

fixed Point Representation:

$$X = 1 (d_1 d_2 \dots d_{K-1} d_K \dots d_n)_{\beta} \cdot (s b_1 b_2 \dots b_{\eta})_{\beta} = 7$$

Where  $d_1, d_2, d_3 \in \{0, 1, 2, 3, \dots, \beta-1\}$

$\beta$  = Base

$$(12345)_{10} \Rightarrow \beta = 10 \text{ and } \eta = 5 \text{ digits}$$

Evaluating fixed point numbers in Base 10:  $0 \leq sb \leq 10$

$$(101.01)_2$$

$$\frac{1}{2} \quad 0 \quad 1 \quad \cdot \quad \frac{0}{-1} \quad \frac{1}{-2}$$

$$= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2}$$

$$= 4 + 0 + 1 + 0 + \frac{1}{4}$$

$$= 5 + \frac{1}{4}$$

$$= \frac{21}{4}$$

Evaluating fixed point numbers in Base 9:  $0 \leq sb \leq 8$

$$(34.7)_9$$

$$\frac{3}{1} \quad 4 \quad \cdot \quad \frac{7}{-1}$$

$$= 3 \times 9^1 + 4 \times 9^0 + 7 \times 9^{-1}$$

$$= 27 + 4 + \frac{7}{9}$$

$$= \frac{286}{9}$$

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## Floating Point Representation:

$$F = \left\{ \pm \left( 0. d_1 d_2 \dots d_n \right) \beta^e \right\}_b \quad \text{where } d_1 \neq 0$$

$\beta$  is base  
 $e$  is exponent  
 $d_1, d_2, \dots, d_n$  are digits  
 $b$  is base

Here,  $\beta = \text{Base}$

$e = \text{exponent}$

$0 \leq d_i \leq \beta - 1$  and in machine floating point format exponent is stored in binary form

$\beta, d_i, e \in \mathbb{Z}$  (integers)

$e_{\min} \leq e \leq e_{\max}$

Example:

$$(123.45)_{10}$$

$$\frac{1}{2} \quad 0 \quad \cdot \quad \frac{1}{0} \quad 0 \quad 1$$

$$\begin{aligned}
 &= (12.345 \times 10) + 1 \cdot 5 \times 0 + 1 \cdot 5 \times 1 + 1 \cdot 5 \times 0 + 1 \cdot 5 \times 1 = \\
 &= 1.2345 \times 10^3 \\
 &= 0.12345 \times 10^4 \quad \xrightarrow{\beta}
 \end{aligned}$$

Example:

$$(56.2)_7$$

$$\begin{aligned}
 &= 5.62 \times 7^1 \quad \text{and in machine floating point format exponent is stored in binary form} \\
 &= 0.562 \times 7^2 \quad e(F.P.E)
 \end{aligned}$$

Highest Possible Floating Point:

$$\begin{aligned}
 \text{When, } \beta &= 2 & e_{\min} &= -1 \\
 n &= 3 & e_{\max} &= 2
 \end{aligned}$$

$$(0.111)_2 \times 2^2$$

$$\frac{F}{1} \quad \cdot \quad \frac{1}{0} \quad 1$$

$$\begin{aligned}
 &= 0 \times F + 0 \times P + 1 \times E = \\
 &= 0 \times F + 1 \times P + 1 \times E = \\
 &= 1 \times E = 1
 \end{aligned}$$

Denormalization form

$$\pm (0.1 d_1 d_2 \dots d_n) \beta^e$$

$$\beta = 2$$

$$n = 2$$

$$e_{\min} = -1$$

$$e_{\max} = 2$$

$$\text{When } e = -1,$$

$$(0.100) \times 2^{-1} \text{ for no 1 digit after decimal point}$$

$$(0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3}) \times \frac{1}{2} = \frac{1}{2} = 0.5$$

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$(0.101) \times 2^{-1}$$

$$= (0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}) \times \frac{1}{2} =$$

$$= \left(\frac{1}{2} + \frac{1}{8}\right) \times \frac{1}{2}$$

$$= \frac{5}{16}$$

$$(0.110) \times 2^{-1}$$

$$= (0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3}) \times 2^{-1} =$$

$$= \left(\frac{1}{2} + \frac{1}{4}\right) \times \frac{1}{2}$$

$$= \frac{3}{8}$$

$$(0.111) \times 2^{-1}$$

$$= (0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}) \times \frac{1}{2}$$

$$= \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) \times \frac{1}{2}$$

$$= \frac{7}{16}$$

$$= 16 \text{ cases.}$$

0.1	$d_2$	$d_3$
fixed	↓	↓
0	0	
0	1	
1	0	
1	1	

total 4 possible cases here.

$e$  can be from

$$-1$$

$$0$$

$$1$$

$$2$$

4 possible cases

$$\text{total} = 4 \times 4$$

$$= 16 \text{ cases.}$$

Normalized form:

Dear visitors from all over the world

$$\pm (1, d_1 d_2 d_3 d_4 \dots) \cdot \beta^{\ell} \cdot s_{b, b} \cdot 1.0 \pm$$

Here,  $d_i$  can be 1 or not 1.

$s \times (001 \cdot 0)$

$$S = 9$$

$$\Sigma = \infty$$

$$f = \min \mathcal{D}$$

$\tilde{S} = \text{geom } D$

21  $\alpha$ (G)

$L \rightarrow \text{rest} \omega$

$$\text{Height possible floating Point} = (1.111)_2 \times 2^2$$

Smallest:  $b, c \in \mathbb{Z}$  (integer)

$$(0.100)_2 \times 2^{-1} \leq q_{\max} \quad \text{for } x(101.0)$$

$$= (0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3}) \times \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

Largest :

$(0.111)$ ,  $\times 2^L$

$$= (0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}) \times 10^0 (5 \times 0) =$$

$$= \left( \frac{1}{3} + \frac{1}{4} + \frac{1}{2} \right) \times 4$$

$$= \frac{7}{2} =$$

1-sx (111.0)

$$5 \times (8 - 5 \times 1 + 5 - 5 \times 1 + 1 - 5 \times 1 + 0 \times 0) =$$

$$72 \times (81 + 74 + 52) =$$

24

$$\beta \times \beta = \text{dot dot}$$

$$\cdot (xw) \cdot j_1 =$$

Example:

$$l_{\min} = -1$$

$$l_{\max} = 2$$

$$B = 2$$

$$n = 2$$

for  $l = -1$ ,  ${}^0Sx(101.0)$  = smallest non-negative number

1st smallest non-negative number =  $(0.100) \times 2^{-1}$

$${}^0Sx(011.0) = \text{smallest} \rightarrow (0 \times 2^0 + 1 \times 2^{-1}) \times \frac{1}{2} = \frac{1}{4}$$

2nd smallest non-negative number =  $(0.101) \times 2^{-1}$

$${}^0Sx(111.0) = \text{smallest} \rightarrow (0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-3}) \times \frac{1}{2} = \left(\frac{1}{2} + \frac{1}{8}\right) \times \frac{1}{2} = \frac{5}{16}$$

3rd smallest non-negative number =  $(0.110) \times 2^{-1}$

$${}^0Sx(101.0) = \text{smallest} \rightarrow (0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}) \times \frac{1}{2} = \left(\frac{1}{2} + \frac{1}{4}\right) \times \frac{1}{2} = \frac{3}{8}$$

4th smallest non-negative number =  $(0.111) \times 2^{-1}$

$${}^0Sx(111.0) = \text{smallest} \rightarrow (0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}) \times \frac{1}{2} = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) \times \frac{1}{2} = \frac{7}{16}$$

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When  $e=0$ 

$$1^{\text{st}} \text{ smallest non-negative number} = (0.100) \times 2^0 \\ = \frac{1}{2}$$

$$2^{\text{nd}} \text{ smallest non-negative number} = (0.101) \times 2^0$$

~~$1^{\text{st}} \times (001.0)$  = minimum with 1 trailing 0~~

$$3^{\text{rd}} \text{ smallest non-negative number} = (0.110) \times 2^0 \\ = \frac{3}{4}$$

$$4^{\text{th}} \text{ smallest non-negative number} = (0.111) \times 2^0$$

$$= \frac{7}{8}$$

$$\text{When } e=1 \times (1 + \frac{1}{2}) =$$

$$1^{\text{st}} \text{ smallest non-negative number} = (0.100) \times 2^1$$

~~$1^{\text{st}} \times (011.0)$  = minimum with 1 trailing 0~~

$$2^{\text{nd}} \text{ smallest non-negative number} = (0.101) \times 2^1 \\ = \frac{3}{4}$$

$$3^{\text{rd}} \text{ smallest non-negative number} = (0.110) \times 2^1$$

~~$1^{\text{st}} \times (110.0)$  = minimum with 1 trailing 0~~

$$4^{\text{th}} \text{ smallest non-negative number} = (0.111) \times 2^1$$

$$= \frac{7}{8}$$

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When  $\epsilon=0$ 

$$1^{\text{st}} \text{ smallest non-negative number} = (0.100) \times 2^0$$

$$= \frac{1}{2}$$

Exponents:  
 $1^- = \text{min}$   
 $1^0 = \text{num}$   
 $1^1 = 1$   
 $1^2 = \text{max}$

$$2^{\text{nd}} \text{ smallest non-negative number} = (0.101) \times 2^0$$

$1^0 \times (001.0) = \text{minimum with } 1^0 - \text{non trailing } 1$

$$3^{\text{rd}} \text{ smallest non-negative number} = (0.110) \times 2^0$$

$$= \frac{3}{4}$$

$$4^{\text{th}} \text{ smallest non-negative number} = (0.111) \times 2^0$$

$$= \frac{7}{8}$$

$$\text{When } \epsilon=1 \times (1^1 + 1^1) =$$

$$1^{\text{st}} \text{ smallest non-negative number} = (0.100) \times 2^1$$

$1^0 \times (011.0) = \text{minimum with } 1^0 - \text{non trailing } 1$

$$2^{\text{nd}} \text{ smallest non-negative number} = (0.101) \times 2^1$$

$$= \frac{5}{4}$$

$$3^{\text{rd}} \text{ smallest non-negative number} = (0.110) \times 2^1$$

$1^0 \times (111.0) = \text{minimum with } 1^0 - \text{non trailing } 1$

$$4^{\text{th}} \text{ smallest non-negative number} = (0.111) \times 2^1$$

$$= \frac{7}{4}$$

$$1^0 \times (1^1 + 1^1 + 1^1) =$$

$$1^0 =$$

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When,  $l = 2$ 

IEEE floating for double precision:

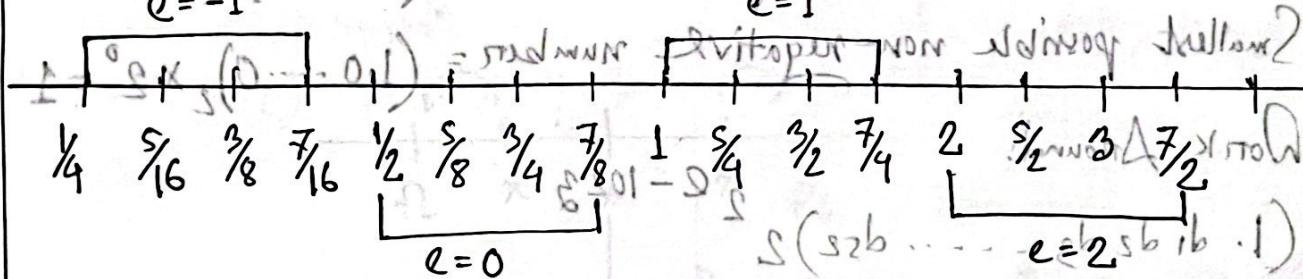
1st smallest non-negative number =  $(0.100) \times 2^2$   $l = 8$   
 fractions not  $\leq 2^{10} - 1$

2nd smallest non-negative number =  $(0.101) \times 2^2$   $l = 8$   
 fractions not  $\leq 2^{10} - 1$

3rd smallest non-negative number =  $(0.110) \times 2^2$   $l = 8$   
 fractions not  $\leq 2^{10} - 1$

4th smallest non-negative number =  $(0.111) \times 2^2$   $l = 8$   
 fractions not  $\leq 2^{10} - 1$

$S \times (1 \dots 11.1) =$  smallest floating non-negative numbers  $l = 8$



$S \times (1 \dots 11.1) =$  smallest floating non-negative numbers  $l = 8$

$0 = S \times (1 \dots 11.1) =$  smallest floating non-negative numbers  $l = 8$

$0 = S \times (0 \dots 001.0) =$  smallest floating non-negative numbers  $l = 8$

$S =$  fractions  $\leq 2^{10} - 1$

$S =$  fractions  $\leq 2^{10} - 1$

IEEE standard for double precision:

$B = 2$

$52$  bits for fraction

$11$  bits for exponent

$1$  bit for sign

Normalized form:

$$(1. d_1 d_2 d_3 \dots d_{52})_2 \times 2^e$$

min value of  $e = 0$  [Where all 11 bits are 0]

max value of  $e = 2^{11} - 1 = 2047$  [Where all 11 bits are 1]

$e_{\min} = 0$

$e_{\max} = 2047$

Largest possible non-negative number =  $(1.11\dots1)_2 \times 2^{2047}$

Smallest possible non-negative number =  $(1.0\dots0)_2 \times 2^0 = 1$

Working Around:

$$(1. d_1 d_2 d_3 \dots d_{52})_2 \times 2^{e-1023}$$

$$(0.1 d_1 d_2 d_3 \dots d_{52})_2 \times 2^{-1022}$$

Now,

$$\text{The highest possible number} = (0.111\dots1)_2 \times 2^{1025} \approx \infty$$

$$\text{The smallest possible number} = (0.100\dots0)_2 \times 2^{-1022} \approx 0$$

$$\text{highest possible exponent} = 2^{1025}$$

$$\text{lowest possible exponent} = 2^{-1022}$$

Now,

The highest possible number =  $(0.111\dots1) \times 2^{1024}$

$$2.1 \text{ Normalized force} = 2.1 \times 10^{308}$$

$$\text{The lowest possible number} = (0.1)_2 \times 2^{-1024}$$

$$= 2.225 \times 10^{-308}$$

31 MAY 1968

if  $x$  is given,  $x$  should always be converted to  $f(x)$

$$6 - \text{rest} = (0x_2 + 1x_1 + 1x_2 + 1x_1) \frac{|x - (0) \cdot 7|}{|x_1|} = 2$$

5. What are the non-negative  $x$  for which  $x - (x) \sqrt{1} = x \cdot \beta$ ?

↳ if perfectly in the middle then round it to the nearest even fl

\* For FL Binary,

if the number ends with 0  $\rightarrow$  Even

if the number ends with 1 → Odd



1.

Lecture form:  $f = \pm (0.d_1 d_2 d_3 \dots d_m) \beta^e$  (1)

Normalized form:  $F = \pm (1.d_1 d_2 d_3 \dots d_m) \beta^e$  (2)

Denormalized form:  $f = \pm (0.1 d_1 d_2 d_3 \dots d_m) \beta^e$  (3)

$\beta = 2, m = 4, e_{\max} = 6, e_{\min} = -3$

(a) What are the maximum numbers that can be stored in the system by the forms defined above? (4)

$$(0.1111)_2 \times 2^6 = (0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4) \times 2^6$$

$$(1.1111)_2 \times 2^6 = (1 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4) \times 2^6$$

$$(0.1111)_2 \times 2^6 = (0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4 + 2 \times 2^5) \times 2^6$$

(b) What are the non-negative minimum numbers that can be stored in the system by 3 forms? (5)

$$(1) (0.1000)_2 \times 2^{-3} = 0(1 \times 2^{-1}) \times 2^{-3} = \frac{1}{16}$$

$$(2) (0.10000)_2 \times 2^{-3} = (1 \times 2^{-1}) \times 2^{-3} = \frac{1}{16}$$

$$(3) (-1.0000)_2 \times 2^{-3} = (1 \times 2^0) \times 2^{-3} = \frac{1}{8}$$

③ Using eq(1), find all the decimal numbers for  $\ell = -1$  plot them in a real line.

given,  $\beta = \frac{1}{2}, m = 4, \ell = -1$

$$\text{① } (0.1000)_2 \times 2^{-1} = (1 \times 2^{-1}) \times 2^{-1} = \frac{1}{4}$$

$$\text{② } (0.1001)_2 \times 2^{-1} = \{ (1 \times 2^{-1}) + (1 \times 2^{-4}) \} \times \frac{1}{2} = \frac{9}{32}$$

$$\text{③ } (0.1010)_2 \times 2^{-1} = \{ (1 \times 2^{-1}) + (1 \times 2^{-3}) \} \times \frac{1}{2} = \frac{5}{16}$$

$$\text{④ } (0.1011)_2 \times 2^{-1} = \{ (1 \times 2^{-1}) + (1 \times 2^{-3}) + (1 \times 2^{-4}) \} \times \frac{1}{2} = \frac{11}{32}$$

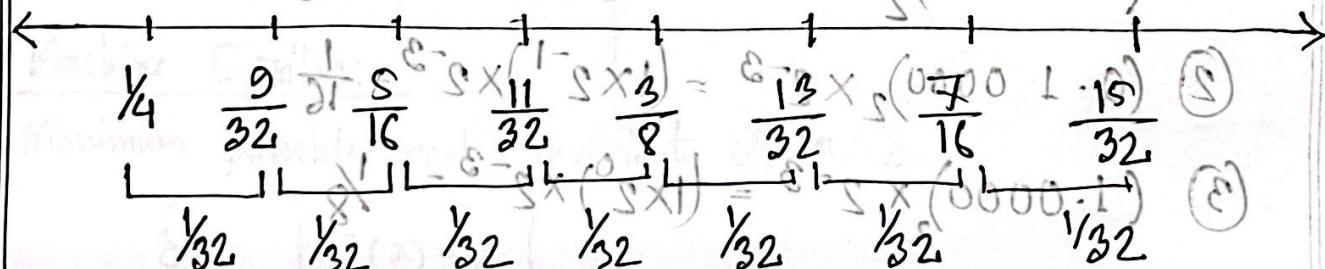
$$\text{⑤ } (0.1100)_2 \times 2^{-1} = \{ (1 \times 2^{-1}) + (1 \times 2^{-2}) \} \times \frac{1}{2} = \frac{3}{8}$$

$$\text{⑥ } (0.1101)_2 \times 2^{-1} = \{ (1 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-4}) \} \times \frac{1}{2} = \frac{13}{32}$$

$$\text{⑦ } (0.1110)_2 \times 2^{-1} = \{ \frac{1}{2} + \frac{1}{4} + \frac{1}{16} \} \times \frac{1}{2} = \frac{7}{16}$$

$$\text{⑧ } (0.1111)_2 \times 2^{-1} = \{ \frac{1}{2} + \frac{1}{4} + \frac{1}{16} + \frac{1}{32} \} \times \frac{1}{2} = \frac{15}{32}$$

$$\frac{1}{32} = \{ (0.1000)_2 \} \times \frac{1}{2} = (0.0001)_2 \quad \text{①}$$



2.  $\beta = 2, m = 4, \ell_{\min} = -1 \Rightarrow \ell_{\max} = 2$

(a) Compute minimum of  $|\lambda|$  for normalized form

$$(1.0000)_2 \times 2^{-1} = (1 \times 2^0) \times \frac{1}{2} = \frac{1}{2}$$

(b) Compute minimum of  $|\lambda|$  for denormalized form

$$(0.10000)_2 \times 2^{-1} = (1 \times 2^{-1}) \times 2^{-1} = \frac{1}{4}$$

(c) Compute machine epsilon for normalized form

$$\epsilon_m = \frac{1}{2} \beta^{-m} = \frac{1}{2} \times 2^{-4} = \frac{1}{32}$$

(d) Compute machine epsilon for denormalized form

$$\epsilon_m = \frac{1}{2} (\beta^{-m} - 1) = \frac{1}{2} \times 2^{-4} = \frac{1}{32}$$

(e) Compute maximum delta value for general form/conversion which is discovered in lecture note.

$$\max \Delta = \epsilon_m = \frac{1}{2} \beta^{1-m}$$

$$\Delta_{s(001.1)} = \frac{1}{2} \times 2^{(1-4)} = \frac{1}{16}$$

$$\Delta_{s(1111.1)} = \frac{1}{2} \times 2^{(1-4)} = \frac{1}{16}$$

$$\Delta_{s(0111.1)} = \frac{1}{16}$$

$$\Delta_{s(1111.1)} = \frac{1}{16}$$

3. Lecture form:  $F = \pm (0.d_1 d_2 d_3 d_4 \dots d_m) \beta^e \cdot S = \beta$

Normalized form:  $F = \pm (1.d_1 d_2 d_3 d_4 \dots d_m) \beta^e$  (a)

Denormalized form:  $F = \pm (0.1 d_1 d_2 d_3 \dots d_m) \beta^e$

$\beta = 2, m = 4, e \in \{-2, -1, 0, 1, 2\}$  (b)

(a) How many numbers (including zero) can be represented by this system? (find for each and abrogate negative numbers)

Lecture form:  $F = \pm (0.d_1 d_2 d_3 \dots d_m) \beta^e$

$(0.1000)_2 2^e$  (0.1001)<sub>2</sub> 2<sup>e</sup> (b)

$(0.1010)_2 2^e$  (0.1011)<sub>2</sub> 2<sup>e</sup>

$(0.1100)_2 2^e$  (0.1101)<sub>2</sub> 2<sup>e</sup> (c)

$(0.1110)_2 2^e$  (0.1111)<sub>2</sub> 2<sup>e</sup> (d)

Normalized form:  $F = \pm (1.d_1 d_2 d_3 d_4 \dots d_m) \beta^e$

$(1.0000)_2 2^e$  (1.0100)<sub>2</sub> 2<sup>e</sup> (1.1000)<sub>2</sub> 2<sup>e</sup> (1.1100)<sub>2</sub> 2<sup>e</sup>

$(1.0001)_2 2^e$  (1.0101)<sub>2</sub> 2<sup>e</sup> (1.1001)<sub>2</sub> 2<sup>e</sup> (1.1101)<sub>2</sub> 2<sup>e</sup>

$(1.0010)_2 2^e$  (1.0110)<sub>2</sub> 2<sup>e</sup> (1.1010)<sub>2</sub> 2<sup>e</sup> (1.1110)<sub>2</sub> 2<sup>e</sup>

$(1.0011)_2 2^e$  (1.0111)<sub>2</sub> 2<sup>e</sup> (1.1011)<sub>2</sub> 2<sup>e</sup> (1.1111)<sub>2</sub> 2<sup>e</sup>

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⑥ For each of the forms (a) & (b) find the smallest, positive number and the largest number representable by the system.

