

# BFO 2020 Temporal Region Axioms

(1) Intervals have no internal gaps

$$\begin{aligned} &\forall i, \text{start}, \text{end} (\text{instanceOf}(i, \text{temporalInterval}, i) \wedge \text{hasFirstInstant}(i, \text{start}) \\ &\quad \wedge \text{hasLastInstant}(i, \text{end}) \\ &\quad \rightarrow \neg (\exists \text{gap}, \text{gapStart}, \text{gapEnd} (\text{hasFirstInstant}(\text{gap}, \text{gapStart}) \\ &\quad \quad \wedge \text{hasLastInstant}(\text{gap}, \text{gapEnd}) \\ &\quad \quad \wedge \text{precedes}(\text{gapEnd}, \text{end}) \wedge \text{precedes}(\text{start}, \text{gapStart}) \\ &\quad \quad \wedge \neg \text{temporalPartOf}(\text{gap}, i)))) \end{aligned}$$

(2) Proper temporal part of is asymmetric

$$\forall a, b (\text{properTemporalPartOf}(a, b) \rightarrow \neg \text{properTemporalPartOf}(b, a))$$

(3) A first instant is either part of an extended region or precedes it

$$\begin{aligned} &\forall l, i (\text{instanceOf}(l, \text{temporalInstant}, l) \wedge \text{instanceOf}(i, \text{temporalRegion}, i) \\ &\quad \wedge \neg \text{instanceOf}(i, \text{temporalInstant}, i) \wedge \text{hasLastInstant}(i, l) \\ &\quad \rightarrow (\neg \text{temporalPartOf}(l, i) \leftrightarrow \text{precedes}(i, l))) \end{aligned}$$

(4) Two intervals are identical if their first and last instants are the same and if an instant is part of one of the intervals it is also part of the other

$$\begin{aligned} &\forall i1, i2 (\text{instanceOf}(i1, \text{temporalInterval}, i1) \wedge \text{instanceOf}(i2, \text{temporalInterval}, i2) \\ &\quad \rightarrow (\exists f, l (\text{hasFirstInstant}(i1, f) \wedge \text{hasFirstInstant}(i2, f) \wedge \text{hasLastInstant}(i1, l) \\ &\quad \quad \wedge \text{hasLastInstant}(i2, l) \wedge (\text{temporalPartOf}(l, i1) \leftrightarrow \text{temporalPartOf}(l, i2)) \\ &\quad \quad \wedge (\text{temporalPartOf}(f, i1) \leftrightarrow \text{temporalPartOf}(f, i2))) \\ &\quad \rightarrow i1 = i2)) \end{aligned}$$

(5) Temporal part of is antisymmetric

$$\forall a, b (\text{temporalPartOf}(a, b) \wedge \text{temporalPartOf}(b, a) \rightarrow a = b)$$

(6) If a has temporal part b then if a is an instance of one dimensional temporal region then b is an instance of one dimensional temporal region or zero dimensional temporal region

$$\begin{aligned} &\forall p, q (\text{hasTemporalPart}(p, q) \\ &\quad \rightarrow (\exists t (\text{instanceOf}(p, \text{oneDimensionalTemporalRegion}, t) \\ &\quad \quad \rightarrow \exists t (\text{instanceOf}(q, \text{oneDimensionalTemporalRegion}, t) \\ &\quad \quad \quad \vee \text{instanceOf}(q, \text{zeroDimensionalTemporalRegion}, t)))))) \end{aligned}$$

(7) Temporal part of and has temporal part are inverse relations

$$\forall a, b (\text{temporalPartOf}(a, b) \leftrightarrow \text{hasTemporalPart}(b, a))$$

