# Security Bounds for Proof-Carrying Data from Straightline Extractors

**Alessandro Chiesa, Ziyi Guan, Shahar Samocha, Eylon Yogev**

EPFL STARKWARE Bar-Ilan University אוניברסיטת בר־אילן

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- We propose an **idealized PCD** that models hash-based PCD in practice.
- We prove that this idealized PCD is **as secure as its underlying SNARK**.

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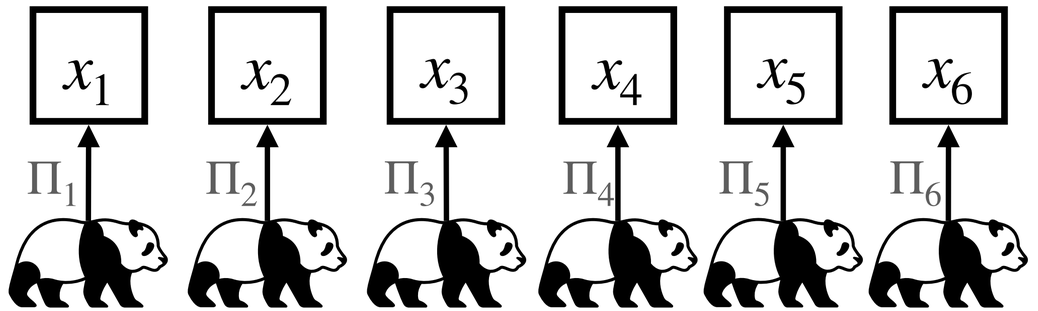

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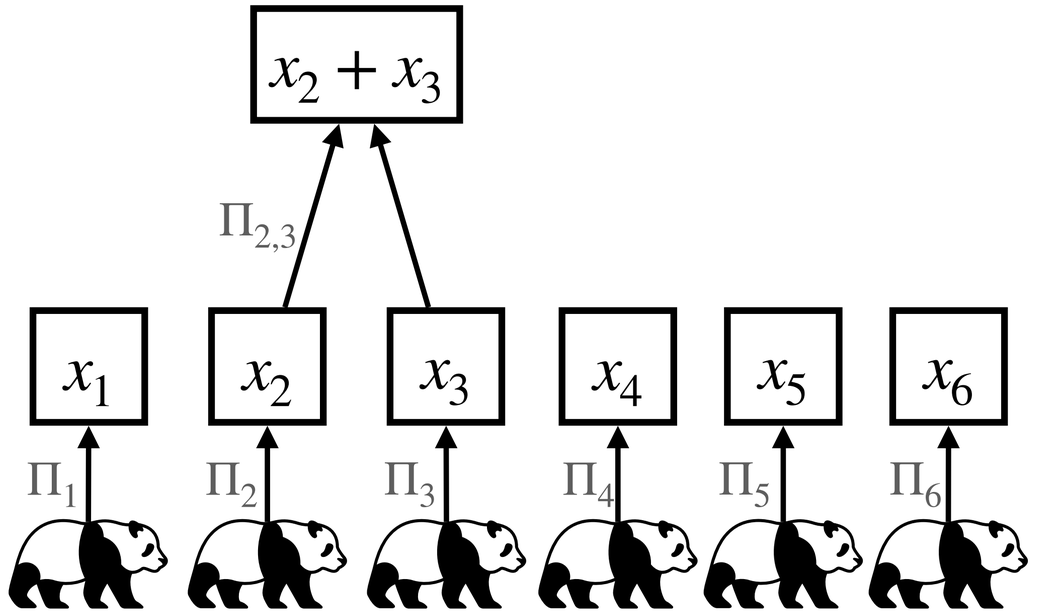

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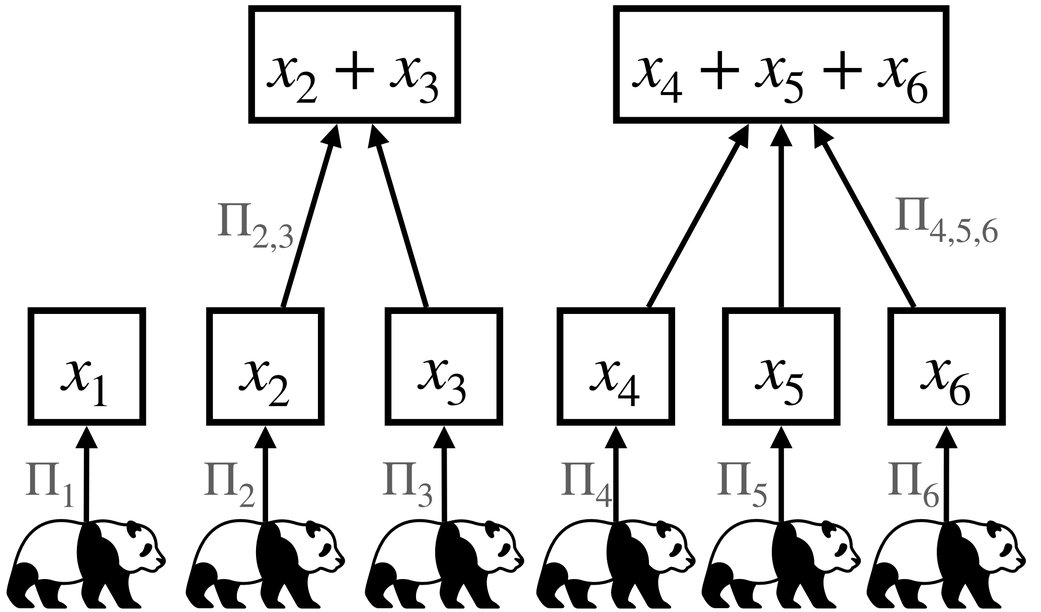

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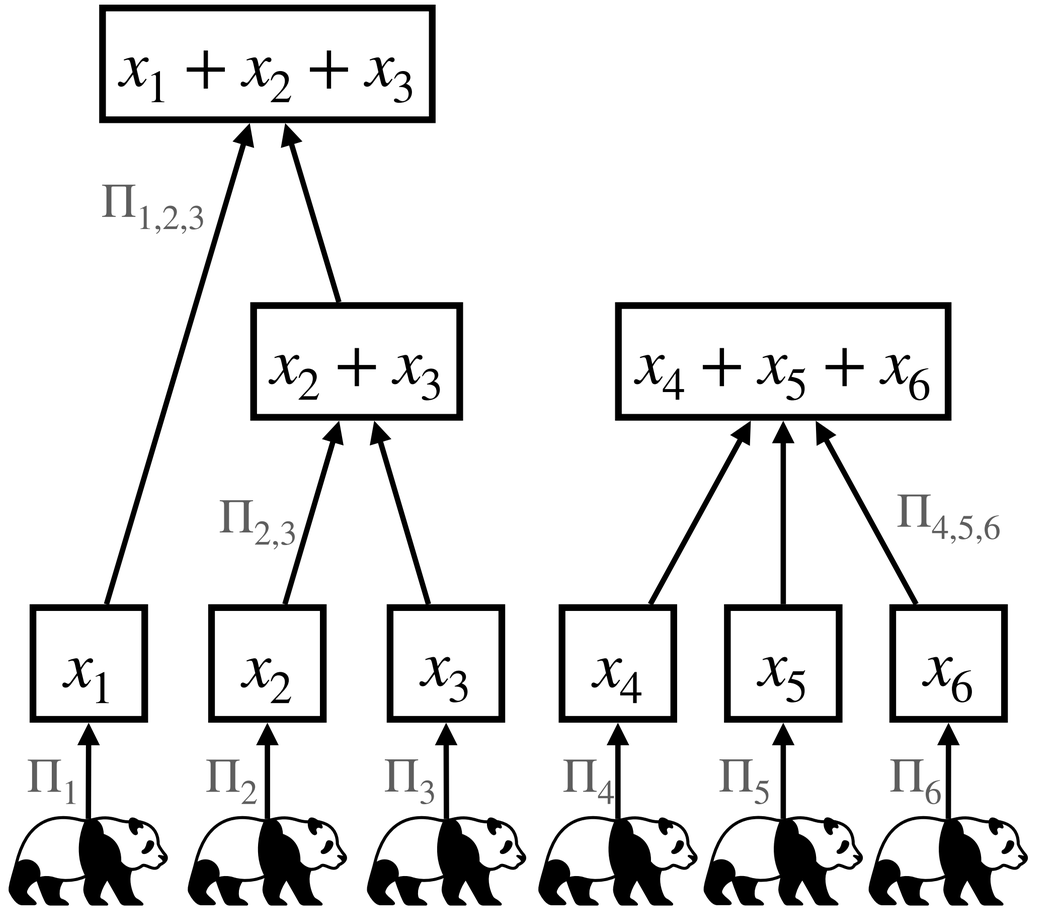

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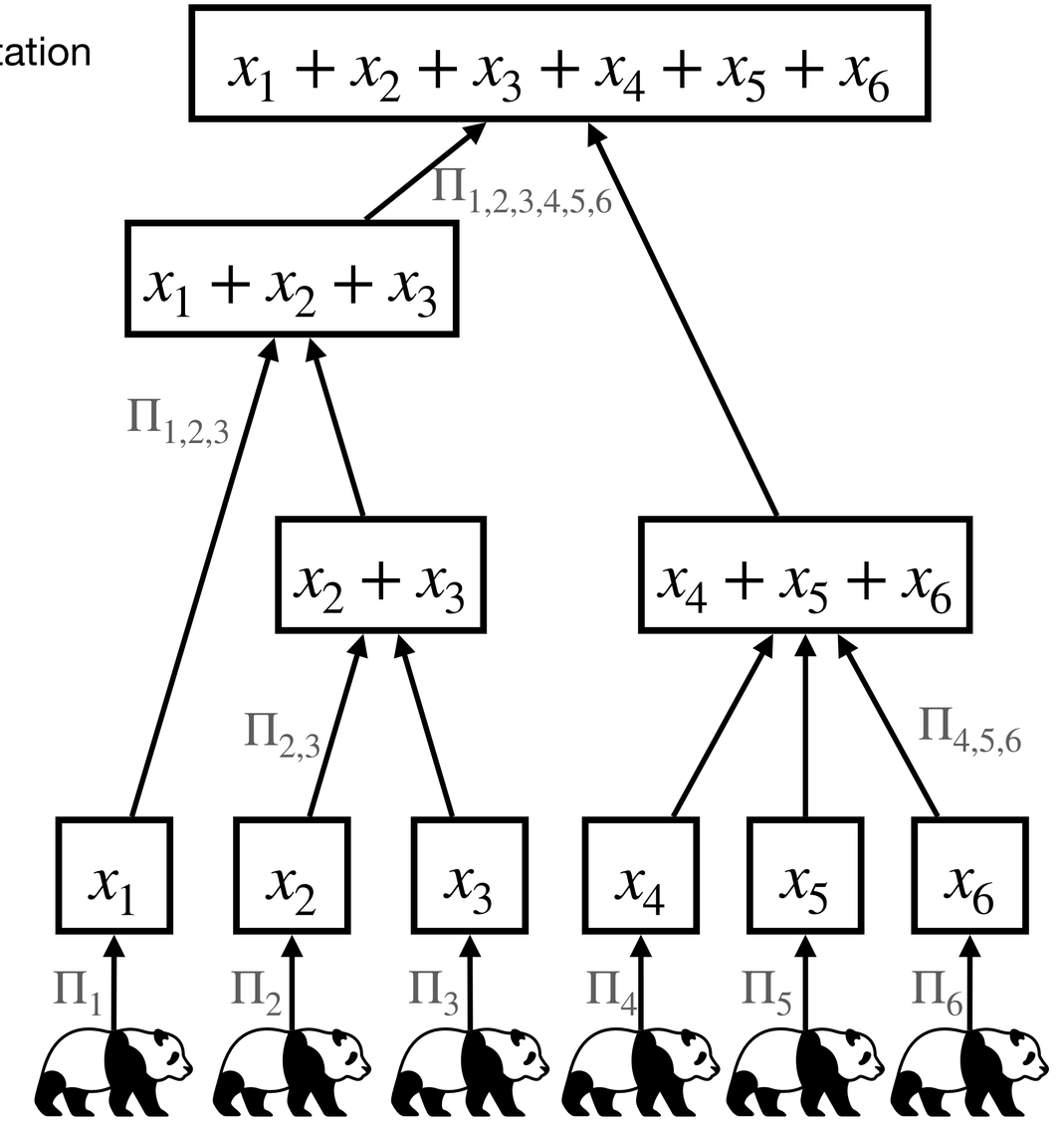

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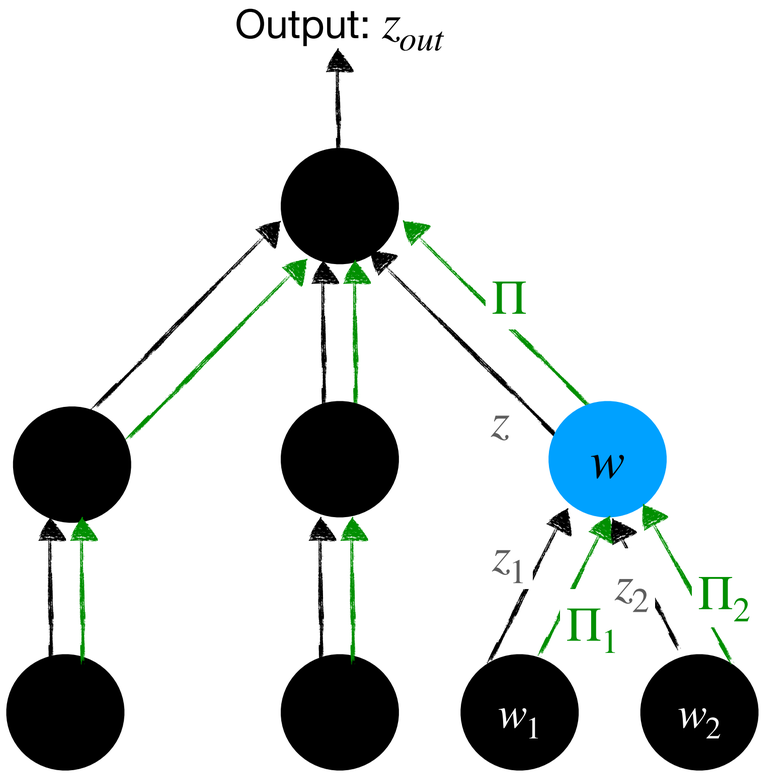


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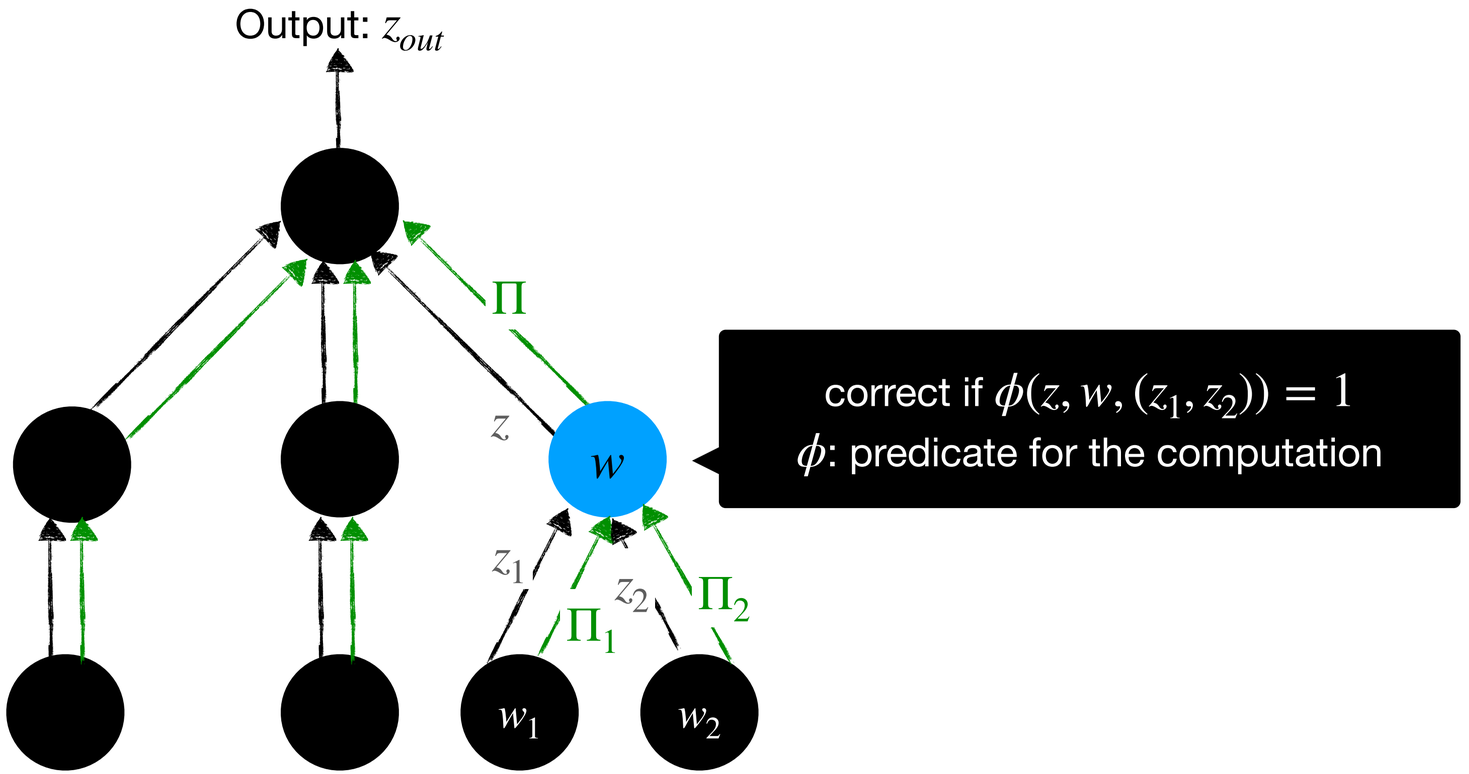


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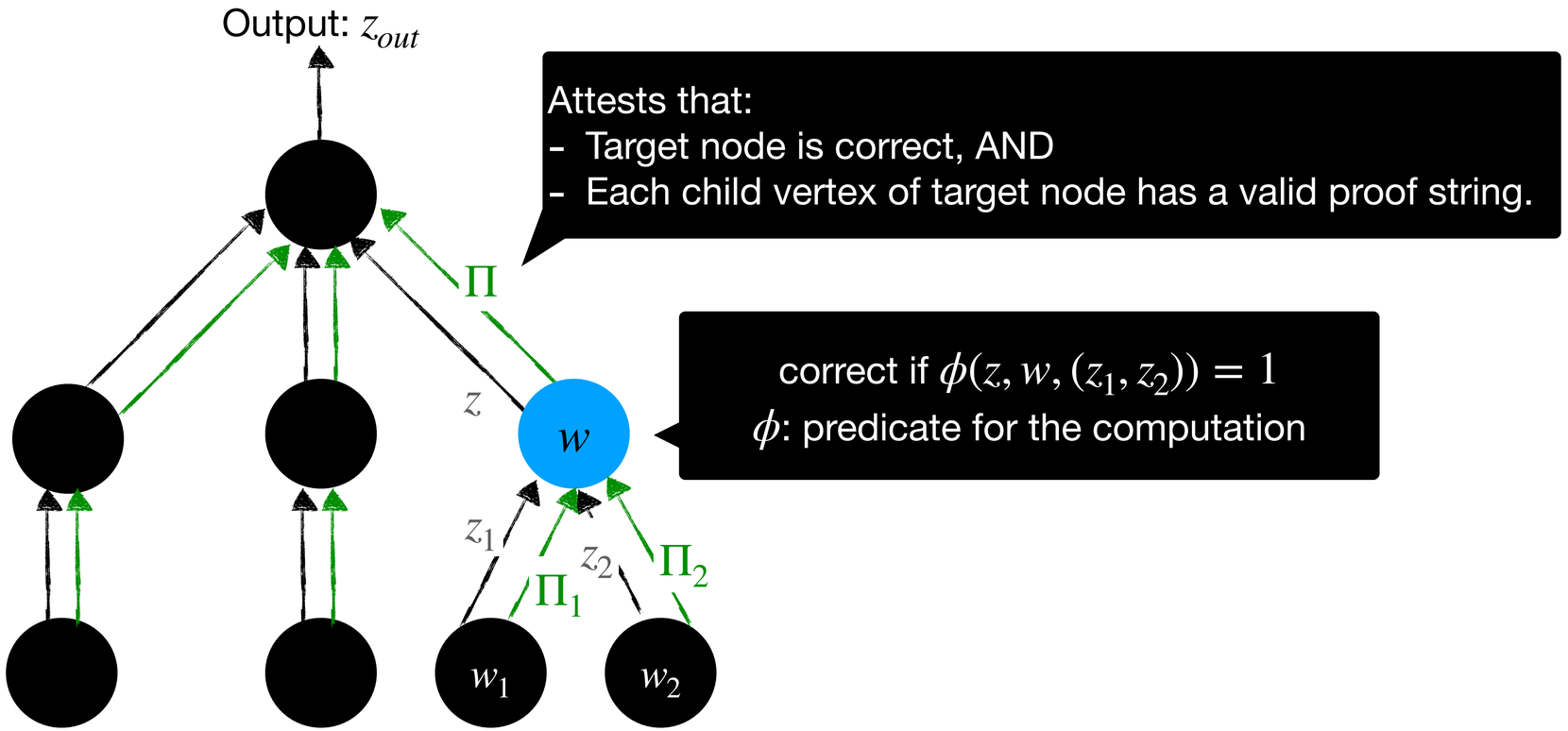


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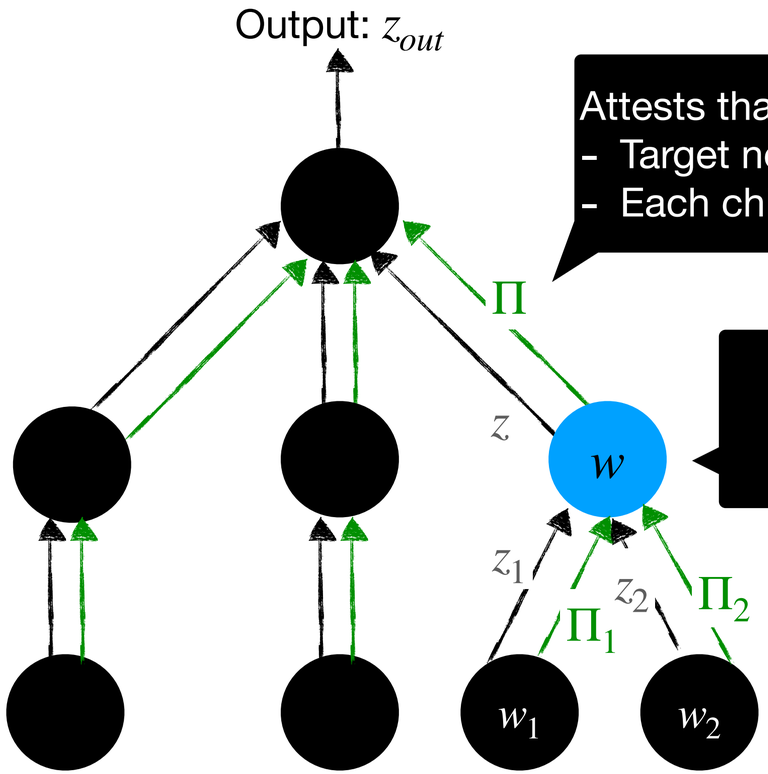


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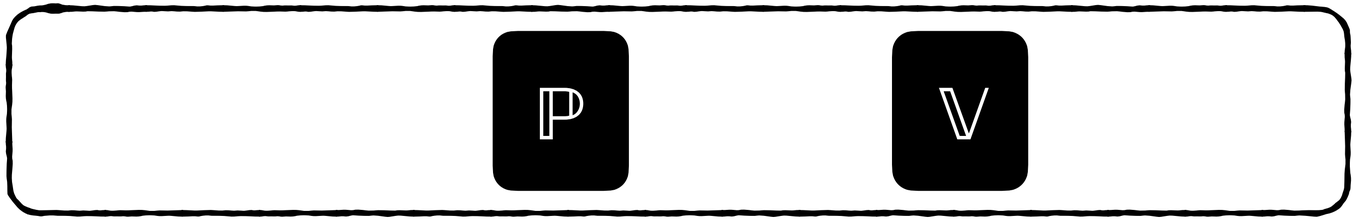

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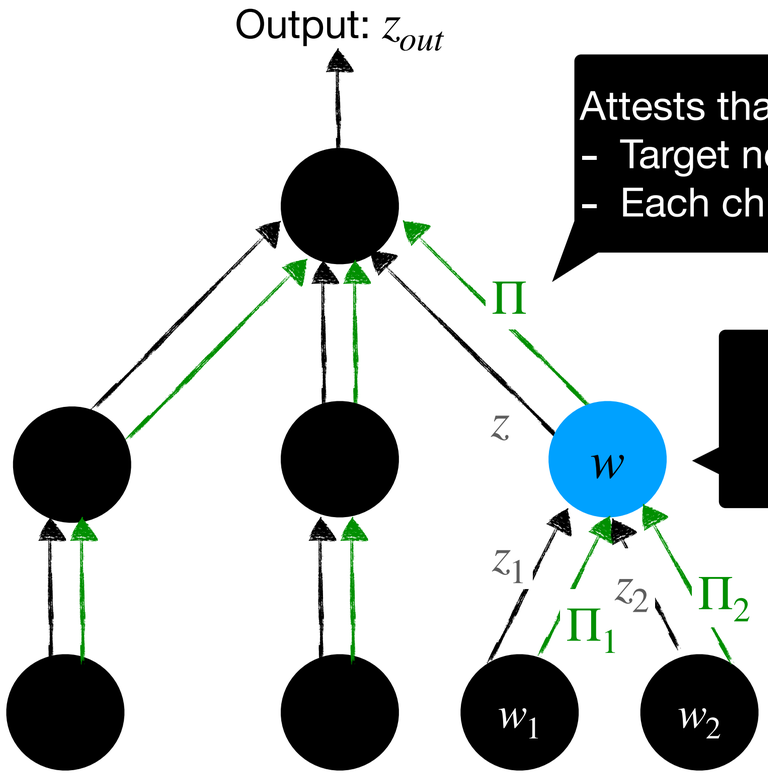


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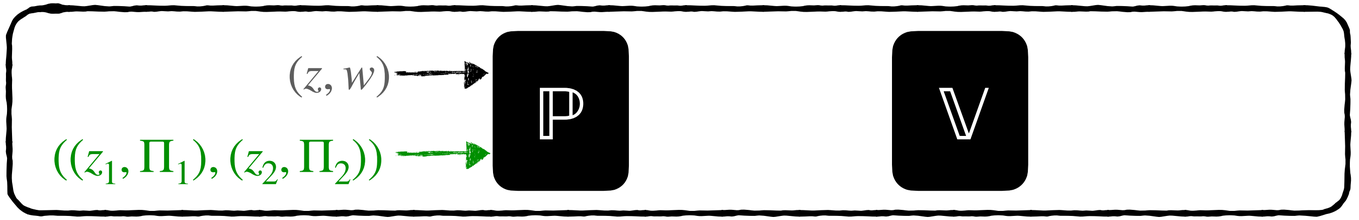

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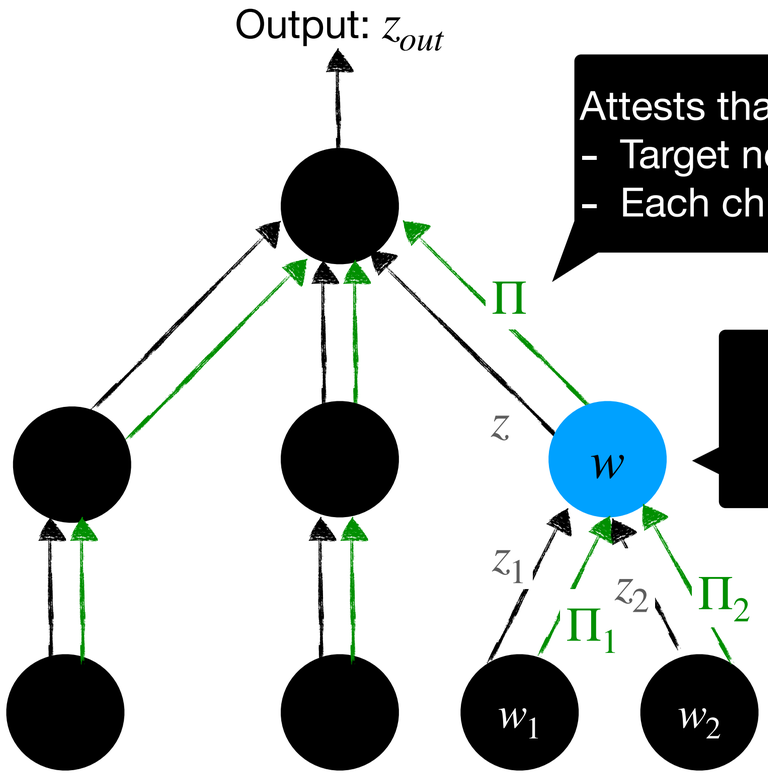


