Confidence Intervals

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Confidence Interals

Two results will help us understand the derivation of confidence intervals:

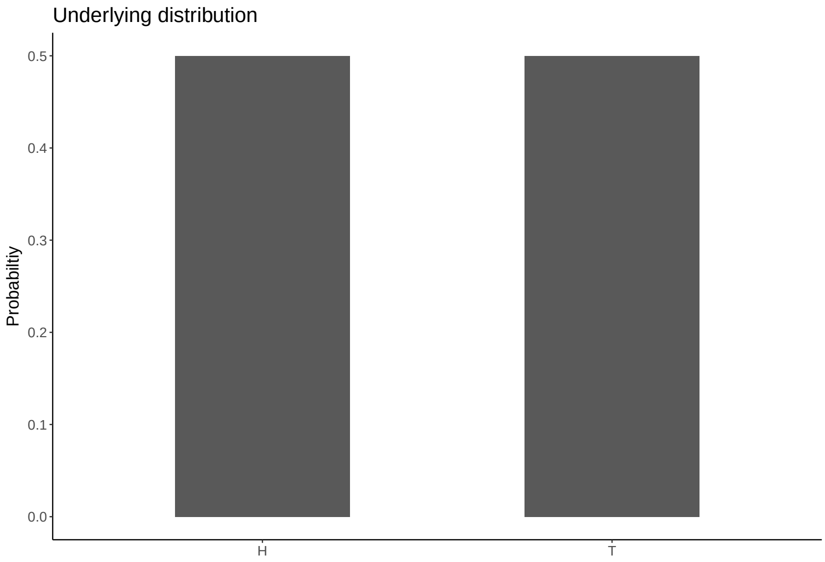
- Central Limit Theorem
- Law of Large Numbers

Central Limit Theorem

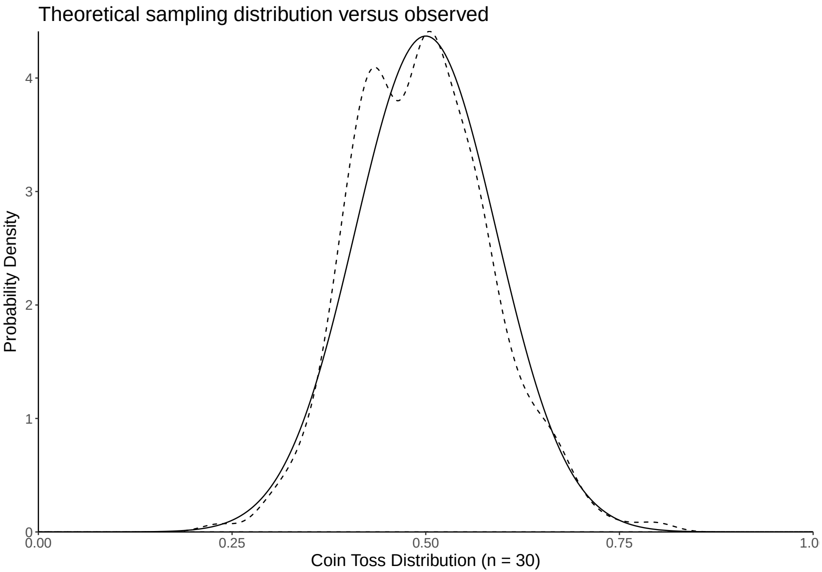
The mean, $\bar{x}$, of n independent, identically distributed random variables, X, with well defined expected value, $\mathrm{E}(X) = \mu$, and variance, $\mathrm{Var}(X) = \sigma^2$, will be approximately normally distributed when n is sufficiently large:

