

Findings from a Computer-supported Analysis of Variants of Gödel's Ontological Proof

Christoph Benzmüller¹, Freie Universität Berlin
jww: Bruno Woltzenlogel-Paleo, TU Vienna

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>
>
> Beweise-mit-Leo2 Notwendigerweise-existiert-Gott.p
Leo-II tries to prove
=====
Goedel's Theorem T3: "Necessarily, God exists"
thf(thmT3,conjecture,
  ( v
    @ ( mbox
      @ ( mexists_ind
        @ ^ [X: mu] :
          ( g @ X ) ) ) )).
Assumptions: D1, C, T2, D3, A5
. searching for proof ..
*****
* Proof found *
*****
% S2S status Theorem for Notwendigerweise-existiert-Gott.p
. generating proof object □
```

¹Supported by DFG Heisenberg Fellowship BE 2501/9-1/2



If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other . . . : Let us calculate.

(Translation by Russell)

Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo . . . dicere: calculemus.
(Leibniz, 1684)



Required:
characteristica universalis and **calculus ratiocinator**



Ontological argument for the existence of God

Focus on Gödel's modern version in higher-order modal logic

Automation with provers for higher-order classical logic (HOL)

- ▶ verification (or falsification) of known results
- ▶ some novel results (by HOL-ATPs)
- ▶ HOL as a universal metalogic via logic embeddings
(characteristica universalis?)

- ▶ Philosophical: Boundaries of Metaphysics & Epistemology
- ▶ Theistic: Successful argument could convince atheists?
- ▶ **Ours:** Can computers (theorem provers) be used ...
 - ... to formalize the definitions, axioms and theorems?
 - ... to verify/falsify the arguments step-by-step?
 - ... to automate (sub-)arguments?
 - ... to generate new knowledge?



Anselm's notion of God (Proslogion, 1078):

“God is that, than which nothing greater can be conceived.”

Gödel's notion of God:

“A God-like being possesses all ‘positive’ properties.”³³

To show by logical, deductive reasoning:

“God exists.”

$$\exists x G(x)$$



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Gödel's Manuscript: 1930's, 1941, 1946-1955, 1970

Ontologischer Beweis

Feb. 10, 1970

$P(\varphi)$ φ is positive ($\Leftrightarrow \varphi \in P$)

Ax 1: $P(p), P(\varphi) \supset P(\varphi \wedge p)$ At 2: $P(p) \supset P(\neg p)$

P_1 $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

P_2 $\varphi_{\text{Ex} \exists x} \equiv (\psi)[\psi(x) \supset N(y)[P(y) \supset \psi(y)]]$ (Existence)

$P \supset_N = N(p \supset q)$ Necessity

Ax 2: $P(\varphi) \supset N P(\varphi)$ } because it follows
 $\neg P(\varphi) \supset N \neg P(\varphi)$ } from the nature of the
 property

Th: $G(x) \supset G_{\text{Ex} \exists x}$

Df: $E(x) \equiv P[\varphi_{\text{Ex} \exists x} \supset N \neg x \varphi(x)]$ necessary Existence

Ax 3: $P(E)$

Th: $G(x) \supset N(\exists y) G(y)$

hence $(\exists x) G(x) \supset N(\exists y) G(y)$

" $M(x) G(x) \supset M N(\exists y) G(y)$ M = possibility
 " $\supset N(\exists y) G(y)$

any two elements of X are nec. equivalent

exclusive or * and for any number of them

$M(x) G(x)$ means all pos. prop. w.r.t. com-
 patible. This is true because of:

Ax 4: $P(\varphi) \cdot q \supset_N \varphi \supset P(\varphi)$ which impl.

$\begin{cases} x=x & \text{is positive} \\ x \neq x & \text{is negative} \end{cases}$

But if a system S of pos. prop. were incons.
 It would mean, that the non-prop. S (which
 is positive) would be $x \neq x$

Positive means positive in the moral aest.
 sense (independently of the accidental structure of
 the world). Only \neg in the ex. form. It is
 also meant "Attribution" as opposed to "negation"
 (or containing negation). This interprets the word

" \neg is positive": $(x) N \neg P(x)$ Otherwise: $P(x) \supset x \neq x$

hence $x \neq x$ positive not $x=x$ negative. At
 the end of proof At 2

X i.e. the normal form in terms of elem. prop. contains
 Members without negation.

Scott's Version of Gödel's Axioms, Definitions and Theorems

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive:

$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Thm. T1 Positive properties are possibly exemplified:

$$\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$$

Def. D1 A God-like being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 The property of being God-like is positive: $P(G)$

Cor. C Possibly, God exists:

$$\Diamond\exists xG(x)$$

Axiom A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2 An essence of an individual is a property possessed by it and necessarily implying any of its properties: $\phi \text{ ess } x \leftrightarrow \phi(x) \wedge \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Thm. T2 Being God-like is an essence of any God-like being:

$$\forall x[G(x) \rightarrow G \text{ ess } x]$$

Def. D3 Necessary existence of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi \text{ ess } x \rightarrow \Box\exists y\phi(y)]$

Axiom A5 Necessary existence is a positive property: $P(NE)$

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Difference to Gödel (who omits this conjunct)

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Modal operators are used

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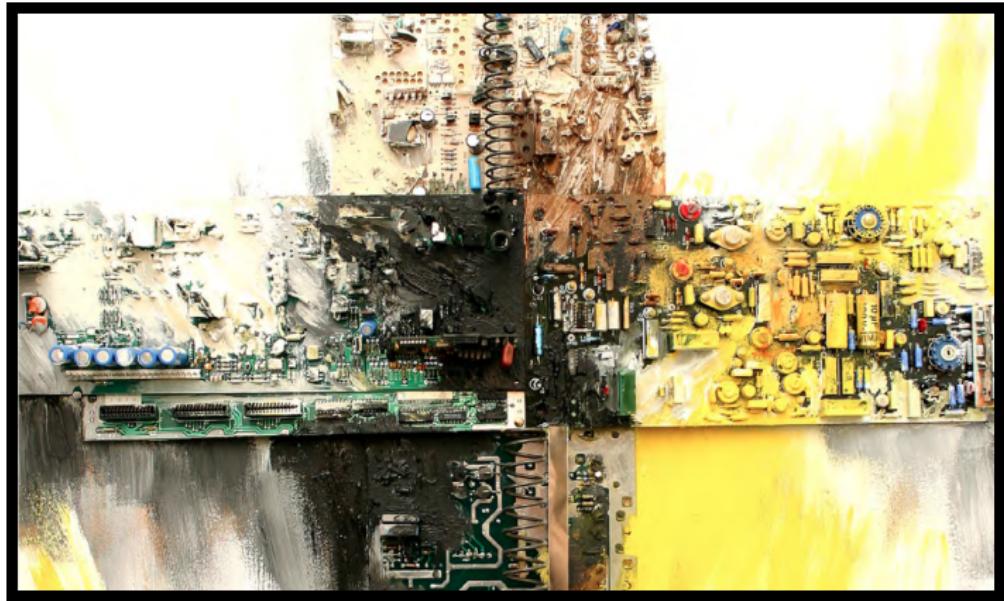
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second-order quantifiers



How to automate Higher-Order Modal Logic?

Challenge: No provers for *Higher-order Modal Logic* (HOML)

Church's Simple Type Theory

Our solution: Embedding in **Higher-order Classical Logic** (HOL)

Then use existing **HOL** theorem provers for reasoning in **HOML**

[BenzmüllerPaulson, Logica Universalis, 2013]

Assumption: Henkin semantics for both **HOML** and **HOL**

Previous empirical findings:

Embedding of *First-order Modal Logic* in **HOL** works well

[BenzmüllerOttenRaths, ECAI, 2012]

[Benzmüller, LPAR, 2013]

HOML $\varphi, \psi ::= \dots | \neg\varphi | \varphi \wedge \psi | \varphi \rightarrow \psi | \Box\varphi | \Diamond\varphi | \forall x_\gamma \varphi | \exists x_\gamma \varphi$

HOL $s, t ::= C_\alpha | x_\alpha | (\lambda x_\alpha s_\beta)_{\alpha \rightarrow \beta} | (s_{\alpha \rightarrow \beta} t_\alpha)_\beta | \neg s_o | s_o \vee t_o | \forall x_\alpha t_o$

HOML in HOL: HOML formulas φ are mapped to HOL predicates $\varphi_{\mu \rightarrow o}$
 (explicit representation of labelled formulas)

\neg	$= \lambda\varphi_{\mu \rightarrow o} \lambda w_\mu \neg \varphi w$
\wedge	$= \lambda\varphi_{\mu \rightarrow o} \lambda\psi_{\mu \rightarrow o} \lambda w_\mu (\varphi w \wedge \psi w)$
\rightarrow	$= \lambda\varphi_{\mu \rightarrow o} \lambda\psi_{\mu \rightarrow o} \lambda w_\mu (\neg \varphi w \vee \psi w)$
\forall	$= \lambda h_{\gamma \rightarrow (\mu \rightarrow o)} \lambda w_\mu \forall d_\gamma h d w$
\exists	$= \lambda h_{\gamma \rightarrow (\mu \rightarrow o)} \lambda w_\mu \exists d_\gamma h d w$
\Box	$= \lambda\varphi_{\mu \rightarrow o} \lambda w_\mu \forall u_\mu (\neg r w u \vee \varphi u)$
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Ax (polymorphic over γ)

The equations in Ax are given as axioms to the HOL provers!

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Ax (polymorphic over γ)

The equations in Ax are given as axioms to the HOL provers!

Example

HOML formula

HOML formula in HOL

expansion, $\beta\eta$ -conversion

expansion, $\beta\eta$ -conversion

expansion, $\beta\eta$ -conversion

$\diamond \exists x G(x)$

valid ($\diamond \exists x G(x)$) $_{\mu \rightarrow o}$

$\forall w_\mu (\diamond \exists x G(x))_{\mu \rightarrow o} w$

$\forall w_\mu \exists u_\mu (rwu \wedge (\exists x G(x))_{\mu \rightarrow o} u)$

$\forall w_\mu \exists u_\mu (rwu \wedge \exists x Gxu)$

Expansion: user or prover may flexibly choose expansion depth

What are we doing?

In order to prove that φ is valid in HOML,

→ we instead prove that valid $\varphi_{\mu \rightarrow o}$ can be derived from Ax in HOL.

This can be done with interactive or automated HOL theorem provers.

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Modal logic axioms

- M: valid $\forall\varphi(\Box\varphi \rightarrow \varphi)$
- B: valid $\forall\varphi(\varphi \rightarrow \Box\Diamond\varphi)$
- D: valid $\forall\varphi(\Box\varphi \rightarrow \Diamond\varphi)$
- 4: valid $\forall\varphi(\Box\varphi \rightarrow \Box\Box\varphi)$
- 5: valid $\forall\varphi(\Diamond\varphi \rightarrow \Box\Diamond\varphi)$

Semantical conditions

- $\forall x(rxy)$
- $\forall x\forall y(rxy \rightarrow ryx)$
- $\forall x\exists y(rxy)$
- $\forall x\forall y\forall z(rxy \wedge ryz \rightarrow rxz)$
- $\forall x\forall y\forall z(rxy \wedge rxz \rightarrow ryz)$

$$\begin{array}{ll} \forall = \lambda h_{\gamma \rightarrow (\mu \rightarrow o)} \lambda w_\mu \forall d_\gamma h d w & \text{(possibilist / constant dom.)} \\ \text{becomes} & \\ \forall^{va} = \lambda h_{\gamma \rightarrow (\mu \rightarrow o)} \lambda w_\mu \forall d_\gamma (\mathbf{ExInW} d w \rightarrow h d w) & \text{(actualist / varying dom.)} \end{array}$$

where \mathbf{ExInW} is an existence predicate.

Additional axioms:

- ▶ domains are non-empty $\forall w_\mu \exists x_\mu \mathbf{exInW} x w$
- ▶ denotation (constants & functions) $\forall w_\mu (\mathbf{exInW} t^1 w \wedge \dots \wedge \mathbf{exInW} t^n w \supset \mathbf{exInW} (f t^1 \dots t^n) w)$

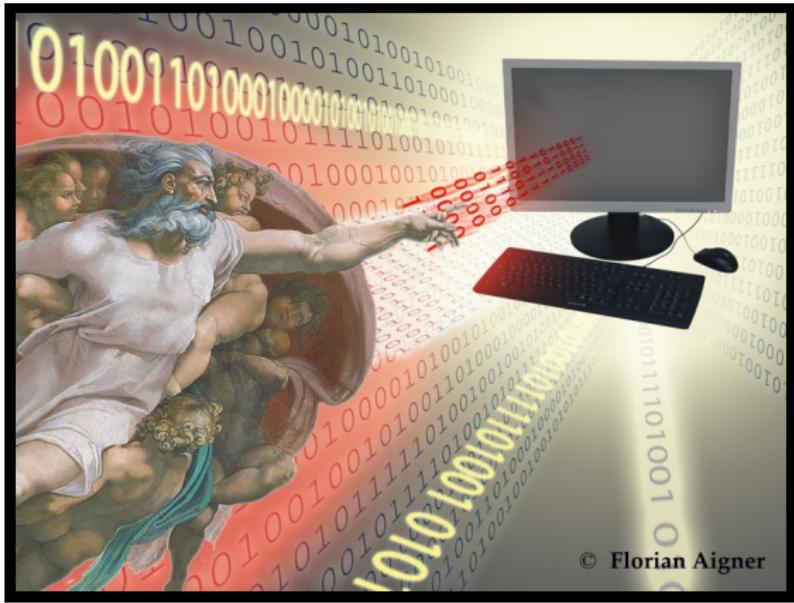
Cumulative domains: $\forall x \forall v \forall w (\mathbf{exInW} x v \wedge r v w \supset \mathbf{exInW} x w)$

Soundness and Completeness (and Cut-elimination)

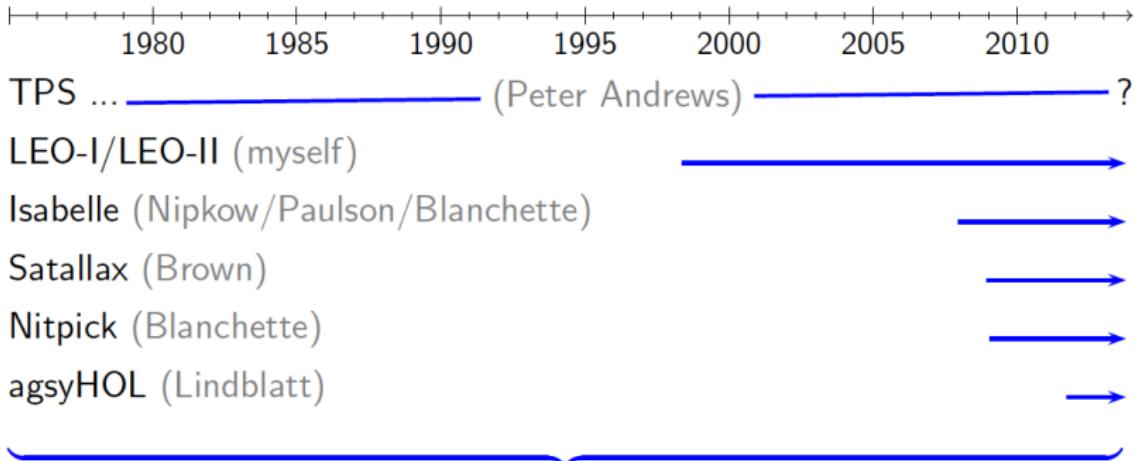
$$\models^L \varphi \text{ iff } \text{Ax} \models_{\text{Henkin}}^{\text{HOL}} \text{valid } \varphi_{\mu \rightarrow o} \text{ (iff } \text{Ax} \vdash_{\text{cut-free}}^{\text{HOL}} \text{valid } \varphi_{\mu \rightarrow o})$$

Logic L:

- ▶ Higher-order Modal Logics
- ▶ First-order Multimodal Logics
- ▶ Propositional Multimodal Logics
- ▶ Quantified Conditional Logics
- ▶ Propositional Conditional Logics
- ▶ Intuitionistic Logics
- ▶ Access Control Logics
- ▶ Logic Combinations
- ▶ ...more is on the way ... including:
 - ▶ Description Logics
 - ▶ Nominal Logics
 - ▶ Multivalued Logics (SIXTEEN)
 - ▶ Logics based on Neighborhood Semantics
 - ▶ (Mathematical) Fuzzy Logics
 - ▶ Paraconsistent Logics



Automated and Interactive Theorem Provers for HOL



- all accept TPTP THF Syntax [SutcliffeBenzmüller, J.Form.Reas, 2009]
 - can be called remotely via SystemOnTPTP at Miami
 - they significantly gained in strength over the last years
 - they can be bundled into a combined prover **HOL-P**

Exploit HOL with Henkin semantics as metalogic
Automate other logics (& combinations) via semantic embeddings
— **HOL-P** becomes a **Universal Reasoner** —

