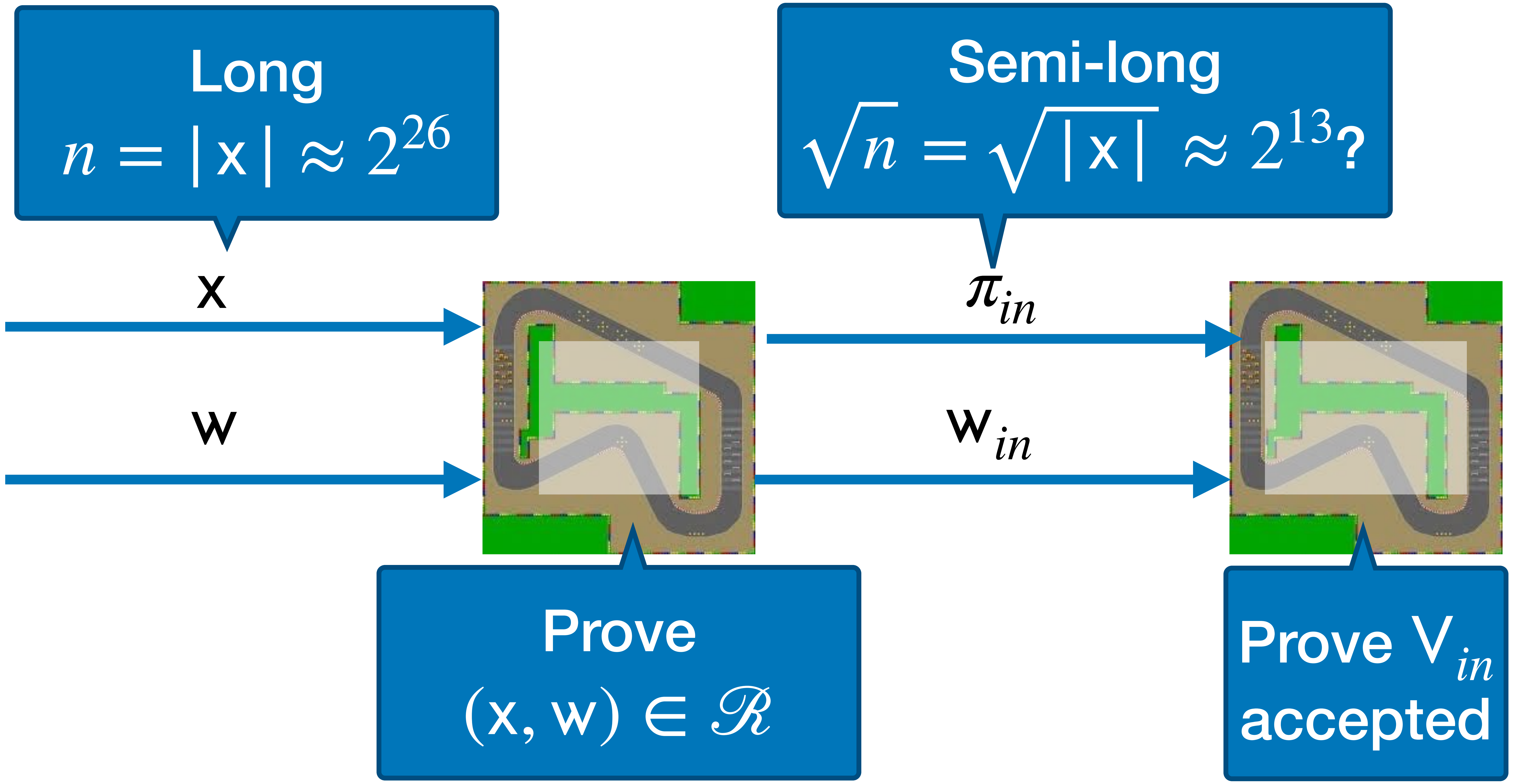

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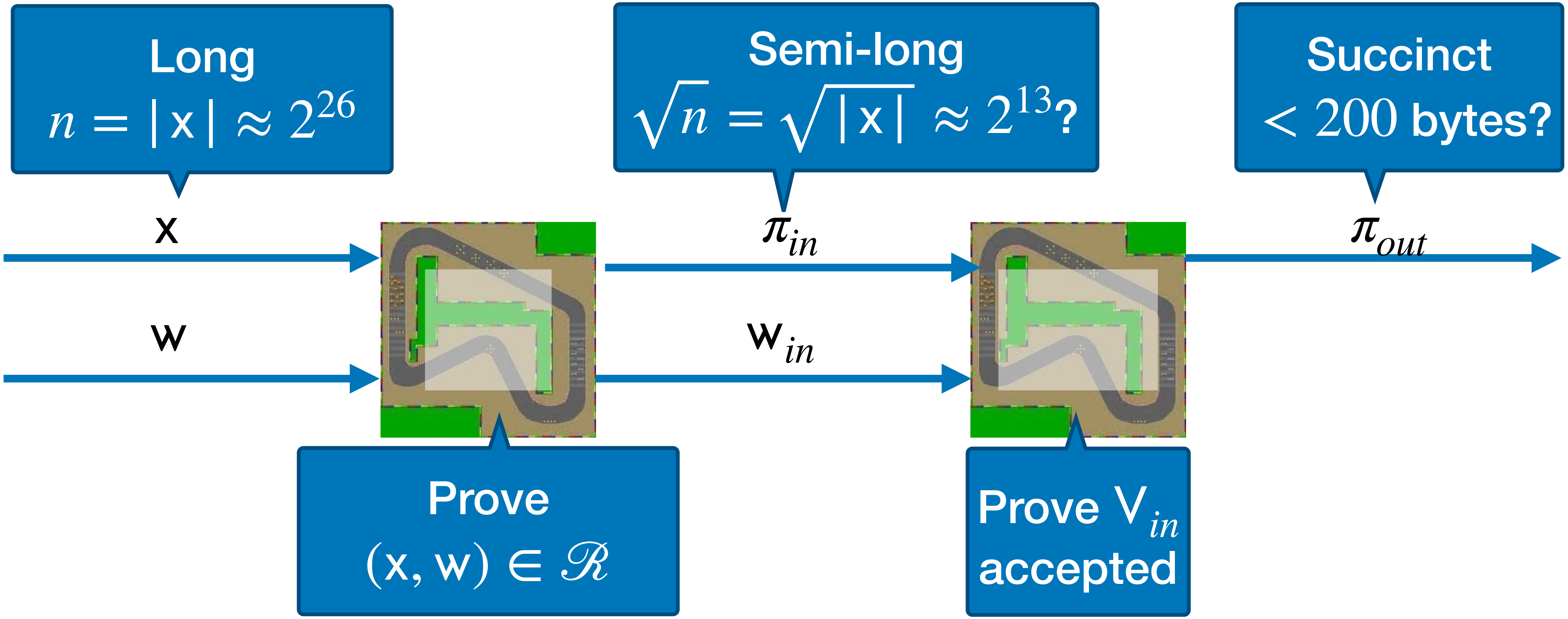

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Succinct
< 200 bytes?

