# Verifiable Computation for Approximate Homomorphic Encryption Schemes

Ignacio Cascudo, Anamaria Costache, **Daniele Cozzo**, Dario Fiore,
Antonio Guimarães, Eduardo Soria-Vazquez

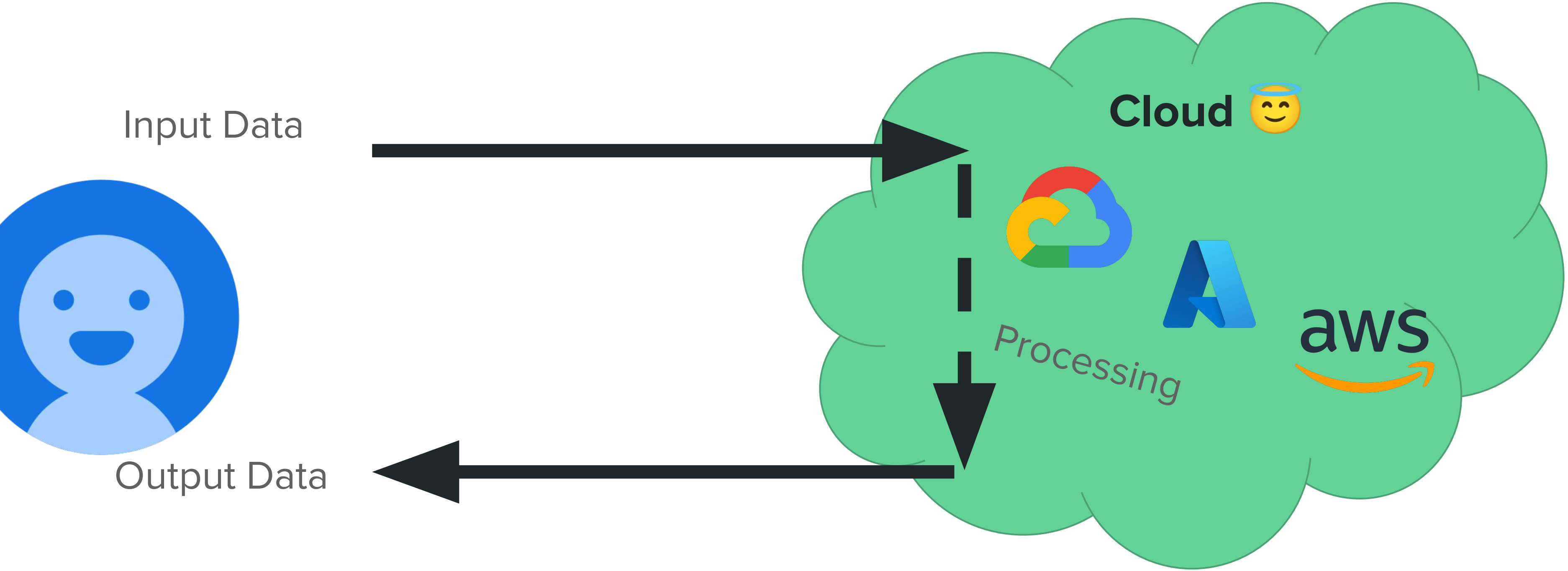
institute IMDEA software

NTNU
Norwegian University of Science and Technology

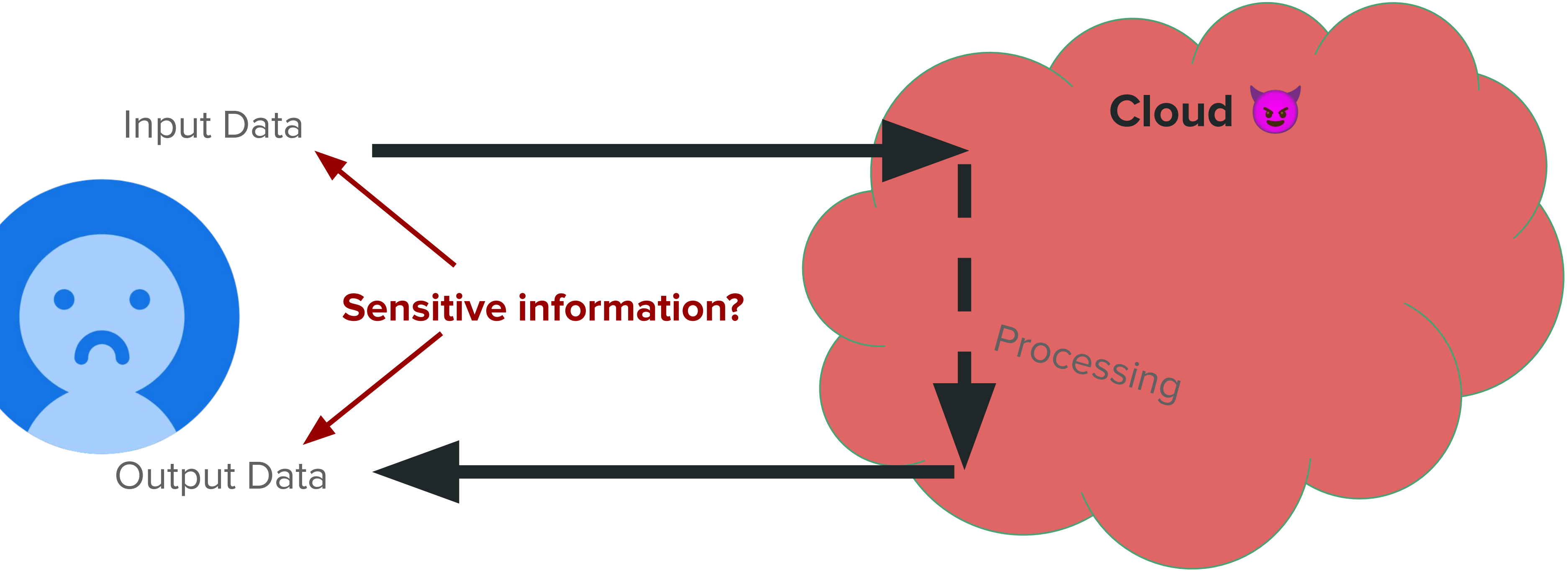
Technology Innovation Institute

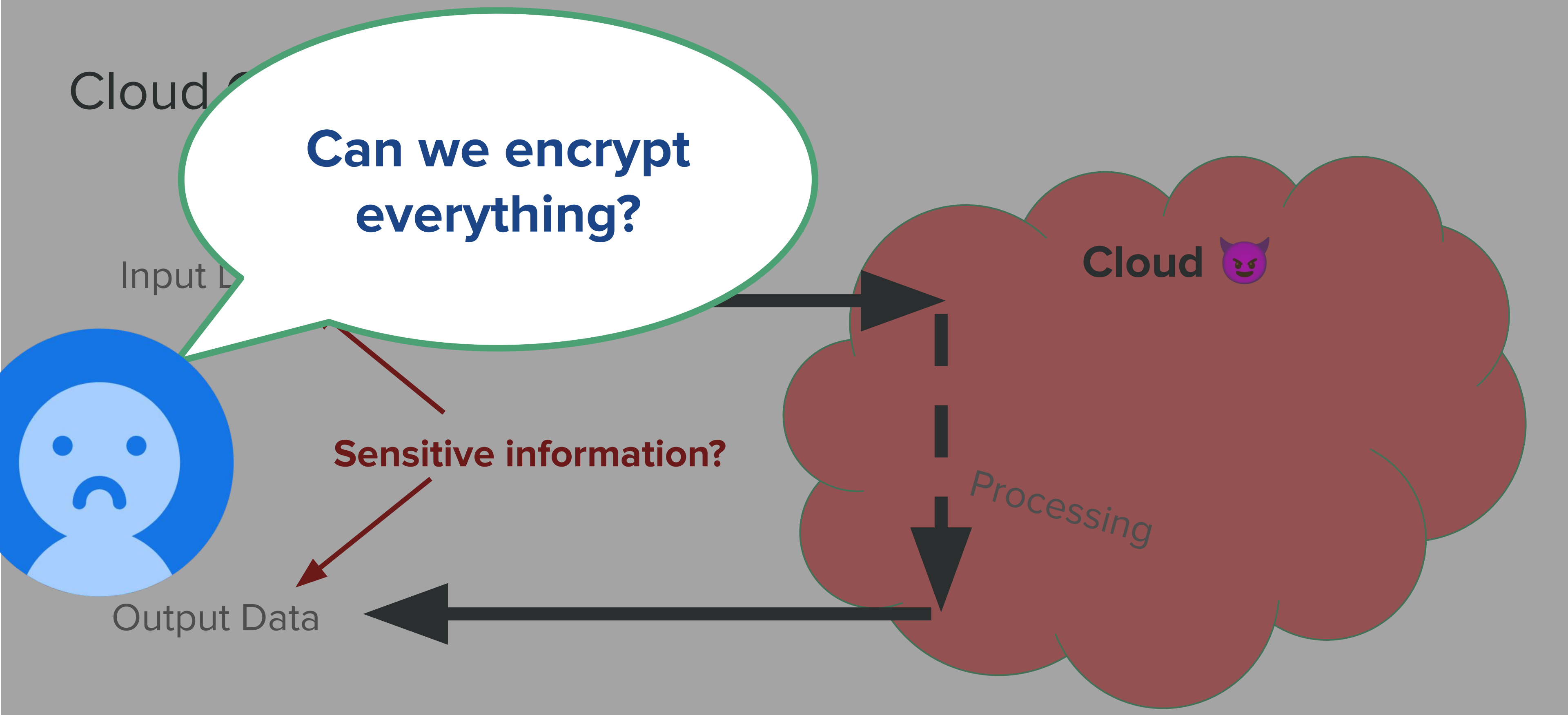
# Context

# Cloud Computing

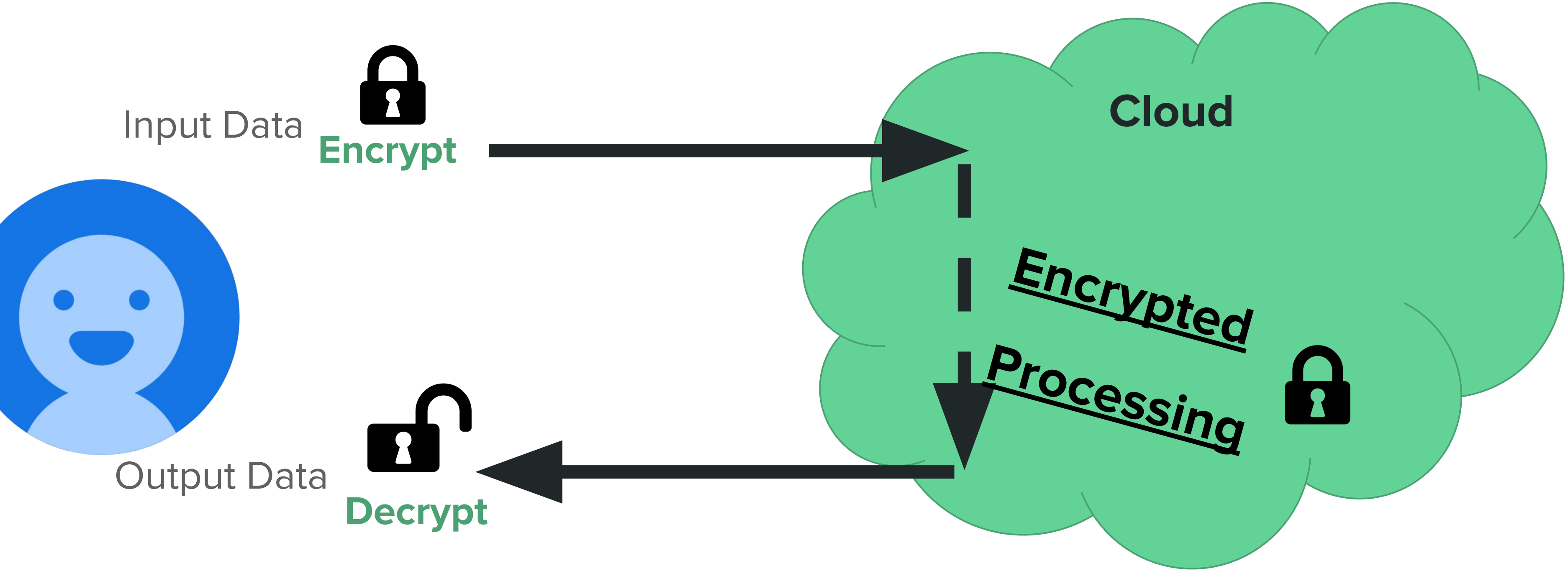


# Cloud Computing



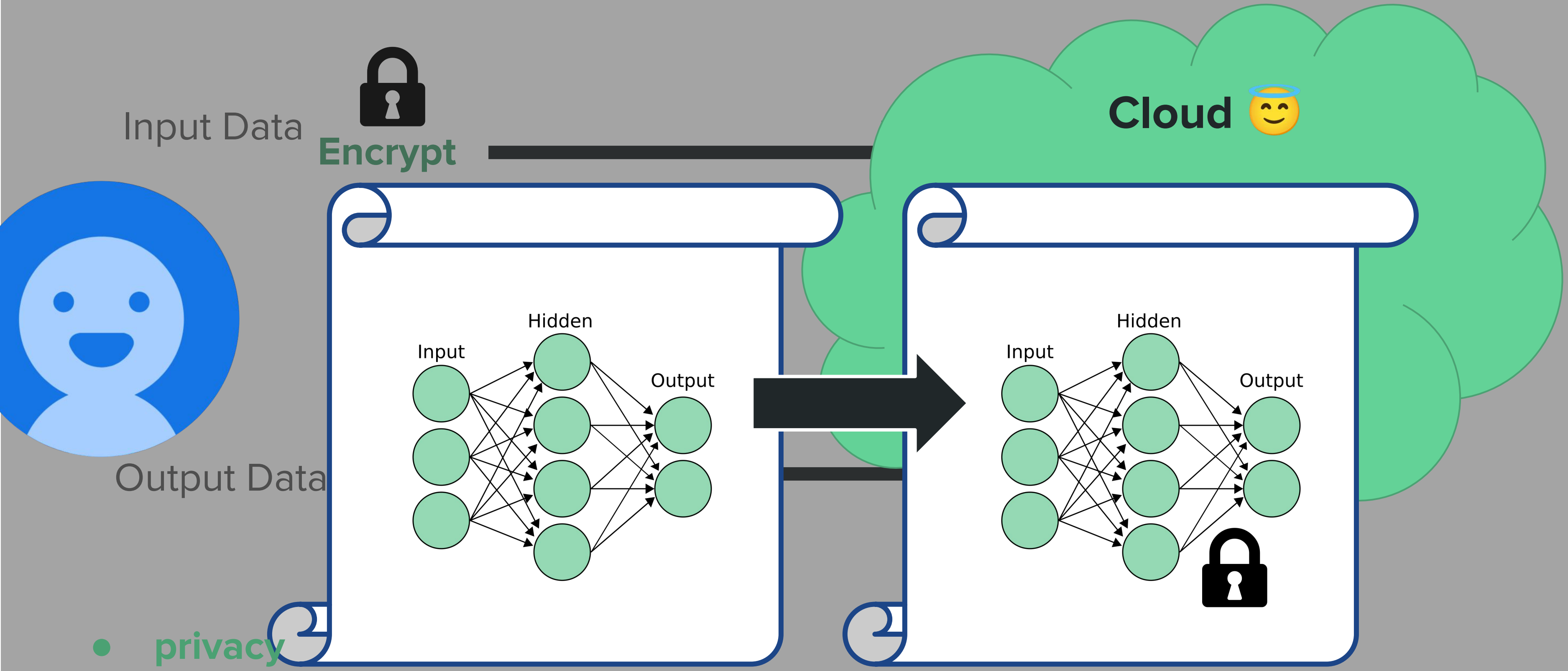

Cloud
Can we encrypt everything?
Input
Cloud 😈
Sensitive information?
Processing
Output Data

