10/10 Rahman

CSE 330 Numerical Methods

SUMMER 2022

Quiz 4

Full Marks - 10

ANSWER ALL THE QUESTIONS

Time: 20mins

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Section 07

[CO4] Consider the following data points

f(-1) = 2

f(0)=4

f(2)= -3

$y = e^{2x} \ln x$

a. Write down the coefficient matrix A.[1 mark]
b. Using Gram-Schmidt, **determine** the orthonormal set of vectors q1 and q2. Hence write down the Q matrix. [5 marks]
c. Using $Rx=Q^Tb$, **solve** the unknowns a_0 and a_1.[4marks]

OR

[CO3] Consider the function $f(x) = e^{2x} + x^2$

For the above function, within the interval [0,2]:

a. **Calculate** the actual integral. [2 marks]
b. **Calculate** the approximate value of the integral using the Trapezium rule. [2 marks]
c. **Calculate** the approximate value of the integral using the Simpson's rule. [2 marks]
d. **Calculate** the approximate value of the integral using Composite Newton Cotes with 3 segments. [4 marks]

$f(1) = 1$

$f(0) = 1$

$f(2) = -3$

$a_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

$\Rightarrow a_1(2) = \Rightarrow a_1(x) = a_0 + a_1 x$

$\Rightarrow a_1(1) = a_0 + a_1 = 2$

$\Rightarrow a_1(0) = a_0 + 0a_1 = 1$

$\Rightarrow a_1(2) = a_0 + 2a_1 = -3$

(a)

(a) $A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$ (θ_1, θ_2)

(b) $P_k = \theta_k - \sum_{i=1}^{k-1} (\theta_k^T \cdot q_i) q_i$

$\Rightarrow P_1 = \theta_1$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$q_k = \frac{P_k}{|P_k|}$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \frac{1}{\sqrt{1^2+1^2+1^2}}$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \frac{\sqrt{3}}{3}$

$P_2 = \theta_2 - \{(\theta_2^T \cdot q_1)\} q_1$

$= \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} - \left\{ \begin{bmatrix} -1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{\sqrt{3}}{3} \right\} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{\sqrt{3}}{3}$

$= \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} - (0.5773) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{\sqrt{3}}{3}$

$= \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} -0.3333 \\ 0.3333 \\ 0.3333 \end{bmatrix} = \begin{bmatrix} -1.3333 \\ -0.3333 \\ 1.6667 \end{bmatrix}$

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③ $Q = [q_1, q_2]$

$$= \begin{bmatrix} 0.57735 & -1.3333 \\ 0.57735 & -0.3333 \\ 0.57735 & 1.6667 \end{bmatrix}$$

④ $$R = \begin{bmatrix} v_1^T q_1 & v_2^T q_1 \\ 0 & v_2^T q_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.7321 & 0.57735 \\ 0 & 4.6667 \end{bmatrix}$$

$\frac{2\sqrt{6}}{3}$

$v_1^T q_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{\sqrt{3}}{3}$

$= [1 \ 1 \ 1] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{\sqrt{3}}{3}$

$= 1.7321$

$v_2^T q_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}^T \begin{bmatrix} -1.3333 \\ -0.3333 \\ 1.6667 \end{bmatrix} \frac{1}{2}$

$= [-1 \ 0 \ 2] \begin{bmatrix} -1.3333 \\ -0.3333 \\ 1.6667 \end{bmatrix}$

$= 4.6667$

⑤ ⑥ $Q^T b$

$$= \begin{bmatrix} 0.57735 & 0.57735 \\ -1.3333 & -0.3333 \\ 0.57735 & 1.6667 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.57735 & 0.57735 & 0.57735 \\ -1.3333 & -0.3333 & 1.6667 \end{bmatrix}$$

$$\times \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -1.73205 \\ -8.999 \end{bmatrix}$$

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(1) $R_n = Q^T B$

$$\Rightarrow \begin{bmatrix} 1.7321 & 0.57735 \\ 0 & 4.6667 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} -1.73205 \\ -8.999 \end{bmatrix}$$

$$\Rightarrow a_1 = \frac{-8.999}{4.6667}$$

$$= -1.928$$

(2)

$$1.7321 a_0 + (-1.928)(0.57735) = 1.7328$$

$$\therefore a_0 = 1.643$$