

BFO 2020 Spatial Axioms

Occurs in and environs are inverse relations [uys-1]

$$\forall a,b(\text{occursIn}(a,b) \leftrightarrow \text{environs}(b,a))$$

Located in and location of are inverse relations [kaw-1]

$$\forall t,a,b(\text{locatedIn}(a,b,t) \leftrightarrow \text{locationOf}(b,a,t))$$

Occurs in is disjunctive on first argument when it is an occurrent [jil-1]

$$\forall p,q,r(\text{occursIn}(p,q) \wedge \text{occurentPartOf}(r,p) \rightarrow \text{occursIn}(r,q))$$

If a process (or process boundary) occurs in a continuant, that continuant exists at least as long as the process does [dxv-1]

$$\forall p,c(\text{occursIn}(p,c) \rightarrow \forall t(\text{existsAt}(p,t) \rightarrow \text{existsAt}(c,t)))$$

Located in is disjunctive on third argument, a temporal region [put-1]

$$\forall p,q,r,s(\text{locatedIn}(p,q,r) \wedge \text{temporalPartOf}(s,r) \rightarrow \text{locatedIn}(p,q,s))$$

Located in is a lower bound on second argument [evu-1]

$$\forall p,q,r,s(\text{locatedIn}(p,q,r) \wedge \text{continuantPartOf}(q,s,r) \rightarrow \text{locatedIn}(p,s,r))$$

Located in is disjunctive on first argument when it is a continuant [wtv-1]

$$\forall p,q,r,s(\text{locatedIn}(p,q,r) \wedge \text{continuantPartOf}(s,p,r) \rightarrow \text{locatedIn}(s,q,r))$$

Occupies spatial region is functional on second argument [zls-1]

$$\forall p,q,r,s(\text{occupiesSpatialRegion}(p,q,r) \wedge \text{occupiesSpatialRegion}(p,s,r) \rightarrow q=s)$$

Occupies spatial region is disjunctive on third argument, a temporal region [mud-1]

$$\begin{aligned} \forall p,q,r,s(\text{occupiesSpatialRegion}(p,q,r) \wedge \text{temporalPartOf}(s,r) \\ \rightarrow \text{occupiesSpatialRegion}(p,q,s)) \end{aligned}$$

Spatially projects onto is disjunctive on third argument, a temporal region [ivt-1]

$$\begin{aligned} \forall p,q,r,s(\text{spatiallyProjectsOnto}(p,q,r) \wedge \text{temporalPartOf}(s,r) \\ \rightarrow \text{spatiallyProjectsOnto}(p,q,s)) \end{aligned}$$

Located in is transitive at a time [ets-1]

$$\begin{aligned} \forall a,b,c,t_1,t_2(\text{locatedIn}(a,b,t_1) \wedge \text{locatedIn}(b,c,t_2) \wedge \text{temporalPartOf}(t_1,t_2) \\ \rightarrow \text{locatedIn}(a,c,t_1)) \end{aligned}$$

If a location of b then if a is an instance of continuant fiat boundary then b is an instance of continuant fiat boundary [wte-1]

$$\begin{aligned} \forall p,q,t(\text{locationOf}(p,q,t) \wedge \text{instanceOf}(p,\text{continuantFiatBoundary},t) \\ \rightarrow \text{instanceOf}(q,\text{continuantFiatBoundary},t)) \end{aligned}$$

All spatial regions are part of a 3 dimensional spatial region [xcx-1]

$$\begin{aligned} \forall s,t(\text{instanceOf}(s,\text{spatialRegion},t) \\ \rightarrow \exists s_3(\text{instanceOf}(s_3,\text{threeDimensionalSpatialRegion},t) \wedge \text{continuantPartOf}(s,s_3,t))) \end{aligned}$$

Occurs in is lower bound location [czc-1]

$$\begin{aligned} \forall p,c_1,c_2(\text{occursIn}(p,c_1) \\ \wedge (\forall t(\text{existsAt}(p,t) \leftrightarrow \text{existsAt}(c_2,t) \wedge \text{continuantPartOf}(c_1,c_2,t))) \\ \rightarrow \text{occursIn}(p,c_2)) \end{aligned}$$

If something is located in something else then the region of the first is part of the region of the second [uas-1]

$$\begin{aligned} \forall a,b,t(\text{locatedIn}(a,b,t) \\ \rightarrow \exists r_1,r_2,t_2(\text{temporalPartOf}(t_2,t) \wedge \text{occupiesSpatialRegion}(a,r_1,t_2) \\ \wedge \text{occupiesSpatialRegion}(b,r_2,t_2) \wedge \text{continuantPartOf}(r_1,r_2,t_2))) \end{aligned}$$

