

Practice Problems: Chapter 1

1. The floating point representation can be expressed in any of the following forms:

Standard Form	:	$F = (\pm 0.d_1d_2d_3 \cdots d_m)_\beta \times \beta^e; \quad d_1 \neq 0.$
IEEE Normalized Form	:	$F = (\pm 0.1d_1d_2d_3 \cdots d_m)_\beta \times \beta^e.$
IEEE Denormalized Form	:	$F = (\pm 1.d_1d_2d_3 \cdots d_m)_\beta \times \beta^e.$

a) Consider a system with $\beta = 2$, $m = 4$, and $-3 \leq e \leq 4$. Find out the **maximum** and **minimum** numbers this system can store with and without **negative support**. Express the numbers both in binary and decimal digits for all three forms.

b) How many numbers can this system represent or store in all these forms?

c) Using **Standard Form**, find all the decimal numbers without **negative support**, plot them on a real line, and show if the number line is **equally spaced** or not.

d) For the **IEEE standard** for double-precision (64-bit) arithmetic, find the **smallest positive number** and the **largest number** representable by a system that follows this standard. Do not find their decimal values, but simply represent the numbers in the following format:

$$(\pm 0.1d_1d_2d_3 \cdots d_m)_\beta \times \beta^{e-\text{exponentBias}}.$$

Be mindful of the conditions for representing $\pm\infty$ and ± 0 in this IEEE standard.

e) In the above IEEE standard, if the exponent bias were to be altered to $\text{exponentBias} = 500$, what would the **smallest positive number** and the **largest number** be? Write your answers in the same format as in part (d). Note that the conditions for representing $\pm\infty$ and ± 0 are still maintained as before.

2. If $x = 3/8$ and $y = 5/8$, find $fl(x \times y)$ where $m = 4$. Also check whether $x \times y = fl(x \times y)$. If not, find the **rounding error** of the product of these two numbers.
3. Consider the quadratic equation, $x^2 - 60x + 1 = 0$. Working to **6 significant figures**, compute the **roots** of the quadratic equation and check that there is a **loss of significance**. Find the **correct roots** such that loss of significance does not occur.
4. Given $\beta = 2$, $m = 5$, $-100 \leq e \leq 100$. Using the **IEEE Normalized form**, answer the following:
 - a) Compute the Machine Epsilon (ϵ_M).
 - b) Compute the minimum of $|x|$.
 - c) How many non-negative numbers can you represent using this system?

5. Consider the quadratic equation $x^2 - 16x + 3 = 0$. Explain how the loss of significance occurs in finding the roots of the quadratic equation if we restrict to **4 significant figures**. Discuss how to avoid this and find the roots.

6. Given a system parameterized by $\alpha = 2$, $\beta = 3$, and $\alpha_{\min} = -1 \leq \alpha \leq \alpha_{\max} = 3$, where

$\alpha \in \mathbb{Q}$. For this system answer the following:

- Find the floating-point representation of the numbers $(6.25)_{10}$ and $(6.875)_{10}$ in the Normalized Form. That is, find $fl(6.25)_{10}$ and $fl(6.875)_{10}$.
- What are the rounding errors ϵ_1, ϵ_2 in part (a)?
- Can the values $(6.25)_{10}$ and $(6.875)_{10}$ be represented in the Denormalized Form? If so, find the floating-point representations. If not, then concisely explain why?
- Find the rounding error for Standard Form, Normalized and Denormalized Form.

7. Consider the **real number** $x = (8.235)_{10}$

- First convert the decimal number x in binary format at least up to 8 binary places.
- What will be the binary value of x [Find $fl(x)$] if you store it in a system with $m = 6$ using the **Denormalized** form of floating point representation.
- Now convert back to decimal form the stored values you obtained in the previous part, and calculate the **rounding error of both numbers**.

8. Consider the quadratic equation:

$$x^2 - 12x + 5 = 0$$

- Compute the roots of the quadratic equation while keeping to **four significant figures**.
- Explain how **loss of significance** occurs in this case due to the subtraction of nearly equal numbers.
- Discuss an alternative approach to computing the roots to **avoid loss of significance**, and use this method to determine the correct roots.

9. Consider a computing system with base $\beta = 2$, $m = 3$, and $e_{\min} = -3 \leq e \leq e_{\max} = 2$

- In the Standard form of this system, determine the total number of representable values including support for negative numbers. Also, compute the maximum value of delta.
- Express the floating-point representations (binary format) for the numbers $x = 4/8$ and $y = 7/8$ in this system.
- Compute $fl(x \times y)$ and determine whether this value can be stored within the given floating-point system.