## **BFO 2020 Temporal Region Axioms**

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Has last instant and last instant of are inverse relations [wal-1]
    \forall a,b (hasLastInstant(a,b) \leftrightarrow lastInstantOf(b,a))
Has first instant and first instant of are inverse relations [bon-1]
    \forall a,b (hasFirstInstant(a,b) \leftrightarrow firstInstantOf(b,a))
Temporal part of and has temporal part are inverse relations [boo-1]
    \forall a,b (temporalPartOf(a,b) \leftrightarrow hasTemporalPart(b,a))
Temporal part of for occurrents implies occurrent part of [bal-1]
    \forall a,b (temporalPartOf(a,b) \rightarrow occurrentPartOf(a,b))
Temporal part of is reflexive [dbj-2]
    \forall a (instanceOf(a,temporalRegion,a) \rightarrow temporalPartOf(a,a))
Proper temporal part of and has proper temporal part are inverse relations [dbc-1]
    \forall a,b (properTemporalPartOf(a,b) \leftrightarrow hasProperTemporalPart(b,a))
If something is an instance of temporal region at t, then t is part of that temporal region [njq-1]
    \forall ti,t(instanceOf(ti,temporalRegion,t) \rightarrow temporalPartOf(t,ti))
Temporal part of is reflexive [bvr-1]
    \forall a (\exists t \text{ instanceOf}(a, occurrent, t) \rightarrow temporalPartOf(a, a))
Temporal part of is antisymmetric [zdq-1]
    \forall a,b (temporalPartOf(a,b) \land temporalPartOf(b,a) \rightarrow a=b)
Proper temporal part of is asymmetric [aqu-1]
    \forall a,b (properTemporalPartOf(a,b) \rightarrow \negproperTemporalPartOf(b,a))
Has last instant is functional on second argument [ogd-1]
    \forall p,q,r (hasLastInstant(p,q) \land hasLastInstant(p,r) \rightarrow q=r)
Has first instant is functional on second argument [fwf-1]
    \forall p,q,r (hasFirstInstant(p,q) \land hasFirstInstant(p,r) \rightarrow q=r)
A proper temporal part of b means a is a temporal part of b and b a is not the same as b [aeu-1]
    \forall x,y (properTemporalPartOf(x,y) \leftrightarrow temporalPartOf(x,y) \land x \neq y)
The only part of a temporal instant is itself [pir-2]
    \forall p,q (instanceOf(p,temporalInstant,p) \land hasTemporalPart(p,q) \rightarrow p=q)
Instance of is dissective on third argument, a temporal region [qaf-1]
    \forall p,q,r,s (instanceOf(p,q,r) \land temporalPartOf(s,r) \rightarrow instanceOf(p,q,s))
Temporal part of is transitive [bfq-1]
    \forall a,b,c (temporalPartOf(a,b) \land temporalPartOf(b,c) \rightarrow temporalPartOf(a,c))
The first and last time points for an instant are the instant itself [nfo-1]
    \forall i (instanceOf(i, temporalInstant, i) \leftrightarrow hasFirstInstant(i, i) \land hasLastInstant(i, i))
Temporal regions are instances at themselves [tvx-2]
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\forall a, u (\exists t (instanceOf(a, temporalRegion, t) \land instanceOf(a, u, t)) \leftrightarrow instanceOf(a, u, a))
If a temporal part of b then if a is an instance of temporal region then b is an instance of temporal region, and vice versa
[mjn-2]
     \forall p,q \text{ (temporalPartOf(p,q))}
           \rightarrow (instanceOf(p,temporalRegion,p) \leftrightarrow instanceOf(q,temporalRegion,q)))
Has last instant has domain temporal region and range temporal instant [jtk-2]
    \foralla,b (hasLastInstant(a,b)
          \rightarrow instanceOf(a,temporalRegion,a) \land instanceOf(b,temporalInstant,b))
Proper temporal part of is transitive [mns-1]
    \forall a,b,c (properTemporalPartOf(a,b) \land properTemporalPartOf(b,c) \rightarrow properTemporalPartOf(a,c))
Has first instant has domain temporal region and range temporal instant [fwk-2]
    \foralla,b (hasFirstInstant(a,b)
          \rightarrow instanceOf(a,temporalRegion,a) \land instanceOf(b,temporalInstant,b))
If the last instant of a temporal region precedes the first instant of another, then the first region precedes the second [qqv-1]
    \foralli1,i2,l1,f2 (hasLastInstant(i1,l1) \land hasFirstInstant(i2,f2) \land precedes(l1,f2)
                 \rightarrow precedes(i1,i2))
Any temporal region has a first and last instant [daf-1]
    ∀i (instanceOf(i,temporalRegion,i)
        \rightarrow \exists t1,t2 (hasFirstInstant(i,t1) \land hasLastInstant(i,t2)))
All temporal regions are part of a temporal interval [mvd-1]
    \forallt (instanceOf(t,temporalRegion,t)
        \rightarrow \exists i (instanceOf(i,temporalInterval,i) \land temporalPartOf(t,i)))
Temporal part of has domain occurrent and range occurrent [ruj-1]
    \forall a,b (temporalPartOf(a,b) \rightarrow \exists t instanceOf(a,occurrent,t) \land \exists t instanceOf(b,occurrent,t))
First instant of a temporal region that is not an instant precedes last instant [rzv-1]
