Team 1 Project Model

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Miller-Tucker-Zemlin (MTZ) formulation for Traveling Salesperson Problem (TSP)

$$\min \quad \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} c_{ij} x_{ij}, \tag{1}$$

$$\text{subject to} \tag{2}$$

$$\sum_{i=1, i\neq j}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n, \tag{3}$$

$$\sum_{j=1, j\neq i}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n, \tag{4}$$

$$u_i - u_j + n x_{ij} \leq n - 1, \quad 2 \leq i \neq j \leq n, \tag{5}$$

$$x_{ij} \in \{0, 1\} \quad i, j = 1, 2, \ldots, n, \quad i \neq j, \tag{6}$$

$$u_i \in \mathbb{R}^{+} \quad i = 1, 2, \ldots, n. \tag{7}$$

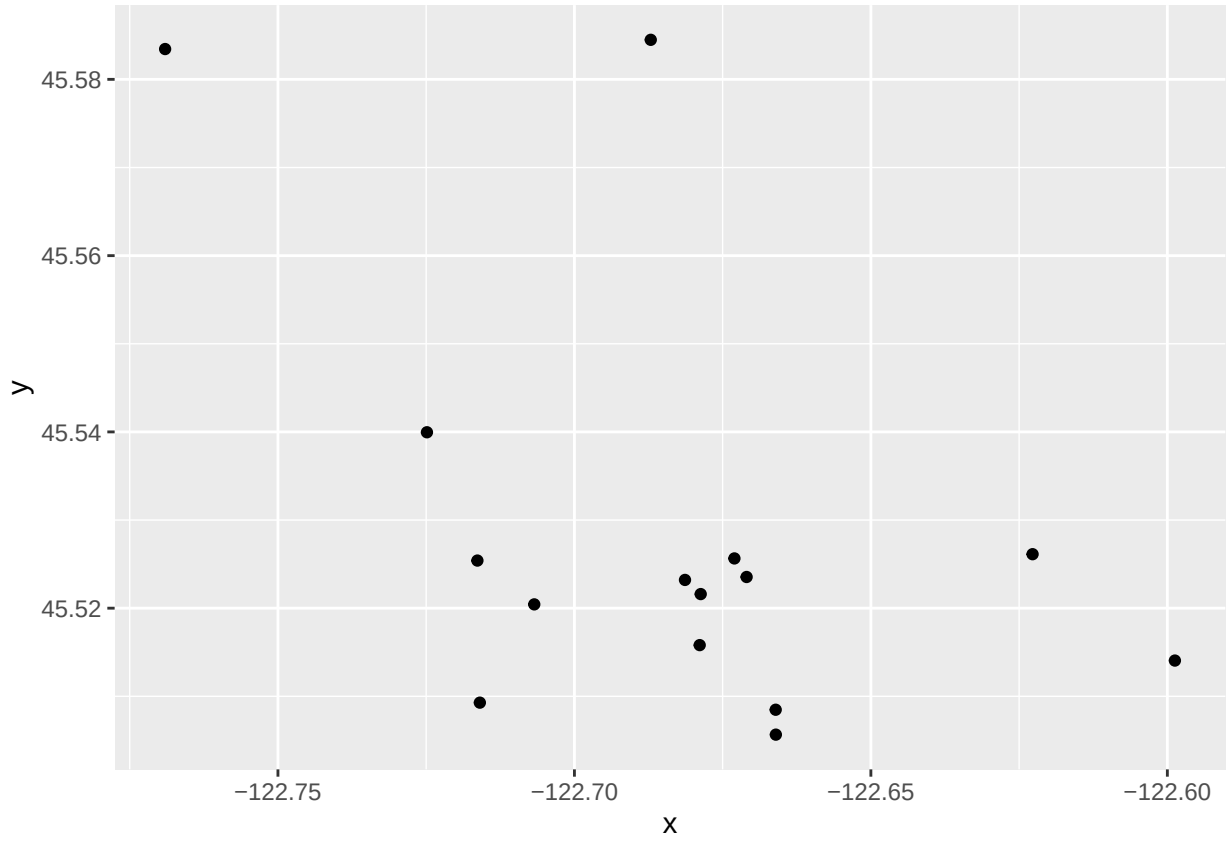
Base Traveling Salesman Problem ompr Model Code to Work From

```
setwd("G:/My Drive/FALL-2021/ETM640/Project/Code/") # SET WORKING DIR

refined_locations <- read.csv("TEST_portland_location_data_2.csv") # LOAD DATA FROM FILE

n <- nrow(refined_locations) # NUMBER OF LOCATIONS TO VISIT (replace with number of data matrix rows)

#Longitude = x, Latitude = y
locations <- data.frame(id = 1:n, x = refined_locations[,7], y = refined_locations[,6])

pander(locations)
```