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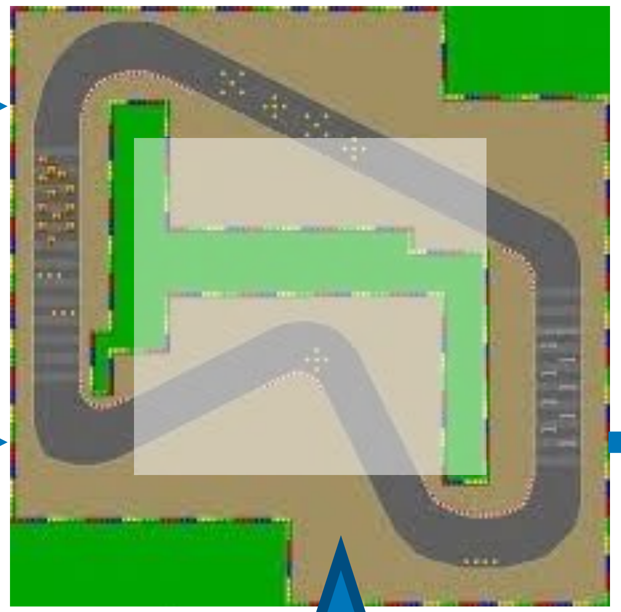

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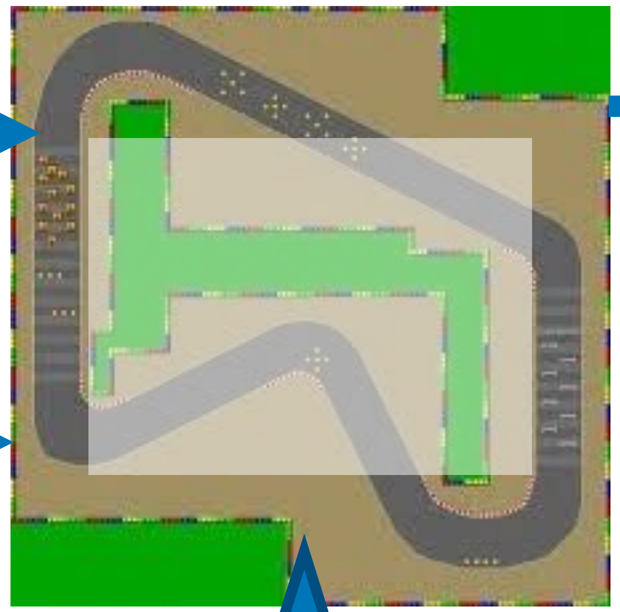

π_{out}

Prove
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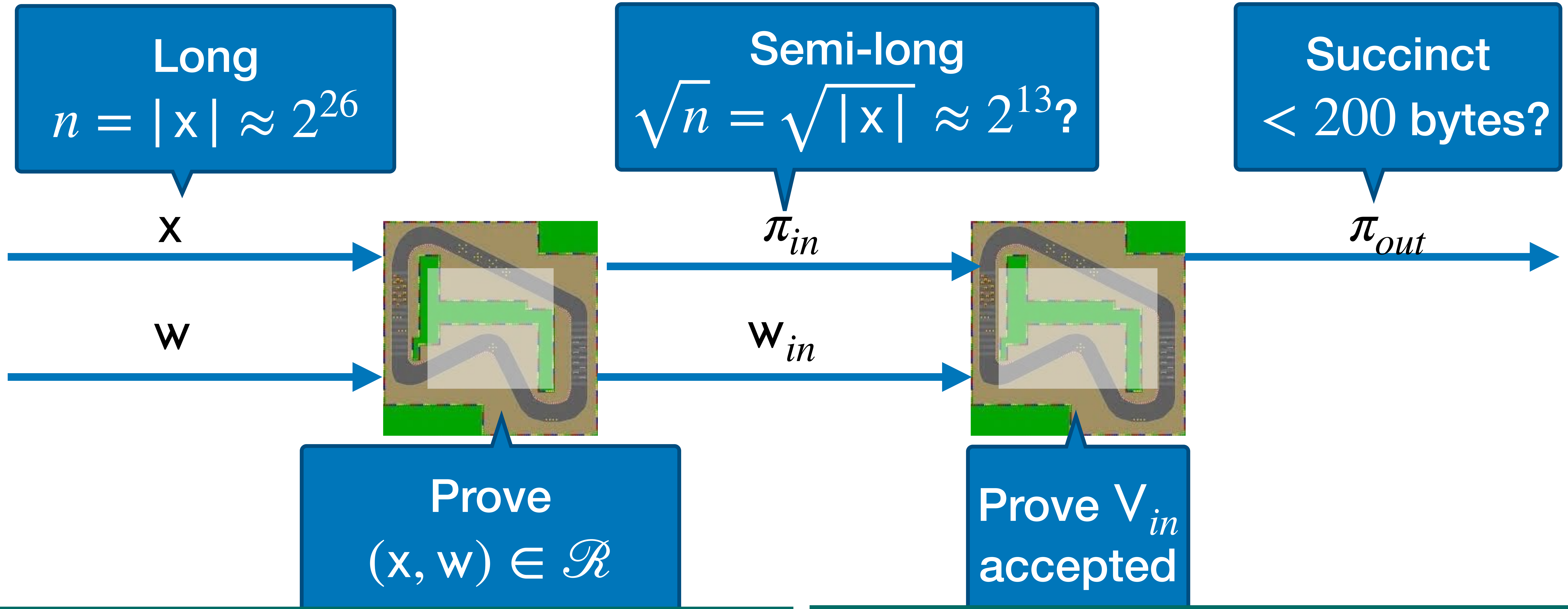
Prove V_{in} accepted

- Long input => need fast prover
- Proof has to be “short enough”
- Verifier’s circuit should be simple
- GKR, FRI, Brakedown, Bulletproofs, …

Composition (Intuition)



- Long input => need fast prover
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- Semi-long input => need decent prover
- Proof has to be “super succinct”
- Groth16!

Composition (Intuition)

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$\sqrt{n} = \sqrt{|x|} \approx 2^{13}$?

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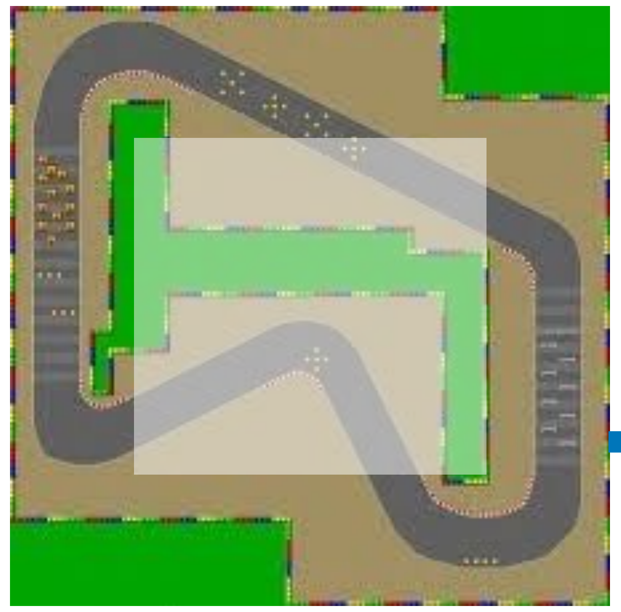

