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Folding over lattices: Performance and implementation

Ilia Vlasov, Marko Čupić, Matthew Klein, Antonio Larriba,
Emanuel Viera, Isaac Villalobos, Albert Garreta

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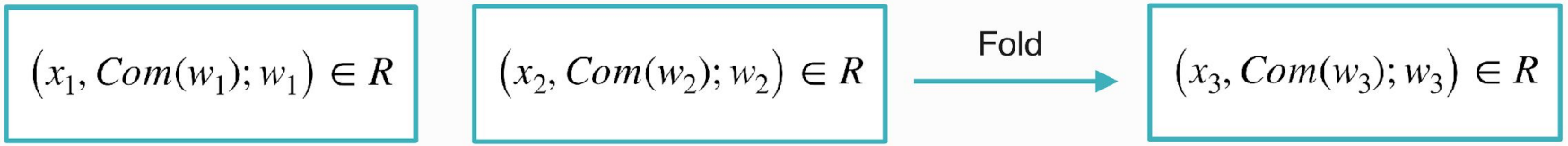

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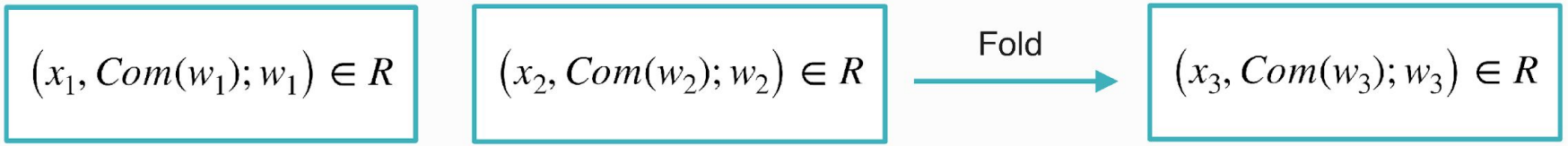

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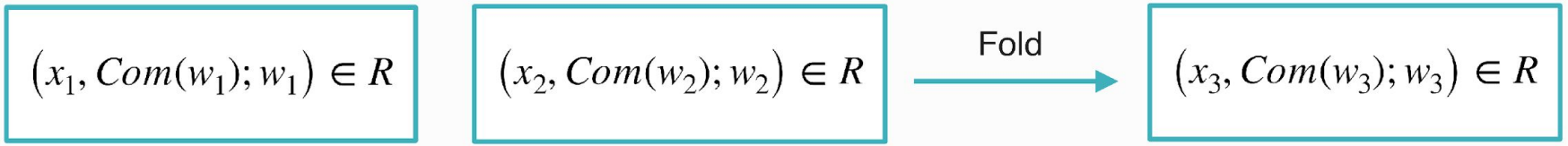

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*Not in, e.g. Arc (Bünz, Mishra, Nguyen, Wang, 2024)

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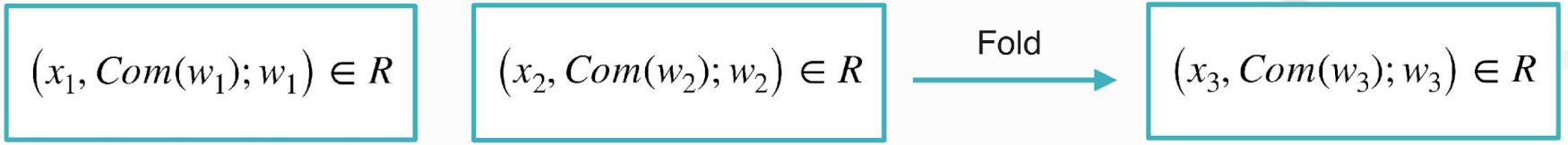

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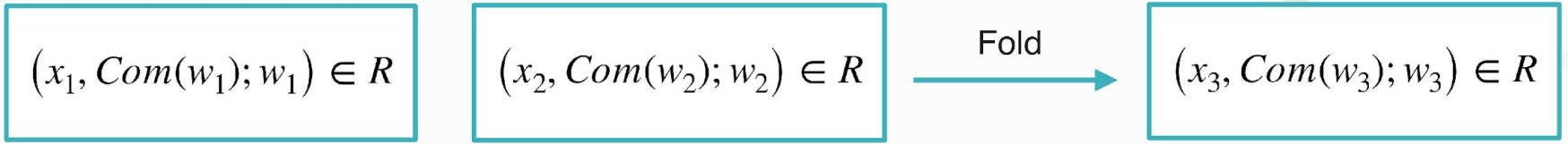

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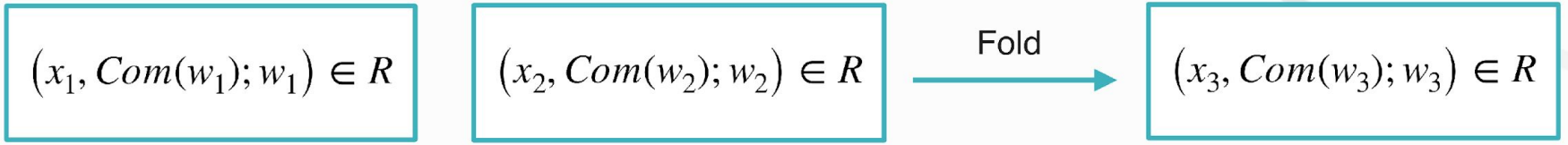

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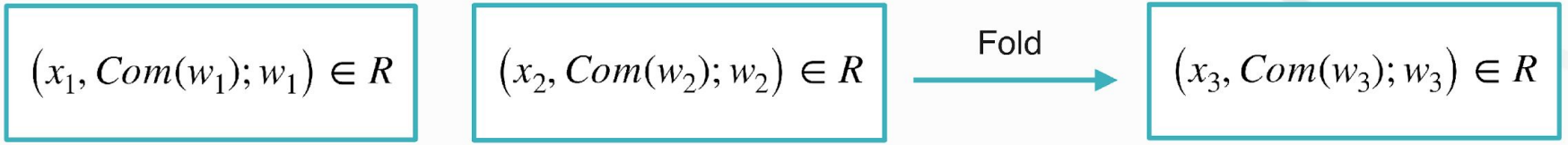

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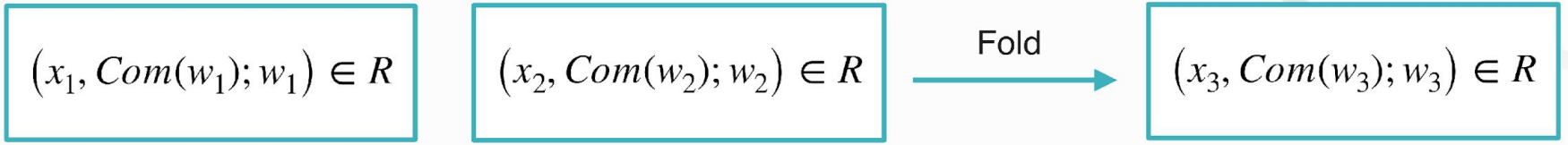

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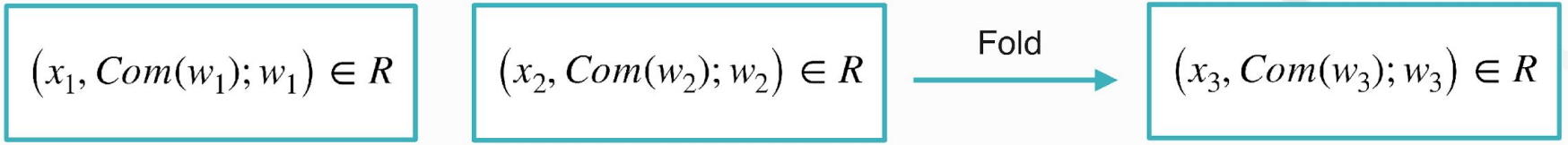

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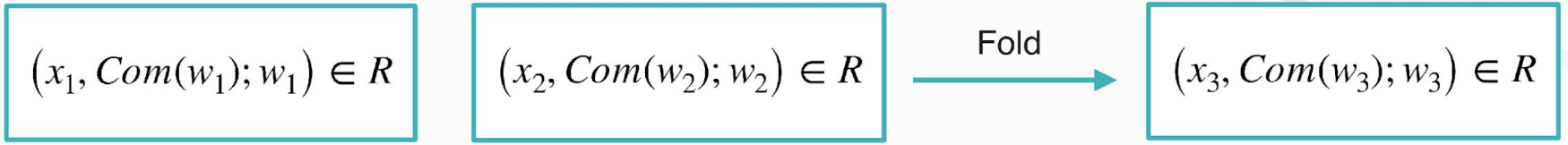

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  - Witness lives in a big field: ≥ 250 bits.
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  - Expensive to recurse over: Proving that folding was performed correctly is complex due to the need of arithmetizing foreign field.

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- Relevant for assessing performance of lattice-based folding

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E.g. in Latticefold+ (Boneh, Chen, 2025) there's commitments to vectors full of elements of the form X^u.

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$A \cdot (v+u)^\mathsf{T} = A \cdot v^\mathsf{T} + A \cdot u^\mathsf{T}$, but maybe $\|v+u\| > B$.

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$\mathbb{F}_q$ can be a small field: $q \approx 32,64$ bits, including STARK primes (Goldilocks, Babybear).

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Boons of Ajtai commitments:

$\mathbb{F}_q$ can be a small field: $q \approx 32,64$ bits, including STARK primes (Goldilocks, Babybear).

Each ring element can pack τ field elements.

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Commitments can be cheap (see next).

Efficiency of Ajtai

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- $B = 2^8$, so committing to a vector in R potentially requires committing to 8 vectors.

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- Note MT is usually computed on an encoding of v. May make sense using $\rho^{-1} |v|$ size as reference for them.

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Two scenarios.

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If v has small norm:

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Two scenarios.

If v has small norm:

- About 2x faster than MT-Keccak

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MT (Poseidon)	2^16	5s	-	-
MT (Poseidon)	2^19	43s	-	-

Intel 12700F 4.9GHz, 32 GB of RAM

Two scenarios.

If v has small norm:

- About 2x faster than MT-Keccak (Depending if we use $\rho^{-1}|v|$ as reference)

NETHERMIND RESEARCH

Efficiency of Ajtai

Configuration

$\mathbb{F}_q$ = Goldilocks field (64 bits), $q \approx 2^{64}$.

$\mathscr{R} = \mathbb{F}_q[X]/(X^{24} - X^{12} + 1) \cong \mathbb{F}_{q^3} \times \overset{8}{\ldots} \times \mathbb{F}_{q^3}$

Set $\kappa = 10$ and $B = 2^8$.

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- Parameters: $A \in \mathscr{R}^{\kappa \times m}$
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Intel 12700F 4.9GHz, 32 GB of RAM

Two scenarios.

If v has small norm:

- About 2x faster than MT-Keccak (Depending if we use $\rho^{-1}|v|$ as reference)
- >500x faster than MT-Poseidon

NETHERMIND RESEARCH

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If v has large norm:

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Intel 12700F 4.9GHz, 32 GB of RAM

Two scenarios.

If v has large norm:

- >1.5-3x slower than MT-Keccac

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Efficiency of Ajtai

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Intel 12700F 4.9GHz, 32 GB of RAM

Two scenarios.

If v has large norm:

- >1.5-3x slower than MT-Keccac
- >70x faster than MT-Poseidon

NETHERMIND RESEARCH

Efficiency of Ajtai

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NETHERMIND RESEARCH

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	|v|	Commit time
Ajtai	2^16	76ms (0.6s)
Pedersen (small entries)	2^16	41ms
Pedersen (small entries)	2^19	289ms

Ajtai commitment:

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Efficiency of Ajtai

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Set $\kappa = 10$ and $B = 2^8$.

	|v|	Commit time
Ajtai	2^16	76ms (0.6s)
Pedersen (small entries)	2^16	41ms
Pedersen (small entries)	2^19	289ms
Pedersen (large entries)	2^16	533ms

Ajtai commitment:

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If v small norm:

- 7-30x faster than Pedersen

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If v small norm:

- 7-30x faster than Pedersen

If v large norm:

- 0.5-4.3x faster than Pedersen

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Latticefold

Latticefold

Latticefold (Boneh, Chen, 2024)

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- Folding scheme for CCS relations over $\mathscr{R}$

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To fold two witnesses w_1, w_2, one usually takes a linear combination $w_1 + \alpha w_2$

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LF solution: Split w_1, w_2 into vectors of small norm, commit to them, and take linear combination of these.

Latticefold

Experiments

$|w_1|, |w_2| = 2^{16}$

LF cost: 3s

LF+ expected cost: 0.4s

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To fold two witnesses w_1, w_2, one usually takes a linear combination $w_1 + \alpha w_2$

Problem: We need extractor to output w_1, w_2 with $\|w_1\|, \|w_2\| \leq B$

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Range check performed by running a sumcheck per chunk committed in the decomposition step

Latticefold

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$|w_1|, |w_2| = 2^{16}$

LF cost: 3s

LF+ expected cost: 0.6-1s

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Linearization

Transforms CCS into a claim about evaluations of certain multilinear polynomials

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Linearization

Transforms CCS into a claim about evaluations of certain multilinear polynomials

Analogous to Hypernova's linearization

Latticefold

Experiments

$|w_1|, |w_2| = 2^{16}$

LF cost: 170ms

LF+ expected cost: 170ms

Linearization

Transforms CCS into a claim about evaluations of certain multilinear polynomials

Analogous to Hypernova's linearization

Latticefold

Configuration

$\mathbb{F}_q$ = Goldilocks field (64 bits), $q \approx 2^{64}$.

$\mathcal{R} = \mathbb{F}_q[X]/(X^{24} - X^{12} + 1) \cong \mathbb{F}_{q^3} \times \overset{8}{\ldots} \times \mathbb{F}_{q^3}$

Set $\kappa = 10$ and $B = 2^8$.

Experiments

$|w_1|, |w_2| = 2^{16}$

LF cost: 170ms

LF+ expected cost: 170ms

Linearization

Transforms CCS into a claim about evaluations of certain multilinear polynomials

Analogous to Hypernova's linearization

Overall costs

NETHERMIND RESEARCH

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Overall costs

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NETHERMIND RESEARCH

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Overall costs

Latticefold

Configuration

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Latticefold

CCS witness
2^13

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Latticefold

CCS witness	|w_1|, |w_2|
2^13	2^16

NETHERMIND RESEARCH

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

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Latticefold

CCS witness	|w_1|, |w_2|	Linearize
2^13	2^16	170ms

NETHERMIND RESEARCH

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Overall costs

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Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp
2^13	2^16	170ms	3s

NETHERMIND RESEARCH

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Overall costs

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Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound
2^13	2^16	170ms	3s	3s

NETHERMIND RESEARCH

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

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Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

NETHERMIND RESEARCH

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

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Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

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AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

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CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

CCS witness
2^13

NETHERMIND RESEARCH

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Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

CCS witness	|w_1|, |w_2|
2^13	2^16

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AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

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Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

CCS witness	|w_1|, |w_2|	Linearize
2^13	2^16	170ms

NETHERMIND RESEARCH

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Overall costs

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Set $\kappa = 10$ and $B = 2^8$.

Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

CCS witness	|w_1|, |w_2|	Linearize	LF+ decomp
2^13	2^16	170ms	~400ms

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AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Overall costs

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Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

CCS witness	|w_1|, |w_2|	Linearize	LF+ decomp	LF norm bound
2^13	2^16	170ms	~400ms	~600ms

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AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

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Set $\kappa = 10$ and $B = 2^8$.

Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

CCS witness	|w_1|, |w_2|	Linearize	LF+ decomp	LF norm bound	Total
2^13	2^16	170ms	~400ms	~600ms	~1.17s

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2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

CCS witness	|w_1|, |w_2|	Linearize	LF+ decomp	LF norm bound	Total
2^13	2^16	170ms	~400ms	~600ms	~1.17s

Hypernova (from Sonobe implementation)

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AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

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Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

CCS witness	|w_1|, |w_2|	Linearize	LF+ decomp	LF norm bound	Total
2^13	2^16	170ms	~400ms	~600ms	~1.17s

Hypernova (from Sonobe implementation)

CCS witness	Total
2^16	1.2s

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AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Overall costs

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Latticefold

CCS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

CCS witness	|w_1|, |w_2|	Linearize	LF+ decomp	LF norm bound	Total
2^13	2^16	170ms	~400ms	~600ms	~1.17s

Hypernova (from Sonobe implementation)

CCS witness	Total
2^16	1.2s

Neo (Nguyen, Setty, 2025)

WIP! (Very promising as well)

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AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Overall costs

Thanks

. bits), $q \approx 2^{64}$.

$-X^{12}+1) \cong \mathbb{F}_{q^3} \times \overset{8}{\ldots} \times \mathbb{F}_{q^3}$

and $B = 2^8$.

Latticefold

CCS witr

	LF norm bound	Total
3s	3s	6.17s

..ed, speculative)

	..nearize	LF+ decomp	LF norm bound	Total
..16	170ms	~400ms	~600ms	~1.17s

..nova (from Sonobe implementation)

CCS witness	Total
2^16	1.2s

Benchmarks

Repo

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

A toy folding scheme

A toy folding scheme

$R = \{(cm; w) \mid Com(w) = cm\}$, Com homomorphic commitment scheme.

