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CSE 330 Numerical Methods

SUMMER 2022

Quiz 2

ANSWER ALL THE QUESTIONS Time: 20mins

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Section 07

[CO4] Consider the following data points

x_0 **f(-1)=2**

x_1 **f(1)= 3**

x_2 **f(2)=-2**

x_3 **f(3)=4**

a. Set a 2nd degree polynomial to create an over-determined system and show the coefficient matrix. [2]
b. Solve the overdetermined system with the help of Least Square Approximation method and Gaussian Elimination. [5]
c. Suppose you have a 70X70 coefficient matrix and you are applying Gaussian elimination row operations. How many elements of the matrix will be zero after 20 row operations. [3]

b. $P_2(x) = a_0 + a_1(x) + a_2(x)^2$

$\Rightarrow P_2(-1) = a_0 - a_1 + a_2 = 2$

$\Rightarrow P_2(1) = a_0 + a_1 + a_2 = 3$

$\Rightarrow P_2(2) = a_0 + 2a_1 + 4a_2 = -2$

$\Rightarrow P_2(3) = a_0 + 3a_1 + 9a_2 = 4$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$$

(b) $Ax = b$

$\Rightarrow A^T \cdot Ax = A^T b$

$\Rightarrow A^T \cdot A$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 2 & 3 \\ 1 & 1 & 4 & 9 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 15 \\ 5 & 15 & 35 \\ 15 & 35 & 99 \end{bmatrix} \Rightarrow A$$

$A^T b$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 2 & 3 \\ 1 & 1 & 4 & 9 \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 33 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 5 & 15 \\ 5 & 15 & 35 \\ 15 & 35 & 99 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 33 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 4 & 5 & 15 & 7 \\ 5 & 15 & 35 & 9 \\ 15 & 35 & 99 & 33 \end{array}\right] \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \quad \begin{matrix} R_2 = R_2 - (\frac{5}{4}) R_1 \\ R_3 = R_3 - (\frac{15}{4}) R_1 \end{matrix} \Rightarrow \left[\begin{array}{ccc|c} 4 & 5 & 15 & 7 \\ 0 & 35/4 & 65/4 & 1/4 \\ 0 & 65/4 & \frac{171}{4} & 27/4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 4 & 5 & 15 & 7 \\ 0 & 35/4 & 65/4 & 1/4 \\ 0 & 65/4 & 171/4 & 27/4 \end{array}\right] \quad R_3 = R_3 - \left(\frac{65/4}{35/4}\right) R_2$$

$$\left[\begin{array}{ccc|c} 4 & 5 & 15 & 7 \\ 0 & 35/4 & 65/4 & 1/4 \\ 0 & 0 & 98/7 & 44/7 \end{array}\right]$$

$$98/7 x_3 = 44/7$$

$$\boxed{x_3 = 1/2}$$

$$\frac{35}{4} x_2 + \frac{65}{4} x_3 = \frac{1}{4}$$

$$\Rightarrow \frac{35}{4} x_2 + \frac{65}{4}(1/2) = \frac{1}{4}$$

$$\boxed{\therefore x_2 = -9/10}$$

$$4x_1 + 5x_2 + 15x_3 = 7$$

$$\Rightarrow 4x_1 + 5(-9/10) + 15(1/2) = 7$$

$$\boxed{\therefore x_1 = 1}$$

(c)

~~69 + 68 = 137 sorus~~

1190 sorus.

20241037