# Homomorphic Encryption (HE)




- privacy

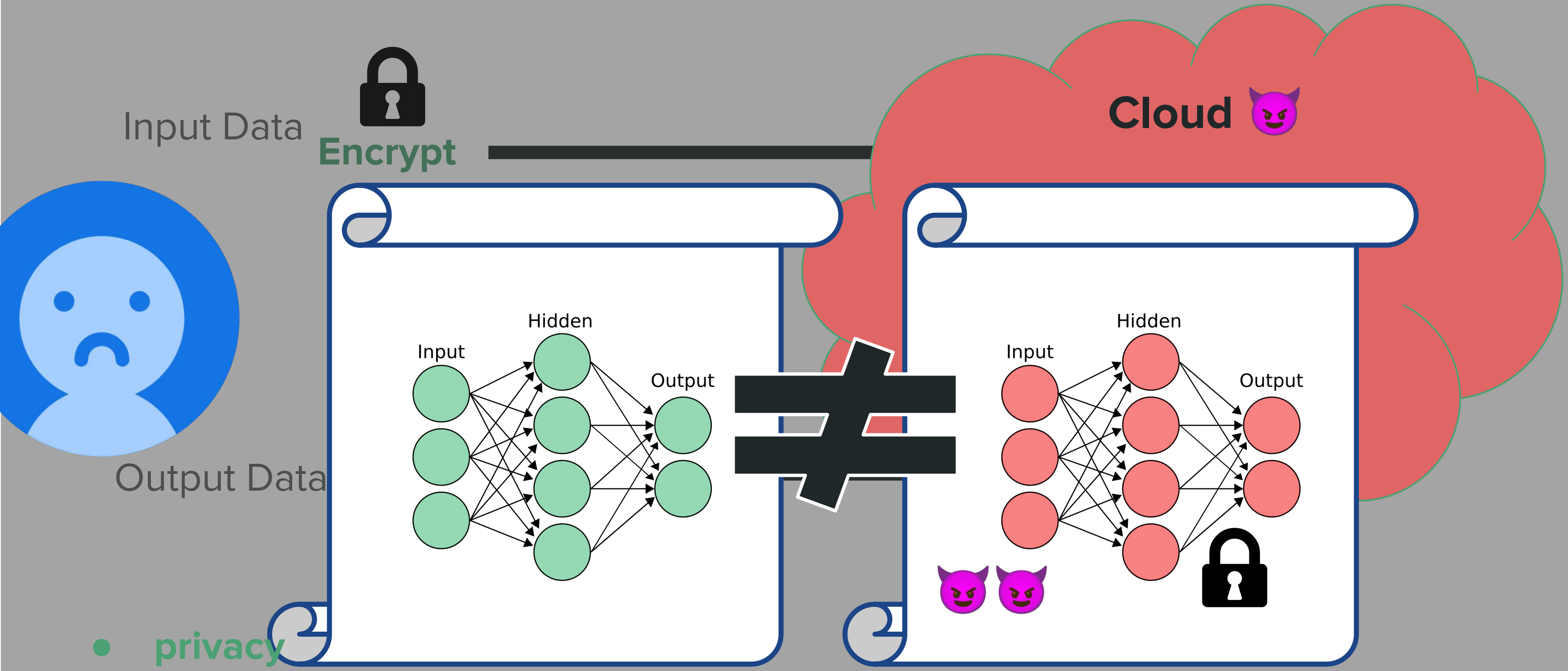
# Homomorphic Encryption (HE)




- privacy

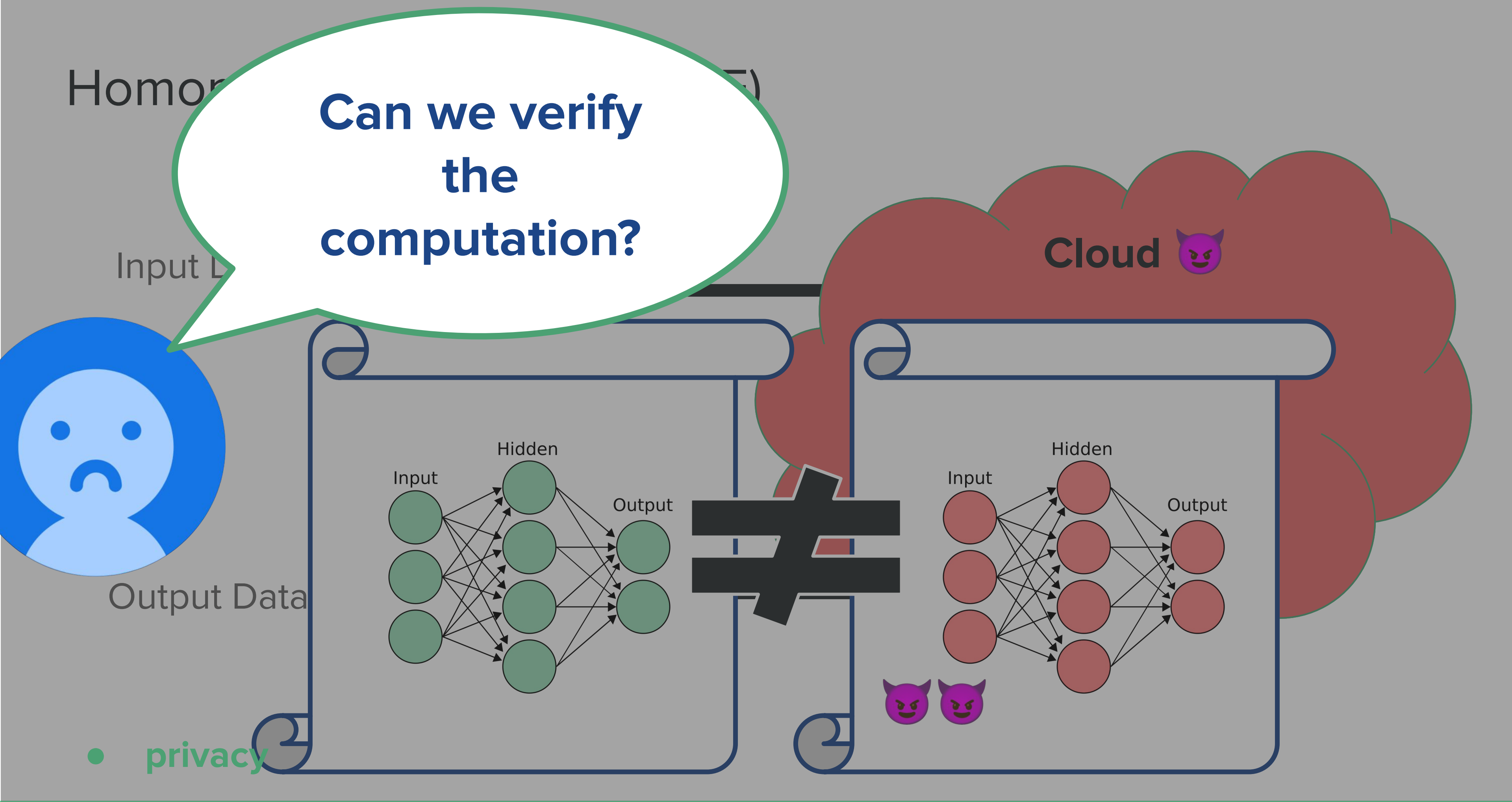
# Homomorphic Encryption (HE)

Input Data

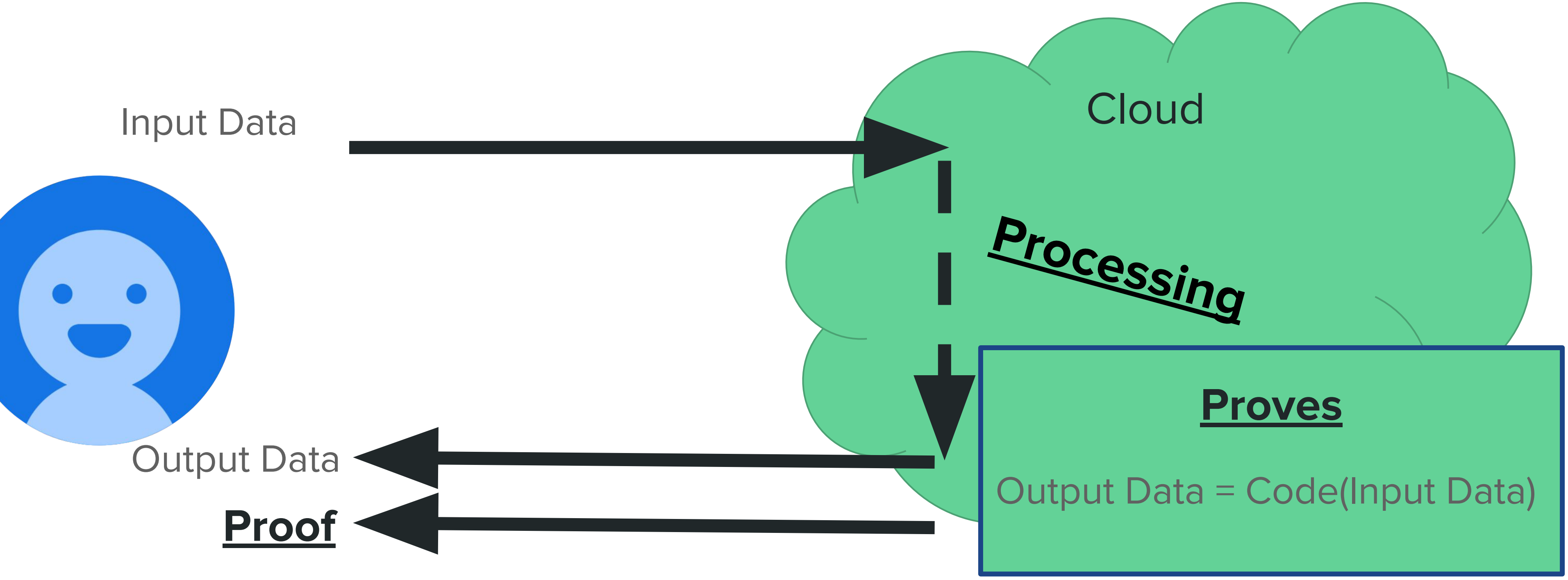



Output Data

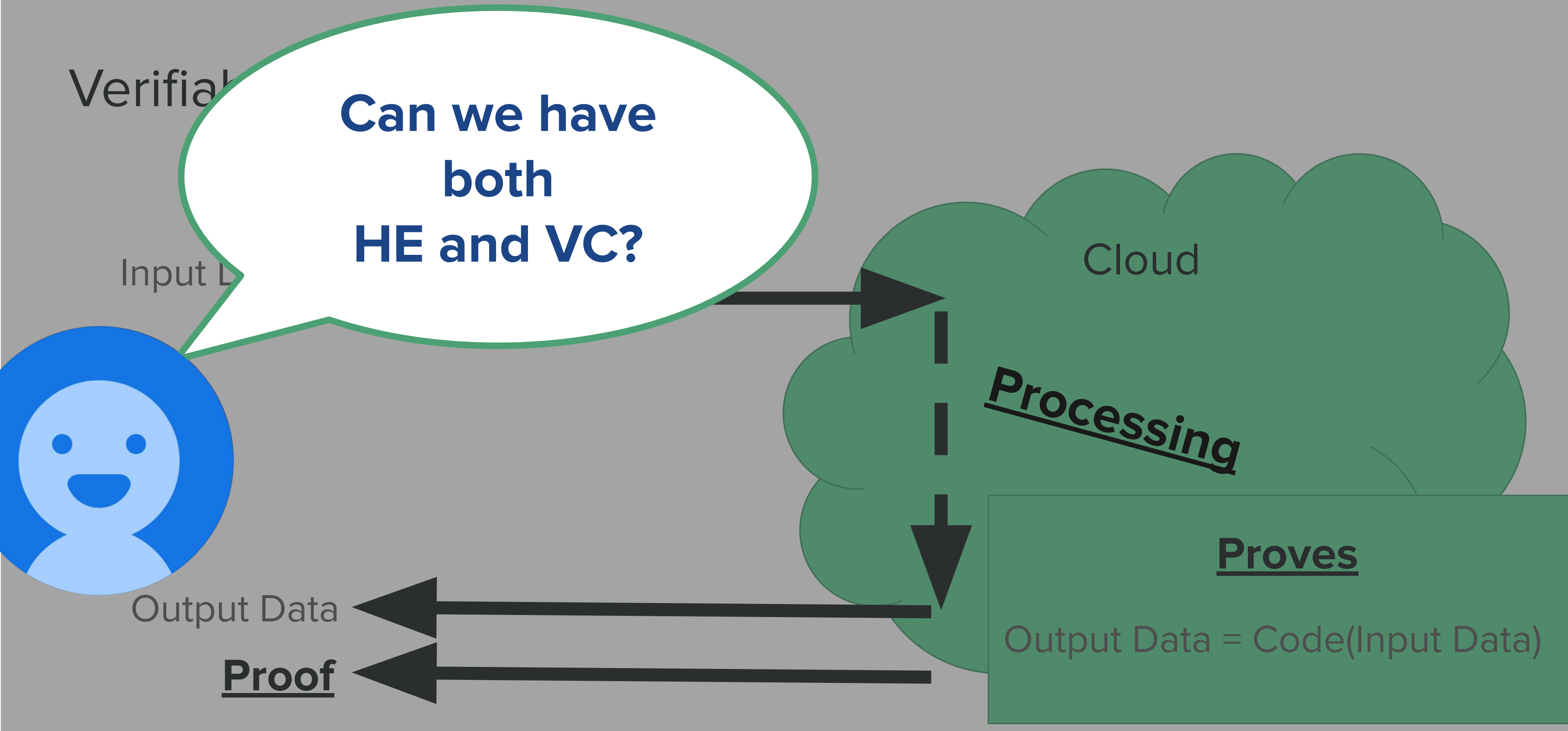
- privacy


Homor
E)
Can we verify the computation?
Input L
Cloud
Hidden
Input
Output
Hidden
Input
Output
Output Data
privacy

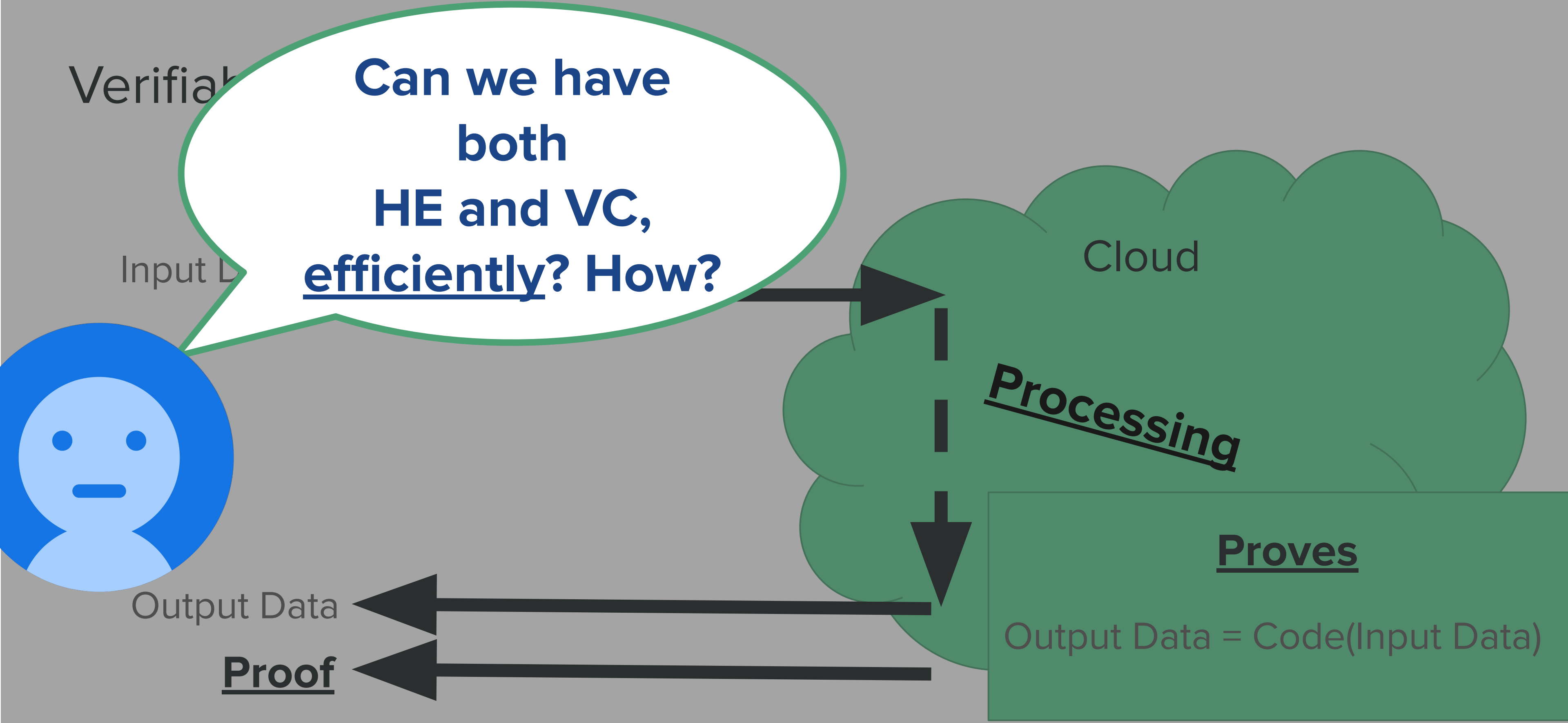
# Verifiable computation (VC)