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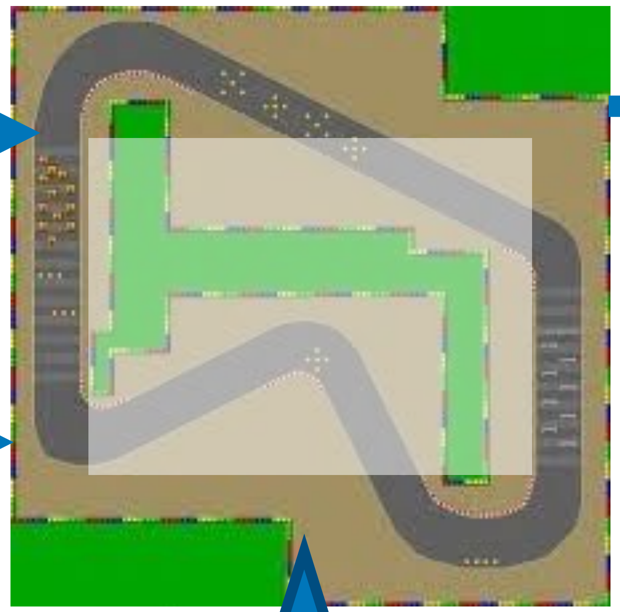

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  - "V_{in} accepts" is a fixed language
  - Non-universality is ok 😀
- Groth16 slow prover?
  - We apply Π_{out} to semi-long input 😀
- **But can we improve on $|\pi_{out}|$ and V speed?** 🤔

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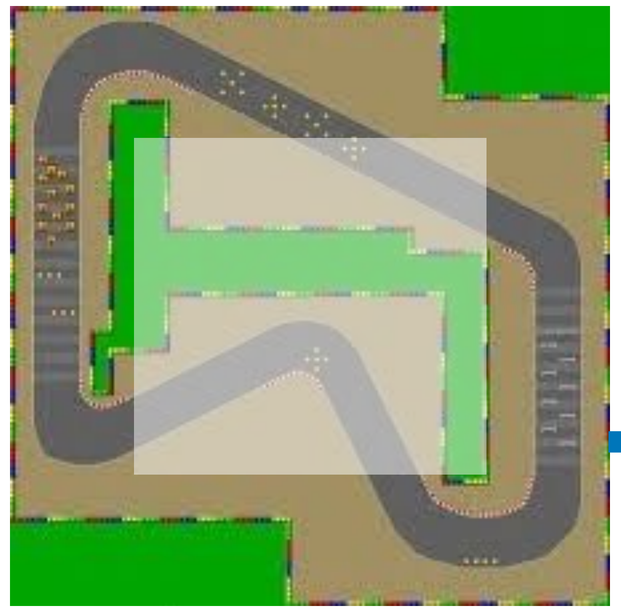

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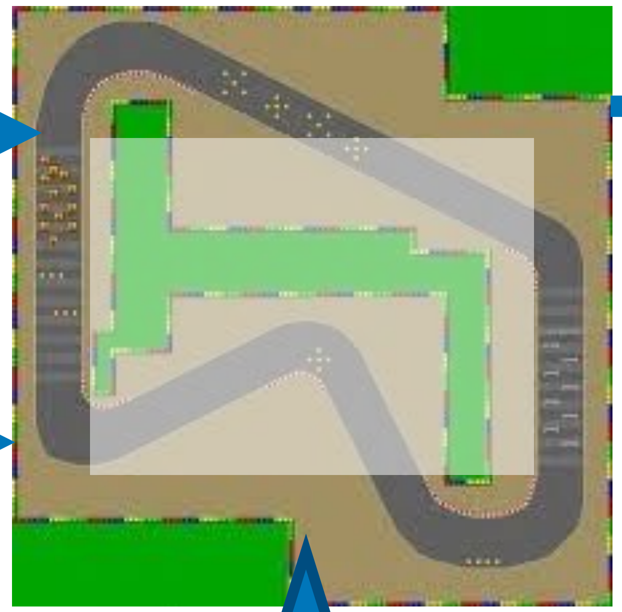

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  - Not known how to batch pre-existing proofs

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On Optimality

$\pi = ([a]_1, [b]_2, [c]_1)$

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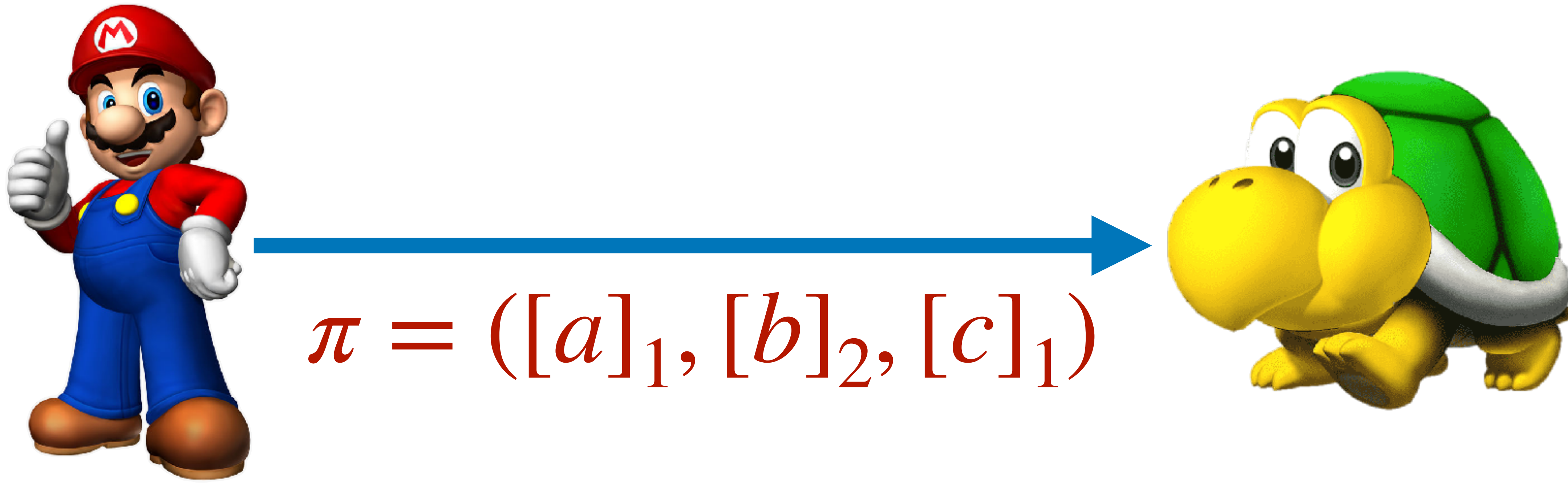

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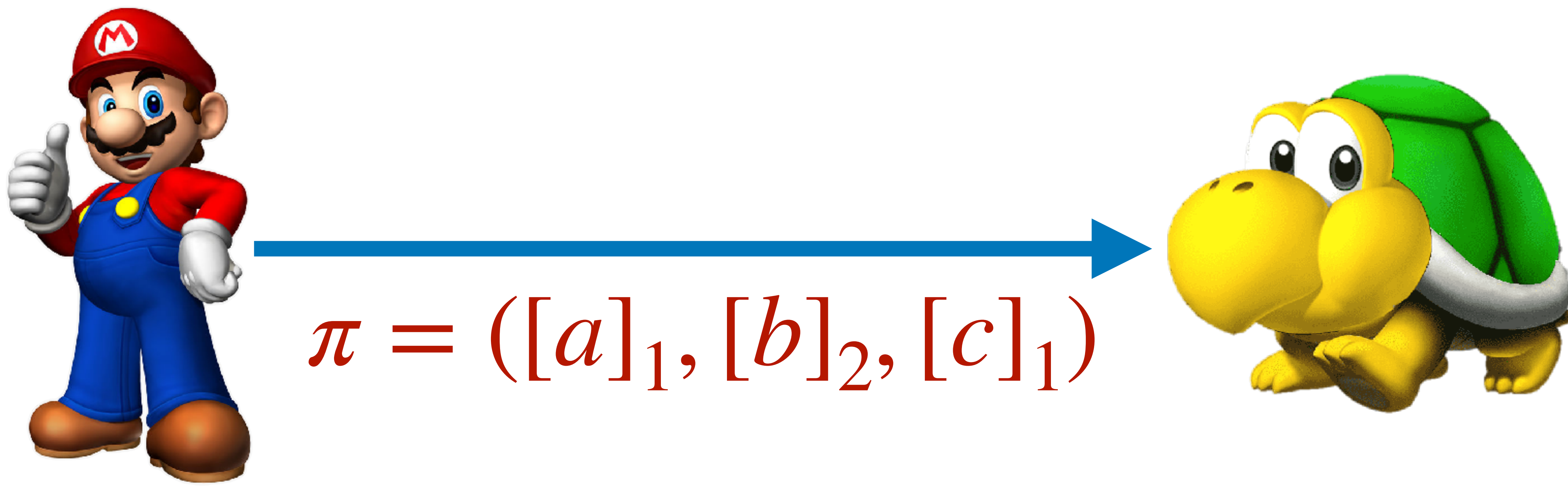