(8) Temporal part of has unique product

$$\begin{aligned} &\forall x, y (\text{instanceOf}(x, \text{temporalRegion}, x) \wedge \text{instanceOf}(y, \text{temporalRegion}, y) \\ &\quad \rightarrow (\exists o (\text{temporalPartOf}(o, x) \wedge \text{temporalPartOf}(o, y)) \\ &\quad \rightarrow \exists z (\text{instanceOf}(z, \text{temporalRegion}, z) \\ &\quad \quad \wedge (\forall w (\text{instanceOf}(w, \text{temporalRegion}, w) \wedge \text{instanceOf}(z, \text{temporalRegion}, z) \\ &\quad \quad \rightarrow (\text{temporalPartOf}(w, z) \\ &\quad \quad \quad \leftrightarrow \text{temporalPartOf}(w, x) \wedge \text{temporalPartOf}(w, y))))))) \end{aligned}$$

(9) Temporal part of has weak supplementation

$$\begin{aligned} &\forall x, y (\text{instanceOf}(x, \text{temporalRegion}, x) \wedge \text{instanceOf}(y, \text{temporalRegion}, y) \\ &\quad \rightarrow (\text{properTemporalPartOf}(x, y) \\ &\quad \rightarrow \exists z (\text{properTemporalPartOf}(z, y) \\ &\quad \quad \wedge \neg (\exists \text{overlap} (\text{instanceOf}(\text{overlap}, \text{temporalRegion}, \text{overlap}) \\ &\quad \quad \quad \wedge \text{temporalPartOf}(\text{overlap}, x) \wedge \text{temporalPartOf}(\text{overlap}, z)))))) \end{aligned}$$

(10) Proper temporal part of is transitive

$$\forall a, b, c (\text{properTemporalPartOf}(a, b) \wedge \text{properTemporalPartOf}(b, c) \rightarrow \text{properTemporalPartOf}(a, c))$$

(11) If two temporal intervals do not overlap then one of them precedes the other

$$\begin{aligned} &\forall t1, t2 (\text{instanceOf}(t1, \text{temporalInterval}, t1) \wedge \text{instanceOf}(t2, \text{temporalInterval}, t2) \\ &\quad \wedge \neg (\exists \text{part} (\text{temporalPartOf}(\text{part}, t1) \wedge \text{temporalPartOf}(\text{part}, t2))) \\ &\quad \rightarrow \text{precedes}(t1, t2) \vee \text{precedes}(t2, t1)) \end{aligned}$$

(12) Has first instant is functional on second argument

$$\forall p,q,r(\text{hasFirstInstant}(p,q) \wedge \text{hasFirstInstant}(p,r) \rightarrow q=r)$$

(13) Instance of is disjunctive on third argument, a temporal region

$$\forall p,q,r,s(\text{instanceOf}(p,q,r) \wedge \text{temporalPartOf}(s,r) \rightarrow \text{instanceOf}(p,q,s))$$

(14) Temporal instants are totally ordered

$$\forall t_1,t_2(\text{instanceOf}(t_1,\text{temporalInstant},t_1) \wedge \text{instanceOf}(t_2,\text{temporalInstant},t_2) \\ \rightarrow \text{precedes}(t_1,t_2) \vee \text{precedes}(t_2,t_1) \vee t_1=t_2)$$

(15) Has last instant is functional on second argument

$$\forall p,q,r(\text{hasLastInstant}(p,q) \wedge \text{hasLastInstant}(p,r) \rightarrow q=r)$$

(16) If a has temporal part b then if a is an instance of zero dimensional temporal region then b is an instance of zero dimensional temporal region

$$\forall p,q(\text{hasTemporalPart}(p,q) \\ \rightarrow (\exists t \text{instanceOf}(p,\text{zeroDimensionalTemporalRegion},t) \\ \rightarrow \exists t \text{instanceOf}(q,\text{zeroDimensionalTemporalRegion},t)))$$

(17) All temporal regions are part of a temporal interval

$$\forall t(\text{instanceOf}(t,\text{temporalRegion},t) \\ \rightarrow \exists i(\text{instanceOf}(i,\text{temporalInterval},i) \wedge \text{temporalPartOf}(t,i)))$$