- **Privacy**
- **Integrity**
- **Succinctness**




- **Privacy**
- **Integrity**
- **Succinctness**




- Privacy
- Integrity
- Succinctness

# VC-HE

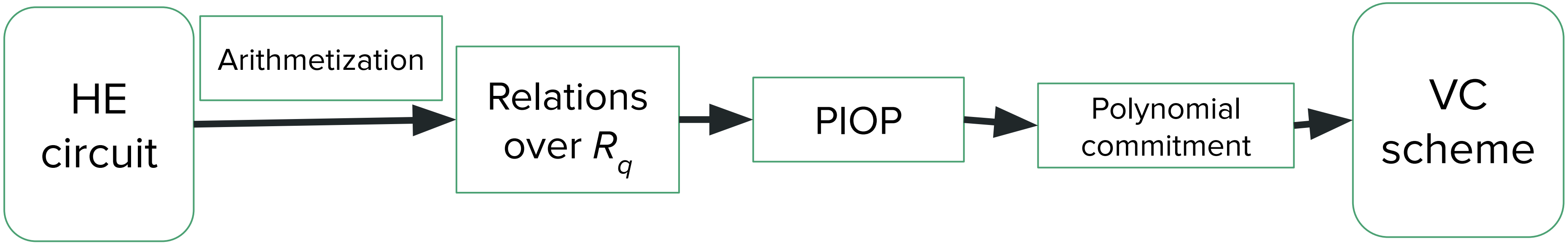
# VC-HE

| | Public Verification | Native $R_q$ Arithmetic | Efficient Key Switching / Rescale | Efficient Bootstrapping | CKKS (approximate schemes) |
|---|---|---|---|---|---|
| Generic SNARK[1] | ✓ | ❌ | ❌ | ❌ | ✓ |
| Rinocchio[2] | ❌ | ✓ | ❌ | ❌ | ✓ |
| HE-IOPs[3] | ❌ | ✓ | ✓ | ✓ | ❌ |
| **Our Work** | ✓ | ✓ | ✓ | ? | ✓ |


[1] A. Viand, C. Knabenhans, and A. Hithnawi, "*Verifiable Fully Homomorphic Encryption*" arXiv:2301.07041
[2] C. Ganesh, A. Nitulescu, and E. Soria-Vazquez, "*Rinocchio: SNARKs for Ring Arithmetic*" Journal of Cryptology, 2023
[3] D. F. Aranha, A. Costache, A. Guimarães, and E. Soria-Vazquez, "*HELIOPOLIS: Verifiable Computation over Homomorphically Encrypted Data from Interactive Oracle Proofs is Practical*" ASIACRYPT 2024

# Our contributions

- VC-HE for **CKKS**
- **Modular** solution




- **Proof-friendly CKKS**
- **AC satisfiability**
- **Range check**
- **Custom GKR**
- **Range proofs for polynomial rings**
- **Improved Brakedown**

# Proof-friendly CKKS

The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$$q \approx 2^{300} \qquad N \approx 2^{14}$$

The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$$R_q$$

# The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$$R_q$$

- Efficient HE computations
    - RNS

# The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$$q = \prod_{i=1}^{L} p_i$$

$$R_q \cong \begin{array}{|c|} \hline R_{p_1} \\ \hline R_{p_2} \\ \hline R_{p_3} \\ \hline \end{array}$$

- Efficient HE computations
  - RNS

# The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$$R_q \overset{q = \prod_{i=1}^{L} p_i}{\cong} \begin{array}{|c|}\hline R_{p_1} \\ \hline R_{p_2} \\ \hline R_{p_3} \\ \hline \end{array}$$

- Efficient HE computations
  - RNS
- Soundness
  - Large exceptional set

# The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N+1)$

$$R_q \underset{q=\prod_{i=1}^{L} p_i}{\cong} \begin{array}{c} R_{p_1} \\ R_{p_2} \\ R_{p_3} \end{array}$$

$$R_{p_1} \underset{X^N+1=\prod_{i=1}^{k}(X^d-\zeta^{2i-1}) \mod p_1}{\cong} R_{11} \quad R_{12} \quad R_{13} \quad R_{14}$$

$$R_{p_2} \underset{X^N+1=\prod_{i=1}^{k}(X^d-\zeta^{2i-1}) \mod p_2}{\cong} R_{21} \quad R_{22} \quad R_{23} \quad R_{24}$$

$$R_{p_3} \underset{X^N+1=\prod_{i=1}^{k}(X^d-\zeta^{2i-1}) \mod p_3}{\cong} R_{31} \quad R_{32} \quad R_{33} \quad R_{34}$$

- Efficient HE computations
  - RNS
- Soundness
  - Large exceptional set

# The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$$R_q \cong \quad q = \prod_{i=1}^{L} p_i$$

$$R_{p_1} \quad X^N + 1 = \prod_{i=1}^{k}(X^d - \zeta^{2i-1}) \mod p_1 \quad \cong \quad \mathbb{F}_{p_1^d} \; \mathbb{F}_{p_1^d} \; \mathbb{F}_{p_1^d} \; \mathbb{F}_{p_1^d}$$

$$R_{p_2} \quad X^N + 1 = \prod_{i=1}^{k}(X^d - \zeta^{2i-1}) \mod p_2 \quad \cong \quad \mathbb{F}_{p_2^d} \; \mathbb{F}_{p_2^d} \; \mathbb{F}_{p_2^d} \; \mathbb{F}_{p_2^d}$$

$$R_{p_3} \quad X^N + 1 = \prod_{i=1}^{k}(X^d - \zeta^{2i-1}) \mod p_3 \quad \cong \quad \mathbb{F}_{p_3^d} \; \mathbb{F}_{p_3^d} \; \mathbb{F}_{p_3^d} \; \mathbb{F}_{p_3^d}$$

- Efficient HE computations
  - RNS
- Soundness
  - Large exceptional set

# The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$R_q$

$q = \prod_{i=1}^{L} p_i$

$\cong$

$R_{p_1}$

$R_{p_2}$

$R_{p_3}$

$X^N + 1 = \prod_{i=1}^{k} (X^d - \zeta^{2i-1}) \mod p_1$

$\cong$

$\mathbb{F}_{p_1^d}$ $\mathbb{F}_{p_1^d}$ $\mathbb{F}_{p_1^d}$ $\mathbb{F}_{p_1^d}$

$X^N$

$\mathbb{F}_{p_2^d}$

$X^N$

$\mathbb{F}_{p_3^d}$

**Size of our exceptional set!**

- Efficient HE computations
  - RNS
- Soundness
  - Large exceptional set

# The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$$R_q \; \underset{\cong}{q = \prod_{i=1}^{L} p_i} \; \begin{array}{c} R_{p_1} \\ R_{p_2} \\ R_{p_3} \end{array}$$

$X^N + 1 = \prod_{i=1}^{k} (X^d - \zeta^{2i-1}) \mod p_1$

$\cong$

$\mathbb{F}_{p_1^d}$ $\mathbb{F}_{p_1^d}$ $\mathbb{F}_{p_1^d}$ $\mathbb{F}_{p_1^d}$

$X^N$ ... $\mathbb{F}_{p_2^d}$

$X$ ... $\mathbb{F}_{p_3^d}$

**Size of our exceptional set!**

To get **128** bits of security:
$|p_i| = 32$, $d = 4$
or
$|p_i| = 64$, $d = 2$

- Efficient HE computations
  - RNS
- Soundness
  - Large exceptional set

# The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$$R_q \overset{q = \prod_{i=1}^{L} p_i}{\cong} R_{p_1} \times R_{p_2} \times R_{p_3}$$

$$X^N + 1 = \prod_{i=1}^{k} (X^d - \zeta^{2i-1}) \mod p_1$$

$$\cong \quad \mathbb{F}_{p_1^d} \quad \mathbb{F}_{p_1^d} \quad \mathbb{F}_{p_1^d} \quad \mathbb{F}_{p_1^d}$$

$X^N$ … $\mathbb{F}_{p_2^d}$

$X^N$ … $\mathbb{F}_{p_3^d}$

**Size of our exceptional set!**

To get **128** bits of security:
$|p_i| = 32$, $d = 4$
or
$|p_i| = 64$, $d = 2$

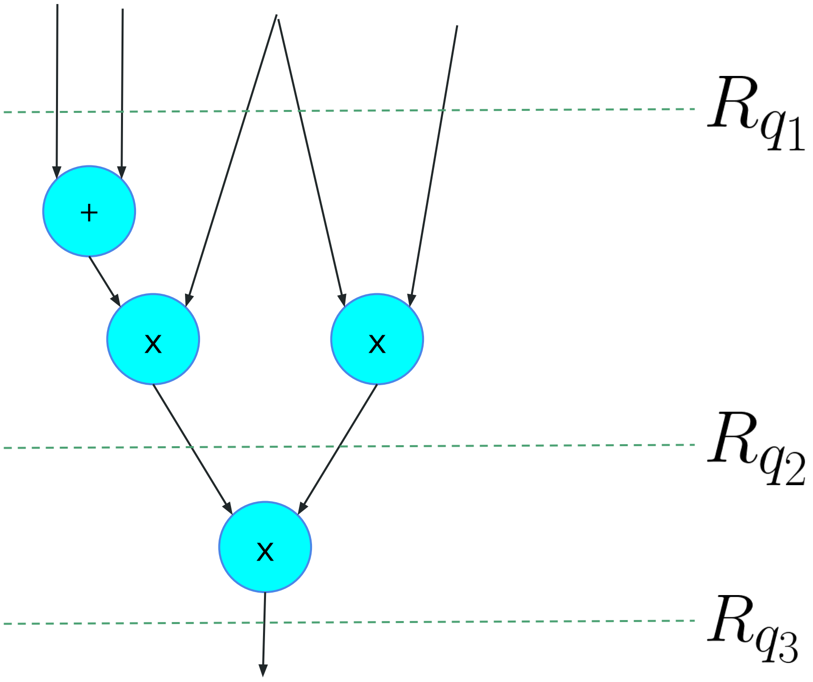
**Optimal performance/security**:
$|p_i| = 49$, $d = 4$

- Efficient HE computations
  - RNS
- Soundness
  - Large exceptional set

# CKKS

$$\begin{cases} q_1 = q \\ q_i = q_{i-1}/p_i \end{cases}$$

$$q = q_1 > q_2 > \cdots > q_L$$



# CKKS

- An approximate scheme:


- RLWE ciphertext:

$$(a, b) \in R_q^2$$

- RNS representation with 3 components:

| | | | | | | |
|---|---|---|---|---|---|---|
| $a_{01}$ | $a_{02}$ | $a_{03}$ | | $b_{01}$ | $b_{02}$ | $b_{03}$ |

# CKKS

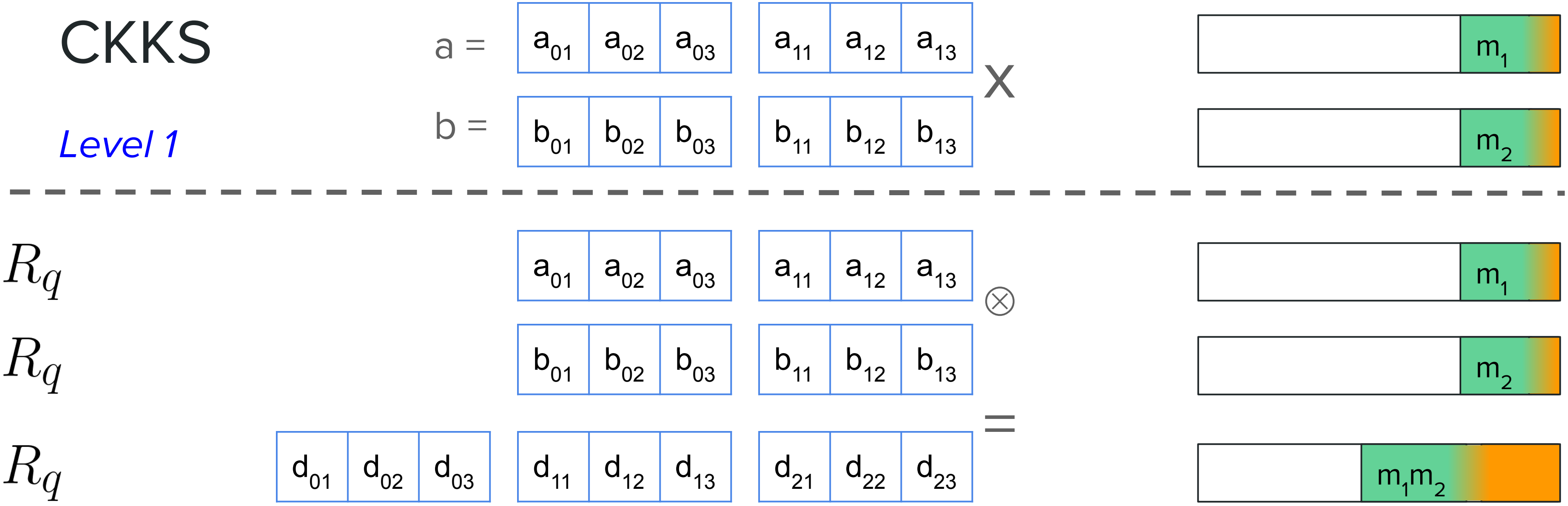
*Level 1*

$$a = \begin{array}{|c|c|c|}\hline a_{01} & a_{02} & a_{03} \\ \hline\end{array} \quad \begin{array}{|c|c|c|}\hline a_{11} & a_{12} & a_{13} \\ \hline\end{array}$$

$\times$

$$b = \begin{array}{|c|c|c|}\hline b_{01} & b_{02} & b_{03} \\ \hline\end{array} \quad \begin{array}{|c|c|c|}\hline b_{11} & b_{12} & b_{13} \\ \hline\end{array}$$

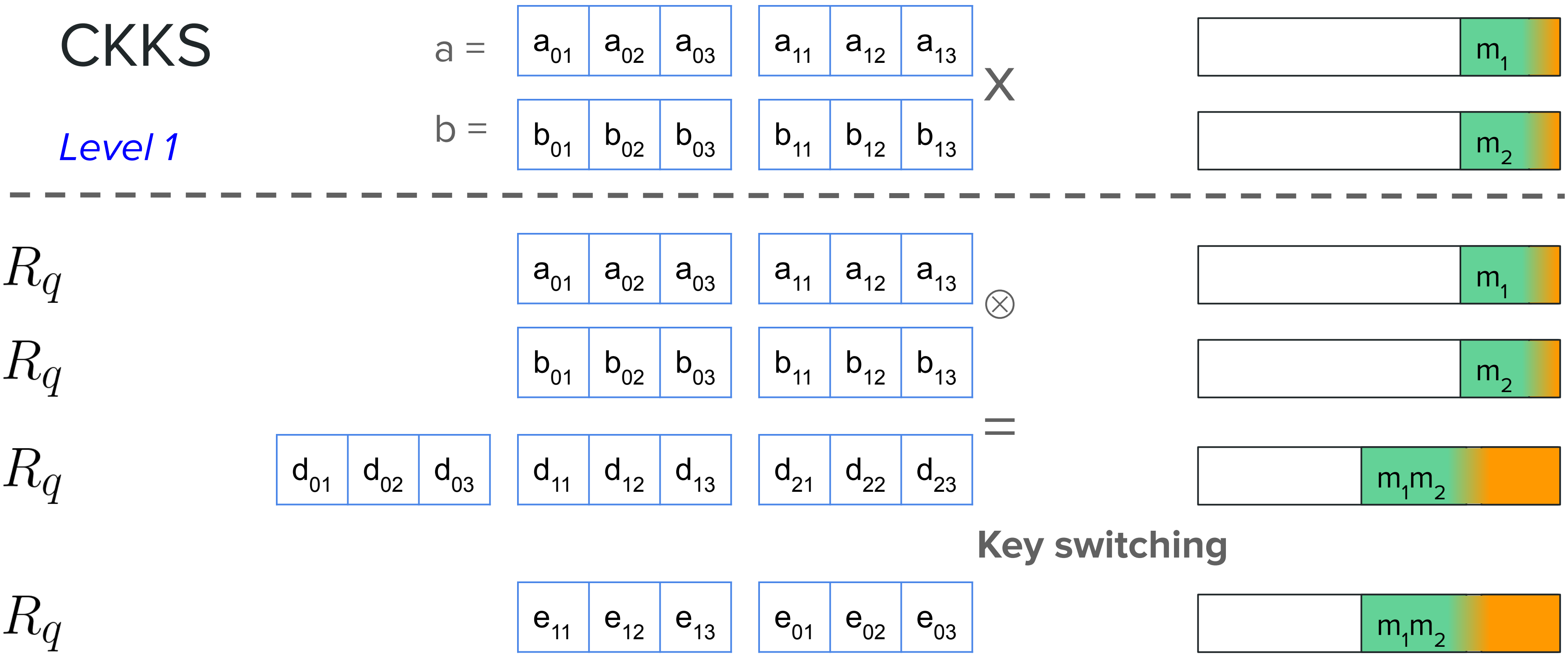
# CKKS



Level 1
a =
$a_{01}$ $a_{02}$ $a_{03}$ $a_{11}$ $a_{12}$ $a_{13}$
X
b =
$b_{01}$ $b_{02}$ $b_{03}$ $b_{11}$ $b_{12}$ $b_{13}$
$m_1$
$m_2$
$R_q$
$a_{01}$ $a_{02}$ $a_{03}$ $a_{11}$ $a_{12}$ $a_{13}$
$\otimes$
$R_q$
$b_{01}$ $b_{02}$ $b_{03}$ $b_{11}$ $b_{12}$ $b_{13}$
=
$R_q$
$d_{01}$ $d_{02}$ $d_{03}$ $d_{11}$ $d_{12}$ $d_{13}$ $d_{21}$ $d_{22}$ $d_{23}$
$m_1$
$m_2$
$m_1m_2$