```
>  
>  
> Beweise-mit-Leo2 Notwendigerweise-existiert-Gott.p
```

Leo-II tries to prove

=====

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```
thf(thmT3,conjecture,  
     ( v  
       @ ( mbox  
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                 ( g @ X ) ) ) ) ).
```

Assumptions: D1, C, T2, D3, A5

. searching for proof ..

* Proof found *

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. generating proof object □

Provers can be called remotely in Miami — no local installation needed!

Download our experiments from

<https://github.com/FormalTheology/GoedelGod/tree/master/Formalizations/THF>



The screenshot shows the official Isabelle website at <http://isabelle.in.tum.de/index.html>. The page features a header with the Isabelle logo, search bar, and navigation links. A sidebar on the left includes links for Home, Overview, Installation, Documentation, Community, and Site Mirrors. The main content area highlights "Isabelle 2013" and "Now available: Isabelle2013". It provides download links for Mac OS X and Linux/Windows, along with some highlights of the new features. The distribution & support section details the BSD license, documentation, and various mailing lists for users and developers.

Isabelle is a generic proof assistant. It allows mathematical formulas to be expressed in a formal language and provides tools for proving those formulas in a logical calculus. Isabelle is developed at University of Cambridge (Larry Paulson), Technische Universität München (Tobias Nipkow) and Université Paris-Sud (Makarius Wenzel). See the [Isabelle overview](#) for a brief introduction.

Now available: Isabelle2013

Download for Mac OS X

Download for Linux - Download for Windows

Some highlights:

- Improvements of Isabelle/Scala and Isabelle/jEdit Prover IDE.
- Advanced build tool based on Isabelle/Scala.
- Updated manuals: isar-ref, implementation, system.
- PolyML support for block-structured specification contexts.
- HOL tool enhancements: Sledgehammer, Lifting, Quickcheck.
- HOL library enhancements: HOL-Library, HOL-Probability, HOL-Cardinals.
- HOL: New BNF-based (co)datatype package.
- Improved performance thanks to Poly/ML 5.5.0.

See also the cumulative [NEWS](#).

Distribution & Support

Isabelle is distributed for free under the BSD license. It includes source and binary packages and documentation, see the detailed [Installation instructions](#). A vast collection of Isabelle examples and applications is available from the [Archive of Formal Proofs](#).

Support is available by [document](#), the [Isabelle community Wiki](#), and the following mailing lists:

- isabelle-users@cl.cam.ac.uk provides a forum for Isabelle users to discuss problems, exchange information, and make announcements. Users of official Isabelle releases should [subscribe](#) or see the [archive](#) (also available via [Google groups](#) and [Narrows](#)).
- isabelle-dev@in.tum.de covers the Isabelle development process, including intermediate repository versions, and administrative issues concerning the website or testing infrastructure. Early adopters of [repository versions](#) should [subscribe](#) or see the [archive](#) (also available at [mail-archive.com](#) or [gmane.org](#)).

