# Lagrange Form

Previously we learn about,

$$P_n(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \dots + a_n x^n$$

Now using lagrange basis:

[Given in the question]

$$P_n(x) = \boxed{l_0(x)} f(x_0) + \boxed{l_1(x)} f(x_1) + \boxed{l_2(x)} f(x_2) + \dots + \boxed{l_n(x)} f(x_n)$$

We need to calculate these

Calculation:

$$l_0(x) = \frac{(x-x_1)}{(x_0-x_1)} \times \frac{(x-x_2)}{(x_0-x_2)} \times \dots \times \frac{(x-x_n)}{(x_0-x_n)}$$

$$l_1(x) = \frac{(x-x_0)}{(x_1-x_0)} \times \frac{(x-x_2)}{(x_1-x_2)} \times \dots \times \frac{(x-x_n)}{(x_1-x_n)}$$

$$l_2(x) = \frac{(x-x_0)}{(x_2-x_0)} \times \frac{(x-x_1)}{(x_2-x_1)} \times \dots \times \frac{(x-x_n)}{(x_2-x_n)}$$

## Example 1:

| $x$ | $f(x)$ |
|---|---|
| $x_0 = 2$ | $30 = f(x_0)$ |
| $x_1 = 5$ | $45 = f(x_1)$ |
| $x_2 = 7$ | $25 = f(x_2)$ |

$$P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2) \quad \dots \text{(i)}$$

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$$l_0(x) = \frac{x-x_1}{x_0-x_1} \times \frac{x-x_2}{x_0-x_2}$$

$$= \frac{(x-5)}{(2-5)} \times \frac{x-7}{(2-7)}$$

$$= \frac{(x-5)\times(x-7)}{15}$$

$$l_1(x) = \frac{x-x_0}{x_1-x_0} \times \frac{x-x_2}{x_1-x_2}$$

$$= \frac{x-2}{5-2} \times \frac{x-7}{5-7}$$

$$= \frac{(x-2)(x-7)}{-6}$$

$$l_2(x) = \frac{(x-x_0)}{(x_2-x_0)} \times \frac{(x-x_1)}{(x_2-x_1)}$$

$$= \frac{(x-2)}{7-2} \times \frac{(x-5)}{7-5}$$

$$= \frac{(x-2)(x-5)}{10}$$

Put all the values in equation (i)

$$P_2(x) = l_0(x)f(x_0) + l_1(x)\,f(x_1) + l_2(x)\,f(x_2)$$

$$= \boxed{\frac{(x-5)(x-7)}{15}} \times \boxed{30} + \boxed{\left(-\frac{(x-2)(x-7)}{6}\right)} \times \boxed{45} + \boxed{\frac{(x-2)(x-5)}{10}} \times 25$$

Example 2:

Give nodes 3. Degree will be (nodes−1) = (3−1) = 2

| Time $(x)$ | Velocity $f(x)$ |
|---|---|
| $x_0 = 15$ | $227.04 = f(x_0)$ |
| $x_1 = 20$ | $362.78 = f(x_1)$ |
| $x_2 = 22.5$ | $517.35 = f(x_2)$ |

$$P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$$

$$l_0(x) = \frac{x-x_1}{x_0-x_1} \times \frac{x-x_2}{x_0-x_2} = \frac{(x-20)}{15-20} \times \frac{(x-22.5)}{15-22.5} = \frac{2(x-20)(x-22.5)}{75}$$

$$l_1(x) = \frac{x-x_0}{x_1-x_0} \times \frac{x-x_2}{x_1-x_2} = \frac{(x-15)}{20-15} \times \frac{(x-22.5)}{20-22.5} = -\frac{2(x-15)(x-22.5)}{25}$$

$$l_2(x) = \frac{x-x_0}{x_2-x_0} \times \frac{x-x_1}{x_2-x_1} = \frac{(x-15)}{22.5-15} \times \frac{(x-20)}{22.5-20} = \frac{4(x-15)(x-20)}{75}$$

$$P_2(x) = \frac{2}{75}(x-20)(x-22.5) \times 227.04 - \frac{2}{25}(x-15)(x-22.5) \times 362.78 + \frac{4}{75}(x-15)(x-20) \times 517.35 .$$

What will be the value of $P_2(17)$?