# CKKS

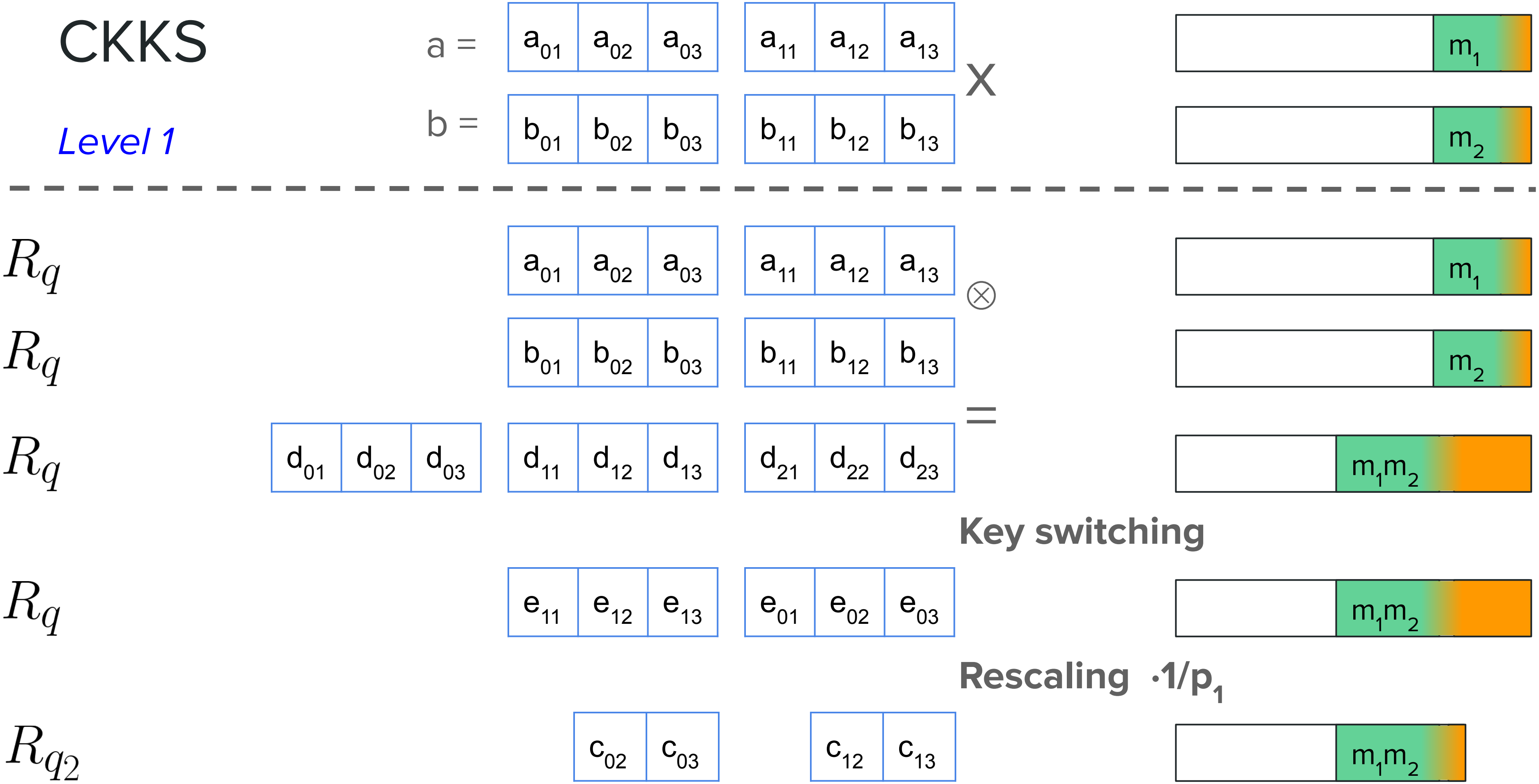
*Level 1*



a =
$a_{01}$ $a_{02}$ $a_{03}$ $a_{11}$ $a_{12}$ $a_{13}$
X
b =
$b_{01}$ $b_{02}$ $b_{03}$ $b_{11}$ $b_{12}$ $b_{13}$
$m_1$
$m_2$
$R_q$
$a_{01}$ $a_{02}$ $a_{03}$ $a_{11}$ $a_{12}$ $a_{13}$
$\otimes$
$m_1$
$R_q$
$b_{01}$ $b_{02}$ $b_{03}$ $b_{11}$ $b_{12}$ $b_{13}$
$m_2$
=
$R_q$
$d_{01}$ $d_{02}$ $d_{03}$ $d_{11}$ $d_{12}$ $d_{13}$ $d_{21}$ $d_{22}$ $d_{23}$
$m_1m_2$
Key switching
$R_q$
$e_{11}$ $e_{12}$ $e_{13}$ $e_{01}$ $e_{02}$ $e_{03}$
$m_1m_2$

# CKKS

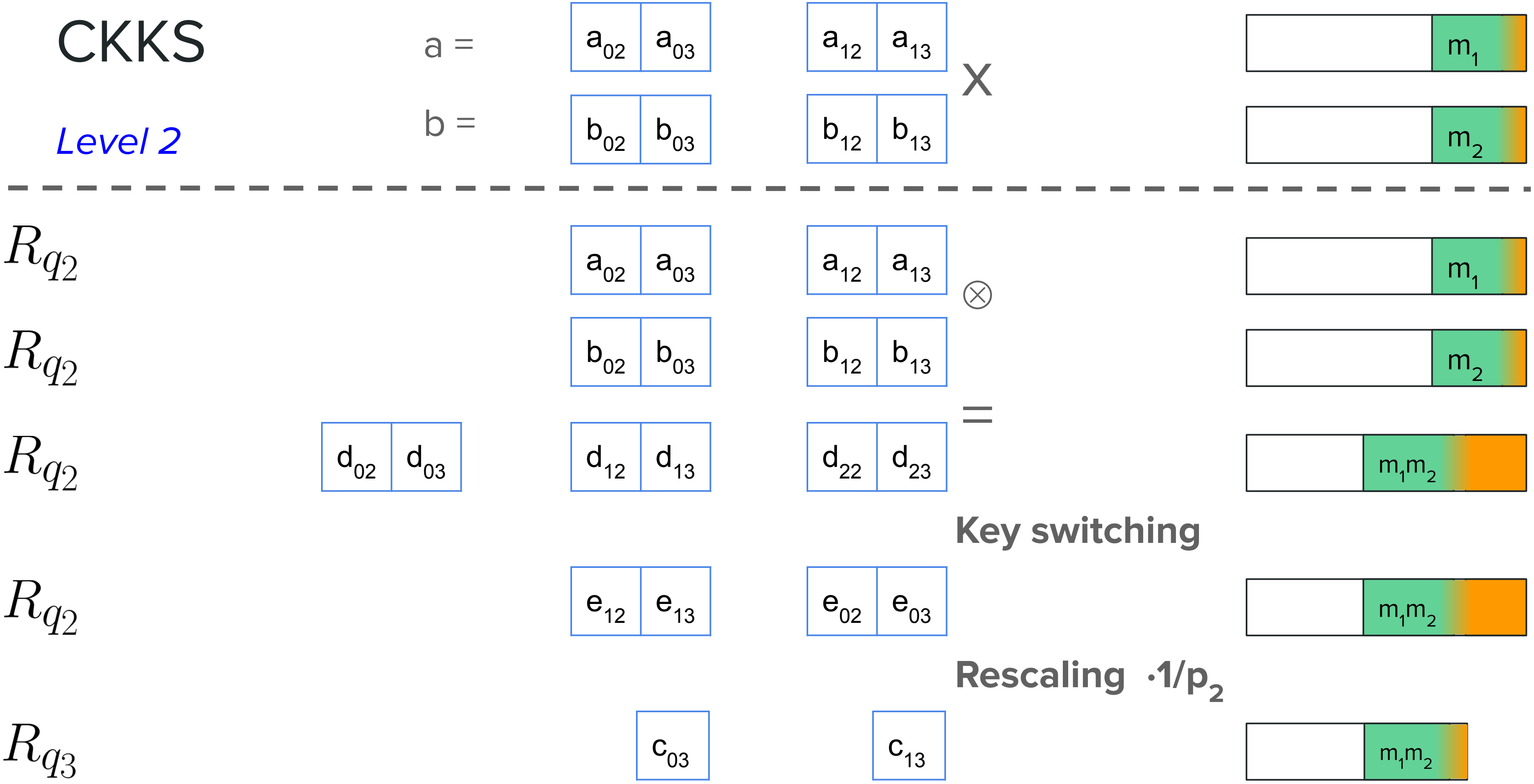
*Level 1*



a =
a01 a02 a03
a11 a12 a13
X
b =
b01 b02 b03
b11 b12 b13
m1
m2
Rq
a01 a02 a03
a11 a12 a13
⊗
m1
Rq
b01 b02 b03
b11 b12 b13
m2
=
Rq
d01 d02 d03
d11 d12 d13
d21 d22 d23
m1m2
Key switching
Rq
e11 e12 e13
e01 e02 e03
m1m2
Rescaling ·1/p1
Rq2
c02 c03
c12 c13
m1m2

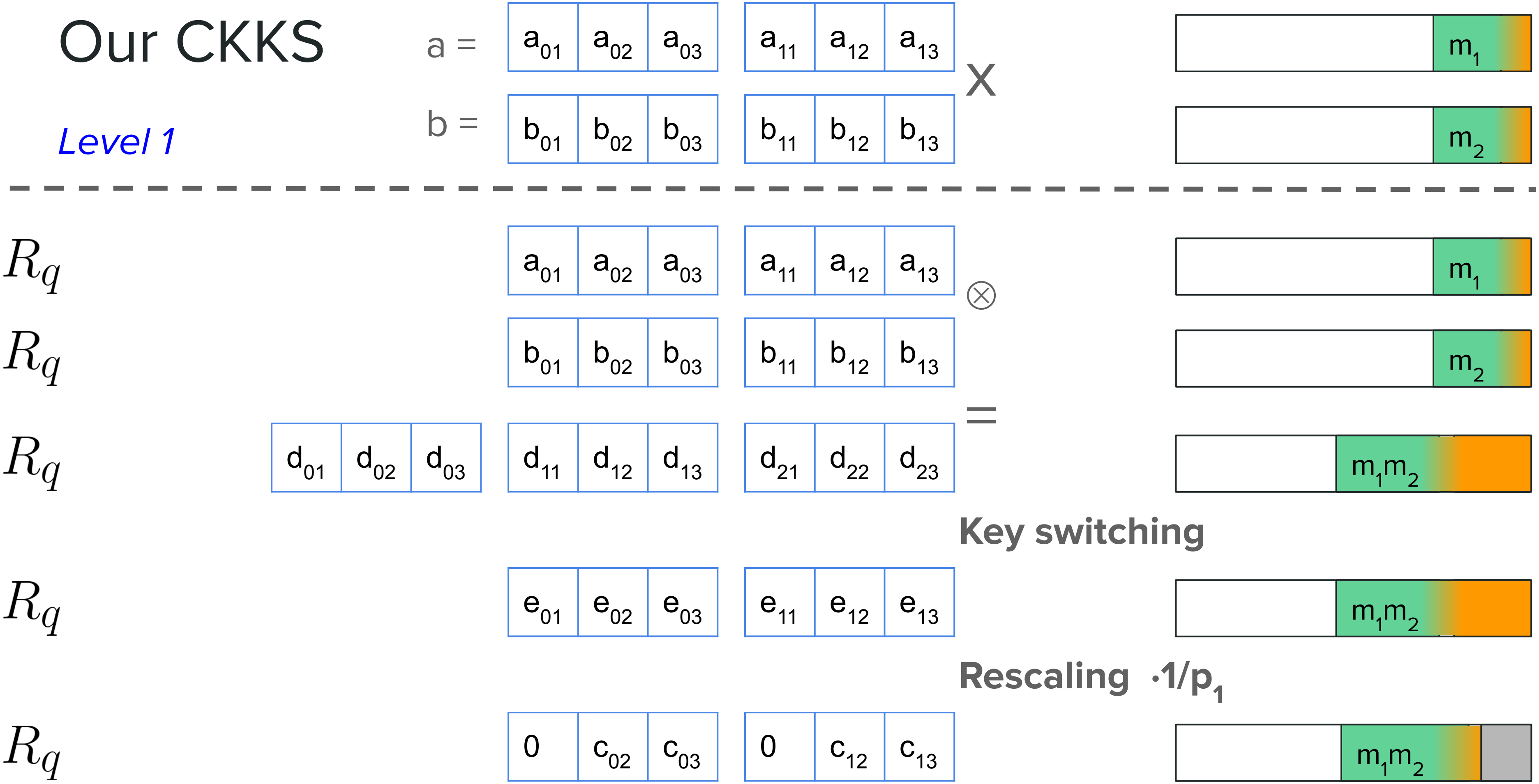
# CKKS

*Level 2*


a =
b =
a02 a03
a12 a13
b02 b03
b12 b13
X
m1
m2
Rq2
Rq2
Rq2
Rq2
Rq3
⊗
=
d02 d03
d12 d13
d22 d23
m1m2
Key switching
e12 e13
e02 e03
Rescaling ·1/p2
c03
c13

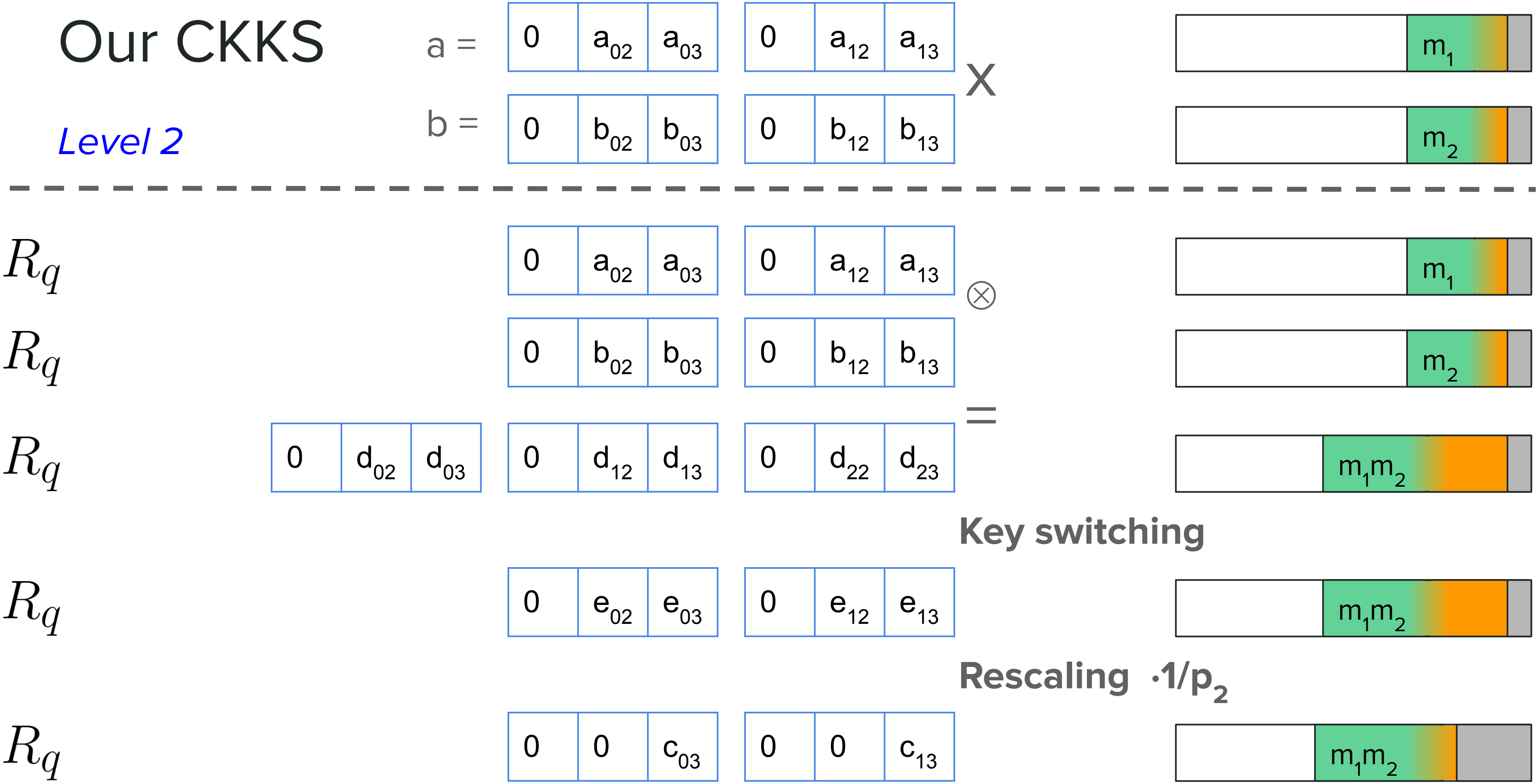
# Our CKKS

*Level 1*



# Our CKKS

*Level 2*



# Proof-friendly CKKS vs CKKS

| | Proof-friendly CKKS | | CKKS |
|---|---|---|---|
| | d = 2 | d = 4 | HEXL |
| CKKS multiplication | 7.394ms | 8.457ms | 7.197ms |