Last updated: 2013-03-09 12:21:39

See verifiable Isabelle/HOL document (Archive of Formal Proofs) at:
<http://afp.sourceforge.net/entries/GoedelGod.shtml>

```

(* Constant predicate that distinguishes positive properties *)
Parameter Positive : (u -> o) -> o.

(* Axiom A1: either a property or its negation is positive, but not both *)
Axiom axiomia : V (mforall p, (Positive (fun x: u => m-(p x))) m-> (m- (Positive p))). 
Axiom axiomib : V (mforall p, (m- (Positive p)) m-> (Positive (fun x: u => m- (p x)))). 

(* Axiom A2: a property necessarily implied by a positive property is positive *)
Axiom axiom2 : V (mforall p, mforall q, Positive p m\> (box (mforall x, (p x) m-> (q x)))). 

(* Theorem T1: positive properties are possibly exemplified *)
Theorem theorem1 : V (mforall p, (Positive p) m-> dia (mexists x, p x)).
Proof.
intro.
intro p.
intro H1.
proof_by_contradiction H2.
apply not_dia_box_not_in H2.
assert (H3: (box (mforall x, m- (p x))) w). (* Lemma from Scott's notes *)
box_intro w1 R1.
intro x.
assert (H4: ((m- (mexists x : u, p x)) w1)).
box_elim H2 w1 R1 G2.
exact G2.

clear H2 R1 H1 w.
intro H5.
apply H4.
exists x.
exact H5.

assert (H6: ((box (mforall x, (p x) m-> m- (x m= x))) w)). (* Lemma from Scott's notes *)
box_intro w1 R1.
intro x.
intro H7.
intro H8.
box_elim H3 w1 R1 G3.
annTu G3 with /w .:= w1

```

See verifiable Coq document at:

<https://github.com/FormalTheology/GoedelGod/tree/master/Formalizations/Coq>

“God is dead.”

- Nietzsche, 1883

“Nietzsche is dead.”

- God, 1900

Findings from our study

[BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

Main Findings

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda X_\mu. \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p \phi)$						
A2	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} \dot{\forall} X_\mu^* (\phi X \dot{\wedge} \psi X) \dot{\wedge} p \psi)$						
T1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Diamond} \exists X_\mu^* \phi X$	A1(○), A2 A1, A2	K	THM	0.1/0.1 0.1/0.1	0.0/0.0 0.0/5.2	—/— —/—
D1	$g_{\mu \rightarrow \sigma} = \lambda X_\mu^*. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \phi X$						
A3	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} g_{\mu \rightarrow \sigma}]$						
C	$[\dot{\Diamond} \exists X_\mu^* g_{\mu \rightarrow \sigma} X]$	T1, D1, A3 A1, A2, D1, A3	K	THM	0.0/0.0 0.0/0.0	0.0/0.0 5.2/31.3	—/— —/—
A4	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} p \phi$						
D2	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma^*} \lambda X_\mu^*. \phi X \dot{\wedge} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (\psi X \dot{\wedge} \dot{\Box} \dot{\forall} Y_\mu^* (\phi Y \dot{\wedge} \psi Y))$						
T2	$\dot{\forall} X_\mu^* g_{\mu \rightarrow \sigma} X \dot{\wedge} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} g X)$	A1, D1, A4, D2 A1, A2, D1, A3, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
			K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\text{NE}_{\mu \rightarrow \sigma} = \lambda X_\mu^*. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} (\text{ess } \phi X \dot{\wedge} \dot{\Box} \exists Y_\mu^* \phi Y)$						
A5	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \text{NE}_{\mu \rightarrow \sigma}]$						
T3	$[\dot{\Box} \exists X_\mu^* g_{\mu \rightarrow \sigma} X]$	D1, C, T2, D3, A5 A1, A2, D1, A3, A4, D2, D3, A5 D1, C, T2, D3, A5 A1, A2, D1, A3, A4, D2, D3, A5	K K KB KB	CSA CSA THM THM	—/— —/— 0.0/0.1 —/—	—/— —/— 0.1/5.3 —/—	3.8/6.2 8.2/7.5 —/— —/—
MC	$[s_\sigma \dot{\wedge} \dot{\Box} s_\sigma]$	D2, T2, T3 A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	17.9/—	3.3/3.2	—/—
			KB	THM	—/—	—/—	—/—
FG	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} X_\mu^* (g_{\mu \rightarrow \sigma} X \dot{\wedge} (\dot{\neg}(p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi) \dot{\wedge} \dot{\neg}(\phi X)))$	A1, D1 A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	16.5/— 12.8/15.1	0.0/0.0 0.0/5.4	—/— —/—
MT	$\dot{\forall} X_\mu^* \dot{\forall} Y_\mu^* (g_{\mu \rightarrow \sigma} X \dot{\wedge} (g_{\mu \rightarrow \sigma} Y \dot{\wedge} X \dot{\equiv} Y))$	D1, FG A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	—/— —/—	0.0/3.3 —/—	—/— —/—
CO	∅ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma^*} \lambda X_\mu^*. \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (\psi X \dot{\wedge} \dot{\Box} \dot{\forall} Y_\mu^* (\phi Y \dot{\wedge} \psi Y))$						
CO'	∅ (no goal, check for consistency)	A1(○), A2, D2', D3, A5 A1, A2, D1, A3, A4, D2', D3, A5	KB KB	UNS UNS	7.5/7.8 —/—	—/— —/—	—/— —/—

Main Findings

	HOL encoding	dependencies	logic	status	LEO-II const/varv	Satallax const/varv	Nitpick const/varv
A1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda X_\mu. \dot{\neg}(\phi X)) \dot{=} \dot{\neg}(p \phi)$						
A2	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} X_\mu^* (\phi X \dot{\wedge} \psi X)) \dot{\wedge} p \psi$						
T1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Diamond} \exists X_\mu^* \phi X$	A1(○), A2 A1, A2	K	THM	0.1/0.1 0.1/0.1	0.0/0.0 0.0/5.2	—/— —/—
D1	$g_{\mu \rightarrow \sigma} = \lambda X_\mu^*. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \phi X$						
A3	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} g_{\mu \rightarrow \sigma}]$						
C	$[\dot{\Diamond} \exists X_\mu^* g_{\mu \rightarrow \sigma} X]$	T1, D1, A3 A1, A2, D1, A3	K	THM	0.0/0.0 0.0/0.0	0.0/0.0 5.2/31.3	—/— —/—
A4	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} p \phi$						
D2	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma^*} \lambda X_\mu^* \phi X \dot{\wedge} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (\psi X \dot{\wedge} \dot{\Box} Y_\mu^* (\phi Y \dot{\wedge} \psi Y))$						
T2	$\dot{\forall} X_\mu^* g_{\mu \rightarrow \sigma} X \dot{\wedge} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} g X)$	A1, D1, A4, D2 A1, A2, D1, A3, A4, D2	K	THM	19.1/18.3 12.9/14.0	0.0/0.0 0.0/0.0	—/— —/—
D3	$\text{NE}_{\mu \rightarrow \sigma} = \lambda X_\mu^* \dot{\forall} \phi_{\mu \rightarrow \sigma^*} (\text{ess } \phi X \dot{\wedge} \dot{\Box} \exists Y_\mu^* \phi Y)$						
A5	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \text{NE}_{\mu \rightarrow \sigma}]$						
T3	$[\dot{\Box} \exists X_\mu^* g_{\mu \rightarrow \sigma} X]$	D1, C, T2, D3, A5 A1, A2, D1, A3, A4, D2, D3, A5 D1, C, T2, D3, A5 A1, A2, D1, A3, A4, D2, D3, A5	K K KB KB	CSA CSA THM THM	—/— —/— 0.0/0.1 —/—	—/— —/— 0.1/5.3 —/—	3.8/6.2 8.2/7.5 —/— —/—
MC	$[s_\sigma \dot{\wedge} \dot{\Box} s_\sigma]$	D2, T2, T3 A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	17.9/— —/—	3.3/3.2 —/—	—/— —/—
FG	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} X_\mu^* (g_{\mu \rightarrow \sigma} X \dot{\wedge} (\dot{\neg}(p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi) \dot{\wedge} \dot{\neg}(\phi X)))$	A1, D1 A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	16.5/— 12.8/15.1	0.0/0.0 0.0/5.4	—/— —/—
MT	$\dot{\forall} X_\mu^* \dot{\forall} Y_\mu^* (g_{\mu \rightarrow \sigma} X \dot{\wedge} (g_{\mu \rightarrow \sigma} Y \dot{\wedge} X \dot{=} Y))$	D1, FG A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/— —/—	0.0/3.3 —/—	—/— —/—
CO	∅ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma^*} \lambda X_\mu^* \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (\psi X \dot{\wedge} \dot{\Box} Y_\mu^* (\phi Y \dot{\wedge} \psi Y))$						
CO'	∅ (no goal, check for consistency)	A1(○), A2, D2', D3, A5 A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	7.5/7.8 —/—	—/— —/—	—/— —/—

Main Findings

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda X_\mu. \dot{\neg}(\phi X)) \dot{=} \dot{\neg}(p \phi)$						
A2	$\dot{\forall} \psi_{\mu \rightarrow \sigma^*} q_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda Y_\mu. \dot{\neg} \dot{\forall} Y_\mu. (\dot{\forall} Y \dot{\neg} \dot{\forall} Y) \dot{=} p Y)$						
T1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{=} \dot{\diamond} \exists X_\mu. \phi X$	A1(\supset), A2 A1, A2	K	THM	0.1/0.1	0.0/0.0	—/—
D1	$g_{\mu \rightarrow \sigma} = \lambda X_\mu. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \supset \phi X$						
A3	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} g_{\mu \rightarrow \sigma}]$						
C	$[\dot{\diamond} \exists X_\mu. g_{\mu \rightarrow \sigma} X]$	T1, D1, A3 A1, A2, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
A4	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{=} \dot{\diamond} p \phi$						
D2	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma^*} \lambda X_\mu. \phi X \dot{\wedge} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (\psi X \dot{=} \dot{\forall} Y_\mu. (\phi Y \dot{=} \psi Y))$						
T2	$\dot{\forall} X_\mu. g_{\mu \rightarrow \sigma} X \dot{=} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} g X)$	A1, D1, A4, D2 A1, A2, D1, A3, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
D3	$\text{NE}_{\mu \rightarrow \sigma} = \lambda X_\mu. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} (\dots)$						
A5	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \text{NE}_{\mu \rightarrow \sigma}]$						
T3	$[\dot{\diamond} \exists X_\mu. g_{\mu \rightarrow \sigma} X]$						
MC	$[s_\sigma \dot{=} \dot{\exists} s_\sigma]$						
FG	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} X_\mu. (s_\sigma \rightarrow X) \dot{=} (s_\sigma \rightarrow X)$						
MT	$\dot{\forall} X_\mu. \dot{\forall} Y_\mu. (g_{\mu \rightarrow \sigma} X \dot{=} (g_{\mu \rightarrow \sigma} Y \dot{=} X))$						
CO	0 (no goal, check for const)						
D2'	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma^*} \lambda$						
CO'	0 (no goal, check for const)						

Automating Scott's proof script

T1: "Positive properties are possibly exemplified" proved by LEO-II and Satallax

- ▶ in logic: K
- ▶ from assumptions:
 - ▶ A1 and A2
 - ▶ A1(\supset) and A2
- ▶ notion of quantification
 - ▶ possibilist quantifiers (constant dom.)
 - ▶ actualist quantifiers for individuals (varying dom.)

