# Lecture 8

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## 1.Knapsack Greedy algorithm

Prove that the greedy algorithm can be made to perform arbitrarily bad.

**Prove**

Let the capacity bound B = 1, we have two items $a_1$ and $a_2$. $size(a_1) = x$ and $profit(a_1) = 2x$, $size(a_2) = 1$ and $profit(a_2) = 1$, here $0 < x < 1$.

We have $\frac{profit(a_1)}{size(a_1)} = 2 > \frac{profit(a_2)}{size(a_2)} = 1$, according to the greedy algorithm, we should pick $a_1$.The total profit is $2x$, can be arbitrarily small when $x \to 0$.

However, if we choose $a_2$, the total profit is 1.

## 2.Improvement for Greedy algorithm

Show the approximation ratio of the modifeid algorithm is $\frac{1}{2}$

**Prove**

Assume that

$$\frac{p(a_1)}{s(a_1)} \geq \frac{p(a_2)}{s(a_2)} \geq ... \geq \frac{p(a_n)}{s(a_n)}$$

Denote by $OPT$ the optimal result, now only need to prove

$$\sum_{i=1}^{k+1} profit(a_i) \geq OPT$$

Which is obvious because $\sum_{i=1}^{k+1} size(a_i) > B$, and they are chosen in a greedy way.

Therefore

$$max\{\{a_1, a_2, ..., a_k\}, a_{k+1}\} \geq \frac{1}{2} OPT$$

## 3.Steiner Tree

Let G=<V,E> be a graph with nonnegative edge costs. S,the senders and R, the receivers, are disjoint subsets of V . The problem is to find a minimum cost subgraph of G that has a path connecting each receiver to a sender (any sender suffices). Partition the instances into two cases: $S \cup R = V$ and $S \cup R \neq V$ . Show that these two cases are in P and NP-hard,respectively. For the second case, give a factor 2 approximation algorithm.

Solve:

$S \cup R = V$ instance:

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1 Polynomial Algorithm:
2 T = NULL
3 while R not empty do
4     find a shortest edge e connect a vertex in R (say v) and a vertex in S
5     add edge e in T, remove v from R, add v in S
6 endwhile
```

In *line 3* of each loop, we pick a shortest edge, and this edge must be in the optimal set. This guarantees that the algorithm produces the optimal result.

$S \cup R \neq V$ instance:
First, we show that this problem is NP-hard by reducing original Steiner Tree problem to it.
For a Steiner Tree problem with G=<V,E>, $V = R \cup S$. Select a random vertex in R, say v, let $S' = v$, $R' = R - v$, S in Steiner Tree problem denotes (V-S'-R') in the new problem.

2-approximation algorithm:
Add a new vertex which is connected to each sender by a zero cost edge. Consider the new vertex and all receivers as required and the remaining vertices as Steiner, and find a minimum cost Steiner tree.

$$result = result(SteinerTree) \leq 2 \cdot OPT(SteinerTree) = 2 \cdot OPT$$