Odds Ratios

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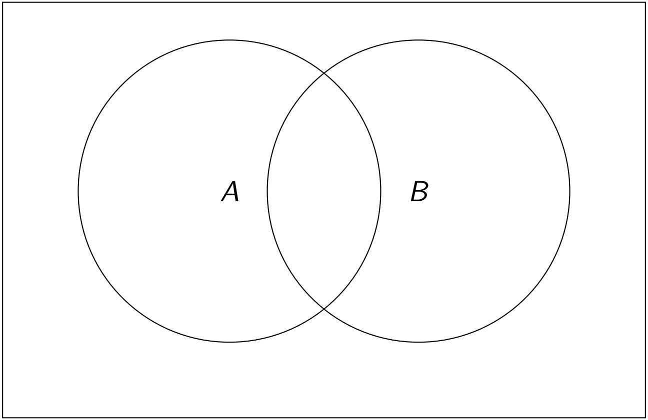
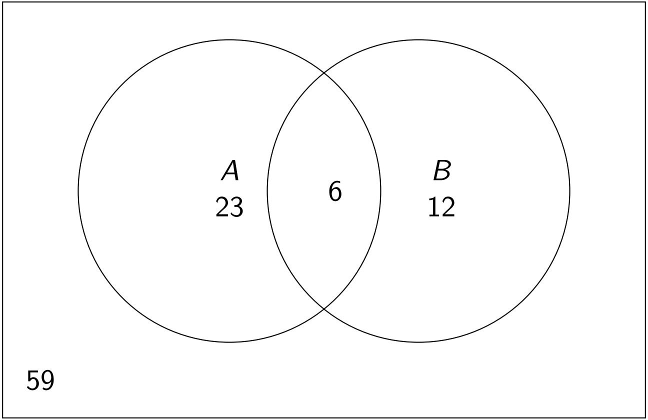
Quick review of probability

This Venn diagram represents the event space for two events, A and B. The area inside of the rectangle is 1.