(18) Temporal regions are instances at themselves

$$\forall a,u(\exists t(\text{instanceOf}(a,\text{temporalRegion},t) \wedge \text{instanceOf}(a,u,t)) \rightarrow \text{instanceOf}(a,u,a))$$

(19) The first and last time points for an instant are the instant itself

$$\forall i(\text{instanceOf}(i,\text{temporalInstant},i) \leftrightarrow \text{hasFirstInstant}(i,i) \wedge \text{hasLastInstant}(i,i))$$

(20) Any temporal instant that precedes the last instant of an interval and which is preceded by the first instant is part of the interval

$$\forall t,r(\text{instanceOf}(t,\text{temporalInstant},t) \wedge \text{instanceOf}(r,\text{temporalInterval},r) \\ \rightarrow \text{hasFirstInstant}(r,t) \vee \text{hasLastInstant}(r,t) \\ \vee (\exists f,l(\text{instanceOf}(r,\text{temporalInterval},r) \wedge \text{hasFirstInstant}(r,f) \\ \wedge \text{hasLastInstant}(r,l) \wedge \text{precedes}(t,l) \wedge \text{precedes}(f,t)) \\ \leftrightarrow \text{properTemporalPartOf}(t,r)))$$

(21) A proper temporal part of b means a is a temporal part of b and b a is not the same as b

$$\forall x,y(\text{properTemporalPartOf}(x,y) \leftrightarrow \text{temporalPartOf}(x,y) \wedge x \neq y)$$

(22) A one dimensional temporal region has at least one interval as part

$$\forall t(\text{instanceOf}(t,\text{oneDimensionalTemporalRegion},t) \\ \rightarrow \exists p(\text{temporalPartOf}(p,t) \wedge \text{instanceOf}(p,\text{temporalInterval},p)))$$

(23) The only part of a temporal instant is itself

$$\forall p,q(\exists t \text{instanceOf}(p,\text{temporalInstant},t) \wedge \text{hasTemporalPart}(p,q) \rightarrow p=q)$$

(24) Temporal part of is reflexive

$$\forall a(\exists t \text{instanceOf}(a,\text{occurrent},t) \rightarrow \text{temporalPartOf}(a,a))$$

(25) Temporal part of is reflexive

$$\forall a(\exists t \text{instanceOf}(a,\text{temporalRegion},t) \rightarrow \text{temporalPartOf}(a,a))$$

(26) If a temporal part of b then if a is an instance of one dimensional temporal region then b is an instance of one dimensional temporal region

$$\forall p,q(\text{temporalPartOf}(p,q) \\ \rightarrow (\exists t \text{instanceOf}(p,\text{oneDimensionalTemporalRegion},t) \\ \rightarrow \exists t \text{instanceOf}(q,\text{oneDimensionalTemporalRegion},t)))$$

(27) Proper temporal part of and has proper temporal part are inverse relations

$$\forall a,b (\text{properTemporalPartOf}(a,b) \leftrightarrow \text{hasProperTemporalPart}(b,a))$$

(28) If the last instant of a temporal region precedes the first instant of another, then the first region precedes the second

$$\forall i1,i2,l1,f2 (\text{hasLastInstant}(i1,l1) \wedge \text{hasFirstInstant}(i2,f2) \wedge \text{precedes}(l1,f2) \rightarrow \text{precedes}(i1,i2))$$

(29) Temporal part of has domain occurrent and range occurrent

$$\forall a,b (\text{temporalPartOf}(a,b) \rightarrow \exists t \text{instanceOf}(a,\text{occurrent},t) \wedge \exists t \text{instanceOf}(b,\text{occurrent},t))$$

(30) If something is an instance of temporal region at t, then t is part of that temporal region

$$\forall ti,t (\text{instanceOf}(ti,\text{temporalRegion},t) \rightarrow \text{temporalPartOf}(t,ti))$$