$$\bar{x} \sim \mathrm{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

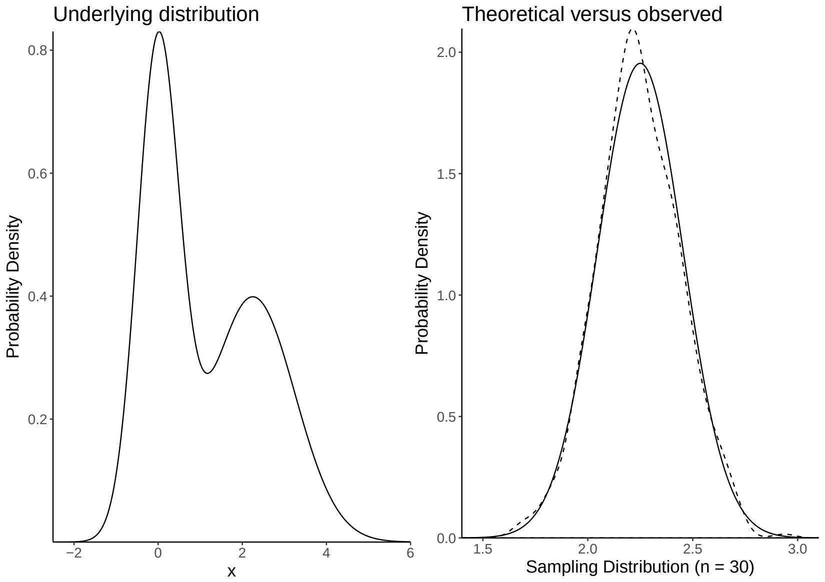
Central Limit Theorem: Simulation - Tossing Coins



Central Limit Theorem: Simulation - Tossing Coins



Central Limit Theorem: Simulation - Bimodal Distn

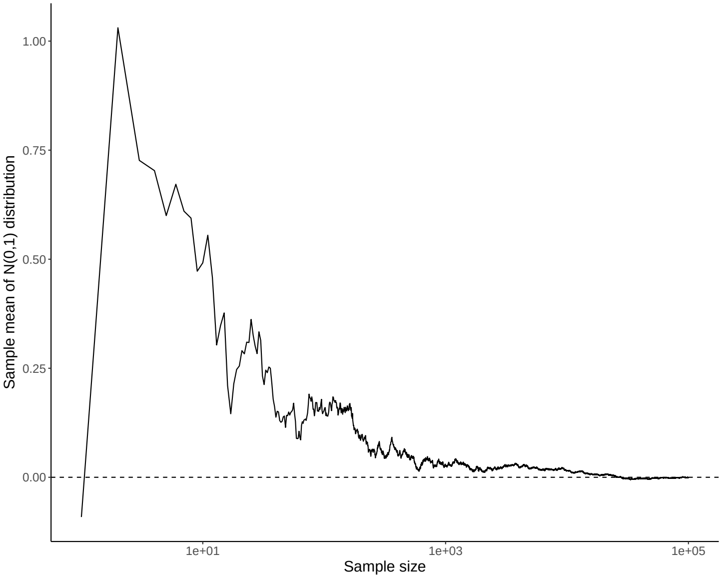


Law of Large Numbers

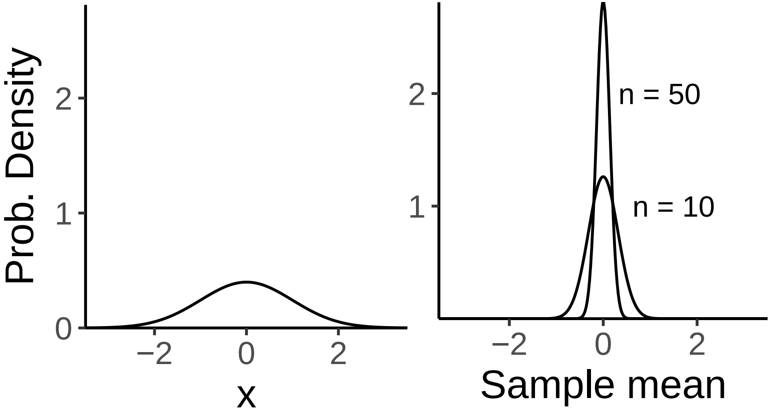
Given our sample mean, $\bar{x}$ will converge to the true population mean as the sample size increases, assuming the sample, X, are independent, identically distributed random variables.