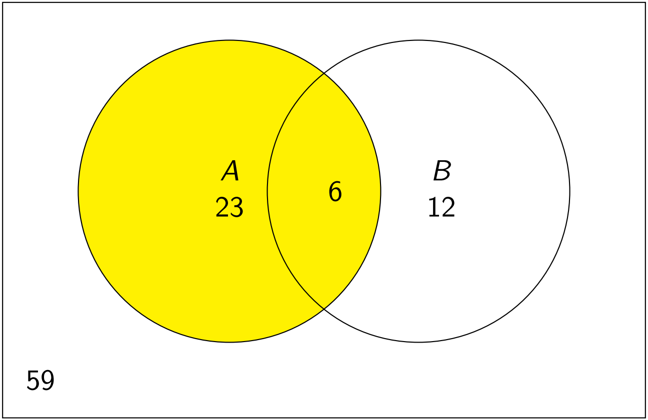


Quick review of probability



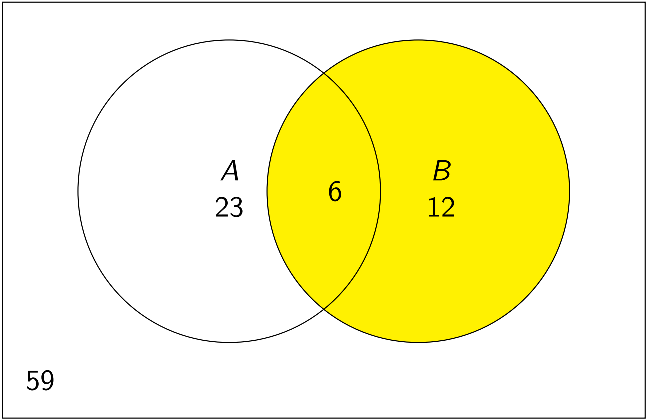
Quick review of probability

$$P(A) = \frac{23 + 6}{100} = 0.29$$



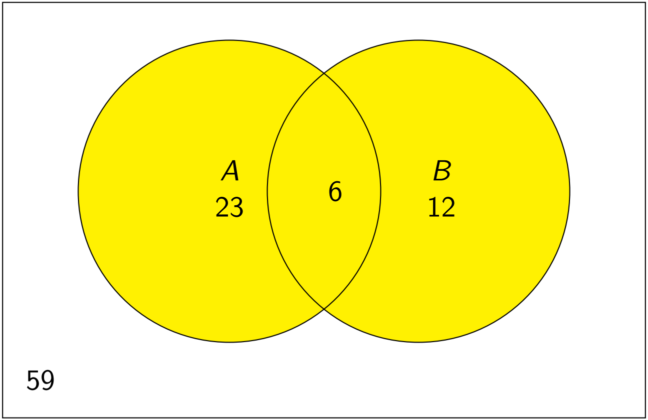
Quick review of probability

$$\mathsf{P}(B) = \frac{12+6}{100} = 0.18$$



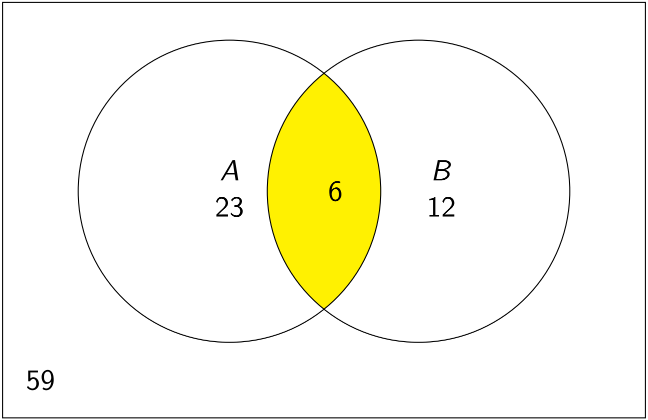
Quick review of probability

$$P(A \cup B) = \frac{23 + 6 + 12}{100} = 0.41$$



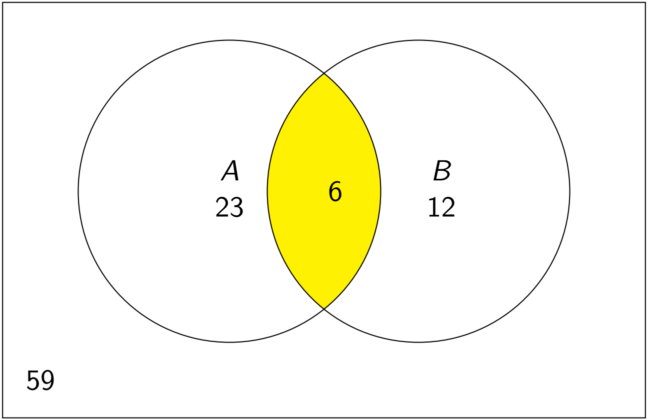
Quick review of probability

$$P(A \cap B) = \frac{6}{100} = 0.06$$



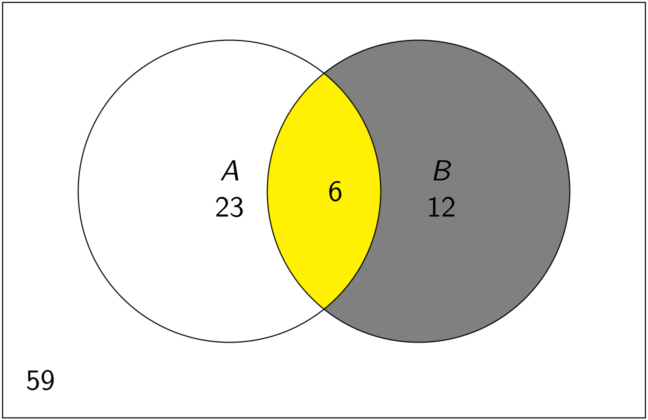
Quick review of probability

$$P(A \cap B) = \frac{6}{100} = 0.06$$



Quick review of conditional probability

$$\mathrm{P}(A|B) = \frac{6}{18} = 0.33 = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$$



Definition: Odds of D given E

Given a disease outcome, D, an exposure, E, and

	$D = 1$	$D = 0$	row total
$E = 1$	$n_{1,1}$	$n_{1,0}$	$n_{1,\bullet}$
$E = 0$	$n_{0,1}$	$n_{0,0}$	$n_{0,\bullet}$
col total	$n_{\bullet,1}$	$n_{\bullet,0}$	N

the odds of event D given exposure are calculated from

$$
\begin{aligned}
\mathrm{P}(D = 1 | E = 1) \\
&= \frac{\mathrm{P}(D = 1 \cap E = 1)}{\mathrm{P}(E = 1)} \\
&= \frac{n_{1,1}}{\cancel{N}} \Big/ \frac{n_{1,\bullet}}{\cancel{N}} \\
&= \frac{n_{1,1}}{n_{1,\bullet}}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(D = 0 | E = 1) \\
&= \frac{\mathrm{P}(D = 0 \cap E = 1)}{\mathrm{P}(E = 1)} \\
&= \frac{n_{1,0}}{\cancel{N}} \Big/ \frac{n_{1,\bullet}}{\cancel{N}} \\
&= \frac{n_{1,0}}{n_{1,\bullet}}
\end{aligned}
$$

Definition: Odds of D given E

Given a disease outcome, D, an exposure, E, and

	$D=1$	$D=0$	row total
$E=1$	$n_{1,1}$	$n_{1,0}$	$n_{1,\bullet}$
$E=0$	$n_{0,1}$	$n_{0,0}$	$n_{0,\bullet}$
col total	$n_{\bullet,1}$	$n_{\bullet,0}$	N

the odds of event D given exposure are

$$\begin{aligned} odds(D=1|E=1) &= \frac{\mathrm{P}(D=1|E=1)}{\mathrm{P}(D=0|E=1)} \\ &= \frac{n_{1,1}}{\cancel{n_{1,\bullet}}} \bigg/ \frac{n_{1,0}}{\cancel{n_{1,\bullet}}} \\ &= \frac{n_{1,1}}{n_{1,0}} \end{aligned}$$