    \forall t,ft,lt (\neginstanceOf(t,temporalInstant,t) \land hasFirstInstant(t,ft) \land hasLastInstant(t,lt)
            \rightarrow precedes(ft,lt))
If one temporal region precedes another then the first last time point precedes the second first time point [miz-1]
     \forallt1,t2,l1,f2 (precedes(t1,t2) \land hasLastInstant(t1,l1) \land hasFirstInstant(t2,f2) \land l1\neqf2
                  \rightarrow precedes(l1,f2))
If a temporal part of b then if a is an instance of one dimensional temporal region then b is an instance of one dimensional
temporal region [mei-2]
     \forall p,q (temporalPartOf(p,q)
           \rightarrow (instanceOf(p,oneDimensionalTemporalRegion,p)
              \rightarrow instanceOf(q,oneDimensionalTemporalRegion,q)))
A one dimensional temporal region has at least one interval as part [jhe-1]
    ∀t (instanceOf(t,oneDimensionalTemporalRegion,t)
        \rightarrow \exists p \text{ (temporalPartOf}(p,t) \land \text{instanceOf}(p,\text{temporalInterval},p)))}
If a has temporal part b then if a is an instance of zero dimensional temporal region then b is an instance of zero dimensional
temporal region [bnt-2]
    \forall p,q (hasTemporalPart(p,q))
           \rightarrow (instanceOf(p,zeroDimensionalTemporalRegion,p)
              \rightarrow instanceOf(q,zeroDimensionalTemporalRegion,q)))
Temporal instants are totally ordered [qnf-1]
    \forall t1,t2 (instanceOf(t1,temporalInstant,t1) \land instanceOf(t2,temporalInstant,t2)
            \rightarrow precedes(t1,t2) \lor precedes(t2,t1) \lor t1=t2)
Temporal region is the union of zero dimensional temporal region and one dimensional temporal region [hgs-1]
    ∀i,t (instanceOf(i,temporalRegion,t)
         \rightarrow instanceOf(i,zeroDimensionalTemporalRegion,t)
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∨instanceOf(i,oneDimensionalTemporalRegion,t))

If the last instant of a temporal region is the first instant of another, the first region precedes the second [suk-1]

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∀i1,i2,l1,f2 (¬instanceOf(i1,temporalInstant,i1) ∧ ¬instanceOf(i2,temporalInstant,i2)
                  \land hasLastInstant(i1,l1) \land hasFirstInstant(i2,f2) \land l1=f2
                  \rightarrow precedes(i1,i2))
If a has temporal part b then if a is an instance of one dimensional temporal region then b is an instance of one dimensional
temporal region or zero dimensional temporal region [eeg-2]
     \forall p,q (hasTemporalPart(p,q))
           \rightarrow (instanceOf(p,oneDimensionalTemporalRegion,p)
               \rightarrow instanceOf(q,oneDimensionalTemporalRegion,q)
                ∨instanceOf(q,zeroDimensionalTemporalRegion,q)))
A last instant is either part of an extended region or is preceded by it [acg-1]
     \forall l,i (instanceOf(l,temporalInstant,l) \land instanceOf(i,temporalRegion,i)
          \land \neg instanceOf(i,temporalInstant,i) \land hasLastInstant(i,l)
          \rightarrow (\negtemporalPartOf(l,i) \leftrightarrow precedes(i,l)))
A first instant is either part of an extended region or precedes it [qga-1]
     \forall f, i (instanceOf(f,temporalInstant,f) \land instanceOf(i,temporalRegion,i)
         \land \neg instanceOf(i,temporalInstant,i) \land hasFirstInstant(i,f)
         \rightarrow (\negtemporalPartOf(f,i) \leftrightarrow precedes(f,i)))
If two temporal intervals do not overlap then one of them precedes the other [owb-1]
     \forall t1,t2 (instanceOf(t1,temporalInterval,t1) \land instanceOf(t2,temporalInterval,t2)
            \land \neg (\exists part(temporalPartOf(part,t1) \land temporalPartOf(part,t2)))
             \rightarrow precedes(t1,t2) \vee precedes(t2,t1))
The first temporal instant is such that it precedes every part of the interval that doesn't have the first instant as part [ixz-1]
     \forall fi,i (instanceOf(fi,temporalInstant,fi) \land instanceOf(i,temporalRegion,i)
          \land \neg instanceOf(i, temporalInstant, i)
          \rightarrow (hasFirstInstant(i,fi)
              \rightarrow \forall ip(temporalPartOf(ip,i) \land \neg temporalPartOf(fi,ip) \rightarrow precedes(fi,ip))))
The last temporal instant is such that every part of the interval that doesn't have the last instant as part precedes it [nhd-1]
     ∀li,i (instanceOf(li,temporalInstant,li) ∧ instanceOf(i,temporalRegion,i)
          \land \neg instanceOf(i,temporalInstant,i)
          \rightarrow (hasLastInstant(i,li)
              \rightarrow (\forall ip (temporalPartOf(ip,i) \land \neg temporalPartOf(li,ip) \rightarrow precedes(ip,li)))))
Intervals have no internal gaps [ekm-1]
     \forall i, start, end (instanceOf(i, temporalInterval, i) \land has FirstInstant(i, start)
                   ∧ hasLastInstant(i,end)