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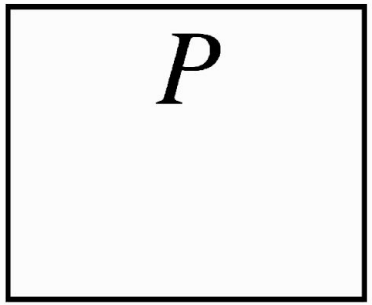

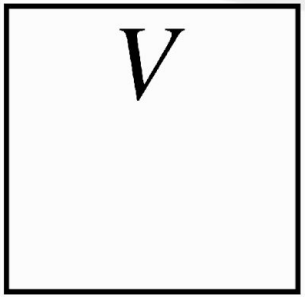

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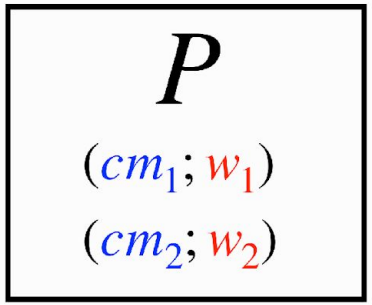

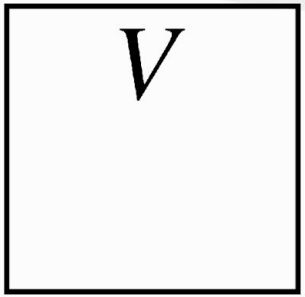

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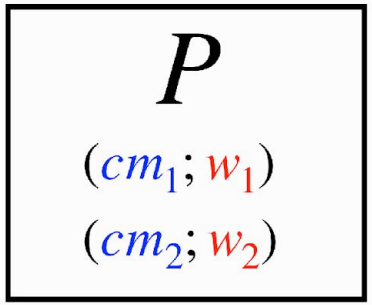

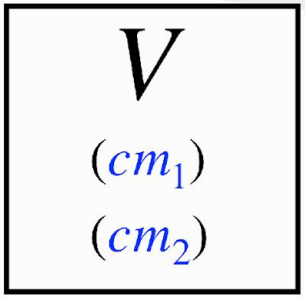

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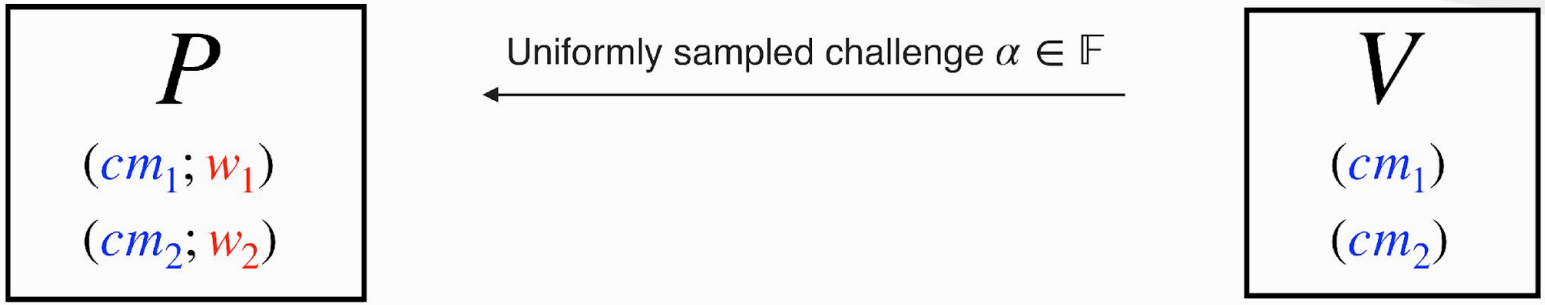

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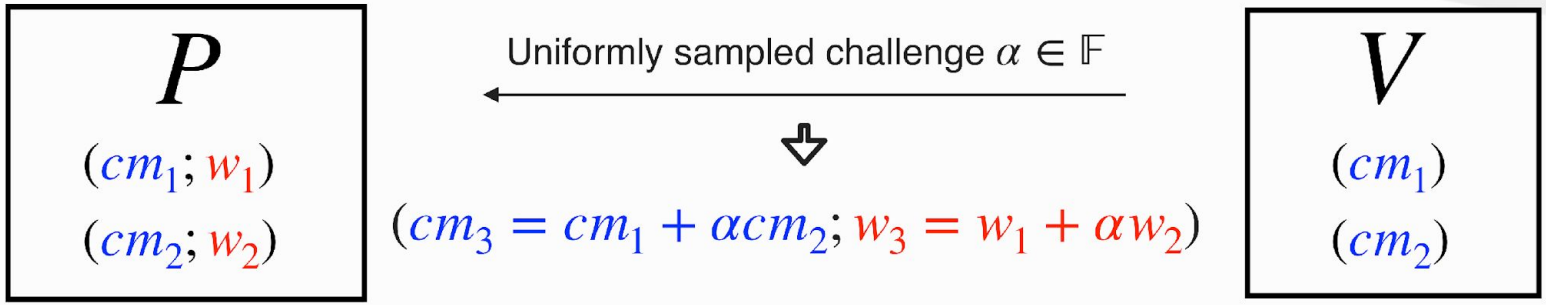

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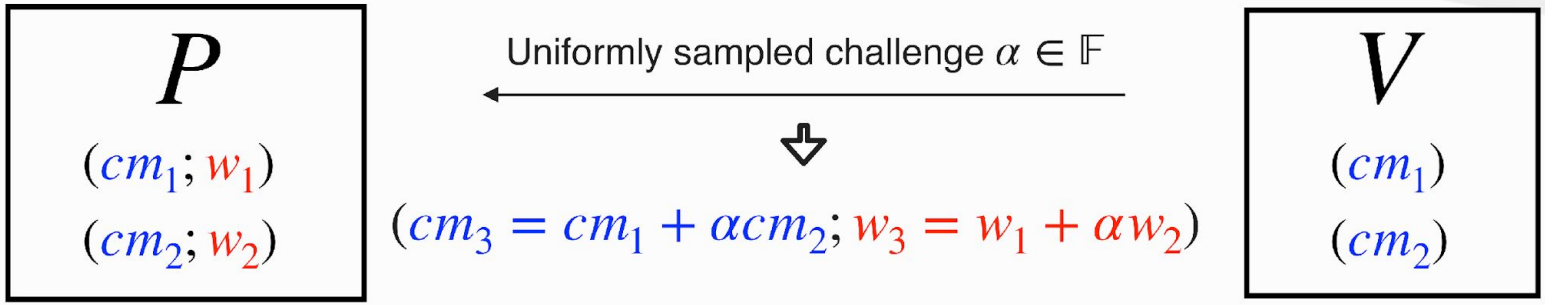

Does not work when Com is the Ajtai commitment.

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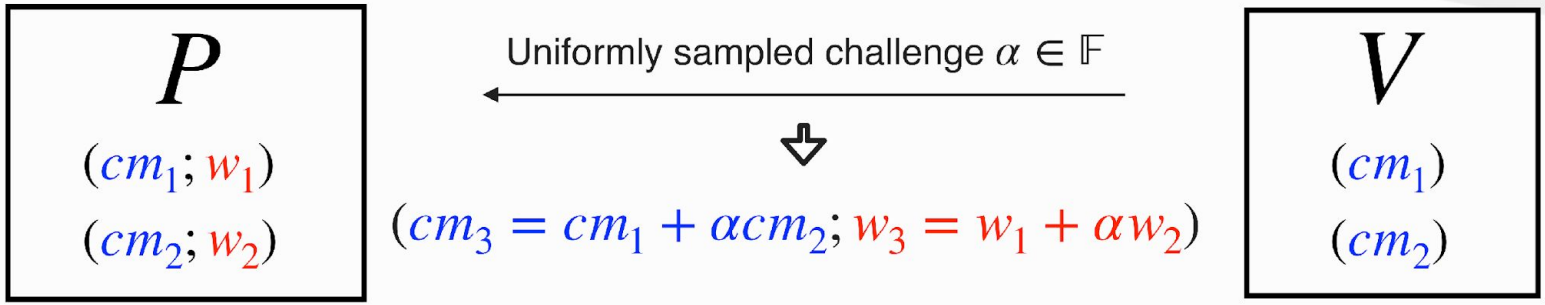

Does not work when Com is the Ajtai commitment.

Problem 1

A toy folding scheme

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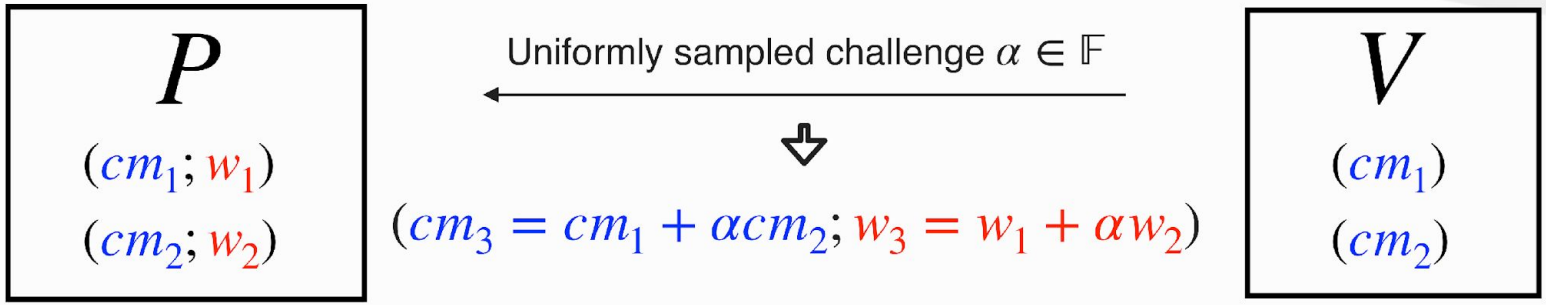

Does not work when Com is the Ajtai commitment.

Problem 1

Need to ask that $\|w\| \leq B$.

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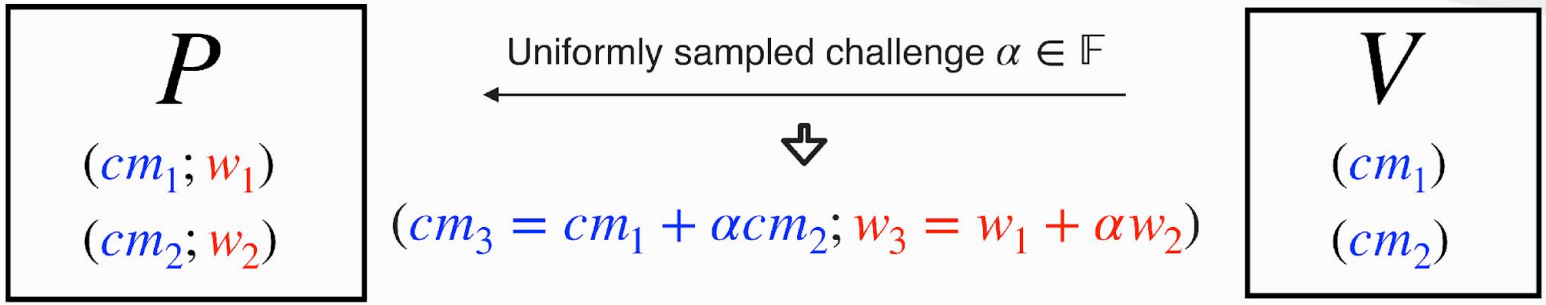


Does not work when Com is the Ajtai commitment.

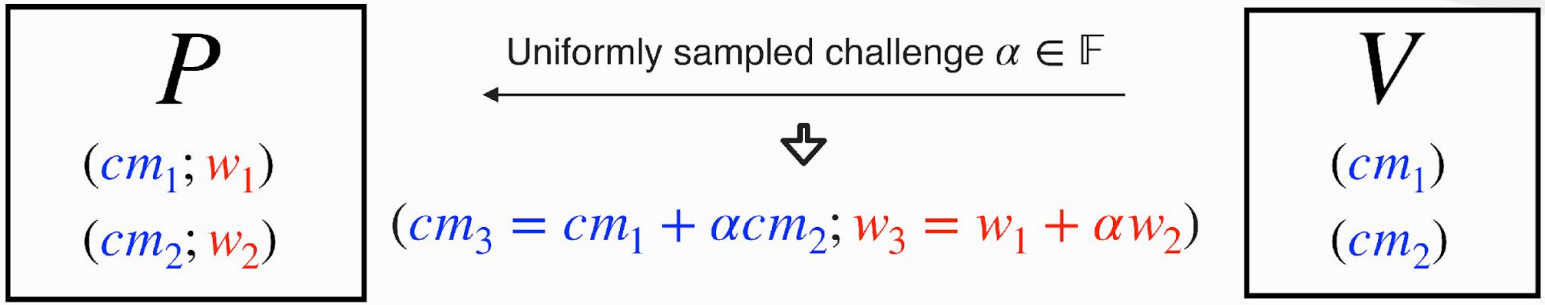
Problem 1

Need to ask that $\|w\| \leq B$.

$R_B = \{(cm; w) \mid Com(w) = cm, \ \|w\| \leq B\}$

A toy folding scheme

$R = \{(cm; w) \mid Com(w) = cm\}$, Com homomorphic commitment scheme.



Does not work when Com is the Ajtai commitment.

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Need to ask that $\|w\| \leq B$.

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Problem 2

A toy folding scheme

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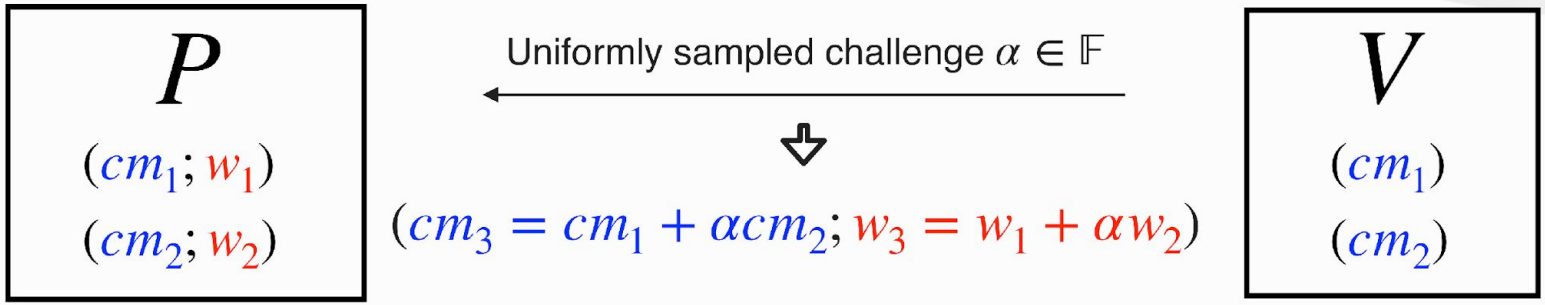


Does not work when Com is the Ajtai commitment.

Problem 1

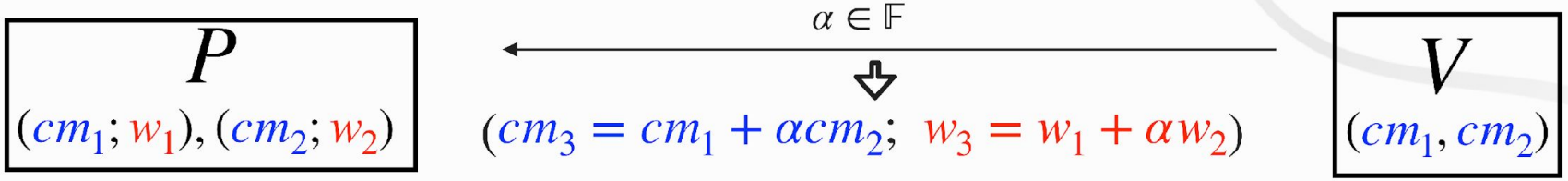
Need to ask that $\|w\| \leq B$.

$R_B = \{(cm; w) \mid Com(w) = cm, \ \|w\| \leq B\}$

Problem 2

Maybe $\|w_1 + \alpha w_2\| > B$.

Solving norm increases



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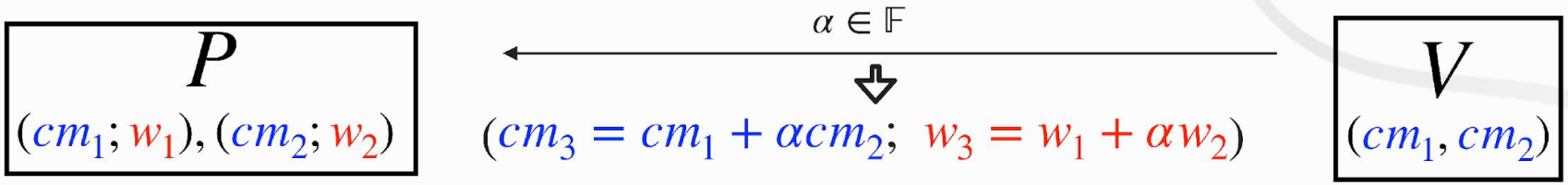
Solving norm increases



Problem 2 Maybe $\|w_1 + \alpha w_2\| > B$

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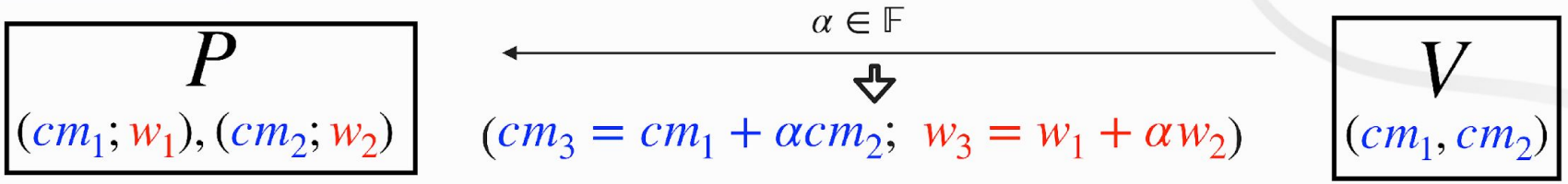
Solving norm increases



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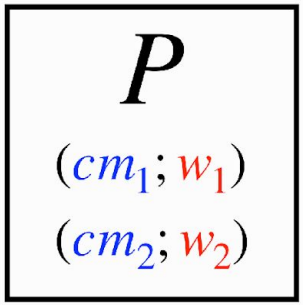

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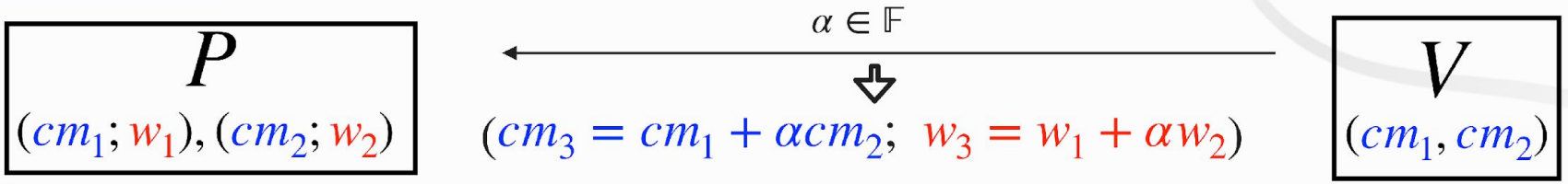
Solving norm increases



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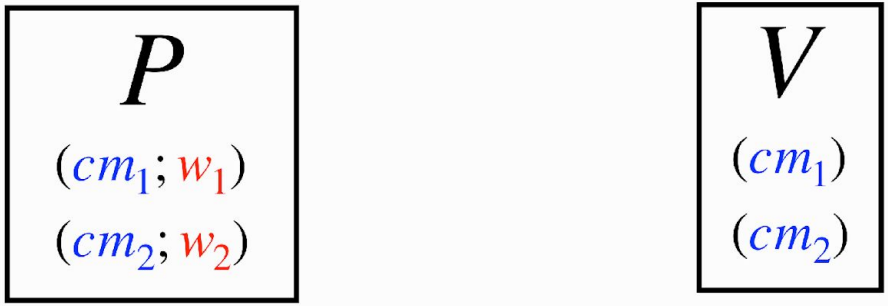
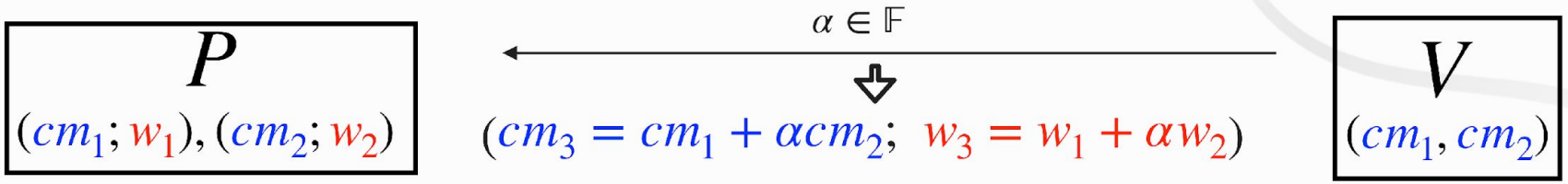