(31) If one temporal region precedes another then the first last time point precedes the second first time point

$$\forall t1,t2,l1,f2 (\text{precedes}(t1,t2) \wedge \text{hasLastInstant}(t1,l1) \wedge \text{hasFirstInstant}(t2,f2) \wedge l1 \neq f2 \rightarrow \text{precedes}(l1,f2))$$

(32) Intervals have no gaps strong version every two instants without another in between bound an interval

$$\begin{aligned} \forall i,\text{start},\text{end} (\text{instanceOf}(i,\text{temporalInterval},i) \wedge \text{hasFirstInstant}(i,\text{start}) \\ \wedge \text{hasLastInstant}(i,\text{end}) \\ \rightarrow \forall t1,t2 (\text{temporalPartOf}(t1,i) \wedge \text{temporalPartOf}(t2,i) \\ \wedge \text{instanceOf}(t1,\text{temporalInstant},t1) \\ \wedge \text{instanceOf}(t2,\text{temporalInstant},t2) \wedge \text{precedes}(t1,t2) \\ \wedge \neg (\exists t3 (\text{instanceOf}(t3,\text{temporalInstant},t3) \wedge \text{precedes}(t1,t3) \\ \wedge \text{precedes}(t3,t2))) \\ \rightarrow \exists \text{fill} (\text{instanceOf}(\text{fill},\text{temporalInterval},\text{fill}) \\ \wedge \text{hasFirstInstant}(\text{fill},t1) \wedge \text{hasLastInstant}(\text{fill},t2) \\ \wedge \text{temporalPartOf}(\text{fill},i)))) \end{aligned}$$

(33) Has last instant has domain temporal region and range temporal instant

$$\forall a,b (\text{hasLastInstant}(a,b) \rightarrow \exists t \text{instanceOf}(a,\text{temporalRegion},t) \wedge \exists t \text{instanceOf}(b,\text{temporalInstant},t))$$

(34) If a temporal part of b then if a is an instance of temporal region then b is an instance of temporal region, and vice versa

$$\forall p,q (\text{temporalPartOf}(p,q) \rightarrow (\exists t \text{instanceOf}(p,\text{temporalRegion},t) \leftrightarrow \exists t \text{instanceOf}(q,\text{temporalRegion},t)))$$

(35) Temporal part of is transitive

$$\forall a,b,c (\text{temporalPartOf}(a,b) \wedge \text{temporalPartOf}(b,c) \rightarrow \text{temporalPartOf}(a,c))$$

(36) An interval has no gaps

$$\begin{aligned} \forall i,\text{start},\text{end} (\text{instanceOf}(i,\text{temporalInterval},i) \wedge \text{hasFirstInstant}(i,\text{start}) \\ \wedge \text{hasLastInstant}(i,\text{end}) \\ \rightarrow \neg (\exists \text{gap},\text{gapStart},\text{gapEnd} (\neg \text{instanceOf}(\text{gap},\text{temporalInstant},\text{gap}) \\ \wedge \text{hasFirstInstant}(\text{gap},\text{gapStart}) \\ \wedge \text{hasLastInstant}(\text{gap},\text{gapEnd}) \\ \wedge (\text{precedes}(\text{gapEnd},\text{end}) \\ \vee (\text{temporalPartOf}(\text{end},i) \wedge \text{gapEnd}=\text{end})) \\ \wedge (\text{precedes}(\text{start},\text{gapStart}) \\ \vee (\text{temporalPartOf}(\text{start},i) \wedge \text{gapStart}=\text{start})) \\ \wedge \neg \text{temporalPartOf}(\text{gap},i)))) \end{aligned}$$

(37) First instant of a temporal region that is not an instant precedes last instant

$$\forall t,ft,lt (\neg \text{instanceOf}(t,\text{temporalInstant},t) \wedge \text{hasFirstInstant}(t,ft) \wedge \text{hasLastInstant}(t,lt) \rightarrow \text{precedes}(ft,lt))$$