Lecture form:  $f = \pm v(0.d_1 d_2 d_3 \dots d_m) \beta^e$

Smallest positive number:  $(0.1000)_2 \times 2^{-2} = (1 \times \frac{1}{2}) \times \frac{1}{4} = \frac{1}{8}$

Largest positive number:  $(0.1111)_2 \times 2^{-2} = (\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}) \times 4 = \frac{15}{4}$

Normalized form:  $f = \pm (0.1 \cdot d_1 d_2 d_3 \dots d_m) \beta^e$

Smallest positive number:  $(1.0000)_2 \times 2^{-2} = (2^0) \times \frac{1}{4} = \frac{1}{4}$

Largest positive number:  $(1.1111)_2 \times 2^{-2} = \frac{3}{4}$

Denormalized form:

Smallest positive number:  $(0.10000)_2 \times 2^{-2} = \frac{1}{8}$

Largest positive number:  $(0.11111)_2 \times 2^{-2} = \frac{31}{8}$

$(0.1 \dots 010)_2 =$

$(1 \dots 110)_2$  (smallest negative number)

$(1 \dots 110)_2 =$

① For the IEEE standard (1985) for double-precision (64-bit) arithmetic, find the smallest, positive number and the largest number representable by a system that follow this standard. Do not find their decimal values; instead, follow the below format:

$$S = \pm (0.1d_1 d_2 d_3 \dots d_m) \times 2^{e - \text{exponent Bias}}$$

$$S = \pm (0.1111 \dots 1) \times 2^{1111 \dots 1} \text{ : maximum shifting digits}$$

Smallest positive number:

$$(0.10 \dots 0)_2 \times 2^{1-1022} \text{ : just before } 0.1$$

$$= (0.1)_2 \times 2^{-1021} \text{ : } 0.10 \dots 0 \pm 7$$

$$S = \pm (0.1 \dots 1) \times 2^{0000 \dots 1} \text{ : maximum shifting digits}$$

Largest positive number:

$$(0.11 \dots 1)_2 \times 2^{2046-1022} \text{ : maximum shifting digits}$$

$$= (0.11 \dots 1)_2 \times 2^{2046-1022} \text{ : just before } 1.0$$

$$= (0.11 \dots 1)_2 \times 2^{2046-1022} \pm 7$$

② If the bias is 500, what is the smallest and largest positive number?

Smallest positive number:

$$(0.10 \dots 0)_2 \times 2^{1-2046-500}$$

$$= (0.1)_2 \times 2^{-499}$$

Largest positive number:

$$(0.11 \dots 1)_2 \times 2^{2046-500}$$

$$= (0.11 \dots 1)_2 \times 2^{1546}$$

4.  $\beta = 2$ ,  $m = 300$  ni sākumā pribinotās vērtības fāns (d)

$$L_{\min} = -1, L_{\max} = 2, \quad \frac{|x - (x) \sqrt{t}|}{\sqrt{t}} = 1$$

① find the floating-point representation of the number  $(6.25)_{10}$  and  $(6.875)_{10}$  in the normalized form. That is, find  $fl[6.25]$  and  $fl[6.875]$ .

$$\begin{aligned}
 (6.25)_{10} &= (110.01)_2 \times 2^0 \\
 &= (1.1001)_2 \times 2^1 \\
 &= (1.100)_2 \times 2^2 \quad [\text{As } m=3] \\
 \therefore \text{fl } [6.25] &= (1.100)_2 \times 2^2
 \end{aligned}$$

$$\begin{aligned}
 (6.875)_{10} &= (110.111)_2 \times 2^0 & 2 \overline{)6} \\
 &= (1.10111)_2 \times 2^2 & 2 \overline{)3-0} \\
 &= (1.101)_2 \times 2^2 & 2 \overline{)1-1} \\
 &= (1.110)_2 \times 2^4 & 0-1
 \end{aligned}$$

$0.75 \times 2 = 1.5$   
 $0.5 \times 2 = 1.0$

⑥ What are the rounding errors  $\delta_1, \delta_2$  in part ⑤?  $S=9$  . P

$$S_1 = \left| \frac{|f(x) - x|}{x} \right| = \left| \frac{f[6.25] - 6.25}{6.25} \right| = \text{value} = \text{approx.}$$

01 (28.2) Struktur ist so mitbewertet und prägt mit best. 6.25

$$\text{Ansatz: } 28.2 \text{ ist breit zu kurz} \Rightarrow \frac{6.00 - 6.25}{6.00} \text{ ist zu } 28.2$$

$$0 = 5 \times 28.0 \quad \boxed{5} \quad \boxed{28.0} \quad \boxed{140} \quad \boxed{6.25} \quad [28.0] \text{ Jf}$$

$$S_2 = \left| \frac{f_L(x) - x}{x} \right| = \left| \frac{f_L[6.875] - 6.875}{6.875} \right| = \left| \frac{25.5(0.011)}{6.875} \right| = 0.1 (25.5)$$

$$\left[ \vec{F} = m \vec{a} \right] = \frac{Sx(001 \cdot 1)}{7 - 6.875} = [2S]$$

$$\frac{0.1}{1-11} = \frac{1}{55} \times (111.011) = 0.1(2F8.0)$$

an the value represented in the Denormalized form?

$$(6.25)_{10} = (110.01)_2 \times 2^0 = (0.11001)_2 \times 2^3$$

$$(6.875)_{10} = (110.111)_2 \times 2^0 = (0.110111)_2 \times 2^3$$

Here  $\ell = 3$  but the  $\ell_{\max} = 2$

∴ Cannot be representable in denormalized form.

② find the upper-bound of the rounding errors for Lecture Note.

Normalized and Denormalized form.

Given that,

$$\beta = 2$$

$$m = 3$$

$$\text{Lecture Note: } \frac{1}{2} \beta^{-m} = \frac{1}{2} (2)^{-3} = \frac{1}{8}$$

$$\text{Normalized: } \frac{1}{2} \beta^{-m} = \frac{1}{2} (2)^{-3} = \frac{1}{8}$$

$$\text{Denormalized: } \frac{1}{2} \beta^{-m} = \frac{1}{2} (2)^{-3} = \frac{1}{8}$$

5.

$$\text{Given, } \beta = 2, m = 4, e_{\min} = -4, e_{\max} = 2$$

$$\text{Lecture Note form: } f = \pm (0.d_1 d_2 d_3 d_4 \dots d_m) \beta^e \quad \textcircled{a}$$

$$\text{Normalized form: } f = \pm (1.d_1 d_2 d_3 \dots d_m) \beta^e \quad \textcircled{b}$$

$$\text{Denormalized form: } f = \pm (0.d_1 d_2 d_3 \dots d_m) \beta^e \quad \textcircled{c}$$

① From the eq. ① find all the decimal numbers for  $e = -3$  and plot it into the dual line

$$\text{From eq. ①: } f = \pm (0.d_1 d_2 d_3 \dots d_m) \beta^e$$

$$\text{Here, } \beta = 2 \text{ and } m = 4, e = -3$$

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$$1. (0.1000)_2 \times 2^{-3}$$

$$= (0 \times 2^0 + 1 \times 2^{-1}) \times \frac{1}{8}$$

$$= \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$$

$$2. (0.1001)_2 \times 2^{-3} = \{ (0 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-4}) \} \times \frac{1}{8}$$

$$= \frac{9}{128}$$

$$3. (0.1010)_2 \times 2^{-3} = \{ (0 \times 2^0) + (1 \times \frac{1}{2}) + (1 \times \frac{1}{8}) \} \times \frac{1}{8}$$

$$= \frac{5}{64}$$

$$4. (0.1011)_2 \times 2^{-3} = \{ (0 \times 2^0) + (\frac{1}{2}) + (\frac{1}{4}) + (\frac{1}{16}) \} \times \frac{1}{8}$$

$$= \frac{11}{128}$$

$$5. (0.1100)_2 \times 2^{-3} = \{ \frac{1}{2} + \frac{1}{4} \} \times \frac{1}{8} = \frac{3}{32}$$

$$6. (0.1101)_2 \times 2^{-3} = \{ \frac{1}{2} + \frac{1}{4} + \frac{1}{16} \} \times \frac{1}{8} = \frac{13}{128}$$

$$7. (0.1110)_2 \times 2^{-3} = \{ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \} \times \frac{1}{8} = \frac{7}{64}$$

$$8. (0.1111)_2 \times 2^{-3} = \{ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \} \times \frac{1}{8} = \frac{15}{128}$$

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Loss of Significance (To avoid rounding error)

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$88.18 + 88 = 10$$

$$88.18 + 88 =$$

$$88.22 =$$

$$50.0 - 88.18 + 88.10.0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Again,  $\alpha$  and  $\beta$  roots

$$ax^2 + bx + c = 0$$

$$\text{or, } x^2 + (\alpha + \beta)x + \alpha\beta = 0$$

$$\text{Here, } \alpha + \beta = -\frac{b}{a} \quad \text{--- (1)}$$

$$50.0 - 88.18 + 88.10.0 = \frac{\alpha\beta}{88.22} = \frac{0}{88.22} = 0$$

Problem:

$$x^2 - 56x + 1 = 0, \text{ for 4 significant figure}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-56) \pm \sqrt{(-56)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$x_1 = \frac{56 + \sqrt{3132}}{2}$$

$$= 28 + 3\sqrt{87}$$

$$= 55.98213716$$

$$= 55.98$$

$$x_2 = \frac{56 - \sqrt{3132}}{2}$$

$$= 28 - 3\sqrt{87}$$

$$= 0.01786284073$$

$$= 0.01786$$

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Computation of Computer floating to 801

$$\alpha = 28 + \sqrt{783}$$

$$= 28 + 27.98$$

$$= 55.98$$

$$\beta = 28 - \sqrt{783}$$

$$= 28 - 27.98$$

$$= 0.02$$

$$28 - 27.98213716$$

$$= 0.01786284073$$

$$= 0.01786284073$$

$$= 0.01786284073$$

Relative error =

$$\left| \frac{f(x) - x}{x} \right| = \left| \frac{0.01786284073 - 0.02}{0.01786284073} \right|$$

$$= 0.1196427432$$

$$\alpha \beta = \frac{c}{a}$$

$$\beta = \frac{c}{\alpha \cdot a} = \frac{1}{55.98 \times 1} = 0.01786352269$$

$$\therefore \alpha = 55.98$$

$$\beta = 0.01786$$

$$= 0.01786$$

Ans

$$= 0.01786$$

$$\frac{561813840.00}{5} = 11236268.00$$

$$\frac{561813840.00}{5} = 11236268.00$$

$$\frac{561813840.00}{5} = 11236268.00$$

$$\frac{561813840.00}{5} = 11236268.00$$

$$561813840.00 =$$

$$561813840.00 =$$

$$11236268.00 =$$

$$11236268.00 =$$

5.

① Standard form:  $f = \pm (0.1d_1d_2d_3 \dots d_m) \beta^e$

Normalized form:  $f = \pm (1.1d_1d_2d_3 \dots d_m) \beta^e$

Denormalized form:  $f = \pm (0.1d_1d_2d_3 \dots d_m) \beta^e$

a) What are the maximum numbers in decimal values?

$$\beta = 2, m = 4, e_{\max} = 2, e_{\min} = -3$$

$$\text{Standard form: } (0.1111)_2 \times 2^2 = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) \times \frac{1}{4} = \frac{15}{64}$$

$$\text{Normalized form: } (1.1111)_2 \times 2^2 = \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) \times \frac{1}{4} = \frac{31}{64}$$

$$\text{Denormalized form: } (0.1111)_2 \times 2^2 = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}\right) \times \frac{1}{4} = \frac{31}{128}$$

b) What are the non-negative minimum numbers?

$$\text{Standard form: } (0.1000)_2 \times 2^{-3} = \left(1 \times \frac{1}{2}\right) \times \frac{1}{8} = \frac{1}{16}$$

$$\text{Normalized form: } (1.0000)_2 \times 2^{-3} = \left(1 \times 2^0\right) \times \frac{1}{8} = \frac{1}{8}$$

$$\text{Denormalized form: } (0.10000)_2 \times 2^{-3} = \left(1 \times \frac{1}{2}\right) \times \frac{1}{8} = \frac{1}{16}$$

$$\text{Normalized form: } (1.1111)_2 \times 2^{-3} = \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) \times \frac{1}{8} = \frac{31}{128}$$

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## 1- floating point

Q) What are the maximum and minimum numbers that can be stored in the system by the three forms defined above if the system has negative support?

Standard form:  $f = \pm (0.d_1 d_2 d_3 \dots d_m) \beta^e$

maximum:  $(0.1111)_2 \times 2^2$

non-negative minimum:  $(0.1000)_2 \times 2^{-3}$

$\frac{1}{2} \times \left( \frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \right) = \frac{1}{2} \times \frac{15}{32} = 0.1000$

negative minimum: - maximum

$$\frac{21}{16} = -(0.1111)_2 \times 2^2$$

Normalized form:  $f = \pm (1.d_1 d_2 d_3 \dots d_m) \beta^e$

maximum:  $(1.1111)_2 \times 2^2$

non-negative minimum:  $(1.0000)_2 \times 2^{-3}$

negative minimum: - maximum

$$= -(1.1111)_2 \times 2^2$$

Denormalized form:  $f = \pm (0.1d_1 d_2 d_3 \dots d_m) \beta^e$

maximum:  $(0.1111)_2 \times 2^2$

non-negative minimum:  $(0.10000)_2 \times 2^{-3}$

negative minimum:  $-(0.1111)_2 \times 2^2$

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2.

Consider the real number  $x = (3.1416)_{10}$  and do the following (Q)

(a) first convert the decimal number  $x$  in binary format at least up to 9 decimal/binary places

$$x = (3.1416)_{10}$$

Integer part:

$$\begin{array}{r} 2 | 3 | 1 \\ \underline{2} \quad \underline{1} \\ 1 \end{array}$$

$$\begin{array}{r} 2 | 3 \\ 2 | \underline{1} - 1 \\ \underline{2} \quad \underline{0} \\ 0 \end{array} \xrightarrow{\text{S}} (000100100.1) =$$

$$(11)_2$$

Fractional part: 0011.0

$$\xrightarrow{\text{S}} (0011.0) =$$

$$; 2 = m \text{ not}$$

$$0.1416 \times 2 = 0.2832 \xrightarrow{\text{S}} (000100100.1) =$$

$$0.2832 \times 2 = 0.5664 \xrightarrow{\text{S}} (3.1416) = (11.001001000)_2$$

$$0.5664 \times 2 = 1.1328 \xrightarrow{\text{S}} (00010010101.1)$$

$$0.1328 \times 2 = 0.2656 \xrightarrow{\text{S}} (00010010011.0) =$$

$$0.2656 \times 2 = 0.5312 \xrightarrow{\text{S}}$$

$$0.5312 \times 2 = 1.0624 \xrightarrow{\text{S}} (10011.0) =$$

$$0.0624 \times 2 = 0.1248 \quad \vdots$$

$$0.1248 \times 2 = 0.2496 \quad \vdots$$

$$0.2496 \times 2 = 0.4992 \quad \text{0th}$$

$$0.4992 \times 2 = 0.9984 \quad \vdots$$

$$0.9984 \times 2 = 1.9968 \quad \vdots$$

$$0.9968 \times 2 = 1.9936 \quad \vdots$$

Q6) What will be the (binary) value of  $x$  [Find  $f_7(n)$ ] if you store it in a system with  $m=4$  and  $m=5$  using the standard form of floating point representation? Q. of Q8 show

for  $m=4$ :

$$(3.1416)_{10} \rightarrow (11.001001000)_2 \times 2^0$$

$$= (1.1001001000)_2 \times 2^1$$

$$= (0.11001001000)_2 \times 2^2$$

$$= (0.1100)_2 \times 2^2$$

$$0.1(21P1.2) = 10$$

: from right

$$\begin{array}{r} 1 \\ | \\ 1 \\ | \\ 0 \\ | \\ 0 \\ \hline \end{array}$$

11

$$0.1100 : \text{from } 10.1101$$

for  $m=5$ :

$$(3.1416)_{10} \rightarrow (11.001001000)_2 \times 2^0$$

$$= (0.001100100 \cdot 11)_2 = (21P1.2)_2 \times 2^0$$

$$= (1.1001001000)_2 \times 2^1$$

$$= (0.11001001000)_2 \times 2^2$$

$$= (0.11001)_2 \times 2^2$$

$$= 0.1100 \times 2^2 = 0.1100 \times 4 = 0.1100 \times 2^1 = 0.1100 \times 2 = 0.1100 \times 1 = 0.1100$$

$$= 0.1100 \times 2^0 = 0.1100 \times 1 = 0.1100$$

$$= 0.1100 \times 2^1 = 0.1100 \times 2 = 0.1100 \times 1 = 0.1100$$

$$= 0.1100 \times 2^0 = 0.1100 \times 1 = 0.1100$$

$$= 0.1100 \times 2^1 = 0.1100 \times 2 = 0.1100 \times 1 = 0.1100$$

$$= 0.1100 \times 2^0 = 0.1100 \times 1 = 0.1100$$

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③ Error,  $S_4 = \left| \frac{f(x) - x}{x} \right|$  (to not with two bits)

When  $m=4$ ,

$$\begin{aligned} (0.1100)_2 &= (0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 0 \times 2^{-4}) \times 4 \\ &= (0 + \frac{1}{2} + \frac{1}{4} + 0 + 0) \times 4 \\ f(x)_4 &= 3 \end{aligned}$$

When  $m=5$ ,

$$(0.11001)_2 \times 2^5 = (0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}) \times 4^1$$

$$0.11001 - (0.11001 - 0.125) = 3.125$$

$$S_4 = \left| \frac{f(x) - x}{x} \right| = \left| \frac{3 - 3.1416}{3.1416} \right| = 0.0450725$$

$$S_5 = \left| \frac{f(x) - x}{x} \right| = \left| \frac{3.125 - 3.1416}{3.1416} \right| = 0.005283931755$$

(to not with two bits)

$$1118 - 28 = 110$$

$$1118 + 28 = 110$$

$$1118 \cdot 10 - 28 =$$

$$1118 \cdot 10 + 28 =$$

$$1118 \cdot 10 =$$

$$1118 \cdot 10 =$$

2. (a)  $x^2 - 70x + 9 = 0$ . Below calculate up to 6 significant figures.

(a) Find out where the loss of significance occurs.

