Computer-Assisted Analysis of the Anderson-Hájek Ontological Controversy

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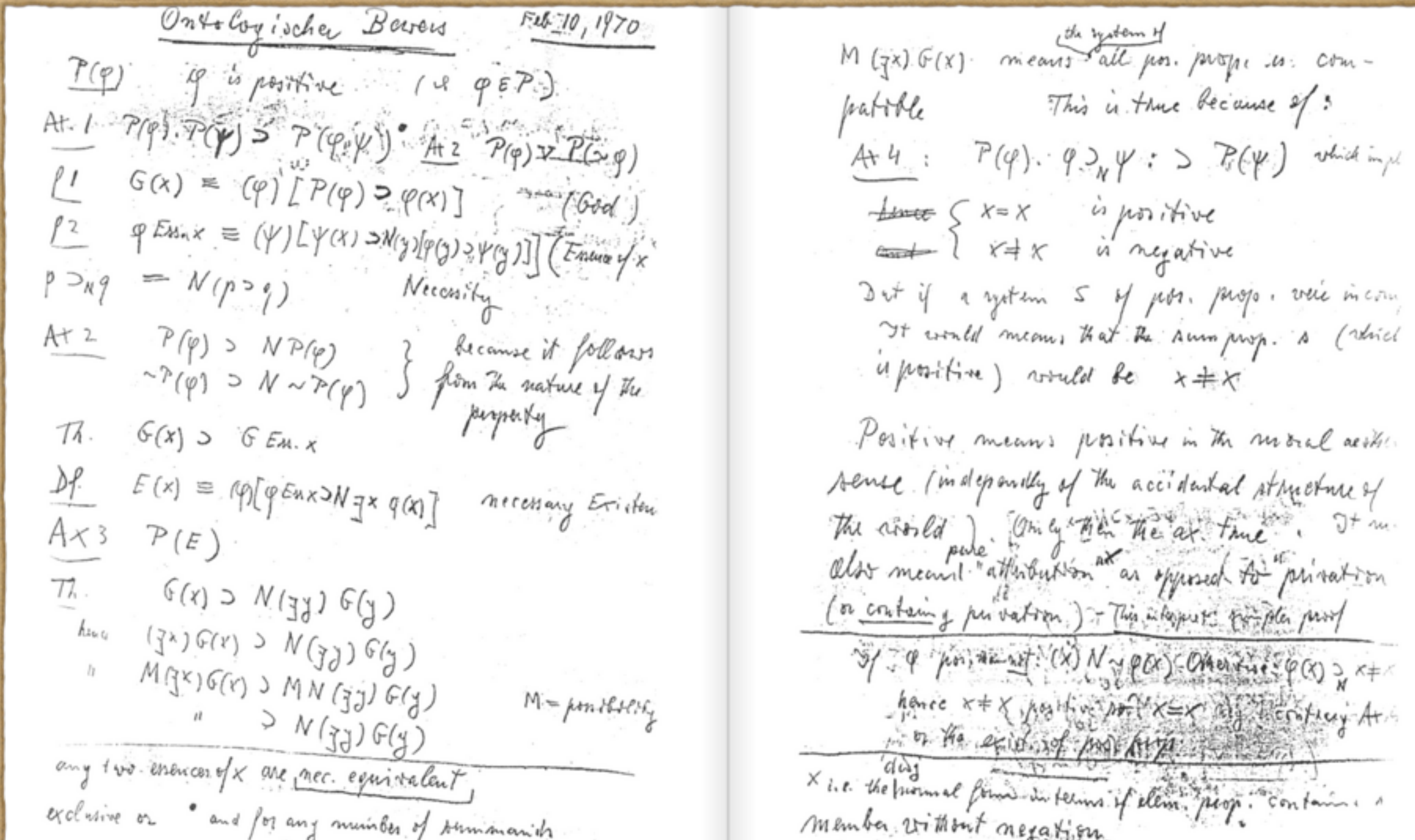

Ontologischer Beweis Feb 10, 1970

$P(\varphi)$ φ is positive (or $\varphi \in P$)

At. 1 $P(\varphi) . P(\psi) \supset P(\varphi . \psi)$* At 2 $P(\varphi) \vee P(\sim\varphi)$

P1 $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

P2 $\varphi \text{ Ess } x \equiv (\psi)[\psi(x) \supset N(y)[\varphi(y) \supset \psi(y)]]$ (Essence of x)

