```
(* doing some practical calculations with the lethargy concept *)

Clear["Global`*"];
source[x_] := (1 / (d * Sqrt[\pi])) * Exp[-(x-20)^2 / d^2];

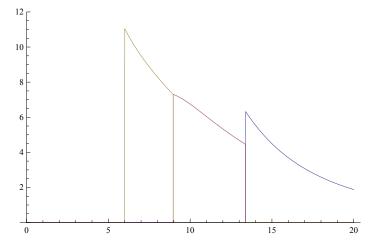
d = 0.3;
a = 10;
alp = (a-1)^2 / (a+1)^2;
e0 = 20;
(* delay differential equation *)
eqn = D[Integrate[F[x] / ((1-N[alp]) * x), {x, e, e / N[alp]}] - F[e], e] == 0

-\frac{3.025 F[e]}{e} + \frac{3.025 F[1.49383 e]}{e} - F'[e] == 0

(* [alp*e0,e0] *)
```

```
eqn1 = D[Integrate[F1[x] / ((1 - N[alp]) * x), {x, e, e0}] - F1[e], e] = 0;
sol1 = NDSolve[{eqn1, F1[20] == source[20]}, F1, {e, N[alp] * e0, e0}];
g1[e_] := If[(N[alp] * e0) \le e \le e0, F1[e] /. sol1[[1]][[1]], 0.0001];
(* [alp^2*e0,alp*e0] *)
eqn2 = D[Integrate[F2[x] / ((1 - N[alp]) * x), \{x, e, N[alp] * e0\}] +
      Integrate[g1[x] / ((1 - N[alp]) * x), {x, N[alp] * e0, e / N[alp]}] - F2[e], e] == 0;
sol2 = NDSolve[{eqn2, F2[N[alp] * e0] = Integrate[g1[x] / ((1 - N[alp]) * x),}
       {x, N[alp] * e0, e0}]}, F2, {e, N[alp] ^2 * e0, N[alp] * e0}];
g2[e] := If[N[alp]^2 * e0 \le e \le N[alp] * e0, F2[e] /. sol2[[1]][[1]], 0.0001];
(* [alp^3*e0,alp^2*e0] *)
eqn3 = D[Integrate[F3[x] / ((1-N[alp])*x), {x, e, N[alp]^2*e0}] + Integrate[
       g2[x]/((1-N[alp])*x), \{x, N[alp]^2*e0, e/N[alp]\}]-F3[e], e] == 0;
sol3 = NDSolve[{eqn3, F3[N[alp]^2 * e0]} = Integrate[g2[x] / ((1 - N[alp]) * x),
       \{x, N[alp]^2 * e0, N[alp]^1 * e0\}\}, F3, \{e, N[alp]^3 * e0, N[alp]^2 * e0\}\};
g3[e_] := If[N[alp]^3 * e0 \le e \le N[alp]^2 * e0, F3[e] /. sol3[[1]][[1]], 0.0001];
```

 $Plot[\{g1[e], g2[e], g3[e]\}, \{e, 0.001, 20\}, PlotRange \rightarrow \{0, 12\}]$ 



 $Plot[\{g1[e] * e, g2[e] * e, g3[e] * e\}, \{e, 0.001, 20\}, PlotRange \rightarrow \{\{0, 20\}, \{30, 100\}\}\}]$ 