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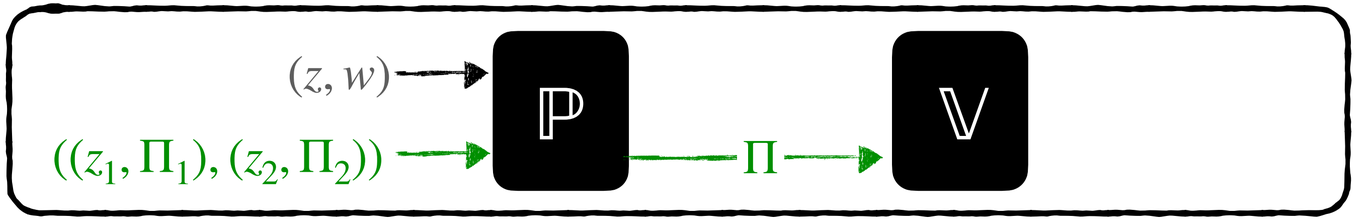

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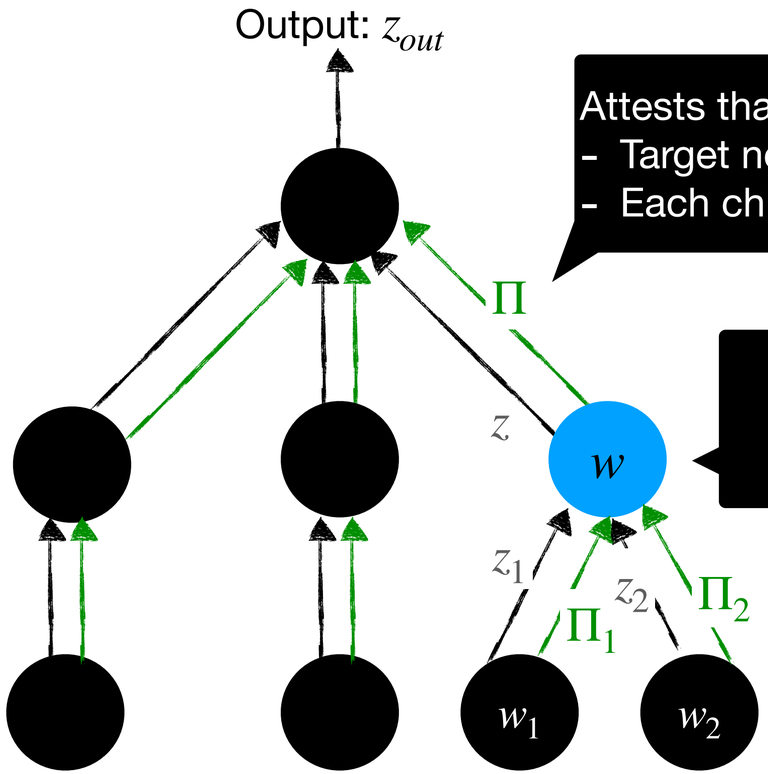


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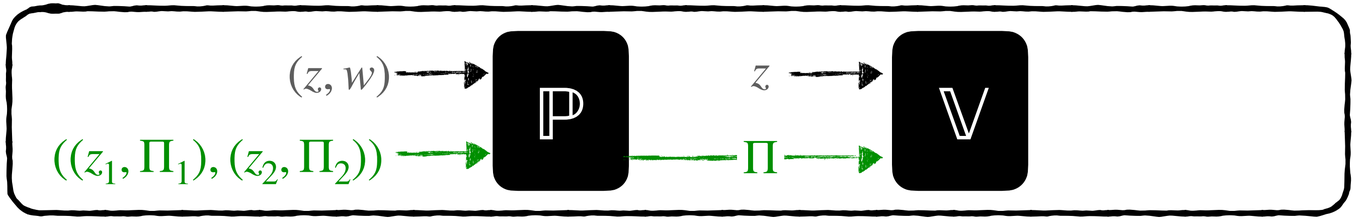

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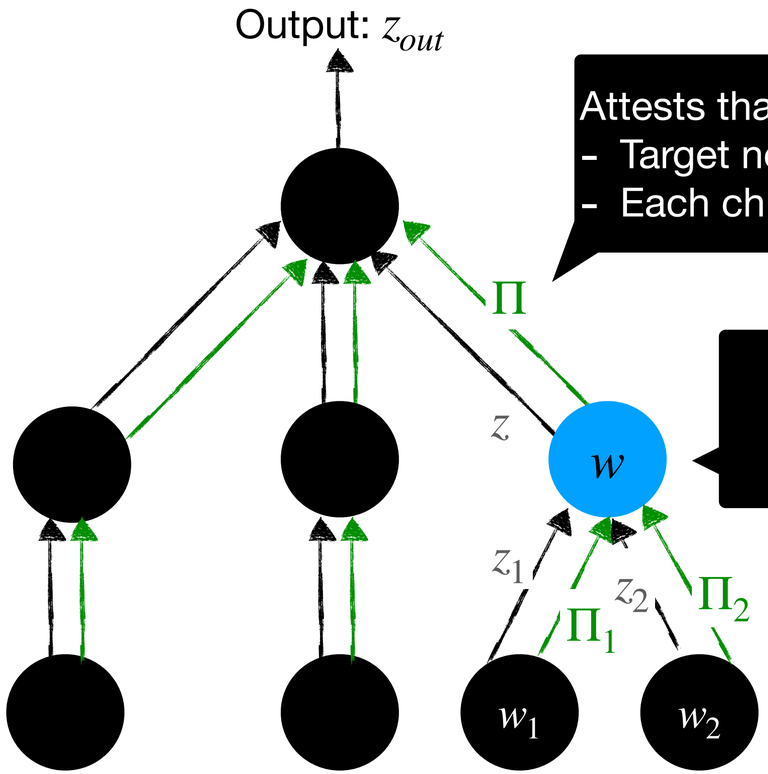


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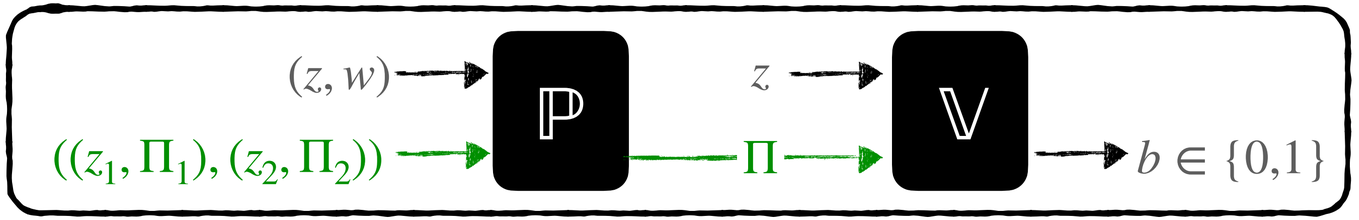

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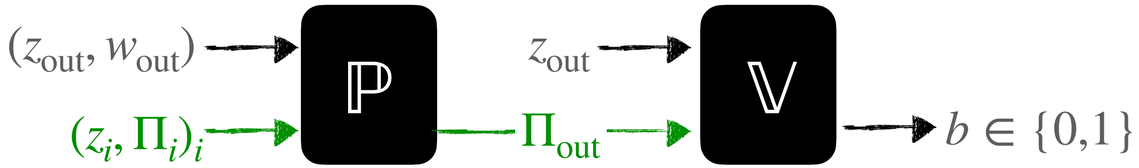

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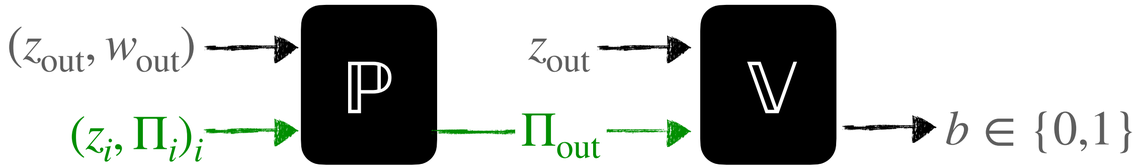


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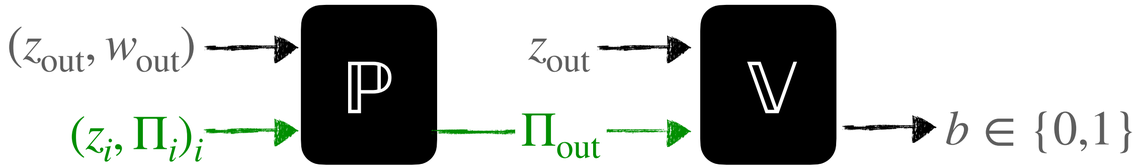


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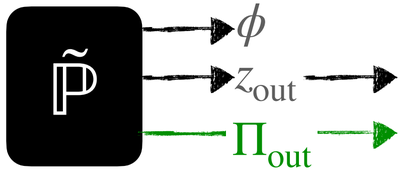

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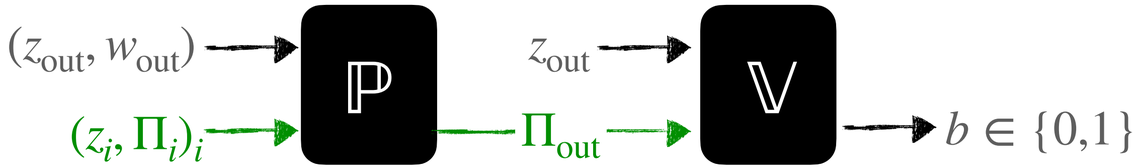


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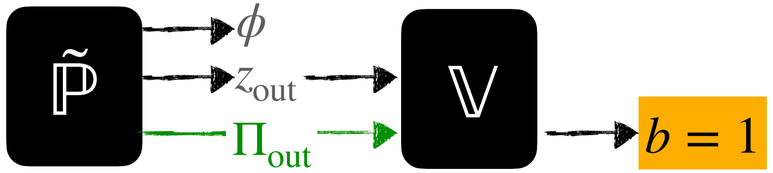

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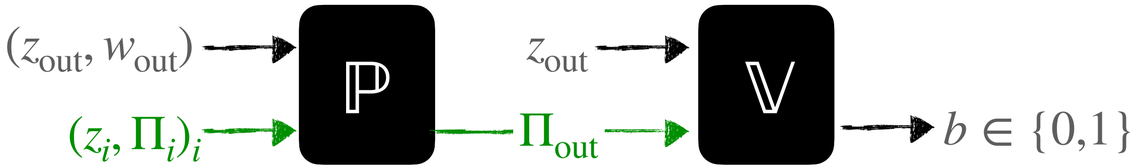


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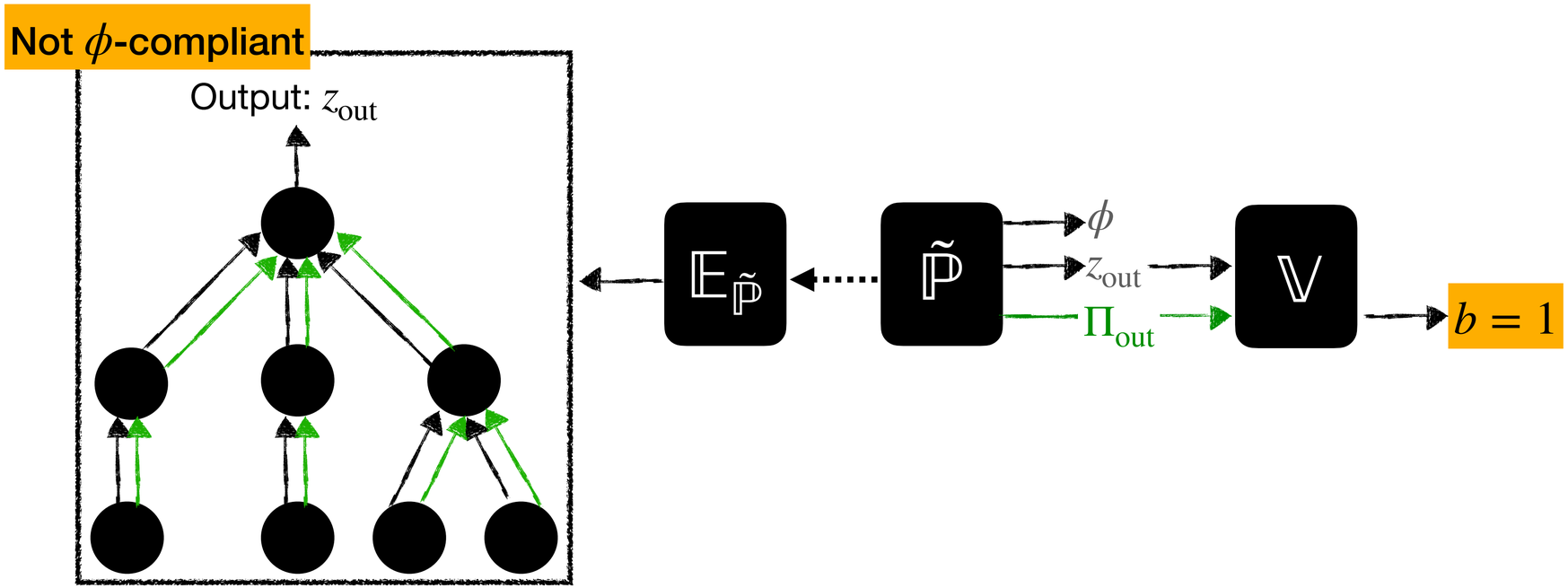

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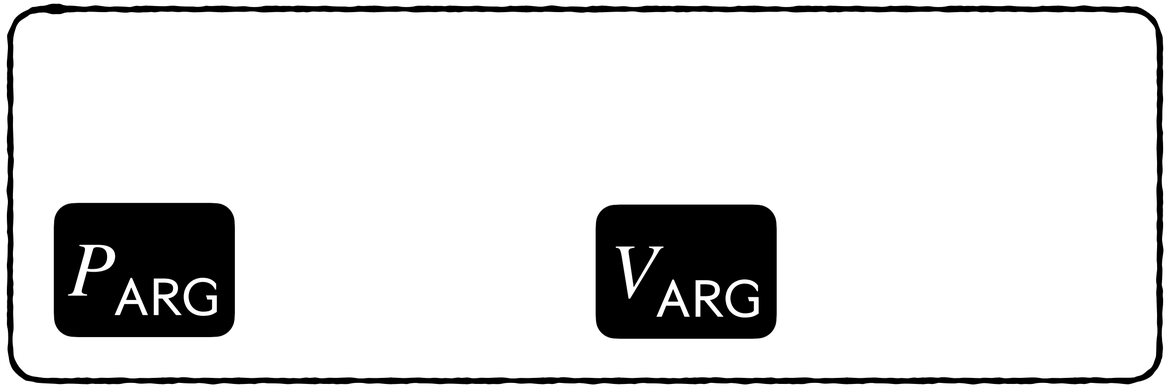

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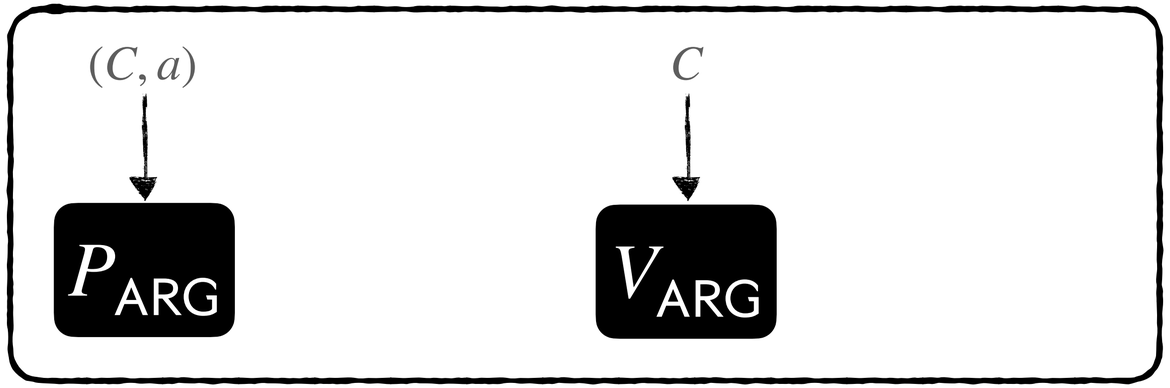

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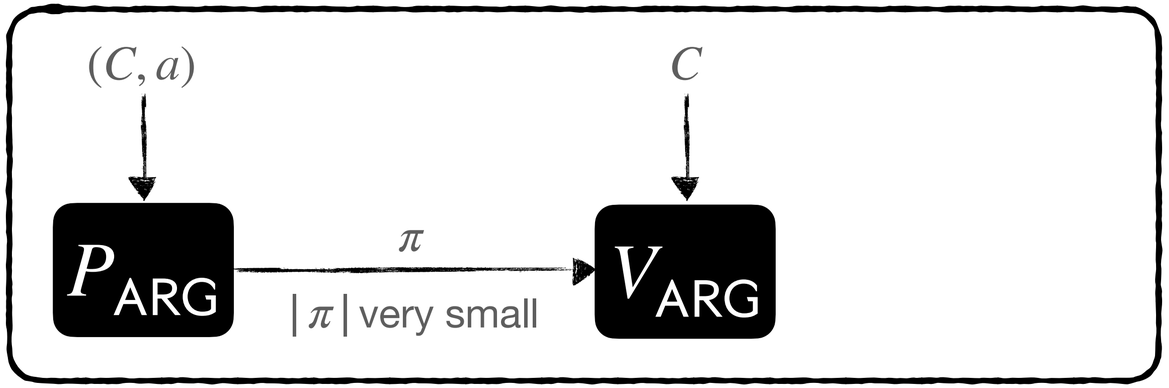

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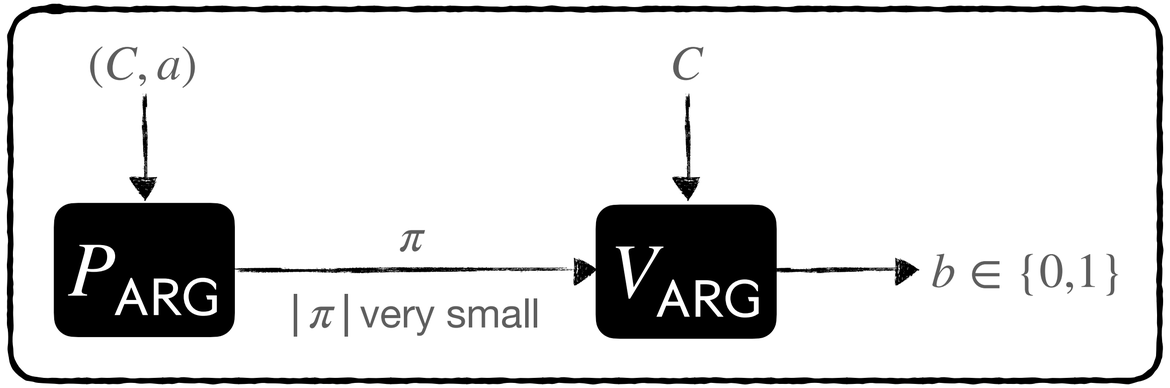

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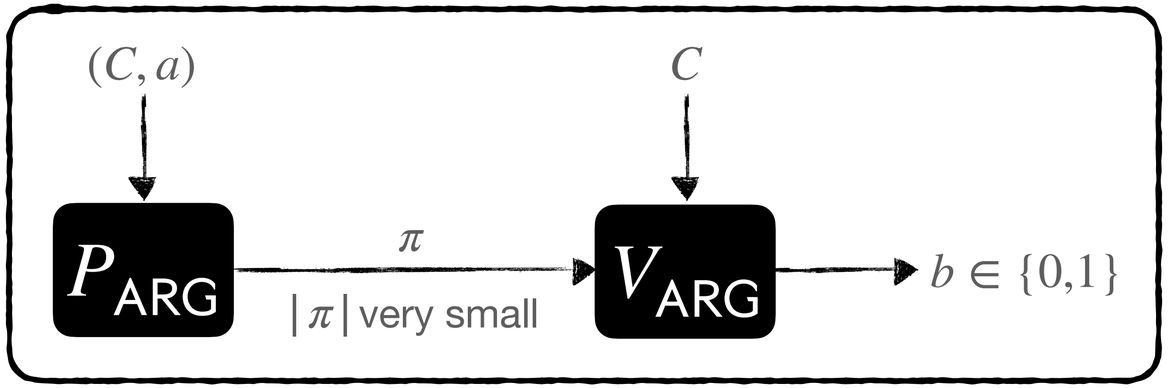


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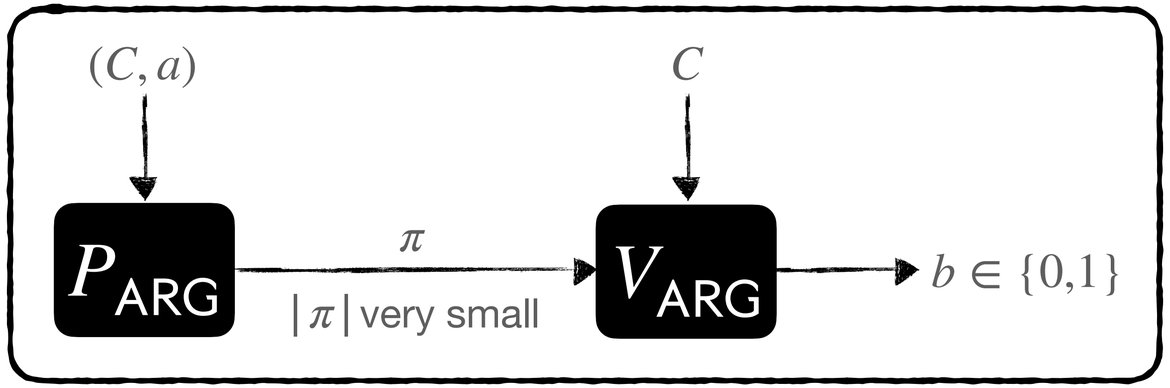


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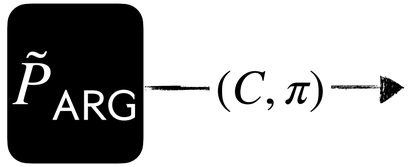
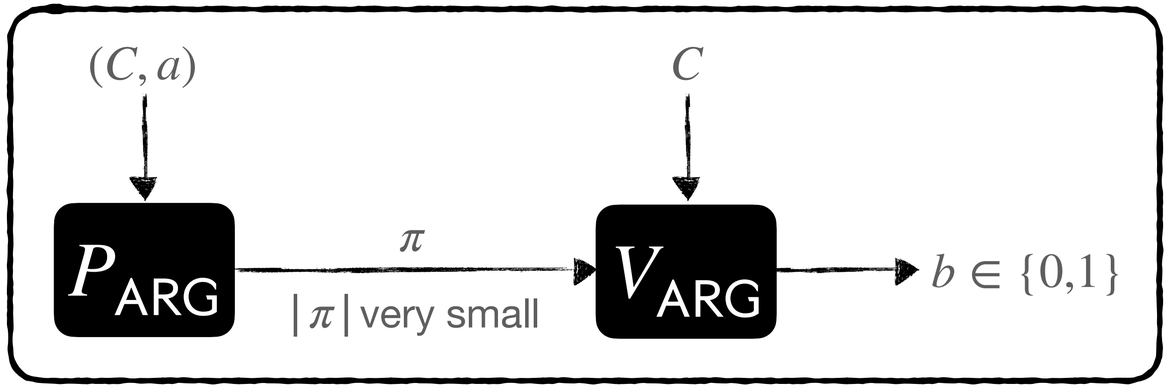


# Review: SNARK

PCD can be constructed from a SNARK (e.g., for CSAT).

$$\text{CSAT} := \{(C, a) : C(a) = 1\}$$

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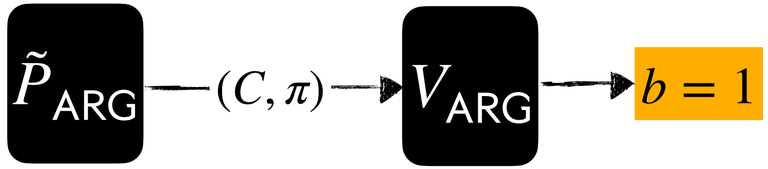

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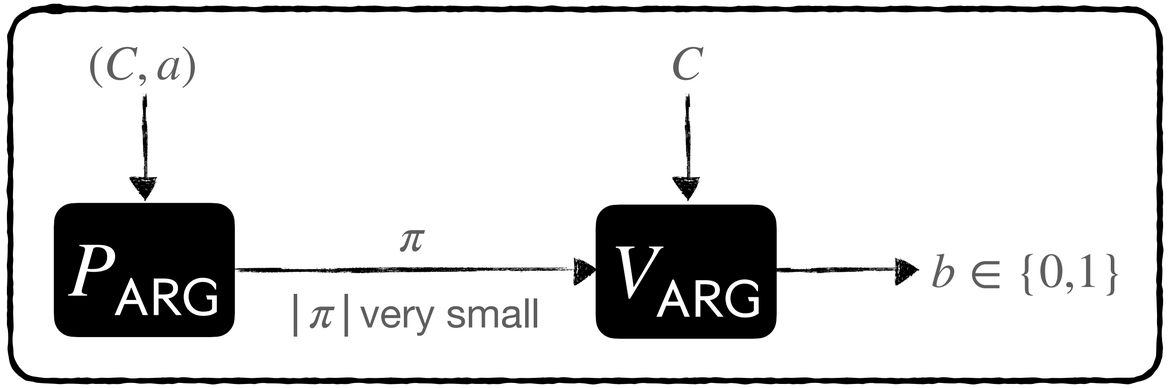