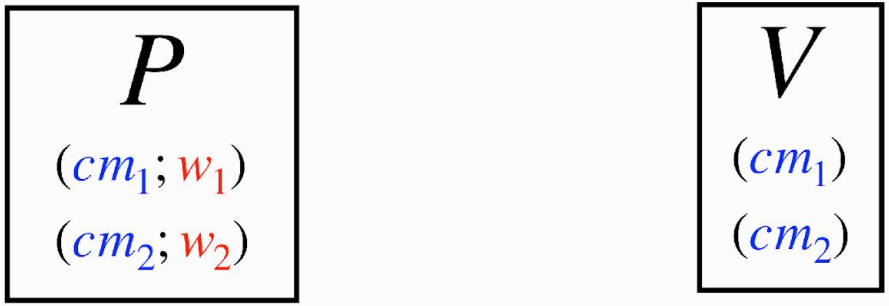
NETHERMIND RESEARCH

Solving norm increases



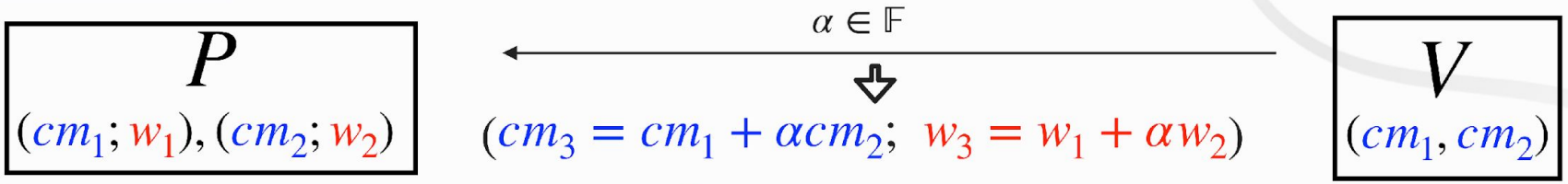
Problem 2 Maybe $\|w_1 + \alpha w_2\| > B$

Solution (Latticefold)



NETHERMIND RESEARCH

Solving norm increases



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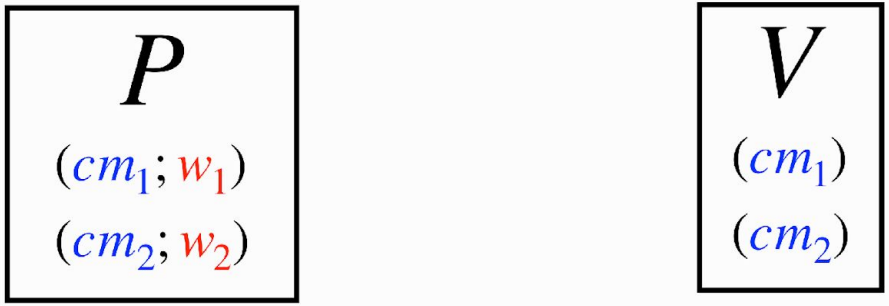
Solution (Latticefold)

P writes $b^2 = B$ and



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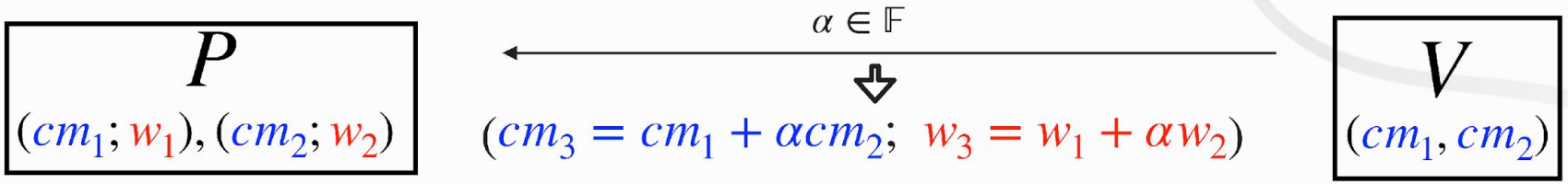

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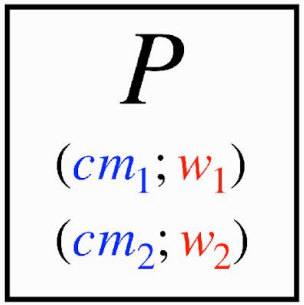

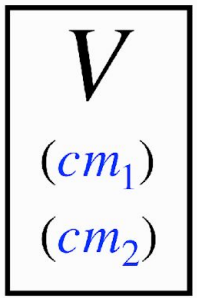

Solution (Latticefold)

P writes $b^2 = B$ and

$$w_1 = w_{11} + w_{12} b$$
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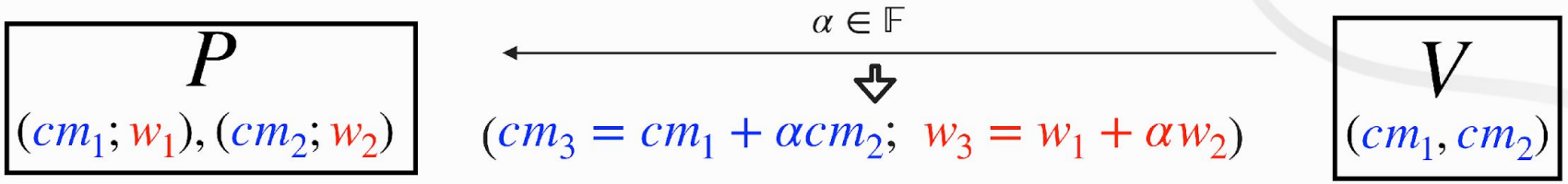
Solving norm increases



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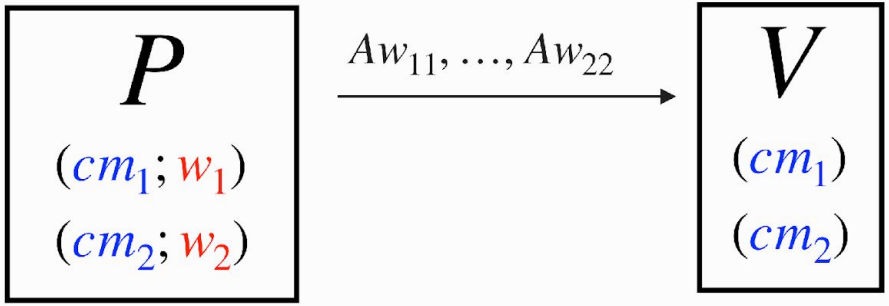

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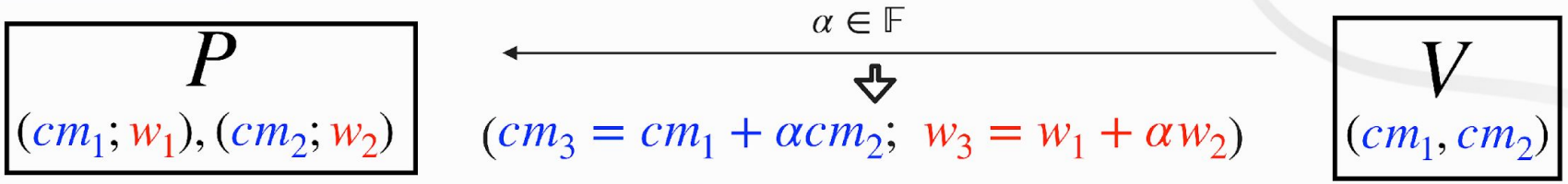

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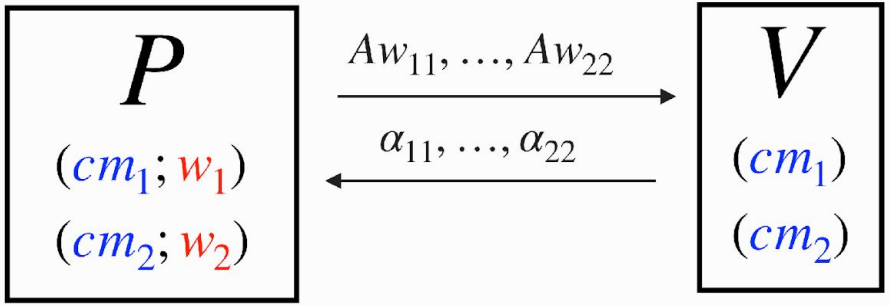

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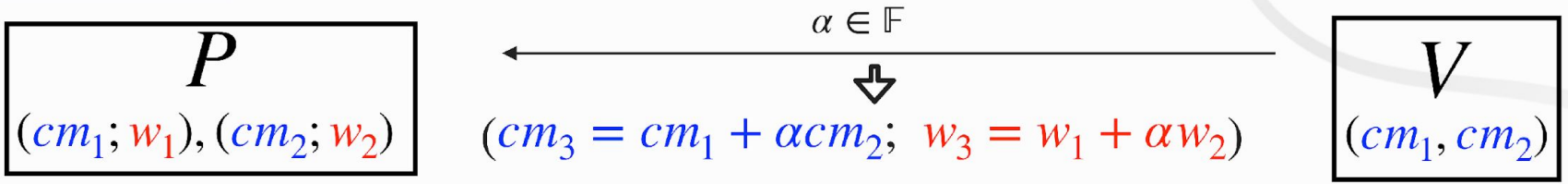

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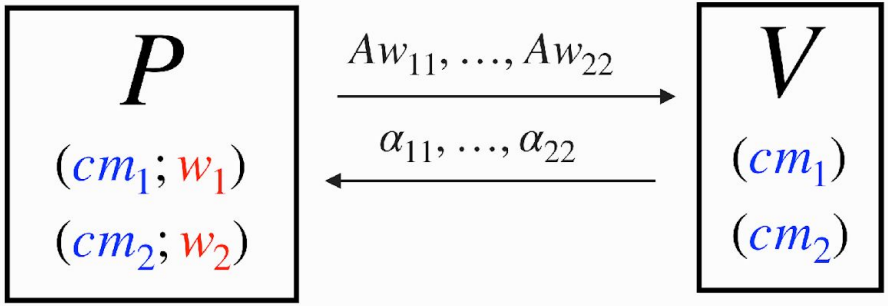

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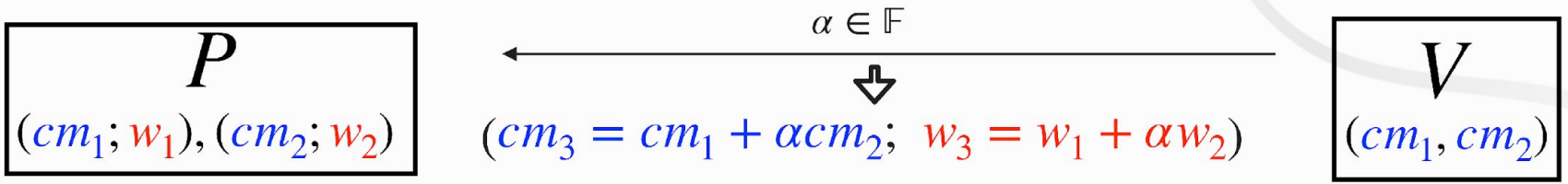
Solving norm increases



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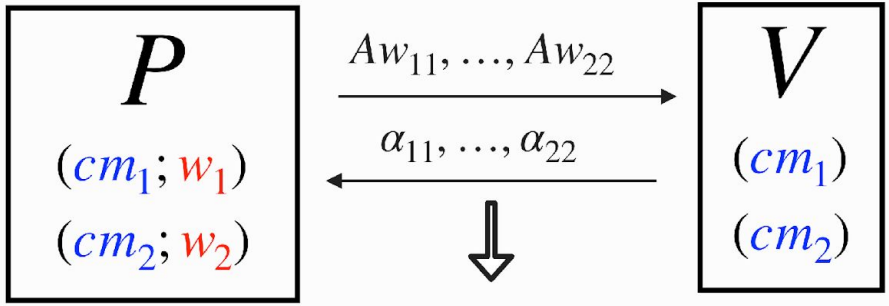

$$\left(\begin{array}{c} cm_3 = \alpha_{11} A w_{11} + \ldots + \alpha_{22} A w_{22}; \\ w_3 = \alpha_{11} w_{11} + \ldots + \alpha_{22} w_{22} \end{array}\right)$$

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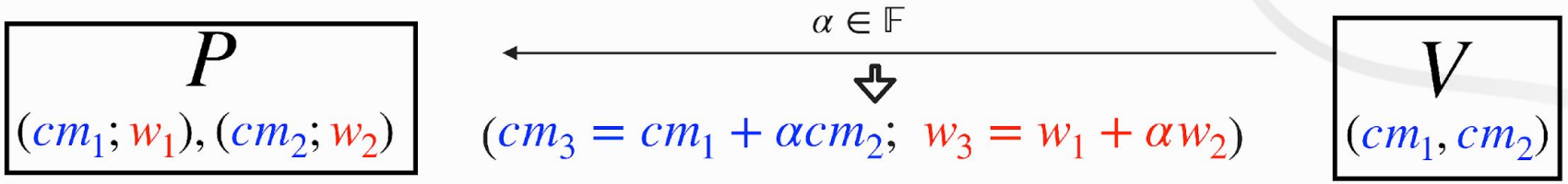
NETHERMIND RESEARCH

Solving norm increases



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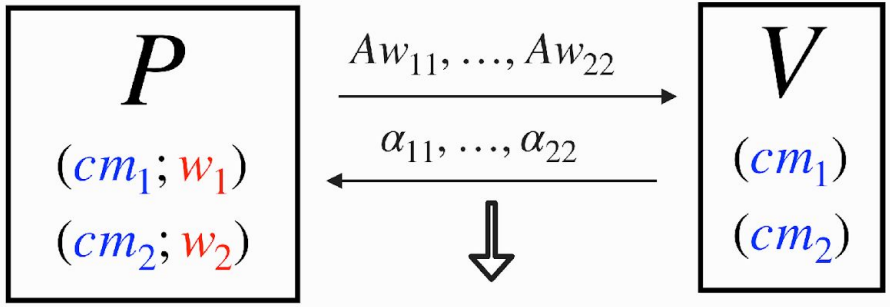

$$\left(\begin{array}{c} cm_3 = \alpha_{11} A w_{11} + \ldots + \alpha_{22} A w_{22}; \\ w_3 = \alpha_{11} w_{11} + \ldots + \alpha_{22} w_{22} \end{array}\right)$$

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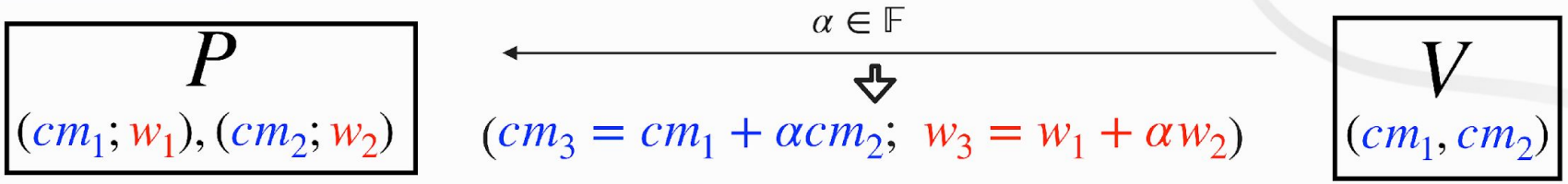
Call the w_{ij} chunks

NETHERMIND RESEARCH

Solving norm increases



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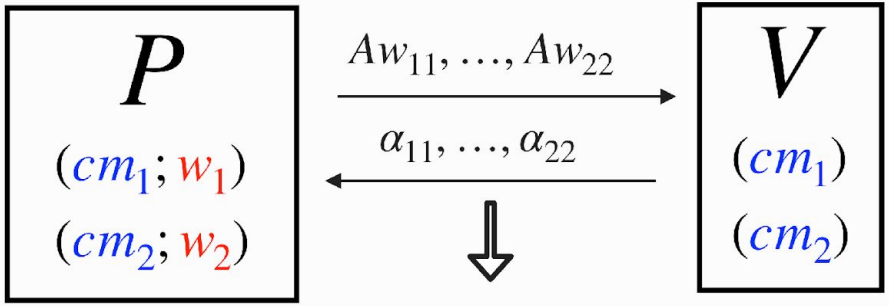
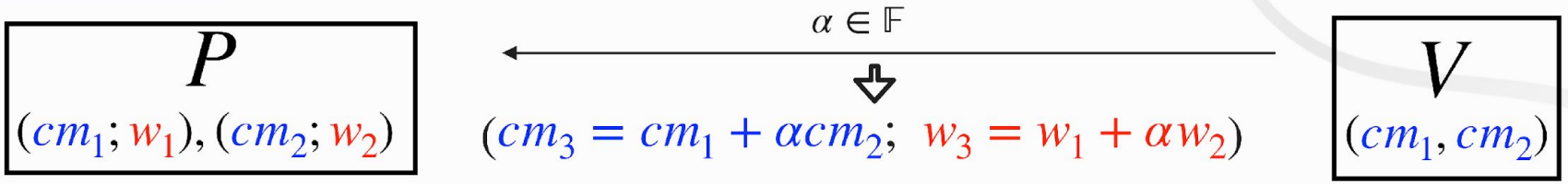


$$\begin{pmatrix} cm_3 = \alpha_{11} Aw_{11} + \ldots + \alpha_{22} Aw_{22}; \\ w_3 = \alpha_{11} w_{11} + \ldots + \alpha_{22} w_{22} \end{pmatrix}$$

Call the w_{ij} chunks

NETHERMIND RESEARCH

Solving norm increases



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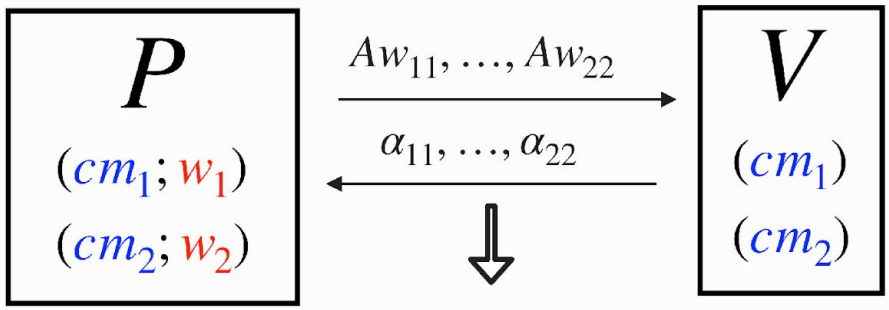


Solution (Latticefold)

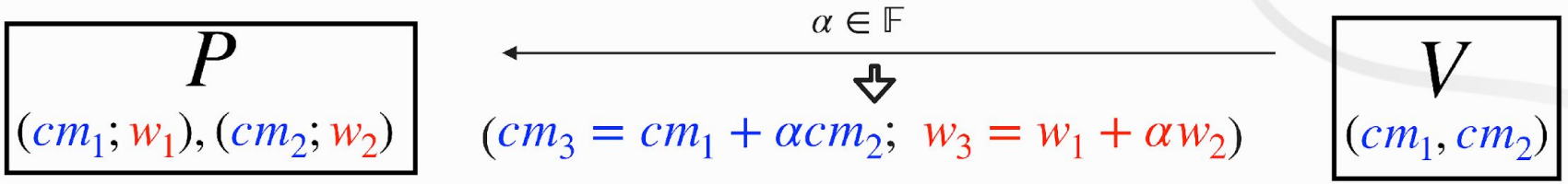
Called decomposition step

$$\begin{pmatrix} cm_3 = \alpha_{11} Aw_{11} + \ldots + \alpha_{22} Aw_{22}; \\ w_3 = \alpha_{11} w_{11} + \ldots + \alpha_{22} w_{22} \end{pmatrix}$$

Call the w_{ij} chunks

NETHERMIND RESEARCH

Solving norm increases



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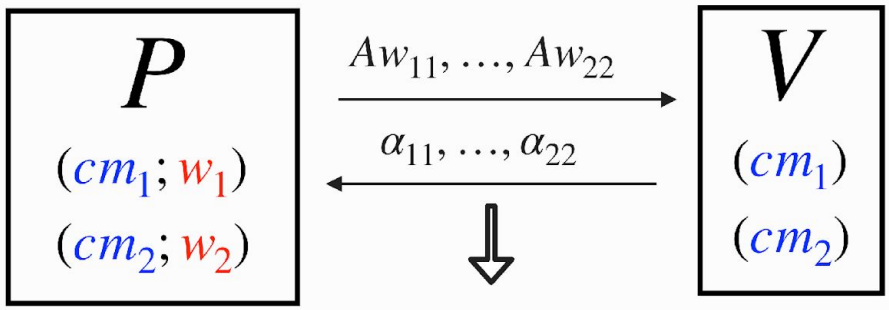


$$\begin{pmatrix} cm_3 = \alpha_{11} A w_{11} + \ldots + \alpha_{22} A w_{22}; \\ w_3 = \alpha_{11} w_{11} + \ldots + \alpha_{22} w_{22} \end{pmatrix}$$

Solution (Latticefold)

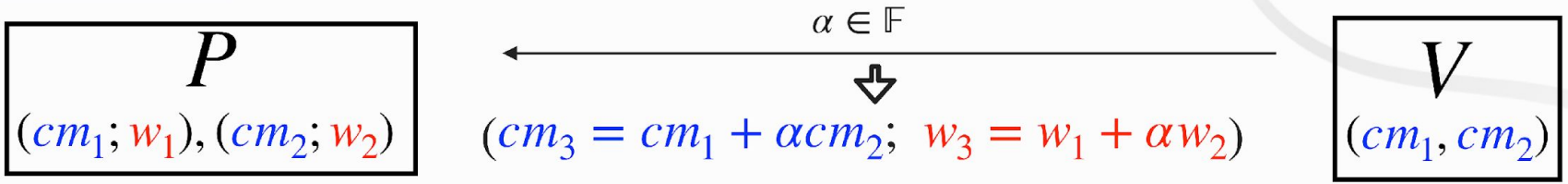
Called decomposition step

In LF, it requires splitting each w_i in many more than 2 chunks.