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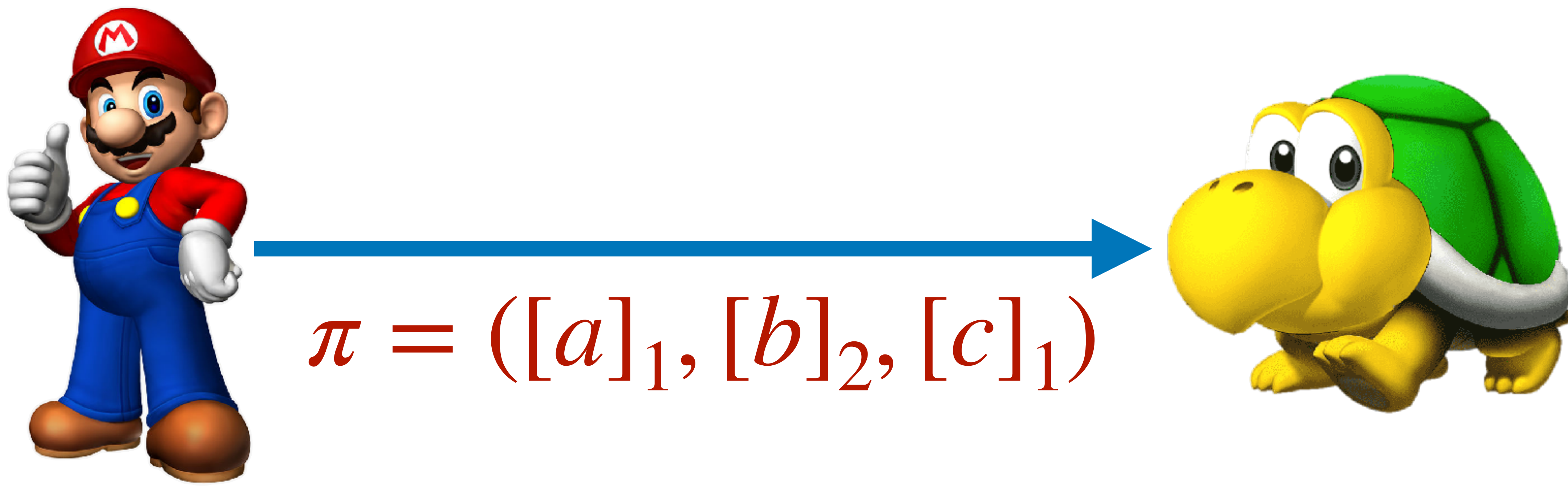

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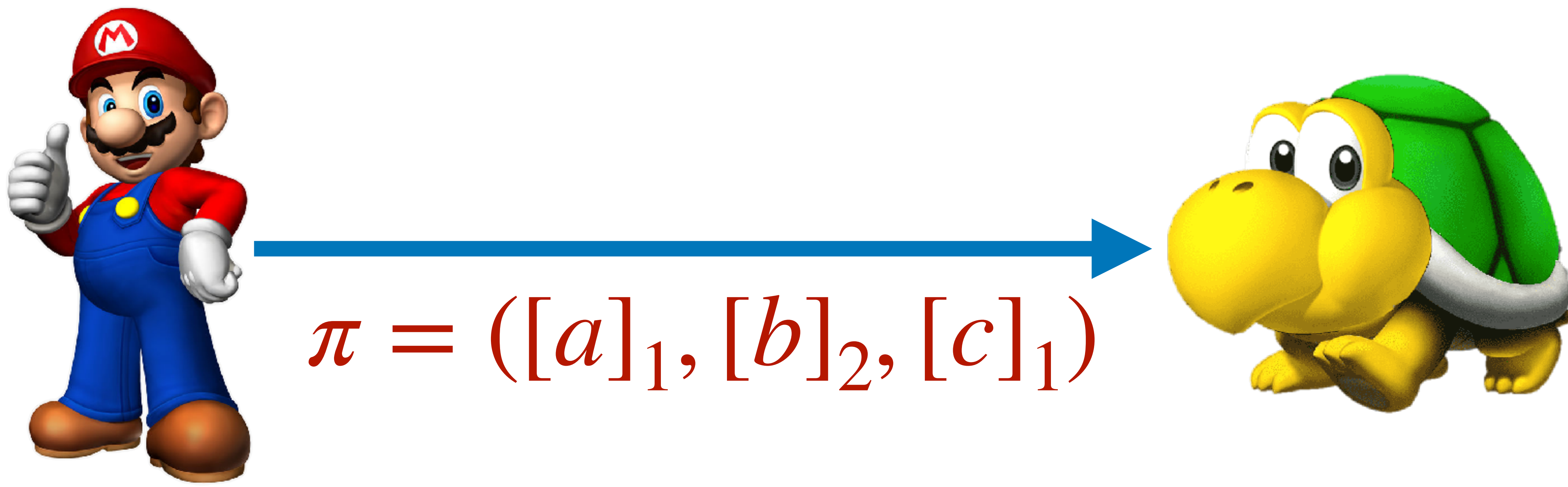

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- The lower bound works in the **standard model** (no RO)

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  - **SAP** (Square Arithmetic Programming) instead of **R1CS**
  - However, then one relies on **symmetric** pairings
  - Group elements will be considerably longer => **worse in practice**

- The lower bound works in the **standard model** (no RO)
- It talks about **#group elements**, not **bit-length**

