Final Exam

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Read the following instructions/rules before you start answering the questions:

- **You MUST keep a Backup Data plan, in case your internet service is interrupted. NO EXCUSE.**
- **The exam time is 1 hour 15 minutes and an additional 15 minutes is for submission of answer script and uploading. So total is 1 hour 30 minutes. No more time time extension, and NO EMAIL SUBMISSION will be accepted.**
- Prepare a Title page containing a) Your Name, b) Your BRACU ID #, c) Your Theory class Section # and d) Date and e) Exam Name.
- Prepare the solution file in orderly fashion: first question first, then maintain the serial.
- Always start an answer to a question from a fresh page.
- Write legibly and neatly, FOLLOW the significant rule for all your calculations.
- YOU MUST WORK ALONE. INVOLVEMENT IN UNFAIR MEANS WILL BE REPORTED TO THE AUTHORITY.

Question # 11: The numerical derivative involves both the truncation and the rounding errors. Let ϵ_M be the machine epsilon of a system. The total error of a derivative of a function $f(x)$ for $x \in [x-h, x+h]$ is given by

$$\text{Error} \leq \frac{h^2}{6}|f'''(\xi)|_{\max} + \frac{\epsilon_M}{h}|f(\xi)|_{\max},$$

where $\xi \in [x-h, x+h]$. The first term on the right-hand side is the contribution from truncation and the second term is the contribution from rounding.

1. [2 Marks] Show that the error is extremum if $h = \left(3\epsilon_M \frac{|f(\xi)|_{\max}}{|f'''(\xi)|_{\max}}\right)^{1/3}$.
2. [2 Marks] Compute h upto five decimal places if $\epsilon_M = 1.0 \times 10^{-10}$, $f(x) = \sin(x)$ for the interval $I = [-\pi, \pi]$

Problem # 22: Consider the function $f(x) = x^2 - 25x + 150$ with $x_\star \in [7, 14]$.

1. [2 Marks] Starting from $f(x) = 0$ and using Newton's method, construct the iteration formula $x_{k+1} = g(x_k)$ where $k = 0, 1, 2, \cdots$
2. [2 Marks] Starting from $x_0 = 9$, compute up to x_3 using the iteration formula found in the previous part. Express the values up to five decimal places.

Question # 32: An upper-triangular system $Ux = b$ can be solved by backward substitution formula

$$x_j = \frac{b_j - \sum_{k=j+1}^{n} u_{jk}x_k}{u_{jj}}, \quad j = n, \ldots, 1.$$

1. [3 marks] Derive the formula for the number of floating-point operations required to find the upper triangular matrix.
2. [1 mark] If the matrix U is of order 18×18 what would be the total number of operations needed to solve the system.

Question # 41:Answer the following: (Show ALL the steps)

1. [2 Marks] Use the Gram-Schmidt process to orthogonalize the following vectors

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad u_3 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

2. [2 Marks] Explain the difference between orthogonal and orthonormal vectors.

Question # 53: A function is given by $f(x) = e^{2x+4} + 5x^2$Answer the following:

1. [3 Marks] Calculate $f'(3)$ using the central difference method with step size 0.5.
2. [3 Marks] Calculate $f'(3)$ using the central difference method with step size 0.25.
3. [3 Marks] Using the results in the previous two parts, calculate $D_{0.5}^{(1)}$ using Richardson Extrapolation formula.
4. [3 Marks] Calculate the exact value of $f'(3)$ and find the actual error of the first derivative $D_{0.5}^{(1)}$.

Question # 66: The following data points has been collected from an over-determined linear system,

$$f(-2) = 4, \quad f(-1) = 2, \quad f(1) = 2 \quad \text{and} \quad f(2) = 4\,.$$

All questions below are related to the QR-decomposing method and finding the approximate polynomial p_2 that represent the data points. Also all symbols use here has the usual meanings. Now, answer the following:

1. [1 Mark] From the given data points, construct the matrices A and b of the over-determined system. Also identify the linearly independent column vectors u_1, u_2 and u_3 from A
2. [1 Mark] Now From u_1, write down the vector p_1 and compute the first orthonormal vector q_1.
3. [2 Marks] Use Gram-Schmidt procedure to construct the vector p_2 from u_2 and q_1. And then compute the second orthonormal vector q_2.
4. [3 Marks] Use Gram-Schmidt procedure to construct the vector p_3 from u_3 , q_1 and q_2. And then compute the third orthonormal vector q_3.
5. [3 Marks] Now identify and write the matrices Q and R
6. [2 Marks] Using Q, $R$and b, find the coefficients a_0, a_1 and a_2 of the polynomial p_2 , and express the solution of the system in standard polynomial form.

FINAL EXAM SUBMISSION

Status

You have completed this assignment. Your final grade will be available when the assessments of your response are complete.

COMPLETE

▸ 1 | Your Response due Sep 22, 2021 17:00 +06 (in 8 minutes)

NOT AVAILABLE

2 | Staff Grade

Waiting for a Staff Grade

Check back later to see if a course staff member has assessed your response. You will receive your grade after the assessment is complete.

▾ Your Grade: Waiting for Assessments

You have completed your steps in the assignment, but some assessments still need to be done on your response. When the assessments of your response are complete, you will see feedback from everyone who assessed your response, and you will receive your final grade.

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