$$P_2(17) = \frac{2}{75}(17-20)(17-22.5) \times 227.04 - \frac{2}{25}(17-15)(17-22.5) \times 362.78 + \frac{4}{75}(17-15)(17-20) \times 517.35$$

$$= 197.228$$

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## Example 3:

$$f(x) = \cos(x)$$

| $x$ | $f(x)$ |
|---|---|
| $x_0 = -\pi/4$ | $1/\sqrt{2}$ $f(x_0)$ |
| $x_1 = 0$ | $1$ $f(x_1)$ |
| $x_2 = \pi/4$ | $1/\sqrt{2}$ $f(x_2)$ |

$P_n(x) = ?$

$$P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} \times \frac{x - x_2}{x_0 - x_2} = \frac{x - 0}{-\pi/4 - 0} \times \frac{x - \pi/4}{-\pi/4 - \pi/4} = \frac{8x}{\pi^2}(x - \pi/4)$$

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} \times \frac{x - x_2}{x_1 - x_2} = \frac{x + \pi/4}{\pi/4} \times \frac{x - \pi/4}{-\pi/4} = \frac{-16}{\pi^2}(x + \pi/4)(x - \pi/4)$$

$$l_2(x) = \frac{x - x_0}{x_2 - x_0} \times \frac{x - x_1}{x_2 - x_1} = \frac{x + \pi/4}{\pi/4 + \pi/4} \times \frac{x}{\pi/4} = \frac{8x}{\pi^2}(x + \pi/4)$$

$$P_2(x) = \frac{8x}{\pi^2}(x - \pi/4)\frac{1}{\sqrt{2}} - \frac{16}{\pi^2}(x + \pi/4)(x - \pi/4) \times 1$$

$$+ \frac{8x}{\pi^2}(x + \pi/4)\frac{1}{\sqrt{2}}$$

$$= \frac{8x}{\pi^2}(x - \pi/4)\frac{1}{\sqrt{2}} + \frac{8x}{\pi^2}(x + \pi/4)\frac{1}{\sqrt{2}}$$

$$- \frac{16}{\pi^2}(x + \pi/4)(x - \pi/4)$$

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Another Example:

$x=19 \xrightarrow{19} 0$, $x=19 \xrightarrow{9} \boxed{10}$, $x=19 \xrightarrow{4} \boxed{15}$, $x=19 \xrightarrow{1} \boxed{20}$, $x=19 \xrightarrow{3.5} \boxed{22.5}$, $x=19 \xrightarrow{11} 30$

| Time, $x$ | Velocity $f(x)$ |
|---|---|
| 0 | 0 |
| $\boxed{10}$ | 227.4 |
| $\boxed{15}$ | 362.8 |
| $\boxed{20}$ | 517.35 |
| $\boxed{22.5}$ | 602.97 |
| 30 | 90.67 |

Here we have 6 node but we have to work with 4 nodes (those have less differance)

$P_3(19) = ?$

Now we work with 4 nodes, $x = 10, 15, 20, 22.5$

if there are 2 nodes with same differance, we will use any one.

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| $x$ | $f(x)$ |
|---|---|
| $x_0$ | $f(x_0)$ |
| $x_1$ | $f(x_1)$ |

nodes $= 2$

degree $= n-1$
$= 2-1 = 1$

$$P_1(x) = l_0(x) f(x_0) + l_1(x) f(x_1)$$

$$l_0(x) = \frac{x-x_1}{x_0-x_1} \longrightarrow \begin{cases} l_0(x_0) = 1 \\ l_0(x_1) = 0 \end{cases}$$

$$l_1(x) = \frac{x-x_0}{x_1-x_0} \longrightarrow \begin{cases} l_1(x_0) = 0 \\ l_1(x_1) = 1 \end{cases}$$

We can write it as:

$$l_0(x_i) \begin{cases} 0 & i=1 \\ 1 & i=0 \end{cases}$$

$$l_1(x_i) \begin{cases} 0 & i=0 \\ 1 & i=1 \end{cases}$$

$\therefore\ l_n(x_j) = \delta_{nj}$ → kroneckar delta

$$\boxed{\begin{matrix} \delta_{nj} = 0 & n \neq j \\ \delta_{nj} = 1 & n = j \end{matrix}}$$

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## Advantage

We donot need to inverse a matrix here like Vandermonde.

## Disadvantage

If we want to add new nodes, we need to do the whole calculation from the beginning.

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