Overdetermined System

$$5x_1 + 2x_2 + 3x_3 = 7$$
$$2x_1 + 7x_2 + 8x_3 = 5$$
$$3x_1 + 9x_2 + 2x_3 = 6$$

From here we can see

the number of equations = number of unknown variables

We can solve this using guassian Elimination, LU Decomposition and inverse matrix. However, we add another equation with the previous set of equations. We will get.

$$\left[\begin{array}{l} 5x_1 + 2x_2 + 3x_3 = 7 \\ 2x_1 + 7x_2 + 8x_3 = 5 \\ 3x_1 + 9x_2 + 2x_3 = 6 \\ \boxed{4x_1 + 2x_2 + 5x_3 = 10} \end{array}\right.$$

We add this new equation

coefficient matrix from this

$$A = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 7 & 8 \\ 3 & 9 & 2 \\ 4 & 2 & 5 \end{bmatrix}$$

This is a 4×3 matrix. It is not a square matrix

$m \times n$ matrix $\quad m > n$

We cannot directly solve this using Gaussian Elimination, LU Decomposition and Inverse matrix because it is not a square matrix.

The number of equation $\neq$ The number of unknown variables.

This is known as $\boxed{\text{overdetermined system}}$ where the number of equations are greater than the number of variables. (which means the matrix is not the square matrix).

Orthonormality

Orthogonal

Normal (Norm)

Orthogonality:

$$x^T y = 0$$

Normality (Norm)

$$x^T x = 1$$

$$y^T y = 1$$

Example: check if the following set is $\boxed{\text{orthonormal}}$.

$$S = \left\{ \underbrace{\frac{1}{\sqrt{5}} (2, 1)^T}_{u^T}, \underbrace{\frac{1}{\sqrt{5}} (1, -2)^T}_{v^T} \right\}$$

* Since the question asks us to check for

orthonormality, we need to verify both orthogonality and normality.

First we check normal:

$$u^T u = 1 \quad , \quad v^T v = 1$$

$$u^T u = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= 5/5 = 1$$

$$\boxed{u^T u = 1} \longrightarrow \text{True}$$

$$v^T v = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & -2 \end{bmatrix} \times \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$= \frac{1}{5} \times 5 = 1$$

$$\boxed{v^T v = 1} \longrightarrow \text{True}.$$

Now we check orthogonality:

$$\boxed{U^T v = 0}$$

$$U^T v = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 \end{bmatrix} \times \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$= \frac{1}{5} (2-2)$$

$$= \frac{1}{5} \times 0 = 0$$

$$\boxed{U^T v = 0} \rightarrow \text{True}$$

Since both normality and orthogonality has been proved so the matrix has orthonormal properties.

Important

* if we have 3 matrices

x, y and z

For normal property we need to prove

$$\boxed{\begin{aligned} x^T x &= 1 \\ y^T y &= 1 \\ z^T z &= 1 \end{aligned}}$$

For orthogonal property we need to prove

$$\boxed{\begin{aligned} x^T y &= 0 \\ y^T z &= 0 \\ z^T x &= 0 \end{aligned}}$$

Krconeckere Delta

$$\delta_{ij} = \begin{cases} 1 & \overset{U}{i} = \overset{U}{j} \quad U^T U \left\{ \text{normal/ norm/ normality} \right. \\ 0 & \overset{u}{i} \neq \overset{V}{j} \quad U^T V \left\{ \text{orthogonality} \right. \end{cases}$$