

Friday practicum: stochastic gravitational wave backgrounds

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I. INTRODUCTION

In today's lectures, you have explored Testing General Relativity with GW observations, [Michele add a blurb about your lectures], and how to detect low frequency GWs using Pulsar Timing Arrays. The following exercises are meant to cement these concepts. These come in at least 2 flavors: general exercises, and advanced ones for those who may already be familiar with these concepts and are up for a challenge!

II. EXERCISES

Do as many of the following exercises as time allows. If you get stuck, please ask a Lecturer or a teaching assistant for help.

A. Pulsar Timing Arrays

Pulsar timing arrays are sensitive to GWs in the nanoHertz frequency regime. Sources here include supermassive black hole binaries, primordial (or relic) GWs from inflation, and cosmic strings. The goal of this exercise is to derive the overlap reduction function for an isotropic stochastic GW background. This is often referred to as the Hellings and Downs curve [1].

1. For a pair of pulsars a and b , define a reference frame. Specifically, write down the unit vectors which point to the pulsars, \hat{p}_a, \hat{p}_b , the direction of GW propagation $\hat{\Omega}$ and the principal axes \hat{m}, \hat{n} , such that $\hat{m} \times \hat{n} = \hat{\Omega}$. *Hint, place pulsar a on the z -axis and b in the x - z plane*

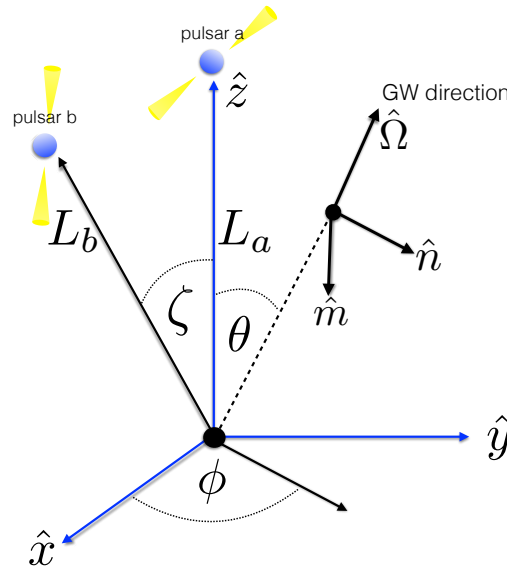


FIG. 1: Suggested geometry: pulsar a is on the z -axis at a distance L_a from the origin (solar system barycentre), pulsar b is in the x - z plane at a distance L_b from the origin making an angle ζ with pulsar a . $\hat{\Omega}$ is the direction of GW propagation with principal axes \hat{m} and \hat{n} such that $\hat{m} \times \hat{n} = \hat{\Omega}$. The polar and azimuthal angles are given by θ and ϕ , respectively.

2. Write down the general expression for the antenna beam pattern,

$$F^A(\hat{\Omega}) = \frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1 + \hat{\Omega} \cdot \hat{p}} e_{ij}^A(\hat{\Omega}), \quad (1)$$

where $A = +, \times$ is the GW polarization. The polarization tensors $e_{ij}^+(\hat{\Omega}) = \hat{m}_i \hat{m}_j - \hat{n}_i \hat{n}_j$, $e_{ij}^\times(\hat{\Omega}) = \hat{m}_i \hat{n}_j + \hat{n}_i \hat{m}_j$. For further reading, see [2, 3].

3. Give expressions for F_a^+ , F_a^\times , F_b^+ , F_b^\times in your chosen coordinate frame
4. The overlap reduction function is given by integrating the antenna beam patterns over the sky, i.e. over all possible GW directions $\hat{\Omega}$:

$$^{(ab)}\Gamma(\zeta) = \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi \sum_A F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega}), \quad (2)$$

where $\zeta \in [0, \pi]$ is the angle between the pulsars. Using your expressions for the antenna beam pattern, you may either integrate the above expression analytically (hint, use contour integration) or open the ipython notebook and do this numerically. Plot the result for all ζ . The result is the Hellings and Downs curve!

5. Which of these steps implicitly assumes an isotropic background? Where would we modify this calculation to take in to account anisotropy in the GW background? More on this in [4].

B. Testing GR

Having built the above tools, Let us move on to explore testing GR in the context of GW background in PTAs. We will do this in two steps: first, we shall compute the overlap reduction function in more general theories of gravity.

6. Suppose GW has a scalar component, i.e., for a gravitational wave propagating along the $\hat{\Omega}$ direction, we have

$$e_{ij}^o(\Omega) = \hat{m}_i \hat{m}_j + \hat{n}_i \hat{n}_j. \quad (3)$$

Write down the antenna beam pattern and compute the overlap reduction function.

7. Suppose, *instead*, that the graviton is actually massive, and that the GW has a dispersion relation

$$\omega^2 = m_g^2 + k^2. \quad (4)$$

In other words, a single-frequency plane GW is given by metric perturbations of

$$h_{ij}(t, \mathbf{x}) = e^{-i\omega t + ik\hat{\Omega} \cdot \mathbf{x}} h_{+} \quad (5)$$

Write down the antenna beam pattern and the overlap reduction function.

As a second step, let us explore how to distinguish between a standard GR background and a non-GR background.

8. Can we distinguish a general anisotropic GR background and a scalar background? What about a massive gravity background?

[1] R. W. Hellings and G. S. Downs, *Astrophysical Journal, Letters* **265**, L39 (1983).
 [2] B. Allen and J. D. Romano, *Phys. Rev. D* **59**, 102001 (1999), gr-qc/9710117.
 [3] M. Anholm, S. Ballmer, J. D. E. Creighton, L. R. Price, and X. Siemens, *Phys. Rev. D* **79**, 084030 (2009), 0809.0701.
 [4] C. M. F. Mingarelli, T. Sidery, I. Mandel, and A. Vecchio, *Phys. Rev. D* **88**, 062005 (2013), 1306.5394.