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda X_\mu. \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p \phi)$						
A2	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} \dot{\forall} X_\mu^* (\phi X \dot{\wedge} \psi X)) \dot{\wedge} p \psi$						
T1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Diamond} \exists X_\mu^* \phi X$	A1(○), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2					
D1	$g_{\mu \rightarrow \sigma} = \lambda X_\mu^*. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \phi X$		K	THM	0.1/0.1	0.0/5.2	—/—
A3	$[g_{\mu \rightarrow \sigma}, g_{\mu \rightarrow \sigma}]$						
C	$[\dot{\Diamond} \exists X_\mu^* g_{\mu \rightarrow \sigma} X]$	T1, D1, A3 A1, A2, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
K			K	THM	0.0/0.0	5.2/31.3	—/—
A4	$\dot{\forall} \psi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \psi \dot{\wedge} \dot{\Box} p \psi$						
D2	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma^*} \lambda X_\mu^* \phi X \dot{\wedge} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (\psi X \dot{\wedge} \dot{\Box} \dot{\forall} Y_\mu^* (\phi Y \dot{\wedge} \psi Y))$						
T2	$\dot{\forall} X_\mu^* g_{\mu \rightarrow \sigma} X \dot{\wedge} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} g X)$	A1, D1, A4, D2 A1, A2, D1, A3, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
D3	$\text{NE}_{\mu \rightarrow \sigma} = \lambda X_\mu^* \dot{\forall} \phi_{\mu \rightarrow \sigma^*} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} \phi X \dot{\wedge} \phi X)$		K	THM	12.9/14.0	0.0/0.0	—/—
A5	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \text{NE}_{\mu \rightarrow \sigma}]$						
T3	$[\dot{\Box} \exists X_\mu^* g_{\mu \rightarrow \sigma} X]$						
MC	$[s_\sigma \dot{\wedge} \dot{\Box} s_\sigma]$						
FG	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} X_\mu^* (g_{\mu \rightarrow \sigma} X \dot{\wedge} (s_\sigma \dot{\wedge} \dot{\Box} s_\sigma))$						
MT	$\dot{\forall} X_\mu^* \dot{\forall} Y_\mu^* (g_{\mu \rightarrow \sigma} X \dot{\wedge} (g_{\mu \rightarrow \sigma} Y \dot{\wedge} (s_\sigma \dot{\wedge} \dot{\Box} s_\sigma)))$						
CO	0 (no goal, check for const.)						
D2'	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma^*} \lambda X_\mu^* \phi X \dot{\wedge} \dot{\forall} Y_\mu^* (\phi Y \dot{\wedge} \psi Y)$						
CO'	0 (no goal, check for const.)						

Automating Scott's proof script

C: "Possibly, God exists"
proved by LEO-II and Satallax

- ▶ in logic: K
- ▶ from assumptions:
 - ▶ T1, D1, A3
- ▶ for domain conditions:
 - ▶ possibilist quantifiers (constant dom.)
 - ▶ actualist quantifiers for individuals (varying dom.)

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$\dot{\forall} \phi_{\mu \rightarrow \sigma} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda X_\mu. \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p \phi)$						
A2	$\dot{\forall} \phi_{\mu \rightarrow \sigma} \dot{\forall} \psi_{\mu \rightarrow \sigma} (p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} \dot{\forall} X_\mu. (\phi X \dot{\wedge} \psi X)) \dot{\wedge} p \psi$						
T1	$\dot{\forall} \phi_{\mu \rightarrow \sigma} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} \exists X_\mu. \phi X$	A1(○), A2 A1, A2	K	THM	0.1/0.1 0.1/0.1	0.0/0.0 0.0/5.2	—/— —/—
D1	$g_{\mu \rightarrow \sigma} = \lambda X_\mu. \dot{\forall} \phi_{\mu \rightarrow \sigma} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \phi X$						
A3	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} g_{\mu \rightarrow \sigma}]$						
C	$[\dot{\Box} \exists X_\mu. g_{\mu \rightarrow \sigma} X]$	T1, D1, A3 A1, A2, D1, A3	K	THM	0.0/0.0 0.0/0.0	0.0/0.0 5.2/31.3	—/— —/—
A4	$\dot{\forall} \phi_{\mu \rightarrow \sigma} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} p \phi$						
D2	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma}. \lambda X_\mu. \phi X \dot{\wedge} \dot{\forall} \psi_{\mu \rightarrow \sigma} ((\phi X \dot{\wedge} \dot{\Box} \psi X) \dot{\wedge} (\psi X \dot{\wedge} \dot{\Box} \psi Y))$						
T2	$\dot{\forall} X_\mu. g_{\mu \rightarrow \sigma} X \dot{\wedge} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} g X)$	A1, D1, A4, D2 A1, A2, D1, A3, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
D3	$\text{NE}_{\mu \rightarrow \sigma} = \lambda X_\mu. \dot{\forall} \phi_{\mu \rightarrow \sigma} (\phi X \dot{\wedge} \dot{\Box} \phi X)$						
A5	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \text{NE}_{\mu \rightarrow \sigma}]$						
T3	$[\dot{\Box} \exists X_\mu. g_{\mu \rightarrow \sigma} X]$						

MC	$\dot{\exists} s_\sigma. \dot{\wedge} \dot{\Box} s_\sigma$
FG	$\dot{\forall} \phi_{\mu \rightarrow \sigma} \dot{\forall} X_\mu. (g_{\mu \rightarrow \sigma} X \dot{\wedge} (\phi X \dot{\wedge} \dot{\Box} \phi X))$
MT	$\dot{\forall} X_\mu. \dot{\forall} Y_\mu. (g_{\mu \rightarrow \sigma} X \dot{\wedge} (g_{\mu \rightarrow \sigma} Y \dot{\wedge} (\phi Y \dot{\wedge} \dot{\Box} \phi Y)))$
CO	0 (no goal, check for const)
D2'	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma}. \lambda X_\mu. \phi X \dot{\wedge} \dot{\forall} Y_\mu. (\phi X \dot{\wedge} \dot{\Box} \psi Y) \dot{\wedge} (\psi X \dot{\wedge} \dot{\Box} \psi Y)$
CO'	0 (no goal, check for const)

Automating Scott's proof script

T2: "Being God-like is an ess. of any God-like being" proved by LEO-II and Satallax

- ▶ in logic: K
- ▶ from assumptions:
 - ▶ A1, D1, A4, D2
- ▶ for domain conditions:
 - ▶ possibilist quantifiers (constant dom.)
 - ▶ actualist quantifiers for individuals (varying dom.)

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda X_\mu. \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p \phi)$						
A2	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\forall} X_\mu^* (\phi X \dot{\wedge} \psi X)) \dot{\wedge} p \psi$						
T1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\exists} X_\mu^* \phi X$	A1(○), A2 A1, A2	K	THM	0.1/0.1	0.0/0.0	—/—
D1	$g_{\mu \rightarrow \sigma} = \lambda X_\mu^*. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \phi X$		K	THM	0.1/0.1	0.0/5.2	—/—
A3	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} g_{\mu \rightarrow \sigma}]$						
C	$[\dot{\forall} \exists X_\mu^* g_{\mu \rightarrow \sigma} X]$	T1, D1, A3 A1, A2, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
A4	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\forall} p \phi$						
D2	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma^*} \lambda X_\mu^* \phi X \dot{\wedge} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (\psi X \dot{\wedge} \dot{\forall} Y_\mu^* (\phi Y \dot{\wedge} \psi Y))$						
T2	$\dot{\forall} X_\mu^* g_{\mu \rightarrow \sigma} X \dot{\wedge} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} g X)$	A1, D1, A4, D2 A1, A2, D1, A3, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
D3	$\text{NE}_{\mu \rightarrow \sigma} = \lambda X_\mu^*. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} (e$						
A5	$[n_{\mu \rightarrow \sigma} \text{NE}_{\mu \rightarrow \sigma}]$						
T3	$[\dot{\forall} \exists X_\mu^* g_{\mu \rightarrow \sigma} X]$						

Automating Scott's proof script

T3: "Necessarily, God exists"
proved by LEO-II and Satallax

- ▶ in logic: KB
- ▶ from assumptions:
 - ▶ D1, C, T2, D3, A5
- ▶ for domain conditions:
 - ▶ possibilist quantifiers (constant dom.)
 - ▶ actualist quantifiers for individuals (varying dom.)

