Practice Problems: Chapter 2

1. Evaluate the interpolation polynomial of a function $f(x) = sin\ (x)$ at the nodes $\{-\pi/2, 0, \pi/2\}$ by using the Vandermonde method. Note you need to show a detailed calculation to get full credit.
2. Consider the function $f(x) = xe^x$. Using Taylor Expansion a polynomial of degree 3 is calculated.
   a. Using Taylor Expansion, write f(x) as an infinite series.
   b. Find the coefficients a_0, a_1, a_2 and a_3 for the polynomial $P_3(X)$.
   c. Compute $f(0.1)$ and $P(0.1)$ up to seven significant figures.
   d. Find the percentage of error using the answers got from Question (c)
3. Consider the function $f(x) = ln(1+x)$. Using Taylor Expansion a polynomial of degree 3 is calculated.
   a. Using Taylor Expansion, write f(x) as an infinite series.
   b. Find the coefficients $a_0, a_1\ and\ a_2$ for the polynomial $P_2(X)$.
   c. Compute $f(0.2)$ and $P(0.2)$ up to seven significant figures.
   d. Find the percentage of error using the answers got from Question (c)
4. A function, $f(x) = x\ sin\ (x)$, is to be interpolated at three nodes: $x_0 = -\pi/2$, $x_1 = 0$ and $x_2 = \pi/2$. Answer the following:
   a. Draw a sketch of the function $f(x)$ and identify the nodes. Briefly explain what kind of interpolation polynomial is expected and discuss the properties of the polynomial.
   b. Compute Lagrange bases.
   c. Write the interpolation polynomial and compare the properties of the interpolation polynomial with the expectation in Part-(a).

5. Read the following and answer accordingly:
   a. Find an interpolating polynomial of appropriate degree using Newton's divided-difference method for $f(x) = x\ cos\ (x)$. Consider the nodes $[-\pi/2, 0, \pi/2]$.
   b. Use the interpolated polynomial to find an approximate value at $\pi/4$, and compute the relative error at $\pi/4$.
   c. Add a new node π to the above nodes, and find the interpolating polynomial.

6. Given, $f(x) = sin(x)$ and Consider the nodes $[\,0, \pi/2,\ \pi]$. Use Newton's Divided-Difference method for the following questions-
   a. Find the values of $a_0, a_1\ and\ a_2$.
   b. Write down the interpolating polynomial.
   c. Add a new node $3\pi/2$ to the existing nodes and find a new interpolating polynomial.
   d. Estimate the upper bound of the interpolating error for the polynomial P_3(x).

7. Consider the following table of data points/nodal points:

Time (sec) t	Velocity (ms^{-1}) v(t)
2	10
4	20
6	25

a. Find an interpolating polynomial of velocity that goes through the above data points by using Vandermonde Matrix method. Also compute an approximate value of acceleration at Time, t = 7 sec.
b. Find an interpolating polynomial of velocity that goes through the above data points by using Lagrange method.
c. If a new data point is added in the above scenario, which method should you use in finding a new interpolating polynomial? Also what will be the degree of that new polynomial?

8. Use a Hermite polynomial that agrees with the data listed below to find the approximation of f(1.5).

k	x_k	$f(x_k)$	$f(x_k)$
0	1.3	0.6200860	-0.5220232
1	1.6	0.4554022	-0.5698959
2	1.9	0.2818186	-0.5811571

9. Find the value of y for x = 2.1 using 2nd order Lagrange polynomials with the appropriate data.

SI.	x	y
1	-1	2.2
2	0	10.6
3	1	17.0
4	2	22.4
5	3	25.8

10. Let $f(x) = cos(x)$ and the nodes are $(-\pi/4, 0, \pi/4)$. Use Cauchy's theorem to find the upper bound of the error.

11. a. Compute the upper bound of error using Cauchy's Theorem for $f(x) = \sin^2 (x/2)$ with nodes $\{-\pi/3, 0, \pi/3\}$ within the interval [-1.2, 1.2].
b. Why are Chebyshev nodes an optimal choice in Interpolation?

12. The following nodes come from the function $f(x) = \ln(5x+9)$:

x	f(x)
-0.5	1.87
0	2.20
0.5	2.44

a. Using Newton's divided difference method, find the equation of a second degree polynomial which fits the above data points.
b. Expand the function $f(x) = \ln(5x + 9)$ using Taylor Series, centered at 0. Include till the x^2 term of the Taylor series.
c. Should the equation which you found in part (a) and part (b) match? Comment on why, or why not.

13. Consider the following nodes:

x	f(x)
0	5
3	9.5
6	5

a. If an equation of a polynomial which fits through the above nodes is found using both the Vandermonde Matrix approach and the Lagrange approach, will both the equations match?
b. Find the equation of a polynomial which fits through the above nodes using the Vandermonde matrix approach.
c. Find the equation of a polynomial which fits through the above nodes using the Lagrange approach.

14. Given a Runge Function, $f(x) = \frac{8}{5+16□^2}$ between the interval [-6,6] where n=3
a. Calculate the equal angled points of □'s.
b. Calculate the values of the Chebychev's nodes.
c. Find the Lagrange basis $l_2(x)$.

15. Consider the following dataset:

x	$f(x)$	$f'(x)$
0.1	-0.62050	3.58502
0.2	-0.28340	3.14033

Answer the following based on the above data:

a. Compute the Hermite bases: $h_0(x)$, $h_1(x)$, $\hat{h}_0(x)$ and $\hat{h}_1(x)$.

b. Write the Hermite polynomial and find the value at $x = 0.15$