id	x	y
1	-122.7	45.52
2	-122.7	45.51
3	-122.7	45.53
4	-122.7	45.52
5	-122.7	45.51
6	-122.7	45.53
7	-122.7	45.54
8	-122.7	45.52
9	-122.6	45.51
10	-122.8	45.58

id	x	y
11	-122.6	45.53
12	-122.7	45.51
13	-122.7	45.58
14	-122.7	45.52
15	-122.7	45.52

```
ggplot(locations, aes(x, y)) +
  geom_point()
```




```
distance <- as.matrix(stats::dist(select(locations, x, y), diag = TRUE, upper = TRUE))
dist_fun <- function(i, j) {
  vapply(seq_along(i), function(k) distance[i[k], j[k]], numeric(1L))
}

model <- MIPModel() %>%
  # we create a variable that is 1 iff we travel from location i to j
  add_variable(x[i, j], i = 1:n, j = 1:n,
               type = "integer", lb = 0, ub = 1) %>%
  # a helper variable for the MTZ formulation of the TSP
  add_variable(u[i], i = 1:n, lb = 1, ub = n) %>%
  # minimize travel distance
  set_objective(sum_expr(dist_fun(i, j) * x[i, j], i = 1:n, j = 1:n), "min") %>%
  # you cannot go to the same location
  set_bounds(x[i, i], ub = 0, i = 1:n) %>%
```

```
  # leave each location
  add_constraint(sum_expr(x[i, j], j = 1:n) == 1, i = 1:n) %>%
  # visit each location
  add_constraint(sum_expr(x[i, j], i = 1:n) == 1, j = 1:n) %>%
  # ensure no sub-tours are used (arc constraints)
  add_constraint(u[i] >= 2, i = 2:n) %>%
  add_constraint(u[i] - u[j] + 1 <= (n - 1) * (1 - x[i, j]), i = 2:n, j = 2:n)

result <- solve_model(model, with_ROI(solver = "glpk", verbose = TRUE))
```

```
## <SOLVER MSG>  ----
## GLPK Simplex Optimizer, v4.47
## 240 rows, 240 columns, 1024 non-zeros
##       0: obj =  0.000000000e+000  infeas = 4.400e+001 (30)
## *    48: obj =  8.464042858e-001  infeas = 0.000e+000 (1)
## *   111: obj =  3.242544448e-001  infeas = 3.172e-017 (1)
## OPTIMAL SOLUTION FOUND
## GLPK Integer Optimizer, v4.47
## 240 rows, 240 columns, 1024 non-zeros
## 225 integer variables, 210 of which are binary
## Integer optimization begins...
## +   111: mip =     not found yet >=              -inf        (1; 0)
## +   235: >>>>>  4.586377765e-001 >=  3.247925508e-001  29.2% (18; 0)
## +  1079: >>>>>  4.483590483e-001 >=  3.457391379e-001  22.9% (95; 10)
## +  1381: >>>>>  4.404220551e-001 >=  3.467124149e-001  21.3% (111; 18)
## + 61762: >>>>>  4.401394852e-001 >=  4.217957546e-001   4.2% (660; 2812)
## + 73961: mip =  4.401394852e-001 >=     tree is empty   0.0% (0; 5747)
## INTEGER OPTIMAL SOLUTION FOUND
## <!SOLVER MSG> ----
```

```
solution <- get_solution(result, x[i, j]) %>%
  filter(value > 0)
kable(head(solution, 3))
```

variable	i	j	value
x	4	1	1
x	1	2	1
x	2	3	1

```
paths <- select(solution, i, j) %>%
  rename(from = i, to = j) %>%
  mutate(trip_id = row_number()) %>%
  tidyr::gather(property, idx_val, from:to) %>%
  mutate(idx_val = as.integer(idx_val)) %>%
  inner_join(locations, by = c("idx_val" = "id"))
kable(head(arrange(paths, trip_id), 4))
```

trip_id	property	idx_val	x	y
1	from	4	-122.6814	45.52321
1	to	1	-122.7068	45.52043
2	from	1	-122.7068	45.52043

trip_id	property	idx_val	x	y
2	to	2	-122.7159	45.50928

```
ggplot(locations, aes(x, y)) +
  geom_point() +
  geom_line(data = paths, aes(group = trip_id)) +
  ggtitle(paste0("Optimal route with cost: ", round(objective_value(result), 2)))
```