BFO 2020 Spatial Axioms

(1) Occurs in is lower bound location

$$\forall p,c1,c2\,(occursIn(p,c1) \\ \wedge(\forall t\,(existsAt(p,t) \leftrightarrow existsAt(c2,t) \wedge continuantPartOf(c1,c2,t))) \\ \rightarrow occursIn(p,c2))$$

(2) Occurs in has domain process or process boundary and range material entity or site

$$\forall a,b\,(occursIn(a,b) \\ \rightarrow(\exists t\,(instanceOf(a,process,t) \vee instanceOf(a,processBoundary,t))) \\ \wedge(\exists t\,(instanceOf(b,materialEntity,t) \vee instanceOf(b,site,t))))$$

(3) Occupies spatial region is functional on second argument

$$\forall p,q,r,s\,(occupiesSpatialRegion(p,q,r) \wedge occupiesSpatialRegion(p,s,r) \rightarrow q=s)$$

(4) If a process (or process boundary) occurs in a continuant, that continuant exists at least as long as the process does

$$\forall p,c\,(occursIn(p,c) \rightarrow \forall t\,(existsAt(p,t) \rightarrow existsAt(c,t)))$$

(5) Spatial regions don't change what they are part of.

$$\forall s,sp\,(\exists t\,(instanceOf(s,spatialRegion,t) \wedge continuantPartOf(sp,s,t)) \\ \rightarrow \forall t\,(\exists sPrime\,continuantPartOf(sPrime,s,t) \rightarrow continuantPartOf(sp,s,t)))$$

(6) All spatial regions are part of a 3 dimensional spatial region

$$\forall s,t\,(instanceOf(s,spatialRegion,t) \\ \rightarrow \exists s3\,(instanceOf(s3,threeDimensionalSpatialRegion,t) \wedge continuantPartOf(s,s3,t)))$$

(7) Located in is a lower bound on second argument

$$\forall p,q,r,s\,(locatedIn(p,q,r) \wedge continuantPartOf(q,s,r) \rightarrow locatedIn(p,s,r))$$

(8) Occupies spatial region is dissective on third argument, a temporal region

$$\forall p,q,r,s\,(occupiesSpatialRegion(p,q,r) \wedge temporalPartOf(s,r) \\ \rightarrow occupiesSpatialRegion(p,q,s))$$

(9) If a location of b then if a is an instance of continuant fiat boundary then b is an instance of continuant fiat boundary

$$\forall p,q,t\,(locationOf(p,q,t) \wedge instanceOf(p,continuantFiatBoundary,t) \\ \rightarrow instanceOf(q,continuantFiatBoundary,t))$$

(10) Located in and location of are inverse relations

$$\forall t,a,b\,(locatedIn(a,b,t) \leftrightarrow locationOf(b,a,t))$$

(11) Occupies spatial region is time indexed and has domain: independent continuant but not spatial region and range: spatial region

$$\forall a,b,t\,(occupiesSpatialRegion(a,b,t) \\ \rightarrow instanceOf(a,independentContinuant,t) \wedge \neg instanceOf(a,spatialRegion,t) \\ \wedge instanceOf(b,spatialRegion,t) \wedge instanceOf(t,temporalRegion,t))$$

(12) Occurs in is dissective on first argumentwhen it is an occurrent

$$\forall p,q,r\,(occursIn(p,q) \wedge occurrentPartOf(r,p) \rightarrow occursIn(r,q))$$

(13) If something is located in something else then the region of the first is part of the region of the second

$$\forall a,b,t\,(locatedIn(a,b,t) \\ \rightarrow \exists r1,r2,t2\,(temporalPartOf(t2,t) \wedge occupiesSpatialRegion(a,r1,t2) \\ \wedge occupiesSpatialRegion(b,r2,t2) \wedge continuantPartOf(r1,r2,t2)))$$

(14) At all times t, there's a part of t when c occupies spatial region r iff every part of c occupies a part of r, and there isn't a smaller part of r that c occupies.

$\forall$c,r,t (instanceOf(c,independentContinuant,t) $\wedge \neg$instanceOf(c,spatialRegion,t)
$\wedge$ instanceOf(r,spatialRegion,t)
$\rightarrow \exists$t2 (temporalPartOf(t2,t)
$\wedge$ (occupiesSpatialRegion(c,r,t2)
$\leftrightarrow$ ($\forall$cp (continuantPartOf(cp,c,t2)
$\rightarrow \forall$rp (occupiesSpatialRegion(cp,rp,t2)
$\rightarrow$ continuantPartOf(rp,r,t2))))
$\wedge \neg(\exists r'(r' \neq r \wedge$ continuantPartOf(r′,r,t2)
$\wedge$ occupiesSpatialRegion(c,r′,t2))))))

(15) No two material entities occupy the same space unless they coincide

$\forall$m1,m2,s,t (instanceOf(m1,materialEntity,t) $\wedge$ occupiesSpatialRegion(m1,s,t)
$\wedge$ instanceOf(m2,materialEntity,t) $\wedge$ occupiesSpatialRegion(m2,s,t)
$\rightarrow$ (continuantPartOf(m2,m1,t) $\wedge$ continuantPartOf(m1,m2,t)) $\vee$ m1=m2)

(16) Located in is dissective on third argument, a temporal region

$\forall$p,q,r,s (locatedIn(p,q,r) $\wedge$ temporalPartOf(s,r) $\rightarrow$ locatedIn(p,q,s))

(17) Occurs in and environs are inverse relations

$\forall$a,b (occursIn(a,b) $\leftrightarrow$ environs(b,a))

(18) If there are two independent continuants that are not spatial regions, and one is part of the other, then it is located in the other

$\forall$a,b,t (continuantPartOf(a,b,t) $\wedge$ instanceOf(a,independentContinuant,t)
$\wedge \neg$instanceOf(a,spatialRegion,t) $\wedge$ instanceOf(b,independentContinuant,t)
$\wedge \neg$instanceOf(b,spatialRegion,t)
$\rightarrow$ locatedIn(a,b,t))

(19) Located in is dissective on first argumentwhen it is a continuant

$\forall$p,q,r,s (locatedIn(p,q,r) $\wedge$ continuantPartOf(s,p,r) $\rightarrow$ locatedIn(s,q,r))

(20) Spatially projects onto is dissective on third argument, a temporal region

$\forall$p,q,r,s (spatiallyProjectsOnto(p,q,r) $\wedge$ temporalPartOf(s,r)
$\rightarrow$ spatiallyProjectsOnto(p,q,s))

(21) Located in is transitive at a time

$\forall$a,b,c,t,t2 (locatedIn(a,b,t) $\wedge$ locatedIn(b,c,t2) $\wedge$ temporalPartOf(t,t2)
$\rightarrow$ locatedIn(a,c,t))

(22) Located in is time indexed and has domain: independent continuant but not spatial region and range: independent continuant but not spatial region

$\forall$a,b,t (locatedIn(a,b,t)
$\rightarrow$ instanceOf(a,independentContinuant,t) $\wedge \neg$instanceOf(a,spatialRegion,t)
$\wedge$ instanceOf(b,independentContinuant,t) $\wedge \neg$instanceOf(b,spatialRegion,t)
$\wedge$ instanceOf(t,temporalRegion,t))

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