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CSE 330 Numerical Methods

SUMMER 2022

Quiz 3 - *Makeup Quiz*

Total Marks-10

Time: 20 mins

ANSWER ALL THE QUESTIONS

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1. Consider the following system of linear equation:

$$\begin{aligned}
 x_1 + x_2 + x_3 &= 1 \\
 4x_1 + 25x_2 - 3x_3 &= 6 \\
 7x_1 + 4x_2 + 9x_3 &= 17
 \end{aligned}
 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 4 & 25 & -3 \\ 7 & 4 & 9 \end{bmatrix} = A^{(1)}$$

Solve the matrix using LU Decomposition [10 marks]

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 25 & -3 \\ 7 & 4 & 9 \end{bmatrix} \xrightarrow{\substack{R_2 = R_2 - (4)R_1 \\ R_3 = R_3 - (7)R_1}} \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix} = F^{(1)}$$

$$A^{(2)} = F^{(1)} \times A^{(1)}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 4 & 25 & -3 \\ 7 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 21 & -7 \\ 0 & -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 21 & -7 \\ 0 & -3 & 2 \end{bmatrix} \xrightarrow{R_3 = R_3 - (-3/21)R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/7 & 1 \end{bmatrix} = F^{(2)}$$

$$A^{(3)} = F^{(2)} \times A^{(2)}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/7 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 0 & 21 & -7 \\ 0 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 21 & -7 \\ 0 & 0 & 1 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 1/7 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & -1/7 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 17 \end{bmatrix}$$

$$\boxed{a_1 = 1} \quad \checkmark$$

$$4a_1 + a_2 = 6$$

$$\Rightarrow 4 + a_2 = 6$$

$$\boxed{\therefore a_2 = 2} \quad \checkmark$$

$$7a_1 - \frac{1}{7}a_2 + a_3 = 17$$

$$\Rightarrow 7 - \left(\frac{1}{7} \times 2\right) + a_3 = 17$$

$$\therefore a_3 =$$

$$\Rightarrow 7 - 2/7 + a_3 = 17$$

$$\therefore a_3 = 72/7$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 21 & -7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 72/7 \end{bmatrix}$$

$$\boxed{x_3 = \frac{72}{7}}$$

$$21x_2 - 7x_3 = 2$$

$$\Rightarrow 21x_2 - 7 \times \frac{72}{7} = 2$$

$$\therefore x_2 =$$

$$\Rightarrow 21x_2 - 72 = 2$$

$$\boxed{\therefore x_2 = \frac{74}{21}}$$

$$x_1 + \frac{74}{21} + \frac{72}{7} = 1$$

$$\therefore x_1 = -\frac{269}{21}$$