BFO 2020 Continuant Mereology Axioms

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(1) If a has continuant part b then if a is an instance of site then b is an instance of site or continuant fiat boundary
     \forall p,q,t \text{ (hasContinuantPart}(p,q,t) \land \text{instanceOf}(p,\text{site},t)
            \rightarrow instanceOf(q,site,t) \lor instanceOf(q,continuantFiatBoundary,t))
(2) If at all times that two object aggreates exist each is part of the other, then they are identical
     \forall a,b ((\exists t (instanceOf(a,objectAggregate,t) \land continuantPartOf(a,b,t)))
               \land continuantPartOf(b,a,t)))
          \land (\forall t (continuantPartOf(a,b,t) \leftrightarrow continuantPartOf(b,a,t)))
          \rightarrow a=b)
(3) If a has continuant part b then if a is an instance of one dimensional spatial region then b is an instance of
one dimensional spatial region or zero dimensional spatial region
     \forall p,q,t \text{ (hasContinuantPart}(p,q,t) \land \text{instanceOf}(p,\text{oneDimensionalSpatialRegion},t)
            \rightarrow instanceOf(q,oneDimensionalSpatialRegion,t)
             ∨instanceOf(q,zeroDimensionalSpatialRegion,t))
(4) If a material entity has a proper part, then at least one of its proper parts is not a material entity
     \forall m,t (instanceOf(m,materialEntity,t) \land (\exists mp (continuantPartOf(mp,m,t) \land mp\neqm))
           \rightarrow \exists mp(mp \neq m \land continuantPartOf(mp,m,t) \land \neg instanceOf(mp,immaterialEntity,t)))
(5) Continuant part of and has continuant part are inverse relations
     \forall t,a,b (continuantPartOf(a,b,t) \leftrightarrow hasContinuantPart(b,a,t))
(6) If a has continuant part b then if a is an instance of fiat point then b is an instance of fiat point
     \forall p,q,t (hasContinuantPart(p,q,t) \land instanceOf(p,fiatPoint,t) \rightarrow instanceOf(q,fiatPoint,t))
(7) Proper continuant part of is transitive at a time
     \forall a,b,c,t,t2 (properContinuantPartOf(a,b,t) \land properContinuantPartOf(b,c,t2)
                \land temporalPartOf(t,t2)
                \rightarrow properContinuantPartOf(a,c,t))
(8) X proper continuant part of y means x is a continuant part of y but y is not continuant part of x
     \forall x,y,t (properContinuantPartOf(x,y,t)
            \leftrightarrow continuantPartOf(x,y,t) \land \neg continuantPartOf(y,x,t))
(9) Continuant part of is time indexed and has domain: continuant and range: continuant
     \foralla,b,t (continuantPartOf(a,b,t)
            \rightarrow instanceOf(a,continuant,t) \land instanceOf(b,continuant,t)
             \land instanceOf(t,temporalRegion,t))
(10) Continuant part of is dissective on third argument, a temporal region
     \forall p,q,r,s (continuantPartOf(p,q,r) \land temporalPartOf(s,r) \rightarrow continuantPartOf(p,q,s))
(11) Continuant part of has a unique product at a time
     \forall x,y,t (instanceOf(x,continuant,t) \land instanceOf(y,continuant,t)
            ∧ instanceOf(t,temporalRegion,t)
            \rightarrow (\exists overlap (instanceOf(overlap,continuant,t) \land continuantPartOf(overlap,x,t)
                          \land continuantPartOf(overlap,y,t))
               \rightarrow \exists overlap (instanceOf(overlap,continuant,t)
                            \wedge (\forall w (instanceOf(\overline{w},continuant,t)
                                      \rightarrow (continuantPartOf(w,overlap,t)
                                        \leftrightarrow continuantPartOf(w,x,t) \land continuantPartOf(w,y,t))))))
(12) If a has continuant part b then if a is an instance of material entity then b is an instance of site or continuant
fiat boundary or material entity
     \forall p,q,t \text{ (hasContinuantPart}(p,q,t) \land \text{instanceOf}(p,materialEntity,t)
             \rightarrow instanceOf(q,site,t) \lor instanceOf(q,continuantFiatBoundary,t)
             \vee instanceOf(q,materialEntity,t))
(13) Exists at is dissective on first argumentwhen it is a continuant
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\forall p,q,r (existsAt(p,q) \land continuantPartOf(r,p,q) \rightarrow existsAt(r,q))
(14) Continuant part of is transitive at a time
     \forall a,b,c,t,t2  (continuantPartOf(a,b,t) \land continuantPartOf(b,c,t2) \land temporalPartOf(t,t2)
                \rightarrow continuantPartOf(a,c,t))
(15) Proper continuant part of and has proper continuant part are inverse relations
     \forallt,a,b(properContinuantPartOf(a,b,t) \leftrightarrow hasProperContinuantPart(b,a,t))
(16) Proper continuant part of is time indexed and has domain: continuant and range: continuant
     \forall a,b,t (properContinuantPartOf(a,b,t)
           \rightarrow instanceOf(a,continuant,t) \land instanceOf(b,continuant,t)
             \land instanceOf(t,temporalRegion,t))
(17) If a has continuant part b then if a is an instance of zero dimensional spatial region then b is an instance of
zero dimensional spatial region
     \forall p,q,t \text{ (hasContinuantPart}(p,q,t) \land \text{instanceOf}(p,zeroDimensionalSpatialRegion,t)}
            \rightarrow instanceOf(q,zeroDimensionalSpatialRegion,t))
(18) If at any time that two non object aggreates exist each is part of the other, then they are identical
     \forall a,b \ (\exists t (instanceOf(a,independentContinuant,t) \land \neg instanceOf(a,objectAggregate,t)
              \land instanceOf(b,independentContinuant,t) \land \neg instanceOf(b,objectAggregate,t)
              \land continuantPartOf(a,b,t)\land continuantPartOf(b,a,t))
          \rightarrow a=b)
(19) If a continuant part of b then if a is an instance of spatial region then b is an instance of spatial region, and
vice versa
     \forall p,q,t (continuantPartOf(p,q,t))
            \rightarrow (instanceOf(p,spatialRegion,t) \leftrightarrow instanceOf(q,spatialRegion,t)))
(20) If a has continuant part b then if a is an instance of continuant fiat boundary then b is an instance of
continuant fiat boundary
     \forall p,q,t \text{ (hasContinuantPart}(p,q,t) \land instanceOf(p,continuantFiatBoundary,t)}
            \rightarrow instanceOf(q,continuantFiatBoundary,t))
(21) If a continuant part of b then if a is an instance of material entity then b is an instance of material entity
     \forall p,q,t \text{ (continuantPartOf}(p,q,t) \land \text{instanceOf}(p,\text{materialEntity},t)
            \rightarrow instanceOf(q,materialEntity,t))
(22) If x,y are both part of a whole w, then if x is located in y it is part of y, if y is located in x it is part of w
     \forall x,y,t (\exists w (continuantPartOf(x,w,t) \land continuantPartOf(y,w,t))
           \rightarrow (locatedIn(x,y,t) \rightarrow continuantPartOf(x,y,t))
            \land (locatedIn(y,x,t) \rightarrow continuantPartOf(y,x,t)))
(23) If a continuant part of b then if a is an instance of independent continuant then b is an instance of independent
continuant, and vice versa
     \forall p,q,t (continuantPartOf(p,q,t))
            \rightarrow (instanceOf(p,independentContinuant,t) \leftrightarrow instanceOf(q,independentContinuant,t)))
(24) If a has continuant part b then if a is an instance of three dimensional spatial region then b is an instance of
spatial region
     \forall p,q,t \text{ (hasContinuantPart}(p,q,t) \land instanceOf(p,threeDimensionalSpatialRegion,t)}
            \rightarrow instanceOf(q,spatialRegion,t))
(25) Proper continuant part of is dissective on third argument, a temporal region
     \forall p,q,r,s (properContinuantPartOf(p,q,r) \land temporalPartOf(s,r)
              \rightarrow properContinuantPartOf(p,q,s))
(26) If a has continuant part b then if a is an instance of fiat line then b is an instance of fiat line or fiat point
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 $\forall p,q,t \text{ (hasContinuantPart}(p,q,t) \land instanceOf(p,fiatLine,t)$ $<math>\rightarrow instanceOf(q,fiatLine,t) \lor instanceOf(q,fiatPoint,t))$