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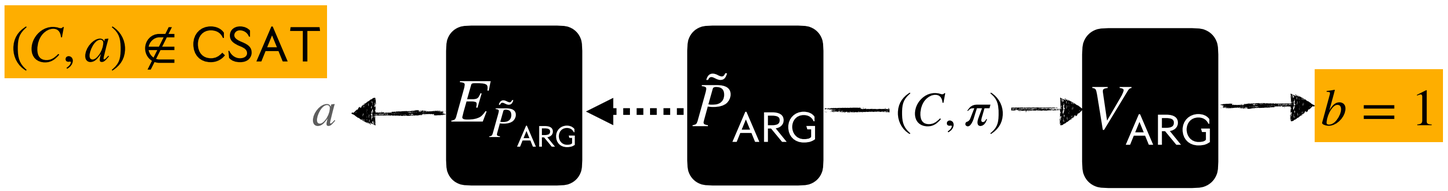
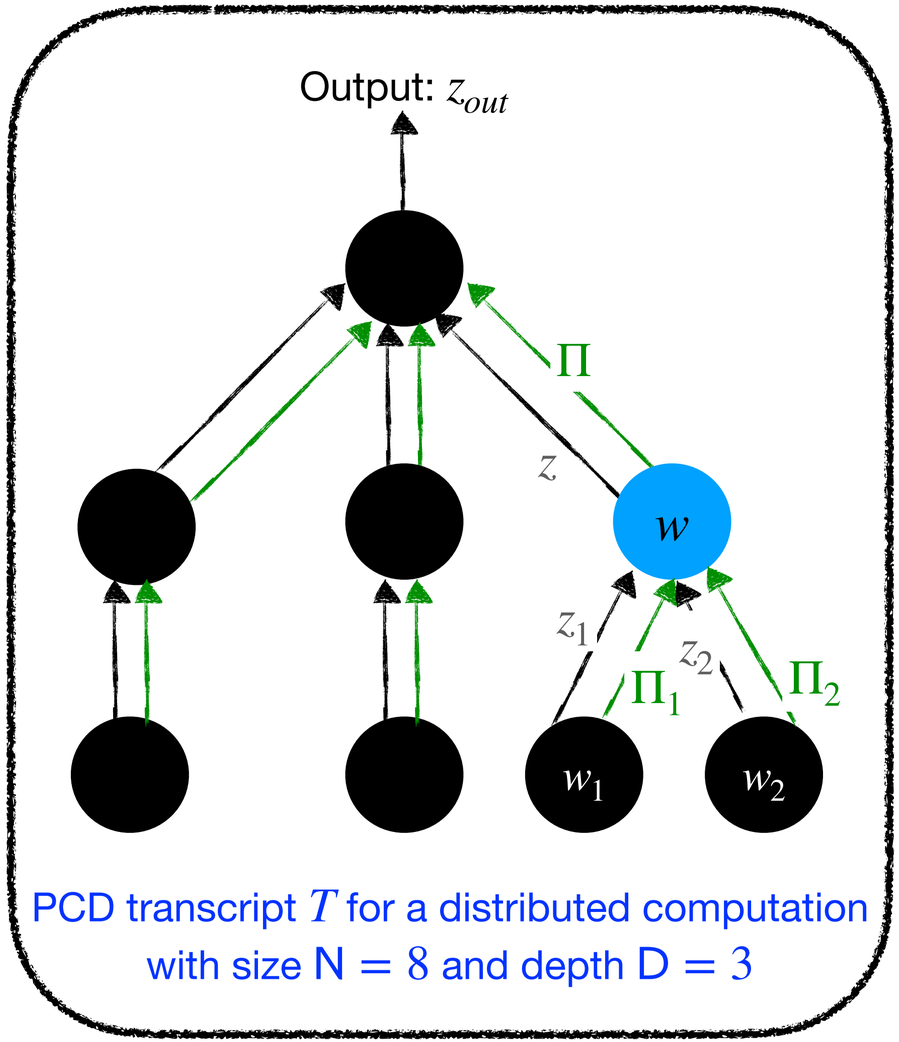


# Naive PCD construction

## Concatenate SNARK proofs




PCD transcript $T$ for a distributed computation
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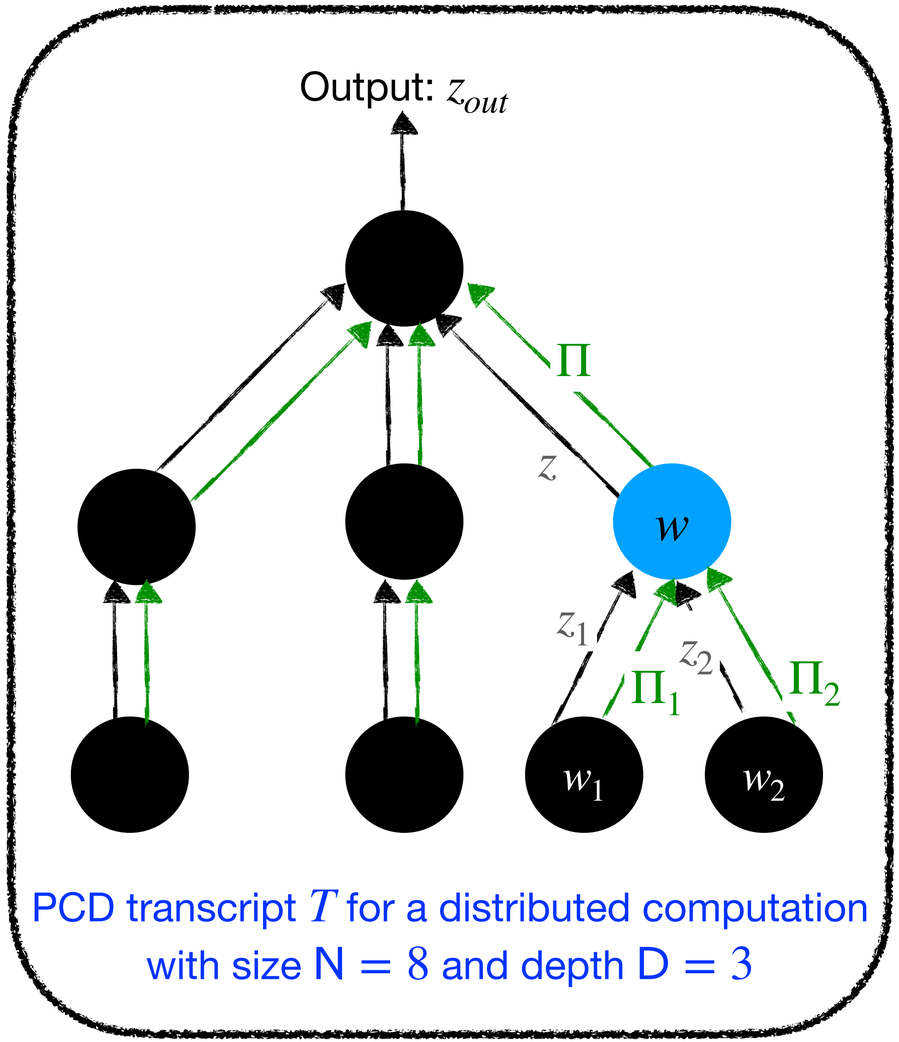


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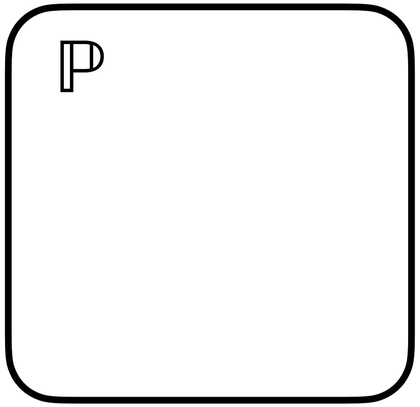

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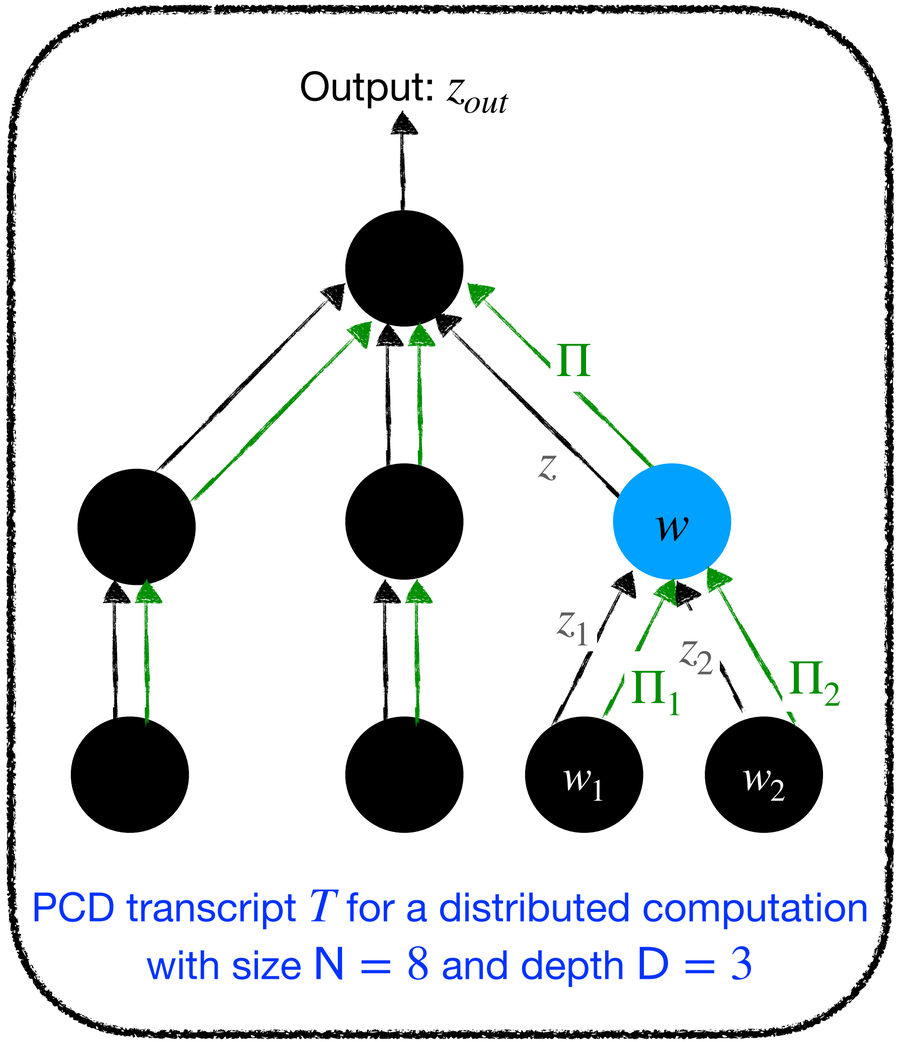


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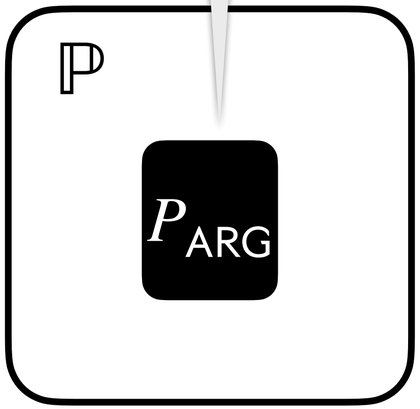

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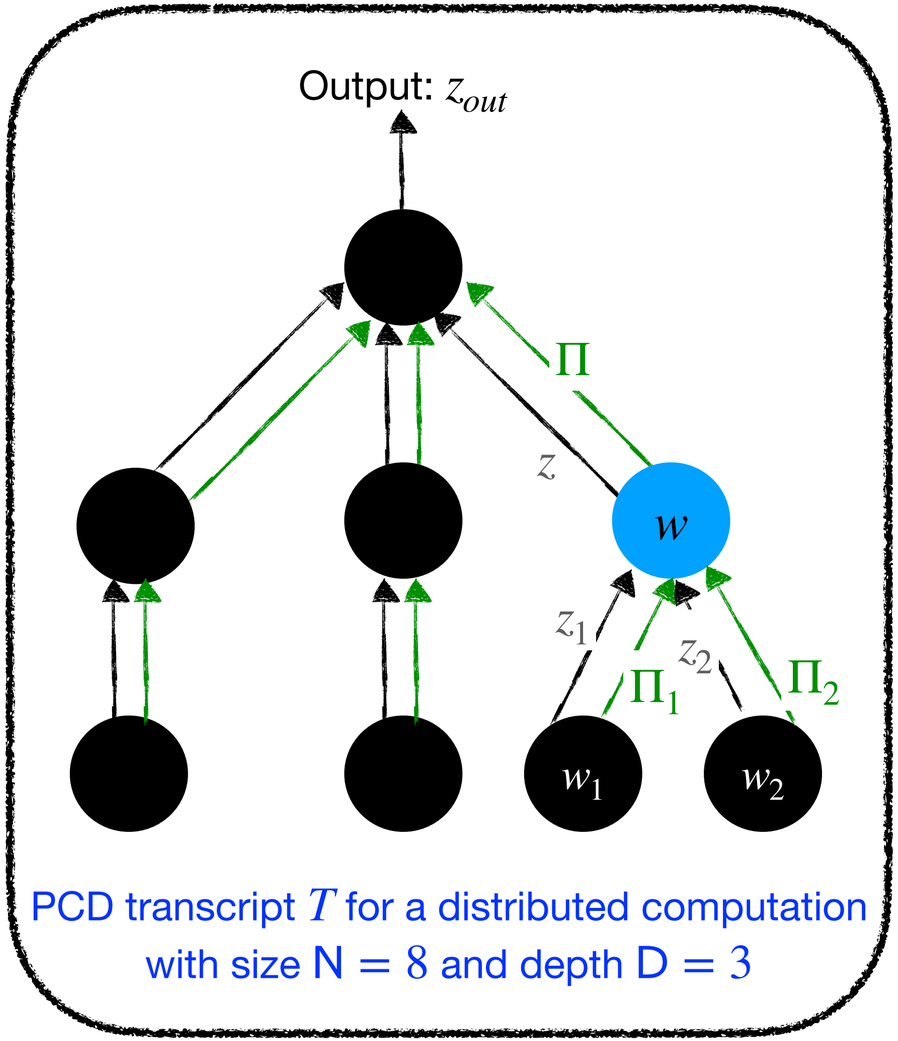


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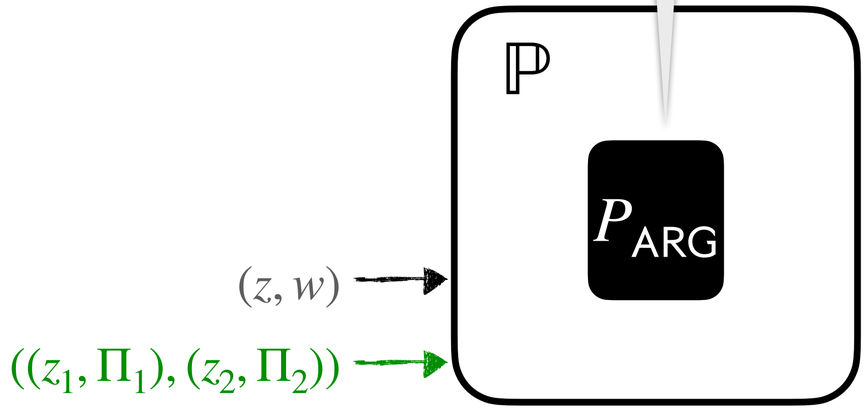

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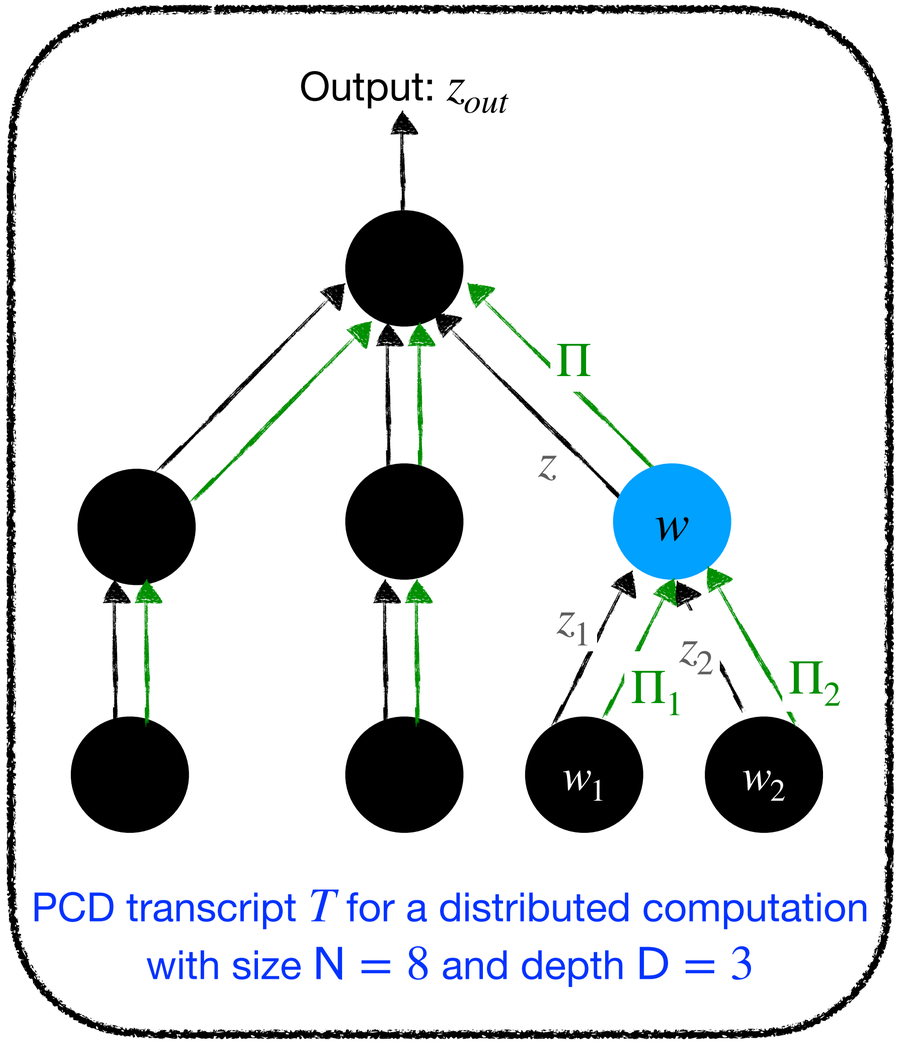


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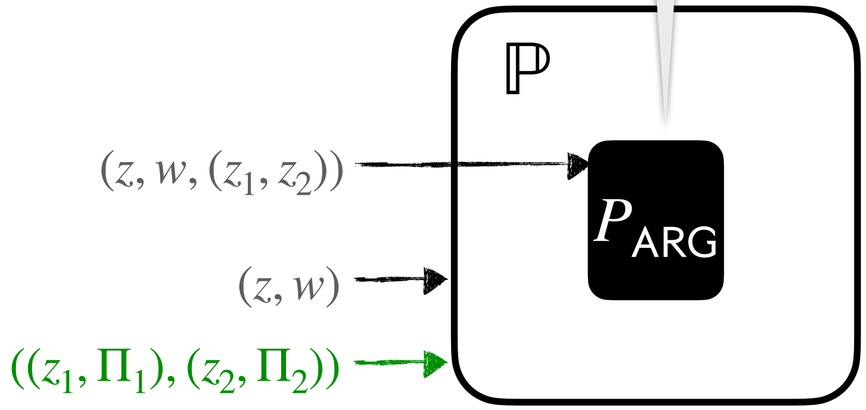

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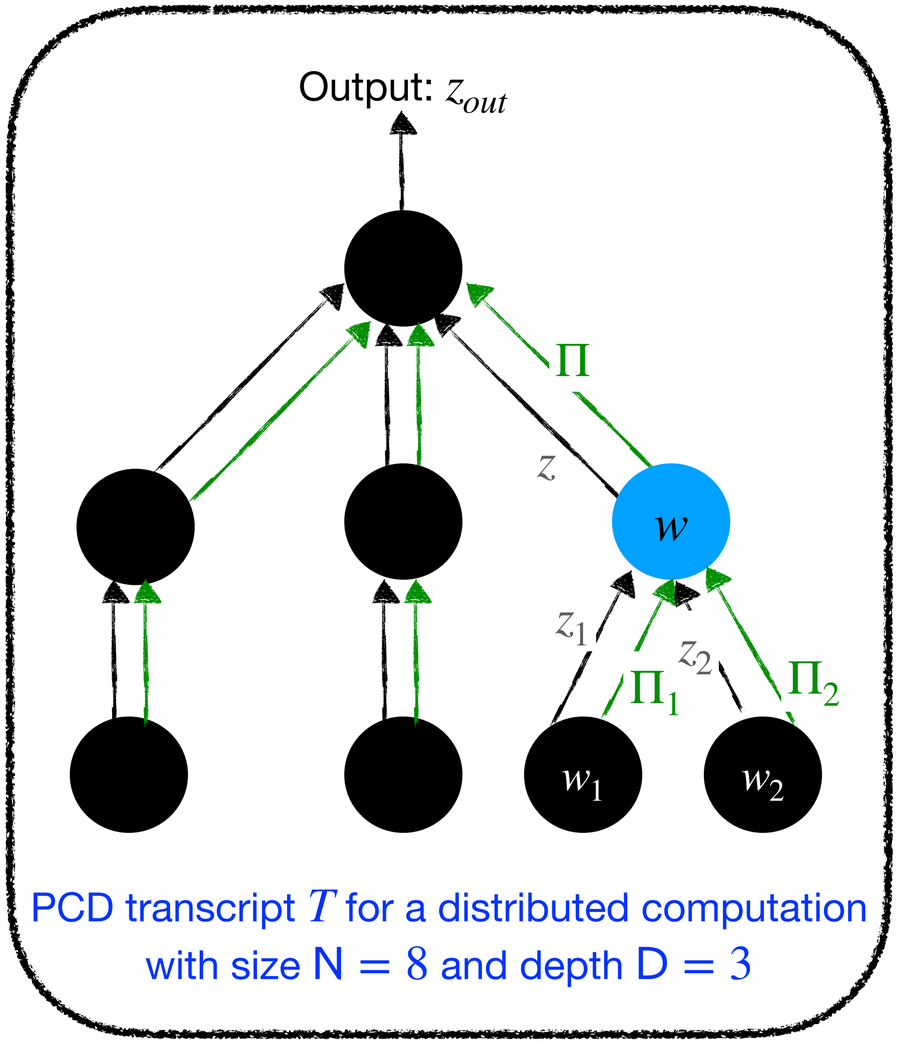


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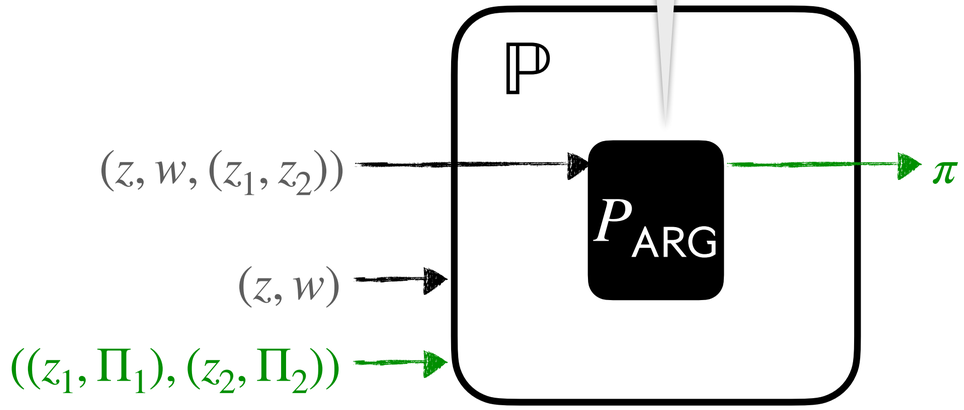

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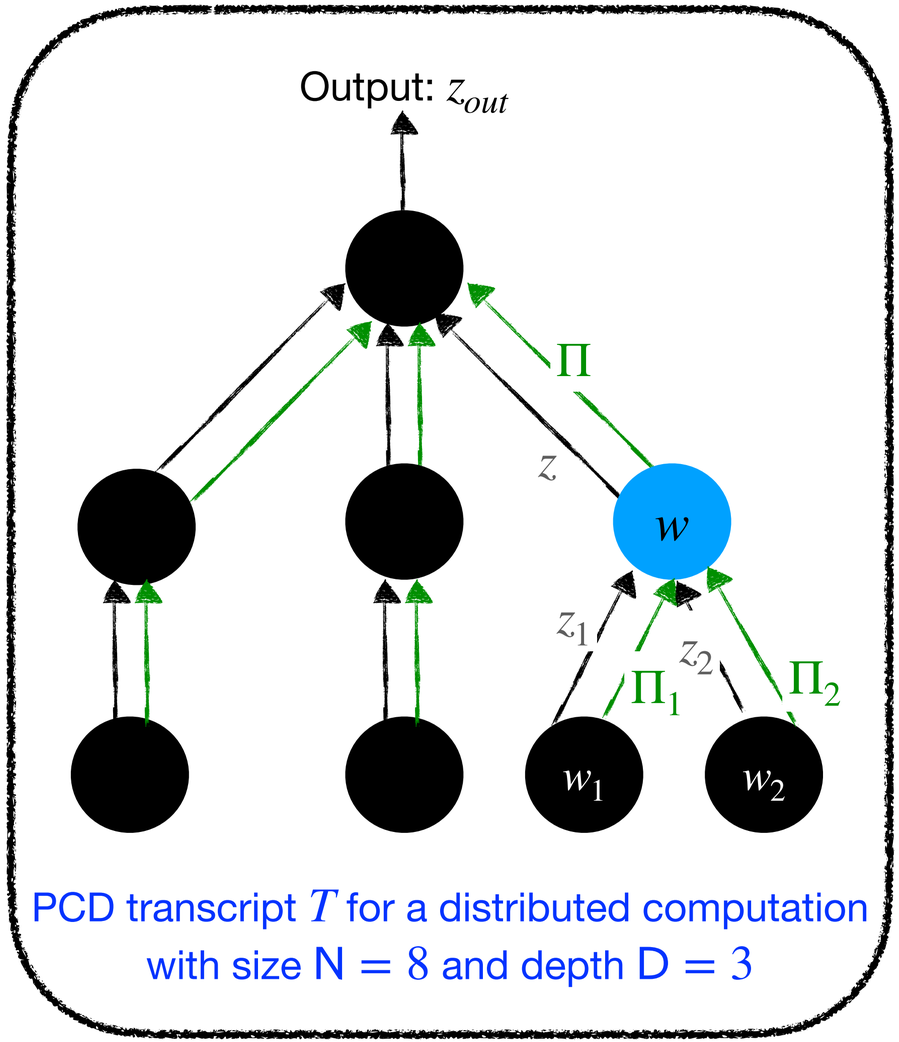


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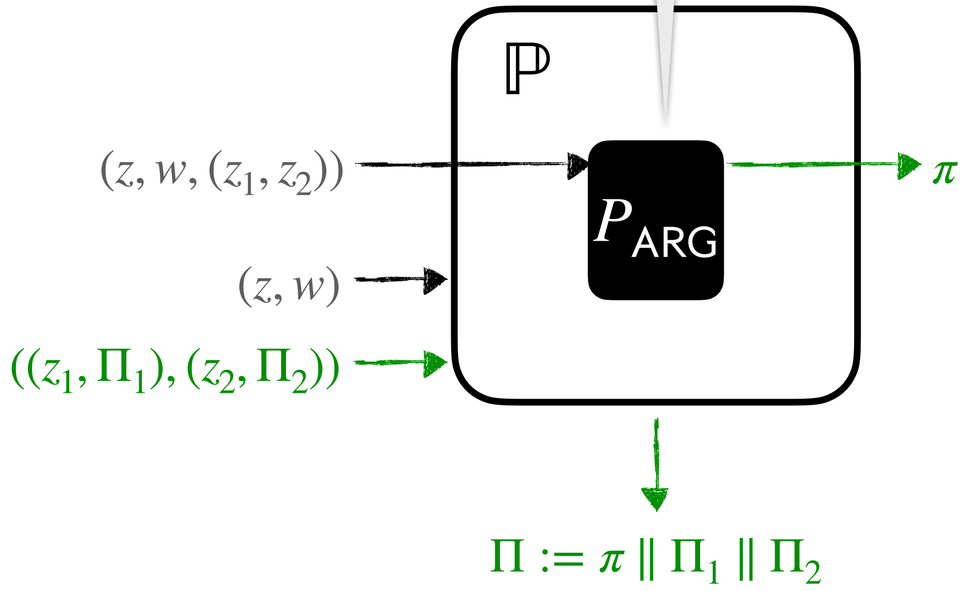