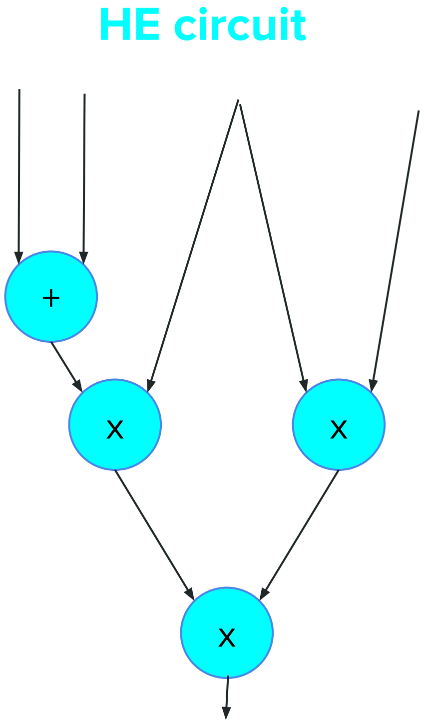
N = 16384
#RNS components (L) = 6

# Proof-friendly CKKS in summary

- Carefully chosen ring setup
  - High **soundness** for proof system
  - **Efficiency** of computations
- Ring does not change
  - Proof system works on **same ring**
- Noise analysis
  - Easier to **prove bounds** on ciphertexts (see later)

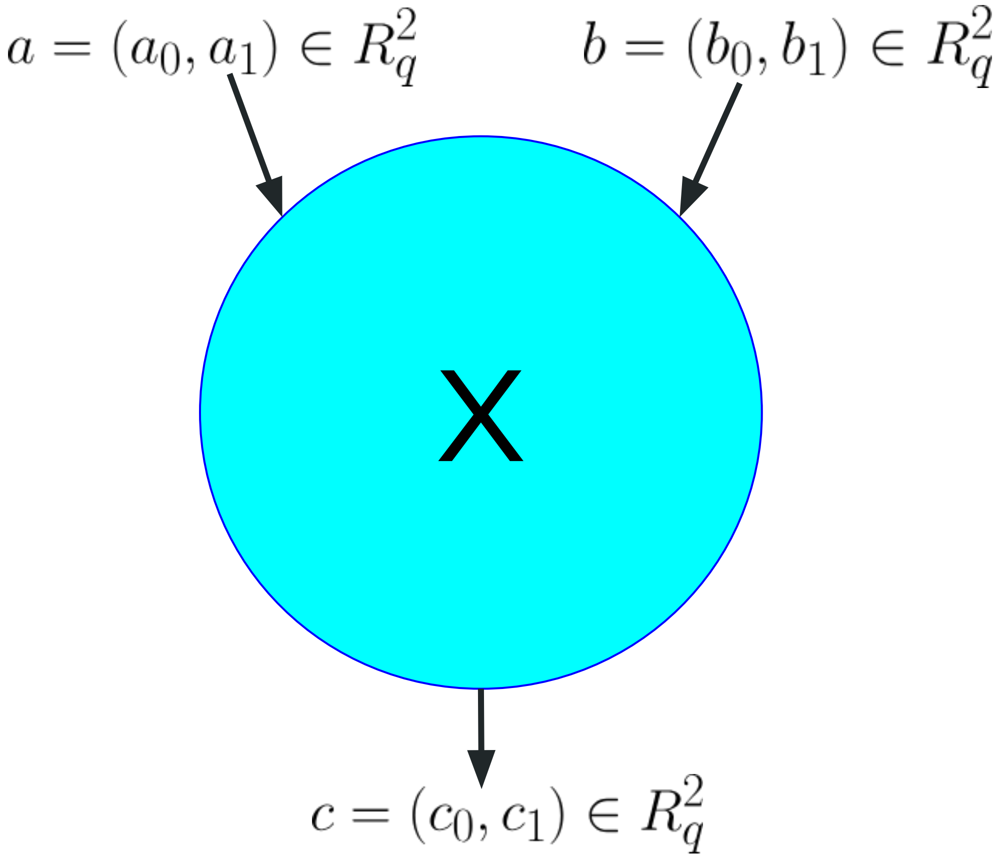
# Arithmetization

# Arithmetization

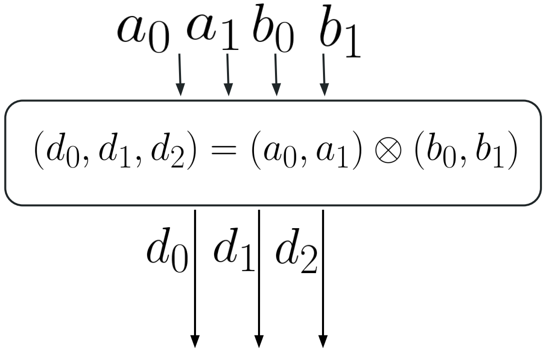



$$\left\{ \begin{array}{c} \text{Relations} \\ \text{over} \\ R_q \end{array} \right\}$$

# Arithmetization



# Arithmetization




**Polynomial mult**

# Arithmetization

$a_0 \quad a_1 \quad b_0 \quad b_1$

$$(d_0, d_1, d_2) = (a_0, a_1) \otimes (b_0, b_1)$$

**Polynomial mult**

$d_0 \quad d_1 \quad d_2$

$$e_0 = d_0 + \langle \text{evk}, CRT^{-1}(d_2) \rangle$$
$$e_1 = d_1 + \langle \text{evk}, CRT^{-1}(d_2) \rangle$$

**Key-switching**

$e_0 \quad e_1$

# Arithmetization

$a_0 \; a_1 \; b_0 \; b_1$

$$(d_0, d_1, d_2) = (a_0, a_1) \otimes (b_0, b_1)$$

**Polynomial mult**

$d_0 \; d_1 \; d_2$

$$e_0 = d_0 + \langle \text{evk}, CRT^{-1}(d_2) \rangle$$
$$e_1 = d_1 + \langle \text{evk}, CRT^{-1}(d_2) \rangle$$

**Key-switching**

$e_0 \quad e_1$

$$c_0 = (e_0 - [e_0]_{p_l}) \cdot p_l^{-1}$$
$$c_1 = (e_1 - [e_1]_{p_l}) \cdot p_l^{-1}$$

**Rescaling**

$c_1 \quad c_0$

# Arithmetization

$a_0 \; a_1 \; b_0 \; b_1$

$$(d_0, d_1, d_2) = (a_0, a_1) \otimes (b_0, b_1)$$

$d_0 \; d_1 \; d_2$

$$e_0 = d_0 + \langle \mathrm{evk}, CRT^{-1}(d_2) \rangle$$
$$e_1 = d_1 + \langle \mathrm{evk}, CRT^{-1}(d_2) \rangle$$

$e_0 \; e_1$

$$c_0 = (e_0 - [e_0]_{p_l}) \cdot p_l^{-1}$$
$$c_1 = (e_1 - [e_1]_{p_l}) \cdot p_l^{-1}$$

$c_1 \; c_0$

# Arithmetization

**Algebraic operation**

$a_0 \; a_1 \; b_0 \; b_1$

$$(d_0, d_1, d_2) = (a_0, a_1) \otimes (b_0, b_1)$$

$d_0 \; d_1 \; d_2$

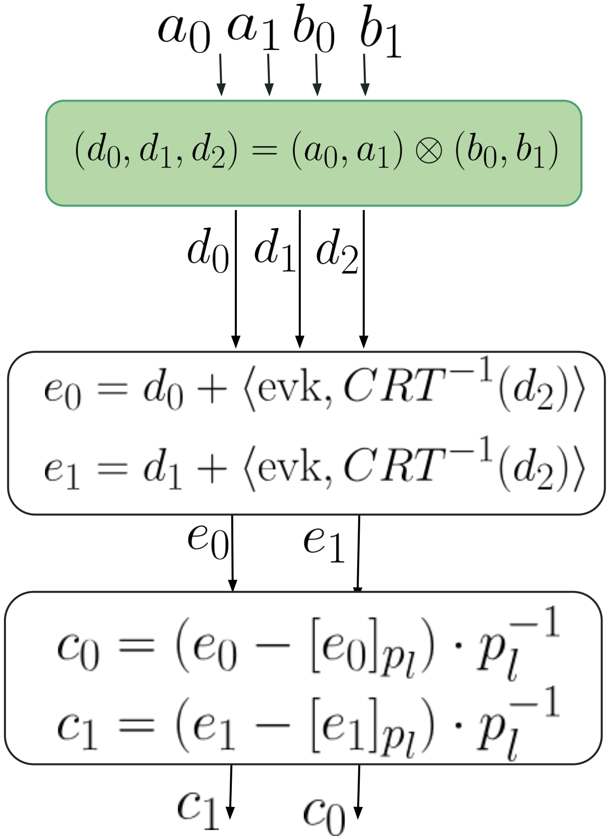
$$e_0 = d_0 + \langle \text{evk}, CRT^{-1}(d_2) \rangle$$
$$e_1 = d_1 + \langle \text{evk}, CRT^{-1}(d_2) \rangle$$

$e_0 \; e_1$

$$c_0 = (e_0 - [e_0]_{p_l}) \cdot p_l^{-1}$$
$$c_1 = (e_1 - [e_1]_{p_l}) \cdot p_l^{-1}$$

$c_1 \; c_0$

# Arithmetization



# Arithmetization

$a_0 \; a_1 \; b_0 \; b_1$

$$(d_0, d_1, d_2) = (a_0, a_1) \otimes (b_0, b_1)$$

$d_0 \; d_1 \; d_2$

$$e_0 = d_0 + \langle \text{evk}, \boxed{CRT^{-1}(d_2)} \rangle$$
$$e_1 = d_1 + \langle \text{evk}, \boxed{CRT^{-1}(d_2)} \rangle$$

**Not algebraic!**

$e_0 \quad e_1$

$$c_0 = (e_0 - [e_0]_{p_l}) \cdot p_l^{-1}$$
$$c_1 = (e_1 - [e_1]_{p_l}) \cdot p_l^{-1}$$