                   \rightarrow \neg (\exists gap, gapStart, gapEnd(hasFirstInstant(gap, gapStart))
                                                  ∧ hasLastInstant(gap,gapEnd)
                                                  ∧ precedes(gapEnd,end) ∧ precedes(start,gapStart)
                                                  \land \neg temporalPartOf(gap,i))))
Temporal part of has weak supplementation [vbw-1]
     \forall x,y \text{ (instanceOf(}x,\text{temporalRegion,}x) \land \text{instanceOf(}y,\text{temporalRegion,}y)
           \rightarrow (properTemporalPartOf(x,y)
              \rightarrow \exists z (properTemporalPartOf(z,y))
                    \land \neg (\exists overlap(instanceOf(overlap,temporalRegion,overlap)))
                                       \land temporalPartOf(overlap,\!x) \land temporalPartOf(overlap,\!z))))))
Any temporal instant that precedes the last instant of an interval and which is preceded by the first instant is part of the
interval [zlp-1]
     \forallt,r (instanceOf(t,temporalInstant,t) \land instanceOf(r,temporalInterval,r)
          \rightarrow hasFirstInstant(r,t) \lor hasLastInstant(r,t)
           \vee (\exists f, l \text{ (instanceOf(r,temporalInterval,r)} \land hasFirstInstant(r,f))
                       \land hasLastInstant(r,l) \land precedes(t,l) \land precedes(f,t))
               \leftrightarrow properTemporalPartOf(t,r)))
Two intervals are identical if their first and last instants are the same and if an instant is part of one of the intervals it is also
part of the other [xkl-1]
     ∀i1,i2 (instanceOf(i1,temporalInterval,i1) ∧ instanceOf(i2,temporalInterval,i2)
             \rightarrow (\exists f,l(hasFirstInstant(i1,f) \land hasFirstInstant(i2,f) \land hasLastInstant(i1,l)
                      \land hasLastInstant(i2,l) \land (temporalPartOf(l,i1) \leftrightarrow temporalPartOf(l,i2))
                      \land (temporalPartOf(f,i1) \leftrightarrow temporalPartOf(f,i2)))
               \rightarrow i1=i2))
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Temporal part of has unique product [wsg-2]
    \forall x,y \text{ (instanceOf(}x,\text{temporalRegion,}x) \land \text{instanceOf(}y,\text{temporalRegion,}y)
           \rightarrow (\exists o (temporalPartOf(o,x) \land temporalPartOf(o,y))
              \rightarrow \exists z (instanceOf(z,temporalRegion,z))
                     \land (\forall w (instanceOf(w,temporalRegion,w) \land instanceOf(z,temporalRegion,z))
                              \rightarrow (temporalPartOf(w,z)
                                    \leftrightarrow temporalPartOf(w,x) \land temporalPartOf(w,y))))))
An interval has no gaps [nui-1]
    \foralli,start,end (instanceOf(i,temporalInterval,i) \land hasFirstInstant(i,start)
                   ∧hasLastInstant(i,end)
                    \rightarrow \neg (\exists gap, gapStart, gapEnd(\neg instanceOf(gap, temporalInstant, gap))
                                                   ∧ hasFirstInstant(gap,gapStart)
                                                   ∧ hasLastInstant(gap,gapEnd)
                                                   ∧ (precedes(gapEnd,end)
                                                       \lor (temporalPartOf(end,i) \land gapEnd=end))
                                                   ∧ (precedes(start,gapStart)
                                                       \lor (temporalPartOf(start,i) \land gapStart=start))
                                                   \land \neg temporalPartOf(gap,i))))
Intervals have no gaps strong version every two instants without another in between bound an interval [cop-1]
     \forall i,start,end (instanceOf(i,temporalInterval,i) \land hasFirstInstant(i,start)
                   ∧hasLastInstant(i,end)
                    \rightarrow \forall t1,t2(temporalPartOf(t1,i) \land temporalPartOf(t2,i)
                               \land instanceOf(t1,temporalInstant,t1)
                               \land instanceOf(t2,temporalInstant,t2) \land precedes(t1,t2)
                               \land \neg (\exists t3 (instanceOf(t3, temporalInstant, t3) \land precedes(t1, t3))
                                           \land precedes(t3,t2)))
                                 \rightarrow \exists fill(instanceOf(fill,temporalInterval,fill)
                                          \land hasFirstInstant(fill,t1) \land hasLastInstant(fill,t2)
                                          ∧ temporalPartOf(fill,i))))
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Alan Ruttenberg, January 8, 2024. The most recent version of this file will always be in the GitHub repository https://github.com/bfo-ontology/bfo-2020

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