Cauchy's Theorem

$$\underbrace{\left| f(x) - P_n(x) \right|}_{\text{Error}} = \left| \frac{f^{n+1}(\xi)}{(n+1)!} \underbrace{(x-x_0)(x-x_1) \cdots (x-x_n)}_{\text{as many nodes as there are/ the number of nodes present}} \right|$$

⊞ Find the maximum possible error / upper bound error.

* We are always concerned with the maximum error and that's why we always find upper bound of error or maximum possible error.

Example:

$f(x) = \cos(x)$, interval $\xi / x \in [-1, 1]$

x	$f(x)$
$x_0 = -\pi/4$	$\frac{1}{\sqrt{2}}$
$x_1 = 0$	1
$x_2 = \pi/4$	$\frac{1}{\sqrt{2}}$

$$\boxed{\begin{aligned} f(x) &= \cos(x) \\ f^1(x) &= -\sin(x) \\ f^2(x) &= -\cos(x) \\ f^3(x) &= \sin(x) \end{aligned}}$$

$$\left| f(x) - P_2(x) \right| = \left| \frac{f^3(\xi)}{3!} (x-x_0)(x-x_1)(x-x_2) \right|$$

$$= \underbrace{\left| \frac{\sin(\xi)}{3!} \right|}_{\substack{\sin(x) \\ max = 1 \\ min = -1}} \underbrace{\left| (x+\pi/4)(x-0)(x-\pi/4) \right|}_{w(x)}$$

[We need to get the max of both of these]

The value of ξ/x is limited to 1 since interval $[-1,1]$ given. Though $\sin(x)$ has max value when $x=\pi/2$.

$\pi/2 = 1.57$ which greater than our max interval 1. So we can't use that we will use 1. However if there is no interval given we will write 1 in the place of $\sin(\xi)$. ~~Since the interval is give~~

So we will take $\sin(1) = 0.8415$.

$$\sin(1) = 0.8415$$
$$\sin(-1) = -0.8415$$

[take the maximum value]

So, $\left| \dfrac{\sin(\xi)}{3!} \right|$

$= \left| \dfrac{\sin(1)}{6} \right|$

$= \left| \dfrac{0.8415}{6} \right|$

To get max value of any function, we know

$$\frac{dy}{dx} = 0$$

Given, $w(x) = (x+\pi/4)\, x\, (x-\pi/4)$

$= (x+\pi/4)(x-\pi/4)\, x$ $\quad [a^2-b^2=(a+b)(a-b)]$

$= \left(x^2 - \dfrac{\pi^2}{16}\right) x$

$= x^3 - \dfrac{\pi^2}{16} x$

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$$\therefore \frac{d\,w(x)}{dx} = \frac{d}{dx}\left(x^3 - \frac{\pi^2}{16}x\right)$$

$$w'(x) = 3x^2 - \frac{\pi^2}{16}$$

$$w'(x) = 0$$

$$3x^2 - \frac{\pi^2}{16} = 0$$

$$x = \pm \frac{\pi}{4\sqrt{3}}$$

x	$w(x)$
$\frac{\pi}{4\sqrt{3}}$	-0.186
$-\frac{\pi}{4\sqrt{3}}$	0.186
1	0.3831
-1	-0.3831

From the give interval

[if the interval is not given, we won't need to do this extra part]

max

Here we have to take the $|W(x)|$ modulous of $w(x)$ value. Mainly we have to take only the maximum number without considering its sign.

$$\therefore \text{Max / upperbound error} = \left|\frac{0.8415}{6} \times 0.3831\right|$$

$$= 0.0537$$

(Ans)

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