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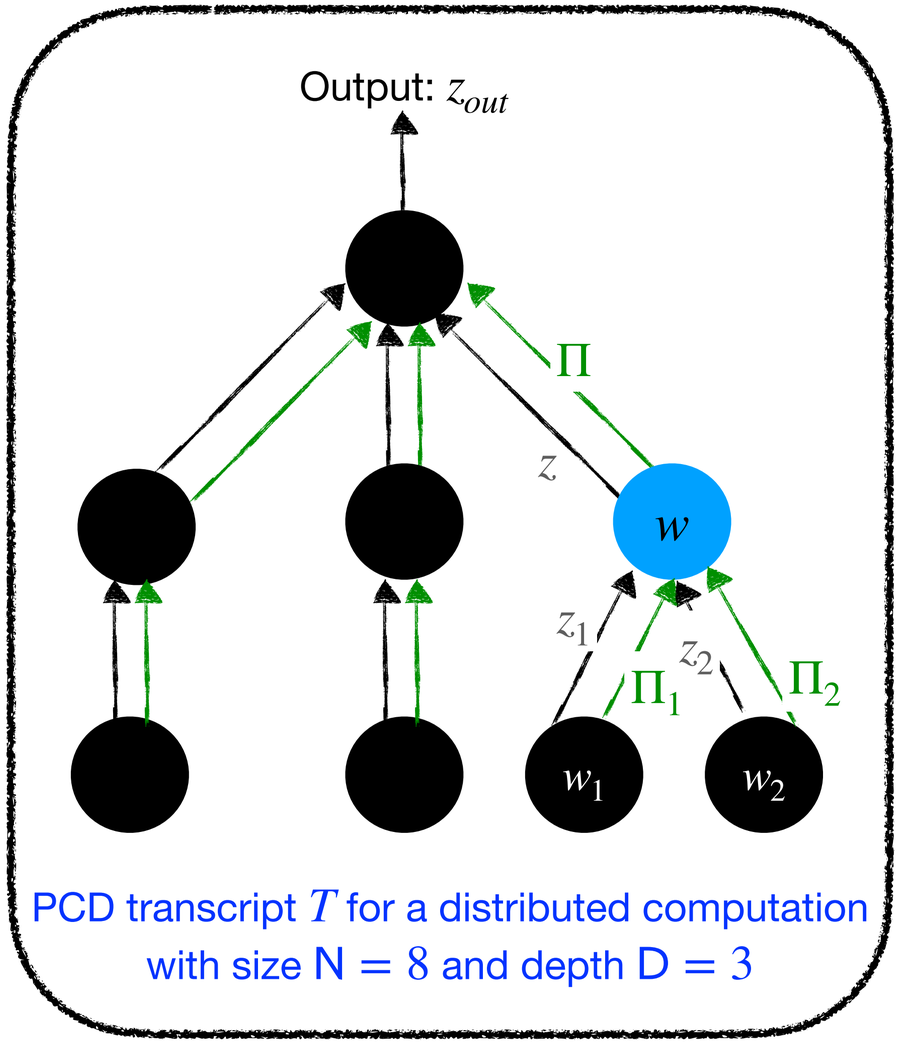


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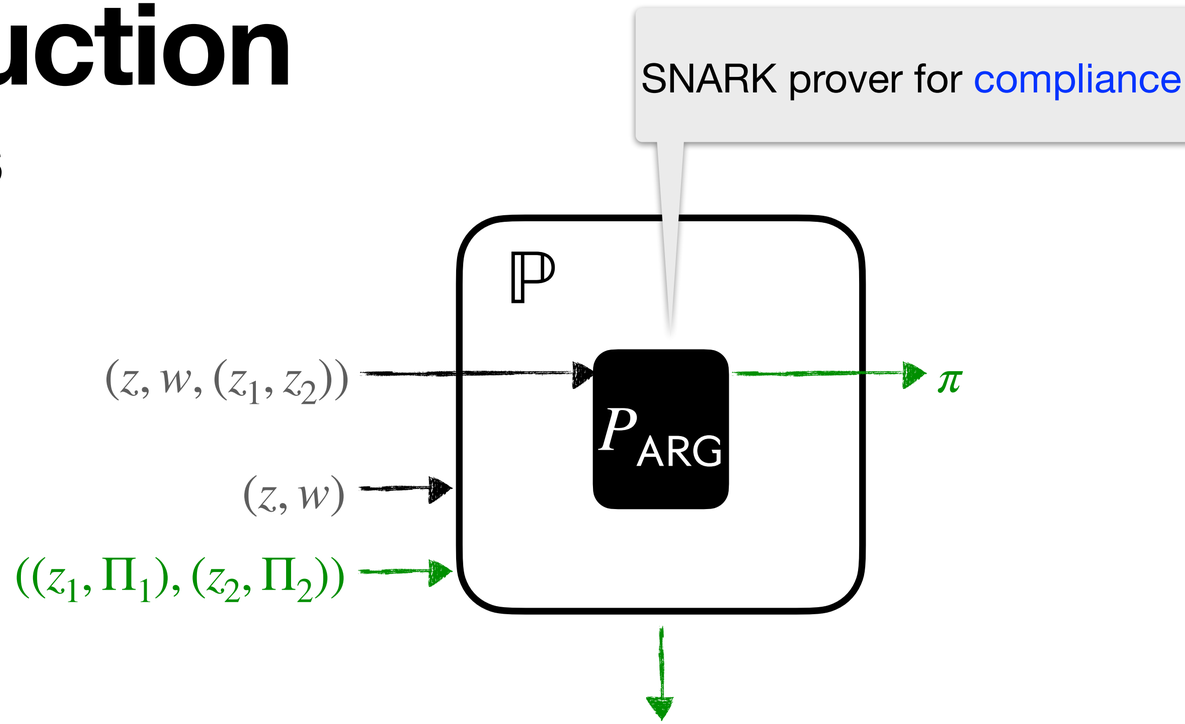


$\phi(z, w, (z_1, z_2)) = 1$

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# Naive PCD construction

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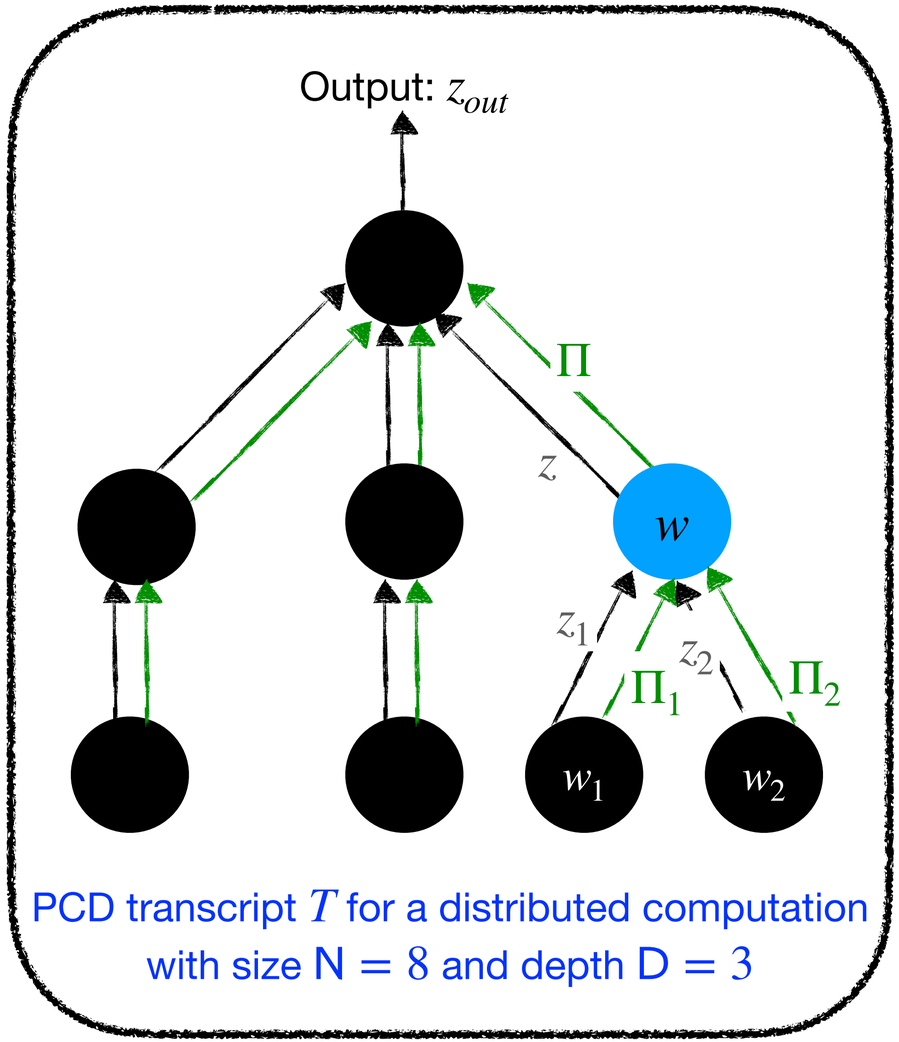


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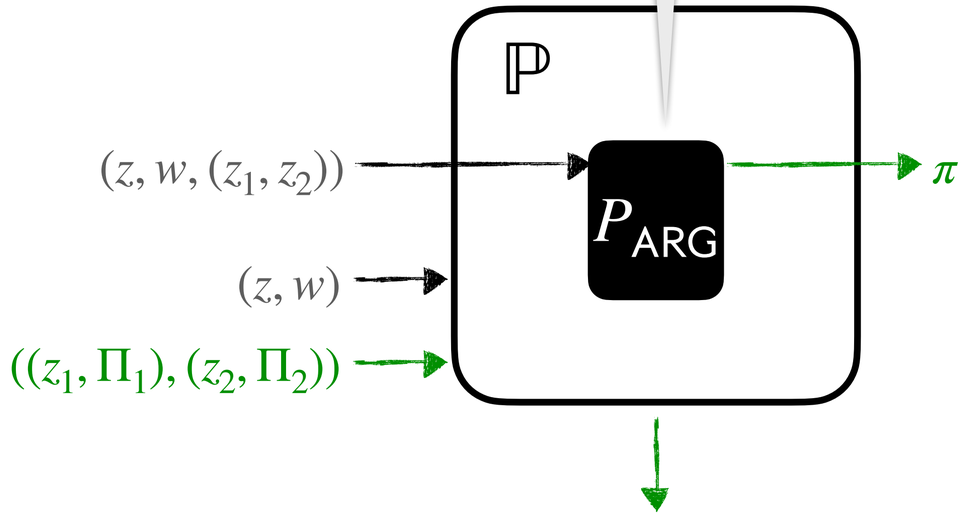


$\Pi := \pi \parallel \Pi_1 \parallel \Pi_2$

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**Pro: as secure as underlying SNARKs!**

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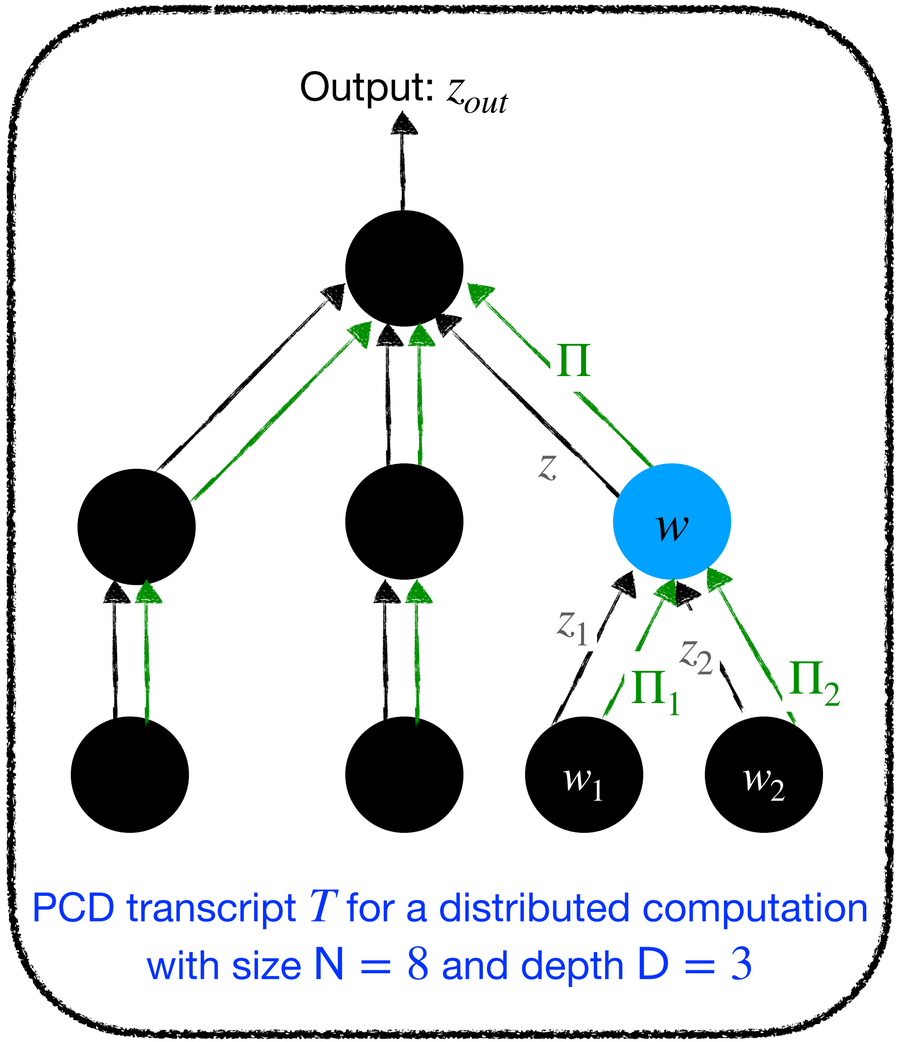


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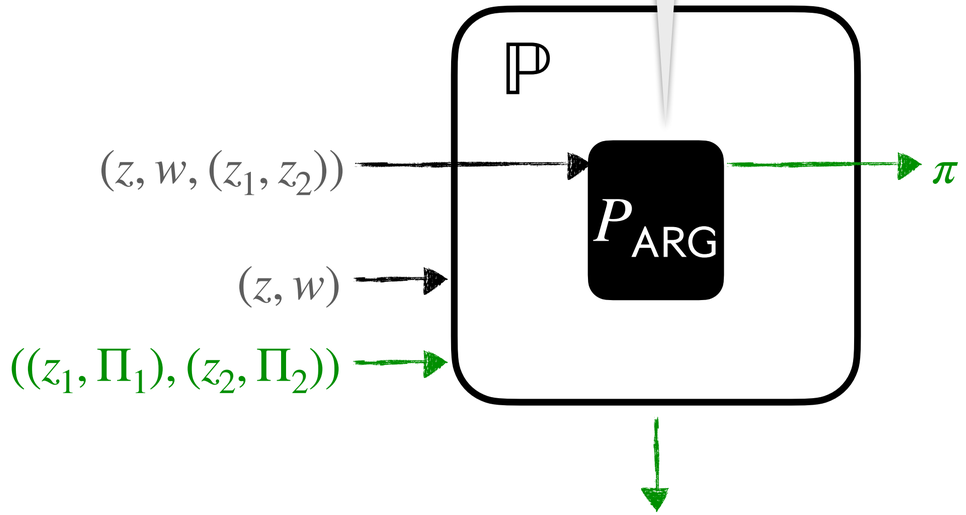


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**Pro: as secure as underlying SNARKs!**

**Con: $\Pi$ is NOT succinct (linear in number of vertices)**

# Recursively compose the SNARK proofs

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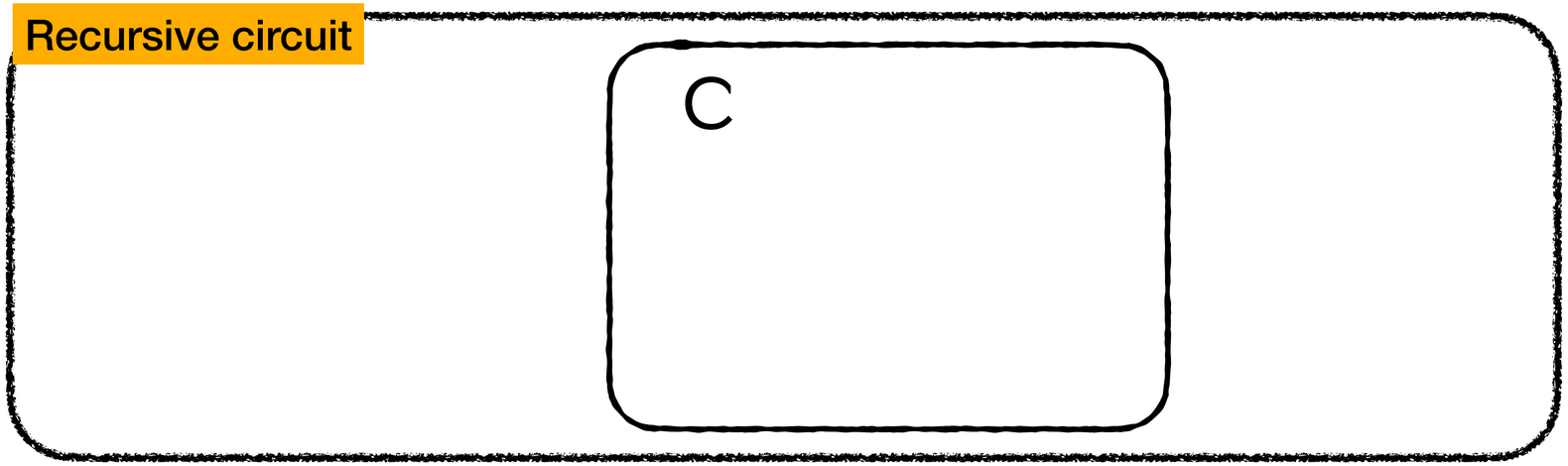

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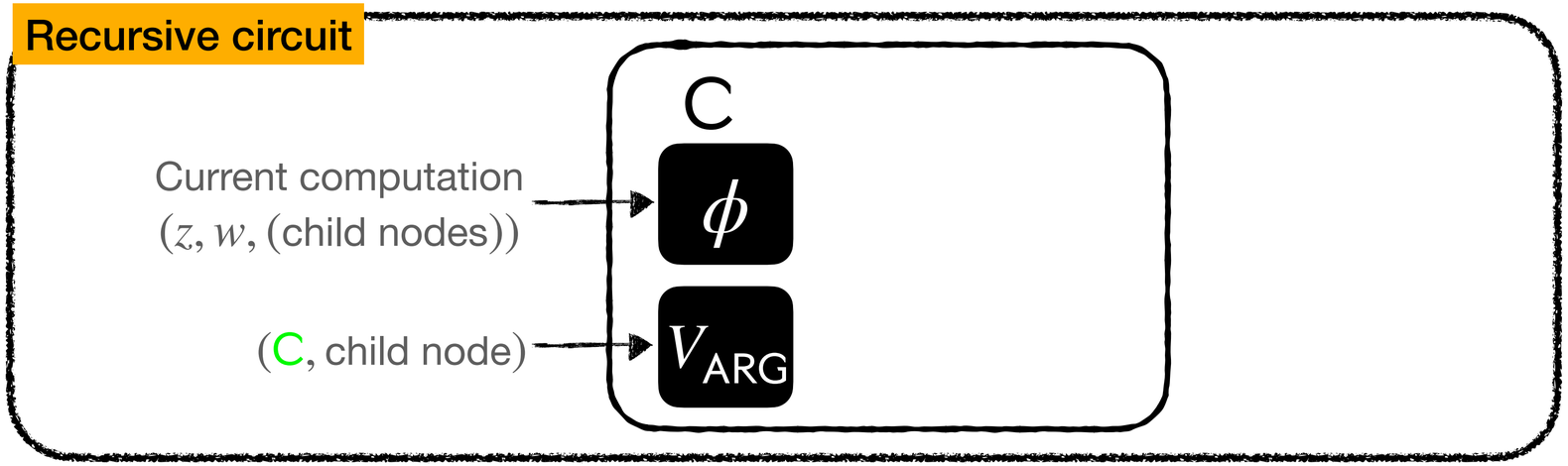

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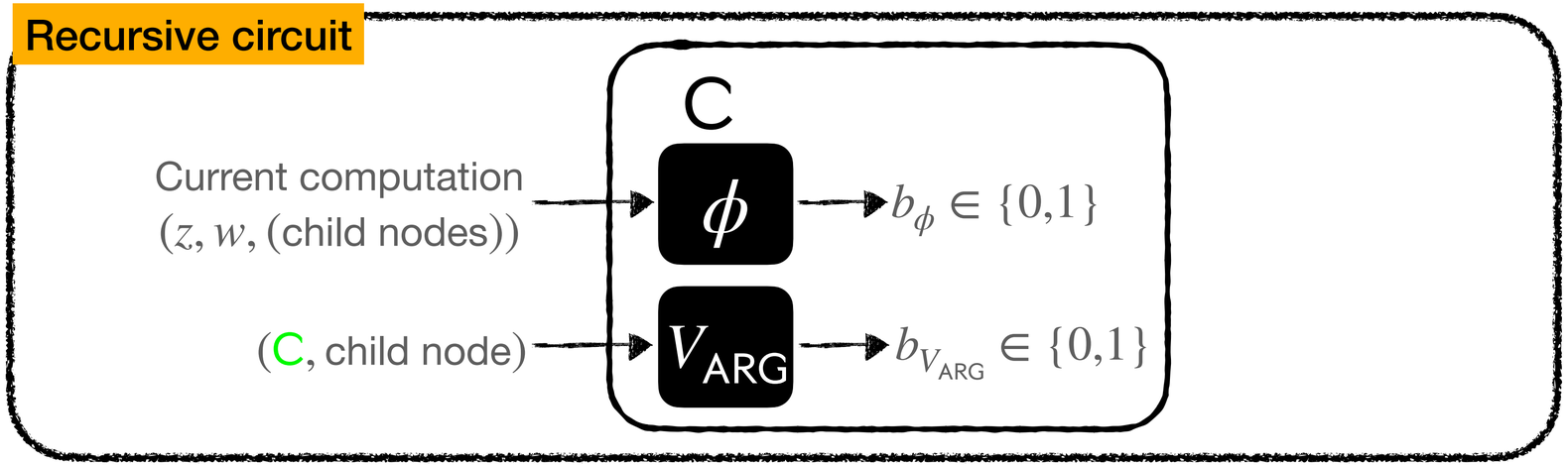

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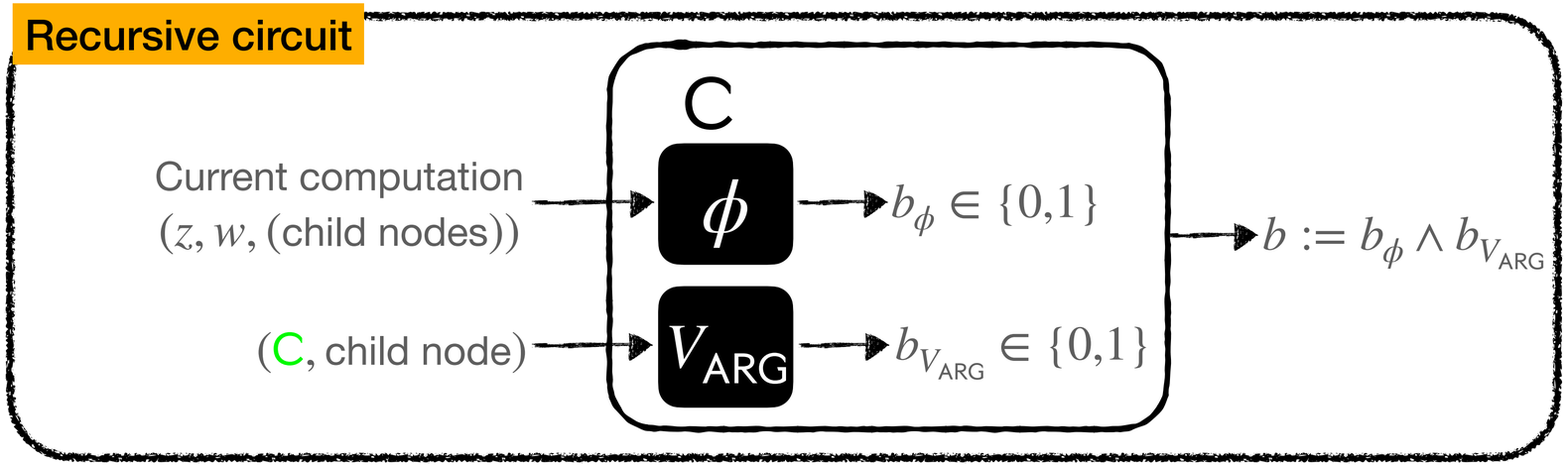

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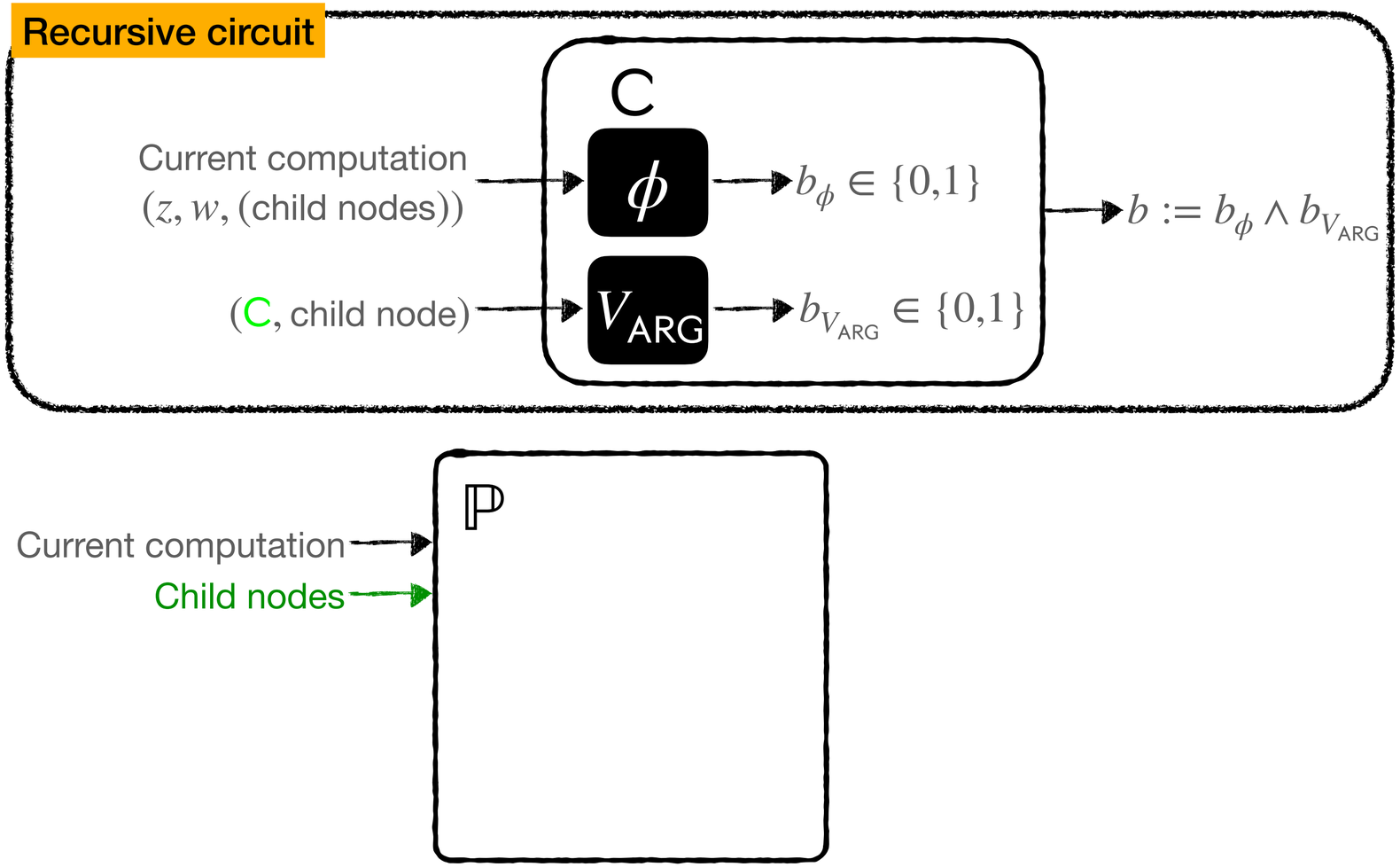

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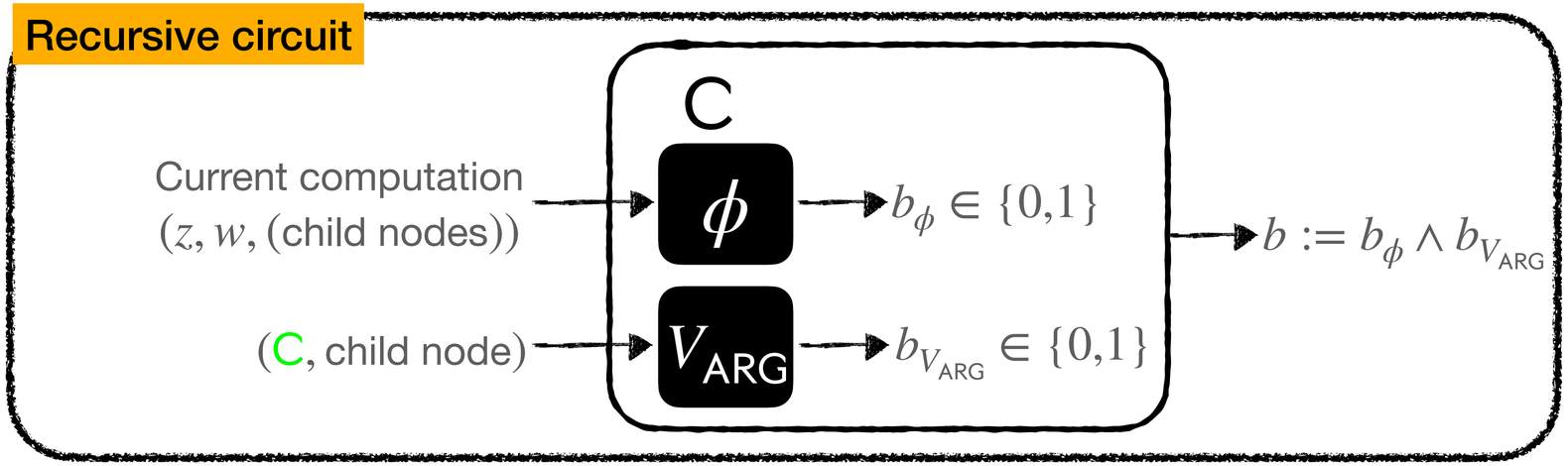