When you calculate the roots?

$$x^2 - 70x + 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$= -(-70) + \sqrt{(-70)^2 - 4 \times 1 \cdot 9}$$

$$= 35 + \sqrt{19 \cdot 1 \cdot 9}$$

$$= 69.871191551 \dots$$

Computation of Computer

$$x_1 = 35 + 8\sqrt{19}$$

$$= 35 + 34.8711$$

$$= 69.8711$$

$$x_2 = 35 - 8\sqrt{19}$$

$$= 35 - 34.8711$$

$$= 0.1289$$

$$\text{Error} = \left| \frac{f(x) - x}{x} \right| = \left| \frac{0.1288084517 - 0.1289}{0.1288084517} \right|$$

$$\begin{aligned} &= 0.0007107320893 \times 10^{-4} \\ &= 0.0007107320893 \end{aligned}$$

$$\textcircled{c} \quad x^2 - 70x + 9 = 0$$

$$\alpha x^2 + (\alpha + \beta)x + \alpha\beta = 0$$

$$\alpha + \beta = -70$$

$$\alpha\beta = 9$$

$$\alpha\beta = \frac{0.1288084517}{\alpha} =$$

$$\beta = \frac{c}{\alpha}$$

$$= \frac{9}{0.1288084517}$$

$$= 0.1288086204 = 0.128808$$

$$\alpha = 0.1288086204$$

$$\alpha x^2 + bx + c = 0 + 10x - 9$$

$$a = 1 \quad b = -70 \quad c = 9$$

$$= 1 \cdot x^2 - (0f-)x + (0f-) -$$

$$\begin{aligned} &= 1 \cdot x^2 - (0f-)x + (0f-) - \\ &= 221011f8.0d = \end{aligned}$$

$$11f8.0d =$$

$$0f - 9 + 0 = 9 + 0$$

$$\frac{(0f-) - }{1} = 808881.0 + 11f8.0d$$

$$0f \neq 808881.0$$

$$\frac{0 - 0f2220000.8}{0}$$

$$\frac{0f - 808881.0d}{0f 808881.0d}$$

$$FFF018P00000.0 - 815288P18100000.0 =$$

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(b) We know,  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 70.00001314285714$

$$ax^2 + bx + c = 0 \text{ where } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + (\alpha + \beta)x + \alpha\beta = 0$$

where,  $\alpha + \beta = -\frac{b}{a}$

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

For,

$$x^2 - 70x + 9 = 0$$

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-(-70) + \sqrt{(-70)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$= \frac{-(-70) + \sqrt{(-70)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$= 69.87119155$$

$$= 69.8711$$

$$\alpha + \beta = -\frac{b}{a}$$

$$69.8711 + 0.128808 = -\frac{(-70)}{1}$$

$$\text{or, } 69.999908 \neq 70$$

$$\left| \frac{69.999908 - 70}{69.999908 - 70} \right|$$

$$= 0.000001314285714$$

$$0 = 0 + 100 - 50 \quad (1)$$

$$0 = 90 + 10(9 + 1) + 50$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-(-70) - \sqrt{(-70)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$= \frac{0.128808}{1} = 0.128808$$

$$\frac{0}{0.128808} = 0$$

$$\frac{0}{100 - 50} =$$

$$808851.0 = \alpha\beta = \frac{0}{100 - 50} =$$

$$8.999956649 \neq 9$$

$$\left| \frac{8.999956649 - 9}{9} \right|$$

$$= 0.000004816777$$

## Polynomial Interpolation

$$\frac{1}{R} = \frac{1}{R} - \frac{1}{R}$$

$$P_n(x) = a_0x^0 + a_1x^1 + a_2x^2 + \dots + a_nx^n$$

Here,  $n =$  degree no. of terms (1)

-  $a_0, a_1, a_2$  = constant / Coefficients

$P_n(x)$  has  $n+1$  coefficients

Bris:

$$P_2(x) = a_0 x^0 + a_1 x^1 + a_2 x^2$$

$$= a_0 1 + a_1 x + a_2 x^2$$

$$\text{Basis} = \{1, x, x^2\}$$

if  $\text{degree} = n$  then;  $\text{basis} = \text{degree} + 1$

## Taylor Series!

$$f(x) = f(x_0) + f^{(1)}(x_0)(x-x_0) + \frac{f^{(2)}(x_0)}{2!} (x-x_0)^2 + \frac{f^{(3)}(x_0)}{3!} (x-x_0)^3 + \dots$$

If we take upto the third term, it becomes a polynomial of degree 2.

This terms are part of

## Taylor's Theorem:

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!} (x-x_0)^k + \frac{f^{(n+1)}(\xi)}{(n+1)!} (x-x_0)^{n+1}$$

Taylor's polynomial's of degree  $n$  in the Lagrange form of the remainder

But we don't know the actual value of  $\frac{(1)(1-x)}{(x)(1-x)} = \frac{1}{x}$

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Rules of Polynomial: Lagrange polynomial

(a) Exponents must be non-negative.

(b) Must be an integer.

Transition Error:  $|f(x) - P_n(x)|$

Coefficients  $a_0, a_1, \dots, a_n$

Vandermonde Matrix

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ 1 & x_2 & x_2^2 & \dots & x_2^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \\ \vdots \\ f(x_n) \end{bmatrix}$$

$$\text{Vandermonde Matrix, } V = \begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ 1 & x_2 & x_2^2 & \dots & x_2^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^n \end{bmatrix}$$

$$\text{on, } A = V^{-1} F \quad [V \text{ matrix must be invertable}]$$

Natural Basis:  $\{1, x, x^2, \dots\}$  | Lagrange:  $\{l_0(x), l_1(x), l_2(x), \dots\}$

Basis:  $\sum_{K=0}^n a_k x^k$

Basis:  $\sum_{K=0}^n f(x_k) l_k(x)$

Basis Calculation of Lagrange:

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3) \dots (x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3) \dots (x_0-x_n)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3) \dots (x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3) \dots (x_1-x_n)}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3) \dots (x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3) \dots (x_2-x_n)}$$

1.  $f(x) = xe^x$ . An interpolating polynomial,  $P_3(x)$  is computed by using Taylor expansion. Write  $f(x)$  as an infinite series (centered at  $x_0 = 1$ )

$$f(x) = xe^x \rightarrow f(x_0) = f(1) = 1e^1 = e \quad \text{as } \frac{(ex)^1}{1!} = 1e$$

$$f'(x) = xe^x + e^x \rightarrow f'(x_0) = f'(1) = 1e^1 + e^1 = 2e \quad \text{as } \frac{(ex)^1}{1!} = 2e$$

$$f''(x) = \frac{d}{dx}(xe^x) + \frac{d}{dx}(e^x) \quad \text{as } \frac{(ex)^2}{2!} = \frac{(ex)^2}{2!} = e^2$$

$$= xe^x + e^x + e^x$$

$$\text{thus } f''(x_0) = xe^x + 2e^x \rightarrow f''(x_0) = f''(1) = 1e^1 + 2e^1 = 3e$$

$$f'''(x) = \frac{d}{dx}(xe^x + 2e^x) \quad \text{as } \frac{(ex)^3}{3!} = \frac{(ex)^3}{3!} = e^3$$

$$= xe^x + e^x + 2e^x$$

$$= 3e^x + xe^x \rightarrow f'''(x_0) = f'''(1) = 3e^1 + 1e^1 = 4e$$

$$\text{OF } 12011.0 = 3(1.0) + (1.0)^3$$

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \dots$$

$$\text{at } x_0 = 1 \quad (1-1.0) + (1-1.0)^2 + (1-1.0)^3 = (1.0)^3$$

$$f(x) = e + 2e(x-1) + \frac{3e}{2 \times 1}(x-1)^2 + \frac{4e}{3 \times 2 \times 1}(x-1)^3 + \dots$$

$$= e + 2e(x-1) + \frac{3e}{2}(x-1)^2 + \frac{4e}{6}(x-1)^3 + \dots$$

(b) find the values of  $a_0, a_1, a_2$  and  $a_3$  if the function is interpolating by a degree 3 polynomial in  $x$ .

$P_3(x)$  is finite we can fit with various polynomials (A)

$$a_0 = f(x_0) = 2$$

$$a_1 = \frac{f'(x_0)}{1!} = 2e \quad \text{as } f'(x) = (1)^2 e = (0x)^2 e \leftarrow x_{00} = (0x)^2$$

$$a_2 = \frac{f''(x_0)}{2!} = \frac{1}{2} e + 1 \cdot 2e = (1)^2 e = (0x)^2 e \leftarrow x_{00} + x_{00} = (0x)^2$$

$$a_3 = \frac{f'''(x_0)}{3!} = \frac{4}{6} e \quad (i) \frac{b}{x^b} + (i_0 x)^b \frac{b}{x^b} = (1)^3 e$$

(c) Compute  $f(0.1)$  and  $P_3(0.1)$  up to seven significant figures.

$$f(x) = 2e^x \quad (1)^2 e = (0x)^2 e \leftarrow x_{00} + x_{00} =$$

$$f(0.1) = (0.1)e = 0.1105170 \quad (0x)^2 e + (0x)^3 e =$$

$$P_3(x) = 2 + 2e(x-1) + \frac{3}{2}e(x-1)^2 + \frac{1}{6}e(x-1)^3 = (1)^3 e$$

$$P_3(0.1) = 2 + 2e(0.1-1) + \frac{3}{2}e(0.1-1)^2 + \frac{1}{6}e(0.1-1)^3 = 0.1929980098 = (1)^3 e$$

$$= -0.1929980098 + (1-0.1) \frac{2e}{2} + (1-0.1) \frac{3e}{6} + 0 =$$

(d) Find the percent error for interpolating  $f(0.1)$  by  $P_3(0.1)$

$$\text{Percent error} = \left| \frac{f(0.1) - P_3(0.1)}{\frac{f(0.1) - P_3(0.1)}{f(0.1)} \times 100} \right|$$

$$= \frac{0.1105170 + 0.1929980}{0.1105170} \times 0.01$$

(e) Find the maximum error using 4th degree ( $P_4(x)$ ) Taylor's Polynomial while approximating  $f(0.2)$  on interval  $[0, 0.2]$

[Given  $x_0=0$ ]

$$f(x) = xe^x$$

$$f'(x) = \frac{d}{dx} (xe^x) = e^x + xe^x$$

$$f''(x) = \frac{d}{dx} (e^x + xe^x) = e^x + e^x + xe^x = 2e^x + xe^x$$

$$f'''(x) = \frac{d}{dx} (2e^x + xe^x) = 2e^x + e^x + xe^x = 3e^x + xe^x$$

$$f''''(x) = \frac{d}{dx} (3e^x + xe^x) = 3e^x + e^x + xe^x = 4e^x + xe^x$$

$$f''''(0.2) = 4e^{0.2} + 0.2e^{0.2} = (4+0.2)e^{0.2} = 4.2e^{0.2} = (4.2)e^{0.2}$$

$$f''''(0.2) = 4e^{0.2} + 0.2e^{0.2} = (4+0.2)e^{0.2} = 4.2e^{0.2} = (4.2)e^{0.2}$$

$$= 6.3513 \quad (\text{max value on interval } [0, 0.2] = (4.2)e^{0.2})$$

(b)  $f(x) = P_4(x) + \frac{f''(x)}{2!} (x-x_0)^2$  fitting with limit (b)

$$|f(x) - P_4(x)| \leq \left| P_4(x) + \frac{f''(x)}{2!} (x-0)^2 - P_4(x) \right|$$

$$10.0 \times \frac{(5e^{-0.2} + 5e^{-0.2})}{120} =$$

at  $x = 0.2$ ,  $f''(x) = \frac{d^2}{dx^2} \sin x$  previous notes maximum with limit (b)

$$|f(0.2) - P_4(0.2)| \leq \frac{6.5 \times 6.3813}{120} \times (0.2)^2 \leq 1.6937 \times 10^{-5} \quad [0 = 0.2 \text{ moving}]$$

2.  $f(x) = \sin x$ . An interpolating polynomial.  $P_3(x)$  is computed by Taylor expansion

(a) Using Taylor expansion write  $f(x)$  as an infinite series.

Centered at  $x_0 = 1$

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!} (x-x_0)^2 + \frac{f'''(x_0)}{3!} (x-x_0)^3 + \dots$$

$$f(x) = \sin x = \sin(x_0) = \sin(1)$$

$$f'(x) = \cos x = \cos(x_0) = \cos(1)$$

$$f''(x) = -\sin x = -\sin(x_0) = -\sin(1)$$

$$f'''(x) = -\cos x = -\cos(x_0) = -\cos(1)$$

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Now, suppose the function  $F$  at  $x_0 = 1.0$  is  $f(1.0) = 1.0$  &  $f'(1.0) = 0$  (given) (i)

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \dots$$

$$f(x) = \sin(1) + \cos(1)(x-1) + \frac{-\sin(1)}{2!}(x-1)^2 + \frac{-\cos(1)}{3!}(x-1)^3 + \dots$$

$$= \sin(1) + \cos(1)(x-1) + -\frac{\sin(1)}{2}(x-1)^2 - \frac{\cos(1)}{6}(x-1)^3 + \dots$$

$$(1-1.0) \frac{(1)20}{2} - (1-1.0) \frac{(1)112}{6} - (1-1.0)120 + (1.0)112 = (1.0) \text{ (i)}$$

(b) find the values of  $a_0, a_1, a_2, a_3$  if the function is interpolating

by a degree 3 polynomial  $P_3(x)$ .

(i) Lagrange not suitable approximating with brief

$$P_3(x) = a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3 \quad (1.0) \text{ (i)}$$

$$a_0 = f(x_0) = \left[ \sin(1) \right] = (1.0) \quad \text{for constant terms}$$

$$a_1 = \frac{f'(x_0)}{1!} = \left[ \cos(1) \right] =$$

$$a_2 = \frac{f''(x_0)}{2!} = \frac{0 - \sin(1)}{2} = 0.0 \quad (1.0) \text{ (i)}$$

$$a_3 = \frac{f'''(x_0)}{3!} = \frac{-\cos(1)}{6} = -0.16666666666666666 \quad (1.0) \text{ (i)}$$

$$-18.01 =$$

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② Compute  $f(0.1)$  and  $P_3(0.1)$  up to 7 significant figures

$$+ (0.1)^2 \frac{f''(x)}{2!} = \sin x + (0.1-x) \frac{(0.1)^2}{2!} + (0.1)^2 = (0.1)^2$$

$$+ (1-x) \frac{(1)^2}{2!} + (1-x) \frac{(1)^2}{2!} + (1-x) \frac{(1)^2}{2!} + (1-x) \frac{(1)^2}{2!} = (0.1)^2$$

$$P_3(x) = \sin(1) + \cos(1)(x-1) - \frac{\sin(1)}{2!}(x-1)^2 - \frac{\cos(1)}{3!}(x-1)^3$$

$$+ (1-x) \frac{(1)^2}{2!} + (1-x) \frac{(1)^2}{2!} + (1-x) \frac{(1)^2}{2!} + (1-x) \frac{(1)^2}{2!} =$$

$$P_3(0.1) = \sin(1) + \cos(1)(0.1-1) - \frac{\sin(1)}{2!}(0.1-1)^2 - \frac{\cos(1)}{3!}(0.1-1)^3$$

$$= 0.08004989 \quad [\text{Ans}]$$

③ find the percentage error for interpolating  $f(0.1)$  by  $P_3(0.1)$ .

$$\frac{(1-x)^2}{2!} + (1-x)^2 + (1-x)^2 + (1-x)^2 = (0.1)^2$$

$$\text{Percentage Error} = \left| \frac{f(0.1) - P_3(0.1)}{f(0.1)} \right| \times 100\% = 0.1$$

$$= \left| \frac{0.09983341 - 0.08004989}{0.09983341} \right| \times 100\%$$

$$= \frac{0.01981653236}{0.09983341} \times 100\% = 19.81653236 \times 100\%$$

$$= 19.81653236 \times 100\%$$

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② find the maximum error using 4th degree Taylor's Polynomial while approximating  $f(0.2)$  on interval  $[0, 0.2]$ . give that  $x_0 = 0$

$$f^4(x) = \frac{f^5(x_0)}{5!} (x-x_0)^5$$

from (a)  $f^5(x) = \frac{d^5}{dx^5} (-\cos x) = \sin x$

$$f^5(0.2) = \frac{f^5(\xi)}{5!} (x_0 - 0)^5 = \frac{f^5(\xi)}{5!} x_0^5$$

from (a):  $f^5(0) = 0$  and  $x_0 = 0.2$

$$f^5(0) = -\cos(0) ; 0 = 1$$

$$f^5(0.2) = \frac{d}{dx} (-\cos x) = \sin x$$

$$f^5(x) = \frac{d^5}{dx^5} (\sin x) \Rightarrow \cos x$$

$$\text{Now, } f^5(0.2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0.98000665$$

$$\begin{bmatrix} f(0) \\ f'(0) \\ f''(0) \\ f'''(0) \\ f''''(0) \end{bmatrix}$$

$$1 + \text{error} = 2 \text{ mod } 5$$

$$1 - \varepsilon = \text{error} -$$

$$\begin{bmatrix} 0.98000665 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = V$$

$$|F(x) - P_4(x)| = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} x^5$$

at  $x = 0.2$

$$|F(0.2) - P_4(0.2)| \leq \frac{0.98000665}{120} \times (0.2)^5$$

$$\leq 2.61331 \times 10^{-6} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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Vandermonde Matrix with basis  $f(x) = xe^x$ . We are going to find interpolating polynomial by using the Vandermonde matrix.

① Construct the vandermonde matrix if  $f(x)$  passes through the nodes  $\{-1, 0, 1\}$

$$x_0 = -1; f(x_0) = -1e^1 = -0.3678$$

Basis = degree + 1

$$\begin{aligned} \text{degree} &= 3 - 1 \\ &= 2 \end{aligned}$$

$$V = \begin{bmatrix} 1 & x_0^1 & x_0^2 \\ 1 & x_1^1 & x_1^2 \\ 1 & x_2^1 & x_2^2 \end{bmatrix} = \begin{bmatrix} 1 & (-1)^1 & (-1)^2 \\ 1 & (0)^1 & (0)^2 \\ 1 & (1)^1 & (1)^2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{bmatrix} = \begin{bmatrix} (0.0) e^0 - (0.0) 1 \\ (0.0) e^1 - (0.0) 1 \\ (0.0) e^2 - (0.0) 1 \end{bmatrix}$$

⑥ Compute  $\det(V)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 1(0-0) + 1(1-0) + 1(0-0) = (25.0) \times 1 + (25.0) \times 25.0 = (25.0) \times 1 + 1 = 25.0 =$$

⑦ Write expression of  $P_2(x)$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{bmatrix} =$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0.5 & -1 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} -0.3678 \\ 221.0 \\ 2.71828 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ 1.575 \\ 1.1752 \end{bmatrix}$$

$$\begin{aligned} \therefore P_2(x) &= a_0 + a_1 x + a_2 x^2 \\ &= 0 + 1.575x + 1.1752x^2 \\ &= 1.575x + 1.1752x^2 \end{aligned}$$

d) Find the Percentage Error for  $0.25$  (v) Job swapped (P)

$$F(0.25) = 0.25e^{0.25} = 0.3210063542$$

$$(0-1)1 + (0-1)1 + (0-0)1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$P_2(0.25) = 1.575 \times (0.25) + 1.1752 (0.25)^2$$

$$= 0.4672$$

$$\text{Error}_P = \left| \frac{F(0.25) - P_2(0.25)}{F(0.25)} \right| \times 100\% \quad \text{To minimize this} \quad (5)$$

$$= \left| \frac{0.3210063542 - 0.4672}{0.3210063542} \right| \times 100\% = A$$

$$= 0.4554229033 \times 100\% = \begin{bmatrix} 0.0 \\ 10 \\ 50 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ -2F1.1 \\ 2F1.1 \end{bmatrix} =$$

$$\therefore x_50 + x_10 + 0 = (15) \quad \dots$$

$$5x2F1.1 + x2F2.1 + 0 =$$

$$\therefore x2F1.1 + x2F2.1 =$$

4. Construct an appropriate polynomial for the following data using Hermite basis by following the question.

	$\alpha$	$f(x)$	$f'(x)$
$x_0 \rightarrow -1$	0	0	$0 + 1(x)_{-1} + 0 + 0(x)_{-1} =$
$x_1 \rightarrow 0$	1	0	$0 + 0 + 0 + 0(x)_{0} =$
$x_2 \rightarrow 1$	0	1	$0 + 0 + 0 + 0(x)_{1} =$

① Find the Lagrange basis from the given data.

