Probability Distribution Formulas

Bernoulli Distribution

(1 event)

Distribution of binary random variable

$PMF = p^k (1-p)^{1-k}$

$1-p$	$k=0$
p	$k=1$

Mean(Estimation)

$\sum k P(k) \quad k = \{0,1\}$

$= p$

Median

$\begin{cases} 0 & p < 1/2 \quad 0 \\ [0,1] & p = 1/2 \quad 0.5 \\ 1 & p > 1/2 \quad 1 \end{cases}$

Mode

$p > q \Rightarrow p$

$p < q \Rightarrow q$

$p = 0.5$ Both 1 & 0 are modes

Variance/SD

$\sigma^2 = qp$

$\sigma = \sqrt{pq} = SD$

Binomial Distribution

$n \rightarrow$ event (yes or no)

Success = p

fail = q

eg: Toss coin 10 times

if $n=1 \Rightarrow$ Bernoulli

$k \Rightarrow$ number of success $\{0, 1, 2, 3 \ldots n\}$

$PMF = {}^{n}C_{k}\, p^k (1-p)^{1-k}$

$SD = \sqrt{nqp}$

Variance = npq

Mean = np

eg: $n=5 \quad p=0.5$ probability of 3 heads

$k=3$

$= {}^{5}C_{3} (0.5)^3 (1-0.5)^{5-3}$

Normal gaussian distribution

$$\boxed{\text{PDF} \quad \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

$\boxed{\text{mean} = \text{median} = \text{mode} = \mu}$

Perfect symmetry
Highest exactly at μ

Intution

$(x-\mu)^2 \Rightarrow$ Symmetric around μ

Exponential $\Rightarrow$ far value become extremely unlikely

$2\pi \rightarrow$ make total one (1)

Area under cover = probability

$$\boxed{\begin{gathered} SD = \sigma = \sqrt{(x-\mu)^2/n} \\ \sigma^2 = \text{variance} \end{gathered}}$$

Zscore table gives area for 0 to Zscore point

Standard normal Distribution

$X = \{1, 2, 3, 4, 5\}$ $\quad \mu = 3 \quad \sigma = 1.414$

Now we apply z score $\quad$ $\text{Zscore} = \frac{x_i - \mu}{\sigma}$

Say for ezy calculation we take $\sigma = 1$
or lets keep 1.414 only

New $x = \{-1.414, -0.707, 0, 0.707, 1.414\}$

Z tells how much SD away from mean

Poissons Distribution

Events happens independently
Average rate is costant
Event occur 1 at time
We count event in a fixed interval of (time, area, length)

$\boxed{PMF = \frac{e^{-\lambda}\lambda^{k}}{k!}}$

$\boxed{Mean = \lambda \quad Variance = \lambda \quad SD = \sqrt{\lambda}}$

$e = 2.718$
λ = average rate
k = chace of happening k events

eg 3 patient coming to cilinic every hour
chance 2 patient coming in ½ hour

avg. Patient coming every half hour = 1.5

$k = 2$

$\lambda = 1.5$

$$P = \frac{e^{-1.5}\lambda^{2}}{2!}$$

Uniform Distribution

Discrete: finite outcome
outcome ki probability equal
eg Fair dice {1,2,3,4,5,6}
lottery number {1 to 100}

$\boxed{PMF = 1/n}$ $\boxed{Mean(\mu) = \frac{n+1}{2}}$ $\boxed{\sigma^2 = \frac{n^2-1}{12}}$

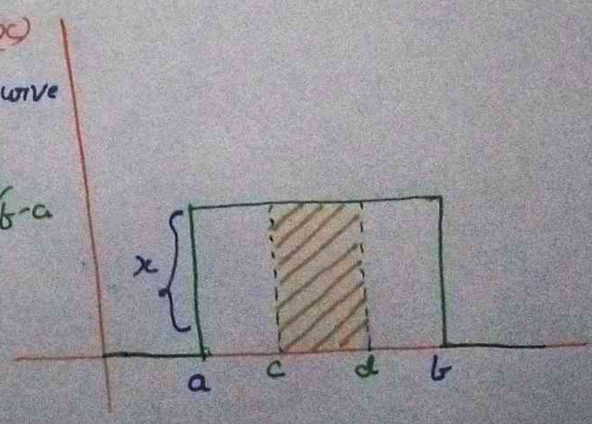
Continuous:

$\boxed{PDF = \frac{1}{(b-a)}}$ $-\infty < a < b < \infty$
else 0

$\Rightarrow$ Proof $x(b-a) = 1$
area under curve
$x = 1/(b-a)$

Continuous $\Rightarrow$ $CDF = x * (d-c)$

$\boxed{CDF = \frac{d-c}{b-a}}$
outside = 0



$\boxed{\sigma = \sqrt{\frac{(b-a)^2}{2}}}$ $\boxed{\frac{(a+b)}{2} = \mu}$