$p \supset_N q = N(p \supset q)$ Necessity

At 2 $P(\varphi) \supset N P(\varphi)$
$\sim P(\varphi) \supset N \sim P(\varphi)$ } because it follows from the nature of the property

Th. $G(x) \supset G \text{ Ess. } x$

Df. $E(x) \equiv (\varphi)[\varphi \text{ Ess } x \supset N \exists x\, \varphi(x)]$ necessary Existence

Ax 3 $P(E)$

Th. $G(x) \supset N(\exists y)\, G(y)$

hence $(\exists x)\, G(x) \supset N(\exists y)\, G(y)$

" $M(\exists x)\, G(x) \supset M N(\exists y)\, G(y)$

" $\supset N(\exists y)\, G(y)$ M = possibility

any two essences of x are nec. equivalent

exclusive or * and for any number of summands

$M(\exists x)\, G(x)$ means the system of all pos. prope. is compatible. This is true because of:

At 4: $P(\varphi) . \varphi \supset_N \psi : \supset P(\psi)$ which impl

$\left\{ \begin{array}{ll} x = x & \text{is positive} \\ x \neq x & \text{is negative} \end{array} \right.$

But if a system S of pos. prop. were incon it would mean that the sum prop. s (which is positive) would be $x \neq x$

Positive means positive in the moral aesth. sense (independently of the accidental structure of the world). Only then the ax. true. It m also mean pure "attribution"x as opposed to "privation" (or containing privation). This interpret. simpler proof

If φ pos. not: $(x) N \sim \varphi(x)$. Otherwise: $\varphi(x) \supset_N x \neq x$ hence $x \neq x$ positive not $x = x$ contrary Ax. or the exist. of pos. prop.

x i.e. the normal form in terms of elem. prop. contains a member without negation.

Gödel's Ontological Proof

1970

Sobel's Modal Collapse

1987

$$\varphi \to \Box \varphi$$

Magari's Claim

1988

4. *Indebolimento dei primi tre assiomi*

Come si è visto i primi tre assiomi e la definizione di G sono sufficienti a dare il teorema principale, ma chiaramente non era questa l'intenzione di A: A intendeva assicurarsi il teorema 1, $\exists m \exists x G_m x$, e ricavarne, usando anche gli altri assiomi e in particolare i concetti di essenza e di esistenza necessaria, il teorema principale. Tutto sommato sembra plausibile immaginare di scegliere i primi tre assiomi, conservando l'assioma definitorio D1. In questo modo, vedendo le cose in un mo-

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"A4 and A5 are superfluous"

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A4 Positive properties are necessarily positive:

$$\forall\varphi[P(\varphi) \to \Box\, P(\varphi)]$$

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D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties:

$$\varphi \ ess\ x \equiv \varphi(x) \wedge \forall\psi(\psi(x) \to \Box\forall y(\varphi(y) \to \psi(y)))$$

D3 *Necessary existence* of an individual is the necessary exemplification of all its essences:

$$\mathrm{NE}(x) \equiv \forall\varphi[\varphi \ ess\ x \to \Box\exists y\varphi(y)]$$

A5 Necessary existence is a positive property:

$$P(\mathrm{NE})$$

Anderson's Emendation

1990

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A:A1 If a property is positive, its negation is not positive:

$$\forall\varphi[P(\varphi) \to \neg P(\neg\varphi)]$$

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A:D2 An *essence* of an individual is a property that necessarily implies those and only those properties that the individual has necessarily:

$$\varphi \ ess_A \ x \equiv \forall\psi[\Box\psi(x) \leftrightarrow \Box\forall y(\varphi(y) \to \psi(y))]$$

Hájek's Claims 1996 - 2001

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are to be assumed? And (3), what *modal axioms* are to be assumed? Sobel [S] and Polívka (cf.[H]) observed that assuming full comprehension leads to *collapse of modalities*, i.e. $(\forall X)(\forall x)(X(x) \equiv \Box X(x))$ become provable, which means that the system trivializes. Magari [M] claims that the first three Gödel's axioms $(A1) - (A3)$ already imply the main theorem; we shall analyze his claim in Section 2 and show that his proof implicitly uses a too strong equality axiom (without which the claim is not true, as shown in [H]). Anderson [A] presented an emended system admitting full comprehension and not suffering by collapse of modalities; I proved in [H] that for Anderson's system Magari's claim becomes true; the analogs of Gödel's $A1 - A3$ prove necessary existence of a godlike individual (in the modified sense), but

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5.3 Theorem: $AO \vdash (A4^*), (C4^*), (A5^*)$.

