

Overdetermined System

$$5x_1 + 2x_2 + 3x_3 = 7$$

$$2x_1 + 7x_2 + 8x_3 = 5$$

$$3x_1 + 9x_2 + 2x_3 = 6$$

→ From here we can see

the number of equations = number of
unknown
variables

We can solve this using gaussian Elimination,
LU Decomposition and inverse matrix. However,
we add another equation with the previous
set of equations. We will get.

coefficient
matrix
from
this

$$\begin{bmatrix} 5x_1 + 2x_2 + 3x_3 = 7 \\ 2x_1 + 7x_2 + 8x_3 = 5 \\ 3x_1 + 9x_2 + 2x_3 = 6 \\ 4x_1 + 2x_2 + 5x_3 = 10 \end{bmatrix}$$

We add this
new equation

$$A = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 7 & 8 \\ 3 & 9 & 2 \\ 4 & 2 & 5 \end{bmatrix}$$

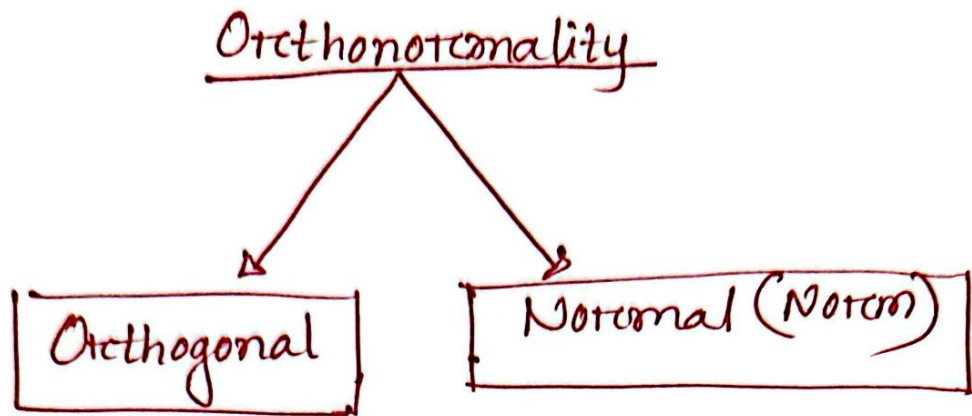
$m \times n$
matrix $m > n$

This is
a 4×3
matrix.
It is not a
square matrix

We cannot directly solve this using Gaussian Elimination, LU Decomposition and Inverse matrix because it is not a square matrix.

The number of equation \neq The number of unknown variables.

This is known as overdetermined system where the number of equations are greater than the number of variables. (which means the matrix is not the square matrix).



orthogonality:

$$x^T y = 0$$

Normality (Norm)

$$x^T x = 1$$

$$y^T y = 1$$

Example: check if the following set is

orthogonal.

$$S = \left\{ \underbrace{\frac{1}{\sqrt{5}} (2, 1)^T}_{u^T}, \underbrace{\frac{1}{\sqrt{5}} (1, -2)^T}_{v^T} \right\}$$

* Since the question asks us to check for

orthonormality, we need to verify both orthogonality and normality.

First we check normal :

$$u^T u = 1 \quad , \quad v^T v = 1$$

$$\begin{aligned} u^T u &= \frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 \end{bmatrix} \times \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \\ &= \frac{1}{5} \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \end{aligned}$$

$$= 5/5 = 1$$

$$\boxed{u^T u = 1} \rightarrow \text{True}$$

$$\begin{aligned} v^T v &= \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & -2 \end{bmatrix} \times \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ -2 \end{bmatrix} \\ &= \frac{1}{5} \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} \end{aligned}$$

$$= \frac{1}{5} \times 5 = 1$$

$$\boxed{v^T v = 1} \rightarrow \text{True} .$$

Now we check orthogonality:

$$\boxed{U^T v = 0}$$

$$\begin{aligned} U^T v &= \frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 \end{bmatrix} \times \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ -2 \end{bmatrix} \\ &= \frac{1}{5} (2 - 2) \\ &= \frac{1}{5} \times 0 = 0 \end{aligned}$$

$$\boxed{U^T v = 0} \rightarrow \text{True}$$

Since both normality and orthogonality has been proved so the matrix has orthonormal properties.

Important

* if we have 3 matrices

x , y and z

For normal property we need to prove

$$\begin{aligned} x^T x &= 1 \\ y^T y &= 1 \\ z^T z &= 1 \end{aligned}$$

For orthogonal property we need to prove

$$\begin{aligned} x^T y &= 0 \\ y^T z &= 0 \\ z^T x &= 0 \end{aligned}$$

Kronecker Delta

$$\delta_{ij} = \begin{cases} 1 & \begin{matrix} u & v \\ i & = & j \end{matrix} & U^T U \begin{cases} \text{normal /} \\ \text{norm /} \\ \text{normality} \end{cases} \\ 0 & \begin{matrix} u & v \\ i & \neq j \end{matrix} & U^T V \begin{cases} \text{orthogonality} \end{cases} \end{cases}$$