Assignment # 2	CSE 330	Due Date: `Ask Your Faculty`
(All Sections)	Summer 2022	(Or see the Discord Channel)

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the pat of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules.
- Start working right away based on whatever you know. **Do not wait for the last moment and ask for time extension**.

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1. A function is given by $f(x) = 6e^{-5x}$. Now answer the following:
   (a) (2 marks) Approximate the derivative of $f(x)$ at $x_0 = 0.2$ with step size $h = 0.5$ using the central difference method up to 6 significant figures. **-26.7079**
   (b) (2 marks) Approximate the derivative of $f(x)$ at $x_0 = 0.2$ with step size $h = 0.5$ using the forward difference method up to 6 significant figures. **-53.4179**
   (c) (3 marks) Calculate the truncation error of $f(x)$ at $x_0 = 2$ using $h = 1, 0.1, 0.01, 0.0001$ in the above mentioned two methods. **CD: 26.7292, 26.7104, 26.7104, 26.7104** **FD: 53.4182, 53.4190, 53.4192, 53.4193**
   (d) (3 marks) Compute $D_{0.5}^{(1)}$ at $x_0 = 0.2$ using Richardson extrapolation method up to 6 significant figures and calculate the truncation error. **-9.95500, 1.08140**

2. During the class, we derived in detail the first order Richardson extrapolated derivative, by using $h \to h/2$,

$$D_h^{(1)} \equiv f'(x_0) - \frac{h^4}{480} f^{(5)}(x_0) + \mathcal{O}(h^6) \ .$$

   (a) (5 marks) Using $h \to h/2$, derive the expression for $D_h^{(2)}$ which is the second order Richardson extrapolation.
   (b) (5 marks) Now starting from the definition of D_h and using $h \to h/3$, derive the expression for $D_h^{(1)}$.

3. Consider the equation $f(x) = x^3 - 2x^2 - 11x + 12$ on the interval $[1.78, 8.25]$.
   (a) (2 marks) Find the minimum number of iterations required to find the root of the given equation within the error bound of 1×10^{-3}.
   (b) (8 marks) Show 5 iterations using the Bisection Method to find the root of the above function within the interval $[1.78, 8.25]$.

4. Consider the function, $f(x) = x^3 - x^2 - 9x + 9$. Answer the following:
   (a) (2 marks) State the exact roots of $f(x)$.
   (b) (2 marks) Construct three different fixed point functions $g(x)$ such that $f(x) = 0$. (Make sure that one of the $g(x)$'s that you constructed converges to at least a root).
   (c) (3 marks) Find the convergence rate/ratio for $g(x)$ constructed in previous part and also find which root it is converging to?
   (d) (3 marks) Find the approximate root, $x_\star$, of the above function using fixed point iterations up to 4 significant figures within the error bound of 1×10^{-3} using $x_0 = 0$ and any fixed point function $g(x)$ from part(b) that converges to the root(s).

5. (10 marks) Use Newton's method to find the root, $x_\star$, of the equation, $f(x) = x^2\, e^{-x} - 0.6$, up to machine epsilon of 1×10^{-4} starting with $x_0 = 0.2$.