Beweis. $(A4^*)$ ist $P(Y) \to \Box P(Y)$. Sei $G^*(g)$; dann ist $P(Y) \equiv \Box Y(g)$, also $\Box P(Y) \equiv \Box\Box Y(g) \equiv \Box Y(g)$, und daher $P(Y) \equiv \Box P(Y)$.

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"Magari is wrong about superfluousness of A4 and A5"

"But A4 and A5' are redundant in Anderson's Theory"

Rebuttal of Anderson and Gettings 1996

[1] Petr Hájek [13] has argued that Anderson's version of the argument has superfluous premisses, but the truth of this claim depends on the details of the underlying second-order modal logic adopted. In the context of the (quite reasonable) version of that logic formulated by Nino Cocchiarella [2] and cited by

Anderson as the underlying logic, Hájek's point does *not* hold. It does indeed follow from the diminished set of premisses that: if THERE IS a Godlike being, then necessarily THERE IS such a being. From this one can easily deduce that THERE IS a Godlike being, as Hajek observes. But the sense of the quantifier indicated by the capitalized phrase here (i.e., as formalized in Cocchiarella's logic) is that of "possible existence" or "subsistence". To express actual existence requires a separate quantifier. So without using the other premisses, nothing yet follows about the actual existence of a Godlike being.

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A simplification of the emendation is defined

Varying Domains

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text {* Technically, varying domains are encoded with
       the help of an  existence relation expressing
       which individuals actually exist in each world.*}

  consts eiw :: "μ ⇒ i ⇒ bool"
  axiomatization where nonempty: "∀w. ∃x. eiw x w"


text {* Actualistic quantifiers are
       quantifiers guarded by the existence relation. *}

  abbreviation mforalle :: "(μ ⇒ σ) ⇒ σ" ("∀e")
           where "∀e Φ ≡ (λw. ∀x. (eiw x w) ⟶ (Φ x w))"

  abbreviation mexistse :: "(μ ⇒ σ) ⇒ σ" ("∃e")
           where "∃e Φ ≡ (λw. ∃x. (eiw x w) ∧ Φ x w)"
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Anderson's Long Footnote 1990

14. One should consult Cocchiarella's article for a detailed and precise account of his semantics for second-order modal logic. But, at least for those having some familiarity with the usual "possible world" approach, the following outline of the semantics may make it possible to see that there is no modal collapse. We are to imagine given a set I of "possible worlds" where each such world has associated with it an "L-model," a triple consisting of a set A (the individuals existing in the world), a set B (the possible individuals; the set of actual individuals must be a subset of this set) and an interpretation function R which assigns appropriate extensions to the constants of the language—possible individuals to individual constants, sets of possible individuals to one-place predicates and so on. It is worth emphasizing that the denotation of a constant (as given by R) at a world and the extension of a predicate (also given by R) at a world need not consist of individuals which are "actual" at the world. And Cocchiarella's logic has two kinds of quantifiers: what we might call "subsistential quantifiers" which (in the semantics) are construed as ranging over all *possible* individuals (the set B), and "e-quantifiers," "existence quantifiers," which are interpreted at a world as ranging only over the entities which exist in that world (the set A). The logic is interpreted by giving a "world system"—a collection of such L-models, one for each possible world i belonging to I, it being required that the possible individuals of each L-model be the same as those of any other (the possible individuals are the same no matter what world you are in). (Technically the L-models are "indexed"; the set I is the domain of a function which yields an L-model for each i belonging to I). And the existing individuals of all the worlds, taken together, must be a subset of the set of possible individuals. A singular attribute is as usual a function which takes as arguments possible worlds and yields as value in each case a set of possible individuals. And we may take the one-place second-order variables as ranging over all the attributes which correspond to a given world system (Cocchiarella has two kinds of second-order quantifiers, but only one is relevant to our present concern). The n-ary attributes are defined analogously. Ignoring for simplicity some not-immediately-relevant complications of Cocchiarella's actual construction, our desired world system may be taken to contain two possible worlds 1 and 2 and corresponding L-models $A_1 = <\{a,b\}, \{a,b\}, R_1>$ and $A_2 = <\{a\}, \{a,b\}, R_2>$. The first set listed in each case contains the actual individuals of the world, the second contains the possible individuals, and R_i assigns extensions (from the second set) at the world to the constants and predicates of the language. The singularly attributes of the world system are therefore all the functions whose ranges consists of just the two worlds and whose values are in each case sets of possible individuals. The n-ary attributes (n > 1) of the world system are defined analo-