$$\bar{x} \xrightarrow{n \to \infty} \mu$$

Law of Large Numbers: Simulation

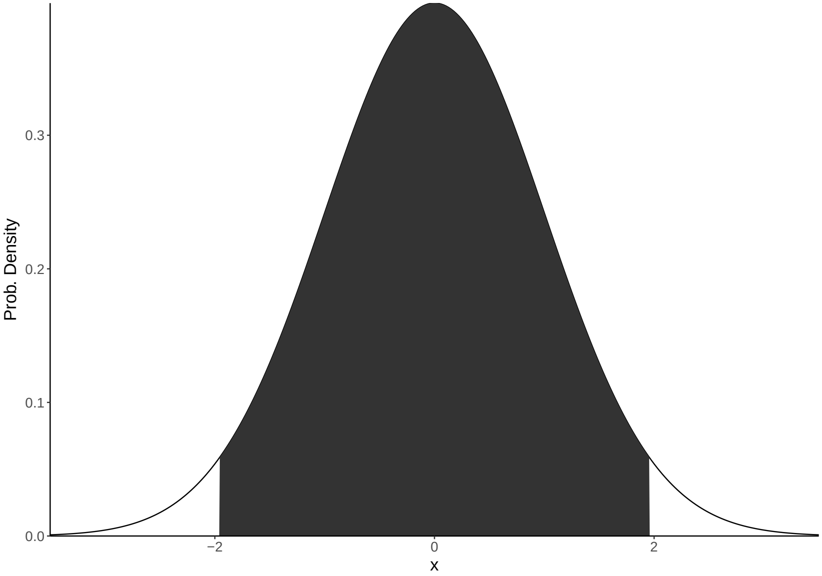


The sampling distribution



95% Confidence Region

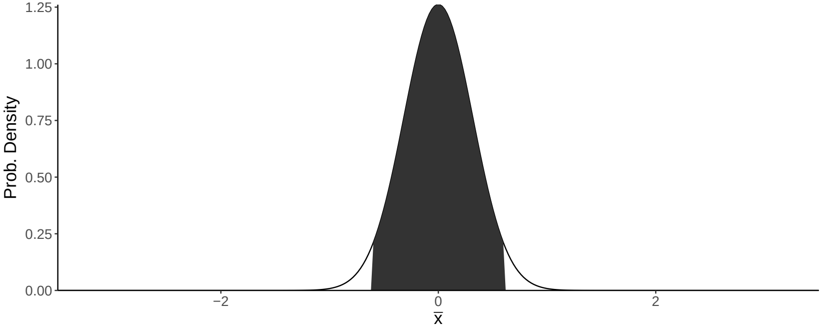
95% of the samples of x we collect will fall in $\mu \pm 1.95\sigma$.



95% Confidence Interval construction

$$P\left(\bar{x} \in \left[\mu \pm \frac{1.95 * \sigma}{\sqrt{n}}\right]\right) = 0.95$$

$$\longrightarrow P\left(\mu \in \left[\bar{x} \pm \frac{1.95 * \text{sd}}{\sqrt{n}}\right]\right) = 0.95$$



95% CI: Example

- Sample size: $n = 100$
- Mean: $\bar{x} = 123$
- SD: $\mathrm{sd} = 12$

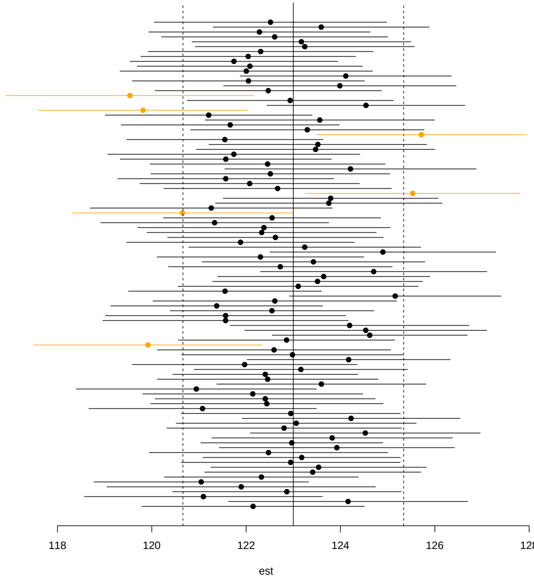
$$\begin{aligned} 95\%\ \mathrm{CI}(\bar{x}) &= 123 \pm \frac{1.95 * 12}{\sqrt{100}} \\ &= (120.66, 125.34) \end{aligned}$$

