Newton divided difference form

$$P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2) + a_4(x-x_0)(x-x_1)(x-x_2)(x-x_3) + \cdots + a_n(x-x_0)(x-x_1)\cdots(x-x_{n-1}).$$

Here

$$a_0 = f[x_0]$$
$$a_1 = f[x_0, x_1]$$
$$a_2 = f[x_0, x_1, x_2]$$
$$a_3 = f[x_0, x_1, x_2, x_3]$$
$$a_4 = f[x_0, x_1, x_2, x_3, x_4]$$
$$\vdots$$
$$a_n = f[x_0, x_1, x_2, x_3, \cdots x_n]$$

$\rightarrow$ We write it directly in the equation.

$$P_n(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1) + f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2) + \cdots + f[x_0,x_1,\cdots,x_n](x-x_0)(x-x_1)\cdots(x-x_{n-1}).$$

$\downarrow$

main polynomial for newton divided difference form.

Example 1

Part 1

	x	$f(x)$	
$x_0 =$	-1	5	$= f(x_0)$
$x_1 =$	0	1	$= f(x_1)$
$x_2 =$	1	3	$= f(x_2)$
$x_3 =$	2	11	$= f(x_3)$

Here nodes 4

$\therefore$ degree $=$ nodes $- 1$
$= 4 - 1$
$= 3$

$$P_3(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$
$$+ f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$

Part 1

$x_0 = -1 \quad f[x_0] = 5$

$f[x_0, x_1] = \dfrac{f[x_1] - f[x_0]}{x_1 - x_0} = \dfrac{1-5}{0-(-1)} = -4$

$x_1 = 0 \quad f[x_1] = 1$

$f[x_0, x_1, x_2] = \dfrac{2-(-4)}{1-(-1)} = 3$

$f[x_1, x_2] = \dfrac{3-1}{1-0} = 2$

$f[x_0, x_1, x_2, x_3] = \dfrac{3-3}{2-(-1)} = 0$

$x_2 = 1 \quad f[x_2] = 3$

$f[x_1, x_2, x_3] = \dfrac{8-2}{2-0} = 3$

$f[x_0, x_1, x_2, x_3, x_4] = \dfrac{-25/24 - 0}{4-(-1)} = -5/24$

$f[x_2, x_3] = \dfrac{11-3}{2-1} = 8$

$f[x_1, x_2, x_3, x_4] = \dfrac{7/6 - 3}{4-0} = -\dfrac{25}{24}$

$x_3 = 2 \quad f[x_3] = 11$

$f[x_2, x_3, x_4] = \dfrac{9/2 - 8}{4-1} = 7/6$

$f[x_3, x_4] = \dfrac{20-11}{4-2} = 9/2$

$x_4 = 4 \quad f[x_4] = 20$

For part 2 we have just modified the previous one with new node

*** [Advantage : Here new data can be incorporated easily we don't need to calculate from the beginning.] ***

$$\therefore P_3(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$$+ f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$

given

given

given

$$= 5 + (-4)(x+1) + 3(x+1)(x-0) + 0 \times (x+1)(x-0)(x-1)$$

$$= 5 - 4(x+1) + 3x(x+1)$$

Part 2:

Now if we asked to add another node then what it will look like

x	$f(x)$
$x_0 = -1$	$5 = f(x_0)$
$x_1 = 0$	$1 = f(x_1)$
$x_2 = 1$	$3 = f(x_2)$
$x_3 = 2$	$11 = f(x_3)$
$x_4 = 4$	$20 = f(x_4)$

→ new node add

$$P_4(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$$+ f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2) + f[x_0, x_1, x_2, x_3, x_4]$$

$$(x - x_0)(x - x_1)(x - x_2)(x - x_3)$$

$$= 5 + (-4)(x+1) + 3(x+1)(x-0) + 0 \times (x+1)(x-0)(x-1)$$

$$+ (-5/24)(x+1)(x-0)(x-1)(x-2)$$

$$= 5 - 4(x+1) + 3x(x+1) - \frac{5}{24}x(x+1)$$