For logic K we got a countermodel by Nitpick

	HOL encoding	dependencies	logic	status	LEO-II const/varv	Satallax const/varv	Nitpick const/varv
A1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda X_\mu. \dot{\neg}(\phi X)) \dot{=} \dot{\neg}(p \phi)$						
A2	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} \dot{\forall} X_\mu. (\phi X \dot{\wedge} \psi X)) \dot{\wedge} p \psi$						
T1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Diamond} \exists X_\mu. \phi X$	A1(○), A2 A1, A2	K	THM	0.1/0.1	0.0/0.0	—/—
D1	$g_{\mu \rightarrow \sigma} = \lambda X_\mu. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \phi X$		K	THM	0.1/0.1	0.0/5.2	—/—
A3	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} g_{\mu \rightarrow \sigma}]$						
C	$[\dot{\Diamond} \exists X_\mu. g_{\mu \rightarrow \sigma} X]$	T1, D1, A3 A1, A2, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
A4	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} p \phi$						
D2	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma}. \lambda X_\mu. \phi X \dot{\wedge} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (\psi X \dot{\wedge} \dot{\Box} \dot{\forall} Y_\mu. (\phi Y \dot{\wedge} \psi Y))$						
T2	$\dot{\forall} X_\mu. g_{\mu \rightarrow \sigma} X \dot{\wedge} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} g X)$	A1, D1, A4, D2 A1, A2, D1, A3, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
D3	$\text{NE}_{\mu \rightarrow \sigma} = \lambda X_\mu. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} (e^{-\lambda V \dot{\wedge} \dot{\Box} V - \lambda V \dot{\wedge} \dot{\forall} V \dot{\wedge} V})$		K	THM	12.9/14.0	0.0/0.0	—/—
A5	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \text{NE}_{\mu \rightarrow \sigma}]$						
T3	$[\dot{\Box} \exists X_\mu. g_{\mu \rightarrow \sigma} X]$						
MC	$[s_\sigma \dot{\wedge} \dot{\Box} s_\sigma]$						
FG	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} X_\mu. (g_{\mu \rightarrow \sigma} X \dot{\wedge} (g_{\mu \rightarrow \sigma} \phi X))$						
MT	$\dot{\forall} X_\mu. \dot{\forall} Y_\mu. (g_{\mu \rightarrow \sigma} X \dot{\wedge} (g_{\mu \rightarrow \sigma} Y \dot{\wedge} (g_{\mu \rightarrow \sigma} X)))$						
CO	0 (no goal, check for const)						
D2'	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma}. \lambda X_\mu. \phi X \dot{\wedge} \dot{\forall} Y_\mu. (\phi Y \dot{\wedge} \psi Y)$						
CO'	0 (no goal, check for const)						

Automating Scott's proof script

Summary

- ▶ proof verified and automated
- ▶ KB is sufficient (criticized logic S5 not needed!)
- ▶ possibilist and actualist quantifiers (individuals)
- ▶ exact dependencies determined experimentally
- ▶ ATPs have found alternative proofs
e.g. self-identity $\lambda x(x = x)$ is not needed

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda X_\mu. \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p \phi)]$						
A2	$[\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \wedge \dot{\square} \dot{\forall} X_\mu^* (\phi X \dot{\wedge} \psi X) \dot{\wedge} p \psi)]$						
T1	$[\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\square} \exists X_\mu^* \phi X]$	A1(?) A2	K	THM	0.1/0.1	0.0/0.0	—/—
D1	$g_{\mu \rightarrow \sigma} = \lambda X_\mu^*. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma}$						
A3	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} g_{\mu \rightarrow \sigma}]$						
C	$[\dot{\square} \exists X_\mu^* g_{\mu \rightarrow \sigma} X]$						
A4	$[\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\square} p \text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma} \lambda$						
D2	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma} \lambda$						
T2	$[\dot{\forall} X_\mu^* g_{\mu \rightarrow \sigma} X \dot{\wedge} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma} \lambda)]$						
D3	$\text{NE}_{\mu \rightarrow \sigma} = \lambda X_\mu^*. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma} \lambda)$						
A5	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \text{NE}_{\mu \rightarrow \sigma}]$						
T3	$[\dot{\square} \exists X_\mu^* g_{\mu \rightarrow \sigma} X]$						
Consistency check: Gödel vs. Scott							
<ul style="list-style-type: none"> ▶ Scott's assumptions are consistent; shown by Nitpick ▶ Gödel's assumptions are inconsistent; shown by LEO-II (new philosophical result?) 							
A1, A2, D1, A3, A4, D2, D3, A5 K CSA / / o.z./r.s.							
D1, C, T2, D3, A5 KB THM 0.0/0.1 0.1/5.3 —/—							
A1, A2, D1, A3, A4, D2, D3, A5 KB THM —/— —/— —/—							
MC [s _σ ⊢ □s _σ] D2, T2, T3 KB THM 17.9/— 3.3/3.2 —/—							
FG [dot{forall} phi_{mu -> sigma^*} dot{forall} X_mu^* (g_{mu -> sigma} X -> (dot{neg}(p_{(mu -> sigma) -> mu -> sigma} phi) -> dot{neg}(phi X)))] A1, D1 KB THM 16.5/— 0.0/0.0 —/—							
FG [dot{forall} phi_{mu -> sigma^*} dot{forall} X_mu^* (g_{mu -> sigma} X -> (g_{mu -> sigma} Y -> X -> Y))] A1, A2, D1, A3, A4, D2, D3, A5 KB THM 12.8/15.1 0.0/5.4 —/—							
MT [dot{forall} X_mu^* dot{forall} Y_mu^* (g_{mu -> sigma} X -> (g_{mu -> sigma} Y -> X -> Y))] D1, FG KB THM —/— 0.0/3.3 —/—							
A1, A2, D1, A3, A4, D2, D3, A5 KB THM —/— —/— —/—							
CO Ø (no goal, check for consistency) A1, A2, D1, A3, A4, D2, D3, A5 KB SAT —/— —/— 7.3/7.4							
D2' ess_{(mu -> sigma) -> mu -> sigma} = lambda phi_{mu -> sigma}. lambda X_mu^*. dot{forall} psi_{mu -> sigma^*} (psi X -> dot{square} dot{forall} Y_mu^* (phi Y -> psi Y)) A1(?) A2, D2', D3, A5 KB UNS 7.5/7.8							
CO' Ø (no goal, check for consistency) A1, A2, D1, A3, A4, D2', D3, A5 KB UNS —/— —/— —/—							

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda X_\mu. \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p \phi)$						
A2	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \wedge \dot{\square} \dot{\forall} X_\mu^* (\phi X \dot{\neg} \psi X) \dot{\neg} p \psi)$						
T1	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\neg} \dot{\exists} X_\mu^* \phi X$	A1(2), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2					
D1	$g_{\mu \rightarrow \sigma} = \lambda X_\mu. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\neg} \dot{\exists} X_\mu^* \phi X$		K	THM	0.1/0.1	0.0/5.2	—/—
A3	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} g_{\mu \rightarrow \sigma}]$						
C	$[\dot{\square} \dot{\exists} X_\mu^* g_{\mu \rightarrow \sigma} X]$						
A4	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\neg} \dot{\square} p$						
D2	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma} \lambda$						
T2	$\dot{\forall} X_\mu^* g_{\mu \rightarrow \sigma} X \dot{\neg} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} X)$						
D3	$\text{NE}_{\mu \rightarrow \sigma} = \lambda X_\mu. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} (\phi$	D1, C, T2, D3, A5	K	CSA	—/—	—/—	3.8/6.2
A5	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \text{NE}_{\mu \rightarrow \sigma}]$	A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
T3	$[\dot{\square} \dot{\exists} X_\mu^* g_{\mu \rightarrow \sigma} X]$	D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \dot{\neg} \dot{\square} s_\sigma]$	D2, T2, T3 A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	17.9/— /	3.3/3.2 /	—/— —/—
FG	$\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} X_\mu^* (g_{\mu \rightarrow \sigma} X \dot{\neg} (\dot{\neg}(p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi) \dot{\neg} \dot{\neg}(\phi X)))$	A1, D1 A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	16.5/— 12.8/15.1	0.0/0.0 0.0/5.4	—/— —/—
MT	$\dot{\forall} X_\mu^* \dot{\forall} Y_\mu^* (g_{\mu \rightarrow \sigma} X \dot{\neg} (g_{\mu \rightarrow \sigma} Y \dot{\neg} X \dot{\equiv} Y))$	D1, FG A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	—/— —/—	0.0/3.3 —/—	—/— —/—
CO	∅ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma} \lambda X_\mu^* \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (\psi X \dot{\neg} \dot{\square} \dot{\forall} Y_\mu^* (\phi Y \dot{\neg} \psi Y))$	A1(2), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
CO'	∅ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

Further Results

- ▶ Monotheism holds
- ▶ God is flawless

Main Findings

	HOL encoding
A1	$[\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} (\lambda X_\mu \dot{\vdash}$
A2	$[\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \dot{\wedge}$
T1	$[\dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\exists}$
D1	$g_{\mu \rightarrow \sigma} = \lambda X_\mu. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi$
A3	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} g_{\mu \rightarrow \sigma}]$
C	$[\dot{\forall} \exists X_\mu. g_{\mu \rightarrow \sigma} X]$
A4	$[\dot{\forall} \phi_{\mu \rightarrow \sigma} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \dot{\wedge} \dot{\Box} p]$
D2	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma}. \lambda$
T2	$[\dot{\forall} X_\mu. g_{\mu \rightarrow \sigma} X \dot{\wedge} (\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma})]$
D3	$\text{NE}_{\mu \rightarrow \sigma} = \lambda X_\mu. \dot{\forall} \phi_{\mu \rightarrow \sigma^*} (e$
A5	$[p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \text{NE}_{\mu \rightarrow \sigma}]$
T3	$[\dot{\Box} \exists X_\mu. g_{\mu \rightarrow \sigma} X]$

Modal Collapse (Sobel)

$$\forall \varphi (\varphi \supset \Box \varphi)$$

- ▶ proved by LEO-II and Satallax
- ▶ for possibilist and actualist quantification (ind.)