Scenic Route to Polymath

For non-muggles

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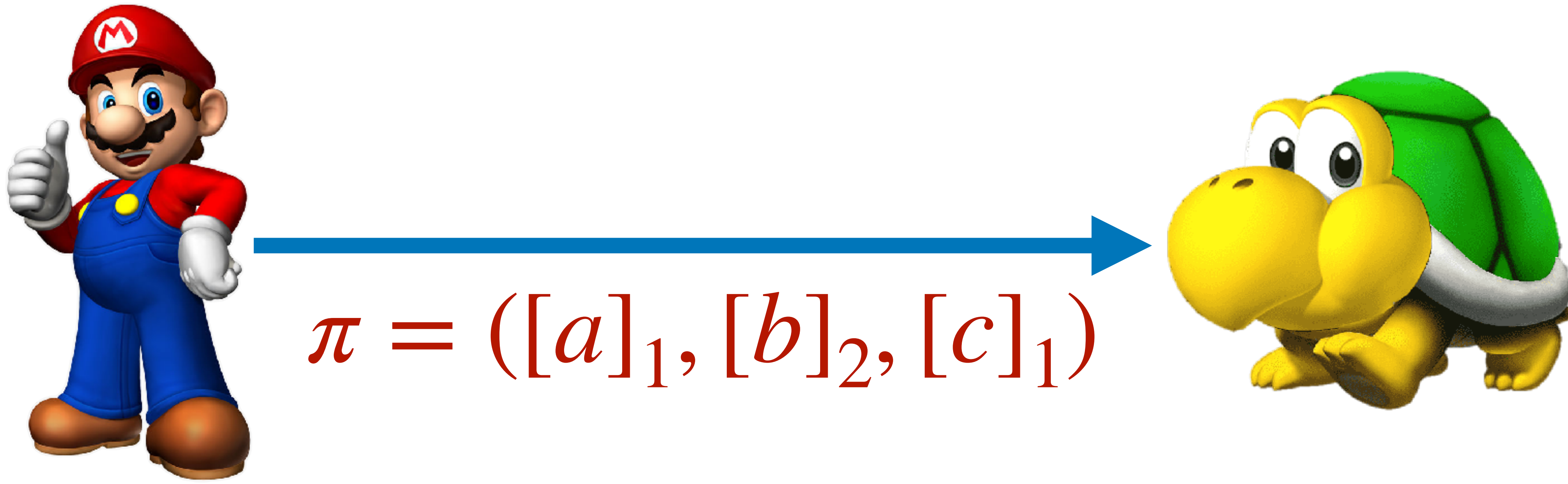

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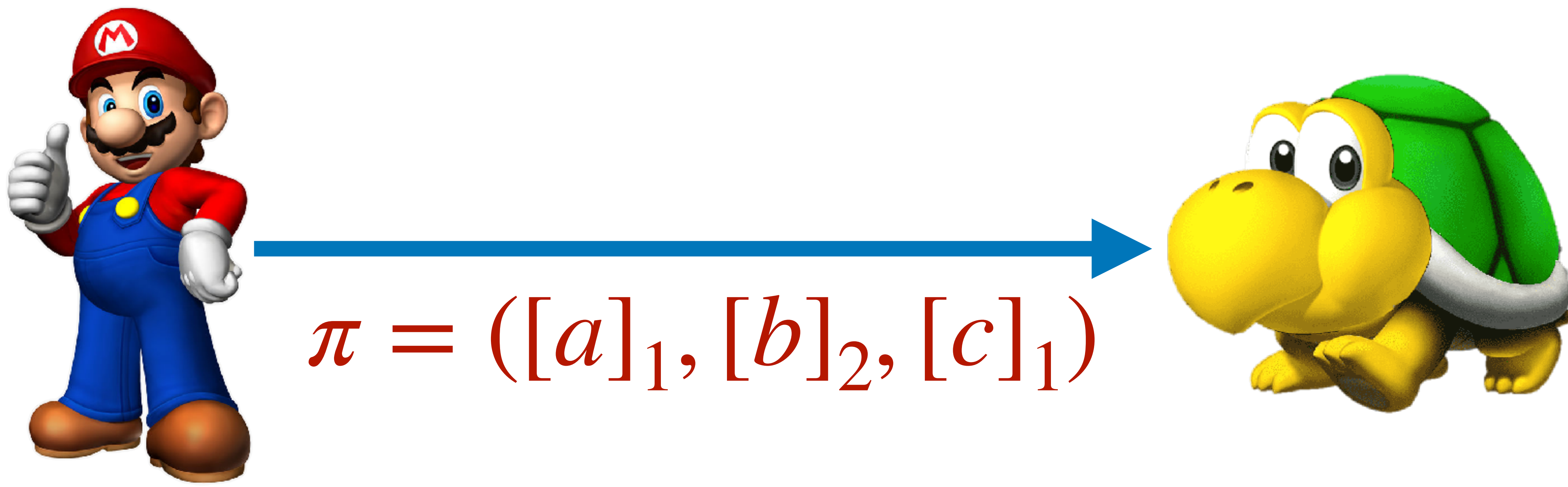

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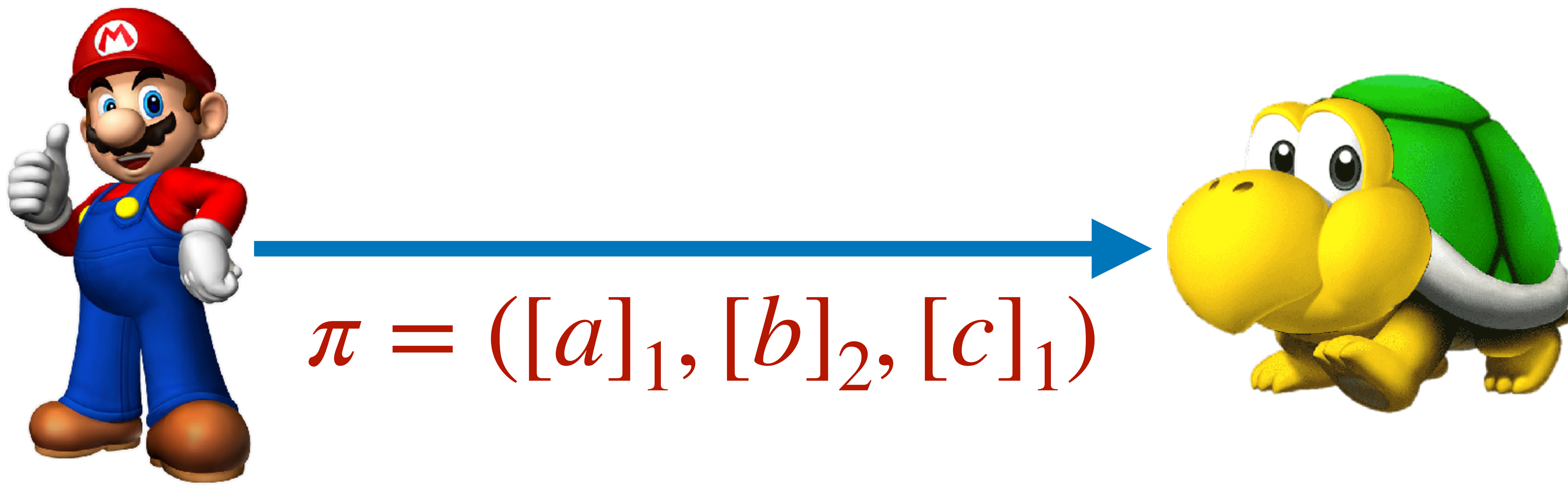

