

Signed Distance from a Hyperplane (Vector Form)

Let w be a normal vector and x be a point vector in \mathbb{R}^n .

Hyperplane (through origin):

$$w^T x = 0$$

Signed Distance:

$$d_s = (w^T x) / \|w\|$$

General Hyperplane:

$$w^T x + b = 0$$

Signed distance of point x from hyperplane:

$$d_s = (w^T x + b) / \|w\|$$

Interpretation:

Positive value -> point lies in direction of normal vector w

Zero -> point lies on the hyperplane

Negative value -> point lies opposite to normal (below the plane)

Geometric Meaning:

$(w^T x) / \|w\|$ represents the scalar projection of vector x onto the unit normal vector.

Machine Learning (SVM):

Sign gives class label, magnitude gives confidence (margin).

This explains why distance becomes negative when a point lies on the opposite side of the hyperplane.