Example

Time, t	Velocity, v(t)
3	11
5	21
7	26

nodes = 3

degree = 3 - 1 = 2

[Question: Find an interpolating polynomial of velocity that goes through these data points by using Vandermonde Matrix. Also, Find the approx value of acceleration at Time, t = 7 sec]

$$P_2(x) = a_0 x^0 + a_1 x^1 + a_2 x^2$$

$$P_2(3) = a_0 3^0 + a_1 3^1 + a_2 3^2$$

$$P_2(5) = a_0 5^0 + a_1 5^1 + a_2 5^2$$

$$P_2(7) = a_0 7^0 + a_1 7^1 + a_2 7^2$$

We know,

$$V \cdot A = Y$$

$$\begin{bmatrix} 3^0 & 3^1 & 3^2 \\ 5^0 & 5^1 & 5^2 \\ 7^0 & 7^1 & 7^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 11 \\ 21 \\ 26 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3^0 & 3^1 & 3^2 \\ 5^0 & 5^1 & 5^2 \\ 7^0 & 7^1 & 7^2 \end{bmatrix}^{-1} \begin{bmatrix} 11 \\ 21 \\ 26 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 9 \\ 1 & 5 & 25 \\ 1 & 7 & 49 \end{bmatrix}^{-1} \begin{bmatrix} 11 \\ 21 \\ 26 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 9 \\ 1 & 5 & 25 \\ 1 & 7 & 49 \end{bmatrix}^{-1} \begin{bmatrix} 11 \\ 21 \\ 26 \end{bmatrix}$$

↓ [using calculator]

$$= \begin{bmatrix} 2.5 & -1.5 & 0 \\ -0.607 & 0.7142 & -0.107 \\ 0.0357 & -0.071 & 0.0357 \end{bmatrix} \begin{bmatrix} 11 \\ 21 \\ 26 \end{bmatrix}$$

$$= \begin{bmatrix} -4 \\ 5.5392 \\ -0.17 \end{bmatrix}$$

$$P_2(x) = a_0 + a_1 x^1 + a_2 x^2$$

$$= -4 + 5.5392x - 0.17x^2$$

$$P_2(7) = -4 + 5.5392 \times 7 - 0.17 \times 7^2$$

$$= 26.44 \text{ ms}^{-1}$$

$$\text{Accelaration} = \frac{d}{dx}(P_2(x)) = \frac{d}{dx}(-4 + 5.5392x - 0.17x^2)$$

$$= 5.5392 - 0.34x$$

$$P_2'(x) = 5.5392 - 0.34x$$

$$= 5.5392 - 0.34 \times 7$$

$$= 3.1592 \text{ ms}^{-2}$$