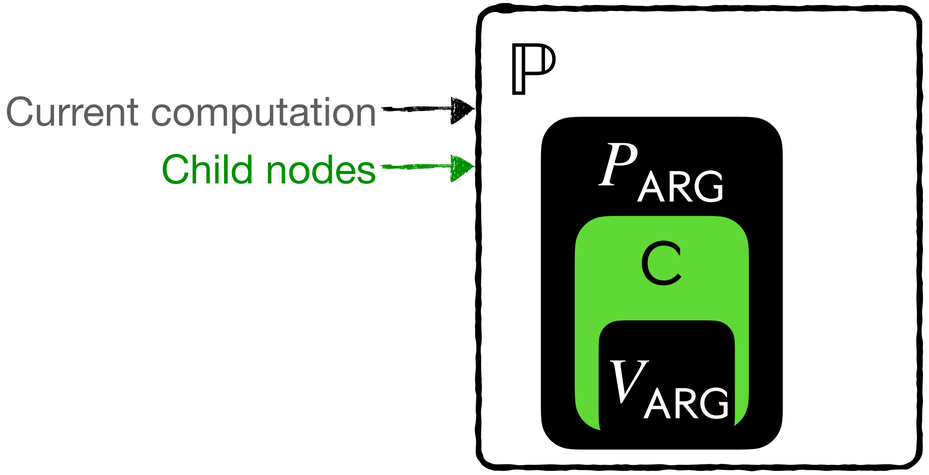

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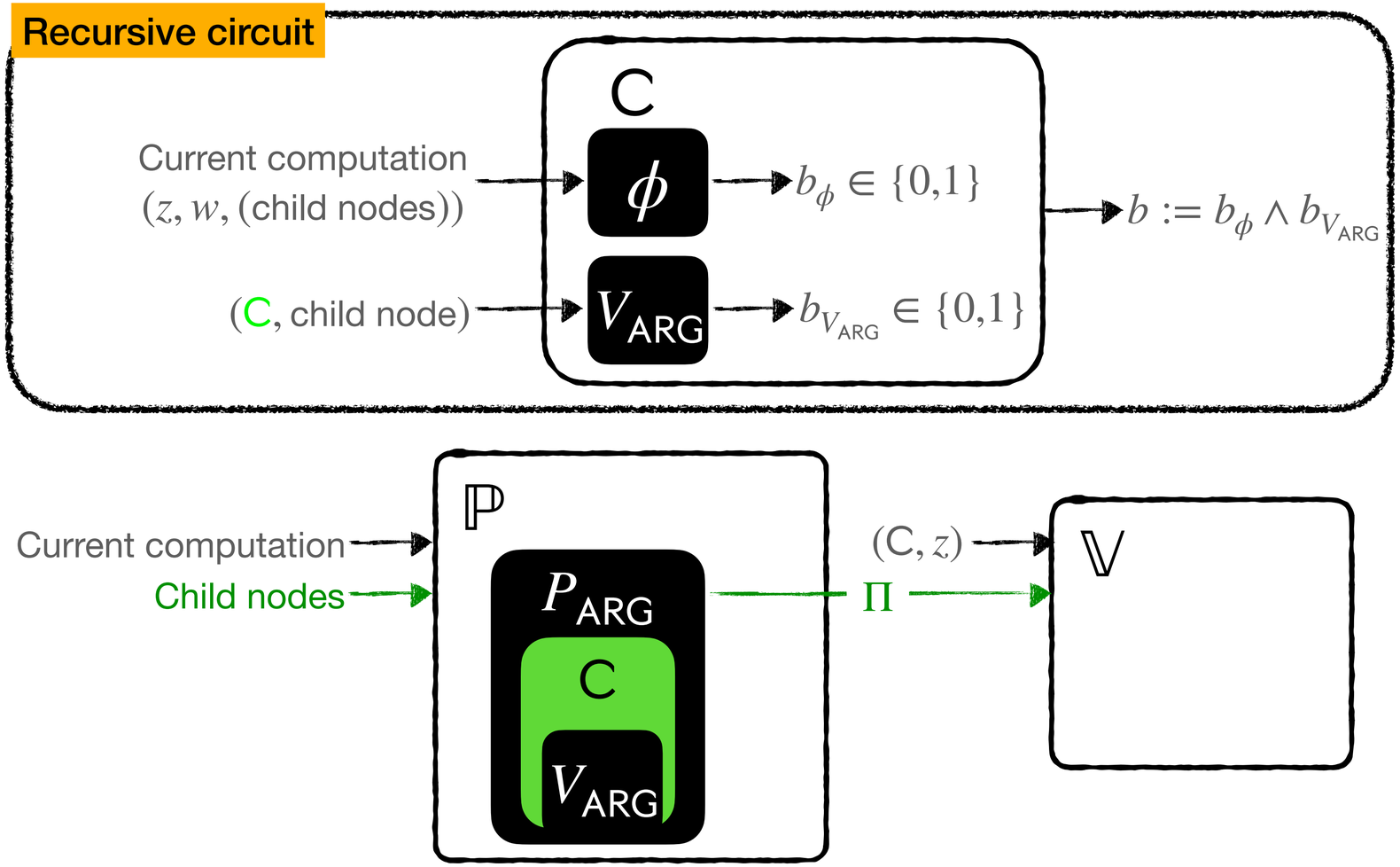

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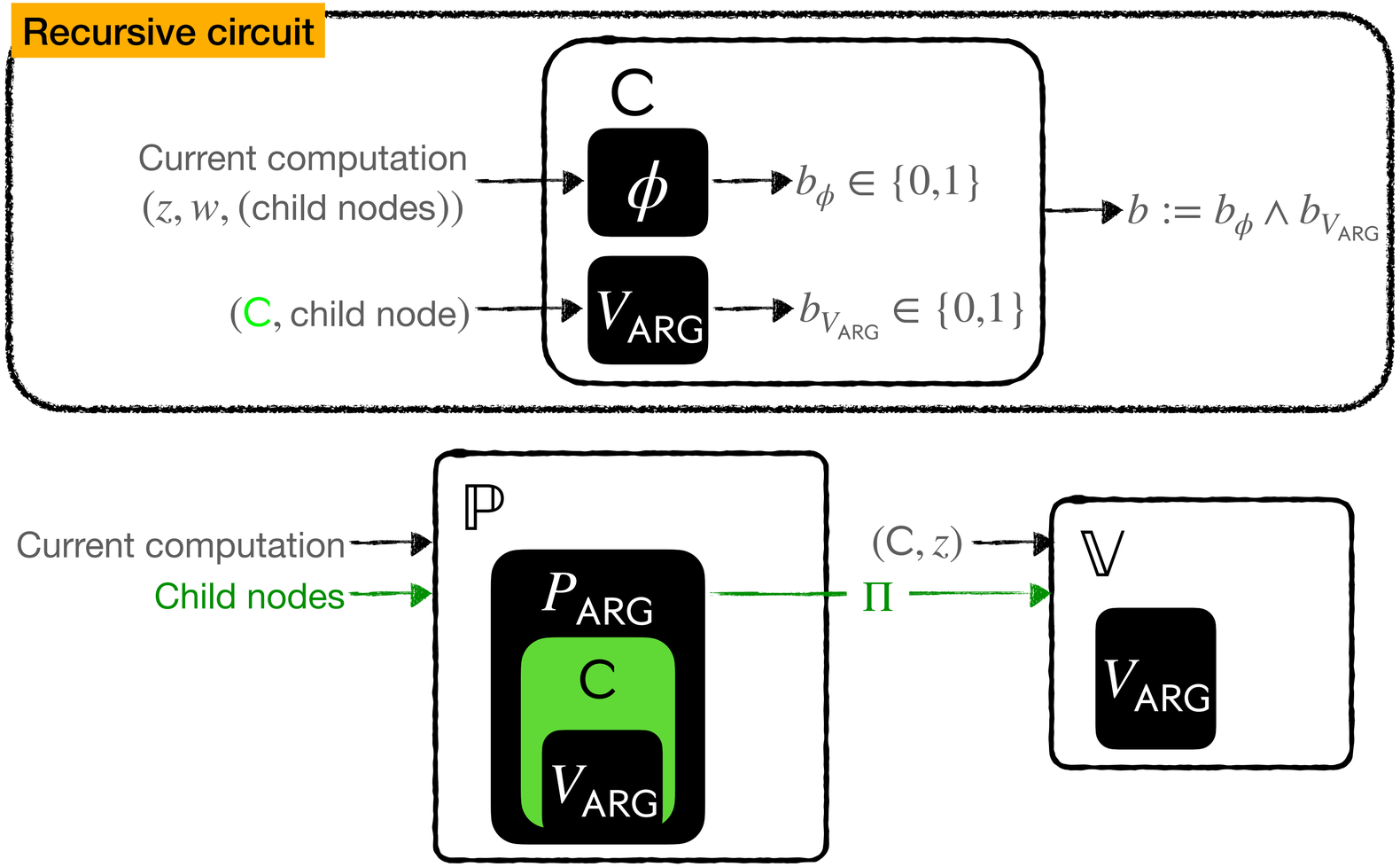

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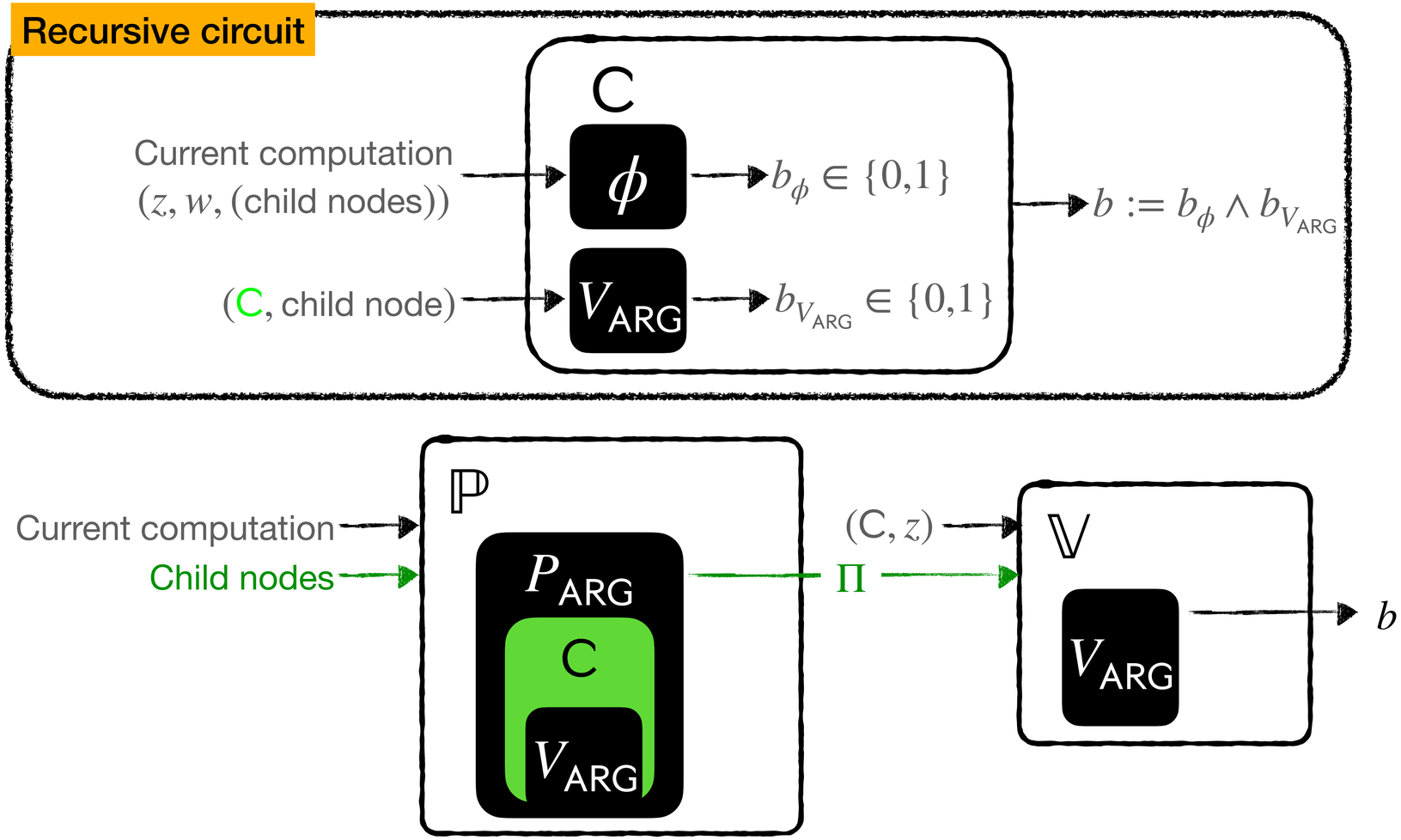

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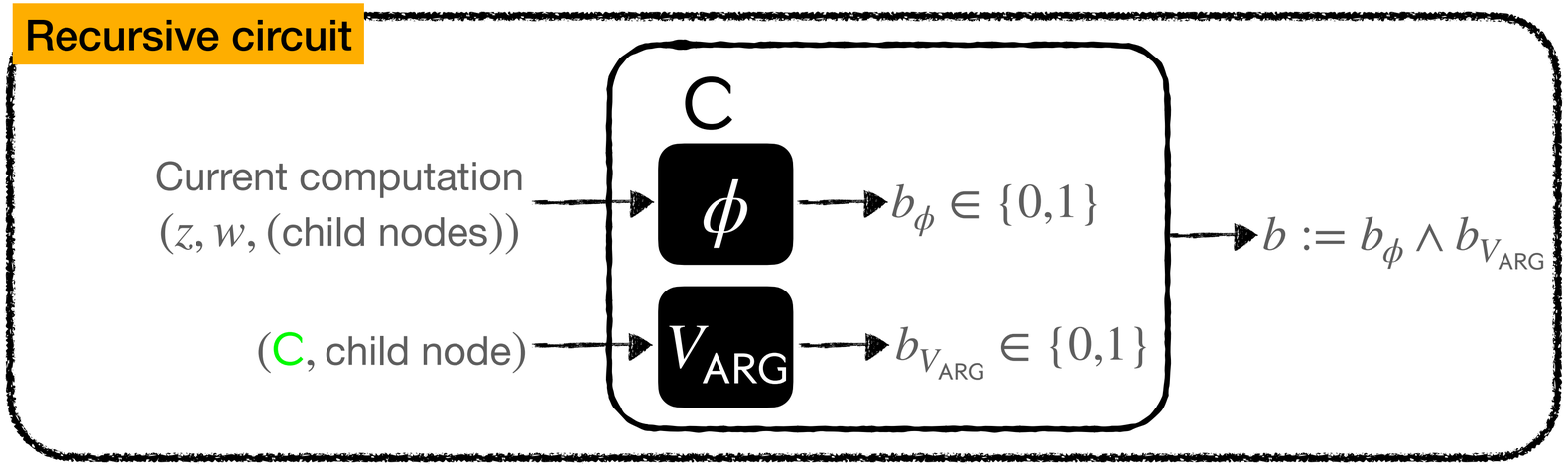


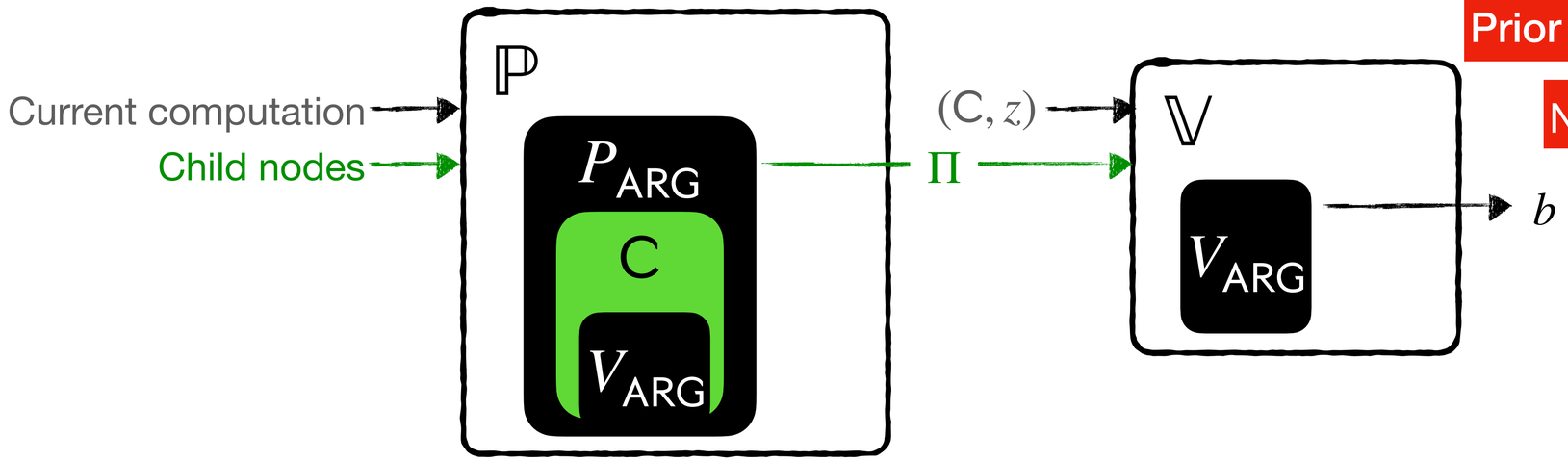


**Prior works:** $\kappa(\lambda, q, D, N) \leq \exp(D) \cdot \kappa_{\text{ARG}}(\lambda, q, N)$

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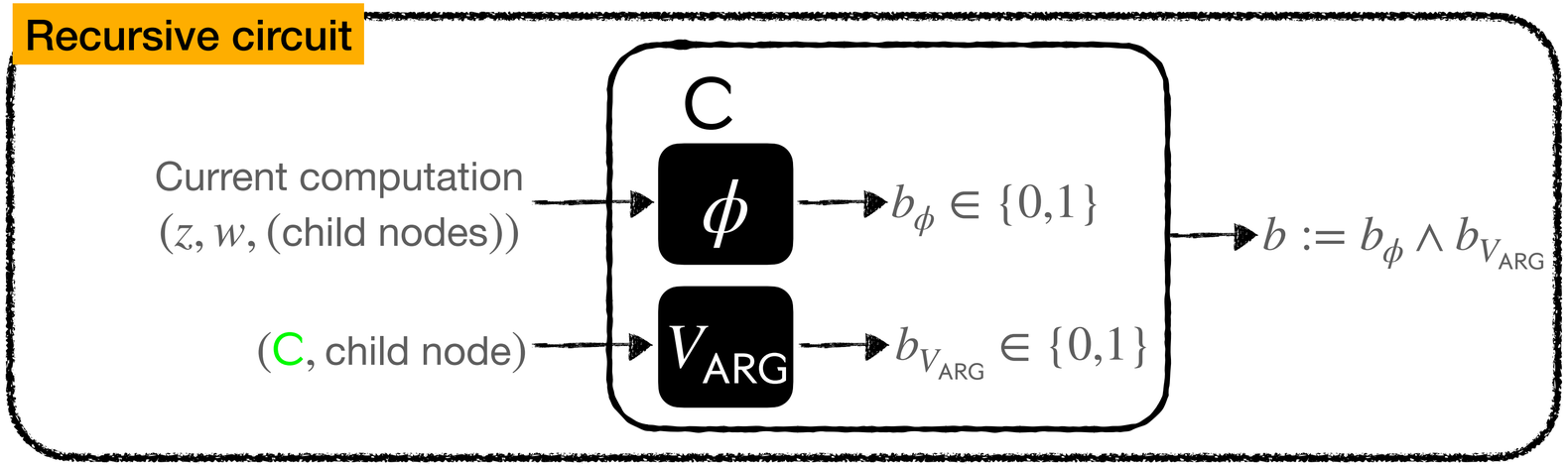


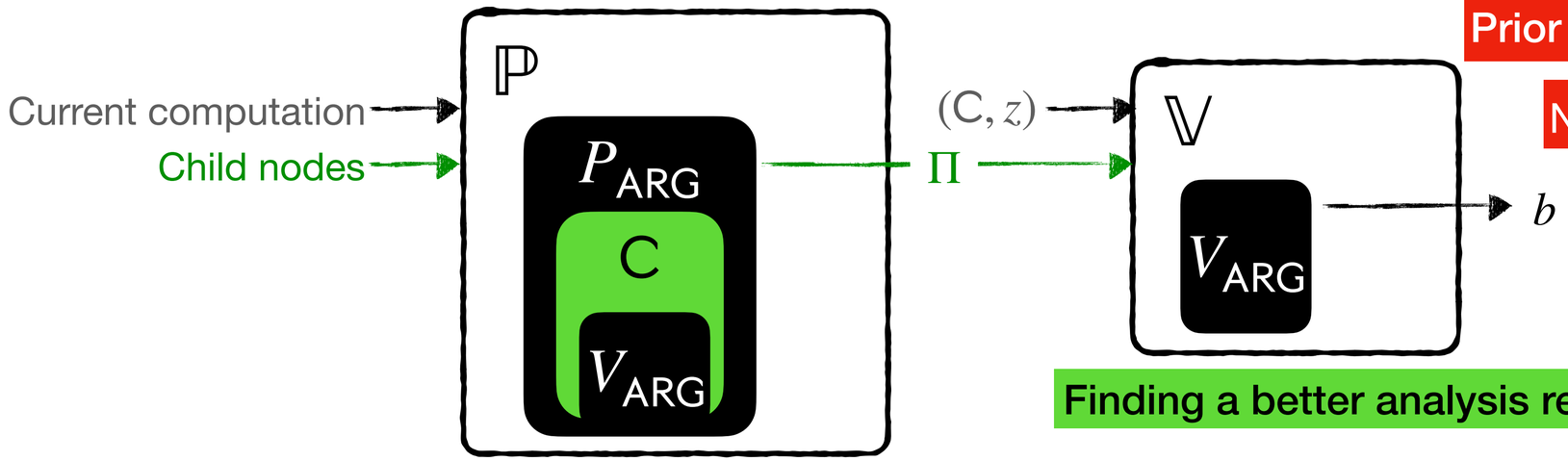


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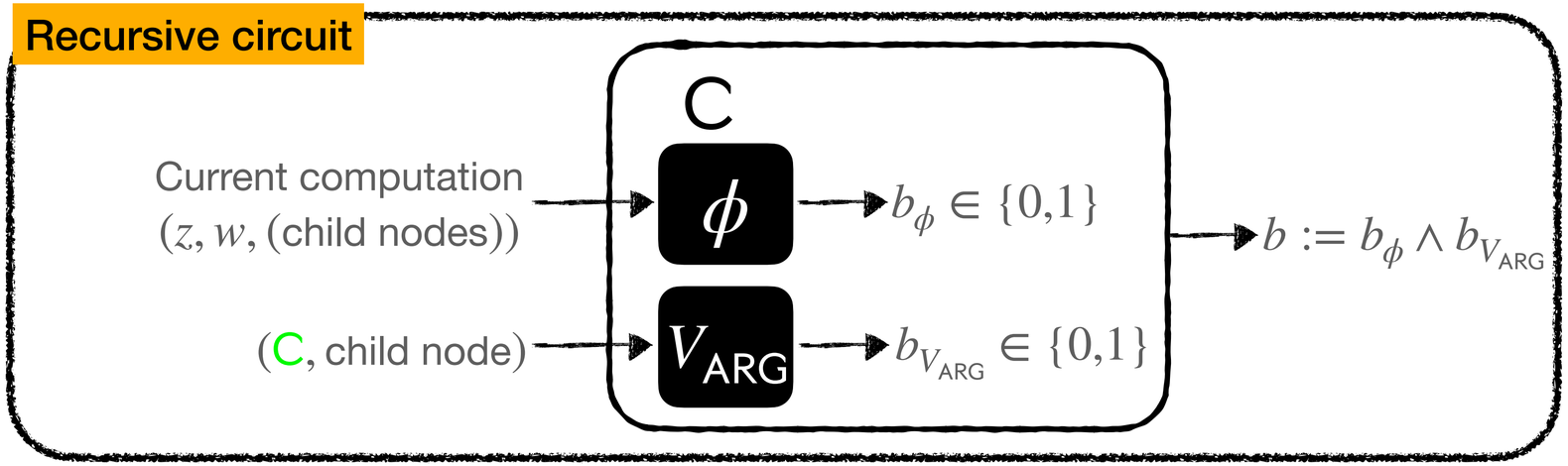


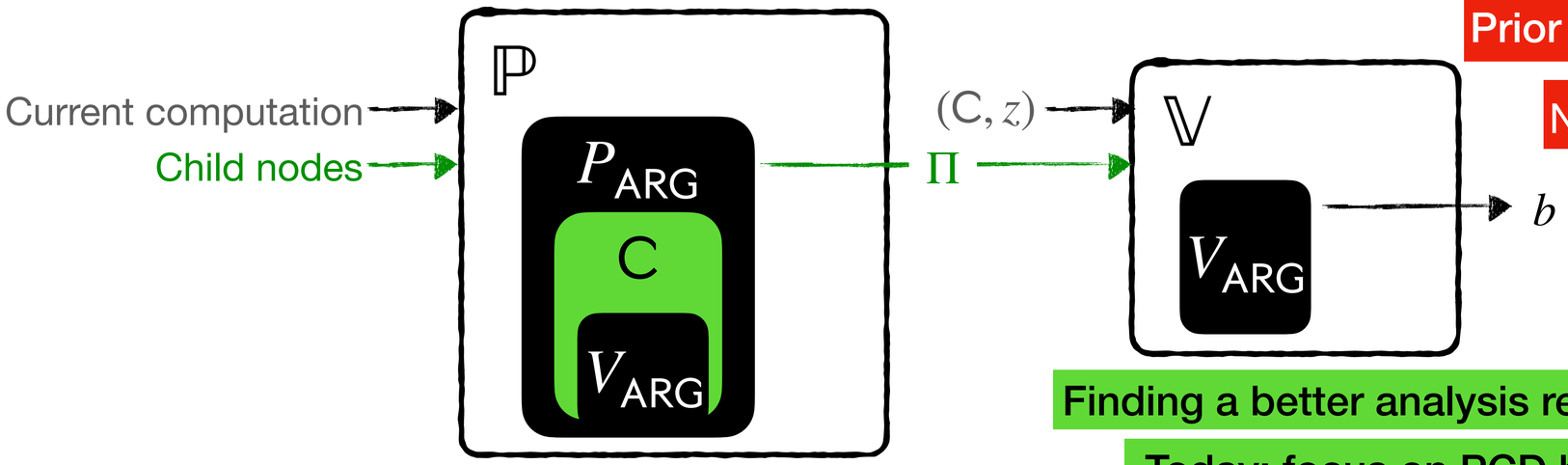


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Today: focus on PCD based on SNARKs with "strong" extraction.