(27) Continuant part of has weak supplementation

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\forall \mathsf{t}, \mathsf{x}, \mathsf{y} \text{ (instanceOf}(\mathsf{x}, \mathsf{continuant}, \mathsf{t}) \land \mathsf{instanceOf}(\mathsf{y}, \mathsf{continuant}, \mathsf{t}) \\ \land \mathsf{instanceOf}(\mathsf{t}, \mathsf{temporalRegion}, \mathsf{t}) \\ \rightarrow (\mathsf{continuantPartOf}(\mathsf{x}, \mathsf{y}, \mathsf{t}) \land \mathsf{x} \neq \mathsf{y} \\ \rightarrow \exists \mathsf{z} (\mathsf{instanceOf}(\mathsf{z}, \mathsf{continuant}, \mathsf{t}) \land \mathsf{continuantPartOf}(\mathsf{z}, \mathsf{y}, \mathsf{t}) \land \mathsf{z} \neq \mathsf{y} \\ \land \neg (\exists \mathsf{overlap}(\mathsf{instanceOf}(\mathsf{overlap}, \mathsf{continuant}, \mathsf{t}) \\ \land \mathsf{continuantPartOf}(\mathsf{overlap}, \mathsf{x}, \mathsf{t}) \\ \land \mathsf{continuantPartOf}(\mathsf{overlap}, \mathsf{z}, \mathsf{t}))))))
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(28) If a has continuant part b then if a is an instance of fiat surface then b is an instance of continuant fiat boundary

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\forall p,q,t (hasContinuantPart(p,q,t) \land instanceOf(p,fiatSurface,t) \rightarrow instanceOf(q,continuantFiatBoundary,t))
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(29) If a has continuant part b then if a is an instance of two dimensional spatial region then b is an instance of two dimensional spatial region or one dimensional spatial region or zero dimensional spatial region

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\label{eq:pqt} \begin{array}{l} \forall p,q,t \, (hasContinuantPart(p,q,t) \land instanceOf(p,twoDimensionalSpatialRegion,t) \\ \rightarrow instanceOf(q,twoDimensionalSpatialRegion,t) \\ \lor instanceOf(q,oneDimensionalSpatialRegion,t) \\ \lor instanceOf(q,zeroDimensionalSpatialRegion,t)) \end{array}
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(30) If a continuant part of b then if a is an instance of site then b is an instance of site or material entity

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\forall p,q,t \ (continuantPartOf(p,q,t) \land instanceOf(p,site,t) \\ \rightarrow instanceOf(q,site,t) \lor instanceOf(q,materialEntity,t))
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(31) Continuant part of is reflexive at a time

 $\forall a,t (instanceOf(a,independentContinuant,t) \rightarrow continuantPartOf(a,a,t))$