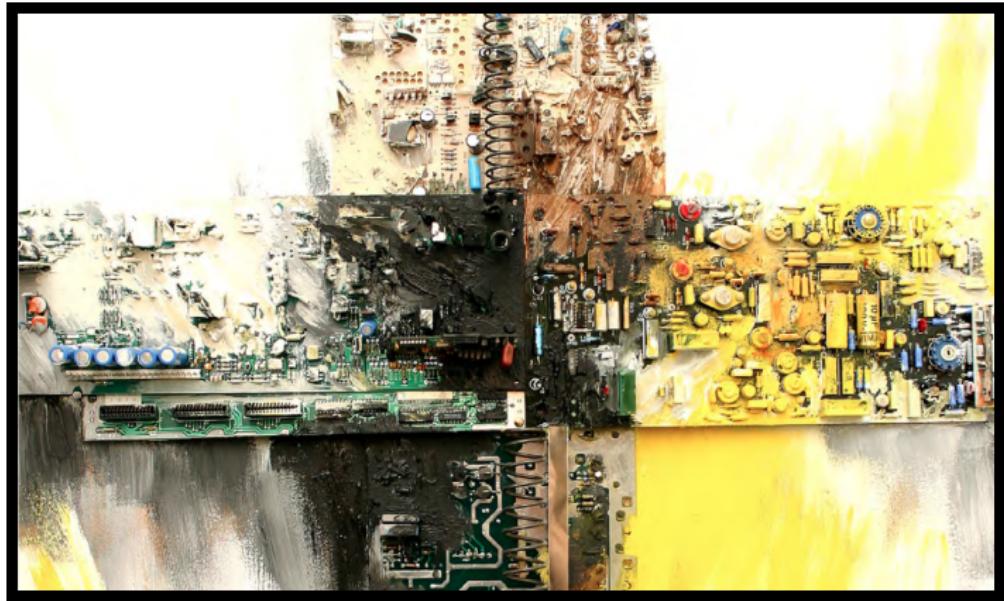
Main critique on Gödel's ontological proof:

- ▶ there are no contingent truths
- ▶ everything is determined / no free will

MC	$[s_\sigma \dot{\wedge} \dot{\Box} s_\sigma]$	D2, T2, T3 A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	17.9/— —/—	3.3/3.2 —/—	—/— —/—
FG	$[\dot{\forall} \phi_{\mu \rightarrow \sigma^*} \dot{\forall} X_\mu. (g_{\mu \rightarrow \sigma} X \supset (\neg(p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi) \supset \neg(\phi X)))]$	A1, D1 A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	16.5/— 12.8/15.1	0.0/0.0 0.0/5.4	—/— —/—
MT	$[\dot{\forall} X_\mu. \dot{\forall} Y_\mu. (g_{\mu \rightarrow \sigma} X \dot{\wedge} (g_{\mu \rightarrow \sigma} Y \dot{\wedge} X \dot{\equiv} Y))]$	D1, FG A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	—/— —/—	0.0/3.3 —/—	—/— —/—
CO	∅ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\text{ess}_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma}. \lambda X_\mu. \dot{\forall} \psi_{\mu \rightarrow \sigma^*} (\psi X \dot{\wedge} \dot{\Box} \dot{\forall} Y_\mu. (\phi Y \dot{\wedge} \psi Y))$	A1, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
CO'	∅ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

Observation

- ▶ good performance of ATPs
 - ▶ excellent match between argumentation granularity in papers and the reasoning strength of the ATPs



Reconstruction of the Inconsistency of Gödel's Axioms

Inconsistency (Gödel): Proof by LEO-II in KB

***** no. of clauses in derivation: 97 *****
***** clause counter: 113 *****

```
|| S2S status Unsatisfiable for ConsistencyWithoutFirstConjunction02.p : (r:0,a:axioms:16,p:3,u:6,ude:false,rleibE0:true,rAndE0:true,use_choice:true,use_extunI:true,use_extcnf_combined:true,expand_extuni:false,foapt:e,atp_timeout:25,atp_calls_frequency:10,ordering:none,proof_output:1,clause_count:113,loop_count:8,foapt_calls:2,translatation:f0,full)
ontoleo:DemoMaterial cbenzmuellers ||
```

Inconsistency (Gödel): Reconstruction of Informal Argument (KB)

(special thanks to Chad Brown for a fruitful discussion)

Axiom A1(\supset)

$$\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$$

Axiom A2

$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

(special thanks to Chad Brown for a fruitful discussion)

Axiom A1(\supset)

$$\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$$

Axiom A2

$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Theorem 1 Positive Properties are possibly exemplified. $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$
by A1(\supset), A2

Inconsistency (Gödel): Reconstruction of Informal Argument (KB)

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by A1(\supset), A2

Def. D2*

$$\phi \text{ ess } x \leftrightarrow \cancel{\Diamond(x)} \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$$

Inconsistency (Gödel): Reconstruction of Informal Argument (KB)

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Def. D2*

$$\phi \text{ ess } x \leftrightarrow \cancel{\Diamond(x)} \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$$

Lemma 1 The empty property is an essence of every entity.

$$\forall x(\emptyset \text{ ess } x)$$

by D2*

(special thanks to Chad Brown for a fruitful discussion)

Axiom A1(\supset)

$$\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$$

Axiom A2

$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

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Lemma 1 The empty property is an essence of every entity.

$$\forall x(\emptyset \text{ ess } x)$$

by D2*

Def. D3

$$NE(x) \leftrightarrow \forall\phi[\phi \text{ ess } x \rightarrow \Box\exists y\phi(y)]$$

Axiom B

$$\forall\varphi(\varphi \rightarrow \Box\Diamond\varphi) \quad (\text{resp. } \forall x\forall y(rxy \rightarrow ryx))$$

(special thanks to Chad Brown for a fruitful discussion)

Axiom A1(\supset)

$$\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$$

Axiom A2

$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Theorem 1 Positive Properties are possibly exemplified. $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$
by A1(\supset), A2

Def. D2*

$$\phi \text{ ess } x \leftrightarrow \cancel{\Diamond(x)} \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$$

Lemma 1 The empty property is an essence of every entity.

$$\forall x(\emptyset \text{ ess } x)$$

by D2*

Def. D3

$$NE(x) \leftrightarrow \forall\phi[\phi \text{ ess } x \rightarrow \Box\exists y\phi(y)]$$

Axiom B

$$\forall\varphi(\varphi \rightarrow \Box\Diamond\varphi) \quad (\text{resp. } \forall x\forall y(rxy \rightarrow ryx))$$

Lemma 2 Exemplification of necessary existence is not possible. $\neg\Diamond\exists x NE(x)$
by B, D3, Lemma1

(special thanks to Chad Brown for a fruitful discussion)

Axiom A1(\supset)

$$\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$$

Axiom A2

$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Theorem 1 Positive Properties are possibly exemplified. $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$
by A1(\supset), A2

Def. D2*

$$\phi \text{ ess } x \leftrightarrow \cancel{\Diamond(x)} \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$$

Lemma 1 The empty property is an essence of every entity.

$$\forall x(\emptyset \text{ ess } x)$$

by D2*

Def. D3

$$NE(x) \leftrightarrow \forall\phi[\phi \text{ ess } x \rightarrow \Box\exists y\phi(y)]$$

Axiom B

$$\forall\varphi(\varphi \rightarrow \Box\Diamond\varphi) \quad (\text{resp. } \forall x\forall y(rxy \rightarrow ryx))$$

Lemma 2 Exemplification of necessary existence is not possible. $\neg\Diamond\exists x NE(x)$
by B, D3, Lemma1

Axiom A5

$$P(NE)$$

(special thanks to Chad Brown for a fruitful discussion)

Axiom A1(\supset)

$$\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$$

Axiom A2

$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Theorem 1 Positive Properties are possibly exemplified. $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$
by A1(\supset), A2

Def. D2*

$$\phi \text{ ess } x \leftrightarrow \cancel{\Diamond(x)} \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$$

Lemma 1 The empty property is an essence of every entity.

$$\forall x(\emptyset \text{ ess } x)$$

by D2*

Def. D3

$$NE(x) \leftrightarrow \forall\phi[\phi \text{ ess } x \rightarrow \Box\exists y\phi(y)]$$

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$$\forall\varphi(\varphi \rightarrow \Box\Diamond\varphi) \quad (\text{resp. } \forall x\forall y(rxy \rightarrow ryx))$$

Lemma 2 Exemplification of necessary existence is not possible. $\neg\Diamond\exists x NE(x)$
by B, D3, Lemma1

Axiom A5

$$P(NE)$$

Inconsistency

$$\perp$$

by A5, T1, Lemma2

Inconsistency (Gödel): Verification in Isabelle/HOL (KB)

The screenshot shows the Isabelle/HOL interface with a theory file named "GoedelGodWithoutConjunctInEss_KB.thy". The code defines various properties and proves their consistency or inconsistency.

```
theory GoedelGodWithoutConjunctInEss_KB imports QML
begin
  consts P :: "(μ ⇒ σ) ⇒ σ"
  axiomatization where A1a: "[∀(λΦ. P (λx. m¬ (Φ x)) m→ m¬ (P Φ))]"
    and A2: "[∀(λΦ. ∀(λΨ. (P Φ m ∧ □ (∀(λx. Φ x m→ Ψ x))) m→ P Ψ))]"

  -- {* Positive properties are possibly exemplified. *}
  theorem T1: "[∀(λΦ. P Φ m→ ◇ (exists Φ))]" by (metis A1a A2)

  definition ess (infixr "ess" 85) where "Φ ess x = ∀(λΨ. Ψ x m→ □ (forall(λy. Φ y m→ Ψ y)))"

  -- {* The empty property is an essence of every individual. *}
  lemma Lemma1: "[(forall(λx.( λy.λw. False) ess x))]" by (metis ess_def)

  definition NE where "NE x = ∀(λΦ. Φ ess x m→ □ (exists Φ))"
  axiomatization where sym: "x r y —> y r x"

  -- {* Exemplification of necessary existence is not possible. *}
  lemma Lemma2: "[m¬ (◇ (exists NE))]" by (metis sym Lemma1 NE_def)

  axiomatization where A5: "[P NE]"

  -- {* Now the inconsistency follows from A5, T1 and Lemma2 *}
  lemma False by (metis A5 T1 Lemma2)
end
```

Output Query Sledgehammer Symbols

11,1 (477/1095)

(isabelle,sidekick,UTF-8-Isabelle)Nr. 263/347 MB 17:18

Inconsistency (Gödel): Verification in Isabelle/HOL (K)