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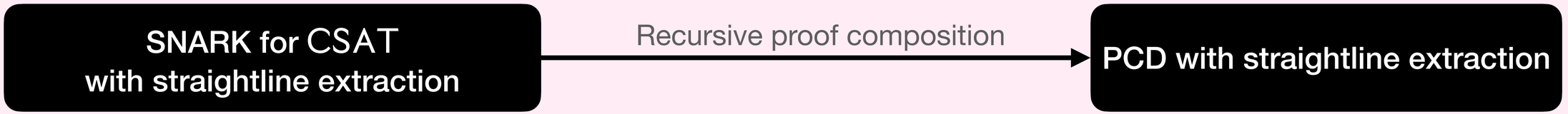

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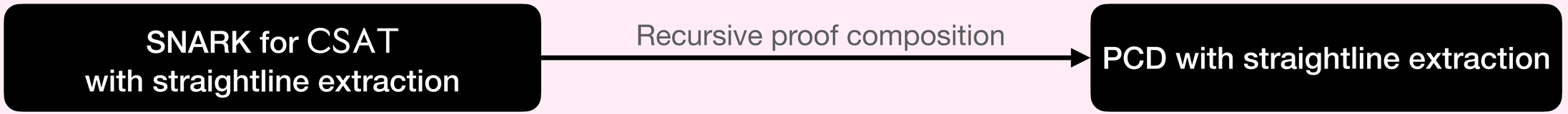


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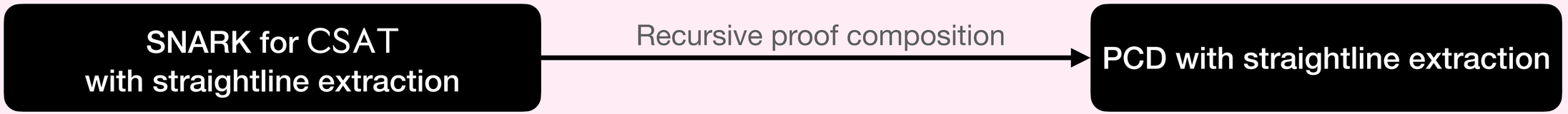


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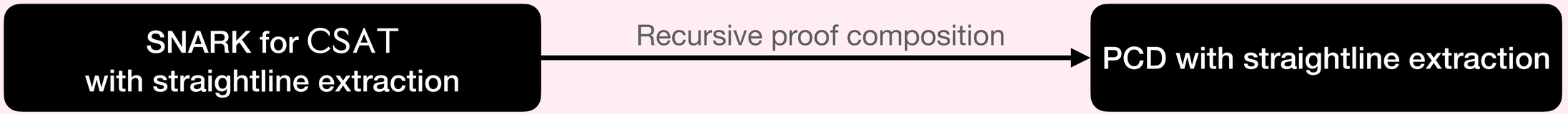


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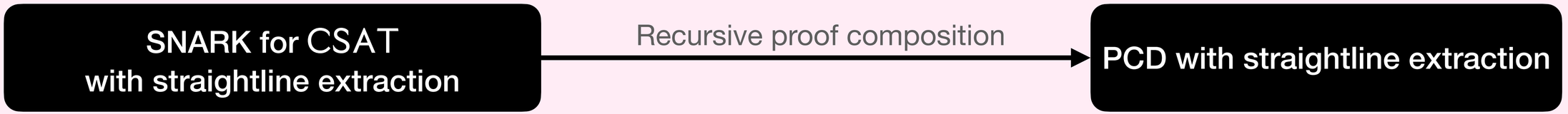


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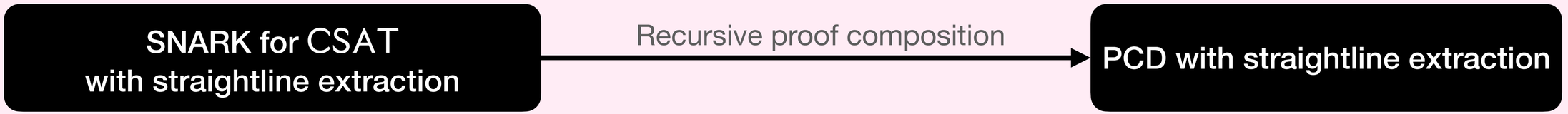


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**How can we apply our theorem in practice then?**

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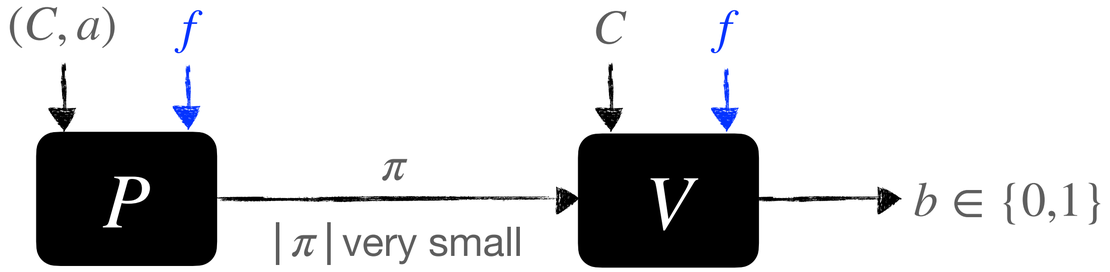

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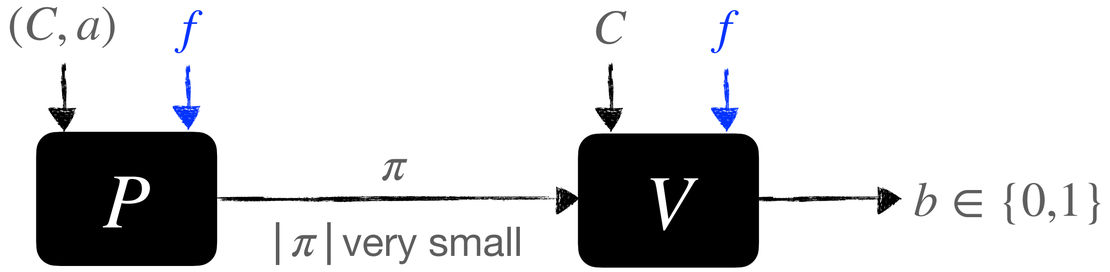


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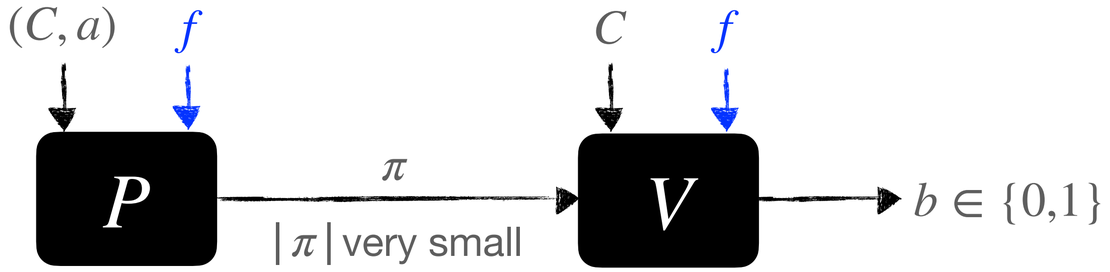


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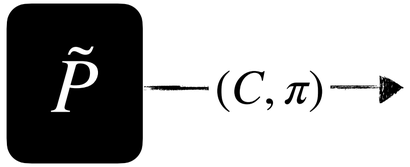

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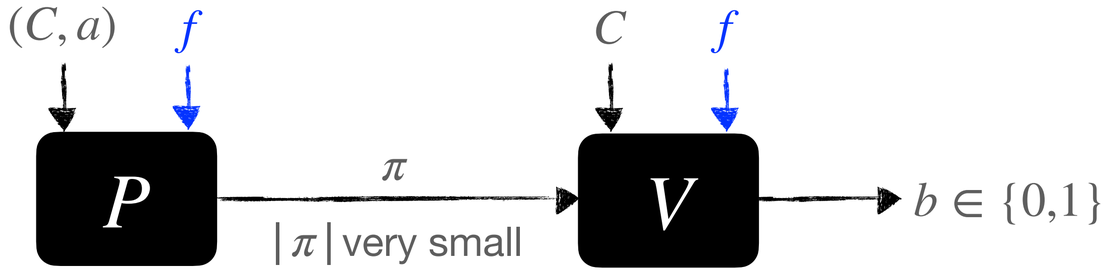


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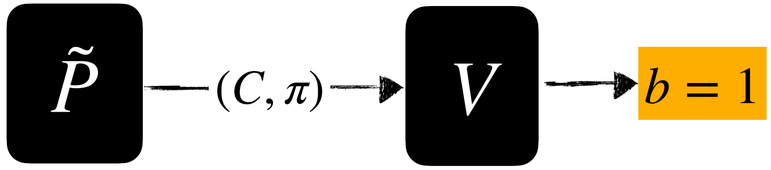

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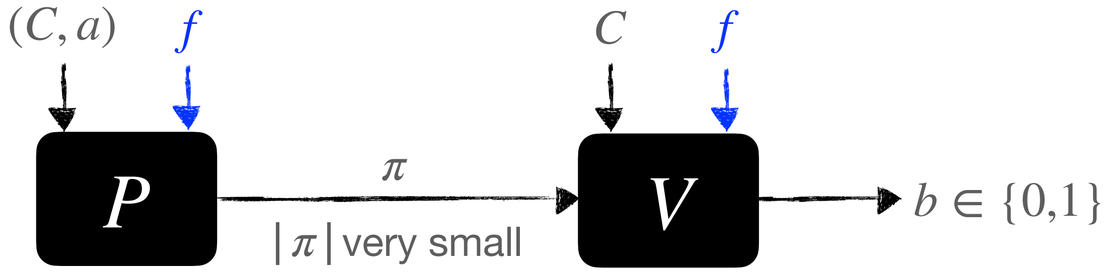


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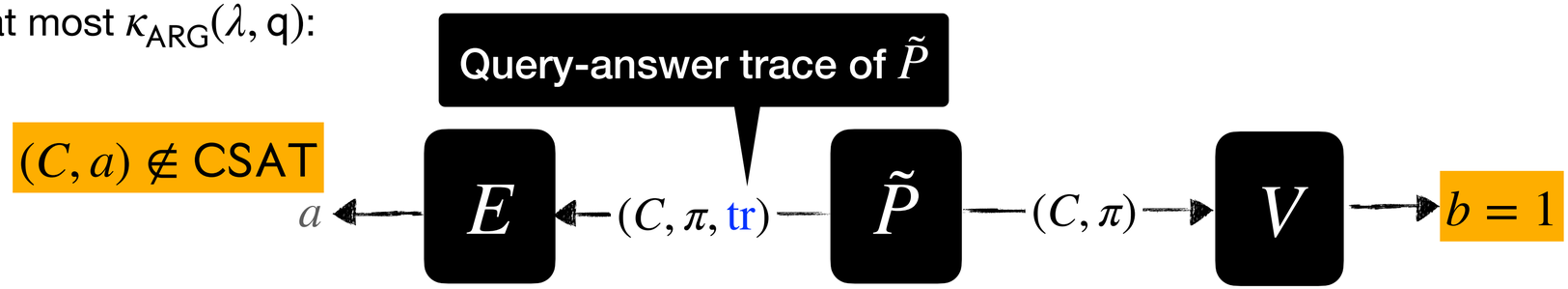

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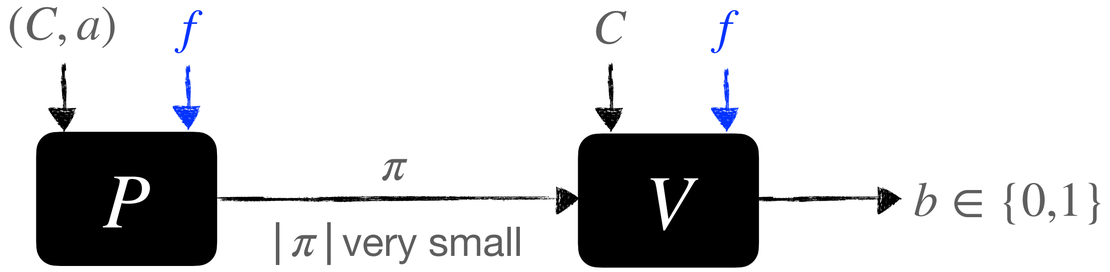


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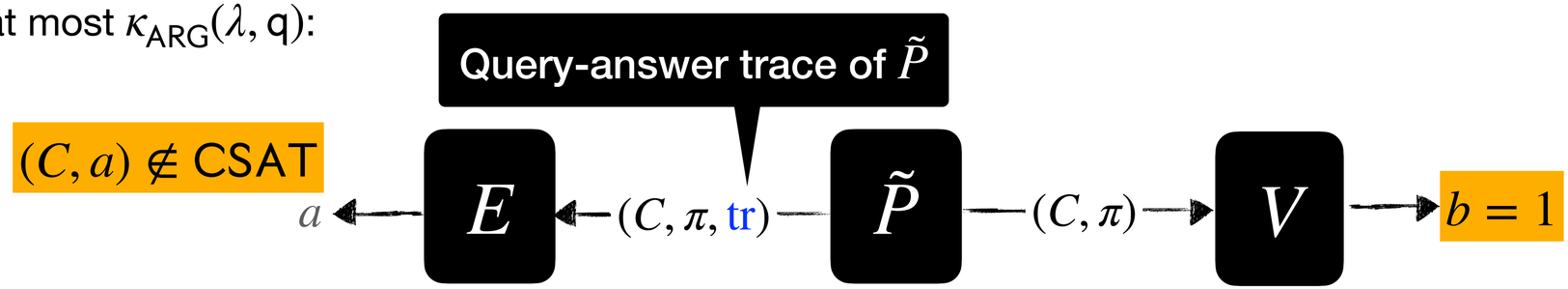


Wonderful Fact: in the ROM (and other interesting oracle models) there are SNARKs of interest with straightline extraction!

(E.g., the Micali SNARK and BCS SNARK and related constructions.)

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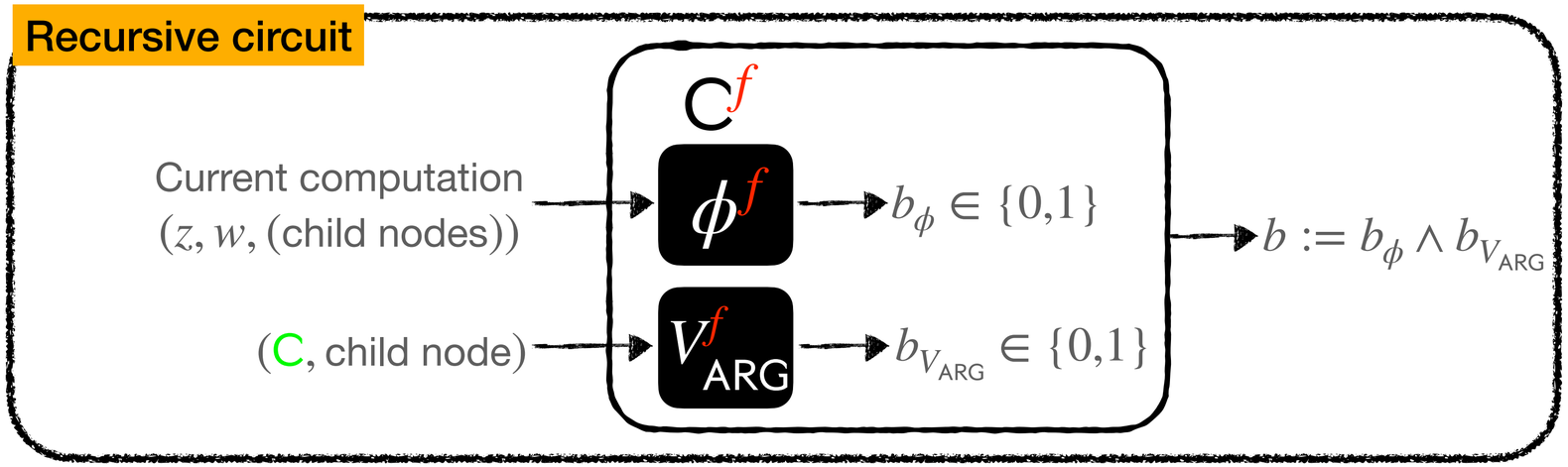

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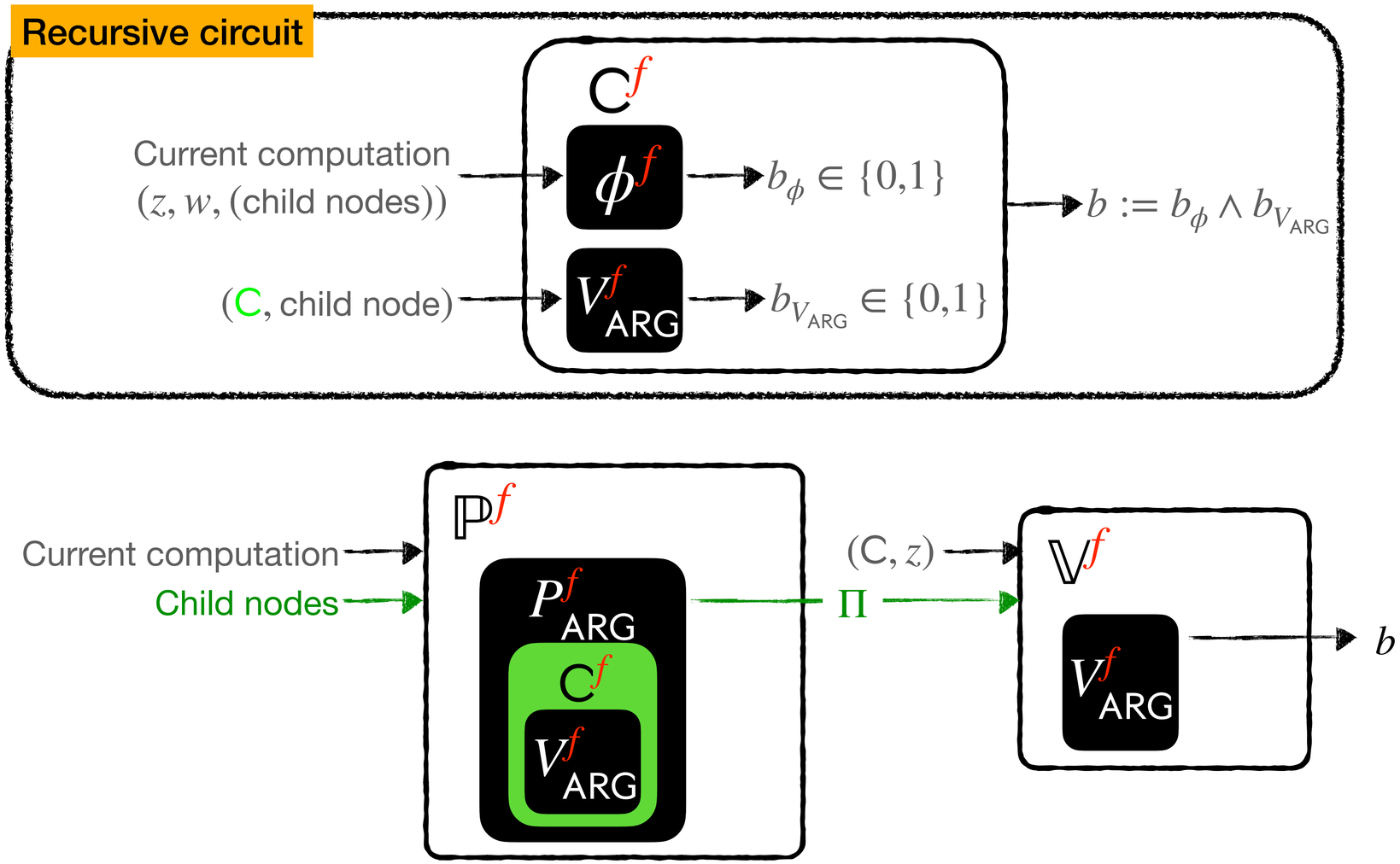

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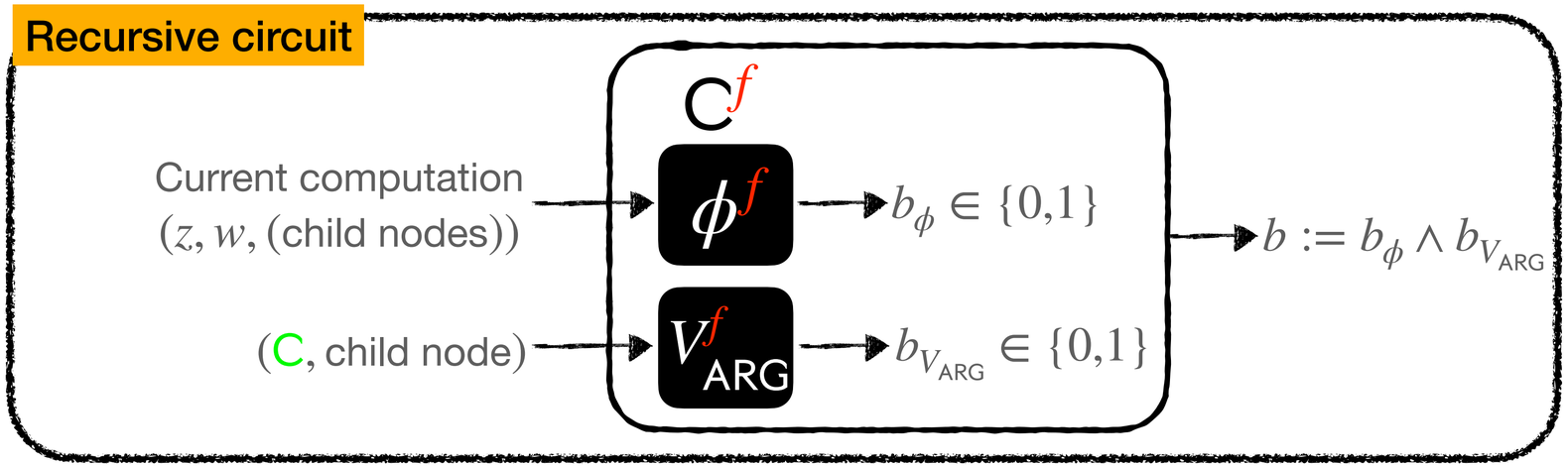


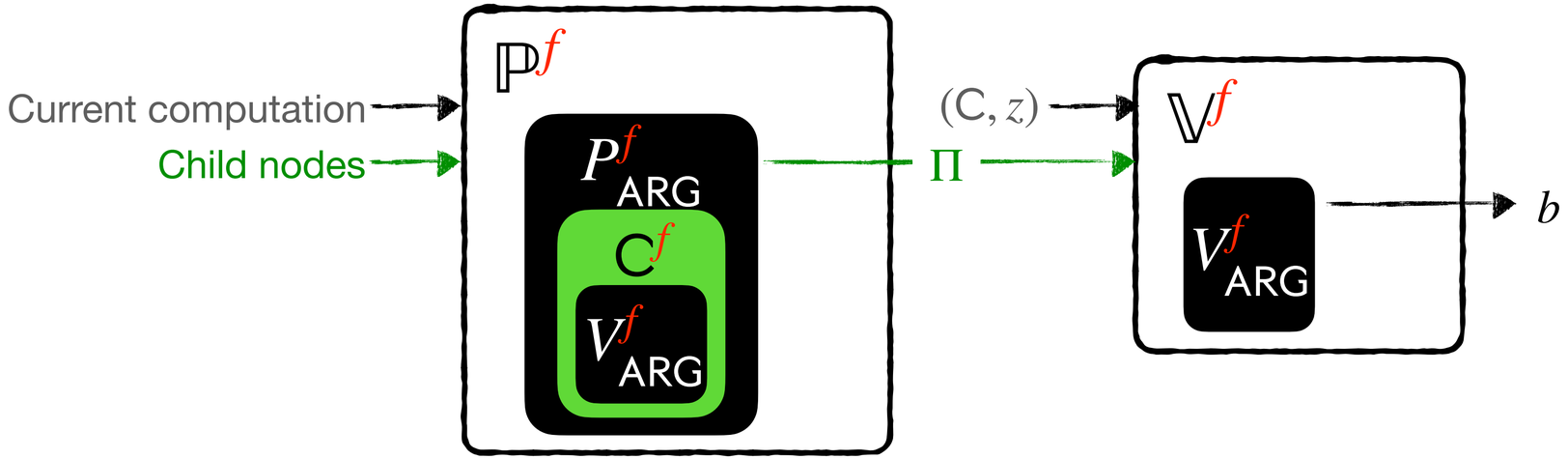


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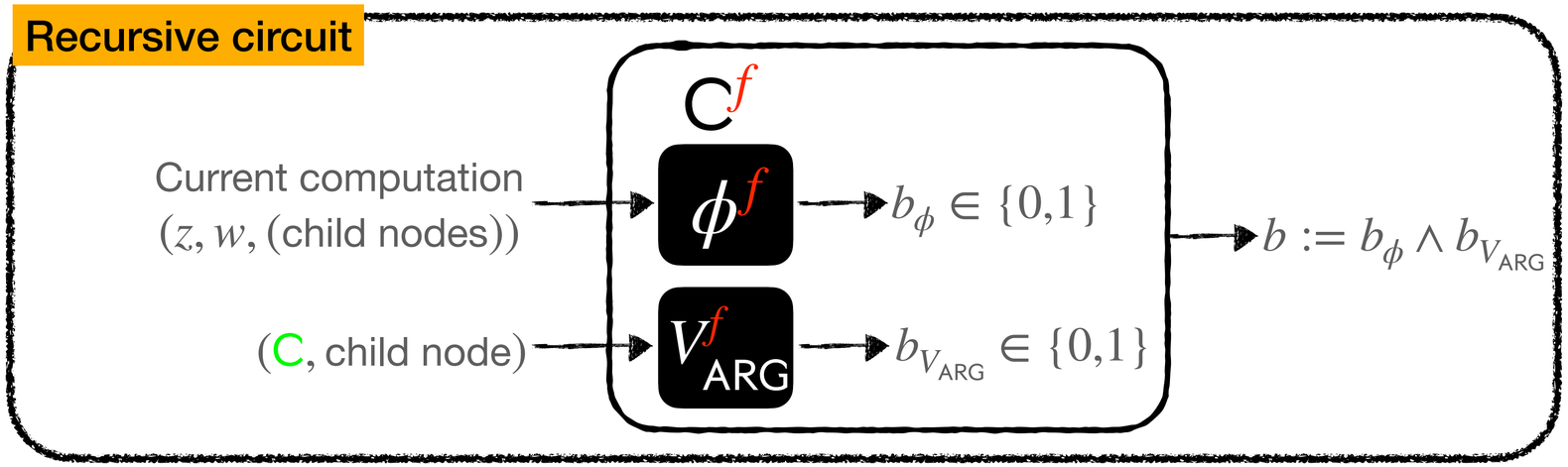