$c_1 \quad c_0$

# Arithmetization

$a_0 \quad a_1 \quad b_0 \quad b_1$

$$(d_0, d_1, d_2) = (a_0, a_1) \otimes (b_0, b_1)$$

$d_0 \quad d_1 \quad d_2$

$$e_0 = d_0 + \langle \text{evk}, \boxed{CRT^{-1}(d_2)} \rangle$$

$$e_1 = d_1 + \langle \text{evk}, \boxed{CRT^{-1}(d_2)} \rangle$$

$e_0 \quad e_1$

$$c_0 = (e_0 - [e_0]_{p_l}) \cdot p_l^{-1}$$

$$c_1 = (e_1 - [e_1]_{p_l}) \cdot p_l^{-1}$$

$c_1 \quad c_0$

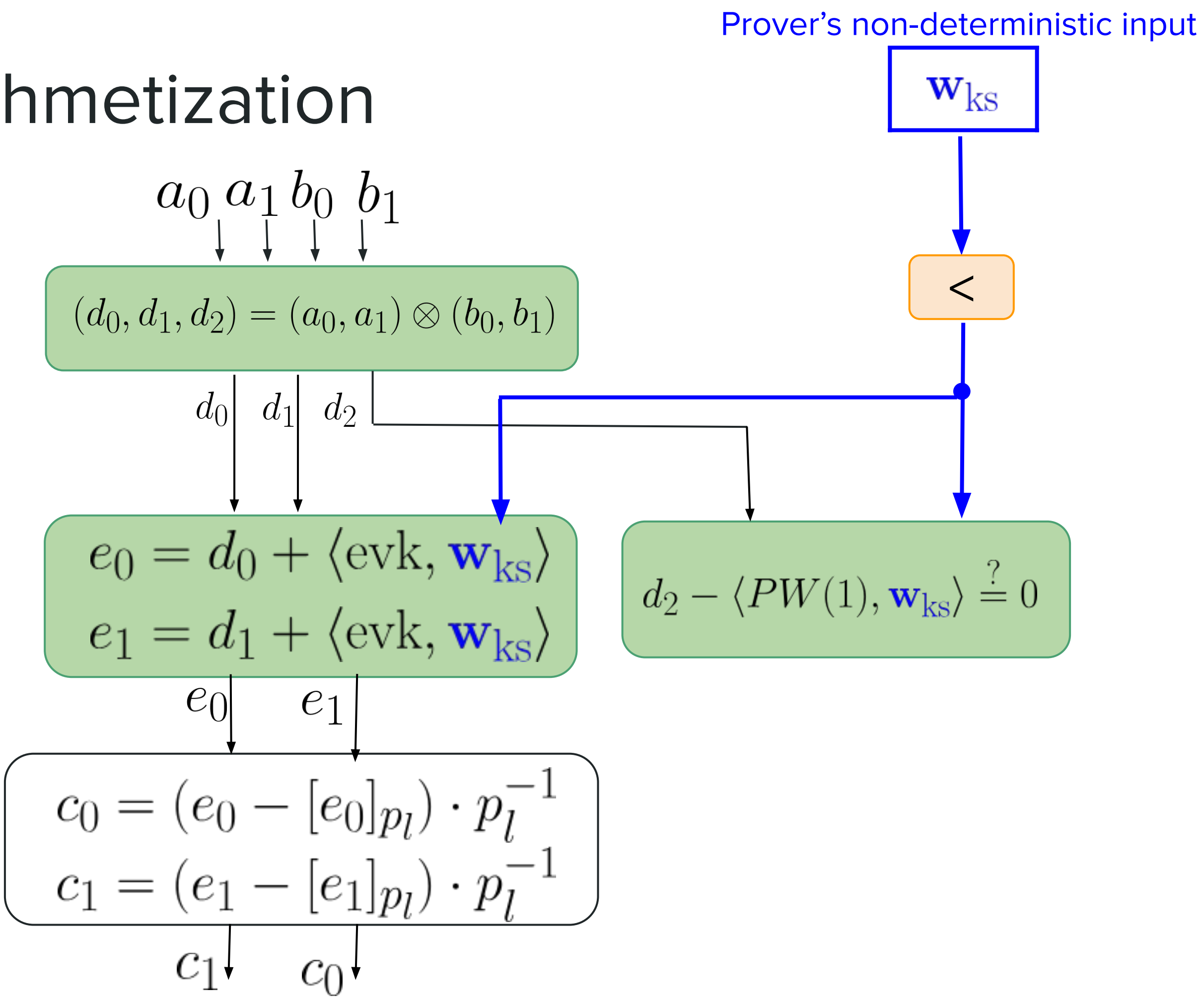
**Not algebraic!**

Prover inputs $\mathbf{w}_{\text{ks}}$

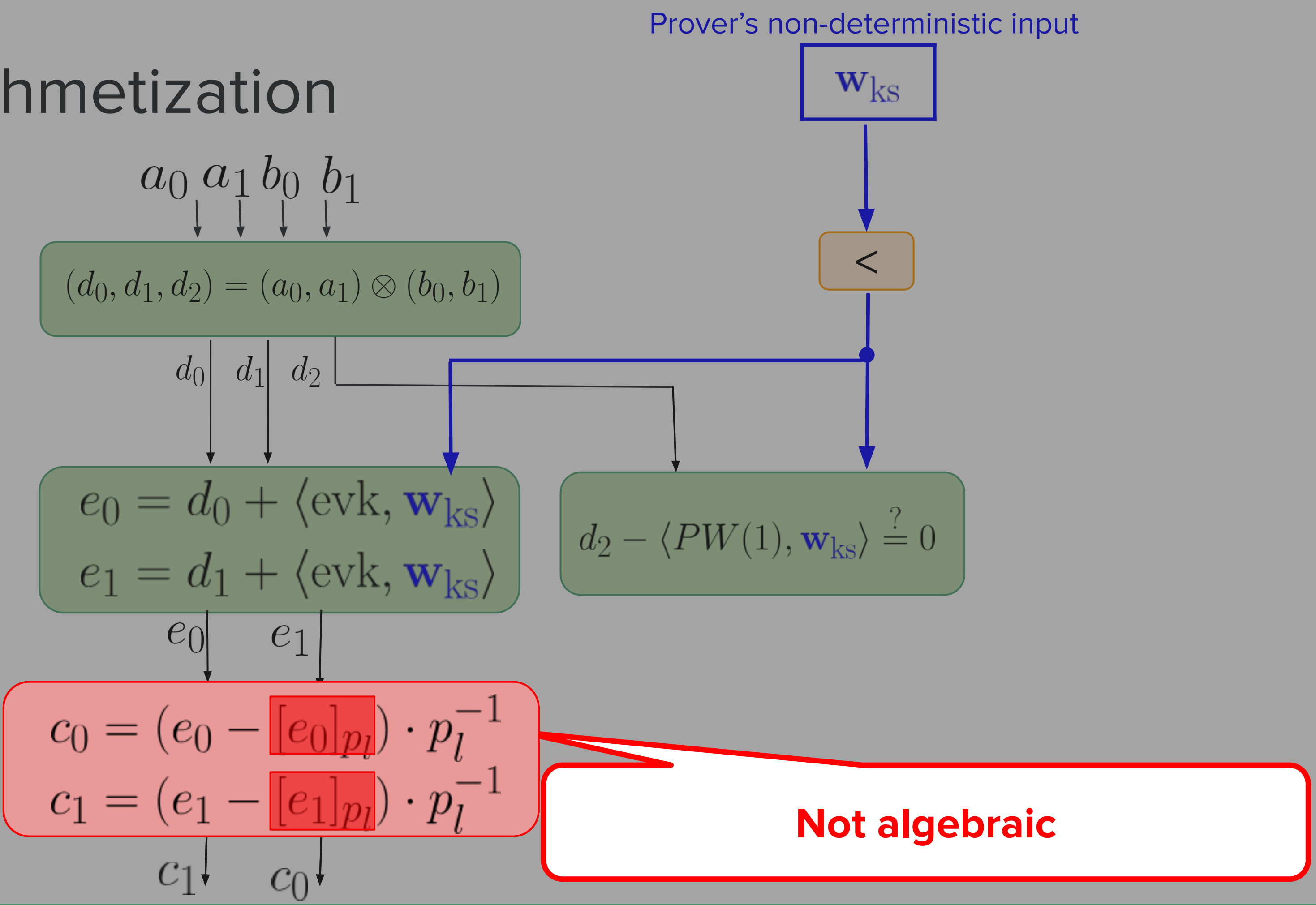
$$\|\mathbf{w}_{\text{ks}}[i]\| < p_i$$

$$d_2 = \langle PW(1), \mathbf{w}_{\text{ks}} \rangle$$

# Arithmetization



# Arithmetization



# Arithmetization

$a_0 \quad a_1 \quad b_0 \quad b_1$

$$(d_0, d_1, d_2) = (a_0, a_1) \otimes (b_0, b_1)$$

$d_0 \quad d_1 \quad d_2$

$$e_0 = d_0 + \langle \mathrm{evk}, \mathbf{w}_{\mathrm{ks}} \rangle$$
$$e_1 = d_1 + \langle \mathrm{evk}, \mathbf{w}_{\mathrm{ks}} \rangle$$

$e_0 \quad e_1$

$$c_0 = (e_0 - \boxed{[e_0]_{p_l}}) \cdot p_l^{-1}$$
$$c_1 = (e_1 - \boxed{[e_1]_{p_l}}) \cdot p_l^{-1}$$

$c_1 \quad c_0$

Can be rewritten as **Euclidean division**

$$e_i = c_i \cdot p_l + [e_i]_{p_l}$$

**Not algebraic**

# Arithmetization

$a_0 \quad a_1 \quad b_0 \quad b_1$

$$(d_0, d_1, d_2) = (a_0, a_1) \otimes (b_0, b_1)$$

$d_0 \quad d_1 \quad d_2$

$$e_0 = d_0 + \langle \mathrm{evk}, \mathbf{w}_{\mathrm{ks}} \rangle$$
$$e_1 = d_1 + \langle \mathrm{evk}, \mathbf{w}_{\mathrm{ks}} \rangle$$

$e_0 \quad e_1$

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$c_1 \quad c_0$

**Not algebraic**

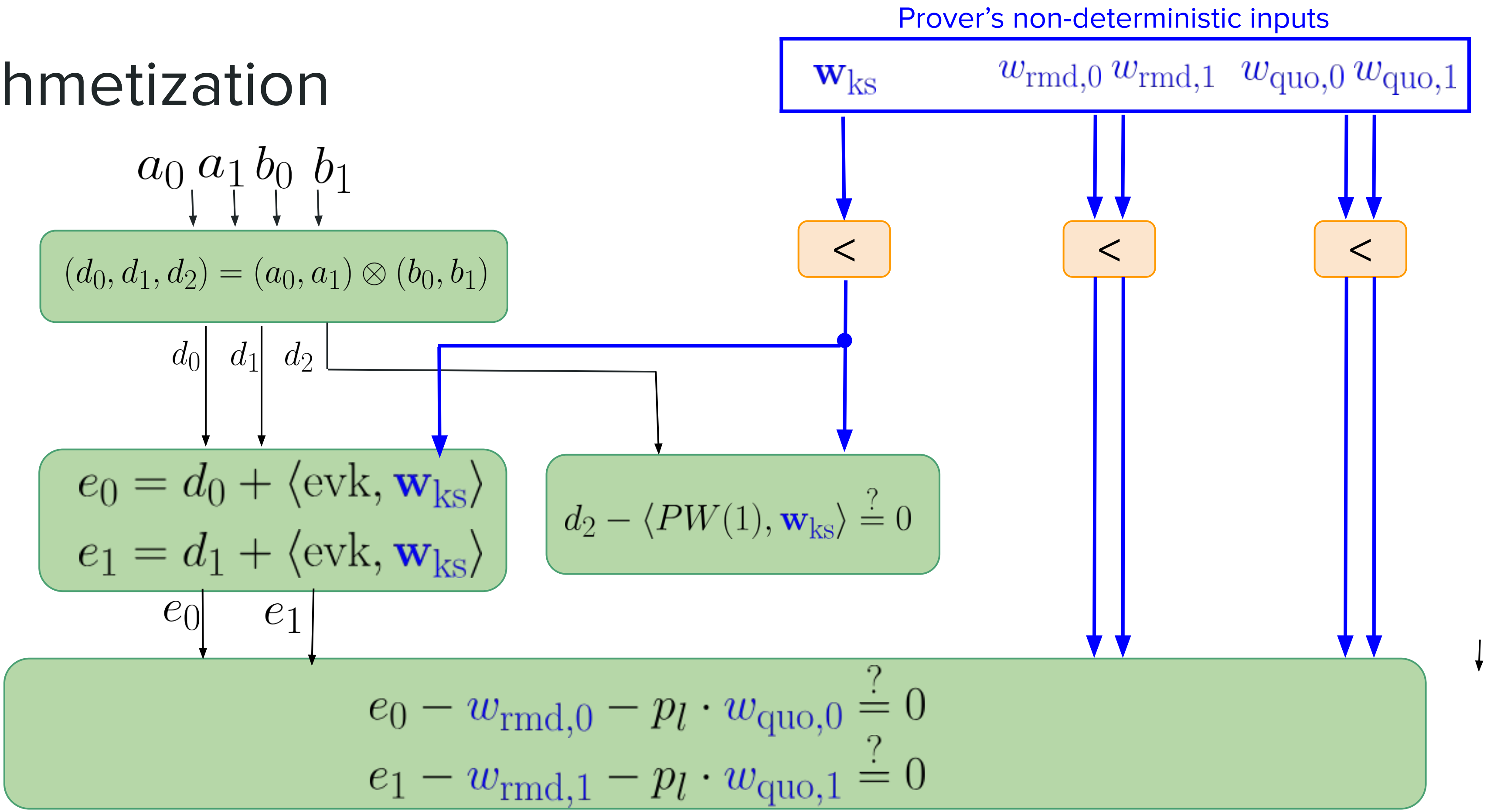
Can be rewritten as **Euclidean division**

$$e_i = c_i \cdot p_l + [e_i]_{p_l}$$

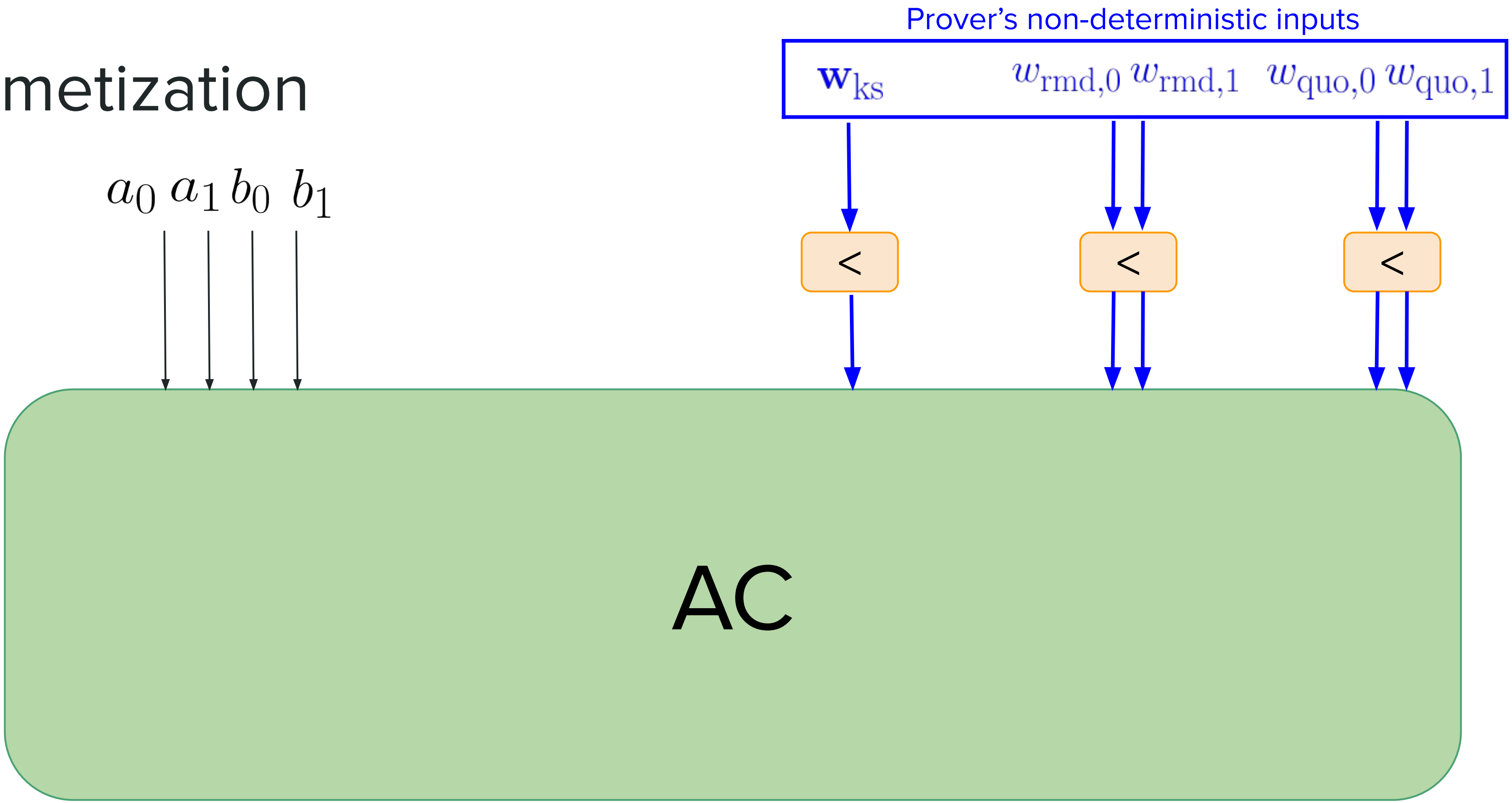
Prover inputs $w_{\mathrm{quo},i} \quad w_{\mathrm{rmd},i}$

$$\|w_{\mathrm{quo},i}\| \leq q_l / p_l$$
$$\|w_{\mathrm{rmd},i}\| < p_l$$
$$e_i = w_{\mathrm{quo},i} \cdot p_l + w_{\mathrm{rmd},i}$$

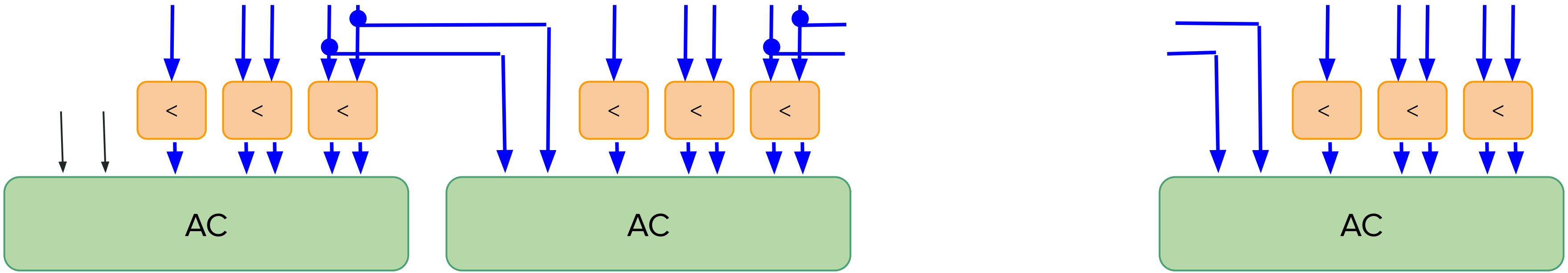
# Arithmetization



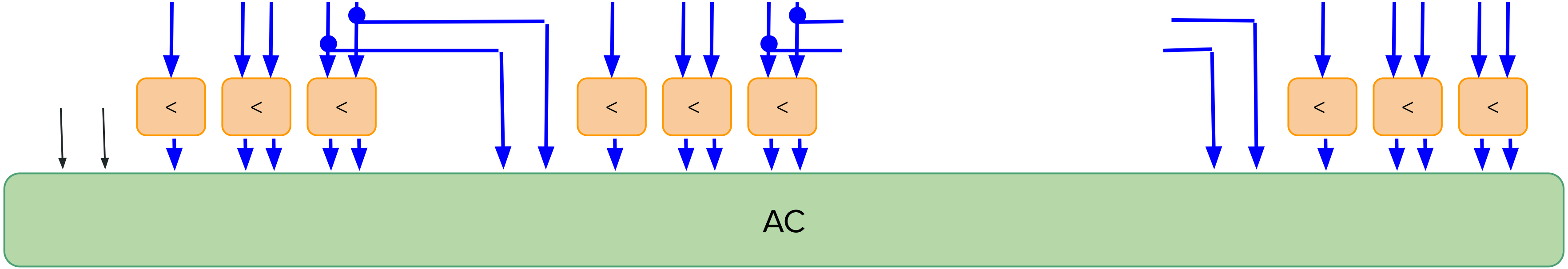
# Arithmetization



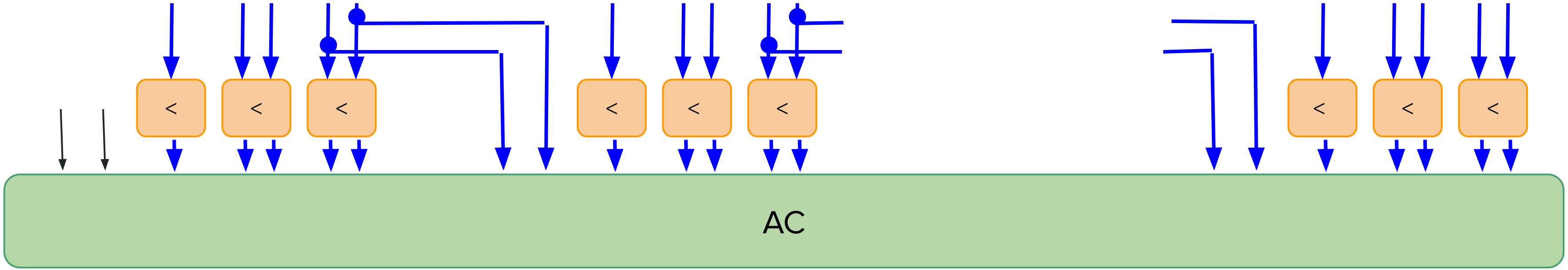
# Flattening the circuit



# Flattening the circuit



# Final set of relations




**HE circuit** satisfiability reduced to **two** main **relations**:

$$\mathcal{R}_{AC} := \{(C; \mathbf{x}, \mathbf{w}) : C(\mathbf{x}, \mathbf{w}) = \mathbf{0}\}$$