The screenshot shows the Isabelle/HOL interface with a proof script for Gödel's Ontological Proof. The script includes definitions of 'ess' and 'NE', an axiomatization with rules A1a and A2, and theorems T1 and Lemmal. It concludes with an inconsistency proof involving rule A5 and lemma Lemmal.

```
theory GoedelGodWithoutConjunctInEss_K imports QML
begin
  consts P :: " $(\mu \Rightarrow \sigma) \Rightarrow \sigma$ "
  definition ess (infixr "ess" 85) where " $\Phi \text{ ess } x = \forall(\lambda\Psi. \Psi x \rightarrow \square (\forall(\lambda y. \Phi y \rightarrow \Psi y)))$ "
  definition NE where " $\text{NE } x = \forall(\lambda\Phi. \Phi \text{ ess } x \rightarrow \square (\exists \Phi))$ "
  axiomatization where A1a: "[ $\forall(\lambda\Phi. P (\lambda x. m \rightarrow (\Phi x)) \rightarrow m \rightarrow (P \Phi))$ ]"
    and A2: "[ $\forall(\lambda\Phi. \forall(\lambda\Psi. (P \Phi \wedge \square (\forall(\lambda x. \Phi x \rightarrow \Psi x)) \rightarrow m \rightarrow P \Psi))$ ]"

  -- {* Positive properties are possibly exemplified. *}
  theorem T1: "[ $\forall(\lambda\Phi. P \Phi \rightarrow \diamond (\exists \Phi))$ ]" by (metis A1a A2)

  -- {* The empty property is an essence of every individual. *}
  lemma Lemmal: "[ $(\forall(\lambda x. (\lambda y. \lambda w. \text{False}) \text{ ess } x))$ ]" by (metis ess_def)

  axiomatization where A5: "[P NE]"

  -- {* Now the inconsistency follows from A5, Lemmal, NE_def and T1 *}
  lemma False
    -- {* sledgehammer [remote_leo2] *}
    by (metis A5 Lemmal NE_def T1)
end
```

Output Query Sledgehammer Symbols

21,7 (980/982)

(isabelle,sidekick,UTF-8-Isabelle)Nr o UG 302/343MB 11:37

Ontologischer Beweis

Feb. 10, 1970

$P(\varphi)$ φ is positive ($\Leftrightarrow \varphi \in P$)

Ax 1: $P(p), P(\varphi) \supset P(\varphi \wedge p)$ • Ax 2: $P(p) \supset P(\neg p)$

P1 $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

P2 $\varphi_{\text{Exn } x} \equiv (\psi)[\psi(x) \supset N(y)[P(y) \supset \psi(y)]]$ (Existence)

$P \supset_N q = N(p \supset q)$ Necessity

Ax 2 $P(\varphi) \supset N P(\varphi)$ } because it follows
 $\neg P(\varphi) \supset N \neg P(\varphi)$ } from the nature of the property

Th. $G(x) \supset G_{\text{Exn } x}$

Df. $E(x) \equiv P[\varphi_{\text{Exn } x} \supset N \neg x \supset \varphi(x)]$ necessary Existence

Ax 3 $P(E)$

Th. $G(x) \supset N(\exists y) G(y)$

hence $(\exists x) G(x) \supset N(\exists y) G(y)$

" $M(x) G(x) \supset M N(\exists y) G(y)$ M = possibility
 $\Rightarrow N(\exists y) G(y)$

any two elements of X are nec. equivalent

exclusive or * and for any number of them

$M(x) G(x)$ means all pos. prop. w.r.t. com-
patible. This is true because of:

Ax 4: $P(\varphi) \cdot q \supset_N \varphi : \supset P(\varphi)$ which impl.

$\begin{cases} x=x & \text{is positive} \\ x \neq x & \text{is negative} \end{cases}$

But if a system S of pos. prop. were incon-
sistent it would mean that the non-prop. S (which
is positive) would be $x \neq x$

Positive means positive in the moral aesthe-
tical sense (independently of the accidental structure of
the world). Only in the ex. True. It is
also meant "Attribution" as opposed to "Platification
(or containing negation)." This interprets the word

" \supset " (positive) $(x) \neg P(x)$ Otherwise $P(x) \supset x \neq x$

hence $x \neq x$ positive not $x=x$ negative. At
the end of proof Ax 4

X i.e. the normal form in terms of elem. prop. contains
members without negation.

Ontologischer Beweis

Feb. 10, 1970

$P(\phi)$ ϕ is positive ($\Leftrightarrow \phi \in P$)

At 1: $P(\phi), P(\psi) \supset P(\phi \wedge \psi)$ • At 2: $P(\phi) \supset P(\neg \phi)$

$P_1 G(x) \equiv (\phi) [P(\phi) \supset \phi(x)]$ (Ged.)

$P_2 \phi \text{ Em. } x \equiv (\psi) [\psi(x) \supset N(y)[P(y) \supset \psi(y)]]$ (Em. of x)

$P = N$ $N(p \circ q)$ Necessity

At 2 $P(\phi) \supset N P(\phi)$ } because it follows
 $\neg P(\phi) \supset N \neg P(\phi)$ } from the nature of the
 property

Th. $G(x) \supset G \text{ Em. } x$

Df. $E(x) \equiv P[\phi \text{ Em. } x \supset N \neg x \phi(x)]$ necessary Existence

At 3 $P(E)$

Th. $G(x) \supset N(\exists y) G(y)$

then $(\exists x) G(x) \supset N(\exists y) G(y)$

" $M(x) G(x) \supset M N(\exists y) G(y)$

" $\supset N(\exists y) G(y)$

any two instances of x are nec. equivalent

exclusive or * and for any number of them

$M(x) G(x)$ means all pos. prop. w. com-
 patible
This is true because of:

At 4: $P(\phi), \phi \supset \psi \supset P(\psi)$ which impl.

$\begin{cases} x=x & \text{is positive} \\ x \neq x & \text{is negative} \end{cases}$

But if a system S of pos. prop. were incon-
 sistent it would mean that the non-prop. S (which
 is positive) would be $x \neq x$

Positive means positive in the moral aesti-
 tive sense (independently of the accidental structure of
 the world). Only \neg in the aest. sense. It is
 also pure.

Inconsistency

Scott

$\forall \phi [P(\neg \phi) \rightarrow \neg P(\phi)]$

A1(▷)

$\forall \phi \forall \psi [(P(\phi) \wedge \square \forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$

A2

$\phi \text{ ess } x \leftrightarrow \forall \psi (\psi(x) \rightarrow \square \forall y (\phi(y) \rightarrow \psi(y)))$

D2*

$NE(x) \leftrightarrow \forall \phi [\phi \text{ ess } x \rightarrow \square \exists y \phi(y)]$

D3

$P(NE)$

A5

Gödel's version	K		KB		S5	
	constant	varying	constant	varying	constant	varying
Consistency	✗	✗	✗	✗	✗	✗
T1	obsolete	obsolete	obsolete	obsolete	obsolete	obsolete
C	obsolete	obsolete	obsolete	obsolete	obsolete	obsolete
T2	obsolete	obsolete	obsolete	obsolete	obsolete	obsolete
T3	obsolete	obsolete	obsolete	obsolete	obsolete	obsolete
Flawless God	obsolete	obsolete	obsolete	obsolete	obsolete	obsolete
Monotheism	obsolete	obsolete	obsolete	obsolete	obsolete	obsolete
Modal Collapse	obsolete	obsolete	obsolete	obsolete	obsolete	obsolete

Further logic details

- ▶ Henkin semantics
- ▶ full comprehension
- ▶ rigid constant symbols

Question:

Has this inconsistency be reported before?

If not, then LEO-II deserves (part of) the credit!

Scott's version	K		KB		S5	
	constant	varying	constant	varying	constant	varying
Consistency	✓	✓	✓	✓	✓	✓
T1	✓	✓	✓	✓	✓	✓
C	✓	✓	✓	✓	✓	✓
T2	✓	✓	✓	✓	✓	✓
T3	✗	✗	✓	✓	✓	✓
Flawless God	✓	✓	✓	✓	✓	✓
Monotheism	✓	✓	✓	✓	✓	✓
Modal Collapse	✓	✓	✓	✓	✓	✓

Further logic details

- ▶ Henkin semantics
- ▶ full comprehension
- ▶ rigid constant symbols

SOME EMENDATIONS OF GÖDEL'S ONTOLOGICAL PROOF

C. Anthony Anderson

Kurt Gödel's version of the ontological argument was shown by J. Howard Sobel to be defective, but some plausible modifications in the argument result in a version which is immune to Sobel's objection. A definition is suggested which permits the proof of some of Gödel's axioms.

Der Mathematiker und die Frage der Existenz Gottes (betreffend Gödels ontologischen Beweis)

Es ist gut, daß wir nicht wissen,
andern glauben, daß ein Gott sei.
(Kant, Nachleid)

1. Einführung

Gödels zu Lebzeiten unveröffentlichter Beweis für die notwendige Existenz eines Gott-ähnlichen Wesens hat sowohl philosophisches als auch mathematisches Interesse geweckt. Zweck der vorliegenden Arbeit ist es, zu einer Deutung des Gödelischen Textes beizutragen, durch eine Emendierung des einschlägigen Literatur und 2. durch Beiträge von verschiedenen Modelltheoreten. Die Arbeit enthält keinen philosophischen Beitrag. Anlässlich der letzten Jahre habe ich etliche Male über Gödels Ontologie vorgetragen, insbesondere auf dem Symposium zur Freiheit von Professor Gerhard Müller (Heidelberg, Januar 1997), doch habe ich niemals beobachtigt, eine Voröffentlichung über die Theorie zu machen. Da ich wiederholt um eine schriftliche Version gebeten wurde, entschloß ich mich, schnell eine „geweihte Kurzfassung“¹ zu schreiben, ohne aus ihr einen

Gödel's Ontological Proof Revisited *

C. Anthony Anderson and Michael Gettings
University of California, Santa Barbara
Department of Philosophy

Gödel's version of the modal ontological argument for the existence of God has been criticized by J. Howard Sobel [5] and modified by C. Anthony Anderson [1]. In the present paper we consider the extent to which Anderson's emendation is defeated by the type of objection first offered by the Monk Gaumilo to St. Anselm's original Ontological Argument. And we try to push the analysis of this Gödelian argument a bit further to bring it into closer agreement with the details of Gödel's own formulation. Finally, we indicate what seems to be the main weakness of this emendation of Gödel's attempted proof.

