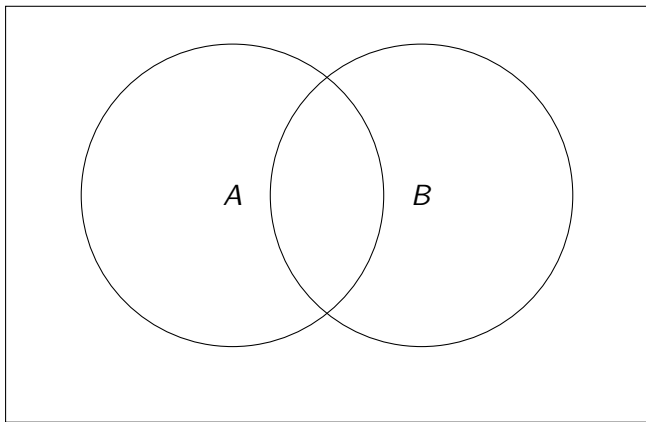# Odds Ratios

Randy Johnson

3/9/2017
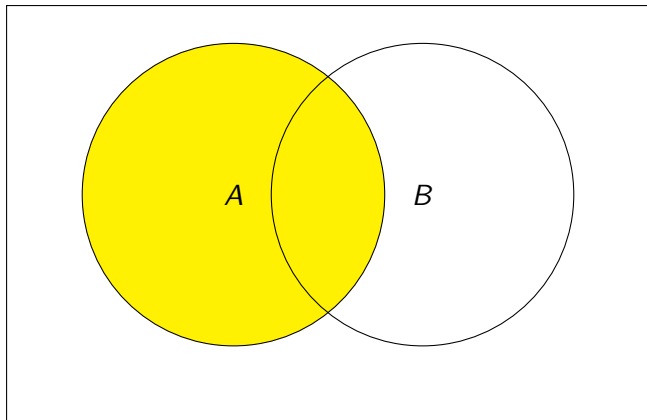
Quick review of probability

This Venn diagram represents the event space for two events, A and B. The area inside of the rectangle is 1.
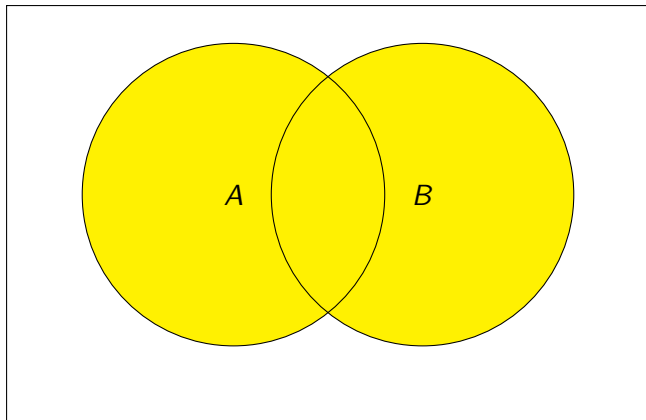
Quick review of probability
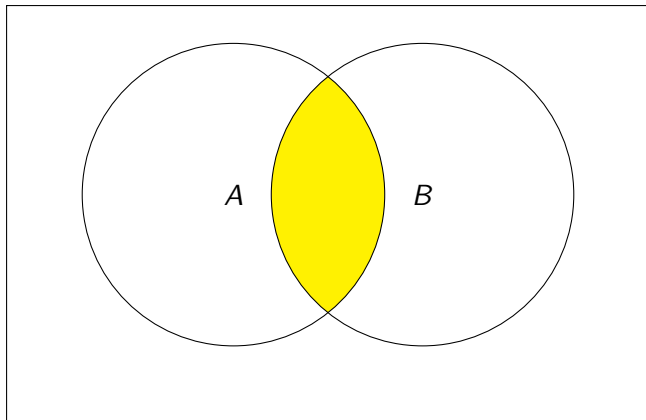
This represents P(A).

Quick review of probability: Union

This represents P($A \cup B$).

Quick review of probability: Intersection

This represents P($A \cap B$).