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-0)(x-1)}{(x-(-1)-0)(x-1-0)} = \frac{x(x-1)}{2}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x+1)(x-1)}{(0+1)(0-1)} = \frac{x^2-1}{-1}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x+1)(x+0)}{(1+1)(1-0)} = \frac{x(x+1)}{2}$$

② Compute Hermite basis

Total nodes = 3

$$\therefore \text{degree} = 2 \cdot n = 3-1 = 2$$

We know  $(2n+1)$  degree should be made

$$\text{for } n=2 \text{ degree} = (2 \times 2 + 1) = 5 \quad \frac{(1-x)(-x)}{1^3} =$$

Lagrange Polynomial

$$f(x) = f(x_0)l_0 + f(x_1)l_1 + f(x_2)l_2 = \frac{x^2-1}{-1}$$

$$\begin{aligned}
 P_5(x) &= f(x_0) h_0(x) + f(x_1) h_1(x) + f(x_2) h_2(x) + f'(x_0) \hat{h}_0(x) + \\
 &\quad f'(x_1) \hat{h}_1(x) + f'(x_2) \hat{h}_2(x) \\
 &= 0 h_0(x) + 1 h_1(x) + 0 h_2(x) + 1 \hat{h}_0(x) + 0 \hat{h}_1(x) + 1 \hat{h}_2(x) \\
 &= h_1(x) + \hat{h}_0(x) + \hat{h}_2(x)
 \end{aligned}$$

We know,

$$\begin{aligned}
 h_K(x) &= \frac{L_K^2(x)}{(1-x)(0-x)} \left[ 1 - 2(x-x_K) L_K'(x) \right] \quad \text{from 2nd row} \quad \text{③} \\
 \hat{h}_K(x) &= L_K^2(x) (x-x_K) \quad (x-x_K) (1-x)
 \end{aligned}$$

$$\begin{aligned}
 h_1(x) &= -L_1^2(x) \left[ 1 - 2(x-x_1) L_1'(x) \right] \frac{(x-0)(0-x)}{(x-1)(0-x)} = (x)_1 \\
 &= \frac{\left(\frac{x^2-1}{x-1}\right)^2 \left[ 1 - 2x(-2) \right]}{(x-1)(0-x)} = (x)_1 \\
 &= (1-x^2)^2 [1-4x^2] \quad [\text{Ans}]
 \end{aligned}$$

$$\hat{h}_0(x) = L_0^2(x) (x-x_0) \quad \text{2nd row 2nd column} \quad \text{④}$$

$$\begin{aligned}
 &= L_0^2(x) (x-x_0) \\
 &= \left[ \frac{x(x-1)}{2} \right]^2 (x+1) \\
 &= \frac{x^2(x-1)^2}{4} (x+1) \quad [\text{Ans}] = \text{Ans} \quad \text{S = m not}
 \end{aligned}$$

$$\frac{1-x}{1-x} = \sqrt{m} + \sqrt{m} + 0 \sqrt{m}$$

$$\begin{aligned}
 \hat{h}_2(n) &= l_2^2(n) (n-x_2) \\
 &= \left[ \frac{n(n+1)}{2} \right]^2 (n-1) \\
 &= \frac{n^2(n+1)^2}{4} (n-1) \quad [Ans]
 \end{aligned}$$

③ Find expression of interpolating Hermite polynomial

$$\begin{aligned}
 P_S(x) &= h_1(n) + \hat{h}_0(n) + \hat{h}_2(n) \\
 &= (1-x^2)^2 \left[ 1 - \frac{4n^2}{(n-x)(n+x)} \right] + \frac{x^2(n-1)^2}{4} (n+1) + \frac{x^2(n+1)^2}{4} (n-1)
 \end{aligned}$$

④ Use expression to determine the approximate value at  $n=0.5$

$$P_S(n) = (1-x^2)^2 \left[ 1 - \frac{4n^2}{(n-x)(n+x)} \right] + \frac{x^2(n-1)^2}{4} (n+1) + \frac{x^2(n+1)^2}{4} (n-1)$$

$$\begin{aligned}
 P_S(0.5) &= (1-0.25)^2 \left[ 1 - \frac{0.25(0.25)}{(-0.5)+0.5} \right] + \frac{0.25(0.25)^2}{4} (1+1) + \frac{0.25(1.25)^2}{4} (-1) \\
 &= -\frac{1}{128} - \frac{9}{128}
 \end{aligned}$$

$$x = -0.078125 = \frac{[0.25]7 - [1.25]7}{0.25 - 1.25} = [1.25, 0.25]7$$

$$0.25 - [1.25]7 = [0.25, 1.25]7$$

5. Construct an appropriate polynomial for the data using  
Newton's divided difference method:

①  $f(x) = \sin x$  and nodes are  $\{0, \frac{\pi}{2}, \pi\}$ . find the values  
of  $a_0, a_1, a_2$ .

$$f(x) = \sin x$$

$$x_0 = f(x_0) = \sin x_0 = \sin 0 = 0$$

$$x_1 = f(x_1) = \sin(x_1) = \sin(\frac{\pi}{2}) = 1 + (x)_0 \cdot \frac{1}{1!} + (x)_1 \cdot \frac{1}{2!} = (x)_2$$

$$(L) \frac{x_2 - x_1 + f(x_2)}{P} = \frac{\sin(x_2) - \sin(x_1)}{x_2 - x_1} + \frac{1}{1!} + \frac{1}{2!} = 0$$

$$P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

~~$$f[x_0] + f[x_0, x_1] + f[x_0, x_1, x_2]$$~~

$$(L) \frac{f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)}{P} = (x)_2$$

$$(2) \frac{x_0 - (2.0)}{2.0} \frac{f(x_0)}{P} = 0 \quad \frac{(2.0) - 2.0}{P} + [1-1] \frac{(2.0-1)}{1!} = (2.0)$$

$$x_1 = \frac{\pi}{2} \quad f[x_1] = 1$$

$$x_2 = \pi \quad f[x_2] = 0$$

$$\frac{0}{180} - \frac{1}{180} = 0$$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{1 - 0}{\frac{\pi}{2} - 0} = \frac{2}{\pi}$$

$$f[x_0, x_1, x_2] = \frac{f[x_2] - f[x_0]}{x_2 - x_0} = 0$$

$$\text{forward} = \frac{f(x+h) - f(x)}{h} + \frac{f''(x)}{2!} (-h)^2$$

note: If  $h = 2.0$  then  $f(x+h) = f(x+2)$  To avoid this we use  $h = 0.5$  for numerical differentiation.

$$\text{central} = \frac{f(x+h) - f(x-h)}{2h} + \frac{f'''(x)}{3!} (h^3)$$

note: If  $h = 2.0$  then  $f(x+h) = f(x+2)$  and  $f(x-h) = f(x-2)$  which is not possible.

$$\text{forward} = \frac{f(x+h) - f(x)}{h} = \frac{\ln(3) - \ln(2)}{1} = 0.182 \quad \{ h \propto \text{error} \}$$

$$\text{Rounding Error} = \frac{f(x+h) - f(x-h)}{2h} = f'(x) \quad \left. \begin{array}{l} 2.0 \\ 0.5 \end{array} \right\} \begin{array}{l} \text{higher value of } h \text{ is} \\ \text{the cause of higher} \end{array} \text{error.}$$

Error:

$$\left| \text{Actual value of diff} - \text{Value of diff by numerical} \right| = \frac{(x+h) - (x-h)}{2h} = \text{(error) forward}$$

$\{ h^2 \propto \text{error} \}$

$$= \left| \frac{f(x_1+h) - f(x_1-h)}{2h} - \frac{f''(x_1)h^2}{2!} - \frac{f''(x_1)h^2}{2h} \right| \quad \{ \text{for central} \}$$

$$= \left| \frac{f(x_1+h) - f(x_1-h)}{2h} - \frac{f''(x_1)h^2}{2!} - \frac{(1+\delta_1)f(x_1+h) - (1+\delta_2)f(x_1-h)}{2h} \right|$$

$$= \left| -\frac{f''(x_1)h^2}{2!} - \frac{\delta_1 f(x_1+h) - \delta_2 f(x_1-h)}{2h} \right| \quad \left. \begin{array}{l} |a+b| \leq |a| + |b| \\ |\delta_1|, |\delta_2| \leq \epsilon_h \end{array} \right\}$$

$$\leq \left| \frac{f''(x_1)h^2}{2!} \right| + \left| \frac{\delta_1 f(x_1+h) - \delta_2 f(x_1-h)}{2h} \right| =$$

$$\leq \left| \frac{f''(x_1)h^2}{2!} \right| + \epsilon_h \left| \frac{f(x_1+h) + f(x_1-h)}{2h} \right|$$

$h \propto \text{rounding error}$

$h \propto \frac{1}{\text{rounding error}}$

$\left\{ h \propto \frac{1}{\text{rounding error}} \right.$

1. (a) given by  $f(x) = 2x - e^{-6x}$
- Approximate the derivative of  $f(x)$  at  $x_0 = 0.5$  with step size  $h = 0.2$  using forward difference method up to 5 significant figures.

$$f'(x) = \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^f - (x)^f}{h} = \frac{h^f - (h-x)^f}{h} = \frac{h^f - (1-h)^f}{h}$$

$$x_0 = 0.5 \quad (x)^f = \frac{(x-h)^f - (x)^f}{h} = \text{forward difference}$$

$$h = 0.2$$

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}$$

$$\text{forward diff} \quad f'(x_0) = \frac{f(0.5+0.2) - f(0.5)}{0.2} = \frac{(1.2)^f - (0.8)^f}{0.2} = \frac{(1.2)^f - (1-0.2)^f}{0.2}$$

$$= \frac{f(0.7) - f(0.5)}{0.2} = \frac{[2(0.7) - e^{-6(0.7)}] - [2(0.5) - e^{-6(0.5)}]}{0.2}$$

$$= \frac{[1.4 - e^{-4.2}] - [1 - e^{-3}]}{0.2} = \frac{[1.4 - e^{-4.2}] - [1 - e^{-3}]}{0.2}$$

$$= \frac{[1.4 - e^{-4.2}] - [1 - e^{-3}]}{0.2} = \frac{[1.4 - e^{-4.2}] - [1 - e^{-3}]}{0.2} = \frac{[1.4 - e^{-4.2}] - [1 - e^{-3}]}{0.2}$$

$$= \frac{[1.4 - e^{-4.2}] - [1 - e^{-3}]}{0.2} = \frac{[1.4 - e^{-4.2}] - [1 - e^{-3}]}{0.2} = \frac{[1.4 - e^{-4.2}] - [1 - e^{-3}]}{0.2}$$

$$= 2.173957457$$

round off to 5 significant figures

(b) Approximate the derivative of  $f(x)$  at  $x_0 = 0.5$  with step size  $h = 0.2$  using central difference method up to 6 significant figure.

$$f(x) = 2x - e^{-6x}$$

$$x_0 = 0.5$$

$$h = 0.2$$

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} = \frac{f(0.5+0.2) - f(0.5-0.2)}{2 \times 0.2} = 4.799993443$$

$$f'(x_0) = \frac{f(0.5+0.2) - f(0.5-0.2)}{2 \times 0.2} = 2 \times (0.2) [f(0.5+0.2) - f(0.5-0.2)]$$

$$= \frac{f(0.7) - f(0.3)}{0.4} = \frac{f(0.7) - f(0.3)}{0.4}$$

$$= \frac{[2 \times (0.7) - e^{-6(0.7)}] - [2 \times (0.3) - e^{-6(0.3)}]}{(0.4)} = \frac{[2 \times (0.7) - e^{-6(0.7)}] - [2 \times (0.3) - e^{-6(0.3)}]}{0.4}$$

$$= \frac{[1.4 - e^{-4.2}] - [0.6 - e^{-1.8}]}{0.4} = \frac{[1.4 - e^{-4.2}] - [0.6 - e^{-1.8}]}{0.4}$$

$$= \frac{1.385004423 - 0.4347011118}{0.4} = \frac{1.385004423 - 0.4347011118}{0.4}$$

$$= \frac{1.385004423 - 0.4347011118}{0.4} = \frac{1.385004423 - 0.4347011118}{0.4} = 2.375758278$$

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② Calculate the upper bound of truncation error of  $f(x)$  at  $x_0 = 2$  using  $h = 0.1$  in both of the above mentioned methods for the interval  $[2.4, 2.7]$ .

$$f(x) = 2x - e^{-6x} \quad \frac{d}{dx} [2x - e^{-6x}] = 2 - (-6)e^{-6x}$$

$$f'(x) = \frac{d}{dx} [2x - e^{-6x}] = 2 - (-6)e^{-6x} \xrightarrow{\text{at } x=2} = (0.1)^{-1}$$

$$f'(x) = \frac{(2.4+6e^{-6x})^{\frac{1}{2}} - (2.0+6e^{-6x})^{\frac{1}{2}}}{(2.4-2.0)^{\frac{1}{2}}} =$$

For interval  $[2.4, 2.7]$  (with  $h = 0.1$ ) we will get

$$x = 2.4, 2.5, 2.6, 2.7 \quad \frac{(2.4+6e^{-6x})^{\frac{1}{2}} - (2.0+6e^{-6x})^{\frac{1}{2}}}{(2.4-2.0)^{\frac{1}{2}}} =$$

question ① forward method  $\xrightarrow{P.0}$

$$\left[ f(x+h) - f(x) \right] = \left[ \frac{(2.5+6e^{-6 \times 2.4})^{\frac{1}{2}} - (2.4+6e^{-6 \times 2.4})^{\frac{1}{2}}}{(0.1)^{\frac{1}{2}}} \right] =$$

Actual value:

$$\left[ \frac{(2.5+6e^{-6 \times 2.4})^{\frac{1}{2}} - (2.4+6e^{-6 \times 2.4})^{\frac{1}{2}}}{(0.1)^{\frac{1}{2}}} \right] = \frac{14.41 - 14.49}{0.1} =$$

$$= 2.00000557 \approx 2.000003344$$

Forward value:

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$$f'(2.4) = \frac{f(2.4+0.1) - f(2.4)}{0.1} =$$

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$$\begin{aligned}
 \text{Backward Value} &= \frac{f(2.5) - f(2.4)}{0.1} \\
 &= \frac{[2(2.5) - e^{-6(2.5)}] - [2(2.4) - e^{-6(2.4)}]}{0.1} \\
 &= \frac{[5 - e^{-15}] - [4.8 - e^{-14.4}]}{0.1} \\
 &= \frac{4.099999694 - 4.799999443}{0.1} \\
 &= 2.00000251
 \end{aligned}$$

Since  $(2.4 - h)$  is out-of-limit there is no backward value.

Truncation Error:  $1.0$

$$\text{Error} = \left| \frac{3 - (2.8)S}{1.0} - \frac{3 - (2.5)S}{1.0} \right|$$

$$\left| \frac{3 - 2.8 - [2.8 - 2.7]}{1.0} - \frac{3 - 2.5 - [2.5 - 2.4]}{1.0} \right| = \left| \frac{2.000000557 - 2.00000251}{1.0} \right|$$

$$\begin{aligned}
 &= \left| 2.000003344 - 2.00000251 \right| \\
 &= 8.3434221 \times 10^{-7}
 \end{aligned}$$

$$83100000.0 =$$

$$\text{Error} = \left| 83100000.0 - 83810000.0 \right|$$

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$$\begin{aligned}
 \text{Backward Value} &= \frac{f(2.5) - f(2.4)}{0.1} \\
 &= \frac{[2(2.5) - e^{-5(2.5)}] - [2(2.4) - e^{-5(2.4)}]}{0.1} \\
 &= \frac{[5 - e^{-15}] - [4.8 - e^{-14.4}]}{0.1} \\
 &= \frac{4.0000000000000005 - 4.799999443}{0.1} \\
 &= 2.00000251
 \end{aligned}$$

Since  $(2.4 - h)$  is out-of-limit there is no backward value.

2. Truncation Error:  $1.0$

$$\text{Error} = \left| \frac{f(2.5) - f(2.4)}{0.1} - \frac{f(2.5) - f(2.5)}{0.1} \right| = \left| \frac{2.000000557 - 2.00000251}{0.1} \right|$$

$$\begin{aligned}
 &= \left| 2.000003344 - 2.00000251 \right| \\
 &= 8.3434221 \times 10^{-7}
 \end{aligned}$$

$$8.3434221 \times 10^{-7}$$

$$F_{0.1 \times 22.1} = \left| 8.3434221 \times 10^{-7} - 8.3434221 \times 10^{-7} \right|$$

Error:

For  $x = 2.5$ 

$$(1.8) + (2.8) =$$

Actual value:

$$f(2.5) = 2 + 6e^{-6 \times 2.5} =$$

$$1.0 = 2.000001835$$

Forward Value:

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$$f(2.5+0.1) - f(2.5) =$$

$$f'(2.5) = \frac{f(2.5+0.1) - f(2.5)}{0.1}$$

$$12800000.8$$

or browser on 2.6 went =  $f(2.6) - f(2.5) (1 - 1.8)$ 

$$0.1$$

$$f(2.6) - f(2.5) = \frac{[2(2.6) - e^{-6 \times 2.6}] - [2(2.5) - e^{-6 \times 2.5}]}{0.1}$$

Actual value:

$$f(2.6) - f(2.5) = \frac{[5.2 - e^{-15.6}] - [5 - e^{-15}]}{0.1}$$

$$12800000.8 - 1220000.8 = 5.19999832 - 4.999999694$$

$$f(2.6) - f(2.5) = 0.1$$

$$= 2.00000138$$

Error:

$$|2.000001835 - 2.00000138| = 4.88 \times 10^{-7}$$

Backward Value =

$$f(2.5) = 10$$

$$(2.5) f(n) = \frac{f(n) - f(n-h)}{h} \text{ Backward}$$

$$f(2.5) = \frac{f(2.5) - f(2.5 - 0.1)}{0.1}$$

$$= \frac{f(2.5) - f(2.4)}{0.1} = \frac{f(2.5) - [2(2.4) - e^{-6(2.4)}]}{0.1}$$

$$= \frac{f(2.5) - [2(2.4) - e^{-14.4}]}{0.1} = \frac{f(2.5) - [5 - e^{-14.4}]}{0.1}$$

$$= \frac{f(2.5) - [5 - e^{-14.4}]}{0.1} = \frac{4.099999694 - 4.799999443}{0.1} = -7.99999777$$

$$\text{Error: } [5 - e^{-14.4}] - [5 - e^{-15.2}] = 2.00000251$$

$$| 2.000001835 - 2.00000251 |$$

$$= 6.75 \times 10^{-7}$$

$$= 6.75 \times 10^{-7}$$

Error:

$$| 6.75 \times 10^{-7} - 6.75 \times 10^{-7} |$$

$$= 0 \times 10^{-7}$$

for  $x = 2.6$ ;

By using Taylor

Actual Value:  $\frac{f(x+h) - f(x)}{h} = f'(x)$

$$f'(2.6) = 2 + 6e^{-6(2.6)}$$

$$\frac{(2.0 - 2.6) f - (2.6) f}{1.0} = 2.000001007$$

Forward Value:

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$$\frac{[2.7] - [2.6]}{1.0} = \frac{f(2.7) - f(2.6)}{0.1}$$

$$\frac{[2(2.7) - e^{-6(2.7)}] - [2(2.6) - e^{-6(2.6)}]}{0.1}$$

$$= [5.4 - e^{-16.2}] - [5.2 - e^{-15.6}]$$

$$\frac{[5.4 - e^{-16.2}] - [5.2 - e^{-15.6}]}{0.1}$$

$$= \frac{5.399999908 - 5.19999832}{0.1}$$

$$= 2.000000759$$

Error:

$$\left| 2.000001007 - 2.000000759 \right|$$

$$= 2.48 \times 10^{-7}$$

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Backward Value:

$$f'(n) = \frac{f(n) - f(n-h)}{h} \quad \text{if } h = 0.1$$

We know that

$$f'(2.6) = \frac{f(2.6) - f(2.6-0.1)}{0.1}$$

$$822000000.1 =$$

$$f(2.6) - f(2.5)$$

$$= [2(2.6) - e^{-6(2.6)}] - [2(2.5) - e^{-6(2.5)}]$$

$$\frac{(2.6) - (2.5)}{0.1} = [5.2 - e^{-15.6}] - [5 - e^{-15}]$$

$$= \frac{0.1}{0.1} = 5.199999832 - 4.999999694$$

$$\frac{[2.6] - [2.5]}{0.1} = \frac{[5.2] - [5 - 1.2]}{0.1} = 2.00000138$$

$$2.000001007 - 2.0000138$$

$$= 3.73 \times 10^{-7}$$

$$= 8.0346518821$$

$$F_{01} \times F_{0.1} =$$

For  $x = 2.7$ ;

$$\frac{\text{Actual Value} - \text{fit}}{(x-h)^1 - (x)^1} = (x)^1 - 6(2.7)$$

$$\frac{(1.0 - 2.5) + - (2.5)^1(2.7)}{1.0} = (2.5)^1 + 6e$$

$$= 2.000000553$$

Since  $(2.7+h)$  is out of limit, there is no forward value

Backward Value:

$$\frac{f(x) - f(x-h)}{h} = \frac{f(2.7) - f(2.6)}{h}$$

$$= \frac{[5.4 - e^{-16.2}] - [5.2 - e^{-15.6}]}{0.1}$$

$$= 5.3999999908 - 5.19999832$$

Error:

$$\left| 2.000000553 - 2.00000076 \right|$$

$$= 2.07 \times 10^{-7}$$

(d) given that,  $(S(N-0.5))^\frac{1}{2} - (S(N+0.5))^\frac{1}{2} = 25.0$

$$x_0 = 0.2$$

We know that,

$$D_{n,0} = \frac{f(n+h) - f(n-h)}{2h}$$

$$D_{n,0} = \frac{f(n_0+h_2) - f(n_0-h_2)}{(20.0)^\frac{1}{2} 2(h_2) \cdot 0} =$$

