LU Decomposition

Example 1:

$$x_1 + 2x_2 + x_3 = 0$$
$$x_1 - 2x_2 + 2x_3 = 4$$
$$2x_1 + 12x_2 - 2x_3 = 4$$

Step 1: Find coefficient matrix. Find $A^{(1)}$ {coeff matrix}

$$A^{(1)} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{bmatrix}$$

To make this value 0

to make this value 0.

$$R_2 = R_2 - \left(\tfrac{1}{1}\right) R_1$$
$$R_3 = R_3 - \left(\tfrac{2}{1}\right) R_1$$

Step 1

multiplier

Step 2: Find Frobenius matrix $F^{(1)}$

$$F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Negative values of multipliers. We get this by making 0 in the first column

diagonally the values should be 1.

In step 1, we are not getting any multiplier for this. So for the time being we are considering this value as 0.

Step 3: find A^2

$$A^2 = \underbrace{F^{(1)} \times A^{(1)}}_{\text{Do not change this sequence.}} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 8 & -4 \end{bmatrix}$$

$R_3 = R_3 - \left(\frac{8}{-4}\right) R_2$

$= R_3 - (-2) R_2$

to make this value 0

multiplier

Step 4: Find Frobenius matrix $F^{(2)}$

$$F^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

In step 3 we are not getting any value (multiplier). So for this time being we are considering this value as 0.

diagonally the values should be 1

From step (3) we get the multiplier but here we use negative value of the multiplier.

Step 5 : Find the upper triangular matrix. [U] or $A^{(3)}$

$$[U] = A^{(3)} = \boxed{F^{(2)} \times A^{(2)}}$$

→ maintaining this sequence is very important

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 8 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

upper triangular matrix.

Step 6 : Create lower triangular matrix

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix}$$

→ diagonally the values should be 1.

Actual values of multipliers

Step 7: Compute $L\,a = b$ → y values or f(x)

a → temporary values

$$L\,a = b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$$

$$\boxed{a_1 = 0}$$

$$a_1 + a_2 = 4$$

$$\boxed{a_2 = 4}$$

$$2a_1 - 2a_2 + a_3 = 4$$

$$\boxed{a_3 = 4 + 2 \times 4 = 12}$$

Step 8: Compute $U\,x = a$

U → upper triangular matrix

x → real unknown variables

a → temporary values from step 7

$$U\,x = a$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 12 \end{bmatrix}$$

$$\boxed{x_3 = 12/-2 = -6}$$

$$\boxed{-4x_2 + x_3 = 4 \Rightarrow x_2 = -2.5}$$

$$\boxed{x_1 + 2x_2 + x_3 = 12 \Rightarrow x_3 = 11.}$$

Advantage: if the values of b ever changes we don't have to do the task from the beginning.