gously (although we do not actually care about them for the present purpose). Now use the notation 'Φ_{aV}' to stand for the attribute that picks out singleton *a* at the world 1 (corresponding to A_1) and picks out the set of all possible individuals $V = \{a,b\}$ at world 2 (corresponding to A_2), with analogous notation for the other fifteen attributes which exist in this world system. Then take $R_1(`\Delta') = R_2(`\Delta') = \Phi_{bb}$. It is easy to check that the only positive attributes (those whose negations entail Φ_{bb}) are Φ_{aa}, Φ_{VV}, Φ_{Va}, and Φ_{aV}, and that the first of these is the attribute corresponding to 'G' and 'NE' and is the unique essence of *a*. Note well that the appropriate translation of the argument into Cocchiarella's notation will take the existential quantifier in the definition of necessary existence to be an e-quantifier (an "existence," rather than "subsistence," quantifier) and so too the quantifier in the conclusion of the argument. All others individual quantifiers may be taken to be subsistential and hence to range over all possibles. It is a purely combinatorial task to show that the modified Gödelian axioms all come out true in this model and that there is no modal collapse. And not everything is a necessary existent. Cocchiarella proves that his axioms for second-order S5 are complete in the "Henkin-sense." Given the definition of "positive property," one can completely formalize the present ontological proof in that system. Thus, there is at least one formalization of the present proof using a reasonably adequate logic in which no modal collapse is demonstrable.

17 footnotes

4 pages of footnotes

in an 8-page paper

Anderson's Long Footnote 1990

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a strange variant with mixed quantifiers is indicated

Hájek's Reaction to the Rebuttal 2002

DEFINITION 8. $\mathcal{AO}$ is the system of axioms (A1)–(A5) and definitions of $\mathtt{H}$, $\mathtt{Ess}$, $\mathtt{NE}$ just presented (Anderson ontological). $\mathcal{AO}_0$ is the subsystem of axioms (A1)–(A3) plus the definition of $\mathtt{H}$.

In [9, 8] I showed that assuming our logic as above, (A4)–(A5) are redundant since (A1)–(A3) prove both $\Box(\exists x)\mathtt{H}(x)$ and also (A4) and (A5) (using the definitions of $\mathtt{Ess}$, $\mathtt{NE}$). In other words, $\mathcal{AO}$ and $\mathcal{AO}_0$ are equivalent in the logic $\mathcal{L}_{\mathrm{ont}}$. But in [2] the authors stress that Anderson's proof works in a finer logic in which all the axioms are needed. We shall discuss this in Section 4; the next section is devoted to the analysis and emendation of $\mathcal{AO}_0$ in our present logic.

Hájek's Reaction to the Rebuttal 2002

Our main results are the following:

1. In the logic $\mathcal{L}_{\mathrm{ont}}$ with fixed domains, the system $\mathcal{AO}'_0$ of axioms

$$\text{(A12)} \qquad (\mathsf{P}(X) \mathbin{\&} \Box(X \subseteq Y)) \rightarrow \neg\mathsf{P}(\neg Y)$$

$$\text{(def 1)} \qquad \mathtt{H}(x) \equiv (\forall Y)(\Box Y(x) \equiv (\exists Z)(\mathsf{P}(Z) \mathbin{\&} \Box(Z \subseteq Y))$$

$$\text{(A3)} \qquad \mathsf{P}(\mathtt{H})$$

proves $\Box(\exists x)\mathtt{H}(x)$. (Modified fragment of Anderson.)

2. In the logic $\mathcal{L}_{ont,E}$ with the existence predicate E, the system $\mathcal{AOE}'_0$ of axioms

$$\text{(A12)}^{\mathrm{E}} \qquad (\mathsf{P}(X) \mathbin{\&} \Box(X \subseteq^{\mathrm{E}} Y)) \rightarrow \neg\mathsf{P}(\neg Y)$$

$$\text{(def 1)} \qquad \mathtt{H}(x) \equiv (\forall Y)(\Box Y(x) \equiv (\exists Z)(\mathsf{P}(Z) \mathbin{\&} \Box(Z \subseteq^{\mathrm{E}} Y))$$

$$\text{(A3)}^{\mathrm{E}} \qquad \mathsf{P}(H \mathbin{\&} E).$$

proves $\Box(\exists^{\mathrm{E}} x)\mathtt{H}(x)$. (Modified fragment of Anderson with existence.)

