ECN 280 | Fall 2022

Isocosts and the Cost Min Problem

Professor McGavock *Module 5.3*

Contents

1 Learning Outcomes for Module 5.3

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After completing this module, you will be able to:

- *Calculate* and *describe* the slope of the isocost line.
- *Describe* and solve for the **tangency condition** that defines optimal cost-minimizing production (if the solution is an interior one).

2 Assignments for Module 5.3

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- Watch:
  - 5B Isocosts and the Tangency Condition
- Read:
  - No additional reading (see 6.12-6.15)
- Turn in: [Turn in problems here.]
  - 5.3.1

3 Lecture Notes

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3.1 Setting up the Cost Minimization Problem

3.1.1 Assumptions

- There are two inputs to (factors of) production: labor (L) and capital (K).
- All inputs are hired in perfectly competitive input markets, meaning that the firm takes the prices of inputs as given.
- The cost of a unit (hour used in production) of labor is the wage of laborers, which is the same for all. We'll use w (for wage) to denote this.
- The cost of a unit (hour used in production) of capital is the rental rate of capital (how much you would get from renting the capital out for one hour instead of using it for your own production). We'll use r (for rental rate) to denote this.
- The firm also takes its technology as given, meaning that it has production function $y = f(K, L)$.

Given the above assumptions, the total costs of the firm if L and K units of the inputs are used becomes:

$$C = wL + rK$$

3.1.2 Where does the firm minimize costs?

- Goal of the firm: produce some quantity at the lowest possible cost. This means they want to choose the combination of inputs (K, L) that minimizes the cost of producing some quantity $y = \bar{y}$. This can be represented mathematically by the following:

$$\begin{aligned} \underset{(K,L)}{\text{minimize}} \quad & wL + rK \\ \text{subject to} \quad & f(K, L) = \bar{y} \end{aligned}$$

- This should look and feel familiar: we talked about maximizing utility subject to a budget constraint. This instead minimizes cost subject to a quantity constraint, the requirement that quantity be $\bar{y}$. What did we do last time? Graphed the functions and found the point of tangency! Same thing here...

3.1.3 Isocost lines

Note that the cost function above can be rearranged so that it more clearly shows the equation of a line in LK-space, which will make it fit on our graph of the isoquant which shows production for given levels of inputs of K and L.

$$rK = C - wL \rightarrow K = \frac{C}{r} - \frac{w}{r}L$$

For fixed input prices (the firm is a price taker in the K and L markets), this means that the slope of the isocost line is **constant**. The market will always let you trade in workers for capital at a rate of $\frac{w}{r}$ because that ratio tells you how valuable a worker is in terms of capital (in the market).

3.2 Solving the Cost Minimization Problem

3.2.1 Method 1: Tangency Condition

Given: all of the above. The firm wants to produce some fixed quantity $\bar{y}$ of output.

Step 1: We know the optimal mix of inputs will happen where the isocost and isoquant curves are tangent to one another. This means:

$$\begin{aligned}\text{Slope of Isoquant} &= \text{Slope of Isocost} \\ -MRTS &= -\frac{w}{r} \\ \rightarrow \frac{MP_L}{MP_K} &= \frac{w}{r}\end{aligned}$$

Step 2: Solve the tangency condition for K (or for L) by rearranging this condition. It will give you an equation that says $K = g(L)$ where $g()$ is just some function. This relates K and L when they are at the optimal production mix.

Step 3: You solved for one of the inputs in terms of the other. Suppose you solved for K. Now plug that into the production function instead of K. Then, you will have $\bar{y}$ and L in the production function. Rearrange and solve for L^* which is the **optimal input of L given some quantity $\bar{y}$**.

Step 4: Plug your expression for L^* into the equation you found in Step 2, NOT back into the production function. This will give you an expression for K^* too! These are called the *conditional input demand functions.*

Step 5: To find the *cost function*, simply plug your conditional input demands into the cost equation. This will give you the minimum cost of producing quantity $\bar{y}$.

4 Problems

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***5.3.1 *Cost Minimization*

Suppose a firm's production function is $y = 10K^{\frac{1}{2}}L^{\frac{1}{2}}$. The cost of a unit of labor is \$20 and the cost of a unit of capital is \$80. They have no other inputs or costs.

(a) The firm is currently producing 100 units of output and has determined that the cost-minimizing quantities of labor and catpial are 20 and 5, respectively. Graphically illustrate this using isocost lines and isoquants and write a description of what your graph is showing. Show at least two isocost lines as you explain what the firm has chosen.

(b) The firm now wants to increase production to 140 units. If capital is fixed in the short run, how much labor will the firm require? On a graph showing labor and ouptut (y), show this as movement along the firm's production function (remember capital is fixed, so the production function is only in terms of labor here).

(c) What is the change in the firm's total cost?

(d) On your graph from part a), graphically show the optimal level of capital and labor in the *long run* if the firm wants to produce 140 units of output.

(e) Use Method 1 (the tangency condition) to solve for numerical values of the optimal quantities of capital and labor for this level of production.