Now,

$$D_{0.5} = \frac{f(0.2+0.5) - f(0.2-0.5)}{2(0.5)} =$$

$$= \frac{f(0.7) - f(-0.3)}{2(0.5)} =$$

$$= \frac{2 + 6e^{-6(0.9)}}{2(0.5)} - \frac{2(-0.3) - e^{-6(-0.3)}}{2(0.5)} =$$

$$= \frac{808828e^{1.8} + 888142e^{-5.4}}{2} =$$

$$= 1.385004423 + 6.649647464$$

$$= \frac{8.034651882}{1 - e^{-5.4}} = D_{n,0}$$

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$$D_{0.25} = \frac{f(x_0 + \frac{h}{2}) - f(x_0 - \frac{h}{2})}{2(\frac{h}{2})} \quad \text{start now}$$

$$= \frac{f(0.2 + 0.25) - f(0.2 - 0.25)}{2(0.25)} \quad \text{start now}$$

$$= \frac{(0.45) f - (0.05) f}{2(0.25)} = \frac{0.4 f}{0.5} = 0.8 f$$

$$= \frac{f(0.45) - f(-0.05)}{0.5}$$

$$= \frac{(2.0 - 0.0) f - \frac{0.5}{6(0.45) - 0.0} f}{0.5} = \frac{[2(0.45) - e^{(2.0)}] f}{0.5} - \frac{[2(0.05) - e^{-0.05}] f}{0.5}$$

$$= \frac{[0.9 - e^{-2.7}] f - [-0.1 - e^{0.3}] f}{0.5} =$$

$$= \frac{[0.9 - e^{-2.7}] f - [-0.1 - e^{0.3}] f}{0.5} =$$

$$= \frac{0.8327944873 f + 1.449858808 f}{0.5} =$$

$$= 4.565306591 f + 6.599003061 =$$

$$D_h^1 = \frac{(2)^2 D_{h/2} - D_h 88122160.8}{(2)^2 - 1} = \frac{22160.8}{22160.8} = \textcircled{a}$$

$$\text{OR, } D_{0.5}^{(1)} = \frac{(2)^L D_{0.25} - D_{1/2-N}}{(2)^2 - 1} (1r)^T + (\text{const}) = (1r)^T$$

$$= 4 (4.565306501) - (8.03465)$$

$$= 10.22657636$$

$$(P_N) 0 + (N) \frac{(1r)^T}{12} + \frac{(N) (1r)^T}{3} + (N) (1r)^T + (1r)^T =$$

$$= 3.408858787$$

Now,

$$f(n) = 2n - e^{(1r)^T}$$

$$f'(n) = \frac{d}{dn} [2n - e^{(1r)^T}] = \frac{(n-1r)^T - (n+1r)^T}{12} = 0$$

$$f'(0.2) = 2 + 6e^{-(0.2)} = 3.807165271$$

$$\text{Error} = 3.807165271 - 3.408858787$$

$$= 0.3983064845$$

$$= 0.398306$$

$$(P_N) 0 + (N) \frac{(1r)^T}{12} + (N) \frac{(1r)^T}{3} + (N) (1r)^T = 0$$

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots \quad (1)$$

$$f(x_1) = f(x_0) + f'(x_0)(x_1-x_0) + \frac{f''(x_0)}{2!}(x_1-x_0)^2 + \dots$$

$$f(x_1+h) = f(x_1) + f'(x_1)(x_1+h-x_1) + \frac{f''(x_1)}{2!}(x_1+h-x_1)^2 + \dots$$

$$= f(x_1) + f'(x_1)h + \frac{f''(x_1)}{2!}h^2 + \frac{f'''(x_1)}{3!}h^3 + O(h^4)$$

$$f(x_1-h) = f(x_1) + f'(x_1)(x_1-h-x_1) + \frac{f''(x_1)}{2!}(x_1-h-x_1)^2 + \dots$$

$$D_h = \frac{f(x_1+h) - f(x_1-h)}{2h} \left[ \frac{f''(x_1)}{2!} + \frac{f''(x_1)}{2!}h^2 \right] = (x_1)^2$$

$$= \frac{1}{2h} f(x_1+h) - f(x_1-h)$$

$$= f'(x_1) + \frac{f''(x_1)h^2}{2!} + \frac{f'''(x_1)h^4}{4!} + O(h^6)$$

$$\boxed{\text{Value}} \quad \boxed{\text{Error}}$$

$$\text{or, } D_h = f'(x_1) + \frac{f''(x_1)}{2!}h^2 + \frac{f'''(x_1)}{4!}h^4 + O(h^6)$$

$$\text{or, } D_{h/2} = f'(x_1) + \frac{f''(x_1)}{2!}(h/2)^2 + \frac{f'''(x_1)}{4!}(h/2)^4 + O(h^6)$$

$$D_h = \frac{f(x+h) - f(x-h)}{2h}$$

$$f(x \pm h) = f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \frac{f''''(x)}{4!}h^4 + \frac{f''''''(x)}{5!}h^5 + O(h^6)$$

$$f(x-h) = f(x) - f'(x)h + \frac{f''(x)}{2!}h^2 - \frac{f'''(x)}{3!}h^3 + \frac{f''''(x)}{4!}h^4 - \frac{f''''''(x)}{5!}h^5 + O(h^6)$$

Now,

$$D_h = \frac{f(x+h) - f(x-h)}{2h}$$

$$= \frac{1}{2h} \left[ 2f'(x)h + 2 \frac{f''(x)}{3!}h^3 + 2 \frac{f''''(x)}{5!}h^5 + O(h^7) \right]$$

$$D_h = f'(x) + \left[ \frac{f''(x)h^4}{3!} + \frac{f''''(x)h^4}{5!} \right] + O(h^6)$$

$$D_{h/2} = f'(x) + \frac{f''(x)}{3!} \left( \frac{h}{2} \right)^2 + \frac{f''''(x)}{5!} \left( \frac{h}{2} \right)^4 + O(h^6)$$

$$= f'(x) + \left[ \frac{f''(x)}{3!} \frac{h^2}{4} \right] + \frac{f''''(x)}{5!} \frac{h^4}{16} + O(h^6)$$

$$4D_{h/2} - D_h = 3f'(x) + 3 \frac{f''''(x)}{5!} \frac{h^4}{16} + O(h^6)$$

$$\frac{4D_{h/2} - D_h}{2} = f'(x) + \frac{f''''(x)}{5!} \frac{h^4}{8} + O(h^6) \rightarrow D_n^{(1)}$$

2. (a) given that

$$D_h^{(1)} = f'(x_0) - \frac{h^4}{480} f^{(5)}(x_0) + O(h^6)$$

$$D_{h/2}^{(1)} = f'(x_0) - \frac{(h/2)^4}{480} f^5(x_0) + O(h^6)$$

$$(D_N) = f'(x_0) - \frac{f^5(x_0)}{480} \frac{h^4}{16} + O(h^6) = f'(x_0) + O(h^6)$$

Now,  $D_h^{(1)} - D_{h/2}^{(1)} = 16 f'(x_0) - \frac{f^5(x_0) h^4}{480} + O(h^6) - f'(x_0) + \frac{h^4 f^5(x_0)}{480}$

$$16 D_{h/2}^{(1)} - D_h^{(1)} = 15 f'(x_0) + O(h^6)$$

$$\frac{16 D_{h/2}^{(1)} - D_h^{(1)}}{15} = f'(x_0) + O(h^6) = D_h^{(2)}$$

(b)  $D_h = \frac{f(x+h) - f(x-h)}{2h}$

$$f(x+h) = f(x) + f'(x)h + \frac{f^2(x)h^2}{2!} + \frac{f^3(x)h^3}{3!} + \frac{f^4(x)h^4}{4!} + \frac{f^5(x)h^5}{5!} + \dots$$

$$f(x-h) = f(x) - f'(x)h + \frac{f^2(x)h^2}{2!} - \frac{f^3(x)h^3}{3!} + \frac{f^4(x)h^4}{4!} - \frac{f^5(x)h^5}{5!} + \dots$$

$$D_h = \frac{1}{2h} \left[ 2f'(n)h + \frac{2f''(n)h^3}{3!} + \frac{2f'''(n)h^5}{5!} + O(h^7) \right] \quad \text{Ansatz ①}$$

$$D_h = f'(n)h + \frac{f''(n)h^2}{2!} + \frac{f'''(n)h^4}{4!} + O(h^6) = \frac{(1)D_h}{h^2} \quad \text{Ansatz ②}$$

$$D_{h/3} = f'(n) + \frac{f''(n)}{3!} \left(\frac{h}{3}\right)^2 + \frac{f'''(n)}{5!} \left(\frac{h}{3}\right)^4 + O(h^6)$$

$$D_{h/3} = f'(n) + \frac{f''(n)}{3!} \frac{h^2}{9} + \frac{f'''(n)}{5!} \frac{h^4}{81} + O(h^6) \quad \text{Ansatz ③}$$

$$9D_{h/3} = 9f'(n) + \frac{f''(n)}{3!} \frac{h^2}{9} + \frac{f'''(n)}{5!} \frac{h^4}{81} + O(h^6) \quad \text{Ansatz ④}$$

Now,

$$9D_{h/3} - D_h = 9f'(n) - f'(n) + \frac{f''(n)h^2}{3!} - \frac{f''(n)h^2}{3!} + \frac{(f'''(n)h^4) - (f'''(n)h^4)}{5!} + O(h^6) = O(h^6)$$

$$9D_{h/3} - D_h = 8f'(n) - \frac{8}{5!} f'''(n)h^4 + O(h^6) \quad \text{Ansatz ⑤}$$

$$9D_{h/3} - D_h = 8 \left[ f'(n) - \frac{1}{5!} f'''(n)h^4 + O(h^6) \right]$$

$$9D_{h/3} - D_h = \frac{8f'(n)h^2}{12} + \frac{8f''(n)h^4}{120} + \frac{8f'''(n)h^6}{720} + \frac{8f''''(n)h^8}{5040} = (n+1)h^2$$

$$\frac{9D_{h/3} - D_h}{8} = f'(n) - \frac{1}{5!} f'''(n)h^4 + O(h^6)$$

$$D_h^{(1)} = f'(n) - \frac{1}{5!} f'''(n)h^4 + O(h^6) = (n+1)h^2$$

$$D_h^{(1)} = f'(n) - \frac{1}{5!} f'''(n)h^4 + O(h^6)$$

① from question ⑥;

$$D_h^{(1)} = f'(x) - \frac{1}{6} \frac{f^{(5)}(x) h^4}{5!} + O(h^6)$$

Here error term is:  $-\frac{1}{6} \frac{f^{(5)}(x) h^4}{5!} + O(h^6)$  where the leading error is  $-\frac{1}{6} \frac{f^{(5)}(x) h^4}{5!}$

Error bound is the absolute value of the error term

$$\left| \frac{P(x) - f(x)}{2x} \right| = \left| \frac{1}{6} \frac{f^{(5)}(x) h^4}{5!} + O(h^6) \right|$$

$$\text{on, } \left| \frac{1}{6} \frac{f^{(5)}(x) h^4}{5!} + O(h^6) \right| \leq \frac{P(x)}{2x} = \frac{1}{2(1)} = (1)^{27} \cdot \sqrt{10}$$

② given that,

$$f(x) = \ln(x)$$

$$x_0 = 1$$

$$h = 0.1$$

Error term from the question ⑥:  $\left| \frac{1}{6} \frac{f^{(5)}(x) h^4}{5!} + O(h^6) \right|$

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x} \quad (x) 0 + \frac{P_N(x)^{(2)}}{12} \cdot 1 - (x)^{1/2} = (x)^{1/2}$$

$$f''(x) = \frac{d}{dx} \left( \frac{1}{x} \right) = -\frac{1}{x^2} \quad \text{2i most part here}$$

$$f''(x) = \frac{d}{dx} \left( -\frac{1}{x^2} \right) = \frac{d}{dx} \left( -x^{-2} \right) = -[-2x^{-3}] = 2x^{-3} = \frac{2}{x^3}$$

$$f''(x) = \frac{d}{dx} \left( \frac{2}{x^3} \right) = \frac{d}{dx} \left( 2x^{-3} \right) = 2(-3)x^{-4} = -6x^{-4} = -\frac{6}{x^4}$$

$$f''(x) = \frac{d}{dx} \left( -\frac{6}{x^4} \right) = \frac{d}{dx} \left( -6x^{-4} \right) = -6x^{-4} \cdot x^{-5} = 24x^{-5} = \frac{24}{x^5}$$

$$f''(x) = \frac{24}{x^5}$$

$$\text{or, } f''(1) = \frac{24}{(1)^5} = 24 \quad \left| \frac{P_N(x)^{(2)}}{12} \right| \quad \text{most part}$$

Now,

$$\frac{1}{9} \left| \frac{f''(x)h^4}{5 \times 4 \times 3 \times 2 \times 1} \right|$$

$$(x) M = (x)^{1/2} \quad \text{most part} \quad \text{⑥}$$

$$L = 0.1$$

$$1 \cdot 0 = 1$$

$$= \frac{1}{9} \left| \frac{24(0.1)^4}{120} \right|$$

most part with most most most

$$= \frac{1}{9} \left| \frac{1}{50000} \right|$$

$$= \frac{1}{450000} \quad \text{or} \quad 2.2223 \times 10^{-6}$$

All Equation item establish

Machine Epsilon:

Normalized  $\rightarrow \frac{1}{2} \beta^{-m}$

Demormalized  $\rightarrow \frac{1}{2} \beta^{-m}$

Maximum Delta value:  $\frac{1}{2} \beta^{1-m}$

Lagrange Basis:  $L_0 = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$   $L_1 = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$   $L_2 = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$

Lagrange Polynomial:  $f(x_0)L_0 + f(x_1)L_1 + f(x_2)L_2$

Relative Error,  $(\%) = \left| \frac{f(x) - P(x)}{f(x)} \right| \times 100$  for  $n = n$

Truncation Error:

given  $\rightarrow h = 1, 0.1, 0.01, 0.001$

$$f(x) = 6e^{-5x}, x_0 = 2$$

Value h	forward; $x_0 = 2$	Error
0.1	$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}$	$f(x) = 6e^{-5x}$ $f'(x) = -30e^{-5x}$ $f'(2) = -30e^{-10}$ error = $[f'(2) - \text{forward}]$
0.01		
0.001	where $x_0 = 2$	

Value h	Central; $x_0 = 2$	Error
0.1	$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$	$f(x) = 6e^{-5x}$ $f'(x) = -30e^{-5x}$ $f'(2) = -30e^{-10}$ error = $[f'(2) - \text{central}]$
0.01		
0.001	where $x_0 = 2$	

Newton's divided difference method

$$P_2(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1)$$

$$= f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1)$$

$$f(x) = \sin x \rightarrow \text{nodes } \{0, \frac{\pi}{2}, \pi\}$$

$$x_0 = 0; f[x_0] = \sin 0 = 0$$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{1 - 0}{\frac{\pi}{2} - 0} = \frac{2}{\pi}$$

$$x_1 = \frac{\pi}{2}; f[x_1] = \sin \frac{\pi}{2} = 1$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{0 - 1}{\pi - \frac{\pi}{2}} = \frac{2}{\pi}$$

$$x_2 = \pi; f[\pi] = \sin \pi = 0$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{2}{\pi}$$

$$\begin{array}{|c|c|} \hline \text{Value} & \text{Value} \\ \hline \frac{2}{\pi} & 1.0 \\ \frac{1}{\pi} = \frac{1}{3.14} & 10.0 \\ \frac{1}{\pi^2} = \frac{1}{9.86} & 100.0 \\ \frac{1}{\pi^3} = \frac{1}{31.4} & \\ \hline \end{array}$$

$$a_0 = f[x_0] = 0$$

$$a_1 = f[x_0, x_1] = \frac{2}{\pi}$$

$$a_2 = f[x_0, x_1, x_2] = -\frac{4}{\pi^2}$$

Polynomial Formulas for equi-spaced nodes

$$a_0 + a_1 \frac{(x-x_0)^1}{1!} + a_2 \frac{(x-x_0)^2}{2!} + \dots + a_N \frac{(x-x_0)^N}{N!} + f(x)^1 + f(x)^2 + \dots + f(x)^N = (N+1) f$$

$$0 + \frac{2}{\kappa} [x-0] + -\frac{4}{\kappa^2} [x-0] [x-\frac{\pi}{2}]$$

$$+ \frac{2}{\kappa} \frac{(x)^2 f}{1!} - \frac{4}{\kappa^2} \frac{(x)^3 f}{2!} + \frac{2}{\kappa} \frac{(x)^4 f}{3!} - \frac{4}{\kappa^2} \frac{(x)^5 f}{4!} + \dots - (x)^N f = (N+1) f$$

$$= \frac{2\pi}{\kappa} - \frac{4\pi^2}{\kappa^2} \left[ x^2 - \frac{3\pi^2}{2} \right] \quad (-)$$

Adding a new node:  $\left\{ 0, \frac{\pi}{2}, \frac{3\pi}{2} \right\}$  (from the previous):

$$x_0 = 0 \quad \frac{2}{\kappa} (x)^2 f + \frac{2}{\kappa} (x)^4 f + f(x)^2 f = (N+1) f - (N+2) f$$

$$[x_0, x_1] = \frac{2}{\kappa} \frac{1}{12}$$

$$x_1 = \pi \cdot \frac{1}{3} \quad [x_0, x_1, x_2] = \frac{-\frac{2}{\kappa} - \frac{2}{\kappa}}{x_1 - x_0} f = \frac{4}{\kappa^2} f$$

$$[x_1, x_2] = \frac{-2}{\kappa} \frac{1}{12} \quad [x_1, x_2, x_3] = \frac{-\frac{2}{\kappa} - \frac{2}{\kappa}}{\frac{3\pi}{2} - \frac{\pi}{2}} f = \frac{4}{\kappa^2} f$$

$$x_2 = 0 \quad [x_2, x_3] = \frac{f(x_3) - f(x_2)}{x_3 - x_2} f = \frac{-1 - 0}{\frac{3\pi}{2} - \pi} f = \frac{2}{\kappa} f$$

$$x_3 = \sin \frac{3\pi}{2} \cdot \theta = \frac{1}{3} (\pi) \frac{(x)^2 f}{12} + \frac{1}{3} (\pi) \frac{(x)^4 f}{16} + f(x)^2 f = \frac{1}{3} \pi f \quad (5)$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0} =$$

$$\frac{(\frac{1}{3}\pi)0 + \frac{1}{3}\pi \frac{(-\frac{4}{\kappa^2} + \frac{4}{\kappa^2})f}{12}}{\frac{3\pi}{2} - 0} = \frac{\frac{1}{3}\pi \frac{2}{\kappa^2} f}{\frac{3\pi}{2}} = -\frac{16}{3\kappa^3}$$

## Richardson Extrapolation

$$f(x+h) = f(x) + f'(x) \cdot h + \frac{f''(x)}{2!} h^2 + \frac{f'''(x)}{3!} h^3 + \frac{f''''(x)}{4!} h^4 + \frac{f''''''(x)}{5!} h^5 + \dots$$

$$f(x-h) = f(x) - f'(x)h + \frac{f''(x)}{2!} h^2 - \frac{f'''(x)}{3!} h^3 + \frac{f''''(x)}{4!} h^4 - \frac{f''''''(x)}{5!} h^5 + \dots$$

(-)

$$f(x+h) - f(x-h) = 2f'(x)h + 2\frac{f''(x)}{3!} h^3 + 2\frac{f''''(x)}{5!} h^5$$

$$f(x+h) - f(x-h) = 2\frac{f'(x)}{1!} h + 2\frac{f''(x)}{3!} h^3 + 2\frac{f''''(x)}{5!} h^5 \quad 0 = 0$$

Now,

$$D_h = \frac{f(x+h) - f(x-h)}{2h} = \left[ \text{error term} \right] \quad L = 10$$

$$\frac{1}{2h} \left[ 2f'(x)h + 2\frac{f''(x)}{3!} h^3 + 2\frac{f''''(x)}{5!} h^5 \right] = \left[ \text{error term} \right]$$

$$BD_h = f'(x) + \frac{f''(x)}{3!} h^2 + \frac{f''''(x)}{5!} h^4 + \left[ \text{error term} \right] \quad \text{--- (a)}$$

$$D_{h/2} = f'(x) + \frac{f''(x)}{3!} \left( \frac{h}{2} \right)^2 + \frac{f''''(x)}{5!} \left( \frac{h}{2} \right)^4 + \left[ \text{error term} \right] = f'(x) + \frac{f''(x)}{3!} \frac{h^2}{4} + \frac{f''''(x)}{5!} \frac{h^4}{16} + \left[ \text{error term} \right] \quad \text{--- (b)}$$

$$4D_{h/2} = 4f'(x) + \frac{f''(x)}{3!} h^2 + \frac{f''''(x)}{5!} \frac{h^4}{4} + \left[ \text{error term} \right] \quad \text{--- (c)}$$

$$\frac{D_h - D_{h/2}}{3!} = \frac{0 - \frac{f''''(x)}{5!} h^4}{8}$$

$$4D_{h_2} - D_n = \frac{3}{2} f'(n) - \frac{3}{4} \frac{f^3(n)}{5!} h^4 + O(h^6)$$

Applying formula for 3rd order difference

$$\frac{4D_{h_2} - D_n}{3} = f'(n) - \frac{1}{4} \frac{f^3(n)}{5!} h^4 + O(h^6)$$

Equation for 2nd order difference

$$\frac{4D_{h_2} - D_n}{(2)^2 - 1} = f'(n) - \frac{1}{4} \frac{f^3(n)}{5!} h^4 + O(h^6)$$

