BFO 2020 Spatiotemporal Axioms

(1) Exists at is a lower bound on first argument

```
\forall p,q,r (existsAt(p,q) \land temporalPartOf(p,r) \rightarrow existsAt(r,q))
```

(2) Occupies temporal region is functional on second argument

```
\forall p,q,r (occupies Temporal Region(p,q) \land occupies Temporal Region(p,r) \rightarrow q=r)
```

(3) Spatially projects onto is time indexed and has domain: spatiotemporal region and range: spatial region

```
\forall a,b,t (spatiallyProjectsOnto(a,b,t) 
 \rightarrow instanceOf(a,spatiotemporalRegion,t) \land instanceOf(b,spatialRegion,t) 
 \land instanceOf(t,temporalRegion,t))
```

(4) Process p (or boundary) occupies spatiotemporal region st iff every part of p occupies spatiotemporal region a part of st, and there isn't a smaller part of st that p occupies.

```
\begin{tabular}{l} $\forall o,st ((\exists t1 instanceOf(o,process,t1) \lor \exists t1 instanceOf(o,processBoundary,t1)) $$ \land \exists t1 instanceOf(st,spatiotemporalRegion,t1) $$ \rightarrow (occupiesSpatiotemporalRegion(o,st) $$ \leftrightarrow (\forall op (occurrentPartOf(op,o) $$ \rightarrow \forall stp (occupiesSpatiotemporalRegion(op,stp) $$ \rightarrow occurrentPartOf(stp,st)))) $$ \land \neg (\exists st'(st' \neq st \land occurrentPartOf(st',st) $$ \land occupiesSpatiotemporalRegion(o,st'))))) $$
```

(5) Occurs in is a lower bound on second argument

```
\forall p,c1,c2(occursIn(p,c1) \land (\forall t(existsAt(p,t) \leftrightarrow locatedIn(c1,c2,t))) \rightarrow occursIn(p,c2))
```

(6) If one occurrent is part of another, then the temporal region of the first is part of the temporal region of the second

```
 \begin{tabular}{l} $\forall o1,o2,t1,t2 ((\exists t (instanceOf(o1,process,t) \lor instanceOf(o1,processBoundary,t))) $$ $$ $\land \exists t \ instanceOf(o2,process,t) \land occurrentPartOf(o1,o2) $$ $\land occupiesTemporalRegion(o1,t1) \land occupiesTemporalRegion(o2,t2) $$ $\rightarrow temporalPartOf(t1,t2)) $$ $$ $$
```

(7) If something occupies a temporal region, then it exists at that region

```
\foralla,t(occupiesTemporalRegion(a,t) \rightarrow existsAt(a,t))
```

(8) Process or process boundary p occupies temporal region t iff every part of p temporally occupies a part of t, and there isn't a smaller part of t that p occupies.

```
\begin{tabular}{l} $\forall o,t ((\exists t1 \ instanceOf(o,process,t1) \lor \exists t1 \ instanceOf(o,processBoundary,t1))$ $$ \land instanceOf(t,temporalRegion,t)$ $$ \rightarrow (occupiesTemporalRegion(o,t))$ $$ \leftrightarrow (\forall op(occurrentPartOf(op,o))$ $$ \rightarrow \forall tp(occupiesTemporalRegion(op,tp) \rightarrow occurrentPartOf(tp,t))))$ $$ \land \neg(\exists t'(t'\neq t \land occurrentPartOf(t',t))$ $$ \land occupiesTemporalRegion(o,t')))))$ $$ \land occupiesTemporalRegion(o,t')))))$
```

(9) The temporal region during which a process occurs is the same as that which the spatiotemporal region the process occupies temporally projects onto

```
\label{eq:pt} \begin{split} \forall p,t & (occupiesTemporalRegion(p,t) \\ & \leftrightarrow \exists st (occupiesSpatiotemporalRegion(p,st) \land temporallyProjectsOnto(st,t))) \end{split}
```

(10) Temporally projects onto is functional on second argument

```
\forall p,q,r (temporally ProjectsOnto(p,q) \land temporally ProjectsOnto(p,r) \rightarrow q=r)
```

(11) Every temporal region is a projection from a spatiotemporal region

```
\forall tr (\exists t \, instanceOf(tr, temporalRegion, t)) \\ \rightarrow \exists st (\exists t \, instanceOf(st, spatiotemporalRegion, t) \land temporallyProjectsOnto(st, tr)))
```