Call the w_{ij} chunks

NETHERMIND RESEARCH

Solving norm increases



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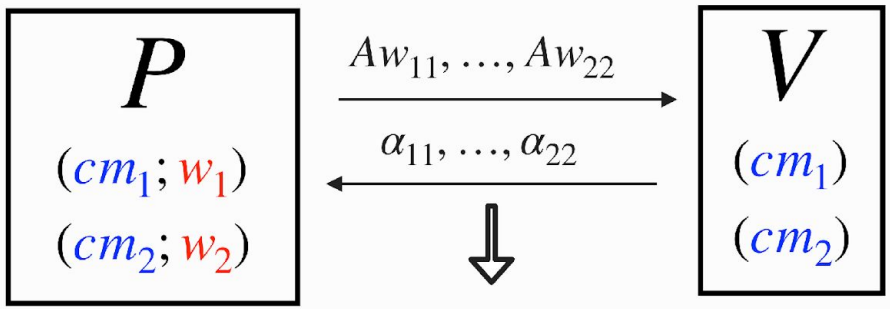

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Solution (Latticefold)

Called decomposition step

In LF, it requires splitting each w_i in many more than 2 chunks.

LF+, manages to make this step almost costless.

Call the w_{ij} chunks

NETHERMIND RESEARCH

Solving norm increases

Configuration

$\mathbb{F}_q$ = Goldilocks field (64 bits), $q \approx 2^{64}$.

$\mathscr{R} = \mathbb{F}_q[X]/(X^{24} - X^{12} + 1) \cong \mathbb{F}_{q^3} \times \overset{8}{\ldots} \times \mathbb{F}_{q^3}$

Set $\kappa = 10$ and $B = 2^8$.

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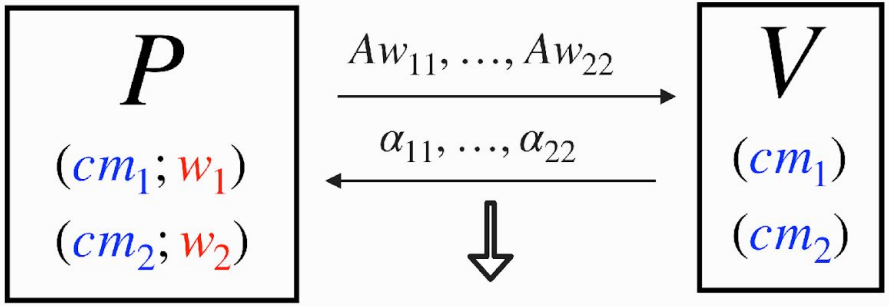

$$\left(\begin{array}{c} cm_3 = \alpha_{11} A w_{11} + \ldots + \alpha_{22} A w_{22}; \\ w_3 = \alpha_{11} w_{11} + \ldots + \alpha_{22} w_{22} \end{array}\right)$$

Call the w_{ij} chunks

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

NETHERMIND RESEARCH

Solving norm increases

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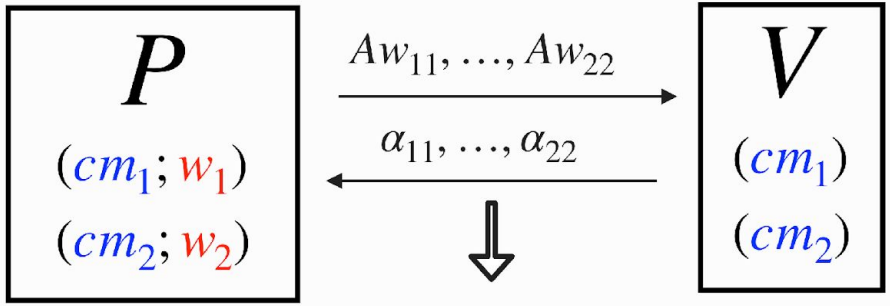
Experiments



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Call the w_{ij} chunks

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NETHERMIND RESEARCH

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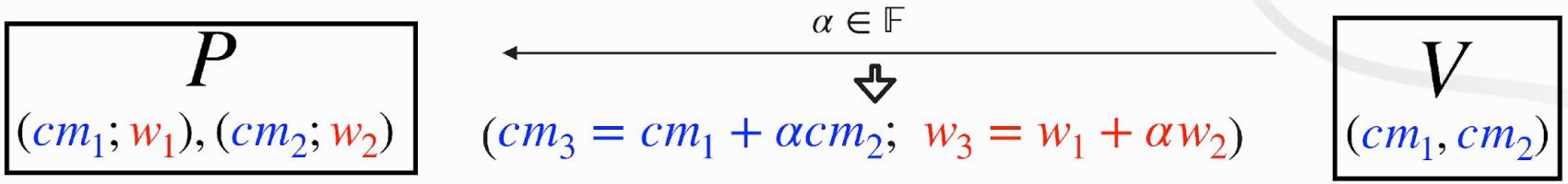

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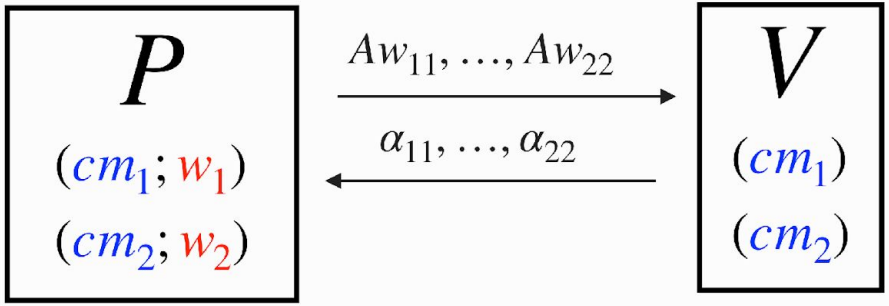


Solution (Latticefold)

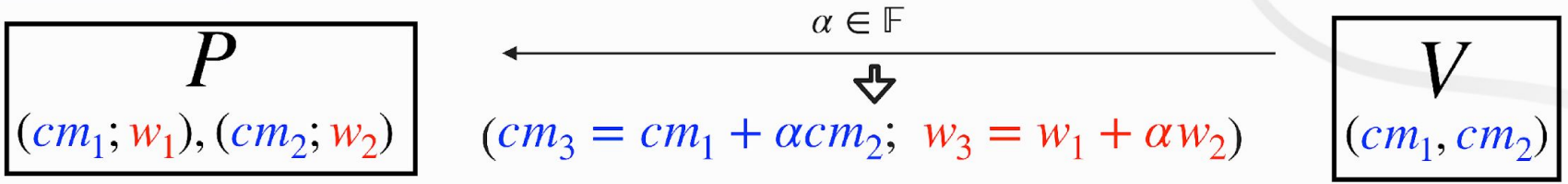
P writes $b^2 = B$ and

$$w_1 = w_{11} + w_{12}b$$

$$w_2 = w_{21} + w_{22}b$$

V sends $\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}$ sampled in certain subset of $\mathcal{R}$ that guarantees

Solving norm increases



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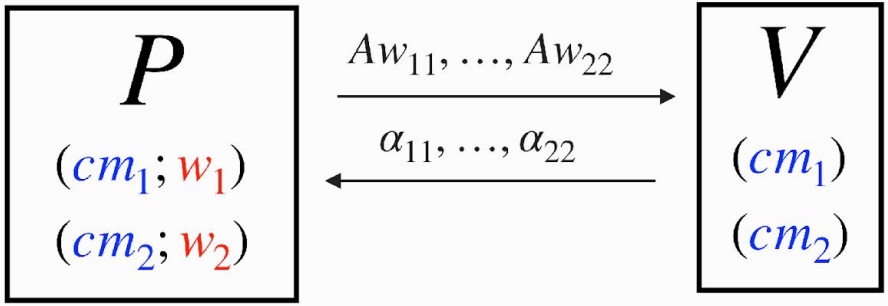

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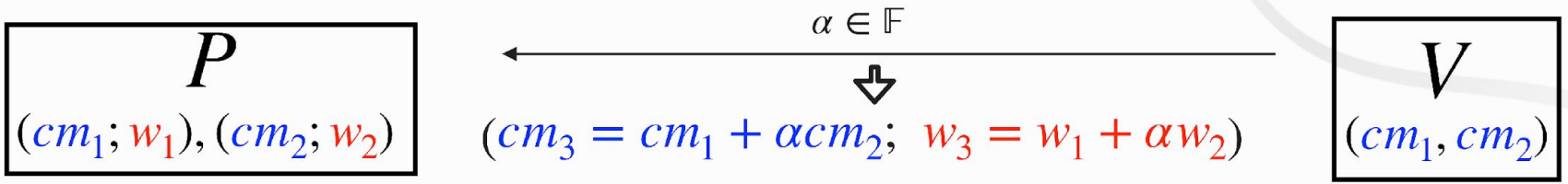

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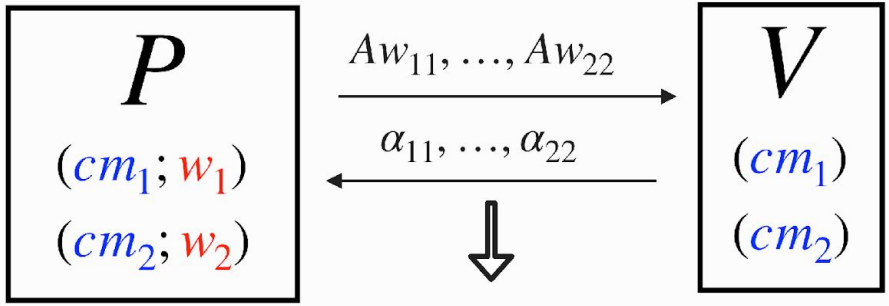

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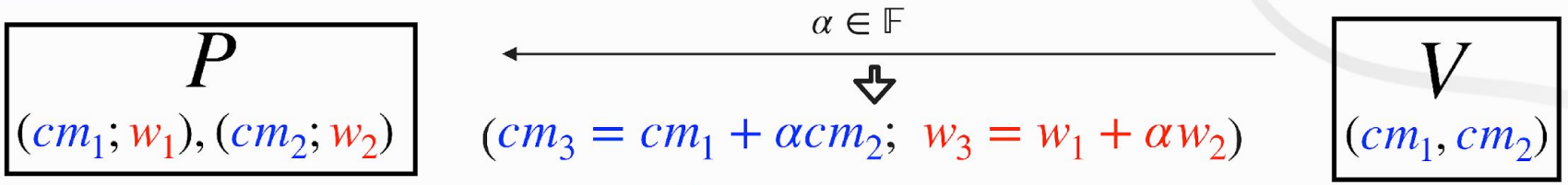
NETHERMIND RESEARCH

Solving norm increases



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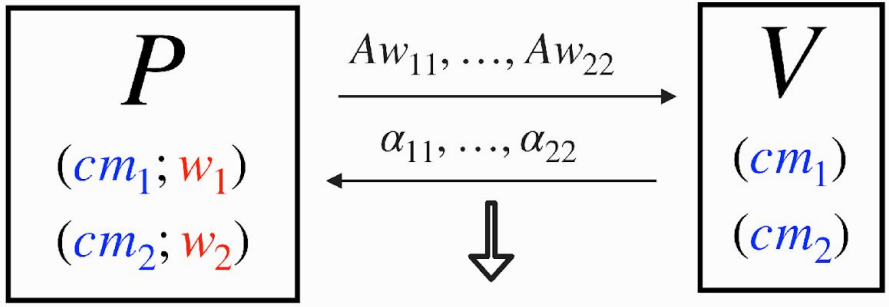

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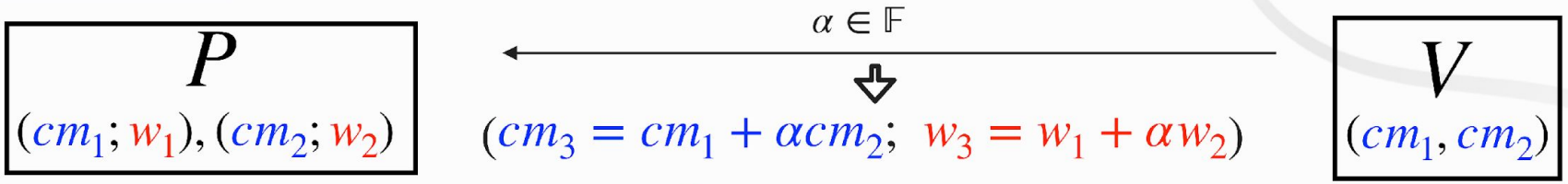
Call the w_{ij} chunks

NETHERMIND RESEARCH

Solving norm increases



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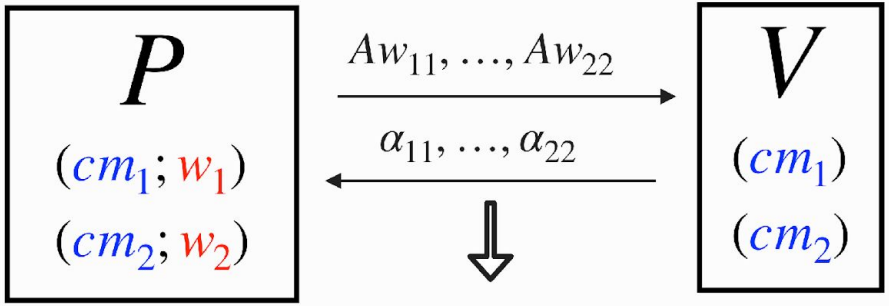
Solution (Latticefold)

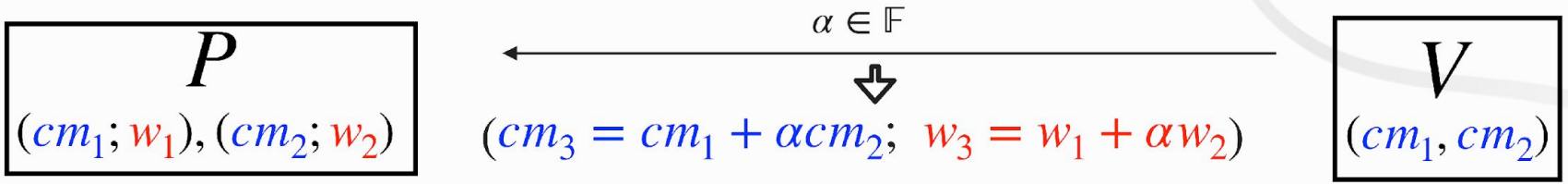


$$\begin{pmatrix} cm_3 = \alpha_{11} Aw_{11} + \ldots + \alpha_{22} Aw_{22}; \\ w_3 = \alpha_{11} w_{11} + \ldots + \alpha_{22} w_{22} \end{pmatrix}$$

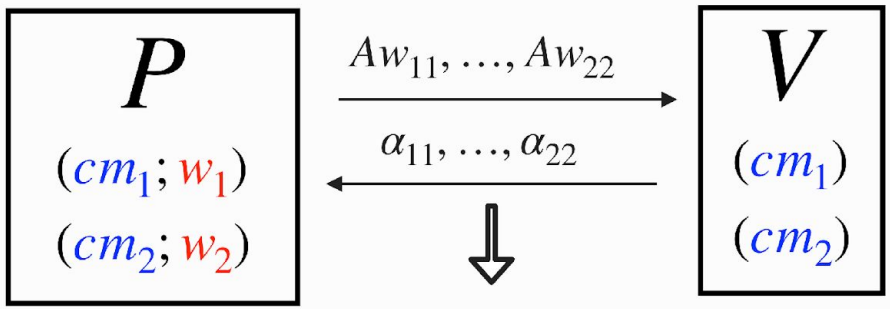
Call the w_{ij} chunks

NETHERMIND RESEARCH

Solving norm increases



Problem 2 Maybe $\|w_1 + \alpha w_2\| > B$



Solution (Latticefold)

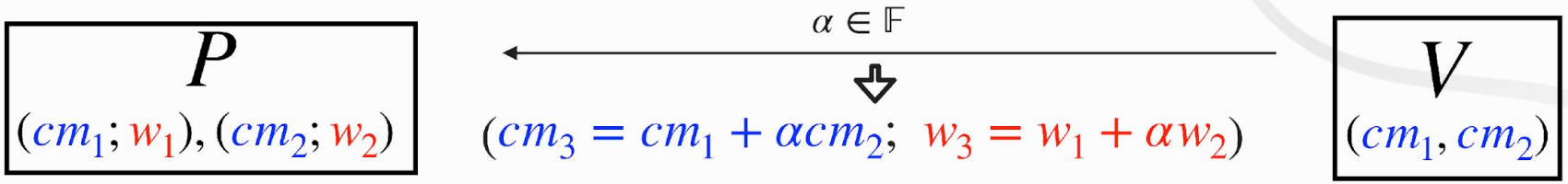
Called decomposition step

$$\begin{pmatrix} cm_3 = \alpha_{11} Aw_{11} + \ldots + \alpha_{22} Aw_{22}; \\ w_3 = \alpha_{11} w_{11} + \ldots + \alpha_{22} w_{22} \end{pmatrix}$$

Call the w_{ij} chunks

NETHERMIND RESEARCH

Solving norm increases



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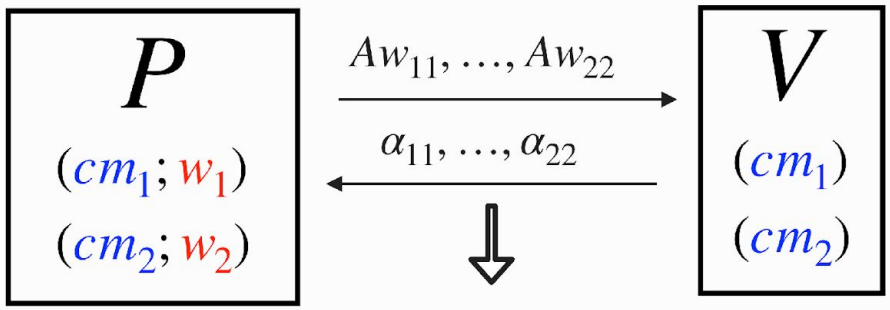


$$\begin{pmatrix} cm_3 = \alpha_{11} A w_{11} + \ldots + \alpha_{22} A w_{22}; \\ w_3 = \alpha_{11} w_{11} + \ldots + \alpha_{22} w_{22} \end{pmatrix}$$

Solution (Latticefold)

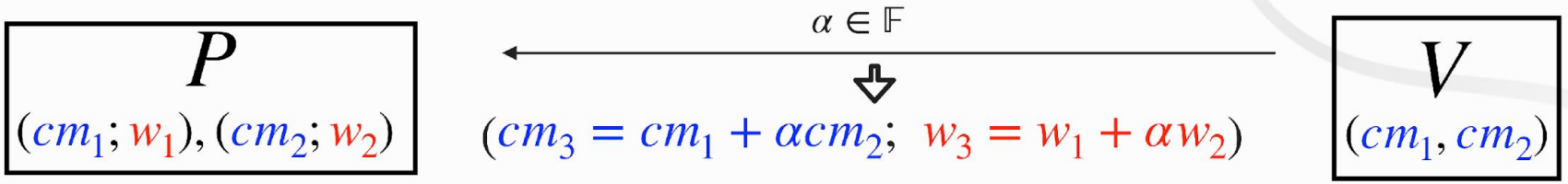
Called decomposition step

In LF, it requires splitting each w_i in many more than 2 chunks.