(38) The first temporal instant is such that it precedes every part of the interval that doesn't have the first instant as part

$$\begin{aligned} \forall fi,i (\text{instanceOf}(fi,\text{temporalInstant},fi) \wedge \text{instanceOf}(i,\text{temporalRegion},i) \\ \wedge \neg \text{instanceOf}(i,\text{temporalInstant},i) \\ \rightarrow (\text{hasFirstInstant}(i,fi) \\ \rightarrow \forall ip (\text{temporalPartOf}(ip,i) \wedge \neg \text{temporalPartOf}(fi,ip) \rightarrow \text{precedes}(fi,ip)))) \end{aligned}$$

(39) If the last instant of a temporal region precedes the first instant of another, the first region precedes the second

$$\begin{aligned} &\forall i1,i2,l1,f2 (\neg \text{instanceOf}(i1,\text{temporalInstant},i1) \wedge \neg \text{instanceOf}(i2,\text{temporalInstant},i2) \\ &\quad \wedge \text{hasLastInstant}(i1,l1) \wedge \text{hasFirstInstant}(i2,f2) \wedge l1=f2 \\ &\quad \rightarrow \text{precedes}(i1,i2)) \end{aligned}$$

(40) Temporal part of for occurrents implies occurrent part of

$$\forall a,b (\text{temporalPartOf}(a,b) \rightarrow \text{occurrentPartOf}(a,b))$$

(41) The last temporal instant is such that every part of the interval that doesn't have the last instant as part precedes it

$$\begin{aligned} &\forall li,i (\text{instanceOf}(li,\text{temporalInstant},li) \wedge \text{instanceOf}(i,\text{temporalRegion},i) \\ &\quad \wedge \neg \text{instanceOf}(i,\text{temporalInstant},i) \\ &\quad \rightarrow (\text{hasLastInstant}(i,li) \\ &\quad \rightarrow (\forall ip (\text{temporalPartOf}(ip,i) \wedge \neg \text{temporalPartOf}(li,ip) \rightarrow \text{precedes}(ip,li)))))) \end{aligned}$$

(42) Has last instant and last instant of are inverse relations

$$\forall a,b (\text{hasLastInstant}(a,b) \leftrightarrow \text{lastInstantOf}(b,a))$$

(43) Has first instant and first instant of are inverse relations

$$\forall a,b (\text{hasFirstInstant}(a,b) \leftrightarrow \text{firstInstantOf}(b,a))$$

(44) Any temporal instant that precedes the last instant of an interval and which is preceded by the first instant is part of the interval

$$\begin{aligned} &\forall i (\text{instanceOf}(i,\text{temporalRegion},i) \\ &\quad \rightarrow \exists t1,t2 (\text{hasFirstInstant}(i,t1) \wedge \text{hasLastInstant}(i,t2))) \end{aligned}$$

(45) A first instant is either part of an extended region or precedes it

$$\begin{aligned} &\forall f,i (\text{instanceOf}(f,\text{temporalInstant},f) \wedge \text{instanceOf}(i,\text{temporalRegion},i) \\ &\quad \wedge \neg \text{instanceOf}(i,\text{temporalInstant},i) \wedge \text{hasFirstInstant}(i,f) \\ &\quad \rightarrow (\neg \text{temporalPartOf}(f,i) \leftrightarrow \text{precedes}(f,i))) \end{aligned}$$

(46) Has first instant has domain temporal region and range temporal instant

$$\begin{aligned} &\forall a,b (\text{hasFirstInstant}(a,b) \\ &\quad \rightarrow \exists t \text{instanceOf}(a,\text{temporalRegion},t) \wedge \exists t \text{instanceOf}(b,\text{temporalInstant},t)) \end{aligned}$$