Complete Probability & Bayes Theorem – Step■by■Step Learning Notes

These notes compile all probability-related concepts discussed in the conversation, starting from fundamentals and gradually moving to advanced Bayes theorem problems. They are written in a simple, exam■ready, and intuitive manner.

1. Basic Probability Concepts

An event is a set of outcomes of a random experiment.

Probability of an event A is denoted by P(A) and lies between 0 and 1.

2. Intersection and Union of Events

Intersection ($A \cap B$) means both events A and B occur.

Union ($A \cup B$) means at least one of the events A or B occurs.

Key formulas:

• $P(A \cap B) = P(A|B)\ P(B)$

• $P(A \cup B) = P(A) + P(B) – P(A \cap B)$

3. Overlapping vs Non■Overlapping Events

Non■overlapping (mutually exclusive) events cannot occur together.

Overlapping events share common outcomes and require subtraction of the intersection to avoid double counting.

4. Conditional Probability

Conditional probability measures the probability of an event given that another event has already occurred.

Formula: $P(A|B) = P(A \cap B) / P(B)$

5. Understanding Intersection Formulas

The probability of intersection can be written in two equivalent ways:

• $P(A \cap B) = P(A|B)\ P(B)$

• $P(A \cap B) = P(B|A)\ P(A)$

Both represent the same joint occurrence and are the foundation of Bayes' theorem.

6. Law of Total Probability

If events A1, A2, … An form a partition of the sample space:

$P(B) = \Sigma\ P(B|Ai)\ P(Ai)$

This law ensures all possible non■overlapping cases are considered.

7. Bayes' Theorem

Bayes' theorem allows us to reverse conditional probabilities.

Formula:

P(A|B) = [P(B|A) P(A)] / P(B)

Interpretation: Posterior ∝ Likelihood × Prior

8. Ball Drawing Examples

Example 1:

Box with 5 red and 3 blue balls. Given first ball is red, probability second ball is red = 4/7.

Example 2 (Reverse Bayes):

Box with 4 red and 6 blue balls. Given second ball is red, probability first was red = 1/3.

9. Two■Box Bayes Problem (Advanced)

Box 1: 2 red, 3 blue

Box 2: 4 red, 1 blue

A box is chosen at random and two balls are drawn without replacement.

Given both balls are red, probability Box 2 was chosen = 6/7.

10. Key Takeaways

- Bayes theorem updates belief after observing evidence.
- Always split problems into non■overlapping cases.
- Without replacement means the denominator reduces.
- P(A|B) is not equal to P(B|A).
- Use intuition to sanity■check answers.

End of Notes

These notes cover the complete probability and Bayes■theorem discussion from basics to advanced interview■level problems.