Call the w_{ij} chunks

NETHERMIND RESEARCH

Solving norm increases



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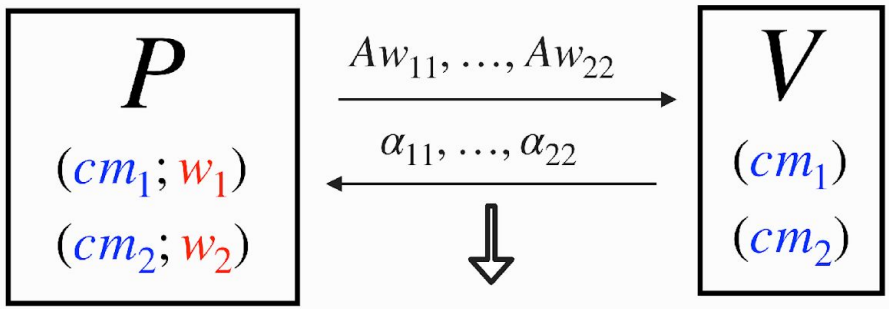

$$\begin{pmatrix} cm_3 = \alpha_{11} Aw_{11} + \ldots + \alpha_{22} Aw_{22}; \\ w_3 = \alpha_{11} w_{11} + \ldots + \alpha_{22} w_{22} \end{pmatrix}$$

Solution (Latticefold)

Called decomposition step

In LF, it requires splitting each w_i in many more than 2 chunks.

LF+, manages to make this step almost costless.

Call the w_{ij} chunks

NETHERMIND RESEARCH

Solving norm increases

Configuration

$\mathbb{F}_q$ = Goldilocks field (64 bits), $q \approx 2^{64}$.

$\mathscr{R} = \mathbb{F}_q[X]/(X^{24} - X^{12} + 1) \cong \mathbb{F}_{q^3} \times \overset{8}{\ldots} \times \mathbb{F}_{q^3}$

Set $\kappa = 10$ and $B = 2^8$.

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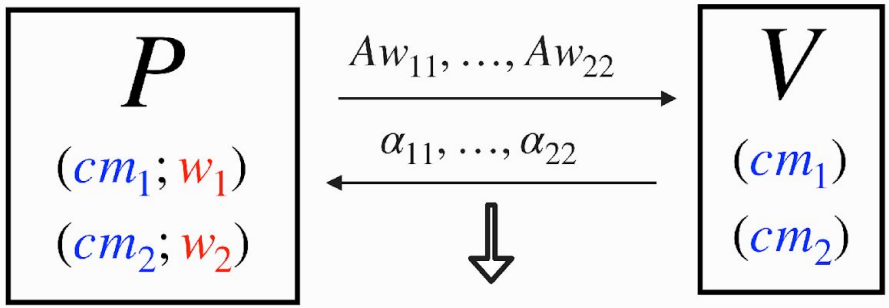

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Call the w_{ij} chunks

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

NETHERMIND RESEARCH

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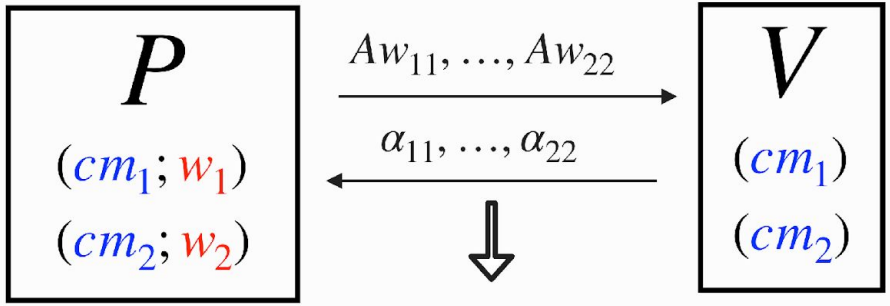
Experiments



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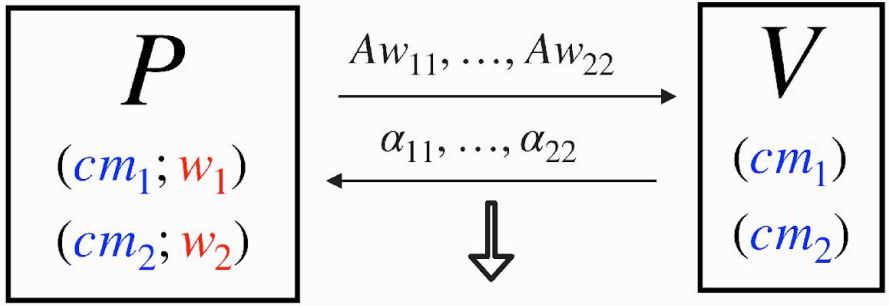

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Experiments

$|w_1|, |w_2| = 2^{16}$

Call the w_{ij} chunks

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

NETHERMIND RESEARCH

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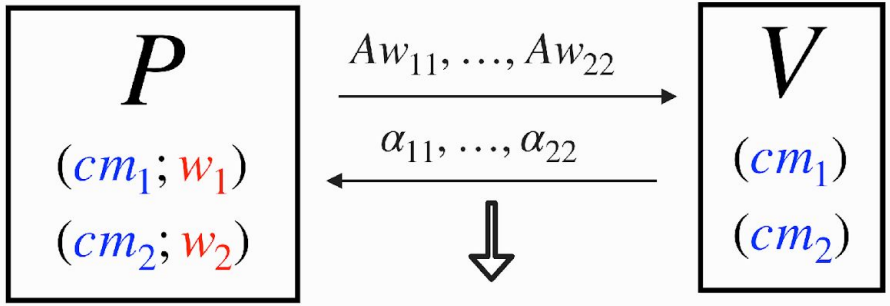

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Experiments

$|w_1|, |w_2| = 2^{16}$

Use 8 chunks per w_i

Call the w_{ij} chunks

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

NETHERMIND RESEARCH

Solving norm increases

Configuration

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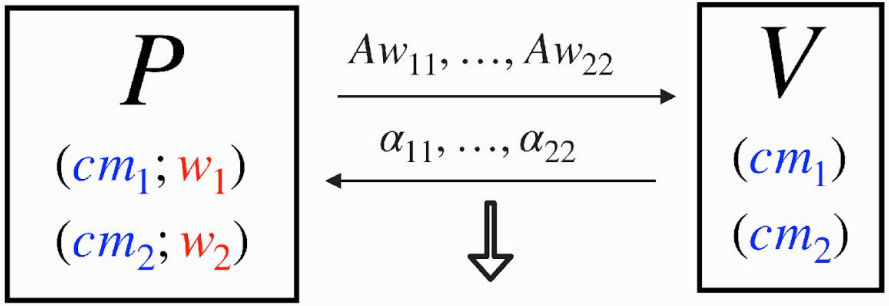

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Use 8 chunks per w_i

LF cost: 3s

Call the w_{ij} chunks

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

NETHERMIND RESEARCH

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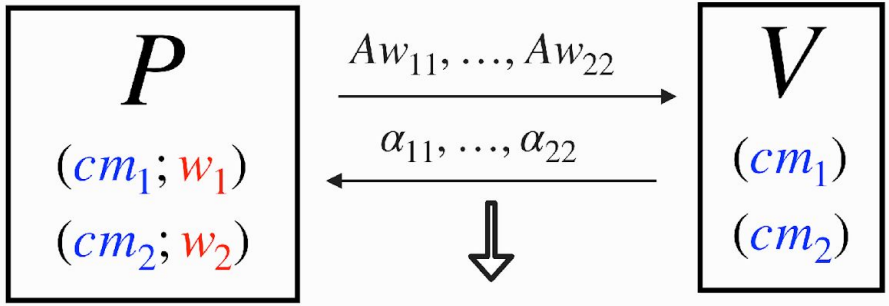

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Experiments

$|w_1|, |w_2| = 2^{16}$

Use 8 chunks per w_i

LF cost: 3s

LF+ needs only 2 chunks in total

Call the w_{ij} chunks

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

NETHERMIND RESEARCH

Solving norm increases

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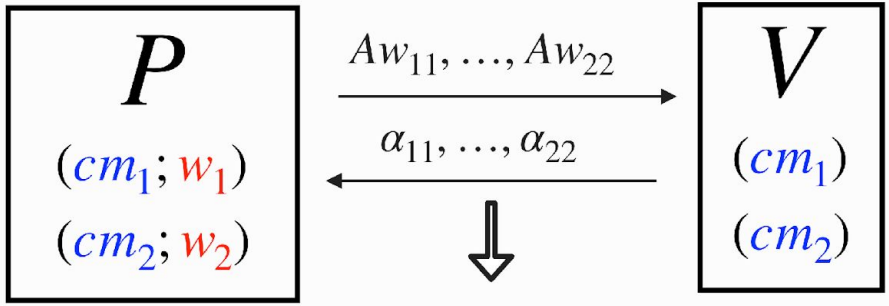

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Experiments

$|w_1|, |w_2| = 2^{16}$

Use 8 chunks per w_i

LF cost: 3s

LF+ needs only 2 chunks in total

LF+ expected cost: 0.4s

Call the w_{ij} chunks

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

NETHERMIND RESEARCH

Solving norm increases

Configuration

$\mathbb{F}_q$ = Goldilocks field (64 bits), $q \approx 2^{64}$.

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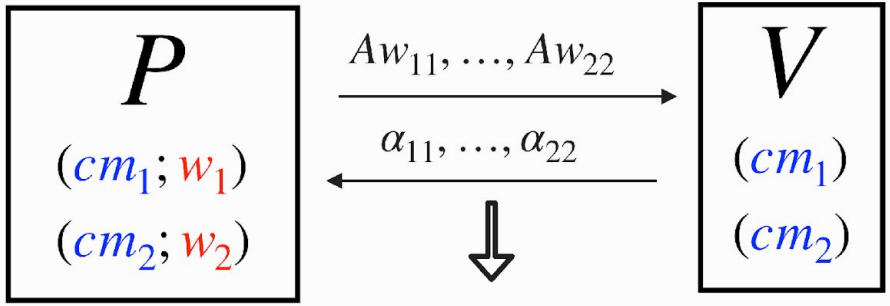

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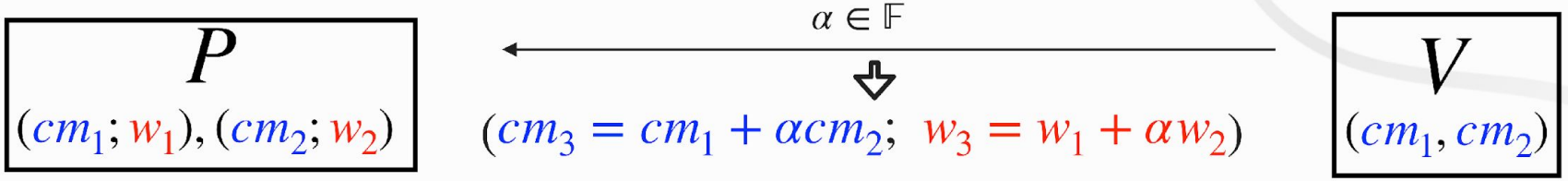
Call the w_{ij} chunks

NETHERMIND RESEARCH

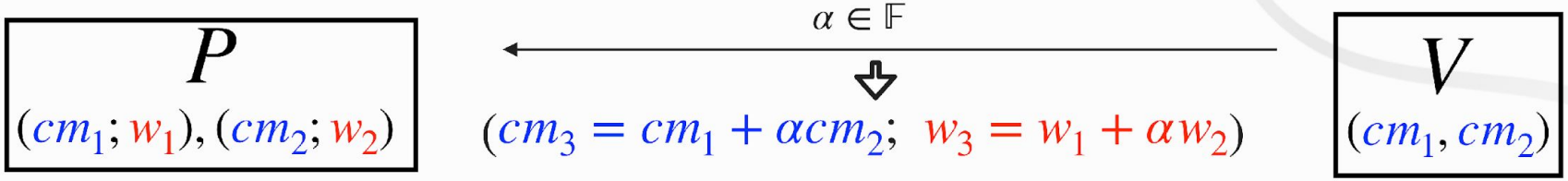
AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Solving norm bounds



NETHERMIND RESEARCH

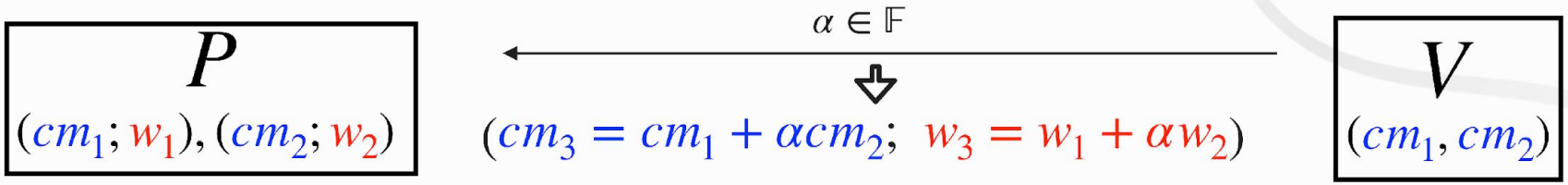
Solving norm bounds



Problem 2 Need to ask that $\|w\| \leq B$. $R_B = \{(cm; w) \mid Com(w) = cm,\ \|w\| \leq B\}$

NETHERMIND RESEARCH

Solving norm bounds



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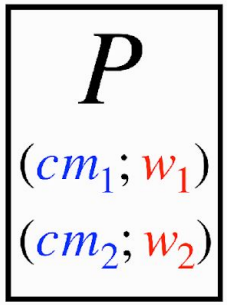
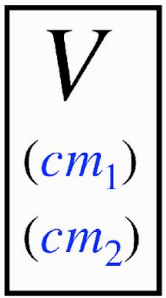
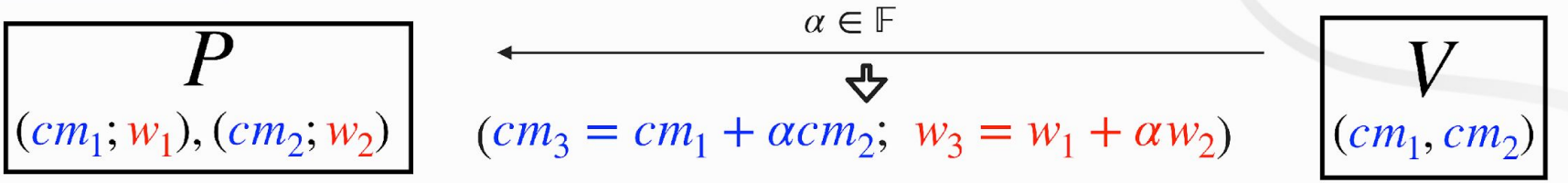
Solution (Latticefold)





NETHERMIND RESEARCH

Solving norm bounds



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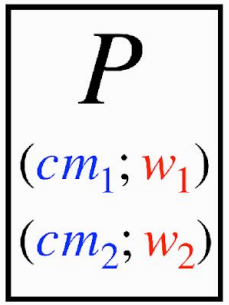
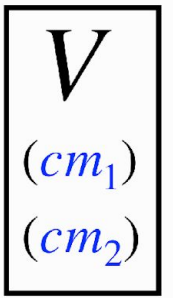
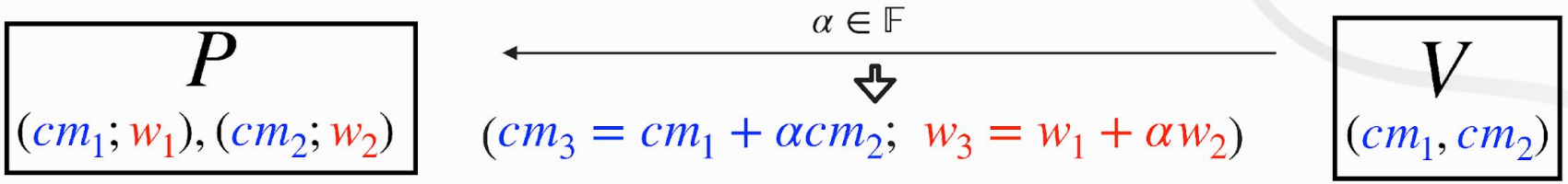




Solution (Latticefold)

Replace $\|w_i\| < B$ by

Solving norm bounds



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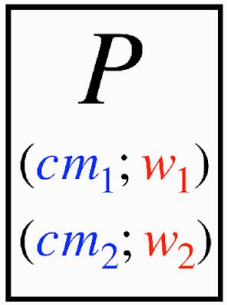
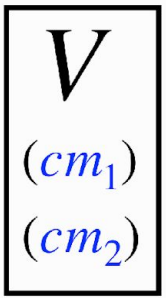




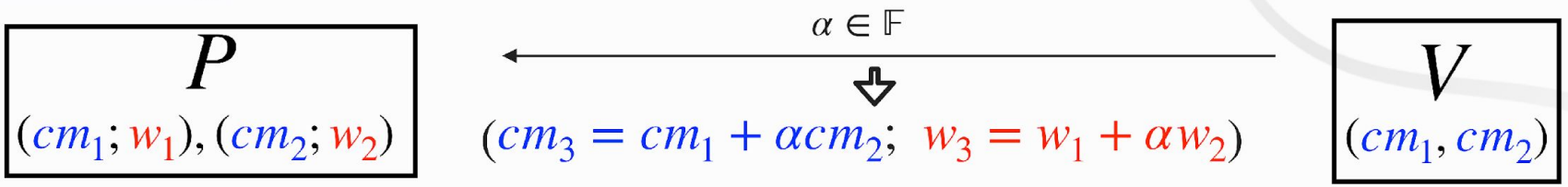
Solution (Latticefold)

Replace $\|w_i\| < B$ by

$\Pi_{-B<j<B} \left(\text{mle}[w_i](x) - j\right) = 0 \quad \forall\ x \in \{0,1\}^{\log(m)}$

NETHERMIND RESEARCH

Solving norm bounds



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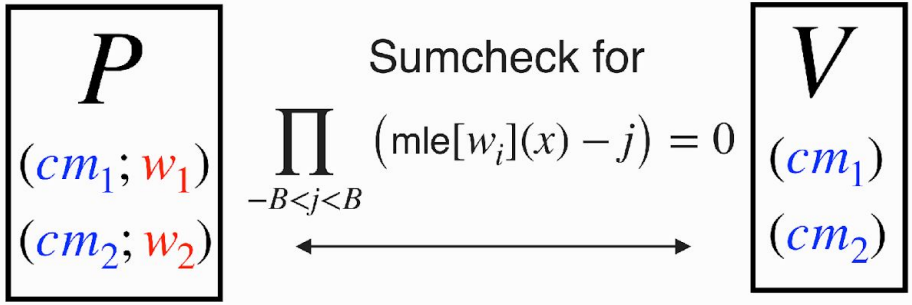

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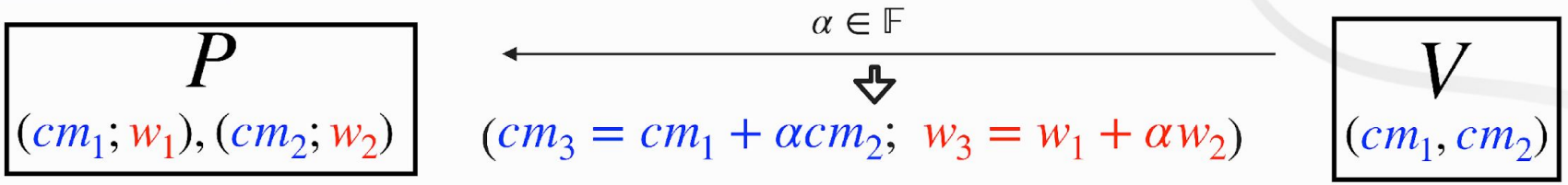
Run sumcheck and reduce it to a multilinear polynomial evaluation claim.

