Quasi-Newton method (Secant form)

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

→ The computational cost is higher when we need to evaluate two different functions.

So, We can easily replace $f'(x_k)$ by computable function g_k.

$$f'(x) \approx g_k$$

To improve newton's method, we introduced Quasi-Newton's method. It can be done in diffinent approach.

1) Secant Method
2) Steffensen's method ✗ [This is not included in our syllabus avoid it]

To remove or replace $f'(x_k)$, we use backward difference formula.

Prove of Secant Method formula

backward difference, $f'(x) = \dfrac{f(x) - f(x-h)}{h}$ --- (i)

Let

$$x_k = x$$

$$x_{k-1} = x - h$$

plug these values in equation (i)

$$f'(x) = \frac{f(x) - f(x-h)}{x - x + h}$$

$$f'(x) = \frac{f(x) - f(x-h)}{x - (x-h)}$$

$$f'(x_k) = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}} \quad \text{--- (ii)}$$

We know,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= x_k - \frac{f(x_k)}{\dfrac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}}$$

$$= \boxed{x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}}$$

(Proved)

Formula of Secant Method

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}}$$

Here we can see we have to compute only one function. For these reason computational cost will be less.

Example :

$x_0 = 0.03$ and $x_{-1} = 0.02$

Show three iterations for $\boxed{f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-3}}$

$f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-3}$

iterations 1:

$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}$$

$$= 0.03 - \frac{f(0.03)(0.03 - 0.02)}{f(0.03) - f(0.02)}$$

$$= 0.03 - \frac{3.8715 \times 10^{-5}}{-6.35 \times 10^{-5}}$$

$$= 0.639685$$

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$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$= 0.639685 - \frac{f(0.639685)(0.639685 - 0.03)}{f(0.639685) - f(0.03)}$$

$$= -156.84$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= -156.84 - \frac{f(-156.84)(-156.84 - 0.639685)}{f(-156.84) - f(0.639685)}$$

$$= 637.5856$$

[calculate these values for double check]