Hájek's Reaction to the Rebuttal 2002

DEFINITION 13. $\mathcal{AOE}''$ in the following modification of $\mathcal{AOE}'$:

- axioms $(\mathrm{A12})^{\mathrm{E}}$, (A3),
- definition of $\mathtt{H}$ as in $\mathcal{AOE}'$,
- definition $\mathsf{P}^{\#}(Y) \equiv (\exists Z)(\mathsf{P}(Z) \,\&\, \Box(Z \subseteq Y))$
  (thus the definition of $\mathtt{H}$ can be written as

$$\mathtt{H}(u) \equiv (\forall Y)(\Box Y(u) \equiv \mathsf{P}^{\#}(Y))),$$

$(\mathrm{A4})^{\#}$ $\quad \mathsf{P}(Y) \to \Box\mathsf{P}^{\#}(Y)$

$(\mathrm{A5})^{\#}$ $\quad \mathsf{P}^{\#}(\mathtt{NE})$.

Bjørdal's Proof

1998

B:D1 A property is positive iff it is necessarily possessed by every God-like being.

$$P_B(\phi) \equiv \Box\forall x(G_B(x) \rightarrow \phi(x))$$

Bjørdal's Proof

1998

B:D1 A property is positive iff it is necessarily possessed by every God-like being.

$$P_B(\phi) \equiv \Box\forall x(G_B(x) \rightarrow \phi(x))$$

B:D2 a *maximal composite* of an individual's positive properties is a positive property possessed by the individual and necessarily implying every positive property possessed by the individual.

$$\mathrm{MCP}(\phi, x) \equiv (\phi(x) \wedge P_B(\phi)) \wedge \forall\psi((\psi(x) \wedge P_B(\psi)) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$$

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$$\mathrm{MCP}(\phi, x) \equiv (\phi(x) \land P_B(\phi)) \land \forall\psi((\psi(x) \land P_B(\psi)) \to \Box\forall y(\phi(y) \to \psi(y)))$$

B:D3 *Necessary existence* of an individual is the necessary exemplification of all its maximal composites.

$$\mathrm{NE}_B(x) \equiv \forall\phi(\mathrm{MCP}(\phi, x) \to \Box\exists y\phi(y))$$

Bjørdal's Proof

1998

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$$\mathrm{NE}_B(x) \equiv \forall \phi(\mathrm{MCP}(\phi, x) \to \Box \exists y \phi(y))$$

A:A1' If a property is positive, its negation is not positive:

$$\forall \varphi [P_B(\varphi) \to \neg P_B(\neg \varphi)]$$

A5' Necessary existence is a positive property.

$$P_B(\mathrm{NE}_B)$$

Bjørdal's Proof

1998

I have later been able to improve upon the result reported here. By making use of a result by Petr Hájek,[3] which he made we aware of at the Liblice-conference, and presupposing certain recursive definition-clauses for *divine* (positive) and *godly being*, we may show that even Ax. 2 is eliminable if we presuppose a reasonable second order comprehension principle for the predicate *godly being*. And so it in fact turns out that, modulo the (reasonable, I think) logical apparatus presupposed, only Ax. 1 is needed in order to derive the theistic conclusion. I hope to be able to publish this improved result, alongside with certain remarks, in a future paper.

Bjørdal's Proof

1998

A5'

I have later been able to improve upon the result reported here. By making use of a result by Petr Hájek,[3] which he made we aware of at the Liblice-conference, and presupposing certain recursive definition-clauses for *divine* (positive) and *godly being*, we may show that even Ax. 2 is eliminable if we presuppose a reasonable second order comprehension principle for the predicate *godly being*. And so it in fact turns out that, modulo the (reasonable, I think) logical apparatus presupposed, only Ax. 1 is needed in order to derive the theistic conclusion. I hope to be able to publish this improved result, alongside with certain remarks, in a future paper.

Bjørdal's Proof

1998

A5'

I have later been able to improve upon the result reported here. By making use of a result by Petr Hájek,[3] which he made we aware of at the Liblice-conference, and presupposing certain recursive definition-clauses for *divine* (positive) and *godly being*, we may show that even Ax. 2 is eliminable if we presuppose a reasonable second order comprehension principle for the predicate *godly being*. And so it in fact turns out that, modulo the (reasonable, I think) logical apparatus presupposed, only Ax. 1 is needed in order to derive the theistic conclusion. I hope to be able to publish this improved result, alongside with certain remarks, in a future paper.