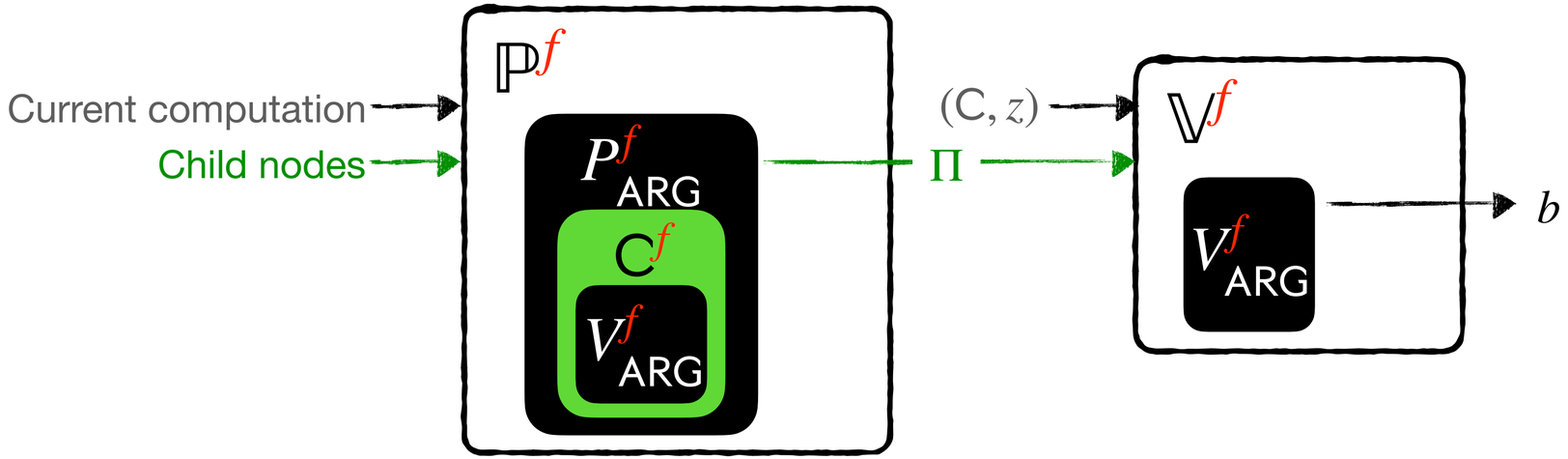


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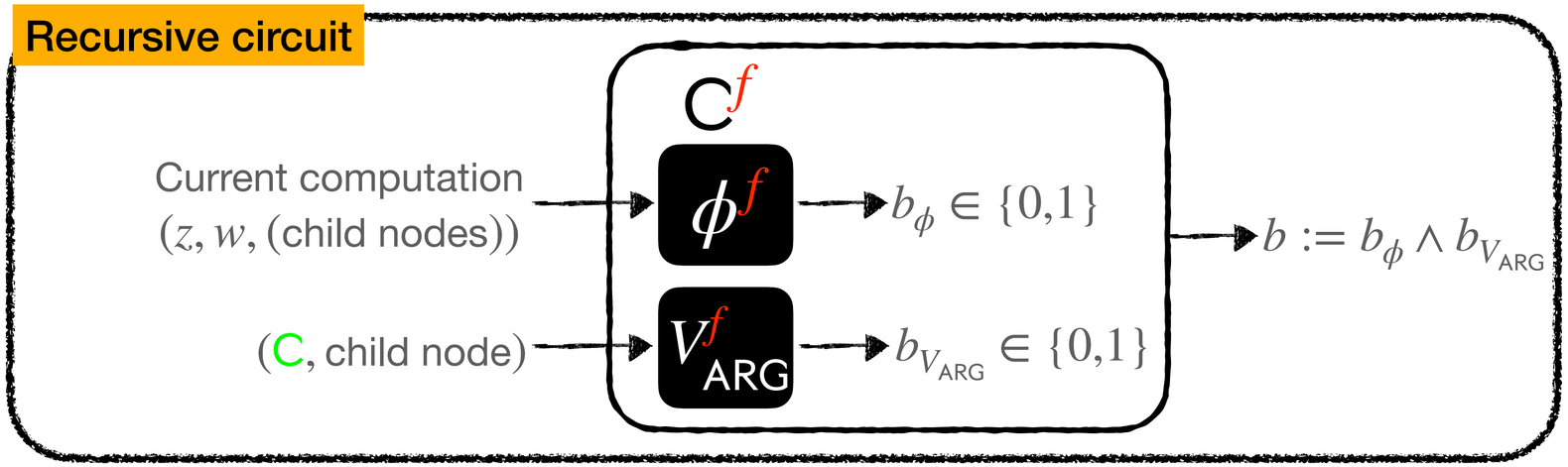


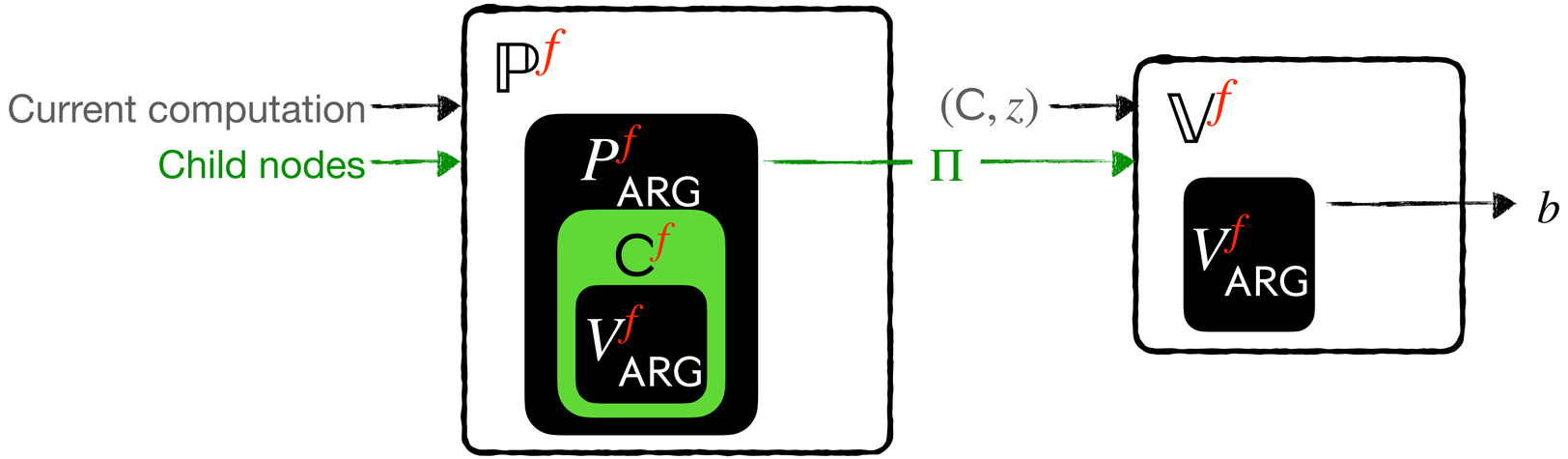


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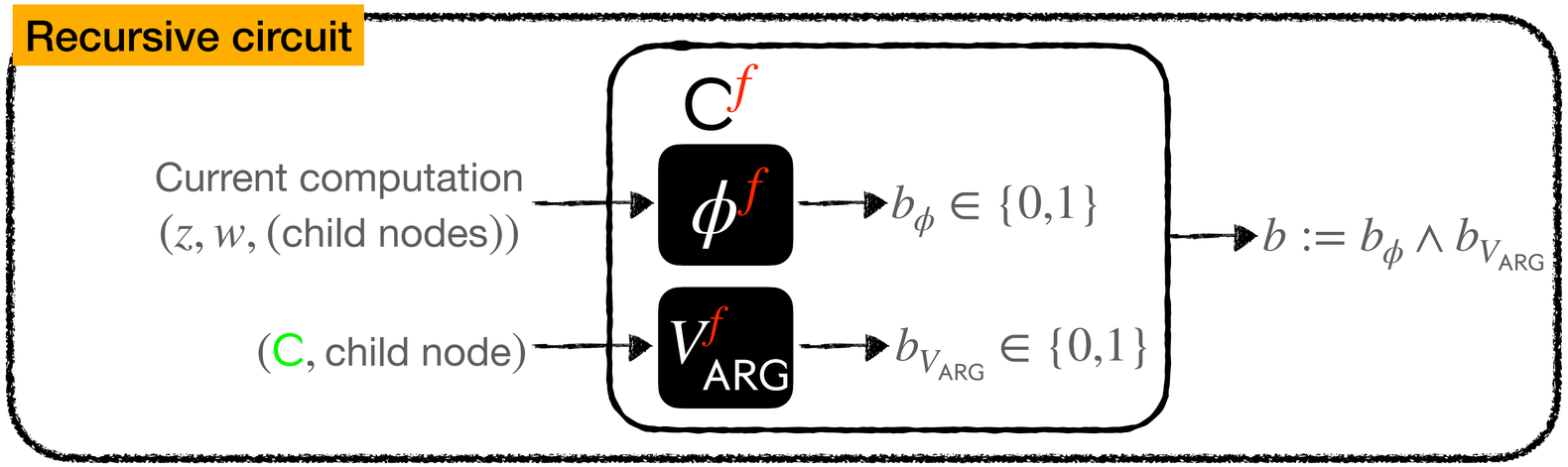


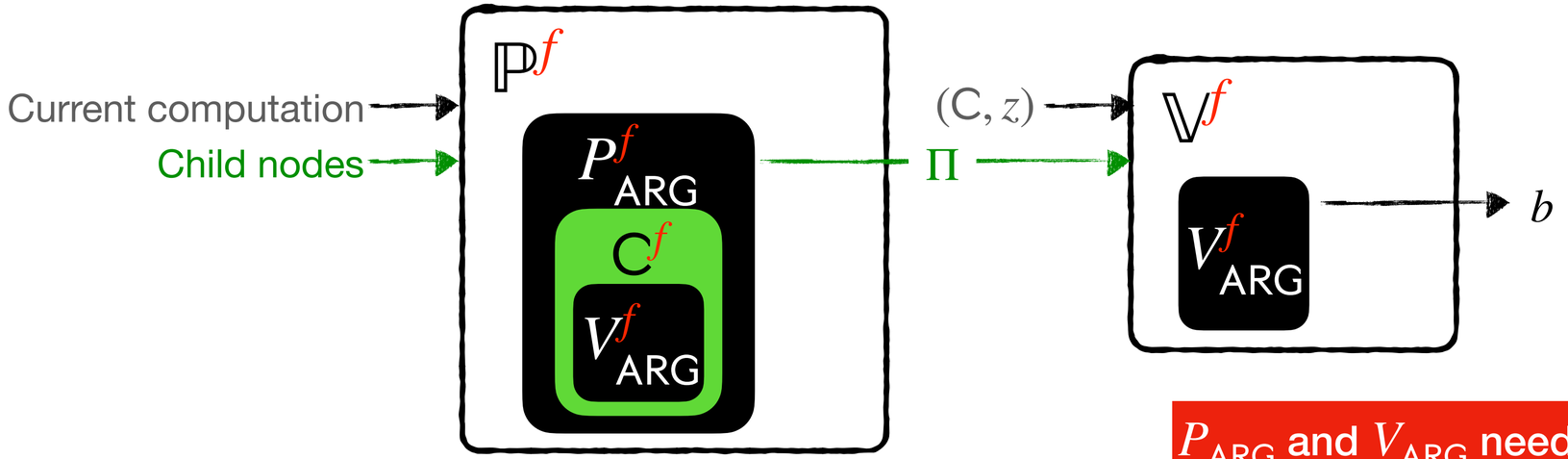


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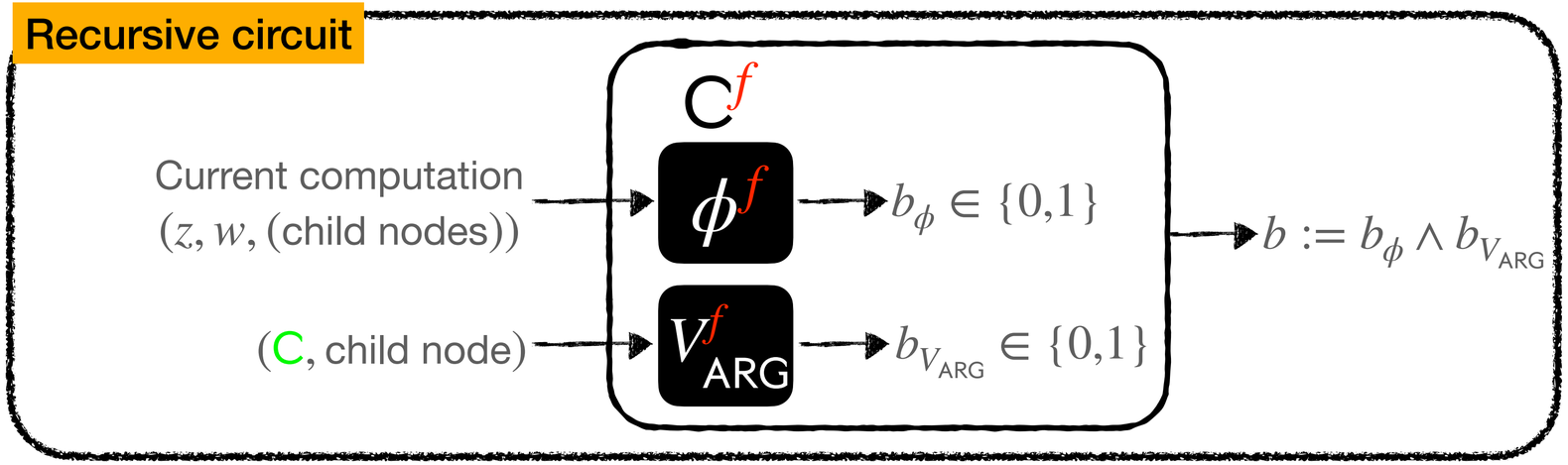


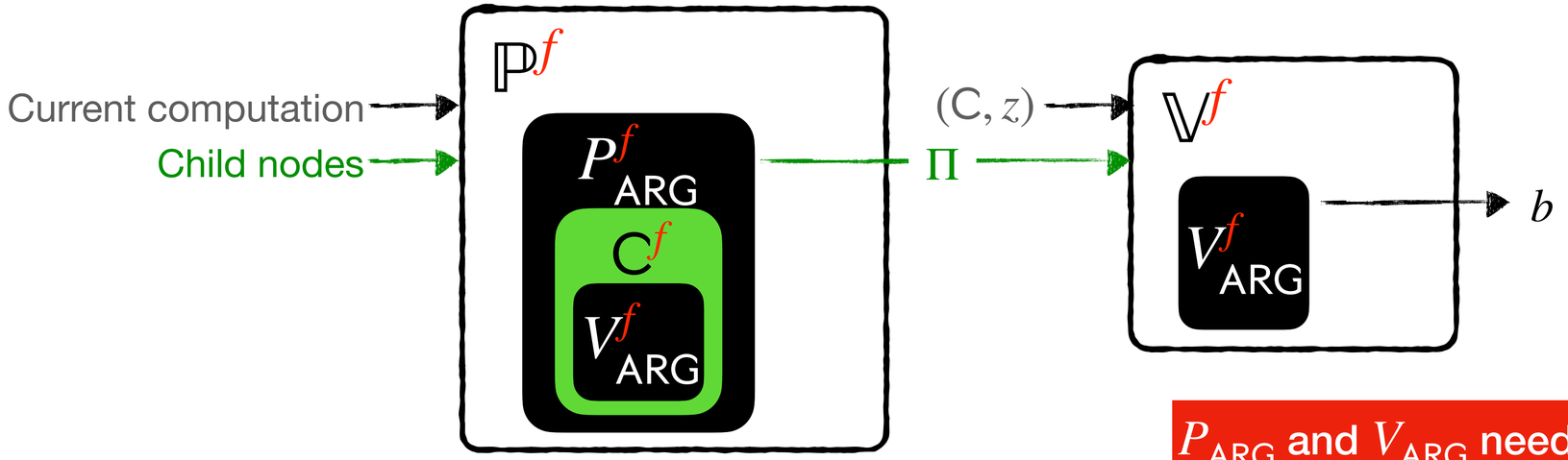


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**Solution: Relativized SNARKs!**

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**Relativized SNARK for $\mathsf{CSAT}^f$** → Recursive proof composition → **PCD with straightline extraction**

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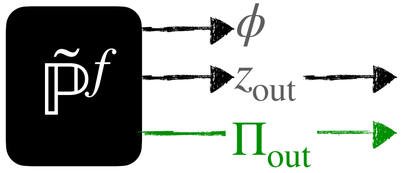

# Relativized SNARKs in an oracle model

$\mathsf{CSAT}^f := \{(C, a) : C^f(a) = 1\}$

**Relativized SNARK for $\mathsf{CSAT}^f$** → Recursive proof composition → **PCD with straightline extraction**

PCD straightline knowledge soundness: ∃ deterministic extractor $\mathbb{E}$, ∀ bounded adversary $\tilde{P}$, the following happens with probability at most $\kappa(\lambda, \mathsf{q}, \mathsf{N})$.

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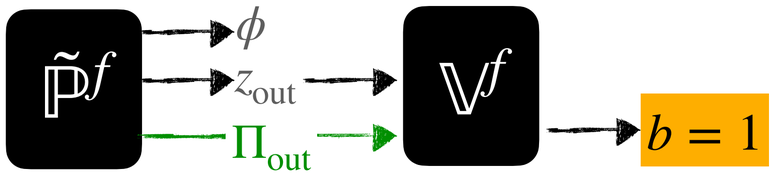

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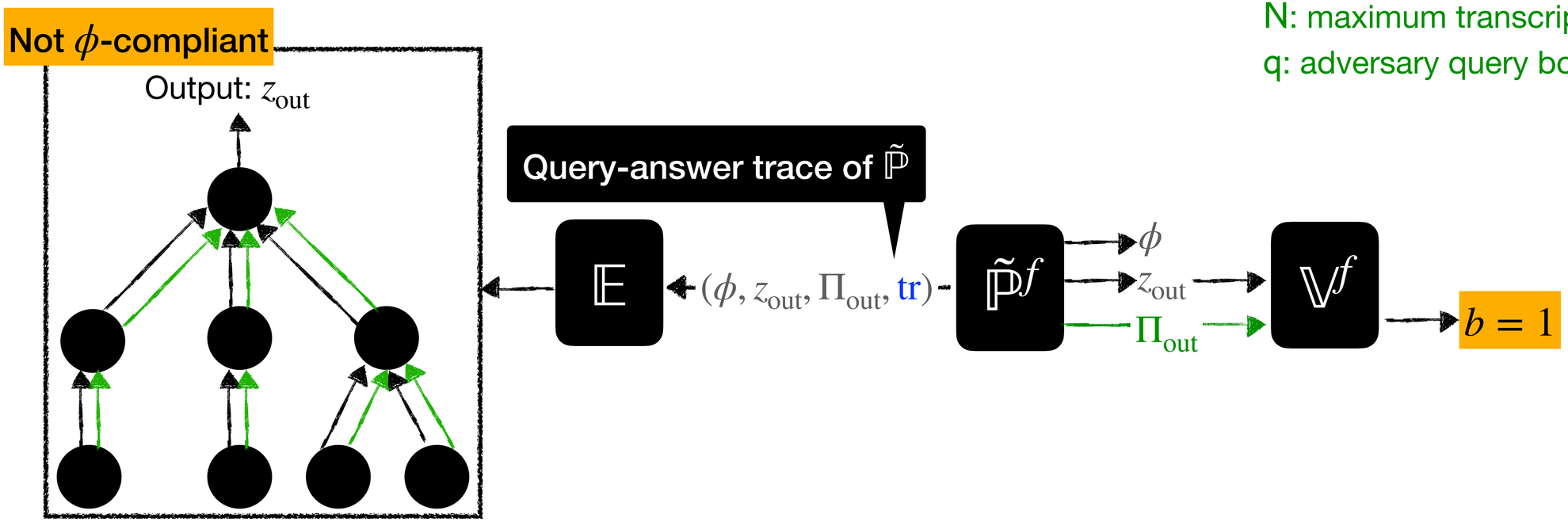

# Application:
## Set security for hash-based PCD

# Warm-up: analyzing hash-based SNARKs

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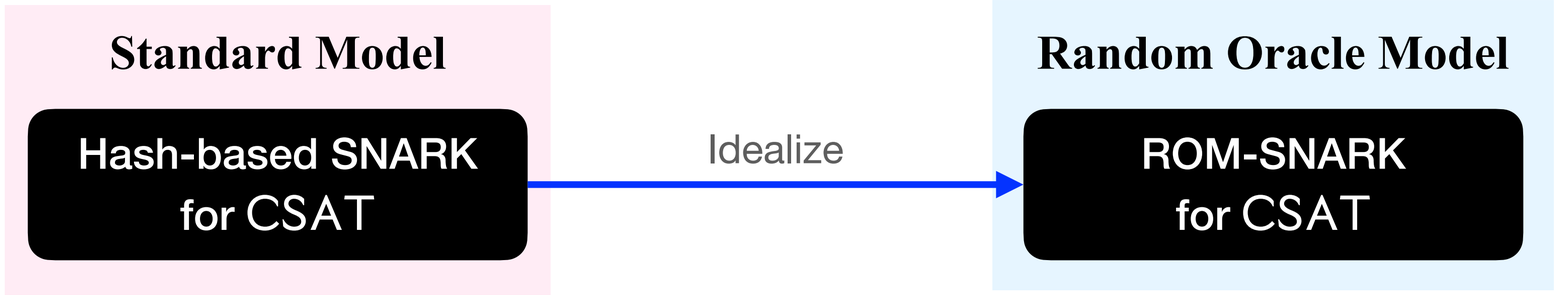

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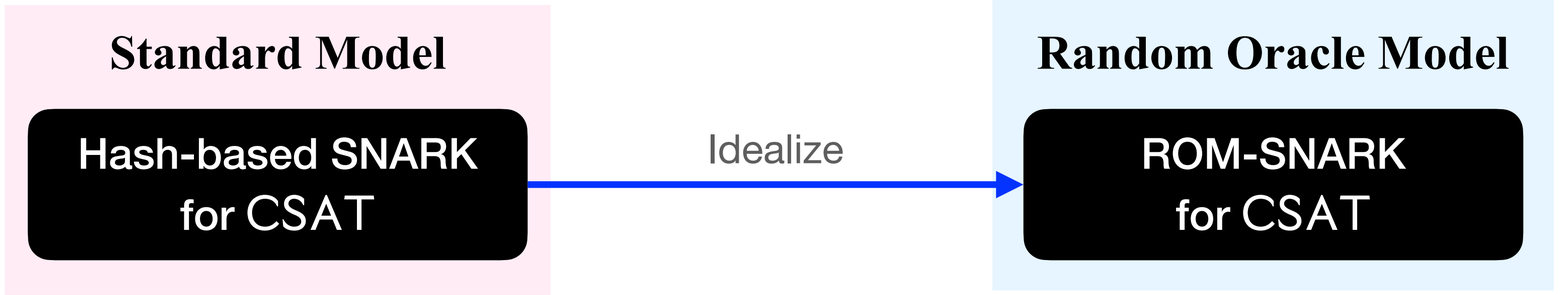

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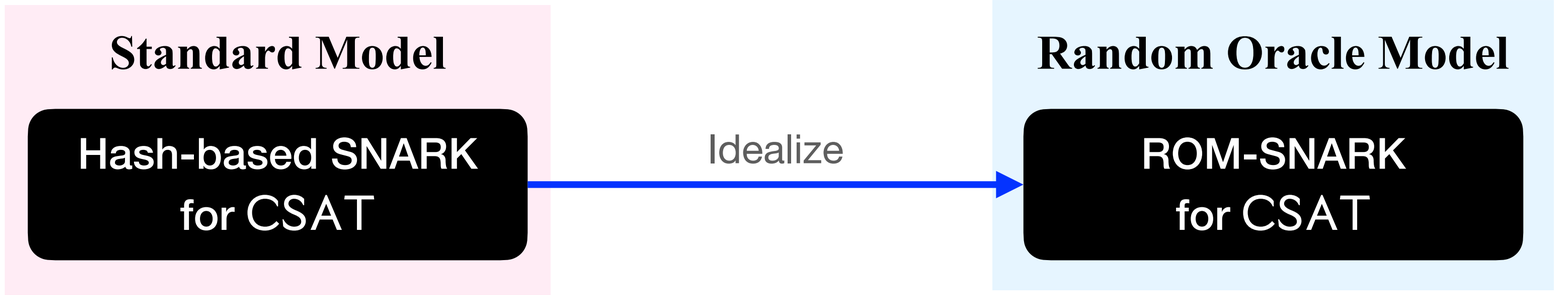

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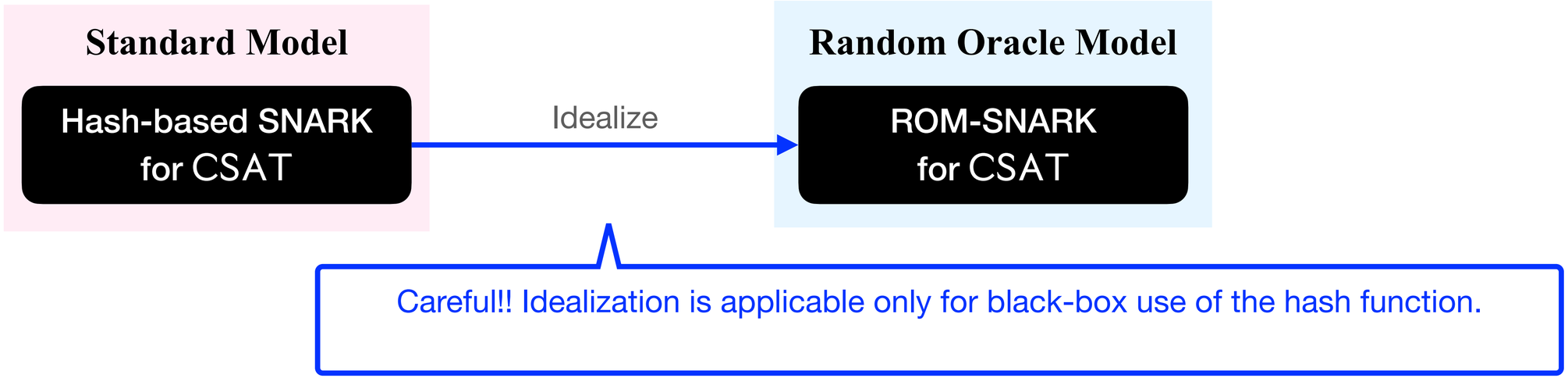

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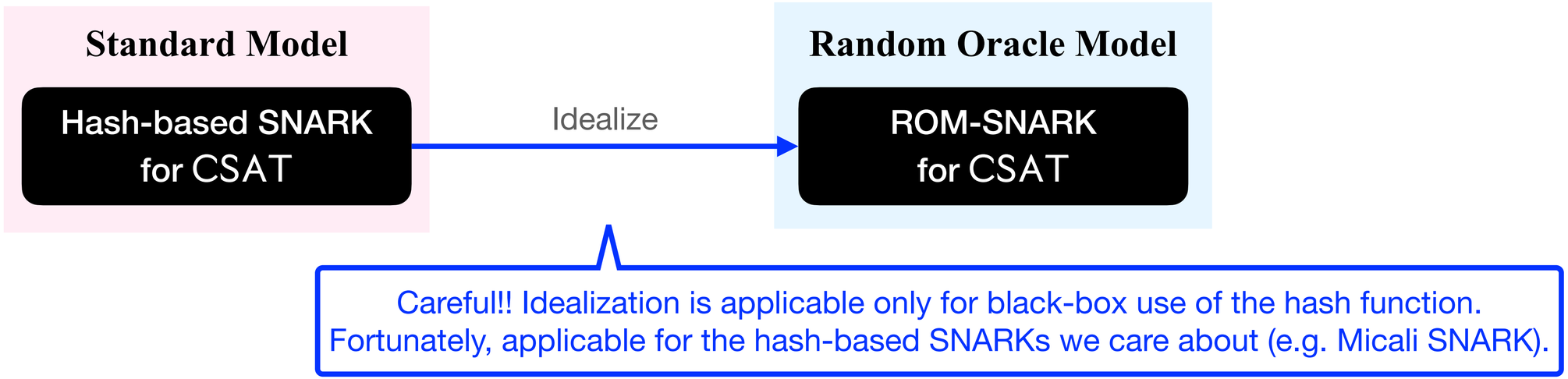

# First attempt for idealization of hash-based PCD

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PCDs are deployed based on various approaches. A popular approach is **hash-based PCD**.