Applying formula for 1st order difference

$$D_{h_3} = f'(n) + \frac{f^3(n)}{3!} (h_3)^2 + \frac{f^5(n)}{5!} (h_3)^4 + O(h^6)$$

$$= f'(n) + \frac{f^3(n)}{3!} \frac{h^2}{9} + \frac{f^5(n)}{5!} \frac{h^4}{81} + O(h^6) \quad \frac{(n-x)}{18}$$

$$9D_{h_3} - D_n = 9f'(n) + \frac{f^3(n)}{3!} h^2 + \frac{f^5(n)}{5!} h^4 + O(h^6)$$

$$9D_{h_3} - D_n = 8f'(n) - \frac{8}{9} \frac{f^5(n)}{5!} h^4 + O(h^6)$$

$$\frac{9D_{h_3} - D_n}{8} = f'(n) - \frac{1}{9} \frac{f^5(n)}{5!} h^4 + O(h^6)$$

$$\frac{9D_{h_3} - D_n}{(3)^2 - 1} = f'(n) - \frac{1}{9} \frac{f^5(n)}{5!} h^4 + O(h^6)$$

FPOS

$$FPOS = 1 - S = 9/16$$

1. SX(1 ... 111) - minimum without max error

2. SX(0 ... 001) without without max error

$$\frac{4D_{\gamma_2} - D_n}{4} = 5f(n) - \frac{2}{3} \frac{f''(n)}{5!} h^4 + O(h^6)$$

$$\frac{4D_{\gamma_2} - D_n}{5} = f(n) - \frac{1}{3} \frac{f''(n)}{5!} h^4 + O(h^6)$$

$$\frac{4D_{\gamma_2} - D_n}{(2)^2 - 1} = f(n) - \frac{1}{3} \frac{f''(n)}{5!} h^4 + O(h^6)$$

$$\textcircled{6} \quad D_{\gamma_2} = f(n) + \frac{f''(n)}{3!} (\gamma_2)^2 + \frac{f''(n)}{5!} (\gamma_2)^4 + O(h^6)$$

$$= f(n) + \frac{f''(n)}{3!} \frac{h^2}{9} + \frac{f''(n)}{5!} \frac{h^4}{81} + O(h^6)$$

$$9D_{\gamma_2} = 9f(n) + \frac{9f''(n)}{3!} h^2 + \frac{9}{5!} \frac{f''(n)}{5!} h^4 + O(h^6)$$

$$9D_{\gamma_2} - D_n = 8f(n) - \frac{2}{3} \frac{f''(n)}{5!} h^4 + O(h^6)$$

$$\frac{9D_{\gamma_2} - D_n}{8} = f(n) - \frac{1}{3} \frac{f''(n)}{5!} h^4 + O(h^6)$$

$$\frac{9D_{\gamma_2} - D_n}{(3)^2 - 1} = f(n) - \frac{1}{3} \frac{f''(n)}{5!} h^4 + O(h^6)$$

Upper Bound problem:  $f(x) = e^x + e^{-x}$  | nodes  $\{-2, 0, 2\}$

Evaluate the upper bound of interpolation error for the given function; interval  $[-1, 1]$

$$f(x) = e^x + e^{-x} + P_N(x) \quad \text{for upper bound: } \frac{f''(x) - f''(x_0)}{1 - e^{-5}}$$

$$f'(x) = e^x - e^{-x}$$

$$f''(x) = e^x + e^{-x}$$

$$f'''(x) = e^x - e^{-x} \quad f'''(1) = e^1 - e^{-1}$$

$$f'''(x) = e^x - e^{-x} \quad f'''(1) = e^1 - e^{-1}$$

$$\frac{f'''(x)}{3!} (x - x_0)^3 = \frac{f'''(x_0)}{3!} (x - x_0)^3 = \frac{f'''(1)}{3!} (x - 1)^3 =$$

$$(x - (-2)) (x - 0) (x - 2) =$$

$$(x - (-2)) (x - 0) (x - 2) = 1.1752 \times 10^{-8}$$

IEEE Maths:

$$\beta = 2$$

52 bits for fraction

11 bits for exponent

1 bit for sign

$$e_{\min} = 0$$

$$e_{\max} = 2^{11} - 1 = 2047$$

max non-negative number -  $(1.11 \dots 1) \times 2^{2047}$

min non-negative number  $(1.00 \dots 0) \times 2^0$

$$f(x) = x \ln(x)$$

$$x_0 = 1.0, h = 0.1$$

$\Delta = x_0 \text{ to } 2x_0$

$$0 = 1 + \log x = (N)^{\frac{1}{h}}$$

upper bound for the central difference.

$$\text{OKE} = 1 + \log x \leq$$

$$x_0 = 1 + \log x$$

$$\text{Truncation Error} = \frac{h^2}{6} f'''(c)$$

$$f(x) = x \ln x$$

$$f'(x) = \frac{1}{x} + \ln x$$

$$= 1 + \ln x$$

$$f''(x) = \frac{1}{x^2}$$

$$f'''(x) = -\frac{1}{x^3}$$

$$c \in [x_0 - h, x_0 + h] = [0.9, 1.1]$$

$$|f'''(c)| = \left| -\frac{1}{(0.9)^3} \right| = 1.234567$$

$$|f'''(c)| = \left| -\frac{1}{(1.1)^3} \right| = 0.82644$$

$$\begin{aligned} \text{Truncation Error} &= \frac{h^2}{6} \max |f'''(c)| \\ &= \frac{(0.1)^2}{6} \max |f'''(c)| \\ &= \frac{0.01}{6} \times 1.23456 \\ &= 2.0575 \times 10^{-3} \end{aligned}$$

If node exist:

$$E(x) \leq \frac{|f'''(c)|}{3!} \max \left\{ (x-x_0)(x-x_1)(x-x_2) \right\}, \text{ here } x = \text{intm}$$

(a)

$$x^3 - 2x^2 - x + 2 = 0$$

$$x = -1, x = 2$$

(b)

given that,  $2x^2 = (0)^2$

$$x^3 - 2x^2 - x + 2 = 0 \quad (i)$$

$$x(x^2 - 2x - 1) + 2 = 0$$

$$\begin{aligned} m &\leftarrow x^2 \leftarrow x^2 - 2x - 1 \quad (0)^2 \\ m &\leftarrow d \leftarrow 0 \quad (0)^2 \end{aligned} \quad \left( \frac{d+1}{d} \right) = Q(x) \quad (a)$$

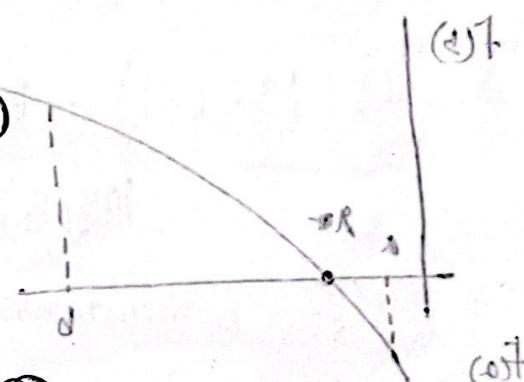
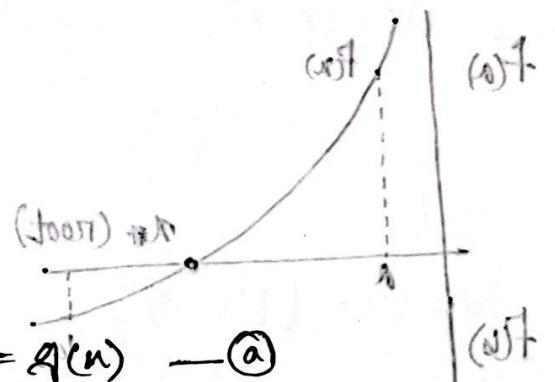
$$x^3 - 2x^2 - x + 2 = 0$$

$$x = (x^2 - 2x - 1) \quad (b)$$

$$x^3 - 2x^2 - x + 2 = 0$$

$$x^3 - x^2 + 2 = 2x$$

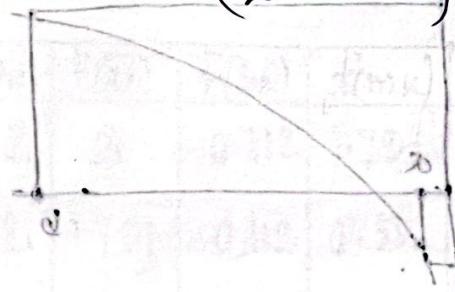
$$\text{On, } x = \sqrt{\frac{x^3 - x^2 + 2}{2}} \quad (c)$$



(c)

$$x = \sqrt{\frac{x^3 - x^2 + 2}{2}} \quad \left[ \frac{d}{dx} \left( \frac{x^3 - x^2 + 2}{2} \right) \right] = -2 \frac{d}{dx} (x^2 - 2x - 1)^{-1}$$

$$\begin{aligned} \frac{d}{dx} &= 0 \\ x_0 &= 0 \\ g(x_0) &= 0 \end{aligned}$$



$$g'(x) = \frac{(4x-4)}{(x^2 - 2x - 1)^2} \quad (i) \text{ and } (ii)$$

$$g'(0) = 0 \quad 0 \cdot 24489 \quad (iii) \text{ and } (iv)$$

$$g'(0.24489) = -1.47745 \quad (v) =$$

$$g'(-1.47745) = -0.57880 \quad (vi) \text{ and } (vii)$$

Tool not keen (smaller)

$$= 2 \cdot (x^2 - 2x - 1)^{-2} \cdot (2x - 2)$$

$$= \frac{(4x-4)}{(x^2 - 2x - 1)^2} \quad (i)$$

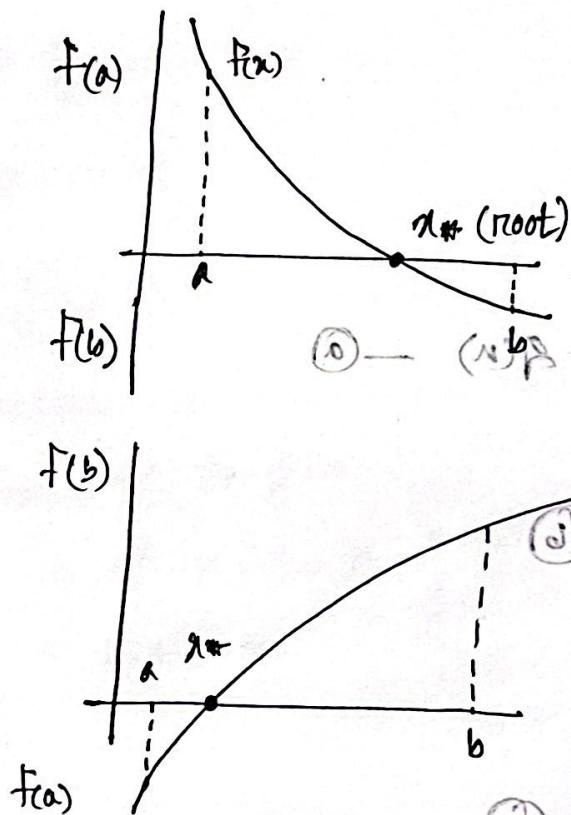
$$(iii) \text{ and } (iv) =$$

$$(v) =$$

Tool to work tool

# Root finding

## Root finding of non-linear equations



$$f(a) = \text{true}$$

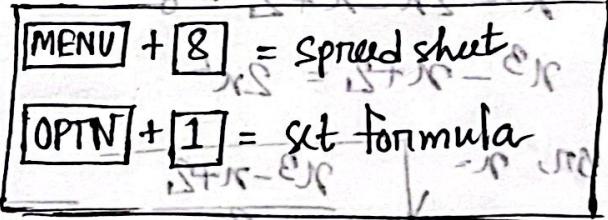
$$f(b) = \text{false}$$

$$m = \frac{a+b}{2}$$

$$\begin{cases} f(a) \times f(m) < 0 \rightarrow a \leftrightarrow m \\ f(a) \times f(m) > 0 \rightarrow b \leftrightarrow m \end{cases}$$

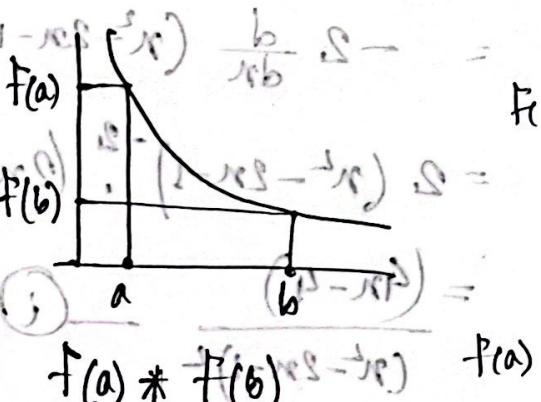
$$f(a) = \text{false}$$

$$f(b) = \text{true}$$



### Existence Check for Root

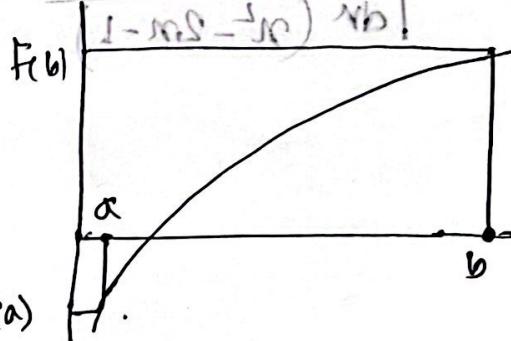
$$[f(a) \neq f(b) \neq 0] \rightarrow \text{Exist}$$



$$= (+ve) \neq (+ve)$$

$$= (+ve)$$

∴ Root does not exist



$$f(a) \neq f(b)$$

$$= (-ve) \neq (+ve)$$

$$= (-ve)$$

Exist Root

$$\frac{(P-1)(P)}{2(P-1)} = (P)^2$$

$$\frac{(P-1)(P)}{2(P-1)} = (P)^2$$

$$\frac{(P-1)(P)}{2(P-1)} = (P)^2$$

$$\frac{(P-1)(P)}{2(P-1)} = (P)^2$$

Subject:

Date:

Date:

Subject:

1.  $f(x) = \frac{1}{x} - 0.5$  Interval =  $[1.5, 3]$  moitorati to minimum problem

$$x^2 - x^2 - 9x + 9 = 0 \quad \frac{(3) \text{ pol} - (1.5 - 1) \text{ pol}}{(1.5) \text{ pol}} \leq 0$$

$$\Rightarrow x^2(2-1) - 2(x-1) = 0 \quad \text{start numbers 2.5}$$

$$[8, 2, 1] = \text{JawutneP}$$

$$81 - 0.1 \times 1.1 = (\text{mulus minima}) \cdot 3$$

$$\frac{(81 - 0.1 \times 1.1) \text{ pol} - (12.1 - 81) \text{ pol}}{(1.5) \text{ pol}} \leq 0$$

$$[81 - 0.1 \times 1.1] = \text{moitorati to minimum} \quad 0.2 \cdot 0.2 \leq 0$$

$$\text{moitorati 0.2} \leq 0$$

moitorati to minimum tridiagonal matrix

2.  $f(x) = x^3 + 7x^2 + 14x - 6 = 0$  Interval =  $[1, 3.2]$  general problem

K	$a_k$	$m_k$	$b_k$	$f(a_k)$	$f(b_k)$	$f(m_k)$	$x_k \in [a_k, b_k]$	$ f(m_k)  \leq 10^{-3}$
0	1	2.1	3.2	2	-0.112	1.791	$[2.1, 3.2]$	No
1	2.1	2.65	3.2	1.791	-0.112	0.552	$[2.65, 3.2]$	No
2	2.65	2.925	3.2	0.5521	-0.112	0.0858	$[2.925, 3.2]$	No
3	2.925	3.062	3.2	0.0858	-0.112	-0.054	$[2.925, 3.062]$	No
4	2.998	3.000	3.002	$1.96 \times 10^{-3}$	$-2.3 \times 10^{-3}$	$-1.95 \times 10^{-4}$	diverged	Yes

finding number of iterations to find root:  $2.0 - \sqrt{5} = 0.707$

formula:

$$n \geq \frac{\log(1b-a) - \log(\epsilon)}{\log(2)} - 1$$

lets assume that,

$$\text{Interval} = [1.5, 3]$$

$$\epsilon, (\text{machine epsilon}) = 1.1 \times 10^{-16}$$

$$n \geq \frac{\log(13-1.5) - \log(1.1 \times 10^{-16})}{\log(2)} - 1$$

$$n \geq 52.59 \quad [\text{minimum number of iteration}]$$

$$n \geq 53 \text{ iterations}$$

fixed Point Iterations.

Convergence Rate,  $\lambda = \left| \frac{g'(x_{\text{root}})}{g'(x_{\text{new}})} \right| = 2 - \lambda^2 + \lambda F - f^2 \lambda = 0.07$

Here, if  $\lambda < 1$ , Converge

if  $\lambda \geq 1$ , Diverge

$\lambda = 0 \rightarrow$  Super Linear Convergence

$0 < \lambda < 1 \rightarrow$  Linear Convergence

$\lambda \geq 1 \rightarrow$  Diverge

1. Consider the function  $f(x) = x^3 - x^2 - 9x + 9$ .

(a) State the exact roots of  $f(x)$ .

$$x^3 - x^2 - 9x + 9 = 0$$

$$\Rightarrow x^2(x-1) - 9(x-1) = 0 \quad | \text{ Factor out } (x-1)$$

$$\Rightarrow (x^2 - 9)(x-1) = 0$$

$$x_k = -3, 1, 3$$

(b) Construct three different fixed point functions  $g(x)$  such that  $f(x) = 0$ ; Make sure that one of the  $g(x)$ 's that you constructed converges to at least a root.

1st choice:

$$\begin{aligned} L &= x^3 - x^2 - 9x + 9 \\ g(x) &= x^3 - x^2 + 9 \end{aligned}$$

2nd choice:

$$\begin{aligned} L &= x^3 - x^2 - 9x + 9 \\ x &= \frac{-9}{x^2 - x - 9} \\ x &= g(x) \end{aligned}$$

3rd choice:

$$\begin{aligned} L &= x + x^3 - x^2 - 9x + 9 \\ x &= x^3 - x^2 - 8x + 9 \\ x &= g(x) \end{aligned}$$

③ find the Convergence rate/ratio for  $g(x)$  constructed in previous part and ratio also find which root is it converging to?

$$\text{Convergence rate: } \lambda = \left| g'(x_*) \right| = \left| \frac{dg}{dx} \Big|_{x=x_*} \right|$$

$$g(x) = (1-\kappa)x - (1-\kappa)^2 x^2$$

$$g'(x) = (1-\kappa) - 2(1-\kappa)^2 x$$

$$g'(x_*) = (1-\kappa) - 2(1-\kappa)^2 x_*$$

for 1st case: it is not linear but trans. with formula (1)

$$g(x) = \frac{1}{3}(x^3 - x^2 + 9)$$

$$g'(x) = \frac{1}{3}(3x^2 - 2x)$$

$$\lambda = (1-\kappa) \left| g'(x_*) \right| = \begin{cases} \frac{1}{3} < 1 \text{ for } x_* = 1 \text{ (Linear Convergence)} \\ \frac{11}{9} > 1 \text{ for } x_* = -3 \text{ (Divergence)} \\ \frac{7}{3} > 1 \text{ for } x_* = 3 \text{ (Divergence)} \end{cases}$$

∴  $g(x)$  is Converging to  $x_* = 1$  for 1st case

$$(1-\kappa)x - (1-\kappa)^2 x^2 = 0$$

$$(1-\kappa) = 0$$

For 2nd Case:

$$g(x) = \frac{-9}{x^2 - x - 9}$$

$$g'(x) = -9 \frac{1}{(x^2 - x - 9)^2} (2x - 1)$$

root of  $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$  to broad states with

$$= -9 \frac{(2x-1)}{(x^2 - x - 9)^2}$$

$$\Delta = |g'(x_*)| = \begin{cases} \frac{1}{9} < 1 \text{ for } x_* = 1 \text{ (Linear Convergence)} \\ 7 > 1 \text{ for } x_* = -3 \text{ (Divergence)} \\ 5 > 1 \text{ for } x_* = 3 \text{ (Divergence)} \end{cases}$$

 $-g(x)$  is Converging to  $x_* = 1$  for second case

For 3rd Case:

$$g(x) = x^3 - x^2 - 8x + 9$$

$$g'(x) = 3x^2 - 2x - 8$$

$$\Delta = |g'(x_*)| = \begin{cases} 7 > 1 \text{ for } x_* = 1 \text{ (Divergence)} \\ 13 > 1 \text{ for } x_* = 3 \text{ (Divergence)} \\ 25 > 1 \text{ for } x_* = -3 \text{ (Divergence)} \end{cases}$$

not Convergence for 3rd case.

$$1 - \frac{(|g(x_*)|_{\text{spot}} - (|g(1)|_{\text{spot}} - |g(3)|_{\text{spot}}))_{\text{spot}}}{|g(1)|_{\text{spot}}} \leq m$$

② find the approximate root,  $x_*$ , of the above function using fixed point iterations up to 4 significant figures within the error bound of  $1 \times 10^{-3}$  using  $x_0 = 0$  and any fixed point function  $g(x)$  from part (b) that converges to root.

error Bound,  $\epsilon_0 = 1 \times 10^{-3}$

$$x_0 = 0$$

$$g(x) = \frac{1}{3} (x^3 - x^2 + 9)$$

$$g(0) = 1.000$$

$$g(1.000) = 1.000$$

∴ 1.000 is the fixed point of  $g(x)$  and it is also the root of  $f(x)$ .