Bjørdal will talk about this on Saturday at 10:45

Our Goal

Our Goal

Follow Leibniz's dictum!

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Follow Leibniz's dictum!

Controversy between Anderson and Hájek.

Our Goal

Follow Leibniz's dictum!

Controversy between Anderson and Hájek.

Who is right?

Our Goal

Follow Leibniz's dictum!

Controversy between
Anderson and Hájek.

Who is right?

Let the "calculus ratiocinator"
compute and decide!

What we did…

What we did...

Computer-Assisted Analysis of
11 variants of
Gödel's Ontological Argument

What we did...

Computer-Assisted Analysis of
11 variants of
Gödel's Ontological Argument

Discovery of flawed claims

What we did...

Computer-Assisted Analysis of
11 variants of
Gödel's Ontological Argument

Discovery of flawed claims

Strengthening of correct claims

Redundancy of A4 and A5 in Anderson's Emendation in "Isabelle"

Anderson_var.thy

Anderson_var.thy (~/Dropbox/Pesquisa/Projects/GoedelGod/Formalizations/Isabelle/Anderson/)

```
  theorem A4:  "[∀(λΦ. P Φ m→ □ (P Φ))]"
  (* sledgehammer [remote_satallax remote_leo2] *)
  by (metis A3 G_def sym trans T3)

  definition ess :: "(μ ⇒ σ) ⇒ μ ⇒ σ" where
           "ess = (λΦ. λx. (( (∀(λΨ. ((□ (Ψ x )) m≡  □(∀e(λy. Φ y m→ Ψ y))))))))"

  definition NE :: "μ ⇒ σ" where
           "NE = (λx. ∀(λΦ. ess Φ x m→ (□ (∃e(λy. Φ y)))))"

  theorem A5: "[P NE]"
  by (metis A2 A3 ess_def NE_def)

subsection {* Consistency again (now with sym and trans) *}
subsection {* Immunity to Modal Collapse *}

  lemma MC: "[∀(λΦ.(Φ m→ (□ Φ)))]"
  nitpick [user_axioms] oops
```

Documentation Sidekick Theories

Output Query Sledgehammer Symbols

66,34 (1797/2570) (isabelle,sidekick,UTF-8-Isabelle)Nmro UG 162/271MB 8:17 PM

Computer-Assisted Analysis

Proof	D1'	A:A1'	A2'	A3'	A4	A4'	H:A4	A5	A5'	H:A5	T3	T3'	MC
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

P = provable; CS = counter-satisfiable;

R = redundant; I = independent;

S = superfluous; N = not superfluous

Computer-Assisted Analysis

Proof	D1'	A:A1'	A2'	A3'	A4	A4'	H:A4	A5	A5'	H:A5	T3	T3'	MC
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Proof	D1'	A:A1'	A2'	A3'	A4	A4'	H:A4	A5	A5'	H:A5	T3	T3'	MC
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Proof	D1'	A:A1'	A2'	A3'	A4	A4'	H:A4	A5	A5'	H:A5	T3	T3'	MC
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Proof	D1'	A:A1'	A2'	A3'	A4	A4'	H:A4	A5	A5'	H:A5	T3	T3'	MC
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Proof	**D1'**	**A:A1'**	**A2'**	**A3'**	**A4**	**A4'**	**H:A4**	**A5**	**A5'**	**H:A5**	**T3**	**T3'**	**MC**
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Proof	D1'	A:A1'	A2'	A3'	A4	A4'	H:A4	A5	A5'	H:A5	T3	T3'	MC
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Proof	D1'	A:A1'	A2'	A3'	A4	A4'	H:A4	A5	A5'	H:A5	T3	T3'	MC
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Proof	**D1'**	**A:A1'**	**A2'**	**A3'**	**A4**	**A4'**	**H:A4**	**A5**	**A5'**	**H:A5**	**T3**	**T3'**	**MC**
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Proof	D1'	A:A1'	A2'	A3'	A4	A4'	H:A4	A5	A5'	H:A5	T3	T3'	MC
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Proof	**D1'**	**A:A1'**	**A2'**	**A3'**	**A4**	**A4'**	**H:A4**	**A5**	**A5'**	**H:A5**	**T3**	**T3'**	**MC**
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	$\boxed{\text{P (KB)}}$	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	$\boxed{\text{P (KB)}}$	CS

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Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
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Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
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Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
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Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (KB)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS (S5)
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS (S5)
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS (S5)
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS (S5)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Future Work

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Conclusion

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Please use our tools!