$$\mathcal{R}_{range} := \{(B; \mathbf{w}) : \|\mathbf{w}\| < B\}$$

# Proof of AC satisfiability

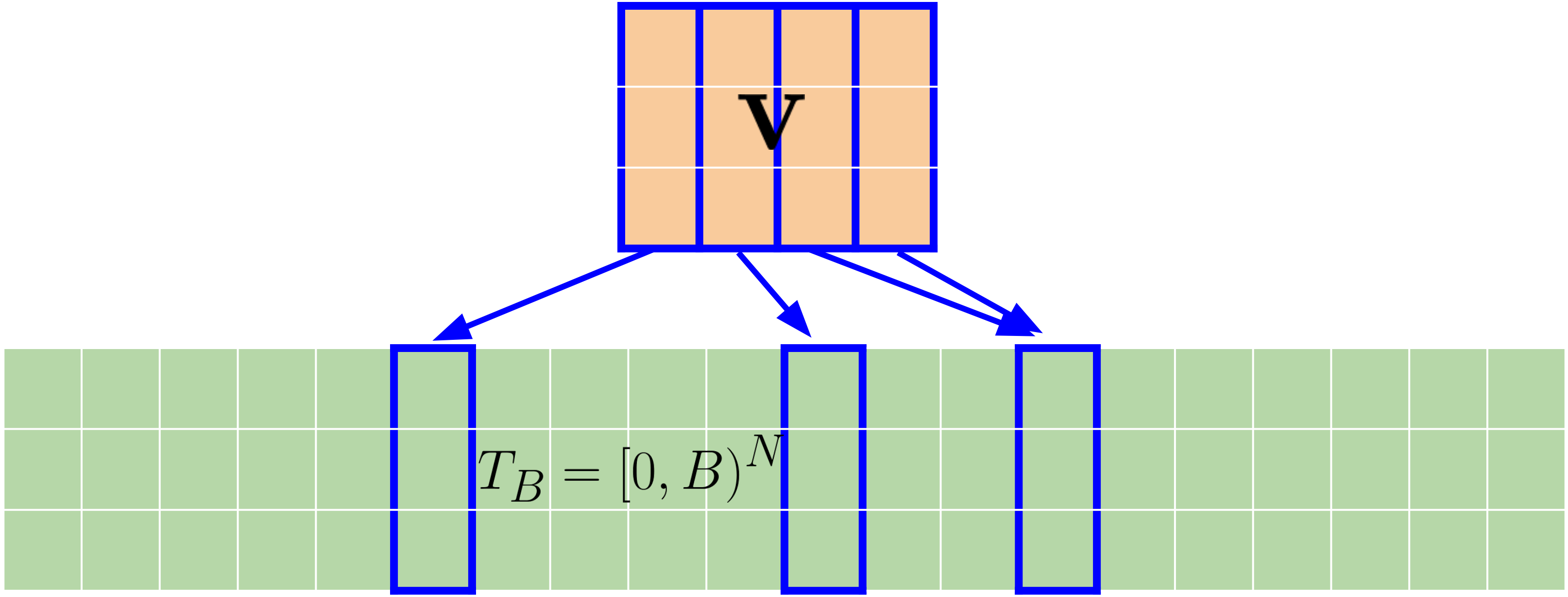
# Proving AC satisfiability

- GKR-style proof system
    - AC satisfiability => layers of equations
    - Consistency of layer *i*-th reduced to that of layer *i+1*-th
    - Works over $\boldsymbol{R_q}$
    - **Custom gates** (rescon, bdcon, ...)
    - Flattened system of relations => **constant depth 4**

# Range checks

# Proving ranges

- Prove that vector **v** of *m* elements in $R_q$ has coeffs bounded by B (e.g B = $q_l$ )
- Can be seen as a look-up argument



# Proving ranges

- Prove that vector **v** of *m* elements in $R_q$ has coeffs bounded by B (e.g B = $q_l$ )
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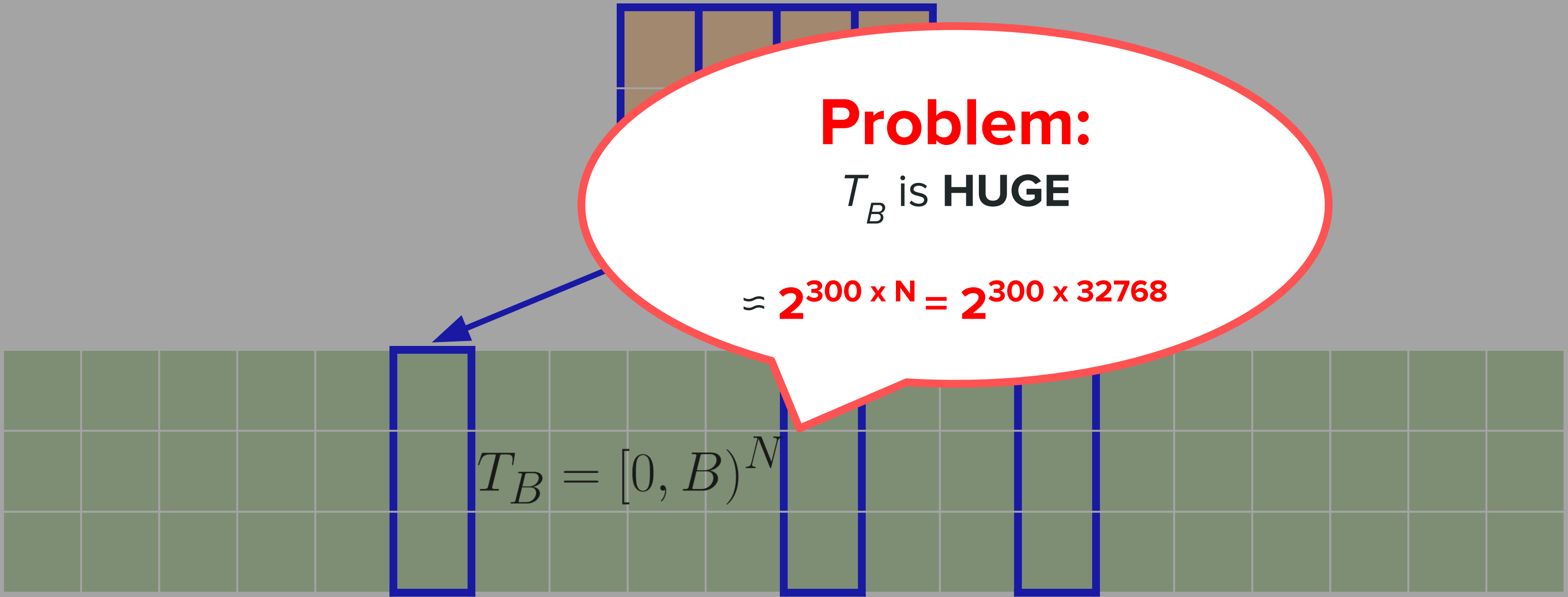
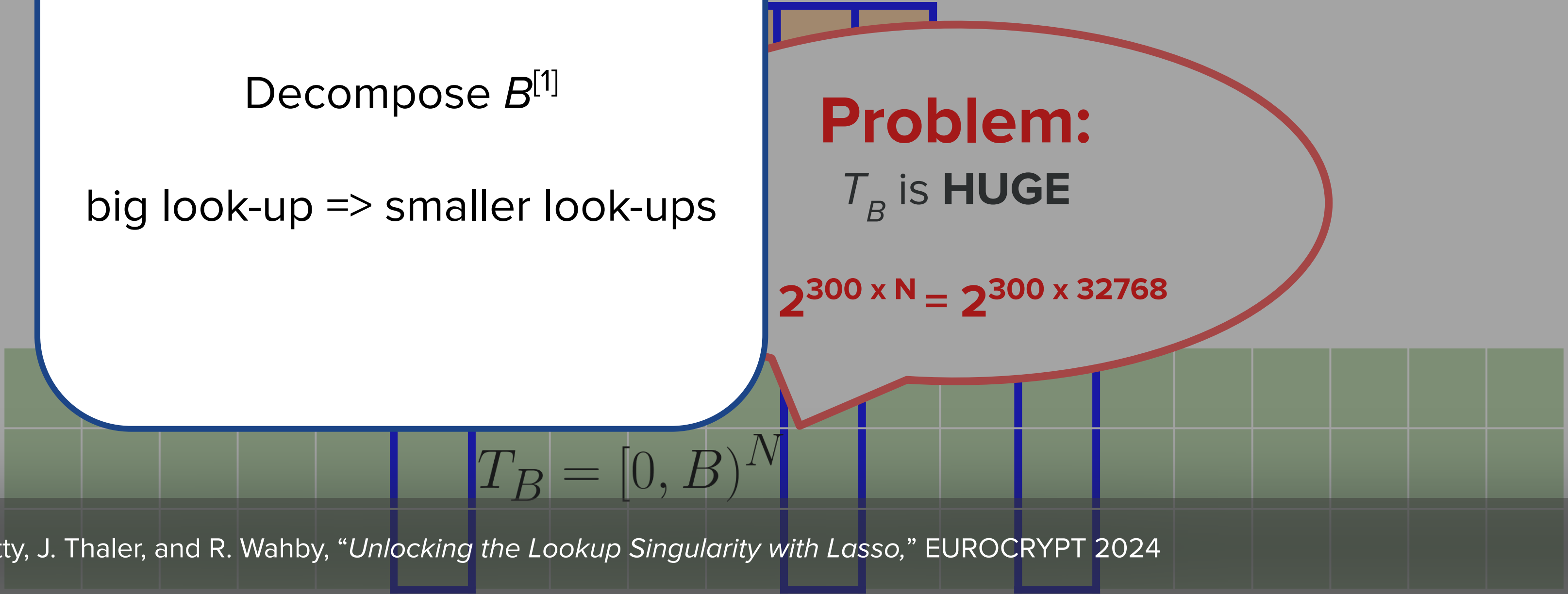


# Proving ranges

- Prove that vector **v** of *m* elements in $R_l$ has coeffs bounded by B (e.g B = $q_l$ )
- C

Decompose $B$[1]

big look-up => smaller look-ups




**Problem:**

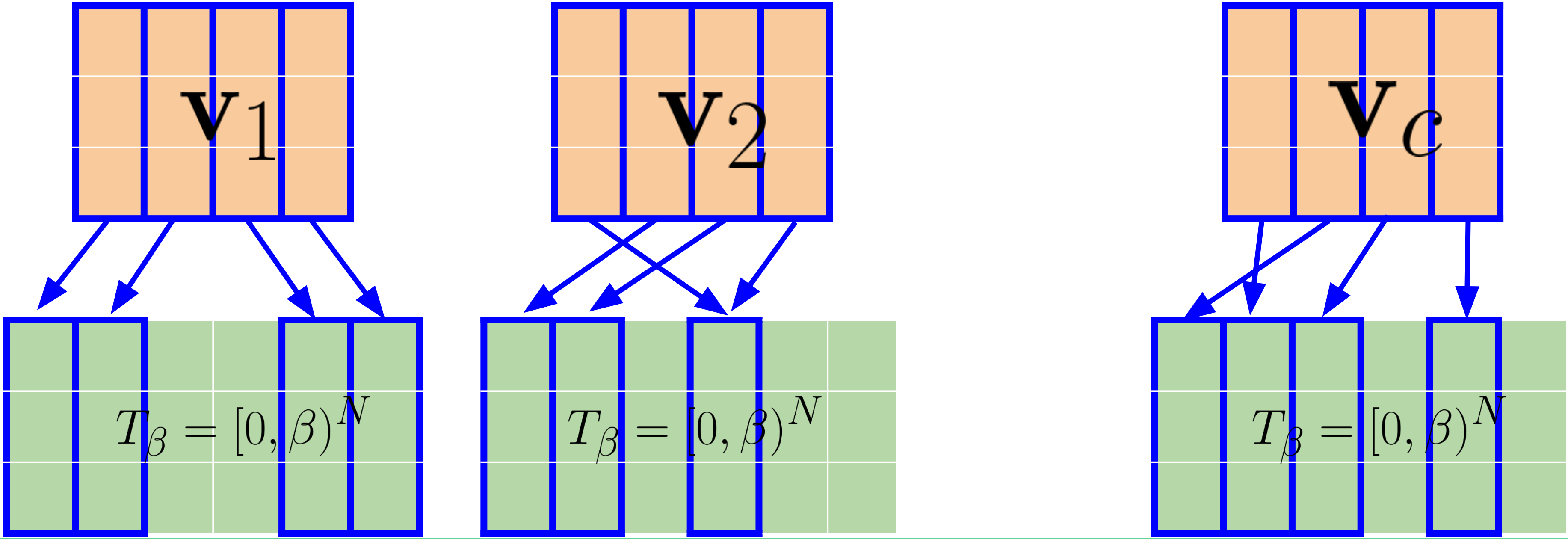
$T_B$ is **HUGE**

$2^{300 \times N} = 2^{300 \times 32768}$

[1] S. Setty, J. Thaler, and R. Wahby, "*Unlocking the Lookup Singularity with Lasso,*" EUROCRYPT 2024

# Proving ranges

- Set $\beta = B^{1/c}$
- Prove that $c$ vectors $\mathbf{v}_1, \ldots, \mathbf{v}_c$ of $m$ elements in $R_q$ have coeffs bounded by $\beta$



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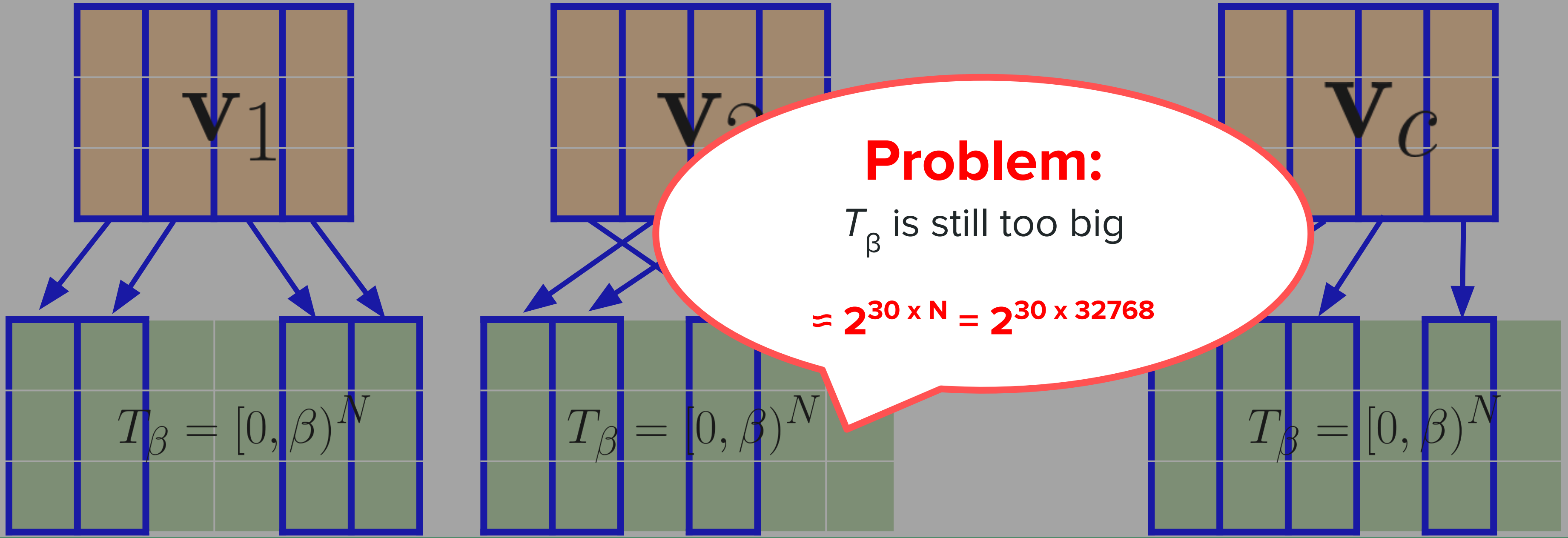
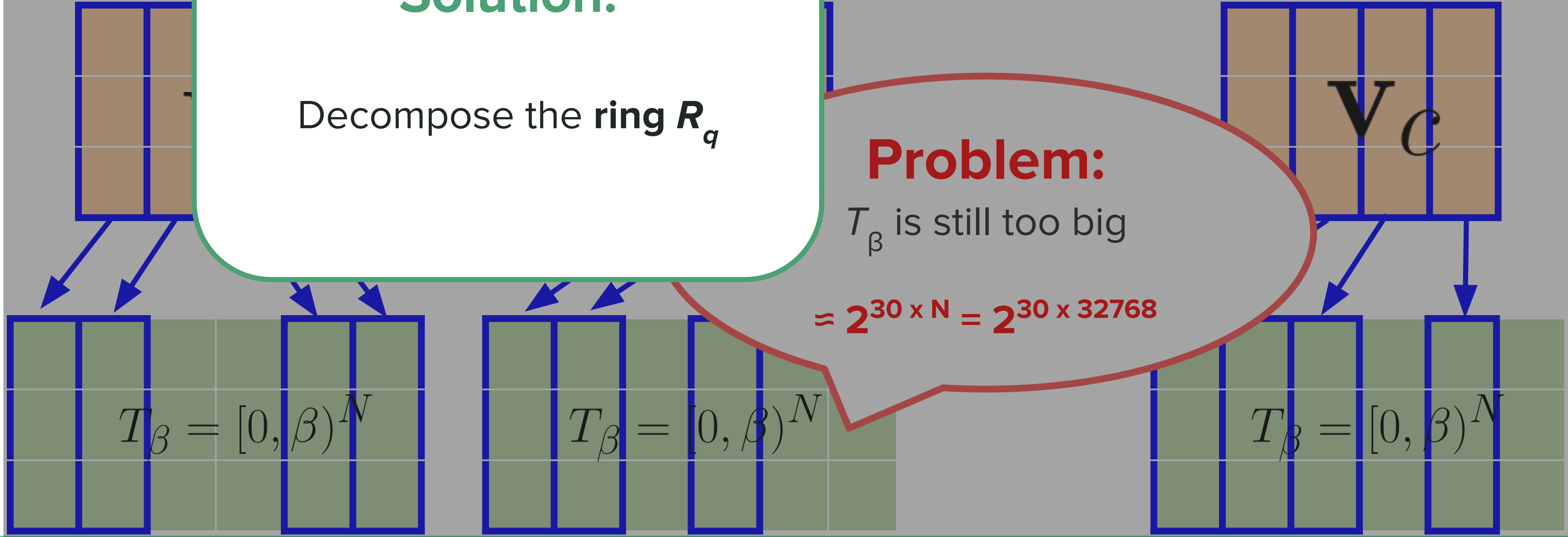