PETR HÁJEK

A New Small Emendation of Gödel's Ontological Proof

Keywords: Ontological proof, Gödel, modal logic, comprehension, positive properties.

1. Introduction

Gödel's ontological proof of necessary existence of a godlike being was finally published in the third volume of Gödel's collected works [7]; but it became known in 1970 when Gödel showed the proof to Dana Scott and Scott presented it (in fact a variant of it) at a seminar at Princeton. Detailed history is found in Adams' introductory remarks to the ontological proof in [7]. The proof uses modal logic and its analysis is an exciting exercise in systems of formal modal logic. Needless to say, formal modal logic has found several

Magari and others on Gödel's ontological proof

Petr Hájek

Institute of Computer Science, Academy of Sciences
182 07 Prague, Czech Republic
e-mail: hajek@iuivt.cas.cz

1 Introduction

This paper is a continuation of my paper [H] and concentrates almost exclusively to mathematical properties of logical systems underlying Gödel's ontological proof [G] and its variants by Anderson [A], with special care paid to Magari's criticism [M]. Since [H] is written in German, we shall try to summarize its content in such a way that knowledge of [H] will be not obligatory for reading the present paper (even it remains advantageous). Here we describe

Understanding Gödel's Ontological Argument

FRODE BJØRDAL

In 1970 Kurt Gödel, in a hand-written note entitled "Ontologischer Beweis", put forward an ontological argument for the existence of God, making use of second-order modal logical principles. Let the second-order formula $P(F)$ stand for "the property F is positive", and let "God" signify the property of being God-like. Gödel presupposes the following definitions:

SOME EMENDATIONS OF GÖDEL'S ONTOLOGICAL PROOF

C. Anthony Anderson

Kurt Gödel's version of the ontological argument was shown by J. Howard Sobel to be defective, but some plausible modifications in the argument result in a version which is immune to Sobel's objections. A definition is suggested which permits the proof of some of Gödel's axioms.

Der Mathematiker und die Frage der Existenz Gottes

(betreffend Gödels ontologischen Beweis)

wie er glaubt, daß Gott sei.
(Kant, Nachhall)

1. Einführung

Gödel zu Lebzeiten eindeutiger Beweis für die notwendige Existenz eines Gödel-Beweises? Wieso hat solches philosophisches als auch mathematisches Interesse geweckt und warum? Von den liegenden Möglichkeiten ist die von Deutsch (1999) vorgeschlagene Lösung durch den Kontakt mit der englischen Philosophie und Literatur zu unterscheiden. Eine Begründung besteht aus der Darstellung eines Modelltheorems. Die Arbeit enthält leichten philosophischen Beiring. Während der letzten Jahre habe ich etliche Male über Gödel's Theorem vorgetragen, insbesondere auf dem Symposium zur Feier von Professor Gerhard Heinzberg (Heidelberg, Januar 1999), doch habe ich nennenswerte Veröffentlichungen über das Thema zu machen. Da ich wiederoberholt eine schriftliche Veröffentlichung über das Thema möchte, schenkt eine „erweiterter Kurzaufzug“¹ zu schreiben, über was ich gehalten wurde, entschuldigt mich, schnell eine „erweiterte Kurzaufzug“¹ zu schreiben, über was ich gehalten wurde,

Petr Hájek

A New Small Emendation of Gödel's Ontological Proof

Keywords: Ontological proof, Gödel modal logic, comprehension, positive properties.

1. Introduction

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Magari and others on Gödel's ontological proof

Petr Hájek

Institute of Computer Science, Academy of Sciences of the Czech Republic

182 07 Prague, Czech Republic

e-mail: hauck@uivt.cas.cz

1 Introduction

This paper is a continuation of my paper [H] and concentrates almost exclusively to mathematical properties of logical systems underlying Gödel's ontological proof [G] and its variant by Anderson [A], with special care paid to Magari's criticism [M]. Since [H] is written in German, we shall try to summarize its content in such a way that knowledge of [H] will not be obligatory for reading the present paper (even it remains advantageous). Here we describe

Understanding Gödel's Ontological Argument

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In 1970 Kurt Gödel, in a hand-written note entitled "Ontologischer Beweis", put forward an ontological argument for the existence of God, making use of second-order modal logical principles. Let the second-order formula $\mathrm{P}(\mathrm{F})$ stand for "the property F is positive", and let " GOD " signify the property of being God-like. Gödel presupposes the following definitions:

Computer-supported Clarification of Controversy see next talk by Bruno Wolzenlogel-Paleo

Overall Achievements

- ▶ significant contribution towards a **Computational Metaphysics**
- ▶ **novel results** contributed by **HOL-ATPs**
- ▶ infrastructure can be adapted for **other logics and logic combinations**
- ▶ **our technology is sufficiently mature** for use by philosophers

Relevance (wrt foundations and applications)

- ▶ Philosophy, AI, Computer Science, Computational Linguistics, Maths

Little related work: only for Anselm's simpler argument

- ▶ first-order ATP PROVER9 [OppenheimerZalta, 2011]
- ▶ interactive proof assistant PVS [Rushby, 2013]

Ongoing/Future work

see next talk by Bruno Woltzenlogel-P.

- ▶ Landscape of verified/falsified ontological arguments
- ▶ You may consider to contribute:
<https://github.com/FormalTheology/GoedelGod.git>

(Interim) Culmination of two decades of related own research

- ▶ Theory of classical higher-order logic (HOL) (since 1995)
- ▶ Automation of HOL / own LEO provers (since 1998)
- ▶ Integration of interactive and automated theorem proving (since 1999)
- ▶ International TPTP infrastructure for HOL (since 2006)
- ▶ HOL as a universal logic via semantic embeddings (since 2008)
- ▶ jww Bruno Woltzenlogel-Paleo:
Application in Metaphysics: Ontological Argument (since 2013)

... success story (despite strong criticism/opposition on the way!) ...
... huge media attention ...

(Interim) Own standpoint

- ▶ I am not fully convinced (yet) by the ontological argument.
- ▶ However, it seems to me that **the belief in a (God-like) supreme being is at least not necessarily irrational/inconsistent.**

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Nachrichten > Wissenschaft > Mensch > Mathematik > Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Von Tobias Hüller



Kurt Gödel (um das Jahr 1935): Der Mathematiker hält seinen Gottesbeweis Jahrzehntlang geheim

Ein Wesen existiert, das alle positiven Eigenschaften in sich vereint. Das bewies der legendäre Mathematiker Kurt Gödel mit einem komplizierten Formelgebilde. Zwei Wissenschaftler haben diesen Gottesbeweis nun überprüft - und für gültig befunden.

Montag, 09.09.2013 – 12:03 Uhr
[Drucken](#) | [Versenden](#) | [Marken](#)

Germany

- Telepolis & Heise
- Spiegel Online
- FAZ
- Die Welt
- Berliner Morgenpost
- Hamburger Abendpost
- ...

Austria

- Die Presse
- Wiener Zeitung
- ORF
- ...

Italy

- Repubblica
- IlSussidario
- ...

India

- DNA India
- Delhi Daily News
- India Today
- ...

US

- ABC News
- ...

International

- Spiegel International
- Yahoo Finance
- United Press Intl.
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SPIEGEL ONLINE INTERNATIONAL

Front Page | World | Europe | Germany | Business | Zeitgeist | Newsletter

English Site | Germany > Science > Scientists Use Computer to Mathematically Prove Gödel God Theorem

Holy Logic: Computer Scientists 'Prove' God Exists

By David Knight



Austrian mathematician Kurt Gödel kept his proof of God's existence a secret for decades. Now two scientists say they have proven it mathematically using a computer.

Two scientists have formalized a theorem regarding the existence of God penned by mathematician Kurt Gödel. But the God angle is somewhat of a red herring -- the real step forward is the example it sets of how computers can make scientific progress simpler.

Germany

- Telepolis & Heise
- Spiegel Online
- FAZ
- Die Welt
- Berliner Morgenpost
- Hamburger Abendpost
- ...

Austria

- Die Presse
- Wiener Zeitung
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Italy

- Repubblica
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- ...

India

- DNA India
- Delhi Daily News
- India Today
- ...

US

- ABC News
- ...

International

- Spiegel International
- Yahoo Finance
- United Press Intl.
- ...

The image shows the front page of the German newspaper 'DIE WELT' from Friday, October 18, 2013. The masthead 'DIE WELT' is prominently displayed at the top left, with a small globe icon between 'DIE' and 'WELT'. Below the masthead is a large photograph of three people: a man in a suit, a man in a blue shirt, and a woman in a dark jacket, all smiling and taking a selfie with a smartphone. To the left of the photo is a column titled 'Zippert zappt' with several short articles. To the right is a column titled 'Teuer erkaufte Harmonie' with an article about political negotiations. The main headline 'Mindestlohn lockt SPD in dritte Ehe mit der Union' is in the center. Below it, a sub-headline reads 'Mittwoch soll über eine große Koalition verhandelt werden. Votum des SPD-Konvents steht aus'. The page also features sections for 'THEMEN' (with a photo of Angela Merkel), 'Sport', 'Wirtschaft' (with a photo of a Boeing plane), 'Karriere', 'Aus aller Welt' (with a photo of Paris), and 'Das im Minus' (with a photo of a balance sheet). At the bottom right is an advertisement for 'UNION GLASHUTTE/SA' featuring a wristwatch.

Austria

- Die Presse
- Wiener Zeitung
- ORF

- ...

Italy

- Repubblica
- IlSussidario

- ...

India

- DNA India
- Delhi Daily News
- India Today

- ...

US

- ABC News

- ...

International

- Spiegel International
- Yahoo Finance
- United Press Intl.

- ...

SCIENCE NEWS

HOME / SCIENCE NEWS / RESEARCHERS SAY THEY USED MACBOOK TO PROVE GOEDEL'S GOD THEOREM

Researchers say they used MacBook to prove Goedel's God theorem

Oct. 23, 2013 | 8:14 PM | 1 comments

See more serious and funny news links at
<https://github.com/FormalTheology/GoedelGod/tree/master/Press>