NETHERMIND RESEARCH

Solving norm bounds



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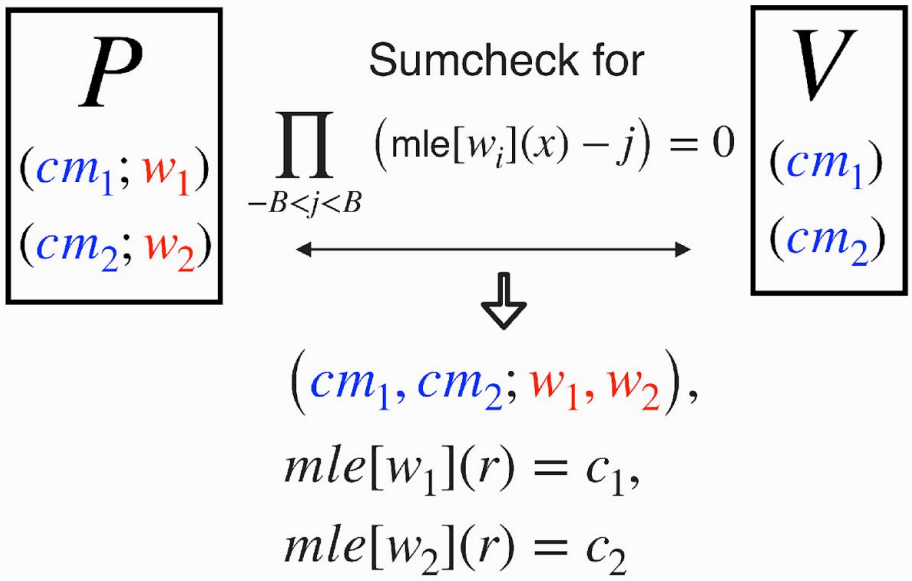

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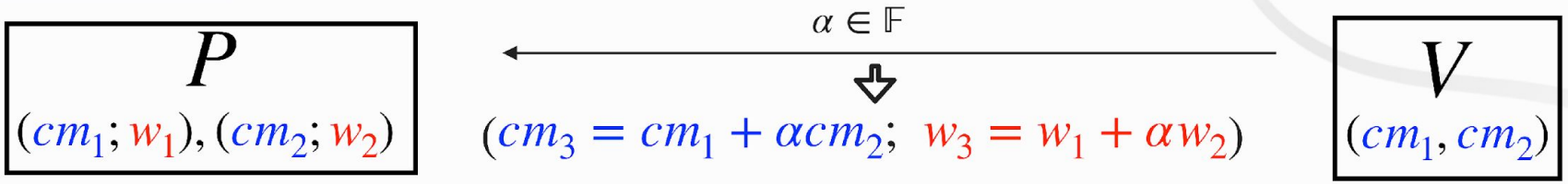
$\Pi_{-B<j<B} (\text{mle}[w_i](x) - j) = 0 \quad \forall\, x \in \{0,1\}^{\log(m)}$

Run sumcheck and reduce it to a multilinear polynomial evaluation claim.

Add this claim to the folded relation.

NETHERMIND RESEARCH

Solving norm bounds



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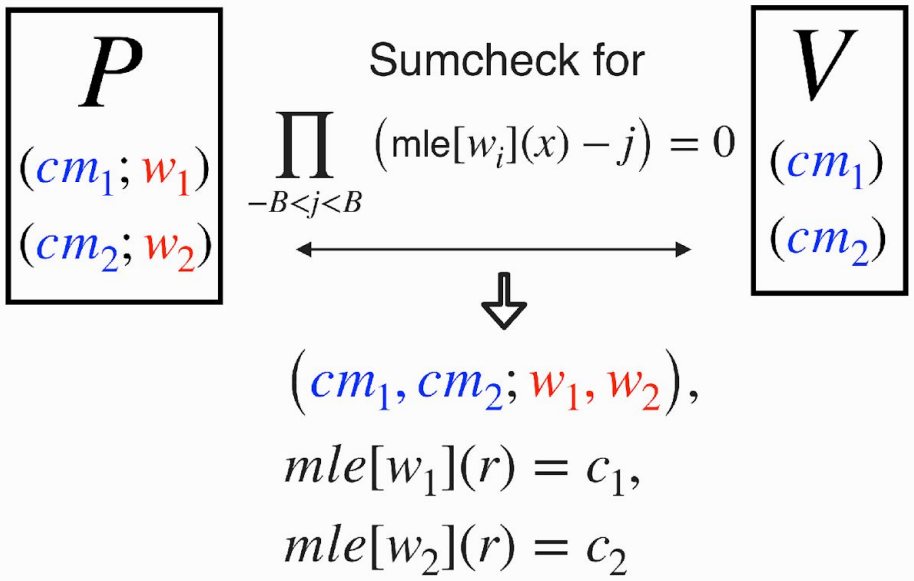


Solution (Latticefold)

Replace $\|w_i\| < B$ by

$\Pi_{-B<j<B} (\text{mle}[w_i](x) - j) = 0 \quad \forall\ x \in \{0,1\}^{\log(m)}$

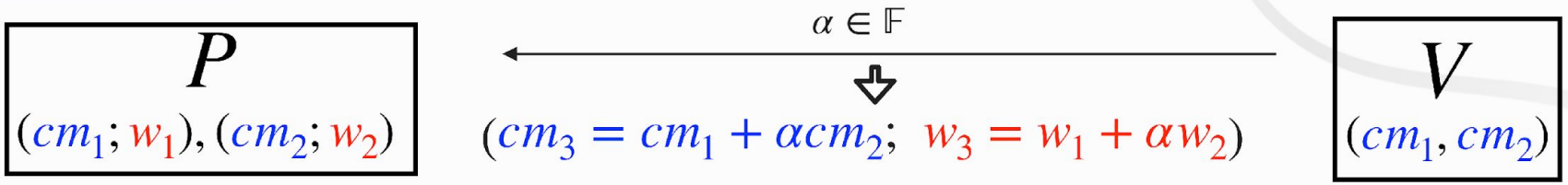
Run sumcheck and reduce it to a multilinear polynomial evaluation claim.

Add this claim to the folded relation.

In LF, there's a sumcheck per chunk w_{ij}

NETHERMIND RESEARCH

Solving norm bounds



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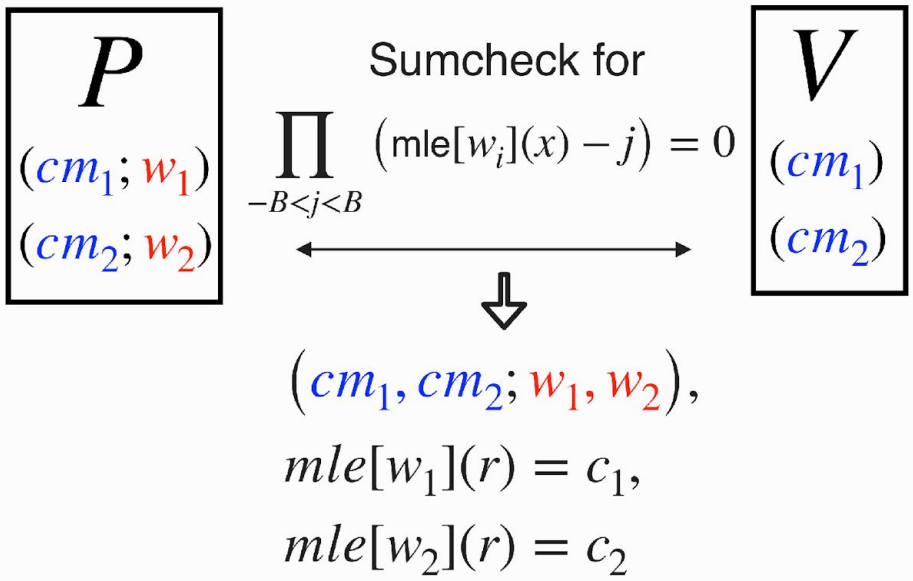

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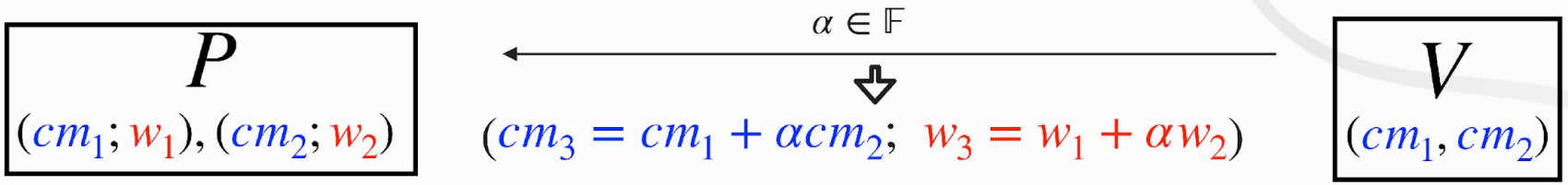

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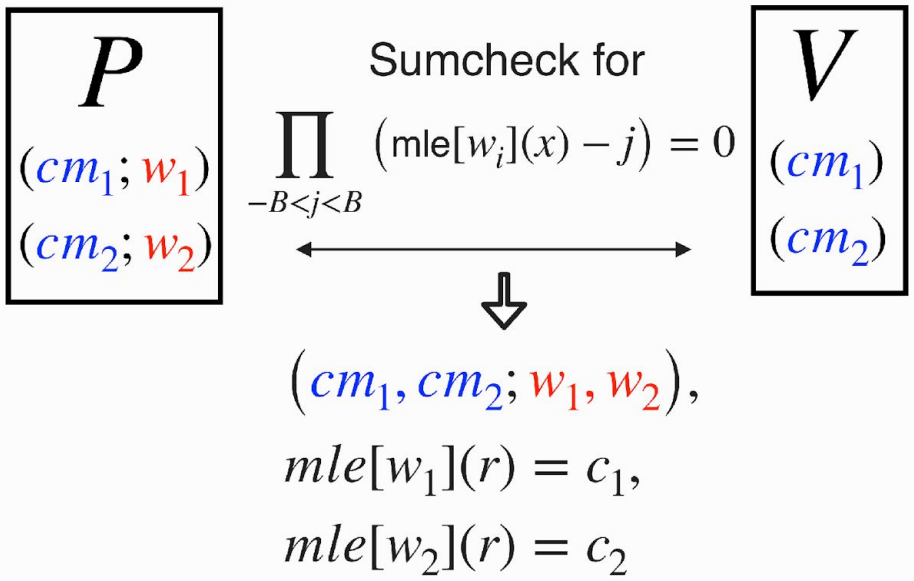

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(These are batched)

And B is replaced by b

NETHERMIND RESEARCH

Solving norm bounds

Configuration

$\mathbb{F}_q$ = Goldilocks field (64 bits), $q \approx 2^{64}$.

$\mathscr{R} = \mathbb{F}_q[X]/(X^{24} - X^{12} + 1) \cong \mathbb{F}_{q^3} \times \overset{8}{\ldots} \times \mathbb{F}_{q^3}$

Set $\kappa = 10$ and $B = 2^8$.

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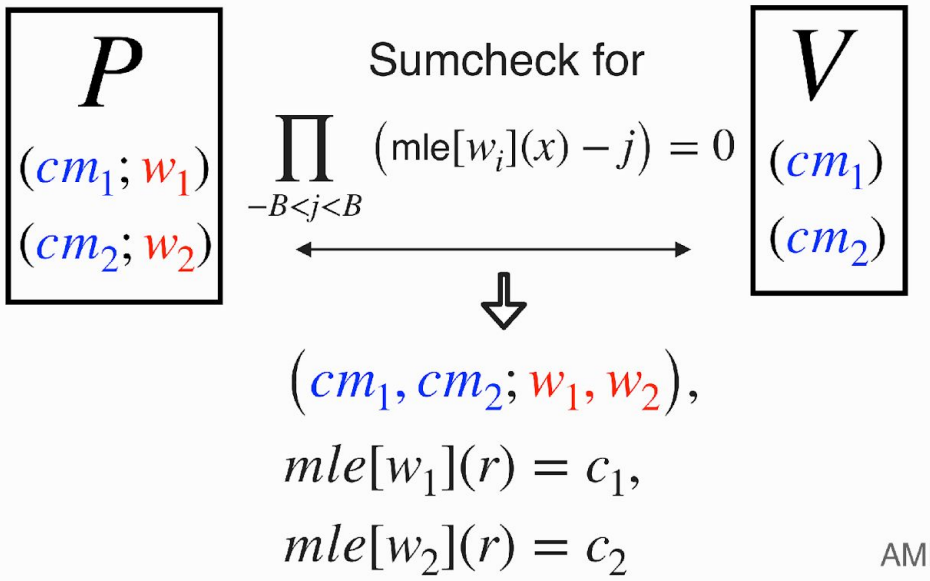

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

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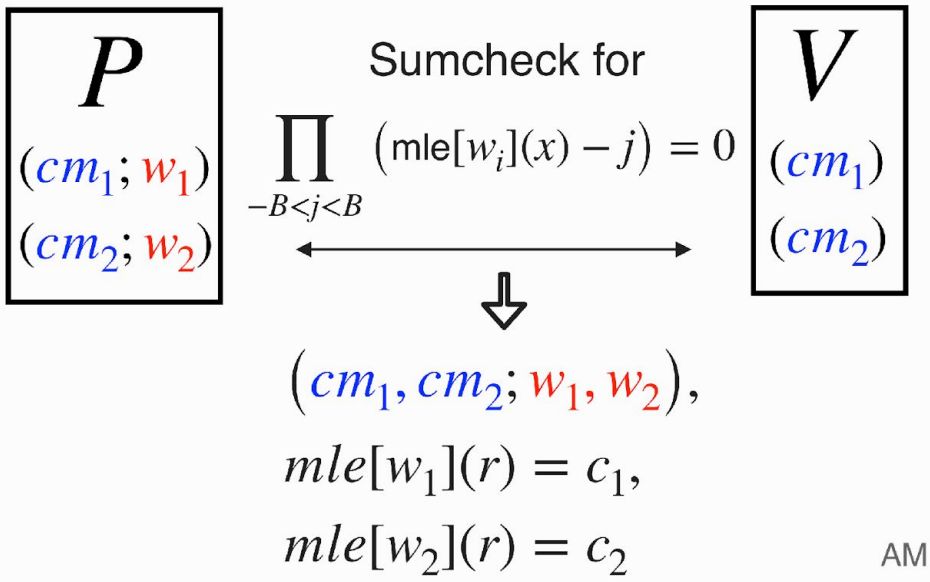

Experiments

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P

$(cm_1; w_1)$

$(cm_2; w_2)$

Sumcheck for

$$\prod_{-B<j<B} \left(\text{mle}[w_i](x) - j\right) = 0$$

V

(cm_1)

(cm_2)

$$\left(cm_1, cm_2; w_1, w_2\right),$$

$$mle[w_1](r) = c_1,$$

$$mle[w_2](r) = c_2$$

Experiments

$|w_1|, |w_2| = 2^{16}$

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Solving norm bounds

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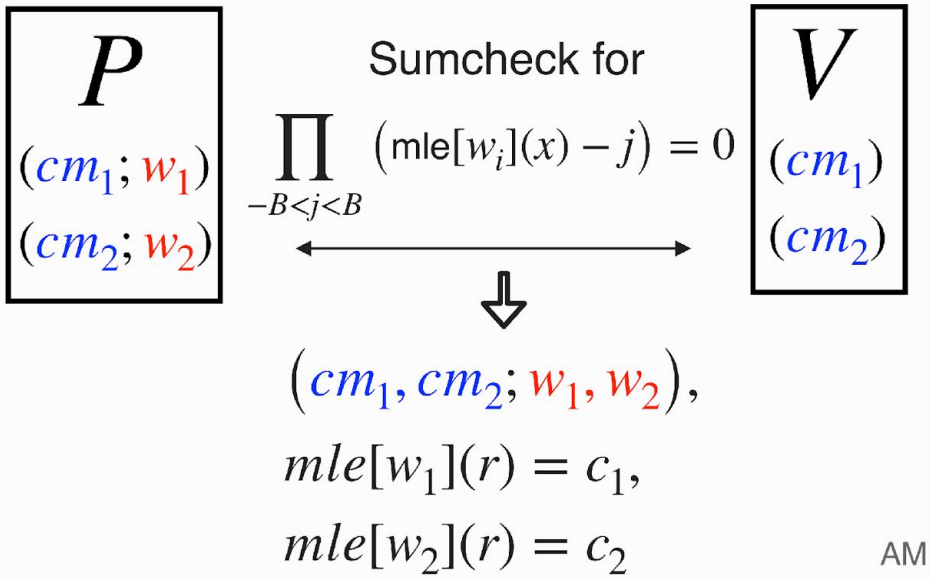

