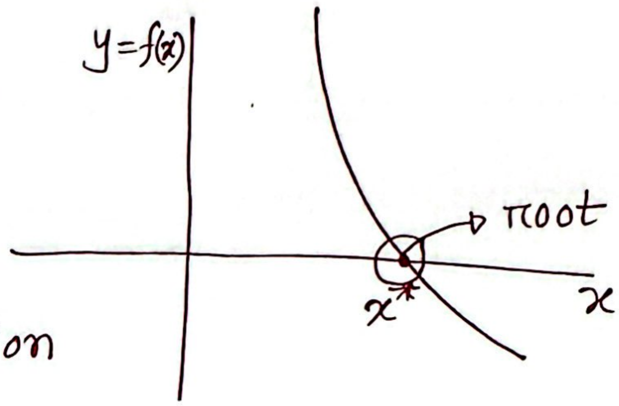
Fixed point iteration

It's an iterative method

Root Finding algorithm

[The algorithm will continue to run until it reaches the root]

The root of a function, $f(x)$ is the point where $f(x) = 0$ which means the point where the function intersects the x-axis.



(*) Given $f(x) = x^2 - 2x - 3$. Find the root for this function?

$$f(x) = 0$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1, 3$$

⊞ Why we use root Finding algorithm?

For lower-degree polynomials, we can easily find the root by using the middle term breaking method. However, for higher-degree polynomials, this process become quite complicated, that's why we use root Finding algorithms to find the roots in such cases.

Fixed point Iteration

$\boxed{f(x) = 0}$ → For fixed point iteration we try to manipulate this function

mathemetically manipulate this

→ $\boxed{g(x) = x}$ *****

→ x is the root of $f(x)$. if it follows the condition

Example: Given $f(x) = x^2 - 2x - 3$

Construct three $g(x)$ from $f(x)$.

$$f(x) = 0$$
$$x^2 - 2x - 3 = 0$$
$$x^2 = 2x + 3$$
$$x = \sqrt{2x+3}$$

$$\boxed{g_1(x) = \sqrt{2x+3}}$$

$$x^2 - 2x - 3 = 0$$
$$x^2 - x - x - 3 = 0$$
$$-x = -x^2 + x + 3$$
$$x = x^2 - x - 3$$

$$\boxed{g_2(x) = x^2 - x - 3}$$

$$x^2 - 2x - 3 = 0$$
$$2x^2 - x^2 - 2x - 3 = 0$$
$$2x^2 - 2x = x^2 + 3$$
$$x(2x - 2) = x^2 + 3$$
$$x = \frac{x^2+3}{2x-2}$$

$$\boxed{g_3(x) = \frac{x^2+3}{2x-2}}$$

Example: show that $g_3(x) = \frac{x^2+3}{2x-2}$ where $f(x) = x^2-2x-3$.

* step 1 do it reversly

rough work

$$g_3(x) = \frac{x^2+3}{2x-2}$$

$$x = \frac{x^2+3}{2x-2}$$

$$x(2x-2) = x^2+3$$

$$2x^2-2x = x^2+3$$

$$2x^2-2x-x^2-3 = 0$$

$$x^2-2x-3 = 0$$

Main work

$$x^2-2x-3 = 0$$

$$2x^2-2x-x^2-3 = 0$$

$$2x^2-2x = x^2+3$$

$$x(2x-2) = x^2+3$$

$$x = \frac{x^2+3}{2x-2}$$

$$g_3(x) = \frac{x^2+3}{2x-2}$$

Example : Now find the root using fixed point iteration. Initial point $x_0 = 0$. [use 3 significant figure]

$$\boxed{g_1(x) = x}$$

$$g_1(x) = \sqrt{2x+3}$$

$$g_2(x) = x^2 - x - 3$$

$$g_3(x) = \frac{x^2+3}{2x-2}$$

we get it, From the previous example

$$g_1(x) = \sqrt{2x+3}$$

[work with 3 significant figure]

$$g_1(0) = 1.73$$

[It's very important to maintain the mentioned significant figure]

$$g_1(1.73) = 2.54$$

$$g_1(2.54) = 2.84$$

$$g_1(2.84) = 2.95$$

Convergence [we have reached the root]

$$g_1(2.95) = 2.98$$

$$g_1(2.98) = 3.00$$

$$g_1(3.00) = 3.00$$

$$\downarrow \qquad \downarrow$$

$$x \qquad x$$

[Fixed point reach]

*

$g_2(x) = x^2 - x - 3$

$g_2(0) = -3.00$

$g_2(-3) = 9.00$

$g_2(9) = 69.0$

$g_2(69.0) = 4.69 \times 10^3$

Divergence (As we didn't reach to the root)

$$g_3(x) = \frac{x^2+3}{2x-2}$$

$g_3(0) = -1.50$

$g_3(-1.50) = -1.05$

$g_3(-1.05) = -1.00$

$g_3(-1.00) = -1.00$

convergence [We have reached the root]

$$\underbrace{-1.00}_{x} \quad \underbrace{-1.00}_{x}$$

$$\boxed{g_3(x) = x}$$

[Fixed point reach]

if the initial point $x_0 = 42$, then

$$g_3(x) = \frac{x^2+3}{2x-2}$$

$g_3(42) = 21.6$

$g_3(21.6) = 11.4$

$g_3(11.4) = 6.39$

$g_3(6.39) = 4.07$

$g_3(4.07) = 3.19$

$g_3(3.19) = 3.01$

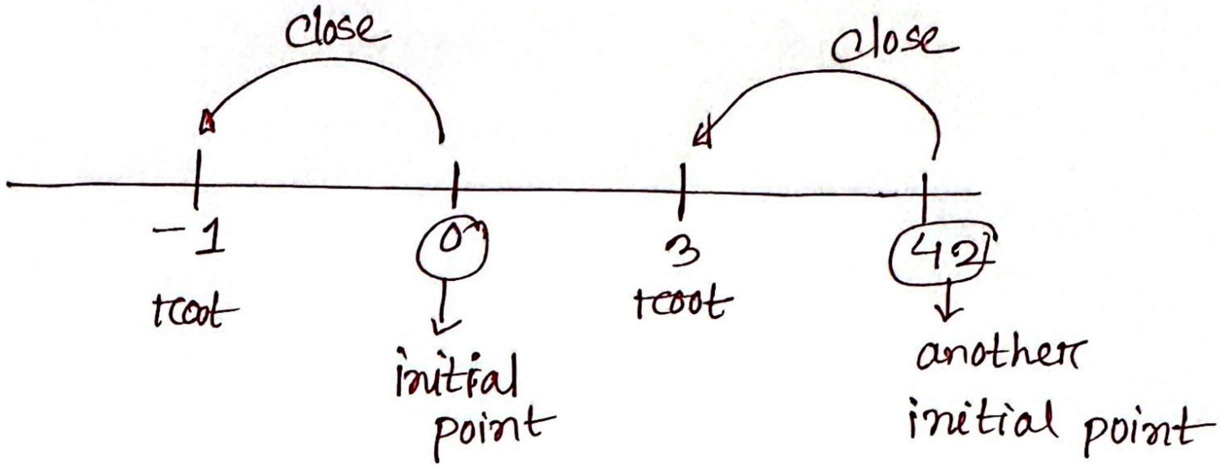
$g_3(3.01) = 3.00$

$g_3(3.00) = 3.00$

$\boxed{g(x) = x}$

[Fixed point reached]

** We have seen the root we converge to depends on the choice of the initial point
* important *



** The root we converge to depends on the choice of the initial point.