Numerical Integration Part II

Simpson's Rule

Newton's cotes formula for $n=2$

→ This is known as Simpson's rule

$$I_2(f) = \int_a^b P_2(x)\,dx$$

$$P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$$

For $n=2$ [Simpson's rule]

range $[a,b]$

$$I_2(f) = \sum_{K=0}^{2} \delta_K f(x_K)$$

As $n=2$ there will be 3 nodes

$$\boxed{h = \frac{b-a}{2}}$$

$$x_0 = a$$

$$x_0 + h = x_1 = \frac{a+b}{2}$$

$$x_2 = b$$

$$\boxed{\text{Let } x_1 = \boxed{\frac{a+b}{2}} = m}$$

$$h = \frac{b-a}{2}$$

$$\boxed{x_0 + h = x_1}$$

$$\Rightarrow a + \frac{b-a}{2}$$

$$\Rightarrow \frac{2a+b-a}{2} \Rightarrow \frac{a+b}{2}$$

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-m)(x-b)}{(a-m)(a-b)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-a)(x-b)}{(m-a)(m-b)}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-a)(x-m)}{(b-a)(b-m)}$$

Now,

$$S_0 = \int_a^b l_0(x)\,dx$$

$$= \int_a^b \frac{(x-m)(x-b)}{(a-m)(a-b)}\,dx$$

$$= \frac{1}{(a-m)(a-b)} \int_a^b (x-m)(x-b)\,dx$$

$$= \frac{1}{6}(b-a)$$

$$S_1 = \int_a^b \frac{(x-a)(x-b)}{(m-a)(m-b)}\,dx$$

$$= \frac{1}{(m-a)(m-b)} \int_a^b (x-a)(x-b)\,dx$$

$$= \frac{2}{3}(b-a)$$

$$\delta_2 = \int_a^b l_2(x)\, dx$$

$$= \int_a^b \frac{(x-a)(x-m)}{(b-a)(b-m)}\, dx$$

$$= \frac{1}{(b-a)(b-m)} \int_a^b (x-a)(x-m)\, dx$$

$$= \frac{1}{6}(b-a)$$

$$\therefore\ I_2(F) = \delta_0 f(x_0) + \delta_1 f(x_1) + \delta_2 f(x_2)$$

$$= \underbrace{\frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]}_{\text{Formula}}$$

P.T.O

Example :

Given Interval $[0,2]$

$$f(x) = e^{0.5x} + \sin x$$

[*] Find the numerical Integral $I_2(f)$.

Simpson's Rule

$$\boxed{I_2(f) = \frac{b-a}{2}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]}$$

$$f(0) = e^{0.5\times(0)} + \sin(0) = 1$$

$$f(2) = e^{0.5\times 2} + \sin(2) = 3.6276$$

$$f\left(\frac{2+0}{2}\right) = f(1) = e^{0.5\times 1} + \sin(1) = 2.4902$$

$$I_2(f) = \frac{b-a}{6}\left[f(a) + 4\times f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$= \frac{2-0}{6}\left[1 + 4\times(2.4902) + 3.6276\right]$$

$$= 4.8628$$