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**Standard Model**

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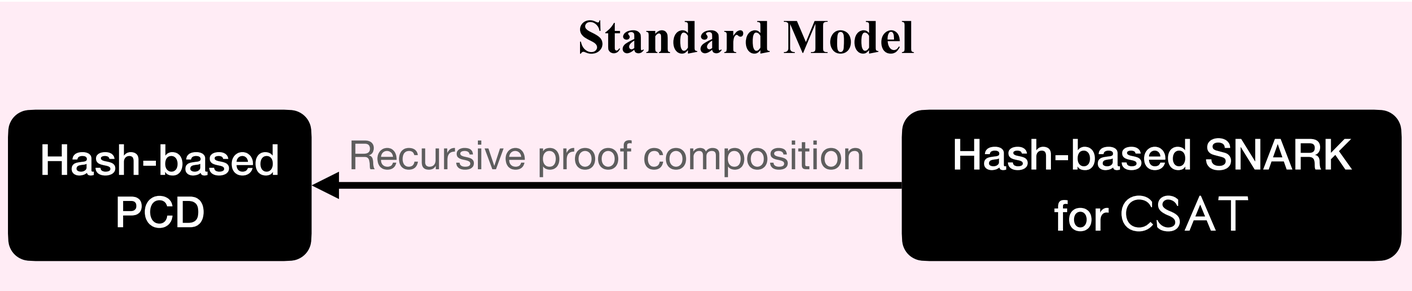

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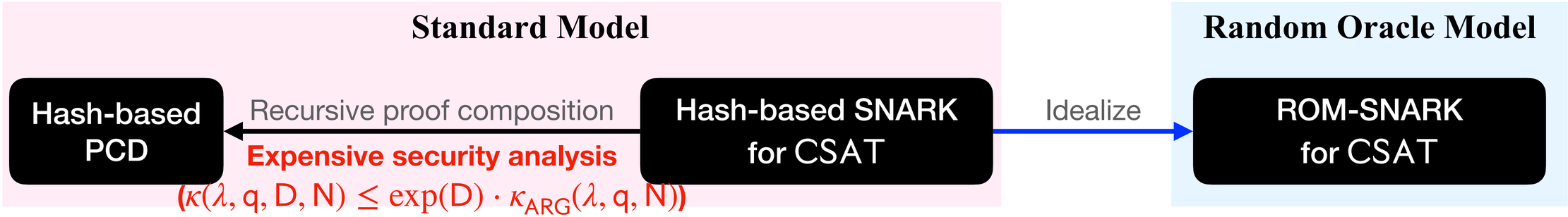

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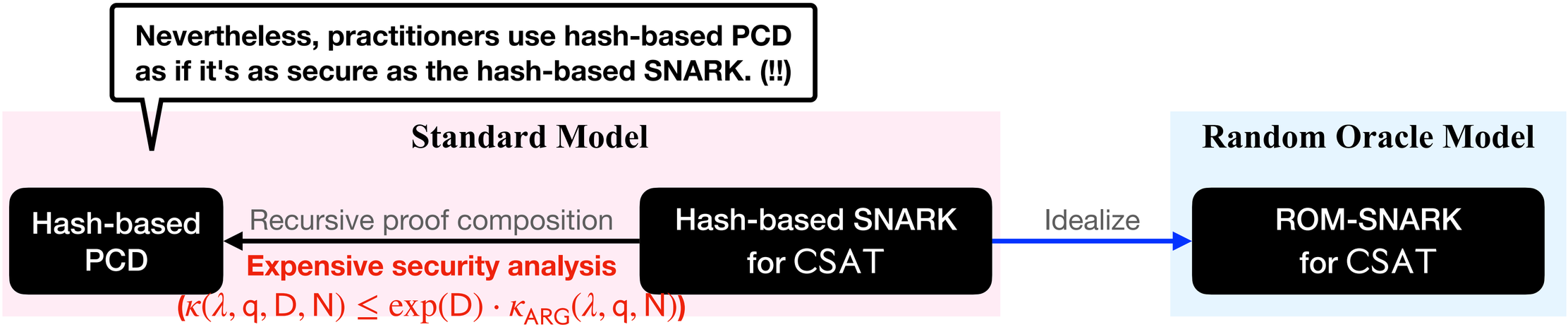

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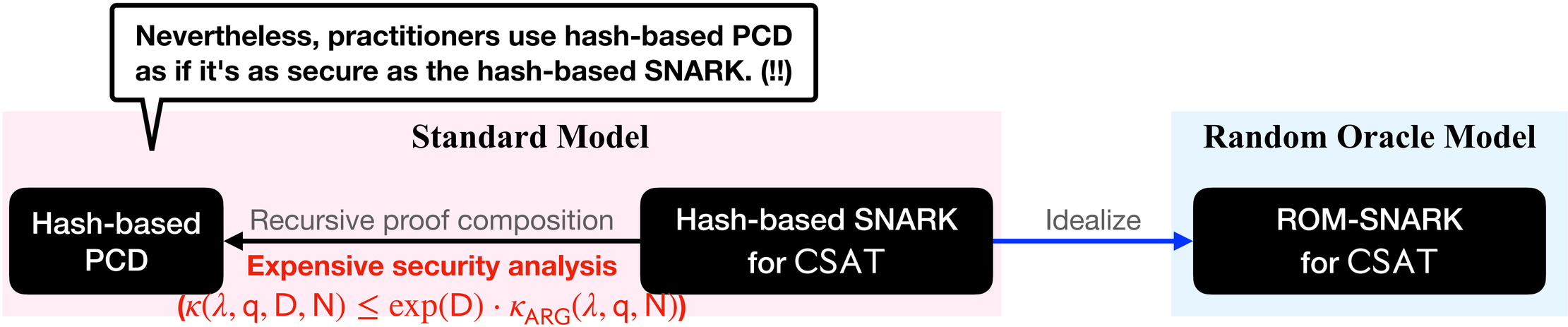


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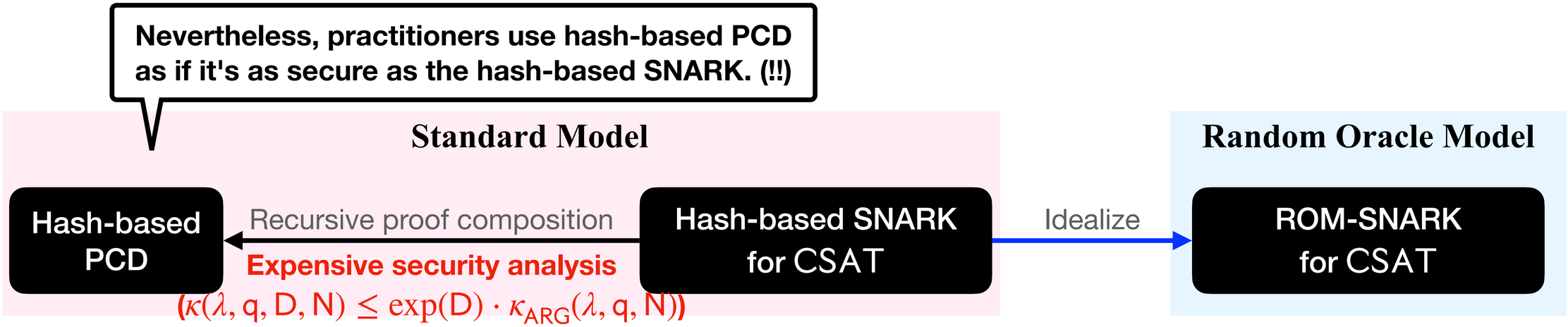


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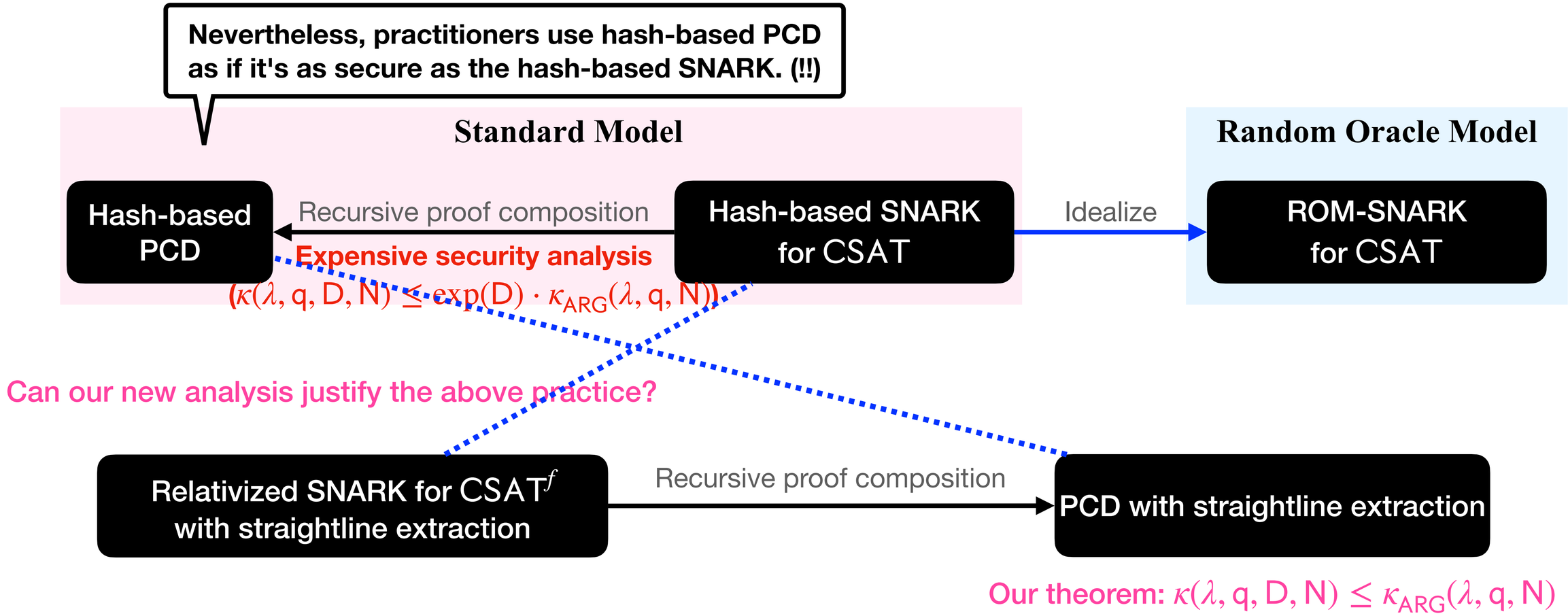

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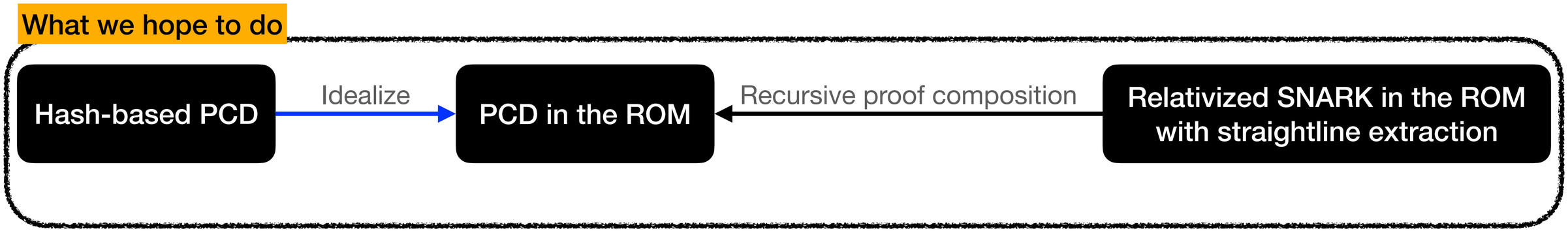

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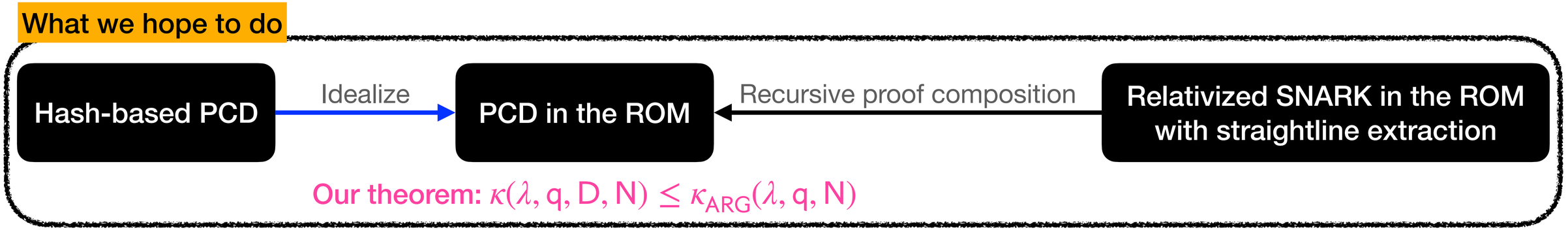

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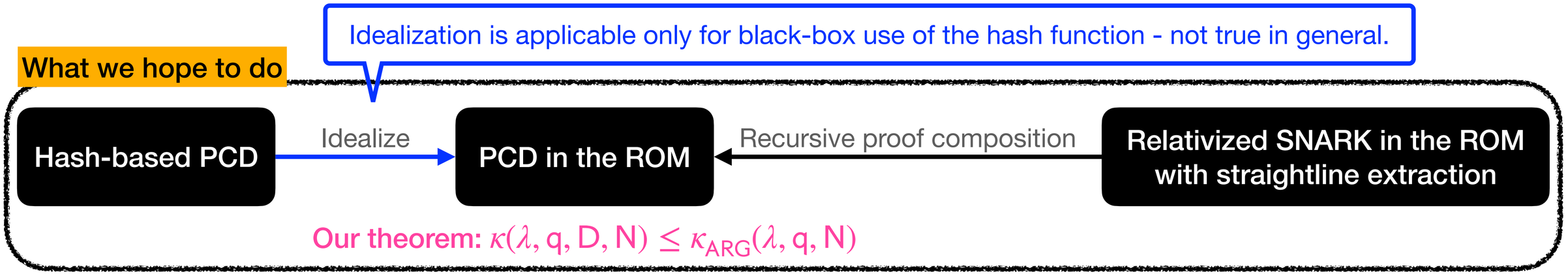

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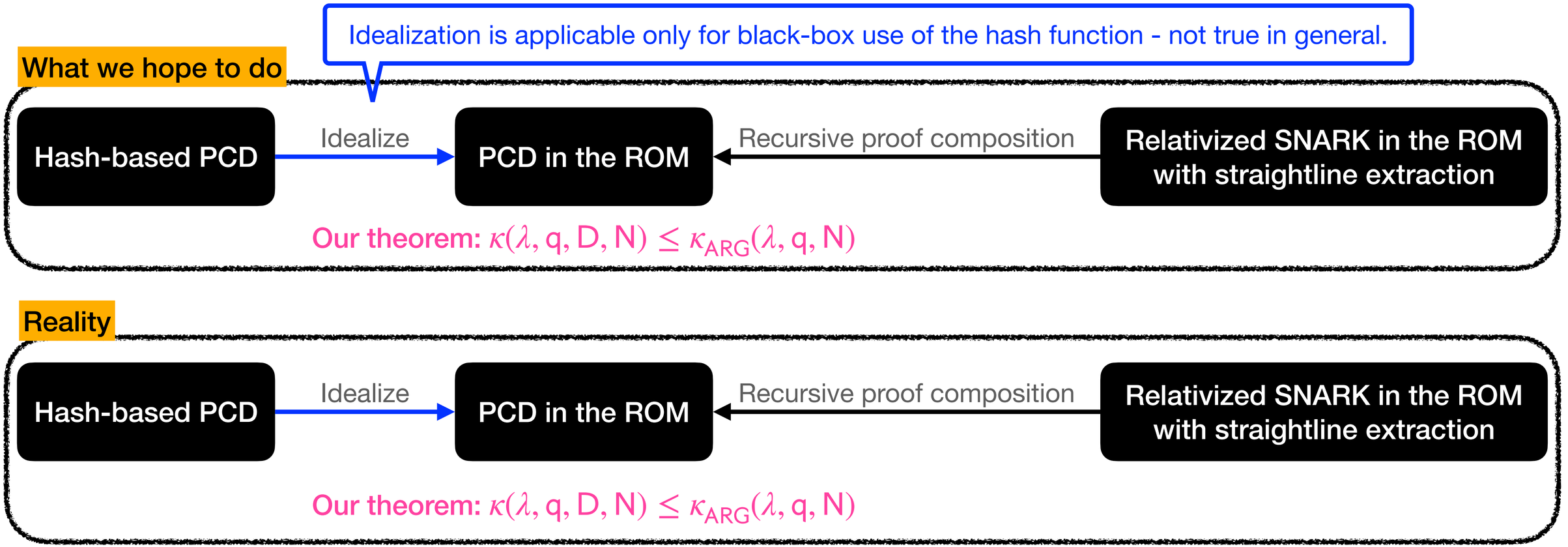

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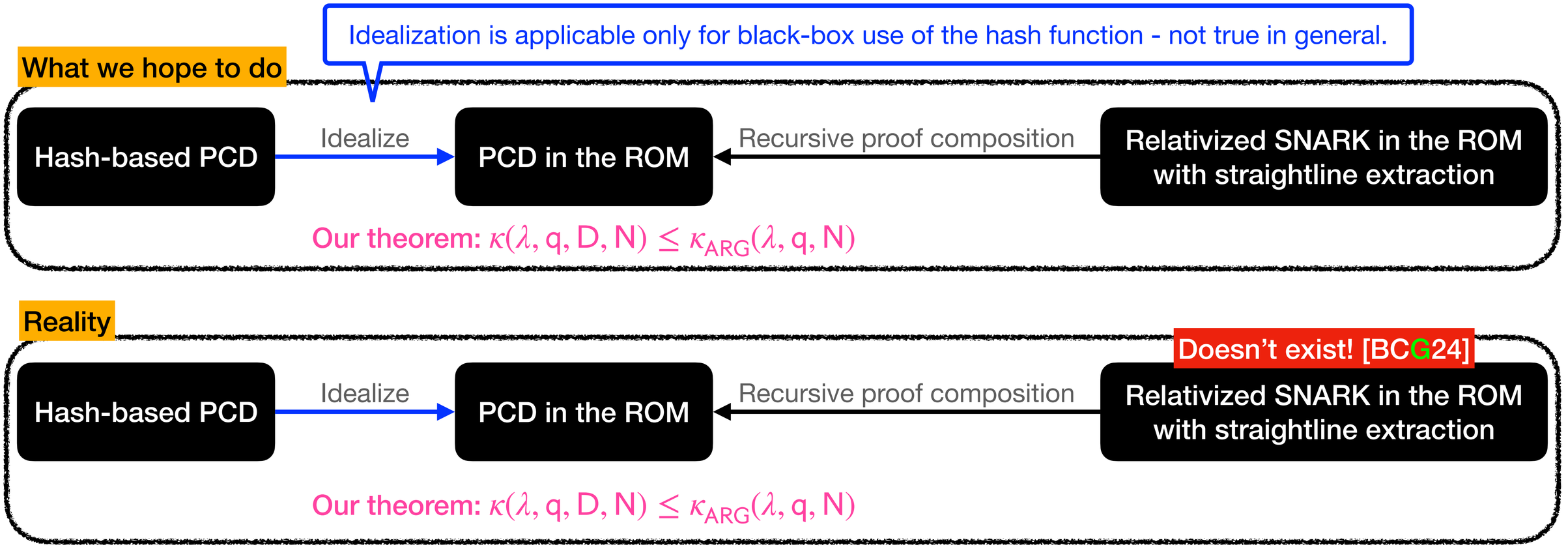

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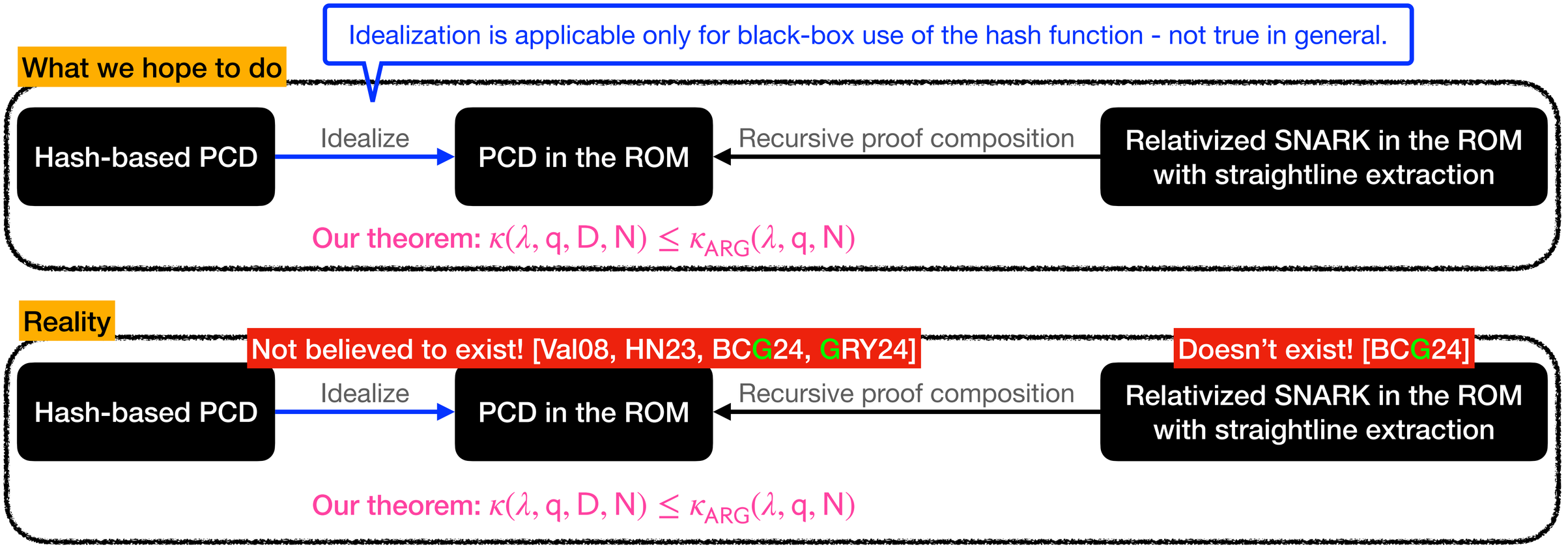

# Second attempt for idealization of hash-based PCD

Idealization is applicable only for black-box use of the hash function - not true in general.

**What we hope to do**

Hash-based PCD —Idealize→ PCD in the ROM ←Recursive proof composition— Relativized SNARK in the ROM with straightline extraction

Our theorem: $\kappa(\lambda, q, D, N) \leq \kappa_{ARG}(\lambda, q, N)$

**Reality**

Not believed to exist! [Val08, HN23, BCG24, GRY24]

Doesn't exist! [BCG24]

Hash-based PCD —Idealize→ PCD in the ROM ←Recursive proof composition— Relativized SNARK in the ROM with straightline extraction

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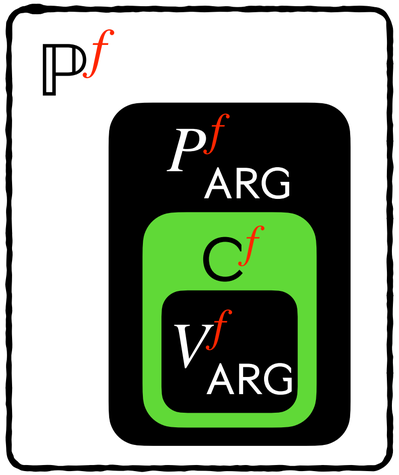


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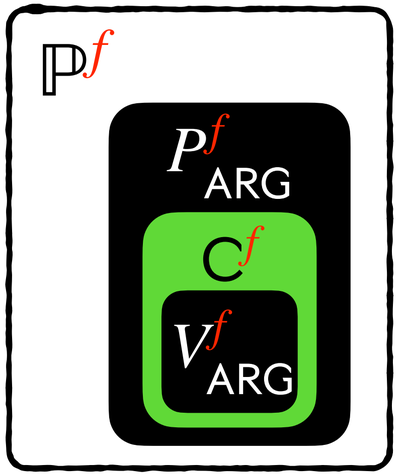


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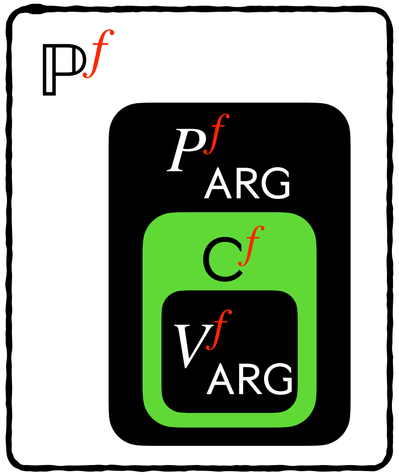


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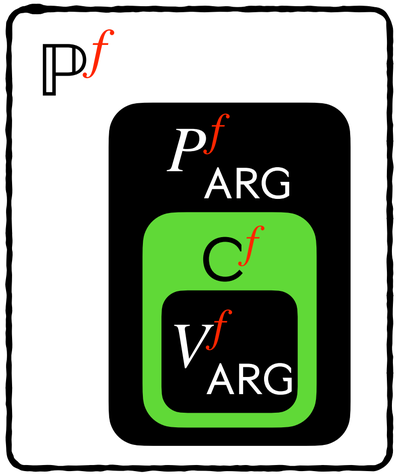


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**Hash-based PCD**

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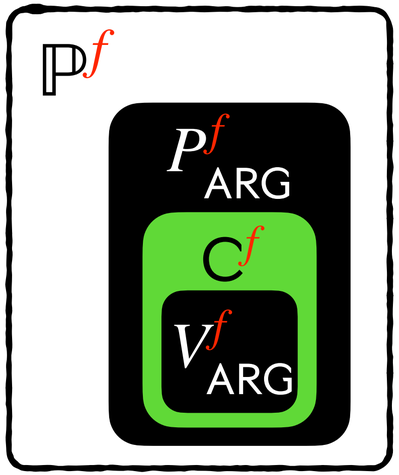


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Hash-based PCD → Idealize

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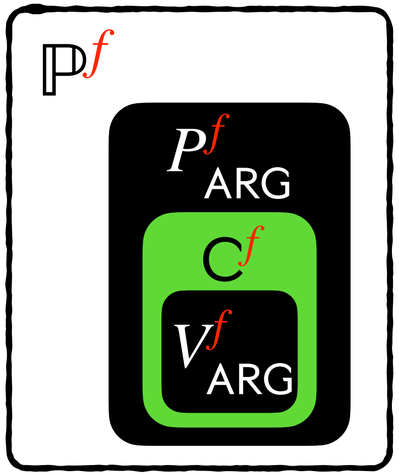


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Hash-based PCD → Idealize → **NON-SUCCINCT** PCD in the ROM with straightline extraction

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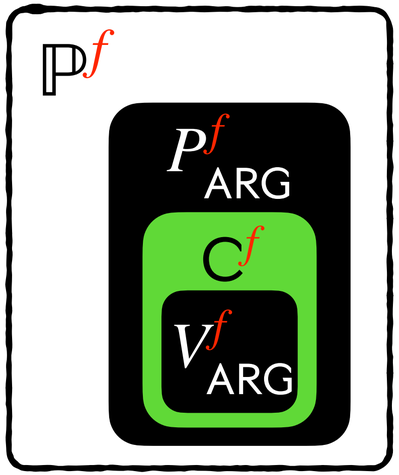


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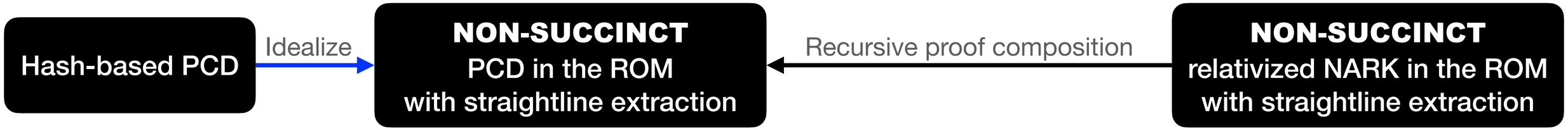

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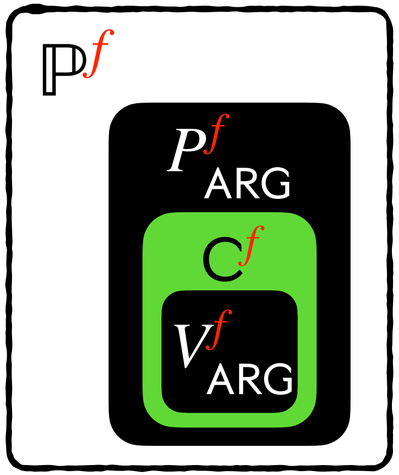


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Hash-based PCD → (Idealize) → **NON-SUCCINCT** PCD in the ROM with straightline extraction ← (Recursive proof composition) ← **NON-SUCCINCT** relativized NARK in the ROM with straightline extraction

Our theorem: $\kappa(\lambda, \mathsf{q}, \mathsf{D}, \mathsf{N}) \leq \kappa_{\mathsf{ARG}}(\lambda, \mathsf{q}, \mathsf{N}) = \kappa_{\mathsf{ARG}}(\lambda, \mathsf{q})$

# TL;DR

What is *proof-carrying data* (PCD)?
- Recursive compositions of SNARKs.
- It's useful for efficiently verifying distributed computations.

**Problem:**
- PCD is deployed under the assumption "security of PCD" = "security of underlying SNARK".
- BUT existing security analyses show a huge gap in security ("PCD is far less secure than underlying SNARK").

**This work:**
- We propose **an idealized PCD** that models hash-based PCD in practice.
- We prove that this idealized PCD is **as secure as its underlying SNARK**.

**Thank you!**