95% CI: Simulation

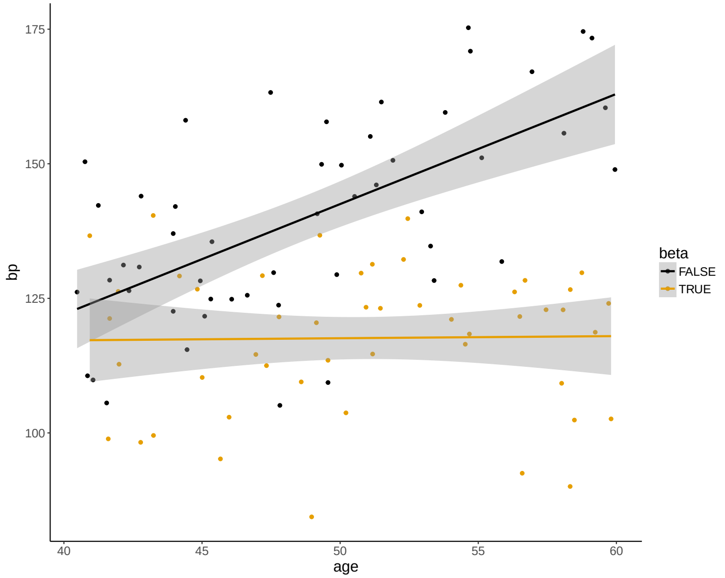
Let's simulate a similar data set in R and use the gmodels package to calculate the CI.

```
##   Estimate   CI lower   CI upper Std. Error
## 124.023961 121.599383 126.448540   1.221932
```

95% CI: Simualtion



95% Confidence Interval: Practice



95% Conficence Interval: Practice

```
##
## Call:
## lm(formula = bp ~ age * beta, data = dat)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -33.147  -9.743   2.708   9.836  27.018
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   40.1783    17.1898   2.337 0.021499 *
## age            2.0467     0.3508   5.835 7.26e-08 ***
## betaTRUE      75.4380    24.8314   3.038 0.003067 **
## age:betaTRUE  -2.0073     0.4957  -4.050 0.000104 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 14.17 on 96 degrees of freedom
## Multiple R-squared:  0.4983, Adjusted R-squared:  0.4826
## F-statistic: 31.78 on 3 and 96 DF,  p-value: 2.346e-14
```

95% Confidence Interval: Practice

```
ci(lm0)[,-4]
```

```
##                 Estimate  CI lower   CI upper      p-value
## (Intercept)    40.178325  6.056900  74.299750 2.149891e-02
## age             2.046744  1.350478   2.743011 7.261338e-08
## betaTRUE       75.437980 26.148032 124.727928 3.067416e-03
## age:betaTRUE   -2.007271 -2.991179  -1.023363 1.040238e-04
```

```
estimable(lm0, c(0, 1, 0, 1), conf.int = 0.95)[,c(1,6,7,5)]
```

```
##               Estimate   Lower.CI  Upper.CI Pr(>|t|)
## (0 1 0 1) 0.03947368 -0.6557158 0.7346631 0.910496
```

95% Confidence Interval: Practice

```
estimable(lm0, c(1, 52, 0, 0), conf.int = 0.95)[,c(1,6,7)]
```

```
##              Estimate Lower.CI Upper.CI
## (1 52 0 0)  146.609 142.0423 151.1757
```