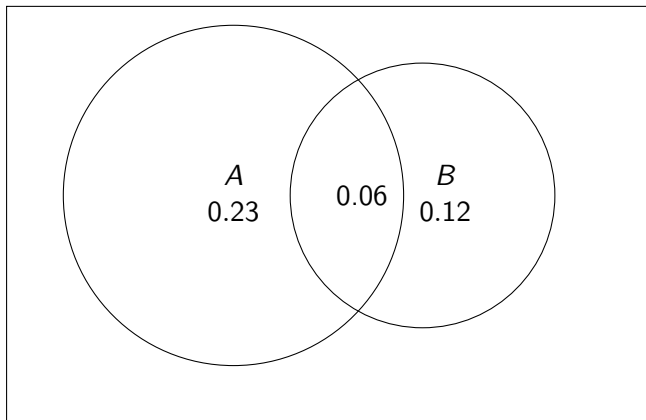# Quick review of probability: Practice

$$P\left(!\left(A \cup B\right)\right) = \qquad\qquad P(A \cap B) =$$
$$P(A) = \qquad\qquad P(A|B) =$$
$$P(A \cap !B) = \qquad\qquad P(A|!B) =$$

A
0.23   0.06   B
0.12

Definition: Odds

Given a disease outcome, D, an exposure, E, and

	$D = 1$	$D = 0$	row total
$E = 1$	$n_{1,1}$	$n_{1,0}$	$n_{1,\bullet}$
$E = 0$	$n_{0,1}$	$n_{0,0}$	$n_{0,\bullet}$
col total	$n_{\bullet,1}$	$n_{\bullet,0}$	N

the odds of event D given exposure / no exposure are

$$odds(D = 1|E = 1) = \frac{P(D = 1|E = 1)}{P(D = 0|E = 1)}$$
$$= \frac{n_{1,1}}{n_{1,\bullet}} \bigg/ \frac{n_{1,0}}{n_{1,\bullet}}$$
$$= \frac{n_{1,1}}{n_{1,0}}$$

$$odds(D = 1|E = 0) = \frac{P(D = 1|E = 0)}{P(D = 0|E = 0)}$$
$$= \frac{n_{0,1}}{n_{0,\bullet}} \bigg/ \frac{n_{0,0}}{n_{0,\bullet}}$$
$$= \frac{n_{0,1}}{n_{0,0}}$$

Definition: Odds Ratio

Given a disease outcome, D, an exposure, E, and

	$D = 1$	$D = 0$	row total
$E = 1$	$n_{1,1}$	$n_{1,0}$	$n_{1,\bullet}$
$E = 0$	$n_{0,1}$	$n_{0,0}$	$n_{0,\bullet}$
col total	$n_{\bullet,1}$	$n_{\bullet,0}$	N

the ratio of the odds of disease comparing exposure vs no exposure is

$$
OR(D|E) = \frac{odds(D = 1|E = 1)}{odds(D = 1|E = 0)}
$$
$$
= \frac{n_{1,1}}{n_{1,0}} \bigg/ \frac{n_{0,1}}{n_{0,0}}
$$
$$
= \frac{n_{1,1} n_{0,0}}{n_{1,0} n_{0,1}}.
$$

Definition: se(log OR)

Given a disease outcome, D, an exposure, E, and

	$D = 1$	$D = 0$	row total
$E = 1$	$n_{1,1}$	$n_{1,0}$	$n_{1,\bullet}$
$E = 0$	$n_{0,1}$	$n_{0,0}$	$n_{0,\bullet}$
col total	$n_{\bullet,1}$	$n_{\bullet,0}$	N

the log OR is normally distributed, and its standard error is

$$se(\log OR) = \sqrt{\frac{1}{n_{1,1}} + \frac{1}{n_{1,0}} + \frac{1}{n_{0,1}} + \frac{1}{n_{0,0}}}.$$

Putting everything together

Given a disease outcome, D, and exposure, E, and

	$D = 1$	$D = 0$	row total
$E = 1$	62	45	
$E = 0$	23	89	
col total			

what is the OR and 95% CI for the exposre, E? Is this statistically significant?