Experiments

$|w_1|, |w_2| = 2^{16}$

When $B = 2$, one single grand product sumcheck costs 0.18s

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Solving norm bounds

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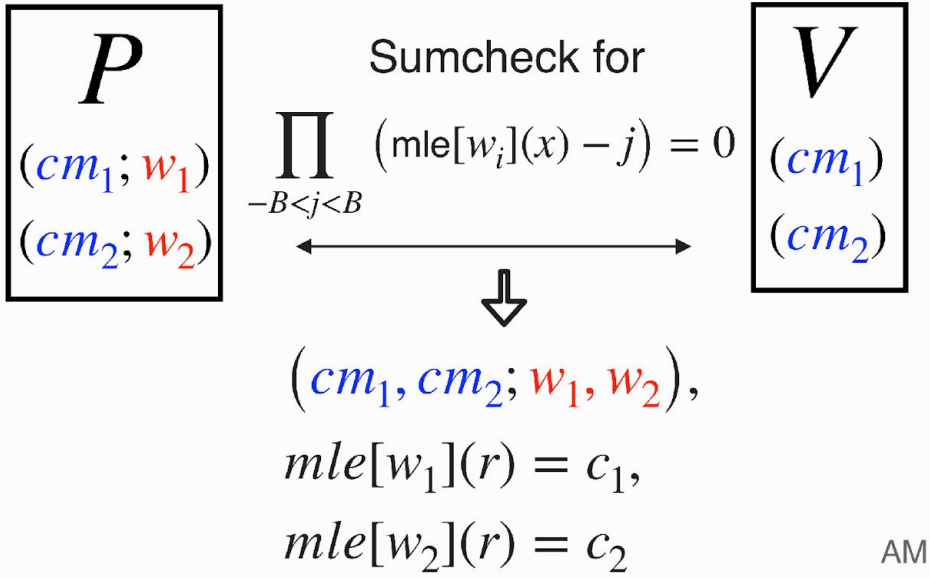

Experiments

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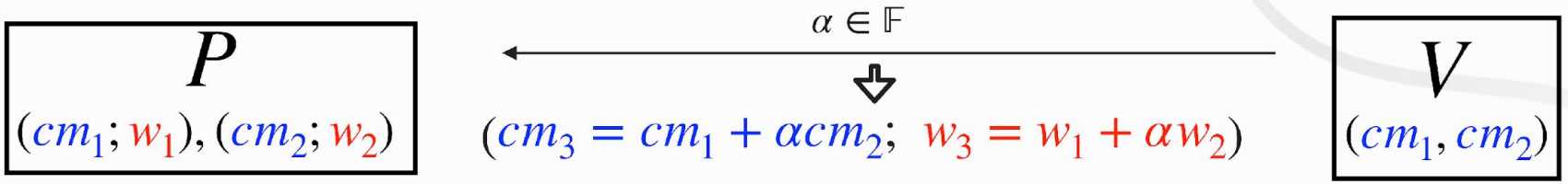
LF performs one sumcheck per chunk, so total cost is 3s

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Solving norm bounds



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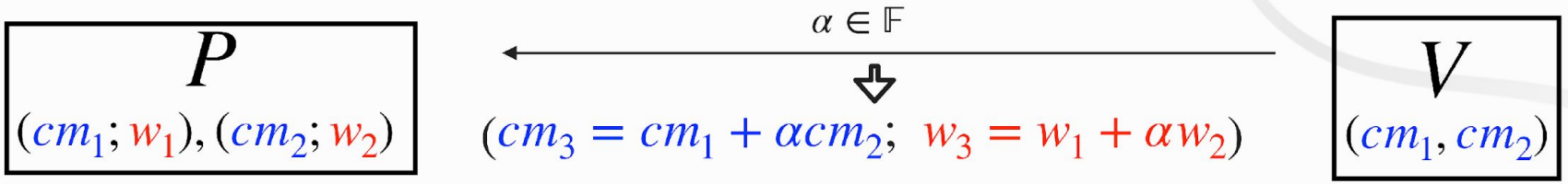
Solution (Latticefold+ w/o double commits)

Solving norm bounds



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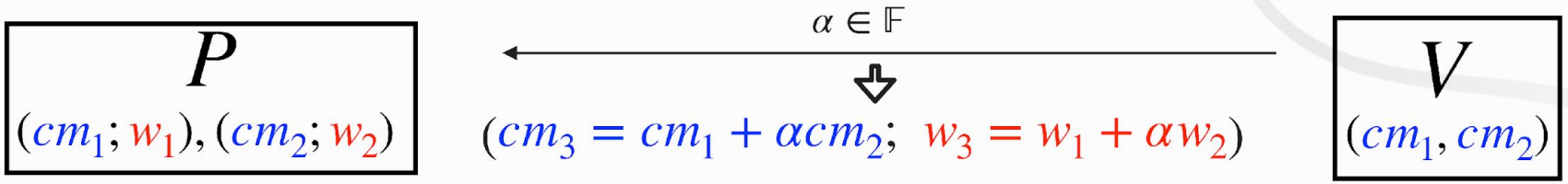

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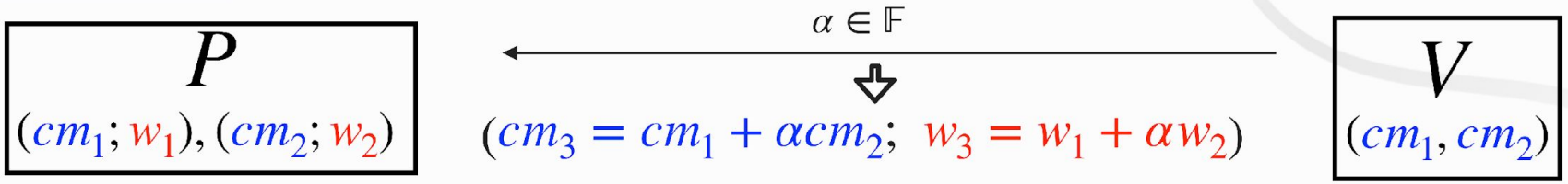

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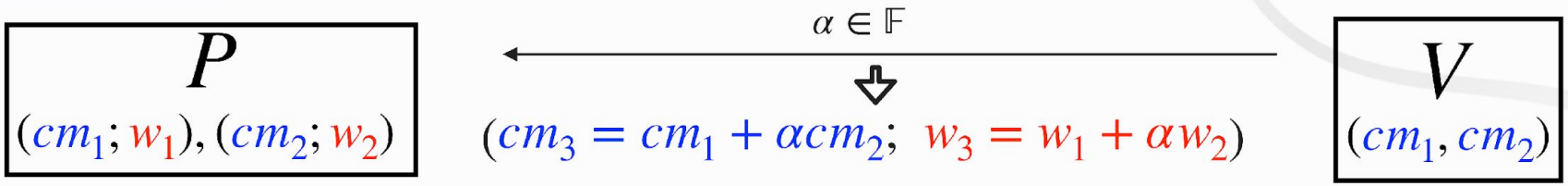

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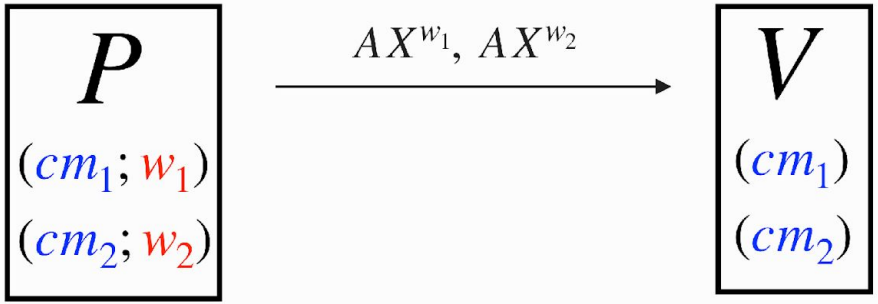

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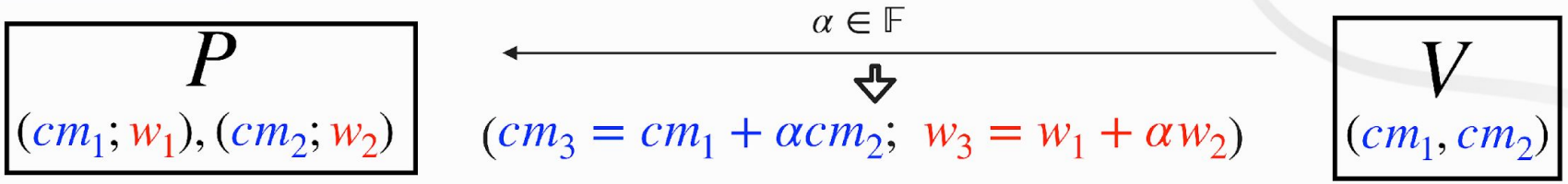

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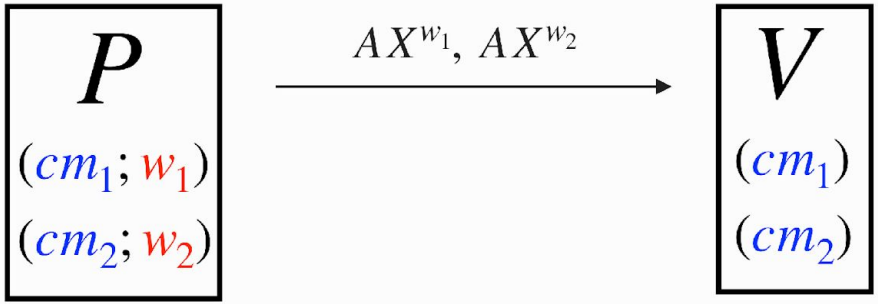

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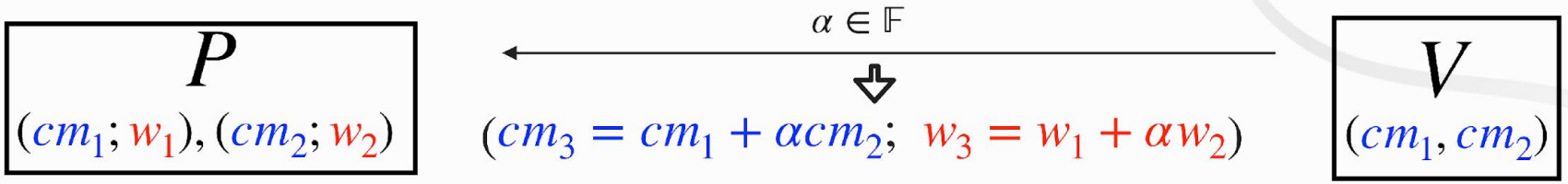
Solving norm bounds



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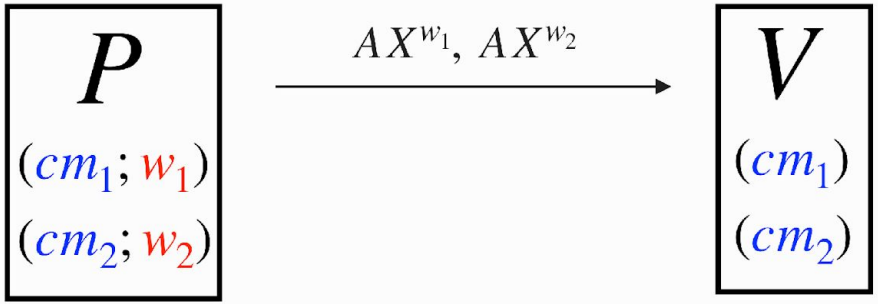

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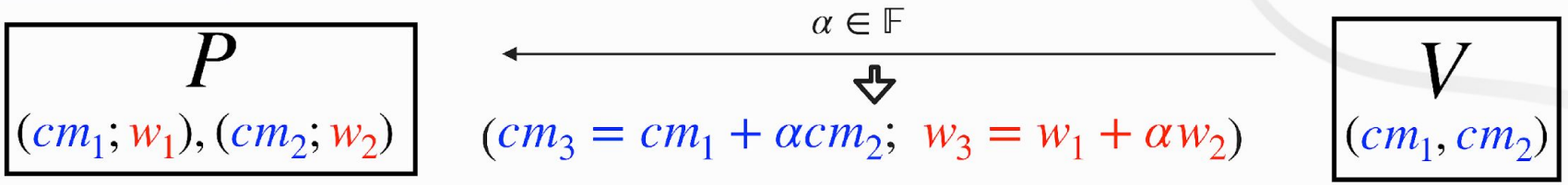
- Only multiplication by monomials

Solving norm bounds



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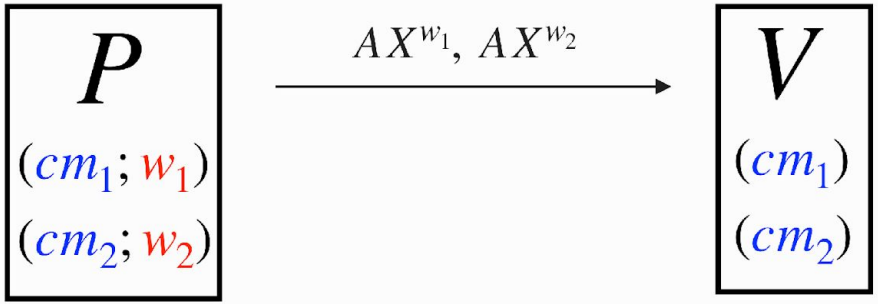

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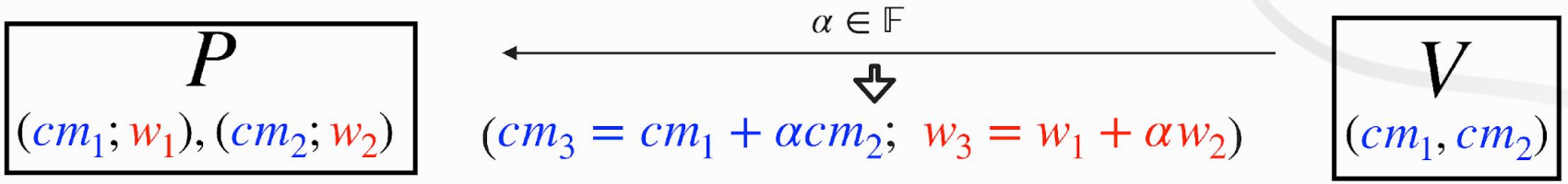
Solving norm bounds



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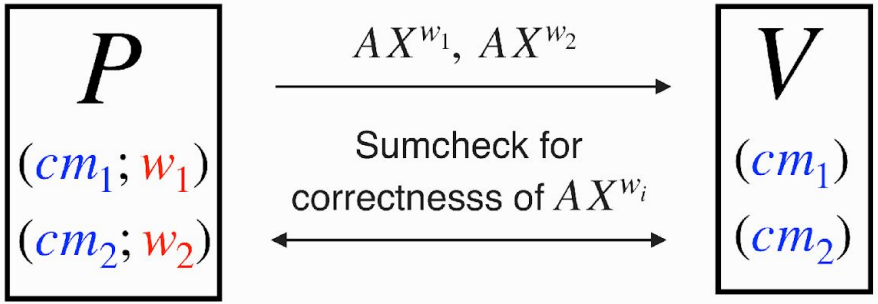

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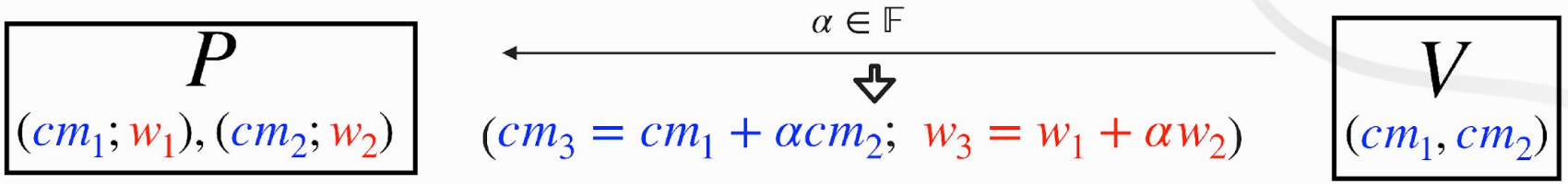
Sumcheck of correctness of AX^{w_i}

Solving norm bounds



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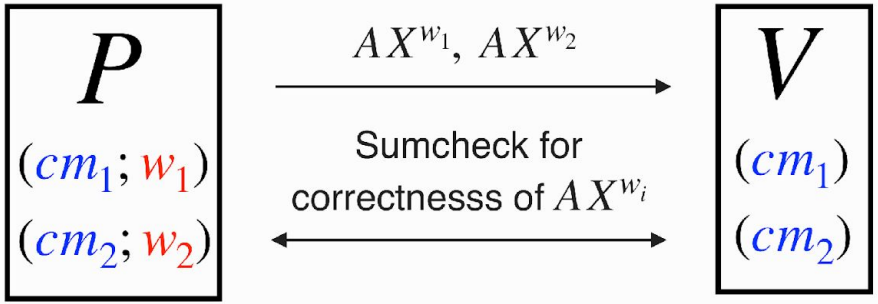

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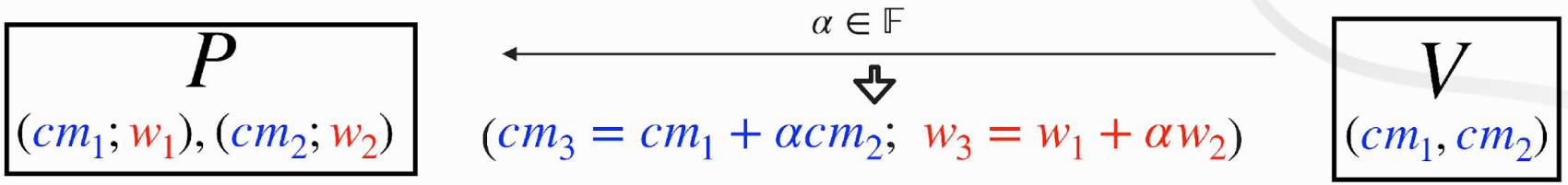
Sumcheck of correctness of AX^{w_i}

= $\deg(d)$ sum checks over $\mathbb{F}_q$

Solving norm bounds



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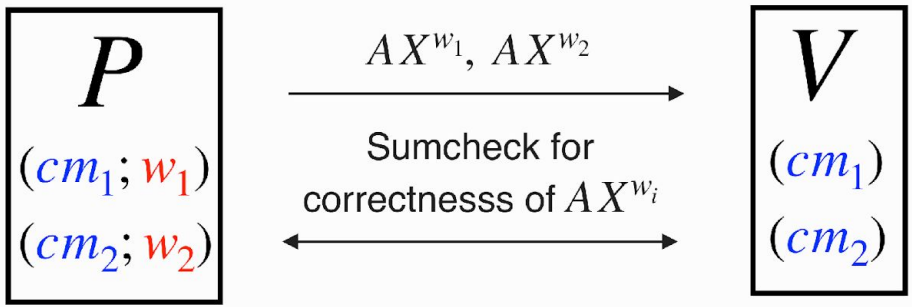

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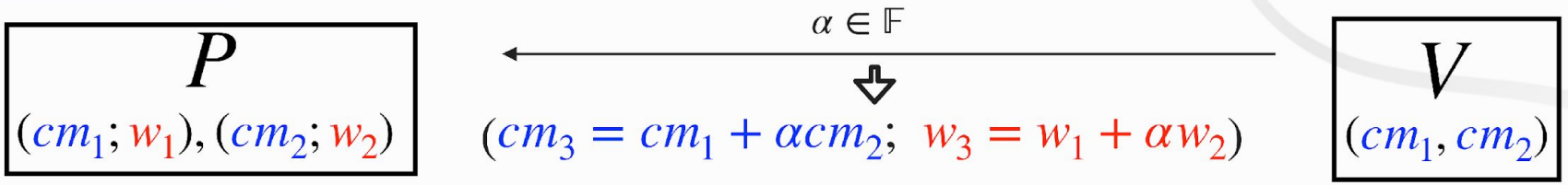