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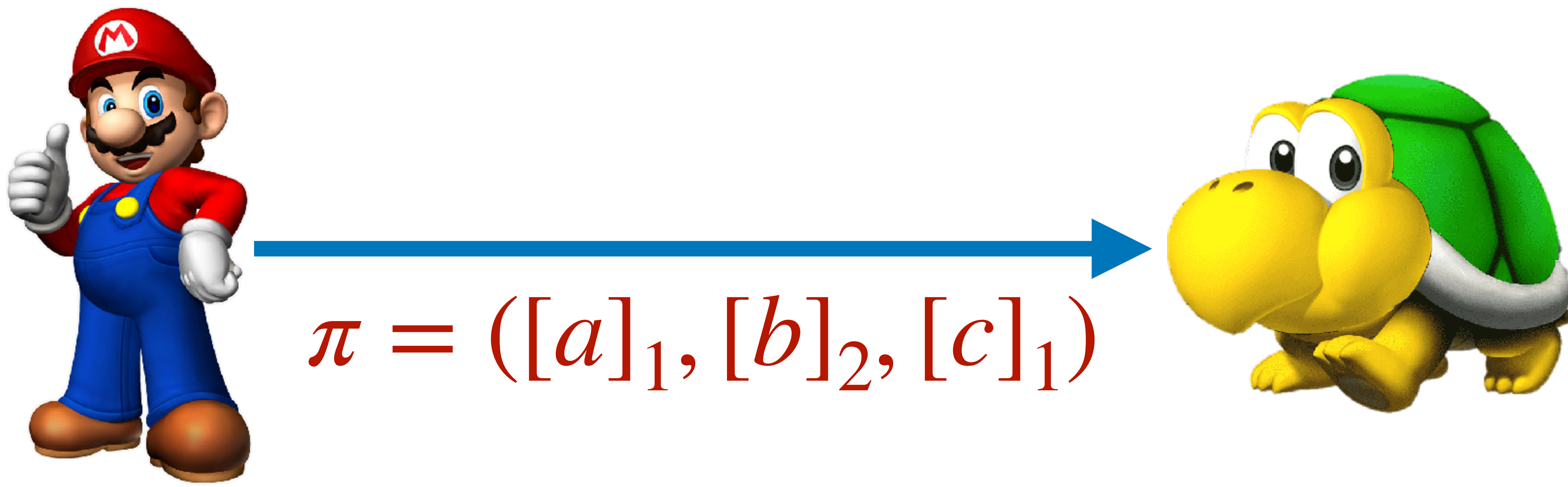

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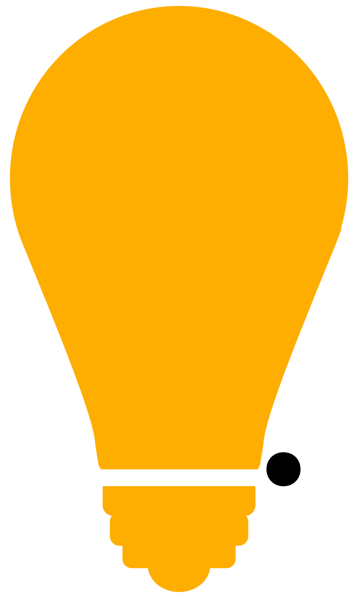

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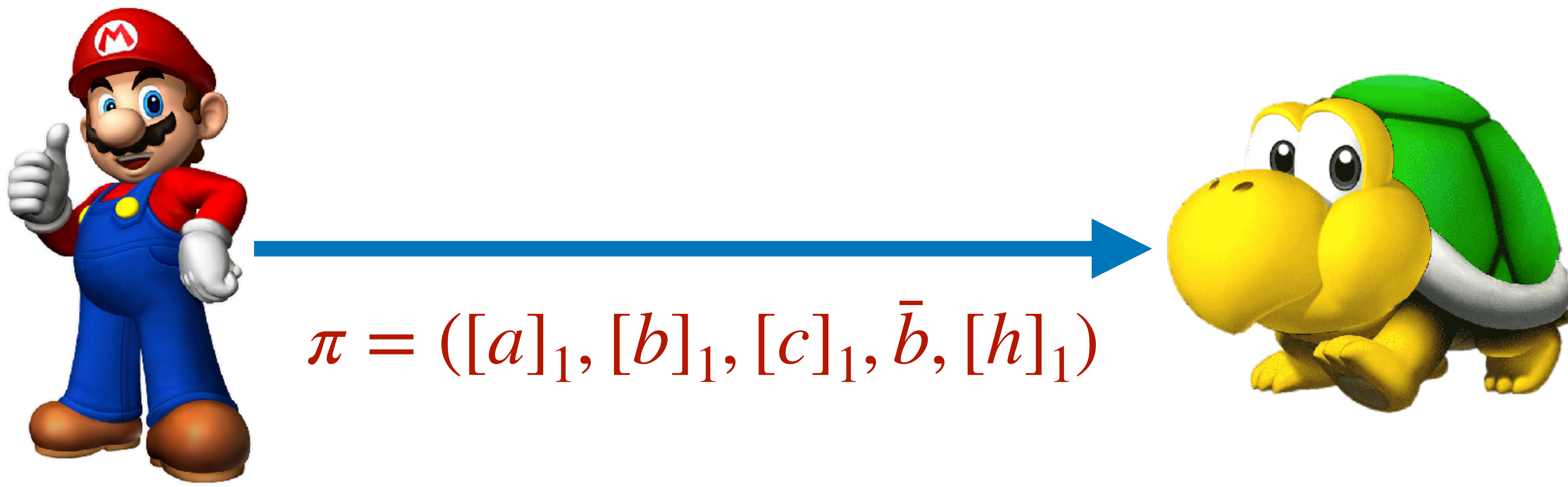
Problem:

- we still have $[b]_1$ in the argument!
- $\ell([b]_2) < \ell([b]_1) + \ell(\bar{b}) + \ell([h]_1)$ in 128-bit level

Scenic Route to Polymath



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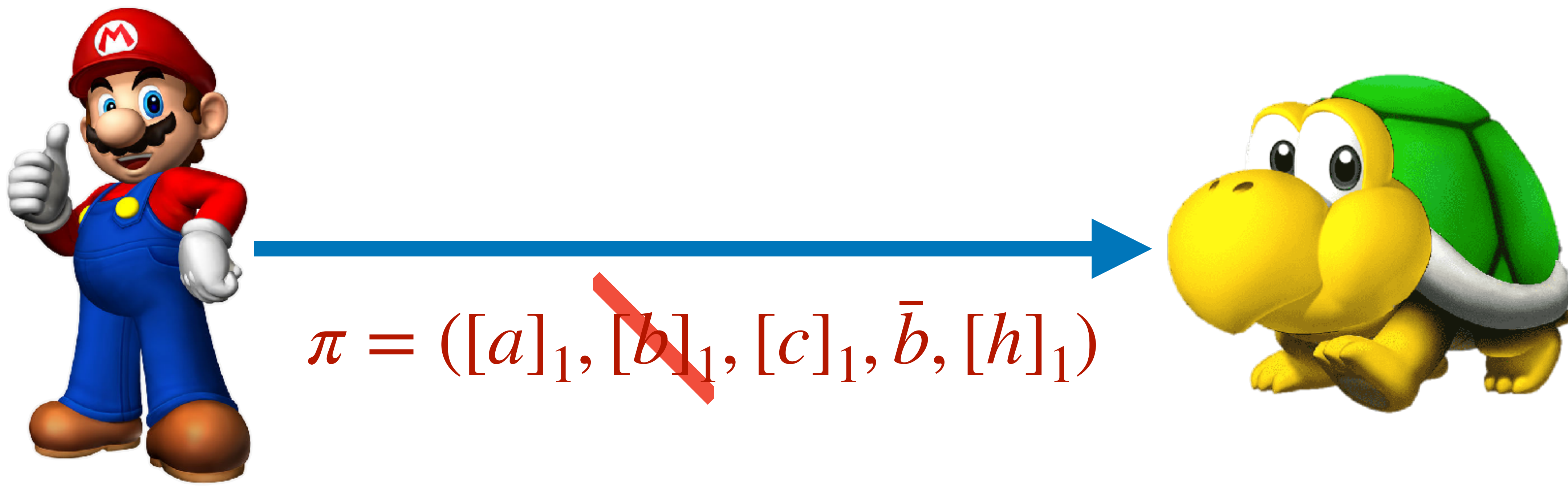