(12) Occupies temporal region has domain process or process boundary and range temporal region

```
\foralla,b (occupiesTemporalRegion(a,b)
           \rightarrow (\existst(instanceOf(a,process,t)\lorinstanceOf(a,processBoundary,t)))
           \land \exists t instanceOf(b, temporalRegion, t))
(13) Spatiotemporal regions always project on to some temporal region
     \forall st (\existst instanceOf(st,spatiotemporalRegion,t)
         \rightarrow \exists t (instanceOf(t,temporalRegion,t) \land temporallyProjectsOnto(st,t)))
(14) If one process or process boundary is part of another, then their corresponding temporal regions are also in
a parthood relation
     \forall o1,o2,st1,st2 ((\existst(instanceOf(o1,process,t)\lor instanceOf(o1,processBoundary,t)))
                    \land (\exists t (instanceOf(o2,process,t) \lor instanceOf(o2,processBoundary,t)))
                    \land occurrentPartOf(o1,o2)\land occupiesSpatiotemporalRegion(o1,st1)
                    ∧ occupiesSpatiotemporalRegion(o2,st2)
                    \rightarrow occurrentPartOf(st1,st2))
(15) Occupies spatiotemporal region has domain process or process boundary and range spatiotemporal region
     \foralla,b (occupiesSpatiotemporalRegion(a,b)
          \rightarrow (\existst(instanceOf(a,process,t)\lorinstanceOf(a,processBoundary,t)))
           \land \exists t \text{ instanceOf(b,spatiotemporalRegion,t))}
(16) Occupies spatiotemporal region is functional on second argument
     \forall p,q,r (occupiesSpatiotemporalRegion(p,q) \land occupiesSpatiotemporalRegion(p,r) \rightarrow q=r)
(17) A process occupies at least a temporal interval
     \forall proc,tr(\existst instanceOf(proc,process,t) \land occupiesTemporalRegion(proc,tr)
              \rightarrow \exists interval (instanceOf(interval,temporalInterval,interval)
                            ∧ temporalPartOf(interval,tr)))
(18) If a process or process boundary is part of another, their spatiotemporal regions are part too
     \forall p1,p2 ((\exists t instanceOf(p1,process,t)) \lor \exists t instanceOf(p1,processBoundary,t))
             \land (\exists t \text{ instanceOf}(p2,process,t) \lor \exists t \text{ instanceOf}(p2,processBoundary,t))
             \rightarrow (occurrentPartOf(p1,p2)
                 \leftrightarrow \existsst1,st2(occupiesSpatiotemporalRegion(p1,st1)
                             ∧occupiesSpatiotemporalRegion(p2,st2)
                             \land occurrentPartOf(st1,st2))))
(19) A process boundary occupies a spatiotemporal instant
     \forall pb, tr (\exists t instanceOf(pb, processBoundary, t) \land occupiesTemporalRegion(pb, tr)
             \rightarrow instanceOf(tr,temporalInstant,tr))
(20) Temporally projects onto has domain spatiotemporal region and range temporal region
     \forall a,b (temporallyProjectsOnto(a,b)
           \rightarrow \exists t \text{ instanceOf(a,spatiotemporalRegion,t)} \land \exists t \text{ instanceOf(b,temporalRegion,t)})
(21) If a occupies spatial region b then if a is an instance of site then b is an instance of three dimensional spatial
region
     \forall p,q,t (occupiesSpatialRegion(p,q,t) \land instanceOf(p,site,t)
             → instanceOf(q,threeDimensionalSpatialRegion,t))
(22) Spatiotemporal regions always project on to some spatial region at any time
     ∀st,t (instanceOf(st,spatiotemporalRegion,t)
           \rightarrow \exists s,tp (temporalPartOf(tp,t) \land instanceOf(s,spatialRegion,tp)
                   ∧ spatiallyProjectsOnto(st,s,tp)))
(23) Spatially projects onto is functional on second argument
     \forall p,q,r,s (spatiallyProjectsOnto(p,q,r) \land spatiallyProjectsOnto(p,s,r) \rightarrow q=s)
(24) Every spatial region is a projection from a spatiotemporal region
     \forall sr (\existst instanceOf(sr,spatialRegion,t)
         \rightarrow \exists st (\existst instanceOf(st,spatiotemporalRegion,t) \land \existst spatiallyProjectsOnto(st,sr,t)))
(25) For every process there's a corresponding spatiotemporal region
     \forall p (\exists t (instanceOf(p,process,t) \lor instanceOf(p,processBoundary,t))
         \rightarrow \existss occupiesSpatiotemporalRegion(p,s))
```

Alan Ruttenberg, December 5, 2019

This work is licensed under a Creative Commons "Attribution 4.0 International" license.