Occurs in has domain process or process boundary and range material entity or site [tfw-1]

$$\begin{aligned} \forall a,b(\text{occursIn}(a,b) \\ \rightarrow (\exists t(\text{instanceOf}(a,\text{process},t) \vee \text{instanceOf}(a,\text{processBoundary},t))) \\ \wedge (\exists t(\text{instanceOf}(b,\text{materialEntity},t) \vee \text{instanceOf}(b,\text{site},t)))) \end{aligned}$$

Spatial regions don't change what they are part of. [mlb-1]

$$\forall s, sp (\exists t (\text{instanceOf}(s, \text{spatialRegion}, t) \wedge \text{continuantPartOf}(sp, s, t)) \\ \rightarrow \forall t (\exists sPrime \text{continuantPartOf}(sPrime, s, t) \rightarrow \text{continuantPartOf}(sp, s, t)))$$

Occupies spatial region is time indexed and has domain: independent continuant but not spatial region and range: spatial region [lzw-1]

$$\forall a, b, t (\text{occupiesSpatialRegion}(a, b, t) \\ \rightarrow \text{instanceOf}(a, \text{independentContinuant}, t) \wedge \neg \text{instanceOf}(a, \text{spatialRegion}, t) \\ \wedge \text{instanceOf}(b, \text{spatialRegion}, t) \wedge \text{instanceOf}(t, \text{temporalRegion}, t))$$

If there are two independent continuants that are not spatial regions, and one is part of the other, then it is located in the other [bao-1]

$$\forall a, b, t (\text{continuantPartOf}(a, b, t) \wedge \text{instanceOf}(a, \text{independentContinuant}, t) \\ \wedge \neg \text{instanceOf}(a, \text{spatialRegion}, t) \wedge \text{instanceOf}(b, \text{independentContinuant}, t) \\ \wedge \neg \text{instanceOf}(b, \text{spatialRegion}, t) \\ \rightarrow \text{locatedIn}(a, b, t))$$

Spatial region is the union of zero dimensional spatial region, one dimensional spatial region, two dimensional spatial region, and three dimensional spatial region [wnm-1]

$$\forall i, t (\text{instanceOf}(i, \text{spatialRegion}, t) \\ \rightarrow \text{instanceOf}(i, \text{zeroDimensionalSpatialRegion}, t) \\ \vee \text{instanceOf}(i, \text{oneDimensionalSpatialRegion}, t) \\ \vee \text{instanceOf}(i, \text{twoDimensionalSpatialRegion}, t) \\ \vee \text{instanceOf}(i, \text{threeDimensionalSpatialRegion}, t))$$

No two material entities occupy the same space unless they coincide [scr-1]

$$\forall m1, m2, s, t (\text{instanceOf}(m1, \text{materialEntity}, t) \wedge \text{occupiesSpatialRegion}(m1, s, t) \\ \wedge \text{instanceOf}(m2, \text{materialEntity}, t) \wedge \text{occupiesSpatialRegion}(m2, s, t) \\ \rightarrow (\text{continuantPartOf}(m2, m1, t) \wedge \text{continuantPartOf}(m1, m2, t)) \vee m1 = m2)$$

Located in is time indexed and has domain: independent continuant but not spatial region and range: independent continuant but not spatial region [bge-1]

$$\forall a, b, t (\text{locatedIn}(a, b, t) \\ \rightarrow \text{instanceOf}(a, \text{independentContinuant}, t) \wedge \neg \text{instanceOf}(a, \text{spatialRegion}, t) \\ \wedge \text{instanceOf}(b, \text{independentContinuant}, t) \wedge \neg \text{instanceOf}(b, \text{spatialRegion}, t) \\ \wedge \text{instanceOf}(t, \text{temporalRegion}, t))$$

At all times t, there's a part of t when c occupies spatial region r iff every part of c occupies a part of r, and there isn't a smaller part of r that c occupies. [grv-1]

$$\forall c, r, t (\text{instanceOf}(c, \text{independentContinuant}, t) \wedge \neg \text{instanceOf}(c, \text{spatialRegion}, t) \\ \wedge \text{instanceOf}(r, \text{spatialRegion}, t) \\ \rightarrow \exists t2 (\text{temporalPartOf}(t2, t) \\ \wedge (\text{occupiesSpatialRegion}(c, r, t2) \\ \leftrightarrow (\forall cp (\text{continuantPartOf}(cp, c, t2) \\ \rightarrow \forall rp (\text{occupiesSpatialRegion}(cp, rp, t2) \\ \rightarrow \text{continuantPartOf}(rp, r, t2)))) \\ \wedge \neg (\exists r' (r' \neq r \wedge \text{continuantPartOf}(r', r, t2) \\ \wedge \text{occupiesSpatialRegion}(c, r', t2)))))$$