2. consider the function  $f(x) = x^3 - 2x^2 - 11x + 12$  on the interval  $[1.78, 8.25]$

① find the maximum number of iterations required to find the root of the given equation within the error bound of  $1 \times 10^{-3}$ .

$$n \geq \frac{\log (|b-a|) - \log (\epsilon_0)}{\log 2} - 1$$

$$n \geq \frac{\log (|8.25 - 1.78|) - \log (1 \times 10^{-3})}{\log 2} - 1$$

$$n \geq 11.65955$$

$$n \geq 12 \text{ iterations}$$

- ⑥ Show 5 iterations using the **Bisection Method** to find the root of the above function within the interval  $[1.78, 8.25]$ .

$k$	$a_k$	$b_k$	$m_k$	$f(a_k)$	$f(b_k)$	$f(m_k)$	$\epsilon_0 \in [ ]$	$ f(m)  \leq 10^{-3}$
0	1.78	8.25	5.015	-8.27	346.64	32.66	[1.78, 5.015]	No
1	1.78	5.015	3.397	-8.27	32.66	-9.24	[3.397, 5.015]	No
2	3.397	5.015	4.206	-9.24	32.66	4.759	[3.397, 4.206]	No
3	3.397	4.20	3.801	-9.24	4.765	-3.78	[3.801, 4.20]	No, to left
4	3.801	4.20	4.003	-3.79	4.765	0.076	[3.801, 4.003]	No

3. Using **Newton's method** to find (the) root ( $x_n$ ) of the equation  $f(x)$ .

$f(x) = x^2 e^{-x} - 0.6$ , up to machine epsilon of  $1 \times 10^{-4}$  starting with

$$x_0 = 0.2$$

$$f(x) = x^2 e^{-x} - 0.6$$

$$\begin{aligned} f'(x) &= -x^2 e^{-x} + e^{-x} 2x \\ &= 2x e^{-x} - x^2 e^{-x} \end{aligned}$$

$$x_0 = 0.2$$

$$\epsilon_0 = 1 \times 10^{-4}$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\begin{aligned} 0 &= f(x) \\ A1 &= (A1^{(2)} e^{(-A1)}) - 0.6 \\ &= (2A1 e^{(-A1)} - A1^{(2)} e^{(-A1)}) \end{aligned}$$

K	$x_K$	$f(x_K)$	$ f(x_K)  \leq S$
0	0.2	-0.56725	No
1	2.1245	-0.060	No
2	0.2049	-0.565	No
3	2.0929	-	-
4	-	-	-

4. given that  $f(x) = x^3 - 2x^2 - x + 2$ . This function has three roots and one root is  $x_1 = 1$ .

(a) find the remaining two exact roots of the function

$f(x)$  algebraically.

$$x^3 - 2x^2 - x + 2 = 0$$

$$\text{of roots} \Rightarrow x^2(x-2) + 1(x-2) = 0 \quad \text{[factorisation]}$$

$$\text{of roots} \Rightarrow (x-2)(x^2+1) = 0 \quad \text{of qu. } x^2+1 = 0$$

$$\text{Here, } x-2=0 \quad x^2-1=0 \quad x^2=0$$

$$x=2 \quad x=\pm 1$$

$$x=\pm 1, 1, 2$$

$$x=0$$

$$\frac{(x^2)^2}{(x^2)^2} - x^2 = 1 + x^2$$

⑥ Construct two different fixed point functions  $g(x)$  such that

$$f(x) = 0$$

given that,  $f(x) = x^3 - 2x^2 - x + 2$

$$f(x) = x^3 - 2x^2 - x + 2$$

Here,

$$x^3 - 2x^2 - x + 2 = 0 \quad | \quad (x^3 - 2x^2 - x + 2) = 0$$

$$x(x^2 - 2x - 1) = -2$$

$$x = \left( \frac{-2}{x^2 - 2x - 1} \right) \quad | \quad (x^2 - 2x - 1) = 0$$

Again,

$$x^3 - 2x^2 - x + 2 = 0 \quad | \quad (x^3 - 2x^2 - x + 2) = 0$$

$$\Rightarrow \frac{x^3 - 2x^2 - x + 2}{x} = 0 \quad | \quad (x^3 - 2x^2 - x + 2) = 0$$

$$\Rightarrow x^2 - 2x - 1 = -\frac{2}{x} \quad | \quad (x^2 - 2x - 1) = 0$$

$$\Rightarrow x(x-2) = 1 - \frac{2}{x} \quad | \quad (x-2) = 0$$

$$\Rightarrow x(x-2) = \frac{x-2}{x} \quad | \quad (x-2) = 0$$

$$\Rightarrow x = \frac{x-2}{x(x-2)} \quad | \quad (x-2) = 0$$

$$\Rightarrow x = -\frac{1}{x} \quad | \quad (x-2) = 0$$

$$\text{provided } x \neq 0 \quad | \quad (x-2) = 0$$

$$\text{provided } x \neq 0 \quad | \quad (x-2) = 0$$

① Compute the Convergence rate  $\lambda$ , for each fixed point function  $g(x)$  obtained in the previous part, and state which root it is converging to or diverging.

We know that,

$$\lambda = |g'(x_*)| \quad 0 = S + x - x^2 - e^x$$

for case 1:

$$g(x) = \frac{(-2)}{x^2 - 2x - 1} \rightarrow \left( \frac{S - x}{1 - x^2 - x} \right) = \lambda$$

$$\begin{aligned} g'(x) &= \frac{d}{dx} \left[ \frac{-2}{x^2 - 2x - 1} \right] = S + x - x^2 - e^x \\ &= -2 \frac{d}{dx} \left[ \frac{1}{x^2 - 2x - 1} \right] = S + x - x^2 - e^x \\ &= +2 \frac{\frac{1}{(x^2 - 2x - 1)^2} (2x - 2)}{(x^2 - 2x - 1)^2} = (S - x) \lambda \\ &= \frac{2(2x - 2)}{(x^2 - 2x - 1)^2} \frac{S - x}{(S - x) \lambda} = \lambda \end{aligned}$$

$$\lambda = |g'(x_*)| = \begin{cases} \text{when } x_* = -1, \lambda = 0 < 1 \text{ [Converge]} \\ \text{when } x_* = 1, \lambda = 2 > 1 \text{ [Diverge]} \\ \text{when } x_* = 2, \lambda = 4 > 1 \text{ [Diverge]} \end{cases}$$

for case 2:

$$g(x) = \frac{1}{x}$$

$$g'(x) = \frac{d}{dx} \left( \frac{1}{x} \right)$$

$$g''(x) = -\frac{1}{x^2}$$

$$|g'(x_k)| = \begin{cases} \text{When } x_k = 0; |g'(x_k)| \rightarrow \infty \text{ [Diverge]} \\ \text{When } x_k = 1; |g'(x_k)| = 1 \text{ [Diverge]} \\ \text{When } x_k = 2; |g'(x_k)| = \frac{1}{4} < 1 \text{ [Converge]} \end{cases}$$

4. Compute the fixed point function  $g(x)$  for the Newton's method; and find the root within  $10^{-6}$  starting with  $x_0 = -3.5$

K	$x_k$	$f(x_k)$	$f'(x_k)$	$ f'(x_k)  < 10^{-6}$	$x_k$	$R$
0	-3.5	-61.87	0.1	0.1 > 10 <sup>-6</sup>	-2.8	0
1	-2.256	-17.4	0.1	0.1 > 10 <sup>-6</sup>	-2.828	1
2	-2.752	-31.237	0.1	0.1 > 10 <sup>-6</sup>	-2.802	2
3	-2.5455	-24.907	0.1	0.1 > 10 <sup>-6</sup>	-2.811	3
4	-2.629	-27.3649	0.1	0.1 > 10 <sup>-6</sup>	-2.807	4
5	-2.595	-26.3478	0.1	0.1 > 10 <sup>-6</sup>	-2.801	5
6	-2.6093	-26.7728	0.1	0.1 > 10 <sup>-6</sup>	-2.8005	6
7	-2.6034					7

given that,

$$f(n) = n^3 - 2n^2 - n + 2$$

$$f'(n) = 3n^2 - 4n - 1$$

$$n_1 = -3.5$$

$$C_0 = \text{Not given}$$

$$[ \text{given} ] n_{k+1} = n_k - \frac{f(n_k)}{f'(n_k)} \text{ is root} \quad \left. \begin{array}{l} \\ \end{array} \right\} = \left| (n_k)^P \right| = L$$

$$[ \text{given} ] L = L = L \text{ is root} \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$[ \text{given} ] L = L = \frac{(n_k)^3 - 2(n_k)^2 - (n_k) + 2}{3(n_k)^2 - 4(n_k) - 1}$$

$$-3.5 - \left( A1^3 - 2(A1)^2 - A1 + 2 \right) : \left( 3A1^2 - 4A1 - 1 \right)$$

K	$n_k$	$f(n_k)$	$ f(n_k)  < 10^{-6}$	$n_k$	$x$
0	-3.5	-61.875	No	2.8	0
1	-2.2562	-17.4	No	1.51	1
2	-1.508	-4.469	No	1.012	2
3	-1.13	-0.866	No	0.502	3
4	-1.012	-0.072	No	0.252	4
5	-1	0	Yes	0.125	5
6	-1			0.0625	6

$$\therefore n_{\text{root}} = -1$$

1.

$$2x_1 + x_2 - x_3 + 2x_4 = 5$$

$$4x_1 + 5x_2 - 3x_3 + 6x_4 = 9$$

$$-2x_1 + 5x_2 - 2x_3 + 6x_4 = 4$$

$$4x_1 + 11x_2 - 4x_3 + 8x_4 = 2$$

Q) Explain how to find this system has unique solutions or not

$$A = \begin{pmatrix} 2 & 1 & -1 & 2 \\ 4 & 5 & -3 & 6 \\ -2 & 5 & -2 & 6 \\ 4 & 11 & -4 & 8 \end{pmatrix}$$

This is a square matrix.

$\det(A) = -12 \neq 0$ . Hence A is non-singular matrix. This system

has unique solution.

b) Write down the augmented matrix.

$$\left( \begin{array}{cccc|c} 2 & 1 & -1 & 2 & 5 \\ 4 & 5 & -3 & 6 & 9 \\ -2 & 5 & -2 & 6 & 4 \\ 4 & 11 & -4 & 8 & 2 \end{array} \right)$$

$$\left( \begin{array}{cccc|c} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right) = (U_1)$$

c) find the upper triangular matrix U

$$\left( \begin{array}{cccc|c} 2 & 1 & -1 & 2 & 5 \\ 4 & 5 & -3 & 6 & 9 \\ -2 & 5 & -2 & 6 & 4 \\ 4 & 11 & -4 & 8 & 2 \end{array} \right)$$

$$\left( \begin{array}{cccc|c} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right) = (U_2)$$

2.

$$x_1 + x_2 + x_3 = 6 \text{ (1) } x_1 - x_2 + x_3 = 1 \text{ (2)}$$

$$2x_1 + 3x_2 + 4x_3 = 20 \text{ (3)}$$

$$3x_1 + 4x_2 + 2x_3 = 17 \text{ (4)}$$

② Construct the Frobenius matrix  $F^{(1)}$  and  $F^{(2)}$  from system.  
for no negative signs and numbers left out of first row

$$Ax = b$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 20 \\ 17 \end{bmatrix} = A$$

for no negative signs and numbers left out of first row

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{\text{eliminate } x_1 \text{ from } (2,3)} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \xrightarrow{\text{eliminate } x_2 \text{ from } (2,3)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 17 \end{bmatrix} = A$$

for no negative signs and numbers left out of first row

$$F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{\text{eliminate } x_1 \text{ from } (2,3)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{eliminate } x_2 \text{ from } (2,3)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 17 \end{bmatrix} = A$$

$$F^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

⑬ Compute the unit lower triangular matrix L.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 9 & 2 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 3 & 3/4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 3 & 3/4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 3 & 3/4 & 1 \end{bmatrix} \xrightarrow{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3/4 & 1 \end{bmatrix} \xrightarrow{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3/4 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[1 \cdot 1 - 1 \cdot 1 = 0]$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[2 \cdot 1 - 1 \cdot 1 = 1]$$

$$[0 \cdot 1 - 1 \cdot 1 = -1]$$

1.

Given system of equations (1)

$$2x - 2y + z = -3$$

$$x + 3y - 2z = 1$$

$$3x - y - z = 2$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 \\ 1 & 3 & -2 \\ 3 & -1 & -1 \end{array} \right]$$

Augmented matrix -

$$\left[ \begin{array}{ccc|c} 2 & -2 & 1 & -3 \\ 1 & 3 & -2 & 1 \\ 3 & -1 & -1 & 2 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 2 & -2 & 1 & -3 \\ 1 & 3 & -2 & 1 \\ 3 & -1 & -1 & 2 \end{array} \right] = \left[ \begin{array}{ccc|c} 2 & -2 & 1 & -3 \\ 0 & 4 & -\frac{5}{2} & \frac{5}{2} \\ 0 & 2 & -\frac{5}{2} & \frac{13}{2} \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 2 & -2 & 1 & -3 \\ 0 & 4 & -\frac{5}{2} & \frac{5}{2} \\ 0 & 2 & -\frac{5}{2} & \frac{13}{2} \end{array} \right] \quad [R_2' = R_2 - \frac{1}{2}R_1]$$

$$[R_3' = R_3 - \frac{3}{2}R_1]$$

$$[R_3' = R_3 - \frac{1}{2}R_2] \quad [Ans]$$

2.

Previously;

$$\left[ \begin{array}{ccc|c} 2 & -2 & 1 & -3 \\ 0 & 4 & -\frac{5}{2} & \frac{5}{2} \\ 0 & 0 & -\frac{5}{4} & \frac{21}{4} \end{array} \right]$$

$$2x_1 - 2x_2 + x_3 = -3$$

$$4x_2 - \frac{5}{2}x_3 = \frac{5}{2}$$

$$-\frac{5}{4}x_3 = \frac{21}{4}$$

$$\text{or, } x_3 = -\frac{21}{5} \quad \text{①}$$

$$x_1 = -\frac{7}{5}$$

$$x_2 = -2$$

$$x_3 = -\frac{21}{5}$$

$$\left[ \begin{array}{ccc|c} 0 & 1 & -3 & -3 \\ 0 & 0 & \frac{5}{2} & \frac{5}{2} \\ 0 & 0 & -1 & -\frac{21}{4} \end{array} \right]$$

$$4x_2 - \frac{5}{2} \left( -\frac{21}{4} \right) = \frac{5}{2}$$

$$\text{or, } x_2 = -\frac{8}{5} \quad \text{②}$$

$$2x_1 - \left( 2 \times -\frac{7}{5} \right) = -3$$

$$\text{or, } x_1 = -2$$

$$2x_1 - (2 \times -2) + \left( -\frac{21}{5} \right) = -3$$

$$2x_1 + 4 - \frac{21}{5} = -3$$

$$x_1 = -\frac{7}{5}$$

3.

$$\left[ \begin{array}{ccc|c} 2 & -2 & 1 & -3 \\ 1 & 3 & -2 & 1 \\ 3 & -1 & -1 & 2 \end{array} \right] \xrightarrow{\begin{array}{l} R_1 \leftrightarrow R_2 \\ R_1 \rightarrow R_1 - 2R_2 \\ R_3 \rightarrow R_3 - 3R_2 \end{array}} \left[ \begin{array}{ccc|c} 1 & 8 & -1 & 1 \\ 0 & 7 & -5 & -1 \\ 0 & -10 & 2 & 5 \end{array} \right] \xrightarrow{\begin{array}{l} R_2 \rightarrow R_2 - 7R_1 \\ R_3 \rightarrow R_3 + 10R_1 \end{array}} \left[ \begin{array}{ccc|c} 1 & 8 & -1 & 1 \\ 0 & 0 & -12 & -8 \\ 0 & 0 & 18 & 15 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 8 & 8 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{R_1 \rightarrow R_1 - 8R_2} \left[ \begin{array}{ccc|c} 0 & -2 & 1 & -3 \\ 0 & 7 & -5 & -1 \\ 0 & 0 & 18 & 15 \end{array} \right]$$

$$\begin{aligned} E &= \frac{1}{18}R + 0xR + 0xR \\ \frac{1}{18} &= E \\ \frac{1}{18} &= E \\ -\frac{1}{2} &= E \\ -\frac{1}{2} &= E \end{aligned}$$

$$\begin{aligned} E &= \frac{1}{18} - (7x\frac{1}{18}) - \frac{1}{18} \\ E &= \left[ \begin{array}{ccc|c} 0 & -2 & 1 & -3 \\ 0 & 7 & -5 & -1 \\ 0 & 0 & 18 & 15 \end{array} \right] \\ E &= \left( \frac{1}{18} - \right) + \left( 7x\frac{1}{18} \right) - \frac{1}{18} \end{aligned}$$

$$\begin{aligned} \textcircled{1} &= \left[ \begin{array}{ccc|c} 0 & 0 & 0 & 1 \\ -\frac{1}{2} & 1 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 & 0 \end{array} \right] \\ \textcircled{1} &= 1 \end{aligned}$$

$$F_2 = \left[ \begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 \end{array} \right]$$

$$\begin{aligned} 1 &= 1 \\ 1 &= 1 \end{aligned}$$

$$\text{Total } F = F_2 F_1$$

4. from (3) we get,  $f_1$  map  $= 0$

$$f_1 = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & (1-\frac{1}{2})(1-\frac{1}{2}) & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} = A$$

from b1 map  $b_1 = 0$

from b2 map  $b_2 = 0$

from b3 map  $b_3 = 0$

$$f_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix} = B$$

$$L = (F_1^{-1}) (F_2^{-1})$$

$$F_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & 0 & 1 \end{bmatrix}$$

$$F_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{2} & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{1}{2} & 1 \end{bmatrix}$$

$$1 - x + \frac{1}{2}x^2 - \frac{1}{8}x^3 = 580$$

$$1 - \frac{1}{2}x + \frac{1}{2}x^2 - \frac{1}{8}x^3 = 580$$

$$1 - \frac{1}{2}x + \frac{1}{2}x^2 - \frac{1}{8}x^3 = 580$$

Q) We know,

$$A = LU$$

$$Lc = d$$

$$Ux = c$$

$$Lc = d$$

U = Upper Triangular Matrix

A = Augmented Matrix

$$L = (F_1^{-1}) (F_2^{-1})$$

d = constant of Augmented Matrix

c = from  $Lc = d$  part

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -3 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$$

$$c_1 = -3$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = 1$$

$$\frac{1}{2}c_1 + c_2 = 1$$

$$c_1 = -3$$

$$c_2 = 1 - \left( \frac{1}{2}(-3) - 3 \right) = \frac{5}{2}$$

$$c_2 = \frac{5}{2}$$

$$c_3 = \frac{21}{4}$$

$$\frac{3}{2}c_1 + \frac{1}{2}c_2 + c_3 = 2$$

$$\text{or, } \frac{3}{2}(-3) + \frac{1}{2}\left(\frac{5}{2}\right) + c_3 = 2$$

$$\text{or, } -\frac{9}{2} + \frac{5}{4} + c_3 = 2$$

$$\text{or, } c_3 = \frac{21}{4}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = 1$$

$$Vx = c$$

Want to find ① L

$$\left[ \begin{array}{ccc|c} 2 & 0 & 0 & 1 \\ 0 & 4 & -5/2 & 5/2 \\ 0 & 0 & -5/4 & 21/4 \end{array} \right] \xrightarrow{\text{PSL}} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 1/2 \\ 0 & 1 & -5/8 & 5/8 \\ 0 & 0 & 1 & 21/4 \end{array} \right] \xrightarrow{\text{PSL}} \left[ \begin{array}{ccc|c} x_1 & x_2 & x_3 & 1/2 \\ 0 & 1 & 0 & 5/8 \\ 0 & 0 & 1 & 21/4 \end{array} \right] \xrightarrow{\text{PSL}} \left[ \begin{array}{ccc|c} x_1 & x_2 & x_3 & 1/2 \\ 0 & 1 & 0 & 5/8 \\ 0 & 0 & 1 & 21/4 \end{array} \right] = \left[ \begin{array}{c} -2 \\ 5/2 \\ 21/4 \end{array} \right] = 2$$

$$\left[ \begin{array}{ccc|c} 2 & -2 & 1 & -3 \\ 0 & 4 & -5/2 & 5/2 \\ 0 & 0 & -5/4 & 21/4 \end{array} \right] \xrightarrow{\text{PSL}} \left[ \begin{array}{ccc|c} x_1 & x_2 & x_3 & -3 \\ 0 & 1 & 0 & 5/2 \\ 0 & 0 & 1 & 21/4 \end{array} \right] = \left[ \begin{array}{c} -3 \\ 5/2 \\ 21/4 \end{array} \right] =$$

$$2x_1 - 2x_2 + x_3 = -3$$

$$4x_2 - 5/2 (-21/5) = 5/2$$

$$4x_2 - 5/2 x_3 = 5/2$$

$$4x_2 = -2$$

$$-5/4 x_3 = 21/4$$

$$2x_1 - 2(-2) + (-21/5) = -3$$

$$\text{or, } x_3 = -21/5$$

$$0 =$$

$$\therefore x_1 = -7/5$$

$$x_2 = -2$$

$$x_3 = -21/5$$

$$\text{or, } x_1 = -7/5$$

$$= 1.4$$

$$0.412$$

$$= 8.75$$

$$\left[ \begin{array}{c} -2 \\ 5/2 \\ 21/4 \end{array} \right]$$

$$\left[ \begin{array}{c} -7/5 \\ -2 \\ -21/5 \end{array} \right]$$

1. ② Prove that,

$$S = \mathbb{R}^3$$

$$S = \left\{ \frac{1}{\sqrt{5}} (2, -1, 0)^T, \frac{1}{\sqrt{30}} (1, 2, -5)^T, \frac{1}{\sqrt{24}} (2, 4, 2)^T \right\}$$

is orthonormal

$$S = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{30}} \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}, \frac{1}{\sqrt{24}} \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \\ 0 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{30} \\ 2/\sqrt{30} \\ -5/\sqrt{30} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{24} \\ 4/\sqrt{24} \\ 2/\sqrt{24} \end{bmatrix}$$

$$x = \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \\ 0 \end{bmatrix}, y = \begin{bmatrix} 1/\sqrt{30} \\ 2/\sqrt{30} \\ -5/\sqrt{30} \end{bmatrix}, z = \begin{bmatrix} 2/\sqrt{24} \\ 4/\sqrt{24} \\ 2/\sqrt{24} \end{bmatrix}$$

$$x^T y = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{30} \\ 2/\sqrt{30} \\ -5/\sqrt{30} \end{bmatrix} = 0$$

$$y^T z = \begin{bmatrix} 1/\sqrt{30} & 2/\sqrt{30} & -5/\sqrt{30} \end{bmatrix} \begin{bmatrix} 2/\sqrt{24} \\ 4/\sqrt{24} \\ 2/\sqrt{24} \end{bmatrix} = 0$$

$$\frac{x^T Q}{Q^T x} = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} & 0 \\ -1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2/\sqrt{24} \\ 4/\sqrt{24} \\ 2/\sqrt{24} \end{bmatrix} = 0$$