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Problem:
- Groth16 has five trapdoors, KZG is univariate
- Not clear how to use KZG

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Problem:

- even after exhaustive search, the exponents i are quite large
- KZG prover time Ω(polynomial degree)
  - => Results in high prover complexity

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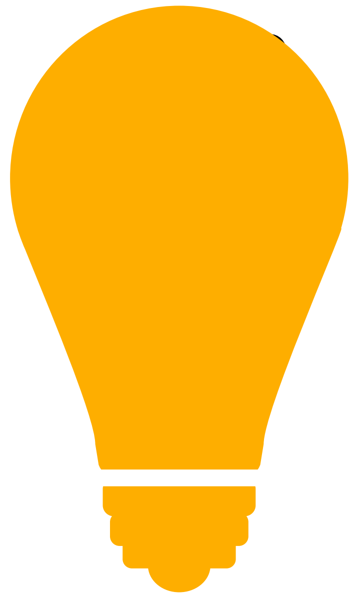

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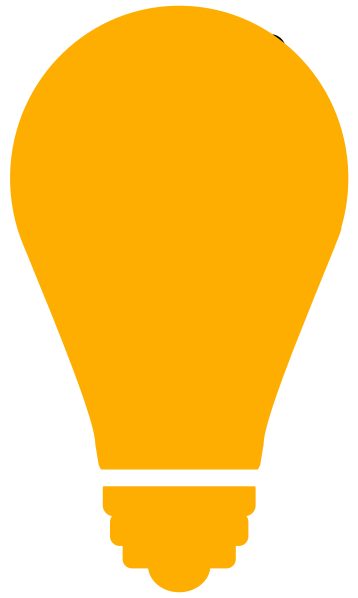

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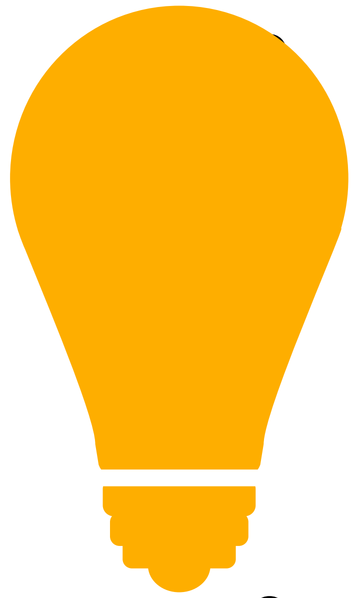

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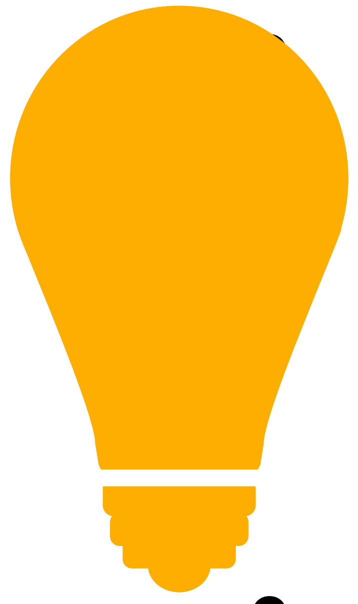

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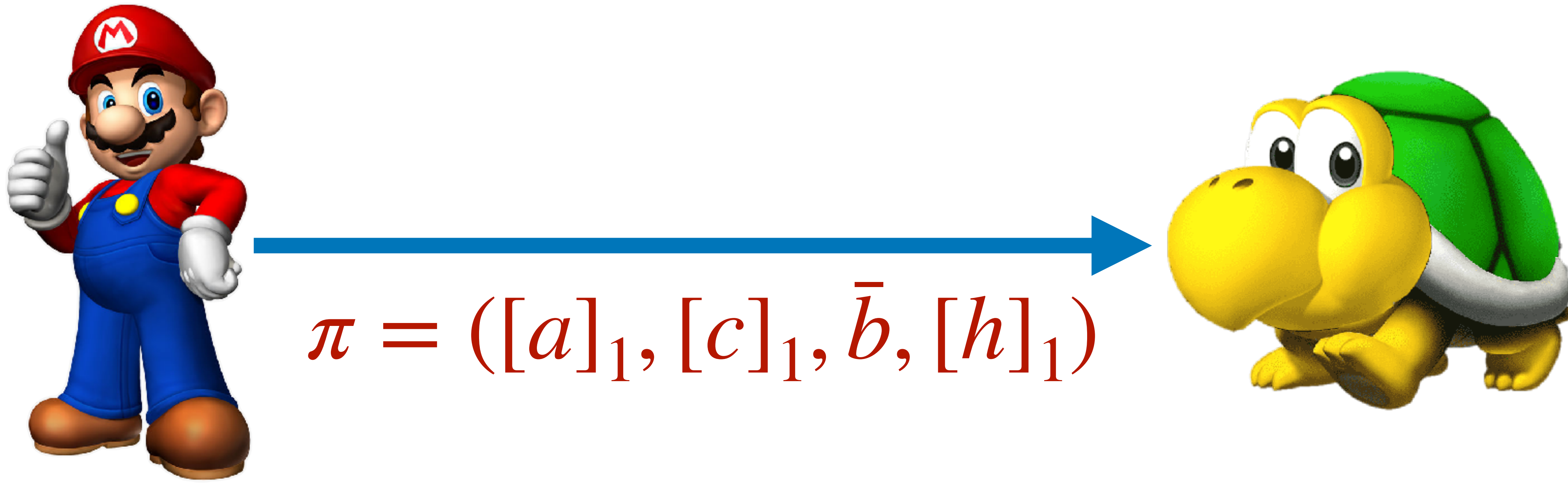

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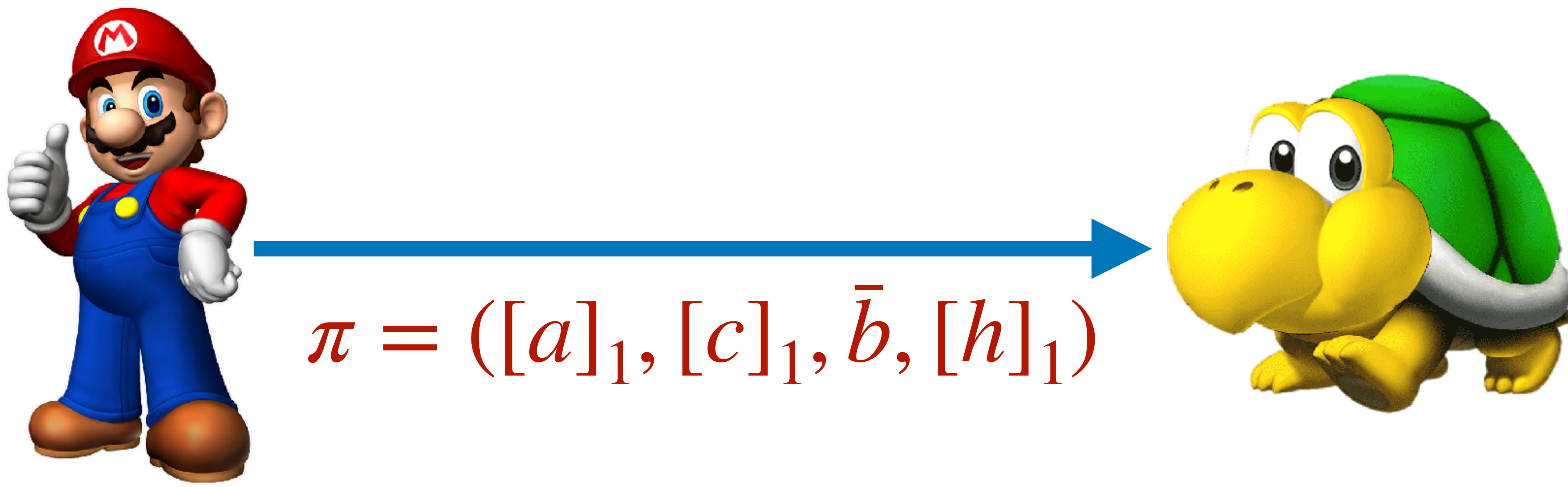