Definition: Odds of D given $!E$

Given a disease outcome, D, an exposure, E, and

	$D = 1$	$D = 0$	row total
$E = 1$	$n_{1,1}$	$n_{1,0}$	$n_{1,\bullet}$
$E = 0$	$n_{0,1}$	$n_{0,0}$	$n_{0,\bullet}$
col total	$n_{\bullet,1}$	$n_{\bullet,0}$	N

the odds of event D given *no* exposure are calculated from

$$
\begin{aligned}
\mathrm{P}(D = 1|E = 0) & \\
&= \frac{\mathrm{P}(D = 1 \cap E = 0)}{\mathrm{P}(E = 0)} \\
&= \frac{n_{0,1}}{\cancel{N}} \bigg/ \frac{n_{0,\bullet}}{\cancel{N}} \\
&= \frac{n_{0,1}}{n_{0,\bullet}}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(D = 0|E = 0) & \\
&= \frac{\mathrm{P}(D = 0 \cap E = 0)}{\mathrm{P}(E = 0)} \\
&= \frac{n_{0,0}}{\cancel{N}} \bigg/ \frac{n_{0,\bullet}}{\cancel{N}} \\
&= \frac{n_{0,0}}{n_{0,\bullet}}
\end{aligned}
$$

Definition: Odds of D given $!E$

Given a disease outcome, D, an exposure, E, and

	$D = 1$	$D = 0$	row total
$E = 1$	$n_{1,1}$	$n_{1,0}$	$n_{1,\bullet}$
$E = 0$	$n_{0,1}$	$n_{0,0}$	$n_{0,\bullet}$
col total	$n_{\bullet,1}$	$n_{\bullet,0}$	N

the odds of event D given *no* exposure are

$$\begin{aligned} odds(D = 1 | E = 0) &= \frac{\mathrm{P}(D = 1 | E = 0)}{\mathrm{P}(D = 0 | E = 0)} \\ &= \frac{n_{0,1}}{\cancel{n_{0,\bullet}}} \Big/ \frac{n_{0,0}}{\cancel{n_{0,\bullet}}} \\ &= \frac{n_{0,1}}{n_{0,0}} \end{aligned}$$

Definition: Odds Ratio

Given a disease outcome, D, an exposure, E, and

	$D = 1$	$D = 0$	row total
$E = 1$	$n_{1,1}$	$n_{1,0}$	$n_{1,\bullet}$
$E = 0$	$n_{0,1}$	$n_{0,0}$	$n_{0,\bullet}$
col total	$n_{\bullet,1}$	$n_{\bullet,0}$	N

the ratio of the odds of disease comparing exposure vs no exposure is

$$
\begin{aligned}
OR(D|E) &= \frac{odds(D = 1|E = 1)}{odds(D = 1|E = 0)} \\
&= \left. \frac{n_{1,1}}{n_{1,0}} \middle/ \frac{n_{0,1}}{n_{0,0}} \right. \\
&= \frac{n_{1,1}\,n_{0,0}}{n_{1,0}\,n_{0,1}}.
\end{aligned}
$$

Definition: $se(\log OR)$

Given a disease outcome, D, an exposure, E, and

	$D = 1$	$D = 0$	row total
$E = 1$	$n_{1,1}$	$n_{1,0}$	$n_{1,\bullet}$
$E = 0$	$n_{0,1}$	$n_{0,0}$	$n_{0,\bullet}$
col total	$n_{\bullet,1}$	$n_{\bullet,0}$	N

the log OR is normally distributed, and its standard error is

$$se(\log OR) = \sqrt{\frac{1}{n_{1,1}} + \frac{1}{n_{1,0}} + \frac{1}{n_{0,1}} + \frac{1}{n_{0,0}}}.$$

Putting everything together

Given a disease outcome, D, and exposure, E, and

	$D=1$	$D=0$	row total
$E=1$	36	54	*90*
$E=0$	32	89	*121*
col total	*68*	*143*	*211*

what is the OR and 95% CI for the exposure, E? Is this statistically significant?

$$
\begin{aligned}
OR(D|E) &= \frac{36*89}{32*54} \\
&= 1.85 \\
\log OR &= 0.617 \\
se(\log OR) &= \sqrt{\frac{1}{36}+\frac{1}{54}+\frac{1}{32}+\frac{1}{89}} \\
&= 0.298
\end{aligned}
$$

Putting everything together

Given a disease outcome, D, and exposure, E, and

	$D=1$	$D=0$	row total
$E=1$	36	54	*90*
$E=0$	32	89	*121*
col total	*68*	*143*	*211*

what is the OR and 95% CI for the exposure, E? Is this statistically significant?

$$
\begin{aligned}
OR &= 1.85 \\
95\%\ CI(\log OR) &= 0.617 \pm 1.96 * 0.298 \\
&= (0.031, 1.199) \\
95\%\ CI(OR) &= (1.03, 3.32)
\end{aligned}
$$