# Proving ranges

- Set
- Prov n $R_q$ have coeffs bounded by β




**Solution:**

Decompose the **ring** $\boldsymbol{R_q}$

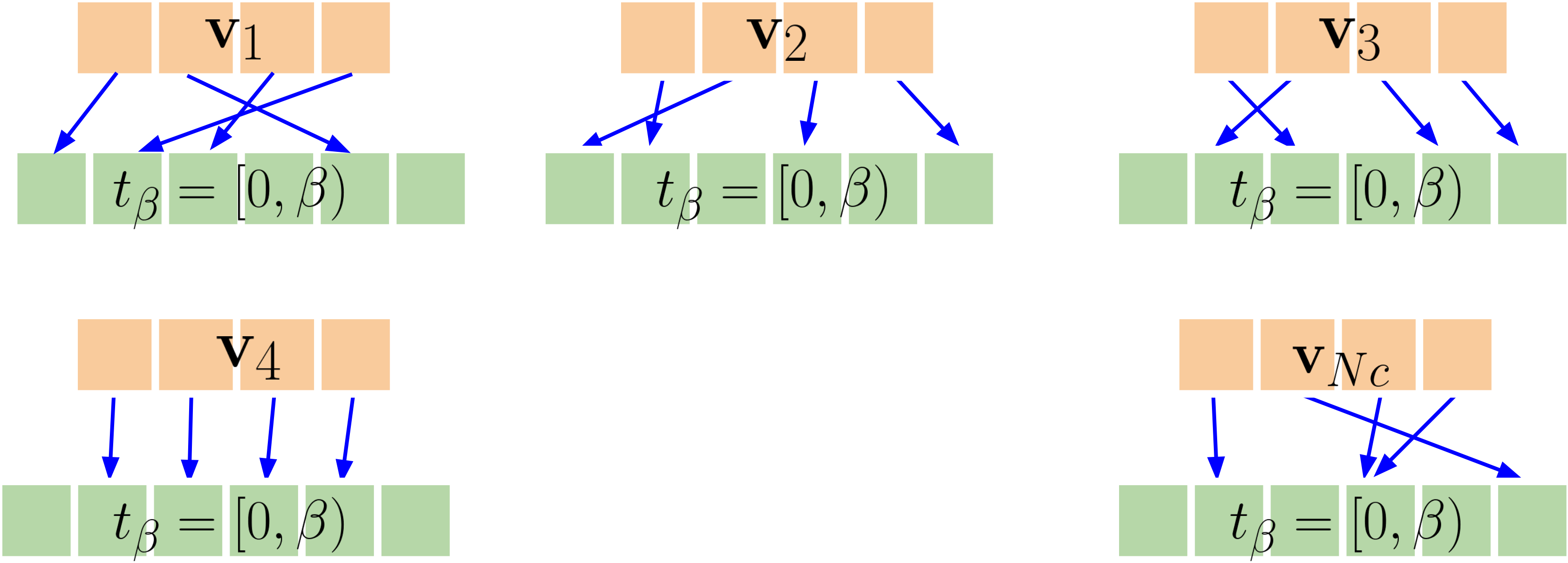
**Problem:**

$T_\beta$ is still too big

$\approx 2^{30 \times N} = 2^{30 \times 32768}$

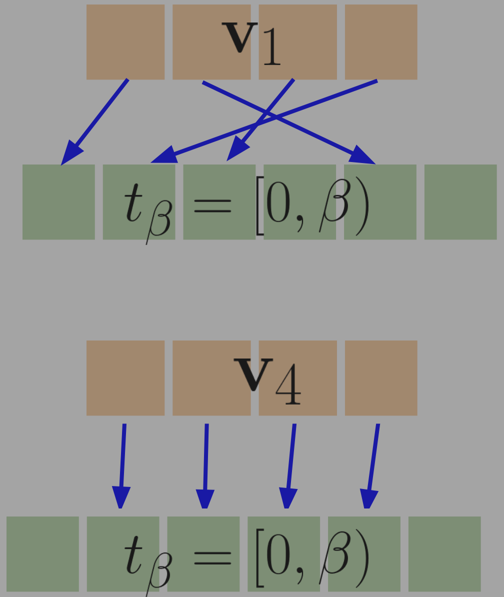
# Proving ranges

- Prove that $cN$ vectors $\mathbf{v}_1, \ldots, \mathbf{v}_{cN}$ of $m$ elements in $Z_q$ have coeffs bounded by β:



# Proving ranges

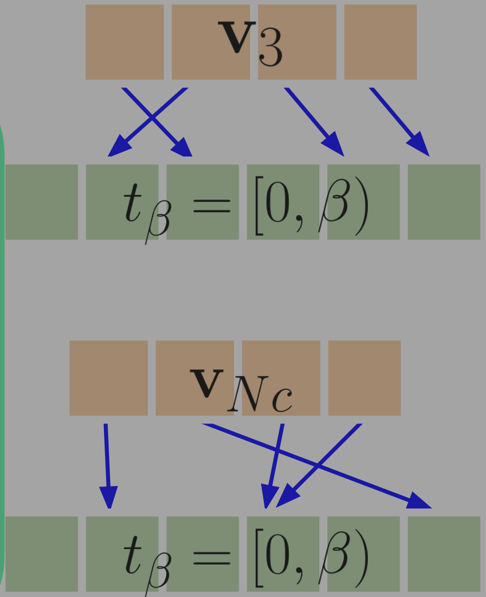
- Prove that $cN$ vectors $\mathbf{v}_1, \ldots, \mathbf{v}_{cN}$ of $m$ elements in $Z_q$ have coeffs bounded by β:




$t_\beta \approx 2^{30}$

This is now a normal look-up argument

Can be done using **memory-check** techniques



# The polynomial commitment

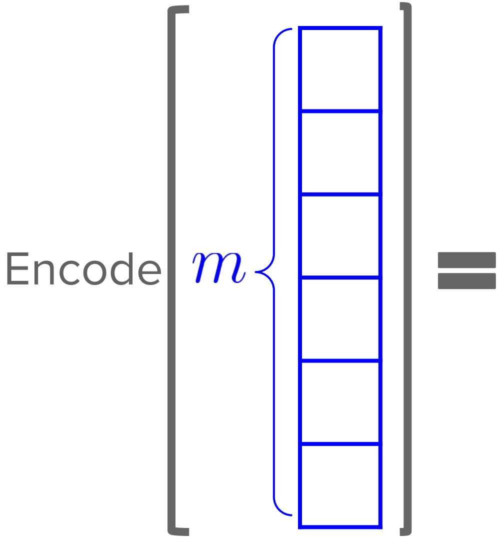
# Polynomial Commitment

Need to commit to elements in $R_q[X_1, \ldots, X_\ell]$ where

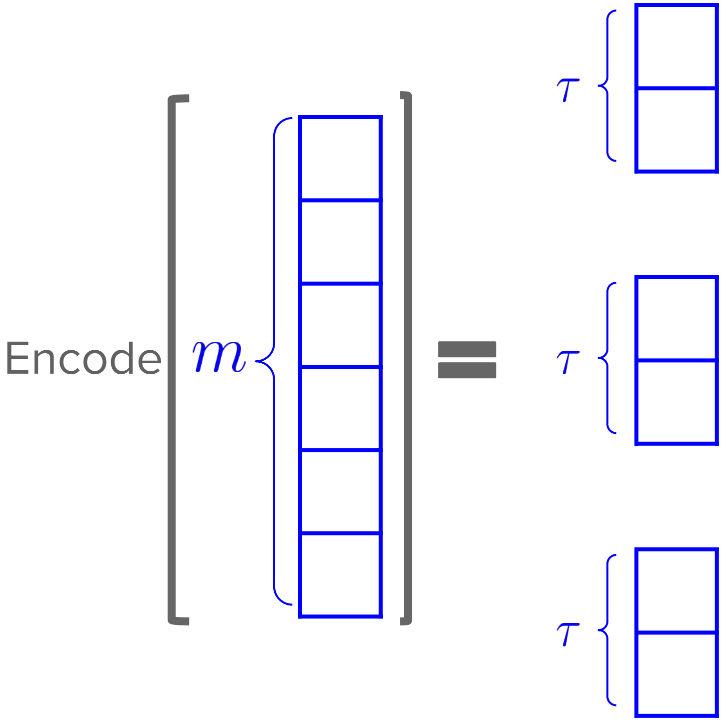
$$R_q \cong \mathbb{F}_{p_0^4} \times \cdots \times \mathbb{F}_{p_L^4}$$

- Reduce MV PC over $R_q$ to MV PC over $\mathbb{F}_{p_i^4}$
- Field-agnostic multivariate PC => **Brakedown**
- $\mathbb{F}_{p_i^4}$ has $N/2$ roots of unity. **Can we use them?**

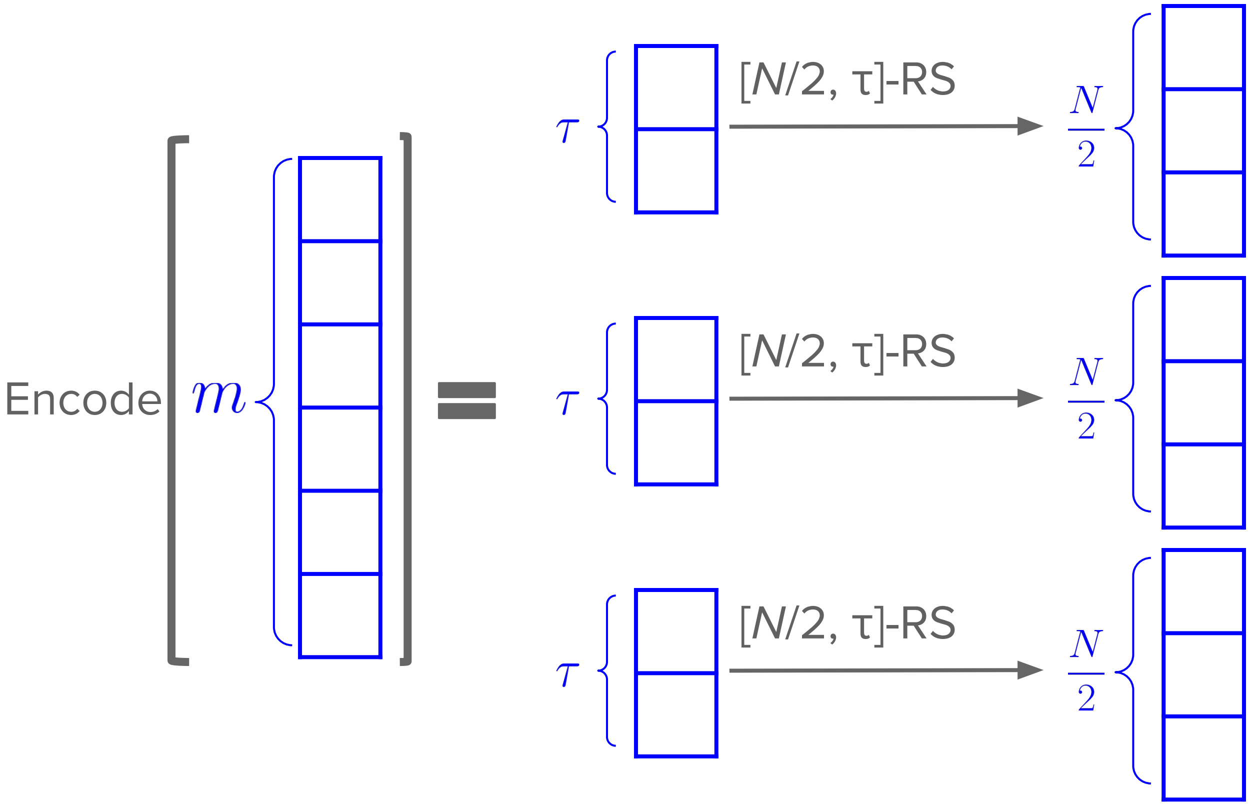
# Piecewise Reed-Solomon code



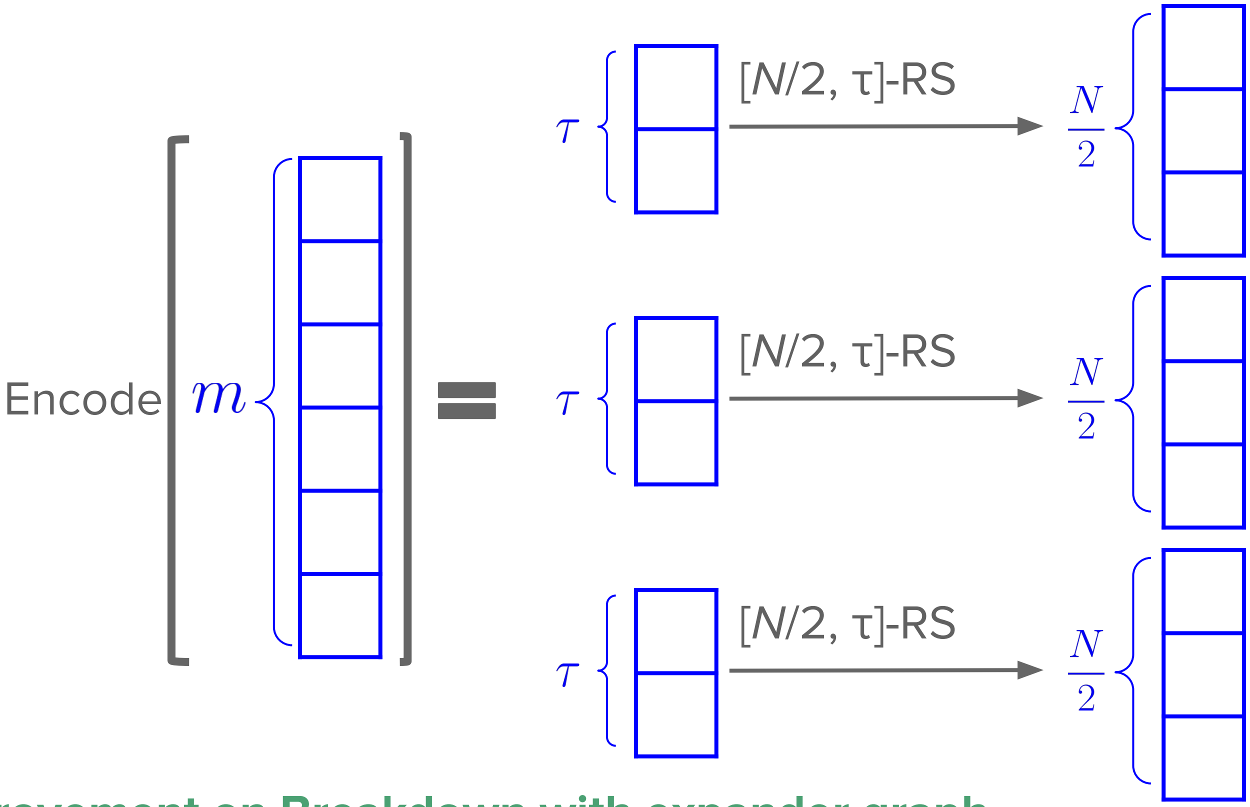
# Piecewise Reed-Solomon code



# Piecewise Reed-Solomon code



# Piecewise Reed-Solomon code




**x10 improvement on Breakdown with expander graph**

# Conclusions

# To summarize

- First practical VC for CKKS
  - Technique extend to FV/BGV
- Description of problem in a modular way (arithmetization)
  - AC satisfiability + range checks
- Design of proof-friendly CKKS
- Design of custom GKR to prove AC over rings
- Design of range proofs for polynomial rings
- Improved Brakedown for medium-sized fields
- Implemented all building blocks

https://eprint.iacr.org/2025/286

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# VC-HE

| | Verification | Arithmetic operations | Ciphertext maintenance | Bootstrapping | Supported HE schemes |
|---|---|---|---|---|---|
| Generic SNARK | Public | emulated | emulated | emulated | Any |
| Rinocchio | Private | Native | emulated | emulated | Any |
| HE-IOPs | Private **VO attacks** | Native | Native | Native | Exact (no CKKS) |
| **Our Work** | Public | Native | Efficiently emulated | ? | Any |