Now,

$$|x| = \sqrt{(2/\sqrt{5})^2 + (-1/\sqrt{5})^2 + 0^2} = \sqrt{\frac{4}{5} + \frac{1}{5} + 0} = \sqrt{1} = 1$$

$$|y| = \sqrt{(1/\sqrt{30})^2 + (2/\sqrt{30})^2 + (-5/\sqrt{30})^2} = \sqrt{\frac{1}{30} + \frac{4}{30} + \frac{25}{30}} = \sqrt{1} = 1$$

$$|z| = \sqrt{(2/\sqrt{24})^2 + (4/\sqrt{24})^2 + (2/\sqrt{24})^2} = \sqrt{\frac{4}{24} + \frac{16}{24} + \frac{4}{24}} = \sqrt{1} = 1$$

Hence  $S$  is orthonormal. [proved]

6)  $f(x) = \sin x$  at the points  $x_0 = 4, x_1 = 9, x_2 = -6$ . Evaluate the best fit straight line using the discrete Square Approximation.

$$x_0 = 4; f(x_0) = \sin(4) = -0.756$$

$$x_1 = 9; f(x_1) = \sin(9) = 0.412$$

$$x_2 = -6; f(x_2) = \sin(-6) = 0.279$$

$$P_1(x) = a_0 + a_1 x$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 9 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} -0.756 \\ 0.412 \\ 0.279 \end{bmatrix}$$

$$\begin{bmatrix} 4^0 \\ 9^0 \\ -6^0 \end{bmatrix} = \begin{bmatrix} 4^1 \\ 9^1 \\ -6^1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} -0.758 \\ 0.412 \\ 0.279 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 1 & 9 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \Rightarrow \begin{bmatrix} -0.756 \\ 0.412 \\ 0.279 \end{bmatrix} = \boxed{\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}} = \boxed{\begin{bmatrix} 0.279 \\ 0.412 \\ -0.756 \end{bmatrix}} = \boxed{\begin{bmatrix} 0.279 \\ 0.412 \\ -0.756 \end{bmatrix}} = \boxed{\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}}$$

$$A^T A x = A^T b$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 9 & -6 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 9 & -6 \end{bmatrix} \begin{bmatrix} -0.75 \\ 0.41 \\ 0.27 \end{bmatrix}$$

standard  $\cdot 2 = 50 \cdot 2 = 100$ ,  $\beta = 100$  trying out to realize = (not) (d)

$$\begin{bmatrix} 3 & 7 \\ 7 & 133 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} -0.065 \\ -0.099 \end{bmatrix} \quad (A) \min_{a_0, a_1} (a_0^2 + a_1^2) \quad (P = 0.01)$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 7 & 133 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 518 \end{bmatrix}; \quad \begin{bmatrix} -0.065 \\ -0.99 \end{bmatrix} \quad \begin{bmatrix} 10 \\ 518 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 19/50 & -1/50 \\ -1/50 & 3/350 \end{bmatrix} \begin{bmatrix} -0.065 \\ -0.99 \end{bmatrix}$$

$$a_0 = -4.9 \times 10^{-3} \quad \text{and} \quad a_1 = -7.18 \times 10^{-3}$$

$$P_1(x) = a_0 + a_1 x$$

$$= -4.9 \times 10^{-3} - 7.18 \times 10^{-3} x$$

$$= -0.0049 - 0.00718 x$$

$$f(0) = 3, \quad f(4) = -2, \quad f(-1) = 2, \quad f(1) = 1$$

⑥ From the given data find out the linearly independent column vectors  $u_1, u_2$  and  $u_3$ .

$$x_0 = 0, x_1 = 4, x_2 = -1 \text{ and } x_3 = 1$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 4 & 16 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array} \right] \xrightarrow{\text{Row operations}} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 4 & 16 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array} \right] \xrightarrow{\text{Row operations}} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 4 & 16 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] = \left[ \begin{array}{c} 0 \\ 4 \\ -1 \\ 1 \end{array} \right]$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \xrightarrow{\frac{P}{P}} \begin{bmatrix} 0 \\ P \\ 1 \\ 1 \end{bmatrix}$$

⑥ Using Gram-Schmidt process, construct the orthonormal column matrices  $q_1, q_2$  and  $q_3$  and then write down the  $Q$  matrix.

from question ⑤ we get;

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad u_2 = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} \quad u_3 = \begin{bmatrix} 0 \\ 16 \\ 1 \end{bmatrix}$$

$$P_1 = u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

1 = (1)  $\hat{1}$   
2 = (1)  $\hat{1}$  3 = (1)  $\hat{1}$  4 = (0)  $\hat{1}$   
initial set two basis sets  $\hat{1}$  and  $\hat{0}$   
so basis sets are written

$$P_2 = u_2 - \frac{u_2 \cdot P_1}{P_1 \cdot P_1} \times P_1$$

$$P_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \frac{\begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} - \frac{4}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 4 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

$$P_3 = u_3 - \frac{u_3 \cdot P_1}{P_1 \cdot P_1} P_1 - \frac{u_3 \cdot P_2}{P_2 \cdot P_2} P_2$$

$$= \begin{bmatrix} 0 \\ 16 \\ -1 \\ 1 \end{bmatrix} - \frac{\begin{bmatrix} 0 \\ 16 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 16 \\ -1 \\ 1 \end{bmatrix} - \frac{18}{4} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} \\ \frac{23}{2} \\ -\frac{7}{2} \\ -\frac{7}{2} \end{bmatrix}$$

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A  $\left[ \begin{array}{cc} 0 & \left[ \begin{array}{cc} 1 & -1 \\ 6 & 1 \\ 8 & -1 \\ 0 & 1 \end{array} \right] \\ 1 & \end{array} \right] = \left[ \begin{array}{cc} -1 & \left[ \begin{array}{cc} 1 & 1 \\ 3 & 1 \\ -2 & 1 \\ 0 & 1 \end{array} \right] \\ 1 & \end{array} \right] \in \left[ \begin{array}{cc} 0 & \left[ \begin{array}{cc} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{array} \right] \\ 1 & \left[ \begin{array}{cc} 1 & -1 \\ 3 & 1 \\ -2 & 1 \\ 0 & 1 \end{array} \right] \end{array} \right] = \left[ \begin{array}{cc} 1 & -1 \\ 3 & 1 \\ -2 & 1 \\ 0 & 1 \end{array} \right] = 8$

B  $\left[ \begin{array}{ccc} -1 & 3 & -2 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right] \xrightarrow{\left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]} \left[ \begin{array}{ccc} -1 & 3 & -2 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right] \xrightarrow{\left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]} \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] = 0$

$= \frac{46}{14} \left[ \begin{array}{ccc} -1 & 1 & 1 \\ 3 & 1 & 1 \\ -2 & 1 & 1 \\ 0 & 1 & 1 \end{array} \right] = \left[ \begin{array}{ccc} 1 & 1 & 1 \\ -2\frac{3}{7} & 3 \times 2\frac{3}{7} & -2 \times 2\frac{3}{7} \\ 8 & 0 & 0 \end{array} \right] = \left[ \begin{array}{ccc} 0 & 0 & 0 \\ -2\frac{3}{7} & 6\frac{9}{7} & -4\frac{6}{7} \\ 0 & 0 & 0 \end{array} \right] = 0$

$P_3 = \left[ \begin{array}{c} -\frac{9}{2} \\ \frac{23}{2} \\ -\frac{7}{2} \\ -\frac{7}{2} \end{array} \right] = \left[ \begin{array}{c} -2\frac{3}{7} \\ 6\frac{9}{7} \\ -4\frac{6}{7} \\ 0 \end{array} \right] = \left[ \begin{array}{c} -\frac{17}{14} \\ \frac{23}{14} \\ \frac{43}{14} \\ -\frac{7}{2} \end{array} \right]$

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Now,

$$q_1 = \frac{P_1}{|P_1|} \frac{1}{\sqrt{1+1+1+1}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{4}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \quad [\text{Ans}]$$

$$q_2 = \frac{P_2}{|P_2|} = \frac{1}{\sqrt{(-1)^2 + (3)^2 + (-2)^2}} \begin{bmatrix} -1 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

$$= \frac{\sqrt{14}}{14} \begin{bmatrix} -1 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{14}}{14} \\ 0 \\ \frac{3\sqrt{14}}{14} \\ -\frac{2\sqrt{14}}{14} \\ 0 \end{bmatrix} \quad [\text{Ans}]$$

[Ans]

$$Q_3 = \frac{P_3}{|P_3|} = \frac{1}{\sqrt{\left(\frac{-17}{14}\right)^2 + \left(\frac{23}{14}\right)^2 + \left(\frac{43}{14}\right)^2 + \left(\frac{-7}{2}\right)^2}} \begin{bmatrix} -17/14 \\ 23/14 \\ 43/14 \\ -7/2 \end{bmatrix}$$

~~$\begin{bmatrix} -17/14 \\ 23/14 \\ 43/14 \\ -7/2 \end{bmatrix}$~~

~~$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$~~

~~$\begin{bmatrix} -0.239 \\ 0.323 \\ 0.604 \\ -0.688 \end{bmatrix}$~~

$$= 0.196657 \begin{bmatrix} -17/14 \\ 23/14 \\ 43/14 \\ -7/2 \end{bmatrix} = \frac{59}{591} = 0.1$$

~~$\begin{bmatrix} -0.239 \\ 0.323 \\ 0.604 \\ -0.688 \end{bmatrix}$~~

~~$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$~~

~~$\begin{bmatrix} \text{Av} \\ \text{Av} \\ \text{Av} \\ \text{Av} \end{bmatrix}$~~

$$Q = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{14}}{14} & -0.239 \\ \frac{1}{2} & \frac{3\sqrt{14}}{14} & 0.323 \\ \frac{1}{2} & -\frac{2\sqrt{14}}{14} & 0.604 \\ \frac{1}{2} & 0 & -0.688 \end{bmatrix}$$

~~$\begin{bmatrix} \text{Av} \\ \text{Av} \\ \text{Av} \\ \text{Av} \end{bmatrix}$~~

① Calculate the matrix elements of  $R$ . and write down the matrix  $R$

$$R = Q^T A$$

from the question ⑥ we get,

$$Q = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{14}}{14} & 0.239 \\ \frac{1}{2} & \frac{3\sqrt{14}}{14} & 0.323 \\ \frac{1}{2} & \frac{-2\sqrt{14}}{14} & 0.604 \\ \frac{1}{2} & 0 & -0.688 \end{bmatrix}$$

$$Q^T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{\sqrt{14}}{14} & \frac{3\sqrt{14}}{14} & -\frac{2\sqrt{14}}{14} & 0 \\ -0.239 & 0.323 & 0.604 & -0.688 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = x$$

$$\begin{bmatrix} 100 + 100 + 0 \\ -100P80.2 + 100P14.8 \\ 100P80.2 \end{bmatrix} = x$$

Q.T.A = R To determine extrema of function

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{-\sqrt{14}}{14} & \frac{3\sqrt{14}}{14} & \frac{-2\sqrt{14}}{14} & 0 \\ 0.239 & 0.323 & 0.604 & -0.688 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.556 & 0 & 0 & 0 \\ 0 & 2 & 9 & 0 \\ 0 & 0 & 3.7416 & 12.2940 \\ 88.0 & 0 & 0 & 5.084 \end{bmatrix}$$

② Compute Rx and Q.T.b.

$$R = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3.7416 & 12.2940 \\ 0 & 0 & 5.084 \end{bmatrix}$$

$$x = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$Rx = \begin{bmatrix} 2a_0 + 2a_1 + 9a_2 \\ 3.7416a_1 + 12.2940a_2 \\ 5.084a_2 \end{bmatrix}$$

$$Q^T b = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{\sqrt{14}}{14} & \frac{3\sqrt{14}}{14} & -\frac{2\sqrt{14}}{14} & 0 \\ -0.239 & 0.323 & 0.604 & -0.688 \end{bmatrix} \begin{bmatrix} d^T A \\ d^T A \\ d^T A \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -3.4743 \\ -0.843 \end{bmatrix}$$

② Let  $x = (a_0, a_1, a_2)^T$  being the coefficients of the polynomial  $P_2(x)$ . Evaluate these coefficients and write down the polynomial  $P_2(x)$ .

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 2 \\ 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 4 & 8 & 1 & 1 \\ 0 & 16 & 16 & 16 & 1 \\ 0 & 828 & 828 & 828 & 828 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 50 \end{bmatrix}$$

$$A^T A x = A^T b \quad = A^T b$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 4 & -1 & 1 \\ 0 & 16 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 16 & 4 & 16 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

Intercept  $\rightarrow$  3 & 4 direction  $\rightarrow$  1 unit period  $\rightarrow 2^T (3, 0, 10, 0) = x + 2$  ③  
 Slope  $\rightarrow$  whole work from direction 2 exist standard. (x) 59  
 . (x) 59

$$\Rightarrow \begin{bmatrix} 4 & 4 & 18 & 64 & 258 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 4 & -9 & -29 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -9 \\ -29 \end{bmatrix} = TA$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 18 & 64 & 258 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -9 \\ -29 \end{bmatrix}$$

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$$(a) \text{ If } x_0 = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -542 \\ -2002 \\ -7986 \end{bmatrix}$$

$$a_0 = -542, a_1 = -2002 \text{ and } a_2 = -7986$$

$$P_2(x) = a_0 + a_1 x + a_2 x^2$$

$$= -542 - 2002x - 7986x^2$$

$$T_2(f) = \int_a^b f(x) \left[ 0.8 + 0.2x + 0.02x^2 \right] dx$$

$I = (a, b)$  एवं  $f(x)$  वक्र वर्तमान ब्रॉड (१)

$$f(x) = x^2 \text{ इंटरवल } = [0, 1]$$

(इन वर्षों का अनुमान)

$$T_2(f) = \int_0^1 x^2 \left[ 0.8 + 0.2x + 0.02x^2 \right] dx = (a, b)$$

$$= \left[ (0.8) + (0.2) + \frac{0.02}{3} \right] \frac{1-0}{0.2} = (f)_1 L$$

$$= [0.8 + 0.2 + \frac{0.02}{3}] \frac{1-0}{0.2} = (f)_1 L$$

$$= [0.8 + 0.2 + \frac{0.02}{3}] \frac{1-0}{0.2} =$$

Absolute Error (अवधारणा)  $802.22 =$

$$= |0.78 - 0.7738| = \text{Absolute Error} = 0.0062$$

$$802.22 =$$

$$\text{Relative Error} = 0.0062$$

## (a) Exact Integration:

Integration

for interval  $[0, 2]$ 

$$f(x) = e^{2x}$$

$$I(f) = \int_0^2 e^{2x} dx$$

$$= \frac{e^{2x}}{2} \Big|_0^2$$

$$= [26.80] \text{ (actual)}$$

$$\frac{d}{dx} (a^x) = a^x \ln(a)$$

$$\int (a^x) = \frac{a^x}{\ln(a)}$$

$$\int \ln(x) = x \ln(x) - x$$

$$R_3 A + R_1 A + \sigma D = (x) S_9$$

$$R_2 S_9 - R_3 S_9 = S_2 =$$

## (b) Closed Newton Cotes formulae with degree(n)=1

(Trapezium Rule)

For degree(n)=1 ; nodes = 2  $\{x_0, x_1\}$ 

Formula:

$$I_1(f) = \frac{b-a}{2} [f(a) + f(b)]$$

$$\text{interval } [0, 2] \quad f(x) = e^{2x}$$

$$I_1(f) = \frac{b-a}{2} [f(a) + f(b)]$$

$$= \frac{2-0}{2} [e^{2(0)} + e^4]$$

$$= 55.598. \text{ (Approximate)}$$

$$\text{Absolute Error} = | \text{Actual} - \text{Approximate} |$$

$$= 28.798$$

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$$\text{Relative Error} = \frac{|\text{Actual} - \text{Approximate}|}{\text{Actual}} \times 100 \text{ %} \quad (1)$$

$$= 1.074 \quad \frac{0.074}{1.000} \times 100 \text{ %}$$

③ Closed Newton Cotes formula with degree (n) = 2  
(Simpson Rule)

1 + m : ab0m

$\int_a^b f(x) dx \approx I_2(f) = f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)$  ; degree (n) = 2 ; node = 3  $\{x_0, x_1, x_2\}$

$I_2(f) = \frac{b-a}{6h} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$

$$f(x) = e^{2x} \quad [\text{interval} = [0, 2]]$$

 $\therefore S = 1+1 = 2$ 

$$I_2(f) = \frac{2-0}{6} \left[ e^{2(0)} + 4e^{2\left(\frac{2+0}{2}\right)} + e^{2(2)} \right]$$

$$= \left[ e^0 + 4e^2 + e^4 \right] \frac{2}{6} = \frac{2}{6} = 0.3333$$

$$= \frac{(85.15437)2}{6} = \frac{170.30874}{6} = 28.38$$

$$\text{Absolute Error} = \left[ \frac{|\text{Actual} - \text{Approximate}|}{\text{Actual}} \right] \times 100 = 5.15$$

$$= \left[ \frac{|26.80 - 28.38|}{28.38} \right] \times 100 =$$

$$= 1.584 \left[ \frac{1}{2} + \frac{1}{3} + 1 \right] \times 100 =$$

$$\text{Relative Error} = 0.0591$$

$$= 1.584 \times 100 =$$

## 4) Composite Newton-Cotes

$[a, b]$  interval is splitted into  $m$  sub-intervals

$$h = \frac{b-a}{m}$$

$\therefore$  Sub-intervals:  $m$  of width  $h$  (G)

nodes:  $m+1$

$$C_1, m = \frac{h}{2} \left[ f(x_0) + 2f(x_1) + 2f(x_2) + \dots + f(x_m) \right]$$

$\left[ (1) f + \left( \frac{1}{2} \right) f + (1) f \right] \xrightarrow{\text{First Node}} \xrightarrow{\text{Middle Node}} \xrightarrow{\text{Last Node}}$   $\uparrow$

Sub-interval = 2 ( $m=2$ )

$$h = \frac{(b-a)}{2m} = \frac{(2-0)}{2} = \frac{1}{2}$$

node =  $m+1 = 3$  nodes

$x_0 = a = 0$   $\xleftarrow{\text{Lower bound}}$

$x_1 = x_0 + h = 0 + 1 = 1$

$x_2 = x_1 + h = 1 + 1 = 2$   $\xrightarrow{\text{Upper bound}}$

$$C_{1,2} = \frac{h}{2} \left[ f(x_0) + 2f(x_1) + f(x_2) \right]$$

$$= \frac{1}{2} \left[ e^{2(0)} + 2e^{2(1)} + e^{2(2)} \right]$$

$$= \frac{1}{2} [ 1 + 2e^2 + e^4 ]$$

$$= 35.18813$$

1020.0 = error estimate