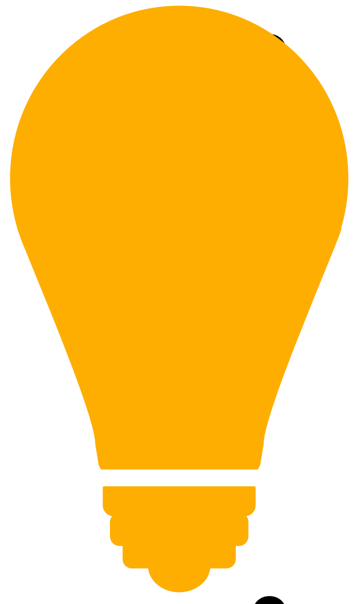

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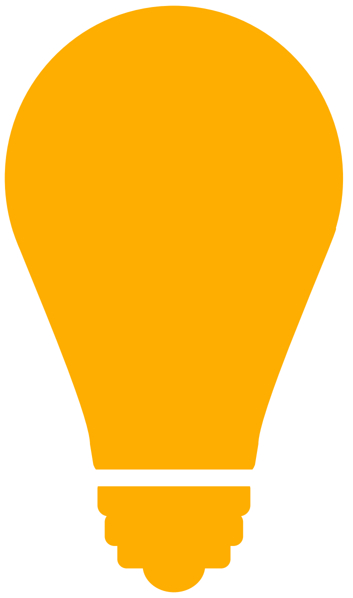

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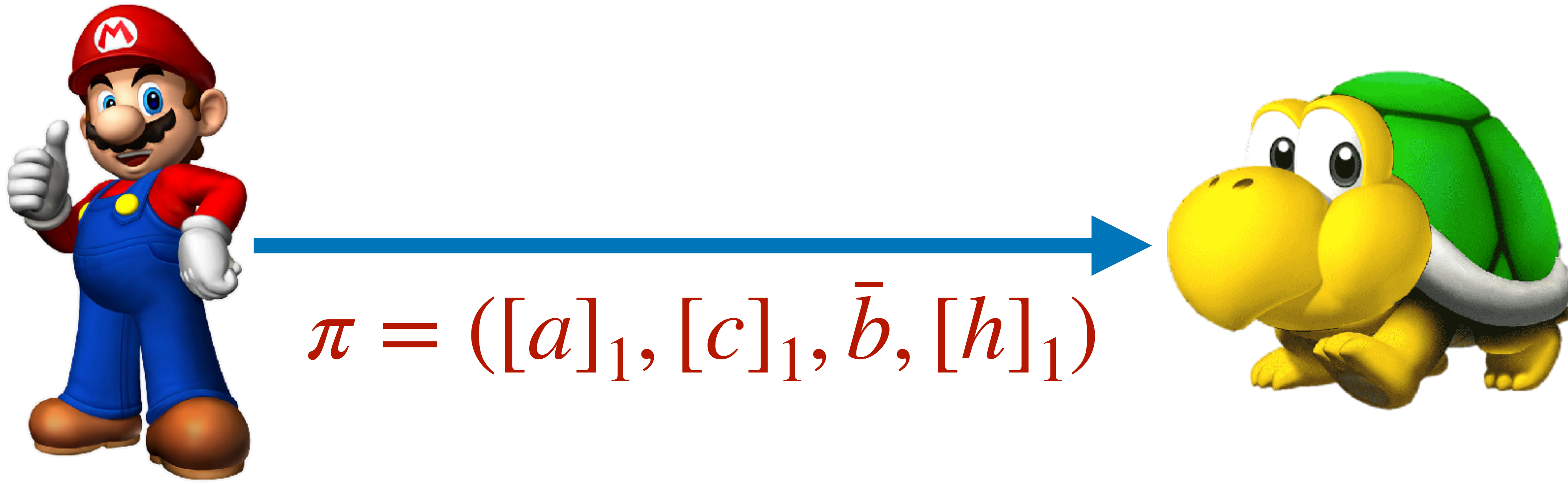

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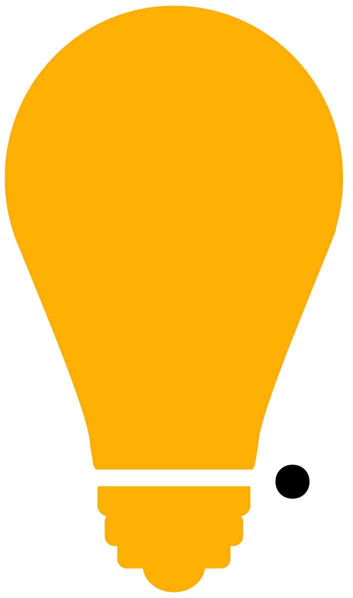

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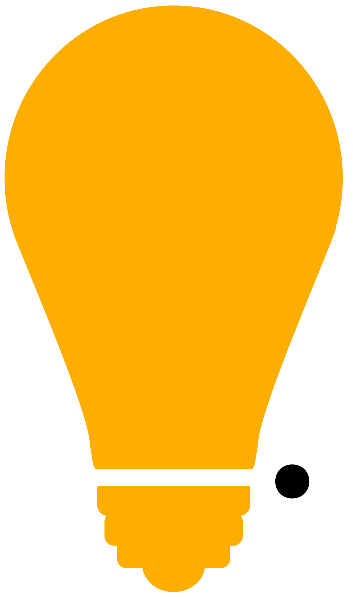

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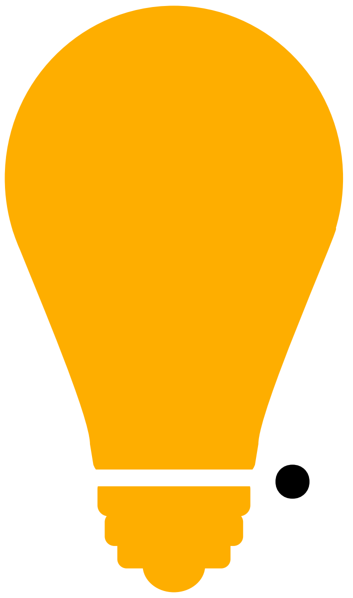

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Problem:

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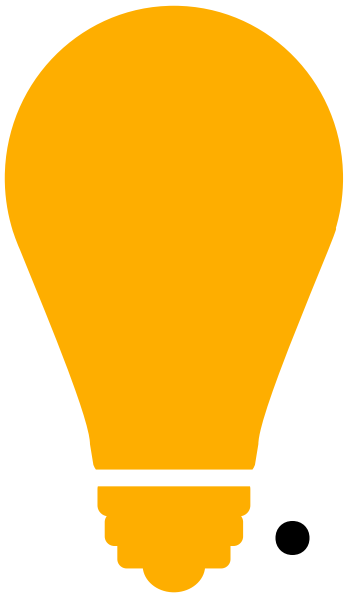

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