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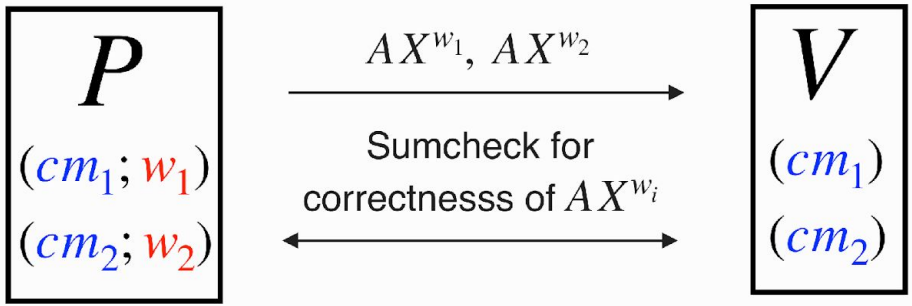
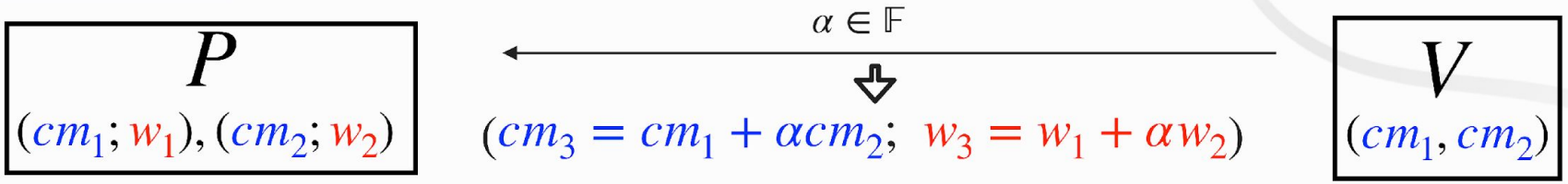


Solution (Latticefold+ w/o double commits)

Once AX^{w_1}, AX^{w_2} are committed,

Solving norm bounds



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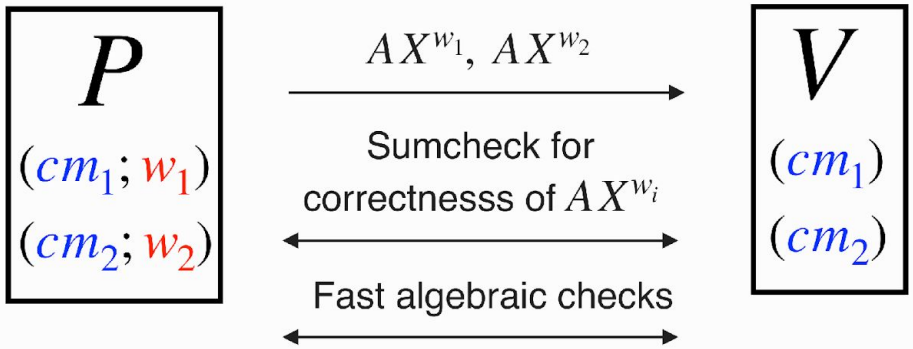
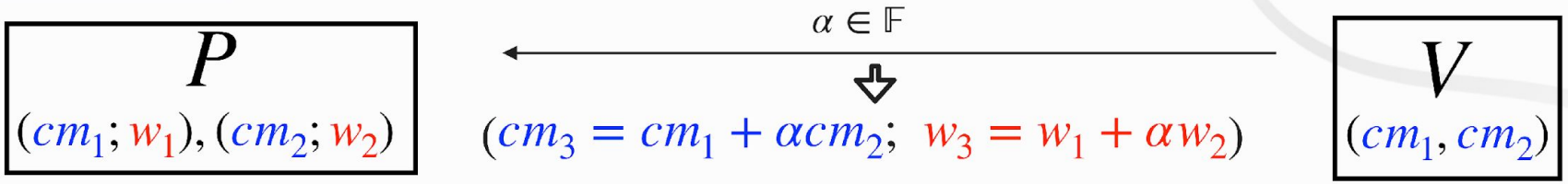


Solution (Latticefold+ w/o double commits)

Once AX^{w_1}, AX^{w_2} are committed,

$\|w_i\| \leq B$ can be ensured with a simple algebraic check

Solving norm bounds



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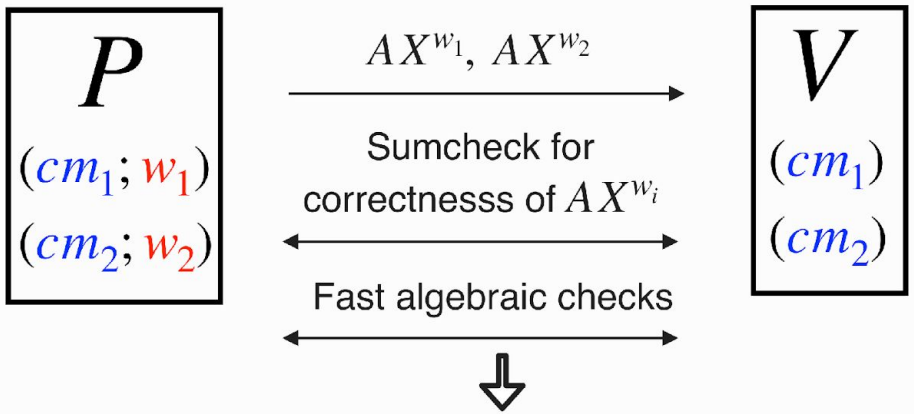

Claims about commitments, norm bounds and evaluations of multilinear polynomials

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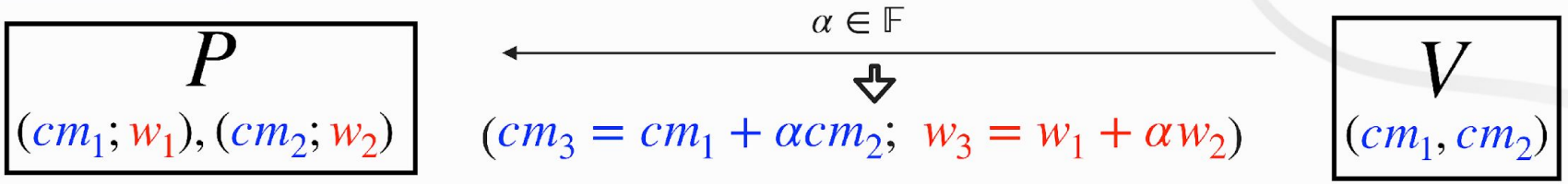
Solving norm bounds



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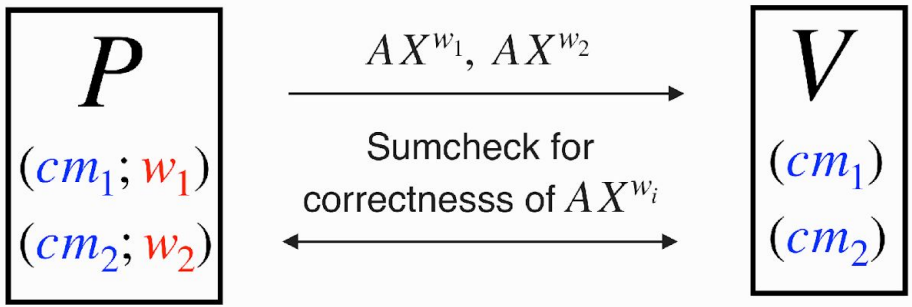
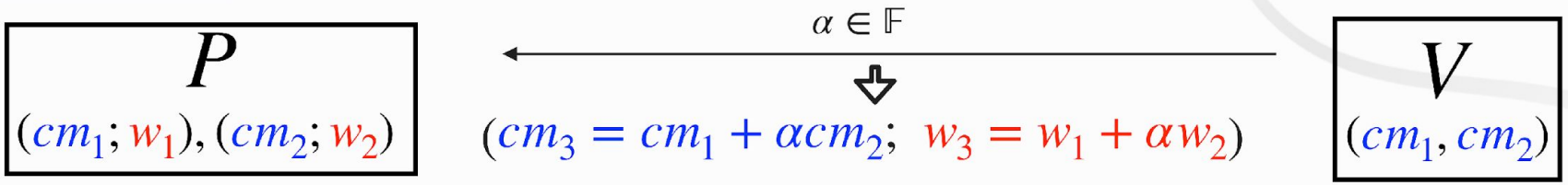


AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Solving norm bounds



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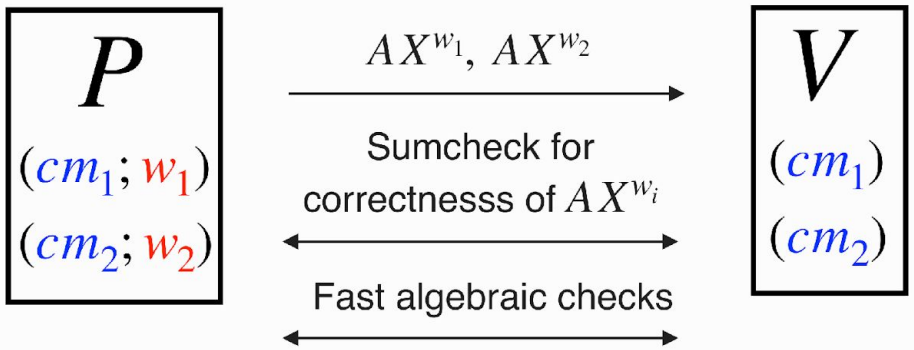

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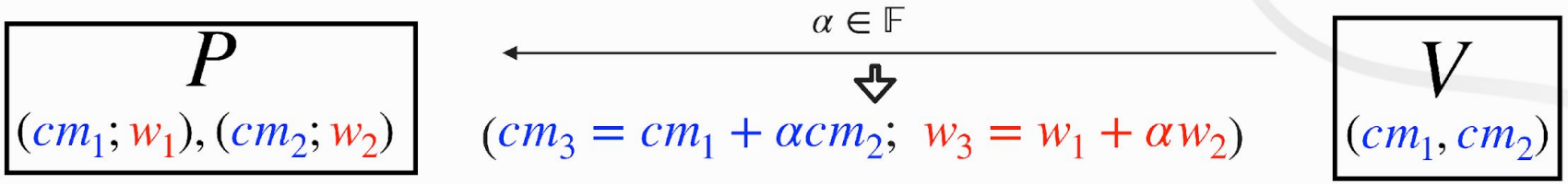

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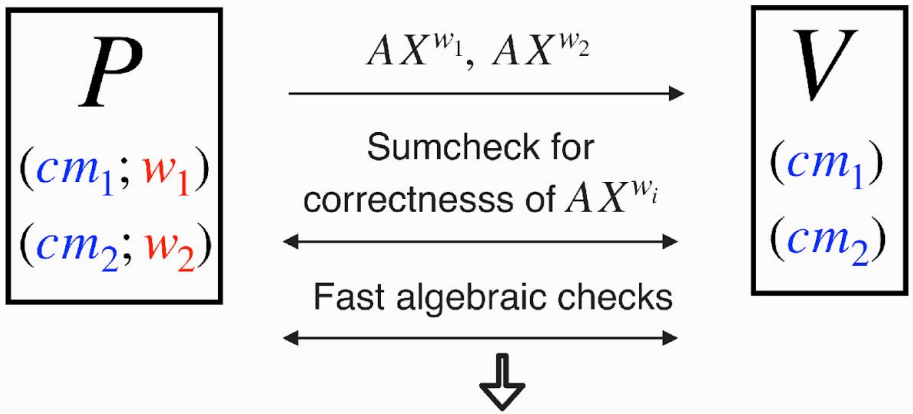
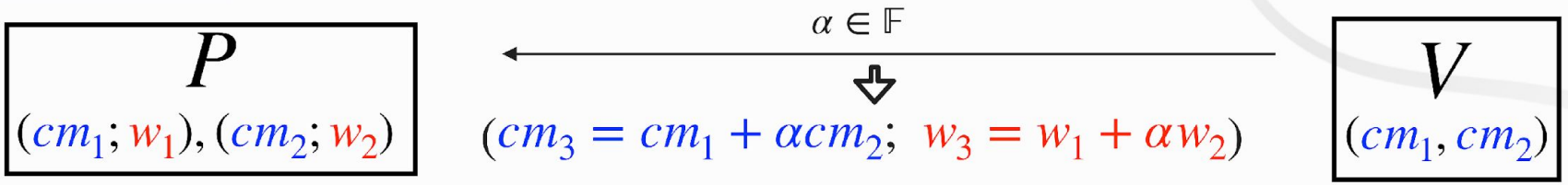


Claims about commitments, norm bounds and evaluations of multilinear polynomials

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Solving norm bounds



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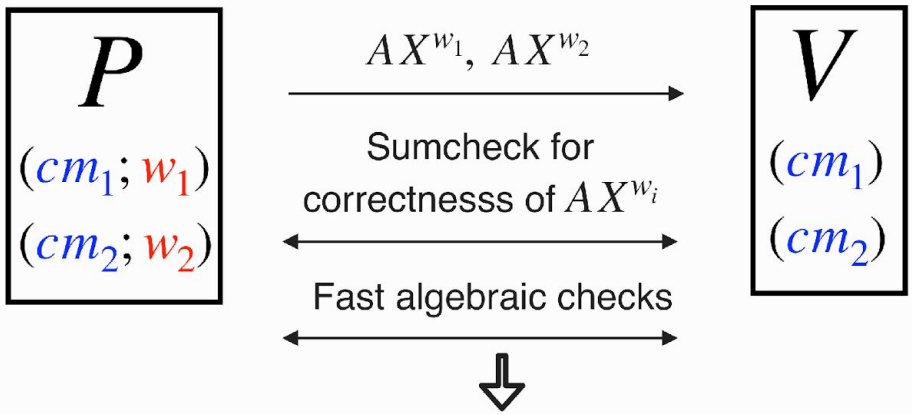

Claims about commitments, norm bounds and evaluations of multilinear polynomials

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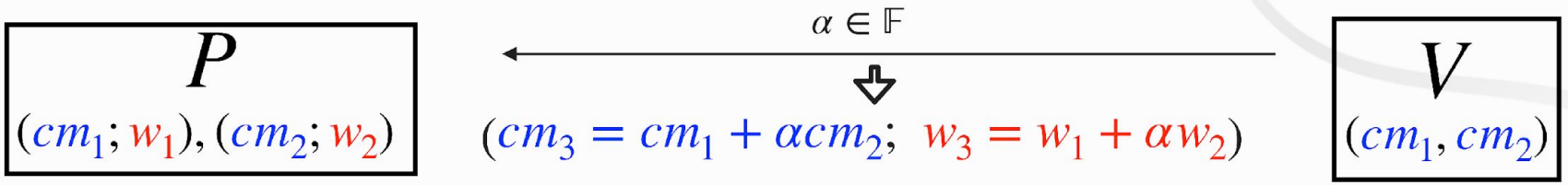
AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Solving norm bounds



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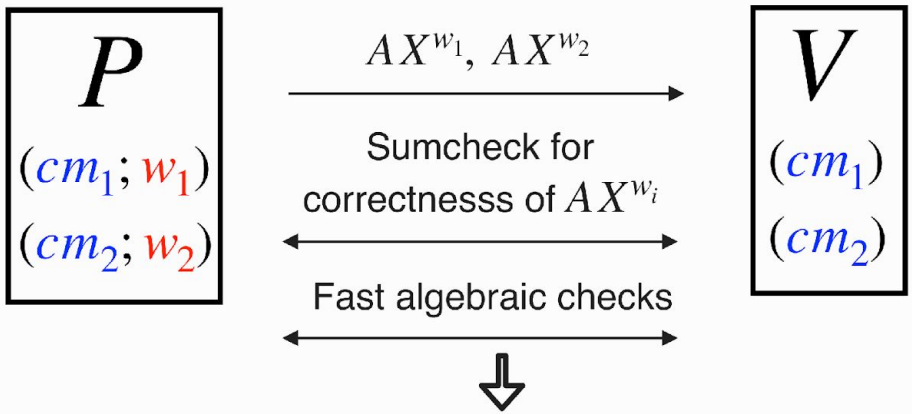

Claims about commitments, norm bounds and evaluations of multilinear polynomials

Estimated runtime

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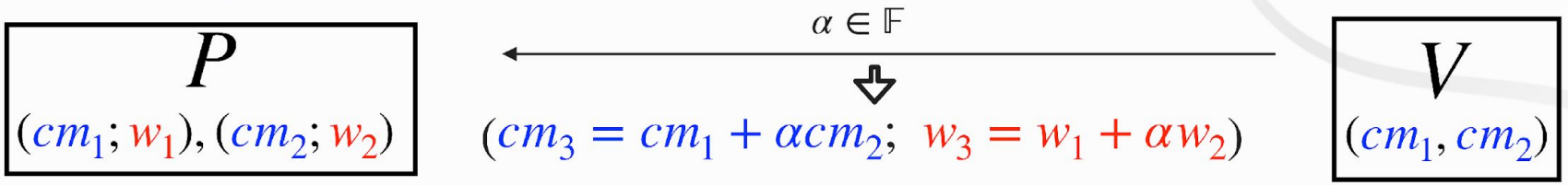
Solving norm bounds



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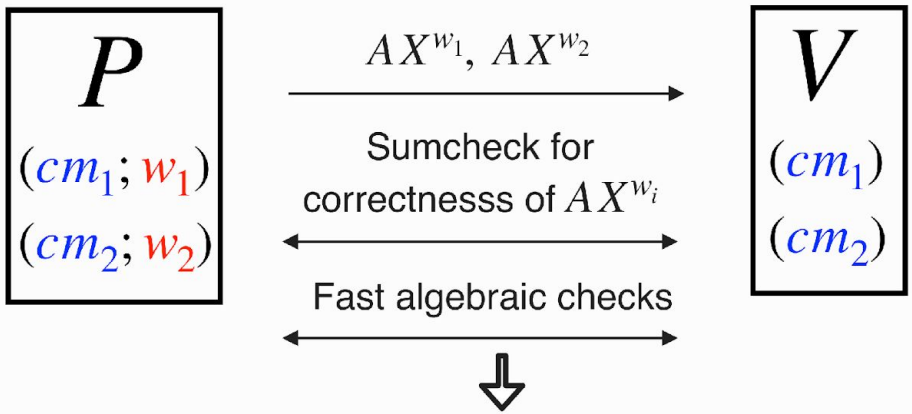

Claims about commitments, norm bounds and evaluations of multilinear polynomials

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Estimate (extrapolation): 0.6-1s

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

Overall costs

The previous techniques
be used to fold CCS
constraints.

One decom
witness
B

Thanks

bits), $q \approx 2^{64}$.

$-X^{12}+1) \cong \mathbb{F}_{q^3} \times \overset{8}{\ldots} \times \mathbb{F}_{q^3}$

and $B = 2^8$.

old

CS witness	|w_1|, |w_2|	Linearize	LF decomp	LF norm bound	Total
2^13	2^16	170ms	3s	3s	6.17s

Latticefold+ (extrapolated, speculative)

CCS witness	|w_1|, |w_2|	Linearize	LF+ decomp	LF norm bound	Total
2^13	2^16	170ms	~400ms	~600ms	~1.17s

or protocols

nova (from Sonobe implementation)

CCS witness	Total
2^16	1.2s

Neo (Nguyen, Setty, 2025)

WIP! (Very promising as well)

NETHERMIND RESEARCH

AMD EPYC 7713 64-Core (with 6 cores available), 16GB of RAM

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